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Essays on Price Formation, Business Cycles,
and Monetary Policy

by

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PhD Dissertation

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Editorial Remarks

This dissertation consists of four chapters; each of them constitutes a self-contained paper and can be read independently of the others. This implies that some repetition of arguments etc. is unavoidable. Also, notation and terminology has been chosen in order to present each paper in the clearest form possible, and consequently some shifts in these may occur between the chapters. Finally, I should note that due to different preferences of my co-authors, the choice of British vs. US English is not uniform across the different chapters. For all these inconveniences I apologise.

Summary

This dissertation consists of four papers, each constituting a chapter. Each of the papers is self-contained and can be read independently. All of them, however, share a common ground as they consider issues related to monetary policy and business cycles and how they interact.

The first two papers are joint work with Jesper Gregers Linaa. The first of these is entitled *What Drives Business Cycles in a Small Open Economy with a Fixed Exchange Rate?* and presents a Bayesian estimation of a dynamic stochastic general equilibrium (DSGE) model on Danish data. That is, the estimation combines likelihood optimization with Bayesian priors in the spirit of Smets and Wouters (2003). The model describes a small open economy and is close to Kollmann (2001, 2002) with some important deviations, however; to avoid stochastic singularity we need to expand his model with a variety of structural shocks to make an estimation feasible. Furthermore, since the scope is to estimate a model on Danish data we drop the Taylor rule and instead introduce an imperfect peg. The peg is postulated “imperfect”; although Denmark successfully has followed a peg, minor movements in the exchange rate have been observed over the years. A variance decomposition reveals that the Danish business cycle is dominated by stochastic movements in the labour supply in the long term, while demand shocks play a major role in the short term. Remarkably, the role of technology is negligible, and foreign factors only contribute little to the Danish business cycle, especially in the long term. With respect to the estimation, we generally find believable estimates although the degree of price stickiness is remarkably high.

The structural nature of the estimated model for the Danish economy allows one to hypothesize the welfare consequences of Denmark abandoning its peg on the euro in favour of an independent monetary policy. That is the scope of the second paper, *Assessing the Welfare Cost of a Fixed Exchange Rate Policy*. In this analysis, we restrict ourselves to focus on operational and implementable monetary rules in the sense of Schmitt-Grohe and Uribe (2004). Specifically, we analyse a specific version of a generalised Taylor (1993) rule, where the central bank sets the interest rate as a function of output growth and inflation.

We conclude that there are sizable benefits to be attained from conducting an independent monetary policy of this kind in favour of maintaining the peg on which current Danish monetary policy is founded. Our estimate suggests that the gain in welfare is equivalent to a permanent increase of 0.8 pct. in the level of consumption. The optimal Taylor rule is found to be characterised by an aggressive interest-rate response to inflation (i.e., attaching a weight of 3, which is the ceiling of our grid search, to inflation) and a moderate response to output growth (i.e., a weight of 0.8). Contrary to Schmitt-Grohe and Uribe (2004) we do not find it beneficial for the central bank to smooth interest rates over time.

With regards to the causes of the higher level of welfare under the Taylor rule, we obtain mixed results: in terms of consumption, the higher welfare is founded in the higher mean of consumption under the Taylor rule, although the volatility of consumption has also increased. For labour this result is reverted; under the peg regime labour is more volatile than under the Taylor rule, while the mean is predicted to be lower under the peg. Overall, agents prefer the higher consumption, despite higher volatility and more labour efforts.

The analysis of the Danish economy raise some key issues that partly inspired the last two papers of this dissertation. The first issue is the very limited impact of the foreign economy on the Danish business cycle. In joint work with Jens Søndergaard I analyse the British economy in a framework similar in nature to that applied to the Danish economy in the first paper, although some technical details differ, partly inspired by the small open economy model of Adolfson et al. (2005). The focus of this analysis, which constitutes the third paper and is entitled *The UK Business Cycle and the Exchange-Rate Disconnect*, is on the determinants of the UK business cycle, and in particular the interaction between the British and the foreign economy. As the UK operates an independent monetary policy, the dynamics of the interest rate and the exchange rate obviously differs from the Danish case which is reflected in some of the changes to the model setup.

Overall, the estimation yields plausible estimates of the structural parameters. According to these estimates, the main source of fluctuations in the British economy is shocks to the technology process. However, our estimation does point towards two areas where the current empirical DSGE literature needs to be reconsidered. The first is specific to the UK economy and regards the inflation dynamics. The (detrended) inflation series for the UK is remarkably less persistent than those for the US and euro area. As much of the workhorse price model of the recent medium-scale DSGE literature has been constructed in a US context, where the main challenge was to account for the high persistence of observed inflation, it is not well suited as a model for UK inflation. The estimated degree of price stickiness is quite low, which is in line with micro-economic evidence for the UK, cf. Hall et al. (2000), yet in contrast to that found for the Danish economy, cf. Chapter 1, and for the euro area (Smets and Wouters, 2003; Adolfson et al., 2005).

Secondly, the foreign economy has no impact on the UK economy whatsoever. This replicates the result for the Danish economy from Chapter 1, and is at first sight quite surprising, why we consider it in detail. Typically, the transmission of foreign real shocks works through the terms-of-trade (TOT) channel. Thus, standard models of the international real business-cycle literature have found that exogenous changes in the TOT account for about 50 percent of domestic output fluctuations, cf. Mendoza (1995). The terms of trade channel also plays a dominant theoretical role in the new open-economy macroeconomic (NOEM) literature (cf. Corsetti and Pesenti, 2001). This could explain why the Danish economy is little exposed to the foreign economy due to the pegged exchange rate, yet one would expect a marked foreign impact on the British economy.

Other recent attempts to estimate DSGE models of the NOEM variety with Bayesian methods, however, suggest that external terms-of-trade shocks have very little effect on domestic output, cf. Lubik and Schorfheide (2003) for the UK, Adolfson et al. (2005) for the euro area, and Martínez-García (2005) in the case of Spain.

In a recent paper, Justiniano and Preston (2006) investigates this puzzling result in a structural analysis of Canadian data. The common result across these different open-economy structural estimations is that the current versions of NOEM models suffer from the *exchange-rate disconnect puzzle*, that is, a volatile behaviour of the real exchange rate that is unrelated to the development of real aggregates (fundamentals) in the respective economies (see Obstfeld and Rogoff (2000) for an articulate presentation of this puzzle). In other words, the NOEM models that have been estimated in this and other papers fail to give an economic explanation of the volatile real exchange rate, and thus they rely on ad-hoc shocks to disconnect the modeled economy from the large movements that we observe in the real exchange rate. Consequently, the estimated economy virtually behaves as a closed economy. Hence, the current DSGE models for open economies are still

unable to provide an adequate description of the interaction with the foreign economy, and further research is certainly warranted.

The fourth and final paper, entitled *Heterogeneous Price Stickiness in Estimated Semi-Structural Models of the US Economy*, is joint work with Carlos Carvalho. It analyses the empirical importance of price contracts with different durations for the overall inflation dynamics. The recent empirical literature on price setting that analyses the datasets underlying the construction of consumer price indices documents a large amount of heterogeneity in the frequency of price changes across different economic sectors, cf. Bils and Klenow (2004) and Nakamura and Steinsson (2007) for the US economy, and Dhyne et al. (2006) and references therein for the euro area.

Through calibration of a structural model, Carvalho (2006) showed how such heterogeneity has dramatic implications for the dynamic response of economies to monetary disturbances. In this paper, we proceed with Bayesian estimation of a model where the supply side is a multi-sector economy with generalised Taylor (1979, 1980) staggered price setting, in which the extent of price rigidity varies across different sectors. We close the model with reduced-form processes for nominal output and an unobserved natural rate of output.

In the estimation, we incorporate the information from the micro data analyzed by Bils and Klenow (2004) and Nakamura and Steinsson (2007) through our prior on the cross-sectional distribution of the frequency of price changes, which in turn affects aggregate dynamics, and estimate the model using aggregate inflation and output as observables. The fact that different cross-sectional distributions of price stickiness imply different aggregate dynamics in principle allows inference about such distribution based solely on time series of macroeconomic variables. However, we had anticipated, and confirm with our results, that identification of this distribution is likely to be weak in this context. Thus the additional value of an approach that makes use of the micro data through the priors.

Our estimation results suggest that heterogeneity of contract lengths is of critical importance for understanding the joint dynamics of inflation and output. When we restrict the models by imposing the same degree of price stickiness across sectors, we obtain results that are significantly worse from a statistical perspective than in the general case with heterogeneity, and that moreover are economically nonsensical. Hence, heterogeneous price stickiness offers an explanation and potential solution to the puzzling findings on price formation for Denmark and the UK in Chapters 1 and 3, respectively, and e.g., for the euro area in Smets and Wouters (2003).

Despite very different empirical methodologies, our results are in line with those obtained by Coenen et al. (2007), who estimate a model with Taylor staggered price setting and heterogeneous contract lengths of up to four quarters. Our estimation results, however, suggest that it is important to allow for sectors in which prices last longer than one year. Neglecting to do so generates too little nominal rigidity relative to the micro-evidence on the one hand, and on the other hand increases the estimated degree of real rigidity way beyond what is found when more heterogeneity is allowed for. Thus, Coenen et al. (2007) find an incredible amount of real rigidity, while our results implied marked decreases in the estimated degree of real rigidity to levels that have been deemed plausible in recent literature (Woodford, 2003, e.g.,).

The results generally conform with the cross-sectional distribution of price contracts that we derive from the work of Nakamura and Steinsson (2007), based on their statistics

on the frequency of *regular* price changes. However, the empirical fit seems on par with that obtained from symmetric priors when contracts of up to two years are considered. This suggests that once one has the average contract length and the extent of heterogeneity in price stickiness right, the specific sectoral masses are not of great importance.

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Chapter 1

What Drives Business Cycles in a Small Open Economy with a Fixed Exchange Rate?^{*,†}

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Abstract

We decompose the Danish business cycle into ten structural shocks using an open-economy DSGE model with infrequent determination of prices and wages which we estimate with Bayesian techniques. Consistent with the Danish monetary policy regime, we formulate an imperfect peg on the foreign exchange rate and analyse the resulting monetary transmission mechanism.

We find that the Danish business cycle is dominated by stochastic movements in the labour supply in the long term, while demand shocks play a major role in the short term. Remarkably, the role of technology is negligible, and foreign factors only contribute little to the Danish business cycle, especially in the long term. With respect to the estimation, we generally find believable estimates although the degree of price stickiness is remarkably high.

Keywords: Open economy, currency peg, business cycles, Bayesian estimation

JEL Classifications: E3, E4, F4

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†Both authors were doctoral students at the University of Copenhagen when this work was conducted. The viewpoints and conclusions stated are the responsibility of the authors and do not necessarily reflect the views of the Danish Economic Councils.

1 Introduction

The aim of this paper is to analyse the determinants of business cycles in a small open economy with a fixed exchange rate. We formulate a dynamic stochastic general equilibrium (DSGE) model for a small open economy and estimate it on Danish data using Bayesian estimation techniques. Hence, our paper belongs to the new open-economy macroeconomics (NOEM) research programme (surveyed by Lane, 2001) which analyses open economies with well-specified microeconomic foundations. Since its inception with the seminal Obstfeld and Rogoff (1995) paper, the NOEM literature has offered many new insights and has proven popular with theorists and central bankers alike. However, empirical work has been relatively scarce within this framework, and thus a secondary aim of this paper is an empirical assessment of the NOEM framework's ability to adequately describe a Scandinavian economy.

We obtain three main conclusions regarding the determinants of the Danish business cycle: First, in the short run demand-side and supply-side shocks both contribute substantially to fluctuations, while long-run cycles are driven almost entirely by supply shocks. Second, even though supply shocks dominate fluctuations in the long run, the influence from technological shocks is almost negligible while shocks to labour supply are the main contributor to long-run volatility. Finally, a surprisingly large share of all cycles appears to be founded in domestic shocks rather than foreign ones. Our model allows for foreign shocks stemming from three channels; foreign prices, foreign demand and changes in the international interest rate level of which the latter channel appears to be the most potent foreign source of fluctuation.

We believe the Danish case to be particularly appealing for a structural estimation for three reasons: First, the dual assumption that the economy is small and open seems uncontroversial for the Danish economy, whereas we find it more problematic for, e.g., the German, British and Japanese economies which have been considered in previous studies; second, the Danish economy has had a clear and unaltered monetary policy regime since 1987 which validates our empirical identification; and third, we have a reliable and consistent data set for the Danish economy covering the entire period (1987-2003) under consideration.

Since our focus is a small open economy, we base our theoretical model on that proposed by Kollmann (2001, 2002). Kollmann (2001) formulates and calibrates a model of a small open economy with imperfect competition and nominal rigidities in order to analyse the responses of nominal and real exchange rates to monetary policy shocks, while Kollmann (2002) elaborates on this calibrated model and analyses welfare consequences of different monetary policies. With respect to the monetary policy, which is of special interest to us, Kollmann (2001) considers a money-growth rule while Kollmann (2002) focuses on a generalised Taylor rule (although a perfect peg is also considered). However, as the Danish monetary policy has consisted of a peg on the euro (and the D-mark before 1999) with a constant parity since 1987, we introduce an imperfect peg regime to describe the monetary policy rule under the implicit assumption that the interest rate is the central bank instrument.¹

¹As Kollmann (2002), we follow the current trend in this literature and consider the cashless limiting economy; that is, we consider an economy where money-based transactions are sufficiently unimportant for the utility of real consumption to be safely ignored. Thus, we ignore money and let the interest rate be the instrument for monetary policy. Woodford (2003) argues convincingly in favour of this approach which we consider to be the empirically relevant one.

We adopt the econometric framework of Smets and Wouters (2003), who successfully pioneered the application of Bayesian estimation techniques to a DSGE model. Thus, they estimated a variant of the complex model describing a closed economy originally constructed by Christiano et al. (2001, henceforth the CEE model) on data for the euro area. As Smets and Wouters, we structurally identify all volatility in the observed data which necessitates expansions of Kollmann's model. Thus, we include richer household preferences and a larger variety of structural shocks in our model.

The estimation of the structural model – the first of its kind that we are aware of on Danish data – yields plausible results. We do find a remarkably high degree of price stickiness, but as we discuss below this is a common feature of the emerging body of empirical evidence on DSGE models, and we discuss different possible explanations.

Bergin (2003) performs a related exercise; he estimates a variant of the Kollmann (2001) model on Australian, British and Canadian data and compares with reduced-form VAR models. In contrast to us, Bergin uses maximum-likelihood estimation and relies on a simple Choleski decomposition for identification of the structural shocks. He finds that the structural model provides a better fit than the VAR model, but is less successful at forecasting the paths of individual variables.

In another related paper, Lindé (2004) analyses the Swedish business cycle in a DSGE model. The focus is different from ours, however, as Lindé excludes preference shocks and nominal rigidities (and thus monetary policy) in his model and estimates it on annual data in accordance with his emphasis on the relative contributions of technology, fiscal policy and foreign factors to economic volatility. In contrast, we emphasise the short-term implications of the monetary policy regime and abstract from fiscal policy in our analysis.

The paper continues as follows; Section 2 presents the model, Section 3 describes the estimation methodology and the results, Section 4 analyses the properties of the estimated model and Section 5 concludes.

2 The Model

In this section we build a modified version of the open-economy DSGE model with staggered price setting presented in Kollmann (2002). Like him, we consider a small open economy that produces a continuum of intermediate goods which are aggregated and sold under imperfect competition to final-good producers at home and abroad. Producers of intermediaries only reoptimise prices infrequently a la Calvo (1983), but can differentiate fully between the domestic and foreign market and price their goods abroad in the local currency. It follows that prices are sticky in the currency of the buyer, an assumption that has been forcefully argued by, e.g., Betts and Devereux (1996, 2000). Recently, Bergin (2003, 2004) has compared local and producer currency pricing in estimated DSGE models and found strong empirical support for local currency pricing. Final goods are produced from aggregates of the intermediate goods from home and abroad and sold in a perfectly competitive market. Thus, all trade takes place in intermediary goods. McCallum and Nelson (1999, 2000) analyse a simpler model based on the same assumption and argue that it is empirically superior to one with trade in final goods.

We replace the homogenous and perfectly competitive labour market of Kollmann (2002) with one of differentiated labour services and rigid wage setting due to Erceg et al. (2000) and Kollmann (2001) which was also implemented in the CEE model. Furthermore,

we follow Smets and Wouters (2003) and assume CRRA preferences and external habit formation; thus, the preferences analysed in Kollmann model are a special case of ours. We maintain, however, the quadratic investment adjustment costs in the relative level of capital, the debt premium on the interest earned on foreign bonds and the UIP shock from the Kollmann (2002) model. Finally, we introduce an imperfect peg regime for monetary policy with a persistent policy shock.

In this section we outline the various components of the rather rich model.²

2.1 Households

Like Erceg et al. (2000) we assume a continuum with unity mass of symmetric households with index j who obtain utility from consumption of the final good $C_t(j)$ and disutility from labour efforts $l_t(j)$. Thus, they are all characterized by the following preferences:

$$E_0 \left[\sum_{t=0}^{\infty} \beta^t U(C_t^*(j), l_t(j)) \right], \quad (1)$$

$$U(C_t^*, l_t(j)) = \zeta_t^b \left[\frac{C_t^*(j)^{1-\sigma_C}}{1-\sigma_C} - \zeta_t^L \frac{l_t(j)^{1+\sigma_L}}{1+\sigma_L} \right], \quad \sigma_C, \sigma_L > 0$$

where ζ_t^b represents a shock to the discount rate and ζ_t^L represents a shock to the labour supply, while the coefficient of relative risk aversion σ_C is also the inverse intertemporal elasticity of substitution, and σ_L represents the inverse Frisch labour supply elasticity; finally, $j \in [0, 1]$ signifies the household. We follow Smets and Wouters (2003) and assume external habit formation in consumption; that is, utility is obtained from

$$C_t^*(j) = C_t(j) - hC_{t-1}, \quad 0 \leq h \leq 1, \quad (2)$$

where hC_{t-1} is the habit stock at time t which is external in the sense that it is proportional to the past aggregate consumption level that is considered exogenous to the individual household. We further assume a security market where households completely diversify their individual income uncertainty, so that consumption is equalised across households; $C_t(j) = C_t, \forall j$.

Each household supplies an idiosyncratic variety of labour service $l_t(j)$. These labour services enter as a Dixit-Stiglitz aggregate in the intermediate-goods firm production; thus, letting $l_t(s, j)$ be the amount of labour service j utilized by firm s we find that firm s uses the following amount of labour services;

$$L_t(s) = \left[\int_0^1 l_t(s, j)^{\frac{1}{1+\gamma_t}} dj \right]^{1+\gamma_t}. \quad (3)$$

Here, γ_t is the net wage markup which is assumed to be an i.i.d. process with mean $\gamma > 0$.

Wage setting is staggered a la Calvo (1983). That is, in each period household j only optimises its wage $w_t(j)$ with probability $1 - D$. The household takes the average wage rate $W_t = \left[\int_0^1 w_t(j)^{-\frac{1}{1+\gamma_t}} dj \right]^{-(1+\gamma_t)}$ as given when it chooses its optimal wage $w_{t,t}$ and

²A technical appendix with a thorough derivation of the model and its steady state is available in Chapter 5.

will meet any demand for the given type of labour;³

$$l_t(j) = \int_0^1 l_t(s, j) ds. \quad (4)$$

In addition to consumption, households can invest in domestic and foreign one-period bonds as well as in domestic capital. Capital K_t earns rental rate R_t and accumulates as follows with δ measuring depreciation;

$$K_{t+1} = K_t(1 - \delta) + I_t - \frac{\Phi(K_{t+1} - K_t)^2}{2K_t}, \quad 0 < \delta < 1, \quad \Phi > 0, \quad (5)$$

where I_t is investment. Here, we have followed Kollmann (2002) and assumed quadratic adjustment costs. Domestic bonds A_t earns net interest i_t , while the interest i_t^f accruing to foreign bonds B_t held by domestic agents deviates from the foreign interest level i_t^* as follows;

$$(1 + i_t^f) = \Omega_t(1 + i_t^*), \quad (6)$$

$$\Omega_t = v_t \exp \left\{ -\lambda \frac{e_t B_{t+1}}{P_t \Xi} \right\}, \quad \Xi = \frac{e P^x Q^x}{P}, \quad (7)$$

where e_t is the nominal exchange rate and P_t is the price of final goods, while Ξ is the steady-state value of export in units of the domestic final good. Thus, the interest on foreign bonds is growing in the foreign debt level which ensures the existence of a unique equilibrium, cf. Schmitt-Grohe and Uribe (2003), while v_t is a stochastic shock which we motivate with the empirically observed departure from the uncovered interest parity. We style v_t a UIP shock but abstain from a deeper explanation of its nature; Bergin (2004) offers a good discussion of UIP shocks in the NOEM literature.

Households own equal shares of domestic firms and thus earn profit from the intermediate-goods firms ($\Delta_t(j)$) in addition to rental rates R_t on the capital, wage income from their labour services and payments from their state-contingent securities ($S_t(j)$). Hence, the budget constraint of household j is

$$\begin{aligned} A_{t+1}(j) + e_t B_{t+1}(j) + P_t(C_t(j) + I_t(j)) &= \\ A_t(j)(1 + i_{t-1}) + e_t B_t(j)(1 + i_{t-1}^f) + R_t K_t(j) + \Delta_t(j) + w_t(j) l_t(j) + S_t(j). \end{aligned} \quad (8)$$

Thus, households decide their consumption, wages and investments in accordance with the solution to the following problem;

$$\begin{aligned} \max_{\{C_t(j), A_{t+1}(j), B_{t+1}(j), K_{t+1}(j), w_{t,t}\}_{t=0}^{\infty}} E_0 \left[\sum_{t=0}^{\infty} \beta^t U(C_t^*(j), l_t(j)) \right], \\ \text{s.t. (1)-(8)}. \end{aligned} \quad (9)$$

³Note that the optimal wage in any period is identical across households, which is the reason why $w_{t,t}$ can be written without index j .

The first-order conditions for domestic and foreign bonds yield regular Euler conditions;

$$(1 + i_t) E_t [\rho_{t,t+1}] = 1, \quad (10)$$

$$\left(1 + i_t^f\right) E_t \left[\rho_{t,t+1} \frac{e_{t+1}}{e_t}\right] = 1, \quad (11)$$

$$\rho_{t,\tau} \equiv \beta^\tau (U_{C,\tau}/U_{C,t}) (P_t/P_\tau), \quad U_{C,t} \equiv \frac{\partial U(C_t^*, L_t)}{\partial C_t}, \quad (12)$$

where $\rho_{t,\tau}$ discounts profits at time τ . One should bear in mind, however, that in this case $U_{C,t}$ depends on C_{t-1} as well as C_t due to our assumption of external habits.

The optimal wage level $w_{t,t}$ is the solution to the following first-order condition;

$$\sum_{\tau=t}^{\infty} (\beta D)^{\tau-t} \frac{\chi_\tau}{\gamma_\tau} w_{t,t}^{-\frac{1+2\gamma_\tau}{\gamma_\tau}} E_t \left[\frac{U_{C,\tau}}{P_\tau} w_{t,t} - (1 + \gamma_\tau) U_{L,\tau} \right] = 0,$$

where $D^{\tau-t}$ is the probability that the chosen wage level $w_{t,t}$ is still in effect in period τ . Thus, the infrequent and stochastic reoptimisation implies that the household must weigh the marginal utility of consumption against the disutility of labour in all future periods when it sets its wage level. Finally, under the Calvo-like assumptions of the wage setting, the aggregate wage level evolves as follows;

$$W_t = \left[D (W_{t-1})^{-\frac{1}{\gamma_t}} + (1 - D) (w_{t,t})^{-\frac{1}{\gamma_t}} \right]^{-\gamma_t}.$$

2.2 Final Goods

Final goods Z_t are produced using intermediate-good bundles from home (Q_t^d) and abroad (Q_t^m) respectively. These intermediary aggregates are combined with a Cobb Douglas technology;⁴

$$Z_t = \left(\frac{Q_t^d}{\alpha^d} \right)^{\alpha^d} \left(\frac{Q_t^m}{\alpha^m} \right)^{\alpha^m}, \quad \alpha^d, \alpha^m < 1 \quad \alpha^d + \alpha^m = 1.$$

Each bundle of intermediate goods is a Dixit-Stiglitz aggregate. Here, we follow the assumptions of the CEE model and let the net markup rate ν_t be an i.i.d. process with mean ν ;

$$Q_t^i = \left[\int_0^1 q^i(s)^{\frac{1}{1+\nu_t}} ds \right]^{1+\nu_t}, \quad i = d, m.$$

Assuming that domestic firms face the problem of minimizing the cost of producing Z_t units of the final good, demands for goods produced domestically and abroad can be written as

$$\begin{aligned} Q_t^i &= \alpha^i \frac{P_t}{P_t^i} Z_t, \quad i = d, m, \\ P_t &= (P_t^d)^{\alpha^d} (P_t^m)^{\alpha^m}, \end{aligned}$$

where the appropriately defined price index P_t is the marginal cost of the final-goods

⁴Bergin (2004) estimates a model where domestic and foreign intermediary goods are combined with the more flexible CES technology. He finds that the special Cobb-Douglas case is in accordance with the data.

producing firm. With perfect competition in the final-goods market, P_t is also the price of one unit of the final consumption good.

2.3 Intermediate Goods

Intermediate goods are produced from labour L_t and capital K_t using Cobb-Douglas technology. Thus, the production function of firm s is

$$y_t(s) = \theta_t K_t(s)^\psi L_t(s)^{1-\psi}, \quad 0 < \psi < 1,$$

where θ_t is the aggregate level of technology. Producers operate in a monopolistic competitive market, where each producer sets the price of her variety, taking other prices as given and supplying whatever amount is demanded at the price set.

Firms rent capital at the rate R_t and compensate labour with wages W_t . Hence, any firm's marginal costs are

$$MC_t = \frac{1}{\theta_t} W_t^{1-\psi} R_t^\psi \psi^{-\psi} (1-\psi)^{-(1-\psi)}. \quad (13)$$

Producers sell their good variety to both domestic and foreign final-goods producers (that is, $y_t(s) = q_t^d(s) + q_t^x(s)$) and are able to price discriminate between the two markets. As is well-known from the Dixit-Stiglitz models, final-good producers demand individual varieties of intermediaries as follows

$$q_t^i(s) = \left(\frac{p_t^i(s)}{P_t^i} \right)^{-\frac{1+\nu_t}{\nu_t}} Q_t^i, \quad i = d, m,$$

and thereby firm profits can be written as

$$\pi^{dx}(p_t^d(s), p_t^x(s)) = (p_t^d(s) - MC_t) q_t^d(s) + (e_t p_t^x(s) - MC_t) q_t^x(s).$$

We furthermore assume that foreign exporters produce at unit costs equivalent to the aggregate foreign price level P_t^* and thus generate the following profits in the domestic market;

$$\pi^m(p_t^m(s)) = (p_t^m(s) - e_t P_t^*) \left(\frac{p_t^m(s)}{P_t^m} \right)^{-\frac{1+\nu_t}{\nu_t}} Q_t^m.$$

Demands from foreign final-goods producers are assumed to be of the Dixit-Stiglitz form as well;

$$q_t^x(s) = \left(\frac{p_t^x(s)}{P_t^x} \right)^{-\frac{1+\nu_t}{\nu_t}} Q_t^x, \quad Q_t^x = \left(\frac{P_t^x}{P_t^*} \right)^{-1} Y_t^*,$$

where the foreign aggregates P_t^*, Y_t^* are exogenous.

As in the case of wages, we follow Calvo (1983) and assume that a firm only reoptimises its prices in any given period with probability $1 - d$. Given that domestic firms seek to maximise profits discounted with a pricing kernel based on household utility (cf. equation (12)), a firm that reoptimises its domestic price faces the following problem;

$$p_{t,t}^d = \arg \max_{\omega} \sum_{\tau=t}^{\infty} d^{\tau-t} E_t [\rho_{t,\tau} \pi^{dx}(\omega, p_t^x(s))].$$

As firms set prices in the domestic and foreign market separately, the constant marginal costs – cf. equation (13) – implies that the two price setting problems are independent. Hence, the optimal price $p_{t,t}^d$ is determined from the following first-order condition;

$$\sum_{\tau=t}^{\infty} d^{\tau-t} E_t \left[\left(p_{t,t}^d - (1 + \nu_{\tau}) MC_{\tau} \right) \rho_{t,\tau} \left(\frac{p_{t,t}^d}{P_{\tau}^d} \right)^{-\frac{1+\nu_{\tau}}{\nu_{\tau}}} \frac{Q_{\tau}^d}{p_{t,t}^d \nu_{\tau}} \right] = 0, \quad (14)$$

which illustrates an essential feature of Calvo pricing; because of the infrequent and stochastic price setting firms must consider expectations of all future levels of marginal costs and demands when calculating their optimal price. The optimal price in the export market ($p_{t,t}^x$) is determined analogously.

Import firms are owned by risk-neutral foreigners who discount future profits at the foreign nominal interest rate $R_{t,\tau} \equiv \prod_{s=t}^{\tau-1} (1 + i_s^*)^{-1}$. Thus, when they reoptimise, they set their prices in order to maximize discounted future profits measured in foreign units;

$$p_{t,t}^m = \arg \max_{\omega} \sum_{\tau=t}^{\infty} d^{\tau-t} E_t [R_{t,\tau} \pi^m(\omega) / e_{\tau}],$$

$$P_t^i = \left[d (P_{t-1}^i)^{-\frac{1}{\nu_t}} + (1 - d) (p_{t,t}^i)^{-\frac{1}{\nu_t}} \right]^{-\nu_t}, \quad i = d, m, x.$$

2.4 Market Clearing

All intermediaries are demanded from either domestic or foreign final goods producers, while final goods can either be consumed or invested in capital. Hence, equilibria in the markets for intermediate and final goods require

$$\begin{aligned} Y_t &= Q_t^d + Q_t^x, \\ Z_t &= C_t + I_t. \end{aligned} \quad (15)$$

Equalising the supply and demand for capital implies

$$K_t = \int K_t(s) ds.$$

Finally, we assume that only domestic agents hold the domestic bond, implying that $A_t = \int A_t(j) dj = 0$ in equilibrium.

Aggregating and manipulating the household budget constraint (8) and using the final-good market equilibrium (15) yields the following equation which simply states that the net foreign assets position (NFA) changes with accruing interest and the net export.

$$\begin{aligned} e_t B_{t+1} + P_t (C_t + I_t) &= e_t B_t \left(1 + i_{t-1}^f \right) + R_t K_t + W_t L_t + P_t^d Q_t^d + e_t P_t^x Q_t^x - (R_t K_t + W_t L_t) \Rightarrow \\ B_{t+1} &= B_t \left(1 + i_{t-1}^f \right) + P_t^x Q_t^x - \frac{P_t^m}{e_t} Q_t^m. \end{aligned}$$

2.5 Monetary Policy

We postulate an imperfect peg against the euro as the monetary policy; in our model the interest rate is the instrument, which is thus used to keep e_t constant up to an exogenous

policy shock ξ_t with unity mean;

$$e_t = e\xi_t. \quad (16)$$

Log-linearizing equations (6) and (7) yields the following relation between the internal foreign interest rate and that paid to domestic holders of foreign bonds;

$$\hat{i}_t^f = \hat{i}_t^* + \hat{v}_t - \lambda \hat{B}_t.$$

Combining this relation with log-linearised versions of equations (10) and (11) yields

$$\begin{aligned} E_t \Delta \hat{e}_{t+1} &= \hat{i}_t - \hat{i}_t^f = \hat{i}_t - \hat{i}_t^* + (\lambda \hat{B}_t - \hat{v}_t), \\ \hat{i}_t &\equiv \log \left(\frac{1 + i_t}{1 + \bar{i}} \right), \quad \hat{v}_t \equiv \log(v_t/v), \quad \hat{B}_t \equiv \frac{B_t}{P^x Q^x}, \end{aligned}$$

which we can combine with (16) to obtain

$$\hat{i}_t = \hat{i}_t^* + (\hat{v}_t - \lambda \hat{B}_t) + E_t \Delta \hat{\xi}_{t+1},$$

that is, the interest rate responds (virtually) one-to-one with the foreign interest rate and the UIP shock and is additionally skewed by the spread and the policy shock.

2.6 Solving the model

We log-linearise the model around its deterministic steady state and solve the resulting linear rational expectation system with the Sims (2002) method. The log-linearised system is summarised in Appendix A, while the method used to solve it is described in Appendix B.

3 Estimation

We now consider the results and underlying assumptions of our estimation. Before we list our specific assumptions and report our estimation results, however, we briefly motivate the Bayesian methodology that we utilise.

3.1 Estimation Methodology

We seek suitable econometric tools to quantify and evaluate our postulated structural model of the Danish economy given our set of observed time series. Building on the seminal analysis in Smets and Wouters (2003), we follow what Geweke (1999) styled the strong econometric interpretation of our DSGE model. This implies that we postulate a full probabilistic characterisation of our observed data which allows us to estimate the structural parameters through classical maximum-likelihood methods; or alternatively—following Bayesian methodology—through combining the likelihood function with prior distributions on the structural parameters and maximise the resulting posterior density.

In this paper we follow the Bayesian approach which allows us to formalise the use of any prior knowledge we may have on the structural parameters. On a more practical level it also helps stabilise the nonlinear minimization algorithm which we use for the estimation. Given the limited length of our sample, reasonable assumptions for the prior

distributions (including restrictions on the support of certain parameters such as, e.g., standard deviations) are likely to be essential for obtaining plausible estimates. On the other hand, we utilise prior distributions we believe to be broad enough in order for the data to inform us on the structural parameters of the theoretical model.

Our model includes ten structural shocks and nine observed variables. Thus, we can proceed on the assumption that there is no measurement error in the data set without facing the problem of stochastic singularity. In other words, we attribute all stochastic volatility to identified structural shock processes. This approach was successfully carried out in the Smets and Wouters (2003) analysis of a close variant of the CEE closed-economy model.

Alternative ways of estimating DSGE models do exist; Christiano et al. (2001), e.g., estimate their model using GMM techniques based on a loss criterion measuring the distance from impulse-response functions of a monetary policy shock generated by an identified VAR and the parameterized DSGE model, respectively. This, however, does not appeal to us; we acknowledge that consistency between the predictions of a VAR and a DSGE model is a strong indication that the predicted outcome of a given experiment is robust. We believe, however, that these predictions should be obtained apart from each other. In particular, the sketched method of estimation depends critically on the right identification of the VAR model – a controversial issue on its own that will have significant effects on the estimation. By maximising a likelihood function combined with priors, we link the DSGE model and its way of propagating shocks directly to the observed patterns in the data, thus avoiding controversial assumptions of the identification of a VAR model.

3.2 Data

We treat Denmark as the home country and a weighted average of a Germany, France and the Netherlands as the foreign country. For Denmark we include observations of real GDP, total real consumption, the GDP deflator, total employment adjusted for variations in hours worked, and a three-month money-market interest rate, corresponding to the theoretical variables Y, C, P, L and i . We are unable to find a satisfactory measure for wages; although a suitable measure is available in the MONA databank, is only observed annually and thus unsuited for our analysis. We use quarterly data for the period 1987-2003 as the last adjustment of the parity between the Danish krone and the D-mark occurred in January 1987.

Due to the data break in German GDP implied by the unification of East and West Germany in 1991, we have manipulated the German real GDP series; specifically we ran an OLS regression including only a linear time trend and a unification step dummy taking the value of one from 1991 onwards. Using the obtained coefficient we shifted the level of GDP and obtained our measure of German real GDP.

Since our model assumes that the home country is pegging the foreign country, and the Danish krone was effectively pegged to the D-mark before the current peg on the euro, our foreign aggregate should at the same time be broad enough to cover as much as possible of the Danish trade and narrow enough that we can plausibly claim that the relevant exchange rate for the foreign area was historically the D-mark. We settled on Germany, France and the Netherlands which constituted 28 percent of Danish exports in 2003. We used their relative weights from the current effective exchange rate for the Danish krone as calculated by Danmarks Nationalbank which are 69, 17 and 14 percent for Germany, France and the Netherlands, respectively. For this EU aggregate we include observations

of geometric averages of real GDP and the GDP deflator, and of the D-mark/euro exchange rate vis-à-vis the Danish krone and a German three-month money-market interest rate, matching the theoretical variables Y^* , P^* , e and i^* . The data and their sources are further detailed in Appendix C.

Our log-linearized model describes stationary deviations from a steady state, so we follow Smets and Wouters (2003) and remove a linear trend from the log of our GDP, consumption and labour supply series. We further adjust the price series for a nominal trend in inflation and remove the same trend from the interest rates.

3.3 Prior Distributions

We fix a subset of key parameters which are likely to be poorly determined in a model that only considers deviations from the steady state. In a Bayesian sense, we assume very fixed prior distributions, namely ones with no variance. Thus, the discount factor β is fixed at 0.99, implying an approximate quarterly return of 1 percent, while the depreciation rate of capital δ is set at 0.025. The capital share ψ is set at 0.33, while the share of domestic intermediates in final production α^d is fixed at 0.7, and the net steady-state mark-up rate ν is fixed at 0.2. Furthermore, we follow Kollmann (2002) and set the capital mobility parameter λ at 0.0019 in accordance with the empirical findings of Lane and Milesi-Ferretti (2002). Finally, we fix η at unity corresponding to a foreign technology equal to that assumed for the home country.

This leaves us with six structural parameters capturing nominal rigidities, preferences and capital adjustment costs, as well as 17 parameters defining the structural shock processes. We assume beta distributions for parameters restricted to the range between 0 and 1, inverse gamma distributions for the standard deviations of the shock innovations, and gamma distributions for the remaining parameters. Thus, all parameters are restricted to take on positive values.

The Calvo parameters d and D are assumed to follow beta distributions with mean 0.75, which would imply that prices and wages are reoptimised every year on average. We keep the distributions tight (standard deviations are set at 0.03) since we consider price and wage contracts lasting more than 2 years on average implausible.

We assume that the inverse intertemporal elasticity of substitution σ_C follows a gamma distribution with mean 1, corresponding to log preferences, while the inverse elasticity of work effort σ_L is assumed to be gamma distributed with mean 2 and a variance broad enough to cover the lower values obtained in the microeconomic studies and the higher values used in the RBC literature.

We further assume that the autocorrelation parameters ρ for the persistent structural shocks all follow a gamma distribution with mean 0.85 and a standard deviation of 0.06; the tight distributions around these high autocorrelations ensure that the persistent structural shocks are distinguishable from the i.i.d. shocks. All variances are assumed to follow inverse gamma distributions (which are the conjugate prior distributions in this case), and we have drawn on the assumptions in Smets and Wouters (2003), the calibrations in Kollmann (2002) and regressions on our data set to determine the mean of each distribution.

All the assumptions on prior distributions are summarised in Table 1.

Table 1: Parameter Estimates

Parameter	Prior distribution			Posterior estimate			Posterior distribution ³		
	Type	Mean	Std. dev. ¹	Mode	Std. error ²	5%	Median	95%	
d	Beta	0.75	0.03	0.941	0.003	0.933	0.940	0.946	
D	Beta	0.75	0.03	0.770	0.017	0.733	0.781	0.824	
h	Beta	0.70	0.10	0.433	0.061	0.310	0.487	0.656	
σ_C	Gamma	1	10	1.709	0.127	1.143	1.583	2.183	
σ_L	Gamma	2	10	1.032	0.064	1.140	1.357	1.729	
Φ	Gamma	15	4	14.422	0.133	14.435	14.907	15.222	
<i>Shocks, persistence</i>									
ϱ^b	Beta	0.85	0.06	0.825	0.077	0.668	0.785	0.874	
ϱ^l	Beta	0.85	0.06	0.962	0.013	0.894	0.950	0.979	
ϱ^t	Beta	0.85	0.06	0.824	0.015	0.753	0.823	0.882	
ϱ^m	Beta	0.85	0.06	0.899	0.030	0.856	0.898	0.935	
ϱ^i	Beta	0.85	0.06	0.877	0.123	0.838	0.876	0.906	
ϱ^P	Beta	0.85	0.06	0.925	0.012	0.878	0.921	0.952	
ϱ^Y	Beta	0.85	0.06	0.912	0.025	0.855	0.914	0.959	
<i>Shocks, volatility</i>									
σ^b	Inv. gamma	0.01	4	0.041	0.010	0.033	0.044	0.060	
σ^l	Inv. gamma	0.01	4	0.161	0.016	0.154	0.199	0.336	
σ^t	Inv. gamma	0.70	4	1.073	0.090	0.953	1.094	1.266	
σ^U	Inv. gamma	0.05	4	0.342	0.029	0.303	0.350	0.407	
σ^{pm}	Inv. gamma	0.15	4	2.095	0.018	1.896	2.070	2.184	
σ^{wm}	Inv. gamma	0.15	4	0.002	0.029	0.001	0.003	0.008	
σ^m	Inv. gamma	0.80	4	0.739	0.059	0.656	0.749	0.863	
σ^i	Inv. gamma	0.10	4	0.102	0.050	0.090	0.104	0.122	
σ^P	Inv. gamma	0.40	4	0.337	0.029	0.300	0.342	0.399	
σ^Y	Inv. gamma	0.80	4	0.786	0.064	0.693	0.797	0.911	

Note: ¹For the gamma and the inverse gamma distributions, we have cited the shape parameter α rather than the standard deviation. ²The standard errors of the posterior-mode estimates are based on the numerically calculated Hessian matrix. ³The posterior distribution is based on a Metropolis-Hastings sampling of 60,000 draws.

3.4 Posterior Estimates

We show our estimation results in Table 1. First, we report the mode of the posterior distribution using a numerical minimisation routine. These estimates are shown along with standard errors derived from the numerically calculated Hessian. Secondly, we report the median and the 5th and 95th percentiles from the posterior distributions. These distributions were simulated with Markov-chain Monte-Carlo methods. In particular, we ran a Metropolis-Hastings algorithm with 60,000 draws from a multivariate Gaussian jumping distribution.⁵

The prior and posterior distributions of the 23 estimated parameters are also illustrated in Figures 1 to 3. Most parameters are estimated to be significantly different from zero. The exceptions are the standard deviations of the innovations to the foreign interest rate and of the wage markup shock. Since we did not have observations of the wage level, it is not too surprising that the wage markup shock is estimated to be insignificant. The absence of a time series for wages is also a likely explanation for the striking similarity between the prior and posterior distribution of the wage rigidity parameter. The autocorrelations of the persistent structural shocks lie in the range from 0.82 (the level of technology) to 0.96 (labour supply); thus, the data have not caused them to diverge much from our prior assumptions.

Our estimates of the preference parameters seem plausible. The labor supply elasticity is approximately one and the intertemporal elasticity of substitution is a half, so that both belong to the range of values regularly applied in similar analyses. The estimate for the external habit stock h lies between a third and a half; this is on the lower side compared with the literature at large, but should be equally uncontroversial.

The estimated nominal rigidities are more problematic. The first thing that stands out is the large discrepancy between our prior beliefs and the posterior distribution of the Calvo price parameter d . The posterior mode is 0.94, corresponding to an average duration of intermediary price contracts of four years. Given a strict interpretation of the Calvo pricing model, our estimate is blatantly implausible. Obviously, our implementation of Calvo pricing is the simplest one possible, and a first attempt to improve it would be to add indexation to past inflation as in the CEE model.⁶ However, when Smets and Wouters (2003) estimate this more elaborate model, they obtain results that are very similar to ours; hence, we are skeptical that this expansion of the pricing model will yield an estimated Calvo parameter in the range we consider plausible. Another obvious extension is to consider separate Calvo parameters for import and export prices,

⁵More accurately, we made 61,000 draws but discarded the first thousand in order to avoid problems with the initial point. We used the inverse Hessian calculated at the posterior mode scaled down by a factor eight as covariance matrix in the jumping distribution.

⁶By indexation to past inflation we mean that prices and wages which are not reoptimized in a given period are adjusted for past aggregate inflation instead. We refer the reader to Woodford (2003, ch. 3) for a thorough discussion of indexation in the Calvo pricing model.

although one would have to include observed time series of these prices to improve the estimation this way.

Smets and Wouters speculate that the high price stickiness is in part due to the assumption of constant returns to scale in the intermediary-goods sector – as confirmed empirically by Galí et al. (2001), this assumption implies an upward bias in the price rigidity if the returns to scale are in fact decreasing. Alternatively, Altig et al. (2005) show that if one replaces the assumption of an economy-wide rental market for capital with one of firm-specific capital, the implied Calvo price parameter is reduced significantly; thus, in their benchmark model (a variant of the closed-economy CEE model) the average time between price reoptimisations is reduced from 5.6 to 1.5 quarters, as they change the assumption regarding the capital market. Both arguments indicate that the cause of the implausibly high estimate of the Calvo parameter may just as well lie outside the specific formulation of the pricing model.

The other troubling result of our estimation is the large variance of the price markup shock. We suspect the high volatility is caused by two factors; first, we have neither public spending nor investment shocks in the model to explain the stochastic nature of demand; second, we believe that the large variance is compensating for the restrictive version of the Calvo pricing model we have implemented.

According to our estimation, the capital adjustment cost parameter Φ follows a tight posterior distribution located very close to the prior mean. Our experience with the estimation algorithm and the impulse-response functions indicate that the model properties change markedly with even small changes in Φ .

We find plausible estimates for the remaining parameters which define the structural shock processes.

The observed time series (solid lines) and the one-step-ahead predictions (dashed lines) from the Kalman filter are shown in Figure 4. Given the simplicity of the presented model, we find the fit to be overall satisfactory, although the restrictive Calvo model yields a somewhat problematic fit for prices, and the model is not quite capable of explaining the large and persistent swings in the observed labour supply.

4 Analysing the Properties of the Estimated Model

4.1 Impulse Response Analysis

We now consider the effects of shocking the exogenous processes of the model. First, we focus on the effects of an expansionary monetary policy shock and a drop in the foreign interest rate level, respectively. Subsequently, we briefly consider the effects of an unanticipated rise in the technology level and in the foreign demand. The impulse-response functions to innovations in these four shocks are shown in Figures 5-8; we depict

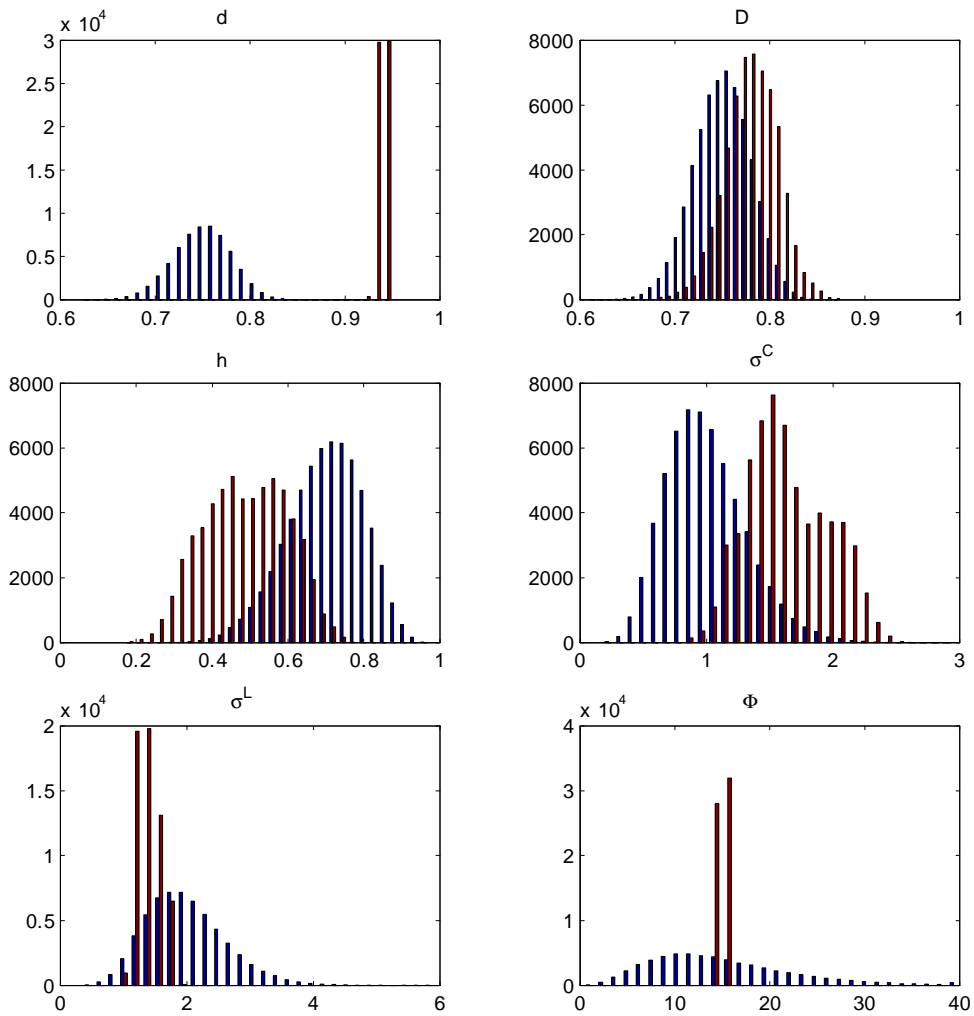


Figure 1: Preference and rigidity parameters

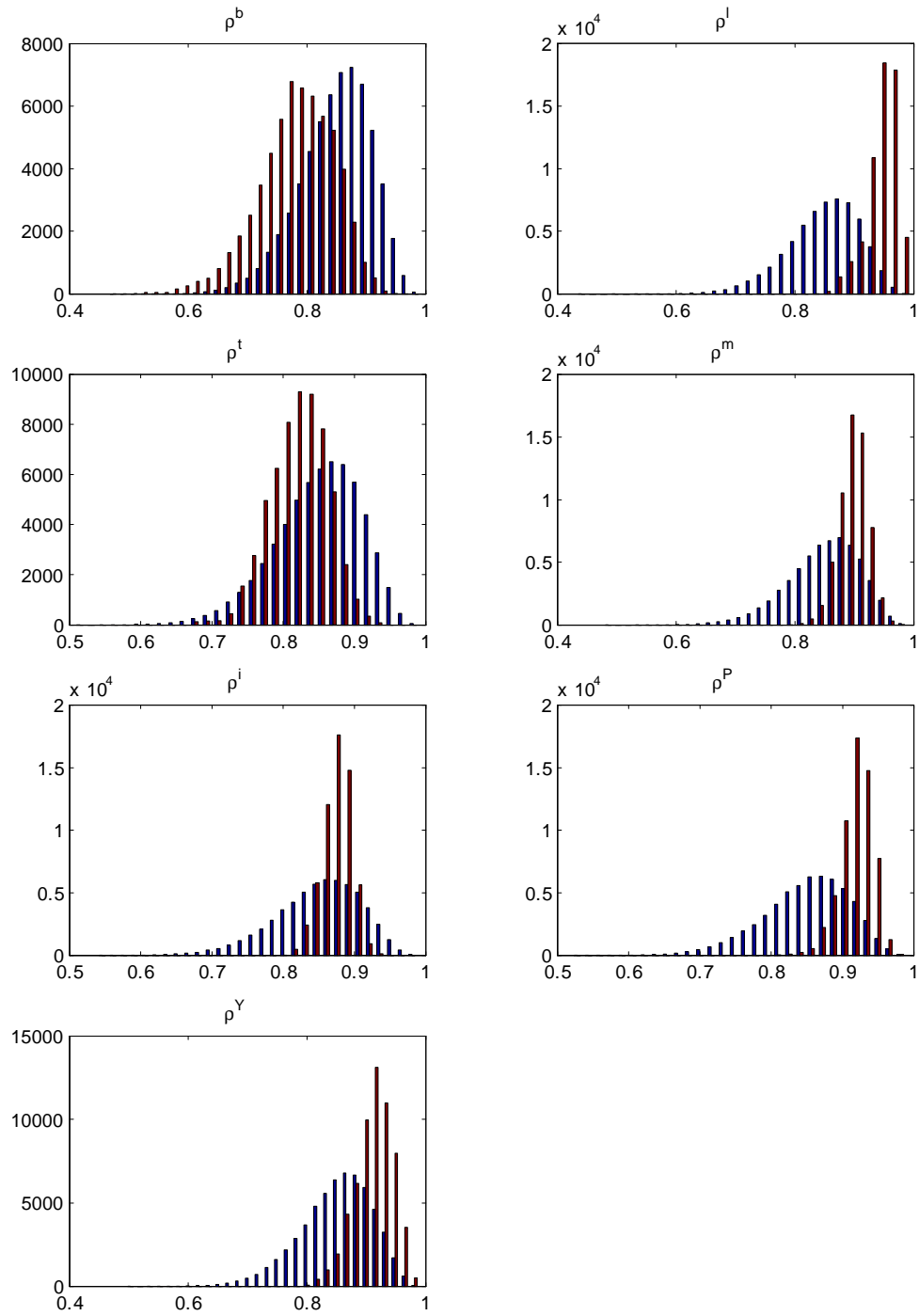


Figure 2: Persistence of structural shocks

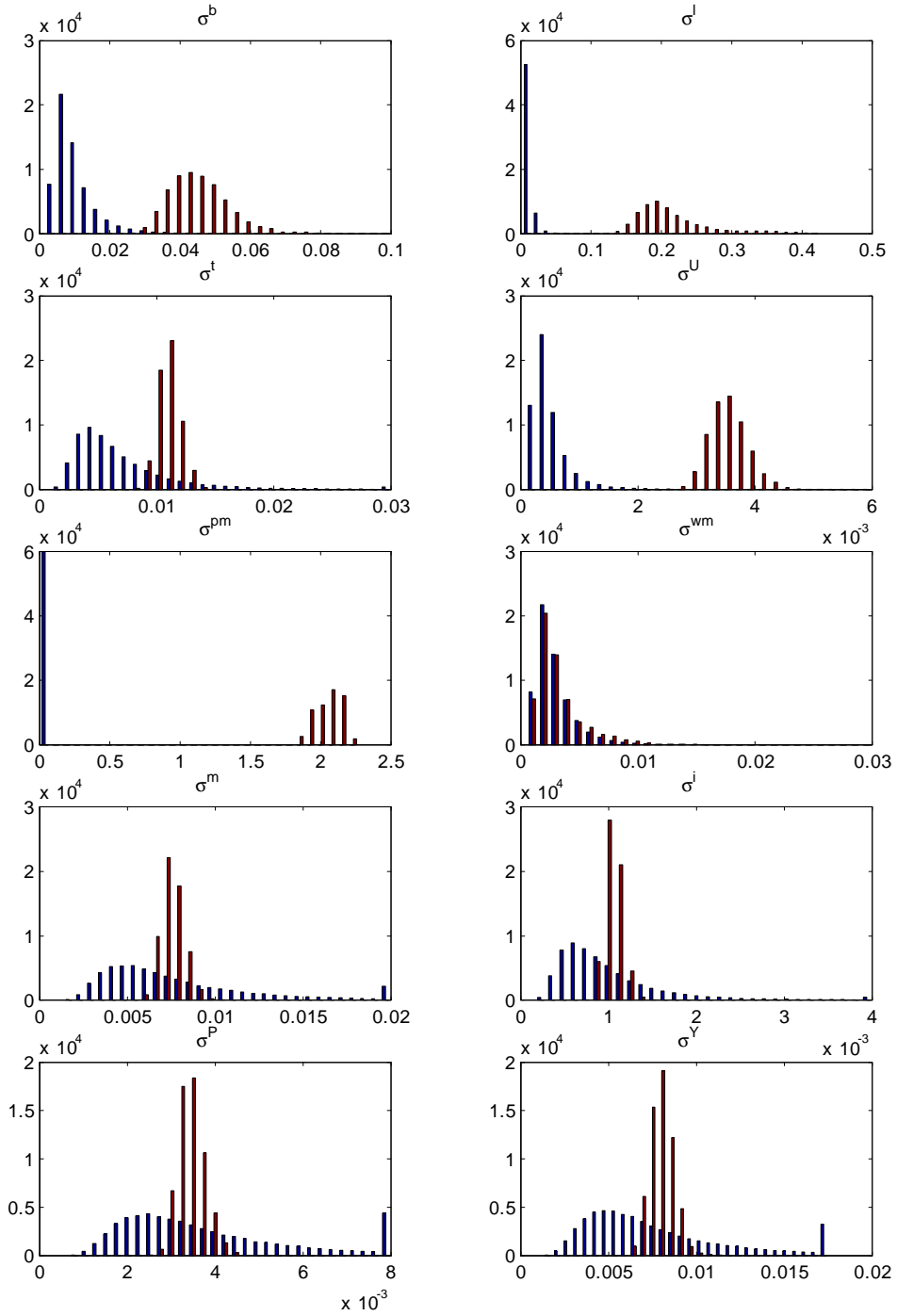


Figure 3: Volatility of structural shocks

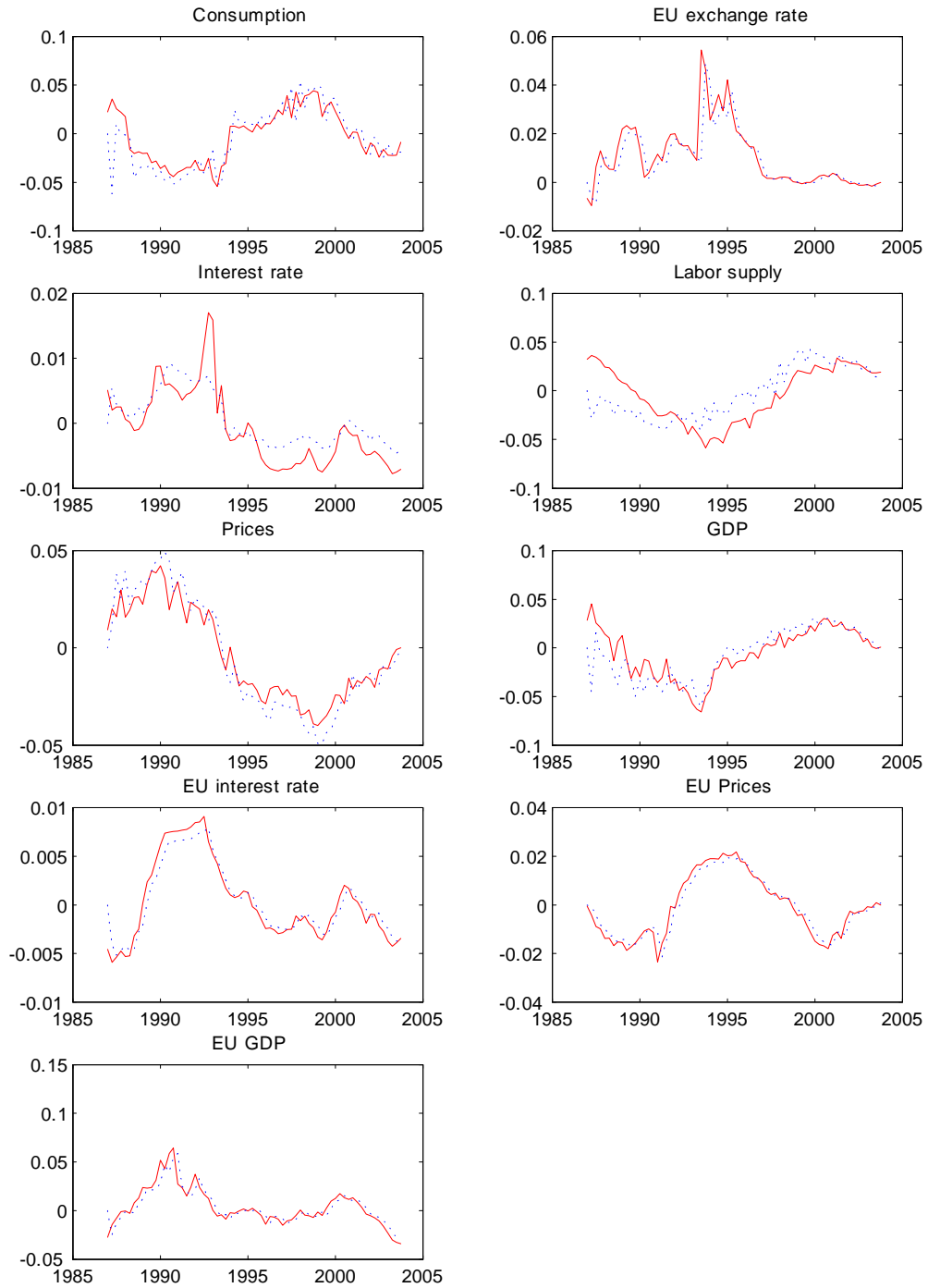


Figure 4: Observed (solid lines) and predicted data

the median (solid line) and the 5th and 95th percentiles (dashed lines) as calculated from 1,000 draws from the posterior distributions.

We begin with an analysis of the monetary transmission mechanism, by which we mean the endogenous responses to a change in the domestic interest rate which is the instrument of the central bank in our model. As discussed in section 2.5, an expansionary monetary policy shock under our peg regime implies setting the interest rate below the level required to keep the exchange rate constant. Thus, technically, the expansive policy shock is equivalent to a temporary devaluation, and we implement it through a positive innovation to ξ_t .

The effects of an expansionary monetary policy shock are shown in Figure 5. We see that a devaluation implies a positive response in consumption and investments and, hence, output. The expansive effect of a devaluation on output peaks one quarter after the devaluation with an increase in output of .8 percent following a 1 percent devaluation. Investments respond more strongly and peaks instantaneously with a 2.9 percent response. The transmission mechanism is as follows; the opportunity cost of consumption decreases with the lower interest rate, as it gets less beneficial to hold domestic bonds. Hence, aggregate demand rises which increases demand for labour and capital. This stimulates investments and pushes up marginal costs as wages and the rental price of capital increase. Since prices on the intermediary goods are determined as a markup on marginal costs, they increase correspondingly. Note, however, that the rise in the price on exported intermediaries (measured in the foreign currency) is less than a third of the rise in the domestic price level. The explanation is that by assumption the firm sets the price in the foreign currency, but seeks to maximise profits measured in local currency; thus, as the devaluation itself provides a sizeable increase in profits from exported intermediaries, maintaining the optimal markup requires a relatively smaller rise in the export price. In total, the devaluation creates inflation, and prices peak after 10 quarters with an estimated rise of .15 percent.

In Figure 6 we consider the effects of shocking the level of the foreign interest rate. The peg regime implies that the central bank lowers the domestic interest rate one-for-one with the foreign decrease in order to keep the nominal exchange rate fixed, and thus the main differences between the former experiment and the present are (i) the positive co-movement in the domestic and foreign interest rate levels; and (ii) the fact that in the present experiment the exchange rate is fixed. The real effects of this shock are similar to the ones above; the nominal effects, however, differ. Notably, the determinants of the import price are unaffected, while the prices set domestically experience long-lasting swings. Domestic producers initially increase their prices because of the higher demand. As capital accumulates and pushes the rental rate down, while the interest rates on bonds return towards the steady-state level, investment and output demand drop below their initial levels, and firms respond by lowering their prices. Thus, while prices rise at first,

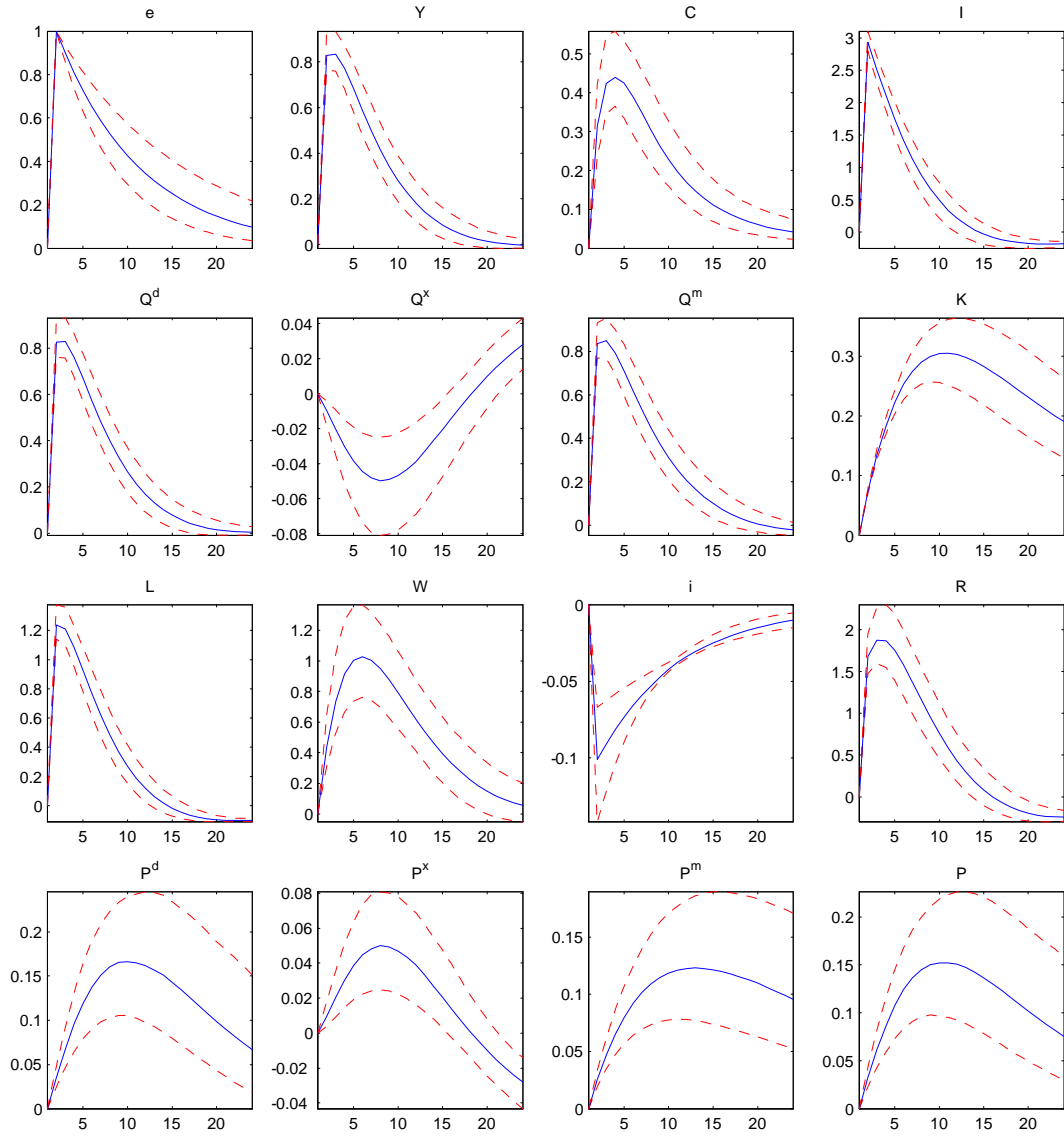


Figure 5: Responses to a monetary policy shock

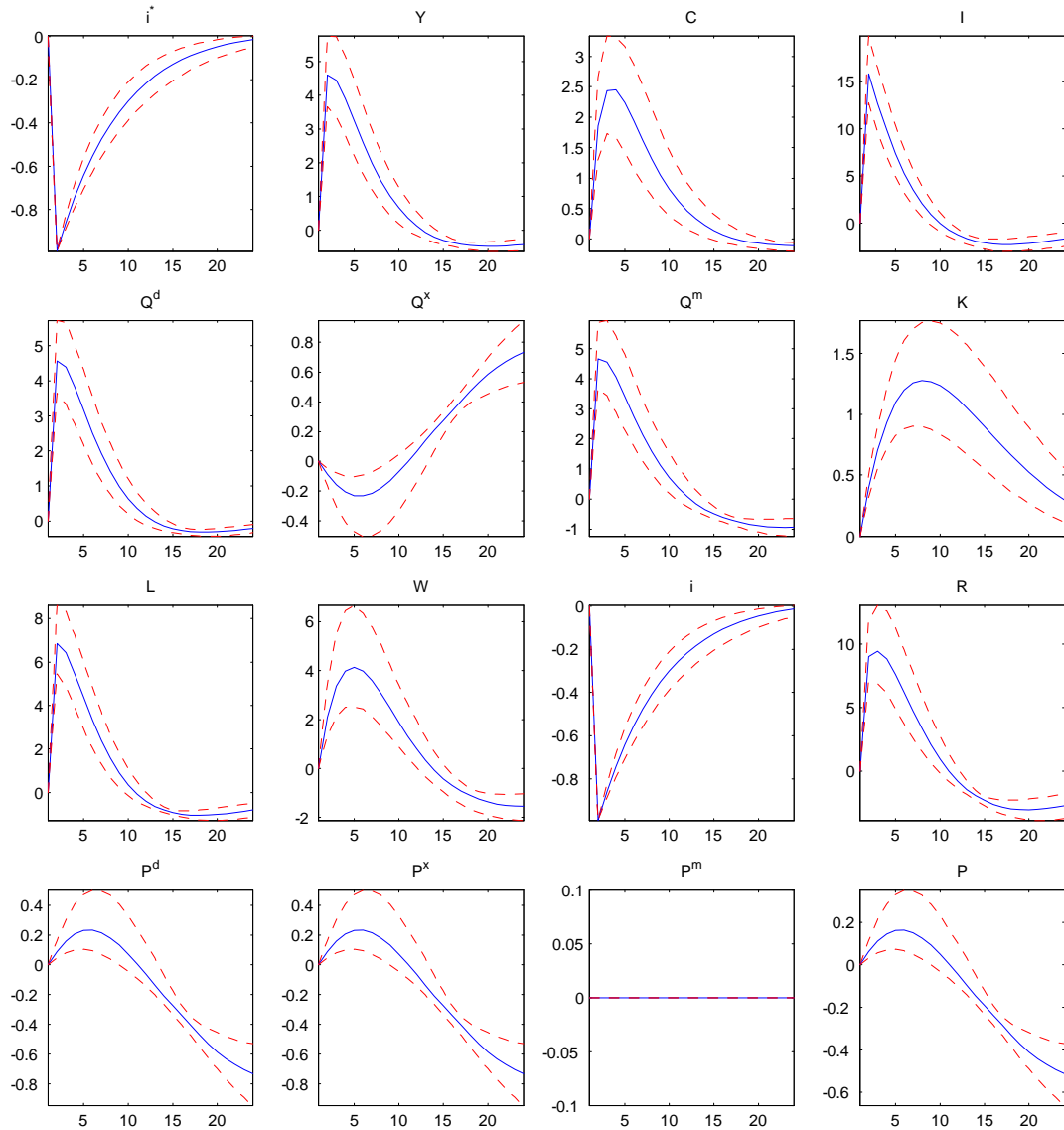


Figure 6: Responses of a shock to the foreign interest rate

they begin to fall after approximately 6 quarters and fall below the initial level after three years; only after some eight years do they begin a slow return towards the long-run equilibrium.

With regards to the technology shock depicted in Figure 7, we see that a positive shock to technology results in an initial drop in output. This is in sharp contrast to a standard RBC model. The reason is the estimated high degree of price stickiness in intermediaries. Thus, even though the positive shock to technology shifts the supply curve of the firms to the right, the price inertia causes the short-run supply curve to be almost horizontal, and thus the direct supply-side effect on output is small. Furthermore, a given level of production can now be reached using fewer production resources due to the higher level of productivity, causing employment as well as capital demand to decrease. In turn, households wish to hold less capital stock and disinvest. Thus, total demand for

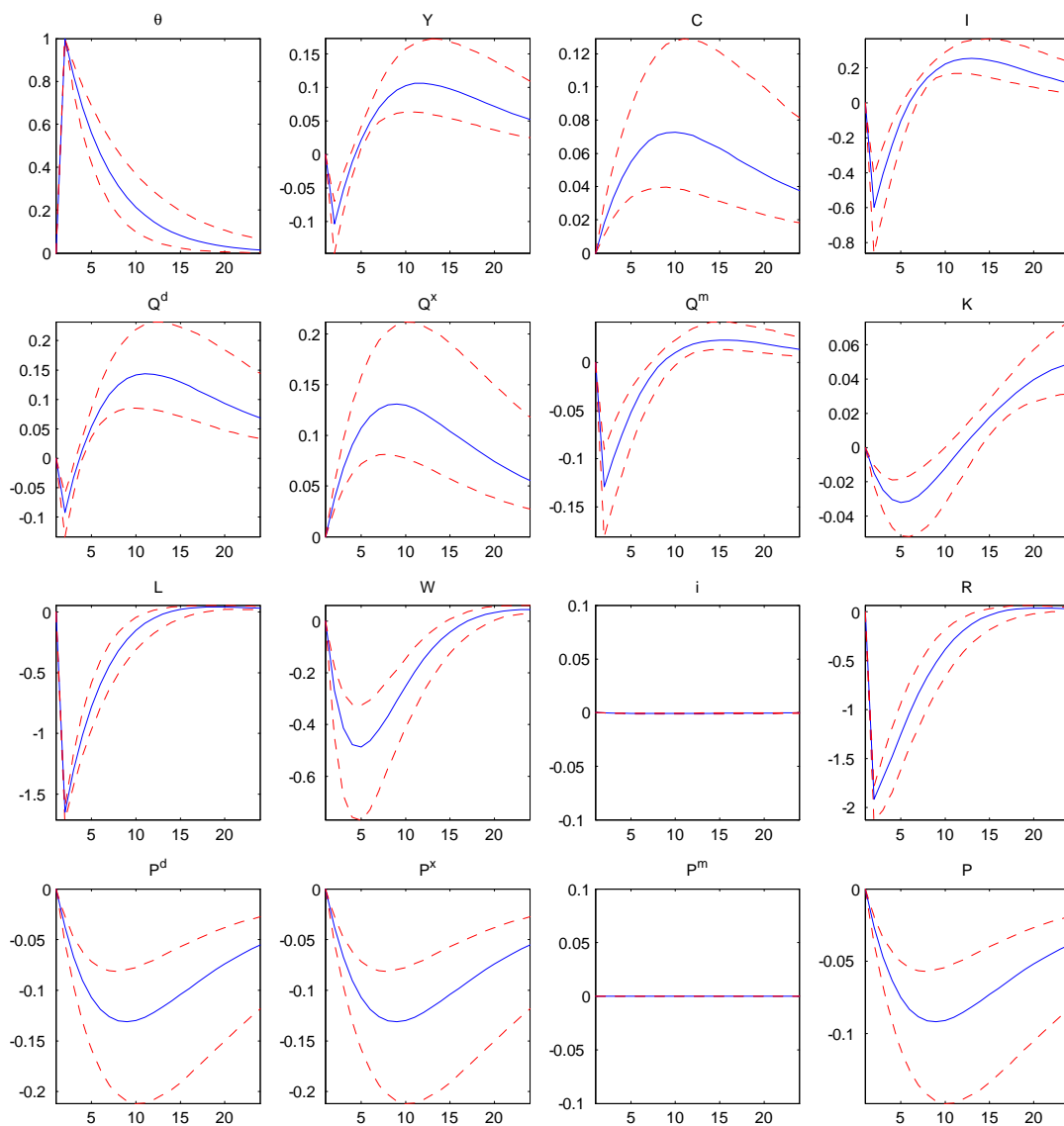


Figure 7: Responses to a shock to technology

final goods has fallen, and in equilibrium this effect dominates the positive supply effect, implying a lower output equilibrium than before the shock. Over time, however, prices do fall because of the highly persistent technology shock that has decreased marginal costs, and as demand responds to the lower prices, capital is accumulated and investments rise.

The effects of shocking the foreign demand are found in Figure 8. No surprises here; an increased demand from abroad increases demands for intermediaries; this raises demand for capital and labour pulling the rental rate and wages up, hence increasing marginal costs. Domestic producers of intermediary goods respond by increasing their prices, and consequently the price of domestic goods goes up. With P_d and P_x increasing, P goes up as well.

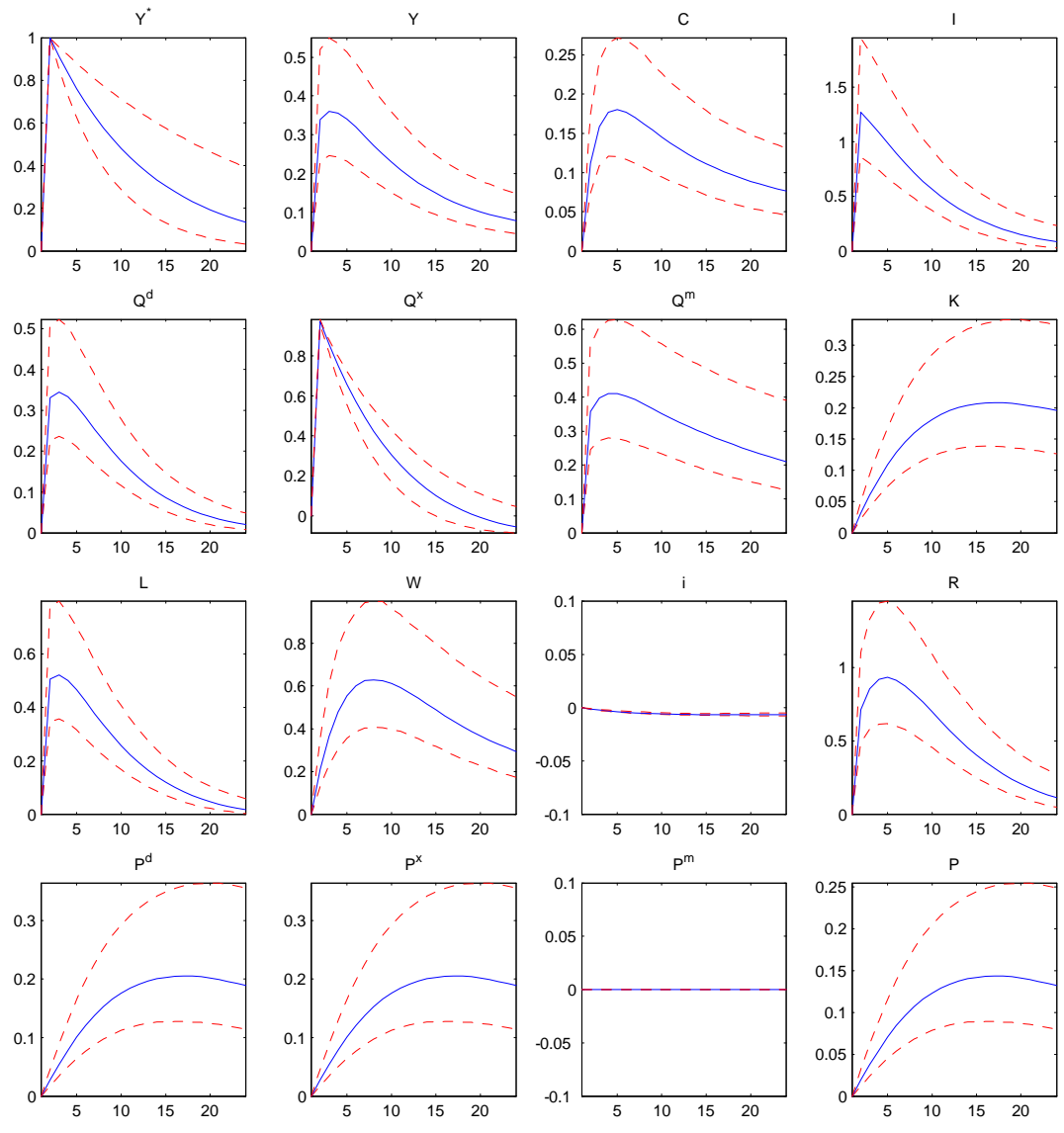


Figure 8: Responses to a foreign demand shock

4.2 Variance Decomposition

In Table 2 and 3 we present a variance decomposition of the forecast mean squared errors (MSE) at various time horizons measured in quarters which is based on 1,000 draws from the posterior distribution. Decomposing the contribution of the individual shocks to the movements in the endogenous variables yields some interesting conclusions. First, we note that technological innovations do not contribute much to the volatility in GDP in neither the short run nor in the long run. This result stands in sharp contrast to the baseline RBC model that relies on technological shocks as the sole driving force behind business cycles. Thus, our results strongly indicate that a model with a more elaborate set of structural shocks is important for understanding the forces that drive the economy. A likely explanation for the modest role of technology shocks follows from the feature of our estimated model documented in Section 4.1 above; namely, that output initially responds negatively to a positive technology shock; in contrast, the initial output response to a positive labour supply shock is positive.⁷ This is likely to support the latter as the dominant supply shock process of the two, as we believe this property to be in better accordance with the correlations of the observed data. We note that Smets and Wouters (2003) find that technology plays a minor role in the business cycle of the euro area and is clearly dominated by shocks to labour supply, and thus we regard our results to be consistent with theirs. In contrast, Bergin (2003) finds that output volatility is dominated by technology shocks, but it is worth noting that he does not consider shocks to labour supply in his analysis.

Second, demand-side shocks – mainly preferences – have a sizeable impact in the short run, while supply shocks drive cycles in the long run; despite the apparently limited role of technological shocks, we note that the supply side do contribute substantially to fluctuations in GDP. Initially, shifts in labour supply account for 23 percent of the volatility in GDP and more than two-thirds of the volatility in investments. In the long run labour supply shocks account for between 80 and 90 percent of all volatility in private consumption, investments and, hence, GDP. In the short run, however, we see that shocks to the discount rate yield the largest impact on GDP. Also, varying price markups contribute heavily. This pattern does not change much in the medium term.

Third, somewhat surprisingly we see that business cycles in Denmark to a large degree appear to be founded domestically. Although Denmark is a highly open economy, less than 10 percent of the volatility in the short and medium term can be directly traced to foreign sources, by which we mean foreign GDP, prices and interest rates. However, shocks to price mark-ups also have a component that is determined abroad, as we have not distinguished between markup shocks from domestic and foreign producers. Taking this into consideration, at most 40 percent of the short run volatility can be attributed

⁷This particular property of the model is not further discussed in the paper. Documentation is available upon request.

Table 2: Variance Decomposition

		<i>C</i>		<i>I</i>		<i>Y</i>				
<i>t</i> = 0	Preferences	0.67	0.82	0.91	0.00	0.00	0.01	0.25	0.32	0.41
	Labour supply	0.02	0.07	0.17	0.58	0.67	0.75	0.16	0.26	0.37
	Technology	0.00	0.00	0.00	0.00	0.01	0.01	0.00	0.00	0.01
	UIP	0.00	0.01	0.01	0.01	0.01	0.02	0.01	0.02	0.03
	Price mark-up	0.02	0.04	0.08	0.11	0.16	0.22	0.12	0.19	0.27
	Wage mark-up	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	Monetary policy	0.02	0.03	0.06	0.06	0.08	0.11	0.08	0.11	0.15
	Foreign int. rate	0.01	0.02	0.04	0.03	0.05	0.07	0.04	0.06	0.1
	Foreign price	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.01
	Foreign demand	0.00	0.00	0.01	0.01	0.02	0.04	0.01	0.02	0.05
	MSE (in pct.)	0.01	0.02	0.02	0.50	0.61	0.77	0.03	0.04	0.04
<i>t</i> = 4	Preferences	0.47	0.63	0.78	0.00	0.01	0.01	0.28	0.39	0.53
	Labour supply	0.12	0.24	0.39	0.54	0.64	0.73	0.08	0.12	0.18
	Technology	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00
	UIP	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.01	0.01
	Price mark-up	0.03	0.06	0.09	0.13	0.19	0.26	0.16	0.24	0.34
	Wage mark-up	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	Monetary policy	0.02	0.04	0.06	0.06	0.08	0.12	0.08	0.12	0.18
	Foreign int. rate	0.01	0.02	0.04	0.02	0.04	0.06	0.04	0.06	0.1
	Foreign price	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.01	0.01
	Foreign demand	0.00	0.01	0.02	0.01	0.02	0.06	0.01	0.03	0.08
	MSE (in pct.)	0.08	0.13	0.19	1.37	1.68	2.09	0.1	0.13	0.18
<i>t</i> = 12	Preferences	0.19	0.31	0.45	0.02	0.04	0.08	0.12	0.19	0.29
	Labour supply	0.44	0.59	0.73	0.51	0.6	0.69	0.4	0.49	0.59
	Technology	0.00	0.00	0.01	0.00	0.01	0.01	0.00	0.00	0.01
	UIP	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00
	Price mark-up	0.03	0.05	0.08	0.14	0.2	0.27	0.12	0.17	0.22
	Wage mark-up	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	Monetary policy	0.01	0.02	0.04	0.05	0.07	0.11	0.04	0.07	0.11
	Foreign int. rate	0.01	0.01	0.02	0.02	0.03	0.05	0.02	0.03	0.05
	Foreign price	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.01
	Foreign demand	0.00	0.01	0.02	0.01	0.03	0.07	0.01	0.02	0.06
	MSE (in pct.)	0.17	0.29	0.48	1.73	2.1	2.61	0.2	0.28	0.4
<i>t</i> = 100	Preferences	0.04	0.11	0.23	0.01	0.03	0.07	0.03	0.06	0.11
	Labour supply	0.67	0.84	0.94	0.7	0.78	0.84	0.72	0.84	0.93
	Technology	0.00	0.00	0.01	0.00	0.00	0.01	0.00	0.00	0.01
	UIP	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	Price mark-up	0.01	0.03	0.06	0.07	0.10	0.15	0.02	0.05	0.1
	Wage mark-up	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	Monetary policy	0.00	0.01	0.02	0.02	0.04	0.06	0.01	0.02	0.04
	Foreign int. rate	0.00	0.00	0.01	0.01	0.02	0.03	0.00	0.01	0.02
	Foreign price	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	Foreign demand	0.00	0.00	0.01	0.01	0.01	0.04	0.00	0.01	0.03
	MSE (in pct.)	0.36	0.84	2.48	3.03	4.12	5.86	0.53	1.03	2.55

Note: The variance decomposition calculation is based on 1,000 draws from the posterior distribution. In the first and third column for each exogenous variable we report the 5th and 95th percentiles respectively, while the middle column in bold states the median.

Table 3: Variance Decomposition, II

		i		W			R			
$t = 0$	Preferences	0.00	0.00	0.00	0.00	0.00	0.00	0.08	0.14	0.21
	Labour supply	0.00	0.00	0.00	0.86	0.92	0.97	0.48	0.61	0.75
	Technology	0.00	0.00	0.00	0.00	0.01	0.01	0.07	0.11	0.16
	UIP	0.83	0.88	0.92	0.00	0.00	0.00	0.00	0.00	0.01
	Price mark-up	0.00	0.00	0.00	0.00	0.01	0.01	0.03	0.06	0.1
	Wage mark-up	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	Monetary policy	0.02	0.04	0.09	0.00	0.01	0.02	0.02	0.04	0.06
	Foreign int. rate	0.05	0.08	0.11	0.00	0.00	0.01	0.01	0.02	0.04
	Foreign price	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	Foreign demand	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.01	0.02
	MSE (in pct.)	0.00	0.00	0.00	0.07	0.14	0.28	0.28	0.41	0.64
$t = 4$	Preferences	0.00	0.00	0.00	0.00	0.00	0.00	0.05	0.11	0.23
	Labour supply	0.00	0.00	0.00	0.89	0.94	0.98	0.61	0.75	0.86
	Technology	0.00	0.00	0.00	0.00	0.00	0.01	0.02	0.04	0.07
	UIP	0.61	0.7	0.77	0.00	0.00	0.00	0.00	0.00	0.00
	Price mark-up	0.00	0.00	0.00	0.00	0.01	0.01	0.02	0.04	0.06
	Wage mark-up	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	Monetary policy	0.05	0.11	0.2	0.00	0.01	0.01	0.02	0.03	0.05
	Foreign int. rate	0.13	0.18	0.26	0.00	0.00	0.01	0.01	0.01	0.02
	Foreign price	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	Foreign demand	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.01	0.02
	MSE (in pct.)	0.00	0.00	0.00	1.65	2.80	4.84	2.08	3.33	5.53
$t = 12$	Preferences	0.00	0.01	0.02	0.00	0.00	0.00	0.03	0.07	0.16
	Labour supply	0.00	0.01	0.01	0.93	0.96	0.98	0.72	0.83	0.9
	Technology	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.03	0.04
	UIP	0.54	0.63	0.71	0.00	0.00	0.00	0.00	0.00	0.00
	Price mark-up	0.00	0.00	0.00	0.00	0.01	0.01	0.02	0.03	0.05
	Wage mark-up	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	Monetary policy	0.08	0.14	0.22	0.00	0.01	0.01	0.01	0.02	0.03
	Foreign int. rate	0.15	0.21	0.29	0.00	0.00	0.00	0.00	0.01	0.01
	Foreign price	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	Foreign demand	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.01	0.02
	MSE (in pct.)	0.00	0.00	0.00	5.90	8.18	11.93	4.14	6.19	9.47
$t = 100$	Preferences	0.01	0.02	0.04	0.00	0.00	0.00	0.03	0.06	0.12
	Labour supply	0.01	0.03	0.06	0.94	0.97	0.98	0.78	0.85	0.9
	Technology	0.00	0.00	0.00	0.00	0.00	0.00	0.01	0.02	0.04
	UIP	0.5	0.59	0.68	0.00	0.00	0.00	0.00	0.00	0.00
	Price mark-up	0.00	0.00	0.00	0.00	0.00	0.01	0.02	0.03	0.04
	Wage mark-up	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	Monetary policy	0.09	0.14	0.22	0.00	0.00	0.01	0.01	0.02	0.03
	Foreign int. rate	0.15	0.2	0.28	0.00	0.00	0.00	0.00	0.01	0.01
	Foreign price	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	Foreign demand	0.00	0.01	0.01	0.00	0.00	0.01	0.00	0.01	0.02
	MSE (in pct.)	0.00	0.00	0.00	8.42	11.3	16.58	5.66	7.41	10.22

Note: The variance decomposition calculation is based on 1,000 draws from the posterior distribution. In the first and third column for each exogenous variable we report the 5th and 95th percentiles respectively, while the middle column in bold states the median.

to foreign impulses dropping to 5-6 percent in the long run. Interestingly, this result conforms with Lindé (2004) who finds that foreign factors play a minor role in the Swedish business cycles, especially in the long run. This weak link to the foreign economies might explain why Denmark seems to have been left relatively unaffected by the international slowdown since 2001.

Recently, there has been an intense debate about the response of labour supply (measured in hours worked) to technological innovations as well as the contribution of these innovations to volatility in activity. On the one hand, Galí (2004) and Galí and Rabanal (2004) argue strongly in favour of a very limited role of technology in this respect, while McGrattan (2004) defends the technology driven business cycle and the RBC model. With regards to the response of hours to technological innovations, Galí (1999) argues that hours fall after a positive shock to technology, while Christiano et al. (2004) find that hours increase. And recently, Uhlig (2004) concluded that the response is slightly positive, but insignificant. On both accounts, our findings are in accordance with those of Galí.

5 Conclusion

In this paper we formulated and estimated a DSGE model of a small open economy with several rigidities in order to facilitate a structural decomposition of the Danish business cycle. We identified ten structural shocks and quantified their relative contributions to the volatility of six central variables; consumption, investments, output, interest rates, wages and capital gains.

Somewhat surprisingly, we find that fluctuations in the Danish economy stem predominantly from domestic shocks, despite the fact that the Danish economy by most standards is considered to be an open economy. In the short run, output cycles are mainly driven by demand shocks, with shocks to preferences being the largest, yet supply-side shocks also play an important role; thus, shocks to labour supply account for one-fourth of the short-term fluctuations, whereas the contribution from technology shocks is almost negligible.

In the longer run we find that cycles are mainly driven by supply shocks. Specifically, labour supply accounts for 85 percent of all output volatility after 25 years. Technology shocks do not contribute at all. The finding that demand factors matter greatly for the short-term cycles, while long-run cycles are driven by the supply side is consistent with the traditional distinction between Keynesian models for short-run modelling and classical model long-run modelling.

We paid special attention to the monetary transmission mechanism when we analysed the properties of our estimated model. We find that a one percent devaluation implies a 0.8 percent rise in output, peaking after two quarters. This experiment leads to a

similar short-run stimulus of the real economy as a negative shock to the international interest rate. In both scenarios the central bank lowers the domestic interest rate; in the first scenario the bank deviates temporarily from the peg with an expansionary monetary policy shock, while it responds to the foreign interest shock in the second scenario in order to maintain the exchange rate peg.

Overall, we consider the estimation to be satisfactory. However, we do acknowledge three critical aspects in relation to the estimation of the model. First, we obtain a very high degree of price stickiness in the intermediate sector, corresponding to firms only being able to re-optimize prices only once every fourth year on average. This is, however, a well-known problem in this literature, and *ex ante* we did not expect to solve it. We did, however, discuss some of the recent explanations that have been suggested recently; of these changing the assumption regarding the capital market seems particularly fruitful, and we should like to implement this in future work.

The second problem concerns the contemporaneous negative reaction in output following a technology shock, and is related to the first problem. Since the short-run supply is almost horizontal, the immediate impact from a technology shock is small. When firms' production technology at the same time have improved, demands for capital decrease *ceteris paribus*. Hence investments fall and more than outweigh the rise in consumption causing output to fall. This implausible property could probably be avoided by introducing variable capital utilisation in the model.

Thirdly, the price markup shock is estimated to be implausibly volatile. We suspect two reasons for this result; (i) we have chosen the most simple and thus inflexible version of the Calvo pricing model; and (ii) we have ignored government spending and investment shock on the demand side. Thus, we leave much of the price dynamics to be accounted for by the markup shock. This result suggests that future work within this framework should consider a more flexible model for the price setting of domestic and foreign firms and include more demand components in the final goods market.

To the best of our knowledge, this is the first attempt to estimate a DSGE model on Danish data. Despite the problems just mentioned, we consider the estimated model to be a major step forward in establishing a suitable framework for the analysis of the Danish business cycle. Not only do we believe to have captured essential features of the Danish economy, we also have a utility-based metric for evaluating the welfare effects of different policies. One obvious question that begs to be answered is the consequences of alternative monetary policy regimes to the current peg. We are currently seeking to implement a generalised Taylor rule in a close variant of the estimated model presented in this paper and quantifying the implied changes in welfare.

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A Log-linearised Model

$$\hat{Q}_t^d = \hat{P}_t + \hat{Z}_t - \hat{P}_t^d, \quad (17)$$

$$\hat{Q}_t^m = \hat{P}_t + \hat{Z}_t - \hat{P}_t^m, \quad (18)$$

$$\hat{Q}_t^x = -\eta \hat{P}_t^x + \eta \hat{P}_t^* + \hat{Y}_t^*, \quad (19)$$

$$\hat{P}_t = \alpha^d \hat{P}_t^d + (1 - \alpha^d) \hat{P}_t^m, \quad (20)$$

$$\hat{L}_t = \hat{R}_t - \hat{W}_t + \hat{K}_t, \quad (21)$$

$$\hat{K}_t = -\hat{\theta}_t - (1 - \psi) \hat{R}_t + (1 - \psi) \hat{W}_t + \hat{Y}_t, \quad (22)$$

$$\widehat{MC}_t = -\hat{\theta}_t + (1 - \psi) \hat{W}_t + \psi \hat{R}_t, \quad (23)$$

$$\hat{\rho}_{t+1} = \hat{U}_{C_{t+1}} - \hat{U}_{C_t} + \hat{P}_t - \hat{P}_{t+1}, \quad (24)$$

$$\hat{P}_t^d - d\hat{P}_{t-1}^d = (1 - d)(1 - d\beta) \left[\widehat{MC}_t + \hat{\nu}_t \right] + d\beta E_t \left[\hat{P}_{t+1}^d - d\hat{P}_t^d \right], \quad (25)$$

$$\hat{P}_t^x - d\hat{P}_{t-1}^x = (1 - d)(1 - d\beta) \left(\widehat{MC}_t - \hat{e}_t + \hat{\nu}_t \right) + d\beta E_t \left[\hat{P}_{t+1}^x - d\hat{P}_t^x \right], \quad (26)$$

$$\hat{P}_t^m - d\hat{P}_{t-1}^m = (1 - d)(1 - d\beta) \left(\hat{e}_t + \hat{P}_t^* + \hat{\nu}_t \right) + d\beta E_t \left[\hat{P}_{t+1}^m - d\hat{P}_t^m \right], \quad (27)$$

$$\hat{W}_t - D\hat{W}_{t-1} = (1 - D)(1 - D\beta) \left(\hat{P}_t + \hat{U}_{L,t} - \hat{U}_{C,t} + \hat{\gamma}_t \right) + D\beta E_t \left[\hat{W}_{t+1} - D\hat{W}_t \right], \quad (28)$$

$$\hat{K}_{t+1} = (1 - \delta) \hat{K}_t + \delta \hat{I}_t, \quad (29)$$

$$\hat{B}_{t+1} = (1 + \bar{v}) \hat{B}_t + \hat{P}_t^x + \hat{Q}_t^x - \hat{P}_t^m + \hat{e}_t - \hat{Q}_t^m, \quad \hat{B}_t \equiv \frac{B_t}{P^x Q^x}, \quad (30)$$

$$\hat{U}_{C,t} = \hat{\zeta}_t^b - \frac{\sigma_C}{(1 - h)} \hat{C}_t + \frac{h\sigma_C}{(1 - h)} \hat{C}_{t-1}, \quad (31)$$

$$\hat{U}_{L,t} = \hat{\zeta}_t^b + \hat{\zeta}_t^L + \sigma_L \hat{L}_t, \quad (32)$$

$$\Phi(1 + \beta) \hat{K}_{t+1} = E_t \hat{\rho}_{t+1} - \hat{P}_t + \beta(1 - \delta) E_t \hat{P}_{t+1} + [1 - \beta(1 - \delta)] E_t \hat{R}_{t+1} + \Phi \hat{K}_t + \beta \Phi E_t \hat{K}_{t+2}, \quad (33)$$

$$\hat{i}_t = -E_t \hat{\rho}_{t+1}, \quad (34)$$

$$\hat{i}_t^f = -E_t \hat{\rho}_{t+1} - E_t \hat{e}_{t+1} + \hat{e}_t, \quad (35)$$

$$\hat{i}_t^f = \hat{i}_t^* + \hat{\nu}_t - \lambda \hat{B}_{t+1}, \quad (36)$$

$$\hat{e}_t = \hat{\xi}_t, \quad (37)$$

$$\hat{Y}_t = \alpha^d \hat{Q}_t^d + (1 - \alpha^d) \hat{Q}_t^x, \quad (38)$$

$$\hat{Z}_t = \frac{C}{Z} \hat{C}_t + \frac{I}{Z} \hat{I}_t. \quad (39)$$

The system has 24 endogenous and 10 exogenous variables. Of the latter we assume that the markup shocks and the UIP shock (ν_t, γ_t, v_t) are i.i.d. and the remaining seven are AR(1) processes;

$$\hat{\zeta}_t^b = \varrho^b \hat{\zeta}_{t-1}^b + \varepsilon_t^b, \quad (40)$$

$$\hat{\zeta}_t^l = \varrho^l \hat{\zeta}_{t-1}^l + \varepsilon_t^l, \quad (41)$$

$$\hat{\theta}_t = \varrho^t \hat{\theta}_{t-1} + \varepsilon_t^t, \quad (42)$$

$$\hat{\xi}_t = \varrho^m \hat{\xi}_{t-1} + \varepsilon_t^m, \quad (43)$$

$$\hat{v}_t^* = \varrho^i \hat{v}_{t-1}^* + \varepsilon_t^i, \quad (44)$$

$$\hat{P}_t^* = \varrho^P \hat{P}_{t-1}^* + \varepsilon_t^P, \quad (45)$$

$$\hat{Y}_t^* = \varrho^Y \hat{Y}_{t-1}^* + \varepsilon_t^Y. \quad (46)$$

B Solving the Log-linearised Model with gensys

We solve the log-linearised system (17)-(46) with the **gensys** method developed by Sims (2002). For this purpose we collect the 23 endogenous variables with 6 lagged variables and 9 exogenous processes (excluding the policy shock ξ_t) in the (38×1) vector Υ_t ;⁸

$$\begin{aligned} \Upsilon_t : & \hat{B}_t, \hat{C}_t, \hat{e}_t, \hat{v}_t, \hat{v}_t^f, \hat{I}_t, \hat{K}_t, \hat{L}_t, \widehat{MC}_t, \hat{P}_t, \hat{P}_t^d, \hat{P}_t^x, \hat{P}_t^m, \hat{Q}_t^d, \hat{Q}_t^x, \hat{Q}_t^m, \hat{R}_t, \hat{\rho}_{t+1}, \hat{U}_{C,t}, \hat{U}_{L,t}, \hat{W}_t, \hat{Y}_t, \hat{Z}_t, \\ & \hat{K}_{t-1}, \hat{P}_{t-1}^d, \hat{P}_{t-1}^x, \hat{P}_{t-1}^m, \hat{W}_{t-1}, \\ & \hat{\zeta}_t^b, \hat{\zeta}_t^l, \hat{\theta}_t, \hat{v}_t, \hat{\gamma}_t, \hat{\xi}_t, \hat{v}_t^*, \hat{P}_t^*, \hat{Y}_t^*. \end{aligned}$$

The i.i.d. shocks are included in the vector $\varepsilon_t \equiv (\varepsilon_t^b, \varepsilon_t^l, \varepsilon_t^t, \hat{v}_t, \varepsilon_t^m, \varepsilon_t^{pm}, \varepsilon_t^{um}, \varepsilon_t^i, \varepsilon_t^P, \varepsilon_t^Y)$ includes the set of i.i.d. shocks, and the seven expectational errors are included in the vector $\eta_t = (\eta_t^d, \eta_t^x, \eta_t^m, \eta_t^W, \eta_t^K, \eta_t^A, \eta_t^B)$ so that we can write the model in the canonical VAR(1) **gensys** form;

$$\Gamma_0 \Upsilon_t = \Gamma_1 \Upsilon_{t-1} + \Psi \varepsilon_t + \Pi \eta_t.$$

⁸Hence, we add six identity equations to the system (17)-(46), corresponding to the six lagged endogenous variables included in Υ_t , and two definitions of the mark-up shocks $(\hat{v}_t = \varepsilon_t^{mp}, \hat{\gamma}_t = \varepsilon_t^{mw})$.

Applying the `gensys` method recasts the system in the solved form

$$\Upsilon_t = \Theta_1 \Upsilon_{t-1} + \Theta_z Z_t.$$

C Data

We use quarterly data for the period 1987-2003. The sources of our data set are specified in Table 4.

Table 4: Data Sources

	Variable	Database	Mnemonic
	GDP		
[1]	Denmark	MONA	fy
[2]	Germany	EO	deu_gdpvq
[3]	France	MEI	fra_expgdp_vnbqrsaq
[4]	Netherlands	MEI	nld_expgdp_lnbqrsaq
	Prices		
[5]	Denmark	MONA	pfy
[6]	Germany	EO	deu_pgdpq
[7]	France	MEI	fra_expgdp_dnbsaq
[8]	Netherlands	MEI	nld_expgdp_dnbsaq
	Interest rates		
[9]	Denmark	EW	dnk14010
[10]	Germany	IFS	s13460c00zfq
	Exchange rate		
[11]	DEM/DKK	EW	dnk19008
	Miscellaneous Danish variables		
[12]	Consumption	MONA	fcp
[13]	Private employment	MONA	qp
[14]	Public employment	MONA	qo
[15]	Self-employed	MONA	qs
[16]	Working hours pr. week (max)	MONA	mactid
[17]	Employment		$([13] + [14] + [15]) \times [16]^{0.7}$

MONA: Model of the Danish Central Bank (cf. Christensen and Knudsen, 1992); EO: OECD Economic Outlook; MEI: OECD Main Economic Indicators; IFS: IMF International Financial Statistics. Data from the latter three sources were provided by EcoWin.

Chapter 2

Assessing the Welfare Cost of a Fixed Exchange-Rate Policy^{*,†}

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Abstract

This paper performs a welfare analysis based on the hypothetical scenario that Denmark gave up its peg and started conducting monetary policy according to a Taylor rule. For this we rely on a dynamic stochastic general equilibrium model for a small open economy that was estimated on Danish data using Bayesian methods. We obtain the result that the gain in welfare is equivalent to a permanent increase of around 0.8 pct in the level of consumption. Examining a range of alternative scenarios does not change this conclusion qualitatively, unless we assume a degree of policy errors under the Taylor rule that is substantially larger than those estimated by other studies.

Keywords: Open economy, Monetary policy, Business cycles, Welfare

JEL Classifications: E3, E4, E5, F4

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†Both authors were doctoral students at the University of Copenhagen when this work was conducted. The viewpoints and conclusions stated are the responsibility of the authors and do not necessarily reflect the views of the Danish Economic Councils.

1 Introduction

With the recent enlargement of the European Union there is now a sizeable number of countries bordering the euro area who are facing a complex question on their future monetary policy. In the longer term, the question will be whether these countries should adopt the euro or conduct an independent monetary policy as Sweden and Great Britain have been doing with considerable success. Recent papers on the optimality of currency areas versus independent monetary policies include Benigno and Benigno (2000) and Benigno (2004).

Yet, it remains an open question how long the new EU members would have to wait before they could fully join the monetary union. Arguably, they could be in a waiting position for years where they will be pegging the euro and thus essentially be passively adopting the monetary policy conducted by the ECB. Thus, it would be of general interest to seek to quantify the welfare consequences of pegging the euro compared with an independent monetary policy regime. Due to the combination of dramatic changes in their economies over the last decade and a very limited set of time series on key aggregate measures, obtaining reliable estimates on the welfare implications of different monetary policy regimes for the new EU members from central and eastern Europe is, alas, a very difficult task.

Incidentally, Denmark offers an interesting case study on this exact question. Although a member of the ERM for years, Denmark has opted out of the third stage of the EMU for political reasons (which mainly has to do with an EU-skeptic population). As a consequence, Denmark has effectively had a fixed exchange-rate policy for decades now; thus, since 1987 the monetary policy has kept a constant parity on the D-mark/euro. This paper seeks to quantify the welfare implications of this peg regime compared with a hypothetical independent monetary policy regime which seeks to stabilise inflation and output volatility. Thus, since the Danes have twice rejected to adopt the euro, this paper provides an answer to the question of which alternative monetary policy is the optimal one.

In order to address this question, we formulate a dynamic stochastic general-equilibrium (DSGE) model for the Danish economy and calculate a second-order approximation around its steady state. We have chosen this solution method since first-order approximations are not adequate for welfare analysis of stochastic models, cf. Kim and Kim (2003) and Schmitt-Grohe and Uribe (2004b).

The model itself builds on the one presented in Kollmann (2002). However, while Kollmann bases his welfare analysis on a calibration of the structural parameters of his model, we rely on the model that was estimated on Danish data in Dam and Linaa (2005). This model makes three important departures from the one in Kollmann (2002). Firstly, in the fixed-exchange rate case we do not consider a peg that is perfect; instead we postulate that the central bank is only able to keep the exchange rate stable up to an exogenous shock, reflecting the (minor) fluctuations observed in the exchange rate around its parity. Secondly, we replace Kollmann's assumption of a competitive labour market with one of differentiated labour and monopolistic competition amongst the households leading them to raise wages above the competitive level; in addition, we impose wage rigidities a la Calvo (1983) by assuming that households are unable to revise their wage demands every period. Thirdly, we generalise the utility function applied in Kollmann (2002) so that the key elasticities as well as habits reflecting household preferences are estimated. All in all, the model underlying our analysis has richer dynamics which *ceteris*

paribus improves its empirical plausibility. This should facilitate the reliability of the quantitative welfare cost that we deduce in this paper.

There are potentially important matters not included in the current analysis; if Denmark decided to adopt a Taylor rule, risk aversion from foreign investors might induce a reduction in direct investment flows into Denmark caused by an increased uncertainty regarding the exchange rate. Furthermore, also Danish exporters face uncertainty regarding the exchange rate and could need to engage in costly arrangements with financial intermediaries in order to eliminate this uncertainty when trading with agents abroad. Finally, we ignore issues related to the potential budget discipline being put on the Government in order to keep a peg credible.

Abstaining from these issues we conclude that there are benefits to be attained from letting monetary policy be conducted according to a Taylor rule (cf. Taylor, 1993; Woodford, 2003) instead of maintaining the peg which is the current goal of Danish monetary policy. Our estimate suggests that the gain in welfare is equivalent to a permanent increase of 0.8 pct in the level of consumption. The optimal Taylor rule is found to be characterised by attaching a weight of 3 (which is the ceiling of our grid search) to inflation and a weight of 0.8 on output growth. Contrary to Schmitt-Grohe and Uribe (2004b) we do not find it beneficial for the central bank to smooth interest rates over time.

With regards to the causes of the higher level of welfare under the Taylor rule, we obtain mixed results: in terms of consumption, the higher welfare is founded in the higher mean of consumption under the Taylor rule, although the volatility of consumption has also increased. For labour this result is reverted; under the peg regime labour is more volatile than under the Taylor rule, while the mean is predicted to be lower under the peg. Overall, agents prefer the higher consumption, despite higher volatility and more labour efforts.

Two related studies are Ambler et al. (2003) and Schmitt-Grohe and Uribe (2004a). Ambler et al. (2003) apply maximum-likelihood techniques to estimate a DSGE model without capital of a small open economy and search for the optimal Taylor rule. They do not, however, consider a fixed exchange-rate regime, as the benchmark model in their study is a Taylor rule estimated on Canadian data. They obtain the result that the gain in welfare is equivalent to a permanent increase of 1.4 pct in the level of consumption compared with the level of welfare under the historical Taylor rule. Schmitt-Grohe and Uribe (2004a) analyse the closed-economy model laid out and estimated by Christiano et al. (2001). Contrary to existing studies, Schmitt-Grohe and Uribe (2004a) find that inflation should be attached a value of just 1, giving room for what they style “a significant degree of optimal inflation volatility”. This is explained by the presence of indexation to past inflation.

This paper goes on as follows: In Section 2 the model is laid out, and in Section 3 it is parameterised. In Section 4 the welfare measure and the solution method is being described and in Section 5 we use this to find the optimal Taylor rule. In Section 6 the results are presented and in Section 7 we analyse the robustness of the results. Section 8 concludes.

2 Model

The model is basically identical to the one used by Dam and Linaa (2005) which again draws heavily on the model presented in Kollmann (2002). Like him, we consider a small

open economy that produces a continuum of intermediate goods which are aggregated and sold under imperfect competition to final-good producers at home and abroad. Producers of intermediaries only reoptimise prices infrequently a la Calvo (1983), but can differentiate fully between the domestic and foreign market and price their goods abroad in the local currency. It follows that prices are sticky in the currency of the buyer, an assumption that has been forcefully argued by, e.g., Betts and Devereux (1996, 2000). Recently, Bergin (2003, 2004) has compared local and producer currency pricing in estimated DSGE models and found strong empirical support for local currency pricing. Final goods are produced from aggregates of the intermediate goods from home and abroad and sold in a perfectly competitive market. Thus, all trade takes place in intermediary goods.

We replace the homogenous and perfectly competitive labour market of Kollmann (2002) with one of differentiated labour services and rigid wage setting due to Erceg et al. (2000) and Kollmann (2001) which was also implemented in the Christiano et al. (2001) model (henceforth the CEE model). Furthermore, we follow Smets and Wouters (2003) and assume CRRA preferences and external habit formation; thus, the preferences analysed in Kollmann's model are a special case of ours. We maintain, however, the quadratic investment adjustment costs in the relative level of capital, the debt premium on the interest earned on foreign bonds and the UIP shock from the Kollmann (2002) model. Finally, we introduce an imperfect peg regime for monetary policy with a persistent policy shock.

An important deviation from Dam and Linaa (2005) is that in this paper we treat mark-up rates as constants rather than allowing them to follow a stochastic process. The reason is a technicality; our method of obtaining a second-order approximation requires that we write the non-linear system as a multivariate first-order expectational difference equation. To our knowledge it is not possible to write a model with stochastically varying markups in this form, and thus we introduce constant markups. Schmitt-Grohe and Uribe (2004a) made an equivalent simplification when they considered optimal monetary policy in the CEE model.

In this section we outline the various components of the model.¹

2.1 Households

Like Erceg et al. (2000) we assume a continuum with unity mass of symmetric households who obtain utility from consumption of the final good and disutility from labour efforts. Thus, they are all characterized by the following preferences:

$$E_0 \left[\sum_{t=0}^{\infty} \beta^t U(C_t^*(j), l_t(j)) \right], \quad (1)$$

$$U(C_t^*, l_t(j)) = \zeta_t^b \left[\frac{C_t^*(j)^{1-\sigma_C}}{1-\sigma_C} - \zeta_t^L \frac{l_t(j)^{1+\sigma_L}}{1+\sigma_L} \right], \quad \sigma_C, \sigma_L > 0$$

where ζ_t^b represents a shock to the discount rate and ζ_t^L represents a shock to the labour supply, while the coefficient of relative risk aversion σ_C is also the inverse intertemporal elasticity of substitution, and σ_L represents the inverse Frisch labour supply elasticity;

¹A technical appendix with a thorough derivation of the model is available in Chapter 5.

finally, $j \in [0, 1]$ signifies the household. We follow Smets and Wouters (2003) and assume external habit formation in consumption; that is, utility is obtained from

$$C_t^*(j) = C_t(j) - hC_{t-1}, \quad 0 \leq h \leq 1, \quad (2)$$

where hC_{t-1} is the habit stock at time t which is external in the sense that it is proportional to the past aggregate consumption level that is considered exogenous to the individual household. We further assume a security market where households completely diversify their individual income uncertainty, so that consumption is equalised across households; $C_t(j) = C_t, \forall j$.

Each household supplies an idiosyncratic variety of labour service $l_t(j)$. These labour services enter as a Dixit-Stiglitz aggregate in the intermediate-goods firm production; thus, letting $l_t(s, j)$ be the amount of labour service j utilized by firm s we find that firm s uses the following amount of composite labour services;

$$L_t(s) = \left[\int_0^1 l_t(s, j)^{\frac{1}{1+\gamma}} dj \right]^{1+\gamma}, \quad \gamma > 1, \quad (3)$$

where γ turns out to be the net wage markup.

As was the case of intermediary prices, wage setting is staggered a la Calvo (1983). That is, in each period household j only optimizes its wage $w_t(j)$ with probability $1 - D$. The household takes the average wage rate $W_t = \left[\int_0^1 w_t(j)^{-\frac{1}{1+\gamma_t}} dj \right]^{-(1+\gamma_t)}$ as given when it chooses its optimal wage $w_{t,t}$ and will meet any demand for the given type of labour;²

$$l_t(j) = \int_0^1 l_t(s, j) ds. \quad (4)$$

In addition to consumption, households can invest in domestic and foreign one-period bonds as well as in domestic capital. Capital K_t earns rental rate R_t and accumulates as follows with δ measuring depreciation;

$$K_{t+1} = K_t(1 - \delta) + I_t - \frac{\Phi(K_{t+1} - K_t)^2}{2K_t}, \quad 0 < \delta < 1, \quad \Phi > 0, \quad (5)$$

where I_t is investment. Here, we have followed Kollmann (2002) and assumed quadratic adjustment costs. Domestic bonds A_t earns net interest i_t , while the interest i_t^f accruing to foreign bonds B_t held by domestic agents deviates from the exogenously given foreign interest level i_t^* as follows;

$$\left(1 + i_t^f\right) = \Omega_t(1 + i_t^*), \quad (6)$$

$$\Omega_t = v_t \exp \left\{ -\lambda \frac{e_t B_{t+1}}{P_t \Xi} \right\}, \quad \Xi = \frac{e P^x Q^x}{P}, \quad (7)$$

where e_t is the nominal exchange rate and P_t is the price of final goods, while Ξ is the steady-state value of export in units of the domestic final good. Thus, the interest on foreign bonds is growing in the foreign debt level which ensures the existence of a unique equilibrium, cf. Schmitt-Grohe and Uribe (2003), while v_t is a stochastic *i.i.d.* shock

²Note that the optimal wage in any period is identical across households, which is the reason why $w_{t,t}$ can be written without index j .

which we motivate with the empirically observed departure from the uncovered interest parity. We style v_t a UIP shock but abstain from a deeper explanation of its nature; Bergin (2004) offers a good discussion of UIP shocks in the new open-economy macroeconomic (NOEM) literature.

Households own equal shares of domestic firms and thus earn profit from the intermediate-goods firms ($\Delta_t(j)$) in addition to rental rates R_t on the capital, wage income from their labour services and payments from their state-contingent securities ($S_t(j)$). Hence, the budget constraint of household j is

$$\begin{aligned} A_{t+1}(j) + e_t B_{t+1}(j) + P_t (C_t(j) + I_t(j)) = \\ A_t(j) (1 + i_{t-1}) + e_t B_t(j) \left(1 + i_{t-1}^f\right) + R_t K_t(j) + \Delta_t(j) + w_t(j) l_t(j) + S_t(j). \end{aligned} \quad (8)$$

Thus, households decide their consumption, wages and investments in accordance with the solution to the following problem;

$$\begin{aligned} \max_{\{C_t(j), A_{t+1}(j), B_{t+1}(j), K_{t+1}(j), w_{t,t}\}_{t=0}^{\infty}} E_0 \left[\sum_{t=0}^{\infty} \beta^t U(C_t^*(j), l_t(j)) \right], \\ \text{s.t. (1)-(8)}. \end{aligned}$$

The first-order conditions for domestic and foreign bonds yield regular Euler conditions;

$$(1 + i_t) E_t [\rho_{t,t+1}] = 1, \quad (9)$$

$$\left(1 + i_t^f\right) E_t \left[\rho_{t,t+1} \frac{e_{t+1}}{e_t} \right] = 1, \quad (10)$$

$$\rho_{t,\tau} \equiv \beta^\tau (U_{C,\tau}/U_{C,t}) (P_t/P_\tau), \quad U_{C,t} \equiv \frac{\partial U(C_t^*, L_t)}{\partial C_t}, \quad (11)$$

where $\rho_{t,\tau}$ discounts profits at time τ . One should bear in mind, however, that in this case $U_{C,t}$ depends on C_{t-1} as well as C_t due to our assumption of external habits.

Having assumed that the household always meets demand for labour at its chosen wage level, the optimal wage rate at time t is

$$w_{t,t} = \left(\frac{\sum (D\beta)^{\tau-t} E_t \left[\zeta_t^b \zeta_t^L (1 + \gamma) W_\tau^{\frac{1+\gamma}{\gamma}(1+\sigma_L)} L_\tau^{1+\sigma_L} \right]}{\sum (D\beta)^{\tau-t} E_t \left[\frac{U_{C,\tau}}{P_\tau} W_\tau^{\frac{1+\gamma}{\gamma}} L_\tau \right]} \right)^{\frac{\gamma}{\gamma+(1+\gamma)\sigma_L}}.$$

where W_t is the aggregate wage level determined as

$$W_t = \left[D (W_{t-1})^{-\frac{1}{\gamma}} + (1 - D) (w_{t,t})^{-\frac{1}{\gamma}} \right]^{-\gamma}.$$

Thus, the infrequent reoptimisation implies that households must consider expectations of all future wage levels and labour supplies when they set their optimal wage.

2.2 Final Goods

Final goods Z_t are produced using intermediate-good bundles from home (Q_t^d) and abroad (Q_t^m) respectively. These intermediary aggregates are combined with a Cobb Douglas

technology;

$$Z_t = \left(\frac{Q_t^d}{\alpha^d} \right)^{\alpha^d} \left(\frac{Q_t^m}{\alpha^m} \right)^{\alpha^m}, \quad \alpha^d + \alpha^m = 1.$$

Each bundle of intermediate goods is a Dixit-Stiglitz aggregate, where ν turns out to be the net markup rate;

$$Q_t^i = \left[\int_0^1 q^i(s)^{\frac{1}{1+\nu}} ds \right]^{1+\nu}, \quad i = d, m.$$

Assuming that domestic firms face the problem of minimizing the cost of producing Z_t units of the final good, demands for goods produced domestically and abroad can be written as

$$\begin{aligned} Q_t^i &= \alpha^i \frac{P_t}{P_t^i} Z_t, \quad i = d, m, \\ P_t &= (P_t^d)^{\alpha^d} (P_t^m)^{\alpha^m}, \end{aligned}$$

where the appropriately defined price index P_t is the marginal cost of the final-goods producing firm. With perfect competition in the final-goods market, P_t is also the price of one unit of the final consumption good.

2.3 Intermediate Goods

Intermediate goods are produced from labour L_t and capital K_t using Cobb-Douglas technology. Thus, the production function of firm s is

$$y_t(s) = \theta_t K_t(s)^\psi L_t(s)^{1-\psi}, \quad 0 < \psi < 1,$$

where θ_t is the exogenously given aggregate level of technology. Producers operate in a monopolistic competitive market, where each producer sets the price of her variety, taking other prices as given and supplying whatever amount is demanded at the price set.

Firms rent capital at the rate R_t and compensate labour with wages W_t . Hence, any firm's marginal costs are

$$MC_t = \frac{1}{\theta_t} W_t^{1-\psi} R_t^\psi \psi^{-\psi} (1-\psi)^{-(1-\psi)}. \quad (12)$$

Producers sell their good variety to both domestic and foreign final-goods producers (that is, $y_t(s) = q_t^d(s) + q_t^m(s)$) and are able to price discriminate between the two markets. As is well-known from the Dixit-Stiglitz models, final-good producers demand individual varieties of intermediaries as follows

$$q_t^i(s) = \left(\frac{p_t^i(s)}{P_t^i} \right)^{-\frac{1+\nu}{\nu}} Q_t^i, \quad i = d, m,$$

and thereby firm profits can be written as

$$\pi^{dx}(p_t^d(s), p_t^x(s)) = (p_t^d(s) - MC_t) q_t^d(s) + (e_t p_t^x(s) - MC_t) q_t^x(s).$$

We furthermore assume that foreign exporters produce at unit costs equivalent to the

aggregate foreign price level P_t^* and thus generate the following profits in the domestic market;

$$\pi^m(p_t^m(s)) = (p_t^m(s) - e_t P_t^*) \left(\frac{p_t^m(s)}{P_t^m} \right)^{-\frac{1+\nu}{\nu}} Q_t^m.$$

Demands from foreign final-goods producers are assumed to be of the Dixit-Stiglitz form as well;

$$q_t^x(s) = \left(\frac{p_t^x(s)}{P_t^x} \right)^{-\frac{1+\nu}{\nu}} Q_t^x, \quad Q_t^x = \left(\frac{P_t^x}{P_t^*} \right)^{-1} Y_t^*,$$

where the foreign aggregates P_t^*, Y_t^* are exogenous.

As in the case of wages, we follow Calvo (1983) and assume that a firm only reoptimises its prices in any given period with probability $1 - d$. Given that domestic firms seek to maximise profits discounted with a pricing kernel based on household utility (cf. equation (11)), a firm that reoptimises its domestic price faces the following problem;

$$p_{t,t}^d = \arg \max_{\omega} \sum_{\tau=t}^{\infty} d^{\tau-t} E_t [\rho_{t,\tau} \pi^{dx}(\omega, p_t^x(s))],$$

As firms set prices in the domestic and foreign market separately, the constant marginal costs – cf. equation (12) – imply that the two price setting problems are independent. Hence, the optimal price $p_{t,t}^d$ is determined from the following first-order condition;

$$p_{t,t}^d = (1 + \nu) \frac{\sum_{\tau=t}^{\infty} d^{\tau-t} E_t \left[\rho_{t,\tau} (P_{\tau}^d)^{\frac{1+\nu}{\nu}} Q_{t+\tau}^d MC_{\tau} \right]}{\sum_{\tau=t}^{\infty} d^{\tau-t} E_t \left[\rho_{t,\tau} (P_{\tau}^d)^{\frac{1+\nu}{\nu}} Q_{\tau}^d \right]}.$$

Import firms are owned by risk-neutral foreigners who discount future profits at the foreign nominal interest rate $R_{t,\tau} \equiv \prod_{s=t}^{\tau-1} (1 + i_s^*)^{-1}$. Thus, they set their prices in order to maximize discounted future profits measured in foreign currency;

$$p_{t,t}^m = \arg \max_{\omega} \sum_{\tau=t}^{\infty} d^{\tau-t} E_t [R_{t,\tau} \pi^m(\omega) / e_{\tau}]$$

which again implies a condition for the optimal price $p_{t,t}^m$ similar to that for $p_{t,t}^d$.

Finally, the aggregate Dixit-Stiglitz prices of the intermediate goods are as follows;

$$P_t^i = \left[d (P_{t-1}^i)^{-\frac{1}{\nu}} + (1 - d) (p_{t,t}^i)^{-\frac{1}{\nu}} \right]^{-\nu}, \quad i = d, m, x.$$

2.4 Market Clearing Conditions

All intermediaries are demanded from either domestic or foreign final goods producers, while final goods can either be consumed or invested in capital. Hence, equilibria in the markets for intermediate and final goods require

$$\begin{aligned} Y_t &= Q_t^d + Q_t^x, \\ Z_t &= C_t + I_t. \end{aligned} \tag{13}$$

Turning to the capital market, aggregate demand for capital is

$$\mathcal{K}_t = \int_0^1 K_t(s) ds = \frac{1}{\theta_t} \left(\frac{\psi}{1-\psi} \frac{W_t}{R_t} \right)^{1-\psi} [q_t^d(s) + q_t^x(s)],$$

and, hence, equilibrium in the capital market ($K_t = \mathcal{K}_t$) implies

$$K_t = \frac{1}{\theta_t} \left(\frac{\psi}{1-\psi} \frac{W_t}{R_t} \right)^{1-\psi} \left[\left(\frac{\mathcal{P}_t^d}{P_t^d} \right)^{-\frac{1+\nu}{\nu}} Q_t^d + \left(\frac{\mathcal{P}_t^x}{P_t^x} \right)^{-\frac{1+\nu}{\nu}} Q_t^x \right],$$

where we introduce

$$\mathcal{P}_t^i \equiv \left[\int_0^1 (p_t^i)^{-\frac{1+\nu}{\nu}} \right]^{-\frac{\nu}{1+\nu}}, \quad i = d, x.$$

Under the assumptions of the Calvo pricing model, these indices of individual prices evolve as follows;

$$\mathcal{P}_t^i = \left[d (\mathcal{P}_{t-1}^i)^{-\frac{1+\nu}{\nu}} + (1-d) (p_{t,t}^i)^{-\frac{1+\nu}{\nu}} \right]^{-\frac{\nu}{1+\nu}}, \quad i = d, x.$$

Finally, we assume that only domestic agents hold the domestic bond, implying that $A_t = 0$ in equilibrium.

Aggregating and manipulating the household budget constraint (8) and using the final-good market equilibrium (13) yields the following equation which simply states that the net foreign assets position (NFA) changes with accruing interest and the net export.

$$\begin{aligned} e_t B_{t+1} + P_t (C_t + I_t) &= e_t B_t \left(1 + i_{t-1}^f \right) + R_t K_t + W_t L_t \\ &\quad + P_t^d Q_t^d + e_t P_t^x Q_t^x - (R_t K_t + W_t L_t) \Rightarrow \\ B_{t+1} &= B_t \left(1 + i_{t-1}^f \right) + P_t^x Q_t^x - \frac{P_t^m}{e_t} Q_t^m. \end{aligned}$$

2.5 Monetary Policy

We have two monetary policy regimes to consider. The first one is a peg regime, as presented in Dam and Linaa (2005), and the second one is a regime in which the central bank conducts monetary policy according to a Taylor rule, first suggested by Taylor (1993) and thoroughly discussed in, e.g., Woodford (2003).

With regards to the peg regime, we postulate that it is impossible for the central bank to keep the exchange rate fully fixed. This is motivated from noting that although the Danish central bank successfully has been able to keep the Danish krone stable vis-a-vis its anchor, minor movements in the exchange rate of first D-mark and then (to a lesser extent) the euro has occurred. Hence, we assume that the central bank can keep the exchange rate fixed around its parity (equal to the steady state value) up to a multiplicative exogenous policy shock ξ_t^{peg} with unity mean;

$$e_t = e \xi_t^{peg}.$$

We assume that $\xi_t^{peg} = \varrho^m \xi_{t-1}^{peg} + \varepsilon_{peg,t}^m$ where $\varepsilon_{peg,t}^m$ is a Gaussian innovation with mean 0 and standard deviation σ_{peg}^m , and $0 \leq \varrho^m < 1$ is the policy error autocorrelation. The intuition of this policy is clearest if we combine it with the Euler equations (9)-(10) and

the equations (6)-(7) describing the wedge on the international interest rate and perform a log-linearisation around the steady state. Then we obtain the following equation for the domestic interest rate;

$$\hat{i}_t = \hat{i}_t^* + \left(\hat{v}_t - \lambda \hat{B}_t \right) + E_t \Delta \hat{\xi}_{t+1}^{peg},$$

where hats indicate a relative deviations from the steady state with proper normalisations.³ Thus, the interest rate responds (virtually) one-to-one with the foreign interest rate and the UIP shock. Furthermore, a positive spread between the foreign and domestic interest rates emerges as the net foreign position of the domestic country becomes negative *et vice versa*. Besides being intuitively appealing, the debt premium also ensures the existence of a unique deterministic steady state.

The alternative Taylor rule is discussed in Section 5 below.

3 Parameterisation

To perform a quantitative welfare analysis and to produce impulse-response functions we need to assign values to the parameters in the model. In Dam and Linaa (2005) we estimated the model using a Bayesian estimation technique; that is, we used the Kalman filter to evaluate the likelihood of a log-linearised version of the model and combined that information with our prior assumptions on the structural parameters in order to obtain the posterior estimates. However, since we had to have the markup rates as constants in this analysis as discussed above, we necessarily have to deviate from the estimation results obtained in that paper. Before justifying the values chosen for the parameters we begin by summarising the parameterisation in Table 1.

Regarding preferences, these posterior estimates imply a labor supply (Frisch) elasticity of approximately one and an intertemporal elasticity of substitution of a half. Thus, our labour supply elasticity is in accordance with a rich body of microeconomic findings, yet in the lower range of the values typically used in the RBC literature. The estimate for the external habit stock h lies between one third and a half; this is on the lower side compared with the literature at large, but should be uncontroversial.

The estimated Calvo parameters imply that prices and wages are updated every four years and one year, respectively. While the latter is plausible, the former implies an implausibly high degree of price rigidity. We discuss potential causes of this puzzling finding in Dam and Linaa (2005) and we return to its implication for the welfare analysis in Section 7.

As mentioned, a difference between the current peg model and the model presented in Dam and Linaa (2005) is the absence of stochastic movements in the mark-up rates in this paper. Hence, we have fixed γ and ν at the values used as means in the markup processes in Dam and Linaa (2005). In that paper we also obtained a value of σ_L which was very high; we have thus attached a new value to this parameter based on obtaining a predicted standard deviation of Y_t (in the peg model) that approximately matches that of its empirical counterpart, GDP.

Apart from this, and apart from the values of $\alpha^d, \beta, \delta, \psi, \lambda$ and η which were kept fixed in the estimation of the model in Dam and Linaa (2005), we use the values obtained as modes in our posterior distribution.

³We refer the reader to Dam and Linaa (2005) for the exact details of a log-linearisation of the model.

Table 1: Parameter Values

Parameter		Value
α^d	Share of domestic intermediaries in final prod.	0.7
β	Discount factor	0.99
δ	Depreciation rate	0.025
ψ	Capital share	0.33
λ	Capital mobility parameter	0.0019
d	Calvo, intermediaries	0.941
D	Calvo, wages	0.770
h	Habit persistence	0.433
σ_C	Household inverse IES	1.709
σ_L	Inverse Frisch elasticity	1.032
Φ	Capital adj. cost	14.422
ν	Wage markup	0.2
γ	Price markup	0.2
<i>Shocks, persistence</i>		
ϱ^b	Discount rate	0.825
ϱ^l	Labor supply	0.962
ϱ^t	Technology	0.824
ϱ^m	Peg	0.899
ϱ^i	Foreign interest rate	0.877
ϱ^P	Foreign price level	0.925
ϱ^Y	Foreign GDP	0.912
<i>Shocks, volatility</i>		
σ^b	Discount rate	0.041
σ^l	Labor supply	0.0295
$\sigma^t \times 100$	Technology	1.073
$\sigma^U \times 100$	UIP	0.342
$\sigma_{peg}^m \times 100$	Peg	0.739
σ_{TR}^m	Monetary policy shock (Taylor rule)	0.08
$\sigma^i \times 100$	Foreign int. rate	0.102
$\sigma^P \times 100$	Foreign price level	0.337
$\sigma^Y \times 100$	Foreign GDP	0.786

4 Welfare Measure and Solution Method

Our measure of welfare is the unconditional expectation of household utility;

$$E \left[\int_0^1 U (C_t^k - hC_{t-1}^k, l_t^k(j)) dj \right],$$

where k refers to the particular policy rule. As discussed thoroughly in Kim et al. (2003), this amounts to comparing welfare in the different stochastic steady states associated with each monetary policy rule under consideration; hence, this measure implicitly disregards any welfare effects stemming from the transition between the initial state of the economy and the stochastic steady state under the considered rule.

Integrating utility over the households is unproblematic with respect to consumption as we have assumed a security market that equates consumption across them, cf. Subsection 2.1. Labour supply, however, has not been smoothed between the households, and thus we need to pay attention to the integral of the disutility of labour. Integrating over the disutility yields

$$\int_0^1 l_t(j)^{1+\sigma_L} dj = L_t^{1+\sigma_L} \left(\frac{\mathcal{W}_t}{W_t} \right)^{-\frac{1+\gamma}{\gamma}(1+\sigma_L)},$$

where

$$\mathcal{W}_t \equiv \left[\int_0^1 w_t(j)^{-\frac{1+\gamma}{\gamma}(1+\sigma_L)} dj \right]^{-\frac{\gamma}{(1+\gamma)(1+\sigma_L)}}.$$

Due to the assumptions of the Calvo-like wage setting, this index of wage dispersion evolves as follows;

$$\mathcal{W}_t = \left[D\mathcal{W}_{t-1}^{-\frac{1+\gamma}{\gamma}(1+\sigma_L)} + (1-D)w_{t,t}^{-\frac{1+\gamma}{\gamma}(1+\sigma_L)} \right]^{-\frac{\gamma}{(1+\gamma)(1+\sigma_L)}}.$$

Thus, the welfare measure can be cast as follows;

$$E \left[\int_0^1 U (C_t^k - hC_{t-1}^k, l_t^k(j)) dj \right] = \frac{\zeta_t^b}{1-\sigma_C} (C_t - h\bar{C}_{t-1})^{1-\sigma_C} - \frac{\zeta_t^b \zeta_t^L}{1+\sigma_L} L_t^{1+\sigma_L} \left(\frac{\mathcal{W}_t}{W_t} \right)^{-\frac{1+\gamma}{\gamma}(1+\sigma_L)}.$$

Given the complexity of our non-linear model, an analytical solution is unattainable. Instead, we obtain a second-order approximation with the DYNARE program.⁴ We have chosen this solution method since first-order approximations are not adequate for welfare analysis of stochastic models. We refer the reader to Kim and Kim (2003) for an example of the inadequacy of first-order approximations, and to Schmitt-Grohe and Uribe (2004b) for a thorough discussion of the merits of second-order approximations.

Application of the DYNARE solution method requires that we write our model in the following general form;

$$E_t [\Upsilon_t, \Upsilon_{t+1}, \varepsilon_t, \varepsilon_{t+1}] = 0, \quad (14)$$

where Υ_t is a vector of the endogenous variables of the model, while ε_t is a vector con-

⁴The DYNARE program is an ongoing project at CEPREMAP and can be downloaded at www.cepremap.cnrs.fr/dynare/ where documentation is also available.

taining the innovations to the structural shock processes.⁵ Thus we recast the model in the iterative form of (14) where we also normalise all nominal variables with the price of domestic (or foreign) final goods. The normalisation is carried out since the Taylor rule will only pin down the inflation rate, not the price level, and we want to work with a stationary system. This version of the model is summarised in Appendix A.⁶

5 Finding the Optimal Taylor Rule

Our alternative to the existing peg is an independent monetary policy rule belonging to the generalised family of Taylor rules. In particular, we restrict ourselves to the following variant of the interest rule;

$$i_t = \rho_i i_{t-1} + (1 - \rho_i) \left(\rho_\pi (\Pi_t - 1) + \rho_y \left(\frac{Y_t}{Y_{t-1}} - 1 \right) \right) + \xi_t^{TR},$$

where the ρ 's are the policy parameters which should be optimised to the economy in question, while ξ_t^{TR} is a Gaussian *i.i.d.* noise term reflecting monetary policy shocks. The standard deviation of this shock cannot be estimated on Danish data since this monetary regime has never existed. Instead the parameter σ_{TR}^m is attached a value equal to 0.0008, which is the posterior mode estimate found by Smets and Wouters (2003) on data for the euro area. We return to this issue in Section 7 below.

We perform a grid search of the policy parameters in the ranges $\rho_i \in [0; 0.9]$, $\rho_\pi \in [0; 3]$ and $\rho_y \in [0; 3]$. We consider increments of 0.15 for the smoothing parameter ρ_i which is usually introduced in order to capture empirically observed policy inertia; we include it in this normative exercise, however, since smoothing can improve welfare in some cases as shown by Schmitt-Grohe and Uribe (2004b). For ρ_π and ρ_y we consider increments of 0.10. Thus, we solve the model for 6727 different configurations of the Taylor rule.

Schmitt-Grohe and Uribe (2004a) formulate three requirements to what they style *operational rules*; they must (i) respond only to a limited set of readily observed variables; (ii) induce a locally unique rational-expectations equilibrium; and (iii) satisfy the non-negativity constraint on nominal interest rates. The first requirement is clearly fulfilled, as we only consider observed variables in the rule in the form of realised levels or growth rates of overall inflation, GDP (and the nominal exchange rate). In light of the controversy regarding the actual calculation of output gaps, we find that this restriction on the functional form of the rule is justified.⁷ To meet the second requirement we only consider configurations of the rule that yield a determinate equilibrium in a radius of 0.2 of the parameters under consideration. This is done in order to avoid configurations close to bifurcation points which tend to invalidate the welfare calculations, cf. Schmitt-Grohe and Uribe (2004b). Thirdly, we follow Schmitt-Grohe and Uribe (2004a) and formulate the non-negativity constraint indirectly through a condition that unconditional expectation of the interest rate should be greater than twice its standard deviation ($E[i_t] > 2\sigma_{i_t}$). This requirement is fulfilled for both the peg and the preferred Taylor rule regime.

⁵See Schmitt-Grohe and Uribe (2004c) for a presentation and derivation of the solution method we apply through DYNARE.

⁶The transformation of the nominal model to real terms is documented in the technical appendix.

⁷Here it could be argued that the central bank does not have information on Y_t at time t when it is to choose i_t . We acknowledge that, but defend our choice by claiming that the central bank should have a relatively reliable forecast regarding Y_t at time t .

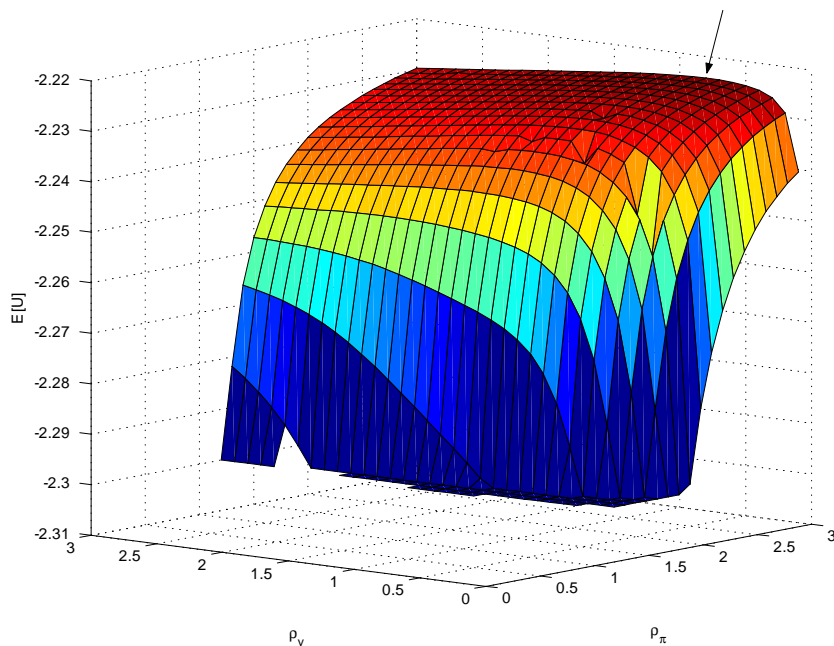


Figure 1: Unconditional utility as a function of the Taylor rule parameters

We first consider the simple version of the Taylor rule, that is, one with no interest smoothing ($\rho_i = 0$). The unconditional utility is shown as a function of the two policy rule parameters in Figure 1.⁸ The maximum utility is obtained for the configuration $(\rho_\pi, \rho_y) = (3, 0.8)$.

Interestingly, the optimal rule does not change when we introduce interest smoothing. That is, unconditional utility is maximised at $(\rho_\pi, \rho_y, \rho_i) = (3, 0.8, 0)$ which is illustrated in Figure 2.⁹ Hence, this rule will be the preferred one in the following section where we compare its merits with those of a fixed exchange rate.

Woodford (2003) establishes the optimality of a Taylor rule in a model similar in spirit to the one we have formulated. However, the optimality requires an output gap measure in the rule based on an economy with no nominal rigidities, while inefficiencies in the economy are assumed to have been eliminated through taxes and subsidies. Hence, optimality of our Taylor rule is unlikely in a wider sense, and thus the welfare gains which we find from an independent monetary policy compared with the existing peg regime only constitute a lower bound on the gains that could be obtained. We do, however, believe that the familiarity and straightforward operationality of the rules we consider is in itself an asset that motivates interest in this particular choice of monetary policy.

⁸Configurations of the policy rule that implies a determinate equilibrium with $E[U] < -2.3$ are assigned that value in Figure 1 for instructive purposes. The calculations have suffered from numerical problems which we have not been able to resolve. Thus, a few of the points have been obtained from interpolation from neighbourhood points within an 0.03 radius. Details are available upon request.

⁹Configurations of the policy rule that implies a determinate equilibrium with $E[U] < -2.25$ are assigned that value in Figure 2 for instructive purposes. The remarks on interpolation in Footnote 8 also applies here.

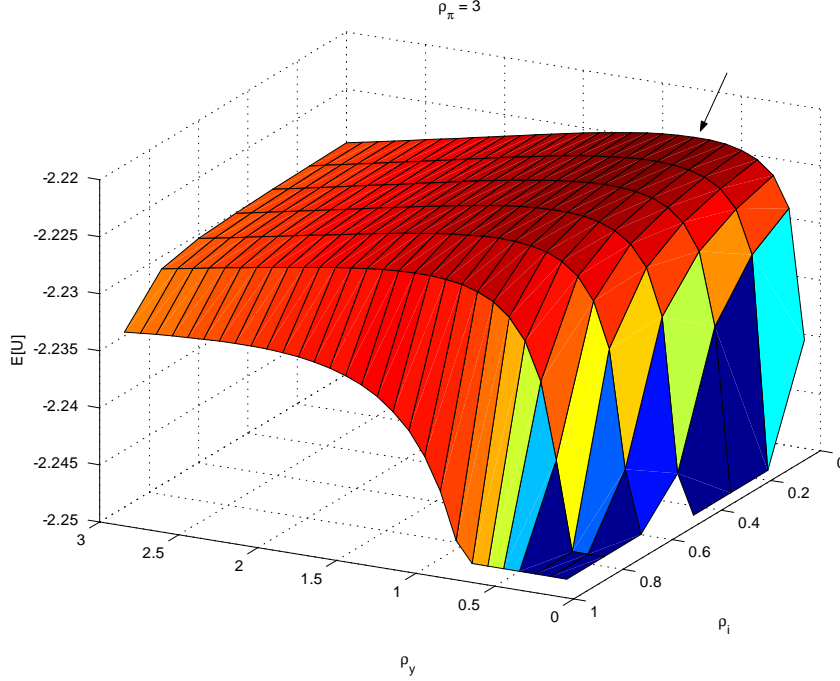


Figure 2: Unconditional utility and interest rate smoothing

6 Results

In this section we analyse the welfare implication of the two monetary policy regimes under consideration as well as their causes.

6.1 Welfare

We measure the welfare gain of a Taylor rule over the existing peg regime through compensating variation. That is, we calculate the relative permanent change in consumption that equates the unconditional utility of households under the peg regime with that obtained under the optimal Taylor rule. Thus, the compensating variation of consumption is defined as the χ that solves the following equation;

$$E \left[\int_0^1 U (C_t^{TR} - hC_{t-1}^{TR}, l_t^{TR}(j)) dj \right] = E \left[\int_0^1 U ((1 + \chi) (C_t^{peg} - hC_{t-1}^{peg}), l_t^{peg}(j)) dj \right].$$

We see from Table 2 that moving from a peg regime to one where the monetary policy is set according to a Taylor rule results in a welfare improvement of 0.79 pct measured in units of consumption goods. On the one hand, we note that although consumption is more volatile under the Taylor regime compared with the peg, the level of consumption has increased. With regards to labour supply this result is reverted; labour supply is more volatile under the peg than in the Taylor rule regime, while the mean of labour supply is lower under the peg. Overall, the household prefers the higher consumption under the Taylor regime even though they need to work more in order to obtain this.

Contrary to Kollmann (2002) we find that volatility in output is higher under a Taylor rule than under the peg. This observation is attributed the existence of the highly persistent labour supply shock we consider in this paper. Decomposing the contribution

Table 2: Welfare Analysis

Std. deviations (in pct)	<i>Peg</i>	<i>Taylor</i>
<i>Y</i>	2.85	3.36
<i>C</i>	3.22	4.99
<i>I</i>	7.03	10.75
<i>L</i>	4.25	4.09
<i>i</i>	0.44	0.19
Π	0.11	0.07
Means (in pct)		
<i>Y</i>	-0.64	-0.25
<i>C</i>	-1.16	-0.44
<i>I</i>	-1.26	0.06
<i>L</i>	-0.11	-0.08
<i>i</i>	-0.01	0.00
Π	0.00	0.00
Welfare equiv. χ (pct of <i>C</i>)		0.792
Note: All reported statistics are relative deviations from the non-stochastic steady state.		

from the shocks reveals the findings reported in Table 3. As stressed by Dam and Linaa (2005), labour supply shocks are the overall dominant source of fluctuations. To verify that this is indeed the main reason behind the increased volatility of the Taylor rule regime compared with the peg, we ran both simulations under the assumption that $\varrho^L = 0.82$ which is the autocorrelation estimated for the technology process. In this case we obtained a standard deviation of output equaling 1.81 pct in the peg regime dropping substantially to 1.00 pct under the Taylor regime, thus re-establishing the findings of Kollmann (2002). In this scenario the welfare gain by leaving the peg and adopting a Taylor rule dropped to 0.28 pct measured in units of consumption goods.

In Figure 3 we compare the unconditional utility as a function of the Taylor parameters (ρ_π, ρ_y) with that obtained under the peg. We see that a rather large set of parameters of ρ_π and ρ_y ensures a level of utility that exceeds the level of utility under the peg.

6.2 Impulse-Response Functions

This section clarifies the important deviations between the economy in which monetary policy is conducted according to a Taylor rule and one in which a constant nominal exchange rate is the monetary policy target. In particular we seek to clarify why volatility in consumption is higher under a Taylor rule than in the peg regime and why volatility in labour is lower. We do so by studying the impulse-responses obtained from both models. Inspecting the consequences of a technological shock, we see from Figure 4 that under the peg output initially drops. This phenomenon was thoroughly analysed in Dam and Linaa (2005); the initial drop in output is a consequence of the very rigid prices; recall the Calvo parameter in the intermediary sector is estimated to be as high as 0.94. Thus, even though the positive shock to technology shifts the supply curve of the firms to the right, the price inertia causes the short-run supply curve to be almost horizontal,

Table 3: Variance Decomposition

Variable		Y	C	I	L	i	Π
Preferences	Peg	0.33	0.90	0.16	0.29	0.02	0.05
	Taylor	0.05	0.28	1.26	0.04	1.54	1.71
Labour supply	Peg	61.17	78.41	42.35	31.79	0.10	75.49
	Taylor	93.97	90.16	81.47	56.31	55.70	48.03
Technology	Peg	2.41	1.33	2.37	39.05	0.00	11.51
	Taylor	0.54	0.15	0.60	41.83	34.18	41.19
UIP	Peg	0.41	0.14	1.56	0.42	62.04	0.02
	Taylor	0.00	0.00	0.00	0.00	0.00	0.00
Monetary policy	Peg	13.57	11.72	28.89	10.20	15.73	0.43
	Taylor	1.57	1.21	2.82	0.98	1.97	2.10
Foreign interest rate	Peg	5.29	3.63	12.85	4.54	21.31	1.97
	Taylor	0.18	0.14	0.44	0.12	0.50	0.46
Foreign price	Peg	2.95	1.12	2.41	2.39	0.15	0.73
	Taylor	0.06	0.17	0.96	0.09	0.86	0.86
Foreign demand	Peg	13.86	2.75	9.41	11.34	0.65	9.79
	Taylor	3.63	7.88	12.46	0.63	5.24	6.65
Note: All shares are in pct.							

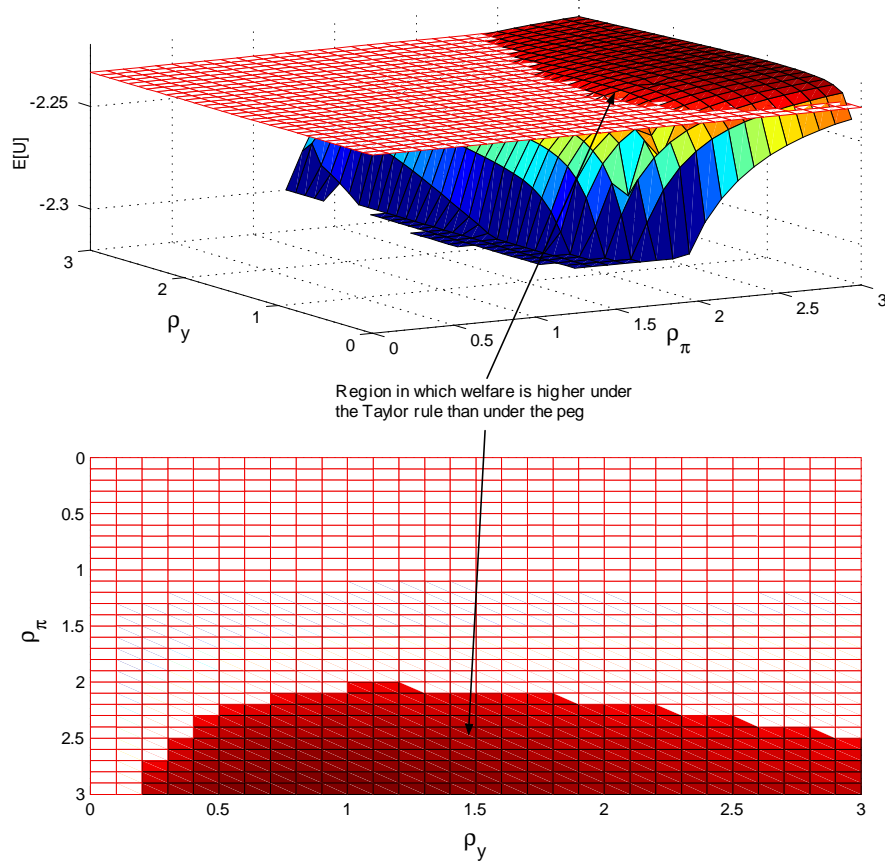


Figure 3: Unconditional utility under the peg and the Taylor rule, resp.

and thus the direct supply-side effect on output is small. Furthermore, a given level of production can now be reached using fewer production resources due to the higher level of productivity, causing employment as well as capital demand to decrease. In turn, households wish to hold less capital stock and disinvest. Thus, total demand for final goods has fallen, and in equilibrium this effect dominates the positive supply effect, implying a lower output equilibrium than before the shock. Over time, however, prices do fall because of the persistent technology shock that has decreased marginal costs, and as demand responds to the lower prices, capital is accumulated and investments rise. A crucial difference between the peg and the Taylor regime is the central bank's reaction to such a shock; under the peg the central bank keeps the interest rate virtually at the pre-shock level because the exchange rate is nearly unaffected by the shock. Over time, however, domestic prices fall since fewer resources are required to produce a given amount of goods; this drop in inflation trickers the central bank under the Taylor regime to lower interest rates. While the response of investments in the models is almost the same, we see that under a Taylor rule consumption initially benefits from the lower interest rates (as the return of holding bonds has declined), thereby bringing total demand into the positive region, ensuring a positive initial response in output.

In Figure 5 we inspect the consequences of an expansionary labour supply shock. The shock represents a shift in the household's relative valuation of consuming and enjoying leisure. Again we observe that under the peg, output initially drops for the same reasons as stated for the technology shock. Responses in consumption to a labour supply shock are far more persistent than the responses following a technology shock are for two reasons; first, the labour supply shock in itself is more persistent than the technology shock is, and second, the labour supply shock changes the relative valuation of consumption relative to leisure. For the same reasons, we also see that persistence in labour responses increase compared to those of a technological shock.

Summarising, we found that three shocks are of great importance for the volatility in labour supply; technology, monetary policy and labour supply shocks. While the response stemming from technological shocks are almost identical in the two models, we just saw that labour supply shocks contribute to generating an aggregated level of volatility in consumption and labour that is higher under a Taylor rule than in the peg regime.

This is reverted, however, when studying expansive shocks to monetary policy, cf. Figure 6. Under the peg, this experiment corresponds technically to shocking ξ_t^{peg} , thereby devaluing e_t . Since this shock is autocorrelated, e_t will remain undervalued compared to its parity for periods to follow. The lower level of interest rates stimulates consumption as well as investments and output rises. Under the Taylor rule the experiment is slightly different; ξ_t^{TR} is negatively shocked, and initially this is expected to induce a fall in i_t . However, when the central bank lowers interest rates households prefer to consume or hold foreign bonds instead of domestic ones; the first effect causes output to increase while the latter effect puts pressure on the exchange rate and inflation rises. The degree of price stickiness is very high, and therefore the central bank immediately reacts by hiking interest rates since also distant future periods is weighted heavily. Additionally, the increase in output causes the central bank to contract monetary policy; this endogenous response in interest rates is larger than the exogenous response stemming from the shocks is, and therefore our experiment of performing an expansionary monetary policy shock results in initially rising interest rates. For the same reason the responses in consumption and investments, and hence output, are more muted than under the peg. In the short term, however, the economy benefits from the expansionary effects stemming from a

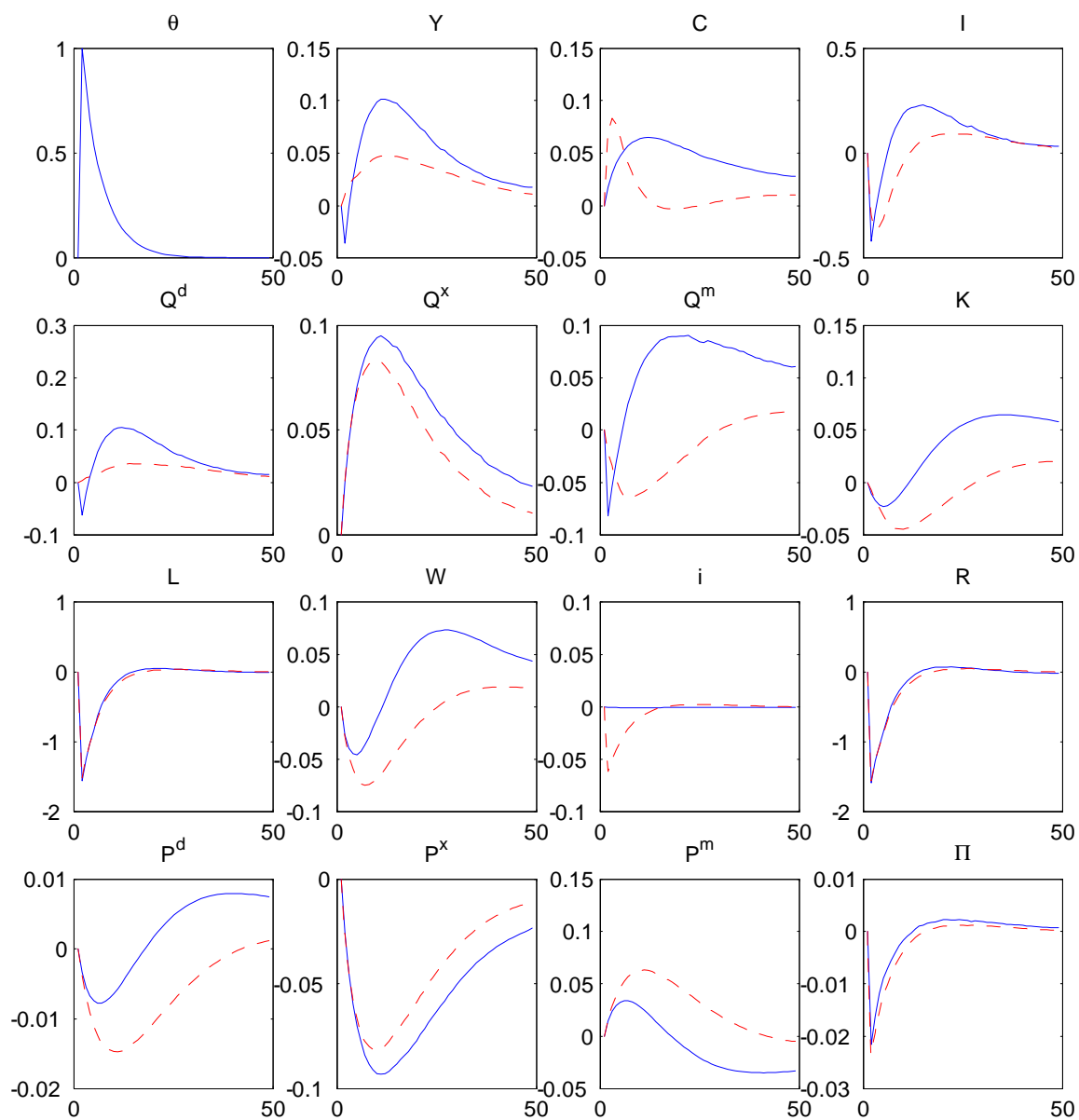


Figure 4: Responses to a technology shock (Peg: Solid lines, Taylor: Dashed lines)

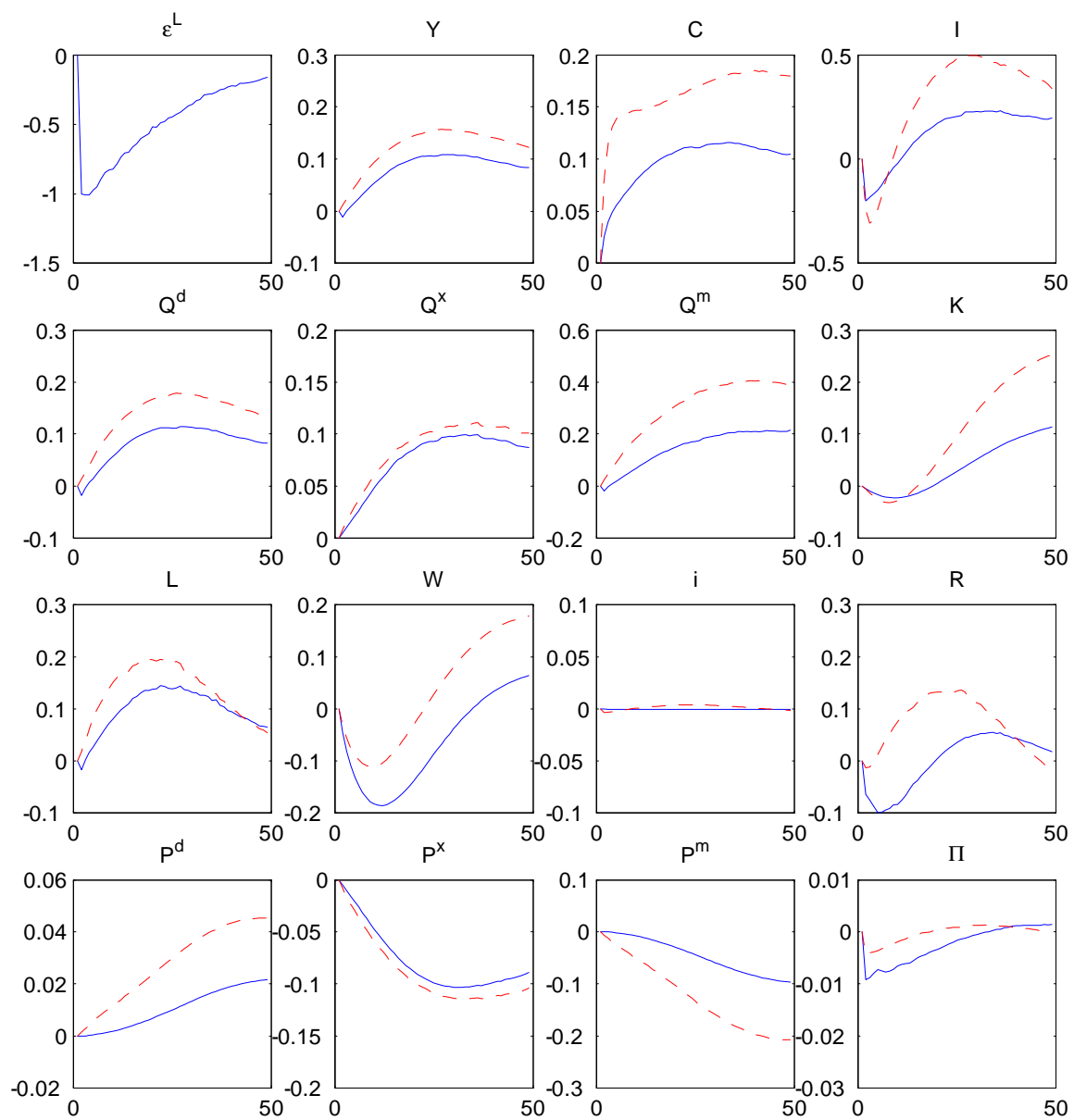


Figure 5: Responses to a labour supply shock. (Peg: Solid lines, Taylor: Dashed lines)

devaluated exchange rate that is higher than under the peg. We finally note that volatility in labour is higher under the peg than in the Taylor rule regime, thereby contributing to the finding that labour is more volatile under the peg.

Finally, in Figure 7 we observe what happens following an exogenous shock to foreign interest rates. Under the peg, the domestic central bank has to follow the direction of the foreign interest rate movements in order to keep the exchange rate fixed. In the Taylor rule regime the central bank hardly reacts as inflation as well as output is unaffected initially by the foreign interest rate shock. For the same reason, only minor movements of all variables are observed in this case. Under the peg, however, the higher level of interest rates dampens consumption as well as investments causing aggregate output to fall. Again, we note that labour is more volatile under the peg, but movements in the foreign interest rate are of less importance for movements in labour than are technology, labour supply and monetary policy shocks.

7 Robustness and Alternative Scenarios

The previous section demonstrated that there are welfare gains from changing the monetary policy from a fixed exchange rate to a Taylor rule. Recall, that the values of σ_{peg}^m and σ_{TR}^m quantify the volatility of the policy errors under the two regimes. Considering the nature and scope of the deregulated foreign-exchange market of today, one should generally find that the central bank's task of assigning the interest rate level that keeps the exchange rate exactly on target is nontrivial, and thus a certain amount of policy errors seems unavoidable. Administering a Taylor rule with fixed intervention dates and infrequent observations of the inflation and output gaps seems like a manageable task in comparison. However, if it is possible for a central bank to obtain a credible peg on a foreign currency, pressure on the exchange rate could plausibly fall to a level where the peg can be maintained with a degree of precision comparable to that of a Taylor rule. Indeed, the recent Danish experience has been one of a very stable exchange rate around the fixed parity, as is evident from Figure 8. It turns out that varying the volatility of the policy shock in the range spanned by the Taylor rule estimate of Smets and Wouters (2003) and that obtained for the Danish peg regime in Dam and Linaa (2005) is of critical importance for the welfare results.

As described in Section 3 we are unable to estimate the volatility of ξ_t^{TR} since this regime has not been in effect in Denmark. Instead we relied on the estimated volatility obtained by Smets and Wouters (2003) as a proxy for "what to expect" if this monetary policy regime was introduced in Denmark. In Figure 9 we show the welfare equivalences χ for varying values of σ_{TR}^m . For a value of σ_{TR}^m slightly above 0.005 we see that this compensation becomes negative meaning that if policy errors under the Taylor rule lies above this level, a regime shift in monetary policy would result in a welfare loss.

We also turned this experiment on its head; we estimated an AR(1) for the exchange rate since 1999. As was seen from Figure 8, volatility in this period has been substantially reduced compared to that of the full sample. This estimation resulted in an autocorrelation, ϱ^m , equal to 0.86, while σ_{peg}^m was estimated to the value of 0.0008, equal to the value of σ_{TR}^m . Working with this process decreased the benefits of adopting the Taylor rule, although it was still advisable, cf. Scenario I in Table 4.

We also carried out two additional simulations of alternative scenarios as is seen from Table 4. Scenario II has already been described in Section 6 and it took the form of

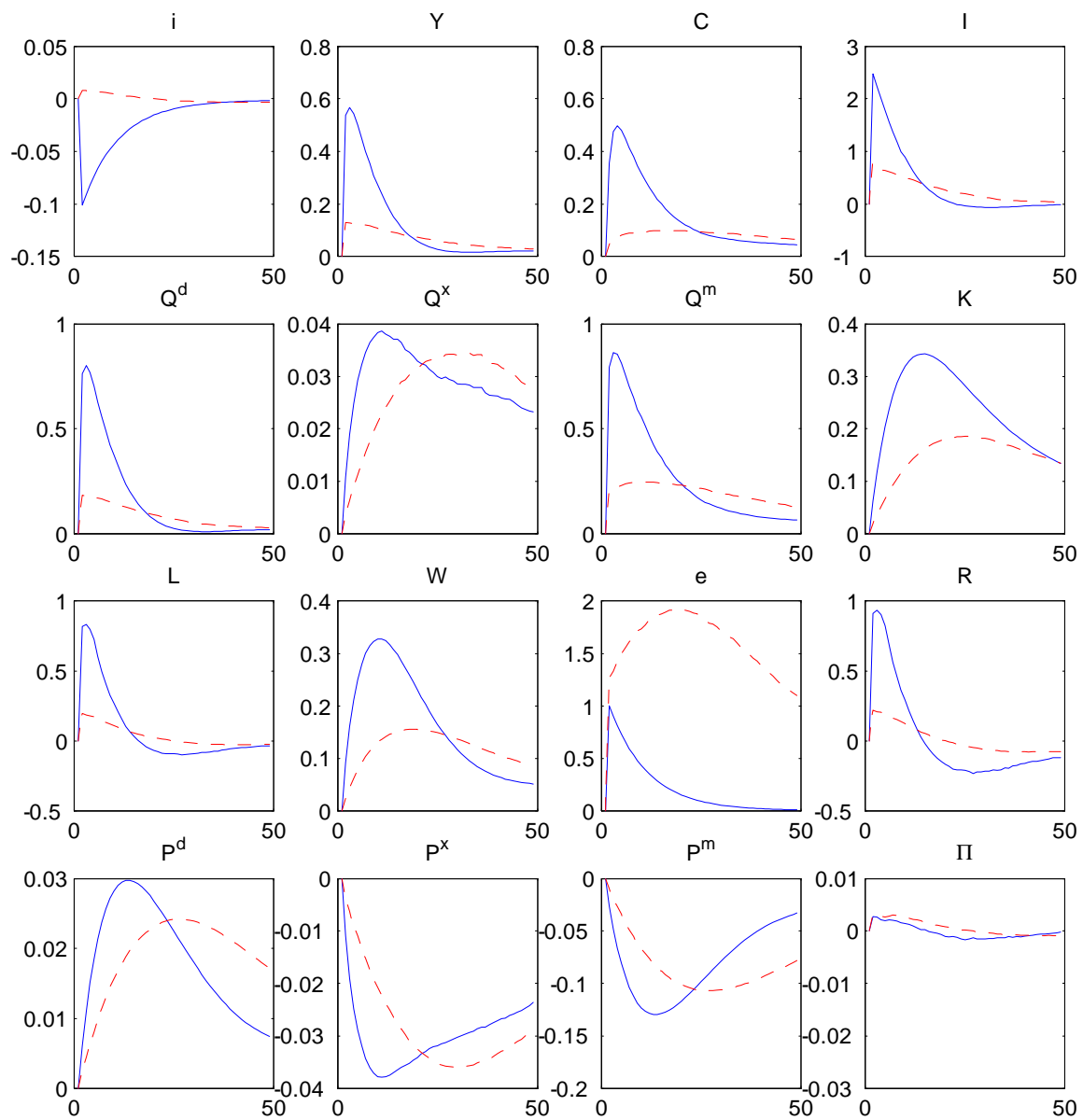


Figure 6: Responses to a monetary policy shock. (Peg: Solid lines, Taylor: Dashed lines)

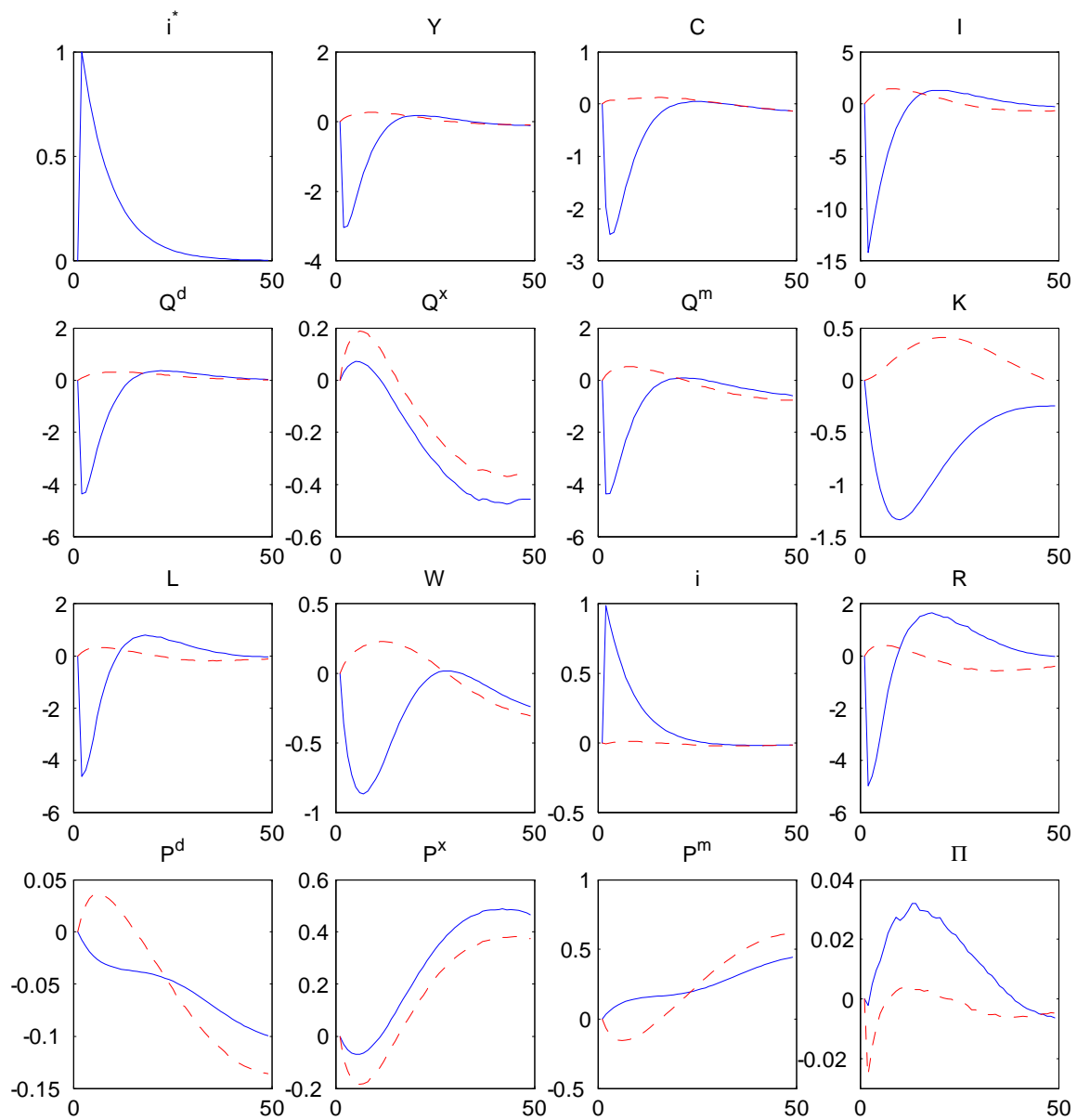


Figure 7: Responses to a foreign interest rate shock. (Peg: Solid lines, Taylor: Dashed lines)

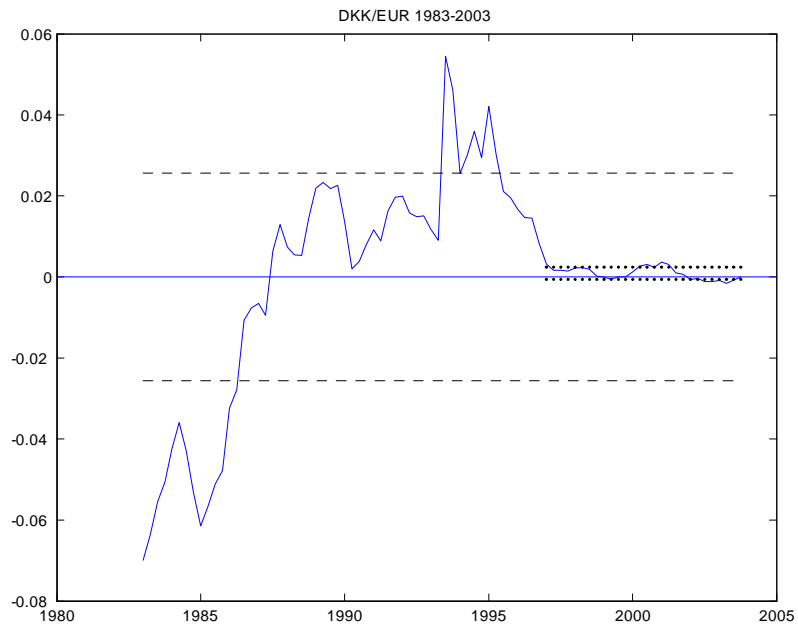


Figure 8: Log of DKK/EUR 1983-2003. Linearly detrended. Dashed and dotted lines are standard deviations for the entire sample and 1997-2003, resp.

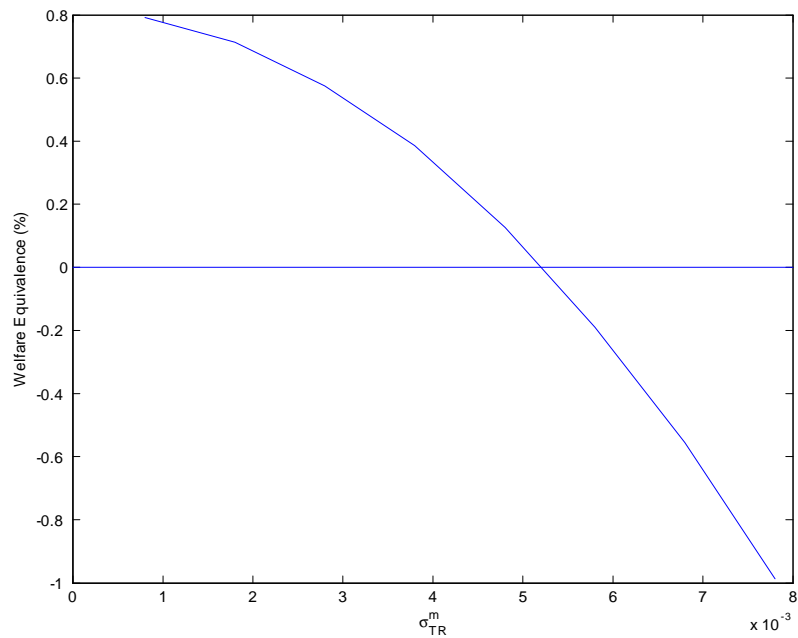


Figure 9: Welfare equivalence and monetary policy volatility

assuming labour supply shocks were no more persistent than technology shocks. In this case ϱ^L was attached a value of 0.82 (equal to ϱ^t) and this reduced the compensation in consumption needed to put household utility under the peg equal to utility in the Taylor rule regime to 0.27 pct.

Table 4: Welfare Analysis - Alternative Scenarios

<i>Scenario</i>	$100 \times \chi$
I Lower exchange rate volatility ($\sigma_{peg}^m = 0.0008$, $\varrho^m = 0.86$)	0.660
II Less persistent labour supply shock ($\varrho^L = 0.82$)	0.277
III Less nominal rigidity ($d = D = 0.75$)	1.149

Note: χ measures the compensating variation of consumption between the peg and the optimal Taylor regime as defined in equation (??).

Scenario III was assuming that prices were less rigid than they were estimated to be; postulating both goods prices as well as wages can be reoptimised once a year increases the compensation that equalises welfare between the two regimes to 1.15 pct. This is the result of two opposing sources; on the one hand, when prices are extremely rigid, an inflation fighting central bank has only limited possibilities to control inflation. This tends to reduce the benefits from leaving the peg and adopt the Taylor rule. On the other hand, however, damages by not controlling inflation are more severe, since they last longer. In this case the first source dominate.

The overall conclusion therefore seems to be that Denmark could potentially benefit from giving up the peg and begin conducting monetary policy according to a Taylor rule. This change in the monetary policy regime seems to be beneficial unless the policy error, σ_{TR}^m , takes a substantially larger value, than Smets and Wouters (2003) estimated as the value relevant for the euro-zone.

8 Conclusion

In this paper we analysed the consequences of Denmark replacing the peg with a Taylor rule. For this purpose we used the model laid out and estimated in Dam and Linaa (2005) in order to quantify the welfare implications in the two regimes. We then dropped the assumption of the central bank following a peg and replaced it with an assumption of the central bank conducting monetary policy according to a Taylor rule.

The models tell us that it is possible to increase the level of welfare by doing so; in fact we find that the benefits can be summerised to 0.79 pct. measured in units of consumption goods. Various alternative scenarios did not change this conclusion although the magnitude of change in welfare, of course, was affected by this. It turned out that welfare under the peg would only exceed welfare in the Taylor regime if policy errors in the Taylor regime are far larger than those estimated for the euro-zone by Smets and Wouters (2003).

Contrary to the related study in Kollmann (2002) we find that volatility of both consumption and output increases when going from a peg to the Taylor rule; the main explanation for this was the existence of a highly volatile and persistent labour supply

shock. Reducing the persistence of this shock puts us back to Kollmann's scenario in which volatility is lower in the Taylor regime.

There are, however, potentially important matters not included in the above mentioned framework. If Denmark decided to adopt a Taylor rule, risk aversion from foreign investors might induce a reduction in direct investment flows into Denmark caused by an increased uncertainty regarding the exchange rate. Furthermore, Danish exporters also face uncertainty regarding the exchange rate and could need to engage in costly arrangements with financial intermediaries in order to eliminate this uncertainty when trading with agents abroad. Finally, we ignore issues related to the potential budget discipline being put on the Government in order to keep a peg credible.

Additionally, a number of obvious extensions of this work lies ahead: Firstly, we are currently considering a more generalised form of the Taylor rule, examining the welfare gains attainable when expanding the Taylor rule to include the exchange rate. We need more work on this issue however, since we discovered a large range of spikes and ridges in the welfare levels derived from different parameterisations of the Taylor rule equipped with changes in the exchange rate. At this point we are unable to explain this, but we will seek to get further insight into this area in our future research. Secondly, we should examine the consequences of focusing on conditional moments rather than using unconditional moments. This might be of great importance for the Danish case, since we currently ignore the transition from the peg regime to the Taylor rule regime.

This paper, however, contributes to the ongoing debate in Denmark whether to stick with the peg, and it contributes to the literature in general by performing a welfare analysis on an estimated small open economy DSGE model with a number of nominal and real rigidities.

A The Non-linear Model

$$Q_t^d = \alpha^d \frac{Z_t}{\tilde{P}_t^d}, \quad (15)$$

$$Q_t^m = (1 - \alpha^d) \frac{Z_t}{\tilde{P}_t^m}, \quad (16)$$

$$Q_t^x = \left(\tilde{P}_t^x\right)^{-\eta} Y_t^*, \quad (17)$$

$$1 = \left(\tilde{P}_t^d\right)^{\alpha^d} \left(\tilde{P}_t^m\right)^{1-\alpha^d}, \quad (18)$$

$$L_t = \frac{1 - \psi}{\psi} \frac{\tilde{R}_t}{\tilde{W}_t} K_t, \quad (19)$$

$$mc_t = \frac{1}{\theta_t} \tilde{W}_t^{1-\psi} \tilde{R}_t^\psi \psi^{-\psi} (1 - \psi)^{-(1-\psi)}, \quad (20)$$

$$\rho_{t,t+1} = \beta (U_{C,t+1}/U_{C,t}) \Pi_{t+1}^{-1}, \quad (21)$$

$$U_{C,t} = \zeta_t^b (C_t - hC_{t-1})^{-\sigma_C}; \quad (22)$$

$$\tilde{\mathcal{N}}_t^d = Q_t^d (1 + \nu) mc_t + dE_t \left[\rho_{t,t+1} \left(\frac{\tilde{P}_{t+1}^d}{\tilde{P}_t^d} \right)^{\frac{1+\nu}{\nu}} \Pi_{t+1}^{\frac{1+2\nu}{\nu}} \tilde{\mathcal{N}}_{t+1}^d \right], \quad (23)$$

$$\tilde{\mathcal{D}}_t^d = Q_t^d + dE_t \left[\rho_{t,t+1} \left(\frac{\tilde{P}_{t+1}^d}{\tilde{P}_t^d} \Pi_{t+1} \right)^{\frac{1+\nu}{\nu}} \tilde{\mathcal{D}}_{t+1}^d \right], \quad (24)$$

$$\tilde{P}_t^d = \left[d \left(\tilde{P}_{t-1}^d / \Pi_t \right)^{-\frac{1}{\nu}} + (1 - d) \left(\tilde{\mathcal{N}}_t^d / \tilde{\mathcal{D}}_t^d \right)^{-\frac{1}{\nu}} \right]^{-\nu}; \quad (25)$$

$$\tilde{\mathcal{N}}_t^x = Q_t^x (1 + \nu) mc_t + dE_t \left[\rho_{t,t+1} \left(\frac{\tilde{P}_{t+1}^x \Pi_{t+1}^*}{\tilde{P}_t^x} \right)^{\frac{1+\nu}{\nu}} \tilde{\mathcal{N}}_{t+1}^x \right], \quad (26)$$

$$\tilde{\mathcal{D}}_t^x = Q_t^x \mathcal{E}_t + dE_t \left[\rho_{t,t+1} \left(\frac{\tilde{P}_{t+1}^x}{\tilde{P}_t^x} \right)^{\frac{1+\nu}{\nu}} \left(\Pi_{t+1}^* \right)^{\frac{1}{\nu}} \Pi_{t+1} \tilde{\mathcal{D}}_{t+1}^x \right], \quad (27)$$

$$\tilde{P}_t^x = \left[d \left(\tilde{P}_{t-1}^x / \Pi_t^* \right)^{-\frac{1}{\nu}} + (1 - d) \left(\tilde{\mathcal{N}}_t^x / \tilde{\mathcal{D}}_t^x \right)^{-\frac{1}{\nu}} \right]^{-\nu}; \quad (28)$$

$$\tilde{\mathcal{N}}_t^m = Q_t^m (1 + \nu) + \frac{d}{1 + i_t^*} E_t \left[\left(\frac{\tilde{P}_{t+1}^m}{\tilde{P}_t^m} \Pi_{t+1} \right)^{\frac{1+\nu}{\nu}} \Pi_{t+1}^* \tilde{\mathcal{N}}_{t+1}^m \right] \quad (29)$$

$$\tilde{\mathcal{D}}_t^m = Q_t^m \mathcal{E}_t + \frac{d}{1 + i_t^*} E_t \left[\left(\frac{\tilde{P}_{t+1}^m}{\tilde{P}_t^m} \right)^{\frac{1+\nu}{\nu}} (\Pi_{t+1})^{\frac{1}{\nu}} \Pi_{t+1}^* \tilde{\mathcal{D}}_{t+1}^m \right], \quad (30)$$

$$\tilde{P}_t^m = \left[d \left(\tilde{P}_{t-1}^m / \Pi_t \right)^{-\frac{1}{\nu}} + (1 - d) \left(\tilde{\mathcal{N}}_t^m / \tilde{\mathcal{D}}_t^m \right)^{-\frac{1}{\nu}} \right]^{-\nu}; \quad (31)$$

$$K_{t+1} = K_t (1 - \delta) + I_t - \frac{1}{2} \frac{\Phi (K_{t+1} - K_t)^2}{K_t}, \quad (32)$$

$$\tilde{B}_{t+1} = (1 + i_{t-1}^f) \tilde{B}_t / \Pi_t^* + \tilde{P}_t^x Q_t^x - \frac{\tilde{P}_t^m}{\mathcal{E}_t} Q_t^m; \quad (33)$$

$$E_t \left[\rho_{t,t+1} \Pi_{t+1} \frac{\left(\tilde{R}_{t+1} + (1 - \delta) - \frac{\Phi}{2} \left(1 - \left(\frac{K_{t+2}}{K_{t+1}} \right)^2 \right) \right)}{\left[1 + \Phi \frac{K_{t+1} - K_t}{K_t} \right]} \right] = 1, \quad (34)$$

$$(1 + i_t) E_t [\rho_{t,t+1}] = 1, \quad (35)$$

$$(1 + i_t^f) E_t \left[\rho_{t,t+1} \frac{\mathcal{E}_{t+1} \Pi_{t+1}}{\mathcal{E}_t \Pi_{t+1}^*} \right] = 1; \quad (36)$$

$$\tilde{\mathcal{N}}_t^w = \zeta_t^b \zeta_t^L (1 + \gamma) \left(\tilde{W}_t \right)^{\frac{1+\gamma}{\gamma} (1+\sigma_L)} L_t^{1+\sigma_L} + d\beta E_t \left[\Pi_{t+1}^{\frac{1+\gamma}{\gamma} (1+\sigma_L)} \tilde{\mathcal{N}}_{t+1}^w \right], \quad (37)$$

$$\tilde{\mathcal{D}}_t^w = U_{C,t} \tilde{W}_t^{\frac{1+\gamma}{\gamma}} L_t + d\beta E_t \left[\Pi_{t+1}^{\frac{1}{\gamma}} \tilde{\mathcal{D}}_{t+1}^w \right], \quad (38)$$

$$\tilde{w}_{t,t} = \left[\frac{\tilde{\mathcal{N}}_t^w}{\tilde{\mathcal{D}}_t^w} \right]^{\frac{\gamma}{\gamma + (1+\gamma)\sigma_L}}, \quad (39)$$

$$\tilde{W}_t = \left[D \left(\tilde{W}_{t-1} / \Pi_t \right)^{-\frac{1}{\gamma}} + (1 - D) \tilde{w}_{t,t}^{-\frac{1}{\gamma}} \right]^{-\gamma}; \quad (40)$$

$$(1 + i_t^f) = (1 + i_t^*) v_t \exp \left\{ -\lambda \mathcal{E}_t \Pi_{t+1}^* \frac{\tilde{B}_{t+1}}{\Xi} \right\}, \quad \Xi = \mathcal{E} \hat{\mathcal{P}}^x Q^x, \quad (41)$$

$$\frac{\mathcal{E}_t \Pi_t}{\mathcal{E}_{t-1} \Pi_t^*} = \frac{\xi_t}{\xi_{t-1}}; \quad (42)$$

$$Y_t = Q_t^d + Q_t^x, \quad (43)$$

$$Z_t = C_t + I_t, \quad (44)$$

$$K_t = \frac{1}{\theta_t} \left(\frac{\psi}{1-\psi} \frac{\tilde{W}_t}{\tilde{R}_t} \right)^{1-\psi} \left[\left(\frac{\tilde{\mathcal{P}}_t^d}{\tilde{P}_t^d} \right)^{-\frac{1+\nu}{\nu}} Q_t^d + \left(\frac{\tilde{\mathcal{P}}_t^x}{\tilde{P}_t^x} \right)^{-\frac{1+\nu}{\nu}} Q_t^x \right], \quad (45)$$

$$\tilde{\mathcal{P}}_t^d = \left[d \left(\tilde{\mathcal{P}}_{t-1}^d / \Pi_t \right)^{-\frac{1+\nu}{\nu}} + (1-d) (\tilde{p}_{t,t}^d)^{-\frac{1+\nu}{\nu}} \right]^{-\frac{\nu}{1+\nu}}, \quad (46)$$

$$\tilde{\mathcal{P}}_t^x = \left[d \left(\tilde{\mathcal{P}}_{t-1}^x / \Pi_t^* \right)^{-\frac{1+\nu}{\nu}} + (1-d) (\tilde{p}_{t,t}^x)^{-\frac{1+\nu}{\nu}} \right]^{-\frac{\nu}{1+\nu}}; \quad (47)$$

$$\tilde{\mathcal{W}}_t = \left[D \left(\tilde{\mathcal{W}}_{t-1} / \Pi_t \right)^{-\frac{1+\gamma}{\gamma}(1+\sigma_L)} + (1-D) \tilde{w}_{t,t}^{-\frac{1+\gamma}{\gamma}(1+\sigma_L)} \right]^{-\frac{\gamma}{(1+\gamma)(1+\sigma_L)}}, \quad (48)$$

$$SW_t = \frac{\zeta_t^b}{1-\sigma_C} (C_t - h\bar{C}_{t-1})^{1-\sigma_C} - \frac{\zeta_t^b \zeta_t^L}{1+\sigma_L} L_t^{1+\sigma_L} \left(\frac{\tilde{\mathcal{W}}_t}{\tilde{W}_t} \right)^{-\frac{1+\gamma}{\gamma}(1+\sigma_L)}. \quad (49)$$

The processes governing the persistent structural shocks are given as

$$\hat{\zeta}_t^b = \varrho^b \hat{\zeta}_{t-1}^b + \varepsilon_t^b, \quad (50)$$

$$\hat{\zeta}_t^l = \varrho^l \hat{\zeta}_{t-1}^l + \varepsilon_t^l, \quad (51)$$

$$\hat{\theta}_t = \varrho^t \hat{\theta}_{t-1} + \varepsilon_t^t, \quad (52)$$

$$\hat{\xi}_t^j = \varrho_j^m \hat{\xi}_{t-1}^j + \varepsilon_t^m, \quad j = peg, TR \quad (53)$$

$$\hat{i}_t^* = \varrho^i \hat{i}_{t-1}^* + \varepsilon_t^i, \quad (54)$$

$$\hat{P}_t^* = \varrho^P \hat{P}_{t-1}^* + \varepsilon_t^P, \quad (55)$$

$$\hat{Y}_t^* = \varrho^Y \hat{Y}_{t-1}^* + \varepsilon_t^Y, \quad (56)$$

where hats denote relative deviations from the steady state, and $0 < \varrho^j < 1$, cf. Table 1. Since, however, the monetary policy shock under the Taylor regime is assumed to be *i.i.d.*, we have $\varrho_{TR}^m = 0$.

Table 5: Variables and Parameters

<i>Variables</i>		<i>Exogenous Variables</i>	
Z_t	Final goods	θ_t	Technology level in intermediary sector
P_t	Price of Z	ζ_t^b	Preference discount rate shock
Q_t^i	Intermediate goods	ζ_t^l	Labor supply shock
P_t^i	Price of Q^i	v_t	UIP shock
$p_{t,\tau}^i$	Intermediary price optimized in period τ	ξ_t	Exchange-rate policy (peg) shock
\mathcal{P}_t^i	Price dispersion measure	Y_t^*	Foreign GDP
Y_t	GDP ($Q^d + Q^x$)	P_t^*	Foreign price level
R_t	Rental rate of capital	i_t^*	Foreign interest rate
MC_t	Marginal cost in intermediary sector		
e_t	Exchange rate		
$\rho_{t,\tau}$	Discount factor between periods t and τ		
$R_{t,\tau}$	Foreign discount factor	<i>Parameters (time invariant)</i>	
C_t	Final consumption	ν	Net price markup (intermediaries)
L_t	Aggregate labor supply	γ	Net wage markup
$w_{t,\tau}$	Wage level optimized in period τ	α^d	Share of Q^d in final output
W_t	Aggregate wage level	ψ	Capital share in intermediate goods
$l_t(s, j)$	Labor of type j supplied to firm s	d	Calvo parameter, intermediaries
K_t	Capital stock	β	Utility discount factor
I_t	Investment	h	Habit persistence
A_t	Domestic bonds (0 in eqm.)	σ_C^{-1}	Household IES
B_t	Foreign bonds in foreign currency	σ_L^{-1}	Work effort elasticity
i_t	Domestic interest rate	δ	Capital depreciation rate
i_t^f	Return on B_t to domestic agents	Φ	Capital adjustment cost
Ω_t	Wedge between i_t^* and i_t^f	Ξ	ss export in units of Z
χ_t	Compound variable in wage eqtn.	λ	Debt premium on foreign bonds
$U_{C,t}$	Marginal utility of consumption	D	Calvo parameter, wages
$U_{L,t}$	Marginal disutility of labor	η	Export demand elasticity
\mathcal{N}_t^i	Auxiliary variable ($p_{t,t}^i$ and $w_{t,t}$)		
\mathcal{D}_t^i	Auxiliary variable ($p_{t,t}^i$ and $w_{t,t}$)		

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Chapter 3

The UK Business Cycle and the Exchange-Rate Disconnect*

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Abstract

We estimate a DSGE model of the British economy which incorporates open-economy elements from the NOEM literature. Generally, we find plausible estimates for the structural parameters characterising the domestic economy. Furthermore, according to our estimates the main source of real fluctuations in the UK are shocks to the technology process.

However, the estimation highlights two problematic features of the model framework. First, the observed inflation process in the UK is less persistent than elsewhere, including the US, and the estimated model is not adequately accounting for the rapid changes in inflation. Secondly, the estimation insulates the domestic economy from the foreign economy. This result corresponds to the findings of other recent papers and implies that the benchmark NOEM assumptions seem incapable of dealing with the exchange-rate disconnect puzzle when confronted with real data.

Keywords: New open-economy macroeconomics, business cycles, Bayesian estimation

JEL Classifications: E3, E4, F4

*The views expressed in this paper do not necessarily reflect those of the Bank of England.

1 Introduction

Our paper seeks answers to the following question: *What are the determinants of the UK business cycle?* For this purpose, we build a dynamic stochastic general-equilibrium (DSGE) model with a sizeable number of structural and ad-hoc shocks and use Bayesian techniques to estimate the structural parameters on quarterly data. This approach has successfully been applied to estimate closed-economy DSGE models, with the work by Smets and Wouters (2003, 2007) on euro area and US data serving as an inspiring benchmark. With the openness of the British economy in mind, we found it natural to incorporate elements of the new open-economy macroeconomic (NOEM) paradigm in the model.

The estimation yielded three main insights. First, shocks to the level of technology in the production sector seem to be the main source of output fluctuations in the British economy during the years 1982-2002. This result is in line with what Bergin (2003) found for Australia, Canada, and the UK, but is in contrast to other related studies of the euro area (Smets and Wouters, 2003) and Denmark (Dam and Linaa, 2005) which find that shocks to preferences and labour supply are more important than shocks to technology.

Secondly, the (detrended) inflation series for the UK are remarkably less persistent than those for the US and euro area. Given a large idiosyncratic element and very little inertia in the observed series, we find that the lagged indexation feature of the sticky-price model of the recent medium-scale DSGE literature has no role to play in the UK context. Furthermore, the Calvo (1983) setup linking inflation to current and future marginal costs seems to be inadequate in accounting for the observed inflation fluctuations, as indicated by a large estimated volatility of the measurement error component in the inflation series.

Thirdly, the foreign economy has no impact on the UK economy whatsoever. At first sight, this is a surprising result, why we consider it in detail. Typically, the transmission of foreign real shocks works through the terms-of-trade (TOT) channel. Thus, standard models of the international real business-cycle (IRBC) literature have found that exogenous changes in the TOT account for about 50 percent of domestic output fluctuations, cf. Mendoza (1995).¹ The terms of trade channel also plays a dominant theoretical role in the NOEM literature (cf. Corsetti and Pesenti, 2001).

Ideally, then, the estimation of our structural model should be able to shed new light on the relative importance of home or foreign disturbances as drivers of the UK business cycle, including output and the real exchange rate. Recent attempts to estimate DSGE models of the NOEM variety with Bayesian methods suggest that external terms-of-trade shocks have a very little effect on domestic output. For instance, Lubik and Schorfheide (2003) find that exogenous terms of trade disturbances (as well as foreign monetary shocks) explain very little of observed volatility in UK output, inflation and exchange rate. Dam and Linaa (2005) find similar results for the Danish economy, as does Martínez-García (2005) in the case of Spain. Obviously, these findings are at odds with the findings of the earlier IRBC literature mentioned above.

In a recent paper, Justiniano and Preston (2006) investigates this puzzling result in a structural analysis of Canadian data. Their model is closely related to ours, although they exclude capital in their setup. They find that the current versions of NOEM models suffer from the *exchange rate disconnect puzzle*, that is, a volatile behaviour of the real exchange rate that is unrelated to the development of real aggregates (fundamentals) in the respective economies (see Obstfeld and Rogoff (2000) for a good introduction to this

¹Kose (2002) found TOT movements explained as much as 90 percent of output volatility in a developing economy.

puzzle). In other words, the NOEM models that have been estimated in this and other papers fail to give an economic explanation of the volatile real exchange rate, and thus they rely on ad-hoc shocks to disconnect the modeled economy from the large movements that we observe in the real exchange rate. Consequently, the estimated economy virtually behaves as a closed economy.

The paper proceeds as follows: in Section 2 we lay out the model, and in Section 3 we present the data. Section 4 accounts for the methodology and results of our estimation, while Section 5 considers the ability of the model to fit the data. In Section 6 we consider the sensitivity of the estimation to variations in the specific model assumptions, while Section 7 analyses the dynamic properties of the benchmark model through impulse-response functions and a variance decomposition. Finally, Section 8 concludes.

2 Model

The model builds on the Swedish Riksbank DSGE model laid out in Adolfson et al. (2005). To this framework we introduce some marked changes to the assumptions underlying household preferences; first, we assume a cashless limiting economy (cf. Woodford, 2003, ch. 2) and thus disregard money altogether; second, we find the felicity function in Adolfson et al. (2005) too restrictive and replace it with that of Smets and Wouters (2003) which allows us to vary the intertemporal elasticity of substitution (IES) as well as the Frisch labour elasticity. Furthermore, as Benigno and Thoenissen (2004) (but in contrast to Adolfson et al., 2005) we follow the lead of McCallum and Nelson (1999, 2000) and assume that all trade takes place in intermediaries. Finally, we simplify the assumptions regarding the labour market to that of perfect competition.

In the following, we present the different components of our model. The derivation of first-order conditions etc. are confined to Appendix A.

2.1 Domestic Households

We assume complete markets in the domestic economy. In accordance with their preferences, agents thus completely diversify their idiosyncratic income risk in a security market. Hence, any agent is fully representative of the economy and we do not use notation to distinguish between agents. The representative agent is assumed to maximise the following utility function;²

$$U_t \equiv E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \zeta_{\tau}^c \left[\frac{(C_{\tau} - h\bar{C}_{\tau-1})^{1-\rho_c}}{1-\rho_c} - \zeta_{\tau}^l \frac{H_{\tau}^{1+\rho_l}}{1+\rho_l} \right],$$

$$\beta, h \in [0; 1], \quad \sigma_c, \sigma_l > 0,$$

where C_t is consumption, H_t is labour, ζ_t^c is a discount shock, and ζ_t^l is a preference shock affecting the disutility of labour; thus, β is a discount factor, ρ_c^{-1} is the IES, ρ_l^{-1} is the Frisch elasticity, and h captures habit formation. We specifically assume external habit formation, implying that the habit stock is formed from aggregate consumption \bar{C}_t .

²As all agents face the same wage in the labour market, and we assume identical levels of wealth in the initial period, there is no need to distinguish between individual households and the average (representative) household.

As households are assumed to own capital and firms in the home country exclusively, the household budget constraint is as follows;

$$B_{t-1} + S_t B_{t-1}^* + W_t H_t + R_t^k K_t + \Delta_t \geq \frac{B_t}{(1 + R_t)} + \frac{S_t B_t^*}{\Omega_t (1 + R_t^*)} + P_t (C_t + I_t), \quad (2.1)$$

where B_t, B_t^* and R_t, R_t^* are domestic and foreign bond holdings and rates of return, respectively, S_t is the nominal exchange rate, W_t is the wage rate, R_t^k is the return to capital K_t , Δ_t includes dividend from intermediary firms as well as payments from state-contingent securities, I_t are investments, P_t is the price of final goods, and Ω_t is a wedge between the foreign interest rate and the return to domestic agents holding foreign bonds;

$$\Omega_t = \exp \left\{ -\omega \frac{S_t B_t^*}{P^D Y^X} + v_t \right\}, \quad \omega > 0,$$

where v_t is an exogenous disturbance to the UIP condition, and $P^D Y^X$ is the steady state value of exports. Thus, the return on foreign bonds is falling in the foreign debt of domestic agents (normalised with the long-run level of exports), a feature that ensures stationarity of the economy, cf. Schmitt-Grohe and Uribe (2003). It follows that we assume incomplete international markets.

Households supply labour in a perfectly competitive labour market where the wage level is trivially determined.

2.1.1 Capital Accumulation

The capital stock accumulates as follows;

$$K_{t+1} = (1 - \delta) K_t + \theta_t^i (1 - S(I_t/I_{t-1})) I_t,$$

where δ is the depreciation rate and S is an investment adjustment cost function with the following properties;

$$S(1) = S'(1) = 0, \quad s \equiv S''(1) > 0,$$

while θ_t^i is an investment-specific technology shock. The investment adjustment cost function is chosen in order to introduce the hump-shaped investment responses to changes in the economic environment that have been found in numerous empirical studies; we refer the reader to Christiano et al. (2001) for further discussion.

2.2 The Intermediate-Goods-Producing Firm

We assume that a continuum of firms supply intermediary goods to final-goods sectors at home as well as abroad in markets of monopolistic competition of the Dixit-Stiglitz variety and set prices in staggered contracts of random duration as developed in Calvo (1983).

Each firm in the intermediary sector has the following production technology;

$$Y_t^D(f) = \theta_t^D K_t(f)^\alpha H_t(f)^{1-\alpha} - \Phi,$$

where Y_t^D is firm f 's production of intermediaries, θ_t^D is a technology shock, and Φ is a fixed cost (which will ensure zero profits in the long run). Solving the cost-minimization

problem of the individual firm f yields the optimal relationship between capital and labor;

$$\frac{R_t^k}{W_t} = \frac{\alpha}{(1-\alpha)} \frac{H_t(f)}{K_t(f)} \iff \frac{r_t^k}{w_t} = \frac{\alpha}{(1-\alpha)} \frac{H_t(f)}{k_t(f)},$$

where $w_t \equiv W_t/P_t$ $r_t^k \equiv R_t^k/P_t$ are real wages and rental rate expressed in units of the final good, respectively. It follows that marginal costs are constant across firms, why we can drop the firm index when defining real (and nominal) marginal costs;

$$mc_t \equiv MC_t/P_t^D = [p_t^D \theta_t^D \alpha^\alpha (1-\alpha)^{1-\alpha}]^{-1} w_t^{1-\alpha} (r_t^k)^\alpha,$$

where $p_t^D \equiv P_t^D/P_t$ expresses the relative product price in units of final goods.

We now turn to the optimal pricing decision of tradable-good firms. Our monopolistic competition assumption implies that each firm, f , in the tradable sector is facing a downward sloping demand curve of the form

$$Y_t^D(f) = \left(\frac{P_t^D(f)}{P_t^D} \right)^{-\frac{\mu_t^d}{\mu_t^d-1}} Y_t^D, \quad (2.2)$$

where $Y_t^D(f)$ is total demand for the intermediate product produced by firm f , while $Y_t^D \equiv Y_t^{D,D} + Y_t^X$ is total demand for domestically produced tradable goods. Here, $Y_t^{D,D}$ is the domestic demand for tradables, while foreign demand for tradable goods is

$$Y_t^X = \left(\frac{P_t^D}{S_t P_t^*} \right)^{-\eta_x} Y_t^*, \quad \eta > 1.$$

As has become established in the literature, we assume that each firm only gets to optimise its price in a given period with constant probability $1 - \xi_d$ (where $\xi_d \in [0; 1]$), implying that price contracts are staggered and of stochastic durations. We follow Smets and Wouters (2003) and expand the simple Calvo pricing model with partial past indexation in the sense that firms not reoptimising their price will adjust partially with the inflation of the previous period. Thus, letting \tilde{P}_t^D be the optimal price of any firm (as the problem is symmetric, the optimal price is identical across firms), the price charged by firm f at time t will be

$$P_t^D(f) = \begin{cases} \tilde{P}_t^D & \text{w.p. } (1 - \xi_d) \\ (\Pi_{t-1}^D)^{\omega_d} P_{t-1}^D(f) & \text{w.p. } \xi_d \end{cases},$$

where $\omega_d \in (0; 1)$ captures the degree of indexation to past inflation, and Π_t^D is the gross inflation rate for domestic tradables.

The optimal price \tilde{P}_t^D is the price that maximizes the net present value of a firm's profits subject to its demand curve in equation (2.2);

$$\max E_t \sum_{\tau=t}^{\infty} (\xi_d \beta)^{\tau-t} \lambda_\tau \pi_\tau^D(f),$$

$$\text{where } \pi_t^D(f) \equiv \left(\tilde{P}_t^D(f) - MC_t^D(f) \right) Y_\tau^D(f) - MC_t^D \Phi,$$

and $\lambda_t = P_t U'_{C_t}$ is the marginal utility of income in nominal terms (cf. Appendix A). The

first-order condition for this problem is

$$\sum_{\tau=t}^{\infty} (\xi_d \beta)^{\tau-t} E_t \left[\frac{\lambda_{\tau} P_{\tau}^D Y_{\tau}^D (f)}{\mu_{\tau}^d - 1} \left(\frac{\tilde{P}_t^D / P_t^D}{P_{\tau}^D / P_t^D} \left(\frac{P_{\tau-1}^D}{P_{t-1}^D} \right)^{\omega_d} - \mu_{\tau}^d m c_{\tau} \right) \right] = 0.$$

It is relatively straightforward to show that aggregate sectoral price index evolve as follows;

$$P_t^d = \left[\xi_d \left((\Pi_{t-1}^d)^{\omega_d} (\bar{\Pi}_t)^{1-\omega_d} P_{t-1}^d \right)^{\frac{1}{1-\mu_t^d}} + (1 - \xi_d) \left(\tilde{P}_t^d \right)^{\frac{1}{1-\mu_t^d}} \right]^{1-\mu_t^d}. \quad (2.3)$$

Log-linearising around a steady state with no net inflation and combining these equations yields the following inflation equation;³

$$\hat{\pi}_t^D = \frac{\omega_d}{1 + \beta \omega_d} \hat{\pi}_{t-1}^D + \frac{\beta}{1 + \beta \omega_d} E_t \hat{\pi}_{t+1}^D + \frac{1 - \xi_d}{\xi_d} \frac{1 - \xi_d \beta}{1 + \beta \omega_d} \left(\hat{\mu}_t^d + \hat{m} c_t^D \right),$$

where hats denote log deviations from the steady state; thus, $\hat{\pi}_t^D$ is the log inflation in the domestic intermediary sector.

2.3 Importing firms

We use the Monacelli (2005) approach to the modeling of import. That is, a continuum of importers buy a homogenous intermediary good abroad at the world market price (so that the law of one price applies at the border) and then brand them individually and sell them in a market of monopolistic competition to the final-good producers. As with the sector of intermediary producers, we assume that the price contracts are staggered and of stochastic durations; in this case, the Calvo parameter determining the average length of contracts becomes a measure of the degree of exchange-rate pass-through in the import prices.⁴ Hence, importers of foreign intermediaries face the following problem;

$$\begin{aligned} & \max E_t \sum_{\tau=t}^{\infty} (\xi_d \beta)^{\tau-t} \lambda_{\tau} \pi_{\tau}^M (f), \\ \text{where } \quad \pi_t^M (f) &= \left(\tilde{P}_t^M (f) - S_t P_t^* \right) Y_t^M (f) - S_t P_t^* \Phi_m, \\ Y_t^M (f) &= \left(\frac{P_t^M (f)}{P_t^M} \right)^{-\frac{\mu_t^m}{\mu_t^m - 1}} Y_t^M. \end{aligned}$$

It follows that marginal costs for all importers are

$$MC_t^M \equiv S_t P_t^* \Rightarrow m c_t^M \equiv S_t P_t^* / P_t^M = Q_t / p_t^M.$$

Having introduced a specific variable for the marginal costs, the problem is analogous to the one solved in Section 2.2 above, as is the definition of the aggregate price level. Thus,

³We do not seek to account for either real or nominal trends in our estimation, and thus settle on the steady state with $\Pi = 1$ for convenience. Throughout, the term steady state refers to this specific case.

⁴Thus, the trade sector is a special case of the *local currency pricing* assumption advocated by, *inter alia*, Betts and Devereux (1996, 2000).

by log-linearisation we can obtain the following import inflation equation;

$$\hat{\pi}_t^M = \frac{\omega_m}{1 + \omega_m \beta} \hat{\pi}_{t-1}^M + \frac{\beta}{1 + \omega_m \beta} E_t \hat{\pi}_{t+1}^M + \frac{1 - \xi_m}{\xi_m} \frac{1 - \xi_m \beta}{1 + \omega_m \beta} \left(\hat{\mu}_t^m + \widehat{m\hat{c}}_t^M \right),$$

where $\widehat{m\hat{c}}_t^M \equiv \hat{q}_t - \hat{p}_t^M$.

Note that as the Calvo stickiness parameter ξ_m approaches zero, the price charged by the importers will fluctuate one-for-one with changes in the world price, that is, we have full exchange-rate pass-through.

2.4 Final-Good Composite

The final consumption good Y_t is a composite of intermediary goods produced domestically and abroad, and it is used for consumption and investment in the capital stock;

$$Y_t = \left[(1 - \iota)^{\frac{1}{\eta}} \left(Y_t^{D,D} \right)^{\frac{\eta-1}{\eta}} + \iota^{\frac{1}{\eta}} Y_t^M \frac{\eta-1}{\eta} \right]^{\frac{\eta}{\eta-1}},$$

$$P_t = \left[(1 - \iota) (P_t^D)^{1-\eta} + \iota (P_t^M)^{1-\eta} \right]^{\frac{1}{1-\eta}},$$

where ι is the quasi share of imported goods. We assume it is traded in a market of perfect competition.

2.5 Market Clearing and Net Foreign Assets

The model is closed by imposing market clearing conditions for the capital and labor markets across all firms in the intermediary sector

$$K_t = \int_0^1 (K_t(f)) df, \quad H_t = \int_0^1 (H_t(f)) df,$$

for the market for final goods

$$Y_t = C_t + G_t + I_t,$$

and for the markets for intermediary goods

$$\int_0^1 Y_t^D(f) df = Y_t^{D,D} + Y_t^X, \quad \int_0^1 Y_t^M(f) df = Y_t^M.$$

Straightforward manipulation of the household budget constraint (2.1) implies that the holding of net foreign assets evolves as follows;

$$S_t B_t^* = \Omega_t (1 + R_t^*) [S_t B_{t-1}^* + P_t^D Y_t^X - S_t P_t^* Y_t^M]; \quad (2.4)$$

that is, they depend on net exports and the foreign interest rate corrected for UIP disturbances and the debt premium.

2.6 The Monetary Authority

Finally we assume that the central bank conducts monetary policy through changes in the nominal interest rate. The policy actions of the monetary authority can be described by a feedback rule of the form

$$\hat{R}_t = \rho_r \hat{R}_{t-1} + (1 - \rho_r) \left[\hat{\pi}_t + \rho_\pi (\hat{\pi}_t - \hat{\pi}_t) + \rho_y \hat{y}_t^D + \rho_s \Delta \hat{S}_t \right] + \epsilon_t^m, \quad (2.5)$$

where \hat{y}_t^D is the log deviation of GDP from its steady-state level, and $\hat{\pi}_t$ is the target value of (log) inflation. As we consider a model with zero steady-state inflation, $\hat{\pi}_t$ is an exogenous mean-zero process. Thus, the central bank changes interest rates, i_t , when its inflation target $\hat{\pi}_t$ changes, or when CPI inflation, $\hat{\pi}_t$, deviates from this target; when real output differs from the long-run growth path; or when the nominal exchange moves. As is common in the literature, we allow for interest rate smoothing ($\rho_r > 0$). Finally, ϵ_t^m is a domestic monetary policy shock assumed to be i.i.d.

2.7 The Foreign Economy

The foreign economy is modeled as an exogenous structural VAR system for the set of observed variables; GDP inflation, output, and the interest rate (all of which have been detrended as described the in the following section);

$$\begin{aligned} F_0 X_t &= F(L) X_t + \epsilon_t^x, & \epsilon_t^x &\sim N(0, \Sigma_x), \\ X_t &= \left(\hat{\pi}_t, \hat{Y}_t^*, \hat{R}_t^* \right)'. \end{aligned}$$

In accordance with the normal battery of tests we fix the lag-length to two periods. Furthermore, we identify the system by assuming that inflation and output are each orthogonal to innovations to the other variables, that is, only the interest rate responds to contemporary innovations to the other variables. This yields one over-identifying restriction which cannot be rejected at conventional levels of significance, and the implied impulse-response functions are broadly consistent with plausible economic models such as the one presented here.

3 Data

We confront the model with eleven time series; the following seven characterise the UK economy: real GDP, aggregate private consumption and investment; GDP and private consumption inflation rates; hours worked; the three-month nominal interest and a measure of changes in the nominal exchange rate.⁵ We use the following time series for the foreign economy: GDP inflation, real output, and the nominal interest rate - all three of which (as well as the exchange rate) are weighted averages of the US and the euro area economies.⁶

⁵The first five series are taken from the databank constructed by the Bank of England for their BEQM model, while we use the short term interest rate and exchange rates from the Economic Outlook databank compiled by the OECD. The real aggregates, including hours worked, have been normalised by the British working-age population from the BEQM dataset.

⁶For the US series we use Economic Outlook data, while the euro area series were taken from the so-called AWM data set constructed by the ECB. The exchange rate of the dollar and the euro are both from the Economic Outlook data base.

We take the logarithm of all time series, and apply a Hodrick-Prescott filter with $\lambda = 1600$ to all except the exchange rate changes. Thus, in terms of the model variables, we observe the following subset; $\hat{Y}_t^D, \hat{C}_t, \hat{I}_t, \hat{\pi}_t^D, \hat{\pi}_t, \hat{H}_t, \hat{R}_t, \Delta \hat{S}_t, \hat{\pi}_t^*, \hat{g}_t^*, \hat{R}_t^*$.

Our sample consists of quarterly observations for the period 1982-2002. The year 1982 was picked as a starting point for a number of reasons: First, the economic policy of the Thatcher government was firmly established by then. Second, we deliberately want to avoid the turbulent crisis years following the 1979 oil crisis. Third, data constraints in particular relating to the availability of artificially constructed euro area data also made us choose 1982 as the starting point. We chose 2002Q4 as our final data point since more recent data is subject to revisions. The properties of the data set are discussed in Section 5.1 below.

We are aware, however, that our sample period spans several monetary regimes in the UK. From 1982 to 1990, UK monetary policy was targeting various monetary aggregates while exchange rate targeting was implemented from 1990 to 1992. Following the ERM crisis in October 1992, monetary policy has been conducted within a formal inflation targeting framework.

4 Estimation

In this paper we make use of Bayesian methods to estimate our structural model.⁷ In short, the Bayesian approach is an application of Bayes' principle; letting θ be the vector of structural parameters, and X the set of observed variables, we can state the Bayesian principle as

$$\begin{aligned} p(\theta|X) &= p(X|\theta)p(\theta)/p(X) \\ &\propto \mathcal{L}(\theta|X)p(\theta), \end{aligned} \tag{4.1}$$

where $p(\theta)$ is the prior density, $\mathcal{L}(\theta|X)$ is the likelihood function, and $p(\theta|X)$ is the posterior density. The latter constitutes the information we can obtain on the distribution of the structural parameters, given what knowledge we have on them *a priori* and the information that the data gives us through our economic model; hence, it is our prime object of interest. The prior density function is a very convenient tool, as it allows us to integrate information from related macroeconomic studies as well as microeconomic estimations that may be more informative on key structural parameters than the aggregate data we make use of; on a more practical level, it also helps stabilise the nonlinear minimization algorithm which we use for the estimation.

We apply the `Dynare` routines in order to estimate the posterior densities through a Markov-chain Monte-Carlo simulation.^{8,9} Specifically, we analytically derive a log-linearisation of the model around the zero-inflation steady state,¹⁰ which `Dynare` solves for all relevant

The indices for prices, GDP, and the exchange rate were geometrically weighted using their relative volume in UK trade (75% for the euro area, and 25% for the US), while the interest rate series is an arithmetic average using the same weights.

⁷A good introduction to the methodology and its application to macroeconomic DSGE models is offered in An and Schorfheide (2005).

⁸The `DYNARE` program is an ongoing project at CEPREMAP and can be downloaded at www.ceprenmap.cnrs.fr/dynare/ where documentation is also available.

⁹Six quarters of data (1980Q3-1981Q4) were used as a training sample to initiate the Kalman filter for each evaluation of the likelihood.

¹⁰The steady state is derived in Appendix B, and the log-linearised model is presented in Appendix C.

parameter configurations using the perturbation methods of Schmitt-Grohe and Uribe (2004). The posterior mode is found with the `csmine1` minimisation, and the simulated posterior densities are obtained from a Metropolis-Hastings MCMC chain of 600,000 draws of which we discard the first half in order to ensure that we only sample from a part of the chain that has converged.¹¹

4.1 Parameterization and Prior Distributions

As part of our Bayesian analysis we have to transform information from prior studies as well as model consistency requirements into assumptions regarding the prior distributions of the structural parameters. For this part of the exercise, we build on the findings for the US, euro area, and Danish economies of the related estimations presented in Smets and Wouters (2003, 2007), Adolfson et al. (2005), and Dam and Linaa (2005).

First, we fix a subset of the structural parameters which we deem to be empirically weakly determined in our analysis, especially with its short-term emphasis in mind. In line with the majority of the literature, we fix the discount factor β at 0.99, implying a long-run real rate of return of 4 percent annually. Furthermore, the capital share in home production α is fixed at $1/3$, and the capital depreciation rate δ is set to 0.025, implying annual capital depreciations of 10 percent; both values are conventional in the literature.

The inverse Frisch labor supply elasticity, ρ_l , is fixed at the value 5. Our motivation for calibrating this parameter is that we have chosen the simplest possible model of the labour market, i.e., perfect competition. As we do not expect such a simple model of the UK labour market to be able to adequately explain the observed comovements in British real wages and hours, we exclude real wages from the observed data set and fix the labour supply elasticity.¹² The value of the Frisch labor supply elasticity has caused a great deal of the debate in the literature. The RBC literature requires a very elastic labor supply to be able to explain the observed variation in output and – in particular – hours worked. For instance, King and Rebello (1999) used $\rho_l^{-1} = 4$ and found that as ρ_l^{-1} dropped to 1, the RBC model could not generate enough volatility in hours worked. On the other hand, labor market economists point to micro-data survey evidence that suggests a very inelastic labor supply schedule. In particular, those studies usually find Frisch elasticities in the range of 0.05 to 0.5. Given the range of estimates, we pick a labor supply elasticity in the middle of the range, $\rho_l^{-1} = 0.2$. This particular choice is consistent with the conclusion of the survey in Card (1994).

Finally, we calibrate the consumption, investment and government spending shares of final domestic demand in accordance with sample averages for the UK. Thus, the C/Y ratio is fixed at 64 percent, I/Y at 14 percent, and G/Y at 22 percent.

We now turn to the parameters with non-degenerate prior distributions, which are presented in Table 1. First we shall consider the group of the parameters that are restricted to lie between zero and unity in order to ensure model consistency, all of which are assumed to follow beta distributions. In accordance with the studies cited above, we pick a mean of

¹¹Gelman et al. (2003, part III) offers a good introduction to the Metropolis-Hastings algorithm and related methods.

¹²The literature has chosen to follow the lead of Erceg et al. (2000) and introduce sticky wages through Calvo (1983) contracts in the labour market. As we consider this to be an inadequate description of the British labour market, we have abstained from including this as an explanation of the wage mechanism. We find that a more promising approach would be to incorporate a search-and-match labour market in the model. However, that is beyond the scope of the present paper; we refer the reader to Christoffel et al. (2006) for a recent study.

0.6 for the habit formation parameter h , and a mean of 0.7 for the smoothness parameter ρ_r of the monetary policy rule that captures the degree of inertia in the realised interest rate. With regards to the parameters of the pricing models, we pick the mean and variance of the prior distribution for the Calvo parameter ξ_d such that we assign 95 percent of the probability mass to an average duration of price contracts between $2\frac{1}{2}$ and 5 quarters, a range we believe to be consistent with the growing body of survey-based micro data.¹³ As already discussed, the Calvo parameter in the import sector captures the degree of exchange-rate pass-through; since we do not want to take a firm stand on this *a priori*, we choose a fairly loose prior with mean 0.5 and standard deviation 0.15. The same prior is chosen for the share of firms in the intermediate and import sectors which index their prices to lagged inflation when they are not reoptimising. The observed import share in final domestic demand has moved from ca. 20 to 30 percent over the sample period we consider, and thus we choose to estimate its long-run value, assigning the intermediate value 0.25 as the mean of our prior. Finally, we pick a mean of 0.8 for the autocorrelation parameters of all the potentially persistent structural processes.

¹³See Hall et al. (2000) for the UK and Álvarez et al. (2005) for the euro area.

Table 1: Parameter Estimates

Parameter	Prior distribution				Posterior distribution				Posterior distribution, no imp. imp			
	Type	Mean ^a	Std.dev. ^a	Mode	Mean	CI ₁₀	CI ₉₀	Mode	Std.dev.	Mean	CI ₁₀	CI ₉₀
h	Habit persistence	0.6	0.1	0.309	0.068	0.349	0.472	0.279	0.059	0.269	0.180	0.360
ρ_c	Household inverse IES	1.5	3	0.624	0.066	0.650	0.537	0.598	0.059	0.611	0.509	0.711
ι	Mean import share	0.25	0.05	0.233	0.049	0.264	0.184	0.230	0.05	0.240	0.157	0.322
η	Import ES	1.5	4	6 [*]	—	6 [*]	—	6 [*]	—	6 [*]	—	—
η_x	Export/foreign output ES	1.5	4	1.367	0.081	1.418	1.256	1.395	0.095	1.491	1.290	1.687
ω	Debt premium	0.01	4	0.012	0.004	0.015	0.006	0.007	0.001	0.007	0.005	0.009
S	Investment adjustment cost	5	4	3.255	0.640	3.557	2.389	3.286	0.653	3.602	2.420	4.779
μ_d	Mean markup, dom. goods	1.2	5	1.190	0.054	1.228	1.133	1.189	0.053	1.223	1.118	1.323
μ_m	Mean markup, imp. goods	1.2	5	1.299	0.080	1.276	1.167	1.384	0.102	1.586	1.285	1.837
ξ_d	Price stickiness, dom. goods	0.7	0.05	0.391	0.032	0.399	0.357	0.396	0.033	0.400	0.357	0.438
ξ_m	Price stickiness, imp. goods	0.5	0.15	0.701	0.067	0.440	0.323	0.940	0.025	0.910	0.843	0.976
ω_d	Price indexation, dom. goods	0.5	0.15	0.152	0.074	0.173	0.052	0.152	0.074	0.181	0.059	0.301
ω_m	Price indexation, imp. goods	0.5	0.15	0.440	0.186	0.405	0.155	0.420	0.148	0.433	0.197	0.662
ρ_r	MP, smoothing	0.7	0.1	0.687	0.047	0.685	0.608	0.664	0.051	0.654	0.575	0.737
ρ_π	MP, inflation response	1.7	0.15	1.861	0.130	1.906	1.690	1.921	0.125	1.939	1.732	2.147
ρ_y	MP, output response	0.125	4	0.037	0.019	0.050	0.016	0.038	0.020	0.047	0.013	0.077
ρ_s	MP, exchange rate response	0.2	4	0.087	0.026	0.097	0.054	0.075	0.026	0.077	0.037	0.118
<i>Shock persistence</i>												
ϱ_c	Preferences	0.8	0.1	0.777	0.065	0.761	0.649	0.806	0.059	0.806	0.714	0.900
ϱ_l	Labour supply	0.8	0.1	0.855	0.093	0.821	0.677	0.855	0.092	0.811	0.669	0.954
ϱ_t	Technology	0.8	0.1	0.968	0.021	0.955	0.920	0.963	0.022	0.954	0.921	0.990
ϱ_i	Investment techn.	0.8	0.1	0.381	0.080	0.418	0.285	0.547	0.079	0.404	0.278	0.525
$\varrho_{\mu d}$	Markup, dom. goods	0.8	0.1	0.884	0.043	0.870	0.799	0.952	0.044	0.863	0.784	0.947
$\varrho_{\mu m}$	Markup, imp. goods	0.8	0.1	0.961	0.021	0.937	0.897	0.981	—	—	—	—
ϱ_g	Govt. spending	0.8	0.1	0.919	0.031	0.921	0.867	0.920	0.027	0.912	0.867	0.962
ϱ_v	up shock	0.8	0.1	0.849	0.099	0.835	0.701	0.980	0.013	0.972	0.951	0.994
ϱ_π	Inflation target	0.8	0.1	0.840	0.099	0.783	0.631	0.840	0.100	0.792	0.646	0.952
<i>Shock volatility</i>												
σ_c	Preferences	0.2	3	0.518	0.093	0.512	0.334	0.560	0.105	0.617	0.410	0.807
σ_l	Labour supply	0.1	3	0.116	0.044	0.199	0.063	0.116	0.044	0.175	0.066	0.300
σ_t	Technology	0.7	3	0.416	0.037	0.414	0.354	0.415	0.037	0.414	0.352	0.473
σ_i	Investment techn.	0.2	3	1.261	0.121	1.247	1.041	1.250	0.120	1.254	1.052	1.447
$\sigma_{\mu d}$	Markup, dom. goods	0.25	3	0.405	0.083	0.378	0.238	0.369	0.078	0.360	0.237	0.482
$\sigma_{\mu m}$	Markup, imp. goods	0.25	3	0.587	0.219	2.218	1.331	3.532	—	—	—	—
σ_g	Govt. spending	0.3	3	0.583	0.064	0.524	0.392	0.553	0.060	0.551	0.453	0.651
σ_v	up shock	0.05	3	0.059	0.023	0.269	0.039	0.377	0.068	0.471	0.319	0.610
σ_π	Inflation target	0.02	3	0.022	0.008	0.033	0.013	0.022	0.008	0.031	0.013	0.050
σ_m	Mon.pol. shock	0.15	3	0.191	0.033	0.201	0.140	0.190	0.035	0.199	0.134	0.258

Note: ^aFor the gamma and inverse gamma distributions, mode and shape parameter are reported in the place of mean and standard deviation. ²The lower bound of the inverse gamma distributions for the intratemporal elasticities and the mark-up levels has been shifted to 1.

The next group of parameters consist of intratemporal elasticities of substitution and the long-run gross mark-up levels which should all be larger than unity, and for all four parameters we settle on an inverse gamma distribution with its lower bound shifted to one. For the elasticity of substitution between domestic and imported intermediaries, η , and the corresponding elasticity of substitution between exports and foreign goods abroad, η_x , we pick a mode of 1.5, which is consistent with numerous macroeconomic studies, including the seminal paper by Chari et al. (2002).¹⁴ As detailed in the next section, however, the estimation calls for an implausibly high value of η , and consequently we prefer to fix this parameter at the value 6. We assign a mode of 1.2 to the gross mark-up parameters μ_d and μ_m , thus centering the prior probability mass around a long-run net markup rate of 20 percent in both sectors.

Economic reasoning and model consistency requirements imply that the remaining parameters should all be non-negative. Turning to the three parameters of the monetary policy rule, we settled on a gamma distribution with mode 0.125 for the output coefficient ρ_y . Note that this corresponds to a coefficient of 0.5 for an annualised inflation rate, in accordance with the original Taylor (1993) result. The response to exchange rate fluctuations was also assigned a gamma distribution with mode 0.2. Finally, the inflation response is assigned a normal distribution centered on 1.7 and leaving only a little less than 10 percent of the mass under 1.5; we settled on this distribution in order to keep problems with indeterminacy to a minimum for the high-dimensional and very non-linear estimation algorithm.

The remaining coefficient priors were assumed to follow inverse gamma distributions. For the inverse intertemporal elasticity of substitution, ρ_c , we picked a mode of 1.5, reflecting the findings in Dam and Linaa (2005) and Smets and Wouters (2003, 2007), but kept the distribution wide enough not to exclude the benchmark value of one *a priori*. In line with the literature, the investment adjustment cost parameter S was assigned a mode of 5. The debt premium parameter was given a mode of 0.01 which is a common value for the literature. The prior modes for the standard deviations of the ten structural shock processes were set in accordance with the assumptions of Smets and Wouters (2003, 2007) and Adolfson et al. (2005), and we refer the reader to these papers for a discussion.

4.2 Posterior Distribution of Estimated Parameters

The posterior parameter distributions are illustrated in Figures 1-3. Generally, the posterior densities have shifted or tightened relative to the prior densities, reflecting a significant extraction of information from the combination of the data with our economic model. However, some parameters were virtually unidentified by the data and model, including the import share ι , the debt premium ω , and the autocorrelation of the labour supply shock and the inflation target process (ϱ_l and ϱ_π).

We find that the posterior mode for the inverse of the intertemporal elasticity of substitution ρ_c equals 0.62, which is somewhat lower than RBC calibrations that traditionally use values between 1 and 2. Thus, our estimate suggests that households are more willing to substitute consumption over time than was traditionally assumed in the RBC literature. As we discuss in Section 5 below, UK aggregate consumption is relatively volatile compared to other countries including the US. Hence, in order to match the relative volatilities of British consumption and GDP we require consumers to be relatively more

¹⁴For all the gamma and inverse gamma distributions we prefer to report the mode rather than the mean, as we find this statistic more informative for these rather skewed distributions.

willing to substitute consumption across time, resulting in the estimated IES of c. 1.6. Less habit persistence would also *ceterius paribus* imply higher willingness to substitute consumption across time; indeed, our estimates for the habit parameter, h , suggest a value around 0.3 which is considerable lower than found elsewhere in the literature. For instance, Smets and Wouters (2007) find a value slightly higher than 0.6 for the US while Smets and Wouters (2003) estimate 0.6 using euro area data. Also, UK investments are quite volatile, which may contribute to our finding of a rather low investment adjustment cost S with a posterior mode of 3.3, where estimates for the US and euro area lie in the range of 6 to 9.

Turning now to our estimates of the price inertia parameters, our estimates of the Calvo parameter for domestic prices is 0.4, implying an average duration of price contracts of $1\frac{2}{3}$ quarters (5 months) which is a considerably lower degree of nominal price rigidity than found in the majority of other related studies; for instance, both Smets and Wouters (2003) and Adolfson et al. (2005) estimate the average duration of euro area prices to be around 10 quarters. However, we think that our results are more in line with microevidence pointing to average contract duration of around 2 quarters, cf. Hall et al. (2000). The degree of domestic price indexation ω_d equals 0.15 which is in line with the 0.17 found by Adolfson et al. (2005) for the euro area, but lower than the estimates in Smets and Wouters who find 0.47 using both US and euro area estimates, and in stark contrast to Christiano et al. (2001) who fix the indexation at 1. Indeed, observed UK inflation is considerably less persistent than US inflation, cf. Section 5 below, implying a little role for the lagged indexation which has been introduced mainly to account for the high US inflation inertia.

For import prices, we estimate an import price indexation parameter ω_m of 0.44. Since our estimate of the average duration of import price contracts is 10 months ($\xi_m = 0.7$), our model implies a fair degree of exchange rate pass-through. Note also that the posterior mean is markedly lower ($\xi_m = 0.44$, implying average contracts of about 5 months).¹⁵ For comparison, the average duration of import price contracts in the BEQM model is 20 months.

The UK interest rate rule is pretty much in line with previous estimates, including Lubik and Schorfheide (2003). We find evidence of a fair degree of interest rate smoothing ($\rho_r = 0.69$) while the inflation targeting coefficient ρ_π equals 1.9. We find only little weight on output gap targeting ($\rho_y = 0.07$) and exchange rate targeting ($\rho_y = 0.1$). The latter results conform with the findings of Lubik and Schorfheide (2003), who use a posterior-odds-based test to establish that the Bank of England did not systematically respond to exchange rate movements over the last two decades.

Our estimates of the steady state markup coefficients are pretty much in line with the existing literature. We find that the steady state markup in the UK domestic goods sector, μ_d , equals 1.19 while the corresponding steady state markup in the import sector, μ_m , equals 1.30.

Turning to the standard deviations of the structural shocks, the thing that stands out is that $\sigma_{\mu m}$ and σ_v are both bimodal with one peak close to the prior mode, and another at a larger value. This is a well-known outcome in Bayesian analysis, reflecting that the data are contradicting the model restrictions under the prior distribution, pulling

¹⁵We believe that this peculiarity stems from a weak identification of the price stickiness in the import sector vis-à-vis the volatility of the import markup shock, $\sigma_{\mu m}$. This suggests that a more parsimonious specification of the import sector could be preferable to the one analysed here. A similar problem with a Canadian estimation is accounted for in Justiniano and Preston (2006),

the posterior shock volatility up relative to the prior assumption. The reason why these particular shocks are inflated in our estimation is discussed in Section 7.2 below.

Finally, we allowed for a small amount of i.i.d. measurement error in the set of observed domestic variables and the exchange rate. These are capturing measurement error in the data as well as model misspecification, as we have made a number of simplifying assumptions and thus cannot hope to fully capture the dynamic properties of the UK economy perfectly. The volatility of each measurement error was estimated using a tight gamma prior distribution with mass centered around 0.1, and the posterior modes of the errors lie between 0.05 and 0.13 except for the two inflation series; for CPI inflation we obtain a mode of 0.24 while the GDP inflation posterior mode is as high as 0.45. This indicates that the inflation dynamics are not adequately described by our model. Plausibly, as the pricing model of this literature was primarily constructed to account for the persistency of US inflation, it is not well suited for the big idiosyncratic element in UK inflation which we now turn to.

5 Assessing the Model Fit

In this section we evaluate how well our model is able to match key business cycle statistics of the UK variables that we introduced in Section 3. The top section of Table 2 contains empirical business cycle statistics for the variables of interest. The statistics were generated from series that have been logged and run through a Hodrick-Prescott filter with a smoothing parameter of 1600, cf. Figures 4 and 5. The only exception is the nominal depreciation rate where we use the first difference of the original series.

5.1 Empirical Moments

The main features of UK data are as follows: UK GDP has a standard deviation of 1.07% while UK consumption and investments are respectively 1.3 and 5.8 times more volatile than UK GDP. Interestingly, both UK consumption and investment exhibit greater fluctuations relative to UK GDP than say US consumption and US investment. According to Canzoneri et al. (2007), US consumption and US investment are only 0.8 and 3 times as volatile as US output.¹⁶ Both UK investment and consumption is positively correlated with UK GDP as well as UK output price inflation. As in US data, all three UK series exhibit a high degree of persistence with a 1 quarter autocorrelation of above 0.85.

The volatility of UK CPI and PPI inflation relative to UK output volatility is $1/3$ and $\frac{1}{2}$, respectively. Both inflation series have a positive unconditional correlation with UK GDP and both series exhibit very little persistence. In fact, the 1 quarter autocorrelation is negative and close to zero. The seeming lack of inflation inertia in the UK filtered inflation series distinguishes the UK inflation dynamics from, e.g., the US where several papers have highlighted the high degree of US inflation persistence.¹⁷ We also note that both UK inflation measures are positively correlated with UK GDP. Likewise, there is

¹⁶Our measure of investment is *Business Investment* which excludes government investment. This is consistent with our theoretical framework where the government does not undertake any investments. However, we found that in the data, the total investment series exhibit lower volatility than the business investment series. This would suggest that government investment is negatively correlated with business investment.

¹⁷The corresponding 1 quarter autocorrelation in HP-filtered US CPI inflation is 0.75.

Table 2: Variable Moments

Variable		Std. deviation	Autocorr.	Corr. w. \hat{Y}_t^D	Corr. w. $\hat{\pi}_t^D$
Data, 1982-2002					
\hat{Y}_t^D	GDP	1.074	0.897	1.000	0.137
\hat{C}_t	Consumption	1.444	0.902	0.377	0.127
\hat{I}_t	Investment	6.277	0.872	0.613	0.307
$\hat{\pi}_t^D$	PPI inflation	0.670	-0.148	0.137	1.000
$\hat{\pi}_t$	CPI inflation	0.394	-0.040	0.120	0.449
\hat{R}_t	Interest rate	0.324	0.828	0.219	0.125
\hat{Q}_t	RER	4.413	0.740	0.024	0.166
$\Delta\hat{S}_t$	Depreciation	3.211	0.171	-0.107	-0.137
Model (evaluated at posterior mode)					
\hat{Y}_t^D	GDP	0.716	0.770	1.000	-0.097
\hat{C}_t	Consumption	1.364	0.751	0.374	-0.202
\hat{I}_t	Investment	5.088	0.822	0.339	0.220
$\hat{\pi}_t^D$	PPI inflation	0.444	-0.024	-0.097	1.000
$\hat{\pi}_t$	CPI inflation	0.420	0.005	-0.096	0.998
\hat{R}_t	Interest rate	0.264	0.553	-0.081	0.460
\hat{Q}_t	RER	4.415	0.764	0.071	-0.096
$\Delta\hat{S}_t$	Depreciation	3.013	-0.038	0.026	0.041
Model (evaluated at posterior mean)					
\hat{Y}_t^D	GDP	0.748	0.777	1.000	-0.132
\hat{C}_t	Consumption	1.365	0.782	0.362	-0.221
\hat{I}_t	Investment	5.465	0.832	0.350	0.193
$\hat{\pi}_t^D$	PPI inflation	0.467	0.036	-0.132	1.000
$\hat{\pi}_t$	CPI inflation	0.454	0.093	-0.124	0.990
\hat{R}_t	Interest rate	0.296	0.600	-0.109	0.483
\hat{Q}_t	RER	4.611	0.757	0.073	-0.121
$\Delta\hat{S}_t$	Depreciation	3.139	-0.037	0.017	-0.023

Note: Theoretical moments are based on the posterior distribution.

also a positive correlation between UK interest rates and inflation rates. The positive correlation between inflation and GDP has also been found in US data as well as in euro area data. (See Canzoneri et al. (2006) for references to empirical work).

Finally, note that the volatility of our measure of the effective Sterling real exchange rate is about 4.5 times as volatile as UK output. The high volatility of the £ real exchange rate is in line with the stylized facts of real exchange rates documented in the international real business cycle (IRBC) literature. For instance, Chari et al. (2002) and Bergin and Feenstra (2001) have documented that the US bilateral real exchange rates against 15 European Union countries are on average about 4-5 times more volatile than GDP.

5.2 Theoretical Moments

How well can our model match key UK business cycle facts? The second part of Table 2 contains the theoretical moments of our model when we evaluate the model at the posterior mode. The general message is that our model does a good job at matching the key moments. In fact, we are able to match the volatility of UK GDP, consumption and investment quite well. The model implies a UK GDP volatility of 0.72% while it is 1.1% in the data. In terms of UK investment, the volatility is 5.1% compared to 6.3% in the data. Our model generates a consumption series with a volatility of 1.36%, only slightly lower than actual volatility of 1.44%. Turning now to persistence, we note that the model generates too little output (0.77) and consumption persistence (0.75) compared to what we observe in the data (0.90 for both). However, our model is capable of matching the observed investment persistence.

Can our framework replicate the UK inflation and interest dynamics? While the model generates too little PPI inflation and interest rate volatility (0.44 and 0.26% compared to 0.67 and 0.32% in the data), it generates a quite volatile CPI inflation series (0.42% in the model compared to 0.39% in the data). Given the low degree of inflation persistence seen in the data, it is not surprising that the theoretical inflation series is as persistent as in the data. On the other hand, our model generates too little persistence in interest rates compared to the data.

Interestingly, our model implies a negative correlation between both UK inflation series and UK GDP. Likewise, the correlation between UK interest rates and GDP in the model is also negative. Recall that the data implies a positive correlation between GDP and inflation and between GDP and interest rates. A positive correlation between GDP, inflation, and the interest rate would indicate that demand shocks are important drivers of cyclical inflation. The intuition is as follows: Think of a government spending shock that would raise output. A temporary increase in output would via the Phillips curve eventually lead to higher UK inflation which would prompt the central bank to raise interest rates. One interpretation of the countercyclical behavior of inflation in our model is that – despite the addition of multiple demand shocks to our setup – the nature of our estimated model is quite close to the RBC and early New Keynesian literature in the sense that productivity shocks are the dominant source of economic fluctuations, and thus the model cannot generate procyclical inflation.

Finally, our model is capable of replicating both the high volatility (4.41%) and high persistence of real exchange rates (0.76%). At first glance, this seems like a remarkable feature of our model in particular considering that earlier papers have been unable to generate the observed real exchange rate dynamics using similar open-economy sticky-price models.¹⁸ However, as we discuss in Section 7.2, the ability of our model to match the moments of the real exchange rate mainly relies on the assumption of a very volatile and persistent exogenous shock process. Earlier papers, (e.g., Kollmann, 2002) have relied on the exogenous UIP shock predominantly driving real exchange movements. While our model also features an exogenous UIP shock, its role is overshadowed by another volatile and persistent exogenous shock namely the import price markup shock. The variance decomposition in Table 4 below illustrates this point.

¹⁸See for instance Chari, Kehoe and McGrattan (2002)

6 Model Robustness Analysis

We now explore the sensitivity of our estimation results to various features of the benchmark model. Table 3 contains estimates for four particular model variations. Column 2 and 3 explores the role of the intratemporal elasticity of substitution η . Column 4 considers the setup with perfect exchange rate pass-through. Finally, column 5 assumes that markup shocks are not persistent.

Table 3: Parameter Estimates

Parameter	Benchmark	Low import ES $\eta = 3$	Estimated η	PCP $\xi_m = \mu_m = 0$	i.i.d markup $e_{\mu m} = \rho d = 0$	Estimated η Linear detrending
	Posterior mode					
Habit persistence	0.309	0.267	0.545	0.227	0.418	0.566
Household inverse IES	0.624	0.609	0.663	0.579	0.626	0.717
Mean import share	0.233	0.197	0.401	0.222	0.276	0.399
Import ES	6*	3*	19.11	6*	6*	16.94
Export/foreign output ES	1.367	1.369	1.388	1.490	1.376	1.385
Debt premium	0.012	0.012	0.009	0.157	0.008	0.009
Investment adjustment cost	3.255	3.245	3.502	3.259	3.248	3.759
Mean markup, dom. goods	1.190	1.192	1.176	1.189	1.177	1.178
Mean markup, imp. goods	1.299	1.741	1.096	2.399	1.223	1.098
Price stickiness, dom. goods	0.391	0.394	0.416	0.402	0.367	0.449
Price stickiness, imp. goods	0.701	0.718	0.387	-	0.925	0.401
Price indexation, dom. goods	0.152	0.157	0.121	0.157	0.127	0.122
Price indexation, imp. goods	0.440	0.496	0.386	-	0.270	0.404
MP, smoothing	0.687	0.667	0.720	0.632	0.740	0.700
MP, inflation response	1.861	1.879	1.855	1.934	1.879	1.830
MP, output response	0.037	0.038	0.040	0.041	0.043	0.029
MP, exchange rate response	0.087	0.083	0.095	0.053	0.121	0.099
<i>Shock persistence</i>						
Preferences	0.777	0.783	0.754	0.800	0.782	0.790
Labour supply	0.855	0.855	0.861	0.856	0.901	0.865
Technology	0.968	0.970	0.957	0.979	0.960	0.978
Investment techn.	0.381	0.391	0.376	0.404	0.388	0.458
Markup, dom. goods	0.884	0.884	0.899	0.969	-	0.938
Markup, imp. goods	0.961	0.970	0.961	-	-	0.963
Govt. spending	0.919	0.927	0.946	0.936	0.933	0.972
UIP shock	0.849	0.850	0.883	0.979	0.960	0.884
Inflation target	0.840	0.839	0.841	0.840	0.841	0.845
<i>Shock volatility</i>						
Preferences	0.518	0.533	0.360	0.579	0.487	0.390
Labour supply	0.116	0.116	0.114	0.116	0.438	0.111
Technology	0.416	0.414	0.418	0.409	0.420	0.477
Investment techn.	1.261	1.255	1.273	1.249	1.245	1.229
Markup, dom. goods	0.405	0.398	0.332	0.348	0.210	0.393
Markup, imp. goods	0.387	0.346	2.482	-	0.793	2.274
Govt. spending	0.583	0.571	0.411	0.530	0.335	0.580
UIP shock	0.059	0.058	0.178	0.793	0.310	0.153
Inflation target	0.022	0.022	0.023	0.022	0.023	0.023
Mon.pol. shock	0.191	0.188	0.190	0.189	0.181	0.216
Log marginal likelihood	-645.62	-654.30	-637.45	-671.51	-652.07	-786.23

First, we consider the role played by the intratemporal elasticity of substitution η . Recall that our model implies a tight connection between import price inflation $\hat{\pi}_t^M$, GDP inflation $\hat{\pi}_t^D$, and CPI inflation $\hat{\pi}_t$, cf. equations (C.13)-(C.14) in Appendix C;

$$\hat{\pi}_t^M = (1 + \varkappa) \hat{\pi}_t - \varkappa \hat{\pi}_t^D,$$

where \varkappa is a constant whose value depends on the steady state import markup μ_m as well as the intratemporal elasticity of substitution across goods η . We are trying to match the volatility of both CPI inflation as well as GDP inflation where our analysis of the data series revealed that GDP inflation is almost twice as volatile as CPI inflation, cf. Section 5. In the estimation, we are not matching the dynamics of import price inflation. Rather, the volatility of import price inflation is determined by the volatility of real exchange rates. Hence, the very volatile real exchange rate in the data implies volatile import price inflation. Given that the above equation imposes a tight link between movements in import price as well as CPI and GDP inflation, \varkappa has to adjust to ensure that our model can

simultaneously match the dynamics of the observed CPI and GDP inflation series as well as generate very volatile import price inflation. A key structural parameter determining \varkappa is the intratemporal elasticity of substitution η .

In our benchmark experiments, we decided to fix the value of η at 6. While the traditional IRBC studies have chosen 1.5 as the appropriate value, a number of recent papers have argued that traded goods are more substitutable than implied by the value 1.5. For instance, Tchakarov (2004) also chose $\eta = 6$ as the appropriate value.

Column 2 contains the estimation results from an experiment where we make traded goods less substitutable. Hence, we set η to equal 3. The results are quite similar to our benchmark results. We estimate a slightly lower import share ι (0.20 versus benchmark 0.23), and our estimate for the steady state import markup is also higher (1.7 versus benchmark value of 1.3). As these parameters are prime determinants of \varkappa along with η , it is hardly surprising that they are the ones to differ markedly from the benchmark case. That is, to ensure that we can still match the dynamics of the two inflation series as well as the real exchange rate for a lower elasticity of substitution, \varkappa has to adjust via changes in the steady state import markup as well as the import share. We note that fixing η at 3 rather than 6 produces markedly lower marginal likelihoods (-654.30 compared to -645.62 in the benchmark).

Rather than fixing the IES, could we not estimate it? Column 3 contains the results for the setup where we allow the model to estimate η . We find that the IES is driven to a very large number ($\eta = 19$). Likewise, the import share ι , is now estimated at 40% while the steady-state import markup share is driven down to 1.1. At the same time, the volatility of the import markup shock is now estimated to be 6 times higher (2.48) than in the benchmark experiment (0.39). Unsurprisingly, this experiment produces a higher marginal likelihood (-637.45) than our benchmark experiment. Given the higher marginal likelihood, why don't we allow η to be estimated as a free parameter in the first place? First of all, we do not regard $\eta = 19$ as a plausible estimate as this would imply a degree of substitutability between domestic and foreign goods which is considerably higher than what microeconomic studies have found (see Tchakarov (2004) for references). Second, the estimated import share is far higher than we see in the data. Finally, we interpret the results of this experiment as evidence that the workhorse one-sector model has difficulty matching the observed real exchange rate volatility. In other words, if we allow η to be freely estimated, the import markup shocks become even more volatile than in the benchmark estimation, which in turn entails very volatile import prices. In order to reconcile volatile import prices with the observed less volatile CPI and GDP inflation series, the model is generating implausible high estimates of η and import shares ι .

While our benchmark model allowed for imperfect exchange rate pass-through, we now consider the case with complete exchange rate pass-through. We therefore estimate a model where there is no import price stickiness and no import markup shocks – that is, a case of *producer currency pricing* (PCP). We find that the persistence of the domestic markup shock has increased considerably (0.97 versus the benchmark 0.88). Likewise, the persistence of the UIP shock has also increased (0.98 versus 0.85 in the benchmark). Finally, the volatility of the UIP shock have increased 10-fold to a standard deviation of 0.79. We also note that PCP setup yields a considerably lower marginal likelihood (-671.51) than the benchmark. How do we interpret these results? We know that matching the volatility and persistence of the real exchange rate is not an easy task. Our benchmark model has a number of features which would help us match the real exchange rate dynamics. First, highly volatile and persistent import markup shocks play a crucial

role in generating both volatile and persistent real exchange rates. Second, as shown by Betts and Devereux (2000), allowing for imperfect exchange rate pass-through would yield more volatile real exchange rates. When we assume no import markup shocks and full exchange-rate pass-through, the model would have to rely on other features in order to be able to match the real exchange rate dynamics. Therefore, it is not surprising that our estimates of the PCP model yield more persistent and more volatile UIP shocks as well as more persistent domestic markup shocks.

The discussion so far has highlighted that persistent markup shocks may play an important role in terms of matching the volatility and persistence of the the real exchange rate. We therefore undertake an experiment where we assume no persistence in the exogenous markup shocks. Our model estimates now imply a higher degree of consumption habit persistence (0.42 versus 0.31 in the benchmark), a higher degree of import price stickiness (0.93 versus 0.70), and twice as volatile a import markup shock (a standard deviation of 0.79 versus 0.39). Finally, we estimate a higher persistence of the UIP shock (0.98 versus 0.85). The results validate our intuitions from the previous experiments: Without persistent markup shocks, the model can only match the dynamics of the real exchange rate by relying more on the other channels that can generate persistent and volatile real exchange rates. Thus, we note that the experiment with i.i.d. markup shocks yield a lower marginal likelihood (-652.07) than the benchmark experiment. Hence, a setup with persistent markup shocks are preferred to a setup with only i.i.d. markup shocks.

Finally, we want to assess the sensitivity of our result to the fact that we have estimated on data, which have been detrended using the HP-filter. Our final experiment therefore re-estimates the model using linear detrended data. We found that our estimation algorithm could not converge when we fix the intratemporal elasticity of substitution, η , to 6. We therefore allow the parameter, η , to be freely estimated. What do we find? Our estimates are remarkably similar to the results in column 3, where we also estimate η freely but using HP-filtered data. We find slightly higher domestic goods price stickiness (0.45 vs old value of 0.42) and slightly higher intertemporal elasticity of substitution (0.72 versus old value of 0.66). But overall there are no real differences between the two experiments and we conclude that our results are independent of whether you estimate on HP-filtered or linear detrended data.

7 Dynamic Properties of the Estimated Model

In this section we evaluate the properties of the estimated model, first through a select set of impulse-response functions, and then through a variance decomposition.

7.1 Impulse-Response Functions

Figure 6 illustrates the effect of a positive domestic technology shock on the main variables of interest. Generally, the effects are fairly similar to those found in the closed-economy literature: A positive domestic productivity shock - *everything else equal* - implies higher domestic output, higher domestic consumption and higher domestic investment. The effect on all three variables is very persistent and we attribute this to the presence of habit persistence in consumption as well as our assumption of investment growth adjustment costs.

A positive domestic productivity shock also implies a fall in domestic marginal costs (in other words domestic producers can produce at a cheaper cost). Thus, we would expect a fall in domestic prices and domestic output price deflation. The IRF for GDP inflation confirms that this is indeed the case.

In addition, the fall in domestic output prices has an external dimension as follows: We would expect lower domestic prices to imply a terms of trade deterioration (domestic goods are relatively cheaper than foreign goods) as well as a real exchange rate depreciation. The impulse responses for the real exchange rate confirm this intuition. Part of the real depreciation is achieved through a nominal depreciation and part is achieved through lower domestic prices.

What is the effect of our technology shock on import price inflation? Recall that import prices are determined in a staggered fashion a la Calvo (1983). Importers are assumed to buy imports on the world markets which they then turnaround and sell on the domestic market. The importers face the traditional downward sloping demand schedules where the demand of imports depends on the price of imports relative to price of domestic produced goods. The optimal import price will be affected by two factors: A nominal depreciation implies a higher world market price (measured in domestic currency) for domestic importers and, hence, tends to imply higher domestic import prices. On the other hand, lower prices for domestic produced goods would force the importer to lower import prices as well in order to protect market shares on the domestic market.

Recall that the positive technology shock implies both a decline in the price of domestic output as well as a nominal depreciation. We find that import prices decline in response which suggests that the effect of a decline in domestic output prices outweigh the inflationary effect stemming from a nominal depreciation. The net effect of a positive productivity shock on CPI inflation is negative which is not surprising given that we both have output price deflation as well as import price deflation.

We note that the technology shock generates very little inflation persistence. One explanation is that our model assumes a very simple labor market with no wage stickiness. Hence, a positive technology shock will lead to a rapid adjustment in nominal wages which implies very little inertia in marginal cost and thus in output price inflation. In other words, with nominal wage contracts persistent technology shocks would imply persistent deviations in marginal costs which would generate more persistence in output price inflation. Given the apparent lack of inflation persistence in UK data during our sample period, we would not attach too much weight to this issue.

We now consider the effects of a positive white-noise shock to domestic interest rates as illustrated in Figure 7.

We note the following: Domestic interest rates rise on impact because of the increase in the i.i.d. monetary policy shock. We can interpret the shock as a negative domestic demand shock which would have the traditional implication. First, higher interest rates lead to a fall in domestic consumption, since higher interest rates encourage higher savings today and less consumption. Lower demand also implies lower investment and hence lower output. Because of our investment adjustment cost assumption, the monetary policy shock implies a hump-shaped response on investment, as discussed in Christiano et al. (2001).

The real exchange rate is appreciating and the terms of trade is improving (the terms-of-trade IRF is not shown here). There are two reasons: First, higher domestic interest rates translates into a nominal appreciation of the domestic currency and hence a real

appreciation when prices are sticky. Second, lower domestic output requires a rise in the price of domestic output relative to foreign output. Hence, a terms of trade improvement is required.

What is the net effect on domestic CPI inflation? Lower domestic output has a deflationary effect. The nominal appreciation also implies import price deflation as seen on the IRF. Combining the two effects, we observe CPI deflation.

Finally, we consider the effects of a positive one-period shock to foreign interest rates, cf. Figure 8. The main international transmission mechanism of foreign shock is through the real exchange rate. Higher foreign interest rates implies – via the UIP condition – a nominal and real depreciation of the domestic currency, which in turn implies higher domestic import prices.

However, the nominal depreciation also implies a domestic competitiveness gain which would increase demand for domestic exports. The rise in domestic exports requires that domestic output is redirected overseas and hence would necessitate a fall in the domestic demand components consumption and investment. The IRFs confirm that domestic consumption and investment both fall but GDP rises. What is the net effect on CPI inflation and domestic interest rates? The rise in GDP would imply higher domestic output inflation and hence higher CPI inflation. The higher import price inflation also lifts CPI inflation. Hence, we would expect the domestic central bank to increase interest rates in response to higher CPI inflation. The IRFs for domestic interest rates suggest that this is in fact occurring.

7.2 A Variance Decomposition of the Estimated Model

In the following, we consider which of the structural shocks that matter for the overall volatility of the estimated model. As documented in Table 4, the unconditional variance of GDP is explained almost exclusively by the two technological shocks, i.e., by the production technology (70%) and the investment adjustment technology (26%) processes.

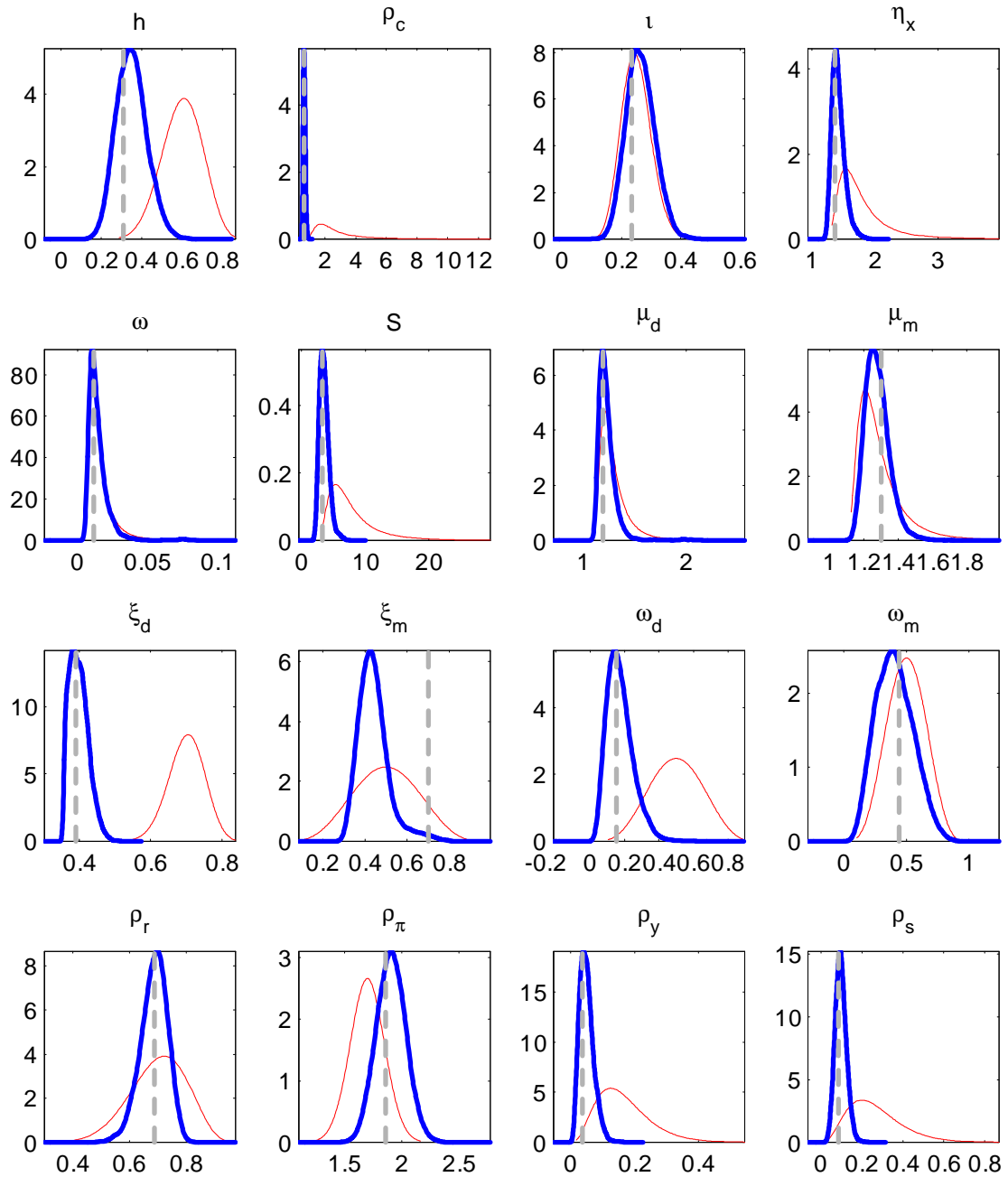


Figure 1: Prior and posterior (bold) densities. Dashed line is posterior mode.

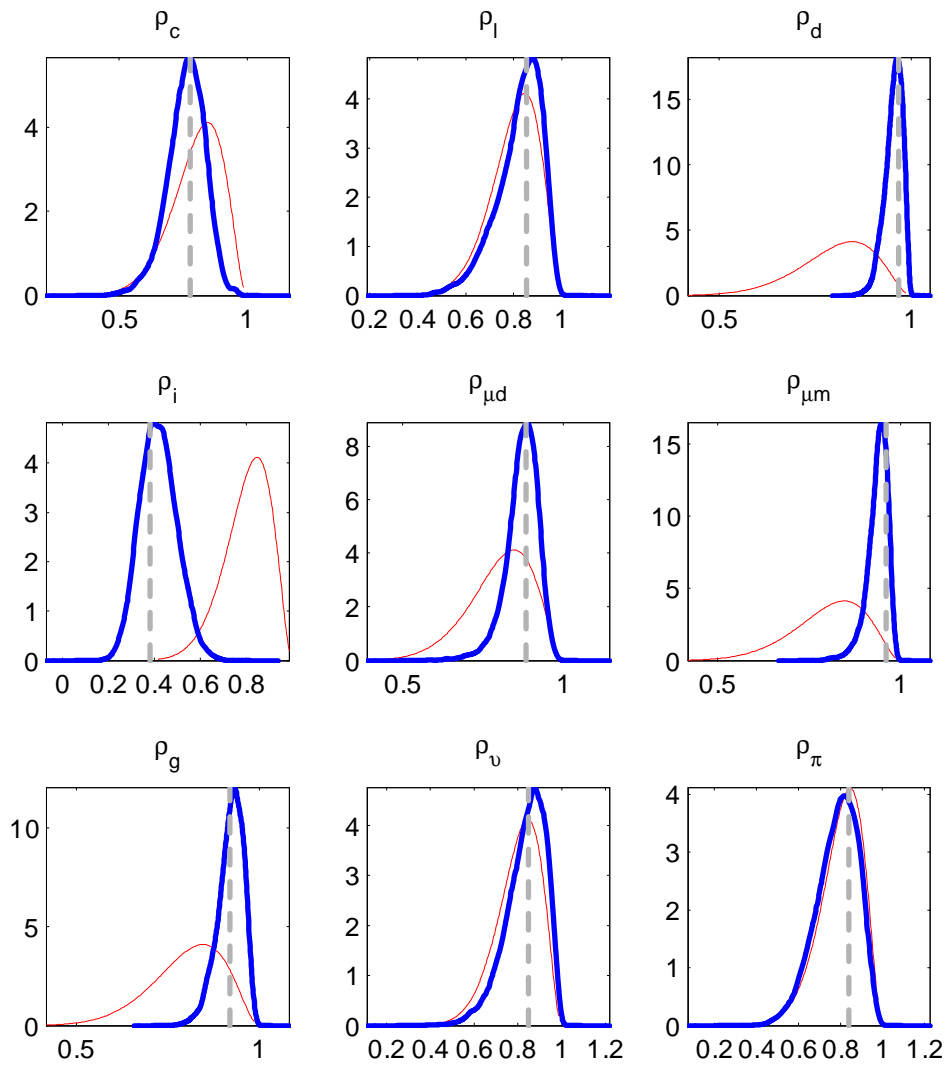


Figure 2: Prior and posterior (bold) densities. Dashed line is posterior mode.

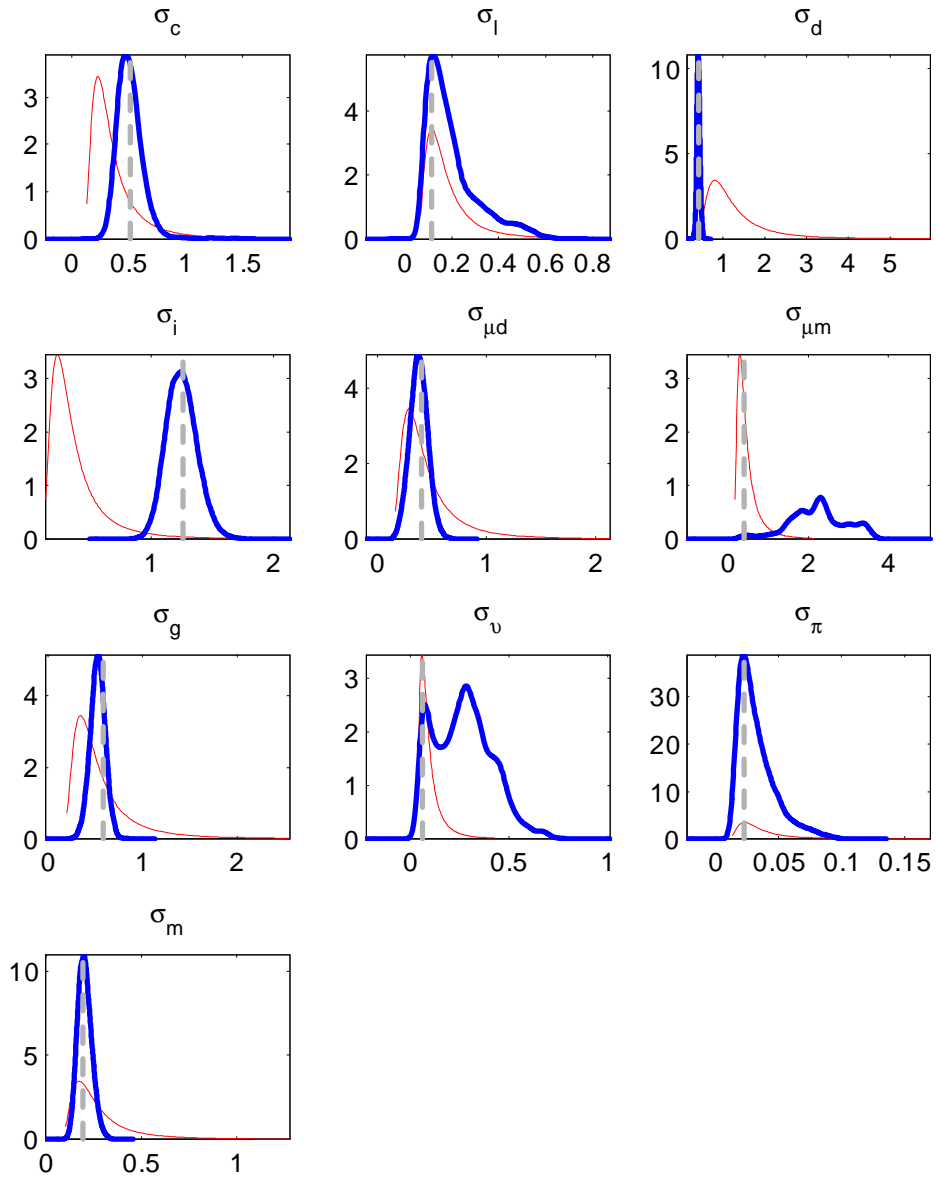


Figure 3: Prior and posterior (bold) densities. Dashed line is posterior mode.

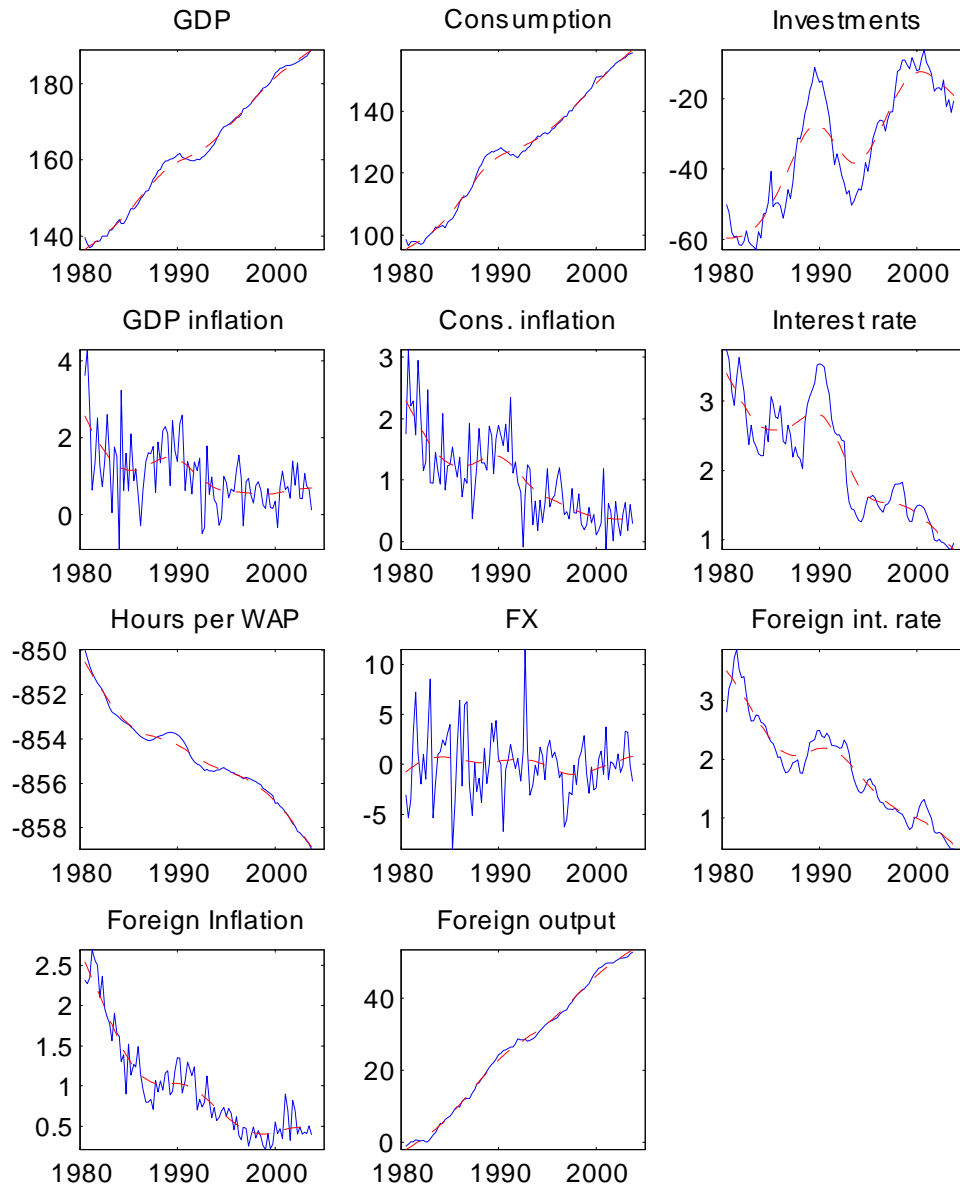


Figure 4: Original time series in logs (except interest rates and inflation) and HP-trend (dashed line)

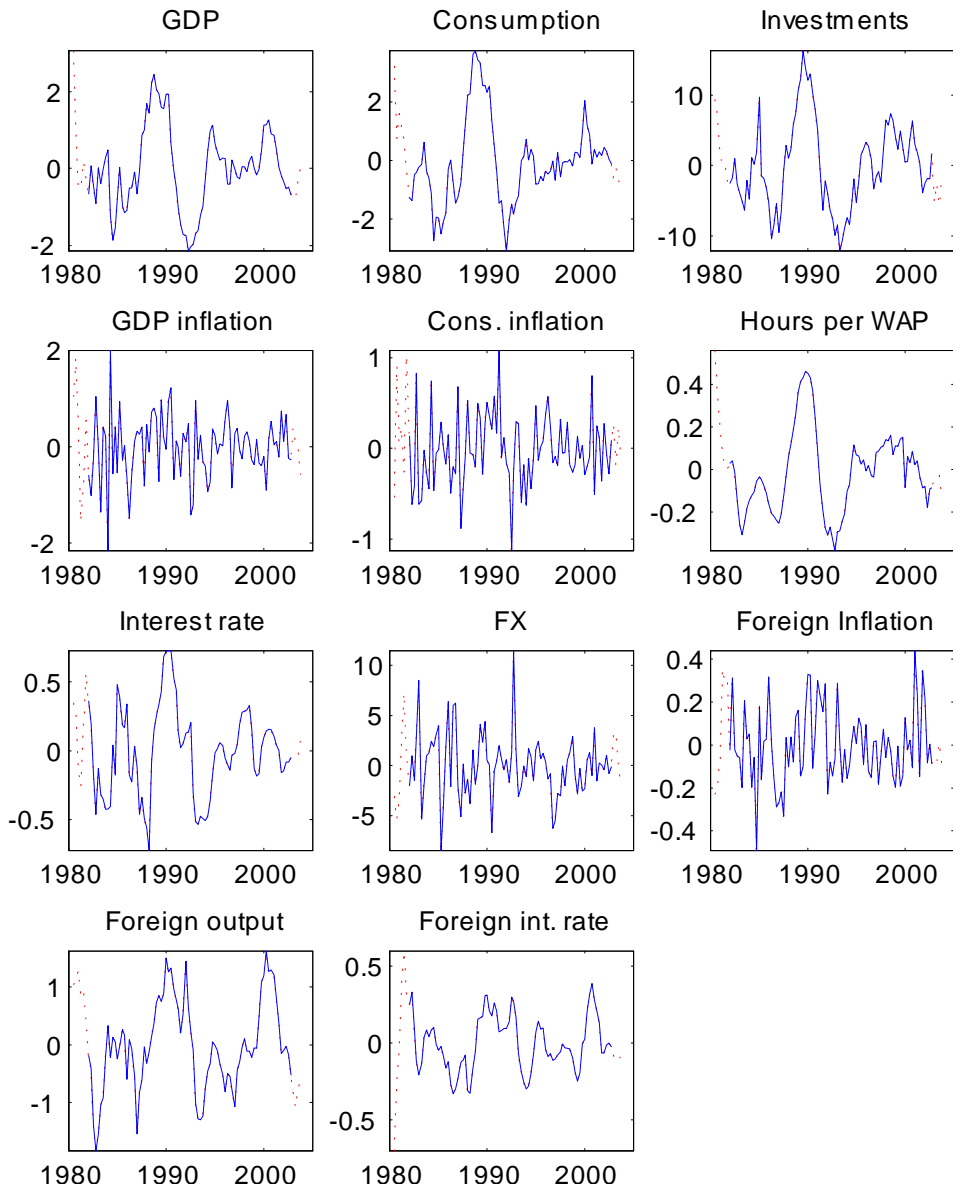


Figure 5: HP-filtered data set, incl. burnin and preliminary data (dotted sections)

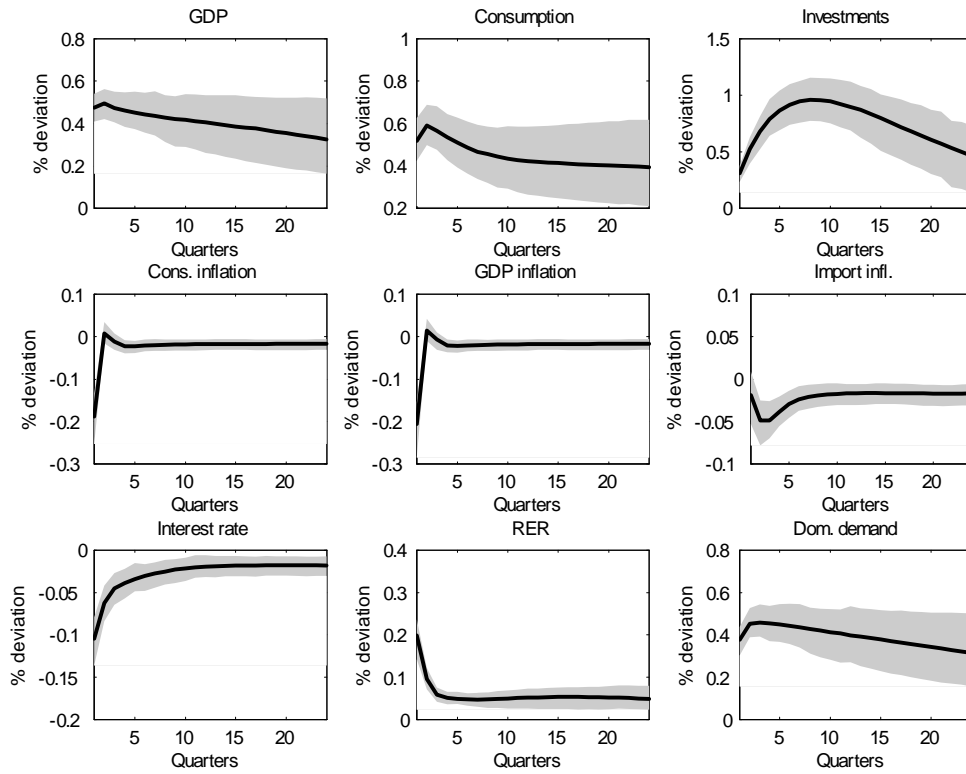


Figure 6: Median response to a technology shock of one standard deviation. Grey area signifies the 90 % HPD interval.

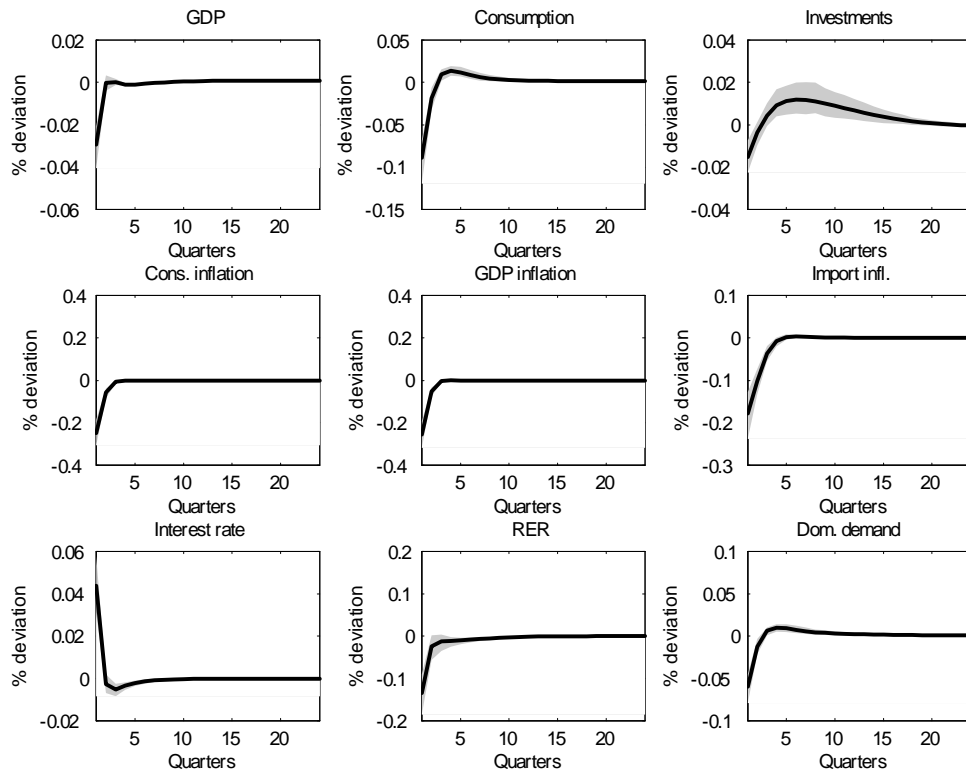


Figure 7: Median response to a monetary policy shock of one standard deviation. Grey area signifies the 90 % HPD interval.

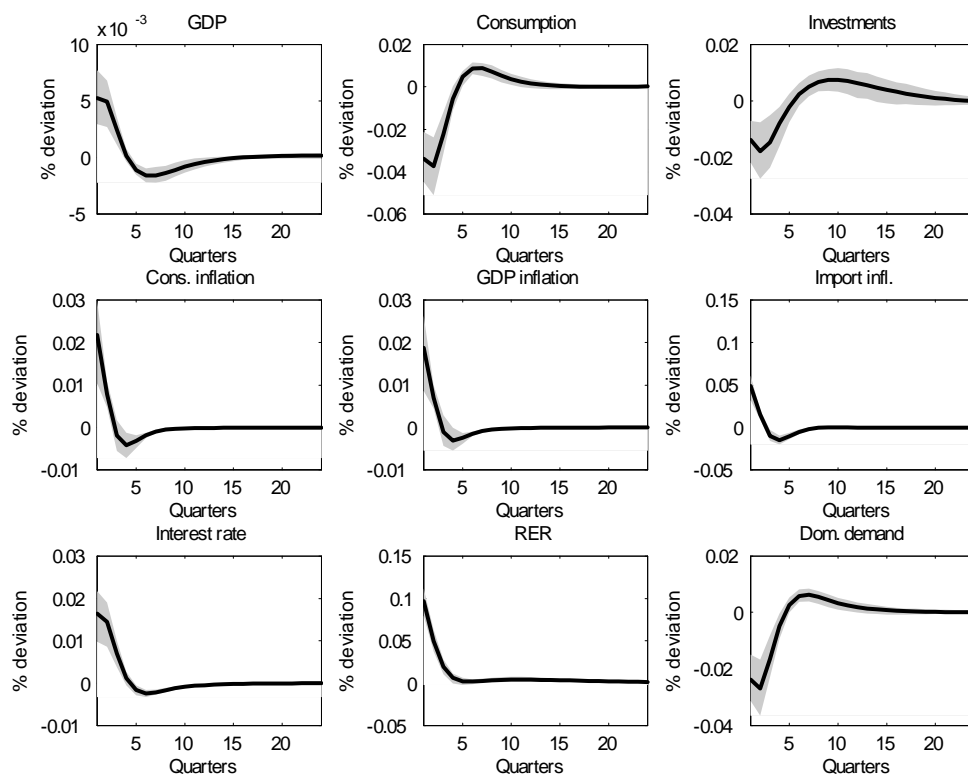


Figure 8: Median response to a foreign interest rate shock of one standard deviation. Grey area signifies the 90 % HPD interval.

Table 4: Variance Decomposition, unconditional, benchmark model

Shock	Preferences	Labour supply	Technology	Investment techn.	Markup, dom. goods	Markup, imp. goods	Govt. spending	up shock	Inflation target	Mon.pol. shock	Foreign inflation	Foreign output	Foreign int. rate	GDP	Consumption	Investments	Cons/dom. inflation	Interest rate	RER	
ε^v	0	0.014	0.028	0.002	0.02	0.04	0.003	0.051	0.101	0.162	0.058	0.109	0.162	0.079	0.174	0.26	0	0	0	0.001
ε^l	0	0.001	0.002	0	0.001	0.001	0	0	0.001	0	0	0.001	0.002	0	0.001	0.002	0	0	0	0
ε^d	0.476	0.701	0.918	0.195	0.471	0.768	0.091	0.192	0.339	0.324	0.107	0.222	0.324	0.102	0.246	0.38	0	0.002	0.003	0
ε^i	0.061	0.255	0.463	0.063	0.228	0.385	0.516	0.678	0.835	0.272	0.122	0.195	0.272	0.19	0.323	0.479	0	0.001	0.003	0
$\varepsilon^{\mu d}$	0	0.011	0.026	0	0.005	0.012	0	0.013	0.031	0.007	0.001	0.004	0.007	0.001	0.004	0.006	0	0	0	0
$\varepsilon^{\mu m}$	0	0.003	0.007	0.002	0.059	0.118	0.002	0.011	0.022	0.014	0.024	0.082	0.014	0.013	0.043	0.067	0.976	0.988	0.998	0
ε^w	0.003	0.013	0.024	0.044	0.202	0.376	0.013	0.049	0.082	0.149	0.039	0.088	0.149	0.036	0.099	0.162	0	0	0	0
ε^v	0	0.001	0.002	0	0.013	0.031	0	0.006	0.014	0.107	0	0.051	0.107	0	0.092	0.182	0	0.008	0.018	0
ε^π	0	0	0	0	0	0	0	0	0	0	0	0.009	0.019	0	0.005	0.011	0	0	0	0
ε^m	0	0	0	0	0.001	0.001	0	0	0	0.132	0.235	0.336	0.004	0.004	0.01	0.017	0	0	0	0
$\varepsilon^{x,\pi}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\varepsilon^{x,y}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\varepsilon^{x,i}$	0	0	0	0	0	0	0	0	0	0.002	0.004	0.001	0.003	0.005	0.005	0	0	0.002	0.002	0

Note: Variance decompositions based on the simulated posterior density: the posterior mean (bold) inside the 90 percent highest posterior density (HPD) interval.

Table 5: Variance Decomposition, unconditional, no import-price markup shock

Shock	Preferences	Labour supply	Technology	Investment techn.	Markup, dom. goods	Govt. spending	up shock	Inflation target	Mon.pol. shock	Foreign inflation	Foreign output	Foreign int. rate	GDP	Consumption	Investments	Cons/dom. inflation	Interest rate	RER		
ε^v	0	0.019	0.043	0.001	0.023	0.05	0.008	0.064	0.13	0.09	0.147	0.203	0.126	0.221	0.321	0.001	0.001	0.018	0.04	
ε^l	0	0	0.001	0	0	0.001	0	0	0	0	0.001	0.002	0	0.001	0.001	0	0	0	0	
ε^d	0.511	0.71	0.924	0.243	0.515	0.768	0.075	0.197	0.317	0.116	0.235	0.357	0.097	0.23	0.36	0.001	0.006	0.006	0.014	0
ε^i	0.083	0.244	0.45	0.095	0.246	0.421	0.525	0.664	0.839	0.147	0.228	0.311	0.214	0.353	0.496	0.005	0.058	0.126	0	0
$\varepsilon^{\mu d}$	0	0.009	0.018	0	0.005	0.009	0	0.011	0.022	0.002	0.004	0.007	0.001	0.003	0.005	0	0	0	0	0
ε^g	0.003	0.013	0.022	0.043	0.194	0.326	0.019	0.047	0.073	0.066	0.123	0.184	0.066	0.12	0.176	0	0.008	0.019	0	0
ε^v	0	0.004	0.008	0	0.016	0.034	0	0.016	0.033	0.009	0.031	0.06	0.018	0.061	0.103	0.717	0.865	0.987	0	0
ε^π	0	0	0	0	0	0	0	0	0	0	0.011	0.025	0	0.006	0.015	0	0.003	0.006	0	0
ε^m	0	0	0	0	0.001	0.001	0	0	0	0.113	0.22	0.33	0.002	0.005	0.009	0.001	0.037	0.083	0.001	0
$\varepsilon^{x,\pi}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\varepsilon^{x,y}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.001	0.002	0.003
$\varepsilon^{x,i}$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.001	0.002	0.002

Note: Variance decompositions based on the simulated posterior density: the posterior mean (bold) inside the 90 percent highest posterior density (HPD) interval.

Remarkably, neither the two shocks in the monetary policy function nor the three foreign processes have any impact on GDP whatsoever. The latter result is quite surprising at first hand; however, two factors are at play. First, the foreign variables in our data set are very smooth compared to those observed for the UK; consequently, the estimation has little foreign volatility to feed into the volatile domestic variables. Secondly, and more importantly, the estimation has essentially rendered the estimated model one of a virtually closed economy.

In this sense, the problem reflects the well-known *exchange-rate disconnect puzzle* in international macroeconomics (discussed, e.g., in the seminal Obstfeld and Rogoff (2000) contribution); that is, whereas the IRBC-based literature predicts a tight link between relative international prices and quantities, we observe that the real exchange rate experiences large movements with no apparent links to the real economy. In terms of our model, the tight link between the real exchange rate and the real domestic economy is apparent once one combines the Euler condition for household consumption growth with the UIP condition, cf. equations (C.1)-(C.2) in Appendix C. Thus, confronted with the restrictions of the model that ties the domestic economy to the foreign, and with the very volatile exchange rate in the data, the estimation responds with inflating the shock processes that relax these model restrictions, namely, the import markup shock and the UIP shock.

Given the nature of the problem, this outcome is not specific to either the UK data or the specific variant of the model analysed in this paper; in a detailed analysis on Canadian data Justiniano and Preston (2006) document precisely how the two shocks just mentioned are required in order to introduce the exchange disconnect that enables a plausible estimation. As they also note, similar effects are at play in the euro area estimation in Adolfson et al. (2005), as they are in the Danish estimation in Dam and Linaa (2005).

In our estimation, the markup shock to import inflation is the key disconnecting factor; it explains virtually all movement in the volatile real exchange rate. This led us to calculate a variance decomposition for the model variant without a markup shock in the import prices, cf. Table 5. The result is quite clearcut; the UIP shock takes over as the factor that moves the real exchange rate and thus retains the domestic economy insulated from the rest of the world. This leaves the volatility contributions to the UK aggregates unchanged from the benchmark model.

Turning to the domestic demand components, investments are - unsurprisingly - driven predominantly by the technology shocks (87%). In the case of consumption, the technology shocks contribute with 70% of the overall volatility while government spending accounts for 20%. Preference shifts, on the other hand, only make up 2% of consumption volatility, and even less for output and investments; in this estimation, their role seem to be dominated by the exogenous government spending process. As with GDP, the foreign variables have no impact on the domestic demand components, while the import markup shock has some impact (5%) on consumption.

The nominal variables are influenced by a larger set of shocks. Again, the technology shocks are important determinants, accounting for 42% of inflation and 57% of interest rate volatility. The preference shock, which played only a minor role for the real aggregates, has a marked impact on the nominal variables (11 and 17% on inflation and the interest rate, respectively), while government spending and the UIP shock also contribute significantly. Finally, the transitory monetary policy error accounts for almost a quarter of the inflation volatility while its contribution to the interest rate itself is miniscule.

Lastly, we note that the domestic markup shock plays virtually no role in the estimated model which corresponds to the euro area findings of Adolfson et al. (2005), but is in stark contrast to the Dam and Linaa (2005) study on Danish data.

8 Conclusion

In this paper, we estimated a DSGE model of the UK economy on quarterly data. We included a rich set of structural and ad-hoc shocks as well as a (reduced-form) foreign economy in order to capture the key determinants of the UK business cycle. According to our estimates, technology shocks were the main determinants of real fluctuations in the UK economy.

Overall, we found plausible estimates of the parameters characterising the domestic economy. However, our estimation did point towards two areas where the current empirical DSGE literature needs to be reconsidered. The first is specific to the UK economy and regards the inflation dynamics. As we describe above, these were rather poorly accounted for by the model. This reflects that the (detrended) time series for UK inflation in our data set are considerably less persistent than their US and euro area counterparts. As much of the workhorse price model of the recent medium-scale DSGE literature has been constructed in a US context, where the main challenge was to account for the high persistence of observed inflation, it is not well suited as a model for UK inflation. Further research into the causes and determinants of the large idiosyncratic component in UK inflation is clearly called for.

Secondly, our estimation yields no role for the foreign economy amongst the determinants of domestic volatility. This result is common with other recent NOEM estimations, including Adolfson et al. (2005), Dam and Linaa (2005), and Justiniano and Preston (2006). As the latter stress, the result reflects that the incarnation of the NOEM models that have been estimated so far do not endogenously account for the large swings in the real exchange rate. This causes the estimation to introduce an exchange-rate disconnect via the included ad-hoc shocks in the UIP and the import price equations.

Consequently, future research into more fruitful ways of linking the dynamic behaviour of the real exchange rate and the terms of trade to the movements of relative outputs is necessary. A promising candidate is to introduce a distinction between traded and nontraded goods. Thus, Benigno and Thoenissen (2004) claim to more or less account for the observed real exchange-rate volatility when they calibrate such a setup. Likewise, Corsetti et al. (2004) obtain remarkable results in a calibrated model where non-traded goods are required for distributing traded goods. It remains to be seen whether such features can reconnect the real exchange rate in a full estimation of an open-economy DSGE model.

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A Model Derivation

A.1 Lagrangian Function for Household Problem

$$\begin{aligned}
\mathcal{L}_t &\equiv E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \zeta_{\tau}^c \left[\frac{(C_{\tau} - h\bar{C}_{\tau-1})^{1-\rho_c}}{1-\rho_c} - \zeta_{\tau}^l \frac{H_{\tau}^{1+\rho_l}}{1+\rho_l} \right] \\
&\quad + \beta^{\tau-t} \tilde{\lambda}_{\tau} \left(B_{\tau-1} + S_{\tau} B_{\tau-1}^* + W_{\tau} H_{\tau} + R_{\tau}^k K_{\tau} + Div_{\tau} - \frac{B_{\tau}}{(1+R_{\tau})} - \frac{S_{\tau} B_{\tau}^*}{\Omega_{\tau}(1+R_{\tau}^*)} - P_{\tau} (C_{\tau} + I_{\tau}) \right) \\
&\quad + \beta^{\tau-t} \tilde{\lambda}_{\tau} P_{\tau} \chi_{\tau} \left((1-\delta) K_{\tau} + \theta_{\tau}^i (1-S(I_{\tau}/I_{\tau-1})) I_{\tau} - K_{\tau+1} \right) \\
&= E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \zeta_{\tau}^c \left[\frac{(C_{\tau} - hC_{\tau-1})^{1-\rho_c}}{1-\rho_c} - \zeta_{\tau}^l \frac{H_{\tau}^{1+\rho_l}}{1+\rho_l} \right] \\
&\quad + \beta^{\tau-t} \lambda_{\tau} \left(\frac{b_{\tau-1}}{\Pi_t} + \frac{S_{\tau}}{S_{\tau-1}} \frac{b_{\tau-1}^*}{\Pi_t} + w_{\tau} H_{\tau} + r_{\tau}^k K_{\tau} + \frac{Div_{\tau}}{P_{\tau}} - \frac{b_{\tau}}{(1+R_{\tau})} - \frac{b_{\tau}^*}{\Omega_{\tau}(1+R_{\tau}^*)} - C_{\tau} - I_{\tau} \right) \\
&\quad + \beta^{\tau-t} \lambda_{\tau} \chi_{\tau} \left[(1-\delta) K_{\tau} + \theta_{\tau}^i (1-S(I_{\tau}/I_{\tau-1})) I_{\tau} - K_{\tau+1} \right], \\
\lambda_{\tau} &\equiv \tilde{\lambda}_{\tau} P_{\tau}, \quad b_t \equiv B_t/P_t, \quad b_t^* \equiv S_t B_t^*/P_t.
\end{aligned}$$

A.2 Household First-Order Conditions:

$$\lambda_t = \zeta_t^c (C_{\tau} - hC_{\tau-1})^{-\rho_c}, \quad (\text{A.1})$$

$$\lambda_t = \beta (1+R_t) E_t [\lambda_{t+1}/\Pi_{t+1}], \quad (\text{A.2})$$

$$\lambda_t = \beta E_t \frac{S_{t+1}}{S_t} \Omega_t (1+R_t^*) \frac{\lambda_{t+1}}{\Pi_{t+1}}, \quad (\text{A.3})$$

$$\lambda_t w_t = \zeta_{\tau}^c \zeta_{\tau}^l H_t^{\rho_l}, \quad (\text{A.4})$$

$$\chi_t = \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \left((1-\delta) \chi_{t+1} + r_{t+1}^k \right) \right], \quad (\text{A.5})$$

$$\begin{aligned}
1 &= \chi_t \theta_t^i \left(1 - S \left(\frac{I_t}{I_{t-1}} \right) - S' \left(\frac{I_t}{I_{t-1}} \right) \frac{I_t}{I_{t-1}} \right) \\
&\quad + \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \chi_{t+1} \theta_{t+1}^i S' \left(\frac{I_{t+1}}{I_t} \right) \left(\frac{I_{t+1}}{I_t} \right)^2 \right].
\end{aligned} \quad (\text{A.6})$$

A.3 Aggregates and Relative Demand Functions

The final-good aggregate, import and export;

$$\begin{aligned}
Y_t^{D,D} &= (1-\iota) (P_t^D/P_t)^{-\eta} Y_t, \\
Y_t^M &= \iota (P_t^M/P_t)^{-\eta} Y_t, \\
P_t^{1-\eta} &= (1-\iota) (P_t^D)^{1-\eta} + \iota (P_t^M)^{1-\eta}, \\
Y_t^X &= \left(\frac{P_t^D}{S_t P_t^*} \right)^{-\eta_x} Y_t^*
\end{aligned}$$

A.4 Intermediate Firms

The profit maximization problem is

$$\max_{\tilde{P}_t^D} E_t \sum_{\tau=t}^{\infty} (\xi_d \beta)^{\tau-t} \lambda_{\tau} \pi_{\tau}^D(f),$$

where $\pi_t^D(f) = \left(\tilde{P}_t^D(f) - MC_t(f) \right) Y_{\tau}^D(f) - MC_t \Phi$.

Since the optimal price is identical across firms, we style the optimal price \tilde{P}_t^D without the subscript f , so the problem becomes

$$\max_{\tilde{P}_t^D} \sum_{\tau=t}^{\infty} (\xi_d \beta)^{\tau-t} E_t \left[\lambda_{\tau} \left(\tilde{P}_t^D \left(\frac{P_{\tau-1}^D}{P_{t-1}^D} \right)^{\omega_d} - MC_{\tau} \right) Y_{\tau}^D(f) - MC_{\tau} \Phi \right]. \quad (\text{A.7})$$

Now, consider the demand facing firm f in period $\tau \geq t$ when the firm last reoptimised in period t ;

$$Y_{\tau}^D(f) = \left(\frac{P_{\tau}^D(f)}{P_{\tau}^D} \right)^{-\frac{\mu_{\tau}^d}{\mu_{\tau}^d - 1}} Y_{\tau}^D = \left(\frac{\tilde{P}_t^D}{P_{\tau}^D} \left(\frac{P_{\tau-1}^D}{P_{t-1}^D} \right)^{\omega_d} \right)^{-\frac{\mu_{\tau}^d}{\mu_{\tau}^d - 1}} Y_{\tau}^D \Rightarrow$$

$$\frac{\partial Y_{\tau}^D(f)}{\partial \tilde{P}_t^D} = -\frac{\mu_t^d}{\mu_t^d - 1} \frac{Y_{\tau}^D(f)}{\tilde{P}_t^D}.$$

It follows that the FOC for the profit maximisation problem (A.7) is

$$\sum_{\tau=t}^{\infty} (\xi_d \beta)^{\tau-t} E_t \left[\frac{\tilde{\lambda}_{\tau} Y_{\tau}^D(f)}{\mu_{\tau}^d - 1} \left(\left(\frac{P_{\tau-1}^D}{P_{t-1}^D} \right)^{\omega_d} \tilde{P}_t^D - \mu_{\tau}^d MC_{\tau} \right) \right] = 0 \iff$$

$$\sum_{\tau=t}^{\infty} (\xi_d \beta)^{\tau-t} E_t \left[\frac{p_{\tau}^D \lambda_{\tau} Y_{\tau}^D(f)}{\mu_{\tau}^d - 1} \left(\frac{(P_{\tau-1}^D / P_{t-1}^D)^{\omega_d}}{P_{\tau}^D / P_t^D} \tilde{p}_t^D - \mu_{\tau}^d mc_{\tau} \right) \right] = 0, \quad (\text{A.8})$$

where

$$\lambda_{\tau} = \tilde{\lambda}_{\tau} P_t, \quad \tilde{p}_t^D \equiv \tilde{P}_t^D / P_t^D, \quad mc_t \equiv MC_t / P_t^D.$$

Log-linearisation of the FOC (A.8) yields

$$\sum_{\tau=t}^{\infty} (\xi_d \beta)^{\tau-t} E_t \left[\hat{\tilde{p}}_t^D + \omega_d \left(\hat{P}_{\tau-1}^D - \hat{P}_{t-1}^D \right) - \left(\hat{P}_{\tau}^D - \hat{P}_t^D \right) - \left(\hat{\mu}_{\tau}^d + \hat{m}c_{\tau} \right) \right] = 0 \Rightarrow$$

$$\hat{\tilde{p}}_t^D = (1 - \xi_d \beta) \sum_{\tau=t}^{\infty} (\xi_d \beta)^{\tau-t} E_t \left[\left(\hat{P}_{\tau}^D - \hat{P}_t^D \right) - \omega_d \left(\hat{P}_{\tau-1}^D - \hat{P}_{t-1}^D \right) + \left(\hat{\mu}_{\tau}^d + \hat{m}c_{\tau} \right) \right] \quad (\text{A.9})$$

Equation (2.3) implies

$$1 = \xi_d \left((\Pi_{t-1}^D)^{\omega_d} / \Pi_t^D \right)^{\frac{1}{1-\mu_t^d}} + (1 - \xi_d) \left(\tilde{p}_t^D \right)^{\frac{1}{1-\mu_t^d}}.$$

Log-linearize this and combine with (A.9) to obtain

$$\begin{aligned}
\hat{\pi}_t^D - \omega_d \hat{\pi}_{t-1}^D &= \frac{1 - \xi_d \widehat{p}_t^D}{\xi_d} \\
&= \frac{1 - \xi_d}{\xi_d} (1 - \xi_d \beta) \sum_{\tau=t}^{\infty} (\xi_d \beta)^{\tau-t} E_t \left[\left(\hat{P}_\tau^D - \hat{P}_t^D \right) - \omega_d \left(\hat{P}_{\tau-1}^D - \hat{P}_{t-1}^D \right) + \left(\hat{\mu}_\tau^d + \widehat{m}c_\tau \right) \right] \\
&= \frac{1 - \xi_d}{\xi_d} \left((1 - \xi_d \beta) \left[\omega_d \left(\hat{\pi}_t^D \right) - E_t \hat{\pi}_{t+1}^D + \left(\hat{\mu}_t^d + \widehat{m}c_t^D \right) \right] + E_t \hat{\pi}_{t+1}^D - \omega_d \hat{\pi}_t^D \right) + \xi_d \beta E_t \left[\hat{\pi}_{t+1}^D - \omega_d \hat{\pi}_t^D \right],
\end{aligned}$$

and thus

$$\begin{aligned}
\hat{\pi}_t^D &= \omega_d \hat{\pi}_{t-1}^D + \beta E_t \hat{\pi}_{t+1}^D - \omega_d \beta \hat{\pi}_t^D + \frac{1 - \xi_d}{\xi_d} (1 - \xi_d \beta) \left(\hat{\mu}_t^d + \widehat{m}c_t \right) \\
&= \frac{\omega_d}{1 + \beta \omega_d} \hat{\pi}_{t-1}^D + \frac{\beta}{1 + \beta \omega_d} E_t \hat{\pi}_{t+1}^D + \frac{1 - \xi_d}{\xi_d} \frac{1 - \xi_d \beta}{1 + \beta \omega_d} \left(\hat{\mu}_t^d + \widehat{m}c_t \right).
\end{aligned}$$

The problem and equations of the importing firms is analogous to the case of the domestic firms just presented, why we exclude it here.

A.5 Summary of Stochastic Shock Processes

Preference shocks;

$$\begin{aligned}
\hat{\zeta}_t^c &= \varrho_c \hat{\zeta}_{t-1}^c + \varepsilon_t^c, \\
\hat{\zeta}_t^l &= \varrho_l \hat{\zeta}_{t-1}^l + \varepsilon_t^l.
\end{aligned}$$

Technology shocks;

$$\begin{aligned}
\hat{\theta}_t^D &= \varrho_d \hat{\theta}_{t-1}^D + \varepsilon_t^d, \\
\hat{\theta}_t^i &= \varrho_d \hat{\theta}_{t-1}^i + \varepsilon_t^i.
\end{aligned}$$

Mark-up shocks;

$$\begin{aligned}
\mu_t^d &= \varrho_{\mu d} \mu_{t-1}^d + (1 - \varrho_{\mu d}) \mu_d + \varepsilon_t^{\mu d}, \\
\mu_t^m &= \varrho_{\mu d} \mu_{t-1}^m + (1 - \varrho_{\mu d}) \mu_m + \varepsilon_t^{\mu m}.
\end{aligned}$$

UIP shock

$$\hat{v}_t = \varrho_v \hat{v}_{t-1} + \varepsilon_t^v.$$

Policy and expenditure shocks;

$$\begin{aligned}
g_t &\equiv G_t/A_t = \varrho_g g_{t-1} + (1 - \varrho_g) \bar{g} + \varepsilon_t^g, \\
\bar{\Pi}_t &= \varrho_\pi \bar{\Pi}_{t-1} + (1 - \varrho_\pi) \bar{\Pi} + \varepsilon_t^\pi,
\end{aligned}$$

and an i.i.d. monetary policy error ε_t^m . Thus, the vector of innovations is

$$\varepsilon_t = \left(\varepsilon_t^c, \varepsilon_t^l, \varepsilon_t^{at}, \varepsilon_t^d, \varepsilon_t^n, \varepsilon_t^i, \varepsilon_t^{\mu d}, \varepsilon_t^{\mu m}, \varepsilon_t^{\mu m}, \varepsilon_t^g, \varepsilon_t^\pi, \varepsilon_t^v, \varepsilon_t^m \right)'$$

B Deriving the Steady State

First note that import prices in the steady state is given as a markup over marginal cost

$$p^M = \mu_m Q$$

Now use the definition of the aggregate price index:

$$\begin{aligned} 1 &= (1 - \iota) (p^D)^{1-\eta_m} + \iota (p^M)^{1-\eta_m} \Rightarrow \\ \left(\frac{1}{p^D}\right) &= \left((1 - \iota) + \iota \left(\frac{p^M}{p^D}\right)^{1-\eta_m} \right)^{\frac{1}{1-\eta_m}} \end{aligned}$$

We can also follow Adolfson et al. (2005) and define

$$\begin{aligned} \left(\frac{1}{p^M}\right)^{1-\eta_m} &= (1 - \iota) \left(\frac{p^D}{p^M}\right)^{1-\eta_m} + \iota \Rightarrow \\ \left(\frac{1}{p^M}\right) &= \left((1 - \iota) \left(\frac{p^D}{p^M}\right)^{1-\eta_m} + \iota \right)^{\frac{1}{1-\eta_m}}. \end{aligned}$$

These can be rewritten using the fact that $p^M = \mu_m Q$;

$$\begin{aligned} \left(\frac{1}{p^M}\right) &= \left((1 - \iota) \left(\frac{p^D}{\mu_m Q}\right)^{1-\eta_m} + \iota \right)^{\frac{1}{1-\eta_m}} \\ &= \left((1 - \iota) (\mu_m)^{\eta_m-1} \left(\frac{p^D}{Q}\right)^{1-\eta_m} + \iota \right)^{\frac{1}{1-\eta_m}}. \end{aligned}$$

We can also rewrite the expression for $1/p^D$ using the fact that $p^M = \mu_m Q$;

$$\begin{aligned} \frac{1}{p^D} &= \left((1 - \iota) + \iota \left(\frac{p^M}{p^D}\right)^{1-\eta_m} \right)^{\frac{1}{1-\eta_m}} = \left((1 - \iota) + \iota \left(\frac{\mu_m Q}{p^D}\right)^{1-\eta_m} \right)^{\frac{1}{1-\eta_m}} \\ &= \left((1 - \iota) + \iota (\mu_m)^{1-\eta_m} \left(\frac{Q}{p^D}\right)^{1-\eta_m} \right)^{\frac{1}{1-\eta_m}}. \end{aligned}$$

We assume that domestic output inflation and foreign inflation grow at the same rate; $\Pi^D = \Pi^*$. Following similar arguments as in Adolfson et al. (2005) it must be that $S = 1$ and initial prices are equal at home and abroad. Then we have that $p^D/Q = 1$.

Using this result, we get

$$\begin{aligned} \left(\frac{1}{p^D}\right)^{(1-\eta_m)} &= ((1 - \iota) + \iota (\mu_m)^{1-\eta_m}), \\ \left(\frac{1}{p^M}\right)^{(1-\eta_m)} &= ((1 - \iota) (\mu_m)^{\eta_m-1} + \iota). \end{aligned}$$

Combine the two above equations and get:

$$\begin{aligned} \left(\frac{P^M}{P^D}\right)^{(1-\eta_m)} &= \frac{((1-\iota) + \iota(\mu_m)^{1-\eta_m})}{((1-\iota)(\mu_m)^{\eta_m-1} + \iota)} \Rightarrow \text{reduce to get} \\ \left(\frac{P^M}{P^D}\right)^{(1-\eta_m)} &= \frac{(\mu_m)^{1-\eta_m} ((1-\iota)(\mu_m)^{\eta_m-1} + \iota)}{((1-\iota)(\mu_m)^{\eta_m-1} + \iota)} = (\mu_m)^{1-\eta_m} \\ \left(\frac{p^M}{p^D}\right) &= \mu_m \end{aligned}$$

Now, use the definition of the aggregate price index for tradables;

$$\begin{aligned} 1 &= (1-\iota)(p^D)^{1-\eta_m} + \iota(p^M)^{1-\eta_m} \Rightarrow \\ \left(\frac{1}{p^D}\right) &= \left((1-\iota) + \iota\left(\frac{p^M}{p^D}\right)^{1-\eta_m}\right)^{\frac{1}{1-\eta_m}} \Rightarrow \\ p^D &= ((1-\iota) + \iota(\mu_m)^{1-\eta_m})^{\frac{-1}{1-\eta_m}} \end{aligned}$$

Consider next the marginal cost condition;

$$\begin{aligned} 1 &= \mu_D mc_t^D \Rightarrow \\ \mu_D [p^D \alpha^\alpha (1-\alpha)^{1-\alpha}]^{-1} w^{1-\alpha} (r^k)^\alpha &= 1. \end{aligned}$$

Use the consumer FOCs

$$\begin{aligned} \frac{1}{\beta} &= (1-\delta) + r^k \Rightarrow \\ r^k &= \frac{1}{\beta} - (1-\delta) = \frac{1-\beta(1-\delta)}{\beta} \end{aligned}$$

Combine the equations for r^k and p^D to solve for wages w :

$$\begin{aligned} \mu_D [p^D \alpha^\alpha (1-\alpha)^{1-\alpha}]^{-1} w^{1-\alpha} (r^k)^\alpha &= 1 \\ w &= \left((\mu_D)^{-1} \left[((1-\iota) + \iota(\mu_m)^{1-\eta_m})^{\frac{-1}{1-\eta_m}} \alpha^\alpha (1-\alpha)^{1-\alpha} \right] \left(\frac{1-\beta(1-\delta)}{\beta} \right)^{-\alpha} \right)^{\frac{1}{1-\alpha}} \end{aligned}$$

We have the result that trade is balanced in the steady state:

$$\begin{aligned} p^D y^X - Q y^M &= 0 \Rightarrow \\ p^D y^X &= Q y^M \\ \frac{y^X}{y^M} &= \frac{Q}{p^D} = 1 \Rightarrow \\ y^X &= y^M = y^* \end{aligned}$$

Go back to the bundler's demand functions and consider the following

$$\begin{aligned}\frac{y^{D,D}}{y^M} &= \frac{1-\iota}{\iota} \left(\frac{p^D}{p^M}\right)^{-\eta_m} \frac{1-\iota}{\iota} (\mu_m)^{\eta_m} \\ y^{D,D} &= \frac{1-\iota}{\iota} (\mu_m)^{\eta_m} y^M = \frac{1-\iota}{\iota} (\mu_m)^{\eta_m} y^*\end{aligned}$$

Use the goods market equilibrium:

$$\begin{aligned}y^D &= y^{D,D} + y^X = \frac{1-\iota}{\iota} (\mu_m)^{\eta_m} y^* + y^* \\ &= \left(\frac{1-\iota}{\iota} (\mu_m)^{\eta_m} + 1\right) y^*.\end{aligned}$$

Hence,

$$\frac{y^{D,D}}{y^D} = \left[\frac{1-\iota}{\iota} (\mu_m)^{\eta_m} + 1\right]^{-1}.$$

Having solved for w and r^k , we can calculate the capital labor ratio using the following equation:

$$\begin{aligned}\frac{k}{H} &= \frac{\alpha}{(1-\alpha)} \frac{w}{r^k} \\ &= \frac{\alpha}{(1-\alpha)} \left((\mu_D)^{-1} \left[((1-\iota) + \iota (\mu_m)^{1-\eta_m})^{\frac{-1}{1-\eta_m}} \alpha^\alpha (1-\alpha)^{1-\alpha} \right] \left(\frac{1-\beta(1-\delta)}{\beta} \right)^{-1} \right)^{\frac{1}{1-\alpha}} \\ &= \alpha^{\frac{1}{1-\alpha}} \left((\mu_D)^{-1} \left[((1-\iota) + \iota (\mu_m)^{1-\eta_m})^{\frac{-1}{1-\eta_m}} \right] \left(\frac{1-\beta(1-\delta)}{\beta} \right)^{-1} \right)^{\frac{1}{1-\alpha}}.\end{aligned}$$

Now use the definition of the production function;

$$y^D = \left(\frac{k}{H}\right)^\alpha H - \Phi^D.$$

The fixed costs in the tradable sector, Φ^D , are given as

$$\begin{aligned}\Phi^D &= (\mu_d - 1) y^D = (\mu_d - 1) \left(\left(\frac{k}{H}\right)^\alpha H - \Phi^D \right) \\ &= \frac{(\mu_d - 1)}{\mu_d} \left(\frac{k}{H}\right)^\alpha H.\end{aligned}$$

The production function can therefore be written as

$$\begin{aligned} y^D &= \left(\frac{k}{H}\right)^\alpha H - \Phi^D = \frac{1}{\mu_d} \left(\frac{k}{H}\right)^\alpha H \\ &= \alpha^{\frac{\alpha}{1-\alpha}} \left((\mu_D)^{-1} \left[((1-\iota) + \iota(\mu_m)^{1-\eta_m})^{\frac{-1}{1-\eta_m}} \right] \left(\frac{1-\beta(1-\delta)}{\beta} \right)^{-1} \right)^{\frac{\alpha}{1-\alpha}} H. \end{aligned}$$

Recall that we have already solved for y^D . Therefore we can solve for H as a function of exogenous parameters including y^* ;

$$\begin{aligned} \left(\frac{1-\iota}{\iota}(\mu_m)^{\eta_m} + 1\right) y^* &= \alpha^{\frac{\alpha}{1-\alpha}} \left((\mu_D)^{-1} \left[((1-\iota) + \iota(\mu_m)^{1-\eta_m})^{\frac{-1}{1-\eta_m}} \right] \left(\frac{1-\beta(1-\delta)}{\beta} \right)^{-1} \right)^{\frac{\alpha}{1-\alpha}} H \Rightarrow \\ H &= \left[\alpha^{\frac{-\alpha}{1-\alpha}} \left((\mu_D)^{-1} \left[((1-\iota) + \iota(\mu_m)^{1-\eta_m})^{\frac{-1}{1-\eta_m}} \right] \left(\frac{1-\beta(1-\delta)}{\beta} \right)^{-1} \right)^{\frac{-\alpha}{1-\alpha}} \right. \\ &\quad \left. \times \left(\frac{1-\iota}{\iota}(\mu_m)^{\eta_m} + 1 \right) y^* \right]. \end{aligned}$$

Having solved for H , the rest is trivial. We can solve for k as follows:

$$\begin{aligned} \frac{k}{H} &= \alpha^{\frac{1}{1-\alpha}} \left((\mu_D)^{-1} \left[((1-\iota) + \iota(\mu_m)^{1-\eta_m})^{\frac{-1}{1-\eta_m}} \right] \left(\frac{1-\beta(1-\delta)}{\beta} \right)^{-1} \right)^{\frac{1}{1-\alpha}} \Rightarrow \\ k &= \alpha^{\frac{1}{1-\alpha}} \left((\mu_D)^{-1} \left[((1-\iota) + \iota(\mu_m)^{1-\eta_m})^{\frac{-1}{1-\eta_m}} \right] \left(\frac{1-\beta(1-\delta)}{\beta} \right)^{-1} \right)^{\frac{1}{1-\alpha}} H \\ &= \left[\alpha \left((\mu_D)^{-1} \left[((1-\iota) + \iota(\mu_m)^{1-\eta_m})^{\frac{-1}{1-\eta_m}} \right] \left(\frac{1-\beta(1-\delta)}{\beta} \right)^{-1} \right) \right. \\ &\quad \left. \times \left(\frac{1-\iota}{\iota}(\mu_m)^{\eta_m} + 1 \right) y^* \right]. \end{aligned}$$

Use the following to solve for i ;

$$\frac{i}{k} = \delta \Rightarrow i = \delta \left[\alpha \left((\mu_D)^{-1} \left[((1-\iota) + \iota(\mu_m)^{1-\eta_m})^{\frac{-1}{1-\eta_m}} \right] \left(\frac{1-\beta(1-\delta)}{\beta} \right)^{-1} \right) \right. \\ \left. \times \left(\frac{1-\iota}{\iota}(\mu_m)^{\eta_m} + 1 \right) y^* \right].$$

We can use the following two equations to solve for λ and c ;

$$\begin{aligned} c(1-h)^{-\rho_c} &= \lambda, \\ w &= H^{\rho_l} / \lambda. \end{aligned}$$

Finally, we can solve for y using the following equation:

$$y = c + g + i.$$

C Log-linearised model

Throughout, hats will signify log deviations from the stationarised steady state, while tildes signify structural shocks that have been rescaled for interpretative reasons as in Smets and Wouters (2003) and Adolfson et al. (2005).

Log-linearising the household's first-order conditions (A.1)-(A.6) and elimination of λ_t and Ω_t yields

$$\begin{aligned}\hat{C}_t &= \frac{h}{1+h}\hat{C}_{t-1} + \frac{1}{1+h}E_t - \frac{1-h}{(1+h)\rho_c}E_t \left[\hat{R}_t - \hat{\pi}_{t+1} + \hat{\zeta}_{t+1}^c - \hat{\zeta}_t^c \right] \\ &= \frac{h}{1+h}\hat{C}_{t-1} + \frac{1}{1+h}E_t\hat{C}_{t+1} - \frac{1-h}{(1+h)\rho_c}E_t \left[\hat{R}_t - \hat{\pi}_{t+1} \right] - \left(E_t\widetilde{\hat{\zeta}_{t+1}^c} - \widetilde{\hat{\zeta}_t^c} \right)\end{aligned}\quad (\text{C.1})$$

$$\hat{R}_t - \hat{R}_t^* = E_t\Delta S_{t+1} - \omega\hat{b}_t^* + v_t, \quad (\text{C.2})$$

$$\hat{w}_t = \hat{\zeta}_t^l + \rho_l\hat{H}_t + \frac{\rho_c}{\gamma-h} \left(\gamma\hat{C}_t - h\hat{C}_{t-1} \right), \quad (\text{C.3})$$

$$\hat{\chi}_t = E_t \left[\beta\frac{1-\delta}{\gamma}\hat{\chi}_{t+1} + \frac{1-\beta(1-\delta)}{1}\hat{r}_{t+1}^k - \left(\hat{R}_t - \hat{\pi}_{t+1} \right) \right], \quad (\text{C.4})$$

$$\begin{aligned}\hat{i}_t &= \frac{\hat{\chi}_t + \hat{\theta}_t^i}{(1+\beta)S''(1)} + \frac{\hat{i}_{t-1}}{1+\beta} + \frac{\beta}{1+\beta}E_t\hat{i}_{t+1} \\ &= \frac{\hat{\chi}_t}{(1+\beta)S''(1)} + \frac{\hat{i}_{t-1}}{1+\beta} + \frac{\beta}{1+\beta}E_t\hat{i}_{t+1} + \widetilde{\hat{\theta}_t^i}.\end{aligned}\quad (\text{C.5})$$

Capital accumulation

$$\begin{aligned}\hat{K}_{t+1} &= (1-\delta)\hat{K}_t + \delta \left(\hat{I}_t + \hat{\theta}_t^i \right) \\ &= (1-\delta)\hat{K}_t + \delta \left(\hat{I}_t + (1+\beta)S''(1)\widetilde{\hat{\theta}_t^i} \right).\end{aligned}\quad (\text{C.6})$$

Domestic intermediary sector;

$$\hat{Y}_t^D = \mu_d \left(\hat{\theta}_t^D + \alpha\hat{K}_t + (1-\alpha)\hat{H}_t \right), \quad (\text{C.7})$$

$$\hat{r}_t^k = \hat{w}_t + \hat{H}_t - \hat{K}_t, \quad (\text{C.8})$$

$$\begin{aligned}\hat{\pi}_t^D - \hat{\pi}_t &= \frac{\beta}{1+\omega_d\beta} \left(E_t\hat{\pi}_{t+1}^D - \varrho_\pi\hat{\pi}_t \right) + \frac{\omega_d}{1+\omega_d\beta} \left(\hat{\pi}_{t-1}^D - \hat{\pi}_t \right) - \frac{\omega_d\beta(1-\varrho_\pi)}{1+\omega_d\beta}\hat{\pi}_t \\ &\quad + \frac{1-\xi_d}{\xi_d} \frac{1-\xi_d\beta}{1+\omega_d\beta} \left(\hat{\mu}_t^d + (1-\alpha)\hat{w}_t + \alpha\hat{r}_t^k - \hat{p}_t^D - \hat{\theta}_t^D \right) \\ &= \frac{\beta}{1+\omega_d\beta} \left(E_t\hat{\pi}_{t+1}^D - \varrho_\pi\hat{\pi}_t \right) + \frac{\omega_d}{1+\omega_d\beta} \left(\hat{\pi}_{t-1}^D - \hat{\pi}_t \right) - \frac{\omega_d\beta(1-\varrho_\pi)}{1+\omega_d\beta}\hat{\pi}_t \\ &\quad + \frac{1-\xi_d}{\xi_d} \frac{1-\xi_d\beta}{1+\omega_d\beta} \left((1-\alpha)\hat{w}_t + \alpha\hat{r}_t^k - \hat{p}_t^D - \hat{\theta}_t^D \right) + \widetilde{\hat{\mu}_t^d}.\end{aligned}\quad (\text{C.9})$$

Intermediary volumes and the relative price of imports to be eliminated in the following;

$$\begin{aligned}
\hat{Y}_t^{D,D} &= \hat{Y}_t - \eta \hat{p}_t^D \\
\hat{p}_t^M &= -\frac{1-\iota}{\iota} (p^D/p^M)^{1-\eta} \hat{p}_t^D, \\
\hat{Y}_t^M &= \hat{Y}_t - \eta \hat{p}_t^M = \hat{Y}_t + \eta \frac{1-\iota}{\iota} (p^D/p^M)^{1-\eta} \hat{p}_t^D, \\
\hat{Y}_t^X &= \hat{Y}_t^* - \eta_x (\hat{p}_t^D - \hat{Q}_t).
\end{aligned}$$

Import prices;

$$\begin{aligned}
\hat{\pi}_t^M - \hat{\pi}_t &= \frac{\beta}{1+\omega_m\beta} (E_t \hat{\pi}_{t+1}^M - \varrho_\pi \hat{\pi}_t) + \frac{\omega_m}{1+\omega_m\beta} (\hat{\pi}_{t-1}^M - \hat{\pi}_t) \\
&\quad - \frac{\omega_m\beta(1-\varrho_\pi)}{1+\omega_m\beta} \hat{\pi}_t + \frac{1-\xi_m}{\xi_m} \frac{1-\xi_m\beta}{1+\omega_m\beta} \left(\hat{\mu}_t^m + \hat{Q}_t + \frac{1-\iota}{\iota} (p^D/p^M)^{1-\eta} \hat{p}_t^D \right) \\
&= \frac{\beta}{1+\omega_m\beta} (E_t \hat{\pi}_{t+1}^M - \varrho_\pi \hat{\pi}_t) + \frac{\omega_m}{1+\omega_m\beta} (\hat{\pi}_{t-1}^M - \hat{\pi}_t) \\
&\quad - \frac{\omega_m\beta(1-\varrho_\pi)}{1+\omega_m\beta} \hat{\pi}_t + \frac{1-\xi_m}{\xi_m} \frac{1-\xi_m\beta}{1+\omega_m\beta} \left(\hat{Q}_t + \frac{1-\iota}{\iota} \mu_m^{\eta-1} \hat{p}_t^D \right) + \tilde{\mu}_t^m,
\end{aligned} \tag{C.10}$$

Net foreign assets and the real exchange rate;

$$\begin{aligned}
\hat{b}_t^* &= (1+R^*) (\hat{b}_{t-1}^* + \hat{p}_t^D + \hat{Y}_t^X - \hat{Q}_t - \hat{Y}_t^M) \\
&= (1+R^*) \left(\hat{b}_{t-1}^* + \hat{p}_t^D + \hat{Y}_t^* + \hat{\theta}_t^* - \eta_x (\hat{p}_t^D - \hat{Q}_t) - \hat{Q}_t - \hat{Y}_t - \eta \frac{1-\iota}{\iota} (p^D/p^M)^{1-\eta} \hat{p}_t^D \right) \\
&= (1+R^*) \left(\hat{b}_{t-1}^* + \hat{Y}_t^* + (1-\eta_x) (\hat{p}_t^D - \hat{Q}_t) - \hat{Y}_t - \eta \frac{1-\iota}{\iota} \mu_m^{\eta-1} \hat{p}_t^D \right),
\end{aligned} \tag{C.11}$$

$$\hat{Q}_t = \hat{Q}_{t-1} + \Delta \hat{S}_t + \hat{\pi}_t^* - \hat{\pi}_t. \tag{C.12}$$

Relative prices;

$$\hat{p}_t^D = \hat{p}_{t-1}^D + \hat{\pi}_t^D - \hat{\pi}_t, \tag{C.13}$$

$$\hat{p}_t^M = \hat{p}_{t-1}^M + \hat{\pi}_t^M - \hat{\pi}_t \iff \hat{\pi}_t^M = \hat{\pi}_t + \Delta \hat{p}_t^M \iff$$

$$\hat{\pi}_t^M = \hat{\pi}_t - \frac{1-\iota}{\iota} \mu_m^{\eta-1} (\hat{p}_t^D - \hat{p}_{t-1}^D) \tag{C.14}$$

Market equilibria;

$$\begin{aligned}
\hat{Y}_t^D &= \frac{Y^{D,D}}{Y^D} \hat{Y}_t^{D,D} + \frac{Y^X}{Y^D} \hat{Y}_t^X \\
&= \frac{Y^{D,D}}{Y^D} [\hat{y}_t - \eta \hat{p}_t^D] + \frac{Y^X}{Y^D} \left[\hat{Y}_t^* + \hat{\theta}_t^* - \eta_x (\hat{p}_t^D - \hat{Q}_t) \right]
\end{aligned} \tag{C.15}$$

$$\hat{Y}_t = \frac{C}{Y} \hat{C}_t + \frac{G}{Y} \hat{G}_t + \frac{I}{Y} \hat{I}_t = \frac{C}{Y} \hat{C}_t + \frac{I}{Y} \hat{I}_t + \tilde{G}_t \tag{C.16}$$

Monetary policy rule;

$$\hat{R}_t = \rho_r \hat{R}_{t-1} + (1-\rho_r) \left[\hat{\pi}_t + \rho_\pi (\hat{\pi}_t - \hat{\pi}_{t-1}) + \rho_y \hat{Y}_t + \rho_s \Delta \hat{S}_t \right] + \epsilon_t^m. \tag{C.17}$$

The domestic exogenous processes can be summarized in the following table as follows:

$$\hat{\zeta}_t^c = \varrho_c \hat{\zeta}_{t-1}^c + \varepsilon_t^c, \quad (\text{C.18})$$

$$\hat{\zeta}_t^l = \varrho_l \hat{\zeta}_{t-1}^l + \varepsilon_t^l, \quad (\text{C.19})$$

$$\hat{\theta}_t^D = \varrho_d \hat{\theta}_{t-1}^D + \varepsilon_t^d, \quad (\text{C.20})$$

$$\hat{\theta}_t^i = \varrho_i \hat{\theta}_{t-1}^i + \varepsilon_t^i, \quad (\text{C.21})$$

$$\hat{\mu}_t^d = \varrho_{\mu d} \hat{\mu}_{t-1}^d + \varepsilon_t^{\mu d}, \quad (\text{C.22})$$

$$\hat{\mu}_t^m = \varrho_{\mu m} \hat{\mu}_{t-1}^m + \varepsilon_t^{\mu m}, \quad (\text{C.23})$$

$$\hat{g}_t = \varrho_g \hat{g}_{t-1} + \varepsilon_t^g, \quad (\text{C.24})$$

$$\hat{\pi}_t = \varrho_\pi \hat{\pi}_{t-1} + \varepsilon_t^\pi, \quad (\text{C.25})$$

$$\hat{v}_t = \varrho_v \hat{v}_{t-1} + \varepsilon_t^v, \quad (\text{C.26})$$

In addition, we also have the exogenous i.i.d interest rate shock ϵ_t^m as well as the three foreign exogenous variables, which are modelled exogenously as a VAR;

$$\begin{aligned} F_0 X_t &= F(L) X_t + \varepsilon_t^x, & \varepsilon_t^x &\sim N(0, \Sigma_x), \\ X_t &= \left(\hat{y}_t^*, \hat{\pi}_t^*, \hat{R}_t^* \right)'. \end{aligned} \quad (\text{C.27})$$

C.1 Summary of Variables

The system has 17 endogenous variables and 13 (10 domestic and 3 foreign) exogenous variables. The endogenous variables are:

$$\hat{Y}_t, \hat{Y}_t^D, \hat{C}_t, \hat{I}_t, \hat{K}_t, \hat{H}_t, \hat{\pi}_t, \hat{\pi}_t^D, \hat{\pi}_t^M, \hat{p}_t^D, \hat{r}_t^k, \hat{\chi}_t, \hat{w}_t, \hat{R}_t, \Delta \hat{S}_t, \hat{Q}_t, \hat{b}_t^*,$$

and the exogenous variables are:

$$\hat{\zeta}_t^c, \hat{\zeta}_t^l, \hat{\theta}_t^D, \hat{\theta}_t^i, \hat{\mu}_t^d, \hat{\mu}_t^m, \hat{G}_t, \hat{\pi}_t, \hat{\epsilon}_t^m, \hat{v}_t, \text{ and } \hat{y}_t^*, \hat{\pi}_t^*, \hat{R}_t^*.$$

Chapter 4

Heterogeneous Price Stickiness in Estimated Semi-structural Models of the U.S. Economy*

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Abstract

We estimate sticky-price models for the U.S. economy in which the degree of price stickiness is allowed to vary across sectors. We employ a Bayesian approach to combine time series data on aggregate inflation and output with information derived from microeconomic data on the cross-sectional distribution of the frequency of price changes in the U.S. economy. Our results show that heterogeneity in price stickiness is of critical importance for understanding the joint dynamics of inflation and output. Moreover, allowing for enough heterogeneity - in particular for prices in some sectors to last beyond one year - is crucial to avoid producing estimates that imply “too little” average nominal rigidity at the expense of “too much” real rigidity.

Keywords: heterogeneity, price stickiness, micro data, macro data, Bayesian estimation

JEL classification codes: E1, E3

*The views expressed in this paper are those of the authors and do not necessarily reflect the position of the Federal Reserve Bank of New York or the Federal Reserve System.

1 Introduction

The recent empirical literature on price setting that analyzes the datasets underlying the construction of consumer price indices documents a large amount of heterogeneity in the frequency of price changes across different economic sectors. Starting with Bils and Klenow (2004), who use data from the Bureau of Labor Statistics for the U.S. economy, shortly thereafter a large number of papers helped establish the fact that such type of heterogeneity is pervasive in modern industrial economies (e.g. Dhyne et al., 2006, and references cited therein document similar facts for the Euro area; Gagnon, 2007 details the evidence for Mexico). Inasmuch as heterogeneity in price stickiness is concerned, this recent literature corroborates and provides better measures supporting the findings of previous work on price setting behavior (e.g. Blinder et al., 1998).

The evidence of substantial heterogeneity in the degree of price stickiness across sectors stands in sharp contrast with the assumption, common to the vast majority of papers on sticky prices, that all firms in the economy change prices with the same frequency. Apart from analytical convenience, the only reason to resort to this assumption and not take heterogeneity explicitly into account in macroeconomic models would be if it did not matter qualitatively in aggregate terms, or at least not quantitatively. However, recent work undermines these arguments. Aoki (2001) and Benigno (2004) show that heterogeneity in price stickiness affects the nature of optimal monetary policy, and Carvalho (2006) shows that it has dramatic implications for the dynamic response of economies to monetary disturbances.¹ In particular, the real effects of such shocks tend to be larger and more persistent than in otherwise identical “one-sector” economies in which all firms face the same degree of price stickiness, and moreover the speed of adjustment to the shock varies during the transition process, as the dynamics are dominated by different sectors at different stages of the process.

The microeconomic evidence on heterogeneity in price stickiness and the theoretical results about its importance for aggregate dynamics jointly underscore the goal of this paper. We combine recently available information about the cross-sectional distribution of the frequency of price changes derived from the microdata with time series of aggregate inflation and output to estimate a series of small “semi-structural” multi-sector models for the U.S. economy. We derive the structure of the supply side of the models from a multi-sector economy with Taylor (1979, 1980) staggered price setting, in which the extent of price rigidity varies across different sectors. Instead of postulating a fully specified economy to obtain the remaining equations to be used in the estimation (the demand side, if you will), we assume exogenous stochastic processes for nominal output and for an unobservable natural rate of output. Thus the “semi-structural” classification. Given that our focus is on estimation of parameters that characterize price setting behavior in the economy in the presence of heterogeneity, our goal in specifying such exogenous time series processes is to close the model with a set of equations that can provide it with some flexibility relative to a fully structural model. This approach allows us to avoid having to

¹The literature that focuses on the implications of this and related forms of heterogeneity continues to grow. Dixon and Kara (2005) study sectoral heterogeneity in wage setting. Carlstrom et al. (2006a,b) analyze the effects of heterogeneity on equilibrium determinacy and aggregate dynamics in two-sector models. Nakamura and Steinsson (2007a) study a multi-sector menu cost model. Sheedy (2007) focuses on how heterogeneity in price stickiness affects inflation persistence. Imbs et al. (2007) study the aggregation of sectoral Phillips curves, and the biases that can arise from not accounting for heterogeneity. Carvalho and Schwartzman (2007) show how heterogeneity in price setting behavior affects aggregate dynamics in a large class of sticky-price and sticky-information models.

find a demand side specification that performs well empirically, which typically requires the introduction of “structural” features which are not the focus of our paper.²

We use our sources of data in two complementary ways, making use of a full-information Bayesian approach. In one direction, we estimate the cross-sectional distribution of price stickiness using only time series data on aggregate inflation and output. We then assess the extent to which the macro data contains information about such cross-sectional distribution, and how the results compare with the recent microeconomic evidence. In particular, we compare the results with empirical cross-sectional distributions of the frequency of price changes derived from Bils and Klenow (2004) and Nakamura and Steinsson (2007b). Alternatively, we incorporate the information from the micro data analyzed by these papers through our prior on the cross-sectional distribution of the frequency of price changes, which in turn affects aggregate dynamics, and estimate the model using aggregate inflation and output as observables. The fact that different cross-sectional distributions of price stickiness imply different aggregate dynamics, as highlighted recently by Carvalho (2006) and Coenen et al. (2007), in principle allows inference about such distribution based solely on time series of macroeconomic variables. However, we had anticipated, and confirmed with our results, that identification of this distribution is likely to be weak in this context. Thus the additional value of an approach that makes use of the micro data.³

Our results strongly support the versions of model with heterogeneity in price stickiness. When we restrict the models by imposing the same degree of price stickiness across sectors, we obtain results that are significantly worse from a statistical perspective than in the general case with heterogeneity, and that moreover are economically nonsensical. In specifications in which the prior distributions imply small differences in price stickiness across sectors, posterior distributions display more heterogeneity.

Despite very different empirical methodologies, our results are in line with those obtained by Coenen et al. (2007), who estimate a model with Taylor staggered price setting and heterogeneous contract lengths of up to four quarters. Our estimation results, however, suggest that it is important to allow for sectors in which prices last longer than one year. Neglecting to do so generates too little nominal rigidity relative to the microevidence on the one hand, and on the other hand increases the estimated degree of real rigidity way beyond what is found when more heterogeneity is allowed for. Thus, Coenen et al. (2007) find an incredible amount of real rigidity. Moreover, for our semi-structural setup, the restriction of at most one year duration price spells comes at significant costs for the empirical validity of the model: across all pairwise comparisons between models with at most one year price contracts, and models in which some prices can last beyond one year, the *lowest* posterior odds ratio in favor of the latter models is around 24 : 1. Allowing prices in some sectors to last one and a half or two years decreases the estimated degree of real rigidity to levels that have been deemed plausible in recent literature (e.g. Woodford 2003), while still leading to an extent of average nominal rigidity that is somewhat in line with the recent evidence based on micro-data for the U.S.

We proceed as follows. Section 2 presents the semi-structural model. Section 3 presents our empirical methodology. We detail the aggregate data used in the estimation,

²Several earlier papers combine structural equations with reduced-form, empirical specifications for other parts of the model. Sbordone (2002), Guerrieri (2006) and Coenen et al (2007) are recent examples.

³Another promising approach, which we do not explore in this paper, involves using sectoral data as well. The natural way to do it is to introduce sectoral shocks and include time-series of sectoral variables as observables. Lee (2007) takes a promising step in that direction.

as well as our method for incorporating the information on price setting behavior at the microeconomic level from recent empirical studies. This section also details our prior assumptions, and our particular estimation algorithm. We follow with the analysis of our results in Section 4, and conclude in the last section.

2 The semi-structural model

There is a continuum of monopolistically competitive firms divided into K sectors that differ in the frequency of price changes. Firms are indexed by their sector, $k \in \{1, \dots, K\}$, and by $j \in [0, 1]$. The distribution of firms across sectors is summarized by a vector $(\omega(1), \dots, \omega(K))$ with $\omega(k) > 0$, $\sum_{k=1}^K \omega(k) = 1$, where $\omega(k)$ gives the mass of firms in sector k . Each firm produces a unique variety of a consumption good, and faces a demand that depends negatively on its relative price.

In any given period, profits of firm j from sector k (henceforth referred to as “firm kj ”) are given by:

$$\Pi_t(k, j) = P_t(k, j) Y_t(k, j) - C(Y_t(k, j), Y_t, \xi_t),$$

where $P_t(k, j)$ is the price *charged* by the firm, $Y_t(k, j)$ is the quantity that it sells at the posted price (determined by demand), and $C(Y_t(k, j), Y_t, \xi_t)$ is the total cost of producing such quantity, which may also depend on aggregate output Y_t , and is subject to shocks (ξ_t) . We assume that the demand faced by the firm depends on its relative price $\frac{P_t(k, j)}{P_t}$, where P_t is the aggregate price level in the economy, and on aggregate output. Thus, we write firm kj ’s profit as:

$$\Pi_t(k, j) = \Pi(P_t(k, j), P_t, Y_t, \xi_t),$$

and make the usual assumption that Π is homogeneous of degree one in the first two arguments, and single peaked at a strictly positive level of $P_t(k, j)$ for any level of the other arguments.⁴

The aggregate price index combines sectoral price indices, $P_t(k)$ ’s, according to the sectoral weights, $\omega(k)$ ’s:

$$P_t = \Gamma\left(\{P_t(k), \omega(k)\}_{k=1, \dots, K}\right),$$

where Γ is an aggregator that is homogeneous of degree one in the $P_t(k)$ ’s. In turn, the sectoral price indices are obtained by applying a symmetric, homogeneous of degree one aggregator Λ to prices charged by all firms in each sector:

$$P_t(k) = \Lambda\left(\{P_t(k, j)\}_{j \in [0, 1]}\right).$$

We assume the specification of staggered price setting inspired by Taylor (1979, 1980). Firms set prices that remain in place for a fixed number of periods, referred to as the “contract length.” The latter is sector-specific, and we save on notation by assuming that firms in sector k set k -period contracts. Firms meet all demand for their products at the posted prices. Finally, across all sectors, adjustments are uniformly distributed over

⁴This is analogous to Assumption 3.1 in Woodford (2003).

time.

When setting its price $X_t(k, j)$ at time t , given that it sets prices for k periods, firm kj solves:

$$\max E_t \sum_{i=0}^{k-1} Q_{t,t+i} \Pi(X_t(k, j), P_{t+i}, Y_{t+i}, \xi_{t+i}),$$

where $Q_{t,t+i}$ is a (possibly stochastic) nominal discount factor. In this context, the first order condition for the firm's problem can be written as:

$$E_t \sum_{i=0}^{k-1} Q_{t,t+i} \frac{\partial \Pi(X_t(k, j), P_{t+i}, Y_{t+i}, \xi_{t+i})}{\partial X_t(k, j)} = 0. \quad (1)$$

Note that all firms from sector k that adjust prices at the same time choose a common price, which we denote $X_t(k)$. Thus, for a firm kj that adjusts at time t and sets $X_t(k)$, the prices charged from t to $t+k-1$ satisfy:

$$P_{t+k-1}(k, j) = P_{t+k-2}(k, j) = \dots = P_t(k, j) = X_t(k).$$

Given the assumptions on price setting, and uniform staggering of price adjustments, with an abuse of notation sectoral prices can be expressed as:

$$P_t(k) = \Lambda \left(\{X_{t-i}(k)\}_{i=0, \dots, k-1} \right).$$

We close the model by specifying a stochastic process for the shock ξ_t , and by positing that nominal output $M_t \equiv P_t Y_t$ also evolves in an exogenous fashion. This is a standard assumption in theoretical work on price setting (e.g. Mankiw and Reis 2002, Woodford 2003, chapter 3). It allows one to focus on the implications of the particular model postulated for pricing behavior for the decomposition of changes in nominal output into purely real and purely nominal effects, without having to specify a full model of the economy. We make use of this assumption in our empirical implementation because it allows us to assess the performance of our heterogeneous price setting model in explaining the dynamics of output and inflation without having to specify the details of the transmission mechanism.

2.1 A loglinear approximation

We assume that the economy has a deterministic zero inflation steady state characterized by $\xi_{t+i} = 0$, $Y_{t+i} = \bar{Y}$, $Q_{t,t+i} = \beta$, and for all (k, j) , $X_t(k, j) = P_t = \bar{P}$, and loglinearize (1) around it to obtain:⁵

$$x_t(k) = \frac{1 - \beta}{1 - \beta^k} E_t \sum_{i=0}^{k-1} \beta^i (p_{t+i} + \zeta (y_{t+i} - y_{t+i}^n)), \quad (2)$$

where lowercase variables denote log-deviations of the respective uppercase variables from the steady-state. The parameter $\zeta > 0$ is the (inverse) Ball and Romer (1990) index of real rigidities. The new variable Y_t^n is the level of output that would prevail in a flexible price economy. It is referred to as the natural level of output, and is defined implicitly as

⁵We write all such expressions as equalities, ignoring higher order terms.

a function of ξ_t by:

$$\left. \frac{\partial \Pi (X_t(k, j), P_t, Y_t^n, \xi_t)}{\partial X_t(k, j)} \right|_{X_t(k, j)=P_t} = 0.$$

In the loglinear approximation, the natural output level moves *pari passu* with $\log(\xi_t)$: $y_t^n = \alpha \log(\xi_t)$.

The definition of nominal output yields:

$$m_t = p_t + y_t. \quad (3)$$

Finally, we postulate that the aggregators that define the overall level of prices P_t and the sectoral price indices give rise to the following loglinear approximations:

$$p_t = \sum_{k=1}^K \omega(k) p_t(k), \quad (4)$$

$$\begin{aligned} p_t(k) &= \int_0^1 p_t(k, j) dj \\ &= \frac{1}{k} \sum_{j=0}^{k-1} x_{t-j}(k). \end{aligned} \quad (5)$$

Large real rigidities (small ζ) reduce the sensitivity of prices to aggregate demand conditions, and thus magnify the non-neutralities generated by nominal price rigidity. In fully specified models, the extent of real rigidities depends on primitive parameters such as the intertemporal elasticity of substitution, the elasticity of substitution between varieties of the consumption good, the labor supply elasticity. It also depends on whether the economy features economy-wide or segmented factor markets, whether there is an explicit input-output structure etc.⁶

In the context of our model, the index itself can be regarded as a primitive parameter. We refer to economies with $\zeta < 1$ as ones displaying *strategic complementarities* in price setting. To clarify the meaning of this expression, replace (3) in (2) to obtain:

$$x_t(k) = \frac{1 - \beta}{1 - \beta^k} E_t \sum_{i=0}^{k-1} \beta^i (\zeta (m_{t+i} - y_{t+i}^n) + (1 - \zeta) p_{t+i}).$$

That is, new prices are set as a discounted weighted average of current and expected future driving variables ($m_{t+i} - y_{t+i}^n$) and prices p_{t+i} . $\zeta < 1$ implies that firms choose to set higher prices if the overall level of current and expected future prices is higher, *ceteris paribus*. On the other hand, $\zeta > 1$ means that prices are *strategic substitutes*, in that under those same circumstances, adjusting firms choose relatively *lower* prices.

2.2 Nominal output m_t and natural output level y_t^n

We postulate an AR(p_1) process for nominal output, m_t :

$$m_t = \rho_0 + \rho_1 m_{t-1} + \dots + \rho_{p_1} m_{t-p_1} + \varepsilon_t^m, \quad (6)$$

⁶For a detailed discussion of sources of real rigidities see Woodford (2003, chapter 3).

and an AR(p_2) process for the natural output level, y_t^n :

$$y_t^n = \delta_0 + \delta_1 y_{t-1}^n + \dots + \delta_{p_2} y_{t-p_2}^n + \varepsilon_t^n, \quad (7)$$

where $\varepsilon_t = (\varepsilon_t^m, \varepsilon_t^n)$ is iid $N(0_{1 \times 2}, \Sigma^2)$, with $\Sigma^2 = \begin{bmatrix} \sigma_m^2 & 0 \\ 0 & \sigma_n^2 \end{bmatrix}$.

2.3 State-space representation

The semi-structural model then consists of equations (2) through (7). We write it in state-space form in the notation of Sims (2002):

$$\Gamma_0 Z_t = \Gamma_1 Z_{t-1} + \mathcal{C} + \Psi \varepsilon_t + \Xi \eta_t,$$

where Z_t is a vector collecting all variables and additional ‘‘dummy’’ variables created to account for leads and lags, \mathcal{C} is a vector of constants, ε_t is as defined before, and η_t is a vector of one period ahead expectational errors. Γ_0 , Γ_1 , Ψ , and Ξ are the appropriate matrices, which are functions of the primitive parameters of the model, collected for notational simplicity in a vector:

$$\theta = (K \ p_1 \ p_2 \ \beta \ \zeta \ \sigma_m^2 \ \sigma_n^2 \ \omega(1) \ \dots \ \omega(K) \ \rho_0 \ \dots \ \rho_{p_1} \ \delta_0 \ \dots \ \delta_{p_2}).$$

We solve the model with Gensys (Sims, 2002), to obtain:

$$Z_t = C(\theta) + G_1(\theta) Z_{t-1} + B(\theta) \varepsilon_t. \quad (8)$$

3 Empirical Methodology

As already mentioned, the main objective of this paper is an empirical assessment of the implications of price setting of firms with different contract lengths for the dynamics of aggregate output and inflation. With the challenges involved in bridging the gap between micro-based information on individual prices and the National Account time series on U.S. real GDP (‘‘output’’) and GDP inflation (‘‘inflation’’), the choice of empirical methodology is of critical importance. We employ a Bayesian approach as this allows us to integrate the microeconomic information in the BLS price data with the macroeconomic time series.⁷

The Bayesian principle can be shortly stated as:

$$\begin{aligned} f(\theta|Z^*) &= f(Z^*|\theta) f(\theta) / f(Z^*) \\ &\propto \mathcal{L}(\theta|Z^*) f(\theta), \end{aligned}$$

where f denotes density functions and Z^* is the vector of observed time series, $\mathcal{L}(\theta|Z^*)$ is the likelihood function and $f(\theta)$ is the joint prior density, while θ is the vector of primitive parameters defined above. The likelihood function is constructed through application of the Kalman filter to the solved log-linear model (8), given that our state vector Z_t includes

⁷The fact that we consider the fundamental axioms and assumptions of Bayesian econometrics better suited to the empirical analysis of aggregate time series than frequentist econometrics is, obviously, comforting. We refer the reader to Lancaster (2004) for a discussion of Bayesian versus frequentist econometrics.

many unobserved variables, such as the natural output level and sectoral prices. Letting H denote the matrix that singles out the observed subspace Z_t^* of the state vector Z_t (i.e., $Z_t^* = HZ_t$), our distributional assumptions can be summarized as:⁸

$$\begin{aligned} Z_t|Z_{t-1} &\sim N(C(\theta) + G_1(\theta)Z_{t-1}, B(\theta)\Sigma B(\theta)') \Rightarrow \\ Z_t^*|\{Z_\tau^*\}_{\tau=1}^{t-1} &\sim N(M(\theta), V(\theta)), \\ \text{where } M(\theta) &\equiv HC(\theta) + HG_1(\theta)\hat{Z}_{t|t-1}, V(\theta) \equiv HB(\theta)\hat{\Sigma}_{t|t-1}B(\theta)'H'. \end{aligned} \quad (9)$$

We use our sources of data in two complementary ways. In one direction, we estimate the cross-sectional distribution of price stickiness using only time series data on aggregate inflation and output. Theoretically, this estimation should be possible due to the marked impact of the cross-sectional distribution on aggregated dynamics as analyzed in Carvalho (2006). We then assess the extent to which the macro data in fact contains information about such cross-sectional distribution, and how the results compare with the recent microeconomic evidence.

Alternatively, we incorporate the information from the micro data analyzed in Bils and Klenow (2004) and Nakamura and Steinsson (2007b) through our prior on the cross-sectional distribution of the frequency of price changes, which in turn affects aggregate dynamics, and estimate the model using aggregate inflation and output as observables. The use of the microeconomic findings from these papers are discussed in Section 3.2 below. While one could have opted to integrate the results of the two papers into one set of priors, we have chosen to utilize them in separate estimations. With this approach, we can investigate how the profile of the cross-sectional distribution affects the estimated aggregate dynamics and thus gain information on which of the two distributions seems more in line with the observed behavior of aggregate output and inflation, and whether they improve on the estimation obtained without using microeconomic evidence in the priors.

When estimating the model, whether using priors based on the micro data or not, we estimate several specifications of the model for different values of K , treating it as fixed in each estimation. We chose this approach over the alternative of specifying a prior over this parameter as well and estimating its distribution due to computational constraints.

In the next subsection we provide details on the macroeconomic time series that constitute the observables in our Bayesian estimation. We then explain how we use the statistics reported by Bils and Klenow (2004) and Nakamura and Steinsson (2007b) to construct empirical cross-sectional distributions of price stickiness. This is followed by a specification of our prior assumptions, with particular emphasis on how we use the cross-sectional information from the micro-data analyzed by these two papers. Finally, we detail the algorithm that we use to simulate the joint posterior distribution of the parameters.

3.1 Macroeconomic time series

The model laid out is tested against the development of quarterly output and inflation. Whereas the assumptions underlying the model include one of an unchanged economic environment, the U.S. economy has undergone profound changes in the recent decades,

⁸Here, we use the notation of Hamilton (1994, ch. 13). Hence, e.g., $\hat{Z}_{t|t-1}$ is the estimate of the endogenous vector given the sequence from period 1 to $t-1$ of the observed variables $Z_t^* = HZ_t$. We refer the reader to Hamilton (1994, ch. 13) for a thorough introduction to the Kalman filter.

including phenomena such as the so-called “Great Moderation” and the Volcker Disinflation. As a consequence, we do not attempt to confront the model with the full sample of post-war output and inflation. More specifically, we exclude entirely the economically volatile period prior to 1960. We use the period from 1960 to 1982 only as a pre-sample, and thus we also exclude the monetary regimes up to and including the Federal Reserve money-supply targeting policy of 1979-1982 from the developments assessed by the likelihood criterion. That is, ultimately we evaluate the model according to its ability to match business cycle developments in output and inflation under the Fed Funds target regime post 1982.

We make use of the pre-sample 1960-1982 by initializing the Kalman filter in the estimation stage with the estimate of Z_t obtained from running a Kalman filter in the pre-sample, and a covariance matrix that is a scaled-up version of the covariance matrix obtained at the end of the pre-sample. In running the Kalman filter for this period, we fix the parameter values as follows. We set $\beta = 0.99$, and $\zeta = 1$. We fix $p_1 = p_2 = 2$, and set the parameters ρ_0, \dots, ρ_2 to the point estimates of an AR(2) estimated on nominal output, and $\delta_0, \dots, \delta_2$ to the point estimates of an AR(2) estimated on *actual* output. Finally, the sectoral masses are set at $1/K$.

Although in principle our empirical specifications can account for trends in the data, they are not suitable to handling *changes* in trends, which might have occurred despite our choice of sample period. Thus, we remove the trend component in both aggregates in a preliminary step, applying a low-pass filter of the Baxter-King (1999) variant that eliminates fluctuations in the time series with periodicities in excess of eight years. The filter eliminates 12 quarters of data in each end of the sample that we do not seek to replace. Hence, our effective estimation sample is 1983-2003. The time series are depicted in Figures 1 and 2.

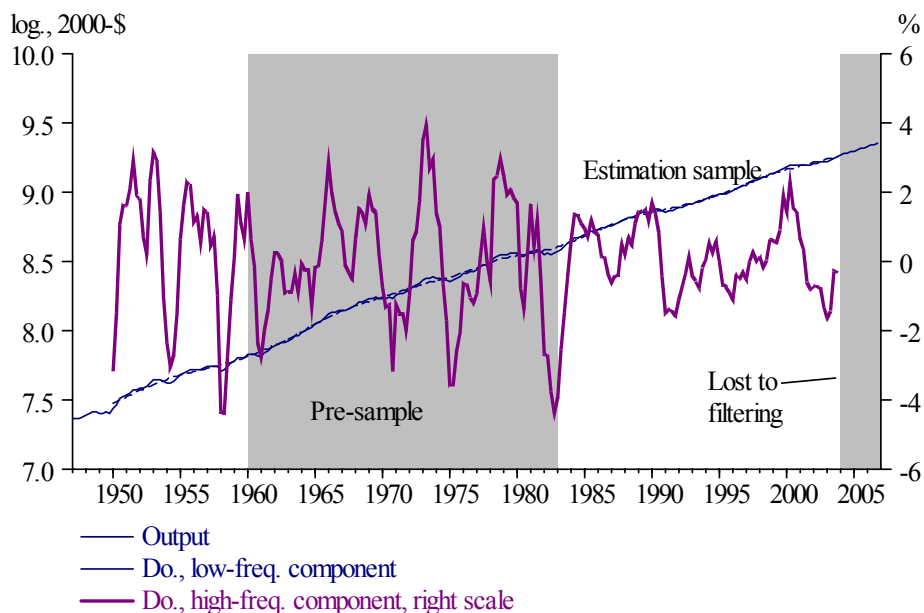


Figure 1: Output

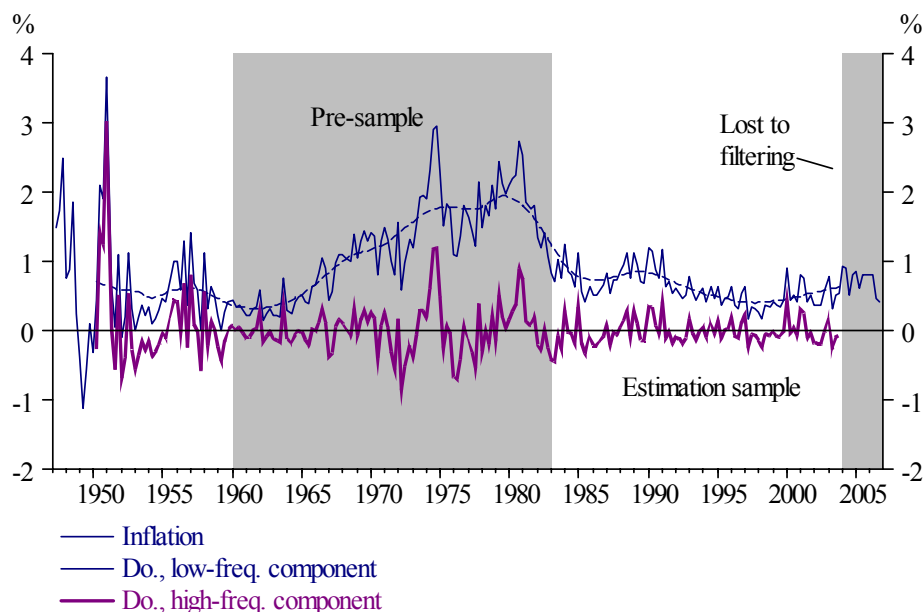


Figure 2: Inflation

3.2 Empirical cross-sectional distributions of price stickiness

We extract our information about the cross-sectional distribution of price stickiness from two recent papers that analyze the frequency of price changes in the U.S. economy using quite disaggregated datasets from the Bureau of Labor Statistics, which underlie the construction of the Consumer Price Index. The first study is Bils and Klenow (2004, henceforth BK), who pioneered the use of such data to analyze the frequency of price changes and other aspects of price setting. The other is Nakamura and Steinsson (2007b, henceforth NS).

The main difference between these two papers is that BK analyze the BLS data at an intermediate level of disaggregation,⁹ whereas NS use the data at its most disaggregated level.¹⁰ Moreover, NS provide statistics with different treatments of sales price observations and product substitutions. When they apply treatments for these events that make their sample close to being a more disaggregated version of the data analyzed by BK, the two papers produce very similar results on the cross-sectional distribution of the frequency of price changes. In contrast, when NS use the observations on what they refer to as *regular* price changes, they find more nominal price rigidity and more dispersion in the frequency of price changes across different goods.¹¹

We work with the statistics on the frequency of price changes for the 350 categories of goods and services (“entry level items”) analyzed by BK, and with the 272 entry level items covered by NS (using the statistics for regular prices). Our goal is to map those statistics into an empirical distribution of sectoral masses over different contract lengths. We work at a quarterly frequency, and consider economies with at most 8 quarters of price stickiness. Thus, we consider contract lengths which are multiples of one quarter,

⁹They used data on so called “entry level items,” from the Commodities and Services Substitution Rate Tables for the period 1995-1997.

¹⁰Their data comes from the CPI Research Database, and covers the period 1988-2005.

¹¹We refer the reader to the two papers for a detailed description of their methodologies.

and for each of the BK and NS data, we aggregate the goods and services categories so that the ones which have an average duration of price spells between zero and one quarter (inclusive) are assigned to the first sector (i.e., the one quarter contract length sector); the ones with an expected duration of price spells between one (exclusive) and two quarters (inclusive) are assigned to the second sector, and so on. The sectoral weights are aggregated accordingly by adding up the corresponding CPI expenditure weights. We proceed in this fashion until the sector with contract lengths of 7 quarters. Finally, we aggregate all the remaining categories, which have mean durations of price rigidity of 8 quarters and beyond, into a sector with 2-year contracts. This gives rise to the empirical cross-sectional distributions of price stickiness that we use in our estimation. We denote the sectoral weight for sector k obtained from this procedure by $\widehat{\omega}_k$. For each of the BK and NS distributions, we also compute the average contract length, $\widehat{k} = \sum_{k=1}^K \widehat{\omega}_k k$, and the standard deviation of contract lengths, $\widehat{\sigma}_k = \sqrt{\sum_{k=1}^K \widehat{\omega}_k (k - \widehat{k})^2}$.

Table 1: Cross-sectional distributions of price stickiness

Parameter	BK	NS
$\widehat{\omega}_1$	0.40	0.27
$\widehat{\omega}_2$	0.24	0.07
$\widehat{\omega}_3$	0.12	0.10
$\widehat{\omega}_4$	0.12	0.11
$\widehat{\omega}_5$	0.04	0.06
$\widehat{\omega}_6$	0.03	0.13
$\widehat{\omega}_7$	0.03	0.06
$\widehat{\omega}_8$	0.03	0.20
$\widehat{k}^{(*)}$	2.54	4.23
$\widehat{\sigma}_k^{(*)}$	1.86	2.66

(*) In quarters. $\sum \widehat{\omega}_k$ might differ from unity due to rounding.

3.3 Assumptions on the prior distributions

The model parameters to be estimated essentially fall in four categories that are dealt with in turn. The first set comprises the ρ 's and δ 's, parameterizing the exogenous AR processes for nominal and natural output, respectively. These are assigned loose Gaussian priors with mean zero. We choose to fix the lag length at 2 for both processes, i.e. $p_1 = p_2 = 2$. The choice of lag lengths was based on LR tests and the Schwarz and Akaike information criteria from univariate estimations (for this purpose, we use actual output in volumes as a proxy for the unobserved natural output process).¹² The second set of parameters consists of the standard deviations of the exogenous processes for nominal output (σ_m) and natural output (σ_n); again, these are strictly positive parameters, and we assign them loose gamma priors with modes of 0.1, in part based on simple univariate estimations of the pre-sample time series.

¹²In principle we could have specified priors over p_1, p_2 and estimated their posterior distribution as well. However, the computational challenge of the set of model configurations analyzed in this paper is already tremendous given our implementation of the estimation algorithm. Therefore, we restrict ourselves to this specification.

The third block of parameters includes only the output gap elasticity of price setting ζ , which should be non-negative. This is captured with a loose gamma prior distribution with mode at unity; hence, any significant degree of strategic complementarity or substitutability in the price setting should be a feature of the estimation rather than of our prior assumptions.

The fourth and final set of parameters are the sectoral masses $\omega = (\omega(1), \dots, \omega(K))$. The combined restrictions of non-negativity and summation to unity of the ω 's are captured through a Dirichlet distribution, which is a multivariate generalization of the beta distribution. Notationally, $\omega \sim D(\alpha)$ with density function:

$$f_{\omega}(\omega|\alpha) = \text{Dirichlet}(\omega|\alpha) \propto \prod_{k=1}^K \omega(k)^{\alpha_k-1}, \quad \forall \alpha_k > 0, \quad \forall \omega(k) \geq 0, \quad \sum_{k=1}^K \omega(k) = 1.$$

The Dirichlet distribution is well known in Bayesian econometrics as the conjugate prior for the multinomial distribution, and the hyperparameters $\alpha_1, \dots, \alpha_k$ are most easily understood in this context, where they can be interpreted as the number of occurrences for each of the K possible outcomes that the econometrician assigns to the prior information.¹³ It follows that for a given K , $\alpha_0 \equiv \sum_k \alpha_k$ captures the overall level of information in the prior distribution. Thus, we control the tightness of our sectoral mass priors through α_0/K , where the normalization by K makes this measure comparable across models with different number of sectors.

For a given level of information in the prior distribution of sectoral masses, the information about the cross-sectional distribution of price stickiness comes from the relative sizes of the α_k 's. The latter also determine the marginal distributions for the $\omega(k)$'s. For example, the expected value of $\omega(k)$ is simply α_k/α_0 . We use the empirical cross-sectional distributions derived from BK and NS described in the previous subsection to assign values for these hyperparameters. For specifications with $K = 8$, we simply set $\alpha_k = \hat{\omega}_k$. For any specification with $K < 8$, we set $\alpha_k = \hat{\omega}_k$ for all $k \leq K - 1$, and $\alpha_K = \sum_{j=K}^8 \hat{\omega}_j$. The $\hat{\omega}_k$'s being taken from the empirical distribution derived from BK gives rise to what we refer to as a *Bils-Klenow prior*, whereas taking the $\hat{\omega}_k$'s from the corresponding distribution based on NS yields a *Nakamura-Steinsson prior*.

In addition to using the BK and NS priors, we also estimated versions of the model with symmetric priors, in which all sectors are assigned identical priors, and $\alpha_k = 1/K$. For these symmetric priors, the case $\alpha_0 = K$ is noninformative in the sense that equal prior density is assigned to all ω vectors in the K -dimensional unit simplex. These noninformative priors allow us to assess the information that the aggregate data contains about the cross-sectional distribution of price stickiness.

Generally, we shall refer to cases with $\alpha_0 = K$ as *loose priors*. As we anticipated weak identification of the sectoral masses without use of prior information on the cross-section, we also use a tighter set of priors. For each configuration of the model (i.e. the number of sectors K , and whether the prior is BK, NS or symmetric) the *tight priors* are defined by setting $\alpha_0 = 5K$. The difference between the loose and the tight priors are illustrated in Figure 3 for the case of symmetric priors, while the tight priors of the BK and NS variants are compared for the case of four sectors in Figure 4 (i.e., $K = 4$ and $\alpha_0 = 20$).

¹³Gelman et al. (2003) offers a good introduction to the use of Dirichlet distribution as a prior distribution for the multinomial model.

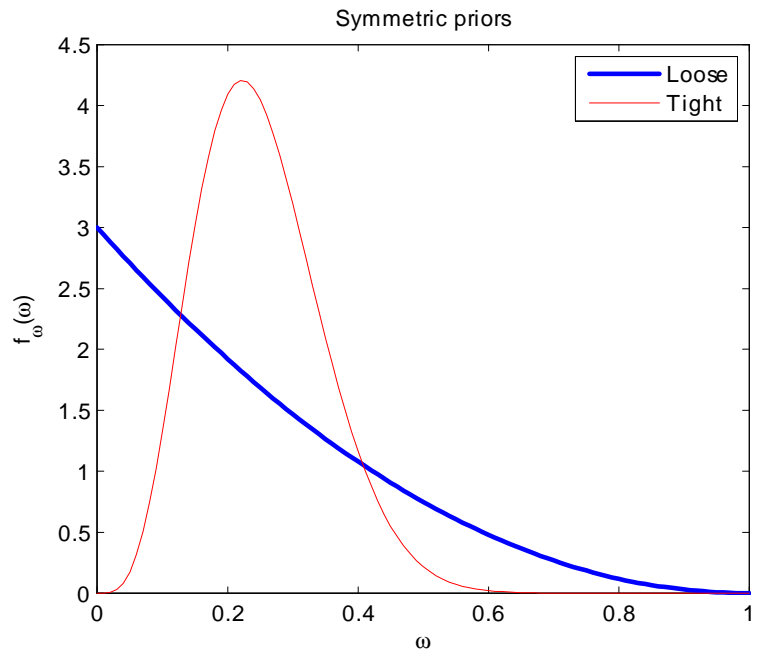


Figure 3: Symmetric sectoral mass priors with different tightness

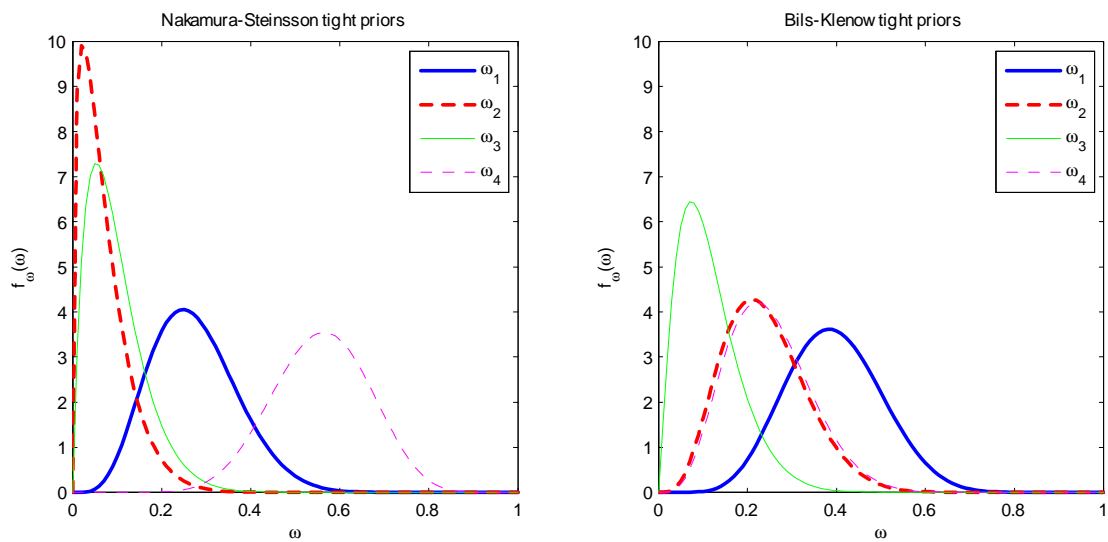


Figure 4: Sectoral mass priors based on BK and NS

3.4 Simulating the posterior distribution

The joint posterior distribution of the model parameters is obtained through application of a Markov-chain Metropolis algorithm. The algorithm produces a simulation sample of the parameter set that converges to the joint posterior parameter distribution under certain conditions.¹⁴ The numerous restrictions and sizeable dimension of the parameter set, especially in relation to our data set consisting of only two aggregate time series, place high demands on the exact implementation of the algorithm in order to obtain efficiency and convergence of the Markov-chain within a manageable number of iterations.

Our specific estimation strategy for each configuration of the model is as follows. We run two numerical optimization routines sequentially in order to determine the starting point of the Markov chain and gain a first crude estimate of the covariance matrix for our Independence Metropolis-Hastings Gaussian jumping distribution.¹⁵ Before running the Markov chains we transform all parameters to have full support on the real line. We use logarithmic transformation of each of $(\zeta, \sigma_m, \sigma_n)$ while $\omega(1), \dots, \omega(K)$ are transformed using a multivariate logistic function, cf. Appendix A. Then we run a so-called *adaptive phase* of the Markov chain, with three sub-phases of 40, 80, and 120 thousand iterations, respectively. At the end of each sub-phase we (potentially) update the estimate of the posterior mode, and compute a sample covariance matrix based on the latter half of the draws, to be used in the jumping distribution in the next sub-phase. Finally, in each sub-phase we rescale the covariance matrix inherited from the previous sub-phase in order to get a *fine-tuned covariance matrix* that yields rejection rates as close as possible to 0.77.¹⁶ Next we run a so-called *fixed phase*. We take the estimate of the posterior mode and sample covariance matrix from the adaptive phase, and run 5 parallel chains of 300,000 iterations each. Again, before making the draws that will form the sample we rescale such covariance matrix in order to get rejection rates as close as possible to 0.77. To initialize each chain we draw from a candidate normal distribution centered on the posterior mode estimate, with covariance matrix given by 9 times the *fine-tuned covariance matrix*. We check for convergence for the latter 2/3s of the draws of all 5 chains by calculating the potential scale reduction¹⁷ (PSR) factors for each parameter and inspecting the histograms across the parallel chains. Upon convergence, the latter 2/3s of the draws of all 5 chains are combined to form a posterior sample of 1 million draws.

Having obtained a sample of the posterior distribution, we can also estimate the marginal density of the data given a model (henceforth MD) as:

$$\text{MD} = f(Z^* | \mathcal{M}_j) = \int \mathcal{L}(\theta | Z^*, \mathcal{M}_j) f(\theta | \mathcal{M}_j) d\theta,$$

where \mathcal{M}_j refers to a specific configuration of the model and prior distribution, and

¹⁴These conditions are discussed in Gelman et al. (2003, part III).

¹⁵The first optimization routine is `csminwel` by Chris Sims, while the second is `fminsearch` from Matlab's optimization toolbox.

¹⁶This is the optimal rejection rate under some conditions. See Gelman et al. (2003, p. 306).

¹⁷For each parameter, the PSR factor is the ratio of (square root of) an estimate of the marginal posterior variance to the average variance within each chain. This factor expresses the potential reduction in the scaling of the estimated marginal posterior variance relative to the true distribution by increasing the number of iterations in the Markov-chain algorithm. Hence, as the PSR factor approaches unity, it is a sign of convergence of the Markov-chain for the estimated parameter. See Gelman et al (2003, p. 294 ff) for more information.

$f(\theta|\mathcal{M}_j)$ denotes the corresponding joint prior distribution. Specifically, we approximate the log MD for each model using Geweke’s (1999) modified harmonic mean. We use this to evaluate the empirical fit of the models relative to one another; the MD ratio of two models constitutes the *Bayes factor*, and – when neither configuration is *a priori* considered more likely – the *posterior odds*. It indicates how likely the two models are relative to one another given the observed data Z^* .

4 Results

The results of the estimations are presented in Tables 3-8 and in Figures 6-23 in terms of marginal distributions for the parameters.¹⁸ A general trend across the estimations is that as we allow for sectors with contracts longer than four quarters, these sectors are assigned significant mass, and usually more than in the prior.¹⁹ By implication, the posterior estimate of the average contract length, \bar{k} , is pulled up relative to its prior level, increasing the average degree of nominal rigidity in the model. Higher estimates of \bar{k} are associated with higher estimates of ζ , i.e., a lower degree of strategic complementarity in price setting. That is, the results push towards a higher degree of nominal rigidity compensated through lowering the degree of real rigidity. Across different specifications, as ζ increases the variance of the innovations to the natural output process, σ_n^2 , decreases. The connection between these key parameters is summarized in Table 2. As prices respond more to movements in the output gap, the estimation requires less volatility in the latter to match the observed dynamics of inflation and output. The estimated AR process (6) for nominal output is quite similar across the configurations of the model. This is not very surprising as both output and inflation were included in the observed data set.

Another key common feature across specifications with heterogeneous firms is that the estimation pushes towards an increased degree of heterogeneity as captured by the standard deviation of contract lengths, σ_k , cf. also Table 2. This is certainly true in the case of the symmetric and Bils-Klenow priors. The Nakamura-Steinsson priors, on the other hand, imply a relatively high level of heterogeneity *a priori*, once we take account of contracts lasting six quarters or more. In this case, the posterior distribution of σ_k is little changed relative to the prior.

While the different priors appear to have a relatively small influence on the overall fit, allowing for more than four quarters of nominal rigidity seems critical. For all specifications with $K = 4$, the posterior distribution for \bar{k} indicates *less* nominal rigidity and only slightly more heterogeneity than in the priors. In particular, in all cases the posterior median for \bar{k} is between 2 and 2.5 quarters, and therefore in line with the estimates for average nominal price rigidity obtained by Bils and Klenow (2004). However, these results are at odds with specifications with $K > 4$, which clearly indicate the presence of both more nominal rigidity *and* heterogeneity.

These discrepancies between specifications with $K = 4$ and $K > 4$ manifest themselves along two different dimensions. First, specifications with $K = 4$ lead to a large degree of real rigidity that is harder to square with models in which such rigidity can be traced back

¹⁸We use a Gaussian kernel density estimation to graph the posterior marginal density for each parameter. The priors on \bar{k} and σ_k are based on 100,000 draws from the prior Dirichlet distribution.

¹⁹In the discussion of estimation results we refer to the marginal posterior medians when nothing else is stated. Note that since the marginal distributions of the ω ’s are skewed, the medians will not sum to one; obviously, the summation restriction still holds for each parameter vector in the simulated joint posterior distribution.

to structural features of the economy. For such specifications, the posterior median for ζ is between 0.03 and 0.04, while for specifications with $K > 4$ the medians for ζ range from 0.06 to 0.17.²⁰ Second, and perhaps more importantly, the fit of specifications with $K = 4$ is clearly worse relative to models with $K > 4$, as the posterior odds overwhelmingly favor the latter. Across all pairwise comparisons between models with $K = 4$ and models with $K > 4$, the *lowest* posterior odds ratio in favor of models with $K > 4$ is around 24 : 1.

From these results we conclude that the evidence favors specifications with more average nominal rigidity and more heterogeneity in price stickiness than can be accommodated in a model with $K = 4$. As a result, estimations with such a specification produce “too much” real rigidity in an attempt to generate sufficiently sluggish dynamics. In specifications with $K > 4$ real rigidity is “traded-off” against longer average contracts and more heterogeneity to yield a better fit.

Despite the very different empirical methodology, our results with $K = 4$ are quite similar to those obtained by Coenen et al. (2007), who also estimate a model with Taylor staggered price setting and heterogeneous contract lengths of up to four quarters. They find an average contract length that is in line with the evidence in Bils and Klenow (2004), but at the same time find an incredible amount of real rigidity. Our findings suggest that specifications that allow for more heterogeneity can produce significantly different results, in particular in terms of the amount of nominal and real rigidities. Thus, results based on these specifications may be more informative for the purpose of identifying the roles of such rigidities in generating monetary non-neutrality.

In the comparison between models with $K > 4$, it seems that the gains from allowing contracts of seven and eight quarters rather than just up to six quarters are limited. Looking closer at the different priors for the sectoral masses, the Nakamura-Steinsson priors have the best fit for $K = 6$, a result that holds true when we tighten the priors around the empirical NS cross-sectional distribution. Turning to $K = 8$, the estimation based on the NS priors still performs better than the Bils-Klenow priors, yet now the symmetric priors yield results on par with the NS priors. The reason is clear from the first and second moments of the distribution of contract lengths, \bar{k} and σ_k : while the BK priors are markedly off the posterior estimates, the symmetric priors happen to produce values for \bar{k} and σ_k that accord well with those generated by the tight NS priors.

In some cases the estimation results for individual sectoral masses seem to suffer from weak identification. This is particularly the case for some sectoral masses, such as ω_2 and ω_3 . It should come as no surprise that the signs of weak identification increase along with the number of included sectors, K . When we only allow contracts of up to four quarters, the estimation pushes mass towards the extremes, namely one-quarter and four-quarter contracts. Note, in particular the NS prior case for $K = 4$ which *a priori* assigns a lot of mass to four-quarter contracts, which is moved to one-quarter contracts. This reflects the move towards increased heterogeneity that has already been discussed. Once we allow for contracts longer than one year, the mass on four-quarter contracts is markedly reduced. This result is in part driven by the preference for increased heterogeneity as it is found uniformly across the different priors. However, in the case of $K = 8$ the estimation moves mass from four-quarter to five-quarter contracts, even though neither are near end-points of the contract-length distribution and thus important for the overall dispersion in contract lengths. This is particularly clear for the BK priors which *a priori* assigns some mass to four-quarter contracts while five-quarter contracts are virtually

²⁰Woodford (2003) argues forcefully that degrees of real rigidity between 0.10 and 0.15 are consistent with reasonably parameterized models. An index of real rigidity of 0.03 is relatively harder to rationalize.

negligible, a result that is negated after confrontation with the model and the aggregate time series. We should stress that our results do not suggest any particularly important role for five-quarter contracts. Rather, the estimation results may be seen as confirming the conclusions from the studies based on BLS micro-data, namely that four-quarter contracts seem to be of no particular empirical relevance. This contrasts with the view based on the earlier empirical literature on price setting, that the typical firm in the U.S. tends to change prices once a year, which was often used to calibrate one sector models with one-year of price rigidity.

Finally, we also estimated models imposing the strong restriction of homogeneous firms, with contracts ranging from two to eight quarters. We kept the same prior distributions for all parameters besides the sectoral masses, and thus each such model with contracts of length k , say, can be seen as a restriction of the more general heterogeneous model, with a prior over the distribution of sectoral masses that puts probability one on $\omega_k = 1$. The results are presented in Table 9 and in Figures 24-28. All such models produce a very poor fit relative to the models with heterogeneity. As an example, picking the *worst* heterogeneous model and the *best* homogeneous model, the posterior odds in favor of the heterogeneous model is of the order of $7^{35} : 1$. For purely illustrative purposes, Figure 5 shows the differences in performance in terms of the one-step ahead forecasts for the best homogeneous model and one of the best heterogeneous models, where in each case we set the parameter values at the point corresponding to the posterior mode.²¹

The homogenous models also produce estimates for some parameter values that are incredible. In particular, they tend to generate large values of ζ coupled with very large variances of the natural output process, σ_n^2 . In our view, these results arise because of the attempt of such models with Taylor pricing and homogeneous contract lengths to generate relatively smooth dynamics. With $\zeta = 1$, such models generate very unrealistic dynamics, with impulse response functions that tend to display sharp kinks at the time when all prices have been reset after the shock hit. Such kinks can be smoothed by moving ζ significantly away from unity, but that has implications for how the economy responds to different types of shocks.²² Overall, we conclude that in the context of our semi-structural specifications, homogeneous models produce very poor fit, and economically nonsensical results.

5 Conclusion

Our estimation results suggest that heterogeneity of contract lengths is of critical importance for understanding the joint dynamics of inflation and output. In our setup, when the prior distributions imply a relatively low dispersion of contract lengths, this is consistently increased in the estimation process. Also, when we restrict the models by imposing homogeneous contract lengths, results are significantly worse than in the general case with heterogeneity in price stickiness.

Furthermore, our estimation results suggest that it is important to allow for prices in some sectors of the economy to last beyond one year. Neglecting to do so will imply too little nominal rigidity, and increase the estimated degree of real rigidity way beyond

²¹This is only illustrative because our measure of fit - the marginal density of the data given model - is not evaluated at any single value of the parameter vector θ . Instead, it depends on the whole joint posterior distribution, since θ is integrated out.

²²Heterogeneous models, on the other hand, naturally tend to display smoother dynamics, and so preclude the need for such large shifts in ζ .

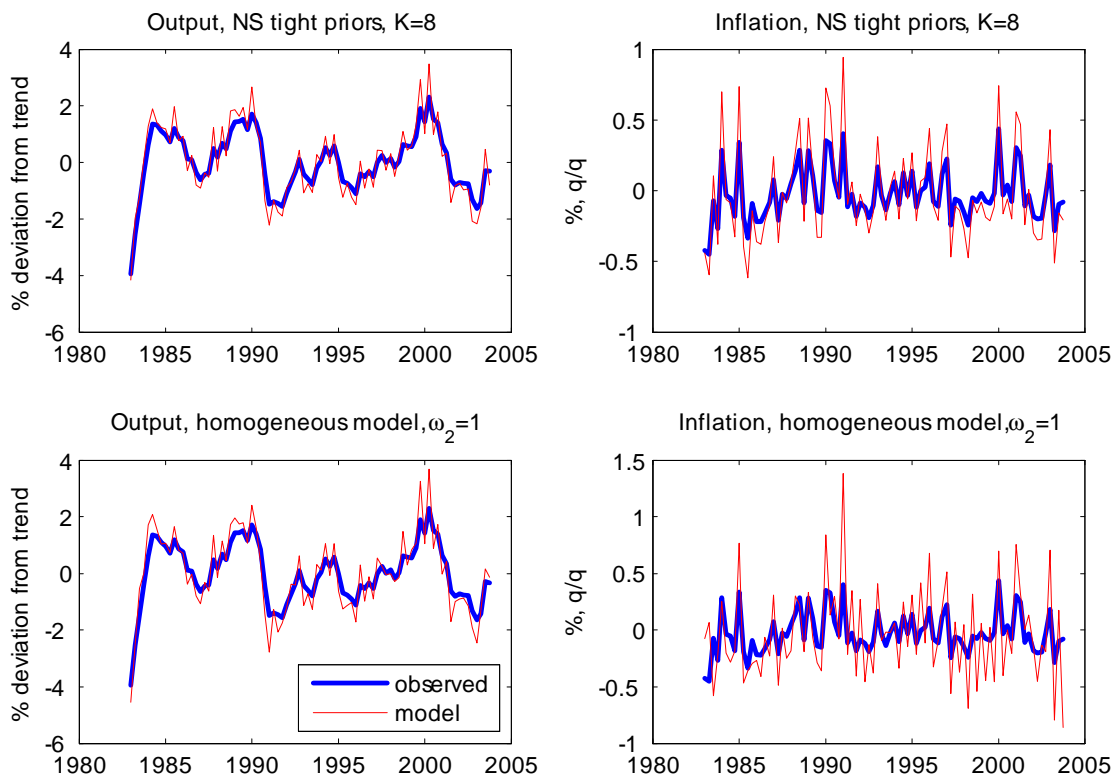


Figure 5: Observed time series and one-step ahead forecast, homogenous vs. heterogeneous models

what is found when more heterogeneity is allowed for. For our semi-structural setup, the restriction of at most one-year-duration price contracts comes at significant costs for the empirical validity of the model. In turn, allowing for contract lengths of one and a half or two years decreased the estimated degree of real rigidity to levels that have been deemed plausible in recent literature (e.g. Woodford 2003), while still leading to an extent of average nominal rigidity that is somewhat in line with the recent evidence based on micro-data for the U.S.

The results generally conform with the cross-sectional distribution of price contracts that we derive from the work of Nakamura and Steinsson (2007b), based on their statistics on the frequency of *regular* price changes. However, the empirical fit seems on par with that obtained from symmetric priors when contracts of up to two years are considered. This suggests that once one has the average contract length and the extent of heterogeneity in price stickiness right, the specific sectoral masses are not of great importance.

We find the results sufficiently compelling to warrant further work. In particular, it would be interesting to evaluate the consequences of allowing for heterogeneous pricing behavior when estimating models of fully structured DSGE economies. The experience with our semi-structural model suggests that combining micro- and macro-data within a Bayesian framework can help us integrate our views on price setting at the microeconomic level, and its implications for aggregate dynamics. In addition, making use of sectoral data as well, along the lines of Lee (2007), can further enhance our understanding of how actual monetary economies work.

Appendix

A Transformation of the sectoral masses

We transform vectors $\omega = (\omega(1), \dots, \omega(K))$ in the K -dimensional unit simplex into vectors $v = (v(1), \dots, v(K))$ in \mathbb{R}^K using the inverse of a restricted multivariate logistic transformation. We want to be able to draw v 's and then use a transformation that guarantees that $\omega = h^{-1}(v)$ is in the K -dimensional unit simplex. For that purpose, we start with:

$$\omega(k) = \frac{e^{v(k)}}{\sum_{k=1}^K e^{v(k)}}, k = 1, \dots, K.$$

The application of exponentials in this way guarantees the non-negativity and summation to unity constraints. However, without additional restrictions on h^{-1} , the mapping is not one-to-one. The reason is that all vectors v along the same ray give rise to the same ω . Therefore, we adopt the restriction $v(K) = 0$ and in effect draw vectors $\tilde{v} = (v(1), \dots, v(K-1))$ in \mathbb{R}^{K-1} . Thus, the transformation becomes $\tilde{\omega} = \tilde{h}^{-1}(\tilde{v})$, with $\tilde{\omega} = (\omega(1), \dots, \omega(K-1))$ and:

$$\begin{aligned} \omega(k) &= \frac{e^{v(k)}}{1 + \sum_{k=1}^{K-1} e^{v(k)}}, k = 1, \dots, K-1 \\ \omega(K) &= \frac{1}{1 + \sum_{k=1}^{K-1} e^{v(k)}}. \end{aligned}$$

If the density $f_\omega(\omega|\alpha)$ is that of the Dirichlet distribution with (vector) parameter α , the density of \tilde{v} is given by:

$$f_{\tilde{v}}(\tilde{v}|\alpha) = |J| f_\omega \left(\frac{e^{v(1)}}{1 + \sum_{k=1}^{K-1} e^{v(k)}}, \dots, \frac{1}{1 + \sum_{k=1}^{K-1} e^{v(k)}} \middle| \alpha \right),$$

where $|J|$ is the determinant of the Jacobian Matrix $\left[\frac{\partial \tilde{h}^{-1}(\tilde{v})}{\partial \tilde{v}} \right]_{ij}$ given by:

$$\begin{bmatrix} \frac{\partial \omega(1)}{\partial v(1)} & \frac{\partial \omega(1)}{\partial v(2)} & \cdots & \frac{\partial \omega(1)}{\partial v(K-1)} \\ \frac{\partial \omega(2)}{\partial v(1)} & \frac{\partial \omega(2)}{\partial v(2)} & \cdots & \frac{\partial \omega(2)}{\partial v(K-1)} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial \omega(K-1)}{\partial v(1)} & \frac{\partial \omega(K-1)}{\partial v(2)} & \cdots & \frac{\partial \omega(K-1)}{\partial v(K-1)} \end{bmatrix},$$

with:

$$\begin{aligned} \frac{\partial \omega(k)}{\partial v(k)} &= \frac{e^{v(k)} \left(1 + \sum_{k=1}^{K-1} e^{v(k)} \right) - e^{v(k)} e^{v(k)}}{\left(1 + \sum_{k=1}^{K-1} e^{v(k)} \right)^2} \\ &= \frac{e^{v(k)}}{1 + \sum_{k=1}^{K-1} e^{v(k)}} - \frac{e^{v(k)} e^{v(k)}}{\left(1 + \sum_{k=1}^{K-1} e^{v(k)} \right)^2}. \end{aligned}$$

So:

$$J = - \begin{bmatrix} \frac{e^{v(1)}}{1 + \sum_{k=1}^{K-1} e^{v(k)}} \\ \vdots \\ \frac{e^{v(K-1)}}{1 + \sum_{k=1}^{K-1} e^{v(k)}} \end{bmatrix} \left[\frac{e^{v(1)}}{1 + \sum_{k=1}^{K-1} e^{v(k)}}, \dots, \frac{e^{v(K-1)}}{1 + \sum_{k=1}^{K-1} e^{v(k)}} \right] + \begin{bmatrix} \frac{e^{v(1)}}{1 + \sum_{k=1}^{K-1} e^{v(k)}} & 0 & \dots & 0 \\ 0 & \ddots & 0 & \vdots \\ \vdots & 0 & \ddots & 0 \\ 0 & \dots & 0 & \frac{e^{v(K-1)}}{1 + \sum_{k=1}^{K-1} e^{v(k)}} \end{bmatrix}$$

To recover the $v(k)$'s from ω simply set:

$$v(k) = \ln(\omega(k)) - \ln(\omega(K)).$$

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Table 2: Selected Parameter Estimates

	$K = 4$		$K = 6$		$K = 8$	
<i>Symmetric priors</i>						
ζ	1.678 (0.355;4.744)	0.039 (0.020;0.077)	1.678 (0.355;4.744)	0.103 (0.054;0.197)	1.678 (0.355;4.744)	0.168 (0.093;0.305)
\bar{k}	2.497 (1.664;3.333)	2.389 (1.998;2.775)	3.498 (2.433;4.559)	3.594 (3.129;4.049)	4.502 (3.239;5.763)	4.577 (4.119;5.030)
σ_k	0.981 (0.620;1.317)	1.158 (0.993;1.294)	1.558 (1.094;2.009)	1.756 (1.579;1.925)	2.136 (1.580;2.679)	2.296 (2.101;2.487)
σ_n	0.017 (0.004;0.047)	0.047 (0.026;0.079)	0.017 (0.004;0.047)	0.037 (0.021;0.064)	0.017 (0.004;0.047)	0.033 (0.019;0.056)
log MD	–	706.128	–	711.854	–	713.388
<i>Loose NS priors</i>						
ζ	1.678 (0.355;4.744)	0.038 (0.015;0.086)	1.678 (0.355;4.744)	0.130 (0.053;0.289)	1.678 (0.355;4.744)	0.171 (0.069;0.395)
\bar{k}	3.009 (1.873;3.786)	2.395 (1.757;2.993)	3.796 (2.435;5.043)	4.159 (3.074;4.968)	4.224 (2.790;5.716)	4.816 (3.550;5.936)
σ_k	1.195 (0.629;1.466)	1.348 (1.095;1.479)	1.955 (1.403;2.336)	1.998 (1.607;2.290)	2.501 (1.888;3.021)	2.430 (1.947;2.878)
σ_n	0.017 (0.004;0.047)	0.035 (0.020;0.065)	0.017 (0.004;0.047)	0.029 (0.016;0.055)	0.017 (0.004;0.047)	0.027 (0.016;0.050)
log MD	–	706.038	–	712.613	–	712.648
<i>Tight NS priors</i>						
ζ	1.678 (0.355;4.744)	0.051 (0.026;0.099)	1.678 (0.355;4.744)	0.110 (0.056;0.214)	1.678 (0.355;4.744)	0.130 (0.067;0.241)
\bar{k}	2.952 (2.447;3.390)	2.702 (2.285;3.078)	3.780 (3.153;4.387)	3.893 (3.286;4.459)	4.232 (3.558;4.923)	4.379 (3.715;5.033)
σ_k	1.293 (1.059;1.427)	1.351 (1.218;1.440)	2.070 (1.847;2.243)	2.076 (1.879;2.234)	2.627 (2.362;2.867)	2.622 (2.380;2.842)
σ_n	0.017 (0.004;0.047)	0.039 (0.022;0.067)	0.017 (0.004;0.047)	0.028 (0.017;0.049)	0.017 (0.004;0.047)	0.025 (0.015;0.043)
log MD	–	706.071	–	712.442	–	712.925
<i>Loose BK priors</i>						
ζ	1.678 (0.355;4.744)	0.030 (0.011;0.070)	1.678 (0.355;4.744)	0.076 (0.028;0.185)	1.678 (0.355;4.744)	0.082 (0.031;0.199)
\bar{k}	2.177 (1.391;3.174)	2.133 (1.551;2.737)	2.371 (1.566;3.575)	3.245 (2.218;4.275)	2.454 (1.677;3.681)	3.381 (2.323;4.641)
σ_k	1.075 (0.637;1.397)	1.215 (0.912;1.414)	1.440 (0.867;2.076)	1.887 (1.531;2.213)	1.629 (1.001;2.544)	2.165 (1.624;2.798)
σ_n	0.017 (0.004;0.047)	0.038 (0.021;0.069)	0.017 (0.004;0.047)	0.030 (0.017;0.055)	0.017 (0.004;0.047)	0.028 (0.016;0.051)
log MD	–	706.612	–	711.835	–	712.184
<i>Tight BK priors</i>						
ζ	1.678 (0.355;4.744)	0.032 (0.016;0.064)	1.678 (0.355;4.744)	0.049 (0.024;0.097)	1.678 (0.355;4.744)	0.053 (0.027;0.100)
\bar{k}	2.211 (1.802;2.671)	2.191 (1.827;2.583)	2.428 (1.995;2.955)	2.635 (2.159;3.188)	2.520 (2.098;3.046)	2.728 (2.259;3.305)
σ_k	1.184 (0.987;1.332)	1.212 (1.042;1.339)	1.584 (1.274;1.897)	1.756 (1.460;2.015)	1.804 (1.431;2.242)	2.002 (1.620;2.410)
σ_n	0.017 (0.004;0.047)	0.040 (0.023;0.069)	0.017 (0.004;0.047)	0.031 (0.018;0.054)	0.017 (0.004;0.047)	0.029 (0.017;0.049)
log MD	–	707.157	–	710.401	–	710.349

Note: For each configuration, the prior (left) and posterior (right) median is reported with the 10th and 90th percentile in parentheses. The logarithm of the Marginal Posterior (log MD) is approximated with Geweke's (1999) modified harmonic mean.

Table 3: Parameter Estimates, symmetric loose priors

	$K = 4$		$K = 6$		$K = 8$	
ζ	1.678 (0.355;4.744)	0.039 (0.020;0.077)	1.678 (0.355;4.744)	0.103 (0.054;0.197)	1.678 (0.355;4.744)	0.168 (0.093;0.305)
ω_1	0.206 (0.017;0.632)	0.312 (0.167;0.487)	0.129 (0.010;0.451)	0.172 (0.090;0.281)	0.094 (0.007;0.348)	0.125 (0.070;0.198)
ω_2	0.206 (0.017;0.632)	0.229 (0.101;0.396)	0.129 (0.010;0.451)	0.157 (0.078;0.265)	0.094 (0.007;0.348)	0.121 (0.066;0.195)
ω_3	0.206 (0.017;0.632)	0.178 (0.071;0.330)	0.129 (0.010;0.451)	0.129 (0.064;0.220)	0.094 (0.007;0.348)	0.104 (0.056;0.168)
ω_4	0.206 (0.017;0.632)	0.254 (0.125;0.412)	0.129 (0.010;0.451)	0.130 (0.063;0.224)	0.094 (0.007;0.348)	0.105 (0.056;0.171)
ω_5	–	–	0.129 (0.010;0.451)	0.198 (0.101;0.321)	0.094 (0.007;0.348)	0.137 (0.075;0.218)
ω_6	–	–	0.129 (0.010;0.451)	0.184 (0.091;0.305)	0.094 (0.007;0.348)	0.127 (0.068;0.206)
ω_7	–	–	–	–	0.094 (0.007;0.348)	0.126 (0.068;0.204)
ω_8	–	–	–	–	0.094 (0.007;0.348)	0.128 (0.070;0.206)
\bar{k}	2.497 (1.664;3.333)	2.389 (1.998;2.775)	3.498 (2.433;4.559)	3.594 (3.129;4.049)	4.502 (3.239;5.763)	4.577 (4.119;5.030)
σ_k	0.981 (0.620;1.317)	1.158 (0.993;1.294)	1.558 (1.094;2.009)	1.756 (1.579;1.925)	2.136 (1.580;2.679)	2.296 (2.101;2.487)
ρ_0	0.000 (–8.224;8.224)	0.000 (–0.002;0.003)	0.000 (–8.224;8.224)	0.001 (–0.002;0.003)	0.000 (–8.224;8.224)	0.000 (–0.002;0.002)
ρ_1	0.000 (–8.224;8.224)	1.150 (0.990;1.306)	0.000 (–8.224;8.224)	1.151 (0.988;1.308)	0.000 (–8.224;8.224)	1.150 (0.988;1.309)
ρ_2	0.000 (–8.224;8.224)	–0.368 (–0.518;–0.214)	0.000 (–8.224;8.224)	–0.356 (–0.506;–0.202)	0.000 (–8.224;8.224)	–0.346 (–0.496;–0.193)
σ_m	0.017 (0.004;0.047)	0.005 (0.004;0.005)	0.017 (0.004;0.047)	0.005 (0.004;0.005)	0.017 (0.004;0.047)	0.005 (0.004;0.005)
δ_0	0.000 (–8.224;8.224)	0.001 (–0.001;0.004)	0.000 (–8.224;8.224)	0.001 (–0.001;0.003)	0.000 (–8.224;8.224)	0.001 (–0.001;0.003)
δ_1	0.000 (–8.224;8.224)	0.199 (–0.036;0.420)	0.000 (–8.224;8.224)	0.250 (0.015;0.480)	0.000 (–8.224;8.224)	0.274 (0.052;0.493)
δ_2	0.000 (–8.224;8.224)	0.029 (–0.111;0.175)	0.000 (–8.224;8.224)	0.085 (–0.074;0.251)	0.000 (–8.224;8.224)	0.107 (–0.053;0.267)
σ_n	0.017 (0.004;0.047)	0.047 (0.026;0.079)	0.017 (0.004;0.047)	0.037 (0.021;0.064)	0.017 (0.004;0.047)	0.033 (0.019;0.056)
log MD	–	706.128	–	711.854	–	713.388

Note: For each configuration, the prior (left) and posterior (right) median is reported with the 10th and 90th percentile in parentheses. The logarithm of the Marginal Posterior (log MD) is approximated with Geweke’s (1999) modified harmonic mean.

Table 4: Parameter Estimates, symmetric tight priors

	$K = 4$		$K = 6$		$K = 8$	
ζ	1.678 (0.355;4.744)	0.043 (0.025;0.076)	1.678 (0.355;4.744)	0.098 (0.057;0.174)	1.678 (0.355;4.744)	0.163 (0.095;0.281)
ω_1	0.242 (0.110;0.419)	0.264 (0.191;0.347)	0.159 (0.070;0.288)	0.169 (0.127;0.216)	0.119 (0.052;0.220)	0.125 (0.097;0.157)
ω_2	0.242 (0.110;0.419)	0.245 (0.174;0.326)	0.159 (0.070;0.288)	0.164 (0.123;0.212)	0.119 (0.052;0.220)	0.124 (0.096;0.156)
ω_3	0.242 (0.110;0.419)	0.232 (0.163;0.311)	0.159 (0.070;0.288)	0.158 (0.117;0.204)	0.119 (0.052;0.220)	0.120 (0.093;0.152)
ω_4	0.242 (0.110;0.419)	0.251 (0.181;0.332)	0.159 (0.070;0.288)	0.158 (0.118;0.205)	0.119 (0.052;0.220)	0.121 (0.093;0.152)
ω_5	–	–	0.159 (0.070;0.288)	0.174 (0.130;0.224)	0.119 (0.052;0.220)	0.127 (0.099;0.161)
ω_6	–	–	0.159 (0.070;0.288)	0.171 (0.128;0.221)	0.119 (0.052;0.220)	0.125 (0.097;0.158)
ω_7	–	–	–	–	0.119 (0.052;0.220)	0.125 (0.097;0.158)
ω_8	–	–	–	–	0.119 (0.052;0.220)	0.126 (0.097;0.158)
\bar{k}	2.499 (2.099;2.901)	2.474 (2.277;2.672)	3.498 (2.997;4.003)	3.520 (3.312;3.729)	4.499 (3.911;5.091)	4.515 (4.305;4.726)
σ_k	1.090 (0.917;1.242)	1.129 (1.048;1.204)	1.678 (1.456;1.889)	1.721 (1.634;1.806)	2.258 (2.000;2.513)	2.294 (2.203;2.383)
ρ_0	0.000 (–8.224;8.224)	0.000 (–0.002;0.003)	0.000 (–8.224;8.224)	0.000 (–0.002;0.003)	0.000 (–8.224;8.224)	0.000 (–0.002;0.002)
ρ_1	0.000 (–8.224;8.224)	1.151 (0.990;1.308)	0.000 (–8.224;8.224)	1.151 (0.989;1.309)	0.000 (–8.224;8.224)	1.148 (0.986;1.307)
ρ_2	0.000 (–8.224;8.224)	–0.370 (–0.520;–0.216)	0.000 (–8.224;8.224)	–0.358 (–0.507;–0.204)	0.000 (–8.224;8.224)	–0.345 (–0.496;–0.192)
σ_m	0.017 (0.004;0.047)	0.005 (0.004;0.005)	0.017 (0.004;0.047)	0.005 (0.004;0.005)	0.017 (0.004;0.047)	0.005 (0.004;0.005)
δ_0	0.000 (–8.224;8.224)	0.001 (–0.001;0.004)	0.000 (–8.224;8.224)	0.001 (–0.001;0.003)	0.000 (–8.224;8.224)	0.001 (–0.001;0.003)
δ_1	0.000 (–8.224;8.224)	0.177 (–0.002;0.361)	0.000 (–8.224;8.224)	0.240 (0.056;0.429)	0.000 (–8.224;8.224)	0.273 (0.090;0.462)
δ_2	0.000 (–8.224;8.224)	0.019 (–0.100;0.145)	0.000 (–8.224;8.224)	0.076 (–0.063;0.223)	0.000 (–8.224;8.224)	0.100 (–0.043;0.246)
σ_n	0.017 (0.004;0.047)	0.050 (0.029;0.082)	0.017 (0.004;0.047)	0.038 (0.022;0.065)	0.017 (0.004;0.047)	0.033 (0.020;0.056)
log MD	–	706.204	–	711.667	–	713.329

Note: For each configuration, the prior (left) and posterior (right) median is reported with the 10th and 90th percentile in parentheses. The logarithm of the Marginal Posterior (log MD) is approximated with Geweke’s (1999) modified harmonic mean.

Table 5: Parameter Estimates, Nakamura-Steinsson loose priors

	$K = 4$		$K = 6$		$K = 8$	
ζ	1.678 (0.355;4.744)	0.038 (0.015;0.086)	1.678 (0.355;4.744)	0.130 (0.053;0.289)	1.678 (0.355;4.744)	0.171 (0.069;0.395)
ω_1	0.233 (0.023;0.659)	0.443 (0.215;0.695)	0.247 (0.046;0.591)	0.211 (0.069;0.443)	0.254 (0.066;0.548)	0.178 (0.061;0.389)
ω_2	0.018 (0.000;0.323)	0.059 (0.000;0.359)	0.031 (0.000;0.278)	0.048 (0.000;0.279)	0.039 (0.001;0.250)	0.047 (0.001;0.238)
ω_3	0.041 (0.000;0.389)	0.028 (0.000;0.209)	0.057 (0.001;0.335)	0.036 (0.001;0.169)	0.066 (0.003;0.302)	0.042 (0.002;0.161)
ω_4	0.569 (0.178;0.902)	0.386 (0.171;0.618)	0.069 (0.002;0.358)	0.034 (0.001;0.178)	0.079 (0.005;0.323)	0.043 (0.003;0.179)
ω_5	–	–	0.021 (0.000;0.252)	0.113 (0.000;0.423)	0.028 (0.000;0.226)	0.139 (0.002;0.398)
ω_6	–	–	0.375 (0.109;0.713)	0.431 (0.164;0.700)	0.099 (0.008;0.355)	0.134 (0.012;0.394)
ω_7	–	–	–	–	0.030 (0.000;0.230)	0.038 (0.000;0.261)
ω_8	–	–	–	–	0.172 (0.030;0.454)	0.207 (0.040;0.453)
\bar{k}	3.009 (1.873;3.786)	2.395 (1.757;2.993)	3.796 (2.435;5.043)	4.159 (3.074;4.968)	4.224 (2.790;5.716)	4.816 (3.550;5.936)
σ_k	1.195 (0.629;1.466)	1.348 (1.095;1.479)	1.955 (1.403;2.336)	1.998 (1.607;2.290)	2.501 (1.888;3.021)	2.430 (1.947;2.878)
ρ_0	0.000 (–8.224;8.224)	0.000 (–0.002;0.003)	0.000 (–8.224;8.224)	0.000 (–0.001;0.002)	0.000 (–8.224;8.224)	0.000 (–0.002;0.002)
ρ_1	0.000 (–8.224;8.224)	1.145 (0.985;1.301)	0.000 (–8.224;8.224)	1.148 (0.988;1.306)	0.000 (–8.224;8.224)	1.150 (0.986;1.308)
ρ_2	0.000 (–8.224;8.224)	–0.354 (–0.506;–0.201)	0.000 (–8.224;8.224)	–0.345 (–0.495;–0.191)	0.000 (–8.224;8.224)	–0.339 (–0.489;–0.186)
σ_m	0.017 (0.004;0.047)	0.005 (0.004;0.005)	0.017 (0.004;0.047)	0.005 (0.004;0.005)	0.017 (0.004;0.047)	0.005 (0.004;0.005)
δ_0	0.000 (–8.224;8.224)	0.001 (–0.001;0.003)	0.000 (–8.224;8.224)	0.001 (–0.001;0.003)	0.000 (–8.224;8.224)	0.001 (–0.001;0.003)
δ_1	0.000 (–8.224;8.224)	0.359 (0.028;0.583)	0.000 (–8.224;8.224)	0.419 (0.005;0.680)	0.000 (–8.224;8.224)	0.435 (0.076;0.694)
δ_2	0.000 (–8.224;8.224)	0.002 (–0.153;0.187)	0.000 (–8.224;8.224)	0.038 (–0.154;0.287)	0.000 (–8.224;8.224)	0.047 (–0.139;0.287)
σ_n	0.017 (0.004;0.047)	0.035 (0.020;0.065)	0.017 (0.004;0.047)	0.029 (0.016;0.055)	0.017 (0.004;0.047)	0.027 (0.016;0.050)
log MD	–	706.038	–	712.613	–	712.648

Note: For each configuration, the prior (left) and posterior (right) median is reported with the 10th and 90th percentile in parentheses. The logarithm of the Marginal Posterior (log MD) is approximated with Geweke’s (1999) modified harmonic mean.

Table 6: Parameter Estimates, Nakamura-Steinsson tight priors

	$K = 4$		$K = 6$		$K = 8$	
ζ	1.678 (0.355;4.744)	0.051 (0.026;0.099)	1.678 (0.355;4.744)	0.110 (0.056;0.214)	1.678 (0.355;4.744)	0.130 (0.067;0.241)
ω_1	0.266 (0.127;0.446)	0.345 (0.214;0.497)	0.268 (0.151;0.413)	0.254 (0.150;0.385)	0.269 (0.165;0.394)	0.250 (0.158;0.364)
ω_2	0.057 (0.008;0.181)	0.073 (0.011;0.206)	0.061 (0.014;0.160)	0.066 (0.016;0.164)	0.064 (0.019;0.147)	0.067 (0.020;0.150)
ω_3	0.085 (0.018;0.223)	0.077 (0.017;0.192)	0.089 (0.028;0.199)	0.079 (0.026;0.166)	0.091 (0.035;0.184)	0.083 (0.033;0.160)
ω_4	0.560 (0.376;0.733)	0.479 (0.328;0.626)	0.101 (0.034;0.215)	0.087 (0.030;0.181)	0.104 (0.042;0.200)	0.092 (0.038;0.175)
ω_5	–	–	0.050 (0.010;0.143)	0.067 (0.013;0.178)	0.053 (0.014;0.131)	0.069 (0.018;0.163)
ω_6	–	–	0.386 (0.248;0.537)	0.408 (0.272;0.552)	0.123 (0.055;0.225)	0.134 (0.061;0.240)
ω_7	–	–	–	–	0.054 (0.014;0.133)	0.057 (0.015;0.139)
ω_8	–	–	–	–	0.193 (0.105;0.308)	0.201 (0.111;0.314)
\bar{k}	2.952 (2.447;3.390)	2.702 (2.285;3.078)	3.780 (3.153;4.387)	3.893 (3.286;4.459)	4.232 (3.558;4.923)	4.379 (3.715;5.033)
σ_k	1.293 (1.059;1.427)	1.351 (1.218;1.440)	2.070 (1.847;2.243)	2.076 (1.879;2.234)	2.627 (2.362;2.867)	2.622 (2.380;2.842)
ρ_0	0.000 (–8.224;8.224)	0.000 (–0.002;0.002)	0.000 (–8.224;8.224)	0.000 (–0.001;0.002)	0.000 (–8.224;8.224)	0.000 (–0.001;0.002)
ρ_1	0.000 (–8.224;8.224)	1.147 (0.986;1.303)	0.000 (–8.224;8.224)	1.146 (0.984;1.306)	0.000 (–8.224;8.224)	1.146 (0.982;1.306)
ρ_2	0.000 (–8.224;8.224)	–0.359 (–0.509;–0.206)	0.000 (–8.224;8.224)	–0.341 (–0.492;–0.187)	0.000 (–8.224;8.224)	–0.330 (–0.482;–0.175)
σ_m	0.017 (0.004;0.047)	0.005 (0.004;0.005)	0.017 (0.004;0.047)	0.005 (0.004;0.005)	0.017 (0.004;0.047)	0.005 (0.004;0.005)
δ_0	0.000 (–8.224;8.224)	0.001 (–0.001;0.003)	0.000 (–8.224;8.224)	0.001 (–0.001;0.003)	0.000 (–8.224;8.224)	0.001 (–0.001;0.003)
δ_1	0.000 (–8.224;8.224)	0.338 (0.124;0.533)	0.000 (–8.224;8.224)	0.421 (0.199;0.632)	0.000 (–8.224;8.224)	0.457 (0.243;0.666)
δ_2	0.000 (–8.224;8.224)	–0.020 (–0.155;0.130)	0.000 (–8.224;8.224)	0.043 (–0.117;0.208)	0.000 (–8.224;8.224)	0.055 (–0.109;0.221)
σ_n	0.017 (0.004;0.047)	0.039 (0.022;0.067)	0.017 (0.004;0.047)	0.028 (0.017;0.049)	0.017 (0.004;0.047)	0.025 (0.015;0.043)
log MD	–	706.071	–	712.442	–	712.925

Note: For each configuration, the prior (left) and posterior (right) median is reported with the 10th and 90th percentile in parentheses. The logarithm of the Marginal Posterior (log MD) is approximated with Geweke's (1999) modified harmonic mean.

Table 7: Parameter Estimates, Bils-Klenow loose priors

	$K = 4$		$K = 6$		$K = 8$	
ζ	1.678 (0.355;4.744)	0.030 (0.011;0.070)	1.678 (0.355;4.744)	0.076 (0.028;0.185)	1.678 (0.355;4.744)	0.082 (0.031;0.199)
ω_1	0.376 (0.072;0.783)	0.457 (0.215;0.725)	0.383 (0.114;0.719)	0.287 (0.105;0.556)	0.386 (0.143;0.679)	0.288 (0.114;0.534)
ω_2	0.194 (0.014;0.619)	0.193 (0.024;0.454)	0.210 (0.032;0.550)	0.177 (0.034;0.396)	0.217 (0.048;0.508)	0.185 (0.046;0.387)
ω_3	0.058 (0.000;0.426)	0.030 (0.000;0.209)	0.075 (0.002;0.368)	0.038 (0.001;0.174)	0.085 (0.006;0.333)	0.048 (0.003;0.174)
ω_4	0.205 (0.017;0.631)	0.259 (0.086;0.492)	0.077 (0.002;0.372)	0.039 (0.001;0.198)	0.087 (0.006;0.336)	0.053 (0.003;0.207)
ω_5	–	–	0.005 (0.000;0.185)	0.154 (0.000;0.490)	0.009 (0.000;0.168)	0.133 (0.000;0.430)
ω_6	–	–	0.054 (0.001;0.329)	0.176 (0.005;0.509)	0.007 (0.000;0.157)	0.021 (0.000;0.300)
ω_7	–	–	–	–	0.005 (0.000;0.146)	0.019 (0.000;0.280)
ω_8	–	–	–	–	0.006 (0.000;0.153)	0.026 (0.000;0.278)
\bar{k}	2.177 (1.391;3.174)	2.133 (1.551;2.737)	2.371 (1.566;3.575)	3.245 (2.218;4.275)	2.454 (1.677;3.681)	3.381 (2.323;4.641)
σ_k	1.075 (0.637;1.397)	1.215 (0.912;1.414)	1.440 (0.867;2.076)	1.887 (1.531;2.213)	1.629 (1.001;2.544)	2.165 (1.624;2.798)
ρ_0	0.000 (–8.224;8.224)	0.000 (–0.002;0.003)	0.000 (–8.224;8.224)	0.001 (–0.001;0.003)	0.000 (–8.224;8.224)	0.000 (–0.002;0.003)
ρ_1	0.000 (–8.224;8.224)	1.147 (0.985;1.303)	0.000 (–8.224;8.224)	1.146 (0.986;1.306)	0.000 (–8.224;8.224)	1.148 (0.987;1.309)
ρ_2	0.000 (–8.224;8.224)	–0.358 (–0.509;–0.202)	0.000 (–8.224;8.224)	–0.343 (–0.495;–0.190)	0.000 (–8.224;8.224)	–0.339 (–0.492;–0.185)
σ_m	0.017 (0.004;0.047)	0.005 (0.004;0.005)	0.017 (0.004;0.047)	0.005 (0.004;0.005)	0.017 (0.004;0.047)	0.005 (0.004;0.005)
δ_0	0.000 (–8.224;8.224)	0.001 (–0.001;0.003)	0.000 (–8.224;8.224)	0.001 (–0.001;0.003)	0.000 (–8.224;8.224)	0.001 (–0.001;0.003)
δ_1	0.000 (–8.224;8.224)	0.251 (–0.049;0.511)	0.000 (–8.224;8.224)	0.317 (–0.028;0.609)	0.000 (–8.224;8.224)	0.328 (0.020;0.608)
δ_2	0.000 (–8.224;8.224)	0.063 (–0.104;0.227)	0.000 (–8.224;8.224)	0.115 (–0.094;0.318)	0.000 (–8.224;8.224)	0.125 (–0.073;0.316)
σ_n	0.017 (0.004;0.047)	0.038 (0.021;0.069)	0.017 (0.004;0.047)	0.030 (0.017;0.055)	0.017 (0.004;0.047)	0.028 (0.016;0.051)
log MD	–	706.612	–	711.835	–	712.184

Note: For each configuration, the prior (left) and posterior (right) median is reported with the 10th and 90th percentile in parentheses. The logarithm of the Marginal Posterior (log MD) is approximated with Geweke’s (1999) modified harmonic mean.

Table 8: Parameter Estimates, Bils-Klenow tight priors

	$K = 4$		$K = 6$		$K = 8$	
ζ	1.678 (0.355;4.744)	0.032 (0.016;0.064)	1.678 (0.355;4.744)	0.049 (0.024;0.097)	1.678 (0.355;4.744)	0.053 (0.027;0.100)
ω_1	0.392 (0.226;0.577)	0.414 (0.265;0.577)	0.393 (0.255;0.544)	0.378 (0.251;0.520)	0.394 (0.273;0.524)	0.381 (0.267;0.504)
ω_2	0.231 (0.102;0.407)	0.223 (0.105;0.373)	0.234 (0.124;0.375)	0.218 (0.120;0.342)	0.235 (0.138;0.356)	0.223 (0.134;0.333)
ω_3	0.103 (0.026;0.249)	0.085 (0.022;0.204)	0.107 (0.038;0.223)	0.087 (0.031;0.177)	0.109 (0.046;0.208)	0.091 (0.039;0.171)
ω_4	0.241 (0.109;0.418)	0.254 (0.136;0.401)	0.109 (0.039;0.226)	0.098 (0.036;0.200)	0.111 (0.047;0.210)	0.104 (0.044;0.193)
ω_5	–	–	0.027 (0.002;0.104)	0.051 (0.005;0.165)	0.029 (0.004;0.095)	0.048 (0.008;0.137)
ω_6	–	–	0.086 (0.026;0.194)	0.126 (0.043;0.251)	0.026 (0.003;0.088)	0.035 (0.004;0.114)
ω_7	–	–	–	–	0.022 (0.002;0.083)	0.032 (0.003;0.108)
ω_8	–	–	–	–	0.024 (0.003;0.086)	0.036 (0.004;0.114)
\bar{k}	2.211 (1.802;2.671)	2.191 (1.827;2.583)	2.428 (1.995;2.955)	2.635 (2.159;3.188)	2.520 (2.098;3.046)	2.728 (2.259;3.305)
σ_k	1.184 (0.987;1.332)	1.212 (1.042;1.339)	1.584 (1.274;1.897)	1.756 (1.460;2.015)	1.804 (1.431;2.242)	2.002 (1.620;2.410)
ρ_0	0.000 (–8.224;8.224)	0.000 (–0.002;0.003)	0.000 (–8.224;8.224)	0.000 (–0.002;0.003)	0.000 (–8.224;8.224)	0.000 (–0.002;0.002)
ρ_1	0.000 (–8.224;8.224)	1.148 (0.987;1.306)	0.000 (–8.224;8.224)	1.147 (0.985;1.305)	0.000 (–8.224;8.224)	1.145 (0.983;1.303)
ρ_2	0.000 (–8.224;8.224)	–0.360 (–0.511;–0.207)	0.000 (–8.224;8.224)	–0.344 (–0.495;–0.190)	0.000 (–8.224;8.224)	–0.337 (–0.488;–0.183)
σ_m	0.017 (0.004;0.047)	0.005 (0.004;0.005)	0.017 (0.004;0.047)	0.005 (0.004;0.005)	0.017 (0.004;0.047)	0.005 (0.004;0.005)
δ_0	0.000 (–8.224;8.224)	0.001 (–0.001;0.004)	0.000 (–8.224;8.224)	0.001 (–0.001;0.003)	0.000 (–8.224;8.224)	0.001 (–0.001;0.003)
δ_1	0.000 (–8.224;8.224)	0.229 (0.017;0.438)	0.000 (–8.224;8.224)	0.290 (0.082;0.500)	0.000 (–8.224;8.224)	0.307 (0.104;0.510)
δ_2	0.000 (–8.224;8.224)	0.058 (–0.082;0.199)	0.000 (–8.224;8.224)	0.116 (–0.037;0.267)	0.000 (–8.224;8.224)	0.123 (–0.027;0.270)
σ_n	0.017 (0.004;0.047)	0.040 (0.023;0.069)	0.017 (0.004;0.047)	0.031 (0.018;0.054)	0.017 (0.004;0.047)	0.029 (0.017;0.049)
log MD	–	707.157	–	710.401	–	710.349

Note: For each configuration, the prior (left) and posterior (right) median is reported with the 10th and 90th percentile in parentheses. The logarithm of the Marginal Posterior (log MD) is approximated with Geweke’s (1999) modified harmonic mean.

Table 9: Parameter Estimates, homogeneous contract lengths

	Prior	$K = 2$	$K = 3$	$K = 4$	$K = 6$	$K = 8$
ζ	1.678 (-8.355;4.744)	9.117 (6.244;12.936)	0.475 (0.345;0.669)	2.477 (1.544;5.529)	7.626 (4.127;14.543)	6.626 (4.009;10.797)
ρ_0	0.000 (-8.224;8.224)	0.000 (-0.002;0.003)	0.000 (-0.004;0.004)	0.000 (-0.003;0.003)	0.000 (-0.002;0.003)	-0.000 (-0.002;0.002)
ρ_1	0.000 (-8.224;8.224)	1.094 (0.915;1.267)	1.103 (0.939;1.260)	1.110 (0.946;1.270)	1.145 (0.979;1.305)	1.142 (0.977;1.301)
ρ_2	0.000 (-8.224;8.224)	-0.288 (-0.456;-0.116)	-0.464 (-0.625;-0.302)	-0.377 (-0.541;-0.214)	-0.373 (-0.539;-0.204)	-0.363 (-0.527;-0.198)
σ_m	0.017 (0.004;0.047)	0.005 (0.004;0.005)	0.005 (0.004;0.006)	0.005 (0.004;0.005)	0.005 (0.004;0.005)	0.005 (0.004;0.005)
δ_0	0.000 (-8.224;8.224)	0.001 (-0.001;0.003)	0.001 (-0.005;0.007)	0.001 (-0.002;0.005)	0.001 (-0.003;0.004)	0.001 (-0.002;0.004)
δ_1	0.000 (-8.224;8.224)	-0.165 (-0.269;-0.068)	-1.181 (-1.231;-1.141)	-0.068 (-0.249;0.275)	-0.112 (-0.577;0.163)	0.557 (0.353;0.740)
δ_2	0.000 (-8.224;8.224)	0.736 (0.642;0.822)	-0.989 (-0.999;-0.960)	-0.505 (-0.713;-0.309)	0.077 (-0.199;0.548)	-0.432 (-0.611;-0.242)
σ_n	0.017 (0.004;0.047)	0.013 (0.011;0.016)	0.268 (0.202;0.349)	0.112 (0.048;0.175)	0.053 (0.032;0.085)	0.064 (0.043;0.097)
log MD	-	623.804	602.118	572.379	601.715	598.079

Note: Prior medians (first column) and posterior medians for each configuration. 10th and 90th percentile in parentheses. The logarithm of the Marginal Posterior (log MD) is approximated with Geweke's (1999) modified harmonic mean.

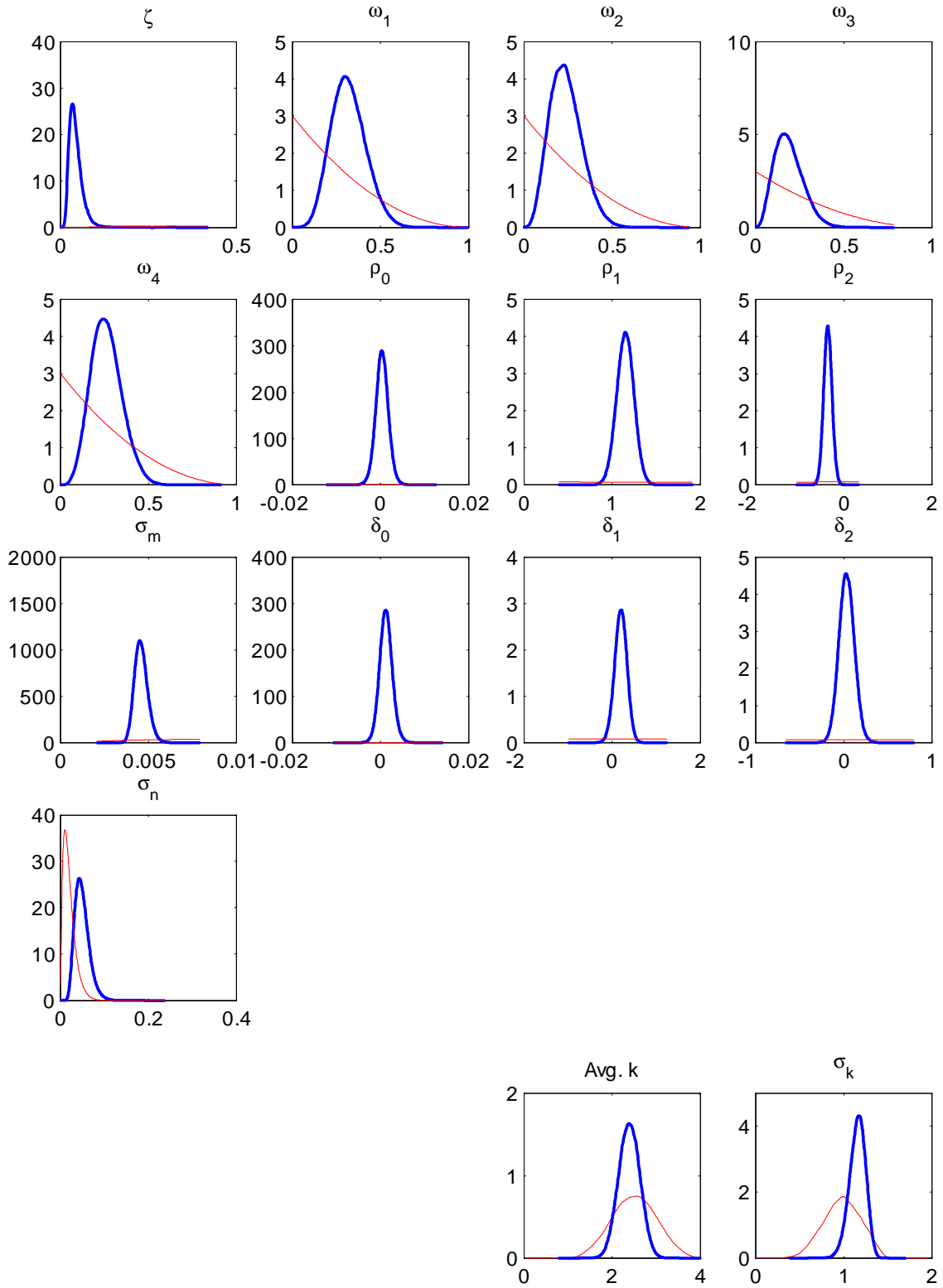


Figure 6: Symmetric loose prior and posterior (bold) distributions, $K = 4$

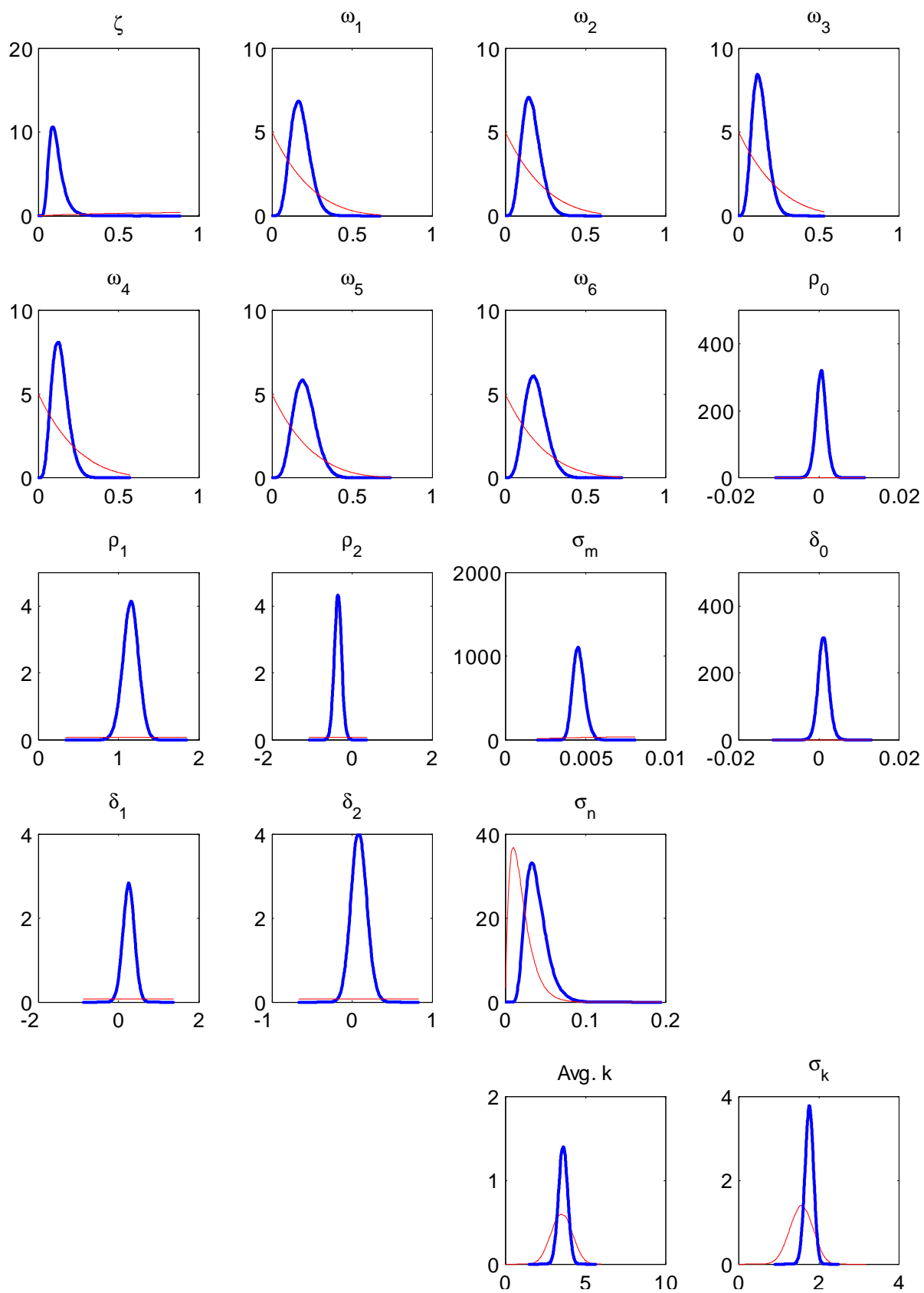


Figure 7: Symmetric loose prior and posterior (bold) distributions, $K = 6$

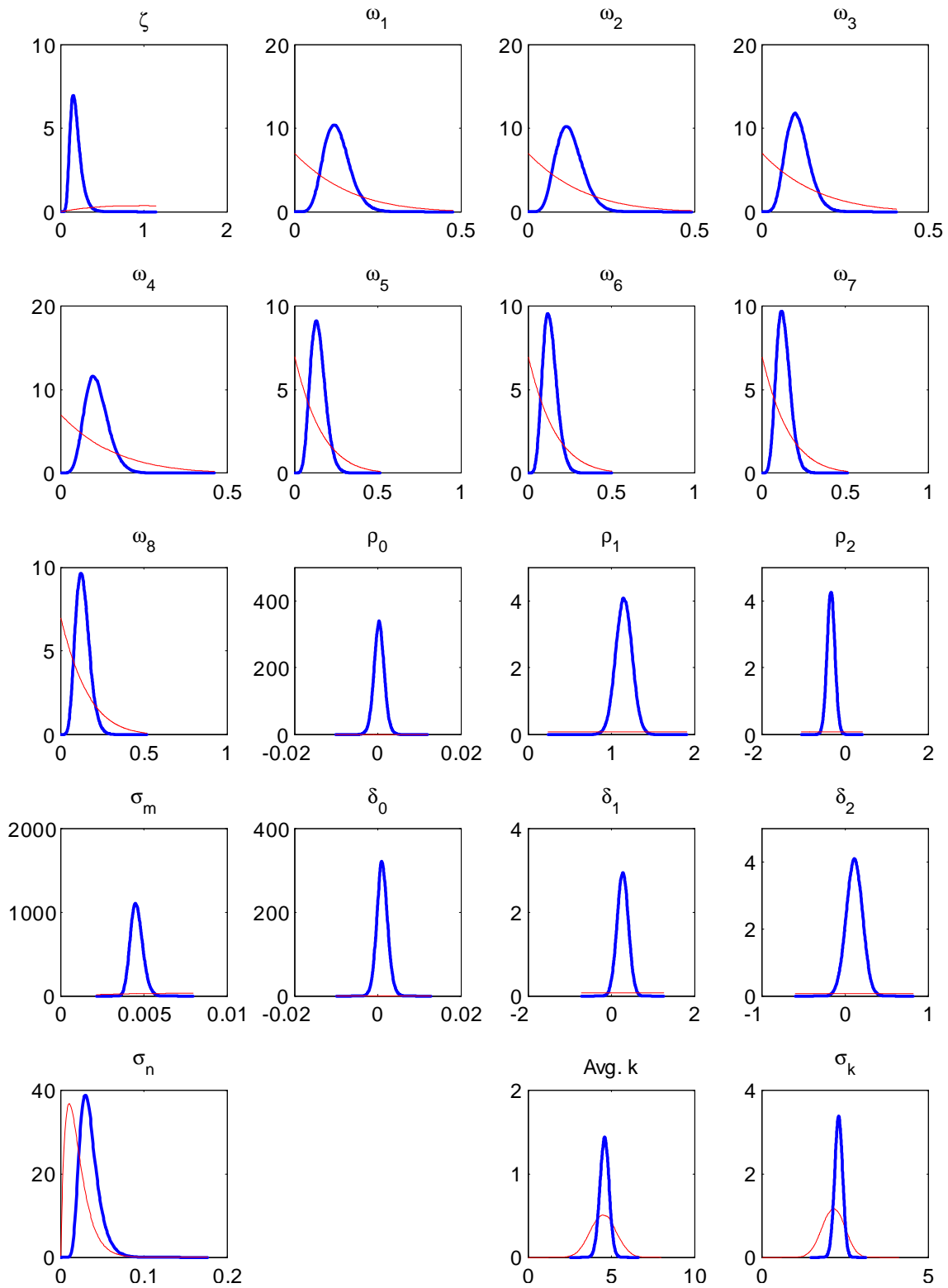


Figure 8: Symmetric loose prior and posterior (bold) distributions, $K = 8$

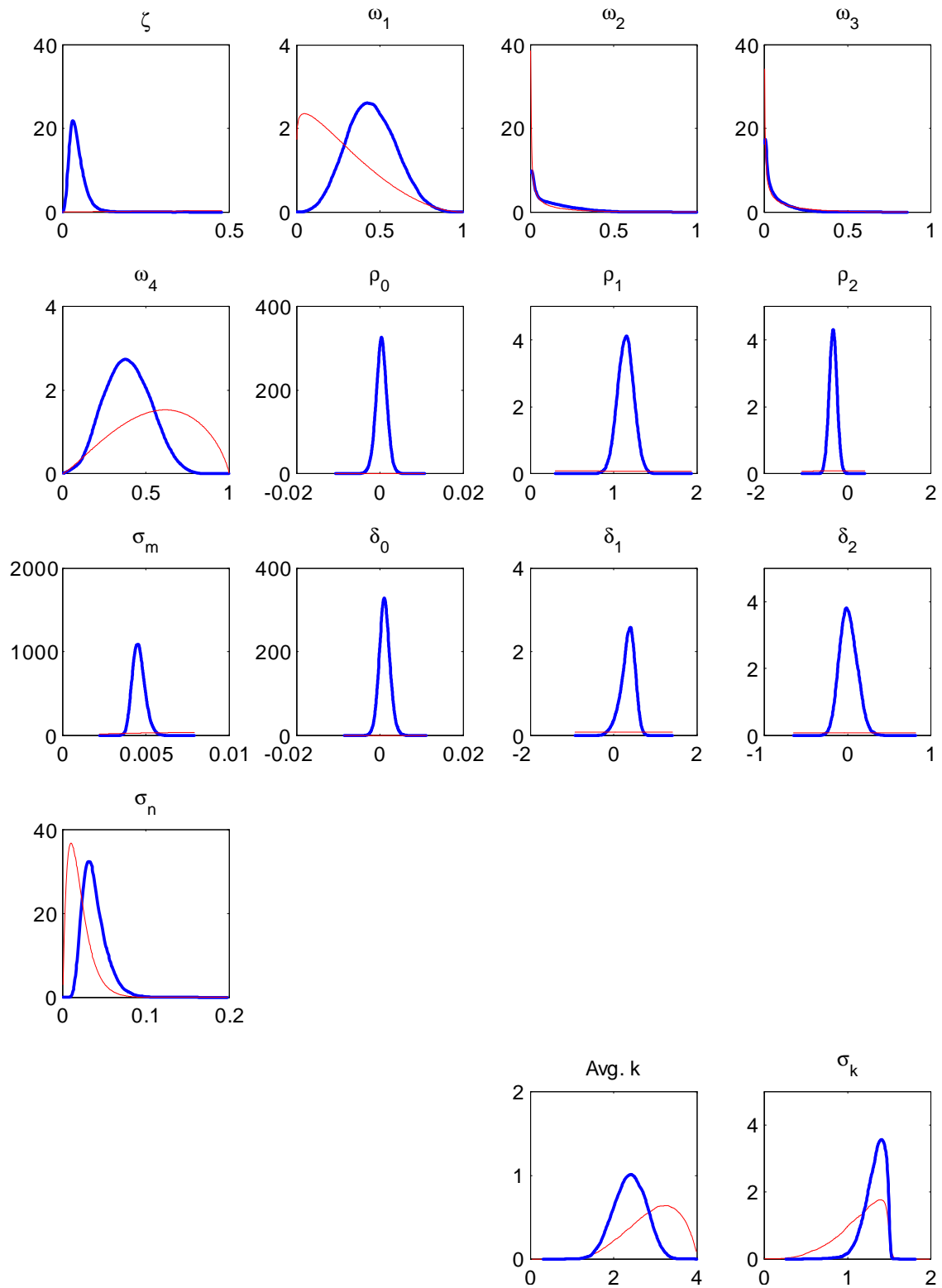


Figure 9: Nakamura-Steinsson loose prior and posterior (bold) distributions, $K = 4$

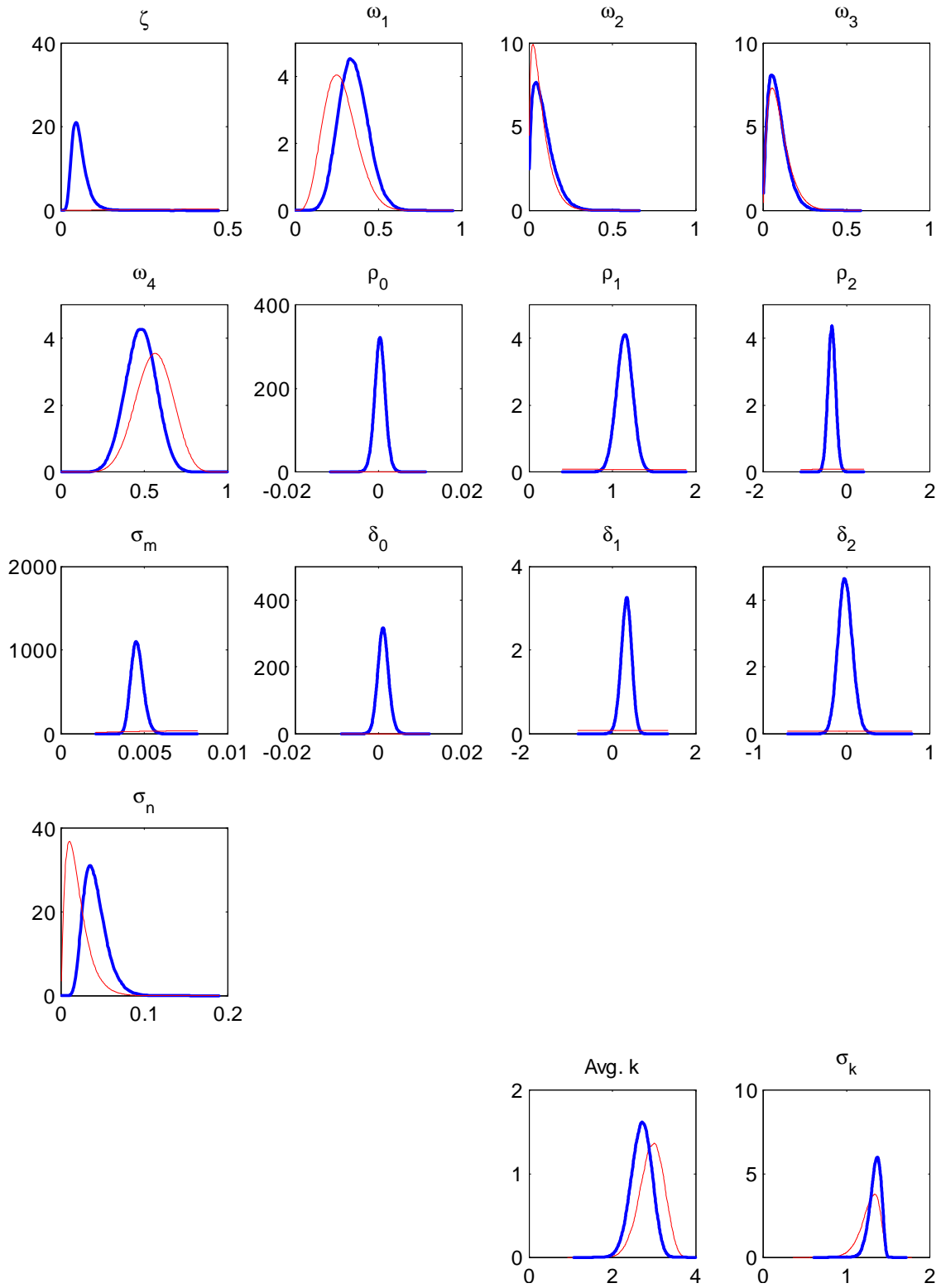


Figure 10: Nakamura-Steinsson tight prior and posterior (bold) distributions, $K = 4$

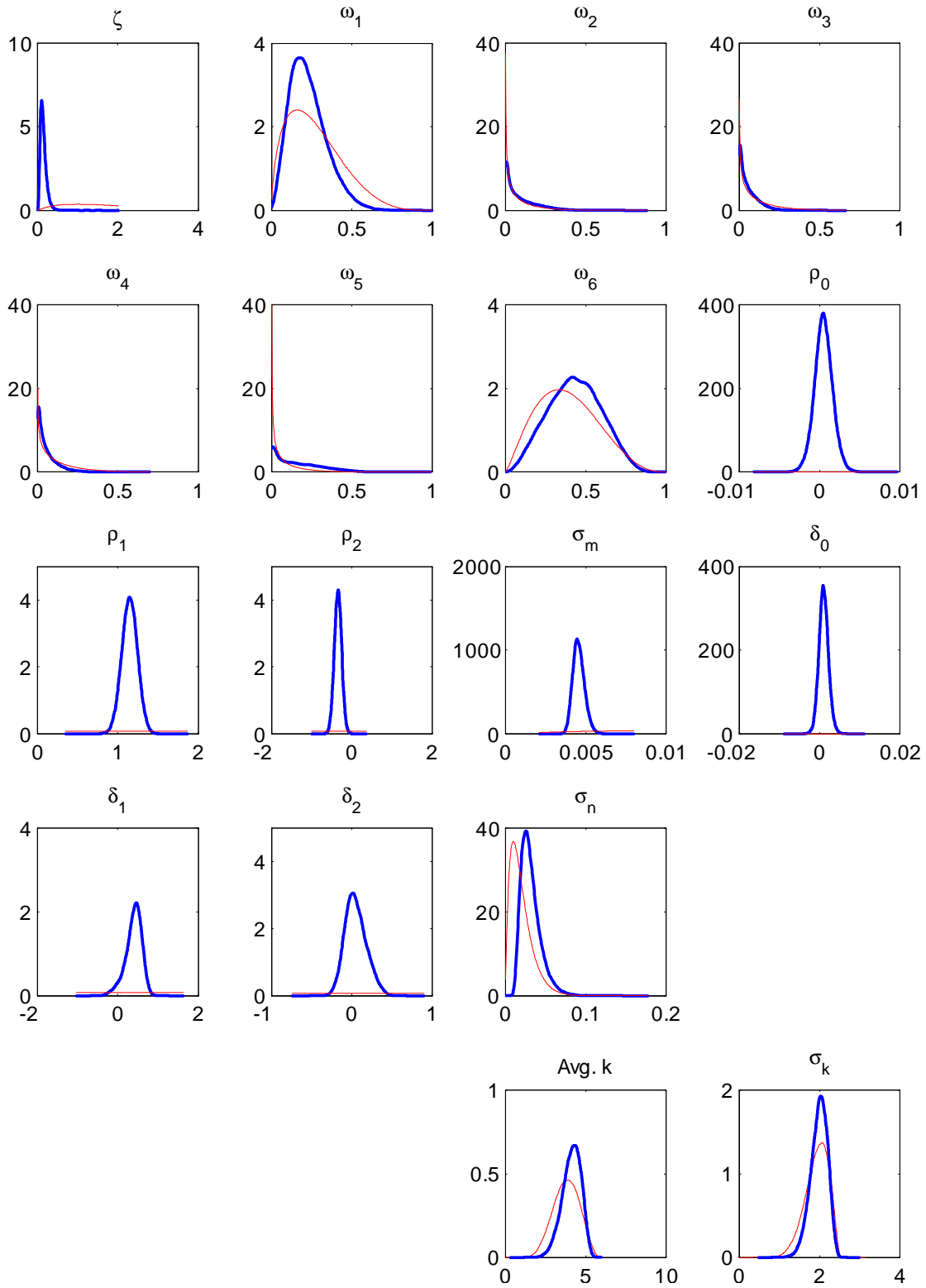


Figure 11: Nakamura-Steinsson loose prior and posterior (bold) distributions, $K = 6$

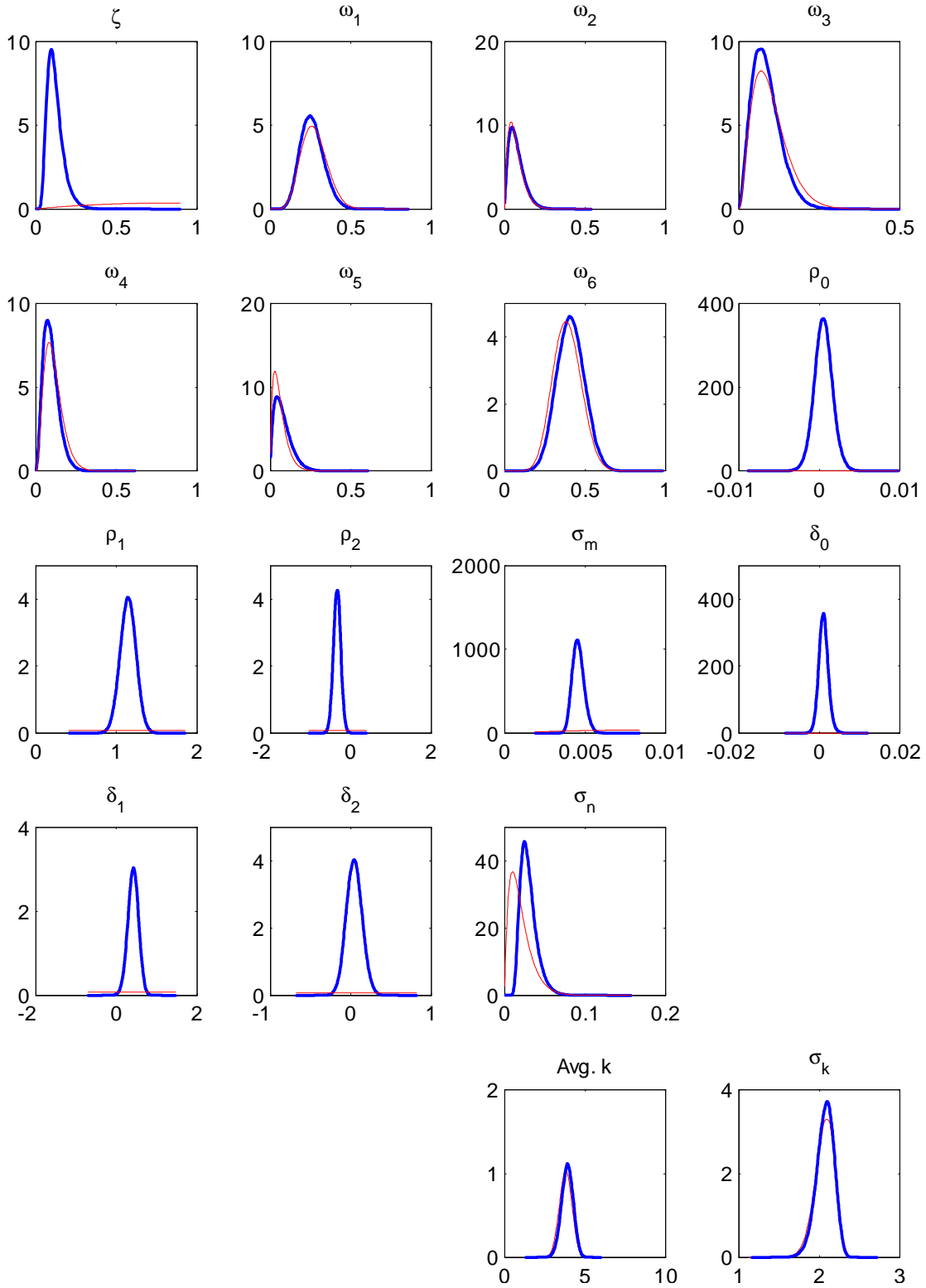


Figure 12: Nakamura-Steinsson tight prior and posterior (bold) distributions, $K = 6$

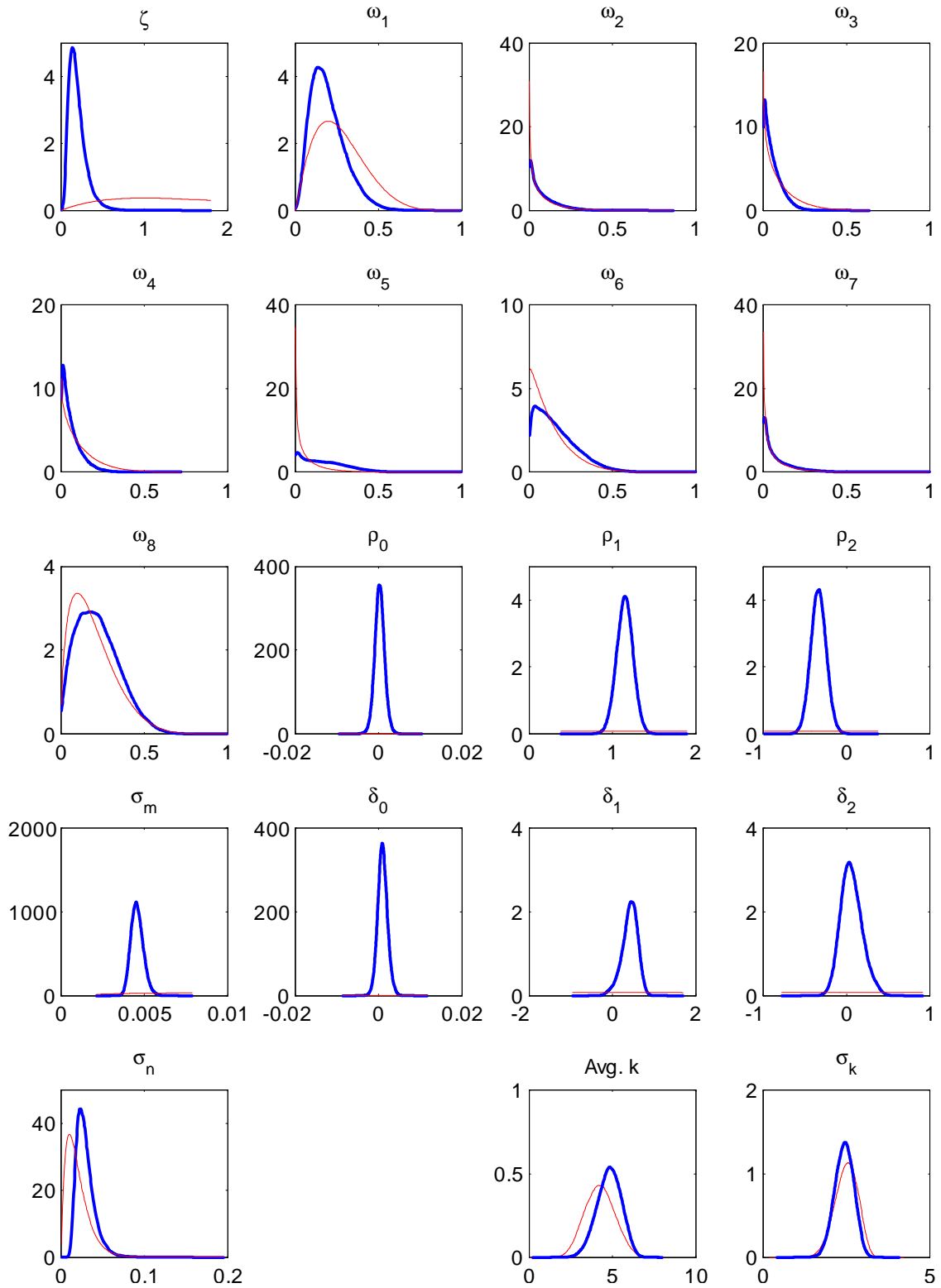


Figure 13: Nakamura-Steinsson loose prior and posterior (bold) distributions, $K = 8$

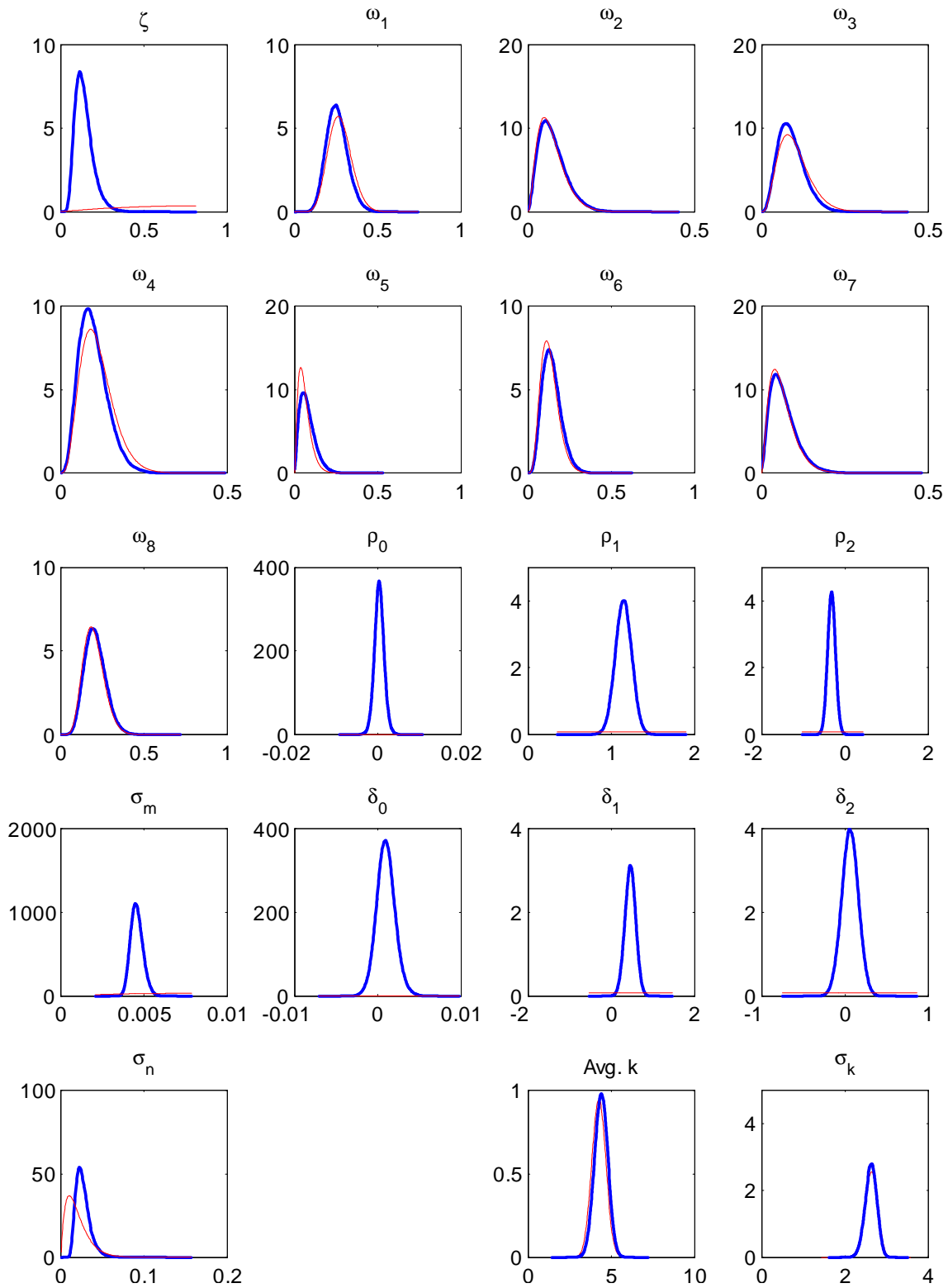


Figure 14: Nakamura-Steinsson tight prior and posterior (bold) distributions, $K = 8$

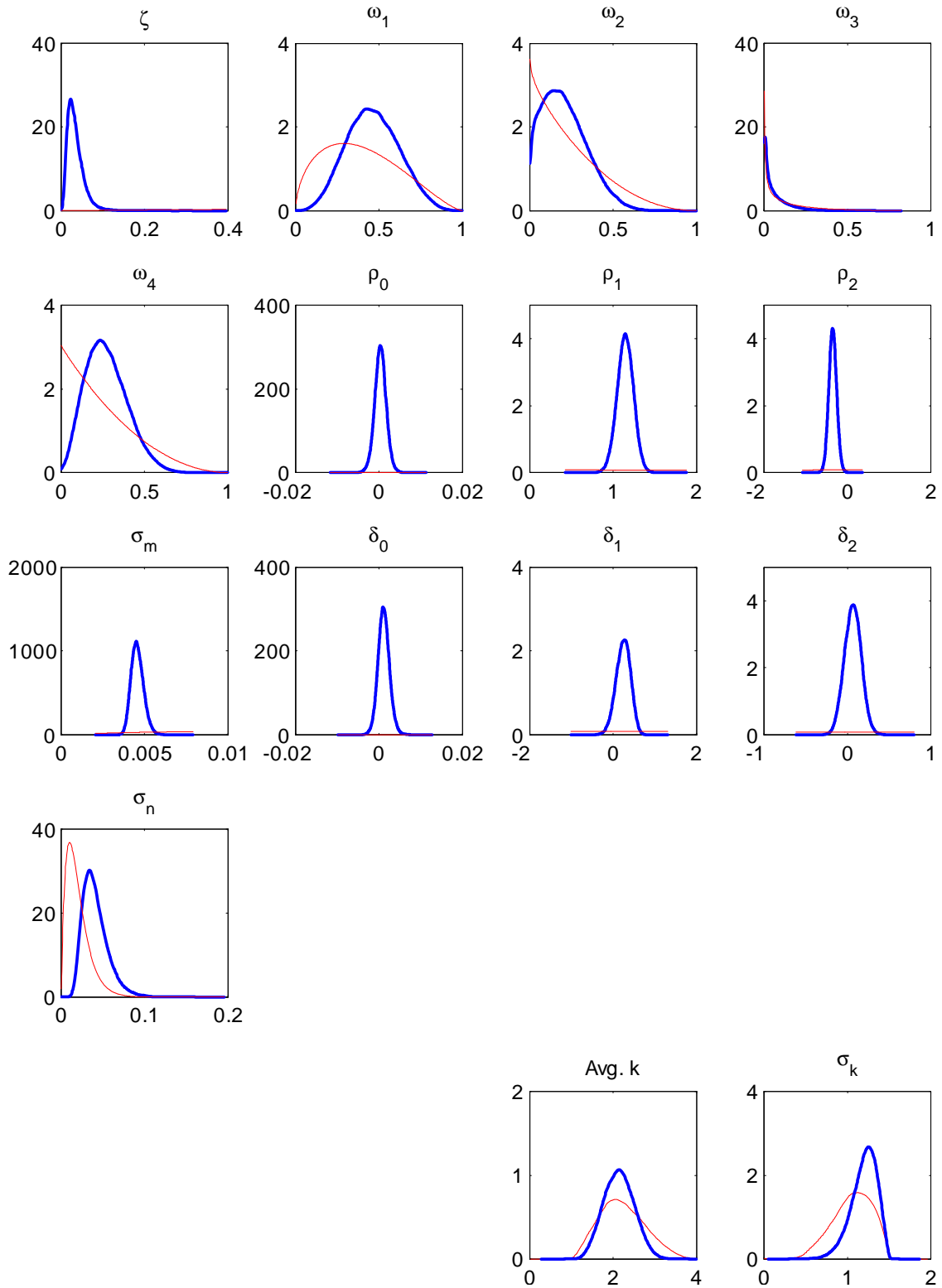


Figure 15: Bilal-Klenow loose prior and posterior (bold) distributions, $K = 4$

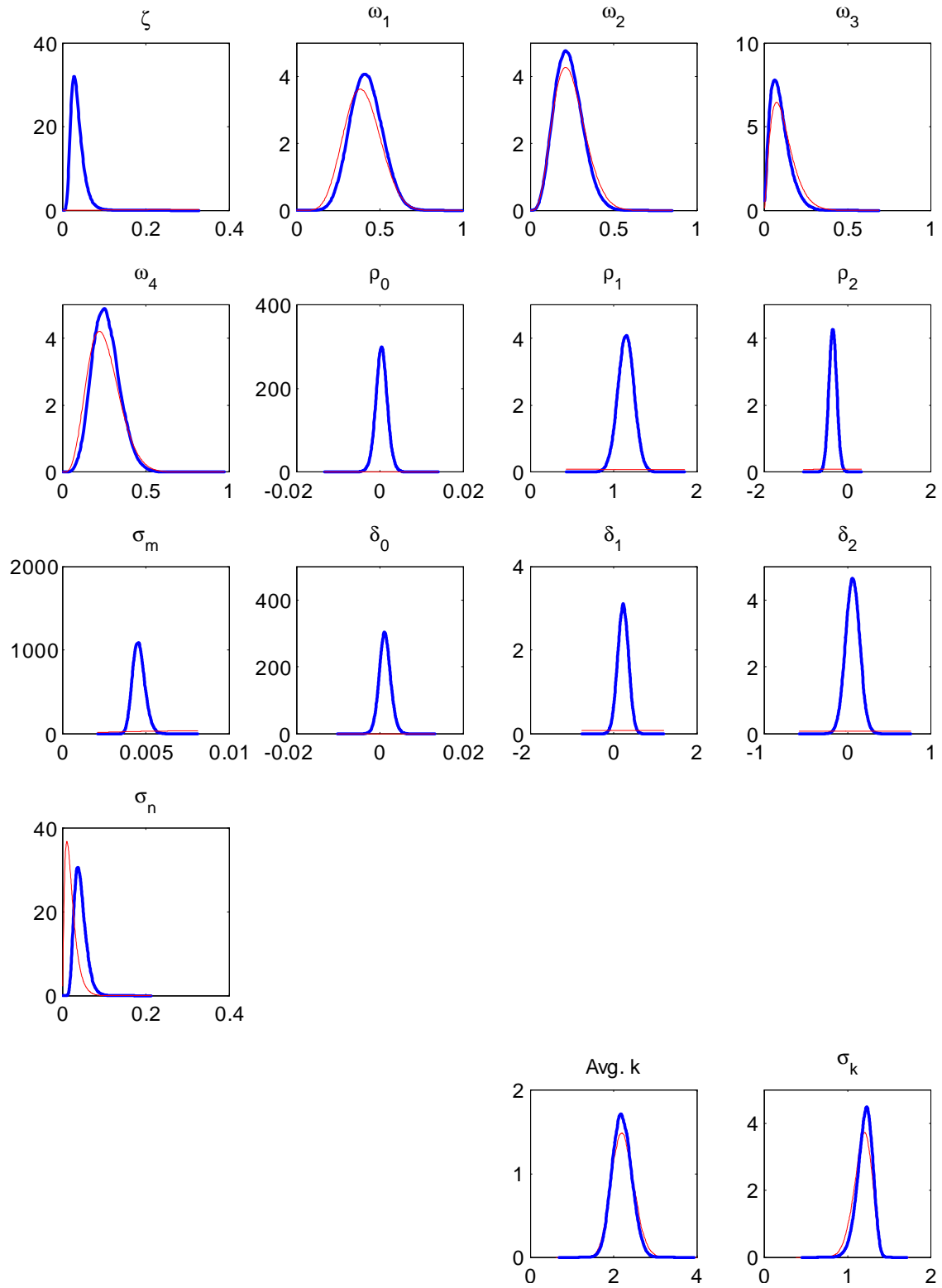


Figure 16: Bilal-Klenow tight prior and posterior (bold) distributions, $K = 4$

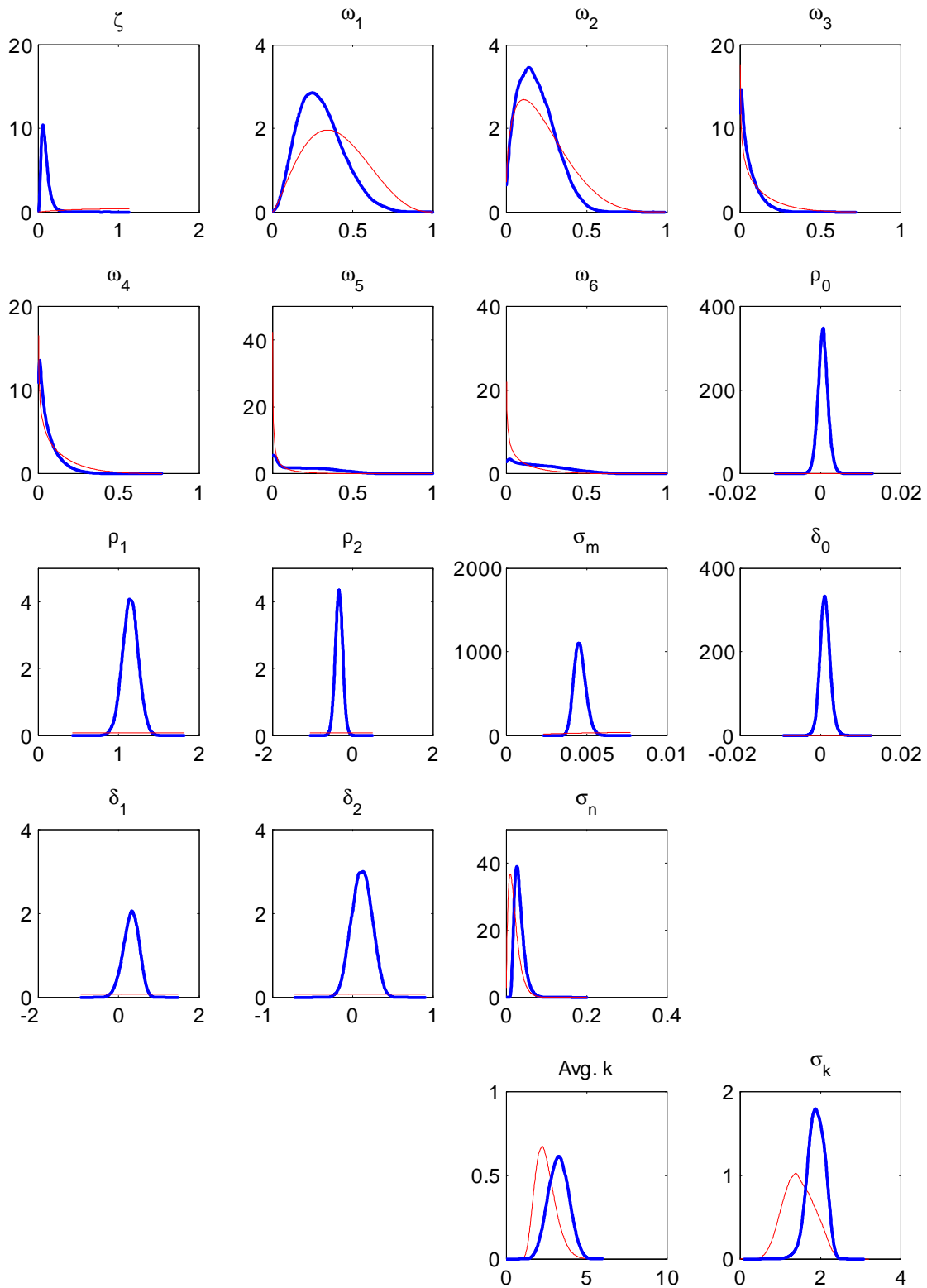


Figure 17: Bils-Klenow loose prior and posterior (bold) distributions, $K = 6$

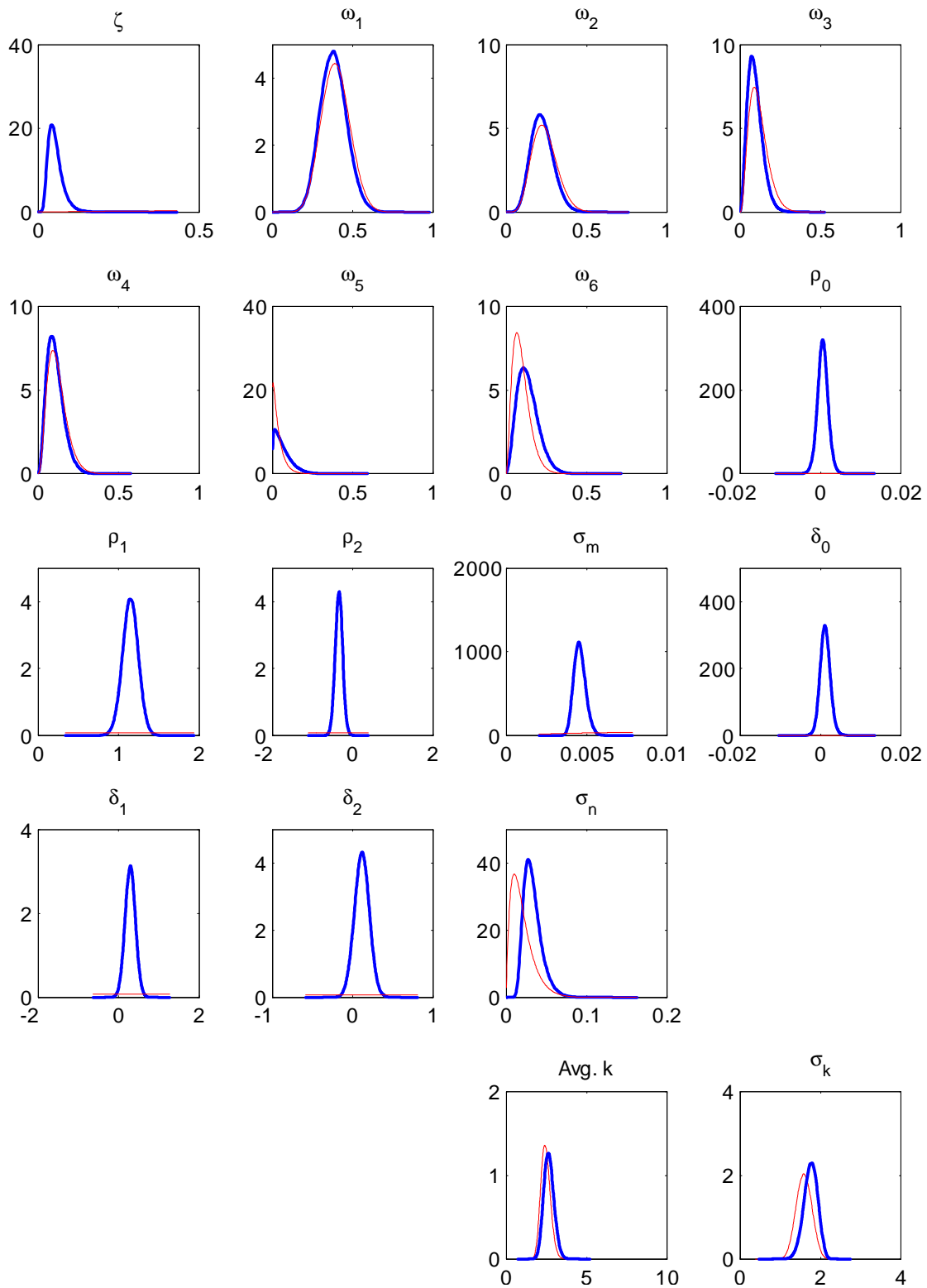


Figure 18: Bils-Klenow tight prior and posterior (bold) distributions, $K = 6$

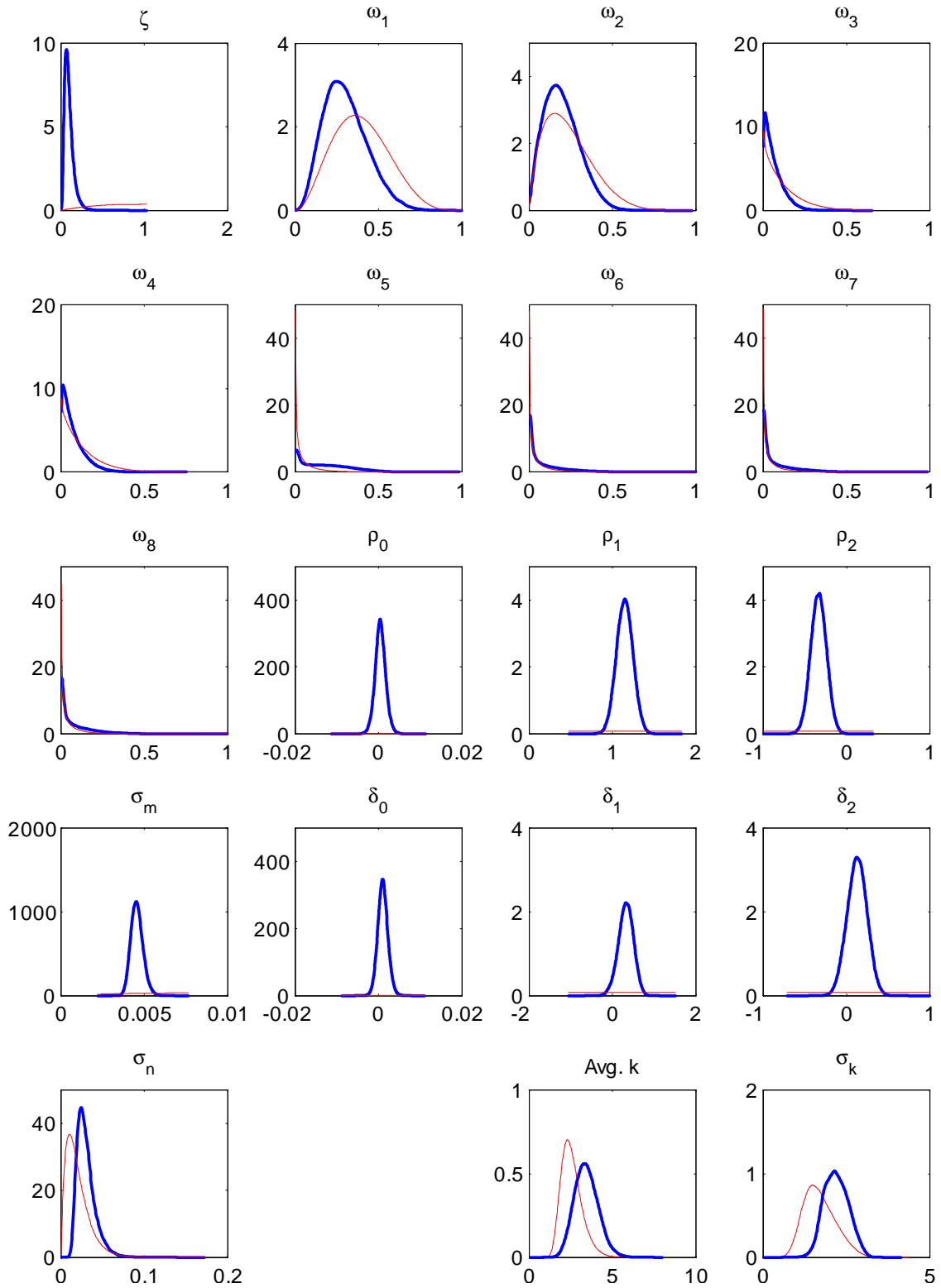


Figure 19: Bils-Klenow loose prior and posterior (bold) distributions, $K = 8$

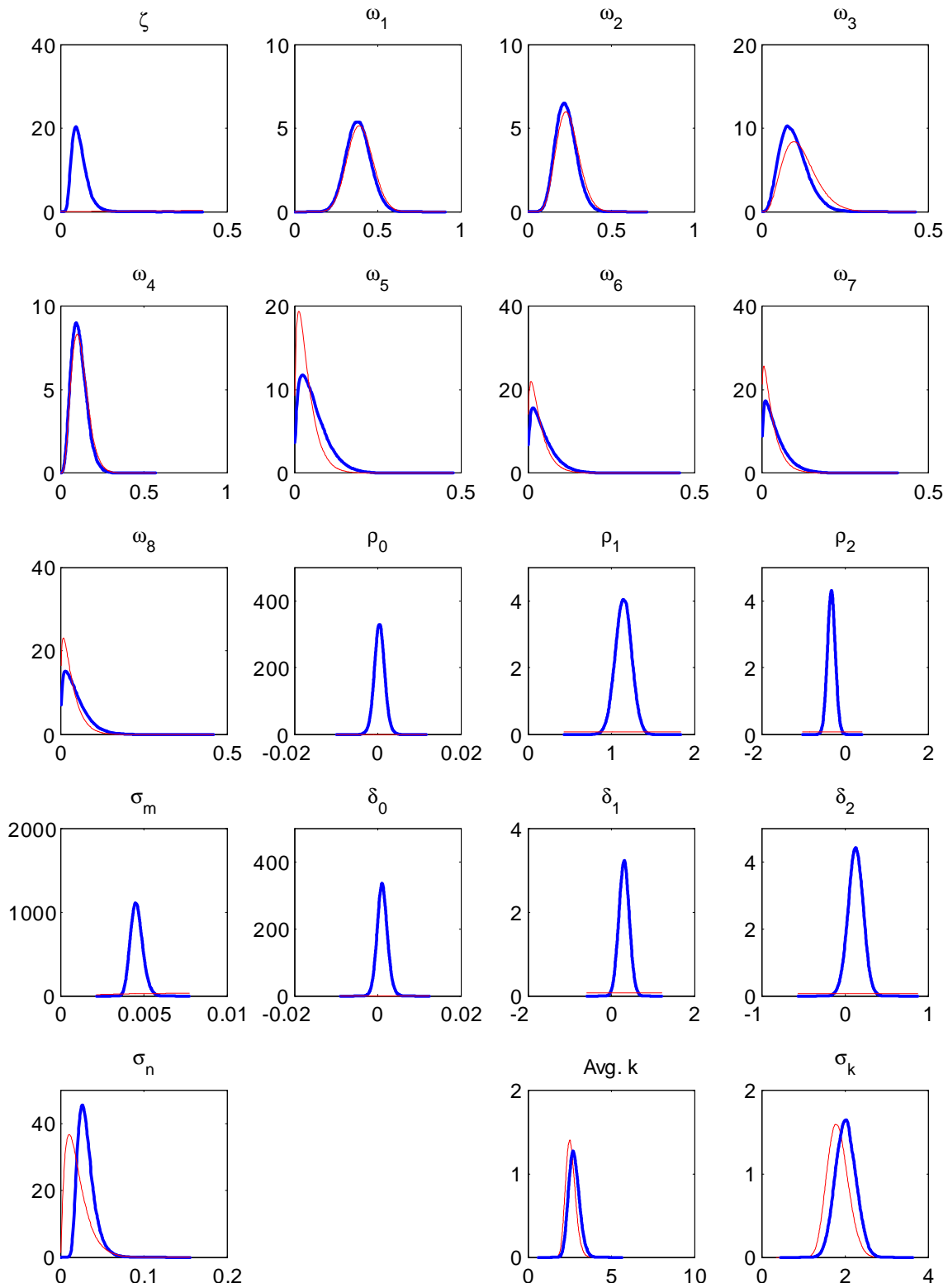


Figure 20: Bils-Klenow tight prior and posterior (bold) distributions, $K = 8$

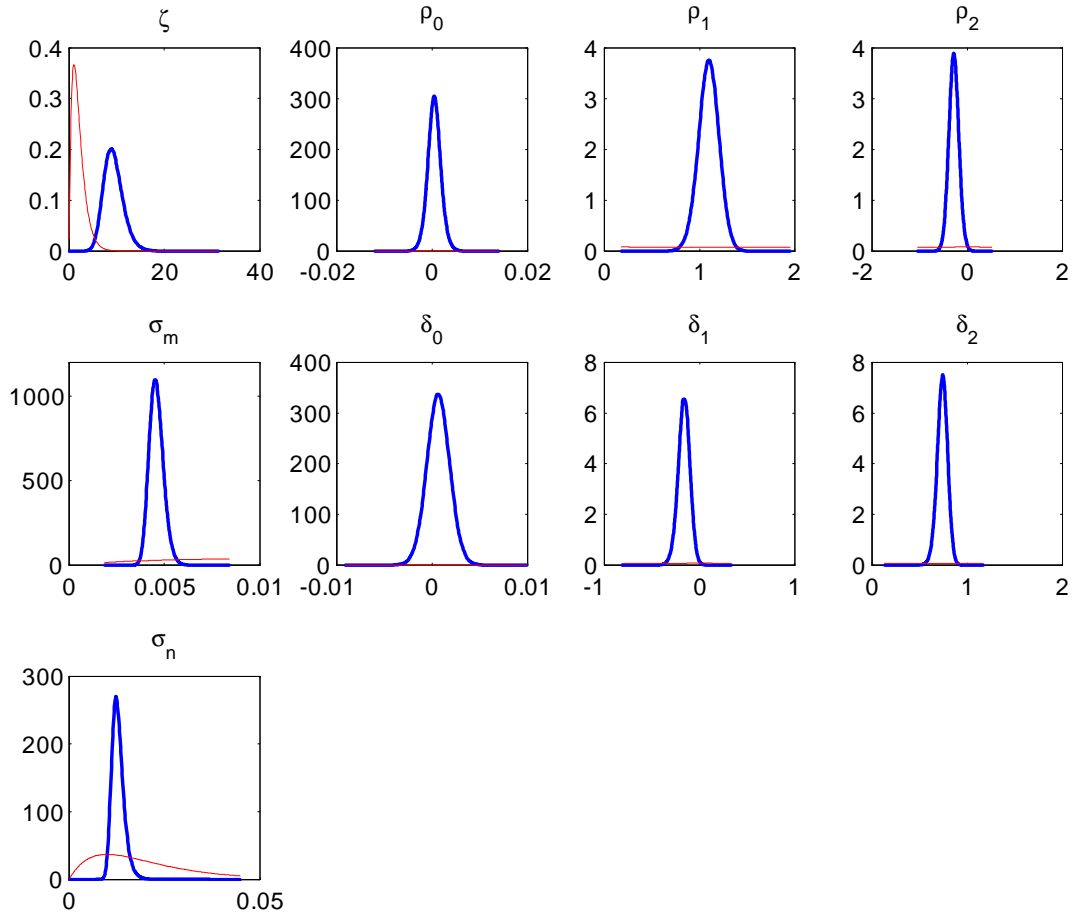


Figure 21: Prior and (bold) posterior distributions, fixed 2-quarter contracts ($\omega_2 = 1$)

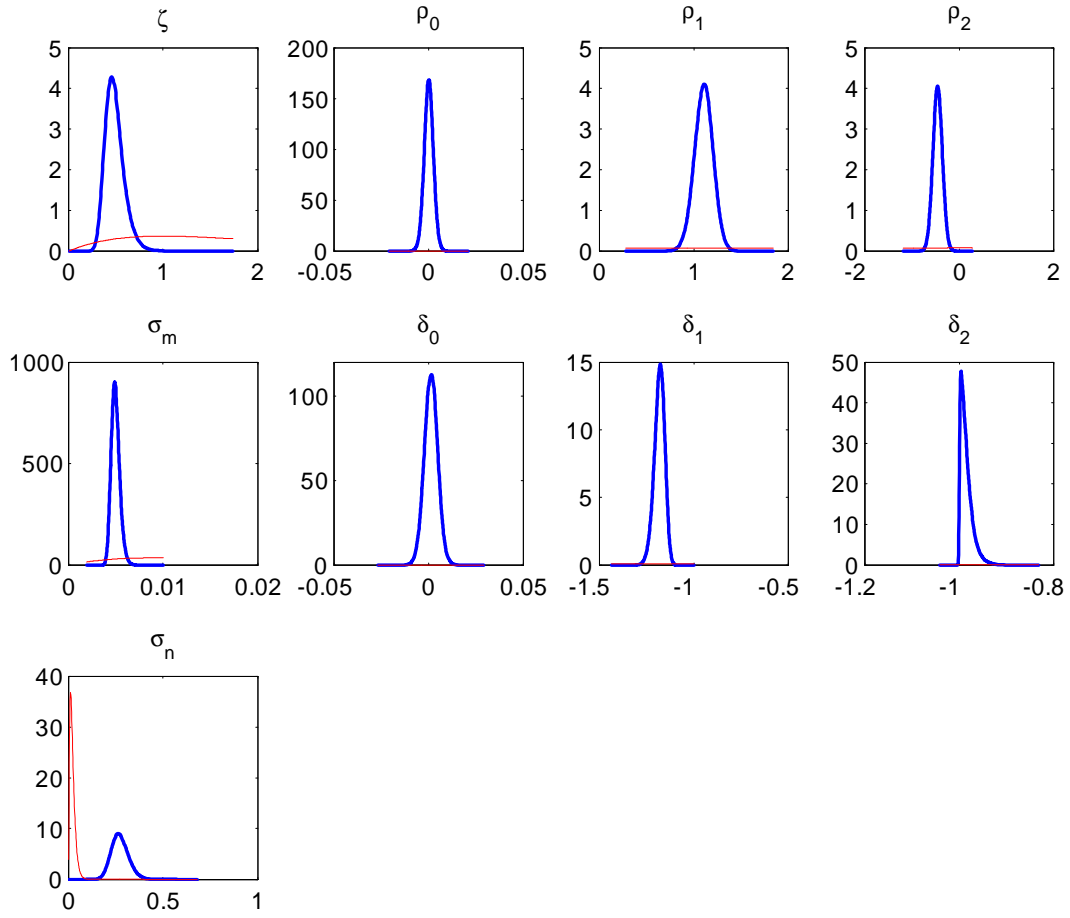


Figure 22: Prior and (bold) posterior distributions, fixed 3-quarter contracts ($\omega_3 = 1$)

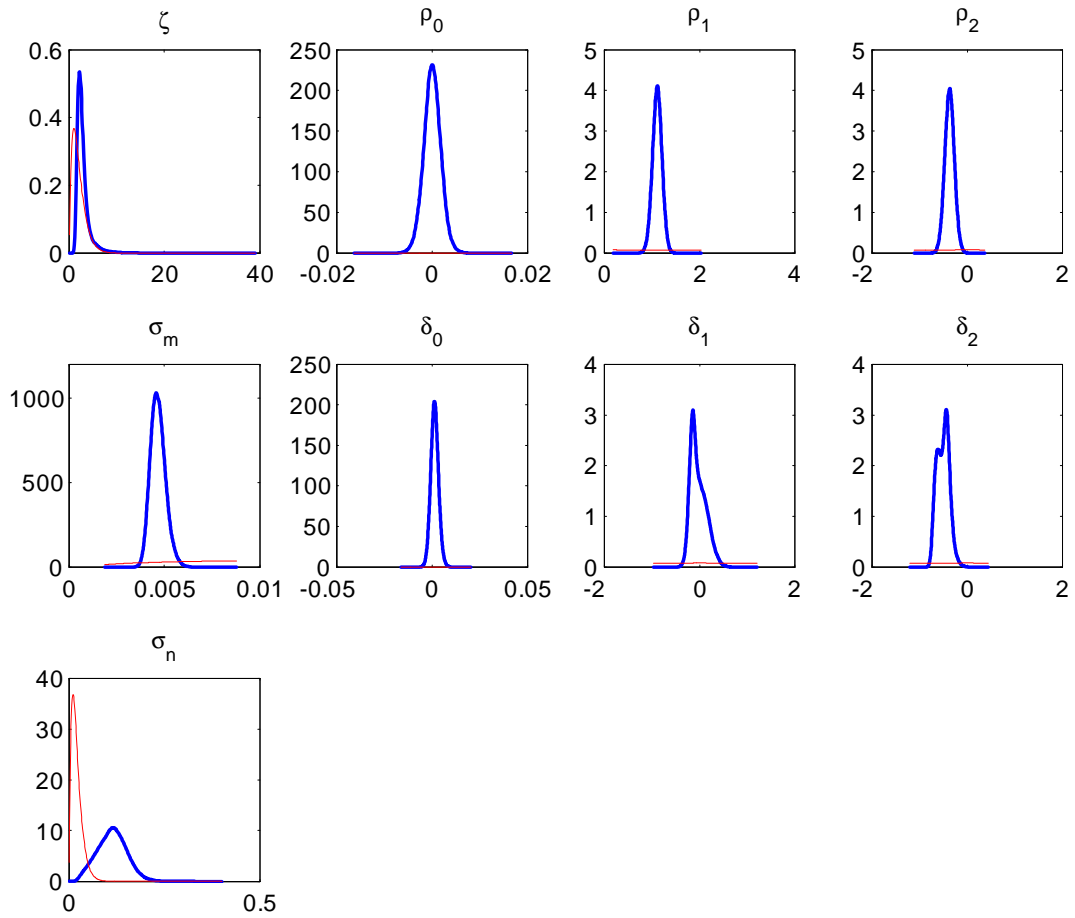


Figure 23: Prior and (bold) posterior distributions, fixed 4-quarter contracts ($\omega_4 = 1$)

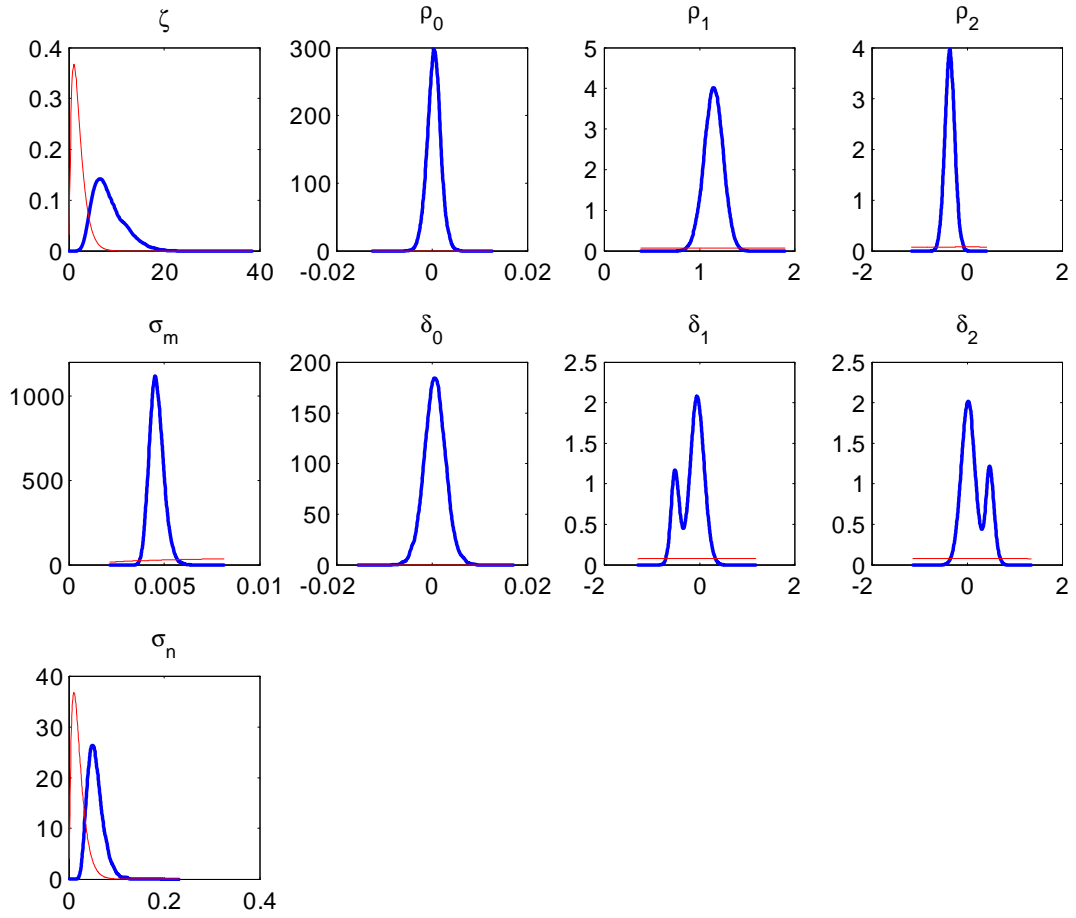


Figure 24: Prior and (bold) posterior distributions, fixed 6-quarter contracts ($\omega_6 = 1$)

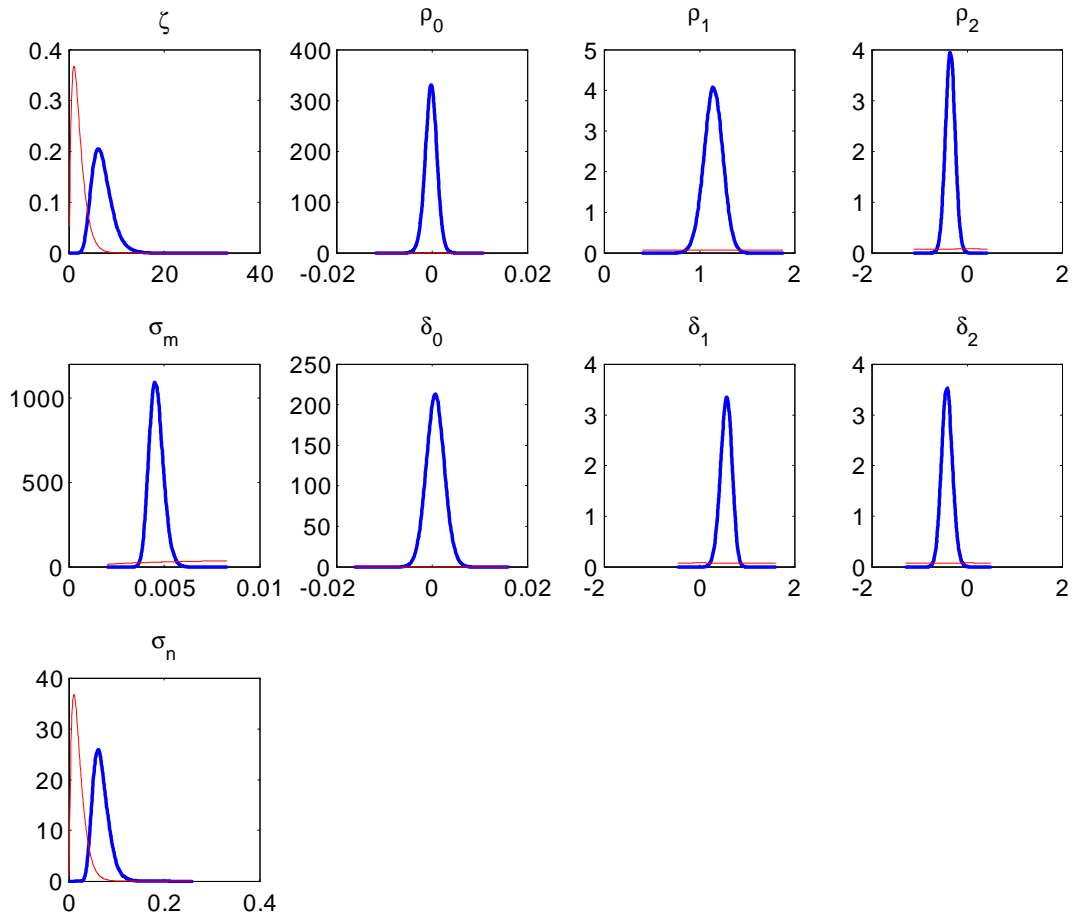


Figure 25: Prior and (bold) posterior distributions, fixed 8-quarter contracts ($\omega_8 = 1$)

Chapter 5

Technical Appendices

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What Drives Business Cycles in a Small Open Economy?

Technical Appendix*

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First version: October 2004

1 Introduction

This note documents the equations obtained in Dam and Linaa (2005). The model is basically identical to Kollmann (2001, 2002) with Calvo pricing on both the product market and on the labour market. Consumption habits are introduced and we focus on a central bank conducting monetary policy according to an imperfect peg. Additionally, a variety of exogenous shocks are added.

2 Derivation of the Model

2.1 Final Goods

Final goods Z_t are produced using intermediate-good bundles from home, Q_t^d , and abroad, Q_t^m respectively. These intermediary aggregates are combined with a Cobb Douglas technology;

$$Z_t = \left(\frac{Q_t^d}{\alpha^d}\right)^{\alpha^d} \left(\frac{Q_t^m}{\alpha^m}\right)^{\alpha^m}, \quad \alpha^d + \alpha^m = 1. \quad (2.1)$$

Each bundle of intermediate goods is a Dixit-Stiglitz aggregate of individual intermediate-goods, $q_t^i(s)$. Here, we follow the assumptions of the CEE model and let the net markup rate ν_t be an i.i.d. process with mean ν ;

$$Q_t^i = \left[\int_0^1 q_t^i(s)^{\frac{1}{1+\nu_t}} ds \right]^{1+\nu_t}, \quad i = d, m.$$

Domestic firms face the problem of minimizing the cost of producing Z_t units of the final good;

*Both authors were doctoral students at the University of Copenhagen when this work was conducted.

$$\min_{Q_t^d, Q_t^m} P_t^d Q_t^d + P_t^m Q_t^m \quad (2.2)$$

$$\text{s.t.} \quad \left(\frac{Q_t^d}{\alpha^d}\right)^{\alpha^d} \left(\frac{Q_t^m}{\alpha^m}\right)^{\alpha^m} = Z_t, \quad (2.3)$$

where individual intermediate-goods prices are $p_t^i(s)$ and appropriate CES price indices are given as

$$P_t^i \equiv \left[\int_0^1 p_t^i(s)^{-\frac{1}{\nu_t}} ds \right]^{-\nu_t}, \quad i = d, m. \quad (2.4)$$

The associated Lagrangian is

$$\begin{aligned} \mathcal{L}_t = & P_t^d Q_t^d + P_t^m Q_t^m \\ & - \lambda_t \left[\left(\frac{Q_t^d}{\alpha^d}\right)^{\alpha^d} \left(\frac{Q_t^m}{1 - \alpha^d}\right)^{1 - \alpha^d} - Z_t \right], \end{aligned}$$

where λ_t is the Lagrange multiplier. The first order condition with respect to Q_t^i gives

$$P_t^i = \lambda_t \alpha^i \frac{Z_t}{Q_t^i} \implies \frac{Q_t^i}{\alpha^i} = \lambda_t \frac{Z_t}{P_t^i} \quad (2.5)$$

Inserting equation (2.5) in the budget constraint (2.2) yields

$$(P_t^d)^{\alpha^d} (P_t^m)^{1 - \alpha^d} = \lambda_t. \quad (2.6)$$

The appropriate aggregate price index P_t is the cost of producing one unit of the final good ($Z_t = 1$). Thus, we use equation (2.5) again to obtain

$$\begin{aligned} \left(\frac{Q_t^d}{\alpha^d}\right)^{\alpha^d} \left(\frac{Q_t^{EU,d}}{1 - \alpha^d}\right)^{1 - \alpha^d} &= 1 \implies \\ P_t \equiv P_t^d Q_t^d + P_t^m Q_t^m & \\ = \lambda_t (\alpha^d + \alpha^m) &= \lambda_t \\ = (P_t^d)^{\alpha^d} (P_t^m)^{\alpha^m} & \end{aligned} \quad (2.7)$$

Profit-maximizing demands are found by combining (2.5) with (2.7);

$$Q_t^i = \alpha^i \lambda_t \frac{Z_t}{P_t^i} = \alpha^i \frac{P_t Z_t}{P_t^i}, \quad i = d, m. \quad (2.8)$$

2.2 Intermediate Goods

Intermediate-goods producers have access to Cobb-Douglas technology

$$y_t(s) = \theta_t K_t(s)^\psi L_t(s)^{1 - \psi}, \quad 0 < \psi < 1,$$

and operate in a monopolistic competitive market, where each producer sets the price of her variety, taking other prices as given and supplying whatever amount is demanded at the price set. Cost minimization implies the following first-order conditions

$$\left. \begin{aligned} (1 - \psi) y_t(s) / L_t(s) &= W_t \\ \psi y_t(s) / K_t(s) &= R_t, \end{aligned} \right\} \Rightarrow \frac{L_t(s)}{K_t(s)} = \frac{1 - \psi}{\psi} \frac{R_t}{W_t},$$

where R_t is the rental price of capital and W_t is the wage rate. Thus,

$$L_t(s) = \frac{1 - \psi}{\psi} \frac{R_t}{W_t} K_t(s) \Rightarrow \quad (2.9)$$

$$y_t(s) = \theta_t K_t(s)^\psi \left(\frac{1 - \psi}{\psi} \frac{R_t}{W_t} K_t(s) \right)^{1 - \psi} = \theta_t K_t(s) \left(\frac{1 - \psi}{\psi} \frac{R_t}{W_t} \right)^{1 - \psi} \Rightarrow$$

$$K_t(s) = \frac{1}{\theta_t} \left(\frac{1 - \psi}{\psi} \frac{R_t}{W_t} \right)^{-(1 - \psi)} y_t(s), \quad (2.10)$$

$$L_t(s) = \frac{1}{\theta_t} \left(\frac{\psi}{1 - \psi} \frac{W_t}{R_t} \right)^{-\psi} y_t(s).$$

Hence, the firm's total and marginal costs are

$$\begin{aligned} TC(y_t(s)) &= W_t L_t + R_t K_t \\ &= \frac{1}{\theta_t} \left[W_t \left(\frac{\psi}{1 - \psi} \frac{W_t}{R_t} \right)^{-\psi} + R_t \left(\frac{1 - \psi}{\psi} \frac{R_t}{W_t} \right)^{-(1 - \psi)} \right] y_t(s) \\ &= \frac{1}{\theta_t} \left[W_t^{1 - \psi} R_t^\psi \left(\frac{\psi}{1 - \psi} \right)^{-\psi} + W_t^{1 - \psi} R_t^\psi \left(\frac{\psi}{1 - \psi} \right)^{1 - \psi} \right] y_t(s) \\ &= \frac{1}{\theta_t} W_t^{1 - \psi} R_t^\psi \left[\left(\frac{\psi}{1 - \psi} \right)^{-\psi} + \left(\frac{\psi}{1 - \psi} \right)^{1 - \psi} \right] y_t(s) \\ &= \frac{1}{\theta_t} W_t^{1 - \psi} R_t^\psi \left[\left(\frac{\psi}{1 - \psi} \right)^{-\psi} \frac{1}{1 - \psi} \right] y_t(s) \\ &= \frac{1}{\theta_t} W_t^{1 - \psi} R_t^\psi \psi^{-\psi} (1 - \psi)^{-(1 - \psi)} y_t(s) \Rightarrow \\ MC_t &= \frac{1}{\theta_t} W_t^{1 - \psi} R_t^\psi \psi^{-\psi} (1 - \psi)^{-(1 - \psi)}. \end{aligned} \quad (2.11)$$

Final-good producers have the following demands for individual varieties of intermediaries

$$q_t^i(s) = \left(\frac{p_t^i(s)}{P_t^i} \right)^{-\frac{1 + \nu_t}{\nu_t}} Q_t^i, \quad i = d, m.$$

Firm profits are thus

$$\begin{aligned}
\pi^{dx}(p_t^d(s), p_t^x(s)) &= (p_t^d(s) - MC_t) q_t^d(s) + (e_t p_t^x(s) - MC_t) q_t^x(s) \quad (2.12) \\
&= (p_t^d(s) - MC_t) \left(\frac{p_t^d(s)}{P_t^d} \right)^{-\frac{1+\nu_t}{\nu_t}} Q_t^d \\
&\quad + (e_t p_t^x(s) - MC_t) \left(\frac{p_t^x(s)}{P_t^x} \right)^{-\frac{1+\nu_t}{\nu_t}} Q_t^x
\end{aligned}$$

with $p_t^x(s)$ being the price of the individual foreign intermediary and where we assumed Dixit-Stiglitz demands from foreign final-goods producers;

$$\begin{aligned}
q_t^x(s) &= \left(\frac{p_t^x(s)}{P_t^x} \right)^{-\frac{1+\nu_t}{\nu_t}} Q_t^x, \\
Q_t^x &= \left(\frac{P_t^x}{P_t^*} \right)^{-\eta} Y_t^*, \quad \eta > 0. \quad (2.13)
\end{aligned}$$

Likewise, foreign exporters generate the following profits in the domestic market;

$$\pi^m(p_t^m(s)) = (p_t^m(s) - e_t P_t^*) \left(\frac{p_t^m(s)}{P_t^m} \right)^{-\frac{1+\nu_t}{\nu_t}} Q_t^m$$

Hence, as we have assumed local currency pricing and infrequent reoptimization according to Calvo (1983), a domestic firm reoptimizing its domestic price faces the following problem:

$$\begin{aligned}
p_{t,t}^d &= \arg \max_{\omega} \sum_{\tau=t}^{\infty} d^{\tau-t} E_t [\rho_{t,\tau} \pi^{dx}(\omega, p_t^x(s))], \\
\rho_{t,\tau} &\equiv \beta^{\tau-t} (U_{C,\tau} / U_{C,t}) (P_t / P_{\tau}) \quad (2.14)
\end{aligned}$$

where $\rho_{t,t+\tau}$ appropriately discounts profits at time $\tau > t$, and $d^{\tau-t}$ is the probability that the current pricing decision is still in effect in period τ . Substituting from the profit expression (2.12) yields

$$\begin{aligned}
&\sum_{\tau=t}^{\infty} d^{\tau-t} E_t \left[\rho_{t,\tau} (p_{t,t}^d - MC_{\tau}) \left(\frac{p_{t,t}^d}{P_{\tau}^d} \right)^{-\frac{1+\nu_{\tau}}{\nu_{\tau}}} Q_{\tau}^d \right] \\
&= \sum_{\tau=t}^{\infty} d^{\tau-t} E_t \left[(p_{t,t}^d - MC_{\tau}) (p_{t,t}^d)^{-\frac{1+\nu_{\tau}}{\nu_{\tau}}} \rho_{t,\tau} (P_{\tau}^d)^{\frac{1+\nu_{\tau}}{\nu_{\tau}}} Q_{\tau}^d \right],
\end{aligned}$$

resulting in the following first-order condition;

$$\begin{aligned}
&\sum_{\tau=t}^{\infty} d^{\tau-t} E_t \left[\left(-\frac{1}{\nu_{\tau}} (p_{t,t}^d)^{-\frac{1+\nu_{\tau}}{\nu_{\tau}}} + \frac{1+\nu_{\tau}}{\nu_{\tau}} (p_{t,t}^d)^{-\frac{1+2\nu_{\tau}}{\nu_{\tau}}} MC_{\tau} \right) \rho_{t,\tau} (P_{\tau}^d)^{\frac{1+\nu_{\tau}}{\nu_{\tau}}} Q_{\tau}^d \right] = 0 \Rightarrow \\
&\sum_{\tau=t}^{\infty} d^{\tau-t} E_t \left[(p_{t,t}^d - (1+\nu_{\tau}) MC_{\tau}) \rho_{t,\tau} \left(\frac{p_{t,t}^d}{P_{\tau}^d} \right)^{-\frac{1+\nu_{\tau}}{\nu_{\tau}}} \frac{Q_{\tau}^d}{p_{t,t}^d \nu_{\tau}} \right] = 0. \quad (2.15)
\end{aligned}$$

Analogously, the optimal price for sales to foreign final-goods producers is determined from the following condition;

$$\sum_{\tau=t}^{\infty} d^{\tau-t} E_t \left[\left(e_{\tau} p_{t,t}^x - (1 + \nu_{\tau}) MC_{\tau} \right) \rho_{t,\tau} \left(\frac{p_{t,t}^x}{P_{\tau}^x} \right)^{-\frac{1+\nu_{\tau}}{\nu_{\tau}}} \frac{Q_{\tau}^x}{p_{t,t}^x \nu_{\tau}} \right] = 0. \quad (2.16)$$

Import firms are owned by risk-neutral foreigners who discount future profits at the foreign nominal interest rate $R_{t,\tau} \equiv \prod_{s=t}^{\tau-1} (1 + i_s^*)^{-1}$, $R_{t,t} \equiv 1$. Thus, when they reoptimize, they set their prices in order to maximize discounted future profits measured in foreign units;

$$\begin{aligned} p_{t,t}^m &= \arg \max_{\omega} \sum_{\tau=t}^{\infty} d^{\tau-t} E_t [R_{t,\tau} \pi^m(\omega) / e_{\tau}] \\ &= \sum_{\tau=t}^{\infty} d^{\tau} E_t [R_{t,\tau} \pi^m(\omega) / e_{\tau}] \\ &= \sum_{\tau=t}^{\infty} d^{\tau} E_t \left[R_{t,\tau} (p_{t,t}^m - e_{\tau} P_{\tau}^*) \left(\frac{p_{t,t}^m}{P_{\tau}^m} \right)^{-\frac{1+\nu_{\tau}}{\nu_{\tau} + \tau}} Q_{\tau}^m / e_{\tau} \right] \\ &= \sum_{\tau=t}^{\infty} d^{\tau} E_t \left[R_{t,\tau} (p_{t,t}^m / e_{\tau} - P_{\tau}^*) (p_{t,t}^m)^{-\frac{1+\nu_{\tau}}{\nu_{\tau}}} (P_{\tau}^m)^{\frac{1+\nu_{\tau}}{\nu_{\tau}}} Q_{\tau}^m \right], \end{aligned}$$

with first-order condition

$$\begin{aligned} \sum_{\tau=t}^{\infty} d^{\tau-t} E_t \left[\left(-\frac{1}{\nu_{\tau}} p_{t,t}^m - \frac{1+\nu_{\tau}}{\nu_{\tau}} / e_{\tau} + \frac{1 + \nu_{\tau}}{\nu_{\tau}} (p_{t,t}^m)^{-\frac{1+2\nu_{\tau}}{\nu_{\tau}}} P_{\tau}^* \right) (P_{\tau}^m)^{\frac{1+\nu_{\tau}}{\nu_{\tau}}} Q_{\tau}^m \right] &= 0 \Rightarrow \\ \sum_{\tau=t}^{\infty} d^{\tau-t} E_t \left[(p_{t,t}^m(s) / e_{\tau} - (1 + \nu_{\tau}) P_{\tau}^*) \left(\frac{p_{t,t}^m}{P_{\tau}^m} \right)^{-\frac{1+\nu_{\tau}}{\nu_{\tau}}} \frac{Q_{\tau}^m}{p_{t,t}^m \nu_{\tau}} \right] &= 0. \end{aligned} \quad (2.17)$$

Considering aggregate Dixit-Stiglitz prices of the intermediate goods (equation (2.4)), we apply to the law of large numbers and the fact that the fraction d of firms that reoptimize is completely random to find that

$$\begin{aligned} (P_t^i)^{-\frac{1}{\nu_t}} &= \int_0^1 p_t^i(s)^{-\frac{1}{\nu_t}} ds = d \int_0^1 (p_{t-1}^i(s))^{-\frac{1}{\nu_t}} ds + (1-d) (p_{t,t}^i)^{-\frac{1}{\nu_t}} \\ &= d (P_{t-1}^i)^{-\frac{1}{\nu_t}} + (1-d) (p_{t,t}^i)^{-\frac{1}{\nu_t}}, \quad i = d, m, x. \end{aligned} \quad (2.18)$$

2.3 Households

A representative household is characterized by the following preferences with external habit formation in consumption:

$$E_0 \left[\sum_{t=0}^{\infty} \beta^t U(C_t^*, L_t) \right], \quad U(C_t^*, L_t) = [u(C_t^*) - v(L_t)], \quad (2.19)$$

with preference shocks ζ_t^b and ζ_t^L . We define

$$C_t^* = C_t - h\bar{C}_{t-1} \quad (2.20)$$

where \bar{C}_t is the average consumption level, which is considered exogenous to the representative household.

Labour enters as a Dixit-Stiglitz aggregate in the intermediate-goods firm production; thus, letting $l_t(s, j)$ be the amount of labour service j utilized by firm s we find that firm s uses the following amount of labour services;

$$L_t(s) = \left[\int_0^1 l_t(s, j)^{\frac{1}{1+\gamma}} dj \right]^{1+\gamma_t}, \quad \gamma_t > 1, \quad (2.21)$$

and total labour is $L_t = \int_0^1 L_t(s) ds$.

The representative household can invest in domestic and foreign one-period bonds as well as in domestic capital. Capital K_t earns rental rate R_t and accumulates as follows;

$$K_{t+1} = K_t(1 - \delta) + I_t - \phi(K_{t+1}, K_t), \quad 0 < \delta < 1, \quad (2.22)$$

where $\phi(K_{t+1}, K_t)$ is an adjustment cost. Domestic bonds A_t earns net interest i_t , while the interest i_t^f accruing to foreign bonds B_t held by domestic agents deviates from the foreign interest level i_t^* as follows;

$$(1 + i_t^f) = \Omega_t^j (1 + i_t^*), \quad (2.23)$$

$$\Omega_t = v_t \exp \left\{ -\lambda \frac{e_t B_{t+1}}{P_t \Xi} \right\}, \quad \Xi = \frac{e P^x Q^x}{P} \quad (2.24)$$

where Ξ is the steady-state value of export in units of the domestic final good.

The household owns domestic firms and thus earns profit from the intermediate-goods firms and rental rates (R_t) on the capital in addition to wage income from its variety of labour services. The budget constraint is thus

$$A_{t+1} + e_t B_{t+1} + P_t (C_t + I_t) = \quad (2.25)$$

$$A_t (1 + i_{t-1}) + e_t B_t (1 + i_{t-1}^f) + R_t K_t + \int_0^1 \pi_t^{dx}(s) ds + \int_0^1 \int_0^1 w_t(j) l_t(s, j) dj ds.$$

Wage setting is staggered a la Calvo (1983). The household takes the average wage rate $W_t = \left[\int_0^1 w_t(j)^{-\frac{1}{1+\gamma_t}} dj \right]^{-(1+\gamma_t)}$ as given when reoptimising the optimal wage $w_{t,t}$, and will meet any demand for the given type of labour;

$$l_t(j) = \int_0^1 l(s, j) ds. \quad (2.26)$$

The household thus faces the following problem

$$\begin{aligned} & \max_{\{C_t, A_{t+1}, B_{t+1}, K_{t+1}, w_{t,t}\}_{t=0}^{\infty}} E_0 \left[\sum_{t=0}^{\infty} \beta^t U(C_t^*, L_t) \right] \\ & \text{s.t. (2.20)-(2.26).} \end{aligned} \quad (2.27)$$

Solving (2.25) with respect to C_t yields

$$\begin{aligned} C_t &= \frac{1}{P_t} \{ A_t (1 + i_{t-1}) + e_t^{EU} B_t^{EU} (1 + i_{t-1}^{EU}) + e_t^{ROW} B_t^{ROW} (1 + i_{t-1}^{ROW}) \\ & \quad + R_t K_t - A_{t+1} - e_t B_{t+1} \} \\ & \quad - I_t + \frac{1}{P_t} \left[\int_0^1 \pi_t^{dx}(s) ds + \int_0^1 \int_0^1 w_t(j) l_t(s, j) dj ds \right] \iff \\ C_t^* &= \frac{1}{P_t} \left[A_t (1 + i_{t-1}) + e_t B_t (1 + i_{t-1}^f) + R_t K_t - A_{t+1} - e_t B_{t+1} \right] \\ & \quad - [K_{t+1} - K_t (1 - \delta) + \phi(K_{t+1}, K_t)] \\ & \quad + \frac{1}{P_t} \left[\int_0^1 \pi_t^{dx}(s) ds + \int_0^1 \int_0^1 w_t(j) l_t(s, j) dj ds \right] - h \bar{C}_{t-1}, \end{aligned}$$

Thus, an interior solution to the household's problem (2.27) yields the following first-order conditions:

Capital,

$$\begin{aligned} \beta^t U_{C,t} [-1 - \phi_{1,t}] + \beta^{t+1} E_t \left[U_{C,t+1} \left(\frac{R_{t+1}}{P_{t+1}} + (1 - \delta) - \phi_{2,t+1} \right) \right] &= 0 \iff \\ \beta^{t+1} E_t \left[U_{C,t+1} \left(\frac{R_{t+1}}{P_{t+1}} + (1 - \delta) - \phi_{2,t+1} \right) \right] &= \beta^t U_{C^*,t} [1 + \phi_{1,t}] \iff \\ \beta \frac{E_t \left[U_{C,t+1} \left(\frac{R_{t+1}}{P_{t+1}} + (1 - \delta) - \phi_{2,t+1} \right) \right]}{U_{C,t} [1 + \phi_{1,t}]} &= 1 \iff \\ E_t \left[\rho_{t,t+1} \frac{P_{t+1}}{P_t} \frac{\left(\frac{R_{t+1}}{P_{t+1}} + (1 - \delta) - \phi_{2,t+1} \right)}{[1 + \phi_{1,t}]} \right] &= 1. \end{aligned} \quad (2.28)$$

Domestic bonds,

$$\begin{aligned} \beta^t U_{C,t} \left[-\frac{1}{P_t} \right] + \beta^{t+1} E_t \left[U_{C,t+1} \left(\frac{1 + i_t}{P_{t+1}} \right) \right] &= 0 \iff \\ (1 + i_t) \beta E_t \left[\frac{U_{C,t+1}}{U_{C,t}} \frac{P_t}{P_{t+1}} \right] &= 1 \iff \\ (1 + i_t) E_t [\rho_{t,t+1}] &= 1. \end{aligned} \quad (2.29)$$

Foreign bonds,

$$\begin{aligned}
\beta^t U_{C^*,t} \left(-\frac{e_t}{P_t} \right) + \beta^{t+1} E_t \left[U_{C^*,t+1} \left(\frac{e_{t+1} (1 + i_t^f)}{P_{t+1}} \right) \right] &= 0 \iff \\
\beta E_t \left[U_{C^*,t+1} \left(\frac{e_{t+1} (1 + i_t^f)}{P_{t+1}} \right) \right] &= U_{C^*,t} \frac{e_t}{P_t} \iff \\
(1 + i_t^f) \beta E_t \left[\frac{U_{C^*,t+1} e_{t+1} P_t}{U_{C^*,t} e_t P_{t+1}} \right] &= 1 \iff \\
(1 + i_t^f) E_t \left[\rho_{t,t+1} \frac{e_{t+1}}{e_t} \right] &= 1, \tag{2.30}
\end{aligned}$$

where $\rho_{t,t+k}$ is defined in equation (2.14) above.

Since the household meets the demand for labour at its chosen wage level, we find the following relations;

$$\begin{aligned}
l_\tau(s, j) &= \left(\frac{w_{t,t}}{W_\tau} \right)^{-\frac{1+\gamma_t}{\gamma_t}} \frac{1-\psi}{\psi} \frac{R_\tau}{W_\tau} K_\tau(s) \Rightarrow \\
\frac{dl_t(s, j)}{dw_t(j)} &= -\frac{1+\gamma_t}{\gamma_t} w_{t,t}^{-\frac{1+2\gamma_t}{\gamma_t}} \frac{1-\psi}{\psi} R_t K_t(s) W_t^{\frac{1}{\gamma_t}} \Rightarrow \\
\int \frac{dl_t(s, j)}{dw_t(j)} ds &= -\frac{1+\gamma_t}{\gamma_t} w_{t,t}^{-\frac{1+2\gamma_t}{\gamma_t}} \chi_t, \\
\text{and } \int \frac{d(l_t(s) w_t(j))}{dw_t(j)} ds &= -\frac{1}{\gamma_t} w_{t,t}^{-\frac{1+\gamma_t}{\gamma_t}} \chi_t, \\
\text{where } \chi_t &\equiv \frac{1-\psi}{\psi} R_t \int K_t(s) ds W_t^{\frac{1}{\gamma_t}} = \frac{1-\psi}{\psi} R_t K_t W_t^{\frac{1}{\gamma_t}}. \tag{2.31}
\end{aligned}$$

Thus, we finally obtain the following first-order condition with respect to the wage rate;

$$\begin{aligned}
\sum_{\tau=t}^{\infty} (\beta D)^{\tau-t} E_t \left[-\frac{U_{C,\tau}}{P_\tau} \frac{1}{\gamma_\tau} w_{t,t}^{-\frac{1+\gamma_\tau}{\gamma_\tau}} \chi_\tau + U_{L,\tau} \frac{1+\gamma_\tau}{\gamma_\tau} w_{t,t}^{-\frac{1+2\gamma_\tau}{\gamma_\tau}} \chi_\tau \right] &= 0 \Rightarrow \\
\sum_{\tau=t}^{\infty} (\beta D)^{\tau-t} \frac{\chi_\tau}{\gamma_\tau} w_{t,t}^{-\frac{1+2\gamma_\tau}{\gamma_\tau}} E_t \left[\frac{U_{C,\tau}}{P_\tau} w_{t,t} - (1 + \gamma_\tau) U_{L,\tau} \right] &= 0. \tag{2.32}
\end{aligned}$$

Analogously to equation (2.18), the aggregate wage level is determined as

$$W_t = \left[D (W_{t-1})^{-\frac{1}{\gamma_t}} + (1 - D) (w_{t,t})^{-\frac{1}{\gamma_t}} \right]^{-\gamma_t}. \tag{2.33}$$

2.4 Steady State

In the steady state we have

$$P^d = p^d = (1 + \nu) MC \implies \frac{P^d}{P} = (1 + \nu) mc \quad (2.34)$$

$$P^x = p^x = (1 + \nu) \frac{MC}{e} \implies P^x = (1 + \nu) mc \left(\frac{P}{e} \right) \quad (2.35)$$

$$P^m = p^m = (1 + \nu) eP^* \implies \frac{P^m}{P} = (1 + \nu) \frac{e}{P}, \quad P^* = 1 \quad (2.36)$$

$$W = w = (1 + \gamma) \frac{U_L}{U_C} P \implies \frac{W}{P} = (1 + \gamma) [(1 - h) C]^{\sigma_C} L^{\sigma_L} \quad (2.37)$$

From (2.14) we have that in the steady state

$$\rho = \beta \quad (2.38)$$

that, combined with (2.29) and (2.30), leads to

$$i = i^f = \frac{1}{\beta} - 1, \quad (2.39)$$

and from (2.28) we get

$$\begin{aligned} \beta \left(\frac{R}{P} + (1 - \delta) \right) &= 1 \iff \\ \frac{R}{P} &= \frac{1}{\beta} - (1 - \delta) \\ &= i + \delta. \end{aligned} \quad (2.40)$$

Hence, marginal costs can be written as

$$\begin{aligned} mc &= \frac{MC}{P} = \left(\frac{W}{P} \right)^{1-\psi} \left(\frac{R}{P} \right)^\psi \psi^{-\psi} (1 - \psi)^{-(1-\psi)} \\ &= [(1 + \gamma) [(1 - h) C]^{\sigma_C} L^{\sigma_L}]^{1-\psi} (i + \delta)^\psi \psi^{-\psi} (1 - \psi)^{-(1-\psi)} \\ &= (1 + \gamma)^{(1-\psi)} [(1 - h) C]^{(1-\psi)\sigma_C} L^{(1-\psi)\sigma_L} (i + \delta)^\psi \psi^{-\psi} (1 - \psi)^{-(1-\psi)}, \end{aligned} \quad (2.41)$$

from which we can get the following expression for consumption;

$$\begin{aligned} [(1 - h) C]^{(1-\psi)\sigma_C} &= (1 + \gamma)^{-(1-\psi)} L^{-(1-\psi)\sigma_L} (i + \delta)^{-\psi} \psi^\psi (1 - \psi)^{1-\psi} mc \iff \\ C &= \frac{1}{1 - h} (1 + \gamma)^{-\frac{1}{\sigma_C}} L^{-\frac{\sigma_L}{\sigma_C}} (i + \delta)^{-\frac{\psi}{(1-\psi)\sigma_C}} \psi^{\frac{\psi}{(1-\psi)\sigma_C}} (1 - \psi)^{\frac{1}{\sigma_C}} mc^{\frac{1}{(1-\psi)\sigma_C}} \end{aligned} \quad (2.42)$$

Note also the following implication of equation (2.41) which will prove useful below;

$$(i + \delta)^{-\psi} \psi^\psi (1 - \psi)^{(1-\psi)} mc = [(1 + \gamma) [(1 - h) C]^{\sigma_C} L^{\sigma_L}]^{1-\psi}.$$

Now, we can determine the relative factor prices as follows;

$$\begin{aligned}\frac{R/P}{W/P} &= \frac{R}{W} = \frac{i + \delta}{(1 + \gamma) [(1 - h) C]^{\sigma_C} L^{\sigma_L}} \\ &= (1 + \gamma)^{-1} (i + \delta) [(1 - h) C]^{-\sigma_C} L^{-\sigma_L}.\end{aligned}\quad (2.43)$$

Since

$$P = (P^d)^{\alpha^d} (P^m)^{1-\alpha^d},$$

we have from equation (2.7) that

$$\begin{aligned}1 &= \left(\frac{P^d}{P}\right)^{\alpha^d} \left(\frac{P^m}{P}\right)^{1-\alpha^d} \\ &= [(1 + \nu) mc]^{\alpha^d} \left((1 + \nu) \frac{e}{P}\right)^{1-\alpha^d} \iff \\ (1 + \nu) \frac{e}{P} &= (1 + \nu)^{-\frac{\alpha^d}{1-\alpha^d}} mc^{-\frac{\alpha^d}{1-\alpha^d}} \iff \\ \frac{e}{P} &= (1 + \nu)^{-\frac{1}{1-\alpha^d}} mc^{-\frac{\alpha^d}{1-\alpha^d}},\end{aligned}\quad (2.44)$$

and we can now normalize P .

Using this, (2.35) and (2.36) we then have

$$\begin{aligned}P^x &= (1 + \nu) mc \left((1 + \nu)^{-\frac{1}{1-\alpha^d}} mc^{-\frac{\alpha^d}{1-\alpha^d}} \right)^{-1} \\ &= (1 + \nu)^{\frac{2-\alpha^d}{1-\alpha^d}} mc^{\frac{1}{1-\alpha^d}},\end{aligned}\quad (2.45)$$

$$\frac{P^m}{P} = (1 + \nu)^{-\frac{\alpha^d}{1-\alpha^d}} mc^{-\frac{\alpha^d}{1-\alpha^d}}. \quad (2.46)$$

In the steady state the NFA accumulation looks like

$$P^x Q^x = \frac{P^m}{e} Q^m,$$

and foreign demands is expressed, cf. (2.13), as

$$\begin{aligned}Q^x &= \left(\frac{P^x}{P^*}\right)^{-\eta} Y^* \\ &= (P^x)^{-\eta},\end{aligned}$$

since in the steady state we assume that $P^* = Y^* = 1$, implying

$$\begin{aligned}P^x (P^x)^{-\eta} &= \frac{P^m}{e} Q^m \iff \\ Q^m &= (P^x)^{1-\eta} \frac{e}{P^m} = \frac{1}{1 + \nu} (P^x)^{1-\eta},\end{aligned}$$

where we used that

$$\frac{e}{Pm} = \frac{\frac{e}{P}}{\frac{Pm}{P}} = \frac{(1+\nu)^{-\frac{1}{1-\alpha^d}} mc^{-\frac{\alpha^d}{1-\alpha^d}}}{(1+\nu)^{-\frac{\alpha^d}{1-\alpha^d}} mc^{-\frac{\alpha^d}{1-\alpha^d}}} = \frac{1}{1+\nu}.$$

Now use (2.45) to obtain

$$Q^x = (1+\nu)^{-\eta\frac{2-\alpha^d}{1-\alpha^d}} mc^{\frac{-\eta}{1-\alpha^d}}, \quad (2.47)$$

$$\begin{aligned} Q^m &= \frac{1}{1+\nu} (1+\nu)^{\frac{(1-\eta)(2-\alpha^d)}{1-\alpha^d}} mc^{\frac{1-\eta}{1-\alpha^d}} \\ &= (1+\nu)^{\frac{(1-\eta)(2-\alpha^d)-1+\alpha^d}{1-\alpha^d}} mc^{\frac{1-\eta}{1-\alpha^d}} \\ &= (1+\nu)^{\frac{1-\eta(2-\alpha^d)}{1-\alpha^d}} mc^{\frac{1-\eta}{1-\alpha^d}} \\ &= (1+\nu)^{\frac{1-\eta}{1-\alpha^d}-\eta} mc^{\frac{1-\eta}{1-\alpha^d}}. \end{aligned} \quad (2.48)$$

From (2.8) we obtain

$$Q^d = \frac{\alpha^d}{1-\alpha^d} \left(\frac{P^d}{P}\right)^{-1} \left(\frac{P^m}{P}\right) Q^m.$$

Now, substitute from (2.34), (2.46) and (2.48) and re-arrange to get

$$\begin{aligned} Q^d &= \frac{\alpha^d}{1-\alpha^d} \left(\frac{P^d}{P}\right)^{-1} \left(\frac{P^m}{P}\right) Q^m \\ &= \frac{\alpha^d}{1-\alpha^d} [(1+\nu)mc]^{-1} \left[(1+\nu)^{-\frac{\alpha^d}{1-\alpha^d}} mc^{-\frac{\alpha^d}{1-\alpha^d}} \right] (1+\nu)^{\frac{1-\eta(2-\alpha^d)}{1-\alpha^d}} mc^{\frac{1-\eta}{1-\alpha^d}} \\ &= \frac{\alpha^d}{1-\alpha^d} (1+\nu)^{\frac{1-\eta(2-\alpha^d)}{1-\alpha^d}-1-\frac{\alpha^d}{1-\alpha^d}} mc^{\frac{1-\eta}{1-\alpha^d}-\frac{\alpha^d}{1-\alpha^d}-1} \\ &= \frac{\alpha^d}{1-\alpha^d} (1+\nu)^{\frac{1-\eta(2-\alpha^d)-(1-\alpha^d)-\alpha^d}{1-\alpha^d}} mc^{\frac{1-\eta-\alpha^d-(1-\alpha^d)}{1-\alpha^d}} \\ &= \frac{\alpha^d}{1-\alpha^d} (1+\nu)^{-\eta\frac{2-\alpha^d}{1-\alpha^d}} mc^{\frac{-\eta}{1-\alpha^d}}. \end{aligned}$$

Furthermore,

$$\begin{aligned} Q^m &= (1-\alpha^d) \frac{PZ}{Pm} \iff \\ Z &= \frac{1}{1-\alpha^d} \frac{P^m}{P} Q^m \\ &= \frac{1}{1-\alpha^d} (1+\nu)^{-\frac{\alpha^d}{1-\alpha^d}} mc^{-\frac{\alpha^d}{1-\alpha^d}} (1+\nu)^{\frac{1-\eta(2-\alpha^d)}{1-\alpha^d}} mc^{\frac{1-\eta}{1-\alpha^d}} \\ &= \frac{1}{1-\alpha^d} (1+\nu)^{\frac{1-\eta(2-\alpha^d)-\alpha^d}{1-\alpha^d}} mc^{\frac{1-\eta-\alpha^d}{1-\alpha^d}} \\ &= \frac{1}{1-\alpha^d} (1+\nu)^{1-\eta\frac{2-\alpha^d}{1-\alpha^d}} mc^{1-\frac{\eta}{1-\alpha^d}}. \end{aligned} \quad (2.49)$$

Given the various quantities of intermediaries we can obtain a steady-state expression for real GDP;

$$\begin{aligned}
Y &= Q^d + Q^x \\
&= \frac{\alpha^d}{1 - \alpha^d} (1 + \nu)^{-\eta \frac{2-\alpha^d}{1-\alpha^d}} mc^{\frac{-\eta}{1-\alpha^d}} + (1 + \nu)^{-\eta \frac{2-\alpha^d}{1-\alpha^d}} mc^{\frac{-\eta}{1-\alpha^d}} \\
&= (1 + \nu)^{-\eta \frac{2-\alpha^d}{1-\alpha^d}} mc^{\frac{-\eta}{1-\alpha^d}} \left[\frac{\alpha^d}{1 - \alpha^d} + 1 \right] \\
&= \frac{1}{1 - \alpha^d} (1 + \nu)^{-\eta \frac{2-\alpha^d}{1-\alpha^d}} mc^{\frac{-\eta}{1-\alpha^d}}. \tag{2.50}
\end{aligned}$$

Turning to labour and capital, we have from (2.10) that

$$\begin{aligned}
K &= Y \left(\frac{1 - \psi}{\psi} \frac{R}{W} \right)^{-(1-\psi)} \\
&= \frac{1}{1 - \alpha^d} (1 + \nu)^{-\eta \frac{2-\alpha^d}{1-\alpha^d}} mc^{\frac{-\eta}{1-\alpha^d}} \left[\frac{1 - \psi}{\psi} \frac{i + \delta}{(1 + \gamma) [(1 - h) C]^{\sigma_C} L^{\sigma_L}} \right]^{-(1-\psi)} \\
&= \frac{1}{1 - \alpha^d} (1 + \nu)^{-\eta \frac{2-\alpha^d}{1-\alpha^d}} mc^{\frac{-\eta}{1-\alpha^d}} \left(\frac{\psi}{1 - \psi} \right)^{(1-\psi)} (i + \delta)^{-(1-\psi)} [(1 + \gamma) [(1 - h) C]^{\sigma_C} L^{\sigma_L}]^{1-\psi} \\
&= \frac{1}{1 - \alpha^d} (1 + \nu)^{-\eta \frac{2-\alpha^d}{1-\alpha^d}} mc^{\frac{-\eta}{1-\alpha^d}} \left(\frac{\psi}{1 - \psi} \right)^{(1-\psi)} (i + \delta)^{-(1-\psi)} (i + \delta)^{-\psi} \psi^\psi (1 - \psi)^{(1-\psi)} mc \\
&= \frac{\psi}{(1 - \alpha^d) (i + \delta)} (1 + \nu)^{-\eta \frac{2-\alpha^d}{1-\alpha^d}} mc^{1 - \frac{\eta}{1-\alpha^d}}, \tag{2.51}
\end{aligned}$$

where we made use of (2.41). Thus, using (2.9) we can solve for labour as follows;

$$\begin{aligned}
L &= \frac{1 - \psi}{\psi} \frac{R}{W} K \\
&= \frac{1 - \psi}{\psi} \frac{i + \delta}{(1 + \gamma) [(1 - h) C]^{\sigma_C} L^{\sigma_L}} \frac{\psi}{(1 - \alpha^d) (i + \delta)} (1 + \nu)^{-\eta \frac{2-\alpha^d}{1-\alpha^d}} mc^{1 - \frac{\eta}{1-\alpha^d}} \\
&= \frac{1 - \psi}{1 - \alpha^d} (i + \delta) \left[(i + \delta)^{\frac{-\psi}{1-\psi}} \psi^{\frac{\psi}{1-\psi}} (1 - \psi) mc^{\frac{1}{1-\psi}} \right]^{-1} (1 + \nu)^{-\eta \frac{2-\alpha^d}{1-\alpha^d}} mc^{1 - \frac{\eta}{1-\alpha^d}} \\
&= \frac{1}{(1 - \alpha^d)} \psi^{-\frac{\psi}{1-\psi}} (i + \delta)^{\frac{\psi}{1-\psi}} (1 + \nu)^{-\eta \frac{2-\alpha^d}{1-\alpha^d}} mc^{1 - \frac{\eta}{1-\alpha^d} - \frac{1}{1-\psi}} \\
&= \frac{1}{(1 - \alpha^d)} \psi^{-\frac{\psi}{1-\psi}} (i + \delta)^{\frac{\psi}{1-\psi}} (1 + \nu)^{-\eta \frac{2-\alpha^d}{1-\alpha^d}} mc^{1 - \frac{\eta}{1-\alpha^d} - \frac{1}{1-\psi}}. \tag{2.52}
\end{aligned}$$

Substituting (2.52) into (2.42) yields

$$\begin{aligned}
C &= \frac{1}{1-h} (1+\gamma)^{-\frac{1}{\sigma_C}} L^{-\frac{\sigma_L}{\sigma_C}} (i+\delta)^{-\frac{\psi}{(1-\psi)\sigma_C}} \psi^{\frac{\psi}{(1-\psi)\sigma_C}} (1-\psi)^{\frac{1}{\sigma_C}} mc^{\frac{1}{(1-\psi)\sigma_C}} \\
&= \frac{1}{1-h} (1+\gamma)^{-\frac{1}{\sigma_C}} (1-\alpha^d)^{\frac{\sigma_L}{\sigma_C}} \psi^{\frac{\sigma_L}{\sigma_C} \frac{\psi}{1-\psi} + \frac{\psi}{(1-\psi)\sigma_C}} (1-\psi)^{\frac{1}{\sigma_C}} (i+\delta)^{-\frac{\sigma_L}{\sigma_C} \frac{\psi}{1-\psi} - \frac{\psi}{(1-\psi)\sigma_C}} (1+\nu)^{\eta \frac{\sigma_L}{\sigma_C} \frac{2-\alpha^d}{1-\alpha^d}} \\
&\quad \times mc^{\frac{1}{(1-\psi)\sigma_C} - \frac{\sigma_L}{\sigma_C} \left(1 - \frac{\eta}{1-\alpha^d} - \frac{1}{1-\psi}\right)} \\
&= \frac{(1+\gamma)^{-\frac{1}{\sigma_C}}}{1-h} (1-\alpha^d)^{\frac{\sigma_L}{\sigma_C}} \left(\frac{\psi}{i+\delta}\right)^{\frac{(1+\sigma_L)\psi}{\sigma_C} \frac{1}{1-\psi}} (1-\psi)^{\frac{1}{\sigma_C}} (1+\nu)^{\eta \frac{\sigma_L}{\sigma_C} \frac{2-\alpha^d}{1-\alpha^d}} \\
&\quad \times mc^{\frac{1}{(1-\psi)\sigma_C} - \frac{\sigma_L}{\sigma_C} \left(1 - \frac{\eta}{1-\alpha^d} - \frac{1}{1-\psi}\right)},
\end{aligned}$$

so defining

$$\Lambda^C \equiv \frac{(1+\gamma)^{-\frac{1}{\sigma_C}}}{1-h} (1-\alpha^d)^{\frac{\sigma_L}{\sigma_C}} \left(\frac{\psi}{i+\delta}\right)^{\frac{(1+\sigma_L)\psi}{\sigma_C} \frac{1}{1-\psi}} (1-\psi)^{\frac{1}{\sigma_C}} (1+\nu)^{\eta \frac{\sigma_L}{\sigma_C} \frac{2-\alpha^d}{1-\alpha^d}},$$

we can write

$$C = \Lambda^C mc^{\frac{1}{(1-\psi)\sigma_C} - \frac{\sigma_L}{\sigma_C} \left(1 - \frac{\eta}{1-\alpha^d} - \frac{1}{1-\psi}\right)}. \quad (2.53)$$

Now, combine the goods market equilibrium and the capital accumulation equations evaluated in the steady state in order to obtain

$$\begin{aligned}
Z &= C + I = C + \delta K \Rightarrow \\
\Lambda^Z mc^{1 - \frac{\eta}{1-\alpha^d}} &= \Lambda^C mc^{\frac{1}{(1-\psi)\sigma_C} - \frac{\sigma_L}{\sigma_C} \left(1 - \frac{\eta}{1-\alpha^d} - \frac{1}{1-\psi}\right)} + \delta \Lambda^K mc^{1 - \frac{\eta}{1-\alpha^d}}, \quad (2.54)
\end{aligned}$$

where we have defined

$$\Lambda^Z \equiv \frac{1}{1-\alpha^d} (1+\nu)^{1 - \eta \frac{2-\alpha^d}{1-\alpha^d}}, \quad \Lambda^K \equiv \frac{\psi}{(1-\alpha^d)(i+\delta)} (1+\nu)^{-\eta \frac{2-\alpha^d}{1-\alpha^d}}. \quad (2.55)$$

Hence, we can now obtain a closed-form solution for the real marginal cost from (2.54);

$$\begin{aligned}
(\Lambda^Z - \delta \Lambda^K) mc^{1 - \frac{\eta}{1-\alpha^d}} &= \Lambda^C mc^{\frac{1}{(1-\psi)\sigma_C} - \frac{\sigma_L}{\sigma_C} \left(1 - \frac{\eta}{1-\alpha^d} - \frac{1}{1-\psi}\right)} \iff \\
mc^{\frac{1}{(1-\psi)\sigma_C} - \frac{\sigma_L}{\sigma_C} \left(1 - \frac{\eta}{1-\alpha^d} - \frac{1}{1-\psi}\right) - \left(1 - \frac{\eta}{1-\alpha^d}\right)} &= \frac{\Lambda^Z - \delta \Lambda^K}{\Lambda^C} \iff \\
mc &= \left(\frac{\Lambda^Z - \delta \Lambda^K}{\Lambda^C}\right)^{\left[\frac{1}{(1-\psi)\sigma_C} - \frac{\sigma_L}{\sigma_C} \left(1 - \frac{\eta}{1-\alpha^d} - \frac{1}{1-\psi}\right) - \left(1 - \frac{\eta}{1-\alpha^d}\right)\right]^{-1}} \\
&= \left(\frac{\Lambda^Z - \delta \Lambda^K}{\Lambda^C}\right)^{\left[\frac{1+\sigma_L}{(1-\psi)\sigma_C} - \frac{\sigma_C + \sigma_L}{\sigma_C} \frac{1-\alpha^d - \eta}{1-\alpha^d}\right]^{-1}}.
\end{aligned}$$

We note that

$$\begin{aligned}
\frac{\Lambda^Z - \delta \Lambda^K}{\Lambda^C} &= \Lambda^C \left[\frac{1}{1-\alpha^d} (1+\nu)^{1 - \eta \frac{2-\alpha^d}{1-\alpha^d}} - \frac{\psi}{(1-\alpha^d)(i+\delta)} (1+\nu)^{-\eta \frac{2-\alpha^d}{1-\alpha^d}} \right] \\
&= \Lambda^C \left[(1+\nu) - \frac{\psi}{(i+\delta)} \right] \frac{(1+\nu)^{-\eta \frac{2-\alpha^d}{1-\alpha^d}}}{1-\alpha^d}.
\end{aligned}$$

2.5 Various steady-state ratios

Using the results above, we can derive the following steady-state ratios for use in the log-linearised system below;

$$\frac{Q^d}{Y} = \frac{\frac{\alpha^d}{1-\alpha^d} (1+\nu)^{-\eta \frac{2-\alpha^d}{1-\alpha^d}} m c^{\frac{-\eta}{1-\alpha^d}}}{\frac{1}{1-\alpha^d} (1+\nu)^{-\eta \frac{2-\alpha^d}{1-\alpha^d}} m c^{\frac{-\eta}{1-\alpha^d}}} = \alpha^d,$$

$$\frac{Q^x}{Y} = 1 - \alpha^d,$$

$$\left. \begin{aligned} Z &= \frac{1}{1-\alpha^d} (1+\nu)^{1-\eta \frac{2-\alpha^d}{1-\alpha^d}} m c^{1-\frac{\eta}{1-\alpha^d}} = \Lambda^Z m c^{1-\frac{\eta}{1-\alpha^d}} \\ K &= \frac{\psi}{(1-\alpha^d)(i+\delta)} (1+\nu)^{-\eta \frac{2-\alpha^d}{1-\alpha^d}} m c^{1-\frac{\eta}{1-\alpha^d}} = \Lambda^K m c^{1-\frac{\eta}{1-\alpha^d}} \end{aligned} \right\} \Rightarrow$$

$$\frac{K}{Z} = \frac{\Lambda^K}{\Lambda^Z} = \frac{\psi}{(1-\alpha^d)(i+\delta)} (1+\nu)^{-\eta \frac{2-\alpha^d}{1-\alpha^d}} \left[\frac{1}{1-\alpha^d} (1+\nu)^{1-\eta \frac{2-\alpha^d}{1-\alpha^d}} \right]^{-1}$$

$$= \frac{\psi}{(i+\delta)(1+\nu)},$$

$$\frac{I}{Z} = \delta \frac{K}{Z} = \frac{\delta \psi}{(i+\delta)(1+\nu)},$$

$$\frac{C}{Z} = 1 - \frac{\delta \psi}{(i+\delta)(1+\nu)}.$$

3 Log-linearised Model

$$\hat{Q}_t^d = \hat{P}_t + \hat{Z}_t - \hat{P}_t^d, \quad (3.1)$$

$$\hat{Q}_t^m = \hat{P}_t + \hat{Z}_t - \hat{P}_t^m, \quad (3.2)$$

$$\hat{Q}_t^x = -\eta \hat{P}_t^x + \eta \hat{P}_t^* + \hat{Y}_t^*, \quad (3.3)$$

$$\hat{P}_t = \alpha^d \hat{P}_t^d + (1 - \alpha^d) \hat{P}_t^m, \quad (3.4)$$

$$\hat{L}_t = \hat{R}_t - \hat{W}_t + \hat{K}_t, \quad (3.5)$$

$$\hat{K}_t = -\hat{\theta}_t - (1 - \psi) \hat{R}_t + (1 - \psi) \hat{W}_t + \hat{Y}_t, \quad (3.6)$$

$$\widehat{MC}_t = -\hat{\theta}_t + (1 - \psi) \hat{W}_t + \psi \hat{R}_t, \quad (3.7)$$

$$\hat{\rho}_{t+1} = \hat{U}_{C_{t+1}} - \hat{U}_{C_t} + \hat{P}_t - \hat{P}_{t+1}, \quad (3.8)$$

$$\hat{P}_t^d - d\hat{P}_{t-1}^d = (1 - d)(1 - d\beta) \left[\widehat{MC}_t + \hat{\nu}_t \right] + d\beta E_t \left[P_{t+1}^d - dP_t^d \right], \quad (3.9)$$

$$\hat{P}_t^x - d\hat{P}_{t-1}^x = (1 - d)(1 - d\beta) \left(\widehat{MC}_t - \hat{e}_t + \hat{\nu}_t \right) + d\beta E_t \left[\hat{P}_{t+1}^x - d\hat{P}_t^x \right], \quad (3.10)$$

$$\hat{P}_t^m - d\hat{P}_{t-1}^m = (1 - d)(1 - d\beta) \left(\hat{e}_t + \hat{P}_t^* + \hat{\nu}_t \right) + d\beta E_t \left[\hat{P}_{t+1}^m - d\hat{P}_t^m \right], \quad (3.11)$$

$$\hat{W}_t - D\hat{W}_{t-1} = (1 - D)(1 - D\beta) \left(\hat{P}_t + \hat{U}_{L,t} - \hat{U}_{C,t} + \hat{\gamma}_t \right) + D\beta E_t \left[\hat{W}_{t+1} - D\hat{W}_t \right], \quad (3.12)$$

$$\hat{K}_{t+1} = (1 - \delta) \hat{K}_t + \delta \hat{I}_t, \quad (3.13)$$

$$\hat{B}_{t+1} = (1 + \bar{i}) \hat{B}_t + \hat{P}_t^x + \hat{Q}_t^x - \hat{P}_t^m + \hat{e}_t - \hat{Q}_t^m, \quad \hat{B}_t \equiv \frac{B_t}{P^x Q^x}, \quad (3.14)$$

$$\hat{U}_{C,t} = \hat{\zeta}_t^b - \frac{\sigma_C}{(1 - h)} \hat{C}_t + \frac{h\sigma_C}{(1 - h)} \hat{C}_{t-1}, \quad (3.15)$$

$$\hat{U}_{L,t} = \hat{\zeta}_t^b + \hat{\zeta}_t^L + \sigma_L \hat{L}_t, \quad (3.16)$$

$$\Phi(1 + \beta) \hat{K}_{t+1} = E_t \hat{\rho}_{t+1} - \hat{P}_t + \beta(1 - \delta) E_t \hat{P}_{t+1} + [1 - \beta(1 - \delta)] E_t \hat{R}_{t+1} + \Phi \hat{K}_t + \beta \Phi E_t \hat{K}_{t+2}, \quad (3.17)$$

$$\hat{i}_t = -E_t \hat{\rho}_{t+1}, \quad (3.18)$$

$$\hat{i}_t^f = -E_t \hat{\rho}_{t+1} - E_t \hat{e}_{t+1} + \hat{e}_t, \quad (3.19)$$

$$\hat{i}_t^f = \hat{i}_t^* + \hat{\nu}_t - \lambda \hat{B}_{t+1}, \quad (3.20)$$

$$\hat{e}_t = \hat{\xi}_t, \quad (3.21)$$

$$\hat{Y}_t = \alpha^d \hat{Q}_t^d + (1 - \alpha^d) \hat{Q}_t^x, \quad (3.22)$$

$$\hat{Z}_t = \frac{C}{Z} \hat{C}_t + \frac{I}{Z} \hat{I}_t. \quad (3.23)$$

The system has 24 endogenous and 10 exogenous variables. Of the latter we assume that the markup shocks and the UIP shock (ν_t, γ_t, v_t) are i.i.d. and the remaining seven are AR(1) processes;

$$\hat{\zeta}_t^b = \varrho^b \hat{\zeta}_{t-1}^b + \varepsilon_t^b, \quad (3.24)$$

$$\hat{\zeta}_t^l = \varrho^l \hat{\zeta}_{t-1}^l + \varepsilon_t^l, \quad (3.25)$$

$$\hat{\theta}_t = \varrho^t \hat{\theta}_{t-1} + \varepsilon_t^t, \quad (3.26)$$

$$\hat{\xi}_t = \varrho^m \hat{\xi}_{t-1} + \varepsilon_t^m, \quad (3.27)$$

$$\hat{v}_t^* = \varrho^i \hat{v}_{t-1}^* + \varepsilon_t^i, \quad (3.28)$$

$$\hat{P}_t^* = \varrho^P \hat{P}_{t-1}^* + \varepsilon_t^P, \quad (3.29)$$

$$\hat{Y}_t^* = \varrho^Y \hat{Y}_{t-1}^* + \varepsilon_t^Y. \quad (3.30)$$

3.1 Solving the Log-linearised Model with gensys

We solve the log-linearised system (3.1)-(3.30) with the **gensys** method developed by Sims (2002). For this purpose we collect the 23 endogenous variables with 6 lagged variables and 9 exogenous processes (excluding the policy shock ξ_t) in the (38×1) vector Υ_t ;¹

$$\begin{aligned} \Upsilon_t : & \hat{B}_t, \hat{C}_t, \hat{e}_t, \hat{v}_t, \hat{v}_t^f, \hat{I}_t, \hat{K}_t, \hat{L}_t, \widehat{MC}_t, \hat{P}_t, \hat{P}_t^d, \hat{P}_t^x, \hat{P}_t^m, \hat{Q}_t^d, \hat{Q}_t^x, \hat{Q}_t^m, \hat{R}_t, \hat{\rho}_{t+1}, \hat{U}_{C,t}, \hat{U}_{L,t}, \hat{W}_t, \hat{Y}_t, \hat{Z}_t, \\ & \hat{K}_{t-1}, \hat{P}_{t-1}^d, \hat{P}_{t-1}^x, \hat{P}_{t-1}^m, \hat{W}_{t-1}, \\ & \hat{\zeta}_t^b, \hat{\zeta}_t^l, \hat{\theta}_t, \hat{v}_t, \hat{\gamma}_t, \hat{\xi}_t, \hat{v}_t^*, \hat{P}_t^*, \hat{Y}_t^*. \end{aligned}$$

The i.i.d. shocks are included in the vector $\varepsilon_t \equiv (\varepsilon_t^b, \varepsilon_t^l, \varepsilon_t^t, \hat{v}_t, \varepsilon_t^m, \varepsilon_t^{pm}, \varepsilon_t^{wm}, \varepsilon_t^i, \varepsilon_t^P, \varepsilon_t^Y)$ includes the set of i.i.d. shocks, and the seven expectational errors are included in the vector $\eta_t = (\eta_t^d, \eta_t^x, \eta_t^m, \eta_t^W, \eta_t^K, \eta_t^A, \eta_t^B)$ so that we can write the model in the canonical VAR(1) **gensys** form;

$$\Gamma_0 \Upsilon_t = \Gamma_1 \Upsilon_{t-1} + \Psi \varepsilon_t + \Pi \eta_t.$$

Applying the **gensys** method recasts the system in the solved form

$$\Upsilon_t = \Theta_1 \Upsilon_{t-1} + \Theta_z Z_t.$$

¹Hence, we add six identity equations to the system (3.1)-(3.30), corresponding to the six lagged endogenous variables included in Υ_t , and two definitions of the mark-up shocks ($\hat{v}_t = \varepsilon_t^{mp}, \hat{\gamma}_t = \varepsilon_t^{mw}$).

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Assessing the Welfare Cost of a Fixed Exchange-Rate Policy

Technical Appendix*

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1 Introduction

This note derives the model analysed in Dam and Linaa (2005a) and cast it in the form

$$E_t [\Upsilon_t, \Upsilon_{t+1}, \varepsilon_t, \varepsilon_{t+1}] = 0,$$

thus facilitating a second-order approximation according to the methodology laid out in Schmitt-Grohe and Uribe (2004).

The structure of the model is basically identical to the one estimated in Dam and Linaa (2005b), which again built on the model in Kollmann (2001, 2002) with Calvo pricing in both the product market and in the labour market. Compared with the Kollmann model we have introduced consumption habits, and we focus on a central bank conducting monetary policy according to an imperfect peg. Additionally, a variety of exogenous shocks are added in line with those utilised by Christiano et al. (2001) and Smets and Wouters (2003). The model is fairly rich in variables and parameters which are summarised in Table 1.

2 The Model

2.1 Final Goods

Domestic final goods are produced from Dixit-Stiglitz aggregates of a continuum of tradable intermediate goods. These are produced domestically and abroad;

$$Q_t^i = \left[\int_0^1 q^i(s)^{\frac{1}{1+\nu}} ds \right]^{1+\nu}, \quad i = d, m.$$

Here, ν turns out to be the net markup rate.

*Both authors were doctoral students at the University of Copenhagen when this work was conducted.

Z_t is the production of final goods using the Cobb Douglas technology

$$Z_t = \left(\frac{Q_t^d}{\alpha^d} \right)^{\alpha^d} \left(\frac{Q_t^m}{\alpha^m} \right)^{\alpha^m}, \quad \alpha^d + \alpha^m = 1.$$

Assuming domestic firms face the problem of minimizing the cost of producing Z_t units of the final good, demands for goods produced domestically and abroad can be written as

$$Q_t^i = \alpha^i \frac{P_t}{P_t^i} Z_t, \quad i = d, m,$$

where P_t is the appropriate price index given by

$$P_t = (P_t^d)^{\alpha^d} (P_t^m)^{\alpha^m}.$$

Thus, P_t is the marginal cost of the final-goods producing firm. With perfect competition in the final-goods market, the price of one unit is also P_t .

2.2 Intermediate Goods

Intermediate goods are produced from labour L_t and capital K_t using Cobb-Douglas technology. Thus, the production function of firm s is

$$y_t(s) = \theta_t K_t(s)^\psi L_t(s)^{1-\psi}, \quad 0 < \psi < 1,$$

where θ_t is the exogenously given aggregate level of technology. Producers operate in a monopolistic competitive market, where each producer sets the price of her variety, taking other prices as given and supplying whatever amount is demanded at the price set.

Cost minimization implies the following first-order conditions

$$\left. \begin{aligned} (1 - \psi) y_t(s) / L_t(s) &= W_t \\ \psi y_t(s) / K_t(s) &= R_t, \end{aligned} \right\} \Rightarrow \frac{L_t(s)}{K_t(s)} = \frac{1 - \psi}{\psi} \frac{R_t}{W_t},$$

where R_t is the rental price of capital and W_t is the wage rate. Therefore we have

$$L_t(s) = \frac{1 - \psi}{\psi} \frac{R_t}{W_t} K_t(s) \Rightarrow$$

and

$$K_t(s) = \frac{1}{\theta_t} \left(\frac{1 - \psi}{\psi} \frac{R_t}{W_t} \right)^{-(1-\psi)} y_t(s).$$

As a consequence, the firm's marginal costs are

$$MC_t = \frac{1}{\theta_t} W_t^{1-\psi} R_t^\psi \psi^{-\psi} (1 - \psi)^{-(1-\psi)}.$$

Following Calvo (1983), we assume that the firm only reoptimizes its prices in any given period with probability $1 - d$. Producers sell their good variety to both domestic and foreign final-goods producers; $y_t(s) = q_t^d(s) + q_t^m(s)$ and are able to price discriminate between the two markets. As is well-known from the Dixit-Stiglitz models, final-good

the following profits in the domestic market;

$$\pi^m(p_t^m(s)) = (p_t^m(s) - e_t P_t^*) \left(\frac{p_t^m(s)}{P_t^m} \right)^{-\frac{1+\nu}{\nu}} Q_t^m.$$

Hence, a domestic firm reoptimizing its domestic price faces the problem:

$$\begin{aligned} p_{t,t}^d &= \arg \max_{\omega} \sum_{\tau=t}^{\infty} d^{\tau-t} E_t [\rho_{t,\tau} \pi^{dx}(\omega, p_t^x(s))], \\ \rho_{t,\tau} &\equiv \beta^{\tau} (U_{C,\tau}/U_{C,t}) (P_t/P_{\tau}), \end{aligned} \quad (2.2)$$

where $\rho_{t,\tau}$ discounts profits at time τ , and $d^{\tau-t}$ is the probability that the current pricing decision is still in effect in period τ . Substituting from the profit expression (2.1) yields

$$\begin{aligned} &\sum_{\tau=t}^{\infty} d^{\tau-t} E_t \left[\rho_{t,\tau} (p_{t,t}^d - MC_{\tau}) \left(\frac{p_{t,t}^d}{P_{\tau}^d} \right)^{-\frac{1+\nu}{\nu}} Q_{\tau}^d \right] \\ &= \sum_{\tau=t}^{\infty} d^{\tau-t} E_t \left[(p_{t,t}^d - MC_{\tau}) (p_{t,t}^d)^{-\frac{1+\nu}{\nu}} \rho_{t,\tau} (P_{\tau}^d)^{\frac{1+\nu}{\nu}} Q_{\tau}^d \right]. \end{aligned}$$

The first-order condition is

$$\begin{aligned} &\sum_{\tau=t}^{\infty} d^{\tau-t} E_t \left[(p_{t,t}^d - (1+\nu) MC_{\tau}) \rho_{t,\tau} (P_{\tau}^d)^{\frac{1+\nu}{\nu}} Q_{\tau}^d \right] = 0 \iff \\ p_{t,t}^d &= (1+\nu) \frac{\sum_{\tau=t}^{\infty} d^{\tau-t} E_t \left[\rho_{t,\tau} (P_{\tau}^d)^{\frac{1+\nu}{\nu}} Q_{\tau}^d MC_{\tau} \right]}{\sum_{\tau=t}^{\infty} d^{\tau-t} E_t \left[\rho_{t,\tau} (P_{\tau}^d)^{\frac{1+\nu}{\nu}} Q_{\tau}^d \right]}, \end{aligned}$$

which we can cast as

$$\begin{aligned} p_{t,t}^d &= \frac{\mathcal{N}_t^d}{\mathcal{D}_t^d}, \\ \text{where } \mathcal{N}_t^d &\equiv \sum_{\tau=t}^{\infty} d^{\tau-t} E_t \left[\rho_{t,\tau} (P_{\tau}^d)^{\frac{1+\nu}{\nu}} Q_{\tau}^d (1+\nu) MC_{\tau} \right], \\ \mathcal{D}_t^d &\equiv \sum_{\tau=t}^{\infty} d^{\tau-t} E_t \left[\rho_{t,\tau} (P_{\tau}^d)^{\frac{1+\nu}{\nu}} Q_{\tau}^d \right]. \end{aligned}$$

Since

$$\begin{aligned}
\mathcal{N}_t^d &= (P_t^d)^{\frac{1+\nu}{\nu}} Q_t^d (1+\nu) MC_t + \sum_{\tau=t+1}^{\infty} d^{\tau-t} E_t \left[\rho_{t,\tau} (P_\tau^d)^{\frac{1+\nu}{\nu}} Q_\tau^d (1+\nu) MC_\tau \right] \\
&= (P_t^d)^{\frac{1+\nu}{\nu}} Q_t^d (1+\nu) MC_t \\
&\quad + dE_t \left[\rho_{t,t+1} \sum_{\tau=t+1}^{\infty} d^{\tau-(t+1)} E_{t+1} \left[\rho_{t+1,\tau} (P_\tau^d)^{\frac{1+\nu}{\nu}} Q_\tau^d (1+\nu) MC_\tau \right] \right] \\
&= (P_t^d)^{\frac{1+\nu}{\nu}} Q_t^d (1+\nu) MC_t + dE_t [\rho_{t,t+1} \mathcal{N}_{t+1}^d], \\
\mathcal{D}_t^d &= (P_t^d)^{\frac{1+\nu}{\nu}} Q_t^d + dE_t [\rho_{t,t+1} \mathcal{D}_{t+1}^d],
\end{aligned}$$

and aggregate prices evolve according to

$$\begin{aligned}
P_t^i &= \left[\int_0^1 p_t^i(s)^{-\frac{1}{\nu}} ds \right]^{-\nu} = \left[d \int_0^1 (p_{t-1}^i(s))^{-\frac{1}{\nu}} ds + (1-d) (p_{t,t}^i)^{-\frac{1}{\nu}} \right]^{-\nu} \\
&= \left[d (P_{t-1}^i)^{-\frac{1}{\nu}} + (1-d) (p_{t,t}^i)^{-\frac{1}{\nu}} \right]^{-\nu}, \quad i = d, x, m,
\end{aligned} \tag{2.3}$$

we can establish the following system of equations for the price of domestic intermediaries;

$$\begin{aligned}
\mathcal{N}_t^d &= (P_t^d)^{\frac{1+\nu}{\nu}} Q_t^d (1+\nu) MC_t + dE_t [\rho_{t,t+1} \mathcal{N}_{t+1}^d], \\
\mathcal{D}_t^d &= (P_t^d)^{\frac{1+\nu}{\nu}} Q_t^d + dE_t [\rho_{t,t+1} \mathcal{D}_{t+1}^d], \\
P_t^d &= \left[d (P_{t-1}^d)^{-\frac{1}{\nu}} + (1-d) (\mathcal{N}_t^d / \mathcal{D}_t^d)^{-\frac{1}{\nu}} \right]^{-\nu}.
\end{aligned}$$

Analogously, the optimal price for sales to foreign final-goods producers is

$$p_{t,t}^x = (1+\nu) \frac{\sum_{\tau=t}^{\infty} d^{\tau-t} E_t \left[\rho_{t,\tau} (P_\tau^x)^{\frac{1+\nu}{\nu}} Q_\tau^x MC_\tau \right]}{\sum_{\tau=t}^{\infty} d^{\tau-t} E_t \left[\rho_{t,\tau} (P_\tau^x)^{\frac{1+\nu}{\nu}} Q_\tau^x e_\tau \right]}.$$

Thus, we obtain the following system of equations for the export price;

$$\begin{aligned}
\mathcal{N}_t^x &= (P_t^x)^{\frac{1+\nu}{\nu}} Q_t^x (1+\nu) MC_t + dE_t [\rho_{t,t+1} \mathcal{N}_{t+1}^x], \\
\mathcal{D}_t^x &= (P_t^x)^{\frac{1+\nu}{\nu}} Q_t^x e_t + dE_t [\rho_{t,t+1} \mathcal{D}_{t+1}^x], \\
P_t^x &= \left[d (P_{t-1}^x)^{-\frac{1}{\nu}} + (1-d) (\mathcal{N}_t^x / \mathcal{D}_t^x)^{-\frac{1}{\nu}} \right]^{-\nu}.
\end{aligned}$$

Import firms are owned by risk-neutral foreigners who discount future profits at the foreign nominal interest rate $R_{t,\tau} \equiv \prod_{s=t}^{\tau-1} (1+i_s^*)^{-1}$. Thus, they set their prices in order

to maximize discounted future profits measured in foreign currency;

$$\begin{aligned} p_{t,t}^m &= \arg \max_{\omega} \sum_{\tau=t}^{\infty} d^{\tau-t} E_t [R_{t,\tau} \pi^m(\omega) / e_{\tau}] \\ &= (1 + \nu) \frac{\sum_{\tau=t}^{\infty} d^{\tau-t} E_t \left[R_{t,\tau} (P_{\tau}^m)^{\frac{1+\nu}{\nu}} Q_{\tau}^m e_{\tau} P_{\tau}^* \right]}{\sum_{\tau=0}^{\infty} d^{\tau-t} E_t \left[R_{t,\tau} (P_{\tau}^m)^{\frac{1+\nu}{\nu}} Q_{\tau}^m \right]}, \end{aligned}$$

which follows from the first-order condition

$$\sum_{\tau=t}^{\infty} d^{\tau} E_t \left[(p_{t,t}^d - (1 + \nu) e_{\tau} P_{\tau}^*) R_{t,\tau} (P_{\tau}^m)^{\frac{1+\nu}{\nu}} Q_{\tau}^m \right] = 0.$$

Hence, we obtain the following pricing equations;

$$\begin{aligned} \mathcal{N}_t^m &= (P_t^m)^{\frac{1+\nu}{\nu}} Q_t^m (1 + \nu) e_t P_t^* + \sum_{\tau=t+1}^{\infty} d^{\tau-t} E_t \left[R_{t,\tau} (P_{\tau}^m)^{\frac{1+\nu}{\nu}} Q_{\tau}^m (1 + \nu) e_{\tau} P_{\tau}^* \right] \\ &= (P_t^m)^{\frac{1+\nu}{\nu}} Q_t^m (1 + \nu) e_t P_t^* + \\ &\quad + d E_t \left[R_{t,t+1} \sum_{\tau=t+1}^{\infty} d^{\tau-(t+1)} E_{t+1} \left[R_{t+1,\tau} (P_{\tau}^m)^{\frac{1+\nu}{\nu}} Q_{\tau}^m (1 + \nu) e_{\tau} P_{\tau}^* \right] \right] \Rightarrow \\ \mathcal{N}_t^m &= (P_t^m)^{\frac{1+\nu}{\nu}} Q_t^m (1 + \nu) e_t P_t^* + \frac{d}{1 + i_t^*} E_t [\mathcal{N}_{t+1}^m], \\ \mathcal{D}_t^m &= (P_t^m)^{\frac{1+\nu}{\nu}} Q_t^m + \frac{d}{1 + i_t^*} E_t [\mathcal{D}_{t+1}^m], \\ P_t^m &= \left[d (P_{t-1}^m)^{-\frac{1}{\nu}} + (1 - d) (\mathcal{N}_t^m / \mathcal{D}_t^m)^{-\frac{1}{\nu}} \right]^{-\nu}. \end{aligned}$$

2.3 Households

Like Erceg et al. (2000) we assume a continuum with unity mass of symmetric households who obtain utility from consumption of the final good and disutility from labour efforts. Thus, they are all characterized by the following preferences:

$$E_0 \left[\sum_{t=0}^{\infty} \beta^t U(C_t^*(j), l_t(j)) \right], \quad U(C_t^*, l_t(j)) = \zeta_t^b \left[\frac{C_t^*(j)^{1-\sigma_C}}{1-\sigma_C} - \zeta_t^L \frac{l_t(j)^{1+\sigma_L}}{1+\sigma_L} \right], \quad (2.4)$$

where ζ_t^b represents a shock to the discount rate and ζ_t^L represents a shock to the labour supply, while $j \in [0, 1]$ signifies the household. We assume external habit formation in consumption; thus, utility is obtained from

$$C_t^*(j) = C_t(j) - h \bar{C}_{t-1}, \quad (2.5)$$

where \bar{C}_t is the aggregate consumption level, which is considered exogenous to each household. We further assume a security market where households completely diversify their individual income uncertainty, so that consumption is equalised across households; $C_t(j) = C_t, \forall j$.

Each household supplies an idiosyncratic variety of labour service $l_t(j)$. These labour services enter as a Dixit-Stiglitz aggregate in the intermediate-goods firm production; thus, letting $l_t(s, j)$ be the amount of labour service j utilized by firm s we find that firm s uses the following amount of labour services;

$$L_t(s) = \left[\int_0^1 l_t(s, j)^{\frac{1}{1+\gamma}} dj \right]^{1+\gamma}, \quad \gamma > 0, \quad (2.6)$$

where γ turns out to be the net wage markup.

As was the case of intermediary prices, wage setting is staggered a la Calvo (1983). That is, in each period household j only optimizes its wage $w_t(j)$ with probability $1 - D$. The household takes the average wage rate $W_t = \left[\int_0^1 w_t(j)^{-\frac{1}{1+\gamma}} dj \right]^{-(1+\gamma)}$ as given when it chooses its optimal wage $w_{t,t}$ and will meet any demand for the given type of labour;¹

$$\begin{aligned} l_t(j) &= \int_0^1 l_t(s, j) ds = \int_0^1 \left(\frac{w_t(j)}{W_t} \right)^{-\frac{1+\gamma}{\gamma}} L_t(s) ds \\ &= \left(\frac{w_t(j)}{W_t} \right)^{-\frac{1+\gamma}{\gamma}} L_t \end{aligned} \quad (2.7)$$

In addition to consumption, households can invest in domestic and foreign one-period bonds as well as in domestic capital. Capital K_t earns rental rate R_t and accumulates as follows;

$$K_{t+1} = K_t(1 - \delta) + I_t - \frac{\Phi}{2} \frac{(K_{t+1} - K_t)^2}{K_t}, \quad 0 < \delta < 1. \quad (2.8)$$

Here, we have followed Kollmann (2002) and assumed quadratic adjustment costs. Domestic bonds A_t earns net interest i_t , while the interest i_t^f accruing to foreign bonds B_t held by domestic agents deviates from the exogenously given foreign interest level i_t^* as follows;

$$(1 + i_t^f) = \Omega_t^j (1 + i_t^*), \quad (2.9)$$

$$\Omega_t = v_t \exp \left\{ -\lambda \frac{e_t B_{t+1}}{P_t \Xi} \right\}, \quad \Xi = \frac{e P^x Q^x}{P}, \quad (2.10)$$

where Ξ is the steady-state value of export in units of the domestic final good. Thus, the interest on foreign bonds is growing in the foreign debt level which ensures the existence of a unique equilibrium, cf. Schmitt-Grohe and Uribe (2003), while v_t is a UIP shock.

Households own equal shares of domestic firms and thus earn profit from the intermediate-goods firms ($\Delta_t(j)$) in addition to rental rates R_t on the capital, wage income from their labour services and payments from their state-contingent securities ($S_t(j)$). Hence, the budget constraint of household j is

$$\begin{aligned} A_{t+1}(j) + e_t B_{t+1}(j) + P_t(C_t(j) + I_t(j)) = \\ A_t(j)(1 + i_{t-1}) + e_t B_t(j) \left(1 + i_{t-1}^f \right) + R_t K_t(j) + \Delta_t(j) + w_t(j) l_t(j) + S_t(j). \end{aligned} \quad (2.11)$$

¹Note that the optimal wage in any period is identical across households, which is the reason why $w_{t,t}$ can be written without index j .

Thus, households face the following problem

$$\begin{aligned} & \max_{\{C_t(j), A_{t+1}(j), B_{t+1}(j), K_{t+1}(j), w_{t,t}\}_{t=0}^{\infty}} E_0 \left[\sum_{t=0}^{\infty} \beta^t U(C_t^*(j), l_t(j)) \right] \\ & \text{s.t. (2.4)-(2.11),} \end{aligned} \quad (2.12)$$

and the first-order conditions with respect to capital, domestic bonds and foreign bonds can be written as

$$E_t \left[\rho_{t,t+1} \frac{P_{t+1}}{P_t} \frac{\left(\frac{R_{t+1}}{P_{t+1}} + (1 - \delta) - \frac{\Phi}{2} \left(1 - \left(\frac{K_{t+2}}{K_{t+1}} \right)^2 \right) \right)}{\left[1 + \Phi \frac{K_{t+1} - K_t}{K_t} \right]} \right] = 1, \quad (2.13)$$

$$(1 + i_t) E_t [\rho_{t,t+1}] = 1, \quad (2.14)$$

$$(1 + i_t^f) E_t \left[\rho_{t,t+1} \frac{e_{t+1}}{e_t} \right] = 1, \quad (2.15)$$

where $\rho_{t,t+k}$ is defined in equation (2.2) above.

Having assumed that the household always meets demand for labour at its chosen wage level, we finally arrive at the first-order condition with respect to the wage rate;

$$\begin{aligned} & \sum_{\tau=t}^{\infty} (D\beta)^{\tau-t} E_t \left[-\frac{U_{C,\tau}}{P_\tau} \frac{1}{\gamma} \left(\frac{w_{t,t}}{W_\tau} \right)^{-\frac{1+\gamma}{\gamma}} L_\tau + \zeta_t^b \zeta_t^L \frac{1+\gamma}{\gamma} \left(\frac{w_{t,t}}{W_\tau} \right)^{-\frac{1+\gamma}{\gamma}(1+\sigma_L)} \frac{L_\tau^{1+\sigma_L}}{w_{t,t}} \right] = 0 \Rightarrow \\ & w_{t,t}^{1+\frac{1+\gamma}{\gamma}\sigma_L} \sum_{\tau=t}^{\infty} (D\beta)^{\tau-t} E_t \left[\frac{U_{C,\tau}}{P_\tau} W_\tau^{\frac{1+\gamma}{\gamma}} L_\tau \right] = \sum_{\tau=t}^{\infty} (\beta D)^{\tau-t} E_t \left[\zeta_t^b \zeta_t^L (1+\gamma) W_\tau^{\frac{1+\gamma}{\gamma}} L_\tau^{1+\sigma_L} \right] \Rightarrow \\ & w_{t,t}^{1+\frac{1+\gamma}{\gamma}\sigma_L} = \frac{\sum (D\beta)^{\tau-t} E_t \left[\zeta_t^b \zeta_t^L (1+\gamma) W_\tau^{\frac{1+\gamma}{\gamma}(1+\sigma_L)} L_\tau^{1+\sigma_L} \right]}{\sum (D\beta)^{\tau-t} E_t \left[\frac{U_{C,\tau}}{P_\tau} W_\tau^{\frac{1+\gamma}{\gamma}} L_\tau \right]} \equiv \frac{\mathcal{N}_t^w}{\mathcal{D}_t^w}, \end{aligned}$$

where

$$\begin{aligned}
\mathcal{N}_t^w &= \sum_{\tau=t}^{\infty} (D\beta)^{\tau-t} E_t \left[\zeta_\tau^b \zeta_\tau^L (1+\gamma) W_\tau^{\frac{1+\gamma}{\gamma}(1+\sigma_L)} L_\tau^{1+\sigma_L} \right] \\
&= \zeta_t^b \zeta_t^L (1+\gamma) W_t^{\frac{1+\gamma}{\gamma}(1+\sigma_L)} L_t^{1+\sigma_L} \\
&\quad + \sum_{\tau=t+1}^{\infty} (D\beta)^{\tau-t} E_t \left[\zeta_\tau^b \zeta_\tau^L (1+\gamma) W_\tau^{\frac{1+\gamma}{\gamma}(1+\sigma_L)} L_\tau^{1+\sigma_L} \right] \\
&= \zeta_t^b \zeta_t^L (1+\gamma) W_t^{\frac{1+\gamma}{\gamma}(1+\sigma_L)} L_t^{1+\sigma_L} \\
&\quad + D\beta E_t \left[\Pi_{t+1}^{\frac{1+\gamma}{\gamma}(1+\sigma_L)} \sum_{\tau=t+1}^{\infty} (D\beta)^{\tau-(t+1)} E_{t+1} \left[\zeta_\tau^b \zeta_\tau^L (1+\gamma) W_\tau^{\frac{1+\gamma}{\gamma}(1+\sigma_L)} L_\tau^{1+\sigma_L} \right] \right] \\
&= \zeta_t^b \zeta_t^L (1+\gamma) W_t^{\frac{1+\gamma}{\gamma}(1+\sigma_L)} L_t^{1+\sigma_L} + D\beta E_t [\mathcal{N}_{t+1}^w], \\
\mathcal{D}_t^w &= \sum_{\tau=t}^{\infty} (D\beta)^{\tau-t} E_t \left[\frac{U_{C,\tau}}{P_\tau} W_\tau^{\frac{1+\gamma}{\gamma}} L_\tau \right] \\
&= \frac{U_{C,t}}{P_t} W_t^{\frac{1+\gamma}{\gamma}} L_t + D\beta E_t \left[\sum_{\tau=t}^{\infty} (D\beta)^{\tau-(t+1)} E_{t+1} \left[\frac{U_{C,\tau}}{P_\tau} W_\tau^{\frac{1+\gamma}{\gamma}} L_\tau \right] \right] \\
&= \frac{U_{C,t}}{P_t} W_t^{\frac{1+\gamma}{\gamma}} L_t + D\beta E_t [\mathcal{D}_{t+1}^w].
\end{aligned}$$

Analogously to equation (2.3), the aggregate wage level is determined as

$$W_t = \left[D(W_{t-1})^{-\frac{1}{\gamma}} + (1-D)(w_{t,t})^{-\frac{1}{\gamma}} \right]^{-\gamma}.$$

Combining these, we obtain a four-equation system for aggregate wages similar to the pricing equations above;

$$\begin{aligned}
\mathcal{N}_t^w &= \zeta_t^b \zeta_t^L (1+\gamma) W_t^{\frac{1+\gamma}{\gamma}(1+\sigma_L)} L_t^{1+\sigma_L} + D\beta E_t [\hat{\mathcal{N}}_{t+1}^w], \\
\mathcal{D}_t^w &= \frac{U_{C,t}}{P_t} W_t^{\frac{1+\gamma}{\gamma}} L_t + D\beta E_t [\hat{\mathcal{D}}_{t+1}^w], \\
w_{t,t} &= (\mathcal{N}_t^w / \mathcal{D}_t^w)^{\frac{\gamma}{\gamma+(1+\gamma)\sigma_L}}, \\
W_t &= \left[DW_{t-1}^{-\frac{1}{\gamma}} + (1-D)w_{t,t}^{-\frac{1}{\gamma}} \right]^{-\gamma}.
\end{aligned}$$

2.4 Market Clearing Conditions

All intermediaries are demanded from either domestic or foreign final goods producers

$$Y_t = Q_t^d + Q_t^x.$$

In the final goods market equilibrium requires

$$Z_t = C_t + I_t, \tag{2.16}$$

Turning to the capital market, we consider first the capital demand of firm s ;

$$\begin{aligned} K_t(s) &= \frac{1}{\theta_t} \left(\frac{1-\psi}{\psi} \frac{R_t}{W_t} \right)^{-(1-\psi)} [q_t^d(s) + q_t^x(s)] \\ &= \frac{1}{\theta_t} \left(\frac{1-\psi}{\psi} \frac{R_t}{W_t} \right)^{-(1-\psi)} \left[\left(\frac{p_t^d(s)}{P_t^d} \right)^{-\frac{1+\nu}{\nu}} Q_t^d + \left(\frac{p_t^x(s)}{P_t^x} \right)^{-\frac{1+\nu}{\nu}} Q_t^x \right]. \end{aligned}$$

It follows directly that aggregate demand for capital is

$$\begin{aligned} \mathcal{K}_t &= \int_0^1 K_t(s) ds \\ &= \frac{1}{\theta_t} \left(\frac{\psi}{1-\psi} \frac{W_t}{R_t} \right)^{1-\psi} \int_0^1 \left[\left(\frac{p_t^d(s)}{P_t^d} \right)^{-\frac{1+\nu}{\nu}} Q_t^d + \left(\frac{p_t^x(s)}{P_t^x} \right)^{-\frac{1+\nu}{\nu}} Q_t^x \right] ds \\ &= \frac{1}{\theta_t} \left(\frac{\psi}{1-\psi} \frac{W_t}{R_t} \right)^{1-\psi} \left[\left(\frac{\mathcal{P}_t^d}{P_t^d} \right)^{-\frac{1+\nu}{\nu}} Q_t^d + \left(\frac{\mathcal{P}_t^x}{P_t^x} \right)^{-\frac{1+\nu}{\nu}} Q_t^x \right], \end{aligned}$$

where we introduce

$$\mathcal{P}_t^i \equiv \left[\int_0^1 (p_t^i)^{-\frac{1+\nu}{\nu}} \right]^{-\frac{\nu}{1+\nu}}, \quad i = d, x.$$

Under the assumptions of the Calvo pricing model, these indices of individual prices evolve as follows;

$$\mathcal{P}_t^i = \left[d (\mathcal{P}_{t-1}^i)^{-\frac{1+\nu}{\nu}} + (1-d) (p_{t,t}^i)^{-\frac{1+\nu}{\nu}} \right]^{-\frac{\nu}{1+\nu}}, \quad i = d, x.$$

Hence, equilibrium in the capital market ($K_t = \mathcal{K}_t$) implies

$$K_t = \frac{1}{\theta_t} \left(\frac{\psi}{1-\psi} \frac{W_t}{R_t} \right)^{1-\psi} \left[\left(\frac{\mathcal{P}_t^d}{P_t^d} \right)^{-\frac{1+\nu}{\nu}} Q_t^d + \left(\frac{\mathcal{P}_t^x}{P_t^x} \right)^{-\frac{1+\nu}{\nu}} Q_t^x \right].$$

Finally, we assume that only domestic agents hold the domestic bond, implying that $A_t = 0$ in equilibrium.

2.5 The Household Budget Constraint and Net Foreign Assets

Aggregating and manipulating the household budget constraint (2.11) and using the final-good market equilibrium (2.16) yields the following equation which simply states that the net foreign assets position (NFA) changes with accruing interest and the net export.

$$\begin{aligned} e_t B_{t+1} + P_t (C_t + I_t) &= e_t B_t \left(1 + i_{t-1}^f \right) + R_t K_t + W_t L_t \\ &\quad + P_t^d Q_t^d + e_t P_t^x Q_t^x - (R_t K_t + W_t L_t) \Rightarrow \\ B_{t+1} &= B_t \left(1 + i_{t-1}^f \right) + P_t^x Q_t^x - \frac{P_t^m}{e_t} Q_t^m. \end{aligned}$$

2.6 Monetary Policy

We postulate an imperfect peg against the euro as the monetary policy; in our model the interest rate is the instrument, which is thus used to keep e_t constant up to an exogenous policy shock ξ_t^{peg} with unity mean;

$$e_t = e\xi_t^{peg}. \quad (2.17)$$

Log-linearizing equations (2.9) and (2.10) yields the following relation between the internal foreign interest rate and that paid to domestic holders of foreign bonds;

$$\hat{i}_t^f = \hat{i}_t^* + \hat{v}_t - \lambda\hat{B}_t.$$

Combining this relation with log-linearised versions of equations (2.14) and (2.15) yields

$$\begin{aligned} E_t\Delta\hat{e}_{t+1} &= \hat{i}_t - \hat{i}_t^f = \hat{i}_t - \hat{i}_t^* + (\lambda\hat{B}_t - \hat{v}_t), \\ \hat{i}_t &\equiv \log\left(\frac{1+i_t}{1+\bar{i}}\right), \quad \hat{v}_t \equiv \log(v_t/v), \quad \hat{B}_t \equiv \log\frac{(B_{t+1}/P_t^*)}{\Xi}, \end{aligned}$$

which we can combine with (2.17) to obtain

$$\hat{i}_t = \hat{i}_t^* + (\hat{v}_t - \lambda\hat{B}_t) + E_t\Delta\hat{\xi}_{t+1}^{peg},$$

that is, the interest rate responds (virtually) one-to-one with the foreign interest rate and the UIP shock and is additionally skewed by the spread and the policy shock.

As an alternative to the peg rule (2.17) we can close the model with a Taylor (1993) like rule;

$$i_t = \rho_i i_{t-1} + (1 - \rho_i) \left(\rho_\pi (\Pi_t - 1) + \rho_y \frac{Y_t - Y_{t-1}}{Y_{t-1}} \right) + \xi_t^{TR}, \quad (2.18)$$

where ξ_t^{TR} is a policy shock, and the ρ 's are restricted to imply a determinate equilibrium, cf. the discussion in Dam and Linaa (2005a).

2.7 Welfare and the Dispersion of Wages and Labour

We use the unconditional expectation of the average household utility in a given period as our measure of welfare. This amounts to the following;

$$\begin{aligned} SW_t &= \int_0^1 U(C_t^*(j), l_t(j)) dj \\ &= \int_0^1 \left(\zeta_t^b \frac{(C_t^*)^{1-\sigma_C}}{1-\sigma_C} - \frac{\zeta_t^b \zeta_t^L}{1+\sigma_L} l_t(j)^{1+\sigma_L} \right) dj \\ &= \frac{\zeta_t^b}{1-\sigma_C} (C_t - h\bar{C}_{t-1})^{1-\sigma_C} - \frac{\zeta_t^b \zeta_t^L}{1+\sigma_L} \int_0^1 l_t(j)^{1+\sigma_L} dj, \end{aligned}$$

implying that we need an expression for $\int_0^1 l_t(j)^{1+\sigma_L} dj$. This illustrates how the wage dispersion only affects the economy through the unequal labour supply across the house-

holds. Since

$$\begin{aligned} l_t(s, j) &= (w_t(j)/W_t)^{-\frac{1+\gamma}{\gamma}} L_t(s) \Rightarrow \\ l_t(j) &= \int_0^1 l_t(s, j) ds = (w_t(j)/W_t)^{-\frac{1+\gamma}{\gamma}} L_t, \end{aligned}$$

we obtain

$$\begin{aligned} \int_0^1 l_t(j)^{1+\sigma_L} dj &= L_t^{1+\sigma_L} \int_0^1 \left(\frac{w_t(j)}{W_t} \right)^{-\frac{1+\gamma}{\gamma}(1+\sigma_L)} dj \\ &= L_t^{1+\sigma_L} \left(\frac{\mathcal{W}_t}{W_t} \right)^{-\frac{1+\gamma}{\gamma}(1+\sigma_L)}, \end{aligned}$$

where

$$\mathcal{W}_t \equiv \left[\int_0^1 w_t(j)^{-\frac{1+\gamma}{\gamma}(1+\sigma_L)} dj \right]^{-\frac{\gamma}{(1+\gamma)(1+\sigma_L)}}.$$

Due to the assumptions of the Calvo-like wage setting, this index of wage dispersion evolves as follows;

$$\mathcal{W}_t = \left[D \mathcal{W}_{t-1}^{-\frac{1+\gamma}{\gamma}(1+\sigma_L)} + (1-D) w_{t,t}^{-\frac{1+\gamma}{\gamma}(1+\sigma_L)} \right]^{-\frac{\gamma}{(1+\gamma)(1+\sigma_L)}}.$$

Thus, the welfare measure can be cast as follows;

$$SW_t = \frac{\zeta_t^b}{1-\sigma_C} (C_t - h\bar{C}_{t-1})^{1-\sigma_C} - \frac{\zeta_t^b \zeta_t^L}{1+\sigma_L} L_t^{1+\sigma_L} \left(\frac{\mathcal{W}_t}{W_t} \right)^{-\frac{1+\gamma}{\gamma}(1+\sigma_L)}.$$

3 Model in Real Terms

We recast the model in real terms, meaning that all prices are stated relative to the price of domestic (or foreign) final goods, e.g., $\tilde{P}_t^d \equiv P_t^d/P_t$. Consider now the optimal intermediary prices;

$$p_{t,t}^d = \frac{\mathcal{N}_t^d}{\mathcal{D}_t^d} \Rightarrow \tilde{p}_{t,t}^d = \frac{\mathcal{N}_t^d/P_t}{\mathcal{D}_t^d} = \frac{(\mathcal{N}_t^d/P_t) / (P_t^d)^{\frac{1+\nu}{\nu}}}{\mathcal{D}_t^d / (P_t^d)^{\frac{1+\nu}{\nu}}} \equiv \frac{\tilde{\mathcal{N}}_t^d}{\tilde{\mathcal{D}}_t^d},$$

where,

$$\begin{aligned}
\tilde{\mathcal{N}}_t^d &\equiv \sum_{\tau=t}^{\infty} d^{\tau-t} E_t \left[\rho_{t,\tau} \left(\frac{P_\tau^d}{P_t^d} \right)^{\frac{1+\nu}{\nu}} Q_\tau^d (1+\nu) \frac{MC_\tau}{P_t} \right] \\
&= Q_t^d (1+\nu) mc_t + E_t \left[\sum_{\tau=t+1}^{\infty} d^{\tau-t} \rho_{t,\tau} \left(\frac{P_\tau^d}{P_t^d} \right)^{\frac{1+\nu}{\nu}} Q_\tau^d (1+\nu) \frac{MC_\tau}{P_t} \right] \\
&= \left(\tilde{P}_t^d \right)^{\frac{1+\nu}{\nu}} Q_t^d (1+\nu) mc_t \\
&\quad + dE_t \left[\rho_{t,t+1} \left(\frac{P_{t+1}^d}{P_t^d} \right)^{\frac{1+\nu}{\nu}} \frac{P_{t+1}}{P_t} \sum_{\tau=t+1}^{\infty} d^{\tau-(t+1)} E_{t+1} \left[\rho_{t+1,\tau} \left(\frac{P_\tau^d}{P_{t+1}^d} \right)^{\frac{1+\nu}{\nu}} Q_\tau^d (1+\nu) \frac{MC_\tau}{P_{t+1}} \right] \right] \\
&= \left(\tilde{P}_t^d \right)^{\frac{1+\nu}{\nu}} Q_t^d (1+\nu) mc_t \\
&\quad + dE_t \left[\rho_{t,t+1} \sum_{\tau=t+1}^{\infty} d^{\tau-(t+1)} E_{t+1} \left[\rho_{t+1,\tau} \left(\frac{P_\tau^d}{P_{t+1}^d} \right)^{\frac{1+\nu}{\nu}} Q_\tau^d (1+\nu) \frac{MC_\tau}{P_{t+1}} \right] \right] \\
&= Q_t^d (1+\nu) mc_t + dE_t \left[\rho_{t,t+1} \left(\frac{\tilde{P}_{t+1}^d}{\tilde{P}_t^d} \right)^{\frac{1+\nu}{\nu}} \Pi_{t+1}^{\frac{1+2\nu}{\nu}} \tilde{\mathcal{N}}_{t+1}^d \right],
\end{aligned}$$

$$\begin{aligned}
\tilde{\mathcal{D}}_t^d &\equiv \sum_{\tau=t}^{\infty} d^{\tau-t} E_t \left[\rho_{t,\tau} \left(\frac{P_\tau^d}{P_t^d} \right)^{\frac{1+\nu}{\nu}} Q_\tau^d \right] \\
&= Q_t^d + dE_t \left[\rho_{t,t+1} \left(\frac{\tilde{P}_{t+1}^d}{\tilde{P}_t^d} \Pi_{t+1} \right)^{\frac{1+\nu}{\nu}} \sum_{\tau=t+1}^{\infty} d^{\tau-(t+1)} \rho_{t+1,\tau} \left(\frac{P_\tau^d}{P_{t+1}^d} \right)^{\frac{1+\nu}{\nu}} Q_\tau^d \right] \\
&= Q_t^d + dE_t \left[\rho_{t,t+1} \left(\frac{\tilde{P}_{t+1}^d}{\tilde{P}_t^d} \Pi_{t+1} \right)^{\frac{1+\nu}{\nu}} \tilde{\mathcal{D}}_{t+1}^d \right].
\end{aligned}$$

Turning to the optimal price on exports and introducing the real exchange rate $\mathcal{E}_t \equiv e_t P_t^*/P_t$, we get

$$p_{t,t}^x = \frac{\mathcal{N}_t^x}{\mathcal{D}_t^x} \Rightarrow \tilde{p}_{t,t}^x = \frac{\mathcal{N}_t^x}{\mathcal{D}_t^x P_t^*} = \frac{(\mathcal{N}_t^x/P_t) / (P_t^x)^{\frac{1+\nu}{\nu}}}{\left(\mathcal{D}_t^x \frac{P_t^*}{P_t} \right) / (P_t^x)^{\frac{1+\nu}{\nu}}} \equiv \frac{\tilde{\mathcal{N}}_t^x}{\tilde{\mathcal{D}}_t^x},$$

where

$$\begin{aligned}
\tilde{\mathcal{N}}_t^x &= \sum_{\tau=t}^{\infty} d^{\tau-t} E_t \left[\rho_{t,\tau} \left(\frac{P_\tau^x}{P_t^x} \right)^{\frac{1+\nu}{\nu}} Q_\tau^x (1+\nu) \frac{MC_\tau}{P_t} \right] \\
&= Q_t^x (1+\nu) mc_t + \sum_{\tau=t+1}^{\infty} d^{\tau-t} E_t \left[\rho_{t,\tau} \left(\frac{P_\tau^x}{P_t^x} \right)^{\frac{1+\nu}{\nu}} Q_\tau^x (1+\nu) \frac{MC_\tau}{P_t} \right] \\
&= Q_t^x (1+\nu) mc_t \\
&\quad + dE_t \left[\rho_{t,t+1} \left(\frac{\hat{P}_{t+1}^x}{\hat{P}_t^x} \Pi_{t+1}^* \right)^{\frac{1+\nu}{\nu}} \sum_{\tau=t+1}^{\infty} d^{\tau-(t+1)} E_{t+1} \left[\rho_{t+1,\tau} Q_\tau^x (1+\nu) \frac{MC_\tau}{P_{t+1}} \right] \right] \\
&= Q_t^x (1+\nu) mc_t + dE_t \left[\rho_{t,t+1} \left(\frac{\tilde{P}_{t+1}^x}{\tilde{P}_t^x} \Pi_{t+1}^* \right)^{\frac{1+\nu}{\nu}} \tilde{\mathcal{N}}_{t+1}^x \right], \\
\tilde{\mathcal{D}}_{t+1}^x &= \sum_{\tau=t}^{\infty} d^{\tau-t} E_t \left[\rho_{t,\tau} \left(\frac{P_\tau^x}{P_t^x} \right)^{\frac{1+\nu}{\nu}} Q_\tau^x \frac{e_\tau P_t^*}{P_t} \right] \\
&= Q_t^x \mathcal{E}_t + dE_t \left[\rho_{t,t+1} \left(\frac{P_{t+1}^x}{P_t^x} \right)^{\frac{1+\nu}{\nu}} \Pi_{t+1} / \Pi_{t+1}^* \sum_{\tau=t+1}^{\infty} d^{\tau-(t+1)} E_t \left[\rho_{t+1,\tau} \left(\frac{P_\tau^x}{P_{t+1}^x} \right)^{\frac{1+\nu}{\nu}} Q_\tau^x \frac{e_\tau P_{t+1}^*}{P_{t+1}} \right] \right] \\
&= Q_t^x \mathcal{E}_t + dE_t \left[\rho_{t,t+1} \left(\frac{\tilde{P}_{t+1}^x}{\tilde{P}_t^x} \right)^{\frac{1+\nu}{\nu}} (\Pi_{t+1}^*)^{\frac{1}{\nu}} \Pi_{t+1} \tilde{\mathcal{D}}_{t+1}^x \right].
\end{aligned}$$

Finally, the optimal relative import price evolves as follows;

$$p_{t,t}^m = \frac{\mathcal{N}_t^m}{\mathcal{D}_t^m} \Rightarrow \tilde{p}_{t,t}^m = \frac{\mathcal{N}_t^m}{\mathcal{D}_t^m P_t} = \frac{(\mathcal{N}_t^m / P_t^*) / (P_\tau^m)^{\frac{1+\nu}{\nu}}}{(\mathcal{D}_t^m P_t / P_t^*) / (P_\tau^m)^{\frac{1+\nu}{\nu}}} \equiv \frac{\tilde{\mathcal{N}}_t^m}{\tilde{\mathcal{D}}_t^m},$$

where

$$\begin{aligned}
\tilde{\mathcal{N}}_t^m &= \sum_{\tau=t}^{\infty} d^{\tau-t} E_t \left[R_{t,\tau} \left(\frac{P_\tau^m}{P_t^m} \right)^{\frac{1+\nu}{\nu}} Q_\tau^m (1+\nu) \frac{P_\tau^*}{P_t^*} \right] \\
&= Q_t^m (1+\nu) \\
&\quad + d E_t \left[R_{t,t+1} \left(\frac{\hat{P}_\tau^m}{\hat{P}_t^m} \Pi_{t+1} \right)^{\frac{1+\nu}{\nu}} \Pi_{t+1}^* \sum_{\tau=t+1}^{\infty} d^{\tau-(t+1)} E_{t+1} \left[R_{t+1,\tau} \left(\frac{P_\tau^m}{P_{t+1}^m} \right)^{\frac{1+\nu}{\nu}} Q_\tau^m (1+\nu) \frac{P_\tau^*}{P_{t+1}^*} \right] \right] \\
&= Q_t^m (1+\nu) + \frac{d}{1+i_t^*} E_t \left[\left(\frac{\tilde{P}_\tau^m}{\tilde{P}_t^m} \Pi_{t+1} \right)^{\frac{1+\nu}{\nu}} \Pi_{t+1}^* \tilde{\mathcal{N}}_{t+1}^m \right], \\
\tilde{\mathcal{D}}_t^m &= \sum_{\tau=t}^{\infty} d^{\tau-t} E_t \left[R_{t,\tau} \left(\frac{P_\tau^m}{P_t^m} \right)^{\frac{1+\nu}{\nu}} Q_\tau^m \frac{P_t}{P_t^* e_\tau} \right] \\
&= \frac{Q_t^m}{\mathcal{E}_t} + \sum_{\tau=t+1}^{\infty} d^{\tau-t} E_t \left[R_{t,\tau} \left(\frac{P_\tau^m}{P_t^m} \right)^{\frac{1+\nu}{\nu}} Q_\tau^m \frac{P_t}{P_t^* e_\tau} \right] \\
&= \frac{Q_t^m}{\mathcal{E}_t} + d E_t \left[R_{t,t+1} \left(\frac{P_{t+1}^m}{P_t^m} \right)^{\frac{1+\nu}{\nu}} \Pi_{t+1}^* / \Pi_{t+1} \sum_{\tau=t+1}^{\infty} d^{\tau-(t+1)} E_{t+1} \left[R_{t+1,\tau} \left(\frac{P_\tau^m}{P_{t+1}^m} \right)^{\frac{1+\nu}{\nu}} Q_\tau^m \frac{P_{t+1}}{P_{t+1}^* e_\tau} \right] \right] \\
&= \frac{Q_t^m}{\mathcal{E}_t} + \frac{d}{1+i_t^*} E_t \left[\left(\frac{\tilde{P}_{t+1}^m}{\tilde{P}_t^m} \right)^{\frac{1+\nu}{\nu}} (\Pi_{t+1})^{\frac{1}{\nu}} \Pi_{t+1}^* \tilde{\mathcal{D}}_{t+1}^m \right].
\end{aligned}$$

Turning to the wage setting, we have

$$\begin{aligned}
w_{t,t}^{1+\frac{1+\gamma}{\gamma}\sigma_L} &= \frac{\sum (d\beta)^{\tau-t} E_t \left[\zeta_t^b \zeta_t^L (1+\gamma) W_\tau^{\frac{1+\gamma}{\gamma}(1+\sigma_L)} L_\tau^{1+\sigma_L} \right]}{\sum (d\beta)^{\tau-t} E_t \left[\frac{U_{C,\tau}}{P_\tau} W_\tau^{\frac{1+\gamma}{\gamma}} L_\tau \right]} \equiv \frac{\mathcal{N}_t^w}{\mathcal{D}_t^w}, \\
\hat{w}_{t,t} &\equiv \frac{w_{t,t}}{P_t} \Rightarrow \\
\tilde{w}_{t,t}^{1+\frac{1+\gamma}{\gamma}\sigma_L} &= \left(\frac{w_{t,t}}{P_t} \right)^{1+\frac{1+\gamma}{\gamma}\sigma_L} = \frac{\mathcal{N}_t^w}{\mathcal{D}_t^w P_t^{1+\frac{1+\gamma}{\gamma}\sigma_L}} = \frac{\mathcal{N}_t^w / P_t^{\frac{1+\gamma}{\gamma}(1+\sigma_L)}}{\mathcal{D}_t^w / P_t^{\frac{1}{\gamma}}} \equiv \frac{\tilde{\mathcal{N}}_t^w}{\tilde{\mathcal{D}}_t^w},
\end{aligned}$$

where

$$\begin{aligned}
\tilde{\mathcal{N}}_t^w &= \sum_{\tau=t}^{\infty} (d\beta)^{\tau-t} E_t \left[\zeta_\tau^b \zeta_\tau^L (1+\gamma) \left(\frac{W_\tau}{P_t} \right)^{\frac{1+\gamma}{\gamma}(1+\sigma_L)} L_\tau^{1+\sigma_L} \right] \\
&= \zeta_t^b \zeta_t^L (1+\gamma) \left(\tilde{W}_t \right)^{\frac{1+\gamma}{\gamma}(1+\sigma_L)} L_t^{1+\sigma_L} \\
&\quad + \sum_{\tau=t+1}^{\infty} (d\beta)^{\tau-t} E_t \left[\zeta_\tau^b \zeta_\tau^L (1+\gamma) \left(\frac{W_\tau}{P_t} \right)^{\frac{1+\gamma}{\gamma}(1+\sigma_L)} L_\tau^{1+\sigma_L} \right] \\
&= \zeta_t^b \zeta_t^L (1+\gamma) \left(\tilde{W}_t \right)^{\frac{1+\gamma}{\gamma}(1+\sigma_L)} L_t^{1+\sigma_L} \\
&\quad + d\beta E_t \left[\Pi_{t+1}^{\frac{1+\gamma}{\gamma}(1+\sigma_L)} \sum_{\tau=t+1}^{\infty} (d\beta)^{\tau-(t+1)} E_{t+1} \left[\zeta_\tau^b \zeta_\tau^L (1+\gamma) \left(\frac{W_\tau}{P_{t+1}} \right)^{\frac{1+\gamma}{\gamma}(1+\sigma_L)} L_\tau^{1+\sigma_L} \right] \right] \\
&= \zeta_t^b \zeta_t^L (1+\gamma) \left(\tilde{W}_t \right)^{\frac{1+\gamma}{\gamma}(1+\sigma_L)} L_t^{1+\sigma_L} + d\beta E_t \left[\Pi_{t+1}^{\frac{1+\gamma}{\gamma}(1+\sigma_L)} \tilde{\mathcal{N}}_{t+1}^w \right], \\
\tilde{\mathcal{D}}_t^w &= P_t^{-\frac{1}{\gamma}} \sum_{\tau=t}^{\infty} (d\beta)^{\tau-t} E_t \left[\frac{U_{C,\tau}}{P_\tau} W_\tau^{\frac{1+\gamma}{\gamma}} L_\tau \right] = \sum_{\tau=t}^{\infty} (d\beta)^{\tau-t} E_t \left[U_{C,\tau} \left(\frac{P_\tau}{P_t} \right)^{\frac{1}{\gamma}} \tilde{W}_\tau^{\frac{1+\gamma}{\gamma}} L_\tau \right] \\
&= U_{C,t} \tilde{W}_t^{\frac{1+\gamma}{\gamma}} L_t + d\beta E_t \left[\Pi_{t+1}^{\frac{1}{\gamma}} \sum_{\tau=t+1}^{\infty} (d\beta)^{\tau-(t+1)} E_{t+1} \left[U_{C,\tau} \left(\frac{P_\tau}{P_{t+1}} \right)^{\frac{1}{\gamma}} \tilde{W}_\tau^{\frac{1+\gamma}{\gamma}} L_\tau \right] \right] \\
&= U_{C,t} \tilde{W}_t^{\frac{1+\gamma}{\gamma}} L_t + d\beta E_t \left[\Pi_{t+1}^{\frac{1}{\gamma}} \tilde{\mathcal{D}}_{t+1}^w \right].
\end{aligned}$$

Thus,

$$\begin{aligned}
\tilde{\mathcal{N}}_t^w &= \zeta_t^b \zeta_t^L (1+\gamma) \left(\tilde{W}_t \right)^{\frac{1+\gamma}{\gamma}(1+\sigma_L)} L_t^{1+\sigma_L} + d\beta E_t \left[\Pi_{t+1}^{\frac{1+\gamma}{\gamma}(1+\sigma_L)} \tilde{\mathcal{N}}_{t+1}^w \right], \\
\tilde{\mathcal{D}}_t^w &= U_{C,t} \tilde{W}_t^{\frac{1+\gamma}{\gamma}} L_t + d\beta E_t \left[\Pi_{t+1}^{\frac{1}{\gamma}} \tilde{\mathcal{D}}_{t+1}^w \right], \\
\tilde{w}_{t,t} &= \left[\frac{\tilde{\mathcal{N}}_t^w}{\tilde{\mathcal{D}}_t^w} \right]^{\frac{\gamma}{\gamma+(1+\gamma)\sigma_L}}, \\
\tilde{W}_t &= \left[D \left(\tilde{W}_{t-1} / \Pi_t \right)^{-\frac{1}{\gamma}} + (1-D) \tilde{w}_{t,t}^{-\frac{1}{\gamma}} \right]^{-\gamma},
\end{aligned}$$

Ultimately, we turn to the monetary policy rule. Straightforward manipulation of the imperfect peg rule (equation (2.17) above) gives

$$\frac{\mathcal{E}_t}{\mathcal{E}_{t-1}} \frac{\Pi_t}{\Pi_t^*} = \frac{\xi_t}{\xi_{t-1}},$$

where, again, ξ_t is an AR(1) policy shock with Gaussian innovations.

4 Model Summary

This section simply summarises the model in real terms;

$$Q_t^d = \alpha^d \frac{Z_t}{\tilde{P}_t^d}, \quad (4.1)$$

$$Q_t^m = (1 - \alpha^d) \frac{Z_t}{\tilde{P}_t^m}, \quad (4.2)$$

$$Q_t^x = \left(\tilde{P}_t^x\right)^{-\eta} Y_t^*, \quad (4.3)$$

$$1 = \left(\tilde{P}_t^d\right)^{\alpha^d} \left(\tilde{P}_t^m\right)^{1-\alpha^d}, \quad (4.4)$$

$$L_t = \frac{1 - \psi}{\psi} \frac{\tilde{R}_t}{\tilde{W}_t} K_t, \quad (4.5)$$

$$mc_t = \frac{1}{\theta_t} \tilde{W}_t^{1-\psi} \tilde{R}_t^\psi \psi^{-\psi} (1 - \psi)^{-(1-\psi)}, \quad (4.6)$$

$$\rho_{t,t+1} = \beta (U_{C,t+1}/U_{C,t}) \Pi_{t+1}^{-1}, \quad (4.7)$$

$$U_{C,t} = \zeta_t^b (C_t - hC_{t-1})^{-\sigma_C}; \quad (4.8)$$

$$\tilde{\mathcal{N}}_t^d = Q_t^d (1 + \nu) mc_t + dE_t \left[\rho_{t,t+1} \left(\frac{\tilde{P}_{t+1}^d}{\tilde{P}_t^d} \right)^{\frac{1+\nu}{\nu}} \Pi_{t+1}^{\frac{1+2\nu}{\nu}} \tilde{\mathcal{N}}_{t+1}^d \right], \quad (4.9)$$

$$\tilde{\mathcal{D}}_t^d = Q_t^d + dE_t \left[\rho_{t,t+1} \left(\frac{\tilde{P}_{t+1}^d}{\tilde{P}_t^d} \Pi_{t+1} \right)^{\frac{1+\nu}{\nu}} \tilde{\mathcal{D}}_{t+1}^d \right], \quad (4.10)$$

$$\tilde{P}_t^d = \left[d \left(\tilde{P}_{t-1}^d / \Pi_t \right)^{-\frac{1}{\nu}} + (1 - d) \left(\tilde{\mathcal{N}}_t^d / \tilde{\mathcal{D}}_t^d \right)^{-\frac{1}{\nu}} \right]^{-\nu}; \quad (4.11)$$

$$\tilde{\mathcal{N}}_t^x = Q_t^x (1 + \nu) mc_t + dE_t \left[\rho_{t,t+1} \left(\frac{\tilde{P}_{t+1}^x}{\tilde{P}_t^x} \Pi_{t+1}^* \right)^{\frac{1+\nu}{\nu}} \tilde{\mathcal{N}}_{t+1}^x \right], \quad (4.12)$$

$$\tilde{\mathcal{D}}_t^x = Q_t^x \mathcal{E}_t + dE_t \left[\rho_{t,t+1} \left(\frac{\tilde{P}_{t+1}^x}{\tilde{P}_t^x} \right)^{\frac{1+\nu}{\nu}} \left(\Pi_{t+1}^* \right)^{\frac{1}{\nu}} \Pi_{t+1} \tilde{\mathcal{D}}_{t+1}^x \right], \quad (4.13)$$

$$\tilde{P}_t^x = \left[d \left(\tilde{P}_{t-1}^x / \Pi_t^* \right)^{-\frac{1}{\nu}} + (1 - d) \left(\tilde{\mathcal{N}}_t^x / \tilde{\mathcal{D}}_t^x \right)^{-\frac{1}{\nu}} \right]^{-\nu}; \quad (4.14)$$

$$\tilde{\mathcal{N}}_t^m = Q_t^m (1 + \nu) + \frac{d}{1 + i_t^*} E_t \left[\left(\frac{\tilde{P}_{t+1}^m}{\tilde{P}_t^m} \Pi_{t+1} \right)^{\frac{1+\nu}{\nu}} \Pi_{t+1}^* \tilde{\mathcal{N}}_{t+1}^m \right] \quad (4.15)$$

$$\tilde{\mathcal{D}}_t^m = Q_t^m \mathcal{E}_t + \frac{d}{1 + i_t^*} E_t \left[\left(\frac{\tilde{P}_{t+1}^m}{\tilde{P}_t^m} \right)^{\frac{1+\nu}{\nu}} (\Pi_{t+1})^{\frac{1}{\nu}} \Pi_{t+1}^* \tilde{\mathcal{D}}_{t+1}^m \right], \quad (4.16)$$

$$\tilde{P}_t^m = \left[d \left(\tilde{P}_{t-1}^m / \Pi_t \right)^{-\frac{1}{\nu}} + (1 - d) \left(\tilde{\mathcal{N}}_t^m / \tilde{\mathcal{D}}_t^m \right)^{-\frac{1}{\nu}} \right]^{-\nu}; \quad (4.17)$$

$$K_{t+1} = K_t (1 - \delta) + I_t - \frac{1}{2} \frac{\Phi (K_{t+1} - K_t)^2}{K_t}, \quad (4.18)$$

$$\tilde{B}_{t+1} = (1 + i_{t-1}^f) \tilde{B}_t / \Pi_t^* + \tilde{P}_t^x Q_t^x - \frac{\tilde{P}_t^m}{\mathcal{E}_t} Q_t^m; \quad (4.19)$$

$$E_t \left[\rho_{t,t+1} \Pi_{t+1} \frac{\left(\tilde{R}_{t+1} + (1 - \delta) - \frac{\Phi}{2} \left(1 - \left(\frac{K_{t+2}}{K_{t+1}} \right)^2 \right) \right)}{\left[1 + \Phi \frac{K_{t+1} - K_t}{K_t} \right]} \right] = 1, \quad (4.20)$$

$$(1 + i_t) E_t [\rho_{t,t+1}] = 1, \quad (4.21)$$

$$(1 + i_t^f) E_t \left[\rho_{t,t+1} \frac{\mathcal{E}_{t+1} \Pi_{t+1}}{\mathcal{E}_t \Pi_{t+1}^*} \right] = 1; \quad (4.22)$$

$$\tilde{\mathcal{N}}_t^w = \zeta_t^b \zeta_t^L (1 + \gamma) \left(\tilde{W}_t \right)^{\frac{1+\gamma}{\gamma} (1+\sigma_L)} L_t^{1+\sigma_L} + d\beta E_t \left[\Pi_{t+1}^{\frac{1+\gamma}{\gamma} (1+\sigma_L)} \tilde{\mathcal{N}}_{t+1}^w \right], \quad (4.23)$$

$$\tilde{\mathcal{D}}_t^w = U_{C,t} \tilde{W}_t^{\frac{1+\gamma}{\gamma}} L_t + d\beta E_t \left[\Pi_{t+1}^{\frac{1}{\gamma}} \tilde{\mathcal{D}}_{t+1}^w \right], \quad (4.24)$$

$$\tilde{w}_{t,t} = \left[\frac{\tilde{\mathcal{N}}_t^w}{\tilde{\mathcal{D}}_t^w} \right]^{\frac{\gamma}{\gamma + (1+\gamma)\sigma_L}}, \quad (4.25)$$

$$\tilde{W}_t = \left[D \left(\tilde{W}_{t-1} / \Pi_t \right)^{-\frac{1}{\gamma}} + (1 - D) \tilde{w}_{t,t}^{-\frac{1}{\gamma}} \right]^{-\gamma}; \quad (4.26)$$

$$(1 + i_t^f) = (1 + i_t^*) v_t \exp \left\{ -\lambda \mathcal{E}_t \Pi_{t+1}^* \frac{\tilde{B}_{t+1}}{\Xi} \right\}, \quad \Xi = \mathcal{E} \hat{\mathcal{P}}^x Q^x, \quad (4.27)$$

$$\frac{\mathcal{E}_t \Pi_t}{\mathcal{E}_{t-1} \Pi_t^*} = \frac{\xi_t}{\xi_{t-1}}; \quad (4.28)$$

$$Y_t = Q_t^d + Q_t^x, \quad (4.29)$$

$$Z_t = C_t + I_t, \quad (4.30)$$

$$K_t = \frac{1}{\theta_t} \left(\frac{\psi}{1-\psi} \frac{\tilde{W}_t}{\tilde{R}_t} \right)^{1-\psi} \left[\left(\frac{\tilde{\mathcal{P}}_t^d}{\tilde{P}_t^d} \right)^{-\frac{1+\nu}{\nu}} Q_t^d + \left(\frac{\tilde{\mathcal{P}}_t^x}{\tilde{P}_t^x} \right)^{-\frac{1+\nu}{\nu}} Q_t^x \right], \quad (4.31)$$

$$\tilde{\mathcal{P}}_t^d = \left[d \left(\tilde{\mathcal{P}}_{t-1}^d / \Pi_t \right)^{-\frac{1+\nu}{\nu}} + (1-d) \left(\tilde{p}_{t,t}^d \right)^{-\frac{1+\nu}{\nu}} \right]^{-\frac{\nu}{1+\nu}}, \quad (4.32)$$

$$\tilde{\mathcal{P}}_t^x = \left[d \left(\tilde{\mathcal{P}}_{t-1}^x / \Pi_t^* \right)^{-\frac{1+\nu}{\nu}} + (1-d) \left(\tilde{p}_{t,t}^x \right)^{-\frac{1+\nu}{\nu}} \right]^{-\frac{\nu}{1+\nu}}; \quad (4.33)$$

$$\tilde{\mathcal{W}}_t = \left[D \left(\tilde{\mathcal{W}}_{t-1} / \Pi_t \right)^{-\frac{1+\gamma}{\gamma}(1+\sigma_L)} + (1-D) \tilde{w}_{t,t}^{-\frac{1+\gamma}{\gamma}(1+\sigma_L)} \right]^{-\frac{\gamma}{(1+\gamma)(1+\sigma_L)}}, \quad (4.34)$$

$$SW_t = \frac{\zeta_t^b}{1-\sigma_C} (C_t - h\bar{C}_{t-1})^{1-\sigma_C} - \frac{\zeta_t^b \zeta_t^L}{1+\sigma_L} L_t^{1+\sigma_L} \left(\frac{\tilde{\mathcal{W}}_t}{\tilde{W}_t} \right)^{-\frac{1+\gamma}{\gamma}(1+\sigma_L)}; \quad (4.35)$$

and the model is closed with a monetary policy rule; either in the form of the imperfect peg rule

$$\frac{\mathcal{E}_t}{\mathcal{E}_{t-1}} \frac{\Pi_t}{\Pi_t^*} = \frac{\zeta_t^{peg}}{\zeta_{t-1}^{peg}}, \quad (4.36)$$

or the generalised Taylor rule

$$i_t = \rho_i i_{t-1} + (1-\rho_i) \left(\rho_\pi (\Pi_t - 1) + \rho_y \frac{Y_t - Y_{t-1}}{Y_{t-1}} \right) + \xi_t^{TR}. \quad (4.37)$$

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