ESSAYS
IN
TWO-SIDED MARKETS
AND OPTIMAL CONTRACTING

PHD THESIS

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Contents

Acknowledgements

Summary

This dissertation consists of three chapters, each of which can be read separately. The three chapters focus on two different fields within microeconomic theory. The first two chapters are closely related and contribute to the literature on industrial organization. Both chapters analyze so-called two-sided markets. In a two-sided market, a platform enables consumers of two distinct groups to interact and thereby obtain the benefits from network externalities between them. Two-sided markets include old-economy industries such as newspapers and shopping malls as well as new-economy industries such as web platforms, video game consoles and software platforms. The rise of the internet has greatly facilitated the emergence and visibility of two-sided platforms and these markets are increasingly important in today's economy. Chapter 1 focuses on a monopoly platform and analyzes how vertical mergers affect prices and welfare when markets are two-sided. Chapter 2 presents a model of differentiated platform competition where consumers have the option of purchasing from one or both platforms. The last chapter contributes to the literature on contract theory. The paper studies how economic actors can construct contractual arrangements in the presence of asymmetric information. In particular, the focus is on a principal-agent relationship with subjective performance evaluation and reciprocal agents. While the literature on contract theory traditionally assumes that agents care only about their monetary pay-off, the experimental literature finds that agents tend to reciprocate unfair behavior. This is incorporated by allowing the agent to create costly conflict whenever she receives a performance appraisal and associated payment below what she feels entitled to. We analyze how the agent's personality and the cost of conflict affect optimal contracts.

Chapter 1, "Vertical Mergers in Two-Sided Markets", analyzes welfare and price effects of vertical mergers in a two-sided market. In a classic one-sided market it is well known that a vertical structure gives rise to the problem of "double marginalization." Supplying consumers through a downstream firm results in higher prices and lower demand than what an upstream firm would prefer. If a vertical merger eliminates the double marginalization problem it will result in lower prices and higher welfare for consumers and producers.

In two-sided markets on the other hand, optimal pricing structures differ in important aspects from those in one-sided markets. As a result, we cannot expect conventional wisdom on vertical mergers to translate directly to two-sided markets. Indeed, I show that a vertical merger may result in higher prices both on the side where it takes place and on the other side of the market. While the presence of a downstream firm still leads to a double marginalization problem for the platform in a two-sided market, it also affects the platform's ability to get the right price balance between the two sides. Consequently, whenever the two sides are not symmetric, the latter effect may dominate and prices can increase as the result of a vertical merger. Furthermore, consumers can be made worse off by a vertical merger. This can be due to higher prices or because the size of the network on the other side of the market changes following a merger.

Chapter 2, "Platform Competition with Endogenous Multihoming Decision" (joint with Carsten Søren Nielsen), analyzes competition between horizontally differentiated platforms. While network effects are in general thought to induce concentration, industries based on two-sided platforms rarely consist of monopolies. A reason might be product differentiation. Furthermore, when two platforms are present consumers often have the option of purchasing from both platforms ("multihoming") instead of buying at one platform only ("singlehoming"). We analyze a model of differentiated platform competition when consumers have the option of multihoming. Importantly, while the existing literature on platform competition assumes that either one or both sides are always singlehoming or multihoming for exogenous reasons we allow the choice of whether to multihome to be completely endogenous. Furthermore, by introducing a love for variety in consumers' utility we allow for the realistic outcome of multihoming on both sides of the market, which tends to be assumed away in the existing literature. We solve for four different equilibria: one in which all consumers on both sides are singlehoming, one in which at least one consumer on both sides multihomes, and two asymmetric equilibria with singlehoming on one side and multihoming on the other. Contrary to the findings in the existing literature, we find that prices are lower when consumers are multihoming compared to when they are singlehoming. Furthermore, a singlehoming side pays a higher price when the other side is

multihoming compared to when the other side is also singlehoming. Lastly, we show that prices under competition are lower than under joint ownership of the platforms and, due to the network externality, the market delivers a suboptimal level of multihoming in the eyes of a social planner.

Chapter 3, "Personality and Conflict in Principal-Agent Relations Based on Subjective Performance Evaluations" (joint with Alexander Sebald), analyzes the role of conflict in principal-agent environments with subjective performance evaluations, reciprocal agents, and endogenous feelings of entitlements. We investigate how certain personality traits affect the level of conflict and what implications this has for optimal recruitment policies and the principal's choice of evaluation procedure. We find that higher conflict costs can actually increase welfare since a higher level of potential conflict enables the principal to commit to a higher wage. Further, we formally characterize situations in which is it optimal for the principal to hire agents who are very sensitive to reciprocity or agents who are likely to have an own opinion. Finally, we extend the framework to allow the principal to choose the evaluation procedure. More precisely, we allow the principal to choose the quality of the process used to evaluate the agent. We show that even if it is costless for the principal to choose a high quality evaluation procedure, he might not find it optimal to do so.

Resumé (Summary in Danish)

Denne afhandling består af tre kapitler, som alle kan læses individuelt. De tre kapitler er omhandler alle emner indenfor teoretisk mikroøkonomi. De to første kapitler bidrager til litteraturen i industriøkonomi, mens fokus i det sidste kapitel ligger indenfor kontraktteori.

Industriøkonomi er den gren indenfor mikroøkonomi, der fokuserer på virksomheders strategiske adfærd på markeder samt hvorledes disse markeder fungerer. Konsekvenserne af en branches struktur på virksomheders adfærd og samfundets velfærd er det primære fokusområde. Kapitel 1 og 2 i denne afhandling omhandler tosidede markeder. På et tosidet marked eksisterer to adskilte grupper, f.eks. købere og sælgere, og der er netværkseksternaliteter mellem grupperne. De to grupper interagerer via en platform, og opnår derved nytten fra netværkseksternaliteten. Eksempler på tosidede markeder inkluderer både klassiske økonomiske markeder såsom aviser, indkøbscentre og ugeblade, og nye økonomiske markeder såsom internetplatforme, spillekonsoller og software-platforme. Tosidede markeder er blevet mere synlige og relevante i forbindelse med opfindelsen og udbredelsen af internettet. Amazon, eBay, Netflix og Spotify er alle eksempler på tosidede markeder. En virksomheds optimale adfærd på et tosidet marked afviger fra den optimale adfærd på et klassisk ensidet marked på grund af netværkseksternaliteten. En platform skal ikke blot sætte to priser, men samtidig fokusere på at opnå den optimale balance mellem priserne. Som følge heraf holder de klassiske resultater fra industriøkonomien ikke nødvendigvis på tosidede markeder.

I kontraktteori undersøges hvorledes økonomiske aktører bedst muligt kan konstruere kontrakter mellem sig. Problemer opstår ofte i situationer, hvor der er asymmetrisk information eller moralfare til stede. I klassisk kontraktteori afhænger agenters nytte kun af deres monetære afkast. Den eksperimentelle litteratur på området finder dog, at agenter, udover at reagere på det monetære afkast de modtager, også reagerer på, hvorvidt de føler sig retfærdigt behandlet af principalen. I kapitel 3 af denne afhandling fokuseres på agenter, der skaber konflikt når de modtager en evaluering, og dertil hørende lønsats, der ligger under, hvad de føler sig berettiget til. Denne konflikt giver anledning til omkostninger for både agenten og principalen. Vi kalder sådanne agenter

for "gengældende agenter."

Kapitel 1, "Vertical Mergers in Two-Sided Markets", analyserer pris- og velfærdseffekter af vertikale fusioner i tosidede markeder. I et klassisk etsidet market er det fra litteraturen i industriøkonomi et etableret resultat, at en vertikal struktur giver anledning til et dobbelt marginaliseringsproblem. Når en virksomhed sælger sine varer gennem et detailled, vil den endelige pris overfor forbrugerne være højere end hvad virksomheden ønsker. Som følge af en vertikal fusion vil det dobbelte marginaliseringsproblem forsvinde, priserne vil falde, og både forbrugere og virksomheden er bedre stillet. I et tosidet marked, hvor den optimale prisstrategi afviger fra den optimale strategi i et klassisk marked, kan det resultat ikke nødvendigvis overføres. Jeg viser, at en vertikal fusion kan føre til højere priser, både på den side hvor fusionen finder sted, men også på den anden side af markedet. En vertikal struktur i et tosidet marked giver stadig anledning til det dobbelte marginaliseringsproblem. Derudover forhindrer det også platformen i at opnå den ønskede prisbalance mellem de to sider. Når de to sider er asymmetriske, kan denne sidste effekt medføre, at en vertikal fusion resulterer i højere priser. Mens platformen er bedre stillet efter en fusion, kan forbrugerne være dårligere stillet. Dette kan ske både som følge af en højere pris, men også som følge af en ændret netværksstørrelse på den anden side af markedet.

Kapitel 2, "Platform Competition with Endogenous Multihoming Decision" (skrevet i samarbejde med Carsten Søren Nielsen) omhandler ligeledes tosidede markeder. Vi opstiller en model for platform-konkurrence, hvor platformene er horisontalt differentierede, og hvor forbrugere har mulighed for at købe hos én platform (hvilket kaldes singlehoming) eller hos begge platforme (hvilket kaldes multihoming). I modsætning til den eksisterende litteratur i platform-konkurrence tillader vi multihoming på begge sider af markedet, og beslutningen om multihoming versus singlehoming er endogent bestemt af priser, størrelsen på netværket på den anden side af markedet og forbrugernes præferencer for variation. Der er fire ligevægte i modellen: En, hvor begge sider singlehomer, en hvor mindst én forbruger på begge sider multihomer, og to asymmetriske ligevægte med singlehoming på den ene side og multihoming på den anden. I skarp kontrast til den eksisterende litteratur, hvor den ene side altid singlehomer, finder vi, at priserne er højere, når forbrugerne single-

homer, i forhold til når de multihomer. Efterfølgende viser vi, at konkurrence fører til lavere priser, end når platformene er under fælles ejerskab. Derudover finder vi, at markedet leverer et suboptimalt niveau af multihoming i forhold til det samfundsmæssige optimale niveau, hvilket er en direkte konsekvens af den positive netværkseksternalitet mellem de to grupper .

Kapitel 3, "Personality and Conflict in Principal-Agent Relations Based on Subjective Performance Evaluation" (skrevet i samarbejde med Alexander Sebald), analyserer konsekvenser af konflikt i en teoretisk model med subjektive evalueringer og gengældende agenter, når aflønningen agenten føler sig berettiget til opstår endogent som følge af agentens egen arbejdsindsats. Vi undersøger, hvorledes forskellige personlighedstræk hos agenten påvirker graden af konflikt, samt hvilke implikationer dette har for den optimale rekrutteringspolitik og for principalens valg af evalueringsform. Vi konkluderer, at højere omkostninger i forbindelse med konflikt kan øge velfærd. Årsagen er, at højere potentielle omkostninger ved konflikt gør det muligt for principalen at forpligte sig til en højere bonusudbetaling. Endvidere karakteriserer vi formelt de situationer, hvor principalen har et incitament til at ansætte agenter, der er meget sensitive overfor konflikt, og agenter som ofte danner egen mening om kvaliteten af deres arbejde. Afslutningsvist udvider vi modellen således, at principalen selv kan vælge kvaliteten af evalueringsproceduren. Vi viser, at bedst mulig kvalitet ikke altid er at foretrække, end ikke når det er omkostningsfrit at vælge høj kvalitet.

Chapter 1

Vertical Mergers in Two-sided Markets[*]

Mie la Cour Sonne[†]

Abstract

This paper analyzes welfare and price effects of vertical mergers in a two-sided market. While a vertical merger in a standard one-sided market will remove the double marginalization problem, lower the price, and increase welfare, this need not be the case in a two-sided market. The presence of network externalities affect firms' optimal pricing behavior and as a result we should not expect conventional wisdom on vertical integration to translate directly to two-sided markets. Depending on the size of the network effects between the two sides, a vertical merger will either increase or decrease the price on the side where it takes place. Furthermore, a vertical merger may also increase the price on the other side of the market. While consumers are always better off on the side where the merger takes place, it is possible that consumers on the other side of the market are worse off.

Keywords: Two-sided Markets; Network externalities; Double marginalization; Vertical Integration; Merger Analysis

JEL-Classifications: L13, L4

1 Introduction

It is well known that vertical structures give rise to the problem of "double marginalization." When an upstream firm supplies consumers through a downstream firm, the end-price paid by consumers is higher - and demand subsequently lower - than what the upstream firm would prefer. This is due to the fact that the downstream firm fails to internalize the effects of its price choice on the upstream firm's profit. The issue of double marginalization was first analyzed by Spengler (1950), who warned antitrust authorities against viewing vertical mergers as per se illegal. If a vertical merger eliminates the double marginalization problem, it will result in lower prices and benefit both producers and consumers.

Vertical control has been widely analyzed in the economics literature for the case of standard one-sided markets. However, many of today's industries are based on so-called two-sided platforms. In a two-sided market, a platform enables consumers of two distinct groups to interact with each other, thereby obtaining the benefits of externalities between them. These two-sided markets include old-economy industries such as newspapers, credit cards, and shopping malls as well as new-economy industries such as web platforms, video game consoles, and software platforms. These markets are increasingly important in today's economy and cannot necessarily be understood with the standard one-sided market logic.

The early literature on two-sided markets, Rochet and Tirole (2003), Armstrong (2006) and Rochet and Tirole (2006) established that optimal pricing in these markets differs in important aspects from that in standard markets. In particular, as a direct consequence of the network externalities, optimal prices may involve below-cost pricing on one side of the market, even in the long run. Thus, individual prices on either side of the market do not necessarily track cost or demand on that side.

The literature warns policy-makers against applying standard competition policy guidelines to these markets (Evans (2003), Wright (2004).) The special characteristics of two-sided markets mean that conventional practices of antitrust policy are not directly applicable. At the same time, two-sided markets are increasingly relevant, emphasizing the need for directly applicable antitrust

results in this area.

This paper investigates the effects of vertical mergers in a two-sided market on prices and consumer welfare. The paper builds on and extends the classic model of two-sided markets presented in Armstrong (2006). An upstream platform serves two sides of a market, but it may do so through downstream firms. Optimal prices in three different organizational structures are derived and compared, the first of which is an organization where downstream firms are present on both sides of the market. In the second organization, a vertical structure is only present on one side of the market, which may be the result of a vertical merger in the first organization. Lastly, the third organization is comparable to the standard two-sided market analyzed in Armstrong (2006), where the platform serves both sides directly.

Vertical integration in a two-sided market turns out to differ from vertical integration in a standard one-sided market in important aspects. First, the downstream firms do not only impose a double marginalization problem on the platform. They also negatively affect the platform's ability to strike the right price balance between the two sides. Consequently, the presence of vertical structures affects the platform's optimal price structure in two ways. First, the platform must take into account how a downstream firm sets its price. In particular, the higher the downstream markup, the lower the platform's optimal price, other things equal. This is, in theory, similar to a standard one-sided market. Second, even if the downstream firms serve one side only and do not compete for consumers on the other side, they react to the price choices made by potential downstream firms on the other side because of the network externality. The platform also has to consider this reaction function. That is, changing its price on side 1 will affect the price choice of the downstream firm on side 2, and the platform optimally accounts for this. Hence, the platform's optimal price structure takes into account the presence of vertical structures in addition to the network externality.

Assuming a uniform distribution of the membership benefits, we show that a vertical merger on one side of the market does not necessarily result in a lower price on the side where it takes place. Furthermore, it may increase the price on the other side of the market. When one side benefits relatively more from interaction, this side will typically experience higher prices as the result

of a vertical merger. This can happen both in the case of positive network externalities on both sides or a negative network externality on one side. In the case of advertising, for example, a vertical merger would likely result in higher prices on the advertising side.

Second, consumers can be made worse off by a vertical merger. In a two-sided market, a price decrease on one side of the market does not necessarily improve welfare. A lower price on one side is often accompanied by a higher price on the other side of the market as a result of the seesaw effect mentioned in Rochet and Tirole (2003). Imagine that a vertical merger results in lower prices on side 1 but higher prices on side 2. If demand decreases on side 2 as a result of the increased price, this will affect consumers on the other side of the market negatively due to the network externality (when this is positive). For the consumers on side 1, this negative effect may outweigh the benefits from paying a lower price. Following the same logic, a higher price does not necessarily result in lower welfare. If demand increases on the other side of the market and the network externality is positive (or if demand decreases and the network externality is negative), consumers may be better off even if they pay a higher price. The welfare effects from price changes are therefore different from standard one-sided markets. To understand the effects of a vertical merger, it is important to look beyond the effects on prices. Our welfare analysis shows that consumers are always better off after a vertical merger on their own side of the market, even if this results in higher prices. However, consumers may be worse off as a result of a vertical merger on the other side of the market.

To illustrate the results, take the example of advertising where consumers dislike ads. A vertical merger on the consumer side of the market will result in higher prices for advertisers. If consumers' dislike for ads is sufficiently strong, this price change will result in lower welfare on the advertiser side. Consumers, however, will be better off as a result of the merger since the amount of advertising will go down. On the other hand, a vertical merger on the advertiser side will result in lower prices for advertisers. This will increase the amount of advertising, and consumers will be worse off as a result of the merger.

The results highlight the importance of focusing on the effects on both sides of the market instead of viewing a sub-market in isolation, as also discussed

in Wright (2004) and Evans (2003). Conventional wisdom from one-sided markets cannot be transferred directly to two-sided markets. If one were to focus only on the side where the merger takes place, important aspects would be left out. While a vertical merger raises welfare on the side where it takes place, it may lower welfare on the other side.

This paper adds to the literature analyzing two-sided markets. Weyl (2008) combined the Spengler (1950) logic of double marginalization with the two-sided market model presented in Rochet and Tirole (2003) motivated by the payment card industry. Weyl concludes that eliminating the double marginalization problem always lowers the price on the relevant side, but it has no systematic effect on the price on the other side of the market (where there is no double marginalization problem). Different from Weyl (2008), this paper builds on the model in Armstrong (2006), which focuses on so-called "membership prices." This model is typically thought to be a better description of software platforms, video game consoles, newspapers, etc. The model in this paper further allows for vertical structures on both sides of the market. Lastly, the focus is not only on price effects but also on welfare effects. This dimension is important since - as discussed above - welfare effects may be very different from price effects. In contrast with the results in Weyl (2008), we find that a vertical merger may increase the price on the side where it takes place and furthermore has the systematic effect of increasing the price on the opposite side whenever network externalities are highest there.

Lee (2013) also studies vertical control in two-sided markets. The paper measures the effect of exclusive vertical arrangements on industry structure, competition and welfare in the U.S videogame industry. Here, the focus is not on prices and the double marginalization problem but instead on foreclosure and entry-deterrence (see also Salinger (1988), Hart and Tirole (1990), Rey and Tirole (2007)). Lee finds that prohibiting exclusive contracts would have benefited the incumbent and hurt smaller entrants. As a result, consumers were better off when platforms used exclusive contracts.

That standard economic predictions do not always hold in two-sided markets is further confirmed in Chandra and Collard-Wexler (2009). While the focus here is on horizontal mergers and not vertical mergers, the authors conclude that a higher degree of concentration does not necessarily lead to higher

prices on either side of the market. This conclusion is tested on data for mergers in the Canadian newspaper industry, and the findings support the theoretical results.

The paper proceeds as follows. Section 2 sets up the general model framework and derives optimal prices. Section 3 analyzes the effects from a vertical merger on prices, while section 4 presents the welfare results of vertical mergers. Section 5 concludes.

2 The Model

We consider an organizational structure that involves a vertical structure on both sides of the market, as shown in the figure below.
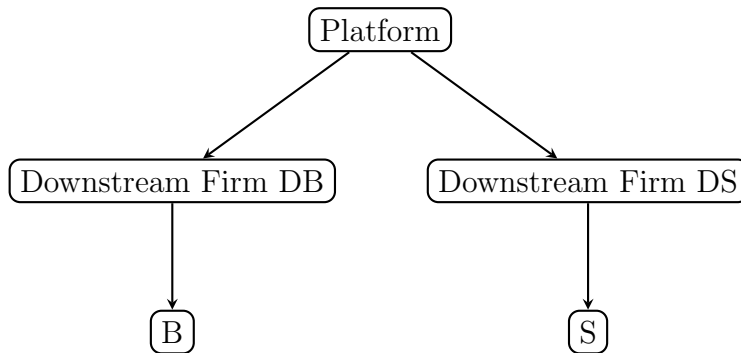
Figure 1: Non-integrated organization

A monopolist platform serves two sides of a market labeled B and S. Imagine, for example, a magazine. On the buyer side of the market, consumers buy their magazine from a downstream firm such as a newsstand or a supermarket. Likewise, on the other side of the market, advertisers often go through agencies to get to the platform. Another example could be videogames, as also described in Lee (2013). A videogame system consists of a hardware platform (the console), which serves on one side gamers and on the other side game developers. Game developers typically go through publishers to reach the platform while gamers buy at downstream retailers.

Each downstream firm behaves as a monopolist on its own side of the market. Consequently, firm DB serves consumers on side B only, and does

not compete with firm DS for side S consumers. Since the downstream firms will in most real life examples serve completely different purposes, it seems fair to assume that they do not compete for consumers in the downstream market. For instance, in the magazine example newsstands do not compete with advertising agencies for readers.

The optimal prices arising from the above organizational structure will be compared to an organizational structure with a downstream firm on one side only (which will be referred to as a "semi-integrated" organization) and to one where the platform simply serves both sides of the market directly (which will be referred to as an "integrated" organization.) These organizations are depicted in Figure 2[1] and Figure 3.

Figure 2: Semi-integrated organization
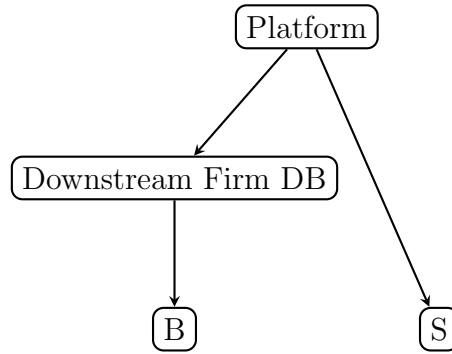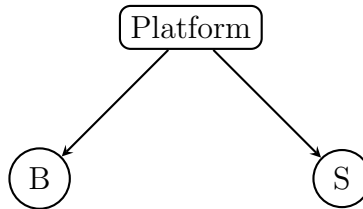
Figure 3: Integrated Organization

Different kinds of vertical integration will be relevant for different kinds of industries. In some situations the initial organization may not be as in Figure 1 but as in Figure 2. In this case a vertical integration will result in the fully

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[1]In Figure 2 the integration has occurred on side S, but it could of course also be on side B.

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integrated organization of Figure 3. For other industries, the non-integrated organization may be the best fitted description and a vertical merger can lead to either the organization in Figure 2 or Figure 3 depending on whether it takes place on one or both sides.

Consumers Consumers derive utility from access to the platform and from usage of the platform through the network externality. First, a consumer k on side i of the market receives a fixed utility of b_i^k from joining the platform. This is the so-called "membership benefit." This fixed benefit is assumed to be weakly positive for all consumers.

In addition to the fixed utility, a consumer on side i receives utility α_i from interacting with members on the other side of the market. As in Armstrong (2006) this "transaction utility" depends simply on the total number of participants on the other side of the market N_j. Hence, the transaction utility for a consumer on side i is given by $\alpha_i \cdot N_j$. It is assumed that all participants on the other side of the market have the same value to a user and the marginal utility from meeting one extra user is constant. As opposed to b_i^k, α_i is not restricted to positive values. There are numerous examples of platforms where agents on one side of the market has a disutility of meeting members on the other side of the market. Typically, this will be the case when one side of the market consists of advertisers. Advertisers like to meet potential consumers, but consumers may dislike ads. However, it will be assumed that at least one group has a positive transaction utility. The nature of two-sided markets is that at least one of the groups should benefit from meeting the other group.

Lastly, consumers pay a price p_i to either a downstream firm or directly to the platform, depending on the organizational structure. This is the so-called "membership-pricing" as presented in Armstrong (2006) and Rochet and Tirole (2006).[2] Similar to Armstrong (2006) and Rochet and Tirole (2006) it is assumed that members are heterogenous with respect to their membership benefits b_i and homogenous with respect to the transaction benefit α_i. More specifically it is assumed that on either side of the market, membership benefits

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[2]As opposed to usage pricing where consumers pay every time a transaction is carried out on the platform.

are distributed according to the function

$$b_i \sim G_i(b_i).$$

When consumer k on side i pays a price of p_i, her total utility is given by

$$U_i^k = b_i^k + \alpha_i N_j - p_i.$$

Given a price p_i and demand on the other side of the market N_j, a consumer on side i will join the platform if she receives (weakly) positive utility from doing so. Hence, all consumers for whom $b_i^k > p_i - \alpha_i N_j$ join the platform. Then, demand is given by the following demand function

$$N_i = 1 - G_i(p_i - \alpha_i N_j), \quad i \in \{B, S\}.$$

Demand on side i depends on the price on this side and the size of the network on side j. Since demand on side j depends on p_j, this price indirectly affects demand on side i as well. Under suitable regularity conditions, the demand system above can be solved for a solution characterizing demands n_B and n_S as functions of prices only:

$$N_B = n_B(p_B, p_S)$$
$$N_S = n_S(p_B, p_S).$$

Firms The timing of the model is as follows: First, the platform chooses prices for both sides of the market, p_B^U and p_S^U. Second, in organizations where a downstream firm is present, this downstream firm chooses its price, p_i^D, and this will be the final price paid by consumers. Consequently, in the integrated organization, demand is given by $n(p_B^U, p_S^U)$, in the non-integrated organization demand is $n(p_B^D, p_B^D)$ and lastly in the semi-integrated organization demand is $n(p_B^D, p_S^U)$ if integration has taken place on side S and $n(p_B^U, p_S^D)$ if it has taken place on side B. Downstream firms are assumed to have no marginal costs except for the price they pay the upstream platform. Furthermore, the platform's marginal costs are assumed to be constant and equal to zero on both sides of the market. Having positive marginal costs complicates the derivations but does not change the results.

Each downstream firm serves one side only. The profit functions of the downstream firms can be written as[3]

$$\pi_B^D = (p_B^D - p_B^U)n_B(p_B^D; p_S^{D/U})$$
$$\pi_S^D = (p_S^D - p_S^U)n_S(p_S^D; p_B^{D/U})$$

and the profit function of the platform is given by

$$\pi_{non-integrated}^P = p_B^U n_B(p_B^D, p_S^D) + p_S^U n_S(p_B^D, p_S^D) \tag{1}$$
$$\pi_{semi-integrated}^P = p_B^U n_B(p_B^D, p_S^U) + p_S^U n_S(p_B^D, p_S^U) \tag{2}$$
$$\pi_{integrated}^P = p_B^U n_B(p_B^U, p_S^U) + p_S^U n_S(p_B^U, p_S^U). \tag{3}$$

2.1 Optimal Prices

In the integrated organization, the platform maximizes (3) with respect to prices which yields the following first order conditions

$$n_i(p_i^U, p_j^U) + p_i^U \frac{\partial n_i(p_i^U, p_j^U)}{\partial p_i^U} + p_j^U \frac{\partial n_j(p_i^U, p_j^U)}{\partial p_i^U} = 0, \quad i \in [B, S].$$

Define the semi-elasticities as $\eta_i \equiv -\frac{n_i(p_i, p_j)}{\partial n_i / \partial p_i}$ and the first order conditions can be rearranged as

$$p_i^U = \eta_i - p_j^U \left[\frac{\partial n_j}{\partial p_i^U} / \frac{\partial n_i}{\partial p_i^U} \right] \quad i \in \{B, S\}. \tag{4}$$

The platform takes into account the own-semi-elasticity of demand as well as the so-called "opportunity cost." Since a higher price leads to lower demand other things equal, this will through the network externality affect demand on the other side of the market as well. If the network externality on side j is positive then the opportunity cost $p_j^U \frac{\partial n_j}{\partial p_i^U} / \frac{\partial n_i}{\partial p_i^U}$ is negative and this will pull in the direction of a lower price on side i. If the network externality is negative, the opportunity cost term is positive, which pulls in the direction of a higher price on side i. The intuition is that a higher price on side i leads to lower

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[3]Here $p^{D/S}$ is a slight abuse of notation but allows us to avoid writing out a profit function for each organizational structure. The price p_S in π_B^D will be given by p_S^D when the organization is as in Figure 1 and by p_S^U when the organization is as in Figure 2.

demand there, but if the network externality is negative on side j the lower demand on side i will lead to higher demand on side j and therefore higher earnings on this side, other things equal.

In the non-integrated organization we solve by backwards induction. The downstream firms maximize their profit resulting in the following first order conditions

$$p_i^D - p_i^U = \eta_i, \quad i \in \{B, S\}. \tag{5}$$

Notice that since demand depends on prices on both sides of the market, so does the semi-elasticity of demand. As a result, even if the downstream firms are not directly competing, their first order conditions in fact define reaction functions. A downstream firm reacts to a change in the price on the other side of the market trough (5).

The platform takes the downstream firms' reactions into account when maximizing its own profit function. With knowledge of (5) the platform indirectly chooses (p_B^D, p_S^D) when setting (p_B^U, p_S^U). Rearrange (1) using (5) to get

$$\pi^P = \left(p_B^D - \eta_B\right) n_B(p_B^D, p_S^D) + (p_S^D - \eta_S) n_S(p_B^D, p_S^D).$$

The platform maximizes profit yielding the following first order condition

$$n_i \left(1 - \frac{\partial \eta_i}{\partial p_i^D}\right) + p_i^U \frac{\partial n_i}{\partial p_i^D} + p_j^U \frac{\partial n_j}{\partial p_i^D} - n_j \frac{\partial \eta_j}{\partial p_i^D} = 0. \tag{6}$$

Notice that if we differentiate (5) with respect to p_i^U we get

$$\frac{\partial p_i^D}{\partial p_i^U} = \frac{1}{1 - \frac{\partial \eta_i}{\partial p_i^D}} \equiv \rho_i$$

where ρ_i measures that rate at which a downstream firm passes on an increase in upstream prices to final consumers. This is the pass-through rate (see Weyl

and Fabinger (2013).) Now, rearrange (6) to get

$$p_i^U = \frac{\eta_i}{\rho_i} - p_j^U \left[\frac{\partial n_j}{\partial p_i^D} / \frac{\partial n_i}{\partial p_i^D} \right] + n_j \left[\frac{\partial \eta_j}{\partial p_i^D} / \frac{\partial n_i}{\partial p_i^D} \right]. \tag{7}$$

This differs from the integrated organization's first order conditions in several ways. First, we see that the non-integrated platform takes the pass-through rate into account on top of the elasticity of demand. The higher the pass-through rate, the more of an increase in upstream prices is passed on to consumers and the lower a price will the platform set. As with the integrated platform, the non-integrated platform also takes into account that a change in price on one side of the market will affect its earnings on the other side of the market due to the network externality. However, there is now a third term to the first order condition which is due to the fact that an upstream price change on one side affects the downstream price on that side of the market, which will in turn affect the price choice of the downstream firm on the other side of the market. If $\frac{\partial \eta_j}{\partial p_i}$ is positive the downstream firm on side j will increase its mark-up when the price is raised on side i. This will pull in the direction of a lower platform price on side i. On the other hand, if $\frac{\partial \eta_j}{\partial p_i}$ is negative, the downstream firm on side j lowers its mark-up when the price on side j is raised and this will pull in the direction of a higher platform price on side i.

Lastly, for the semi-integrated organization[4] the first order conditions are

$$p_B^U = \frac{\eta_B}{\rho_B} - p_S^U \frac{\partial n_S}{\partial p_B^D} / \frac{\partial n_B}{\partial p_B^D} \tag{8}$$

$$p_S^U = \eta_S - p_B^U \frac{\partial n_B}{\partial p_S^U} / \frac{\partial n_S}{\partial p_S^U} + n_B \frac{\partial \eta_B}{\partial p_S^U} / \frac{\partial n_S}{\partial p_S^U}. \tag{9}$$

On side B the platform still takes into account the pass-through rate as opposed to the integrated platform. However, since there is no downstream firm on side S in the semi-integrated organization the third term present in (7) is not taken into account here. On side S the opposite is true. Since there is no downstream firm here, the platform reacts only to the elasticity of demand and not to a pass-through rate. However, since there is a downstream firm on side B the platform takes into account how a changed price on side S affects the optimal

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[4]where integration has without loss of generality been assumed to have taken place on side S

price as chosen by the downstream firm on side B.

To sum up, adding a downstream dimension adds two factors to the platform's optimization problem. When choosing the optimal price on side i, the platform must take into account the mark-up chosen by the downstream firm on side i. Furthermore, it must take into account how the downstream firm on side j reacts to the mark-up chosen by the downstream firm on side i.

In the integrated organization, the final price paid by consumers is given by (4). For the semi-integrated and the non-integrated organizations, final prices paid by consumer can be found by inserting (7), (8), and (9) respectively into (5). For the non-integrated organization this yields the following

$$p_i^D = \eta_i \left[1 + \frac{1}{\rho_i} \right] - p_j^U \left[\frac{\partial n_j}{\partial p_i^D} / \frac{\partial n_i}{\partial p_i^D} \right] + n_j \left[\frac{\partial \eta_j}{\partial p_i^D} / \frac{\partial n_i}{\partial p_i^D} \right].$$

For the semi-integrated organization final prices are given by

$$p_B^D = \eta_i \left[1 + \frac{1}{\rho_i} \right] - p_j^U \left[\frac{\partial n_j}{\partial p_i^D} / \frac{\partial n_i}{\partial p_i^D} \right]$$

$$p_S^U = \eta_i - p_j^U \left[\frac{\partial n_j}{\partial p_i^D} / \frac{\partial n_i}{\partial p_i^D} \right] + n_j \left[\frac{\partial \eta_j}{\partial p_i^D} / \frac{\partial n_i}{\partial p_i^D} \right].$$

The optimal prices are all implicitly given and cannot be compared directly. However, they do provide some intuition about the effects of a vertical merger. If we compare the prices before and after a vertical merger on, say, side S we see that for side S the term $\frac{\eta_S}{\rho_S}$ no longer appears after the merger. This is related to the double marginalization problem. A vertical merger removes the downstream unit and there will no longer be and added mark-up here. The term $\frac{\eta_S}{\rho_S}$ is positive and since it is left out of the optimal price in the semi-integrated organization, this pulls in the direction of a lower price here. For the price on side B, we see that in the semi-integrated organization, the term $n_S \frac{\partial \eta_S}{\partial p_B^D} / \frac{\partial n_B}{\partial p_B^D}$ no longer appears. Since there is no longer a downstream firm present on side S, the downstream firm on side B does not have to take into account how such a firm react to its price choice. The term $\frac{\partial \eta_S}{\partial p_B^D}$ may be either positive or negative and as a result the effects of a merger seem to be able to go in different directions. If $\frac{\partial \eta_S}{\partial p_B^D}$ is negative (positive) a vertical merger on side S will pull in the direction of higher (lower) prices on side B.

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To arrive at a closed form solution, which will enable us to give precise price and welfare effects of vertical mergers, the remaining part of the paper will assume a specific distribution of the membership benefits b_i.

2.2   Uniformly distributed benefits.

We now assume that membership benefits b_i are uniformly distributed on the interval $[0, \beta_i]$:

$$b_i \sim U\left[0, \beta_i\right].$$

For this distribution we get the demand functions

$$N_i = 1 - \frac{1}{\beta_i}\left(p_i - \alpha_i N_j\right), \quad i \in \{B, S\} \tag{10}$$

and solving this system yields

$$n_B\left(p_B, p_S\right) = max\left[0, \frac{\beta_B\beta_S + \alpha_B\beta_S - \beta_S p_B - \alpha_B p_S}{\beta_B\beta_S - \alpha_B\alpha_S}\right] \tag{11}$$

$$n_S\left(p_B, p_S\right) = max\left[0, \frac{\beta_B\beta_S + \alpha_S\beta_B - \beta_B p_S - \alpha_S p_B}{\beta_B\beta_S - \alpha_B\alpha_S}\right]. \tag{12}$$

Some restrictions must be placed on parameters to ensure well behaved profit and demand functions. In particular, the following assumption ensures that second order conditions are satisfied

Assumption 1 $4\beta_B\beta_S - (\alpha_B + \alpha_S)^2 > 0$.

Notice that when Assumption 1 holds it is also the case that $\beta_B\beta_S - \alpha_B\alpha_S > 0$. Hence, under Assumption 1 the demand functions (11) and (12) are well-behaved in the sense that demand is always decreasing in own price and decreasing (increasing) in the price on the other side of the market if the network externality is positive (negative.)

Before proceeding to optimal prices, it will be useful to define the following

constants, which are all positive due to Assumption 1

$$C_1 \equiv 4\beta_B\beta_S - (\alpha_B + \alpha_S)^2$$
$$C_2 \equiv 4\beta_B\beta_S - (\alpha_B + \alpha_S)^2 + 4\left(\beta_B\beta_S - \alpha_B\alpha_S\right)$$
$$C_3 \equiv \beta_B\beta_S\left(4\beta_B\beta_S - (\alpha_B + \alpha_S)^2\right) + 4(\beta_B\beta_S - \alpha_B\alpha_S)(3\beta_B\beta_S - \alpha_B\alpha_S)$$
$$\tilde{\beta} \equiv \beta_B\beta_S - \alpha_B\alpha_S.$$

Using the above demand functions, optimal prices in the three organizations can be solved for and are listed below.

The integrated organization

$$p_B^U = \frac{\beta_B\{2\beta_B\beta_S - \alpha_B\alpha_S + \beta_S(\alpha_B - \alpha_S) - \alpha_S^2\}}{C_1}$$
$$p_S^U = \frac{\beta_S\{2\beta_B\beta_S - \alpha_B\alpha_S + \beta_B(\alpha_S - \alpha_B) - \alpha_B^2\}}{C_1}.$$

The semi-integrated organization with integration on side S

$$p_B^U = \frac{\beta_B(6\beta_B\beta_S - 5\alpha_B\alpha_S) + \beta_B\beta_S(3\alpha_B - \alpha_S) - 2\alpha_S\alpha_B^2 - \beta_B\alpha_S^2}{C_2}$$
$$p_S^D = \frac{\beta_S\left(4\beta_B\beta_S - 3\alpha_B\alpha_S + \beta_B(\alpha_S - \alpha_B) - \alpha_B^2\right)}{C_2}.$$

The non-integrated organization

$$p_B^D = \frac{\beta_B\{12\beta_B^2\beta_S^2 - 15\beta_B\beta_S\alpha_B\alpha_S - \beta_B\beta_S\alpha_S(\alpha_S + \beta_S) + 3\beta_B\beta_S^2\alpha_B + 2\alpha_B^2\alpha_S(2\alpha_S - \beta_S)\}}{C_3}$$
$$p_S^D = \frac{\beta_S\{12\beta_B^2\beta_S^2 - 15\beta_B\beta_S\alpha_B\alpha_S - \beta_B\beta_S\alpha_B(\alpha_B + \beta_B) + 3\beta_S\beta_B^2\alpha_S + 2\alpha_S^2\alpha_B(2\alpha_B - \beta_B)\}}{C_3}.$$

Notice that the denominators in all the prices listed above are positive due to Assumption 1. Notice further that the above prices do not need to be positive. Marginal costs were assumed to be zero. However, in a two-sided market, a price below marginal cost could be optimal (on only one side of the market, naturally) and this possibility should be allowed for.

The demands resulting from optimal prices in the three organizations are listed below

The integrated organization

$$N_B = \frac{\beta_S(2\beta_B+\alpha_B+\alpha_S)}{C_1}, \qquad N_S = \frac{\beta_B(2\beta_S+\alpha_B+\alpha_S)}{C_1}.$$

The semi-integrated organization

$$N_B = \frac{\beta_S(2\beta_B+\alpha_B+\alpha_S)}{C_2}, \qquad N_S = \frac{2\tilde{\beta}+\beta_B(2\beta_S+\alpha_B+\alpha_S)}{C_2}.$$

The non-integrated organization

$$N_B = \frac{\beta_B\beta_S\{2\tilde{\beta}+\beta_S(2\beta_B+\alpha_B+\alpha_S)\}}{C_3}, \qquad N_S = \frac{\beta_B\beta_S\{2\tilde{\beta}+\beta_B(2\beta_S+\alpha_B+\alpha_S)\}}{C_3}.$$

Demand cannot be negative and in order to avoid a tedious analysis with the boundary restrictions, parameters are restricted such that demand is non-negative on both sides. The following is therefore assumed for the remainder of the paper

Assumption 2 $\xi_i \equiv 2\beta_i + \alpha_i + \alpha_j \geq 0 \quad i \in \{B, S\}.$

Assumption 2 is trivially satisfied when network externalities are positive on both sides of the market. However, it is possible that the network externality is negative on one side. In this case, if demand is to be non-negative, the network externality cannot be too negative relative to the other benefits.

3   Consequences of a Vertical Merger

This section compares prices and demand in order to draw conclusions on price and welfare effects of vertical mergers. There are two "kinds" of vertical mergers. One where we go from the non-integrated to the semi-integrated organization. That is, a downstream unit still exists on one side of the market. Further, there is one where we go from the semi-integrated organization to the integrated organization. Here, there is no downstream units left after integration.

From non-integration to semi-integration   The price effects from a vertical merger on side i when a vertical structure still exists on side j are listed below where $\Delta p_i \equiv p_i$ *(before merger)*-p_i *(after merger)*.

$$\Delta p_i = \frac{2\beta_i\tilde{\beta}(2\tilde{\beta}+2\beta_j\xi_i)[4\beta_i\beta_j-\alpha_i(\alpha_i+3\alpha_j)]}{C_3C_2} \tag{13}$$

$$\Delta p_j = \frac{2\tilde{\beta}(2\tilde{\beta}+2\beta_j\xi_i)\left[\beta_j\beta_i(\alpha_i-3\alpha_j)+2\alpha_j^2\alpha_i\right]}{C_3C_2}. \tag{14}$$

From semi-integration to integration   The price effects from a vertical merger on side i when there is no downstream unit on side j are listed below where $\Delta p_i \equiv p_i$ *(before merger)*-p_i *(after merger)*.

$$\Delta p_i = \frac{2\tilde{\beta}\xi_i(2\beta_i\beta_j-\alpha_i(\alpha_i+\alpha_j))}{C_1C_2} \tag{15}$$

$$\Delta p_j = \frac{2\beta_j\tilde{\beta}\xi_i(\alpha_i-\alpha_j)}{C_1C_2}. \tag{16}$$

The first thing to notice is that the sign of the price effects are all ambiguous and will depend on parameters. Hence, a vertical merger does not necessarily lower the price neither on the side where it takes place nor on the other side of the market. This contradicts the standard one-sided market logic of vertical integration where a vertical merger always removes the double marginalization problem and lowers the price.

Focus first on the price effect from a merger on the side where it takes place. Propositions 1 and 2 below present the results when there is still a downstream unit present on the other side and when there is not, respectively.

Proposition 1. A vertical merger on side i when there is a downstream unit present on side j will increase p_i if and only if $4\beta_i\beta_j - \alpha_i(\alpha_i + 3\alpha_j) < 0$. Under Assumption 1, a necessary condition for the price to increase is that the network externality on side i is higher than on side j and both are positive, $\alpha_i > \alpha_j > 0$.

Proof. All terms in (13) are positive except for $4\beta_i\beta_j - \alpha_i(\alpha_i + 3\alpha_j)$ which may be either positive or negative depending on parameters. Consequently, a necessary and sufficient condition for the price to increase following a merger is $4\beta_i\beta_j - \alpha_i(\alpha_i + 3\alpha_j) < 0$.

From Assumption 1 we have $4\beta_i\beta_j > (\alpha_i + \alpha_j)^2$. Hence, if $(\alpha_i + \alpha_j)^2 > \alpha_i(\alpha_i + 3\alpha_j)$ it is not possible that the price increases unless Assumption 1 is violated. We get

$$(\alpha_i + \alpha_j)^2 < \alpha_i(\alpha_i + 3\alpha_j) \Leftrightarrow$$
$$\alpha_j^2 < \alpha_i\alpha_j. \tag{17}$$

Notice that (17) is only satisfied when $\alpha_i > \alpha_j > 0$ and consequently this is a necessary condition for the price to increase under Assumption 1. Notice however that it is not sufficient. We need $4\beta_i\beta_j < \alpha_i(\alpha_i + 3\alpha_j)$ and hence there will always be values of β_i and β_j large enough for this not to hold even if $\alpha_i > \alpha_j > 0$. □

From Proposition 1 we can conclude that in the case where the two sides are symmetric such that $\beta_B = \beta_S$ and $\alpha_B = \alpha_S$ the standard logic of vertical integration applies. A vertical merger removes the double marginalization problem and results in a lower price. However, in the case where the sides are not symmetric and in particular when side i benefits relatively more from meeting side j than vice versa, a vertical merger on side i can lead to a higher price there. In the asymmetric case of $\alpha_i > \alpha_j$, the platform prefers a relatively higher price on side i. The removal of the downstream firm enables this "correct" price balance and when it dominates the effect from the removal of the double marginalization problem the price increases.

The next result deals with the case of a vertical merger when a downstream firm is not present on the other side of the market.

Proposition 2. A vertical merger on side i when there is not a downstream unit present on side j will increase p_i if and only if $2\beta_i\beta_j - \alpha_i(\alpha_i + \alpha_j) < 0$. Under Assumption 1, a necessary condition for the price to increase is that the network externality on side i is larger than on side j in absolute value, $|\alpha_i| > |\alpha_j|$.

Proof. All terms in (15) are positive except for $2\beta_i\beta_j - \alpha_i(\alpha_i + \alpha_j)$ which may be either positive or negative depending on parameters. As a result, $2\beta_i\beta_j - \alpha_i(\alpha_i + \alpha_j) < 0$ is necessary and sufficient for the price to increase following integration.

Assumption 1 still has to hold so we compare those two conditions

$$\frac{\alpha_i(\alpha_i + \alpha_j)}{2} > \frac{(\alpha_i + \alpha_j)^2}{4}$$

$$\alpha_i^2 > \alpha_j^2. \tag{18}$$

Only when (18) holds is it possible for the price effect to be negative without violating Assumption 1. However, this still requires moderate values of β_i and β_j since these can be chosen large enough such that $2\beta_i\beta_j - \alpha_i(\alpha_i + \alpha_j) > 0$. □

Again, when the two sides are symmetric, a vertical merger on side i will lead to a lower price there. However, when the sides are asymmetric, a vertical merger may increase the price. This can happen for positive or negative network externalities. First, if side i benefits relatively more from meeting side j, a vertical merger will increase the price on side i. Second, if the network externality is negative on side i and the disutility here outweighs the positive network externality on side j a vertical merger on side i can increase the price paid on this side.

The effect on p_j from a merger on side i is summarized in the proposition below

Proposition 3. A sufficient condition for the price to increase on side j following a vertical merger on side i is that the network externality on side j is larger than on side i, $\alpha_j > \alpha_i$. This holds regardless of whether a downstream unit is present on side j.

Proof. For the case of no downstream unit on side j the results follows directly from (16). For the case of a downstream unit on side j focus on (14). This is only negative if $\beta_j\beta_i(\alpha_i - 3\alpha_j) + 2\alpha_j^2\alpha_i < 0$. A necessary condition for this is $\alpha_i - 3\alpha_j < 0$. Also notice that it will always be negative if $\alpha_i < 0$ and never be negative if $\alpha_j < 0$. Rewrite the inequality as $\beta_j\beta_i(\alpha_i - 3\alpha_j) < -2\alpha_j^2\alpha_i$. Since β_i and β_j are never negative, a necessary condition for the inequality to be satisfied is $\alpha_i - 3\alpha_j < 0$. Then, the price increases as long as

$$\beta_j\beta_i > \frac{2\alpha_j^2\alpha_i}{\alpha_i - 3\alpha_j}. \tag{19}$$

From Assumption 1 we know that $\beta_i\beta_j > \frac{(\alpha_i+\alpha_j)^2}{4}$ which means that (19) is always satisfied as long as $\alpha_j > \alpha_i$. Hence, $\alpha_i - 3\alpha_j < 0$ is a necessary but not sufficient condition for a price increase while $\alpha_j > \alpha_i$ is sufficient. $\square$

Similarly to the price effects on side i, the price on side j will decrease as the result of a vertical merger when the two sides are symmetric. However, the price on side j following a vertical merger on side i will increase if members here value interaction with side i relatively more than the other way around. In the case where the network externality is negative on side i, a vertical merger here will always lead to higher prices on side j where the network externality is then positive.

Concluding from the results above, the logic of double marginalization does not translate directly to two-sided markets. A vertical merger will not necessarily lead to lower prices. Even if a vertical merger results in a lower price on the side where it takes place it is possible that it leads to a higher price on the other side of the market. An increased price following a merger tends to happen when the network externality on the side in question is large (in absolute value) relative to the network externality on the other side of the market.

In a standard one-sided market, a vertical structure gives rise to the problem of double marginalization, as defined by Spengler (1950). A downstream firm sets a mark-up over the upstream price but fails to internalize the demand effect on the upstream firm's profit when choosing this mark-up. The resulting equilibrium price is too high and demand too low. A vertical merger eliminates the double marginalization problem which ultimately results in lower prices.

In a two-sided market, it turns out that a vertical structure poses more challenges for the upstream firm than a double marginalization problem. It also hinders the platform in choosing the right price structure, that is the balance of prices between the two sides. As is the case for a one-sided market, a downstream firm does not take into account that a higher downstream price leads to lower upstream profit. However, in a two-sided market a downstream firm also fails to internalize the effect its price choice has on the price balance between the two sides. A factor which also affects the upstream firm's profit. As a result, a vertical merger not only removes the double marginalization

problem but also removes the distortionary effect the downstream firm has on the price balance between the two sides. The price effect of a merger depends on the sign and relative sizes of these two effects.

As an example, imagine that the network externality is highest on side S ($\alpha_S > \alpha_B > 0$). A two-sided platform will typically prefer a relatively low price on side B in order to attract members to this "low benefit side." Doing so will enable the platform to set a high price on side S where consumers value interaction with side B members relatively more. In this example a vertical merger should be expected to lead to a lower price on side B and a higher price on side S. While the removal of the double marginalization problem results in lower prices on side B, the platform uses the new organizational structure to get the desired price balance, which involves even higher side S prices. This would be the result we would expect to see in markets where one side values interaction relatively more than the other side. A classic example is advertising supported media where advertisers are thought to value interaction more than consumers, Kaiser and Wright (2004). This could be the case for positive network externalities on both sides or when the network externality is negative on the consumer side.

It is important to notice that, even if the price consumers on a given side has to pay increases as the result of a vertical merger, these consumers are not necessarily worse off. As a direct consequence of the network externality present between the two groups, there are two effects on welfare. One is the standard effect through the price. An increase in price will decrease the utility of a consumer, other things equal. However, with a change in prices comes change in demand on the other side of the market, which in turn affects a consumer's utility as well. For positive network externalities it is possible that consumers on a given side be better off after a price increase as long as there are more members joining on the other side of the market, for example due to a price decrease there. For a negative transaction utility, the opposite is true. A group can be better off after a price increase if there are fewer members joining on the other side of the market, for example as a result of a price increase here.

The next section will investigate the welfare effects of vertical mergers.

4   Welfare

Consumer surplus on side i depends on the price p_i and on the size of the network on the other side of the market N_j. While consumer surplus is decreasing in the price, the effect from changes in N_j will depend on the sign of the network externality. For a positive (negative) network externality α_i, consumer surplus is increasing (decreasing) in N_j. The sign of the change in consumer surplus on side i when going from a pair (p_i', N_j') to (p_i'', N_j'') will be the same as the sign of the change in demand on side i: $N_i(p_i'', N_j'') - N_i(p_i', N_j')$. To see this, rewrite the demand function (10) as

$$p_i = \beta_i - \beta_i N_i + \alpha_i N_j.$$

For the combination of price and network size (p_i^*, N_j^*), consumer surplus, Λ, is given by the area below the demand curve and above the price

$$\Lambda_i = \int_0^{N_i^*} \left(\beta_i - \beta_i N_i + \alpha_i N_j^* - p_i^* \right) dN_i = \tfrac{\beta_i}{2} (N_i^*)^2. \tag{20}$$

From the above derivation of consumer surplus we directly get the result

$$\Lambda_i(N_i') > \Lambda_i(N_i'') \quad \Leftrightarrow \quad N_i' > N_i''.$$

As a consequence, to determine how a vertical merger has affected consumer welfare on side i we focus on the changes in demand on side i following the merger.

From non-integration to semi-integration   The demand effects from a vertical merger on side i when a downstream unit is present on side j are listed below where $\Delta N_i \equiv N_i(\text{after merger})\text{-}N_i(\text{before merger})$.

$$\Delta N_i = \frac{4\tilde{\beta}(2\tilde{\beta} + \beta_j \xi_i)(\beta_j \beta_i + \tilde{\beta})}{C_2 C_3} > 0 \tag{21}$$

$$\Delta N_j = (\alpha_j + \alpha_i) \frac{2\beta_i \tilde{\beta}(2\tilde{\beta} + \beta_j \xi_i)}{C_2 C_3}. \tag{22}$$

From semi-integration to integration   The demand effects from a vertical merger on side i when a downstream unit is not present on side j are listed

below where $\Delta N_i \equiv N_i$ (after merger)-N_i (before merger).

$$\Delta N_i = \frac{4\beta_j \tilde{\beta} \xi_i}{C_1 C_2} > 0 \tag{23}$$

$$\Delta N_j = (\alpha_i + \alpha_j) \frac{2\tilde{\beta} \xi_i}{C_1 C_2}. \tag{24}$$

The consumer welfare effects on the side where integration takes place are unambiguously positive. Even if the price paid by consumers on this side increases, consumers will be better off. Hence, if the price effect on welfare is negative it will always be outweighed by a positive demand effect through the network externality.

The consumer welfare effects following integration on the other side of the market, however, need not be positive. As can be seen from (22) and (24) these will be positive as long as the sum of network externalities is positive, $(\alpha_B + \alpha_S) > 0$. As a result, as long as we have positive network externalities on both sides of the market, integration always increases welfare on both sides of the market even if it raises prices. If instead we have a negative network externality on one side of the market (as in the case of advertising), it is possible that consumers are worse off on one side of the market as a result of integration on the other side.

The welfare results are summed up in the proposition below.

Proposition 4. A vertical merger on side i will always result in increased consumer surplus on this side. It will result in higher consumer surplus on side j if $\alpha_i + \alpha_j > 0$ and lower consumer surplus on side j if $\alpha_i + \alpha_j < 0$.

Proof. Follows directly from (21), (22), (23), and (24) . □

From Proposition 4 it can be concluded that at least one side will always be better off after integration while at most one side will be worse off. The proposition below discusses the overall effects on demand from a vertical merger.

Proposition 5. In the case where a vertical merger on side i results in a decrease in demand on side j, the decrease in demand on side j is always smaller than the increase in demand on side i.

Proof. The relevant case is when demand decreases on side j, which happens when $\alpha_i + \alpha_j < 0$. Focus first on the case of a merger on side i when a vertical

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structure is still present on side j. We are comparing the size of (21) to (22). We want to show that whenever the demand effect on side j is negative, the change in demand on side i is always larger than on side j. Since the change on side j is negative and we are comparing the size of the effects, we multiply (22) by minus 1 and ask when it is larger than the effect on side i

$$\Delta N_i = \frac{4\tilde{\beta}(2\tilde{\beta}+\beta_j\xi_i)(\beta_j\beta_i+\tilde{\beta})}{C_2 C_3} < -(\alpha_j + \alpha_i)\frac{2\beta_i\tilde{\beta}(2\tilde{\beta}+\beta_j\xi_i)}{C_2 C_3} = \Delta N_j$$
$$4\beta_j\beta_i - (\alpha_i + \alpha_j)^2 + 4\tilde{\beta} < 0.$$

Under Assumption 1, the last inequality is never satisfied and as a consequence the increase in demand on side i is always larger than a decrease on side j.

For the case of a merger on side i when there is no vertical structure on side j we focus on (23) and (24) and the argument is identical to the one above

$$\Delta N_i = \frac{4\beta_j\tilde{\beta}\xi_i}{C_1 C_2} < -(\alpha_i + \alpha_j)\frac{2\tilde{\beta}\xi_i}{C_1 C_2} = \Delta N_j$$
$$2\beta_j + \alpha_i + \alpha_j < 0.$$

Under Assumption 2, the last inequality is never satisfied and as a consequence the positive demand effect always outweighs a negative demand effect. $\square$

Even if a vertical merger results in lower demand on one side of the market, it will always increase overall demand defined as the sum of demands on the two sides.

Keep in mind that when the market in question is two-sided, the definition of total consumer surplus is not always clear. The two groups will have to be weighted against each other. However, the question of what weights to use is controversial. The groups could for example be weighted equally or they could be weighted by size. Further, a competition authority may put higher weight on one group compared to another depending on the industry in question for example consumers versus advertisers. Proposition 5 simply states that total demand always increases following a vertical merger.

If we define total consumer surplus as the sum of consumer surpluses on the two sides we see that as long as the sum of network externalities is positive, total consumer surplus always increases as the result of a merger. This is true since demand increases on both sides of the market and consumer surplus is

given by (20). Hence, in the case where the two sides are symmetric, vertical mergers always increase total consumer surplus. Further, in the asymmetric case with positive network externalities on both sides, or with a negative network externality on one side but a positive sum of network externalities, vertical mergers always increase total consumer surplus. The one case where total consumer surplus may decrease is when the sum of network externalities is negative. In this case, consumer surplus decreases on one side and increases on the other. When we label demand in the before merger case as N_i' and in the after merger case as N_i'', the overall effect is negative when

$$\frac{\beta_i\left((N_i'')^2-(N_i')^2\right)+\beta_j\left((N_j'')^2-(N_j')^2\right)}{2} < 0.$$

For the firms, the standard vertical integration argument applies. Since the integrated platform can always mimic the prices of the non-integrated organization, it cannot be worse off after integration. As a result, total profit in the organization will always be (at least weakly) higher after a vertical merger.

The fact that vertical integration in two-sided markets does not only alleviate a double marginalization problem but affects the platform's ability to get the right price balance means that welfare is not necessarily improved. Vertical mergers do not necessarily leave all parties better off but might result in higher prices and lower demand.

5   Conclusion

Two-sided markets have special characteristics and optimal price structures differ from those of standard markets. As a consequence, standard antitrust results cannot be transferred directly to two-sided markets. The logic needs to be worked out in a two-sided set-up.

This paper analyzed the effects of vertical integration in a two-sided market. Optimal prices were derived, focusing on a general organizational structure which allowed for a downstream firm on both sides of the market. A downstream firm not only imposes a double marginalization problem on the platform but also prevents the platform from getting the desired price balance

between the two sides. Furthermore, a vertical merger in a two-sided market does not unambiguously lead to lower prices as in a one-sided set-up. A vertical merger can increase the price on the side of the market where it takes place if the transaction utility on this side is relatively high, and may raise the price on the other side of the market for both positive and negative transaction utilities.

In addition, a vertical merger will not unambiguously increase welfare as is the case in one-sided markets. Consumers may be worse off even if they face lower prices, since utility also depends on demand on the other side of the market through the network externality.

Throughout the paper it was assumed that downstream firms take the upstream price as given and react by choosing a mark-up over this price. In reality, downstream firms often have some degree of bargaining power and the price formation may be more complex. It would be interesting to see how this would affect results.

Furthermore, it was assumed throughout that the downstream firms and the platform all act as monopolists. It is well-known that if there is perfect competition downstream, the double marginalization problem disappears. Introducing some kind of competition downstream or upstream would be an interesting direction for future research.

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Chapter 2

Platform Competition with Endogenous Multihoming Decision[*]

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Abstract

This paper analyzes competition between horizontally differentiated platforms when consumers on both sides of the market have the option of multihoming. Contrary to the existing literature on platform competition we let the choice of whether to purchase at one or both platforms be endogenously determined by prices and network size. This allows for the realistic outcome where some consumers on a given side multihomes while other consumers on the same side are loyal to their preferred brand. Furthermore, our set-up allows for an outcome where consumers on both sides multihome. This is often observed in reality but assumed away in the existing literature. In contrast to Armstrong (2006) and Armstrong and Wright (2007) we find that prices are lower for multihoming consumers compared to singlehoming consumers. We compare the competitive outcome to the case of joint ownership, which could be the result of a merger of the two platforms, and confirm that prices are lower under competition. Lastly, we show that the amount of multihoming in the competitive outcome is suboptimal. This is a direct effect of the presence of the positive network externality between the groups.

Keywords: Two-sided markets; Price competition; Multi purchase; Preference for variety.

JEL-Classifications: D43; D62; J13.

1 Introduction

Many of today's markets consist of so-called two-sided platforms. These platforms enable agents of two distinct groups to interact. Network externalities exist between the groups, and the benefit enjoyed by an agent in one group depends on how well the platform attracts agents from the other group. Such markets are referred to as two-sided markets (cf. Rochet and Tirole, 2003). Commonly cited examples include payment schemes, media markets, entertainment platforms, computer operating systems, matching markets, credit cards and search engines.

It is well known that network effects induce concentration. Consequently, one might expect industries based on two-sided platforms to tend towards natural monopolies. Product differentiation is an important countervailing force. As observed in Evans and Noel (2005), it is relatively uncommon for industries based on two-sided platforms to be monopolies. Instead, these industries tend to feature differentiated platforms. Consumers can choose between different payment cards, newspapers, computer operating systems, game consoles, etc. Platforms differentiate themselves from each other by choosing features or prices that which appeal to particular groups of consumers. When Microsoft, for example, introduced their video game console Xbox, the price difference between Xbox and Sony's Playstation was negligible. Instead, the two platforms compete on the variety of associated games the offer by courting game developers. While some developers, like Electronic Arts, develop both for Xbox and Playstation, these platforms still differentiate themselves by not offering cross-platform compatibility. Most consumers have preferences for Microsoft Windows versus Apple OS, Nintendo versus Sony, Google Chrome versus Internet Explorer, etc.

Such horizontal platform differentiation may cause consumers on either side of the market to use more than one platform. While consumers will buy at most one copy of a particular newspaper or one game console from a particular brand, some choose to buy newspapers from two different sources or game consoles from two different brands. Consumers might choose to buy from more than one platform in order to benefit from variety and/or from gaining access to a greater network. For example, some consumers have one

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credit card, while others prefer to have two. Many buyers of videogames only have one console. However, some buyers own more than one since this allows for access to a wider variety of games. Some readers only subscribe to one newspaper, while others prefer to subscribe to more than one because it offers them a more nuanced news flow. In all of these cases, agents can choose to join one platform (singlehome) or two platforms (multihome). Behavior can vary between groups; there may be singlehoming on one side of the market and multihoming on the other side. However, it may also vary *within* a group. As mentioned in the example above, some cardholders prefer to hold one card only while others hold more than one.

The choice of whether to multihome or singlehome seems to be endogenously determined by prices, market characteristics, and consumer taste. Indeed, for most markets it is not a given that consumers will always, say, singlehome independently of prices. Typically, for a given set of prices, some consumers will find it worthwhile to multihome while others will not. For another set of prices, we should expect to see a different composition of singlehomers versus multihomers in a given group. In particular, for very high prices it is likely that most consumers will purchase at one firm only, while for very low prices more consumers should find it optimal to multihome, even if this means buying from a less preferred brand in addition to their preferred brand. However, the literature on platform competition traditionally assumes that on one side of the market, all consumers are singlehoming for exogenous reasons. This assumption effectively removes competition on the side where multihoming is possible. Since consumers on one side are always singlehoming (even for very low, possibly negative prices), the platforms can offer exclusive access to these consumers. As a result, the choice on the other side of the market of whether to join platform 1 is completely independent of the choice of whether to join platform 2. This greatly affects the resulting optimal price structure.

In addition, the literature on platform competition has generally focused on multihoming on one side of the market only. Either one side has been assumed to singlehome for exogenous reasons, or multihoming may be possible on both sides ex ante but will never happen in equilibrium. The argument is that once consumers on one side are multihoming, there is no incentive for consumers

on the other side to use more than one platform. In reality, however, some multihoming on both sides is often the norm. Payment cards are one example. Many cardholders carry multiple cards, and merchants accept more than one card. Likewise, advertising-supported media also have multihoming on both sides. Advertisers are often present on more than one platform, while readers buy more than one newspaper the same way viewers watch multiple channels.

Based on the above two observations, we propose a model of platform competition with horizontally differentiated platforms where both sides have the option of multihoming and this choice is endogenously determined by prices, the size of the network, and the degree of differentiation. Furthermore, by allowing consumers to benefit from variety when using more than one platform, we effectively allow for an equilibrium outcome where (some) consumers on both sides choose to multihome. Our model enables us to study how optimal prices vary with the degree of multihoming and to determine which market characteristics, such as degree of differentiation, lead to which market outcomes.

We further compare the results arising from differentiated competition to a market with no competition, and lastly to the outcome preferred by a social planner. This allows us to study whether a competitive two-sided market results in lower prices compared to a monopoly two-sided market, and to analyze how effective the market is at providing a socially optimal outcome.

In our model, two competing platforms are located at the two end points of the unit interval, and users dislike distance from their own location to that of a platform. As the two platforms become more differentiated, the cost of reaching either one increases. It is possible that each agent goes to one platform only. However, if prices and transportation costs are not too high, some consumers may find it worthwhile to purchase on both platforms. The competitive environment when consumers are multihoming is different from when they are singlehoming. When consumers visit one platform only, a classic "business stealing effect" exists. If platform 0 lowers its price, some consumers will choose to go to platform 0 instead of platform 1. When consumers multihome, this changes. A lower price on platform 0 leads to more multihoming consumers but does not lower the demand for platform 1 on the

same side of the market. Furthermore, since consumers on both sides have the option of multihoming no platform can necessarily offer exclusive access to consumers, and thus there is competition on both sides of the market. Allowing for multihoming on both sides of the market changes optimal behavior by firms. Consequently, the optimal prices differ from those in a model where the multihoming decision is not endogenous.

There are four different types of equilibria in the model: one in which all agents on both sides singlehome, one in which at least one agent on both sides multihome, and two asymmetric equilibria with singlehoming on one side and (some) multihoming on the other side. The outcome predicted by our model depends on the parameters. As in Armstrong (2006), strong product differentiation leads to an outcome where both sides choose to singlehome. Strong preference for variety and/or a smaller degree of differentiation leads to a multihoming equilibrium. We show that for a range of parameter values, all four kinds of equilibria exist. Furthermore, for large enough differentiation, only equilibria in which all agents singlehome exist, and for low degrees of differentiation, only equilibria with multihoming exist. Lastly, for some parameter values only the asymmetric equilibria exist. This happens when the degree of differentiation is low and there is not a strong preference for variety.

For the range of parameters where all four kinds of equilibria exist, we can directly compare the equilibrium prices. We find that the price paid by consumers on a multihoming side is lower than the price on a singlehoming side. In addition, the singlehoming price is lower when the other side is also singlehoming, as compared to when the other side is multihoming.

We compare these results to a case of joint platform ownership, hence no competition. Under joint ownership, the optimal price is chosen such that the marginal consumer is left with zero surplus. We confirm the logic from one-sided markets that prices are higher when there is no competition.

Since a positive externality exists between the groups, the competitive equilibrium is bound to be sub-optimal in the eyes of a social planner. More precisely, the competitive equilibrium results in too little multihoming. We solve for the optimal location as preferred by a social planner and confirm that it does indeed involve more multihoming than the market solution. The

social planner trades off benefits from participation with transportation costs, and the optimal level of demand will depend on the parameters. If agents have a strong taste for variety, the social planner prefers full multihoming, whereas for large values of platform differentiation, it is preferred that some agents multihome while others singlehome.

The literature on competition between differentiated two-sided platforms is limited but growing. One of the first papers to analyze differentiated platform competition was Armstrong (2006). Armstrong shows that when agents are allowed to use one platform only, and there is sufficiently strong product differentiation on either side of the market, the price-cost margin for each side will be equal to the product differentiation parameter for that side minus the externality enjoyed by the other side. If attracting one side of agents (say buyers) makes the platform particularly attractive to the other side, then the buyers will be "subsidized." In a set-up where multihoming is allowed on one side while the other side always singlehomes, Armstrong concludes that the multihoming side faces "excessive prices." Armstrong and Wright (2007) elaborate on this idea. Armstrong and Wright study product differentiation on one side only (the buyer side). While buyers always singlehome, sellers are allowed to multihome. In the unique equilibrium, buyers are singlehoming and all sellers are multihoming. Since buyers are singlehoming, platforms can act as monopolists to the sellers who want to interact with their buyers. As a result, the multihoming side faces high prices and buyers have their surplus fully extracted. Our paper advances by allowing for the general case where both sides can choose to multihome and in particular where multihoming on both sides may occur in equilibrium. A direct effect is that platforms cannot act as monopolists to multihoming consumers. Since consumers on the other side can now also multihome, platforms compete on both sides of the market. This greatly affects the optimal strategies of firms. In contrast to Armstrong (2006) and Armstrong and Wright (2007), we find that lower prices are offered to a multihoming side than to a singlehoming side. An equilibrium with a low singlehoming price and a high multihoming price is not sustainable for two reasons. On the singlehoming side, some consumers will start to multihome once prices are low. On the multihoming side in our model the choice of whether to join one platform is directly linked to the choice of whether to join

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the other. As a result, when both platforms offer very high prices, consumers will simply not find it optimal to multihome.

Our results show similarities to Gabszewicz and Wauthy (2004), who study a model of platform competition with vertical differentiation. In their model, the choice of whether to multihome is endogenous, and consumers on both sides of the market are, in theory, allowed to multihome. However, with the assumptions ultimately made about beliefs and consumers' utility, multihoming on both sides can never occur in equilibrium. When the choice of multihoming is endogenous, Gabszewicz and Wauthy also find that the price on a multihoming side is lower than on a singlehoming side in their vertical differentiation model.

A few papers studying conventional "one-sided" markets consider the case where each agent either single- or multihomes. For example, Kim and Serfes (2006) propose a model for spatial competition that allows for multi-purchase (i.e. agents are allowed to buy one unit from each of the two firms). Anderson *et al.* (2012) elaborate on Kim and Serfes' idea and propose a model where multi-purchase is allowed and the firms' choice of quality is more appreciated the closer a good is to a consumer's ideal variety. When consumers multi-purchase, prices are strategically independent. Furthermore, if quality is high, firms prefer to price high to eliminate multi-purchase. Common to these "one-sided" models is that some consumers multi-purchase while others are loyal to their preferred brand. The authors argue that this observation fits many markets such as credit cards and newspapers. However, these markets are notoriously two-sided, which is not taken into account in their model. Our model takes the idea of multi-purchase under differentiated competition to a two-sided market. Some results of competition in our two-sided model resemble the results in Anderson *et al.* (2012). We also find that prices are strategically independent when consumers multi-purchase, in the sense that a price change on side 1 does not affect the other platform on this side. However, it does have an effect *on the other side of the market*. Hence, not all prices are strategically independent once the market is two-sided. Furthermore, we find that when the degree of differentiation is high, platforms will prefer to set high prices and eliminate multihoming in the same way that firms preferred high prices when quality was high in Anderson *et al.* (2012).

Competition in two-sided markets with singlehoming has been successfully applied to the modeling of the television markets. In their paper, Anderson and Coate (2005) consider television viewers who are distributed on a Hotelling line with channels located at the endpoints. Viewers watch only one channel, while advertisers can buy commercials on both television channels. Several papers have extended Anderson and Coate's model. For example, Gabszewicz *et al.* (2004) allow viewers to mix their time between channels, Peitz and Valletti (2008) analyze optimal locations of stations, and Reisinger (2012) considers singlehoming of advertisers. These papers, however, do not allow for the intuitive case where some viewers multihome by switching between channels. Our model adds to this more applied literature by allowing for such switching.

The case where multihoming is possible on both sides has not received much attention. One exception is Choi (2010), who models two-sided multihoming markets by assuming that the amount of multihoming on one side (say seller-side) is exogenously given. Another exception is Ambrus *et al.* (2013), who develop a model of television market provision that allows both advertisers and viewers to multihome. However, in order to do so, Ambrus *et al.* make the (extreme) assumption that viewer demand for one platform does not affect the demand for the other platform. Unlike these authors, we assume that agents on either side derive utility from multihoming in two ways. First, they are exposed to a greater network (this effect is zero if all consumers on the other side are multihoming.) Second, consumers enjoy variety. Buying from two platforms instead of one increases the fixed utility/willingness to pay.

The rest of the paper is organized as follows. Section 2 sets up the model and section 3 presents the results. In section 4 we discuss the case of joint ownership while section 5 focuses on welfare results. Section 6 concludes.

2 The Model

Two platforms are located at the two endpoints of a unit interval $[0, 1]$. The platforms are competing for consumers in two distinct groups, each being uniformly distributed with unit mass on the $[0, 1]$ interval. There is a network externality between groups meaning that consumers in one group care about the number of people joining on the other side of the market. The set-up is illustrated in the figure below.

Group A

0 ———————— 1

Platform 0   Platform 1

0 ———————— 1

Group B
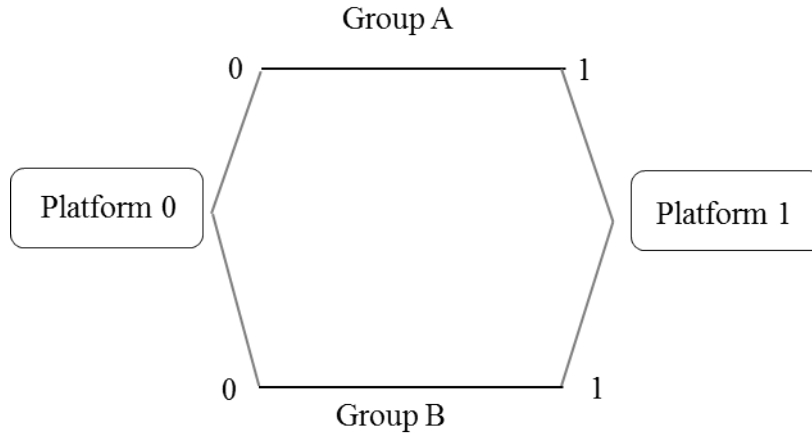
Figure 1: Set-up

Consumers on each side buy at most one good from each platform but can choose to buy from both. The decision of whether to buy at just one platform (i.e. singlehoming) or buy at both platforms (i.e. multihoming) will depend on the prices offered by the platforms, the consumer's location on the line and on the size of the network on the other side.

We label the platforms 0 and 1 respectively and the two groups of consumers A and B. Superscripts are used to refer to a platform where $i \in \{0, 1\}$, $j \in \{0, 1\}$, $j \neq i$ and subscripts are used to refer to a group with index $k \in \{A, B\}$ and $l \in \{A, B\}$, $l \neq k$. In this way p_k^i refers to the price platform i charges members of group k. Each consumer has the option of buying from one or both platforms (or not buy at all).

Consumers on side k of the market receive a fixed utility of β_k when they buy at a platform. If a consumer buys from both platforms, he receives an

additional utility of θ, which we assume to be smaller than β_k to reflect diminishing marginal utility. This is similar to the set-up in Kim and Serfes (2006). The parameter θ can be thought of as capturing the consumers' love for variety. The higher is θ, the more do consumers gain from purchasing at both platforms.

Consumers care about the number of participants on the other side of the market. If a consumer in group k joins platform i and there are n_l^i members of this platform on the other side, then the network externality gives a consumer in group k n_l^i extra utility. Further, if he chooses to join both platforms he will meet all the consumers on the other side of the market, and total utility stemming from the network externality is given by 1. When deciding whether to singlehome or multihome, only the additional agents you will meet by multihoming matter.

Lastly, the two platforms are differentiated and consumers dislike distance to the platforms. This disutility is given by $t \cdot d$ where d is the consumer's distance to the platform. As usual, we refer to t as a transportation cost parameter though we do not necessarily view this disutility as having to do with actual literal transportation. Consequently, the total disutility in case a consumer buys from both platforms is $t \cdot 1$. Throughout the analysis the following assumption will be made

$$t > 1.$$

This assumption ensures that demand functions are well behaved and further that second order conditions are satisfied.

Each consumer has four choices: (1) Buy from platform 0 only; (2) Buy from platform 1 only; (3) Buy from both platforms; (4) Do not buy at all. The utility of an agent at side k of the market located at x in each of these four cases is given by

$$u_k = \begin{cases} \beta_k + n_l^0 - tx - p_k^0 & \text{if the agent buys from 0} \\ \beta_k + n_l^1 - t(1-x) - p_k^1 & \text{if the agent buys from 1} \\ \beta_k + \theta + 1 - t - p_k^0 - p_k^1 & \text{if the agent multihomes} \\ 0 & \text{if the agent does not buy} \end{cases}.$$

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All consumers maximize utility taken prices and participation on the other side of the market as given. Throughout the analysis we assume that the market is covered. To ensure this we assume that

$$\theta + t\beta_k \geq \tfrac{3}{2}(t^2 - 1) \quad k = \{A, B\} \tag{1}$$

$$\beta_k \geq \tfrac{3}{2}(t - 1) \quad k = \{A, B\}. \tag{2}$$

2.1 Demand

To characterize the demand functions facing the two platforms we find the location of the indifferent consumer. First, the consumer who is indifferent between buying only at platform 0 and buying at both platforms is located at x^0

$$x^0 = 1 - \left(\frac{\theta + 1 - n_l^0 - p_k^1}{t} \right).$$

The consumer who is indifferent between buying only at platform 1 and buying at both platforms is located at

$$x^1 = \frac{\theta + 1 - n_l^1 - p_k^0}{t}.$$

These two locations give us a characterization of when multihoming will occur.
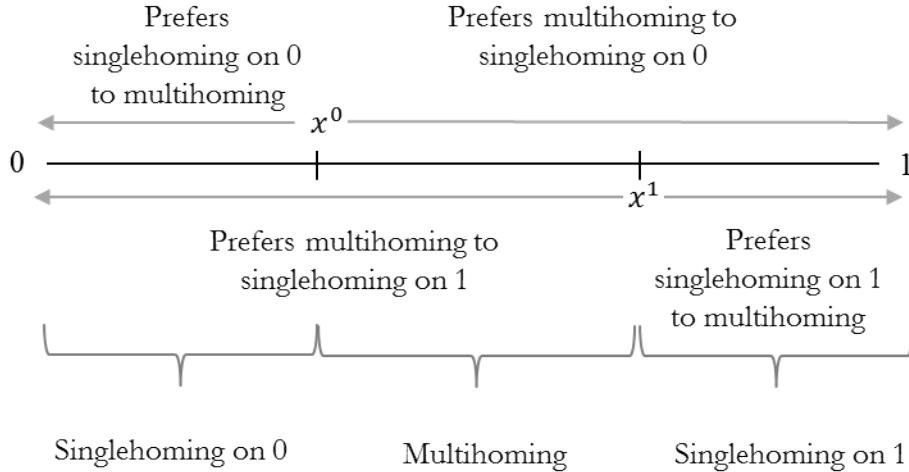

Figure 2: Hotelling line with indifferent consumers

Consumers to the left of x^0 prefer singlehoming on platform 0 over multi-

homing and vice versa for consumers to the right of x^0. Likewise for x^1, consumers to the right of this cut-off prefer singlehoming on platform 1 whereas consumers to the left prefer multihoming over singlehoming on platform 1. All consumers located to the right of x^0 but to the left of x^1 will therefore prefer multihoming over either form of singlehoming. Consequently, a necessary condition for multihoming to occur is that x^0 smaller than x^1. This gives us the following condition

$$p_k^0 + p_k^1 + t \leq 2\theta + \left(1 - n_l^0\right) + \left(1 - n_l^1\right). \tag{3}$$

If some consumers are to choose multihoming the total costs - prices and transportation cost - should be sufficiently low compared to the benefits of joining the platforms.

If (3) is not satisfied, all consumers will choose to singlehome on either platform 0 or platform 1. The indifferent consumer can then be found by equating the relevant utilities

$$\bar{x} = \frac{1}{2} + \frac{p_k^1 - p_k^0 + n_l^0 - n_l^1}{2t}$$

Notice that this agent is located at exactly one half if the platforms set identical prices and the size of the network on the two platforms is identical.

We can now state the demand functions. Group k's demand for platform 0's product is given by

$$n_k^0 = \begin{cases} x_1 & \text{if } p_k^0 + p_k^1 < 2\theta + \left(1 - n_l^0\right) + \left(1 - n_l^1\right) - t \\ \bar{x} & \text{otherwise} \end{cases}$$

Likewise, group k's demand for platform 1's product is given by

$$n_k^1 = \begin{cases} 1 - x_0 & \text{if } p_k^0 + p_k^1 < 2\theta + \left(1 - n_l^0\right) + \left(1 - n_l^1\right) - t \\ 1 - \bar{x} & \text{otherwise} \end{cases}$$

Demand depends both on prices and on number of participants on the other side of the market. Furthermore, notice that the competitive situation is different under multihoming and singlehoming. Under singlehoming, the demand function is similar to the standard Hotelling demand function except that de-

mand on the other side of the market enters as well. If the platforms set the same price and have the same demand on the other side of the market they will share the market equally. A platform can get more than half of the market by pricing lower than the competitor or by having more members on the other side of the market.

Under multihoming we see that the only price entering the demand function is the platform's own price. However, network size on the competing platform enters as well. If platform 0 lowers its price on side A, some consumers who were previously singlehoming on platform 1 will now find it optimal to multihome. As a consequence, demand for platform 0 on side A increases as a result of the lower price, but demand for platform 1 on side A is unchanged. This is the lack of a "business stealing effect" also pointed out in Kim and Serfes (2006). However, there is an indirect business stealing effect on the other side of the market. Since the size of the network on platform 0 on side A has now increased, the users of platform 0 on side B are now meeting more people. For this reason, multihoming is now less attractive to them since the additional number of people they get to meet if they choose to multihome has gone down. Consequently, consumers on side B located close to platform 0 will start to singlehome on platform 0 instead of multihoming as a result of the lower price on side A. This does not affect platform 0's demand on side B since these consumers are purchasing either way. However, it does decrease platform 1's demand. This is an indirect business stealing effect, working through the network externality.

There are four possible equilibrium outcomes for this market. First, it is possible that some consumers will be multihoming on both sides of the market. This case will henceforth be referred to as MultiMulti (sometimes abbreviated to MM.) Notice that not all consumers are necessarily multihoming. We use the term MultiMulti as long as at least one consumer on each side is multihoming. Second, it is possible that all consumers on both sides are singlehoming. This will henceforth be referred to as SingleSingle (sometimes abbreviated to SS.) Lastly, there are the asymmetric cases where one group singlehomes while (at least some) members of the other group multihome. The case where at least one member on side A multihomes and all members on side B singlehomes is labeled "MultiSingle" and the opposite case is labeled "SingleMulti". In each

of these cases we can solve the demand system, which leaves the following demand function by group k for platform i's product

$$
n_k^i\left(p_k^i, p_l^i, p_k^j, p_l^j\right) = \begin{cases} \frac{(\theta+1)(t-1)+p_l^j-tp_k^i}{t^2-1} & \text{if Multi on } k \text{ and Multi on } l \\ \frac{1}{2} + \frac{t(p_k^j-p_k^i)+(p_l^j-p_l^i)}{2(t^2-1)} & \text{if Single on } k \text{ and Single on } l \\ \frac{1}{2t} + \frac{2\theta(t^2-1)-t(p_l^i-p_l^j)+p_k^j-(2t^2-1)p_k^i}{2t(t^2-1)} & \text{if Multi on } k \text{ and Single on } l \\ \frac{1}{2} + \frac{p_l^j-p_l^i+t(p_k^j-p_k^i)}{2(t^2-1)} & \text{if Single on } k \text{ and Multi on } l \end{cases}
$$

$$(4)$$

Notice that the denominators are all positive due to the assumption made on t. This ensures that demand is decreasing in own price.

2.2 Profit

Total profit is given by the sum of profit made on each of the two groups. For simplicity we will assume that marginal costs are constant and normalized to zero. Then, platform i's profit is given by

$$
\pi^i = p_k^i \cdot n_k^i\left(p_k^i, p_l^i; p_k^j, p_l^j\right) + p_l^i \cdot n_l^i\left(p_k^i, p_l^i; p_k^j, p_l^j\right)
$$

Inserting the demand functions we get a complete characterization of the profit function

$$
\pi^i = \begin{cases} \pi^{iMM} = p_A^i\left[\frac{(\theta+1)(t-1)+p_B^j-tp_A^i}{t^2-1}\right] + p_B^i\left[\frac{(\theta+1)(t-1)+p_A^j-tp_B^i}{t^2-1}\right] \\[2mm] \pi^{iSS} = p_A^i\left[\frac{1}{2} + \frac{t(p_A^j-p_A^i)+2(p_B^j-p_B^i)}{2(t^2-1)}\right] + p_B^i\left[\frac{1}{2} + \frac{t(p_B^j-p_B^i)+2(p_A^j-p_A^i)}{2(t^2-1)}\right] \\[2mm] \pi^{iMS} = p_A^i\left[\frac{1}{2t} + \frac{2\theta(t^2-1)-t(p_B^i-p_B^j)+p_A^j-(2t^2-1)p_A^i}{2t(t^2-1)}\right] + p_B^i\left[\frac{1}{2} + \frac{p_A^j-p_A^i+t(p_B^j-p_B^i)}{2(t^2-1)}\right] \\[2mm] \pi^{iSM} = p_A^i\left[\frac{1}{2} + \frac{p_B^j-p_B^i+t(p_A^j-p_A^i)}{2(t^2-1)}\right] + p_B^i\left[\frac{1}{2t} + \frac{2\theta(t^2-1)-t(p_A^i-p_A^j)+p_B^j-(2t^2-1)p_B^i}{2t(t^2-1)}\right] \end{cases}
$$

$$(5)$$

2.3 Best Responses

Each platform maximizes profit taking the prices of its competitor as given. In order to find best responses, we have to consider the four different cases individually. Assume that there is multihoming on each side of the market and

find the solution to the profit maximization problem given p_A^j and p_B^j. Call this solution (p_A^{iMM}, p_B^{iMM}). For this solution to be a best response it necessarily has to be the case that this solution together with p_A^j and p_B^j fulfills (3) on both sides of the market. This is only the case for some (p_A^j, p_B^j) and thus gives us a subset of (p_A^j, p_B^j) for which the solution (p_A^{iMM}, p_B^{iMM}) is a potential best response. In the same way, we use (3) (or the negation of (3)) to check when (p_A^{iSS}, p_B^{iSS}) are potential best responses etc.

Differentiating platform i's profit function with respect to p_A^i and p_B^i in the four possible scenarios yields

$$
\left(p_A^i, p_B^i\right) = \begin{cases} \left(p_A^{iMM}, p_B^{iMM}\right) & = \left(\frac{(\theta+1)(t-1)+p_B^j}{2t}, \frac{(\theta+1)(t-1)+p_A^j}{2t}\right) \\ \left(p_A^{iSS}, p_B^{iSS}\right) & = \left(\frac{1}{2}\left(t-1+p_A^j\right), \frac{1}{2}\left(t-1+p_B^j\right)\right) \\ \left(p_A^{iMS}, p_B^{iMS}\right) & = \left(\frac{\theta}{2}, \frac{t^2-1-\theta+p_A^j+tp_B^j}{2t}\right) \\ \left(p_A^{iSM}, p_B^{iSM}\right) & = \left(\frac{t^2-1-\theta+p_B^j+tp_A^j}{2t}, \frac{\theta}{2}\right) \end{cases}
\tag{6}
$$

The responses are almost all upward sloping, which reflects that the platforms' products are substitutes. The only exceptions are the asymmetric cases where the responses are constant in the case of multihoming since the indirect business stealing effect is absent here.

Now use (3) to find the (p_A^j, p_B^j) for which (p_A^{iMM}, p_B^{iMM}) fulfills (3) etc. This gives us the following conditions.

p_A^{iMM}, p_B^{iMM}, p_A^j, and p_B^j fulfill (3) if and only if

$$
p_A^j \leq \frac{(\theta+1)(3t^2-1)-2t\theta-2t^3}{2t^2-1} + p_B^j \frac{t}{2t^2-1}
$$
$$
p_B^j \leq \frac{(\theta+1)(3t^2-1)-2t\theta-2t^3}{2t^2-1} + p_A^j \frac{t}{2t^2-1}
$$

p_A^{iSS}, p_B^{iSS}, p_A^j, and p_B^j fulfill (3) if and only if

$$
p_A^j > \frac{4}{3}\theta - (t-1)
$$
$$
p_B^j > \frac{4}{3}\theta - (t-1)
$$

p_A^{iMS}, p_B^{iMS}, p_A^j, and p_B^j fulfill (3) if and only if

$$p_A^j \leq \frac{3}{2}\theta - (t-1)$$
$$p_B^j > p_A^j \frac{1}{3t} + \frac{4t(\theta+1) - 3t^2 - 2\theta - 1}{3t}$$

p_A^{iSM}, p_B^{iSM}, p_A^j, and p_B^j fulfill (3) if and only if

$$p_A^j > p_B^j \frac{1}{3t} + \frac{4t(\theta+1) - 3t^2 - 2\theta - 1}{3t}$$
$$p_B^j \leq \frac{3}{2}\theta - (t-1)$$

These requirements are illustrated in the figure below (for a discussion see Appendix A.1.)
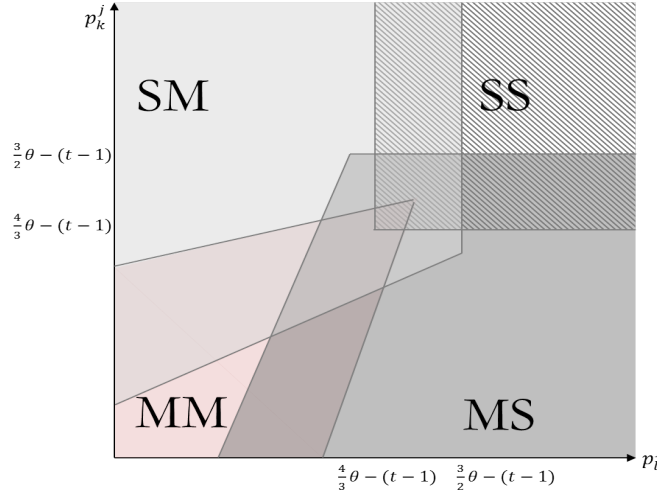
Figure 3: All price requirements

In order to arrive at a full characterization of best responses there is one step left. If two or more responses are valid for the same range of the competitor's prices in the sense that an MM response fulfills (3) and an SS response does not etc., we need to determine which response yields the highest profit in this price range. Hence, for all areas where the price choices overlap we need to compare the profits associated with the price choices. This results in the price requirements listed below (for a discussion see Appendix A.2.)

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MultiMulti responses are best responses if and only if

$$p_A^j \le p_B^j \left(t + \sqrt{2(t^2 - 1)} \right) + (\theta - (t-1)) \left(t - 1 - \sqrt{2(t^2 - 1)} \right) + \theta(t-1)$$
$$p_B^j \le p_A^j \left(t + \sqrt{2(t^2 - 1)} \right) + (\theta - (t-1)) \left(t - 1 - \sqrt{2(t^2 - 1)} \right) + \theta(t-1)$$

SingleSingle responses are best responses if and only if

$$p_A^j > \sqrt{2}\theta - (t-1) \tag{7}$$
$$p_B^j > \sqrt{2}\theta - (t-1) \tag{8}$$

MultiSingle responses are best responses if and only if

$$p_A^j \le \sqrt{2}\theta - (t-1)$$
$$p_B^j > p_A^j \left(t + \sqrt{2(t^2 - 1)} \right) + (\theta - (t-1)) \left(t - 1 - \sqrt{2(t^2 - 1)} \right) + \theta(t-1)$$
$$p_A^j \ge p_B^j$$

SingleMulti responses are best responses if and only if

$$p_A^j > p_B^j \left(t + \sqrt{2(t^2 - 1)} \right) + (\theta - (t-1)) \left(t - 1 - \sqrt{2(t^2 - 1)} \right) + \theta(t-1)$$
$$p_B^j \le \sqrt{2}\theta - (t-1)$$
$$p_A^j \ge p_B^j$$

The above price requirements are illustrated in the figure below
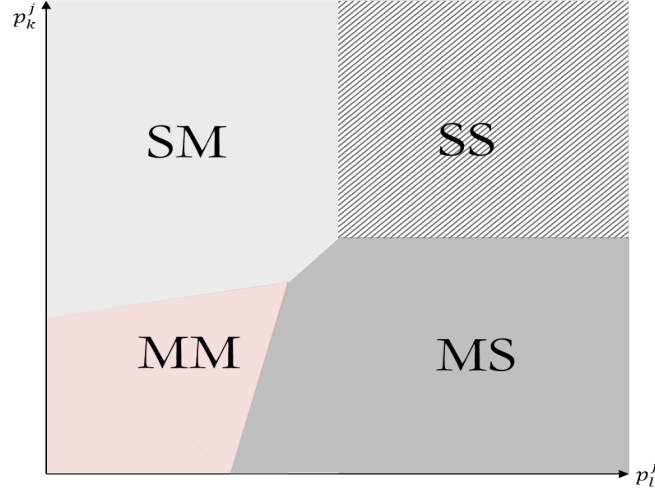
Figure 4: Types of best responses

3 Results

For an equilibrium to exist, the reaction functions should cross in the "right regions". That is, the requirements listed in the previous section should be satisfied. If we at first disregard these restrictions we get the following "four equilibrium candidates"

$$
(p_A^*, p_B^*) = \begin{cases}
\left(\frac{(1+\theta)(t-1)}{2t-1}, \frac{(1+\theta)(t-1)}{2t-1} \right) & \text{if MultiMulti} \\
(t-1, t-1) & \text{if SingleSingle} \\
\left(\frac{\theta}{2}, \frac{2(t^2-1)-\theta}{2t} \right) & \text{if MultiSingle} \\
\left(\frac{2(t^2-1)-\theta}{2t}, \frac{\theta}{2} \right) & \text{if SingleMulti}
\end{cases}
$$

For the SingleSingle outcome to be an equilibrium, (7) and (8) must be satisfied. Inserting $(p_S^*, p_B^*) = (t-1, t-1)$ in (7) and (8) we get that a SingleSingle equilibrium exists if and only if

$$
\theta < \sqrt{2}(t-1) \equiv \overline{\theta}(t) \tag{9}
$$

The love-of-variety parameter θ cannot be too large compared to transportation cost if consumers are to singlehome in equilibrium.

Likewise for the three other equilibrium candidates we get restrictions on the size of θ versus t. A MultiMulti equilibrium exists if and only if

$$\theta > \frac{2t(t-1)\left(t-1+\sqrt{2(t^2-1)}\right)}{(t-1)(3t-1)+t\sqrt{2(t^2-1)}} \equiv \underline{\theta}(t) \tag{10}$$

For a MultiMulti equilibrium to exist we need - as opposed to SingleSingle - θ to be sufficiently large relative to t. The right hand side of 10 is increasing in t such that, the larger is t the larger θ has to be for a MultiMulti equilibrium to exist. For a large t, consumers will be less inclined to travel all the way to both platforms unless the love-of-variety is large.

For the asymmetric cases the requirement is

$$\theta \in \left[\underline{\underline{\theta}}(t) \equiv \frac{1}{\sqrt{2}-1/2}(t-1), \frac{(t-1)\left(4t^2+2\sqrt{2(t^2-1)}(2t+1)\right)}{2t(2t-1)+\sqrt{2(t^2-1)}(2t+1)} \equiv \overline{\overline{\theta}}(t) \right]$$

If we are to have singlehoming on one side but multihoming on the other, θ can be neither too large nor too small compared to t.

The figure below illustrates the requirements for existence of the equilibria. The MultiMulti equilibria exist for all parameter combinations in the area above the $\underline{\theta}(t)$ line. Likewise, the SingleSingle equilibria exist for all parameter combinations in the area below the $\overline{\theta}(t)$ line and asymmetric equilibria exist between the two $\underline{\theta}(t)$ and $\overline{\overline{\theta}}(t)$ lines.
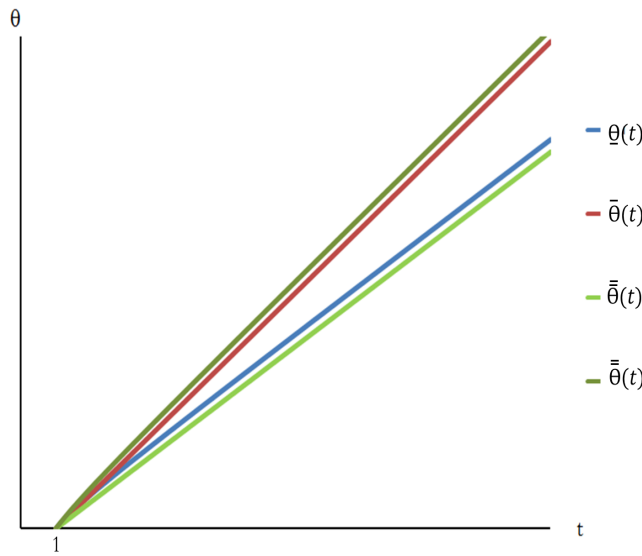
Figure 5: Existence of equilibria

From figure 5 we see that there are values of the parameters t and θ for which all kinds of equilibria exist. This happens in the area between the $\underline{\theta}(t)$ and $\overline{\theta}(t)$ lines which is, generally speaking, when both t and θ take on intermediate values meaning that θ is neither "too" high nor "too" low relative to t. Furthermore, we see that for all parameter values for which both MultiMulti and SingleSingle equilibria exist, asymmetric equilibria also exist.

For values of t close to 1 the $\underline{\theta}$ and $\overline{\theta}$ lines cross, as shown in the figure below, which is identical to Figure 5, however with a zoom around $t = 1$.

Figure 6: Existence of equilibria, t close to 1

This means that there is an area of parameter combinations for which only the asymmetric equilibria exist. When t is very small, singlehoming on both sides of the market cannot be sustained since a low value of t means that multihoming is relatively cheap, and in addition the incentive to multihome is larger when the other side is sing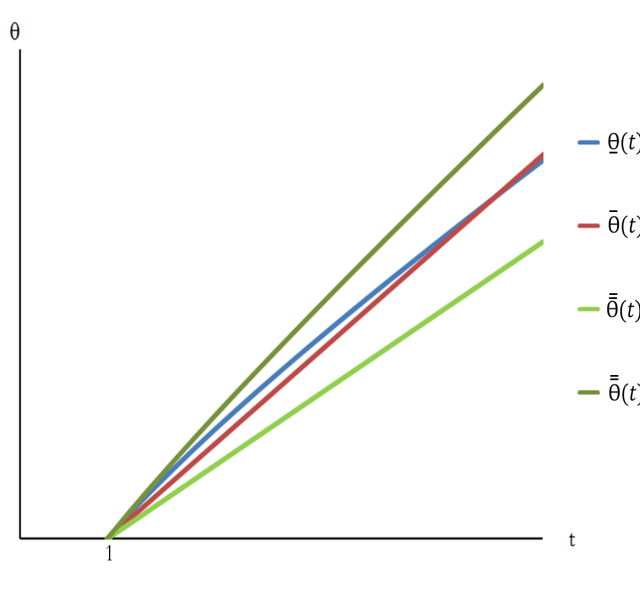lehoming. However, in this area θ is also not large enough to support multihoming on both sides of the market. Consequently, even in our symmetric set-up, there are parameter values for which only asymmetric equilibria exist.

The results above are summarized in the proposition below.

Proposition 1. When $\theta > \bar{\bar{\theta}}(t)$ only MultiMulti equilibria exist. For $\theta \in \left[\bar{\theta}(t), \bar{\bar{\theta}}(t)\right]$ both MultiMulti and asymmetric equilibria exist. For $\theta \in \left[\underline{\theta}(t), \underline{\theta}\right]$ SingleSingle and asymmetric equilibria exist. For $\theta < \underline{\theta}(t)$ only SingleSingle equilibra exists. For $\theta \in \left[\underline{\theta}, \bar{\theta}\right]$) all four kinds of equilibria exist.

In a SingleSingle equilibrium the price is independent of θ and is increasing in the degree of differentiation like is normally the case for one-sided Hotelling models. When consumers are multihoming on both sides they pay a price of $\frac{(1+\theta)(t-1)}{2t-1}$. In this case, both the taste for variety and the degree of differentiation matter. The more consumers love variety, the higher a price they

pay. Likewise, as with the case of the singlehoming equilibrium, the higher the degree of differentiation, the higher a price do the platforms charge in equilibrium. For the asymmetric case, the price on the multihoming side depends only on the love-of-variety parameter θ and not on the degree of differentiation t. On the singlehoming side of the asymmetric equilibrium the price is increasing in the degree of differentiation, however it is *decreasing* in the love-of-variety parameter θ. Thus, in the asymmetric case, a higher value of θ tends to increase the multihoming price relative to the singlehoming price.

For the range of parameters where all equilibria are possible we can directly compare prices in the case of singlehoming to the case of multihoming. The relationship between prices on side k is listed below where $p_k(multi|single)$ refers to the price to side k when there is multihoming here and side l single-homes.

$$p_k(single|multi) > p_k(single|single) > p_k(multi|multi) > p_k(multi|single)$$

The price is always higher if the group is singlehoming as opposed to when they are multihoming. Interestingly, the price a singlehoming side is paying when the other side also singlehomes is lower than the price they pay if the other side is multihoming. Likewise, the price a multihoming side is paying when the other side multihomes is higher than the price they are paying when the other side singlehomes. A singlehoming consumer only meets those consumers on the other side of the market who are active on the same platform as himself. This means that the network a singlehoming consumer meets is larger when consumers on the other side are multihoming as opposed to when they are singlehoming. The willingness to pay for singlehoming consumers is in this sense higher when consumers on the other side are multihoming and as a result the platforms can charge singlehoming consumers a higher price.

In the model of platform competition in Armstrong (2006), in which no multihoming is allowed, the equilibrium price on side k is $p_k = t_k - \alpha_l + f_k$. Here, f_k is the fixed cost of serving a consumer on side k. In our model, this is assumed to be zero. The parameter α_l in the Amstrong model is the network externality on side l. In our model, this is set to 1. Hence, with our assumptions the equilibrium price in Amstrong would be $p_k = t - 1$ which is

exactly the equilibrium price we find in the SingleSingle case.

Armstrong (2006) calls the asymmetric case "competitive bottlenecks" but assumes that one group always singlehomes for exogenous reasons while the other group has the option of multihoming. Amstrong does not offer any closed form solutions but notes that it is a feature of these markets that "the singlehoming side is treated well and the multihoming side's interests are ignored in equilibrium," and that the multihoming side faces "excessive prices".

Armstrong and Wright (2007) analyze a model where the platforms are viewed as differentiated by agents on one side (buyers) but as non-differentiated by agents on the other side (sellers). Furthermore, buyers always singlehome. The authors show that sellers multihome and that the platform competes aggressively to sign up buyers charging them, perhaps, less than cost. The platforms then make their profits on sellers who want to reach these buyers. Again, the singlehoming side is treated favorably.

In our set-up where multihoming is allowed on both sides and is endogenously determined, the consumers' responses to prices differ from Armstrong (2006) and Armstrong and Wright (2007) in important aspects. In particular, for multihoming consumers, the choice of whether to join one platform is not independent of the choice of whether to join the other platform. Furthermore, it is not possible to price arbitrarily low and still have consumers singlehoming. When prices on one side get low enough, consumers here will start to multihome. This will tend to make multihoming less attractive on the other side and demand may decrease here as a result. This effect is not possible when consumers one side are "forced" to singlehome.

Allowing for endogenous multihoming changes the equilibrium price structure compared to models with exogenous multihoming decision. There are two things to note related to the above observations. First, if we focus on the singlehoming side, this group is actually facing a higher price when the other side is multihoming compared to when the other side is singlehoming. This is in direct contrast with Armstrong (2006) and Armstrong and Wright (2007) where the singlehoming side is "treated well" when the other side multihomes. Furthermore, since we already noted that the multihoming price is lower than the singlehoming price, it is not the case that the multihoming side is facing

"excessive prices" compared to the singlehoming side. When the multihoming decision is endogenous, arising from prices and network externalities, the results of the model differ from a model where consumers do not get to choose whether they want to singlehome or multihome but where this is determined exogenously.

It is worth noting that since not all types of equilibria exist for all parameter combinations it is not always possible to directly compare prices. For a very large value of θ for example, only the equilibrium where both sides multihome exists and prices will be relatively high due to the high θ and not directly comparable to the singlehoming prices.

For the range of parameters where we have multiple equilibria, we cannot predict the actual outcome. However, if one outcome consistently yields higher profit for both platforms this could be a good prediction of the actual outcome. In the equilibrium with singlehoming on both sides, profit is increasing in the degree of differentiation. This is in contrast to the case of multihoming on both sides where profit is decreasing in the degree of differentiation. Notice that this observation is in line with the conclusions in Anderson *et al.* (2012) for their one-sided model. Comparing profit under SingleSingle and MultiMulti we see that profit is higher under MultiMulti as long as

$$\theta > \frac{\sqrt{8t^4 - 6t^2 + 2t}}{2t} - 1 > \overline{\theta} \tag{11}$$

However, this is not possible for the values of θ for which both the SingleSingle and the MultiMulti outcomes are possible. Consequently, whenever both equilibria exist, the platforms prefer the SingleSingle outcome.

Likewise, we can compare the SingleSingle equilibrium to the asymmetric outcome MultiSingle or SingleMulti. We see that SingleSingle yields the highest profit as long as

$$\theta < \sqrt{2}(t - 1) = \underline{\theta}$$

which is also the requirement for the SingleSingle equilibrium to exist. We conclude that, whenever multiple equilibria exist, the platforms always prefer the SingelSingle outcome.

4 Joint Ownership

To better understand the effects of platform competition, this section will analyze a situation with less competition. These results will be useful for regulation, for example in the case of a merger. The natural benchmark against which to compare our prices is joint ownership of the platforms, which could for example be the result of a merger between the two platforms. The locations of the platforms are then unchanged but the profit of both platforms is now taken into account when solving for optimal prices.

First we solve for optimal prices in a world where both sides singlehome. In a standard one-sided Hotelling model the case of joint ownership is simple. The firm will always want to divide the market equally between its two divisions. Because of the transportation cost it cannot be optimal to let consumers who are located closer to firm 1 travel to firm 0. These consumers would have to be compensated by a lower price in order to make them travel, and profits would be higher if they simply purchased at their preferred firm. For a two-sided market the intuition is not as straightforward. Due to the network externality it might be optimal for the firm to offer a low price at say platform 0 attracting consumers located closer to platform 1. This will be costly on this side but will enable the firm to charge a higher price on platform 0 on the other side of the market due to the network externality. For this reason, we cannot draw immediate conclusions but have to solve the problem of profit maximization.

In addition to checking whether the firm wants to divide the market equally between the two platforms, we also have to check whether the firm prefers to serve the whole market or only consumers located close to the platforms who have a higher willingness to pay. We will first assume that the market is always covered and later check that it is indeed optimal for the firm to serve the whole market.

Imagine that the firm wants to divide the market at x^k such that platform 0 serves all consumers to the left of x^k and platform 1 serves all consumers to the right of x^k. Platform 0 will choose the highest possible price making the consumer located at x_k exactly indifferent between buying or not buying.

This price is given by

$$p_k^0 = \beta_k + x_l - tx_k \tag{12}$$

and the price for platform 1 is

$$p_k^1 = \beta_k + (1 - x_l) - t(1 - x_l) \tag{13}$$

For each possible division of the line x_k and $1 - x_k$ we get associated prices for the platforms as the ones above. As a result, we can always see the firm's optimization problem as if the firm chooses location of the marginal consumers x_k and then adjust the price according to (12) and (13). The profit of the firm is then given by

$$\pi = x_A \left(\beta_A + x_B - tx_A \right) + (1 - x_A) \left(\beta_A + (1 - x_B) - t(1 - x_A) \right) +$$
$$x_B \left(\beta_B + x_A - tx_B \right) + (1 - x_B) \left(\beta_B + (1 - x_A) - t(1 - x_B) \right)$$

with first order conditions

$$-4tx_A + 4x_B + 2(t - 1) = 0 \tag{14}$$
$$-4tx_B + 4x_A + 2(t - 1) = 0 \tag{15}$$

Second order conditions are satisfied when $t > 1$. From the first order conditions in (14) and (15) we get optimal locations $x_A^* = x_B^* = \frac{1}{2}$ with associated prices

$$p_A^{JO} = \beta_A - \tfrac{1}{2} \left(t - 1 \right)$$
$$p_B^{JO} = \beta_B - \tfrac{1}{2} \left(t - 1 \right)$$

The optimal prices are increasing in β and decreasing in t which is in contrast to the optimal prices under competition. When platforms are competing, a high degree of differentiation means that consumers located close to a platform are very loyal and it allows the platforms to act as local monopolists and charge higher prices. Under joint ownership on the other hand, a high degree of differentiation means that the consumers located in the middle must be offered a relatively low price in order for them to buy. For this reason the

optimal price is decreasing in t.

Lastly, we make sure that the platform finds it optimal to serve the entire market, which was implicitly assumed above. Focus on platform 0. The profit made on this platform is given by

$$\pi = x_A \left(\beta_A + x_B - t x_A \right) + x_B \left(\beta_B + x_A - t x_B \right)$$

Maximizing profit with respect to locations yields optimal locations $x_A = \frac{\beta_B + t \beta_A}{2(t^2-1)}$ and $x_B = \frac{\beta_A + t \beta_B}{2(t^2-1)}$. This of course needs to be restricted to the interval $\left[0, \frac{1}{2}\right]$. Hence, only if $\frac{\beta_k + t \beta_l}{2(t^2-1)} < \frac{1}{2}$ will it be optimal for the firm to not serve the whole market. Consequently, the condition for a covered market on side k is given by

$$\beta_k + t \beta_l \geq \tfrac{1}{2}(t^2 - 1)$$

This inequality is always satisfied under (2). Intuitively, if the β's are very small compared to transportation costs, consumers located around the middle will demand very low prices and may not be optimal to serve. As long as (2) holds however, the firm will find it optimal to serve all consumers.

For the case of multihoming on both sides of the market, the firm chooses two locations on each side of the market, x_k^0 and x_k^1 such that demand for platform 0 on side k is given by x_k^0 and demand for platform 1 on side k is given by $1 - x_k^1$ and furthermore $x_k^0 \in \left[\frac{1}{2}, 1\right]$ and $x_k^1 \in \left[0, \frac{1}{2}\right]$. Given location x_k^0 the maximum price platform 0 can charge is the one that makes the consumer located at x_k^0 indifferent between multihoming and buying at platform 1 only:

$$\beta_k + \theta + 1 - t - p_k^0 - p_k^1 = \beta_k + \left(1 - x_l^1\right) - t(1 - x_k^0) - p_k^1$$
$$p_k^0 = \theta + x_l^1 - t x_k^0$$

Likewise, given a location x_k^1, the maximum price platform 1 can charge is the one making the consumer located at x_k^1 indifferent between multihoming and buying from 0 only:

$$\beta_k + \theta + 1 - t - p_k^0 - p_k^1 = \beta_k + x_l^0 - t x_k^1 - p_k^0$$
$$p_k^1 = \theta + \left(1 - x_l^0\right) - t(1 - x_k^1)$$

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The profit function is then given by

$$\pi = x_A^0\left(\theta + x_B^1 - tx_A^0\right) + \left(1 - x_A^1\right)\left(\theta + (1 - x_B^0) - t(1 - x_A^1)\right)$$
$$+ x_B^0\left(\theta + x_A^1 - tx_B^0\right) + \left(1 - x_B^1\right)\left(\theta + (1 - x_A^0) - t(1 - x_B^1)\right)$$

Differentiating with respect to the four locations and solving the system of equations yields optimal locations

$$x_A^0 = x_B^0 = \frac{\theta + 1}{2(t + 1)}$$
$$x_A^1 = x_B^1 = \frac{2t + 1 - \theta}{2(t + 1)} = 1 - \frac{\theta + 1}{2(t + 1)}$$

These locations result in optimal prices of

$$p_A^{JO} = p_B^{JO} = \frac{\theta + 1}{2}.$$

Lastly, for the asymmetric case of multihoming on side k and singlehoming on side l, by the same argumentation as above, profit is given by

$$\pi = x_k^0\left(\theta + x_l - tx_k^0\right) + \left(1 - x_k^1\right)\left(\theta + (1 - x_l) - t(1 - x_k^1)\right)$$
$$+ x^l\left(\beta_l + x_k^0 - tx_l\right) + (1 - x_l)\left(\beta_l + (1 - x_k^1) - t(1 - x_l)\right)$$

Solving first order conditions yields optimal locations

$$x_B = \frac{1}{2}, \quad x_A^0 = \frac{\theta + 1}{2t}, \quad x_A^1 = \frac{2t - 1 - \theta}{2t}$$

and the associated prices are

$$p_k^{JO} = \frac{\theta}{2}$$
$$p_l^{JO} = \frac{2t\beta_l - (t^2 - 1) + \theta}{2t}$$

The firm will naturally choose the outcome that yields the highest profit. This should be expected to depend on parameters. For example, when θ is large the willingness to pay for multihoming is larger and this would tend to make the multihoming outcome more profitable. The profit associated with

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the four different regimes are listed below.

$$\pi^{MM} = \frac{(\theta + 1)^2}{t + 1}$$

$$\pi^{SS} = \beta_A + \beta_B - (t - 1)$$

$$\pi^{MS} = \frac{2t\beta_B + 2\theta + \theta^2 - (t^2 - 1)}{2t}$$

$$\pi^{SM} = \frac{2t\beta_A + 2\theta + \theta^2 - (t^2 - 1)}{2t}$$

All profit functions depend on t. While profit under MultiMulti is increasing in θ, profit under SingleSingle does not depend on θ but is instead increasing in β_k and β_l. In the asymmetric case, profit is increasing in θ and in the relevant β.

Profit under MultiMulti is larger than under Singlesingle when

$$\theta > \sqrt{(t + 1)(\beta_A + \beta_B) - (t^2 - 1)} - 1 \equiv \underline{\underline{\theta}} \tag{16}$$

While θ can never be larger than β, this inequality will be satisfied when θ is sufficiently large compared to the β's. Furthermore, the larger is t the more likely is the firm to choose the SingleSingle outcome. When t is large it is expensive for the firm to induce consumers to multihome. Travelling is expensive and as a result, a multihoming price must be low. This will not be profitable when t is too large compared to θ.

MultiMulti is preferred to MultiSingle whenever

$$\theta > \frac{\sqrt{t\left(2\beta_B(t^2 - 1) - t^2(t - 1)\right)}}{(t - 1)} - 1 \equiv \breve{\theta}_{MS}$$

and MultiMulti is preferred to SingleMulti whenever

$$\theta > \frac{\sqrt{t\left(2\beta_A(t^2 - 1) - t^2(t - 1)\right)}}{(t - 1)} - 1 \equiv \breve{\theta}_{SM}$$

Intuitively, these are satisfied when β_B or β_A are not too large compared to θ respectively.

Profit under SingleSingle is higher than under MultiSingle when

$$\theta < \sqrt{2t\beta_A - (t^2 - 1)} - 1 \equiv \hat{\theta}_{MS}$$

Notice that only β_A and not β_B is relevant here. When β_A is large relative to θ it is more profitable to let side A singlehome and extract a high price here instead of offering them the low multihoming price. As a result, the firm then prefers SingleSingle for high values of β_A. Similarly, SingleSingle is preferred to SingleMulti when

$$\theta < \sqrt{2t\beta_B - (t^2 - 1)} - 1 \equiv \hat{\theta}_{SM}$$

Lastly, MultiSingle is preferred to SingleMulti when $\beta_B > \beta_A$. When β_B is relatively large, it is more profitable to let side B singlehome and charge them the high singlehoming price, and as a result MultiSingle is preferred to Single-Multi.

The results of joint ownership are summarized below

Proposition 2. Under joint ownership, the firm prices according to MultiMulti when $\theta > \max\{\underline{\theta}, \breve{\theta}_{MS}, \breve{\theta}_{SM}\}$ and according to SingleSingle when $\theta < min\{\underline{\theta}, \hat{\theta}_{MS}, \hat{\theta}_{SM}\}$. An asymmetric outcome with multihoming on side k and singlehoming on side l is preferred when $\theta \in \left[\hat{\theta}_{MS/SM}, \breve{\theta}_{MS/SM}\right]$ and $\beta_l > \beta_k$.

The firm will choose optimal prices leading to multihoming on both sides of the market whenever θ is high relative to β_A and β_B and whenever t is not too large. On the other hand, large values of β_A and β_B will lead to a a singlehoming outcome. The asymmetric outcome with multihoming on side k and singlehoming on side l is predicted when the β's are asymmetric such that $\beta_k < \beta_l$. In this case, the platform can profitably lower prices on side k such that consumers here start to multihome. This enables the platforms to increase their prices on side l, which is the high profit side of the market.

4.1 Comparing prices

Having solved for equilibrium prices under platform competition and joint ownership we move on to compare the resulting prices in order to determine

whether prices are always lower under competition.

For the case of multihoming on both sides of the market, the competitive price is given by $\frac{(\theta+1)(t-1)}{2t-1}$ while the price under joint ownership is given by $\frac{\theta+1}{2}$. Hence, the competitive price is always lower. Under platform competition, the platforms do not take into account the negative effect a lower price will have on the competitor's demand. As a result, the prices will be "too low" in the eyes of the platforms. When there is joint ownership this externality is taken into account and as a direct consequence, prices are higher.

In a market where both sides singlehome, the competitive price is $t-1$ while the price under joint ownership is given by $\beta_k - \frac{1}{2}(t-1)$. The competitive price is lowest as long as $\beta_k > \frac{3}{2}(t-1)$ which always holds under (2). Consequently, we can conclude that for the appropriate range of parameters, the price is lower under competition.

Lastly, for the asymmetric case the price charged on the multihoming side is identical in the two cases. The price on the singlehoming side is lower under competition as long as $t\beta_l + \theta > \frac{3}{2}(t^2 - 1)$ which always holds under (1).

We conclude that, as expected, prices are always (weakly) lower under competition. With joint ownership, the platforms can charge marginal consumers their entire surplus which is also the optimal price strategy. This is not possible under competition since the competing platform will have an incentive to undercut its rival and set lower prices resulting in higher demand. Consequently, a merger of two differentiated platforms should be expected to raise prices other things equal.

5 Welfare

Due to the positive externality between the two groups, the competitive equilibrium will feature too little participation in the eyes of a social planner. Indeed, if we focus on a multihoming equilibrium there are two indifferent consumers. Each of them are indifferent between singlehoming on 0 or 1 respectively or multihoming. Switching from singlehoming to multihoming will have zero effect on their utility since they are by definition indifferent. However, switching to multihoming will have a positive effect on consumers

on the other side of the market due to the network externality. Hence, a social planner would want the indifferent consumer (and consumers located arbitrarily close to him) to multihome instead of singlehome. As a result, the competitive equilibrium must feature too little multihoming.

If we want to maximize overall welfare we need not look at prices as these are merely a transfer between agents. Since the market is covered by assumption the only welfare aspect we need to focus on is whether "enough" agents participate. Imagine that two indifferent consumers are located at $x_k^0 \in [0, \frac{1}{2}]$ and $x_k^1 \in [\frac{1}{2}, 1]$ respectively. Then, demand for platform 0 on side k is given by x_k^1 while demand for platform 1 on side k is given by $1 - x_k^0$. The marginal benefits from having one extra consumer on side k multihoming as opposed to singlehoming on platform 1 are threefold. First, there is the incremental fixed utility θ. Second, the network benefits from meeting additional agents on the other side of the market, which is given by x_l^0. Third, consumers on platform 0 on the other side of the market who did not previously meet the consumer who is now multihoming will all gain from meeting one extra consumer. This amounts to x_l^0. The marginal costs of the agent who is now multihoming is given by his incremental transportation cost, which is equal to $t x_k^1$. The social planner sets marginal benefits equal to marginal costs which leaves us with

$$\theta + 2x_l^0 = t x_k^1$$

Likewise, we can find the benefits and costs of having one extra consumer on side l multihoming instead of singlehoming on platform 1. Equating these yields

$$\theta + 2x_k^1 = t x_l^0$$

Solving the two equations in two unknown results in the following optimal locations

$$x_k^1 = \frac{\theta + 2}{t + 2}$$
$$x_l^0 = 1 - x_l^1 = \frac{t - \theta}{t + 2}$$

Equating benefits and costs from having one extra consumer multihoming

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instead of singlehoming on platform 0 results in symmetric locations.

$$x_1^1 = \frac{\theta + 2}{t + 2}$$

$$x_k^0 = 1 - x_k^1 = \frac{t - \theta}{t + 2}$$

Notice that this means that the social planner prefers multihoming over singlehoming whenever

$$\theta > \frac{1}{2}t - 1$$

When θ is sufficiently large compared to t, the social planner prefers that everybody multihomes since the benefits will always outweigh the transportation costs. For lower values of θ relative to t we get an interior solution where the social planner prefers consumers with intermediate locations to multihome while consumers located at the two extremes of the Hotelling line should be singlehoming. When $\theta < \frac{1}{2}t - 1$, the socially optimal outcome is for all agents on both sides to singlehome.

As already hinted in the beginning of this section, the market fails to deliver enough multihoming. If we compare the socially optimal level of demand to the competitive multihoming equilibrium $n_k^0 = n_k^1 = \frac{t(\theta+1)}{(t+1)(2t-1)}$ we can conclude that demand is lower in the competitive multihoming equilibrium.

When the market solution involves multihoming, it does not involve enough multihoming. However, the market solution may involve singlehoming when the socially optimal outcome would be multihoming. The social planner prefers multihoming whenever $\theta > \frac{1}{2} - 1$. However, the MultiMulti equilibrium exists only when (10) is satisfied, which is a stronger condition. Furthermore, whenever multiple equilibria exist, the platforms prefer singlehoming equilibria and we might not get a multihoming outcome, even if this exists. Indeed, only when $\theta > \bar{\bar{\theta}}(t)$ can we be sure to get multihoming on both sides of the market and this condition is even stronger than (10).

The case of joint ownership is more difficult to compare directly. The parameter β does not play a role for the social planner who focuses on marginal benefits and costs. However, β determines which outcome is most profitable for the jointly owned platforms. The jointly owned platforms deliver multihoming

on both sides of the market when (16) is satisfied, which is whenever $\theta > \sqrt{(t+1)(\beta_A + \beta_B) - (t^2 - 1)} - 1$. Remembering that $\beta_k > \theta \geq t$ it is true that

$$\sqrt{(t+1)(\beta_A + \beta_B) - (t^2 - 1)} - 1 > \frac{1}{2}t - 1$$

and we can conclude that the jointly owned platforms also fail to deliver enough multihoming.

6   Conclusion

This paper set up a model of differentiated platform competition where consumers on both sides of the market had the option of multihoming. The model built on Armstrong (2006) and Kim and Serfes (2006). Different to Armstrong, the decision of whether to buy at one or both platforms was determined endogenously by the agent's location, prices and the network externality. By introducing a love for variety in agents' preferences, the model allowed for the possibility of multihoming on both sides of the market, an outcome not possible in Amstrong.

For strong product differentiation and relatively low love-of-variety, the model predicts an outcome with singlehoming on both sides. The prices in this case are comparable to the equilibrium prices found in Armstrong (2006). However, for a weaker product differentiation and/or increased love-of-variety an equilibrium exists where some consumers multihome on both sides of the market. Multihoming will be preferred by agents located around the middle of the Hotelling line, while agents located at either extreme are more loyal to their preferred brand.

The more agents prefer variety, the higher is the price paid in a multihoming equilibrium. However, for the range of parameters where all four equilibria exist, the multihoming price is always lower than the singlehoming price. This contradicts the results in Armstrong (2006) and Armstrong and Wright (2007) where the multihoming side is found to be facing "excessive prices" in order for the platform to recoup profits foregone on the singlehoming side. Furthermore, in contrast with the results in Amstrong and Amstrong and Wright we conclude that a group, which is is singlehoming, is worse off when the other side

is multihoming compared to when they are singlehoming. In an asymmetric equilibrium, the singlehoming side pays a higher price than in the symmetric equilibrium with singlehoming on both sides.

While the competitive equilibria deliver lower prices than the case of joint ownership, demand is still too low compared to what a social planner would want. Due to the positive network externality there is not enough multihoming in the competitive equilibrium.

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A   Appendix

A.1   Best responses

The four different price choices are valid for the combinations of the competitor's prices listed below.

p_A^{iMM}, p_B^{iMM}, p_A^j and p_B^j fulfill (3) if and only if

$$p_A^j \le \frac{(\theta + 1)(3t^2 - 1) - 2t\theta - 2t^3}{2t^2 - 1} + p_B^j \frac{t}{2t^2 - 1} \tag{17}$$

$$p_B^j \le \frac{(\theta + 1)(3t^2 - 1) - 2t\theta - 2t^3}{2t^2 - 1} + p_A^j \frac{t}{2t^2 - 1} \tag{18}$$

p_A^{iSS}, p_B^{iSS}, p_A^j and p_B^j fulfill the negation of (3) if and only if

$$p_A^j > \frac{4}{3}\theta - (t - 1)$$

$$p_B^j > \frac{4}{3}\theta - (t - 1)$$

p_A^{iMS}, p_B^{iMS}, p_A^j and p_B^j fulfill (3) if and only if

$$p_A^j \le \frac{3}{2}\theta - (t - 1)$$

$$p_B^j > p_A^j \frac{1}{3t} + \frac{4t(\theta + 1) - 3t^2 - 2\theta - 1}{3t}$$

p_A^{iSM}, p_B^{iSM}, p_A^j and p_B^j fulfill (3) if and only if

$$p_A^j > p_B^j \frac{1}{3t} + \frac{4t(\theta + 1) - 3t^2 - 2\theta - 1}{3t}$$

$$p_B^j \le \frac{3}{2}\theta - (t - 1)$$

The price choices of SingleSingle, MultiSingle and SingleMulti are depicted below
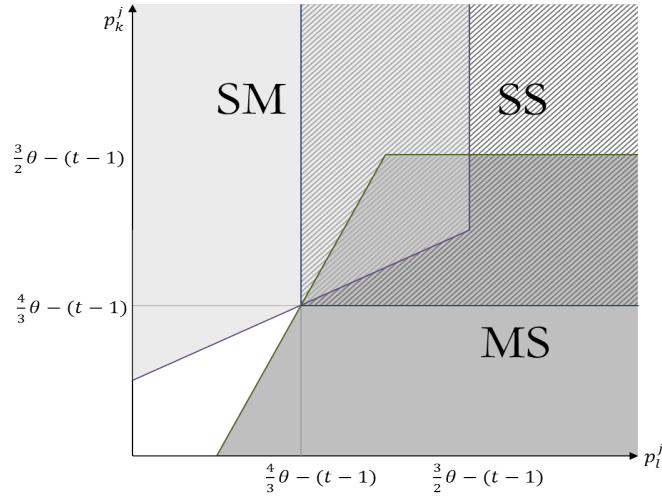
Figure 7: SingleSingle, MultiSingle and SingleMulti

For the case of MultiMulti we see that (17) and (18) cross in the first quadrant of the (p_A^j, p_B^j) space if $\theta > \frac{2t^2-t-1}{3t+1}$ and in the third quadrant otherwise. This means that if $\theta < \frac{2t^2-t-1}{3t+1}$ platform j needs to price below cost on both sides of the market for the MultiMulti best responses to be valid. Consequently, MultiMulti will never be an equilibrium if $\theta < \frac{2t^2-t-1}{3t+1}$. Intuitively, we need θ to be sufficiently large for MultiMulti to be a possibility. For $\frac{2t^3-3t^2+1}{3t^2-2t} > 0$, which will always hold under the restrictions laid upon t and θ for the MultiMulti equilibrium to exist, the case of MultiMulti is depicted below.
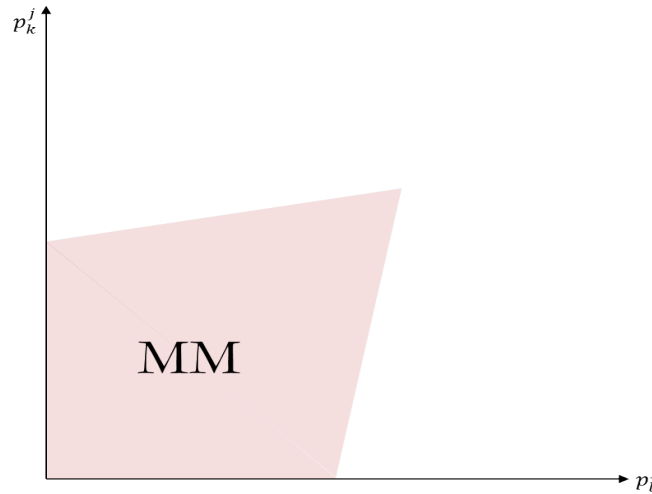

Figure 8: MultiMulti

Lastly, the MultiMulti requirement intersects the MultiSingle requirement for $p_k^j = -(t-1) - (t-2)\theta$. This is always negative when $t \geq 2$ and negative for $t \in]1, 2[$ as long as θ is not too large. Assuming that

$$-(t-1) - \theta(t-2) < 0$$

we get the following picture of price requirements
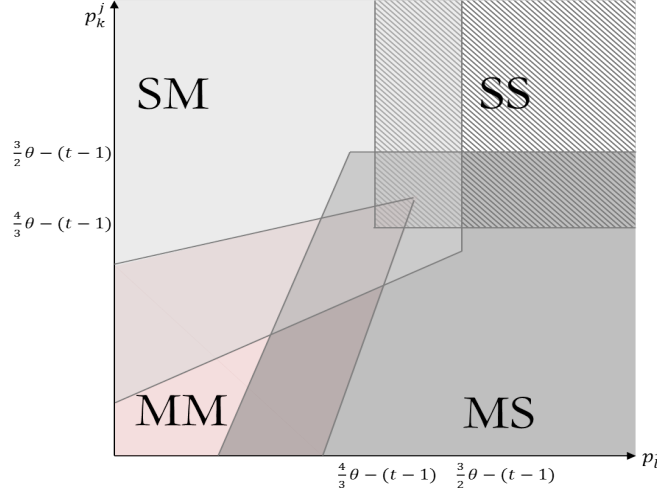

Figure 9: All price requirements

A.2 Comparing profits associated with price choices

In all the areas where price choices overlap, we need to compare the profits associated with the price choices.

First, compare SingleSingle to the choice of MultiSingle. When the platform prices according to SingleSingle the profit, as a function of the other platform's prices, is given by

$$\pi^{SS} = \frac{t\left(p_A^j\right)^2 + t\left(p_B^j\right)^2}{8(t^2-1)} + \frac{p_A^j}{4} + \frac{p_B^j}{4} + \frac{p_A^j p_B^j}{4(t^2-1)} + \frac{t-1}{4}$$

For the choice of MultiSingle profit is

$$\pi^{MS} = \frac{\left(p_A^j\right)^2 + t^2\left(p_B^j\right)^2}{8t(t^2-1)} + \frac{p_A^j}{4t} + \frac{p_B^j}{4} + \frac{p_A^j p_B^j}{4(t^2-1)} + \frac{\theta^2}{4t} + \frac{t^2-1}{8t}$$

Notice that both profit functions are increasing in platform j's prices which reflect that a business stealing effect is present and that the platforms' prices are strategic substitutes. Notice further that the profit functions are increasing

by the same amount in p_B^j while π^{SS} is increasing faster in p_A^j compared to π^{MS}.

Now compare the two profit functions in order to determine what choice yields the highest profit as a function of (p_A^j, p_B^j)

$$\pi^{SS} \geq \pi^{MS} \Rightarrow$$
$$p_A^j \geq \sqrt{2}\theta - (t-1)$$

Remember from figure 7 that the two price choices overlap when $p_k^j \in \left[\frac{4}{3}\theta - (t-1), \frac{3}{2}\theta - (t-1)\right]$. We now know that within this area, the choice of SingleSingle yields the highest profit only when $p_k^j > \sqrt{2}\theta - (t-1)$ as illustrated in the figure below
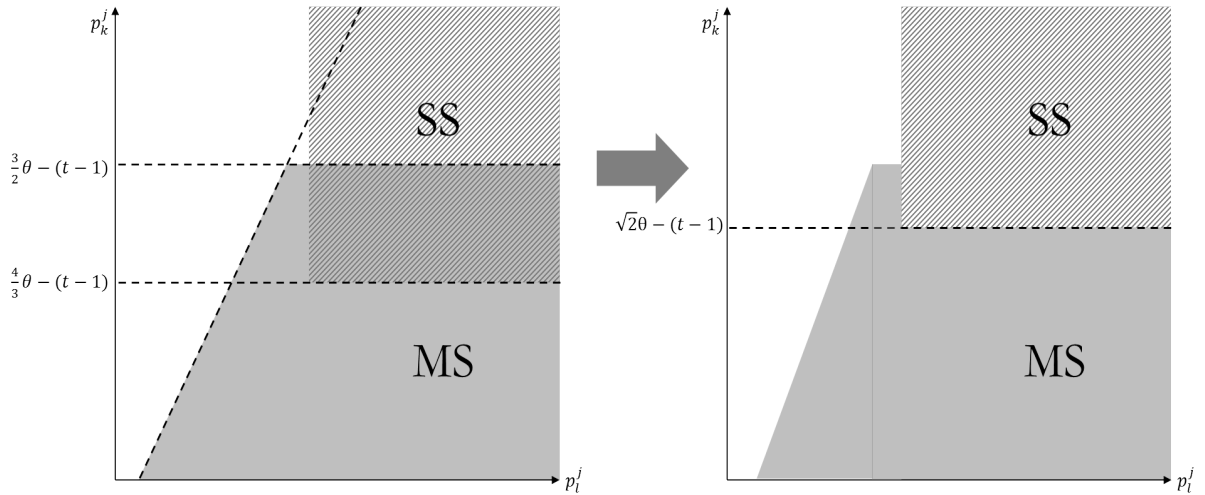
Figure 10: Price choices SS and MS

This comparison is carried out for all combinations of the four price choices and the resulting best responses are stated below.

MultiMulti respones are best responses if and only if

$$p_A^j \leq p_B^j \left(t + \sqrt{2(t^2 - 1)}\right) + (\theta - (t-1))\left(t - 1 - \sqrt{2(t^2 - 1)}\right) + \theta(t-1)$$
$$p_B^j \leq p_A^j \left(t + \sqrt{2(t^2 - 1)}\right) + (\theta - (t-1))\left(t - 1 - \sqrt{2(t^2 - 1)}\right) + \theta(t-1)$$

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SingleSingle responses are best responses if and only if

$$p_A^j > \sqrt{2}\theta - (t-1)$$
$$p_B^j > \sqrt{2}\theta - (t-1)$$

MultiSingle responses are best responses if and only if

$$p_A^j \le \sqrt{2}\theta - (t-1)$$
$$p_B^j > p_A^j \left(t + \sqrt{2(t^2-1)}\right) + (\theta - (t-1))\left(t - 1 - \sqrt{2(t^2-1)}\right) + \theta(t-1)$$
$$p_A^j \ge p_B^j$$

SingleMulti responses are best responses if and only if

$$p_A^j > p_B^j \left(t + \sqrt{2(t^2-1)}\right) + (\theta - (t-1))\left(t - 1 - \sqrt{2(t^2-1)}\right) + \theta(t-1)$$
$$p_B^j \le \sqrt{2}\theta - (t-1)$$
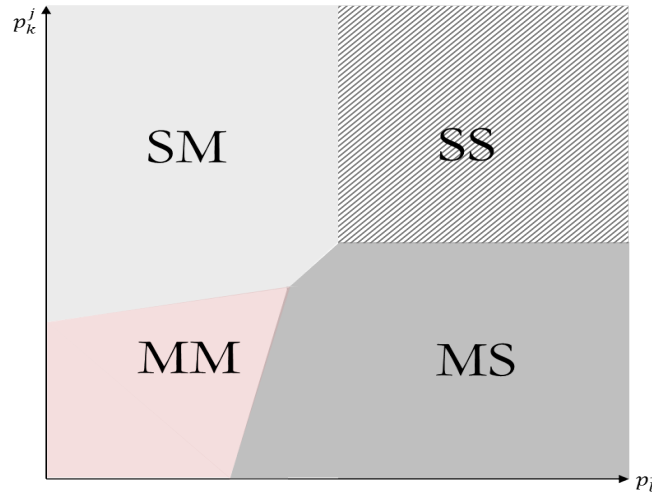$$p_A^j \ge p_B^j$$

This is illustrated in the figure below

Figure 11: Types of best responses

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Chapter 3

Personality and conflict in principal-agent relations based on subjective performance evaluations[*]

Mie la Cour Sonne[†]   Alexander Sebald[‡]

Abstract

We analyze the role of conflict in principal-agent environments with subjective performance evaluations, reciprocal agents and endogenous feelings of entitlements. By explicitly modeling conflict as the reciprocal reaction of agents who feel unkindly treated, we reveal intriguing welfare effects associated with the agents' personality and provide a rational for the widespread use of personality tests in recruitment and promotion processes. Finally, we extend our framework to allow principals to choose their evaluation procedure and show, for example, that even if it is costless to choose a high quality evaluation procedure, principals might not find it optimal to do so.

Keywords: Subjective performance evaluations; Reciprocity; Procedural Concerns.

JEL-Classifications: D01; D02; D82; D86; J41.

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1 Introduction

Evaluating performance and linking rewards such as bonuses and promotions to subjective performance appraisals is an integral and important part of many of today's work relations [see e.g. Bushman *et al.* (1996), Ittner *et al.* (1997), Ittner *et al.* (2003), Murphy and Oyer (2001), Gibbs *et al.* (2004)]. To capture performance in a purely objective way is very costly and often hard to accomplish, since a lot of valuable information about performance is captured by subjective impressions rather than objective measures. As a result, it is often preferred to leave (at least part of the) performance feedback to more holistic subjective appraisals.

However, principal-agent relations involving ex-post asymmetric information in the form of non-verifiable subjective performance evaluations are fragile and prone to conflict. If labor contracts specify rewards on the basis of subjective appraisals, principals have an incentive to claim that performance was poor according to their perception to establish low wages (i.e. ex-post hold-up [see e.g. Macleod (2000)]). In addition, agents might feel shortchanged and create conflict when they receive a performance appraisal and reward from their principal which is lower than what they feel entitled to on the basis of their own subjective performance assessment [see e.g. Sebald and Walzl (2012b)].

In this paper we theoretically analyze the impact and importance of conflict created by ex-post asymmetric information and hold-up in principal-agent environments based on non-verifiable subjective performance evaluations. We investigate factors that mitigate this conflict and describe implications for optimal recruitment policies and the principal's choice of evaluation procedure.

The existing literature analyzing the problem of ex-post asymmetric information and hold-up in principal-agent relations has already highlighted the need to break budget balance through 'money burning' or 'third-party payments' to establish mutual beneficial relations [see e.g. Levin (2003), MacLeod (2003) and Fuchs (2007)]. As shown in this literature: letting principals choose the optimal degree of money burning or third party payments mitigates the potential truthtelling problems inherent in these strategic environments.

Different to this, we characterize and analyze a principal-agent model with

non-verifiable subjective performance evaluations in which we do not model conflict as e.g. third-party payments optimally chosen to ensure truthtelling. Instead, we explicitly formalize conflict as the reciprocal reaction of agents that feel shortchanged and unkindly treated by their principal. In our setting a principal decides upon undertaking a project. The project requires effort of an agent. However, as the project is a complex good or service and its success is non-verifiable, incentive contracts contingent on the successful completion of the project are not feasible. Contracts can only be based on non-verifiable subjective performance evaluations.

We assume that whenever the principal and agent voluntarily agree on a contract before the agent invests effort into the project, the contract shapes the agent's feeling of entitlement which defines the wage she feels entitled to ex-post (see Hart and Moore (2008)). This feeling of entitlement constitutes a benchmark or reference point against which she judges the kindness of the principal's action, i.e. his performance feedback. Whenever the agent receives a performance feedback and, hence, an associated payment that lies below her feeling of entitlement she reacts reciprocally by creating costly conflict.[1]

By now it is a well established theoretical and empirical finding that reciprocity in general is an important motivational driving force mitigating moral hazard in principal-agent relations [see e.g. Fehr *et al.* (1993), Fehr *et al.* (1997), Charness (2004), Kube *et al.* (2011)].[2] However, the existing literature has abstracted from employment relations containing ex-post asymmetric information and hold-up. Analyzing the role of reciprocity in such strategic environments thus requires us to go beyond existing conceptualizations [e.g. Rabin (1993), Dufwenberg and Kirchsteiger (2004) and Hart and Moore (2008)].

Two important dimensions in which our model differs from the existing literature on reciprocity are:

First, the agent's feeling of entitlement in our setting is not exogenous, but

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[1] Note that this type of negative reciprocity is different from the payoff-independent form of reciprocity analyzed in Sebald and Walzl (2012a) in which it is assumed that the agent's feeling of entitlement is solely shaped by his own performance evaluation independent of the payments specified in the contract.

[2] In particular there exists ample evidence showing that negative reciprocity is the empirically more relevant dimension or reciprocal behavior Fehr *et al.* (2009b).

is endogenously shaped by the agent's own performance assessment. If the agent believes she did a good job, she feels entitled to a higher wage, than if she believes she did a bad job. Quite intuitively, the more effort she puts into the project, the more likely it is that she receives a good performance signal and, hence, the more likely it is that she feels entitled to a higher wage. As a consequence, the reciprocal reaction of the agent depends on the agent's own subjective evaluation of her performance. Note that this idea is closely related to Carmichael and MacLeod (2003) in which it is analyzed how caring about sunk costs can help agents achieve efficient investments in a team production environment in which agents bargain about the division of the surplus only after they have made their investment decisions. Also in their setting with symmetric information agents' feelings of entitlement in the ex-post bargaining stage depend on their ex-ante investments, i.e. feelings of entitlement are endogenous.[3]

Second, in line with findings in the psychological literature we assume that the extent to which the agent feels entitled to a reward also depends on the principal's expertise and familiarity with the agent's task [see e.g. Landy and Murphy (1978), Ilgen and Taylor (1979), Greenberg (1986a) and Greenberg (1986b)]. Specifically, we assume that she feels less shortchanged by a low performance evaluation and reward by the principal the greater his familiarity with her task and the greater his expertise in evaluating the success of the project.

Explicitly modeling conflict as originating from the reciprocal reaction of an agent that feels shortchanged and unkindly treated uncovers intriguing welfare effects.

First, we demonstrate that an increase in the principal's cost of conflict can actually enhance welfare if the project is sufficiently valuable. The intuitive explanation is that a higher level of conflict helps the principal commit to a higher wage. This, in turn, helps the principal to achieve a higher effort level from the agent and a higher expected profit.

Second, we find that it might be optimal for the principal to hire an agent with a high emotional sensitivity to reciprocity. A high emotional sensitivity

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[3]Another paper in which the same idea is used is MacLeod (2007).

to reciprocity on the side of the agent expands the range of effort levels the principal can truthfully commit to, which implies a higher expected profit.

Third, we formally characterize situations in which it is optimal for the principal to hire an agent for whom the likelihood of having an own opinion is minimized and situations in which he might prefer an agent who is very opinionated, i.e. likely to have an own opinion. Lastly, the principal might find it optimal to hire an agent who has a high probability of identifying a successful project in case she forms an independent judgement.

Clearly, these findings relate to and complement Prendergast (1993)'s theory of 'Yes Men', i.e. agents that never form an own judgement concerning their performance and always agree with their principals' opinions. Prendergast (1993) analyzes the incentive that agents have to conform to their principals' opinions and the inefficiencies that this behavior creates. He concludes by mentioning that an important incompleteness of his analysis lies in the fact that it does not 'address why managers may wish to have cronies who agree with them' Prendergast (1993, p. 770). Our analysis addresses this issue by clearly characterizing the circumstances under which principals have an incentive to hire 'Yes Men'.

Interestingly, the last three results regarding the agent's 'characteristics' closely link to and extend a fairly recent discussion in the economics literature and a long standing debate in the human resource/organizational behavior literature concerning the importance and effectiveness of 'applicant screenings' and 'personality tests' in recruitment and promotion processes.

The recent economics literature highlights the importance of screening to identify applicants with e.g. high 'work ethics' [see Bartling *et al.* (2012) and Huang and Cappelli (2010)] which is shown to be associated with a lower need to control and higher employee productivity.

The human resource and organizational behavior literature, on the other hand, stresses that personality tests are used by firms to identify applicants whose personal traits (e.g. the applicant's openness, determination and ability to cope with hierarchies and feedback) fit best to the 'culture' of the organization and the 'character' of the vacancy [see e.g. Raymark *et al.* (1997), Kristof (1996), Cable and Judge (1994), Judge and Cable (1997) and Li *et al.* (2008)].

According to this literature, the 'person-organization' and 'person-job' fit are very important for the performance of employees and success of companies [e.g. Barrick and Mount (1991), Tett *et al.* (1999), Chatman *et al.* (1999) and Tett and Christiansen (2007)]. The culture of an organization and the character of a vacancy are determined by the nature of the industry, the character of the projects the organization is involved in and the technologies it uses [e.g. Schein (2004)].

In line with this, our results also highlight that it is vital for the performance and success of firms operating in complex environments preventing the specification of complete contracts to employ agents whose personal traits are in line with the culture of the organization they work for and the character of the job they perform.

Finally, we extend our framework to allow the principal to choose the evaluation procedure. More precisely, we allow the principal to choose the quality of the process used to evaluate the performance of the agent. In reality, the principal often does not only decide upon the contractual arrangements such as bonuses or fixed payments. He also decides upon the acquisition of information on the agent's performance.

It has been suggested in recent experimental and theoretical works that such procedural choices are important in strategic interactions with reciprocal agents [see e.g. Blount (1995), Sebald (2010), Aldashev *et al.* (2010)]. According to this literature procedural choices are important as they influence agents' kindness perceptions. In our setting, procedural choices influence the agent's feeling of entitlement and the a priori probability of conflict which, in turn, influence the agent's reaction to a particular feedback and the 'price' that the principal has to pay to implement a specific effort level.

Interestingly, we show that even if it is costless for the principal to choose a high quality evaluation procedure, he might not always find it optimal to do so. Signal imperfections and, thus, potential conflict might be necessary to implement the principal's preferred effort level. This highlights that the choice of the evaluation procedure constitutes an integral and important part of strategic environments in which agents are motivated by reciprocity.[4]

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[4]Note that a similar result has also been highlighted by Sebald and Walzl (2012a) in which it is assumed that the agent is motivated by a payoff-independent form of reciprocity.

The organization of our analysis is as follows. In section 2, we present our principal-agent environment in which the agent behaves reciprocal and performance can only be measured subjectively. In section 3, we characterize the agent's optimal effort and conflict level, the principal's truthtelling limits, his optimal choice of effort and the associated implications for welfare. The impact of the principal's procedural choice is analyzed in section 4, followed by a conclusion.

2 The model

Consider a principal P who decides upon undertaking a project which might generate a profit ϕ if successful. The project requires effort of an agent A. If the agent invests effort $\tau \in [0, 1]$ the expected profit of the project is $\tau\phi$. The project is a complex good or service and its success is non verifiable.

The Information Technology. The agent's effort is unobservable and, as a result, the principal and the agent are left to subjectively judge the success of the project. That is, the principal and the agent receive private non-verifiable performance signals $s_P \in S_P$ and $s_A \in S_A$ with $S_A = S_P = \{H, L\}$ respectively. These signals are informative with respect to the success of the project. If the project is not successful, the principal and the agent receive the signal $s_P = s_A = L$. On the other hand, if the project is successful, the principal receives the signal $s_P = H$ with probability g, the agent receives the same evaluation as the principal with probability ρ and receives $s_A = H$ as an independent signal with probability x. Hence, g indicates the quality of the principal's signal, $(1 - \rho)$ measures the likelihood with which the agent has an own independent opinion and x quantifies the quality of the agent's signal if she forms an independent judgment [Note, this specification of the information technology coincides with (MacLeod, 2003, p. 228)].[5]

We denote the probability that the principal receives signal k while the agent receives signal l given that the project is a success by γ_{kl}. More specifically, $\gamma_{HH} = g(\rho + (1-\rho)x)$, $\gamma_{HL} = g(1-\rho)(1-x)$, $\gamma_{LL} = (1-g)(\rho + (1-\rho)(1-x))$

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[5]We restrict ourself to a binary signal for expositional ease. The extension to a finer signal structure can be done at a notational cost.

and $\gamma_{LH} = (1-g)(1-\rho)x$.

In this principal-agent environment contracts contingent on the generation of ϕ are not feasible. Instead, a contract Γ specifies payments ω contingent on verifiable events, i.e. $\Gamma = \{\omega_{kl} | k \in S_P, l \in S_A\}$ where k and l respectively are the principal's and the agent's report concerning their subjective performance evaluations. The agent accepts a contract if it is individually rational. We normalize the agent's outside option to zero and as a consequence, the agent accepts a contract whenever her expected utility is weakly positive. The agent then chooses τ so as to maximize her utility. In this case we say that Γ *implements* τ. The principal and the agent report their signal truthfully if and only if they weakly benefit from doing so.

The Agent. We assume that the agent is risk neutral and not only motivated by her material payoffs, but also by reciprocity. More specifically, the agent's utility function is

$$U = \omega - v(\tau) - \theta \cdot \max\{\tilde{\omega} - \omega, 0\} \cdot (1-q) - c(q). \tag{1}$$

where ω is the agent's wage, $v(\tau)$ is the effort cost at effort level τ with $v'(0) = 0$, $v'' > 0$ and $\lim_{\tau \to 1} v(\tau) = \infty$ and $\theta > 0$ is the agent's sensitivity to reciprocity. The agent acts reciprocally whenever her wage ω is below her feeling of entitlement $\tilde{\omega}$, i.e. whenever $(\tilde{\omega} - \omega) > 0$. The reciprocal action consists of creating a conflict q costly to the principal. This conflict could be interpreted as a law suit, stealing from the work place, creating rumors which could hurt the firm's reputation or demotivation in an un-modeled future interaction etc. As work effort, also conflict is costly to the agent. A conflict level of q incurs a cost $c(q) \geq 0$ with $c(0) = 0$, $c'(0) = 0$, $c'' > 0$ and $\lim_{q \to 1} c(q) = \infty$.

The Principal. In contrast to the agent, we assume that the principal only cares about his profit. His expected profit is given by:

$$\Pi = \tau\phi - E\{\omega\} - E\{q\}\psi \tag{2}$$

where $E\{\omega\}$ and $E\{q\}\psi$ are the principal's expected wage costs and expected costs of conflict respectively. The parameter ψ captures the principal's 'sensitivity' to conflict or the agent's ability to impose costs on the principal by causing conflict. Alternatively, as our assumptions on $c(q)$ ensure that $q \in [0,1]$, one can also interpret q as the probability with which the agent creates costs of ψ for the principal.

Contracts. Quite naturally cost-minimizing revelation contracts in our environment have the following basic characteristics

Lemma 1. Suppose there exists a contract Γ which implements $\tau > 0$. Then, there always exists a contract $\hat{\Gamma}$ implementing τ at weakly lower costs which has the following characteristics:

(i) the principal and agent tell the truth,

(ii) wage payments only depend on the principal's report, i.e. $\omega_{kl} = \omega_{km} \equiv \omega_k$ for all $k \in S_P$ and $l, m \in S_A$ and

(iii) wage payments are higher in case the principal reports H than if he reports L, i.e. $\omega_H > \omega_L$.

Proof: Appendix A.1

Since signals are private and non-verifiable, the contract cannot be made contingent on the principal's signal. Instead, the optimal contract depends on the principal's report of his signal, i.e. it depends on the subjective performance evaluations of the principal. Furthermore, the wage payment associated with a good report H has to be strictly higher than the wage payment following a bad review L. We say that the agent is being paid a wage of ω_L if the principal reports L and is being paid $\omega_H > \omega_L$ if the principal reports H.

Feelings of Entitlement. Different feelings of entitlement have been suggested in the literature conceptualizing and analyzing the influence of reciprocity in strategic environments. Rabin (1993) and Dufwenberg and Kirchsteiger (2004), for example, assume that people feel entitled to the average of what they could receive. Translating this into our context means that the agent would feel entitled to the average she could have received independent

of her own signal and irrespective of the contract that the principal and agent agreed upon before she invested effort into the project. On the other hand, Hart and Moore (2008) assume that contracts constitute reference points and that ex-post people feel entitled to the maximum as specified by the contract that all parties had voluntarily agreed upon ex-ante. In our setting this means $\tilde{\omega}(\Gamma) = \omega_H$ independent of the agent's own performance signal.[6]

As in Hart and Moore (2008), also in our context it is natural to assume that a contract that the principal and agent voluntarily agree upon before the agent invests effort into the project shapes the parties feelings of entitlements ex-post. In case there exists a mutual agreement on the terms of the contract, feelings of entitlement arise relative to the possible payments agreed upon in the contract ex-ante.

Interestingly, the existing literature on reciprocity usually abstracts from the question in what situation/how feelings of entitlement arise.[7] Agents either feel entitled e.g. to the high wage as in Hart and Moore (2008) or to the average of what they could have received as e.g. in Dufwenberg and Kirchsteiger (2004). In contrast, we assume that feelings of entitlement are not exogenous, but endogenously shaped by the agent's own performance assessment which is influenced by the agent's own work effort. As mentioned in the introduction, our conceptualization is closely related to Carmichael and MacLeod (2003) in which it is analyzed how caring about sunk costs can help agents achieve efficient investments in a team production environment in which agents bargain about the division of the surplus only after they have made their investment decisions. Also in their setting with symmetric information agents' feelings of entitlement in the ex-post bargaining stage depend on their ex-ante investments, i.e. feelings of entitlement are endogenous.

More specifically, we model feelings of entitlement in the following way: we assume that the agent feels entitled to the high wage ω_H with intensity λ where $0 < \lambda \leq 1$, when she receives the positive performance signal $s_A = H$, but she does not feel entitled to it, if she receives the low performance signal $s_A = L$.

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[6]Note that this seems to imply a strong self-serving bias (the so-called 'Lake Wobegon effect'; see Hoorens (1993)) as the agent feels entitled to the highest possible wage independent of her own perception concerning her performance.

[7]One exception to this is e.g. MacLeod (2007) in which sunk investments trigger reciprocal reactions.

In other words, if the agent believes she did a good job she feels entitled to a higher wage than if she believes she did a bad job. Quite intuitively then, the more effort she puts into the project, the more likely it is that she receives a high own evaluation and, hence, the more likely it is that she feels entitled to a higher wage.

Furthermore, in line with the psychological evidence concerning agents' feelings of entitlements and fairness perceptions [see e.g. Landy and Murphy (1978), Ilgen and Taylor (1979), Greenberg (1986a) and Greenberg (1986b)], we assume that $\lambda(\cdot)$ is a decreasing function of the principal's signal quality g (i.e. $\frac{\partial \lambda(g)}{\partial g} \leq 0$). Intuitively, the more knowledgable the principal is, the less the agent feels shortchanged when she does not get the high wage ω_H.[8] This is, we assume

$$
\tilde{\omega} = \begin{cases} \lambda(g) \cdot \omega_H + (1 - \lambda(g)) \cdot \omega_L & \text{if } s_A = H \\ \omega_L & \text{if } s_A = L \end{cases} \tag{3}
$$

with $\lambda(g) \in (0,1)$ and $\frac{\partial \lambda(g)}{\partial g} \leq 0$.[9]

3 Conflict, truthtelling and welfare

The agent's ability to create conflict has a negative as well as positive effect within our model. On the one hand, the principal's ability to incentivize the agent is burdened by potential future conflict. On the other hand, the risk of conflict enables the principal to commit to a truthful revelation of his signal. In absence of conflict, the principal never finds it optimal to pay out the bonus to the agent. As a result, the agent never finds it optimal to work. Thus, conflict creates room for mutual beneficial relations. Furthermore, this dichotomy also shapes the principal's incentive to hire agents who are not likely to have an

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[8]For notational simplicity we write λ instead of $\lambda(g)$ whenever no confusion might arise.

[9]We choose to model the feelings of entitlement as depending on the agent's own-evaluation. This is in line e.g. with experimental evidence in Sebald and Walzl (2012b). Of course, it is also intuitive to assume that the agent's feelings of entitlement also depend on his own effort choice τ which is unobservable to the principal (or some other state-independent variable). Note though that this leads to technical challenges, but should not effect our results as long as some difference in the agent's feelings of entitlement depending on his own performance evaluation remains.

own opinion concerning their performance (i.e. 'Yes Men') versus agents that are particularly good at evaluating themselves independently.

Optimal Conflict. The principal offers a contract where the agent is paid a higher wage when the principal reports H than when he reports L. We interpret this as a flat wage with a bonus payment following a report of H. That is, $\omega_H = f + b$ and $\omega_L = f$.[10] No conflict arises if the agent's own evaluation is negative, i.e. $s_A = L$, as $\tilde{\omega} - \omega = \omega_L - \omega_L = 0$. However, conflict arises when the principal reports L and the agent believes she did a good job, i.e. $s_A = H$. Following a report L and an own evaluation H, the agent is paid the fixed wage f and $\tilde{\omega} - \omega = \lambda \cdot b$. The agent chooses the level of conflict q to minimize her psychological cost of conflict, i.e.

$$\min_q \ \theta \cdot \lambda \cdot b \cdot (1 - q) + c(q)$$

where the optimal level of conflict q^* is implicitly given by

$$c'(q^*) = \theta \cdot \lambda \cdot b.$$

That is, the optimal level of conflict is a function of the bonus, i.e. $q^*(b)$, with

$$\frac{dq^*}{db} > 0 \tag{4}$$

and it is increasing in the agent's sensitivity to reciprocity θ and in the degree λ to which she feels entitled to the high wage in case she believes she did a good job.

The higher the bonus agreed upon in the contract, the stronger the reciprocal agent's reaction in case of conflict. Intuitively, the higher the bonus that the agent could have earned in case the principal had reported a high signal, the stronger the agent's reaction when she believes she did a good job and does not receive the bonus. Thus, the higher the bonus b, the higher is the potential conflict level q^*.

---

[10]Notice that in principle f can be negative as long as the agent's participation constraints is not violated.

The Agent's Choice of Effort. With knowledge of the potential future conflict level, the agent's optimal effort choice $\hat{\tau}$ can be derived. The agent maximizes utility

$$U = f + \tau \cdot (\gamma_{HH} + \gamma_{HL}) \cdot b - v(\tau) - \tau \cdot \gamma_{LH} \cdot [\theta \cdot \lambda \cdot b \cdot (1 - q^*) + c(q^*)],$$

with respect to τ which yields the following implicit relationship between the bonus offered by the principal and the effort level optimally chosen by the agent

$$(\gamma_{HH} + \gamma_{HL}) \cdot b - \gamma_{LH} \cdot [\theta \cdot \lambda \cdot b \cdot (1 - q^*) + c(q^*)] = v'(\hat{\tau}), \tag{5}$$

The implicit relation between bonus and optimal effort level captured in equation 5 is such that the incentive compatible bonus simultaneously has to overcome effort costs and expected costs of conflict. Thus performance pay creates an endogenous source of conflict if agents behave reciprocal. The principal would like to incentivize the agent to perform high effort, but by doing so he generates potential conflict.

Truthtelling. Since the principal can report either H or L irrespective of his actual signal s_P, he will only choose to report his true signal if his expected profit from doing so is higher than his expected profit from doing otherwise.

Suppose $s_P = H$. Then, the principal tells the truth whenever his expected payoff from doing so (which is given by $\phi - f - b$) exceeds his expected payoff from reporting L (which is given by $\phi - f - pr(s_A = H | s_P = H) \cdot \psi \cdot q^*$). Consequently, the principal reports H if

$$b \leq \frac{\gamma_{HH}}{\gamma_{HH} + \gamma_{HL}} \cdot \psi \cdot q^* = (\rho + (1 - \rho)x) \cdot \psi \cdot q^* \equiv b^{max}. \tag{6}$$

The principal cannot credibly commit to bonuses above b^{max}. The reason is that for very high bonuses the principal has an incentive to report L irrespective of his true signal s_P. In other words, for sufficiently high bonus levels he prefers to face possible costs of conflict rather than paying the bonus. The value of the maximal credible bonus b^{max} is increasing in the quality of the agent's independent signal x and the correlation between the principal's and the agent's signal ρ. Furthermore, b^{max} is increasing in the level of conflict q^*.

If instead the principal receives signal $s_P = L$ he tells the truth whenever the payoff from doing so (which is given by $\frac{(1-g)\tau}{1-g\tau} \cdot \phi - f - pr(s_A = H | s_P = L) \cdot \psi \cdot q^*$) exceeds his payoff from reporting H (which is given by $\frac{(1-g)\tau}{1-g\tau} \cdot \phi - f - b$). Hence, the principal reports L if

$$b \geq \frac{\tau \cdot \gamma_{LH}}{\tau \cdot (\gamma_{LH} + \gamma_{LL}) + (1 - \tau)} \cdot \psi \cdot q^* = \frac{(1 - g)\tau}{1 - g\tau} (1 - \rho) \cdot x \cdot \psi \cdot q^* \equiv b^{min}. \quad (7)$$

From this expression it is clear that the principal can also not credibly commit to very low bonuses. The reason is that for such low bonuses the principal has an incentive to evade conflict by always paying out the bonus regardless of his signal. This, in turn, would be anticipated by the agent who would simply not provide any (costly) effort and still get the bonus. The value of the lowest credible bonus is increasing in τ and x, decreasing in ρ and increasing in the level of conflict. Notice further that for all $\tau \in [0, 1]$ it holds that $b^{max} > b^{min}$.

Importantly, the principal has to offer a bonus $b \in [b^{min}, b^{max}]$ to incentivize the agent. Furthermore, equations 6 and 7 reveal that without conflict, i.e. with $q^* = 0$, the principal cannot truthfully commit to any positive bonus as both $b^{min} = b^{max} = 0$. Hence, potential conflict is crucial for principal-agent environments based on non-verifiable subjective performance evaluations as only bonuses that the principal can truthfully commit to create the basis for any mutually beneficial relation.

As can be concluded from this section, in order to incentivize the agent the principal has to offer a bonus which is credible. In addition to being credible, the bonus also has to sufficiently compensate the agent for his cost of effort and potential cost of conflict [see Appendix A.2 for a complete presentation of the pure moral hazard effect in our principal-agent environment]. In particular, as the contract establishes a reference point, and incentive pay constitutes an endogenous source of conflict, there could be situations in which the agent's optimal choice of effort is unresponsive to increases in the bonus offered by the principal simply because the risk of future conflict outweighs the potential benefit from receiving a bonus. Given this, the question arises which effort levels can and will optimally be implemented by the principal.

Optimal Effort Level. What is the optimal effort level τ^* that the principal implements? Let τ^{min} and τ^{max} be the (incentive compatible) effort levels implemented by bonus b^{min} and b^{max} respectively. That is, τ^{min} is the effort level optimally chosen by the agent when she is offered the bonus level b^{min}. Furthermore, let $\tilde\tau = \arg\max \Pi(\tau)$ be the effort choice that the principal would choose in the absence of the truthtelling limits τ^{min} and τ^{max}.

Remember that the principal has to offer a bonus $b \in \left[b^{min}, b^{max}\right]$ to incentivize the agent to work. Whether the principal finds it worthwhile to offer the agent such a contract depends on whether his expected profit from doing so is positive. This depends, among other things, on the project value.

As it turns out, not all project values are large enough for the principal to find it profitable to induce the agent to work. The bonus required to incentivize the agent may be too large relative to the expected value of the project. The principal will find it profitable to incentivize the agent to work only if the value of the project is such that $\phi > \overline\phi$ where $\overline\phi$ is the value of the project at which the principal's expected profit is zero if τ^{min} is implemented, i.e $\Pi(\tau^{min})\big|_{\phi=\overline\phi} = 0$. [See Appendix A.3 for a complete characterization of the conditions under which a mutually beneficial relationship arises]. When the principal finds it profitable to offer the agent a contract that induces her to work, we say that the principal *implements* a positive effort level.

Suppose the value of the project is such that the principal finds it optimal to induce the agent to work. The following lemma characterizes the optimal effort level τ^* that the principal implements given the lower and upper truthtelling constraint b^{min} and b^{max}. Remember that the relationship between bonus and effort level optimally chosen by the agent is implicitly defined by equation 5.

Lemma 2. The effort level implemented by the principal is described by the following three cases

(1) Binding lower truth-telling constraint: the principal implements $\tau^* = \tau^{min}$ with bonus b^{min} if $0 < \tilde\tau < \tau^{min}$.

(2) Binding upper truth-telling constraint: the principal implements $\tau^* = \tau^{max}$ with bonus b^{max} if $\tilde\tau > \tau^{max}$.

(3) Non-binding truth-telling constraint: the principal implements $\tau^* = \tilde{\tau}$ by paying $b(\tilde{\tau})$ if $\tilde{\tau} \in [\tau^{min}, \tau^{max}]$.

Proof: Follows directly from the shape of the profit function. See Appendix A.2.

The principal implements $\tilde{\tau}$ whenever possible (i.e. Case (3) of lemma 2). However, he is limited to τ^{max} and τ^{min} whenever the bonus associated with the effort level that he actually would like to implement in absence of the truthtelling limits lies above or below the thresholds that he can credibly commit to (i.e. Cases (1) and (2) of lemma 2).

Welfare. What are the welfare implications of conflict costs and agent characteristics such as the agent's sensitivity to reciprocity in our strategic environment?

Before getting to the results, note two things. First, it is useful to define a characteristic of the agent's effort cost function which proves important for some of our welfare results. Generally speaking, any parameter change in our setting has two effects on welfare: a direct and an indirect. The direct effect captures the change in the principal's profit due to a change in the price of effort as a result of the parameter change. The indirect effect, on the other hand, regards the agent's optimal choice of effort which might change in response to a change in parameters. The magnitude of the indirect effect will depend on the curvature of the agent's effort cost function $v(\tau)$. Specifically, it depends on the measure

$$\frac{v'(\tau)}{v''(\tau)}, \tag{8}$$

which captures the degree of 'convexity' of the agent's effort costs.

Second, note that total welfare is given by the principal's profit since the agent does not earn any rent.

Given this, the following results obtain:

Proposition 1. Welfare is increasing in the principal's costs of conflict ψ, if the value of the project ϕ is sufficiently high.

Proof: Appendix A.5

Proposition 1 shows that conflict can have a welfare enhancing impact in principal-agent environments based on subjective performance evaluations. When the upper truthtelling constraint is binding, an increase in the principal's sensitivity to conflict ψ can increase the principal's profit and thus increase welfare. As already hinted at in the beginning of this section, there are two effects of an increase in ψ. First, ignoring the truthtelling problem (i.e. the pure moral hazard case) the direct effect on welfare of an increase in ψ for a given effort level is negative. However, since $\frac{d\tau^{max}}{d\psi} > 0$, an increase in the principal's sensitivity to conflict ψ also relaxes the upper truthtelling constraint. Thus, when the upper truthtelling constraint is binding, an increase in ψ can enable the principal to credibly commit to, and hence implement, higher effort levels. This in turn increases the expected profit. When the potential value of the project ϕ is sufficiently high, the latter effect dominates and welfare is increasing in ψ.

It is not only the principal's cost of conflict that has an impact on welfare. Agent characteristics' such as the agent's sensitivity to reciprocity, the likelihood with which she forms an independent opinion concerning her performance as well as her ability to independently identify a successful project can also influence the principal's profit, and thus welfare, as the following results demonstrate.

First, we focus on the agent's sensitivity to reciprocity θ. Imagine that the principal can choose between two agents who are identical in all respects except for their value of θ.[11] One has a high value of θ and the other a low value. Which type will the principal prefer to hire?

Proposition 2. An increase in the agent's sensitivity to reciprocity can increase welfare if

  (i) the expected value of the project is sufficiently high, and the principal is sufficiently sensitive to conflict.

  (ii) the value of the project is small, the principal is not too sensitive to conflict and the agent's effort costs are not too 'convex' (i.e. the measure 8 is sufficiently large).

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[11]Notice that when we say the agents are *identical* in all respects except for their sensitivity θ that also includes their outside option, which is normalized to zero for both types of agents.

Proof: Appendix A.6

Interestingly, proposition 2 shows that it might be beneficial for the principal to hire an agent who has a high emotional sensitivity to reciprocity even if this agent will potentially impose high conflict costs. An increase in the agent's sensitivity to reciprocity increases the expected cost of conflict. This will make it less tempting for the principal to lie if he receives the high performance signal, which in turn relaxes the upper truthtelling constraint. However, an increase in the agent's sensitivity also makes a given effort level more expensive to implement since the agent must be compensated for potential conflict costs. If the principal is sufficiently sensitive to conflict, the first effect will dominate, and welfare is increasing in θ. Likewise, an increase in the agent's sensitivity makes it more tempting to lie if the principal receives the low performance signal. However, it also increases the price the principal has to pay for a specific effort level which makes it less tempting to lie. If the principal is not very sensitive to conflict this last effect will dominate.

Second, regarding the correlation between the principal's and agent's signal ρ:

Proposition 3. Welfare is decreasing in the correlation of signals ρ, if the value of the project ϕ is sufficiently low, the principal is not too sensitive to conflict and the agent's effort costs are not too 'convex' (i.e. the measure 8 is sufficiently large).

Proof: Appendix A.7

Third, regarding the agent's ability to independently evaluate the success of the project x:

Proposition 4. An increase in the agent's ability to independently identify a successful project can increase welfare in the following two ways:

 (i) if the value of the project is small, the principal is not too sensitive to conflict and the agent's effort cost is not too 'convex' (i.e. the measure 8 is sufficiently large).

 (ii) if the expected value of the project is sufficiently large, and the principal is sufficiently sensitive to conflict.

Proof: Appendix A.8

Imagine two agents, Agent 1 and Agent 2, who are identical except for their ability to independently evaluate the success of the project and the correlation between their own signal and the principal's signal. Assume Agent 2 has a lower value of ρ and a higher value of x compared to Agent 1. That is, Agent 2 will - compared to Agent 1 - more often create conflict. Hiring Agent 2 instead of agent 1 has two effects on welfare. First, it is more expensive to induce Agent 2 to work. Agent 2 requires a higher incentive compatible bonus for every given effort level compared to Agent 1. On the other hand, since conflict is more of a risk with Agent 2, it is possible that the range of effort levels the principal can credibly implement is larger for Agent 2 compared to Agent 1. Naturally, in some cases the first effect will dominate and the principal will find it welfare enhancing to hire Agent 1. In other cases, the principal might be able to implement more desired effort levels with Agent 2, effort levels which would be unfeasible in case the principal decides to hire Agent 1.

Intuitively, propositions 3 and 4 show that if the project value is such that truthtelling constraints are not a concern, the principal should always hire an agent for whom the likelihood of having an own opinion is minimized (i.e. 'Yes Men'), and an agent who is not good in independently evaluating the success of the project. These two 'agent characteristics' or 'personality traits' minimize the potential for conflict and, hence, increase welfare. However, if the project value is sufficiently low, the principal might find it optimal to hire an agent for whom the likelihood of having an own opinion concerning her performance is high. In addition, for sufficiently high and low project values the principal might find it optimal to hire an agent who is very good in independently identify the success of the project.

These welfare effects highlight that personality tests that e.g. assess an applicant's sensitivity to reciprocity or ability to form an own opinion can play an important role in recruitment processes in work environments in which firms cannot write complete contracts that specify all aspects of the employment relation. To form a mutually beneficial and optimal relation the personality of an applicant should fit the character of the vacancy he or she applies to.

4 The choice of evaluation procedures

Until now, we have investigated optimal contract design and welfare implications of an *exogenously* given quality of the principal's signal g. In reality, however, the principal often does not only decide upon the contractual arrangements such as bonuses or fixed payments. He may also decide upon the acquisition of information on the agent's performance. The principal can, for example, decide how much time he spends on supervising the agent in the accomplishment of the project. He could (i) sit next to the agent during the whole project, or (ii) close the door to his office and only have a glance at the result. Arguably, the quality of the signal g is expected to be better under the first evaluation procedure.[12]

Of course, under classical assumptions about preferences the quality of the evaluation procedure has no impact on the effort choice of the agent in our setting. The agent simply does not trust the principal to truthfully reveal his signal and hence provides no effort. In contrast to this, however, it has been suggested in recent experimental and theoretical works that procedural choices are important in strategic interactions with reciprocal agents [see e.g. Blount (1995), Sebald (2010), Aldashev *et al.* (2010)]. Procedural choices are important because reciprocal agents might exhibit procedural concerns and, hence, react differently in outcome-wise identical situation depending on the evaluation/decision-making procedure which led to the outcome. Translated into our setting, the agent's perception concerning the kindness of the principal towards her depends on the evaluation procedure chosen by the principal. The higher the quality of the evaluation process, the kinder the agent perceives the principal and, hence, the kinder the agent's response.

To formally analyze the impact of the quality of the evaluation procedure on the agent's effort choice and the principal's optimal choice of signal quality in our setting, assume that the quality of the signal is costless. This assumption is made (i) to simplify the analysis and (ii) to show that even with costless

---

[12]Note that we explicitly avoid terms like *control* and *(dis)trust* here (as e.g. used in Falk and Kosfeld (2006) and Ellingsen and Johannesson (2008)). The choice of the quality of the evaluation procedure has an influence on how well the principal can observe an acceptable effort given that the project is a success. Therefore, the higher the quality of the principal's evaluation process, the higher the probability that the agent is rewarded in case of success. A higher quality is, hence, not regarded as negative by the agent.

monitoring the principal might not choose a perfect evaluation procedure in our setting with subjective performance evaluations.[13]

Implementable Bonuses Remember the following properties of the relation between the incentive compatible bonus $b(\hat{\tau})$, b^{min}, b^{max} and the quality of the principal's evaluation procedure g:

(i) a bonus b which makes the effort choice of τ incentive compatible only satisfies the upper and lower truthtelling constraint of the principal if $b \in [b^{min}, b^{max}]$,

(ii) the incentive compatible bonus $b(\hat{\tau})$ in our setting is monotonically decreasing in the principal's signal quality g with $\lim_{g \to 0} b(\hat{\tau}) = \infty$ and $\lim_{g \to 1} b(\hat{\tau}) = v'(\hat{\tau})$ [see also Appendix A.2] and

(iii) b^{max} and b^{min} are (weakly) monotonically decreasing in g (because $\frac{dq^*}{dg} \leq 0$) with $\lim_{g \to 0} b^{min} < \infty$, $\lim_{g \to 1} b^{min} = 0$, $\lim_{g \to 0} b^{max} < \infty$ and $\lim_{g \to 1} b^{max} = (\rho + (1 - \rho)x) \psi q^* > 0$.

Properties (i)-(iii) allow us to distinguish the following possible cases describing the optimal choice of g for the implementation of an effort level $\tau > 0$.

Lemma 3. Fix some effort level $\bar{\tau} > 0$. In order to implement that specific effort level, the principal has to offer a bonus $b(\bar{\tau})$. The implementability of that effort level depends on the signal quality. One of the following cases will hold:

(1) $\bar{\tau}$ cannot be implemented regardless of the choice of g if $b(\bar{\tau}) > b^{max}$ for all g.

(2) $\bar{\tau}$ is implemented with the maximal g for which $b(\bar{\tau}) = b^{max}$ if $b(\bar{\tau}) \leq b^{max}$ for some $g < 1$ but $b(\bar{\tau}) > b^{max}$ for $g = 1$. That is, the principal chooses a less than perfect signal quality.

(3) $\bar{\tau}$ is implemented with $b(\bar{\tau}) = v'(\bar{\tau})$ and the principal chooses perfect signal quality (i.e. $g = 1$) if $b(\bar{\tau}) \in [b^{min}, b^{max}]$ for $g = 1$.

---

[13]Assuming that the choice of evaluation procedure is costly to the principal would not change our results, but blur the interplay between the principal's limits, b^{min} and b^{max}, and his choice of g.

Proof: Follows directly from the aforementioned properties (i)-(iii).

In Case (1) effort $\bar{\tau}$ cannot be implemented with any signal quality g because the incentive compatible bonuses are too large to be credible. This situation arises, for example, if the agent is insensitive to reciprocity (i.e. $\theta = 0$) or there are no retaliation opportunities (i.e. $\psi = 0$). Case (3), on the other hand, depicts the situation in which the incentive compatible bonus is credible for signal quality $g = 1$. Cases (2) and (3) of lemma 3 show that, as a better signal quality reduces the probability of conflict and expected psychological costs, the principal will always implement τ with the largest possible signal quality which still ensures truthtelling.[14]

To graphically exemplify Case (2) of lemma 3 consider the following scenario

[Figures 1 here]

Figure 1 shows Case (2) in which the incentive compatible bonus $b(\hat{\tau})$ is lower than b^{max} for some $g < 1$, but higher at $g = 1$. In this case the optimal bonus and signal quality is denoted $\bar{b}^{max}$ and $\bar{g}$.

Welfare Implications. From the above analysis it is clear that welfare is not always increasing in the quality of the principal's signal. What are the precise conditions under which welfare is increasing or decreasing in the quality of the evaluation procedure?

Proposition 5. The welfare effect of the choice of signal quality:

(i) If the principal's preferred choice of effort is unbounded by the truthtelling constraints, welfare is unambiguously increasing in the principal's signal quality g.

(ii) Welfare can be decreasing in the signal quality, if the principal's choice of effort is bounded by his truthtelling constraints:

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[14]Recall that signal quality was assumed to be costless. Whenever costs of information acquisition are increasing in g there is an obvious tradeoff between decreasing effort costs $C(\tau)$ [see Appendix A.2] and increasing costs of quality.

(a) Welfare is decreasing in the signal quality if the project value is sufficiently high such that the principal is bounded by the upper-truthtelling constraint and the principal's cost of conflict is sufficiently high $\psi > \tilde{\psi}$.

(b) Welfare is decreasing in the signal quality if the project value is sufficiently low such that the principal is bounded by the lower-truthtelling constraint, the principal's cost of conflict is not too high $\psi < \tilde{\psi}$, and the agent's effort cost is not too 'convex' (i.e. the measure 8 is sufficiently large).

Proof: Appendix A.9

Intuitively, if the principal is free to implement his most preferred effort level there is only the direct effect on profit from a change in g and this is positive since the 'price' the principal has to pay to implement a certain effort level is decreasing in g.

However, when the lower or upper truthtelling constraint binds it is possible that welfare be decreasing in the quality of the principal's signal. This is because an increase in g affects the highest and lowest implementable effort levels by changing the range of credible bonuses.

As a first example, imagine that the principal is bounded by the upper truthtelling constraint. An increase in g has ambiguous effects on the maximum implementable effort level. First, effort is cheaper for higher levels of signal quality which has an increasing effect on τ^{max}. Second, since a higher value of g also makes conflict less likely, it is more tempting for the principal to lie when he receives signal H. This decreases the maximum credible bonus and pulls towards a lower value of τ^{max}. If the principal is sufficiently sensitive to conflict, the level of b^{max} will change so much that the positive effect on τ^{max} is outweighed by the negative effect. Thus the principal will find himself unable to commit to high bonus levels following an increase in g. If the project is sufficiently valuable and hence requires high effort levels, such a change can decrease welfare since it limits the principal to choose 'too low' effort levels.

As a second scenario, imagine that the principal is bounded by the lower truthtelling constraint. When the principal chooses a better signal quality the agent responds by creating lower potential conflict. This decreases the

minimum credible bonus. However, since an increase in g also makes a given effort level cheaper it is possible that the minimum credible effort level is increasing in g because the principal will more often prefer to evade conflict by paying out the bonus unconditionally. Now, if the principal is bounded by the lower truthtelling constraint and an increase in g tightens this truthtelling constraint, it is possible that welfare decreases overall because the principal has to implement a 'too high' effort level which is costly - even if this effort level comes cheaper as a result of the higher value of g.

5 Conclusion

Our analysis focused on the role and importance of conflict in work environments based on non-verifiable subjective performance evaluations. Contrary to the existing literature we did not model conflict as e.g. third-party payments optimally chosen to ensure truthtelling, but explicitly formalized conflict as the reciprocal reaction of agents that feel shortchanged and unkindly treated by their principal.

In our setting, contracts constitute frames/reference points and performance pay creates an endogenous source of conflict since the agent's feelings of entitlement, and her potential reciprocal reaction, is intensifying in the bonus. In other words, by promising to pay a bonus the principal incentivizes the agent to perform effort while simultaneously generating potential conflict.

We showed that the principal's optimal choice of contract in such an environment is limited by a maximum and minimum bonus that he can credibly commit to. Bonuses above the upper threshold or below the lower threshold fail to fulfill the principal's truthtelling constraint and, hence, lead to an inefficiently low effort provision by the agent. These limits, in turn, influence the optimal effort levels that the principal can actually implement.

Explicitly modeling conflict as originating form the reciprocal reaction of an agent who feels shortchanged revealed interesting welfare effects. In particular, linking up with a fairly recent literature in economics and a long standing debate in the organizational behavior/human resource literature we showed that agent characteristics play a crucial role in principal-agent environments

based on non-verifiable performance evaluations. In this way, our analysis provides one rational for the use of personality tests and applicant screenings in recruitment and promotion processes.

Furthermore, following the recent literature on procedural concerns, we extended our framework to allow the principal to choose between evaluation procedures that differ in terms of the quality of the principal's signal. In our setting the choice of evaluation procedure influences the agent's feeling of entitlement as well as the a priori probability of conflict. Interestingly, our analysis reveals that even if it is costless for the principal to choose a perfect evaluation procedure, he might not choose a perfect evaluation process. The principal may benefit from some 'noise' in the evaluation procedure since this creates a risk of conflict making more desired effort levels implementable.

Finally, we feel that there are at least two important directions for future research. First, in the same way as Fuchs (2007) has extended MacLeod (2003), it is also important to take our ideas to a repeated setting and explore the interplay between personal traits, reciprocity and reputational effects. This seems particularly important in the light of empirical evidence showing the important connection between concerns for reciprocity and reputation [e.g. Fehr *et al.* (2009a) and Gächter and Falk (2002)]. Second, experiments should be conducted that not only test the assumptions we make regarding the agent's reciprocal inclination, but also the theoretical implications of our theory.

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A   Appendix

A.1   Proof of Lemma 1

To save on notation, we denote $\theta \cdot \max\{\tilde{\omega} - \omega, 0\} \cdot (1 - q) - c(q) \equiv Y_{kl}(\tau)$ throughout this proof.

Part (i). For a given contract Γ and signals s_P and s_A, the principal and the agent decide upon their report. Let $\sigma_P : S_P \to \Delta(S_P)$ and $\sigma_A : S_A \to \Delta(S_A)$ be the principal's and agent's reporting strategies (i.e. mappings from the set of signals S_P and S_A to the set of probability distributions over S_P and S_A respectively). Suppose that (σ_P^*, σ_A^*) is the pair of optimal reporting strategies for contract Γ. Then, the revelation principle implies that there exists a contract $\hat{\Gamma}$ which implements the same effort at the same costs and induces truthful reports by principal and agent. We will, henceforth, restrict our analysis to this type of (revelation) contracts.

Suppose that $\Gamma = \{\omega_{kl}\}$ is a revelation contract, i.e. the principal and the agent tell the truth under contract Γ and Γ implements $\tau > 0$. Then the incentive compatibility constraint

$$\Sigma_{k \in S_P, l \in S_A}(\omega_{kl} - Y_{kl}(\tau)) \frac{dPr\{s_P = k, s_A = l\}}{d\tau} = v'(\tau)$$

is satisfied. Consider a contract $\hat{\Gamma}$ which fixes payments of $\hat{\omega}_k = \Sigma_{l \in S_A} \omega_{kl} Pr\{s_P = k, s_A = l\}$ if the principal receives signal $s_P = k$, i.e. payments are independent of s_A. These payments also satisfy the incentive compatibility constraint (see above).[15] Moreover, the agent weakly benefits from telling the truth. Finally, the principal's truth-telling constraint is also satisfied under $\hat{\Gamma}$. To see this observe that the principal reports k given that he has received k under contract Γ if

$$\begin{aligned}
& Pr\{s_A = H | s_P = k\}(\omega_{oH} - \omega_{kH}) + Pr\{s_A = L | s_P = k\}(\omega_{oL} - \omega_{kL}) \quad (9) \\
\geq \ & Pr\{s_A = H | s_P = k\}((q^*\psi)_{kH} - (q^*\psi)_{oH}) \\
& + Pr\{s_A = L | s_P = k\}((q^*\psi)_{kL} - (q^*\psi)_{oL})
\end{aligned}$$

for all $o \in S_P$ (where $(q^*\psi)_{l,k}$ denotes the anticipated conflict costs for a reported configuration (l, k)). This set of inequalities holds because Γ implements truth-telling by assumption. $\hat{\Gamma}$ implements truth-telling if

$$\begin{aligned}
\hat{\omega}_o - \hat{\omega}_k \geq \ & Pr\{s_A = H | s_P = k\}((q^*\psi)_{kH} - (q^*\psi)_{oH}) \quad (10) \\
& + Pr\{s_A = L | s_P = k\}((q^*\psi)_{kL} - (q^*\psi)_{oL}).
\end{aligned}$$

---

[15]Individual rationality is trivially fulfilled as expected payments for the agent are the same under Γ and $\hat{\Gamma}$ and Γ is individually rational by assumption.

holds for all $o, k \in S_P$. Inserting $\hat{\omega}_k$ and $\hat{\omega}_o$ yields

$$
\begin{aligned}
&Pr\{s_A = H | s_P = k\}(w_{oH} - \omega_{kH}) + Pr\{s_A = L | s_P = k\}(\omega_{oL} - \omega_{kL}) \\
\geq\; &Pr\{s_A = H | s_P = k\}((q^*\psi)_{kH} - (q^*\psi)_{oH}) \\
&+ Pr\{s_A = L | s_P = k\}((q^*\psi)_{kL} - (q^*\psi)_{oL}).
\end{aligned}
$$

which coincides with equation 9 and therefore shows that for $\hat{\Gamma}$ the principal's truthtelling constraint is satisfied as well. Hence, any revelation contract Γ can be substituted by a revelation contract $\hat{\Gamma}$ with ω_{kl} independent of l which also implements $\tau > 0$ and leaves the principal weakly better off.

Part (ii). Suppose by contradiction that Γ implements $\tau > 0$ with $\omega_H = z$ and $\omega_L = z + \epsilon$ with $\epsilon \geq 0$. Then, the incentive compatibility constraint of the agent can be written as

$$
\epsilon = \frac{v'(p) + \gamma_{LH} Y_{LH}}{(\gamma_{LH} + \gamma_{LL} - 1)}.
$$

Observe that the numerator of the RHS is strictly positive and the denominator is strictly negative. Hence, the RHS is strictly negative and the incentive compatibility constraint is not satisfied for any $\epsilon \geq 0$. A contradiction.

Not for publication / Online material

A.2 Pure moral hazard

The principal's objective to offer a profit maximizing contract - i.e. an optimal combination of a fixed payment and a bonus - is burdened by (i) a moral hazard problem and (ii) a truthtelling problem as the agent's effort is unobservable, and the principal has to credibly commit himself to a truthful revelation of his own signal. This section of the appendix will analyze the pure moral hazard problem abstracting from the truthtelling problem. That is, we focus on the dynamics between the bonus offered and the effort level optimally chosen by the agent taking truthtelling as given.

The optimal level of conflict q^* is implicitly given by the following expression

$$c'(q^*) = \theta \lambda b.$$

The Agent's Effort Choice. The following result characterizes the relationship between bonus and optimal effort as chosen by the agent.

Result 1. The endogeneity of the conflict creates two cases describing the relation between bonus and optimal effort:

(i) There is a positive relationship between the offered bonus and the optimally chosen effort level for all levels of bonuses if $g - (1-g)(1-\rho)x\theta\lambda \geq 0$.

(ii) There is a positive relationship between the offered bonus and the optimally chosen effort level only for bonuses above $\underline{b} > 0$, where $\underline{b}$ is the bonus level that solves the following equation

$$gb = (1-g)(1-\rho)x\left[\theta\lambda b(1-q_H^*) + c(q_H^*)\right],$$

if $g - (1-g)(1-\rho)x\theta\lambda < 0$. For bonuses below $\underline{b}$ the optimally chosen effort level is zero and thus the effort level is unresponsive to changes in the offered bonus.

Proof. Inserting for γ_{HH}, γ_{HL} and γ_{LH} in equation 5 (for a formal definition of these see section 2 of the paper) and rearranging leads to:

$$gb - (1-g)(1-\rho)x\left[\theta\lambda b(1-q_H^*(b)) + c(q_H^*(b))\right] = v'(\tau). \qquad (11)$$

Note that the *LHS* of equation 11 depends on the bonus b (and is independent of the effort τ), whereas the *RHS* depends on the effort τ (and is independent of the bonus b). Furthermore, as $v'' > 0$, the *RHS* is monotonically increasing in τ and $RHS(\tau) : [0,1] \to [0,\infty]$.

With regard to the LHS note first that $\frac{\partial q^*}{\partial b} > 0$ (see equation 4), $lim_{b \to 0}$ $q^* = 0$ and $lim_{b \to \infty} q^* = 1$.

The derivative of the LHS of equation 11 with respect to b is

$$g - (1 - g)(1 - \rho)x\theta\lambda(1 - q^*(b)) \tag{12}$$

and

$$\frac{\partial LHS}{\partial b}\Big|_{b=0} = g - (1 - g)(1 - \rho)x\theta\lambda \tag{13}$$

(13) is (i) weakly positive if $g \geq ((1 - g)(1 - \rho)x\theta\lambda$ and (ii) negative if $g < ((1 - g)(1 - \rho)x\theta\lambda$. It is important to see that as the exogenous parameters $g, \rho, x, \lambda \in [0, 1]$ and $\theta \in (0, \infty)$, both cases are possible.

If the exogenous parameters, g, ρ, x, λ and θ, are such that (13) is positive then (12) will be positive for all values of b since the conflict level q^* is increasing in the bonus and therefore the negative term in (12) is decreasing in the bonus. That is, if (13) is positive then the LHS of equation 11 is monotonically increasing in b and, hence, Case (i) of Result 1 obtains.

On the other hand, it is possible as we already noted that (13) is negative. Notice however that

$$\lim_{b \to \infty} g - (1 - g)(1 - \rho)x\theta\lambda(1 - q^*(b)) = g > 0.$$

This means that eventually the derivative will be positive and we will have a positive relationship between the offered bonus and the chosen effort level. This happens when $b > \underline{b}$ where $\underline{b}$ is the bonus level that solves

$$gb = (1 - g)(1 - \rho)x\left[\theta\lambda b(1 - q^*(b)) + c(q^*(b))\right]$$

In this case the optimally chosen effort level will be 0 as long as $b \leq \underline{b}$ and the effort level is unresponsive to changes in the offered bonus. For bonus levels $b > \underline{b}$, however, we have a positive relationship between effort and bonus. Hence case (ii) obtains. $\qquad\square$

The following result shows that the non-positive relation between effort and bonus described in Case (ii) can always be overcome. That is, there always exist bonus levels high enough such that the agent will find it optimal to provide effort

Result 2. Irrespective of the information technology (i.e. g, x and ρ), sensitivity to reciprocity (i.e. θ) or the agent's feeling of entitlement, there always exists a bonus level $\hat{b} > \underline{b}$ above which the optimal effort level is positive, $\tau > 0$.

Proof. This follows directly from the proof of Result 1. There always exists a b such that the LHS of equation 11 is positive. This implies that there always exists a $\hat{b} : \underline{b} < \hat{b} < \infty$ for which $\tau > 0$ at $\hat{b}$. $\qquad\square$

Result 2 shows that the counterproductive relation between bonus and conflict can always be overcome by paying a sufficiently high bonus. In other words, sufficiently high bonuses always imply a positive relation between bonus and effort irrespective of the information technology, sensitivity to reciprocity or the agent's feeling of entitlement. Intuitively, for sufficiently large bonuses the monetary incentive associated with a bonus always outweighs the potential conflict that this bonus creates and consequently the agent will choose to work.

Principal's Choice of Contract. We now know that the principal can always pay a bonus high enough such that the agent finds it optimal to work. The next question is whether the principal will always find it profitable to offer the agent such a bonus. That is, given the agent's optimal choice of conflict and effort, we are now interested in the question whether the principal always wants to implement a positive effort level $\tilde{\tau}$ independent of the potential profitability of the project ϕ. For Case (i) in Result 1 we can state the following result:

Result 3. When parameter values are such that Case (i) in Result 1 obtains, the principal will implement a positive effort level $\tilde{\tau}$ for all project values $\phi > 0$.

Proof. The principal's expected profit is given by

$$\Pi = \tau\phi - \tau\psi\gamma_{LH}q^* - C(\tau)$$

where $C(\tau)$ is the expected labor cost given by $f + \tau(\gamma_{HH} + \gamma_{HL})b$. The agent accepts the contract if she receives a weakly positive payoff from doing so. Her participation constraint is therefore given by

$$f + \tau(\gamma_{HH} + \gamma_{HL})b - v(\tau) - \tau\gamma_{LH}(\theta\lambda b(1 - q^*) + c(q^*)) \geq 0$$

The principal will always choose f such that this participation constraint binds. Use this to rewrite $C(\tau)$ such that

$$C(\tau) = v(\tau) + \tau\gamma_{LH}[\theta\lambda b(1 - q^*) + c(q^*)].$$

Inserting this into the principal's profit function yields

$$\Pi = \tau\phi - [v(\tau) + \tau\gamma_{LH}(\psi q^* + \theta\lambda b(1 - q^*) + c(q^*)).$$

We want to show that there always exists an effort level such that the expected profit is positive. Hence we look into the shape of the profit function. The

first term of the profit function $\tau\phi$ is linearly increasing in τ as long as $\phi > 0$. From this we subtract a function of τ given by the term in the square brackets. Label this function $F(\tau)$, i.e.

$$F(\tau) \equiv v(\tau) + \tau\gamma_{LH}(\psi q^* + \theta\lambda b(1-q^*) + c(q^*)).$$

$F(\tau)$ is convex and has the following properties: $F(0) = 0$ and $F'(0) = 0$. As a consequence, it is true that $\pi = \tau\phi - F(\tau)$ is positive for some values of τ, and a positive optimal $\tilde{\tau}$ exists if $\phi > 0$. $\qquad\square$

For Case (ii) in Result 1 matters are different, and the principal does not always want to implement a positive effort level.

Result 4. When parameter values are such that Case (ii) in Result 1 obtains, the principal will implement a positive effort level only if

$$\phi > \underline{\phi}$$
$$= \gamma_{LH}(\psi q^*(\underline{b}) + \theta\lambda\underline{b}(1-q^*(\underline{b})) + c(q^*(\underline{b})))$$

Proof. Again, the principal's expected profit is equal to

$$\Pi = \tau\phi - F(\tau)$$

with

$$F(\tau) \equiv v(\tau) + \tau\gamma_{LH}(\psi q^* + \theta\lambda b(1-q^*) + c(q^*))$$

$F(\tau)$ is convex and $F(0) = 0$. It remains to check the derivative.

$$F'(\tau) = v'(\tau) \quad + \quad \gamma_{LH}(\psi q^* + \theta\lambda b(1-q^*) + c(q^*))$$
$$+ \quad \tau\gamma_{LH}\left(\psi\frac{\partial q^*}{\partial b}\frac{\partial b}{\partial \tau} + \theta\lambda\frac{\partial b}{\partial \tau}(1-q^*)\right).$$

Now, for $\tau = 0$ we have $b = \underline{b}$. From equation (4) we know that the conflict level is increasing in b. Thus, when $b = \underline{b}$ it is not the case that $q^* = 0$. It holds,

$$F'(0) = \gamma_{LH}(\psi q^*(\underline{b}) + \theta\lambda\underline{b}(1-q^*(\underline{b})) + c(q^*(\underline{b}))) \geq 0.$$

The expected profit function has maximum for a positive value of τ as long as the slope of $\tau\phi$ is steeper than the slope of $F(\tau)$ in $\tau = 0$:

$$\phi > F'(0) \Leftrightarrow$$
$$\phi > \gamma_{LH}(\psi q^*(\underline{b}) + \theta\lambda\underline{b}(1-q^*(\underline{b})) + c(q^*(\underline{b}))).$$

Thus, the minimum value of ϕ, which ensures a positive effort level, is

$$\underline{\phi} = \gamma_{LH}\left(\psi q^*(\underline{b}) + \theta\lambda\underline{b}(1 - q^*(\underline{b})) + c(q^*(\underline{b}))\right).$$

For project values below $\underline{\phi}$, the expected value of the project will not exceed the costs of providing the agent with incentives to work. As a result, the principal will not find it profitable to incentivize the agent to work. $\qquad\square$

A.3   Mutually beneficial principal-agent relations

The following lemma characterizes the conditions under which a mutual beneficial principal-agent relationship arises:

Lemma 4. We distinguish between the two cases described in Result 1. For Case (i), the principal finds it profitable to implement a positive effort level, $\tau^* > 0$, if and only if the project is sufficiently valuable, i.e. $\phi > \overline{\phi} > 0$ with $\Pi(\tau^{min})\big|_{\phi=\overline{\phi}} = 0$. For Case (ii), the principal implements a positive effort level under the same condition except if $b^{max} < \underline{b}$ where $\underline{b}$ is the bonus level that solves the following equation

$$gb = (1 - g)(1 - \rho)x\left[\theta\lambda b(1 - q^*(b)) + c(q^*(b))\right]$$

In this case, no credible effort level can induce the agent to work and consequently no principal-agent relationship will be established.

Proof. To establish sufficiency, pick some $\phi' > \overline{\phi}$. Since $\frac{\partial\Pi}{\partial\phi} > 0$ it holds that $\Pi(\tau^{min})\big|_{\phi'} > 0$. Now, $\Pi(\tau = 0) = 0 < \Pi(\tau^{min})\big|_{\phi'}$ and therefore $\tau^* > 0$.

To show necessity, suppose $\tau^* > 0$. Then it must be the case that $\phi > \overline{\phi}$. $\Pi(\tau)$ is continuous in $\tau \geq 0$ and concave with a unique maximum at $\tilde{\tau} > 0$. Now suppose that $\phi = \phi' < \overline{\phi}$. Then, as a consequence, $\Pi(\tau^{min})\big|_{\phi'} < 0$. From this we must conclude that $\tilde{\tau} < \tau^{min}$ and $\Pi(\tau) < 0$ for all $\tau \in [\tau^{min}, \tau^{max}]$, which contradicts $\tau^* > 0$.

For Case (ii) the proof is similar. However, if parameters are such that Case (ii) obtains *and* $b^{max} < \underline{b}$ no credible bonus is high enough to induce the agent to work. $\qquad\square$

A.4 Comparative Statics of τ^{max} and τ^{min}

The bonuses b^{min} and b^{max} put limits on the effort levels that the principal can implement. Denote by τ^{min} and τ^{max} the optimal effort levels as chosen by the agent when presented with a bonus of b^{min} and b^{max} respectively. The following Result summarizes some comparative statics with regard to the lowest and highest possible effort levels, τ^{min} and τ^{max}, which the principal can implement.

Result 5. (i) $\frac{d\tau^{min}}{d\psi} > 0$ and $\frac{d\tau^{max}}{d\psi} > 0$, (ii) $\frac{d\tau^{min}}{dg} > 0$ and $\frac{d\tau^{max}}{dg} > 0$ if ψ is sufficiently small, (iii) $\frac{d\tau^{min}}{dx} > 0$ and $\frac{d\tau^{max}}{dx} > 0$ if ψ is sufficiently large, (iv) $\frac{d\tau^{max}}{d\theta} > 0$ and $\frac{d\tau^{min}}{d\theta} > 0$ if ψ is sufficiently large, (v) $\frac{d\tau^{max}}{d\rho} > 0$ regardless of the size of ψ and $\frac{d\tau^{min}}{d\rho} > 0$ if ψ is sufficiently small.

Proof. A change in parameters has two effects on τ^{min} and τ^{max}. First, there is an effect through the incentive compatible bonus $b(\hat{\tau})$. That is, the price of effort changes. Second, there is an effect through b^{min} and b^{max}. The overall effect will depend on the sign and magnitude of these two effects.

τ^{min} and τ^{max} are implicitly given by

$$b^{max} = \left(\rho + (1-\rho)x\right)\psi q^* = b(\tau^{max})$$
$$b^{min} = \frac{(1-g)\tau}{1-g\tau}(1-\rho)x\psi q^* = b(\tau^{min})$$

These equations will be used to compute the comparative statics of τ^{min} and τ^{max}. Let $F^{min} = b^{min} - b(\hat{\tau})$ and $F^{max} = b^{max} - b(\hat{\tau})$. Then, for some parameter κ, $\frac{d\tau^{min/max}}{d\kappa} = -\frac{\partial F^{min/max}/\kappa}{\partial F^{min/max}/\tau^{min/max}}$. Notice first with respect to the sign of the denominator that $\frac{\partial F^{max}}{\partial \tau^{max}} < 0$ since $b(\hat{\tau})$ is increasing in τ in the relevant range and b^{max} is independent of τ.

For the sign of $\frac{\partial F^{min}}{\partial \tau^{min}}$ we notice that both b^{min} and $b(\hat{\tau})$ are decreasing in τ. To determine the sign of $\frac{\partial F^{min}}{\partial \tau}$ we have to take a closer look at F^{min}. The slope of b^{min} in $\tau = 0$ is positive while the slope of $b(\hat{\tau})$ in $\tau = 0$ is equal to zero. In the limit where $\tau = 1$ we have $b(\tau = 1) = \infty$ while b^{min} takes on a positive finite value. We can conclude that b^{min} starts out above $b(\hat{\tau})$ but eventually crosses $b(\hat{\tau})$ for large enough effort levels. Therefore, F^{min} is increasing for small values of τ and decreasing above a certain value of τ. Consequently, F^{min} will always be decreasing in τ^{min}.

Part (i). The determining part is the sign of $\frac{\partial F^{min/max}}{\partial \psi}$. Since $b(\hat{\tau})$ does not depend on ψ and b^{min} is increasing in ψ it must be the case that $\frac{\partial F^{min}}{\partial \psi} > 0$. Then we can conclude that $\frac{d\tau^{min}}{d\psi} > 0$. The argumentation is the same for $\frac{d\tau^{max}}{d\psi}$. Since $\frac{\partial b^{max}}{\partial \psi} > 0$ it holds that $\frac{\partial F^{max}}{\partial \psi} > 0$ and consequently $\frac{d\tau^{max}}{d\psi} > 0$.

The parameter ψ reflects the principal's cost of conflict. A higher level of ψ increases both the minimum and the maximum credible bonuses, b^{min} and b^{max}, but it does not change the price of effort, $b(\hat{\tau})$. Consequently, if the principal receives signal H, with a higher value of ψ but an unchanged price of effort $b(\hat{\tau})$ he will find it less tempting to lie about his signal relative to a situation with a low value of ψ. Intuitively, τ^{max} is increasing in the principal's cost of conflict ψ. In the case where the principal receives signal L, a higher value of ψ makes it less tempting to tell the truth since conflict is now more costly. Therefore b^{min} increases which pulls in the direction of a higher level of τ^{min} since $b(\hat{\tau})$ is unchanged.

For this reason τ^{min} and τ^{max} are increasing in ψ. Intuitively, when ψ increases the principal will find it less tempting to cheat on the agent by lying since the expected cost of conflict is now higher. He can therefore credibly offer higher bonuses than for lower values of ψ. Likewise, for higher values of ψ the principal will more often prefer to pay out the bonus regardless of state and in this way avoid potential conflict costs. Consequently, there are now some bonuses too low to be credible compared to a situation with a lower value of ψ. Thus, τ^{min} increases.

Part (ii). Because λ is decreasing in g, the value of b^{min} and b^{max} will depend on g through the optimal conflict level. Focus first on b^{min} and differentiate this with respect to g

$$\frac{\partial b^{min}}{\partial g} = -\frac{\tau(1-\tau)(1-g\tau)}{(1-g\tau)^2}(1-\rho)x\psi q^* + \frac{(1-g)\tau}{1-g\tau}(1-\rho)x\psi\frac{\partial q^*}{\partial \lambda}\frac{\partial \lambda}{\partial g} < 0$$

Now we need to sign the effect on the incentive compatible bonus $b(\hat{\tau})$ when g changes. Remember that $b(\hat{\tau})$ is implicitly given by

$$b\left(g - (1-g)(1-\rho)x\theta\lambda(1-q^*)\right) = v'(\tau) + (1-g)(1-\rho)xc(q^*) \qquad (14)$$

Differentiating both sides with respect to g yields

$$\frac{\partial b}{\partial g}\left[g - (1-g)(1-\rho)x\theta\lambda(1-q^*)\right]$$

$$= -b\left(1 + (1-\rho)x\theta\lambda(1-q^*) - (1-\rho)xc\left(q^*\right)\right) + b(1-g)(1-\rho)x\theta\lambda(1-q^*)\frac{\partial \lambda}{\partial g}$$

Since $\frac{\partial \lambda}{\partial g} < 0$ the right hand side is clearly negative. The term in the square brackets on the left hand side is positive for the relevant range of bonuses (see Appendix A.2) and as a result we conclude that $\frac{\partial b}{\partial g}$ is negative for all the relevant bonus levels.

Having determined that both $\frac{\partial b^{min}}{\partial g}$ and $\frac{\partial b}{\partial g}$ are negative, we see that the effects pull in opposite directions (a lower b^{min} pulls in the direction of a lower τ^{min}

while a lower $b(\hat{\tau})$ makes it more tempting for the principal to evade conflict by reporting H when his true signal is L. This pulls in the direction of a higher τ^{min}). However, $\frac{\partial b^{min}}{\partial g}$ is clearly less negative the smaller is the principal's cost of conflict ψ. Hence, there will exist a $\bar{\psi}$ such that for $\psi < \bar{\psi}$, $\frac{\partial F^{min}}{\partial g} > 0$ and consequently $\frac{d\tau^{min}}{dg} > 0$.

The argument is identical for $\frac{d\tau^{max}}{dg}$. $\frac{\partial b^{max}}{\partial g}$ is negative since $\frac{\partial \lambda}{\partial g}$ is negative. Therefore, again the effects pull in opposite directions. However, there will exist a $\bar{\psi}$ such that for $\psi < \bar{\psi}$, $\frac{\partial F^{max}}{\partial g} > 0$ and consequently $\frac{d\tau^{max}}{dg} > 0$.

Part (iii). $\frac{\partial b^{min}}{\partial x} = \frac{(1-g)\tau}{1-g\tau}(1-\rho)\psi q^*$ which is positive. For the effect on $b(\hat{\tau})$ we get

$$\frac{\partial b}{\partial x}\left[g - (1-g)(1-\rho)x\theta\lambda(1-q^*)\right] = (1-g)(1-\rho)\left[\theta\lambda b(1-q^*) + c(q^*)\right]$$

The left hand side is positive for the relevant range of bonuses (see Appendix A.2). The right hand side is positive and hence we conclude that $\frac{\partial b}{\partial x} > 0$.

Again, the effects pull in opposite directions. Now, $\frac{\partial b^{min}}{\partial x}$ is clearly more positive the larger is ψ whereas $\frac{\partial b}{\partial x}$ does not depend on ψ. As a consequence there will exist a $\tilde{\psi}$ such that for all $\psi > \tilde{\psi}$ $\frac{\partial F^{min}}{\partial x} > 0$ and therefore $\frac{\partial \tau^{min}}{\partial x} > 0$.

The argument is identical for the case of τ^{max}.

Part (iv). For the case of θ we have

$$\frac{\partial b^{min}}{\partial \theta} = \frac{(1-g)\tau}{1-g\tau}(1-\rho)x\psi\frac{\partial q^*}{\partial \theta} > 0$$
$$\frac{\partial b^{max}}{\partial \theta} = \left(\rho + (1-\rho)x\right)\psi\frac{\partial q^*}{\partial \theta} > 0$$

Where the inequalities hold since $\frac{\partial q^*}{\partial \theta} > 0$.

For the effect through $b(\hat{\tau})$ we have

$$\frac{\partial b}{\partial \theta}\left[g - (1-g)(1-\rho)x\theta\lambda(1-q^*)-\right] = b(1-g)(1-\rho)x(1-q^*)$$

The term in the square brackets on the left hand side is positive for the relevant range of bonuses (see Appendix A.2). Furthermore, the right hand side is positive. As a result we can conclude that $\frac{\partial b}{\partial \theta} > 0$.

Again, the effects pull in opposite directions. Now, $\frac{\partial b^{min}}{\partial \theta}$ is clearly more positive the larger is ψ whereas $\frac{\partial b}{\partial \theta}$ does not depend on ψ. As a consequence there will exist a $\tilde{\psi}$ such that for all $\psi > \tilde{\psi}$ $\frac{\partial F^{min}}{\partial \theta} > 0$ and therefore $\frac{\partial \tau^{min}}{\partial \theta} > 0$.

The argument is identical for the case of τ^{max}.

Part (v). For the effect of a change in ρ we have

$$\frac{\partial b^{min}}{\partial \rho} = -\frac{(1-g)\tau}{1-g\tau}x\psi q^* < 0$$

$$\frac{\partial b^{max}}{\partial \rho} = (1-x)\psi q_H^* > 0$$

For the effect through the incentive compatible bonus we see that

$$\frac{\partial b}{\partial \rho}\left[g - (1-g)(1-\rho)x\theta\lambda(1-q^*)\right] = -(1-g)xc\left[(\theta\lambda b(1-q^*) + c(q^*))\right]$$

The right hand side is negative. The term in the square bracket on the left hand side is positive for the relevant range of bonus levels (see Appendix A.2). As a result, $\frac{\partial b}{\partial \rho}$ is negative.

For the case of τ^{max} the effects pull in the same direction. A higher level of ρ makes effort cheaper and also makes truthtelling more attractive in the case where the principal receives signal H. Therefore we conclude that $\frac{\partial F^{max}}{\partial \rho} > 0$ and as a result $\frac{\partial \tau^{max}}{\partial \rho} > 0$.

For τ^{min} the result is ambiguous since $\frac{\partial b^{min}}{\partial \rho} < 0$ but also $\frac{\partial b}{\partial \rho} < 0$. However, if $\frac{\partial b^{min}}{\partial \rho}$ is not too negative - which will hold if ψ is not too large - we will have $\frac{\partial F^{max}}{\partial \rho} > 0$ and therefore $\frac{\partial \tau^{min}}{\partial \rho} > 0$.

$\square$

A.5 Proof of Proposition 1

The effect of a change in some parameter κ on equilibrium profits is given by

$$\frac{d\Pi(\tau^*)}{d\kappa} = \frac{\partial\Pi(\tau^*)}{\partial\kappa} + \frac{\partial\Pi(\tau^*)}{\partial\tau}\frac{d\tau^*}{d\kappa} \tag{15}$$

For the direct effect $\frac{\partial\Pi(\tau^*)}{\partial\psi}$ we know that

$$\frac{\partial\Pi}{\partial\psi} = -\tau\gamma_{LH}q^* < 0$$

That is, the direct effect of an increase in ψ is negative.

For the effect through the effort level we now focus on the second term of equation 15, $\frac{\partial\Pi(\tau^*)}{\partial\tau}\frac{d\tau^*}{d\psi}$. This effect will be zero when the chosen effort level is optimal, i.e. $\tau^* = \tilde{\tau}$. However, if the principal is bounded by the upper or lower truthtelling constraint he implements τ^{max} or τ^{min} respectively and $\frac{\partial\Pi(\tau^*)}{\partial\tau}\frac{d\tau^*}{d\psi}$ will be different from zero. Investigating first $\frac{\partial\Pi(\tau)}{\partial\tau}$ we see that

$$\begin{aligned}\frac{\partial\Pi(\tau)}{\partial\tau} &= \phi - v'(\tau) - \gamma_{LH}\left(\psi q^* + \theta\lambda b(1-q^*) + xc(q^*)\right) \\ &\quad -\tau\gamma_{LH}\left[\psi\frac{\partial q^*}{\partial b}\frac{\partial b}{\partial\tau} + \theta\lambda(1-q^*)\frac{\partial b}{\partial\tau}\right].\end{aligned}$$

Notice that for a fixed τ, $\frac{\partial\Pi(\tau)}{\partial\tau}$ is linearly increasing in ϕ. For a fixed value of ϕ the derivative is decreasing in τ.

Now recall from Result 5 that $\frac{d\tau^{max}}{d\psi} > 0$. Fix any $\tau^{max} \in (0,1)$. Then, there exists a ϕ' such that $\frac{\partial\Pi(\tau)}{\partial\tau}\big|_{\tau=\tau^{max}} > 0$ and $\tau^* = \tau^{max}$ for all $\phi > \phi'$. Then the indirect effect is negative, $\frac{\partial\Pi(\tau)}{\partial\tau}\big|_{\tau=\tau^*}\frac{d\tau^*}{d\psi} > 0$. Since $\frac{d\tau^{max}}{d\psi}$ and $\frac{\partial\Pi(\tau)}{\partial\psi}$ do not depend on ϕ and $\frac{\partial\Pi(\tau)}{\partial\tau}$ is linearly increasing in ϕ there exist a ϕ'' such that $\frac{d\Pi(\tau^*)}{d\psi} > 0$ for all $\phi > \tilde{\phi} \equiv \max(\phi',\phi'')$ meaning that the second effect through the implemented effort level dominates and welfare is increasing in the principal's cost of conflict.

A.6 Proof of Proposition 2

Similar to the proof of Proposition 1.

Again, we need to determine the signs of the direct and the indirect effects. For the direct effect $\frac{\partial \Pi(\tau)}{\partial \theta}$ we have

$$\frac{\partial \Pi}{\partial \theta} = -\tau \gamma_{LH} \lambda b \left(1 - q^*(b)\right) - \tau \gamma_{LH} \left(\psi \frac{\partial q^*(b)}{\partial b} \frac{\partial b}{\partial \theta} + \left(1 - q^*(b)\right) \frac{\partial b}{\partial \theta}\right).$$

From equation 4 we know that $\frac{\partial q_H^*(b)}{\partial b} > 0$ and from part (iv) of Appendix A.4 we know that $\frac{\partial b}{\partial \theta} > 0$. From this we conclude that $\frac{\partial \Pi}{\partial \theta} < 0$.

Part (i) For the sign of the indirect effect, fix any $\tau^{max} \in (0,1)$ with a ψ in accordance with Result 5 such that $\frac{d\tau^{max}}{d\theta} > 0$. There exist a project value ϕ' such that $\tau^* = \tau^{max}$. Since $\frac{d\tau^{max}}{d\theta}$ and $\frac{\partial \Pi(\tau)}{\partial \theta}$ do not depend on ϕ and $\frac{\partial \Pi(\tau)}{\partial \tau}$ is linearly increasing in ϕ there exist a ϕ'' such that $\frac{d\Pi(\tau^*)}{d\theta} > 0$ for all $\phi > \tilde{\phi} \equiv \max(\phi', \phi'')$.

Part (ii) If the principal is bounded by the lower truthtelling constraint we have $\frac{\partial \Pi(\tau^*)}{\partial \tau} < 0$. There exist a project value $\tilde{\phi}$ such that $0 < \tilde{\tau} < \tau^{min}$ and therefore $\tau^* = \tau^{min}$ for all $\phi < \tilde{\phi}$. Then fix a ψ in accordance with Result 5 such that $\frac{d\tau^{min}}{d\theta} < 0$.

Now, notice that $\frac{\partial \Pi(\tau^*)}{\partial \tau}$ is decreasing in τ for a fixed ϕ and is more negative the larger is $v'(\tau)$. Further, $\frac{d\tau^{min}}{d\theta}$ is larger, the smaller is $v''(\tau)$. Hence, we have that $\frac{\partial \Pi(\tau)}{\partial \theta}$ is independent of $v(\tau)$ and its derivatives whereas $\frac{\partial \Pi(\tau)}{\partial \tau} \frac{d\tau^{min}}{d\theta}$ is increasing in $\frac{v'(\tau)}{v''(\tau)}$. Fix a positive real number z. Then there exists an effort cost function $v(\tau)$ such that $\frac{v'(\tau)}{v''(\tau)} > z$. Hence, $\frac{d\Pi(\tau)}{d\theta} > 0$ if z is sufficiently large.

A.7 Proof of Proposition 3

Similar to the proof of Proposition 1. First we investigate the sign of $\frac{\partial \Pi(\tau)}{\partial \rho}$.

$$\frac{\partial \Pi}{\partial \rho} = -\tau \frac{d\gamma_{LH}}{d\rho} \left(\psi q^* + \theta \lambda b (1 - q^*) + c(q^*) \right) - \tau \gamma_{LH} \left(\psi \frac{\partial q^*}{\partial b} \frac{\partial b}{\partial \rho} + \theta \lambda (1 - q^*) \frac{\partial b}{\partial \rho} \right) > 0$$

where the inequality holds since $\frac{d\gamma_{LH}}{d\rho}$ is negative, we know from equation 4 that $\frac{\partial q^*}{\partial b}$ is positive and from Appendix A.4 that $\frac{\partial b}{\partial \rho}$ is negative.

Now, fix any $\tau^{min} \in (0,1)$ with a ψ small enough such that $\frac{d\tau^{min}}{d\rho} > 0$ and a positive real number z. There exist a project value $\tilde{\phi}$ such that $\tau^* = \tau^{min}$ and there exists an effort cost function such that $\frac{v'(\tau^{min})}{v''(\tau^{min})} > z$. Notice that $\frac{\partial \Pi(\tau)}{\partial \rho}$ is independent of $v(\tau)$ and its derivatives. Furthermore, $\frac{\partial \Pi(\tau)}{\partial \tau}\big|_{\tau=\tau^{min}} < 0$ and $\frac{\partial \Pi(\tau)}{\partial \tau} \frac{d\tau^{min}}{d\rho}$ is increasingly negative in $\frac{v'(\tau)}{v''(\tau)}$. For this reason there exists a z large enough such that $\frac{d\Pi(\tau)}{d\rho}$ is negative.

A.8 Proof of Proposition 4

Similar to the proof of Proposition 1.

For the direct effect we have

$$\frac{\partial \Pi}{\partial x} = -\tau \frac{d\gamma_{LH}}{dx} \left(\psi q^* + \theta \lambda b (1 - q^*) + c(q^*) \right) - \tau \gamma_{LH} \left(\psi \frac{\partial q^*}{\partial b} \frac{\partial b}{\partial x} + \theta \lambda (1 - q^*) \frac{\partial b}{\partial x} \right) < 0$$

where the inequality holds since $\frac{d\gamma_{LH}}{dx} > 0$ is positive, $\frac{\partial q^*}{\partial b} > 0$ (equation 4) and $\frac{\partial b}{\partial x} > 0$ (Appendix A.4).

Part (i) For the indirect effect fix any $\tau^{min} \in (0,1)$ with a ψ in accordance with Result 5 such that $\frac{d\tau^{min}}{dx} < 0$ and a positive real number z. There exist a project value ϕ' such that $\tau^* = \tau^{min}$ and there exists an effort cost function such that $\frac{v'(\tau^{min})}{v''(\tau^{min})} > z$. Notice that $\frac{\partial \Pi(\tau)}{\partial x}$ is independent of $v(\tau)$ and its derivatives. Furthermore, $\frac{\partial \Pi(\tau)}{\partial \tau}\big|_{\tau=\tau^{min}} < 0$ and $\frac{\partial \Pi(\tau)}{\partial \tau} \frac{d\tau^{min}}{dx}$ is increasingly negative in $\frac{v'(\tau)}{v''(\tau)}$. For this reason there exists a z large enough such that $\frac{d\Pi(\tau)}{dx}$ is positive.

Part (ii) For the indirect effect fix any $\tau^{max} \in (0,1)$ with a ψ in accordance with Result 5 such that $\frac{d\tau^{max}}{dx} > 0$. There exist a project value ϕ' such that $\tau^* = \tau^{max}$. Since $\frac{d\tau^{max}}{dx}$ and $\frac{\partial \Pi(\tau)}{\partial x}$ do not depend on ϕ and $\frac{\partial \Pi(\tau)}{\partial \tau}$ is linearly increasing in ϕ there exist a ϕ'' such that $\frac{d\Pi(\tau^*)}{dx} > 0$ for all $\phi > \tilde{\phi} \equiv \max(\phi', \phi'')$.

A.9 Proof of Proposition 5

The effect of a change in g on equilibrium profits is given by

$$\frac{d\Pi(\tau^*)}{dg} = \frac{\partial\Pi(\tau^*)}{\partial g} + \frac{\partial\Pi(\tau^*)}{\partial\tau}\frac{d\tau^*}{dg}.$$

Part (i) When the principal is not bounded by either truthtelling constraint it holds that $\frac{\partial\Pi(\tau^*)}{\partial\tau} = 0$ and hence the total welfare effect from a change in g will be given by the first term $\frac{\partial\Pi(\tau^*)}{\partial g}$. The sign of this term is investigated below.

$$\frac{\partial\Pi}{\partial g} = -\tau\frac{d\gamma_{LH}}{dg}\left(\psi q^* + \theta\lambda b(1-q^*) + c(q^*)\right)$$
$$-\tau\gamma_{LH}\left(\psi\frac{\partial q^*}{\partial b}\frac{\partial b}{\partial g} + \theta(1-q^*)\frac{\partial b}{\partial g} + \theta b(1-q^*)\frac{d\lambda}{dg}\right).$$

We know that $\frac{\partial\gamma_{LH}}{\partial g} < 0$ and $\frac{\partial q^*}{\partial b} > 0$. From Appendix A.4 we know that $\frac{\partial b}{\partial g} < 0$. Thus we conclude that $\frac{\partial\Pi}{\partial g}$ is unambiguously positive. As a result, welfare is increasing in the quality of the principal's signal when he is not bounded by truthtelling constraints.

Part (ii)(a) We have just shown that $\frac{\partial\Pi}{\partial g} > 0$. Now fix any $\tau^{max} \in (0,1)$ with a ψ in accordance with Result 5 such that $\frac{d\tau^{max}}{dg} < 0$. There exists a project value ϕ' such that $\frac{\partial\Pi(\tau)}{\partial\tau}\big|_{\tau=\tau^{max}} > 0$. Then $\frac{\partial\Pi(\tau)}{\partial\tau}\big|_{\tau=\tau^{max}}\frac{d\tau^{max}}{dg} < 0$. Since $\frac{d\tau^{max}}{dg}$ and $\frac{\partial\Pi(\tau)}{\partial g}$ are independent of ϕ and $\frac{\partial\Pi(\tau)}{\partial\tau}$ is linearly increasing in ϕ for a fixed $\tau = \tau^{max}$, there exists a project value ϕ'' such that $\frac{d\Pi(\tau^*)}{dg} < 0$ for all $\phi > \tilde{\phi} \equiv \max(\phi', \phi'')$.

Part (ii)(b) Fix any $\tau^{min} \in (0,1)$ with a ψ in accordance with Result 5 such that $\frac{d\tau^{min}}{dg} > 0$ and a positive real number z. There exist a project value ϕ' such that $\tau^* = \tau^{min}$ and there exists an effort cost function such that $\frac{v'(\tau^{min})}{v''(\tau^{min})} > z$. Notice that $\frac{\partial\Pi(\tau)}{\partial g}$ is independent of $v(\tau)$ and its derivatives. $\frac{\partial\Pi(\tau)}{\partial\tau}\frac{d\tau^{min}}{dg}$ on the other hand is increasingly negative in $\frac{v'(\tau)}{v''(\tau)}$. For this reason there exists a z large enough such that $\frac{d\Pi(\tau)}{dg}$ is negative.
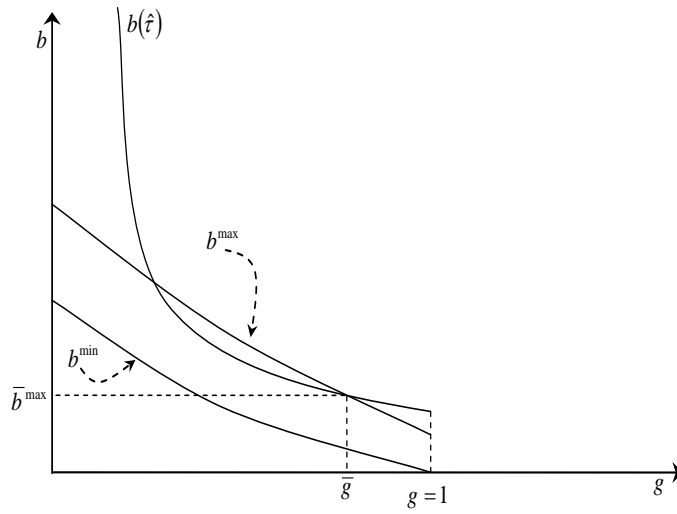
Figure 1: The Quality of the Evaluation Process: *Case (2)*