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Essays in the Economics of Transport

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# **Essays in the Economics of Transport**

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#### **Preface and Acknowledgments**

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Changes since the dissertation was submitted

Since the dissertation was submitted in December 2006 and the public defense which

took place in Marts 2007 a few things have happened. First, a revised version of paper 1

titled 'Externalities, taxation and time allocation' has been accepted for publication in the

International Journal of Transport Economics. Secondly, it has been brought to my

attention that there were some typing mistakes in paper 3 and paper 4. These have been

corrected in the present version of the dissertation.

Jens Erik Nielsen

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#### **Summary and Introduction**

The purpose of this introduction is to summarize the five papers that constitute this thesis. Each paper is self-contained and can be read independently of the others but they all centers around the same theme; the economics of transport. Papers one and five are theoretical dealing with questions related to the theory of taxation. The remaining three are empirical and centers on the estimation of car ownership and the use of cars in Denmark.

The first paper, Taxation, Time Allocation, and Externalities, derives the optimal tax rules in a model of household time allocation and atmospheric externalities based on Becker (1965). In a situation without externalities and without distributional considerations the optimal tax structure in such a setting is derived in Kleven (2004). We extend on his findings in two ways. First, we include atmospheric externalities thus generalizing the model making it more in line with the situation found in the transport sector. Secondly we allow households to differ and implement a more general social welfare function than the one used by Kleven thereby introducing distributional considerations in the setup.

We show that the additivity property derived in Sandmo (1975) survives in the present setup, even though it has to be modified to cope with distributional issues and time allocation. We show that the definition of the net social marginal utility of income defined in Diamond (1975) enters the optimal tax formula in a modified form which includes income effects on the externality. Since the modified net social marginal utility

of income enters the tax formula for both polluting and non-polluting goods the additivity property in the pure form, where the tax on non-polluting goods are independent of the externality fails. We show that the substitution effects still enters the tax formula for the externality generating good in an additive way and the additivity property thus survives in a modified form. Furthermore we find that the factor share for the polluting good also enters the additive term and the corrective term therefore no longer equals the pigouvian marginal cost.

The policy implications of the insights obtained are obvious. As in Kleven (2004) we show that fast modes of transport should be taxed less than slow modes and that the modified additivity property states that a corrective tax should be levied on the polluting good only. The optimal corrective tax is not equal to the pigouvian level found in Sandmo (1975) since the time allocation has to be accounted for. If two modes pollute at the same level the corrective tax on the fastest mode should be set at a lower rate than the one at the slow mode.

The result from this paper addresses the present policy debate regarding taxation of aviation fuel according to the emission of greenhouse gasses (CO2). Lately the Danish minister of the environment, Connie Hedegaard, has argued that airline traffic should be taxed according to the pigouvian principle since it emits large quantities of greenhouse gasses compared to car transport. Using the results presented in paper 1 we know that this argument is not a clear-cut case. It might be true that airline traffic emit higher quantities of greenhouse gasses but it is also true that for many trips it saves time to use airlines

instead of cars. Our result states that if the time savings are high enough there is an argument for having lower emission taxes on airlines than on cars. How the taxes are to be set is an empirical matter which the paper does not address.

The second paper, Estimation of car ownership in Denmark - Discrete choice modeling and repeated cross-section analysis, examines the demand for cars in Denmark by using simple cross-section method. In this paper the problem of parameter instability in such models are addressed and we hypothesizes that omission of a variable for household wealth in the form of real estate values causes the estimates of income elasticities to be upward biased.

To examine this we follow the same path as Pendyala et al. (1995) and use repeated cross-section data to estimate a simple multinomial model for car ownership and examine how the demand for cars evolves over time. The data used comes from the Danish Transport Diary Survey which is an interview based survey conducted on a monthly basis. The problem of parameter instability simple cross-section models is well known and it is also known that the omission of important macro variables can cause estimated parameters and elasticities to be biased. Due to lack of data one often has to relay on this approach even though more sophisticated frameworks should be used. The problems mentioned above therefore remains.

In the paper we find that the estimated income elasticities are non-decreasing over time, which is expected from other studies. Furthermore we find that including a variable for

real estate values in the Danish municipalities reduce the estimated income elasticities. If our hypothesis that the real estate values influence the demand for cars we should see the largest changes in income elasticities for households who live in areas with the highest values of real estate. By showing that the income elasticities for real estate owners in urban areas are affected more than for other households and with real estate owners in general being affected more by the inclusion of a variable for real estate values we conclude that our hypothesis is correct.

The third paper extends on this finding and uses a dynamic model to examine the effect of the housing prices in more detail.

In paper three, Real estate ownership and the demand for cars in Denmark - A pseudopanel analysis, the investigation of the influence of the rising real estate prices and the
falling interest rate, which was started in paper two, continues. Inspired by Dargay and
Vythoulkas (1999) we construct a dynamic partial adjustment model for car ownership in
Denmark. Using the approach described in Deaton (1985) we use data from the Danish
Transport Diary survey to construct a pseudo panel and combine this with aggregate time
series from Statistics Denmark for the development in real estate values in the Danish
municipalities and the development in the long-term interest rate. We hypothesis that the
increasing real estate values have increased the demand for cars and that the falling
interest rate also increase car demand.

With the rising real estate prices and the falling interest rate we have in Denmark a situation where real estate owners can redraw equity from their real estate without increasing their monthly expenses. These households have thus received a capital gain which tenants have not and real estate owners could therefore have increased their demand for cars. If this hypothesis is true we expect that the increasing housing prices influence the demand for cars for real estate owners but not for tenants. The influence of the interest rate is less clear since all households face approximately the same interest rate if differences in capital restrictions are ignored. What we find is that only real estate owners are affected by the increasing real estate prices but all households increase their demand for cars due to the falling interest rate. Our hypothesis is thus confirmed.

Our findings in this paper is important since excluding capital gains from models used for forecasting could cause estimates to be misleading. We have shown that future models should keep this in mind and if possible include variables for capital gains and especially gains originating from the real estate marked. Unfortunately we have not been able to examine the effect of falling real estate prices and increasing interest rates. This should be done since we should not expect the responses to be symmetric; a finding which Dargay (2001) found with regards to income where hysteresis exist in the demand for cars.

**Paper four**, *Demand for car transport in Denmark- Differences between rural and urban car owners*, continues the investigation of differences between car owners in rural and urban areas which was started in Dargay (2002). The paper contributes to the literature in

two ways. Since Dargay only used a constructed value for car travel derived from the UK household expenditure survey one could question the validity of the results. By using a transport survey and obtaining estimates which are comparable with Dargays results we thus show that results obtained from household expenditure surveys can provide credible results about transport behavior. Secondly we show that the purchase price and fuel prices affects rural and urban households differently. This is an important insight since it helps us to understand how different policies affect households living in different areas. We show that rural households respond mostly to changes in the purchase price on cars whereas urban households respond more to changes in fuel price (or variable costs).

The last paper, *Transport tax reforms, two-part tariffs, and revenue recycling*, construct a model for commuting traffic. The consumers consume a composite commodity, leisure and choose between public transport (a metro) and private transport (car) when they commute. The model is thus based on the framework presented in Parry and Bento (2001). The model extends on previous work by incorporating the discrete nature of car purchase in a tax model which (to our knowledge) has not been done before.

#### References

Becker (1965) A Theory of the Allocation of Time, *The Economic Journal* **75**, 493-517.

Dargay, J. M and Vythoulkas, P. C. (1999) Estimation of a Dynamic Car Ownership Model: A Pseudo-panel Approach, Journal of Transport Economics and Policy 33:3, 287-302.

Dargay, J. M. (2001) The effect of income on car ownership: evidence and asymmetry, *Transportation Research Part A* **35**, 807-821.

Dargay, J. M. (2002) Determinants of car ownership in rural and urban areas: a pseudopanel analysis, Transportation Research Part E, 38, 351-366.

Deaton, A. (1985) Panel Data from Time Series of Cross Sections, *Journal of Econometrics* **30**, 109-126.

Diamond, P. A. (1975) A many-person Ramsey tax rule, *Journal of Public Economics* **4**, 335-342.

Kleven, H. J. (2004) Optimum Taxation and the Allocation of Time, *Journal of Public Economics* **88**, 545-557.

Parry, I. W. H. & Bento, A. (2001) Revenue Recycling and the Welfare Effects of Road Pricing, *Scandinavian Journal of Economics* 103:4, 645-671.

Pendyala, R. M., Kostyniuk, L. P. & Goulias, K. G. (1995) A repeated cross-sectional evaluation of car ownership, *Transportation* **22**, 165-184.

Sandmo, A. (1975) Optimal Taxation in the Presence of Externalities, *Swedish Journal of Economics*. 77, 86-97.

## **Taxation, Time Allocation, and Externalities**

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#### **Abstract**

This paper shows that the traditional additivity property of Sandmo (1975) for taxation of atmospheric externalities needs modification when household time allocation is modeled in a Becker (1965) setup and it address the question of distributional consequences in a model with explicit modeling of household time allocation. The insights obtained have important policy implications in the transport sector, since we show that time saving activities should have their externality tax reduced compared to the pure Pigouvian case. The traditional arguments for marginal cost pricing thus looses some of its appeal.

#### 1. Introduction

This paper models household time allocation in a setup with atmospheric externalities. We assume that households require both household time and market produced commodities in the production of consumption goods in line with Becker (1965). This setup is particular relevant when the theory of optimal taxation is applied to the transport sector since one of the main characteristics of transportation is the use of time. Furthermore, personal transport produced by households takes up a large fraction of household time budgets and transport is also a significant source of externalities. In a situation without externalities and without distributional considerations the optimal tax rules for the present setup is derived in Kleven (2004). He derives a so called inverse factor share rule which state that activities with a high time share and thus a low factor share in household production should be taxed at a higher rate than activities which a low time share and a high factor share. The present paper extends this result by including externalities and considering distributional concerns.

It is shown here that the additivity property found in Sandmo (1975) has to be modified when household production and time allocation is modeled explicitly in the presence of externalities. The additivity property states that if the externality generating good can be taxed directly, efficiency requires that this good is taxed by combining the Pigouvian principle and Ramsey taxation and that the externality level only enters the tax formula for the externality generating good. The Pigouvian tax is additive thus making a clear separation between the efficiency part of the tax formula and the part of the tax formula which correct for the externalities. When the allocation of time in household production

is included in the setup we show that the corrective tax has to account not only for the externalities but also for the time cost involved. This insight invalidates the conclusion that the internalizing part of the tax should be set equal to the marginal external damage costs. Furthermore we show that the definition of the net social marginal utility of income defined in Diamond (1975) has to be modified to account for the negative welfare effects of the externalities if distributional issues are included in the model.

Allowing for distributional issues in the economy does not invalidate the additivity property but it has to be modified. We show that since the net social marginal utility of income is modified to include the income effect on the level of the externality the Ramsey part of the tax problem is no longer independent of the externality. The corrective term containing substitution effects still enters additive in the tax formula for the good causing externalities and the additivity property therefore remains valid.

It has almost become the convention in the transport sector that a system of road pricing where the tax is set equal to the marginal external cost is the most efficient way to internalize externalities. In the present setup this no longer holds. The tax should also account for the time allocation inside the households. A direct consequence is that a transport mode which saves time should carry a lower corrective tax *even* if the marginal external damage costs for the different modes are the same. For example even if cars cause more atmospheric pollution than busses the optimal corrective tax on cars might be lower than the tax on busses. This can happen if the time saved from using the car is high enough. The insight can also be applied to the taxation of airlines where politicians are

discussing if a tax should be put on aviation fuel in order to reduce the emission of greenhouse gasses. Since air transport in general saves time compared to other modes of transport our result says that the internalizing tax should be set lower on aviation fuel relative to fuel used in car transport even if the marginal damage caused by the two modes are the same.

Similar insights concerning pigouvian taxation are obtained from a different setup in Parry and Bento (2001). They analyze tax reforms in a simple labor-leisure model with two types of transport to show that when distortions on the labor market are included it might not be welfare improving to internalize externalities by setting the tax equal to the marginal external damage cost. Using their model they investigate different tax reforms in the transport sector and use numerical simulation to illustrate their results. The Parry and Bento result is extended in several recent papers. In Parry and Bento (2002) they extend their analysis and investigate more types of externalities and interactions with other taxes. De Borger and Van Dender (2003) show, using the same framework as Parry and Bento, that the value of time is also affected by the tax system and in Nielsen (2006) the setup is extended to include the discrete choice of car purchase.

None of the above mentioned papers derive optimal tax rules in models with time allocation and externalities. This paper thus contributes to the literature by deriving the optimal tax rule in a model with explicit modeling of household time allocation, atmospheric externalities, and by including distributional considerations.

The paper is organized as follows. Section 2 sets up the model describing the households' time allocation problem and the governments' problem of raising revenue. Section 3 derives the optimal tax rules and the final section concludes.

#### 2. The Model

As in the standard setup the model used here includes a utilitarian social planner and a set of H households together with N+1 commodities. Households do not consume commodities bought in the market directly. Instead they undertake a production where market produced commodity,  $X_i$ , and household time,  $L_i$ , are used to produce consumption good,  $Z_i$ , through a production process,  $f^i$ . Letting h describe the individual household, they each have a utility function given by

$$U^{h} = U^{h}(Z_{0}^{h}, Z_{1}^{h}, ..., Z_{N}^{h}) - \phi^{h}(\overline{Z}_{N}), \quad h = 1, ..., H$$
(1)

where

$$\overline{Z}_N = \sum_{h=1}^H Z_N^h \tag{2}$$

is the total consumption of good N in the economy and

$$Z_i^h = f^i(X_i^h, L_i^h), \quad i = 0, ..., N$$
 (3)

represents the way good  $Z_i^h$  is being produced in household h using market produced commodity,  $X_i^h$ , household time,  $L_i^h$ , and production technology,  $f^i$ . The production technologies,  $f^i$ , are Leontief and every household uses the same set of technologies when producing consumption goods. This means that if a household wants to see a movie at a cinema they have to allocate the time required to see the movie and they have to purchase movie tickets. One could argue that more than one commodity could be

required in the production which would result in  $X_i$  being a vector of these market produced commodities, but to keep things simple we assume that only one commodity goes into the production of every consumption good<sup>1</sup>.

In this setup the consumption of good N generates externalities. It is assumed that total consumption of good N and not only total consumption of market produced commodities that causes externalities. Considering transport as the main example in this paper, this is the intuitive choice and changing it will not affect the conclusions. The function  $\phi^h(\overline{Z}_N)$  is assumed to be increasing in  $\overline{Z}_N$  so that

$$\phi^{h'} = \frac{\partial \phi^{h}(\overline{Z}N)}{\partial \overline{Z}N} > 0 \tag{4}$$

As a result the total consumption of good N decreases the utility of the households. Assuming that H is large the households consider  $\overline{Z}_N$  as fixed when making their choices. We therefore make the standard assumption that the individual household behaves as if  $\frac{\partial \overline{Z}_N}{\partial Z_N^h} = 0$ . This means that a household may realize that it affects the total consumption of good N but regards its contribution as insignificant. Using this we now formulate the optimization problem for household h as

$$\max_{X_0^h, \dots, X_N^h, L_0^h, \dots, L_N^h} U^h(f^0(X_0^h, L_0^h), f^1(X_1^h, L_1^h), \dots, f^N(X_N^h, L_N^h))$$
s.t.
$$\sum_{i=0}^N P_i X_i^h = w^h T^h$$

$$\sum_{i=0}^N L_i^h + T^h = T$$
(5)

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<sup>&</sup>lt;sup>1</sup> A discussion of this can be found in Pollak and Wachter (1975).

where  $T^h$  is the amount of time spent on work for household h,  $P_i$  is the consumer price of commodity i, T is the total time available to the households, and  $w^h$  is the wage rate for household h and given exogenously. Since  $\overline{Z}_N$  is considered to be constant we can leave out  $\phi^h(\overline{Z}_N)$  of the utility maximization problem (5).

The household production problem describing the way in which households choose  $X_i^h$  and  $L_i^h$  in order to produce one unit of consumption good  $Z_i^h$  in the cheapest possible way can be formulated as

$$\min_{X_{i}^{h}, L_{i}^{h}} P_{i}X_{i}^{h} + w^{h}L_{i}^{h} 
s.t.$$
(6)
$$f^{i}(X_{i}^{h}, L_{i}^{h}) = 1$$

Due to the Leontief production technology the solution to (6) gives the constant factor demand coefficients

$$a_{Xi} = \frac{X_i^h}{Z_i^h} \tag{7}$$

$$a_{Li} = \frac{L_i^h}{Z_i^h} \tag{8}$$

These are identical for all households since they use identical technologies. Using (7) and (8) together with the fact that the two constraints in (5) are interdependent through the variable  $T^h$ , we can restate the households maximization problem as

$$\max_{Z_0^h,...,Z_N^h} U^h(Z_0^h, Z_1^h,..., Z_N^h)$$
s.t.
$$\sum_{i=0}^N Q_i^h(P_i, w^h) Z_i^h = I^h(w^h)$$
(9)

where  $I^h(w^h) = w^h T$  is the households total income and  $Q_i^h(P_i, w^h) = P_i a_{Xi} + w^h a_{Li}$ . To see this remember that  $a_{Xi}$  and  $a_{Li}$  are constant and identical for all households. Multiplying the time constraint in (5) with  $w^h$  and adding it to the budget constraint in (5) gives (9). Note that  $P_i a_{Xi}$  is the direct monetary cost of using  $X_i^h$  as input and  $w^h a_{Li}$  is the value of the time used for the production which equals the earnings lost due to lower working time<sup>2</sup>.  $Q_i^h(P_i, w^h)$  is therefore the total opportunity cost of consuming one unit of  $Z_i^h$  for household h.

With (9) being a standard utility maximization problem the solution gives the ordinary demand functions  $Z_i^h(Q_0^h,Q_1^h,...,Q_N^h,I^h)$  and the indirect utility function  $V^h(Q_0^h,Q_1^h,...,Q_N^h,I^h)$ . Note that the households' indirect utility function does not include externalities. We also know that standard results like Roy's Identity, which states that

$$\frac{\partial V^h}{\partial Q_k^h} = -\lambda^h Z_k^h, \quad k = 0, ..., N$$
(10)

will apply where  $\lambda^h$  is the Lagrangian multiplier from the households utility maximization problem (9) and thus equals marginal utility of income for consumer h. We also know that the Slutsky equation

$$\frac{\partial Z_k^h}{\partial Q_j^h} = \frac{\partial \widehat{Z}_k^h}{\partial Q_j^h} - Z_j^h \frac{\partial Z_k^h}{\partial I^h}, \quad j, k = 0, ..., N$$
(11)

holds where  $\hat{Z}_k^h$  is the compensated demand function for good k.

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<sup>&</sup>lt;sup>2</sup> We have assumed that the value of time is equal to the wage rate. A large literature on the value of time exists and it is one of the main research areas in transport economics. A recent Danish publication on the topic of time values is Fosgerau and Pilegaard (2003) which also contains a survey of the time value literature.

Having characterized the households' behavior, we now focus on the government. The government knows that externalities are present and it therefore know that the true indirect utility function for household h is given by

$$\overline{V}^{h}(Q_{0}^{h}, Q_{1}^{h}, ..., Q_{N}^{h}, I^{h}) = V^{h}(Q_{0}^{h}, Q_{1}^{h}, ..., Q_{N}^{h}, I^{h}) - \phi(\sum_{h=1}^{H} Z_{N}^{h}(Q_{0}^{h}, Q_{1}^{h}, ..., Q_{N}^{h}, I^{h}))$$
(12)

We assume that the government seeks to maximize a Bergson-Samuelson type social welfare function  $W = W(\overline{V}^1, \overline{V}^2, ..., \overline{V}^H)$  where  $\frac{\partial W}{\partial \overline{V}^h} > 0$ . Assuming that the production sector operates under constant returns to scale and that all markets are fully competitive the producer price,  $p_i$ , for commodity  $X_i$  is fixed and equal to the marginal cost of production. We assume that good 0 is pure leisure and thus having  $a_{X0} = 0$  making good 0 untaxable since no market produced commodity is used in the production of  $Z_0$ . We define the tax rate on commodity  $X_i$  as

$$t_i = P_i - p_i, \quad i = 1, ..., N$$
 (13)

The government therefore has full control of all commodity prices. Furthermore the government must raise an externally given revenue, G, for some unspecified tasks (e.g. defense, healthcare, infrastructure) resulting in the governmental budget constraint

$$\sum_{i=0}^{N} \left( t_i \sum_{h=1}^{H} X_i^h \right) = G \tag{14}$$

We can now write the governments welfare maximization problem as

$$\max_{P_{1}, P_{2}, \dots, P_{N}} W(\overline{V}^{0}, \overline{V}^{1}, \dots, \overline{V}^{H})$$
s.t.
$$\sum_{i=1}^{N} ((P_{i} - p_{i}) \sum_{h=1}^{H} a_{Xi} Z_{i}^{h}) = G$$
(15)

since  $a_{Xi}Z_i^h = X_i^h$  and  $a_{X0} = 0$ . The Lagrangian function emerging from (15) is given by

$$L = W(\overline{V}^{0}, \overline{V}^{1}, ..., \overline{V}^{H}) + \mu \sum_{i=1}^{N} ((P_{i} - p_{i})(\sum_{h=1}^{H} a_{Xi}Z_{i}^{h}) - G)$$
(16)

where  $\mu$  is the Lagrangian multiplier from (15) and representing marginal value of government funds. The first order conditions for the governments welfare maximization problem (15) can now be written as

$$\frac{\partial L}{\partial P_{k}} = \sum_{h=1}^{H} \frac{\partial W}{\partial \overline{V}^{h}} \left( \frac{\partial V^{h}}{\partial Q_{k}^{h}} \frac{\partial Q_{k}^{h}}{\partial P_{k}} - \frac{\partial \phi^{h}}{\partial \overline{Z}_{N}} \sum_{\bar{h}=1}^{H} \frac{\partial Z_{\bar{N}}^{\bar{h}}}{\partial Q_{k}^{\bar{h}}} \frac{\partial Q_{\bar{k}}^{\bar{h}}}{\partial P_{k}} \right) 
+ \mu \sum_{h=1}^{H} a_{Xk} Z_{k}^{h} + \mu \sum_{i=1}^{N} t_{i} \sum_{h=1}^{H} a_{Xi} \frac{\partial Z_{i}^{h}}{\partial Q_{k}^{h}} \frac{\partial Q_{k}^{h}}{\partial P_{k}} = 0, \quad k = 1, ..., N$$
(17)

which (by using Roy's identity (10) and the fact that  $\frac{\partial Q_k^h}{\partial P_k} = a_{Xk}$ ) can be written as

$$\sum_{h=1}^{H} \frac{\partial W}{\partial \overline{V}^{h}} ((-\lambda^{h} Z_{k}^{h}) - \phi^{h}' \sum_{\overline{h}=1}^{H} \frac{\partial Z_{N}^{\overline{h}}}{\partial Q_{k}^{\overline{h}}}) = -\mu \sum_{h=1}^{H} Z_{k}^{h} - \mu \sum_{i=1}^{N} t_{i} \sum_{h=1}^{H} a_{Xi} \frac{\partial Z_{i}^{h}}{\partial Q_{k}^{h}}, \quad k = 1, ..., N$$
(18)

Applying the Slutsky Equation (11) now gives

$$-\sum_{h=1}^{H} \frac{\partial W}{\partial \overline{V}^{h}} (\lambda^{h} Z_{k}^{h}) - \sum_{h=1}^{H} \frac{\partial W}{\partial \overline{V}^{h}} \phi^{h} \sum_{\overline{h}=1}^{H} (\frac{\partial \overline{Z}_{N}^{\overline{h}}}{\partial Q_{k}^{\overline{h}}} - Z_{k}^{\overline{h}} \frac{\partial Z_{N}^{\overline{h}}}{\partial I^{\overline{h}}})$$

$$= -\mu \sum_{h=1}^{H} Z_{k}^{h} - \mu \sum_{i=1}^{N} t_{i} \sum_{h=1}^{H} a_{Xi} (\frac{\partial \overline{Z}_{i}^{h}}{\partial Q_{k}^{h}} - Z_{k}^{h} \frac{\partial Z_{N}^{h}}{\partial I^{h}}), \quad k = 1, ..., N$$

$$(19)$$

Following Diamond and Mirrlees (1971) and Diamond (1975) we define the *social* marginal utility of income  $\beta^h$  and the net social marginal utility of income  $\overline{\beta}^h$  for household h as

$$\beta^h = \frac{\partial W}{\partial \overline{V}^h} \lambda^h \tag{20}$$

$$\overline{\beta}^h = \beta^h + \mu \sum_{i=1}^N t_i a_{Xi} \frac{\partial Z_i^h}{\partial I^h}$$
(21)

We see that  $\beta^h$  is the social value of increasing the utility of consumer h by increasing his budget. But if his budget is increased he will change his consumption pattern which will result in a change in the tax payments. The second term in the definition of  $\overline{\beta}^h$  captures this effect and it is therefore the net social marginal utility of income for consumer h. By using (20) the first order condition (19) can now be written as

$$\sum_{h=1}^{H} \beta^{h} Z_{k}^{h} + \sum_{h=1}^{H} \frac{\partial W}{\partial \overline{V}^{h}} \phi^{h} \sum_{\overline{h}=1}^{H} \left( \frac{\partial \widehat{Z}_{N}^{\overline{h}}}{\partial Q_{k}^{\overline{h}}} - Z_{k}^{\overline{h}} \frac{\partial Z_{N}^{\overline{h}}}{\partial I^{\overline{h}}} \right)$$

$$= \mu \sum_{h=1}^{H} Z_{k}^{h} + \mu \sum_{i=1}^{N} t_{i} a_{Xi} \sum_{h=1}^{H} \left( \frac{\partial \widehat{Z}_{i}^{h}}{\partial Q_{k}^{h}} - Z_{k}^{h} \frac{\partial Z_{i}^{h}}{\partial I^{h}} \right), \quad k = 1, ..., N$$
(22)

By applying (21) and the symmetry of the derivative of the compensated demand functions,  $\frac{\partial \widehat{Z}_{i}^{h}}{\partial O_{i}^{h}} = \frac{\partial \widehat{Z}_{k}^{h}}{\partial O_{i}^{h}}$ , we can write (22) as

$$\frac{1}{\mu} \frac{\sum_{h=1}^{H} \overline{\beta}^{h} Z_{k}^{h}}{\sum_{h=1}^{H} Z_{k}^{h}} + \frac{1}{\mu} \frac{\sum_{h=1}^{H} \frac{\beta^{h}}{\lambda^{h}} \phi^{h} \cdot \sum_{h=1}^{H} (\frac{\partial \widehat{Z}_{N}^{\overline{h}}}{\partial Q_{k}^{\overline{h}}} - Z_{k}^{\overline{h}} \frac{\partial Z_{N}^{\overline{h}}}{\partial I^{\overline{h}}})}{\sum_{h=1}^{H} Z_{k}^{h}} = 1 + \frac{\sum_{i=1}^{N} \sum_{h=1}^{H} t_{i} a_{X_{i}} \frac{\partial \widehat{Z}_{k}^{h}}{\partial Q_{i}^{h}}}{\sum_{h=1}^{H} Z_{k}^{h}}, \quad k = 1, ..., N$$

We now define the index of discouragement (Mirrlees (1976)) as

$$d_k = \frac{\sum_{i=1}^{N} \sum_{h=1}^{H} t_i a_{xi} \frac{\partial \overline{Z}_k^h}{\partial Q_i^h}}{\sum_{h=1}^{H} Z_k^h}$$
(24)

We see that  $t_i a_{Xi}$  is the increase in generalized price,  $Q_i^h$ , caused by the tax. The numerator therefore describes the total change in consumption of good k caused by changing the price system. The denominator is the total consumption of good k and (24) therefore measures the proportionate reduction in total consumption of good k caused by the tax system. Using (24) we can write (23) as

$$d_{k} = \underbrace{\frac{1}{\mu} \underbrace{\sum_{h=1}^{H} \overline{\beta}^{h} Z_{k}^{h}}_{\sum_{h=1}^{H} Z_{k}^{h}} - 1}_{= \frac{1}{\mu} \underbrace{\sum_{h=1}^{H} \frac{\beta^{h}}{\lambda^{h}} \phi^{h} \cdot \sum_{h=1}^{H} (\frac{\partial \overline{Z}_{N}^{\overline{h}}}{\partial Q_{k}^{\overline{h}}} - Z_{k}^{\overline{h}} \frac{\partial Z_{N}^{\overline{h}}}{\partial I^{\overline{h}}})}_{(b)}, \quad k = 1, ..., N$$
(25)

This formula characterizes the optimal tax system in the economy. Knowing that the compensated cross-price effects are positive the discouragement index is positive (if the tax is positive). The right hand side of (25) capture the three goals of the tax system; efficient generation of revenue, equity considerations, and internalization of externalities. Part (a) of (25) is the standard Ramsey term<sup>3</sup>. The Ramsey result emerges in its most simple form if we take (a) of (25), assume that all households are identical, and that no externalities exist. This gives  $\overline{\beta}^h = \beta$  and  $\phi^h = 0$  for all h and the tax rule can be written as

$$d_k = \frac{\beta}{\mu} - 1, \quad k = 1, ..., N$$
 (26)

saying that the taxes should be set in such a way that the index of discouragement is identical for all goods. The Ramsey term (a) in (25) capture both efficiency and equity

considerations through  $\overline{\beta}^h$ . To see this note that through  $\frac{1}{\mu} \frac{\sum\limits_{h=1}^{H} \beta^h Z_k^h}{\sum\limits_{h=1}^{H} Z_k^h}$  we get that the

reduction in demand should be low when  $\beta^h$  is high, which happens for socially

important households. The second part of (a) given by  $\frac{\sum\limits_{h=1}^{H}Z_{k}^{h}\sum\limits_{i=1}^{N}t_{i}a_{Xi}\frac{\partial Z_{i}^{h}}{\partial I^{h}}}{\sum\limits_{h=1}^{H}Z_{k}^{h}}$  tells us that if

demand is consentrated among households where the tax payments are reduced significantly when income change, then the reduction in demand should be kept low,

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 $<sup>^3</sup>$  See for example Myles (1995) formula (4.40) and (4.44)

since a high reduction in demand will reduce tax revenue significantly thus calling for even higher taxes to be set in order to meet the governments revenue requirement, which will lead to even larger distortions

Part (b) of (25) deals with the externality. To simplify this we define

$$\overline{\beta}^h = \frac{\partial W}{\partial \overline{V}^h} \phi^h \,$$
 (27)

which, to keeping with the terminology, is the *social marginal disutility of the externality* for household h. It measures how much social welfare change due to changes in the utility for household h if the consumption of the externality generating good increases. We see that households who are socially important (having a high value of  $\frac{\partial W}{\partial V^h}$ ) and who are highly sensitive to changes in the externality level (having a high value of  $\phi^h$ ) will have a high value of  $\beta^h$ . In a transport setting this could be the case for low-income households living near the source of pollution (e.g. near large roads or near airports). We see that from (4) and the definition of W, the social marginal disutility of the externality is positive and higher values of  $\beta^h$  thus decrease welfare since it enters negatively in the utility function for the households (1).

As in (2) we define  $\overline{Z}_k = \sum_{h=1}^H Z_k^h$  as the total consumption of good k and use it to write (b) of (25) as

$$\underbrace{\frac{1}{\mu} \underbrace{\sum_{h=1}^{H} \overline{\beta}^{h} \sum_{\bar{h}=1}^{H} (\frac{\partial \overline{Z}_{N}^{\bar{h}}}{\partial Q_{k}^{h}} - Z_{\bar{h}}^{\bar{h}} \frac{\partial Z_{N}^{\bar{h}}}{\partial I^{\bar{h}}})}_{(b) \text{ of } (25)}}_{= \underbrace{\frac{1}{\mu} \sum_{h=1}^{H} \overline{\beta}^{h} \sum_{\bar{h}=1}^{H} \frac{\partial \overline{Z}_{\bar{k}}^{\bar{h}}}{\partial Q_{N}^{\bar{h}}}}_{(c)} - \underbrace{\frac{1}{\mu} \sum_{h=1}^{H} \overline{\beta}^{h} \sum_{\bar{h}=1}^{H} Z_{\bar{k}}^{\bar{h}}}_{\bar{d}I^{\bar{h}}}}_{(d)} \tag{28}$$

remembering that (b) of (25) shown in (28) increase the discouragement index if positive. Since we know that compensated cross-price effects are positive, (c) of (28) is positive. A high value of (c) thus indicates that the reduction in demand will be high. This makes sense since the externality tax not only affect the demand for good N but also increase the demand for good k thus countering negative effects of the tax imposed on good k directly. Due to the symmetry of the compensated demand derivatives it is also possible to reduce the externality by taxing good k if the compensated price effects are large. Again this calls for a larger reduction in demand for good k if the negative effect of this reduction is compensated by the positive effect of the reduction in the externality level. Assuming that good N is a normal good and thus having  $\frac{\partial Z_N^h}{\partial I^{\bar{n}}} > 0$  we see that (d) of (28) is positive and a high value of (d) thus indicate that a low reduction in demand for good k. The structure of (d) is the same as the structure of the last part of the Ramsey term in (25) which we can see if we write it as  $\frac{1}{\mu} \frac{\sum_{h=1}^{H} Z_{k}^{\bar{h}} \sum_{h=1}^{H} \overline{\beta}^{h} \frac{\partial Z_{N}^{h}}{\partial I^{\bar{h}}}}{\overline{Z}_{k}}$  and it now tells us that if demand is concentrated among households where the disutility from the externality are reduced significantly when income change, then the reduction in demand should be high, since a high reduction in demand will reduce the externality level significantly.

#### 3. Tax rules

Since (25) only characterize the optimal tax system implicitly it is difficult to derive clear policy implications from it. This section imposes further restrictions on the model to derive some well known results and some new insights.

#### 3.1 Inverse factor share rule and inverse elasticity rule

To obtain the inverse factor share rule we take (22) as a starting point. We assume that all households are identical (giving us  $\beta^h = \lambda$ ,  $Q_k^h = Q_k$ , and  $Z_k^h = Z_k$  for all household), that no externalities are present ( $\phi^h = 0$  for all households), that the government maximizes the unweighted sum of household utilities, and that no income effects exist since we are not concerned with distributional issues. With these assumptions the first order conditions for (15) reduces to

$$\frac{\lambda - \mu}{\mu} Z_k = \sum_{i=1}^N t_i a_{Xi} \frac{\partial \widehat{Z}_i}{\partial Q_k}, \quad k = 1, ..., N$$
(29)

Assuming that there are no compensated cross price effects between taxable goods in the economy, and defining the constant

$$\theta = \frac{\lambda - \mu}{\mu} \tag{30}$$

we can simplify (29) to

$$\theta = t_k a_{Xk} \frac{\partial \widehat{Z}_k}{\partial Q_k} \frac{1}{Z_k}, \quad k = 1, ..., N$$
(31)

This allow us to write the optimal tax formulas as

$$\frac{t_k}{P_k} = \frac{1}{\alpha_{Yk} \widehat{\varepsilon}_{kk}} \theta, \quad k = 1, ..., N$$
(32)

where  $\alpha_{xk} = \frac{P_k a_{xk}}{Q_k}$  is the factor share of  $X_k$  and  $\hat{\varepsilon}_{kk} = \frac{\partial \bar{Z}_k}{\partial Q_k} \frac{Q_k}{Z_k}$  is the compensated own price elasticity of commodity k (remembering that in optimum the value of compensated and uncompensated demand are equal,  $\hat{Z}_k = Z_k$ ). This is the inverse factor share rule derived in Kleven (2004) saying that goods which use much household time in production and hence have a low value of  $a_{xk}$  (thus having a large time share<sup>4</sup>) should carry a higher tax rate than goods which primarily use market-produced commodities in the household production. The rationale behind this result is that since leisure time can not be taxed directly the tax system causes a distortion on the labor market. If the government taxes goods having a high time share in household production they therefore reduce the distortion by taxing time indirectly through the production process. It is easy to see that the inverse elasticity formula is imbedded in this formulation. Letting  $a_{xk} = 1$  for all households the model reduces to the standard model used in the analysis of optimal taxation<sup>5</sup> resulting in the inverse elasticity formula

$$\frac{t_k}{P_k} = \frac{\theta}{\widehat{\varepsilon}_{kk}}, \quad k = 1, ..., N \tag{33}$$

saying that goods with high compensated own-price elasticity should be taxed less than goods with low compensated own-price elasticity in order to reduce distortions in the consumption patterns. Lifting the assumption about identical treatment of the households allows us to derive a more general version of the inverse factor share rule (32) where distributional considerations are taken into account. We thus relax the assumption about the government allowing it to maximize a more general welfare function and treat the

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<sup>&</sup>lt;sup>4</sup> The time share of  $X_k$  is given by  $\alpha_{Lk} = \frac{wa_{Lk}}{Q_k} = 1 - \alpha_{Xk}$ .

<sup>&</sup>lt;sup>5</sup> See Sandmo (1976) for an introduction.

households as being different (different utility functions and different income). Again we start from (20) we write the first order condition as

$$\sum_{h=1}^{H} \beta^{h} Z_{k}^{h} = \mu \sum_{h=1}^{H} Z_{k}^{h} + \mu \sum_{i=1}^{N} t_{i} a_{Xi} \sum_{h=1}^{H} \left( \frac{\partial \widehat{Z}_{i}^{h}}{\partial Q_{k}^{h}} - Z_{k}^{h} \frac{\partial Z_{i}^{h}}{\partial I^{h}} \right), \quad k = 1, ..., N$$
(34)

We assume that no cross-price effects exist between taxable goods. This allow us to write (34) as

$$\frac{t_k}{P_k} = \frac{\sum_{h=1}^{H} Z_k^h (\frac{\beta^h - \mu}{\mu})}{\sum_{h=1}^{H} Z_k^h \hat{c}_{kk}^h \alpha_{Xk}^h}, \quad k = 1, ..., N$$
(35)

where  $\alpha_{Xk}^h = \frac{P_k a_{Xk}}{Q_k^h}$  is the factor share of good k for household h. The insights from the simple inverse factor share rule (32) still holds but we see that we now have to take account of distributional concerns. It is easily recognized that  $(\frac{\overline{\beta}^h - \mu}{\mu})$  is negative (if G is positive) since  $\mu$  is the Lagrangian from (15). With the compensated own-price elasticity,  $\hat{\varepsilon}_{kk}$ , also being negative we have that  $\frac{t_k}{P_k}$  is positive. Since high-income households have higher generalized price,  $Q_k^h$ , their factor-share,  $\alpha_{Xk}^h$ , will be lower than the factor-share for low-income households. To see this note that  $Q_i^h(P_i, w^h) = P_i a_{Xi} + w^h a_{Li}$  and that a higher value of  $w^h$  gives a higher value of  $Q_i^h$ . In this situation the inverse factor share rule will benefit high-income households which is a general dilemma in the transport sector when the value of time is assumed to be higher for high-income households (and especially if the wage rate is used as a proxy for the value of time as we do here). In such cases time-savings experienced by high-income households weight more than the time savings experienced by low-income households. The distributional considerations working through  $\beta^h$  pulls in the opposite direction and if good k is consumed primarily by socially important households (normally low-income households) the tax should be set at a low since  $\overline{\beta}^h$  will be closer to  $\mu$  for these households thus bringing  $\frac{\overline{\beta}^h - \mu}{\mu}$  closer to zero calling for a lower tax to be set.

### 3.3 The additivity property

When externalities are present we can derive the optimal tax rules by again taking (22) as a starting point. Assuming that all households' are identical and that the government maximizes the unweighted sum of the households utility. This gives the first order conditions

$$\frac{\lambda - \mu}{\mu} = -H \frac{\partial \phi}{\partial Z_N} \frac{\partial Z_N}{\partial Q_k} \frac{1}{Z_k} \frac{1}{\mu} + \sum_{i=1}^N \frac{\partial Z_i}{\partial Q_k} t_k a_{Xi} \frac{1}{Z_k}, \quad k = 1, ..., N$$
(36)

This, by assuming that no compensated cross-price effects exist between taxable goods and ignoring income effects, simplifies to

$$\theta = t_k a_{Xk} \frac{\partial \widehat{Z}_i}{\partial Q_k} \frac{1}{Z_k}, \quad k = 1, ..., N - 1$$

$$\theta = t_k a_{Xk} \frac{\partial \widehat{Z}_i}{\partial Q_k} \frac{1}{Z_k} - \frac{H}{\mu} \frac{\partial \phi}{\partial Z_k} \frac{\partial \widehat{Z}_k}{\partial Q_k} \frac{1}{Z_k}, \quad k = N$$
(37)

giving the optimality conditions

$$\frac{t_k}{P_k} = \frac{\theta}{\alpha_{Xk}\widehat{\varepsilon}_{kk}} = (1 - \xi)(\frac{-1}{\alpha_{Xk}\widehat{\varepsilon}_{kk}}), \quad k = 1, ..., N - 1$$

$$\frac{t_k}{P_k} = \frac{\theta}{\alpha_{Xk}\widehat{\varepsilon}_{kk}} + \frac{H}{\alpha_{Xk}Q_k\mu}\phi' = (1 - \xi)(\frac{-1}{\alpha_{Xk}\widehat{\varepsilon}_{kk}}) + \xi \frac{H}{\alpha_{Xk}Q_k\lambda}\phi', \quad k = N$$
(38)

where  $\xi = \frac{\lambda}{\mu}$ . Note that since  $\lambda$  is the marginal value of private income and  $\mu$  is the marginal value of public income the parameter  $\xi$  can be interpreted as the marginal rate of substitution between private and public income. The formulas for optimal taxation

consist of three elements. The first,  $\frac{1}{\alpha_{Xk}}$ , is the inverse factor share rule found in Kleven (2004) stating that goods which has a low time share and thus a high factor share in the household production process should be taxed less than other goods. The second,  $\frac{1}{\hat{\epsilon}_{kk}}$ , is the well known inverse elasticity rule stating that goods having low compensated ownprice elasticity should be taxed higher than goods with high compensated own-price elasticity. The last element,  $\frac{H\phi'}{\mu Q_k} \frac{1}{a_{Xk}}$ , is the additive term on the externality generating good. With no explicit modeling of time (e.g. having  $a_{xk} = 1$  for all k) it is just the standard additive term found in Sandmo (1975) saying that the extra tax should be set according to the principle for Pigouvian taxation. This no longer holds as the term  $\frac{1}{\alpha_{Xk}}$ enters the formula. The tax rule now states that the social planner has to account for the time allocation by taxing time saving activities at a lower rate even if these activities generate negative externalities. In summary, the tax problem can be separated into two parts. First, a tax is used to internalize externalities taking into consideration the time allocation involved. Hereafter the government uses the inverse factor share rule to satisfy its revenue requirement. The formula also tells us that in the event of  $\xi > 1$  where the revenue generated from internalizing the externalities more than covers the government revenue requirement and funds therefore have to be given back to the households. Note that in the unlikely situation where  $\xi = 1$  the tax system will be first-best. In this situation the government can finance its revenue requirement by using the corrective tax alone and no other taxes are needed.

If we lift the assumption about identical treatment of the households the basic results from above do not change. In this case the formulae combine those previously found and we can write the tax rules as

$$\frac{t_{k}}{P_{k}} = \frac{\frac{1}{\mu} \sum_{h=1}^{H} \widehat{Z}_{k}^{h} (\overline{\beta}^{h} - \mu - \sum_{h=1}^{H} \overline{\beta}^{h} \frac{\partial Z_{N}^{h}}{\partial I^{h}})}{\sum_{h=1}^{H} \widehat{Z}_{k}^{h} \widehat{\varepsilon}_{kk}^{h} \alpha_{Xk}^{h}}, \quad k = 1, ..., N-1$$

$$\frac{t_{k}}{P_{k}} = \frac{\frac{1}{\mu} \sum_{h=1}^{H} \widehat{Z}_{k}^{h} (\overline{\beta}^{h} - \mu - \sum_{h=1}^{H} \overline{\beta}^{h} \frac{\partial Z_{N}^{h}}{\partial I^{h}})}{\sum_{h=1}^{H} \widehat{Z}_{k}^{h} \widehat{\varepsilon}_{kk}^{h} \alpha_{Xk}^{h}} + \frac{\frac{1}{\mu} \sum_{h=1}^{H} \widehat{Z}_{k}^{h} \widehat{\varepsilon}_{kk}^{h} \frac{1}{Q_{k}^{h}} \sum_{h=1}^{H} \overline{\beta}^{h}}{\sum_{h=1}^{H} \widehat{Z}_{k}^{h} \widehat{\varepsilon}_{kk}^{h} \alpha_{Xk}^{h}}}, \quad k = N$$

$$\underbrace{\sum_{h=1}^{H} \widehat{Z}_{k}^{h} \widehat{\varepsilon}_{kk}^{h} \alpha_{Xk}^{h}}_{(f)} + \underbrace{\sum_{h=1}^{H} \widehat{Z}_{k}^{h} \widehat{\varepsilon}_{kk}^{h} \alpha_{Xk}^{h}}_{(g)}}_{(g)}, \quad k = N$$

One difference is that in the Ramsey-term of the tax formulas we now have an income effect from the externality. Changing income for the households will influence the consumption of good N and thus the level of the externality. The term  $\sum_{\bar{h}=1}^{n} \beta^{\frac{-\bar{h}}{2}} \frac{\partial Z_{N}^{h}}{\partial l^{h}}$  capture this effect and it enters in the same way as the revenue effect enters the definition of the net social marginal utility of income defined in (21). It has opposite sign though since the revenue generates positive welfare effects and the externality generates negative welfare effects. If we want the Ramsey parts of (39) to be structurally identical to the Ramsey parts of (35) we let  $\hat{\beta}^{h}$  be the *externality corrected net social marginal utility of income* given by

$$\widehat{\boldsymbol{\beta}}^{h} = \boldsymbol{\beta}^{h} + \mu \sum_{i=1}^{N} t_{i} a_{Xi} \frac{\partial Z_{i}^{h}}{\partial I^{h}} - \sum_{\bar{h}=1}^{H} \frac{\partial W}{\partial \bar{V}^{\bar{h}}} \phi^{\bar{h}} \cdot \frac{\partial Z_{N}^{h}}{\partial I^{h}}$$

$$\tag{40}$$

where the first term is the marginal utility of income for household h, the second term capture the effect of extra tax revenue collected from household h if the income is increased, and the last term is the loss in welfare due to changes in the consumption of good N by household h if the income is increased. Using this we can write (40) as

$$\frac{t_{k}}{P_{k}} = \frac{\sum_{h=1}^{H} \widehat{Z}_{k}^{h} (\overline{\beta}^{h} - \mu)}{\sum_{h=1}^{H} \widehat{Z}_{k}^{h} \widehat{\varepsilon}_{kk}^{h} \alpha_{Xk}^{h}}, \quad k = 1, \dots, N - 1$$

$$\frac{t_{k}}{P_{k}} = \frac{\frac{1}{\mu} \sum_{h=1}^{H} \widehat{Z}_{k}^{h} (\overline{\beta}^{h} - \mu)}{\sum_{h=1}^{H} \widehat{Z}_{k}^{h} \widehat{\varepsilon}_{kk}^{h} \alpha_{Xk}^{h}} + \frac{\frac{1}{\mu} \sum_{h=1}^{H} \widehat{Z}_{k}^{h} \widehat{\varepsilon}_{kk}^{h} \frac{1}{Q_{k}^{h}} \overline{\beta}^{h}}{\sum_{h=1}^{H} \widehat{Z}_{k}^{h} \widehat{\varepsilon}_{kk}^{h} \alpha_{Xk}^{h}}, \quad k = N$$

$$\underbrace{\sum_{h=1}^{H} \widehat{Z}_{k}^{h} \widehat{\varepsilon}_{kk}^{h} \alpha_{Xk}^{h}}_{(i)} + \underbrace{\sum_{h=1}^{H} \widehat{Z}_{k}^{h} \widehat{\varepsilon}_{kk}^{h} \alpha_{Xk}^{h}}_{(i)}}_{(i)}, \quad k = N$$

where (h) and (i) of (40) is the familiar Ramsey term and has the same interpretation. Part (j) of (40) is the corrective term of the tax formula and it still enters additive and this additive term is the only part of (41) which incorporate substitution effects for good N. The additivity property thus survives in the present setup.

#### 4. Conclusion and possible extensions

In this paper we have presented a model of household behavior where time enters the utility function directly as proposed in Becker (1965). Since the consumption of time is very important in the transport sector the approach is a natural extension to the traditional microeconomic setup when this sector is being modeled.

The tax rules found in the previous section help us to understand how the tax system should be designed. If distributional considerations are ignored the inverse factor share rule states that fast transportation should carry a lower rate of tax than slow transportation. This conclusion is less clear when externalities are included because a

faster car might cause externalities that a slow car does not (for example accidents and pollution).

Moving on to the question of distributional issues we see that the tax rules are easily generalized to include these. The results resemble those found elsewhere in the literature and it is worth noting that the corrective part of the tax which targets the externality still enters as an additive term. The additivity property has to be modified though since the marginal external damage enters the optimal tax formula for all taxes through an income effect. The claim that the corrective tax enters additive for the externality generating good still holds and the property prevails but in a more complicated form.

We have extended the results by Kleven and included externalities and distributional concerns in the approach. We showed that the tax formulas emerging resemble those found by Sandmo and we therefore conclude that the additivity property survives in this new setup due to the additive externality in the utility functions. Furthermore we make it possible to see how distributional questions will affect the tax system.

The model presented here can be generalized. The modeling of the externalities in a separable way could be criticized and alternative ways of modeling this will be subject to future research. Furthermore to assume that all households have the same technologies available to them could also be questioned. In spite of this it is believed that the insights from the model are valuable when trying to understand how the tax structure in the transport sector ought to be designed.

# References

Becker (1965) A Theory of the Allocation of Time, *The Economic Journal* **75**, 493-517.

De Borger, B. and Van Dender, K. (2003) Transport tax reform, commuting, and endogenous value of time, *Journal of Urban Economics* **53**, 510-530.

Diamond, P. A. and J. A. Mirrlees (1971) Optimal Taxation and Public Production II: Tax Rules, *The American Economic Review* **61:3**, 261-278.

Diamond, P. A. (1975) A many-person Ramsey tax rule, *Journal of Public Economics* **4**, 335-342.

Fosgerau, M. and Pilegaard, N. (2003) Værdien af tid, Grundlaget for tidsværdier til samfundsøkonomiske vurderinger, *Danish Transport Research Institute*, Notat 2003:2, www.dtf.dk.

Kleven, H. J. (2004) Optimum Taxation and the Allocation of Time, *Journal of Public Economics* **88**, 545-557.

Miles, G. D. (1995) *Public Economics*, Cambridge University Press.

Nielsen, J. E. (2006) Transport tax reform, two-part tariffs, and revenue recycling, *Thesis Paper*.

Parry, I. W. H. and Bento, A. (2001) Revenue Recycling and the welfare effects of Road Pricing, *Scandinavian Journal of Economics* **103:4**, 645-671.

Parry, I. W. H. and Bento, A. (2002) Estimating the Welfare Effect of Congestion Taxes: The Critical Importance of Other Distortions within the Transport System, *Journal of Urban Economics* **51**, 339-365.

Pollak, R. A. and Wachter, M. L. (1975) The Relevance of the Household Production Function and Its Implications for the Allocation of Time, *The Journal of Political Economy* **83:2**, 255-278.

Sandmo, A. (1975) Optimal Taxation in the Presence of Externalities, *Swedish Journal of Economics*. 77, 86-97.

# **Estimation of car ownership in Denmark**

# - Discrete choice modeling and repeated cross-section analysis

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# **Abstract**

We investigate the demand for cars in Denmark by applying a simple discrete choice model for car ownership and estimate it on a series of neighboring cross-section data from Denmark for the years 1995 to 2002. We hypothesize that the increasing real estate prices in Denmark have biased parameter estimates. By including a variable for real estate values we show that only real estate owners and tenants in urban areas and real estate owners in rural areas are affected by the increasing prices on real estate. This we take as an indication that the hypothesis that real estate values influence car ownership in Denmark is true. If so, income elastities estimated on models not including a variable for real estate values might be upward biased for those households most strongly affected. We show that this is exactly what happens since including a variable for real estate values reduces the estimated income elastitites for real estate owners more than for tenants.

## 1. Introduction

Transport demand is determined not only by demand changes on the intensive margin (the choice of how much to travel) but also on the extensive margin (the choice of transport mode)<sup>6</sup> and different approaches have been used to examine transport demand. Some papers look at one of the margins while others combine the two in a unified framework. Which approach to choose depends on the purpose of the analysis but it is important to have reliable estimates of e.g. income elasticities for both car ownership and car transport when forecasting the transport demand and thus deciding how many resources to invest in the transport infrastructure to cope with the expected future demand.

This paper examines car demand by using a simple discrete choice model for car ownership<sup>7</sup> and estimating it on Danish cross-section data. We examine the problem of parameter stability when this method is used which is important since transport planners in many situations are forced to use simple cross-section methods due to data limitations. We show that car demand seems to have changed from 1995 to 2002 and that the income elasticity depends on the year in which the model is estimated. This insight is not new but if we are able to identify a variable which seems to reduce the severity of the problem this will be useful in future work.

<sup>&</sup>lt;sup>6</sup> The terms intensive and extensive margin is borrowed from the labor supply literature where decisions on the intensive margin describes the decision on how much to work and decisions on the extensive margin describes labor force participation decisions.

<sup>&</sup>lt;sup>7</sup> A survey of relevant literature on this subject can be found in De Jong et al (2005).

A problem with the use of simple cross-sectional modeling is that omission of important macro variables might bias the result and perhaps cause the estimated parameters to become unstable. We investigate this and speculate that the omission of a variable for household wealth measured by the value of owner occupied housing cause the parameters for real estate owners to be biased compared with parameters estimated for tenants. Since real estate values in Denmark have increased much more in urban areas than in rural areas with values in urban areas being much higher than values in rural areas we can examine this hypothesis by looking at households in the two regions separately. By further separating the households in 'owners' and 'tenants' we have two samples in each region which should be affected differently by the increasing real estate prices if the hypothesis is true. We show that differences exist between real estate owners and tenants as well as between urban and rural households. We therefore speculate that the development in real estate prices have influenced the demand for cars. To examine this we include a variable for real estate value showing that this has affected real estate owners as well as tenants living in urban areas but not tenants living in rural areas. Unfortunately we are not able to include a variable for the wealth accumulated in real estate but we still believe that the variable used here can be used as a proxy, since our goal is to see if housing prices influence the demand for cars.

We find that the elasticities estimated for real estate owners fluctuate more than those for tenants. To see if real estate values might reduce the fluctuations we include a variable for the real estate value in the different municipalities in Denmark. We then test whether we can restrict the parameter for this variable to zero. For tenants in rural areas this

restriction is accepted indicating that these households do not change their demand for cars due to changing real estate values. For urban households the test for the restriction on the parameter is rejected and we conclude that these households change their demand for cars due to the level of real estate prices. For rural real estate owners the parameter restriction is also rejected indicating that these households have been affected by the changing real estate values. If the hypothesis is correct we would expect urban real estate owners to be affected the most with rural tenants being affected the least (or not at all) and our investigation seems to support this. Ranking rural real estate owners and urban tenants with regards to how much they are expected to be affected is not straight forward since the increasing real estate values could influence tenants indirectly thus making a clear ranking impossible.

Previous models of car demand looking at both the intensive and the extensive margin include Train (1980) who uses a multinomial logit model extimated on cross-section data from 1975 for the San Francisco Bay Area to evaluate the effect of a new train service. His model predicts transport demand and cars per household by estimating the number of cars a household would own and which mode households choose for their commuting trips. Pendyala et al. (1995) use repeated cross-sectional analysis to examine the relationship between car ownership and income over time. They employ an ordered probit model for car ownership estimated on Dutch Panel Survey data showing that large differences between different household types exist. They conclude that when modeling car demand it is important to account for the household structure. They also find that

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<sup>&</sup>lt;sup>8</sup> They focus on differences between couples with children, couples without children, singles with children and singles without children.

income elasticities change with the level of motorization which could cause parameter instability in cross-section models. Their results suggest that the level of motorization should be included in models for car ownership and that some kind of dynamic specification is needed. Even though this is true the transport planners often do not have data available to estimate dynamic models and they are forced to use simple cross-section analysis. The problem of parameter stability therefore remains in applied work and investigating the influence of variables which might reduce this problem is still in demand.

The increasing motorization is also addressed in Jansson (1989) where he describes this phenomenon as a diffusion process where households increase their taste for cars over time. Other papers mention that we might approach a saturation point for car ownership, and we should therefore expect income elasticities to fall over time (Kwon and Preston (2005)). As mentioned in Fosgerau et al. (2004) we do not expect serious saturation effects to be present in Denmark since the number of cars per capita is much lower than in many other countries which is also shown in Dargay et al. (2006).

Another study pointing out interesting aspects of car demand is Dargay (2002) where both a static and a dynamic model is used to estimate car demand in the UK. She concludes that the cross-section income elasticity declines over time. She also constructs a dynamic model of car ownership estimating it on a pseudo-panel<sup>9</sup> from the UK Family Expenditure Survey. She analyses both life-cycle effects, cohort effects, and introduce the

<sup>&</sup>lt;sup>9</sup>See Deaton (1985) for an introduction to construction and estimation of pseudo-panel models. We use a pseudo-panel approach to estimate the demand for cars in Denmark in Nielsen (2006a) and apply the method to estimate demand for car transport in Nielsen (2006b).

possibility of hysteresis effects. Several conclusions are drawn from Dargay's paper. She shows that the life-cycle effects cause car ownership to decrease when the 'head of household' reaches the early 50s. This accord with findings of Jansson (1989), where entry and exit propensities of cars together with cohort data are used to determine car demand. Differences between urban and rural households are also addressed in Dargay (2002) using a dynamic partial adjustment model for the UK. She concludes that large differences exist between the different household groups and that car ownership in urban areas is more sensitive to changes in user cost of transport, fuel cost and car purchase cost than in rural households<sup>10</sup>. In a static model like the one applied here the inclusion of a variable for the general user cost on a national level would not bring anything, since it would enter as a constant in the estimation. We are however able to include a variable for real estate prices since these differ between municipalities. In Dargay (2001) car ownership is also investigated and she concludes that cohort effects may not be described by changes in income alone. Furthermore, she shows that income elasticities are falling when car ownership rises and she finds evidence of hysteresis effects in car demand. She thus confirms findings in Pendyala et al. (1995).

The link between car ownership and geography is also examined in Christens & Fosgerau (2004) who use cross-section data from the Danish Transport Diary Survey to estimate a multinomial logit model for car ownership. They estimate the income elasticities for household car availability in different regions of Denmark and find that households in large cities have higher income elasticities than households living in small cities or in the

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<sup>&</sup>lt;sup>10</sup> In Nielsen (2006a) we show that the increasing real estate prices and the falling interest rate which have taken place in Denmark since 1993 have influenced the demand for cars.

countryside. They find that the income elasticities in the five largest cities in Denmark<sup>11</sup> can be around 2.5 times higher than income elasticities in the countryside. The elasticities found for the five largest cities are between 0.47 and 0.86 whereas the elasticities elsewhere is between 0.31 and 0.54.

Other Danish studies of car demand include Bjørner (1999). He uses a simple dynamic model for personal transport estimated on Danish registers data, and finds short and long run income elasticities for personal transport by car to be 0.21 and 0.57 respectively. He argues that these are low compared to other studies. Birkeland et al. (2000) calculate that the income elasticity for distance traveled by car is 0.19 for the pseudo-panel analysis and for a cross-section analysis in a non-dynamic setting they find income elasticities for car demand between 0.28 and 0.48.

This paper proceeds as follows. Section 2 presents the data and explores some characteristics of car ownership in Denmark. In section 3 a simple multinomial logit model for car ownership is set up and estimated. Section 4 calculates income elasticities for car ownership and discusses the results. The final section concludes.

# 2. The data

We use data from the Danish Transport Diary Survey between 1995 and 2002<sup>12</sup>. It is an interview based survey where a random sample is drawn from the Danish Civil Register once every month. Every person in the sample receives a letter explaining about the

Copenhagen, Århus, Odense, Ålborg and Esbjerg.
 A full description of the Danish Transport Diary Survey can be found at the homepage of the Danish Transport Research Institute, www.dtf.dk.

interview and its purpose. During the month in question people from Statistics Denmark call and ask about the travel behavior on the day before the interview is conducted. Furthermore, information concerning the household, family, car and occupation is collected and some register data is added to the sample. Until 1997 a total of around 1800 persons between the age of 16 and 74 were drawn from the Danish Civil Register every month. In 1998 the survey was extended to 2100 persons between the age of 10 and 84. The response rate is around 65-70%. It is important to note that the variable in the survey we model is called 'car availability' and could potentially include cars not owned by the household. Discrepancies might therefore exist between the number of cars owned by the household and the number of cars 'available' to the households since the latter could include e.g. work cars or cars shared between several households. What we model is thus 'car availability' but we will continue to refer to it as car ownership<sup>13</sup>.

To keep the sample as homogeneous as possible not all data are included. We exclude all people who were not part of the 'head of household couple'. This ensures that we do not include young people living with their parents thus having access to the parents' car but having very low income. Furthermore observations with missing values were excluded and we restrict the sample to interviewees between the age of 18 and 74. With these exclusions subsets of data are constructed for each year. The total number of observations in the different samples are shown in table 1.

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<sup>&</sup>lt;sup>13</sup> Another factor that we have to keep in mind relates to the years 1995 and 1996. In these years the question in the survey concerning car availability differs from the one used in the preceding years and it might cause the number of recorded cars from 1997 and onwards to be slightly larger than in 1995 and 1996.

Year	Real estate	owners	Tenants		Total		
	Urban	Rural	Urban	Rural			
1995	993	2.312	861	668	4.834		
1996	1.644	4.119	1.382	962	8.107		
1997	1.477	3.985	1.384	896	7.742		
1998	1.766	4.600	1.490	1.171	9.027		
1999	1.592	4.434	1.368	924	8.318		
2000	1.498	4.210	1.276	973	7.957		
2001	1.421	3.784	1.115	887	7.207		
2002	599	1.730	559	384	3.272		
Total	10.990	29.174	9.435	6.865	56.464		

Table 1: Number of observations

The average number of cars per household for the entire population as well as for real estate owners and tenants living in urban and rural areas are shown in figure 1. We see that the number of cars in Danish households increases over time conforming to the expectation of motorization (as in Pendyala et al. (1995)). We note that the increase in car ownership happens for households living in rural areas. That the number of cars is expected to increase is also confirmed in Fosgerau et al. (2004). They predict that the car fleet in Denmark will increase further since the number of cars per capita in 2003 is 0.38 which is far from the saturation point of 0.65 estimated in Dargay & Gately (1999). Fosgerau et al. (2004) claim, that the saturation point for Denmark is probably around 0.6 cars per capita and in Dargay et al. (2006) it is shown that the number of cars per capita is

lower in Denmark than in most other European countries<sup>14</sup>. We therefore do not expect serious saturation effects to be present in Denmark.

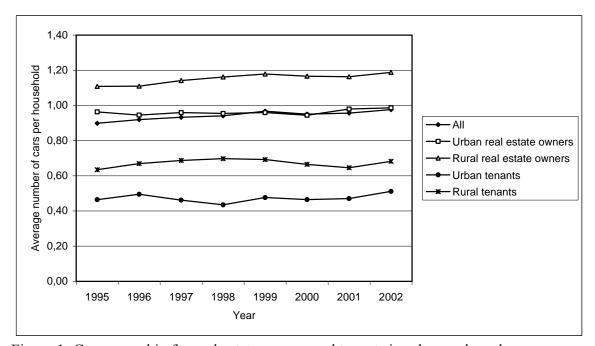


Figure 1: Car ownership for real estate owners and tenants in urban and rural areas.

Source: Danish Transport Diary Survey

The development in after-tax income for households is shown in figure 2 for real estate owners and tenants living in urban and rural areas<sup>15</sup>. We see that not only do real estate owning households have higher income than tenants. The income for real estate owners also increases more than for tenants from 1995 to 2002, both in absolute terms and in relative terms as shown in figure 3. A possible explanation for the increasing car ownership seen in figure 1 is the increasing income shown in figure 2 since we expect cars to be a normal good. One expectation emerging from figure 1 is that the income

<sup>14</sup> Dargay et al. (2006) use data from EuroStat to show this.

<sup>&</sup>lt;sup>15</sup> We define urban areas as the five largest cities in Denmark (Copenhagen and suburbs, Århus, Odense, Ålborg and Esbjerg). Rural areas are defined as the remaining part of Denmark.

elasticity decreases over the period since we expect that increasing car ownership decreases the income elasticity of car ownership as pointed out in Dargay & Gately (1999). We also expect that the increasing income shown in figure 2 will reduce the income elasticity for car ownership.

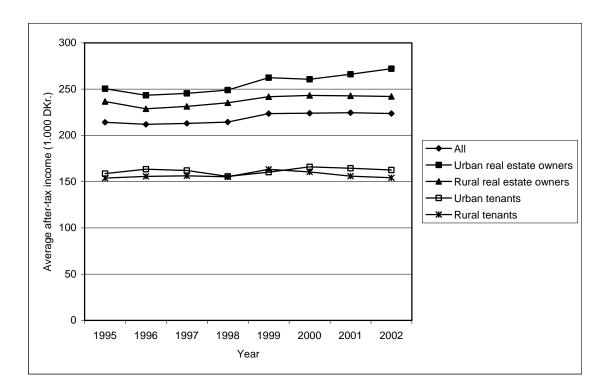


Figure 2: Average after-tax income for real estate owners and tenants deflated to 2000-values (1.000 DKr.)

Source: Danish Transport Diary Survey

With the results from Christens & Fosgerau (2004) showing that the income elasticity in the urban areas is higher than the income elasticity elsewhere, we expect the number of cars to increase more in urban areas if the income rises equally in the two regions. As seen in figure 3 real estate owners have experienced the largest increases in income.

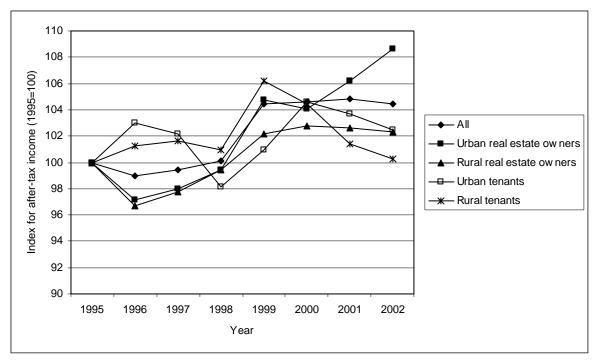


Figure 3: Index for after-tax income deflated to 2000 (1995=100)

Source: Danish Transport Diary Survey

Based on the increases in income we should expect car ownership to increase more for households living in urban areas than elsewhere. But if we look at the regional differences shown in figure 1 this is not the case. It turns out that it is the households not living in the five largest cities who have experienced the largest increase in car ownership even though their income has increased the least.

In this paper we speculate that the value of real estate influence the demand for cars. As seen in Figure 4 the housing prices have increased significantly from 1992 to 2002 and if the hypothesis that real estate values influence car demand is true a model not incorporating this might produced biased forecasts.

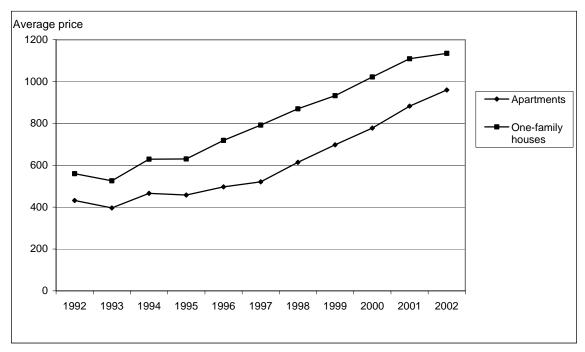


Figure 4: Real estate prices (1.000 DKr.).

Source: Statistics Denmark.

The developments in housing prices have differed much between rural and urban areas as shown in figure 5. In 1995 the difference in average housing prices between urban and rural areas was about 200.000 DKr. This difference increased to around 450.000 DKr. and we thus see that urban households have gained much more wealth in housing than rural households.

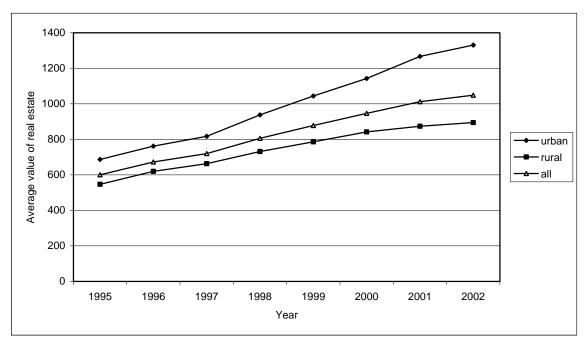


Figure 5: Development of real estate value in urban and rural areas

Source: Statistics Denmark

Figure 6 shows the development in the interest rate from 1992 to 2004 which has dropped from around 10% to around 5% in Denmark. With the falling interest rate and the increasing housing prices, real estate owners will be able to capitalize wealth gains from real estate through the Danish mortgage credit associations. If the wealth gains generate higher demand for cars we expect that omission of this when modeling car ownership will cause the estimated income elastitities to be too high thus explaining some of the observed inconsistency mentioned above. Since tenants do not experience this wealth increase, it is likely that we can find out if the hypothesis that real estate values influence the demand for cars.

With the changing housing prices being a flow variable we examine if the inclusion of a variable for the value of real estate prices (a stock variable) influence real estate owners and tenants differently. We obtain this variable from Statistics Denmark for every municipality in Denmark and link it to the observations from the Danish Transport Diary Survey. Ideally we should have the information for the individual observations but this information is not available. We therefore assume that the average value of real estate in the municipalities can be used as a proxy for the real estate value for households living in that municipality.

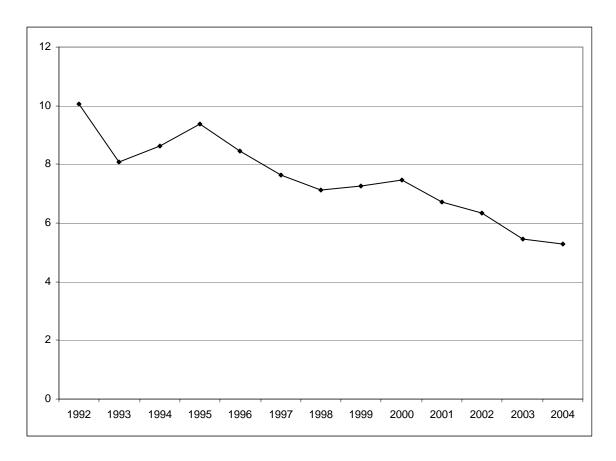


Figure 6: Interest on 30-years bonds.

Source: Statistics Denmark

To examine these things more closely, the next section sets up a multinomial logit model for car ownership and estimated it on a series of neighboring cross-section data.

## 3. Model and estimation

The model used is a simple multinomial logit model<sup>16</sup>. With the choice variable for the households being the number of cars this problem fits very well into a discrete framework. The alternatives the households choose between are 'no cars', 'one car' or 'more than one car'. Letting Y represent the number of cars chosen by the households, and letting  $V_0$ ,  $V_1$ , and  $V_2$  represent household utility when owning 'no cars', 'one car' or 'more than one car', and normalizing 'no cars' to zero we have the following standard choice probabilities for the logit model

$$P(Y = 0) = \frac{1}{1 + e^{V_1} + e^{V_2}}$$

$$P(Y = 1) = \frac{e^{V_1}}{1 + e^{V_1} + e^{V_2}}$$

$$P(Y \ge 2) = \frac{e^{V_2}}{1 + e^{V_1} + e^{V_2}}$$
(1)

and the functions we estimate are

$$V_{1}(X) = \log(\frac{P(Y=1)}{P(Y=0)}) = \beta_{1}X$$

$$V_{2}(X) = \log(\frac{P(Y\geq2)}{P(Y=0)}) = \beta_{2}X$$
(2)

where  $\beta_1$  and  $\beta_2$  are vectors of parameters and X is a vector of observations. The  $\beta$ 's can now be found using standard maximum likelihood procedures. With the specification given above we expect parameters that have a positive effect on car

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<sup>&</sup>lt;sup>16</sup> For an introduction to the multinomial logit model see Ben-Akiva & Lerman (1985) or Train (2003)

ownership to fulfill  $\beta_1 < \beta_2$  and parameters having a negative effect on car ownership to fulfill  $\beta_1 > \beta_2$  since the chosen specification sets parameters relative to having no car<sup>17</sup>.

As noted in Pendyala et al. (1995) it is important to account for differences in household structure when it comes to the number of adults and children. We would like to estimate separate models for different household types (couples with and without children, and singles with and without children) but in order to keep the number of observations 'high' in every estimation, we instead include variables for the number of children under the age of 16 and the number of people over the age of 16 in the household 18. We expect both variables to affect the probability of car ownership positively. Since the proximity (and thus accessibility) of public transport also influences car ownership, we include a variable for the distance between nearest public transport node and the home address and a variable for the distance between nearest public transport node and the workplace. Our expectation is that a longer distance to public transport increases the possibility of car ownership. We also include a dummy variable for the degree of urbanization (e.g. living in one of the 5 largest cities in Denmark or not) together with a variable for the age of the interviewee since a higher degree of urbanization is expected to reduce the demand for cars and people of different age have different demand for cars. We also include a variable for age squared. Age is thus modeled in a polynomial way allowing us to capture life-cycle effects. The income variable included accounts for total after-tax income in the

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$$\overline{V}_2(X) = \log(\frac{P(Y \ge 2)}{P(Y = 1)}) = \overline{\beta}_2 X$$

<sup>&</sup>lt;sup>17</sup> This point becomes clear if we had chosen

in which case  $\overline{\beta}_2$  parameters are relative to having one car.

<sup>&</sup>lt;sup>18</sup> We would like the distinction to be at the age of 18 since this is the legal age in Denmark for acquiring a drivers license. But the data does not allow us to make this distinction.

households deflated to 2000 values. Furthermore we hypothesis, that households living in areas where housing prices are 'high' might have a different demand for cars than households living in areas where real estate values are 'low'. We include a variable for the average real estate value in the different municipalities. Since we do not have information on individual wealth we will use this variable as a proxy for household wealth. A list of all variables used in the model can be seen in table 2.

# 3.1 Estimating the model

The model is estimated allowing parameters to differ between years. We estimate the model on the five different sets of data presented in table 1 and present the parameter estimates and test statistics in table A.1 through A.5 in the appendix<sup>19</sup>. In general we find that higher income gives a higher probability of car ownership and the signs of 'Age' and 'Age2' differ. We also see that poor access to public transport (e.g. longer distance from home to nearest public transport node and longer distance from work to nearest public transport node) gives a higher probability of car ownership. The composition of the family also affects the choice probabilities as expected, since both the number of children and the number of adults in the household has a positive effect on the choice probabilities. Furthermore we see that people living in less urbanized areas have higher probability of owning a car, and we find that not owning real estate gives a lower

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<sup>&</sup>lt;sup>19</sup> The five sets are; urban real estate owners, rural real estate owners, urban tenants, rural tenants, and the entire sample.

probability of owning cars. All five models produce good values for the McFadden  $\rho^2$ statistics<sup>20</sup> between 0.34 and 0.43 which we find to be satisfactory.

Variable	Description
Cars	Number of cars available to the household
Loginc	Logaritm of household yearly after-tax income deflated to 2000 values
Age	Age of the person interviewed
Age2	Squared value of age
Distpub	Distance from home to public transport
Workpub	Distance from workplace to public transport
Child	Number of children younger than 16 in the household
Adults	Number of people not registered as children
Urban	Dummy for living in highly urbanized area (5 largest cities in Denmark)
Owner	Dummy for ownership of real estate (1 if real estate owner, 2 if tenant)
Value	Average value of real estate in the municipality of residence

Table 2: Model variables

We also estimate models where we do not allow parameters to differ between years, effectively pooling all observations together. Estimates and test statistics can also be found in table A.1 through A.5 in the appendix. The signs are as expected and all five models produce McFadden  $\rho^2$  statistics between 0.33 and 0.43 which we find to be

The McFadden  $\rho^2$  values are defined as  $\rho^2(0) = 1 - \frac{L(\beta)}{L(0)}$  and  $\rho^2(c) = 1 - \frac{L(\beta)}{L(c)}$  where L(0), L(c) and L( $\beta$ ) are likelihood values for estimations with zero coefficients, constants and all parameters respectively. A discussion of this measure can be found in Ortúzar & Willumsen (1994) chapter 8.

satisfactory. To examine if the restrictions on the parameters can be accepted we calculate  $\chi^2$ -value in all five models for the restrictions that parameters are identical for all years. The results are shown in table 3.

	Likelihood of	Likelihood of	Degrees of	$\chi^2$ -
	unrestricted model	restricted model	freedom	value
A.1: All households	-37.021,3581	-37.305,5362	140	0,0000
A.2: Urban owners	-7.953,7969	-8.060,4679	140	0,0001
A.3: Rural owners	-18.055,3316	-18.176,7024	140	0,0000
A.4: Urban tenants	-5.846,9292	-5.970,4143	140	0,0000
A.5: Rural tenants	-4.752,4732	-4.817,3043	140	0,7236

Table 3: Test for restricting parameters

The restrictions in the full model (A.1) through (A.4) are strongly rejected but for tenants living in rural areas the restriction is accepted. These households are also those which should be affected the least, if the hypothesis that real estate values affect car demand is correct since they do not own real estate and live in the areas which have experienced the lowest increase in real estate values.

The difference between real estate owners and tenants in rural areas could indicate that there is something causing parameter instability, which is not affecting tenants in rural areas. To investigate further we examine if the inclusion of a variable for real estate values change the results.

#### 3.2 Estimating the model with a variable for real estate values

We now estimate the model where a variable for the real estate values are included. As before we estimate both the restricted model where all parameters are identical for all years and the unrestricted model where parameters are allowed to differ between years. The results are shown in table A.6 through A.10 in the appendix. Table 4 summarizes the statistics for the restrictions. As before we see that the restrictions are rejected in all cases except for tenants living in rural areas. The next step is therefore to see if we can restrict the parameter for real estate values to zero in any of the models used. This will be done next.

	Likelihood of	Likelihood of	Degrees	$\chi^2$ -
	unrestricted	restricted model	of freedom	value
	model			
A.6: All households	-36.916,0413	-37.247,5566	158	0,0000
A.7: Urban owners	-7.841,0372	-8.001,3378	158	0,0000
A.8: Rural owners	-18.041,8511	-18.172,5051	158	0,0000
A.9: Urban tenants	-5.819,2340	-5.961,9033	158	0,0000
A.10: Rural tenants	-4.744,1334	-4.815,1884	158	0,8127

Table 4: Test for restricting parameters in the models with a variable for real estate values

# 3.3 Testing for insignificant effect of real estate value

Table 5 and table 6 present the test results from testing the hypotheses that the variable for real estate values can be set equal to zero. We test this both in the model with different parameters for each year and in the model with identical parameters for all years.

	Likelihood of	Likelihood of	Degrees	$\chi^2$ -
	unrestricted model	restricted model	of freedom	value
A.6: All households (β <sub>value</sub> =0)	-36.916,0413	37.021,3581	20	0,0000
A.7: Urban owners ( $\beta_{value}$ =0)	-7.841,0372	-7.953,7969	20	0,0000
A.8: Rural owners ( $\beta_{\text{value}}=0$ )	-18.041,8511	-18.055,3316	20	0,1364
A.9: Urban tenants ( $\beta_{value}$ =0)	-5.819,2340	-5.846,9292	20	0,0000
A.10: Rural tenants ( $\beta_{value}$ =0)	-4.744,1334	-4.752,4732	20	0,6737

Table 5: Test for restricting parameters in the models with a variable for real estate values

The results of this test are clear. For households living in rural areas the restriction is accepted and we conclude that the real estate values have not significantly affected these households. For urban areas the restriction is not accepted. We believe that this indicates that our hypothesis that real estate values affect car demand could be correct. As we saw in figure 6 the largest increases in real estate values have happened in urban areas and the assumption of no influence of the real estate values are rejected for households living here.

	Likelihood of	Likelihood of	Degrees	χ²-
	unrestricted model	restricted model	of freedom	value
A.6: All households (β <sub>value</sub> =0)	-37.247,5566	-37.305,5362	6	0,0000
A.7: Urban owners ( $\beta_{\text{value}}=0$ )	-8.001,3378	-8.060,4679	6	0,0000
A.8: Rural owners (β <sub>value</sub> =0)	-18.172,5051	-18.176,7024	6	0,2106
A.9: Urban tenants ( $\beta_{value}$ =0)	-5.961,9033	-5.970,4143	6	0,0092
A.10: Rural tenants ( $\beta_{\text{value}}=0$ )	-4.815,1884	-4.817,3043	6	0,6453

Table 6: Test for restricting parameters in the models with a variable for real estate values and identical parameters for all years

# 4. Calculating income elasticities

We now use the unrestricted models to find income elasticities for car ownership for the different household groups. These can be found by using the model to simulate the changing choice probabilities when the income changes. We first calculate income elasticities using the model without a variable for real estate values but with separate parameters for all years. The results are shown in figure 7.

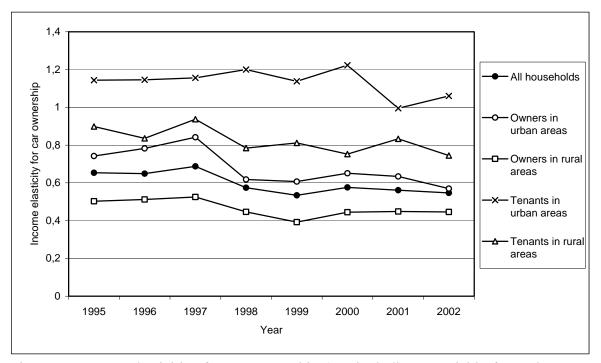


Figure 7: Income elasticities for car ownership (not including a variable for real estate values)

It is clear that in general tenants have higher income elasticities than real estate owners and urban households have higher income elasticities than rural households. We also see that the conventional wisdom that the income elasticity for car ownership should fall over time is not rejected in our model. Looking at figure 7 it also becomes clear that income elasticities are not stable.

The magnitude of the elasticities found for all households are higher than those reported in Birkeland et al. (2000) but in line with those reported by Dargay (2001) using a pseudo-panel analysis. The findings also conform to the differences in income elasticities between urban and rural households reported in Christens & Fosgerau (2004) and the size of the elasticities found here are in line with their findings.

One explanation for the difference in income elasticity between real estate owners and tenants could be that real estate owners have higher car ownership than tenants as shown in figure 1, which reduces the income elasticity for car ownership for real estate owners. Furthermore, real estate owners in general have higher income than tenants which also cause tenants to have higher income elasticities for car ownership than real estate owners.

Now we calculate elasticities using the model where real estate values are included. The results can be seen in figure 8. The same picture as before arises with urban households having higher income elasticities than rural households and with tenants having higher income elasticities than real estate owners. Comparing figure 7 and figure 8 we see that the inclusion of a variable for real estate values have decreased the elasticities for all households with the largest decrease for real estate owners. We take this as an indication that the real estate values have biased the income elasticities upwards. Calculating the best-fit straight lines for the development in income elasticities shown in figure 7 and

figure 8 we can make a comparison of the slopes and the constants to see how the estimates are affected by the inclusion of the variable for real estate value. These are shown in table 8.

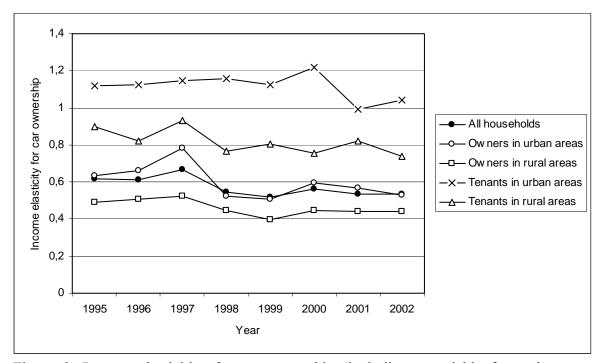


Figure 8: Income elasticities for car ownership (including a variable for real estate values)

It is clear that the inclusion of the variable for real estate values have reduced the slope numerically and the constants for the straight line equations shown. As we see the largest changes have happened for real estate owners with a slope for urban real estate owners increasing from -0.0302 to -0.0212 and for rural real estate owners the slope increased from -0.0185 to -0.0110. This is an increase of 0.0090 and 0.0075. For tenants the increases in the slope have only been 0.0025 and 0.0002. It is thus clear that the largest

impact of including the variable for real estate values have been in the elasticities for real estate owners.

For real estate owners the change in the slope due to the inclusion of a variable for real estate value is 29.8% for urban real estate owners and 40.5% for rural real estate owners. For tenants the change in slope is 17.5% for urban tenants and 1.0% for rural tenants. For the model with all households the change is 15.1%.

For real estate owners the change in constant due to the inclusion of a variable for real estate value is 14.7% for urban real estate owners and 24.9% for rural real estate owners. For tenants the change in constant is 2.3% for urban tenants and 0.8% for rural tenants. For the model with all households the change is 5.4%. We conclude that the real estate values influence real estate owners and tenants differently and we see that omission of a variable for real estate prices have biased the income elastities upwards. Further examination of this topic we address in Nielsen (2006).

Without real estate values	With real estate values
0.6813-0.0185*year	0.6443-0.0157*year
0.8165-0.0302*year	0.6962-0.0212*year
0.6813-0.0185*year	0.5116-0.0110*year
1.1969-0.0143*year	1.1692-0.0118*year
0.9110-0.0192*year	0.9036-0.0190*year
	0.6813-0.0185*year  0.8165-0.0302*year  0.6813-0.0185*year  1.1969-0.0143*year

Table 7: Best-fit linear equations for income elasticities

(year=0 for 1995, year=1 for 1996,..., year=7 for 2002)

# 5. Conclusion

In this paper we addressed the problem of bias in estimated parameters when simple cross-section methods are used to determine car ownership. The problem of parameter instability is not unknown and ideally time series should be used. Unfortunately transport researchers are often forced to use simple methods due to data limitations. Identification of variables which can reduce the problem is therefore in demand. We used repeated cross-section analysis of Danish data to show that the omission of a variable for real estate values bias estimated income elastities upwards. By dividing data into four groups which we expect to be affected differently by real estate values we are able to analyze the hypothesis that real estate values influence car demand. We showed that the inclusion of a variable for real estate values in the Danish municipalities reduce the estimated income elasticities showing us that omission of this variable cause the estimates to be upward biased.

## References

Ben-Akiva, M. & Lerman, S. R. (1985) *Discrete Choice Analysis: Theory and Application to Travel Demand*, The MIT Press, Cambridge, England.

Birkeland, M. E., Brems, C. R. and Kabelmann, T. (2000) Analyse af personers transportarbejde, 1975-1998, Trafikdage på Aalborg Universitet 2000, Conference paper (www.trafikdage.dk)

Bjørner, T. B. (1999) Estimation af elasticiteter for persontransporten med bil, in *Transport Demand and Environmental Regulation in the Transport Sector, Ph.D.* dissertation, Rød Serie 54, Department of Economics, University of Copenhagen.

Christens, P. F. & Fosgerau, M. (2004) Rådighed over bil, En beskrivelse af sammenhængen mellem husstandsindkomst, bilrådighed og geografi, *Notat* **3:2004**, Danish Transport Research Institute.

Dargay, J. M. and Gately, D. (1999) Income's effect on car and vehicle ownership, worldwide: 1960-2015, Transportation Research Part A, 33, 101-138.

Dargay, J. M and Vythoulkas, P. C. (1999) Estimation of a Dynamic Car Ownership Model: A Pseudo-panel Approach, Journal of Transport Economics and Policy 33:3, 287-302.

Dargay, J. M. (2001) The effect of income on car ownership: evidence and asymmetry, *Transportation Research Part A* **35**, 807-821.

Dargay, J. M. (2002) Determinants of car ownership in rural and urban areas: a pseudopanel analysis, Transportation Research Part E, 38, 351-366.

Dargay, J., Hivert, L. and Legros, D. (2006) The dynamics of car ownership in E.U countries: a comparison based on the european household panel survey, *working paper*.

Deaton, A. (1985) Panel Data from Time Series of Cross Sections, Journal of Econometrics 30, 109-126.

De Jong, G., Pieters, M., Miller, S., Daly, A., Plasmeijer, R., Graafland, I., Lierens, A., Baak, J., Walker, W. & Kroes, E. (2005), Uncertainty in traffic Forecasts – Literature Review and New Results for the Netherlands, Rand Europe for the AVV Transport Research Centre.

Fosgerau, M., Holmblad, M. & Pilegaard, N. (2004) ART – En aggregeret prognosemodel for dansk vejtrafik, *Notat* **5:2004**, Danish Transport Research Institute.

Jansson, J. O. (1989) Car demand modeling and forecasting – A New Approach, *Journal* of *Transport Economics and Policy* **13:2**, 125-140.

Kwon, T. and Preston, J. (2005) Driving Forces behind the Growth of Per-capita Car Driving Distance in the UK, 1970-2000, *Transport Reviews* **25:4**, 467-490.

Nielsen, J. E. (2006a) Real estate ownership and the demand for cars in Denmark – A pseudo-panel analysis, *Thesis paper*.

Nielsen, J. E. (2006b) Demand for car transport in Denmark – Differences between rural and urban households, *Thesis paper*.

Orhúzar, J. de D. and Willumsen, L. G. (1994) Modelling Transport, Second Edition, John Wiley & Sons, Chichester.

Oum, T. H., Waters II, W. G. & Yong, J-S. (1992) Concepts of price elasticities of transport demand and recent empirical estimates, An Interpretative Survey, *Journal of Transport Economics and Policy* **26:2**, 139-154.

Pendyala, R. M., Kostyniuk, L. P. & Goulias, K. G. (1995) A repeated cross-sectional evaluation of car ownership, *Transportation* **22**, 165-184.

Train, K. (1980) A Structured Logit Model of Auto Ownership and Mode Choice, *The Review of Economic Studies* **47:2**, 357-370.

Train, K. E. (2003) *Discrete Choice Methods with Simulation*, Cambridge University Press, Cambridge, United Kingdom.

# Appendix A

Estimates for the A.1 model (All households but no variable for real estate values)

	199	5	1990	6	199	7	199	8	199	9	200	0	200	1	200	2	Identi	cal coef
Choice 1	Est.	t-value	Est.	t-value	Est.	t-value	Est.	t-value	Est.	t-value	Est.	t-value	Est.	t-value	Est.	t-alue	Est.	t-value
Constant	-9,8192	-13,73	-9,3776	-16,70	-10,4673	-17,72	-9,3193	-17,98	-9,1949	-16,51	-9,0795	-16,19	-9,4952	-15,89	-7,7651	-9,15	-1,4489	-9,78
Loginc	2,1582	14,98	2,1819	19,05	2,2801	19,23	1,9542	19,29	2,0074	18,39	2,1833	19,30	1,9537	16,85	1,9682	11,55	0,0126	46,92
Age	-0,0120	-0,56	-0,0353	-2,12	0,0067	0,39	0,0227	1,45	-0,0179	-1,03	-0,0361	-2,03	0,0170	0,92	-0,0573	-1,96	0,0061	0,98
Age2	0,0004	1,56	0,0006	3,32	0,0002	1,00	0,0000	0,15	0,0005	2,45	0,0007	3,52	0,0001	0,67	0,0009	2,78	0,0002	2,74
Distpub	0,2758	4,22	0,3005	5,98	0,2731	5,25	0,2714	5,75	0,2394	4,92	0,2475	5,16	0,1278	2,70	0,1817	2,45	0,2235	12,39
Distwork	0,1520	2,85	0,1002	2,37	0,1566	3,48	0,1212	2,77	0,1726	3,72	0,0854	1,83	0,1379	2,76	0,1254	2,04	0,1418	8,48
Child	0,1658	2,49	0,1623	3,22	0,0789	1,51	0,2619	4,94	0,2445	4,57	0,3851	6,53	0,2298	4,01	0,3044	3,17	0,1865	9,09
Adults	0,3061	3,12	0,3859	5,06	0,2563	3,15	0,3861	5,28	0,5219	6,59	0,2762	3,49	0,5446	6,32	0,3871	3,20	0,4646	15,63
Urban	-0,9672	-10,44	-1,0941	-14,93	-1,2366	-16,00	-1,2019	-17,17	-1,1346	-14,94	-1,2247	-15,90	-1,1469	-14,25	-1,0358	-8,58	-1,1307	-40,53
Owner	-0,7659	-7,91	-0,8216	-11,07	-0,8472	-10,92	-0,9129	-12,92	-0,7968	-10,39	-0,9589	-12,53	-0,8515	-10,47	-0,8357	-6,91	-0,8533	-30,09
Choice 2																		
Constant	-28,0344	-19,75	-27,0840	-24,79	-28,5254	-26,40	-25,4912	-27,88	-22,9536	-25,24	-24,2704	-25,20	-24,6226	-24,55	-21,6672	-14,96	-5,7092	-24,05
Loginc	5,0782	19,78	5,1079	25,54	5,4031	27,62	4,4288	27,79	4,2253	26,04	4,6614	27,44	4,5099	25,78	4,2178	16,63	0,0215	67,64
Age	0,0305	0,77	-0,0385	-1,32	-0,0066	-0,22	0,0770	2,90	0,0034	0,12	-0,0198	-0,68	-0,0020	-0,06	-0,0209	-0,45	0,0406	3,84
Age2	-0,0002	-0,45	0,0006	1,75	0,0002	0,58	-0,0006	-2,13	0,0002	0,50	0,0004	1,29	0,0003	0,97	0,0003	0,61	-0,0003	-2,66
Distpub	0,4962	6,10	0,5075	8,00	0,4322	6,81	0,4491	7,89	0,3673	6,38	0,4476	7,68	0,3113	5,25	0,2184	2,39	0,3889	17,58
Distwork	0,2229	3,29	0,1842	3,50	0,1758	3,21	0,1588	3,03	0,1679	3,07	0,1182	2,12	0,1610	2,68	0,0988	1,37	0,1833	9,08
Child	0,1109	1,34	0,1451	2,30	0,0486	0,75	0,3153	5,05	0,3139	5,01	0,3656	5,35	0,2768	4,07	0,3498	3,24	0,2075	8,49
Adults	0,3290	2,58	0,5277	5,45	0,4641	4,60	0,5419	5,98	0,7119	7,43	0,4547	4,72	0,8065	7,57	0,5504	3,75	0,6826	18,69
Urban	-1,2592	-9,22	-1,3741	-12,90	-1,6323	-14,69	-1,6301	-16,28	-1,6561	-15,71	-1,6624	-15,15	-1,5365	-13,44	-1,6632	-9,93	-1,5788	-38,92
Owner	-1,3158	-7,25	-1,2292	-9,03	-1,4167	-10,13	-1,3229	-10,72	-1,3765	-10,39	-1,6210	-11,65	-1,5951	-10,52	-1,2421	-6,17	-1,4477	-28,32
Observations				56.464														56.464
L(0)			-62.0	032,0443													-6	2.032,0443
L(c) (degrees of freed	lom)		-51.	148,2943													-5	1.202,5516
L(β) (degrees of freed	lom)		-37.0	021,3581													-3	7.305,5362
McFaddens $\rho^2$ (0)				0,4032														0,3986
McFaddens $\rho^2$ (c)				0,2762														0,2714

Table A.1: Test statistics for the model with all households. Model estimates with free variables for all years and model estimates with identical parameters for all years. Variable for real estate value not included.

Estimates for the A.2 model (Urban real estate owners but no variable for real estate values)

	199	5	1996		1997		1998		199	9	200	0	200	1	200	)2	Identio	cal coef
Choice 1	Est.	t-value	Est.	t-value	Est.	t-value	Est.	t-value	Est.	t-value	Est.	t-value	Est.	t-value	Est.	t-value	Est.	t-value
Constant	-13,1682	-8,34	-13,6167	-11,70	-14,3422	-11,48	-11,1457	-10,81	-11,2608	-10,04	-11,5683	-10,58	-13,8841	-10,87	-7,6215	-4,13	-4,1225	-13,64
Loginc	2,0563	6,49	2,3969	10,03	2,4116	9,09	1,7169	8,21	1,9496	8,35	1,9376	8,18	2,3117	8,99	1,7115	4,42	0,0112	22,56
Age	0,0400	0,77	0,0142	0,38	0,0461	1,20	0,0143	0,38	-0,0326	-0,79	0,0059	0,14	0,0039	0,09	-0,1535	-1,95	0,0191	1,32
Age2	-0,0001	-0,14	0,0002	0,58	-0,0002	-0,42	0,0003	0,68	0,0008	1,82	0,0004	0,87	0,0005	1,09	0,0023	2,71	0,0002	1,37
Distpub	0,2759	1,66	0,1637	1,23	0,4930	3,48	0,5187	3,71	0,3257	2,35	0,3652	2,69	0,5027	3,29	0,0697	0,28	0,3568	6,89
Distwork	0,2773	2,02	0,0703	0,75	0,1738	1,57	0,0158	0,16	0,1573	1,42	-0,0789	-0,72	0,1299	1,13	0,1646	1,11	0,1003	2,58
Child	0,2177	1,59	0,2445	2,24	-0,0307	-0,30	0,4233	3,79	0,4610	4,01	0,4563	3,89	0,3348	2,77	0,7590	3,76	0,2901	6,80
Adults	0,6365	2,99	0,3190	2,16	0,1179	0,71	0,5894	3,89	0,4921	2,99	0,3312	2,04	0,2887	1,68	0,4379	1,56	0,4290	7,05
Choice 2																		
Constant	-33,3122	-11,00	-35,7628	-14,32	-37,0878	-15,52	-28,3874	-15,30	-27,4226	-14,06	-26,1558	-13,82	-29,6404	-14,62	-22,1481	-7,33	-9,6952	-19,38
Loginc	5,3125	9,71	5,5732	12,83	5,7955	13,76	4,2024	13,27	4,3128	12,66	4,3841	12,98	4,6372	13,15	3,9633	7,26	0,0188	32,70
Age	0,0471	0,57	0,0936	1,38	0,1156	1,69	0,0982	1,65	-0,0270	-0,41	-0,0571	-0,88	0,0026	0,04	-0,1243	-1,13	0,0719	3,04
Age2	-0,0001	-0,14	-0,0005	-0,69	-0,0009	-1,21	-0,0007	-1,13	0,0008	1,08	0,0010	1,47	0,0006	0,85	0,0020	1,70	-0,0004	-1,44
Distpub	0,2945	1,42	0,4651	2,79	0,9114	5,09	0,8760	5,27	0,6585	3,95	0,6468	3,92	0,5969	3,16	0,0447	0,15	0,6128	9,68
Distwork	0,3885	2,24	0,1750	1,44	0,0633	0,45	-0,1435	-1,11	0,0599	0,43	-0,1129	-0,80	0,0404	0,28	0,0434	0,24	0,0753	1,52
Child	0,0841	0,49	0,4020	2,99	0,0381	0,29	0,4529	3,42	0,5414	3,92	0,5696	4,12	0,4664	3,24	0,7461	3,25	0,3689	7,20
Adults	0,6676	2,54	0,3990	2,08	0,2587	1,20	0,5831	3,01	0,8643	4,22	0,5114	2,49	0,7404	3,50	0,4582	1,35	0,6566	8,67
Observations	•			10.990														10.990
L(0)			-12.	073,7491													-12	2.073,7491
L(c) (degrees of freed	dom)		-10.	125,5609													-10	0.132,4230
L(β) (degrees of freed	dom)		-7.	953,7969													-8.060,4679	
McFaddens $\rho^2(0)$				0,3412														0,3324
McFaddens $\rho^2$ (c)			0.2145														0,2025	

Table A.2: Test statistics for the model with all households. Model estimates with free variables for all years and model estimates with identical parameters for all years. Variable for real estate value not included.

Estimates for the A.3 model (Rural real estate owners but no variable for real estate values)

Listifiates 10	199		199		199		199		199		200		200	1	2002		Identi	cal coef
Choice 1	Est.	t-value	Est.	t-value	Est.	t-value	Est.	t-value	Est.	t-value	Est.	t-value	Est.	t-value	Est.	t-value	Est.	t-value
Constant	-10,9887	-7,07	-11,7382	-9,46	-13,1728	-10,24	-9,8357	-8,98	-8,9171	-7,55	-11,0497	-8,35	-10,9490	-8,37	-10,4957	-5,66	-3,2004	-10,43
Loginc	2,0183	6,29	2,1496	8,64	2,5698	10,36	1,7775	8,28	1,4695	6,17	2,2570	8,70	1,8521	7,54	1,9860	5,31	0,0123	20,40
Age	0,0488	1,09	0,0150	0,44	0,0269	0,73	0,0135	0,40	0,0418	1,16	-0,0249	-0,61	0,0554	1,39	0,0099	0,16	0,0354	2,65
Age2	-0,0003	-0,67	0,0002	0,48	0,0000	0,02	0,0001	0,37	-0,0003	-0,73	0,0006	1,37	-0,0003	-0,61	0,0002	0,25	-0,0001	-0,92
Distpub	0,3246	2,92	0,2782	3,48	0,1467	1,84	0,2154	2,70	0,1015	1,42	0,1855	2,41	0,0290	0,40	0,1578	1,35	0,1560	5,51
Distwork	0,0231	0,24	0,0598	0,77	0,0799	0,93	0,0990	1,17	0,2311	2,64	0,0344	0,39	0,1980	2,00	0,1799	1,53	0,1187	3,75
Child	0,1130	0,98	0,3072	2,99	0,2138	1,95	0,4746	3,83	0,1853	1,66	0,4989	3,49	0,1857	1,59	0,1766	0,92	0,2202	5,17
Adults	0,3731	1,80	0,6665	4,03	0,3874	2,40	0,8548	5,48	1,1276	6,34	0,6890	3,70	0,8621	4,76	0,7323	2,76	0,8204	12,99
Choice 2																		
Constant	-30,4935	-13,98	-29,9672	-17,69	-30,4862	-18,13	-26,7399	-18,59	-22,9029	-15,85	-26,8778	-16,54	-27,0654	-16,50	-25,9104	-10,93	-8,1781	-22,00
Loginc	4,8902	11,71	5,1220	15,79	5,4977	17,48	4,2921	16,06	3,6003	12,89	4,7645	15,57	4,3990	14,82	4,3955	9,86	0,0218	33,81
Age	0,1455	2,33	0,0050	0,11	0,0107	0,22	0,0781	1,82	0,0786	1,79	-0,0177	-0,36	0,0555	1,12	0,0555	0,72	0,0756	4,46
Age2	-0,0015	-2,21	0,0002	0,43	0,0000	-0,06	-0,0007	-1,44	-0,0008	-1,63	0,0004	0,70	-0,0003	-0,52	-0,0005	-0,65	-0,0007	-3,95
Distpub	0,5927	4,79	0,4586	5,03	0,2555	2,86	0,3599	4,13	0,1949	2,48	0,3540	4,15	0,2201	2,71	0,1938	1,48	0,3004	9,49
Distwork	0,0705	0,65	0,1262	1,47	0,1015	1,10	0,1793	2,00	0,2258	2,43	0,0896	0,95	0,2461	2,35	0,1880	1,51	0,1696	5,00
Child	0,1098	0,85	0,2851	2,55	0,1434	1,21	0,4974	3,84	0,2638	2,26	0,4519	3,05	0,2354	1,91	0,2332	1,17	0,2333	5,17
Adults	0,3053	1,33	0,8388	4,65	0,6262	3,58	0,9908	5,90	1,2704	6,75	0,9229	4,67	1,0530	5,38	0,9039	3,19	1,0106	14,88
Observations				29.174														29.174
L(0)			-32.	050,9149													-32	2.050,9149
L(c) (degrees of freed	L(c) (degrees of freedom)		-21.	168,3518													-21	.214,8269
L(β) (degrees of freed	$L(\beta)$ (degrees of freedom) -18.055,3316		055,3316													-18	3.176,7024	
McFaddens $\rho^2$ (0)				0,4367														0,4329
McFaddens $\rho^2$ (c)	McFaddens $\rho^2$ (c) 0,1471															0,1432		

Table A.3: Test statistics for the model with all households. Model estimates with free variables for all years and model estimates with identical parameters for all years. Variable for real estate value not included.

Estimates for the A.4 model (Urban tenants but no variable for real estate values)

	199	5	199	96	199	7	199	98	199	9	200	0	200	)1	200	12	Identi	cal coef
Choice 1	Est.	t-value	Est.	t-value	Est.	t-value	Est.	t-value	Est.	t-value	Est.	t-value	Est.	t-value	Est.	t-value	Est.	t-value
Constant	-12,2322	-10,41	-10,6336	-12,00	-11,1991	-12,22	-13,2917	-14,29	-12,4605	-13,24	-11,5224	-12,32	-10,8954	-11,49	-11,5442	-8,00	-3,3886	-14,00
Loginc	2,3956	9,02	2,3504	11,42	2,0526	10,13	2,1779	11,29	2,4414	12,11	2,2995	11,59	1,8349	8,90	2,1013	6,69	0,0132	27,77
Age	-0,0374	-0,92	-0,1226	-3,98	-0,0385	-1,24	0,0486	1,65	-0,0594	-1,83	-0,0739	-2,24	0,0111	0,31	-0,0164	-0,32	-0,0146	-1,26
Age2	0,0006	1,42	0,0015	4,53	0,0006	1,88	-0,0002	-0,73	0,0009	2,50	0,0010	2,84	0,0001	0,20	0,0003	0,62	0,0004	3,06
Distpub	0,1851	1,16	0,2597	2,23	0,3032	2,57	0,3145	2,88	0,3266	2,75	0,4141	3,49	0,0196	0,17	0,3781	2,06	0,2467	5,78
Distwork	-0,0400	-0,36	0,1106	1,24	0,1640	1,84	0,1760	1,90	0,1054	0,98	0,1499	1,51	-0,0543	-0,47	-0,0808	-0,62	0,1016	2,85
Child	0,3117	2,16	0,1467	1,46	0,0608	0,58	0,1145	1,15	0,3775	3,75	0,3894	3,61	0,1621	1,46	0,4882	2,39	0,2039	5,11
Adults	0,0444	0,23	0,1299	0,95	0,2638	1,75	-0,0510	-0,37	0,0614	0,44	-0,0330	-0,24	0,5333	3,22	0,2304	1,02	0,2057	3,77
Choice 2																		
Constant	-31,2998	-5,81	-25,1345	-8,47	-31,9303	-7,75	-31,8202	-9,55	-25,3325	-8,87	-34,3646	-8,83	-28,3631	-7,88	-26,5916	-5,60	-8,5283	-11,48
Loginc	5,1608	5,07	4,3206	7,36	6,2462	8,25	4,7447	7,81	4,1904	7,95	5,8738	8,21	4,9150	7,99	4,6859	5,19	0,0230	26,33
Age	-0,1269	-0,80	-0,0169	-0,19	-0,3122	-2,77	0,0978	0,98	-0,0430	-0,46	-0,0034	-0,03	-0,1193	-0,99	-0,1610	-1,04	-0,0345	-0,92
Age2	0,0015	0,79	0,0000	-0,02	0,0038	2,96	-0,0007	-0,63	0,0005	0,51	0,0004	0,33	0,0013	0,92	0,0016	0,92	0,0005	1,15
Distpub	0,6917	1,56	0,8821	3,48	-0,3416	-0,65	0,7034	2,56	0,6219	2,22	0,5858	1,80	0,4236	1,17	0,5089	1,11	0,5314	4,69
Distwork	-0,0523	-0,18	0,0547	0,27	0,8800	3,84	0,0148	0,06	0,1995	0,91	0,2807	1,16	-0,1235	-0,45	-0,0061	-0,02	0,2122	2,51
Child	0,0372	0,10	-1,0878	-2,89	-0,0008	0,00	-0,4361	-1,40	0,1634	0,73	-0,1225	-0,42	0,0924	0,35	0,3657	1,06	-0,1700	-1,68
Adults	1,3582	2,83	-0,1297	-0,38	0,3345	0,71	0,2844	0,83	0,3734	1,20	-0,8083	-1,81	0,8046	1,82	1,2380	2,34	0,4200	3,34
Observations				9.435														9.435
L(0)			-10	.365,4069													-10	0.365,4069
L(c) (degrees of freed	dom)		-7.	.498,6473													-7	.507,2105
L(β) (degrees of free	dom)		-5.	.846,9292													-5	5.970,4143
McFaddens $\rho^2$ (0)				0,4359														0,4240
McFaddens $\rho^2$ (c)				0,2203														0,2047

Table A.4: Test statistics for the model with all households. Model estimates with free variables for all years and model estimates with identical parameters for all years. Variable for real estate value not included.

Estimates for the A.5 model (Rural tenants but no variable for real estate values)

	199	15	199	6	199	7	199	18	199	9	200	0	200	1	200	2	Identio	cal coef
Choice 1	Est.	t-value	Est.	t-value	Est.	t-value	Est.	t-value	Est.	t-value	Est.	t-value	Est.	t-value	Est.	t-value	Est.	t-value
Constant	-10,0490	-7,92	-10,1520	-9,17	-11,8626	-9,69	-10,3137	-10,53	-9,5764	-8,51	-9,4138	-8,80	-10,4882	-9,20	-7,6365	-4,98	-2,7396	-10,57
Logine	2,1192	7,48	1,8542	7,24	2,2532	8,66	2,0324	9,48	1,9026	7,91	1,9314	8,29	1,9687	7,91	2,0162	5,83	0,0147	21,97
Age	-0,0599	-1,46	0,0090	0,26	0,0037	0,10	0,0011	0,04	-0,0419	-1,20	-0,0418	-1,28	-0,0009	-0,03	-0,1094	-1,93	-0,0060	-0,49
Age2	0,0008	1,81	-0,0001	-0,16	0,0002	0,48	0,0001	0,19	0,0006	1,52	0,0006	1,64	0,0002	0,56	0,0012	1,95	0,0002	1,43
Distpub	0,2676	2,25	0,3893	3,83	0,3192	3,15	0,2271	2,78	0,3271	3,37	0,1734	2,03	0,1507	1,70	0,0728	0,56	0,2281	6,80
Distwork	0,3073	3,12	0,1394	1,70	0,1781	2,13	0,1273	1,57	0,1289	1,55	0,1930	2,25	0,2131	2,45	0,2315	1,87	0,1870	6,07
Child	0,1056	0,67	0,0120	0,12	0,0666	0,57	0,1311	1,22	-0,0573	-0,52	0,2366	1,98	0,2639	2,24	-0,0462	-0,24	0,0782	1,84
Adults	0,1007	0,49	0,3518	1,89	0,0479	0,26	0,1383	0,88	0,4525	2,51	0,2306	1,40	0,2331	1,30	0,0892	0,39	0,2150	3,33
Choice 2																		
Constant	-26,9017	-6,78	-29,3039	-8,40	-31,9003	-9,31	-24,8556	-10,12	-29,5413	-9,09	-26,0697	-8,59	-29,4290	-8,43	-19,7749	-5,18	-8,3348	-13,19
Logine	4,8788	6,29	5,2491	7,65	5,3295	8,52	4,1739	8,83	5,1596	8,32	3,8689	7,56	5,0107	7,74	3,2026	4,69	0,0250	24,64
Age	-0,0758	-0,64	-0,1189	-1,13	0,0576	0,60	-0,0252	-0,36	-0,0602	-0,63	0,1432	1,43	-0,0109	-0,09	0,0949	0,70	0,0115	0,34
Age2	0,0008	0,58	0,0011	0,83	-0,0006	-0,56	0,0003	0,43	0,0007	0,63	-0,0018	-1,47	0,0001	0,04	-0,0012	-0,81	-0,0002	-0,53
Distpub	0,3811	1,61	0,5859	3,20	0,6851	4,31	0,3934	2,90	0,5505	3,44	0,5809	3,85	0,3607	1,92	0,2568	1,20	0,4625	7,90
Distwork	0,4244	2,45	0,4122	2,79	0,1770	1,25	0,1206	0,90	0,2631	1,79	0,0186	0,12	0,0345	0,18	-0,1136	-0,52	0,1959	3,59
Child	-0,4890	-1,42	0,1172	0,59	-0,0095	-0,05	0,4296	2,76	0,0062	0,03	0,1732	0,90	-0,0618	-0,25	0,1053	0,38	0,1085	1,52
Adults	0,2602	0,62	0,8120	2,19	0,1878	0,57	0,7768	2,92	0,5170	1,41	0,3890	1,49	0,6504	1,85	0,1280	0,33	0,5763	5,13
Observations				6.865														6.865
L(0)			-7.	541,9734													-7	.541,9734
L(c) (degrees of freed	dom)		-5.	919,4227													-5	.926,5814
L(β) (degrees of freed	dom)		-4.	752,4732													-4	.817,3043
McFaddens $\rho^2$ (0)				0,3699														0,3613
McFaddens ρ <sup>2</sup> (c)				0,1971														0,1872

Table A.5: Test statistics for the model with all households. Model estimates with free variables for all years and model estimates with identical parameters for all years. Variable for real estate value not included.

Estimates for the A.6 model (All households and a variable for real estate values)

Listimates it	199		199		199		199		199		200	0	200	)1	200	)2	Identic	cal coef
Choice 1	Est.	t-value	Est.	t-value	Est.	t-value	Est.	t-value	Est.	t-value	Est.	t-value	Est.	t-value	Est.	t-value	Est.	t-value
Constant	-10,1048	-14,02	-9,7002	-17,18	-10,5801	-17,90	-9,5156	-18,26	-9,3042	-16,68	-9,1202	-16,26	-9,5093	-15,90	-7,8113	-9,18	-1,6420	-10,84
Loginc	2,1139	14,62	2,1105	18,31	2,2282	18,69	1,9021	18,66	1,9585	17,81	2,1624	18,95	1,9512	16,69	1,9555	11,40	0,0124	45,79
Age	-0,0116	-0,54	-0,0374	-2,24	0,0043	0,25	0,0192	1,22	-0,0195	-1,12	-0,0368	-2,07	0,0167	0,90	-0,0575	-1,97	0,0054	0,86
Age2	0,0004	1,50	0,0006	3,33	0,0002	1,05	0,0000	0,26	0,0005	2,47	0,0007	3,54	0,0001	0,68	0,0009	2,78	0,0002	2,75
Distpub	0,2676	4,10	0,2884	5,75	0,2646	5,09	0,2614	5,55	0,2363	4,87	0,2447	5,10	0,1268	2,68	0,1816	2,45	0,2200	12,20
Distwork	0,1531	2,87	0,1001	2,37	0,1560	3,47	0,1203	2,75	0,1721	3,72	0,0876	1,87	0,1380	2,76	0,1252	2,04	0,1433	8,56
Child	0,1494	2,23	0,1368	2,70	0,0622	1,19	0,2410	4,53	0,2331	4,34	0,3803	6,43	0,2290	3,99	0,3019	3,14	0,1805	8,79
Adults	0,2996	3,03	0,3904	5,08	0,2644	3,24	0,3837	5,23	0,5249	6,61	0,2776	3,50	0,5442	6,31	0,3907	3,23	0,4709	15,79
Urban	-1,0744	-10,90	-1,2346	-15,86	-1,3258	-16,20	-1,3241	-17,63	-1,2407	-14,92	-1,2712	-14,98	-1,1568	-12,62	-1,0807	-7,69	-1,1916	-39,96
Owner	-0,6464	-6,27	-0,6379	-7,93	-0,7336	-8,74	-0,7758	-10,21	-0,7029	-8,60	-0,9214	-11,35	-0,8437	-9,81	-0,8100	-6,36	-0,7972	-26,75
Value	0,0008	3,36	0,0010	5,81	0,0005	3,49	0,0006	4,76	0,0004	3,27	0,0002	1,37	0,0000	0,30	0,0001	0,65	0,0003	6,11
Choice 2																		
Constant	-28,1495	-19,74	-27,1152	-24,72	-28,5041	-26,34	-25,3696	-27,64	-22,9351	-25,19	-24,1350	-25,00	-24,3291	-24,20	-21,5514	-14,85	-6,1339	-25,41
Loginc	4,8584	18,75	4,8732	24,07	5,2782	26,61	4,2330	26,11	4,0972	24,88	4,5572	26,34	4,3540	24,52	4,1333	16,04	0,0209	64,75
Age	0,0337	0,85	-0,0419	-1,43	-0,0090	-0,30	0,0739	2,78	0,0021	0,08	-0,0200	-0,69	-0,0007	-0,02	-0,0207	-0,44	0,0404	3,81
Age2	-0,0003	-0,61	0,0006	1,74	0,0002	0,58	-0,0006	-2,15	0,0001	0,47	0,0004	1,25	0,0003	0,84	0,0003	0,58	-0,0003	-2,84
Distpub	0,4906	6,02	0,4953	7,81	0,4277	6,74	0,4410	7,76	0,3709	6,45	0,4488	7,70	0,3243	5,47	0,2220	2,44	0,3899	17,63
Distwork	0,2248	3,31	0,1911	3,62	0,1757	3,21	0,1614	3,08	0,1704	3,12	0,1245	2,23	0,1635	2,72	0,0992	1,38	0,1874	9,28
Child	0,0796	0,96	0,1106	1,74	0,0249	0,39	0,2841	4,52	0,2962	4,71	0,3539	5,16	0,2638	3,87	0,3475	3,22	0,1934	7,90
Adults	0,3358	2,62	0,5467	5,60	0,4793	4,74	0,5544	6,10	0,7199	7,49	0,4633	4,80	0,8182	7,67	0,5576	3,80	0,6992	19,10
Urban	-1,6364	-10,65	-1,7261	-14,58	-1,8145	-14,87	-1,9381	-17,25	-1,8896	-15,83	-1,8263	-14,75	-1,7865	-13,47	-1,8220	-9,29	-1,7655	-39,78
Owner	-1,0061	-5,31	-0,8678	-6,02	-1,2266	-8,32	-1,0658	-8,24	-1,2038	-8,72	-1,5156	-10,52	-1,4529	-9,28	-1,1642	-5,60	-1,3107	-24,89
Value	0,0017	5,69	0,0017	7,72	0,0008	4,12	0,0011	6,59	0,0007	4,40	0,0004	2,72	0,0004	3,23	0,0003	1,49	0,0006	10,46
Observations				56.464														56.464
L(0)			-62.	.032,0443													-62	.032,0443
L(c) (degrees of free	dom)		-51.148,	2943 (18)													-51.20	2,5516 (2)
L(β) (degrees of free	dom)		-36.916,0	413 (180)													-37.247	,5566 (22)
$McFaddens \ \rho^2 \ (0)$				0,4049														0,3995
McFaddens $\rho^2$ (c)				0,2783														0,2725

Table A.6: Test statistics for the model with all households. Model estimates with free variables for all years and model estimates with identical parameters for all years. Variable for real estate value included.

# Estimates for the A.7 model (Urban real estate owners and a variable for real estate values)

	199	5	199	16	199	7	199	8	199	9	200	0	200	1	200	)2	Identic	cal coef
Choice 1	Est.	t-value	Est.	t-value	Est.	t-value	Est.	t-value	Est.	t-value	Est.	t-value	Est.	t-value	Est.	t-value	Est.	t-value
Constant	-12,7202	-8,07	-12,5693	-10,77	-13,6642	-10,91	-10,8348	-10,50	-10,8919	-9,69	-11,3777	-10,41	-13,7924	-10,78	-7,4517	-4,03	-4,3066	-14,17
Loginc	1,8781	5,78	2,1086	8,63	2,2103	8,17	1,6197	7,66	1,7949	7,62	1,8342	7,66	2,2742	8,77	1,6152	4,13	0,0107	20,96
Age	0,0397	0,76	-0,0065	-0,17	0,0425	1,10	0,0054	0,15	-0,0458	-1,10	0,0006	0,01	0,0005	0,01	-0,1591	-2,03	0,0166	1,14
Age2	-0,0001	-0,24	0,0003	0,79	-0,0002	-0,50	0,0003	0,72	0,0009	1,91	0,0004	0,86	0,0005	1,11	0,0024	2,75	0,0002	1,23
Distpub	0,2460	1,47	0,0432	0,32	0,4349	3,06	0,4442	3,16	0,2835	2,03	0,2985	2,18	0,4888	3,21	0,0251	0,10	0,3235	6,25
Distwork	0,2891	2,09	0,0969	1,02	0,1746	1,57	0,0161	0,16	0,1680	1,51	-0,0568	-0,52	0,1229	1,07	0,1785	1,20	0,1083	2,77
Child	0,1611	1,16	0,1365	1,23	-0,0946	-0,90	0,3318	2,90	0,3733	3,19	0,3945	3,30	0,3191	2,62	0,7310	3,62	0,2530	5,88
Adults	0,6070	2,81	0,2806	1,87	0,1348	0,80	0,5412	3,54	0,4289	2,60	0,3061	1,88	0,2778	1,62	0,4481	1,58	0,4297	7,02
Value	0,0010	2,35	0,0018	5,84	0,0008	3,13	0,0008	3,63	0,0010	4,52	0,0006	2,98	0,0002	1,18	0,0004	1,43	0,0005	6,74
Choice 2																		
Constant	-31,7394	-10,38	-33,3613	-13,18	-35,8590	-14,89	-26,4259	-14,08	-25,4350	-12,91	-25,0223	-13,11	-28,3596	-13,89	-21,4223	-7,05	-10,0625	-19,94
Loginc	4,7132	8,47	4,9049	11,05	5,4350	12,58	3,6939	11,33	3,7540	10,72	4,0567	11,72	4,2597	11,83	3,7243	6,71	0,0173	29,37
Age	0,0571	0,68	0,0706	1,02	0,1141	1,67	0,0888	1,48	-0,0459	-0,69	-0,0680	-1,04	0,0041	0,06	-0,1333	-1,20	0,0699	2,94
Age2	-0,0004	-0,40	-0,0005	-0,62	-0,0010	-1,33	-0,0008	-1,19	0,0008	1,12	0,0010	1,46	0,0005	0,67	0,0021	1,73	-0,0004	-1,73
Distpub	0,2122	1,01	0,2974	1,74	0,8499	4,74	0,7808	4,67	0,6092	3,63	0,5699	3,43	0,5613	2,96	-0,0080	-0,03	0,5701	8,98
Distwork	0,4121	2,34	0,2095	1,69	0,0577	0,41	-0,1489	-1,14	0,1033	0,74	-0,0743	-0,53	0,0167	0,11	0,0544	0,30	0,0864	1,73
Child	-0,0351	-0,20	0,2662	1,94	-0,0501	-0,37	0,2985	2,19	0,3884	2,74	0,4601	3,25	0,4031	2,77	0,7278	3,16	0,2955	5,69
Adults	0,6661	2,50	0,3906	2,00	0,2881	1,32	0,5600	2,87	0,7781	3,76	0,4984	2,42	0,7385	3,48	0,4689	1,37	0,6690	8,79
Value	0,0023	4,58	0,0027	7,40	0,0011	3,56	0,0014	5,72	0,0018	6,58	0,0010	4,14	0,0008	3,55	0,0006	1,93	0,0009	10,51
Observations				10.990														10.990
L(0)			-12.	.073,7491													-12	.073,7491
L(c) (degrees of free	dom)		-10.125,	5609 (18)													-10.132	2,4230 (2)
L(β) (degrees of free	edom)		-7.841,0	372 (180)													-8.001,	,3378 (22)
McFaddens $\rho^2(0)$				0,3506														0,3373
McFaddens $\rho^2$ (c)				0,2256														0,2103

Table A.7: Test statistics for the model with all households. Model estimates with free variables for all years and model estimates with identical parameters for all years. Variable for real estate value included.

# Estimates for the A.8 model (Rural real estate owners and a variable for real estate values)

	199:	5	199	6	199	7	199	8	199	9	200	0	200	1	200	2	Identio	cal coef
Choice 1	Est.	t-value	Est.	t-value	Est.	t-value	Est.	t-value	Est.	t-value	Est.	t-value	Est.	t-value	Est.	t-value	Est.	t-value
Constant	-10,9842	-7,06	-11,7386	-9,44	-13,2030	-10,27	-9,8478	-8,98	-8,9202	-7,55	-11,0389	-8,35	-11,0392	-8,42	-10,4784	-5,64	-3,1744	-10,21
Loginc	2,0336	6,25	2,2000	8,73	2,5037	9,99	1,7687	8,16	1,4820	6,12	2,2960	8,69	1,9819	7,88	2,0136	5,33	0,0124	20,22
Age	0,0488	1,09	0,0164	0,48	0,0237	0,64	0,0134	0,40	0,0418	1,16	-0,0261	-0,64	0,0604	1,51	0,0100	0,16	0,0355	2,66
Age2	-0,0003	-0,66	0,0002	0,47	0,0000	0,07	0,0001	0,37	-0,0003	-0,72	0,0006	1,40	-0,0003	-0,69	0,0002	0,26	-0,0001	-0,92
Distpub	0,3238	2,91	0,2770	3,46	0,1497	1,88	0,2163	2,72	0,1014	1,42	0,1822	2,36	0,0192	0,27	0,1526	1,30	0,1554	5,48
Distwork	0,0235	0,24	0,0603	0,77	0,0861	1,00	0,0991	1,17	0,2310	2,64	0,0356	0,40	0,1885	1,88	0,1816	1,55	0,1186	3,75
Child	0,1146	0,99	0,3185	3,08	0,2017	1,84	0,4739	3,83	0,1864	1,67	0,5014	3,50	0,1973	1,67	0,1792	0,93	0,2207	5,18
Adults	0,3727	1,79	0,6626	4,00	0,4003	2,47	0,8541	5,48	1,1242	6,32	0,6826	3,67	0,8417	4,63	0,7277	2,74	0,8191	12,96
Value	-0,0002	-0,28	-0,0005	-1,37	0,0007	1,80	0,0001	0,29	-0,0001	-0,29	-0,0002	-0,78	-0,0007	-2,98	-0,0002	-0,56	0,0000	-0,48
Choice 2																		
Constant	-30,4213	-13,94	-29,8701	-17,61	-30,5557	-18,16	-26,7402	-18,57	-22,9384	-15,86	-26,8801	-16,53	-27,0133	-16,45	-25,8419	-10,88	-8,2534	-21,90
Loginc	4,8057	11,36	5,1189	15,56	5,4559	17,09	4,2738	15,78	3,6475	12,84	4,8116	15,41	4,4498	14,68	4,3957	9,71	0,0217	33,16
Age	0,1470	2,36	0,0060	0,13	0,0076	0,16	0,0780	1,82	0,0785	1,78	-0,0190	-0,39	0,0606	1,22	0,0563	0,72	0,0759	4,48
Age2	-0,0016	-2,25	0,0002	0,43	0,0000	-0,03	-0,0007	-1,44	-0,0008	-1,61	0,0004	0,74	-0,0003	-0,61	-0,0005	-0,65	-0,0007	-3,98
Distpub	0,6017	4,86	0,4607	5,04	0,2554	2,86	0,3616	4,15	0,1889	2,40	0,3498	4,09	0,2236	2,73	0,1914	1,45	0,3039	9,59
Distwork	0,0686	0,63	0,1298	1,51	0,1070	1,16	0,1797	2,00	0,2245	2,42	0,0904	0,96	0,2389	2,25	0,1901	1,53	0,1706	5,03
Child	0,1068	0,83	0,2951	2,62	0,1327	1,12	0,4969	3,84	0,2657	2,27	0,4544	3,07	0,2442	1,96	0,2359	1,18	0,2326	5,16
Adults	0,3142	1,36	0,8391	4,65	0,6364	3,63	0,9915	5,90	1,2632	6,71	0,9155	4,64	1,0424	5,31	0,9014	3,18	1,0146	14,92
Value	0,0006	0,93	-0,0002	-0,50	0,0006	1,31	0,0001	0,39	-0,0003	-0,85	-0,0002	-0,81	-0,0005	-1,66	-0,0001	-0,27	0,0001	0,99
Observations				29.174														29.174
L(0)			-32.	050,9149													-32	.050,9149
L(c) (degrees of freed	lom)		-21.168,	3518 (18)													-21.21	4,8269 (2)
L(β) (degrees of freed	lom)		-18.041,8	511 (180)													-18.172,	5051 (22)
McFaddens $\rho^2$ (0)				0,4371														0,4330
McFaddens $\rho^2$ (c)				0,1477														0,1434

Table A.8: Test statistics for the model with all households. Model estimates with free variables for all years and model estimates with identical parameters for all years. Variable for real estate value included.

## Estimates for the A.9 model (Urban tenants and a variable for real estate values)

	199	5	199	16	199	7	199	8	199	9	200	0	200	1	200	2	Ide	ntical
																	coef	
Choice 1	Est.	t-value	Est.	t-value	Est.	t-value	Est.	t-value	Est.	t-value	Est.	t-value	Est.	t-value	Est.	t-value	Est.	t-value
Constant	-10,9842	-7,06	-11,7386	-9,44	-13,2030	-10,27	-9,8478	-8,98	-8,9202	-7,55	-11,0389	-8,35	-11,0392	-8,42	-10,4784	-5,64	-3,4162	-13,69
Loginc	2,0336	6,25	2,2000	8,73	2,5037	9,99	1,7687	8,16	1,4820	6,12	2,2960	8,69	1,9819	7,88	2,0136	5,33	0,0132	27,66
Age	0,0488	1,09	0,0164	0,48	0,0237	0,64	0,0134	0,40	0,0418	1,16	-0,0261	-0,64	0,0604	1,51	0,0100	0,16	-0,0145	-1,26
Age2	-0,0003	-0,66	0,0002	0,47	0,0000	0,07	0,0001	0,37	-0,0003	-0,72	0,0006	1,40	-0,0003	-0,69	0,0002	0,26	0,0004	3,05
Distpub	0,3238	2,91	0,2770	3,46	0,1497	1,88	0,2163	2,72	0,1014	1,42	0,1822	2,36	0,0192	0,27	0,1526	1,30	0,2464	5,77
Distwork	0,0235	0,24	0,0603	0,77	0,0861	1,00	0,0991	1,17	0,2310	2,64	0,0356	0,40	0,1885	1,88	0,1816	1,55	0,1016	2,85
Child	0,1146	0,99	0,3185	3,08	0,2017	1,84	0,4739	3,83	0,1864	1,67	0,5014	3,50	0,1973	1,67	0,1792	0,93	0,2041	5,12
Adults	0,3727	1,79	0,6626	4,00	0,4003	2,47	0,8541	5,48	1,1242	6,32	0,6826	3,67	0,8417	4,63	0,7277	2,74	0,2078	3,80
Value	-0,0002	-0,28	-0,0005	-1,37	0,0007	1,80	0,0001	0,29	-0,0001	-0,29	-0,0002	-0,78	-0,0007	-2,98	-0,0002	-0,56	0,0000	0,38
Choice 2																		
Constant	-30,4213	-13,94	-29,8701	-17,61	-30,5557	-18,16	-26,7402	-18,57	-22,9384	-15,86	-26,8801	-16,53	-27,0133	-16,45	-25,8419	-10,88	-8,8793	-11,84
Loginc	4,8057	11,36	5,1189	15,56	5,4559	17,09	4,2738	15,78	3,6475	12,84	4,8116	15,41	4,4498	14,68	4,3957	9,71	0,0224	25,34
Age	0,1470	2,36	0,0060	0,13	0,0076	0,16	0,0780	1,82	0,0785	1,78	-0,0190	-0,39	0,0606	1,22	0,0563	0,72	-0,0404	-1,08
Age2	-0,0016	-2,25	0,0002	0,43	0,0000	-0,03	-0,0007	-1,44	-0,0008	-1,61	0,0004	0,74	-0,0003	-0,61	-0,0005	-0,65	0,0005	1,26
Distpub	0,6017	4,86	0,4607	5,04	0,2554	2,86	0,3616	4,15	0,1889	2,40	0,3498	4,09	0,2236	2,73	0,1914	1,45	0,5054	4,45
Distwork	0,0686	0,63	0,1298	1,51	0,1070	1,16	0,1797	2,00	0,2245	2,42	0,0904	0,96	0,2389	2,25	0,1901	1,53	0,2289	2,70
Child	0,1068	0,83	0,2951	2,62	0,1327	1,12	0,4969	3,84	0,2657	2,27	0,4544	3,07	0,2442	1,96	0,2359	1,18	-0,1778	-1,74
Adults	0,3142	1,36	0,8391	4,65	0,6364	3,63	0,9915	5,90	1,2632	6,71	0,9155	4,64	1,0424	5,31	0,9014	3,18	0,4472	3,55
Value	0,0006	0,93	-0,0002	-0,50	0,0006	1,31	0,0001	0,39	-0,0003	-0,85	-0,0002	-0,81	-0,0005	-1,66	-0,0001	-0,27	0,0008	4,09
Observations			9.435														9.435	
L(0)			-10.365,40	069													-10.365,4	069
L(c) (degrees of free	dom)		-7.498,647	73 (18)													-7.507,21	05 (2)
L(β) (degrees of free	dom)		-5.819,234	40 (180)													-5.961,90	33 (22)
McFaddens $\rho^2$ (0)			0,4386														0,4248	
McFaddens $\rho^2$ (c)			0,2240														0,2058	

Table A.9: Test statistics for the model with all households. Model estimates with free variables for all years and model estimates with identical parameters for all years. Variable for real estate value included.

## Estimates for the A.10 model (Rural tenants and a variable for real estate values)

	199	5	199	16	199	7	199	8	199	9	200	0	200	1	200	)2	Ide	entical
																	coef	
Choice 1	Est.	t-value	Est.	t-value	Est.	t-value	Est.	t-value	Est.	t-value	Est.	t-value	Est.	t-value	Est.	t-value	Est.	t-value
Constant	-10,2408	-8,03	-10,3889	-9,28	-11,8521	-9,69	-10,3785	-10,57	-9,6147	-8,54	-9,4050	-8,78	-10,6433	-9,28	-7,5970	-4,94	-2,8064	-10,71
Loginc	2,1066	7,45	1,8258	7,10	2,2284	8,54	2,0232	9,42	1,8866	7,82	1,9434	8,30	1,9550	7,83	2,0198	5,84	0,0147	21,81
Age	-0,0606	-1,48	0,0120	0,35	0,0010	0,03	-0,0010	-0,03	-0,0432	-1,23	-0,0403	-1,23	-0,0036	-0,10	-0,1084	-1,91	-0,0072	-0,59
Age2	0,0008	1,82	-0,0001	-0,24	0,0002	0,55	0,0001	0,26	0,0006	1,56	0,0006	1,59	0,0002	0,63	0,0012	1,93	0,0002	1,51
Distpub	0,2685	2,24	0,3840	3,77	0,3208	3,15	0,2255	2,75	0,3283	3,38	0,1750	2,05	0,1532	1,72	0,0735	0,57	0,2273	6,77
Distwork	0,3061	3,11	0,1464	1,79	0,1815	2,17	0,1286	1,59	0,1273	1,53	0,1897	2,20	0,2231	2,55	0,2319	1,87	0,1886	6,12
Child	0,0918	0,58	-0,0013	-0,01	0,0631	0,54	0,1225	1,14	-0,0653	-0,59	0,2409	2,02	0,2570	2,20	-0,0411	-0,22	0,0745	1,76
Adults	0,0939	0,46	0,3578	1,91	0,0419	0,23	0,1396	0,88	0,4507	2,50	0,2298	1,39	0,2062	1,14	0,0947	0,41	0,2142	3,31
Value	0,0007	1,51	0,0007	1,92	0,0004	0,97	0,0003	0,94	0,0003	0,82	-0,0002	-0,65	0,0005	1,77	-0,0001	-0,39	0,0002	1,80
Choice 2																		
Constant	-27,2850	-6,78	-29,3952	-8,43	-31,9786	-9,24	-24,5007	-10,02	-29,5657	-9,10	-26,0277	-8,56	-29,5114	-8,43	-19,8121	-5,16	-8,4312	-13,29
Logine	4,9552	6,26	5,1725	7,46	5,3275	8,39	4,0545	8,53	5,1346	8,22	3,8679	7,51	4,9587	7,57	3,1782	4,63	0,0248	24,26
Age	-0,0723	-0,61	-0,1172	-1,11	0,0574	0,59	-0,0385	-0,55	-0,0614	-0,64	0,1429	1,42	-0,0123	-0,11	0,0908	0,67	0,0086	0,25
Age2	0,0008	0,54	0,0010	0,81	-0,0006	-0,55	0,0005	0,59	0,0007	0,64	-0,0018	-1,46	0,0001	0,04	-0,0012	-0,79	-0,0002	-0,47
Distpub	0,3798	1,59	0,5752	3,13	0,6890	4,31	0,3753	2,76	0,5517	3,45	0,5834	3,86	0,3655	1,94	0,2422	1,12	0,4606	7,87
Distwork	0,4148	2,39	0,4199	2,83	0,1785	1,26	0,1375	1,01	0,2626	1,79	0,0191	0,12	0,0500	0,26	-0,1170	-0,53	0,2003	3,66
Child	-0,4906	-1,41	0,1011	0,50	-0,0137	-0,07	0,3979	2,54	-0,0039	-0,02	0,1716	0,88	-0,0714	-0,29	0,0809	0,29	0,1006	1,40
Adults	0,2553	0,61	0,8317	2,24	0,1806	0,55	0,7853	2,96	0,5172	1,41	0,3868	1,48	0,6269	1,78	0,0953	0,24	0,5752	5,12
Value	-0,0001	-0,11	0,0010	1,30	0,0002	0,29	0,0010	1,76	0,0003	0,52	0,0000	-0,08	0,0007	1,06	0,0005	0,80	0,0004	1,67
Observations	•			6.865														6.865
L(0)			-7.	.541,9734													-7	.541,9734
L(c) (degrees of free	dom)		-5.919,	4227 (18)													-5.920	6,5814 (2)
L(β) (degrees of free	edom)		-4.744,1	334 (180)													-4.815,	,1884 (22)
McFaddens $\rho^2(0)$				0,3710														0,3615
McFaddens $\rho^2$ (c)				0,1985														0,1875

Table A.10: Test statistics for the model with all households. Model estimates with free variables for all years and model estimates with identical parameters for all years. Variable for real estate value included.

# Real estate ownership and the demand for cars in Denmark

# - A pseudo-panel analysis

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## **Abstract**

This paper examines how real estate ownership, increasing real estate values and the falling interest rates affect car demand. It uses data from the Danish Transport Diary Survey together with data from Statistics Denmark to estimate a simple partial adjustment model for car ownership in Danish households. We find that car ownership differs among households owning real estate and households not owning real estate and we find that real estate owners have increased their demand for cars as a result of the increasing real estate values and that other households are unaffected by the increasing real estate values. Furthermore we show that both household groups have increased their demand for cars due to the falling interest rate. We also find that long-run income elasticities for car ownership differ between rural and urban households.

# 1. Introduction

The modeling and forecasting of car ownership is often based on cross section data in a discrete model setting (e.g. logit or probit) where it is assumed that the parameters estimated remain constant over time. There are two underlying assumptions behind this. The first is that that the economy is in equilibrium. The other is that observed differences in consumption between, e.g., a high income person and a low income person is a valid description of what would happen if a low income person suddenly received the same income as the high income person, all other things being equal. Both these assumptions are probably invalid. What is needed is a dynamic model which explicitly takes account of the dynamic nature of car demand.

Ideally, panel data should be used but since these are rarely available in the transport sector and since many cross section data exist, the simpler approach of cross-section modeling is often adopted and it is argued that the elasticities found are long-run elasticities<sup>21</sup>. Deaton (1985) shows that it is possible to create panel data from repeated cross-section data named pseudo-panel data. He shows that by using a characteristic that is invariant over time for a given household type (e.g. year of birth of the oldest person in the household) it is possible to create a pseudo panel describing average behavior for the household type in question. The use of pseudo-panel data is an attempt to circumvent some of the shortcomings of the cross-sectional data and use the strength of the time series analysis. The pseudo panel approach also allows for the inclusion of macro

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<sup>&</sup>lt;sup>21</sup> In Goodwin et. al. (2004) a survey of many recent elasticity studies are presented and a discussion of the claim that the elasticities found in simple cross-section studies can be interpreted as long-run elasticities can also be found here.

variables which might affect both the transport behavior (e.g. number of kilometers traveled) and the demand for transport vehicles (e.g. cars). This paper utilizes the Danish Transport Diary Survey together with data from Statistics Denmark to create a pseudopanel data set for the Danish population based on the year of birth for the interviewee in the Danish Transport Diary Survey, real estate ownership status for the interviewees household, and whether the household lives in an urban or rural area. It examines how changing real estate values and a falling interest rate affect the number of cars available in Danish households and to what extent the different types of households differ with regard to income elasticities. To our knowledge this is the first time the effect of the development in real estate prices and the interest rate is linked to the demand for cars.

The approach suggested by Deaton has been utilized in a number of papers. An estimation of dynamic car ownership models is undertaken for the first time in Dargay and Vythoulkas (1999) where the UK Family Expenditure Survey is used. They demonstrate that the pseudo-panel method can be applied and gives credible estimates when it comes to describing the dynamics of transport behavior. They also show that there are large differences between short and long run elasticities with the latter being three times bigger than the former. Birkeland et al. (2000) use data from the Danish Tranport Diary Survey in a pseudo panel analysis of personal transport in Denmark. They use a non-dynamic model to identify cohort effects and life-cycle effects. They also compare income elasticities estimated by simple cross-section analysis with those found by the use of pseudo panel data, showing that the two approaches yield very different results. They conclude that pseudo panel methods are preferable when predicting future

demand for transport. In another paper (Nielsen (2006b)) we use repeated cross-section analysis to investigate the demand for cars in Denmark and we show that housing prices probably influence the demand for cars. The present paper is thus a continuation of the work in Nielsen (2006b). In Dargay (2001) a pseudo-panel for UK is constructed and the approach is used to show that hysteresis effects are present for car ownership. She shows that the elasticities with regard to rising income is higher than the elasticities for falling income. This hysteresis shows that a car after it is purchased becomes a necessity, which is not disposed of as easily as it is acquired. Using the same data as in Dargay (2001) the approach is later used in Dargay (2002) to show that important differences in the elasticities between rural and urban households exists and that rural households have lower income elasticities than urban households.

In Denmark the real estate values have increased steadily and at very high rates since 1993 and at the same time the long run interest rate has dropped from around 10% to around 5%. This is shown in figure 1 and figure 2.

In a situation like the one experienced in Denmark with ricing real estate prices and falling interest rate, households already owning real estate can (after a few years) withdraw equity from their real estate without increasing monthly mortgage payments due to the fall in the long run interest rate. Such an increase in wealth could increase the number of cars in households. For households entering the real estate market the effect is less clear. The fact that the real estate value increases will make it more expensive to purchase a house or an apartment and the mortgage payments will be higher than for

those households who already own real estate. The decreasing interest rate will counter this by reducing the mortgage payments. If the first effect dominates the households will have less income available for consumption which will reduce the number of cars. If the latter effect dominates the mortgage payments will go down and the household will have more income available for consumption which could increase the number of cars in the households.

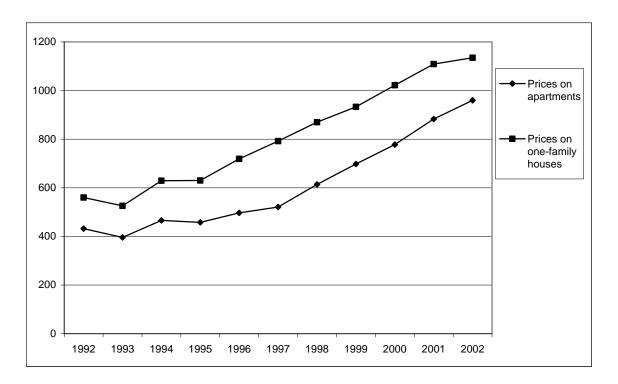


Figure 1: Real estate prices (1.000 DKr.).

Source: Statistics Denmark

Since the interest rate is the same for all households in the country we examine real estate owners and tenants separately. This enables us to see if the changing real estate prices and the changing interest rate has affected the two groups differently. Our expectation is

that the falling interest rate could affect both groups but the increasing real estate values only affect the real estate owners. One problem is that the interest rate and the housing prices are correlated and that non-real estate owners may face different credit market constraints than real estate owners. If these capital restrictions for non-real estate owners are strong we expect that the interest rate has affected the real estate owners more and may even have had no effect on non real estate owners.

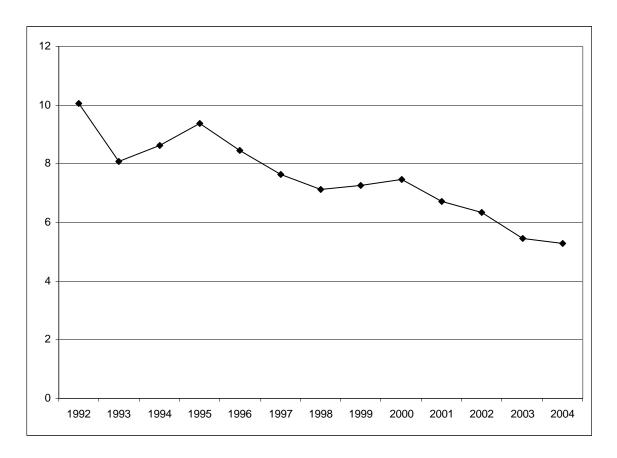


Figure 2: Interest on 30-years bonds.

Source: Statistics Denmark

This paper extends the findings in previous studies by looking at the differences between real estate owners and tenants, thus providing more insight into the behavior of different

household groups. It also identify effects of rising real estate prices and falling interest rates on car demand which is new to the transport literature. Furthermore, we also identify differences between urban and rural households, showing that these groups have different long run income elasticities but fairly identical short run income elasticities with regard to car ownership. The findings show that rural households have lower long run elasticities and thus that they adjust their vehicle stock more slowly than urban households, supporting findings for the UK in Dargay (2002).

The paper proceeds as follows. Section 2 discusses the data and the construction of the pseudo-panel. Section 3 sets up the model and section 4 presents the estimations. Section 5 calculates different elasticities and discusses the results. Section 6 concludes.

# 2. The pseudo-panel data

The data utilized in the present paper come from two sources, the Danish Transport Diary Survey and Statistics Denmark. The people participating in the Danish Transport Diary Survey are selected by random draw from the Danish Civil Register. Data concerning the individual as well as the household are collected and the travel pattern for a single day for the interviewee is recorded. In the years 1992 to 1997 a monthly sample of 1800 was drawn for people between the age of 16 and 74. In 1998 this was extended to 2100 and the age group was extended to 10 to 84. The response rate in the survey is about 65-70%. The variables included in the present analysis are after-tax income, number of adult household members, degree of urbanization (living in a major Danish city or not), car availability (how many cars the household has access to), information about whether the household owns real estate, and if they live in a house or in an apartment. Due to data

limitations on certain variables, the sample used here is restricted to the years 1996 to 2002.

The pseudo panel is constructed by dividing the data into cohorts. Following Deaton (1985) the cohorts have to be based on some characteristic that remain invariant in the period analyzed. In the present study we have used the year of birth of the interviewee as the determining factor together with real-estate ownership status and degree of urbanization. The two latter may not be invariant over time but in this study we exclude moving patterns thus assuming that households do not change residence. We thus assume that real-estate ownership status and the degree of urbanization remains invariant over time for all households. Each of the cohorts' averages for all the variables included are then calculated resulting in a 'representative' observation for the given cohort which in our panel is an interviewee born in a given year, who is either a real estate owners or a tenant, and either lives in an urban or rural area. This means that for a representative person born in e.g. 1945 or in 1960, who is a tenant in an urban area we have a series of observations from 1996 to 2002 describing the behavior of the representative person each year. This data can then be linked to the macro data for the development in housing prices and interest rate obtained from Statistics Denmark giving us the pseudo panel used in the paper.

Car ownership includes both ownership of cars and other cars which the household can use for personal transport<sup>22</sup>. Car ownership is calculated as the total number of cars

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 $<sup>^{22}</sup>$  The appropriate expression would be 'car availability' since the respondents are asked if they have access to a car.

available to the households divided by the number of households for every cohort year. These are shown in figure 3 and figure 4 where the car ownership for different cohorts over time according to age is shown. Figure 3 shows the cohorts for real estate owners and figure 4 shows the cohorts for tenants. It is clear from these figures that there is a huge difference not only between households living in cities and on the countryside but also between real estate owners and tenants.

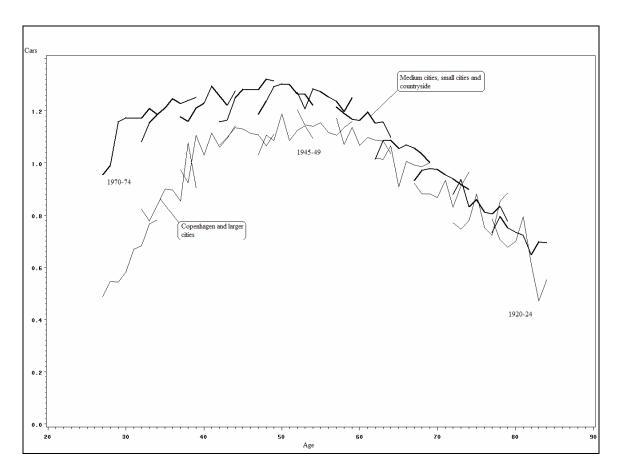


Figure 3: Car ownership by cohort for real estate owners.

Source: Danish Transport Diary Survey.

The figures show that the life-cycle effect is larger for real estate owners. It is also clear that households living in less urbanized areas have higher car ownership than households living in large cities or in Copenhagen. One explanation of this is that the public transport network is better and distances are smaller in cities, thus reducing the need for a car<sup>23</sup>.

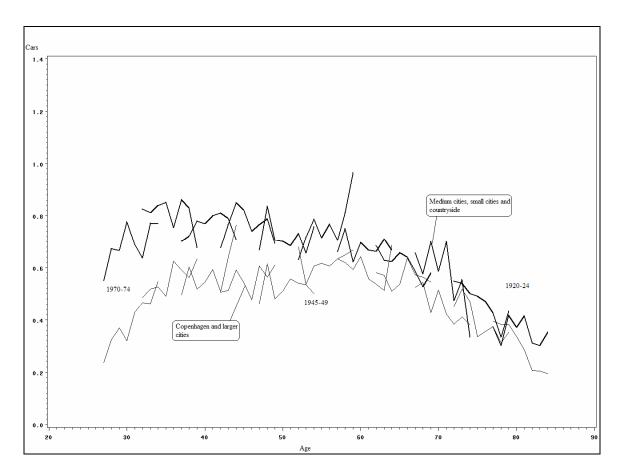


Figure 4: Car ownership by cohort for tenants.

Source: Danish Transport Diary Survey.

Figure 5 gives another picture of a life cycle effect for households. It depicts the number of adults living in a household. As the age of the interviewee increases, the number of

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 $<sup>^{23}</sup>$  See Nielsen (2006a) for further discussion.

adults also increases. This is due to people getting married and having children. When the children reach a certain age they also count as adults<sup>24</sup>. This goes on until the interviewee reaches the age of 50 where the children start to move away from their parents thus reducing the size of the households. We are not able to see if households change residence when these changes happen but it is likely that more adults and more children will increase the demand for cars and induce the household to look for a new (and bigger) home. The size of the households also decreases as a result of divorce and death.

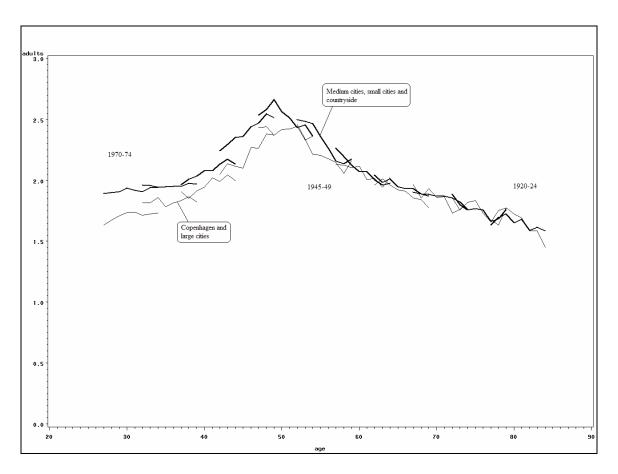


Figure 5: Number of adults in the household by cohort.

Source: Danish Transport Diary Survey.

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<sup>&</sup>lt;sup>24</sup> A problem with the classification of 'adults' in the Danish Transport Diary Survey is that people over the age of 16 are counted as adults but a driving license can not be acquired before the age of 18.

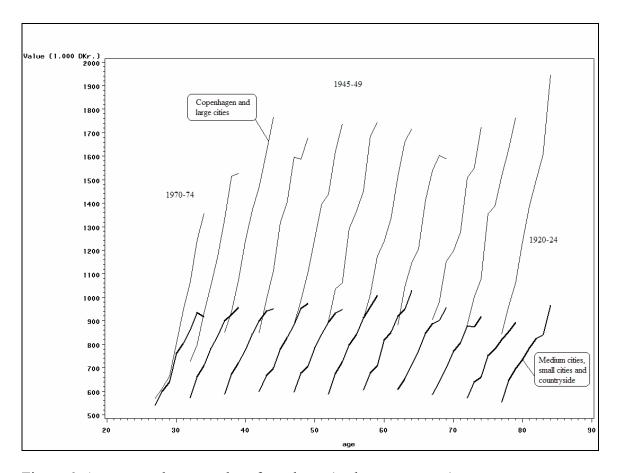


Figure 6: Average real estate values for cohorts (real estate owners).

Source: Statistics Denmark.

Unfortunately the Danish Transport diary Survey does not include information concerning the value of real estate owned by the households. Data for the average housing prices in the separate municipalities can be obtained from Statistics Denmark and these data can be linked to the information in the Danish Transport diary Survey for each household living in a given municipality. We thus assume that these average values are the same for each household in a given municipality<sup>25</sup>. It is well known that the development in housing prices has differed significantly between different regions in

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<sup>&</sup>lt;sup>25</sup> We are also able to distinguish between households living in apartments and households living in houses. This is important since the development in market values for these two types of housing differs.

Denmark, with the largest increases happening in the greater Copenhagen area and the large cities. This can be seen in figure 6 where the average real estate values for different cohorts are shown. Furthermore it seems that for urban households the older cohorts have experienced higher increases than younger cohorts and in rural areas the picture is the opposite with the oldest cohorts having experienced the lowest increases in real estate values.

The interest rate shown in figure 2 is also obtained from Statistics Denmark on an annual basis. Since this is a general macro variable, all households in the economy face the same interest rate. Some households might have limited access to the financial market, but we ignore this and assume that all households have the same opportunities for borrowing money and that they all face the same long run interest rate<sup>26</sup>.

As mentioned earlier it would be preferable to use real panel data but using a pseudopanel also hold some advantages. One advantage is that we do not need to include the same households in each observation. The problem of finding a panel and following the same panel over a long period is thus avoided. A tradeoff has to be made between the number of individuals in a given cohort and the number of cohorts constructed. In our case we have 11 cohorts, 2 degrees of urbanization, and 2 states for real estate ownership. In all we therefore have 44 observations which we track over a period of 7 years. It is also clear that when using a pseudo-panel we loose information about the variation between the individuals within a given cohort. If the variation is large one could argue that the loss

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 $<sup>^{26}</sup>$  We have used the interest rate for the 30-year bond. It could be argued that another interest rate should be used.

of information is large but at the same time if the variation within a given cohort is small, the loss of information is also low. Having these shortcomings in mind the next section presents the dynamic car ownership model used in the analysis.

# 3. The car ownership model

With the examination of the impact of real estate ownership, changing real estate prices, and the changing interest rate as the objective we specify a simple partial adjustment model of car demand inspired by Dargay & Vythoulkas (1999). The data we use were described in section 2. In Dargay (2001) different specifications<sup>27</sup> are tested and compared. She concludes that the semi-log specification dominates and also argues that this specification makes most sense economically. Based on her result we use a semi-log specification. We let  $C_t^i$  represent the number of cars at time t for household i,  $I_t^i$  the number of adults in the household,  $G^{i}$  the cohort number,  $R_{i}$  the long term interest rate at time t,  $Y_t^i$  the annual after tax income for household i at time t,  $E_t^i$  is a dummy indicating if the household is a real estate owner,  $U_t^i$  is a dummy for households living in urban areas, and  $W_t^i$  is the increase in real estate values experienced during the last year<sup>28</sup>. For each household we assume that the number of cars can be described as

$$C_{t}^{i} = \alpha + \beta_{Y} \log(Y_{t}^{i}) + \beta_{W} W_{t}^{i} + \beta_{E} E_{t}^{i} + \beta_{U} U_{t}^{i} + \beta_{R} R_{t} + \beta_{I} I_{t}^{i} + \beta_{G} G^{i} + \beta_{C} C_{t-1}^{i} + \gamma_{t}^{i}$$
(1)

where  $C_{t-1}^i$  is the number of cars in household i in period t-1,  $\alpha$  is a constant, and  $\gamma_t^i$  is an error process which we will describe in more detail below.

Linear, Double-log and Semi-log.
 One could speculate that the level of housing prices should be used instead. A discussion of this possibility can be found in appendix A.

Due to the aggregation each variable has the form of an average for the cohort it comes from. The average at the cohort level is thus given by  $\sum_i \frac{A_i^i}{n_t^c} = \overline{A_i^c}$  where  $n_t^c$  is the number of households in cohort c and  $A_t^c$  is the variable. Using this aggregation we let  $\overline{C_t^i}$  represent the number of cars at time t for cohort i,  $\overline{I_t^i}$  the number of adults in the household,  $\overline{G}^i$  the cohort number,  $\overline{R_t}$  the long term interest rate (which is identical for all cohorts),  $\overline{Y_t^i}$  the annual after tax income,  $\overline{E_t^i}$  indicates if the household is a real estate owner,  $\overline{U_t^i}$  is a dummy for households living in urban areas, and  $\overline{W_t^i}$  is the increase in real estate values experienced during the last year. This gives the general functional form

$$\overline{C}_{t}^{i} = \overline{\alpha} + \beta_{V} \log(\overline{Y}_{t}^{i}) + \beta_{W} \overline{W}_{t}^{i} + \beta_{E} \overline{E}_{t}^{i} + \beta_{U} \overline{U}_{t}^{i} + \beta_{R} \overline{R}_{t} + \beta_{I} \overline{I}_{t}^{i} + \beta_{G} \overline{G}_{t}^{i} + \beta_{C} \overline{C}_{t-1}^{i} + \overline{\gamma}_{t}^{i}$$

$$\tag{2}$$

where  $\overline{C}_{t-1}^i$  is the number of cars in the previous period,  $\alpha$  is a constant, and and  $\overline{\gamma}_t^i$  again is an error process. As shown in figure 7 we note that the increase in real estate value experienced by one cohort does not have to be identical to the increase experienced by other cohorts since we have been able to distinguish between the housing prices in different municipalities. In contrast all cohorts experience the same development in the interest rate. To capture possible saturation effects in income we take the logarithm of  $\overline{Y}_t^i$ . As argued by Dargay & Vythoulkas this type of model can be estimated using standard techniques.

# 4. Estimation

A list of the variables included in the model can be seen in table 1 together with their sources. The hypothesis put forward in the introduction is modeled by the variables 'value increase' and 'interest rate'.

Variable	Source	Description
Cars	DTDS	Number of cars available to the household
Owner	DTDS	Real estate ownership status
Urbanization	DTDS	Living in urban area (Copenhagen or large city)
Value increase	SD	Increase in housing prices during last year
Interest rate	SD	Average 30 years interest rate
Income (log)	DTDS	Household yearly after-tax income
Generation	DTDS	Generation effect (cohort number)
Adults	DTDS	Number of adults in the household

Table 1: Variables used in the model (Statistics Denmark (SD) and Danish Transport Diary Survey (DTDS)).

The number of observations used to construct each of the cohorts can be seen in table 2 divided into groups coming from urban areas (Copenhagen and suburbs together with the 3 largest cities) or rural areas (medium and small cities or the countryside) and owning or not owning real estate<sup>29</sup>. It should be noted that especially for tenants the number of

 $^{29}$  Due to data limitations we have excluded households living on the island of Bornholm (a small Danish island in the Baltic Sea).

observations for some cohorts is quite low. The number could be increased by reducing the number of cohorts and increasing the number of observations within each of these.

#### 4.1 Models for household owners and tenants

Initially we specify three different models. The first model (M-all) includes all variables described in table 1. The second model (M-owner) and the third model (M-tenant) does not include the variable for real estate ownership because we split the data into real estate owners and tenants.

Cohort	Cohort date	Urban	Rural	Urban	Rural
number	of birth	owner	owner	tenants	tenants
1	1920-24	467	1335	483	687
2	1925-29	689	2103	617	767
3	1930-34	866	2605	519	720
4	1935-39	1043	3250	478	712
5	1940-44	1420	4297	513	711
6	1945-49	1709	5247	611	793
7	1950-54	1475	4873	591	695
8	1955-59	1534	4625	615	829
9	1960-64	1532	4519	755	1087
10	1965-69	1482	3892	1031	1607
11	1970-74	1094	2135	1310	2151
Cohort average		1210	3535	684	978

Table 2: Number of observations

Since we have a lagged dependent variable in the specification, we use the Durbin-h statistics to test for the presence of autocorrelation. Table 3 present the results for the Durbin-h statistics in the three models. The tests indicate that autocorrelation is present in two of the models (M-all and M-tenants). Since we wish to keep the specifications identical we proceed under the assumption that autocorrelation is present.

The next step is to identify the order of autocorrelation. To do this we use stepwise autoregression initially specifying an autoregressive model with four lags and then removing autoregressive parameters until we have significant t-tests. This procedure shows that we only need to specify a model with one autoregressive lag.

	M-all	M-owners	M-tenants
Durbin-h statistics	-4.0950	-1.5218	-4.9070
p-value	< 0.0001	0.0640	<0.0001

Table 3: Test for the presence of autocorrelation

Since the number of households in each cohort is not the same, we potentially face the problem of heteroscedasticity. To avoid this problem we weight all observations by the square root of the number of households in the given cohort. To see this note that the variance of the error-term will be given by

$$Var(\overline{\gamma}_t^i) = Var(\frac{1}{n_t^c} \sum_i \gamma_t^i) = (\frac{1}{n_t^c})^2 Var(\sum_i \gamma_t^i) = \frac{1}{n_t^c} Var(\gamma_t^i) = Var(\frac{1}{\sqrt{n_t^c}} \gamma_t^i)$$
(3)

thus depending on the number of observations,  $n_t^c$ . Multiplying with the square root of the number of observations used in the given cohort gives

$$Var(\sqrt{n_t^c} \frac{\gamma_t^i}{\gamma_t^i}) = Var(\sqrt{n_t^c} \frac{1}{n_t^c} \sum_i \gamma_t^i) = (\frac{1}{n_t^c})^2 Var(\sqrt{n_t^c} \sum_i \gamma_t^i) = \frac{1}{n_t^c} Var(\sqrt{n_t^c} \gamma_t^i) = Var(\gamma_t^i)$$

$$\tag{4}$$

thus making the variance of the error-term independent of the number of observations in the different cohorts. Using this procedure we see, that the problem of heteroscedasticity disappears and we therefore assume that all the error terms follow a normal distribution. Based on the Durbin-h statistics, we proceed under the assumption of homoscedasticity in the error process and the full model is now given by

$$\overline{C}_{t}^{i} = \alpha + \beta_{Y} \log(\overline{Y}_{t}^{i}) + \beta_{W} \overline{W}_{t}^{i} + \beta_{E} \overline{E}_{t}^{i} + \beta_{R} \overline{R}_{t} + \beta_{I} \overline{I}_{t}^{i} + \beta_{G} \overline{G}^{i} + \beta_{C} \overline{C}_{t-1}^{i} + \gamma_{t}$$

$$\gamma_{t} = \varepsilon_{t} + \beta_{Y} \gamma_{t-1}$$

$$\varepsilon_{t} \sim N(0, \sigma^{2})$$
(5)

The estimation results are shown in table 5 together with test statistics for tree different models. The models for tenants include the variable for the increasing real estate values. This we do to see if it is significant. If so we should be skeptical about our hypothesis since we do not expect non-real estate owners to benefit from increasing real estate values. A problem with the variable for the increasing real estate values could be that households who have lived in their house for a longer period of time have accumulated more wealth than indicated by this variable. The dynamic model specification is capable of handling this since past increases in real estate values are included. If a given cohort has experienced increases in the housing prices in the past, this is included in the model. Another problem with the present specification is that we do not account for moving patterns. We thus assume that households do not change residence in the selected period. It could be interesting to examine moving patterns and its influence on car demand but we do not have the data to do this and it is also not the main focus of this paper.

Variable	M-all	M-owners	M-tenants
Intercept	-0.1815 (-1.30)	-0.2403 (-0.76)	-0.0239 (-0.08)
Real estate owner	0.0650 (6.10)		
Urbanization	-0.0625 (-5.36)	-0.0968 (-4.17)	-0.0524 (-3.08)
Value increase	0.0003 (2.46)	0.0007 (3.05)	-0.0002 (-0.95)
Interest rate	-0.0458 (-4.84)	-0.0565 (-4.18)	-0.0564 (-3.56)
Income (log)	0.0756 (3.99)	0.1105 (4.20)	0.1040 (2.82)
Generation (cohort)	0.0040 (3.36)	0.0036 (2.27)	0.0030 (1.14)
Adults	0.0659 (4.50)	0.0627 (3.50)	0.0490 (1.31)
Cars (t-1)	0.7221 (21.39)	0.6737 (13.53)	0.6797 (12.09)
AR1 (β <sub>γ</sub> )	0.2872 (4.90)	0.2270 (2.55)	0.3941 (4.91)
$\mathbb{R}^2$	0.9966	0.9958	0.9486
Log Likelihood	-305.8879	-160.8236	-135.0271
SSE	131.3977	72.7733	52.0174
MSE	0.4409	0.5019	0.3587

Table 5: Estimates, t-values and summary statistics<sup>30</sup>

All parameters have the expected sign and from the  $R^2$  values we see that especially the complete model and the model for real estate owners fit the data well. A high degree of urbanization reduces the number of cars which we also saw in figure 3 and 4. This is not surprising since urban households generally have access to better public transport

 $^{30}$  A model using real estate values instead of the yearly increase in real estate values are estimated in appendix A.

facilities, they have access to fewer parking spaces and in general have to travel shorter distances to reach their destination. Higher income affects car availability positively. Again this is expected since cars are assumed to be normal goods. Generation effects are found to be present for real estate owners. Younger generations have a higher tendency to purchase cars. For non-real estate owners the generation effect is also positive but statistically insignificant. This is in line with findings of generation effects in Dargay (2001) and Dargay (2002) where less significant generational effects were found which could be seen as a confirmation of the findings here that the generation effects are not present in all household groups. We also see that the number of adults affect the demand for cars positively but the effects are only statistically significant for real estate owners. Turning to the interest rate, we see that both real estate owners and tenants experience an increase in their demand for cars when the interest rate decreases. Looking at the effect of the increasing real estate values, we get the expected result that only real estate owners are affected and as expected the households have increased their demand for cars as a consequence of the increasing wealth. For tenants, the effect of increasing real estate values is negative but statistically insignificant. Letting  $\phi = (1 - \beta_C)$  we have  $\phi = 0.33$  for real estate owners and  $\phi = 0.32$  for tenants. We thus see that 33% and 32% of the adjustment in car availability for the two household groups happen within the first year. The high degree of significance for the adjustment parameter tells us that the dynamic specification is needed since households in general do not adjust to changes instantaneously.

#### 4.2 Models for urban and rural households

Another possibility we have with the data available is to examine the differences between rural and urban households. As pointed out in Dargay (2002), large differences between these two kinds of households can be expected. To examine this we estimate two models, one for rural households (M-rural) and one for urban households (M-urban). We use a model based on equation (2) in which the difference between households owning real estate and other households are captured by the variable for household ownership status.

	M-rural	M-urban
Durbin-h statistics	-0.8940	-4.1873
p-value	0.1857	>0.0001

Table 6: Test for the presence of autocorrelation

As before, we use the Durbin-h statistics to test for the presence of autocorrelation. The test statistics in table 6 show that autocorrelation is present in the model for urban households but absent in the model for rural households. Using stepwise autoregression to identify the number of lags we find, as before, that one lag is needed. Based on this the model we estimate will be given by

$$\overline{C}_{t}^{i} = \alpha + \beta_{Y} \log(\overline{Y}_{t}^{i}) + \beta_{E} \overline{E}_{t}^{i} + \beta_{R} \overline{R}_{t} + \beta_{I} \overline{I}_{t}^{i} + \beta_{G} \overline{G}^{i} + \beta_{C} \overline{C}_{t-1}^{i} + \gamma_{t}$$

$$\gamma_{t} = \varepsilon_{t} + \beta_{Y} \gamma_{t-1}$$

$$\varepsilon_{t} \sim N(0, \sigma^{2})$$
(6)

and the estimation results together with summary statistics can be found in table 7.

Variable	M-rural	M-urban
Intercept	-0.8894 (-3.32)	-0.6061 (-1.99)
Real estate owner	0.1145 (6.41)	0.0668 (4.30)
Interest rate	-0.0799 (-5.42)	-0.0685 (-4.44)
Value increase	0.0009 (3.33)	0.0000 (0.18)
Income (log)	0.1589 (5.62)	0.1188 (3.64)
Generation (cohort)	0.0075 (4.91)	-0.0028 (-1.02)
Adults	0.0932 (5.37)	0.0656 (2.15)
Cars (t-1)	0.4280 (6.86)	0.6936 (11.93)
AR1 $(\beta_{\gamma})$	0.1426 (1.58)	0.3782 (4.65)
$R^2$	0.9971	0.9800
Log Likelihood	-132.9520	-148.6676
SSE	50.6830	62.1046
MSE	0.3495	0.4283

Table 7: Estimates, t-values and summary statistics

Both models fit the data well with high  $R^2$  values and we see that all statistically significant variables have expected signs. As in the M-all model we find that owning real estate increases the car ownership, and we find that a higher interest rate affects car ownership negatively. One striking difference between rural and urban households is that it seems like only rural households have increased their demand for cars due to the falling interest rate. An explanation could be that better public transport and parking restrictions

reduce the attractiveness of car ownership in urban areas since this is not the case for rural households they exploit falling interest rates and thus falling cost of borrowing more than urban households to purchase cars since cars are necessary due to the poor service of public transport compared to the public transport service in urban areas.

Cohort effects are only found to be present in the model for rural households but it is not significant in the model for urban households. Both the number of adults, increasing income and the car-ownership in the previous period has a positive effect on car ownership. In the model for rural households we see that the  $\beta_{\gamma}$  parameter is insignificant which is not surprising since the Durbin-h statistics rejected the presence of autocorrelation. A large difference between rural and urban households are found in the adjustment parameter  $\phi$  where we for rural households have  $\phi = 0.57$  and for urban households have  $\phi = 0.31$  which tells us that for rural households the adjustment happens faster than for urban households. An explanation for this could be that cars are more necessary in rural areas than in urban areas which cause adjustment to happen faster. Both of these parameters are again highly significant underlining the need for the dynamic specification

# **5 Elasticities**

Short run income elasticities can be calculated directly from the estimated parameters, since we know that the short run income elasticity for car ownership,  $\varepsilon_Y^{sr}$ , given the logarithmic specification, will be given by  $\varepsilon_Y^{sr} = \frac{\partial C}{\partial Y} \frac{Y}{C} = \beta_Y \frac{1}{C}$ . The long run elasticity,  $\varepsilon_Y^{lr}$ , is given by  $\varepsilon_Y^{lr} = \frac{\varepsilon_Y^{sr}}{\phi}$ . With the semi-logarithmic specification we also know that the

elasticities fall as car ownership increases which, as mentioned earlier, seems realistic since some saturation effects are expected even if it is lower in Denmark than in other countries (see Fosgerau et al. (2004) and Dargay et at. (2006))

#### 5.1 Income elasticities for real estate owners and tenants

For the models estimated in section 4.1 (table 5) the income elasticities for car ownership are shown in table 8. What can be seen from table 8 is that real estate owning households in general have slightly higher income elasticity than tenants both in the short run and in the long run if they have the same level of car ownership. But if we account for the differences in car ownership level, we find that the tenants have higher short- and longrun elasticities than real estate owners. We also see that long run income elasticities are three times higher than short run elasticities which are the same order of magnitude as found in Dargay and Vythoulkas (1999). The values for the elasticities are lower than those found for Denmark in Dargay and Gately (1999) where the long run GDP elasticity for cars in Denmark for 1992 was found to be 1.13. Our findings are more in line with findings in Bjørner (1999) where the short run income elasticity for the size of the car fleet was found to be 0.21 and the corresponding long run income elasticity was found to be 0.57. The values found by Bjørner is based on 1991 values and we would thus expect our estimates to be below his since we use more recent data and the effect of motorization (that is, the increasing car ownership in the households) is expected to decrease the income elasticities.

#### 5.2 Income elasticities for urban and rural households

The difference in elasticities for urban and rural households can be found if we calculate the elasticities from the estimates found in section 4.2 (table 7). The results are shown in table 9.

	M-owners		M-tenants	
	Short run	Long run	Short run	Long run
Car availability = 0.5	0.2210	0.6773	0.2080	0.6494
Car availability = 0.75	0.1473	0.4514	0.1387	0.4330
Car availability = 1.00	0.1105	0.3386	0.1040	0.3203
Car availability = 1.25	0.0884	0.2709	0.0832	0.2598
Mean car availability in the group <sup>31</sup>	0.1008	0.3090	0.1952	0.6094

Table 8: Income elasticities for car ownership<sup>32</sup>.

For households living in urban areas we find that long run income elasticities are around three times larger than short run elasticities. This however, does not hold for rural households where the long run elasticities are around twice the size of the short run elasticities. The short run income elasticities found here are lower than the ones reported in Dargay (2002) for the UK where short run income elasticities for rural and urban households were reported to be 0.36 and 0.25. The long run income elasticities found

<sup>&</sup>lt;sup>31</sup> The average for real estate owners is 1.0960 cars and for tenants it is 0.5326.

<sup>&</sup>lt;sup>32</sup> For comparison, a model without a variable for the increasing real estate values and the resulting income elastitities can be found in appendix B.

here for urban households are in line with Dargays finding of a long run income elasticity for urban households around 0,50. Our finding for rural households remain quite low compared with her reported elasticity around 0.34 for rural households.

	M-rural		M-urban	
	Short run	Long run	Short run	Long run
Car availability = 0.5	0.3178	0.5556	0.2376	0,7755
Car availability = 0.75	0.2104	0,3678	0.1584	0.5170
Car availability = 1.00	0.1589	0.2780	0.1188	0.3877
Car availability = 1.25	0.1272	0.2224	0,0950	0,3101
Mean car availability in the group <sup>33</sup>	0.1506	0.2631	0,1653	0.5395

Table 9: Income elasticities for car ownership.

#### 5.3 Elastitities for the real estate value and the interest rate

To calculate the elasticity of car demand with regards to changing real estate values we use the model M-owners where the parameter for changing real estate prices was found to be 0.0007. Short run and long run elasticities can now be calculated as  $\varepsilon_{\overline{W}}^{sr} = \frac{\partial \overline{C}}{\partial \overline{W}} \frac{\overline{W}}{\overline{C}} = \beta_W \frac{\overline{W}}{\overline{C}}$ . The long run elasticity,  $\varepsilon_{\overline{W}}^{lr}$ , is given by  $\varepsilon_{\overline{W}}^{lr} = \frac{\varepsilon_{\overline{W}}^{sr}}{\phi}$ . Similar the short run and long run interest rate elasticities are given by  $\varepsilon_{\overline{R}}^{sr} = \frac{\partial \overline{C}}{\partial \overline{R}} \frac{\overline{R}}{\overline{C}} = \beta_{\overline{R}} \frac{\overline{R}}{\overline{C}}$ . The long run elasticity,  $\varepsilon_{\overline{R}}^{lr}$ , is given by  $\varepsilon_{\overline{R}}^{lr} = \frac{\varepsilon_{\overline{R}}^{sr}}{\phi}$ . To calculate these we use the models M-owners and M-tanents. The results can be seen in table 10.

<sup>33</sup> The total average for rural households is 1.0549 cars and for urban households it is 0.7188.

What can be seen from table 10 is that tenants respond more to changes in the interest rate than real estate owners.

	Interest rate		Real estate values	
	Short run	Long run	Short run	Long run
Real estate owners	-0.2578	-0.7902		
Tenants	-0.5396	-1.6843		
Value increase (wealth elasticity)			0.1277	0.3916

Table 10: Interest rate elasticities and real estate wealth elasticities<sup>34</sup>.

The housing prices' wealth elasticity for car ownership indicate that the number of cars in real estate owning households have increased because of the increasing housing prices. These elasticities is believed to be higher than what could be expected since we have seen increases especially in urban areas which is very high but the development in car ownership does not support the high elasticities found here. Our findings could indicate that the increase in wealth caused by the increasing real estate values and capitalized through the mortgage credit associations induces households to make an instant purchase of a car. If this is the case the household wealth elasticity we find is biased by this. According to the results presented here some of this increase in the number of cars can be explained by the increasing real estate values and the falling interest rate.

<sup>34</sup> The average for real estate owners is 1.0960 cars and for tenants it is 0.5326. The average increase in housing prices in the period has been around 200.000 DKK. per year. The interest rate is assumed to be 5%.

# 6. Conclusions and caveats

We have shown that differences between different household types exist when it comes to car ownership and we have shown that there are differences in the long run income elasticities for urban and rural households as well as between real estate owners and tenants. We also show that households in urban areas in general are slower to adapt to changing situations, confirming findings from earlier studies and explaining why the long run income elasticity for car ownership differs between rural and urban households.

Furthermore, we have shown that real estate owners have benefited from the recent increases in real estate values giving these households large capital gains which have resulted in an increased demand for cars. We have also shown that all households were affected by the falling interest rate and that the decreasing cost of borrowing has increased the demand for cars.

The present study could be improved. The use of the pseudo-panel is not fully satisfactory and real panel data should be obtained in order to test the hypotheses examined here. The exclusion of moving patterns is also not satisfactory and the correlation between interest rate and housing prices should be included in the analysis. However, the findings point in the expected direction and indicate that the hypothesis put forward is correct, namely the possibility to finance the purchase of cars by borrowing against capital gains on real estate. To confirm the hypothesis more strongly, access to register data is needed. These shortcomings will hopefully be addressed in future work.

#### References

Birkeland, M. E., Brems, C. R. and Kabelmann, T. (2000) Analyse af personers transportarbejde, 1975-1998, Trafikdage på Aalborg Universitet 2000, *Conference paper*. Christens, P. F. and Fosgerau, M. (2004) Rådighed over bil - en beskrivelse af sammenhængen mellem husstandsindkomst, bilrådighed og geografi, *Notat* **3 2004**, Note, Danish Transport Research Institute, www.dtf.dk.

Dargay, J. M. and Gately, D. (1999) Income's effect on car and vehicle ownership, worldwide: 1960-2015, *Transportation Research Part A* **33**, 101-138.

Dargay, J. M and Vythoulkas, P. C. (1999) Estimation of a Dynamic Car Ownership Model: A Pseudo-panel Approach, *Journal of Transport Economics and Policy* **33:3**, 287-302.

Dargay, J. M. (2001) The effect of income on car ownership: evidence of asymmetry, *Transportation Research Part A* **35**, 807-821.

Dargay, J. M. (2002) Determinants of car ownership in rural and urban areas: a pseudopanel analysis, *Transportation Research Part E* **38**, 351-366.

Dargay, J., Hivert, L. and Legros, D. (2006) The dynamics of car ownership in E.U countries: a comparison based on the european household panel survey, *unpublished* working paper.

Deaton, A. (1985) Panel Data from Time Series of Cross Sections, *Journal of Econometrics* **30**, 109-126.

Fosgerau, M., Holmblad, M. and Pilegaard, N. (2004) ART – En aggregeret prognosemodel for dansk vejtrafik, *Notat* **5 2004**, Danish Transport Research Institute.

Goodwin, P, Dargay, J. and Hanly, M. (2003) Elasticities of Road Traffic and Fuel Consumption with Respect to Price and Income: A Review, *Transport Reviews* **24:3**, 275-292.

Nielsen, J. E. (2006a) Storbyer har færre biler end de øvrige landsdele, *TU-notits* **Marts 2006**, Danish Transport Research Institute.

Nielsen, J. E. (2006b) Estimation of car ownership in Denmark - Discrete choice modeling and repeated cross-section analysis, *Thesis Paper*.

# **Appendix A**

In this appendix we estimate the model with the actual prices for real estate instead of the annual increase in real estate values. The test for autocorrelation can be found in table A.1.

	M-all	M-owners	M-tenants
Durbin-h statistics	-4.3946	-1.3423	-4.9927
p-value	>0.0001	0.0898	>0.0001

Table A.1: Test for the presence of autocorrelation

Using stepwise autoregression initially specifying an autoregressive model with four lags and then removing autoregressive parameters until we have significant t-tests we find that one lag is required. As before, we weight all observations with the square root of the number of households in the cohort to avoid the problem of heteroscedasticity. The estimation results are seen in table A.2.

As seen in table A.2 the signs of all variables except for the absolute value of real estate are as expected. The variable for the value of real estate are non-significant in all cases and close to zero and negative. The change in the value of real estate was found to be significant in table 5. This seems to support our hypothesis that it is the gain in wealth which has influenced car ownership and not the absolute value of real estate.

Variable	M-all	M-owners	M-tenants
Intercept	-0.0349 (-0.23)	-0.4433 (-1.29)	0.1215 (0.37)
Real estate owner	0.0734 (6.38)		
Urbanization	-0.0330 (-3.56)	-0.0278 (-1.88)	-0.0519 (-3.19)
Real estate value	-0.00002 (-1.32)	-0.000020 (-0.73)	-0.00005 (-1.53)
Interest rate	-0.0432 (-4.44)	-0.0391 (-3.00)	-0.0620 (-3.79)
Income (log)	0.0720 (3.68)	0.0946 (3.37)	0.1177 (3.06)
Generation (cohort)	0.0039 (3.18)	0.002944 (1.60)	0.0032 (1.21)
Adults	0.0590 (3.90)	0.0564 (2.79)	0.0395 (1.04)
Cars (t-1)	0.7551 (23.47)	0.7120 (13.44)	0.6691 (11.83)
AR1 $(\beta_{\gamma})$	0.3069 (5.33)	0.1970 (2.18)	0.3923 (4.95)
$ ight]$ $ m R^2$	0.9944	0.9936	0.9146
Log Likelihood	-308.1442	-165.2860	-134.2893
SSE	133.3314	77.1221	51.5219
MSE	0.4474	0.5319	0.3553

Table A.2: Estimates, t-values and summary statistics

# **Appendix B**

This appendix presents estimates and income elasticities for a model without a variable for real estate values. The test for autocorrelation can be found in table B.1.

	M-all	M-owners	M-tenants
Durbin-h statistics	-4.2815	-1.2593	-4.8138
p-value	>0.0001	0.1040	>0.0001

Table B.1: Test for the presence of autocorrelation

Using stepwise autoregression initially specifying an autoregressive model with four lags and then removing autoregressive parameters until we have significant t-tests we find that one lag is required. We weight all observations with the square root of the number of households in the cohort to avoid the problem of heteroscedasticity. The estimation results are seen in table B.2.

Variable	M-all-A	M-owners-A	M-tenants-A
Intercept	-0.1215 (-0.88)	-0.5626 (-1.75)	-0.0580 (-0.19)
Real estate owner	0.0677 (6.36)		
Urbanization	-0.0408 (-5.43)	-0.0351 (-3.03)	-0.0588 (-3.71)
Interest rate	-0.0386 (-4.29)	-0.0355 (-3.10)	-0.0549 (-3.47)
Income (log)	0.0633 (3.46)	0.0884 (3.46)	0.0983 (2.69)
Generation (cohort)	0.004349 (3.63)	0.003479 (2.11)	0.003051 (1.15)
Adults	0.0648 (4.41)	0.0623 (3.25)	0.0578 (1.57)
Cars (t-1)	0.7461 (23.21)	0.6954 (13.39)	0.6796 (11.99)
AR1 $(\beta_{\gamma})$	0.2922 (5.05)	0.1750 (1.93)	0.3822 (4.80)
$\mathbb{R}^2$	0.9944	0.9936	0.9132
Log Likelihood	-309.0223	-165.5526	-135.5135
SSE	134.0981	77.3939	52.3507
MSE	0.4485	0.5301	0.3586

Table B.2: Estimates, t-values and summary statistics

As seen in table B.2 the signs of all variables are as expected. Calculating income elasticities we get the results presented in table B.3. Comparing the results from table B.3 with the elasticities given in table 8 we see that excluding the variable for the increasing real estate values reduce both short run and long run income elasticities.

	M-owners-A		M-tenants-A	
	Short run	Long run	Short run	Long run
Car availability = 0.5	0.1768	0.6963	0.1966	0.6136
Car availability = 0.75	0.1179	0.3871	0.1311	0.4092
Car availability = 1.00	0.0884	0.2902	0.0983	0.3068
Car availability = 1.25	0.0675	0.2216	0.0786	0.2453
Mean car availability in the group <sup>35</sup>	0.0807	0.2649	0.1846	0.5762

Table B.3: Income elasticities for car ownership.

The average for real estate owners is 1.0960 cars and for tenants it is 0.5326.

# **Demand for car transport in Denmark**

#### - Differences between rural and urban car owners

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#### **Abstract**

We examine how the demand for car travel in Denmark is affected by income, fuel cost, and car purchasing cost. We use two different sources of data; The Danish Transport Diary Survey and time series from Statistics Denmark. A partial adjustment model for daily car transport is estimated. As in recent UK and Danish studies we find that on a national level, car travel is more sensitive to car purchase cost than it is to fuel cost. We offer some insight into this result by showing that car travel by urban households depends mostly on fuel prices and car travel for rural households depend mostly on car purchase cost. This finding is important since it helps us to understand how different transport policies might affect households in different regions thus contributing to the understanding of the distributional impact of the policies.

## 1. Introduction

To plan infrastructure investments and tax policies in the transport sector we need to understand how the demand for transport evolves, what affect the demand and who is affected by changing prices. To do so one often looks at past experiences, extrapolates trends, and examines what cause the trend to increase or decrease. When investigating demand for car transport there are two margins which are of interest. The first is the mode choice, i.e. the choice on the extensive margin. The second is the choice of how much to travel given the mode choice, i.e. the choice on the intensive margin. Investigating behavior on both margins is important since it helps to understand the level of road congestion which is a major concern in almost all major cities. The estimation of car ownership in Denmark we addressed in Nielsen (2006a) and Nielsen (2006b). The present paper is concerned with the choice of how much to travel once households have acquired at least one car and use it. We therefore look at car trips thus focusing on the choice on the intensive margin.

We set up a partial adjustment model for car transport and estimate it using the approach suggested by Dargay & Vythoulkas (1999) where a partial adjustment model for car ownership for the first time is estimated using a pseudo-panel. The approach has since been extended in several ways to analyze car ownership decisions. Dargay (2001) allows for asymmetric income effects in car ownership showing that hysteresis is present. She later examines differences in car ownership between rural and urban households (Dargay (2002)) and in Nielsen (2006a) the influence of changing real estate prices and changing interest rates on car demand is analyzed. The method is also used in Dargay (2004) to

examine the price and income effects on car travel in the UK. She shows that households' car transport is more sensitive to changes in car purchase cost than changes in user cost. She finds short-run elasticites with regards to car purchase cost to be -0.35 and the short-run elasticity with regards to motor fuel prices to be -0.10 with corresponding long-run elasticities of -0.46 and -0.14. This paper extend on her findings and offer some insight into this result by showing that for rural households car travel is sensitive to car purchase cost and not so much to fuel cost. For urban households we find the opposite; that car travel is sensitive to fuel cost and not so much to car purchase cost. One possible explanation for this is that rural households have a higher need for car transport and low possibility of changing to other means of transport. This causes their travel demand to be affected more by the number of cars available to them (one car or more than one car) compared to the changed use of a single car. For urban households the public transport system is better and when fuel prices increase they can more easily change their mode of transport and they also have shorter distances to travel than rural households.

In a Danish setting the demand for car transport has been investigated in Bjørner (1999). He uses aggregated time series to estimate a model where he assumes that the level of transport can be described by a Cobb-Douglas function. He finds long-run elasticities with regard to income, fuel cost and car price to be 0.42, -0.84 and -1.12 respectively. Surprisingly he find that income in some estimations are not significant but point out that this might be due to multicollinearity between income, car ownership and a trend parameter used in his specification, but as he also points out it might be that income plays a minor role in the demand for car transport once a car is purchased. Birkeland et al.

(1999) use both cross-section modeling and a pseudo-panel approach to estimate income elasticities for car transport in Denmark. The cross-section methods they use gives income elasticities between 0.28 and 0.48 whereas the pseudo-panel method (used on a non-dynamic model) results in an income elasticity of 0.19. A recent Danish study also dealing with the demand for car transport is Fosgerau et al. (2004) where an aggregate model for the Danish road transport is presented. Their model consists of several minor models, one of which deals with the demand for car transport. They use gross national product per capita for Denmark together with a relative price index for the running cost of driving to estimate a partial adjustment model finding short- and long-run income elasticities of 0.13 and 0.16 together with short- and long-run cost elasticities of -0.30 and -0.37. They also address the problem of saturation in car ownership in Denmark concluding that saturation effects are not likely to affect the development in car ownership in Denmark, since the car ownership per capita is still well below that of other countries. This is confirmed by figures from EuroStat where Denmark is one of the countries with the lowest amount of cars per 1.000 inhabitants (350 cars per 1.000 inhabitants. See Dargay et al. (2006) for details)

A recent paper dealing with the development in car transport is Kwon and Preston (2005). They use data from the National Travel Survey in the UK to decompose the effect of increased travel distance into what they call 'car ownership effects' (changes caused by changing car ownership) and 'car use effects' (changes in car use dependent on car ownership level)<sup>36</sup>. They find that 'car ownership effects' explain about half of the

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<sup>&</sup>lt;sup>36</sup> In our work we label these effects 'intensive effects' and 'extensive effects' in order to use the same methodology as is used in other economic papers, especially literature dealing with the labor-market where

growth in car trip distance but also that this trend seems to be going down possibly because of saturation effects. They also identify two trends affecting the 'car use effects' namely increasing number of trips and increasing trip lengths. The first of these were dominating in the 1970s and 80s whereas the latter seems to dominate in the 90s. Furthermore they identify several factors affecting these effects and they present a survey of some recent fuel price elasticities for car transport reporting a consensus of -0.15 in the short run and -0.30 in the long run. These findings are higher than those reported by Dargay (2004) and also higher than what we find for long run fuel cost elasticities in the present paper.

Our paper adds to the literature in two ways. It offers new insights into the influence of car purchase prices and fuel prices on car travel found in both Denmark and in the UK. Secondly, since the data used by Dargay in the UK studies did not include information on the distance traveled but calculated a proxy for this variable from household expenditure data, average fuel price data, and data on the average fuel efficiency of cars, one could question the reliability of the UK results. By using a different type of data, namely a transport survey where the needed variables are available, we obtain results comparable to those found for the UK thus verifying the approach used in these studies. This is important since large transport surveys are rarely available and household expenditure surveys are.

<sup>&#</sup>x27;intensive responses' capture effects on the number of working hours and 'extensive responses' describe the labor market participation decisions (see for example Kleven and Kreiner (2006)).

This paper has the following structure. The next section presents the data and describes the construction of the pseudo-panel. Section 3 describes the model, section 4 presents the estimation and section 5 derives price and income elasticities. The final section concludes.

# 2. The data and the pseudo-panel

We use data from the Danish Transport Diary Survey together with time series from Statistics Denmark for the period from 1995 to 2002. The survey is conducted on a monthly basis with 1800 people between 16 and 74 being sampled from the Danish Central Personal Register every month until 1997. From 1998 the sample is extended to include people from age 10 to 84 and the sample size is increased to 2100 persons per month. The response rate is between 65% and 70%. Information on a given day of travel, including transport mode, distance traveled, trip chaining, and time use. Background information about the interviewee and her household is also collected (income, number of adults and children, car ownership, employment, etc.) together with a series of geographical variables<sup>37</sup>. We do not use all the observations in the survey. Since our focus is on the distance traveled by car we exclude all households not owning at least one car. Observations with missing variables are also excluded from the sample. From Statistics Denmark we obtain time series for car purchase costs and fuel costs for the years 1995 to 2002.

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 $<sup>^{37}</sup>$  A full description of the data can be found at the homepage of the Danish Transport Research Institute, www.dtf.dk.

Using the procedure described by Deaton (1985) we construct a pseudo-panel for cohorts for the years 1995 to 2002. We use three characteristics to create our panel; year of birth of the interviewee, real estate ownership status, and urbanization. The year of birth is often used when pseudo-panels are created and we choose the remaining characteristics since we want to examine differences between urban and rural households and since it was shown in Nielsen (2006a) that real estate ownership could influence the demand for cars. Creating the pseudo-panel according to these variables thus reduce the variation within the cohort-group and thus reduce the loss of information caused by the aggregation to cohort levels (See Deaton (1985) for a discussion of this). We thus end up with cohort-observations for four types of households; urban real estate owners, rural real estate owners, urban tenants, and rural tenants. With 11 cohorts we therefore have a total of 44 observations for each of the years between 1995 and 2002.

In figure 1 we see that the distances traveled by car depend on both the degree of urbanization and the cohort. The fact that the urbanization level influences the travel distance by car is not surprising since distances in general are longer in rural areas than in urban areas and since public transport services are better in urban areas. Urban households thus have better substitutes for car travel than rural households. Differences in the traveled distance only seem to be present for the younger cohorts though. For persons over the age of 55 the traveled distance still depends on the cohort but the effect of the level of urbanization seems to disappear.

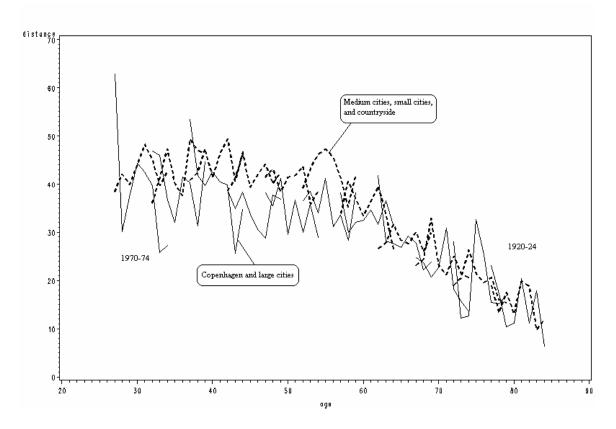


Figure 1: Distance traveled by car in urban and rural areas by different cohorts, contingent on the households having at least one car.

Incomes are also different for urban and rural households with urban households having higher income than rural households as shown in figure 2. We also see that the income differs between cohorts with income rising until a person turns 50 and a large drop in income when the person passes 60 and retires. These differences are to a large extent due to life-cycle effects but some cohort-effects are also present with young generations having higher salaries than the older generations had when they were young.

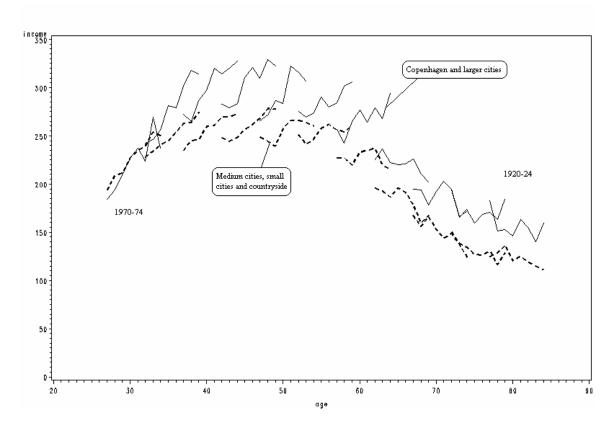


Figure 2: Household income (after tax) for different cohorts in urban and rural areas for car owning households.

The number of adults in the households is shown in figure 3 where a clear life-cycle effect can be seen. One difference between urban and rural households which is shown in figure 3 is that the number of adults peaks a few years earlier for rural households than for urban areas. One explanation for this difference could be that the children in rural households have to move from their parents to get an education whereas children coming from urban households can stay with their parents while studying.

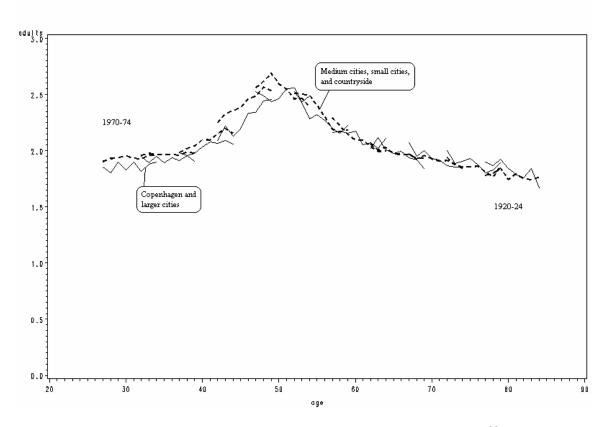


Figure 3: Number of adults in urban and rural areas for different cohorts<sup>38</sup> contingent on car ownership.

Figure 4 shows the average number of cars in the different cohorts for households have at least one car. It thus shows the development in multi-car households. As can be seen there is no large difference between rural and urban households except for a slight tendency for young rural households to own more cars than young urban households. A reason for this difference could be that more students live in the large cities and that they have substantially lower car ownership than other young people.

<sup>&</sup>lt;sup>38</sup> One problem with the data is that people count as adults when they become 16 years old.

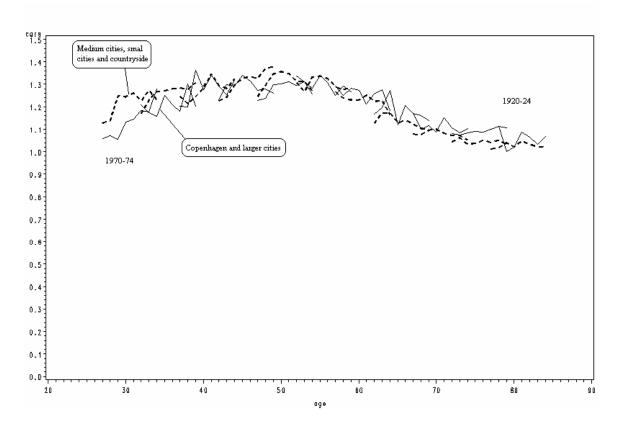


Figure 4: Car ownership for urban and rural households, contingent on households having at least one car.

Figure 5 shows the development in both fuel price and purchase price for cars between 1993 and 2002. We see that car prices have increased slightly in the period whereas fuel prices have fluctuated more with a huge increase from 1998 to 2000.

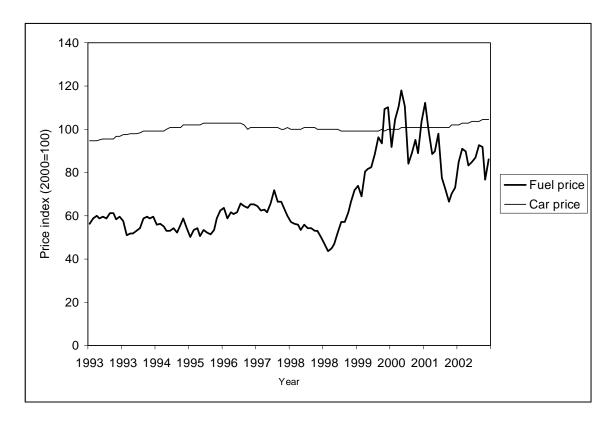


Figure 5: Price index for fuel price and car purchase cost (2000=100).

Source: Statistics Denmark.

# 3. The model

We use a model based on Dargay and Vythoulkas (1999) with a semi-log specification for income. Our choice of semi-log specification is based on the findings of Dargay (2001) where different specifications were tested (Linear, Double-log and Semi-log) concluding that the semi-log specification was the preferred one. Letting  $C_t^i$  represent the number of kilometers driven at time t for household i,  $Y_t^i$  the annual after tax income for household i at time t,  $T_t$  is the average purchase price for cars at time t,  $T_t^i$  is a dummy for households living in urban areas,  $T_t^i$  is the average fuel price at time t,  $T_t^i$  the number

of adults in the household, and with  $G^i$  being the cohort number, the demand for car travel is determined by

$$C_{t}^{i} = \alpha + \beta_{Y} \log(Y_{t}^{i}) + \beta_{T} T_{t} + \beta_{U} U_{t}^{i} + \beta_{F} F_{t} + \beta_{I} I_{t}^{i} + \beta_{G} G^{i} + \beta_{C} C_{t-1}^{i} + \varepsilon_{t}^{i}$$
(1)

where  $\alpha$  is a constant and  $\varepsilon_t^i$  is an term following a normal distribution,  $\varepsilon_t^i \sim N(0, (\sigma_t^i)^2).$ 

Since we use a pseudo-panel approach as described in Deaton (1995) all our variables take the form  $\sum_{i} \frac{A_{t}^{i}}{n_{t}^{c}} = \overline{A}_{t}^{c}$  where  $n_{t}^{c}$  is the number of households in cohort c and  $A_{t}^{c}$  is the variable. We let  $\overline{C}_{t}^{i}$  represent daily car transport in kilometers in year t for cohort i,  $\overline{I}_{t}^{i}$  the number of adults in the household,  $\overline{G}^{i}$  the cohort number,  $\overline{F}_{t}$  the fuel price,  $\overline{Y}_{t}^{i}$  the annual after tax income,  $\overline{U}_{t}^{i}$  is a dummy for households living in urban areas, and  $\overline{T}_{t}$  is the average purchase price for cars.

In order to determine if autocorrelation is present we use the Durbin-h statistics and the Durbin-t statistics. The results for the three models are shown in table 1.

	M-all	M-urban	M-rural
Durbin-h	-0.2776 (0.3907)	-2.8589 (0.0021)	
Durbin-t			1.1436 (0.1274)

Table 1: Test statistics for the presence of autocorrelation.

As we can see in the M-all model and the M-rural model we get a strong indication that we can proceed under the assumption of autocorrelation. For the M-urban model the result is less clear but we have estimated the model with several lags and all parameters are insignificant. We thus conclude that we can proceed under the assumption of autocorrelation.

The statistical model can thus be written as

$$\overline{C}_{t}^{i} = \overline{\alpha} + \beta_{Y} \log(\overline{Y}_{t}^{i}) + \beta_{T} \overline{T}_{t} + \beta_{U} \overline{U}_{t}^{i} + \beta_{F} \overline{F}_{t} + \beta_{I} \overline{I}_{t}^{i} + \beta_{G} \overline{G}^{i} + \beta_{C} \overline{C}_{t-1}^{i} + \overline{\varepsilon}_{t}$$

$$\overline{\varepsilon}_{t} \sim N(0, (\overline{\sigma}_{t}^{i})^{2}) \tag{2}$$

where  $\overline{C}_{t-1}^i$  is the number of cars in the previous period and  $\overline{\varepsilon}_t$  is an error term<sup>39</sup>.

Estimating the model shown in (2) directly on the pseudo-panel described in section 2 would result in the problem of hetehoscedasticity since the variance of the error-run will be given by

$$Var(\overline{\varepsilon_t^i}) = Var(\frac{1}{n_t^c} \sum_{i} \varepsilon_t^i) = (\frac{1}{n_t^c})^2 Var(\sum_{i} \varepsilon_t^i) = \frac{1}{n_t^c} Var(\varepsilon_t^i) = Var(\frac{1}{\sqrt{n_t^c}} \varepsilon_t^i)$$
(3)

and thus be dependent on the number of observations in each of the cohorts. In order to avoid this we simply multiply all cohort-observations with the square root of the number of observations used in the given cohort thus getting

$$Var(\sqrt{n_t^c} \frac{1}{\varepsilon_t}) = Var(\sqrt{n_t^c} \frac{1}{n_t^c} \sum_i \varepsilon_t^i) = (\frac{1}{n_t^c})^2 Var(\sqrt{n_t^c} \sum_i \varepsilon_t^i) = \frac{1}{n_t^c} Var(\sqrt{n_t^c} \varepsilon_t^i) = Var(\varepsilon_t^i)$$

$$\tag{4}$$

<sup>&</sup>lt;sup>39</sup> A test for the presence of autocorrelation showed that in most cases we could proceed with the simple error structure shown in (2).

and thus having a model where heteroscedasticity is eliminated. We therefore assume that all the error terms follow a normal distribution with the same variance  $\sigma^2$  as in (1). As argued by Dargay and Vythoulkas this model can be estimated using standard techniques.

## 4. Estimation

The variables used can be seen in table 2 together with their sources.

Variable	Source	Description
Urbanization	DTDS	Dummy for living in a large city
Income (log)	DTDS	Logarithm of household income
Cohort	DTDS	Cohort number
Adults	DTDS	Number of adults in the household
Car purchase price	SD	Price index for car purchase cost
Fuel price	SD	Price index for fuel cost
Transport demand	DTDS	Daily transport demand in kilometers

Table 2: Variables used in the model (Source: Statistics Denmark (SD), Danish Transport Diary Survey (DTDS))

In table 3 the number of observations used to construct the pseudo-panel is shown. We have defined urban households as households living in one of the five largest cities in Denmark and rural areas are thus the remaining households. Since the number of observations in each cohort should be as large as possible we are faced with a tradeoff since increasing the number of observations reduces the number of cohorts we can

construct. Here we have chosen 5-year bands for each cohort but still the average number of observations for tenants are low.

Cohort	Cohort date	Urban	Rural	Urban	Rural
number	of birth	owner	owner	tentants	tenants
1	1920-24	291	936	198	170
2	1925-29	502	1707	303	288
3	1930-34	695	2293	319	298
4	1935-39	880	3026	372	278
5	1940-44	1237	4080	375	314
6	1945-49	1500	5010	431	390
7	1950-54	1277	4676	335	375
8	1955-59	1314	4430	427	421
9	1960-64	1259	4337	578	509
10	1965-69	1081	3709	803	744
11	1970-74	614	1989	762	812
Cohort average		968	3290	446	418

Table 3: Number of observations

Using the pseudo-panel to estimate the model, remembering that we multiply each cohort-observation with the number of observations used to construct it, we get the results shown in table 4.

Variable	M-all	M-rural	M-urban
Constant	-0.0056 (-0.00)	12.4345 (0.75)	-70.3248 (-3.10)
Urbanization	-6.3271 (-6.39)		
Income (log)	9.4187 (3.67)	10.5508 (2.75)	4.2964 (1.19)
Adults	2.9518 (1.69)	3.2965 (1.44)	1.2177 (0.41)
Car purchase price	-0.3357 (-3.07)	-0.4316 (-2.62)	0.0533 (0.32)
Fuel price	-0.0425 (-2.40)	-0.0256 (-1.08)	-0.0703 (-2.50)
Cohort	1.5164 (7.82)	1.4793 (5.27)	1.6192 (6.33)
Transport demand	0.2025 (3.52)	0.2332 (2.71)	0.0270 (0.35)
$\mathbb{R}^2$	0.9535	0.9580	0.8896
Log Likelihood	-1661.3546	-850.2932	-792.6594
SSE	1213776.56	789693.948	364321.752
MSE	4185	5561	2566

Table 4: Estimates, t-values and summary statistics

Most of the variables are significant but we see that income and the number of adults in the urban households is insignificant. For income this correspond to the findings in Bjørner (1999) where income was found to be insignificant in some instances. Signs of the variables are also as expected with the exception of car purchase prices in the model for urban households but this variable is not statistically significant. For rural households the variable for fuel price is not significant. One possible explanation is that rural households, having poor access to public transport and longer distances to travel are more

dependent on their car for all trips and especially trips which are necessary (like commuting trips, essential shopping trips, and leisure trips where the distances makes other modes of transport than car impractical). Such trips could potentially be a large part of the trips undertaken for rural households. This is not the case for urban households where the distances are shorter and the public transport network is a good substitute for car transport. Even though we do not see great differences between rural and urban households when it comes to the number of cars available in car-owning households (see figure 3) the effect of changing prices on the purchase of cars only seems to affect the car transport of rural households and not that of urban households. The estimation also shows that cohort effects are present and younger generations travel longer than older generations. Except for the urban households the adjustment parameter is also significant but with values of 0.20 and 0.23 we see that around 80% of the adjustment in transport demand happens within the first year. For rural households the adjustment parameter is 0.03 and adjustment is almost instantaneous which also explains why the parameter is not statistically significant.

# 5. Elasticities

To calculate the elasticities for Denmark we use the M-all model. The formula for short run elasticities of car transport with regards to variable  $x_i$  is given by  $\varepsilon_i^{sr} = \frac{\partial C}{\partial x_i} \frac{x_i}{C} = \beta_{x_i} \frac{x_i}{C}$  and the corresponding long run elasticity can be calculated as  $\varepsilon_i^{LR} = \frac{\varepsilon_i^{SR}}{\theta_i}$  where  $\theta_i$ =(1- $\beta_i$ ) is the adjustment parameter. Estimated short- and long-run elasticities are shown in table 5.

Income		Car purchase price		Fuel price	
Short run	Long run	Short run	Long run	Short run	Long run
0.25	0.32	-0.90	-1.13	-0.11	-0.14

Table 5: Elasticities for the complete model<sup>40</sup>

With the adjustment parameter being relatively high ( $\theta_i$ =7975) the difference between short- and long-run elasticities are not large. The income elasticities found here is in line with some findings in Birkeland et al. (1999) but lower than those found in Bjørner (1999) and Fosgerau et al (2004). Our income elasticities are much lower than those reported for UK in Dargay (2004) where values around 1 are found. For the elasticity with regard to car purchase cost our results are comparable with those found in Bjørner (1999) but our fuel price elasticity is substantially below his finding but in line with the findings in Birkeland et al. (1999) and Dargay (2004).

	Rural households		Urban households	
	Short run	Long run	Short run	Long run <sup>43</sup>
Income elasticity	0.27	0.35	0.14	0.14
Car purchase price elasticity <sup>41</sup>	-1.09	-1.42		
Fuel price elasticity <sup>42</sup>			-0.23	-0.23

Table 6: Elasticities for rural and urban households<sup>44</sup>

<sup>&</sup>lt;sup>40</sup> We assume that the number of kilometers traveled is 36 which is the average in 2000. The price index for the purchase price for cars was 100 in 2002 and the price index for fuel was 100.

<sup>&</sup>lt;sup>41</sup> The car purchase price is only significant for rural households.

<sup>&</sup>lt;sup>42</sup> The fuel price is only significant for urban households

<sup>&</sup>lt;sup>43</sup> For urban households the income parameter and the adjustment parameter are insignificant. With an adjustment parameter close to 1 we thus have almost identical short- and long-run elasticities.

Using the models for urban and rural households we get the results shown in table 6. As in Christens and Fosgerau (2004) that urban households have higher income elasticity than rural households but we also keep in mind that their elasticities were for car ownership and our elasticities are for travel by car. For the two groups the elasticities with regard to car purchase price and fuel price increase numerically compared to the elasticities shown in table 5.

#### 6. Conclusion

Other studies have estimated the demand for car transport for Denmark but in this paper we for the first time utilize the Danish Transport Diary Survey to create a pseudo-panel and use this to estimate a dynamic partial adjustment model for car transport. Other dynamic specifications have used aggregate time series or based their estimates on cross-section methods. Elasticities for car transport are found with regard to income, car purchase cost and fuel price.

We show that urban households respond more to changing fuel prices than to changes in the purchase price of cars. The opposite holds for rural households where the demand for car transport is more sensitive to changes in the purchase price of cars than to changing fuel prices. Our result supports the findings of Dargay (2004) where the UK Family Expenditure Survey is used to estimate a dynamic model for car transport. She constructed a proxy for car transport and it is thus interesting to see, if a different type of data gives comparable results. Finding that the different data yield similar results is

<sup>&</sup>lt;sup>44</sup> We assume that the number of kilometers traveled is 39.7 for rural households and 30.9 for urban households which is the average in 2000. The price index for the purchase price for cars was 100 and the price index for fuel was 100 in 2000.

important since many countries have large expenditure surveys but do not have large transport surveys. Using expenditure surveys in future work thus seems to be justified.

## References

Birkeland, M. E., Brems, C. R. and Kabelmann, T. (2000) Analyse af personers transportarbejde, 1975-1998, *Trafikdage på Aalborg Universitet 2000*, Conference paper (www.trafikdage.dk)

Bjørner, T. B. (1999) Transport Demand and Environmental Regulation in the Transport Sector, *Ph.D. dissertation*, University of Copenhagen, Department of Economics, Red Series number 54.

Christens, P. F. and Fosgerau, M. (2004) Rådighed over bil - en beskrivelse af sammenhængen mellem husstandsindkomst, bilrådighed og geografi, *DTF Notat* **3 2004**, Danish Transport Research Institute, www.dtf.dk.

Dargay, J. M. and Gately, D. (1999) Income's effect on car and vehicle ownership, worldwide: 1960-2015, *Transportation Research* **Part A 33**, 101-138.

Dargay, J. M and Vythoulkas, P. C. (1999) Estimation of a Dynamic Car Ownership Model: A Pseudo-panel Approach, *Journal of Transport Economics and Policy* **33:3**, 287-302.

Dargay, J. (2001) The effect of income on car ownership: evidence of asymmetry, Transportation Research Part A 35, 807-821.

Dargay, J. (2002) Car ownership in rural and urban areas: a pseudo-panel analysis, Transportation Research Part B 38, 351-366.

Dargay, J. (2004) The Effect of Prices and Income on Car Travel in the UK, World Conference on Transport Research, Istanbul, Turkey, July 2004.

Dargay, J., Hivert, L. and Legros, D. (2006) The dynamics of car ownership in E.U countries: a comparison based on the european household panel survey, *unpublished* working paper.

Deaton, A. (1985) Panel Data from Time Series of Cross Sections, Journal of Econometrics 30, 109-126.

Fosgerau, M., Holmblad, M. and Pilegaard, N. (2004) ART – En aggregeret prognosemodel for dansk vejtrafik, *Notat* **5 2004**, Danish Transport Research Institute.

Kleven, H. J. and Kreiner, C. T (2006) The Marginal Cost of Public Funds: Hours of Work Versus Labor Force Participation, *The Journal of Public Economics*, Forthcomming.

Kwon, T-H, and Preston, J. (2005) Driving Forces behind the Growth of Per-capita Car Driving Distance in the UK, 1970-2000, *Transport Reviews* **25:4**, 467-490.

Nielsen, J. E. (2006a) Real estate ownership and the demand for cars in Denmark – A pseudo-panel analysis, *Thesis Paper*.

Nielsen, J. E. (2006b) Estimating car ownership in Denmark – Discrete choice modeling and repeated cross-section analysis, *Thesis Paper*.

Nielsen, J. E. (2006c) Transport tax reform, two-part tariffs and revenue recycling, *Thesis Paper*.

# Transport tax reforms, two-part tariffs, and revenue recycling

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#### Abstract

We explore the interaction between taxes on the purchase of cars and on the use of cars when households face a discrete choice of purchasing a car or not. We use a simple labor-leisure model with a logit formulation for the discrete choice of car-ownership to examine how a tax reform which shifts taxes from car ownership to use of cars affects welfare. To our knowledge this is the first time the discrete nature of car purchase is modeled explicitly in a tax-reform model and it is thus the first consistent analysis of a tax reform involving both fixed and variable cost. Car transport is associated with negative externalities and feedback effects. We show that the welfare effect of this type of reform depends on both the choices of car usage and car ownership decisions and that feedback effects exist on both the intensive margin and on the extensive margin. Furthermore the effect of such a tax reform depends on the initial level of the variable tax on car transport relative to the pigouvian level. The findings are illustrated by simulations.

## 1. Introduction

The importance of intervention in the transport sector has become obvious in recent years with externalities, and especially congestion externalities, increasing rapidly in almost every major city. The dilemma facing the transport authorities is that transport, while causing negative externalities, is an essential part of social life and that increasing transport seems to be closely related to economic growth. One aspect of this dilemma is pointed out in Parry & Bento (2001). They show that the implementation of marginal cost pricing to reduce externalities can cause negative welfare effects if the extra costs of transport discourage labor supply. A similar insight is obtained in Nielsen (2006a) where the optimal tax rules in a Becker (1965) framework for household time allocation are derived for a situation with atmospheric externalities. Here it is shown that the additivity property derived by Sandmo (1975) still apply, though in a modified form thus extending the findings of Kleven (2004) where an inverse factor share rule was derived in a Becker framework. An important insight from the Parry and Bento paper is that we have to be careful when we evaluate the welfare effects of tax reforms in the transport sector, since a tax which seems to be optimal when analyzed in isolation (in their case marginal cost pricing) could have negative welfare effects if analyzed in a general equilibrium setting.

In this paper we examine a tax reform in the transport sector involving a fixed cost of purchasing (or owning) cars. A fixed purchase (or ownership) tax on cars is replaced by a variable tax on the use of cars in a budget neutral way. In order to analyze this reform one needs to account for car ownership decisions (e.g. decisions on the extensive margin). Reforms of this kind are frequently suggested in the Danish debate since the marginal tax

on the purchase of cars in Denmark is 180% (DORS (2006)). This type of tax reforms have to our knowledge not yet been analyzed consistently, since the discrete nature of the car ownership decision has been left out of previous analysis.

We build upon several results from the economic literature. The explicit inclusion of time in economic models was first undertaken by Becker (1965), DeSerpa (1971), and others<sup>45</sup>. They include time as a source of utility for households, either directly in the utility functions (DeSerpa), or indirectly through a household production function (Becker). For the modeling of car ownership, we draw upon the results from Small and Rosen (1981). Based on the work of McFadden (1974) they present a framework for modeling welfare effects when discrete consumer choices have to be taken into account. De Borger (2000) demonstrates that the Small & Rosen approach can be implemented in a tax model, and he uses their framework to derive the optimal two-part tariff in a model of discrete choice which he extends to a situation with externalities in De Borger (2001). Unfortunately the time allocation problem and thus the labor-leisure tradeoff is not included in these papers.

The modelling framework used here is based on Parry and Bento (2001). They construct a general equilibrium model for commuting transport, labor supply, and congestion externalities in order to analyze tax reforms involving variable taxes on the use of cars, subsidies to public transport (in their model a metro system), and reduction in labor taxes. They show that interactions with pre-existing taxes are important and that feedback effects influence the welfare effects of the tax reforms. They use the same framework in

4.5

<sup>&</sup>lt;sup>45</sup> A recent survey on the literature on time allocation can be found in Jara-Díaz (2003)

Parry and Bento (2002) to analyze situations with different types of externalities and interactions with different types of taxes. Again they conclude that welfare effects of tax reforms depend on interactions with pre-existing distortions in the economy. De Borger and Van Dender (2003) use the same framework to analyze the effect of a tax reform on the value of time. They show that commuting taxes might reduce the value of time while taxes on non-commuting traffic plausibly raise the value of time. They thus demonstrate that the tax system and time values are closely related; a finding which was also pointed out earlier in Forsyth (1980).

In this paper we show that the welfare effect of a tax reform depends on a combination of several factors, some of which are identified in the papers mentioned above. The new finding is that changes on the extensive margin (e.g., changes in car-ownership) could be large and should be modeled explicitly if correct estimates of welfare effects are to be obtained. With the use of simulation, we demonstrate that it is possible to include discrete effects and we believe that future evaluation of tax reforms in the transport sector should include these or at least reflect on the consequences of not including extensive responses. Furthermore our simulation shows that the optimal tax on the use of cars is equal to the marginal external damage cost, i.e. the pigouvian tax rate.

The paper has the following structure. Section 2 presents the modeling framework in which household decisions on the intensive margin (how much to consume contingent on car-ownership) is analyzed and the choice on the extensive margin (the car-ownership decision) is also described in detail. The production sector and the government

optimization problem are also presented. Section 3 derives an expression for the welfare effect of a tax reform involving a shift from fixed to variable taxation of cars. Section 4 simulate a tax reform to illustrate the theoretical findings. The final section concludes.

#### 2. The model

Inspired by Parry and Bento (2001) we construct a model where households decide on the consumption of goods and leisure and they also choose how much to work and if they want to commute by private transport ('car') or public transport ('metro'). Our model deviates from previous models of this type since we do not assume households to be identical. Instead we assume that all the observable socioeconomic characteristics of the households are identical and it thus seems as if they have identical utility functions. Observing the outcome of the households' actions their choice of car-ownership is not consistent with the households being identical since some households purchase a car and others do not. Their choice thus appears to be random. The households can therefore be divided into two types: those who do not purchase a car and those who purchase a car. We assume that households purchasing a car can commute by either car or metro. Households not purchasing a car can only commute by metro. Furthermore we assume that congestion externalities are present in 'car transport'.

The production sector in this economy is fully competitive and operates under constant returns to scale. All producer prices are therefore constant and equal to the marginal cost of production. The government taxes goods, transport and labor income in order to generate revenue for some unspecified tasks (e.g. national defense or the health care system).

#### 2.1 The households

Households consume 'pure leisure', L, an aggregate consumption commodity, Z, public transport (a metro service), and private transport (car transport). Households also supply labor to the production sector and can go to work by either metro or car. We assume that consumption of the aggregate good and consumption of leisure as well as 'transport' in itself generates household utility and we write the individual households utility function as

$$U(L,Z) + u(Z^b, Z^c) \tag{1}$$

where  $Z^b$  is trips by metro and  $Z^c$  is trips by car. These are defined in detail below. The separability assumption between U and u in (1) separates the choice of transport mode from the decision on how much to work. It also allows for imperfect substitutability between transport by metro and transport by car. This seems like a realistic assumption since some people dislike crowded metros and enjoy the privacy of their own car.

We assume that H households exist and that H is large. All households have the same earning capacity and thus the same wage rate w. Furthermore all households have the same endowment of time,  $\overline{L}$ , and non-labor income, y. Each commuting trip takes up a certain amount of household time which we will denote  $L^b$  for a trip by metro and  $L^c$  for a trip by car. A trip is defined as traveling both to and from work. This means that households either go back and forth by car or by metro. Following Parry & Bento (2001) we assume that the number of working hours in one day is fixed and equal to one. This means that the supply of labor hours given by  $L^w$  is equal to the total number of commuting trips

$$L^{w} = Z^{b} + Z^{c} \tag{2}$$

We let  $\overline{Z}$  be total number of trips by car and assume that, because of congestion, a higher value of  $\overline{Z}$  increases the time requirement for car transport. We thus have that

$$L' = \frac{\partial L^{c}(\overline{Z})}{\partial \overline{Z}} > 0 \tag{3}$$

Assuming that households ignore their own influence on  $\overline{Z}$ , we have an externality problem in car transport. We assume that the metro service is not affected by congestion.

Letting P,  $P^b$ , and  $P^c$  represent consumer prices on goods, metro trips and car trips,  $t_w$  the tax on labor, and  $\overline{P}$  be the fixed cost of purchasing a car. Normalizing the wage rate w to one then allows us to write the constraints which the households face as

$$PZ + P^{b}Z^{b} + P^{c}Z^{c} + \overline{PI}_{\{Z^{c} > 0\}} = (1 - t_{w})L^{w} + y$$
(4)

$$L + L^b Z^b + L^c Z^c + L^w = \overline{L}$$

$$\tag{5}$$

where  $I_{\{Z^c>0\}}$  is an indicator function equal to 1 if  $Z^c>0$  and zero otherwise. It therefore indicate whether the households purchase a car or not. Following DeSerpa (1971) we will label (4) the *budget constraint* and label (5) the *time resource constraint*. The constraints are interdependent through  $L^w$ ,  $Z^b$ , and  $Z^c$ . Apart from the fixed cost,  $\overline{P}$ , and the fact that households can make a choice on the extensive margin with regards to car ownership, this part of the model is similar to the one used in Parry & Bento (2001).

## 2.1.1 The choice at the intensive margin

We now examine how a household behaves conditional on its choice of car ownership status. Since households owning a car and households not owning a car can choose

between different consumption bundles ('non-owners' can not choose to travel by car) and face different budget constraints ('owners' have to pay a fixed fee  $\overline{P}$ ), we analyze the two types of households separately.

#### 2.1.1.1 The choice for car owners

Letting subscript 1 indicate car ownership, using the constraint given in (4) and (5), and remembering that the households ignore their own influence on the level of  $\overline{Z}$  we can specify the utility maximization problem for car owners as

$$\max_{L^{1},Z^{1},Z^{b1},Z^{c1}} U(L^{1},Z^{1}) + u(Z^{b1},Z^{c1})$$
s.t.
$$PZ^{1} + P^{b}Z^{b1} + P^{c}Z^{c1} + \overline{P} = (1 - t_{w})L^{w1} + y \qquad (\lambda^{M1})$$

$$L^{1} + L^{b}Z^{b1} + L^{c}(\overline{Z})Z^{c1} + L^{w1} = \overline{L} \qquad (\lambda^{T1})$$

where  $\lambda^{M1}$  is the marginal utility of income,  $\lambda^{T1}$  is the marginal utility of time as a resource, and labor supply will be given by

$$L^{w1} = Z^{b1} + Z^{c1} (7)$$

With (6) being a standard maximization problem the first order conditions are given by

$$\frac{\partial U}{\partial L^{1}} = \lambda^{T1} \tag{8}$$

$$\frac{\partial U}{\partial \mathcal{T}^{\mathsf{I}}} = \lambda^{M\mathsf{I}} P \tag{9}$$

$$\frac{\partial u}{\partial Z^{b1}} = \lambda^{M1} (P^b - (1 - t_w)) + \lambda^{T1} (L^b + 1)$$
 (10)

$$\frac{\partial u}{\partial Z^{c1}} = \lambda^{M1} (P^c - (1 - t_w)) + \lambda^{T1} (L^c(\overline{Z}) + 1)$$
(11)

together with the budget constraint (4) and the time resource constraint (5).

Following De Serpa (1971) we define the value of time for car owners,  $VoT^1$ , as  $\frac{\partial U}{\partial L^1} / \frac{\partial U}{\partial Z^1}$  and by using (8), (9), and (11) we can write this as

$$VoT^{1} = \frac{\lambda^{T1}}{\lambda^{M1}} = \frac{\frac{\partial u}{\partial Z^{c1}}}{\frac{\lambda^{M1}}{\lambda^{M1}} + ((1-t_{w}) - P^{c})}}{L^{c}(\overline{Z}) + 1}$$

$$(12)$$

or by using (10) instead of (11) we can write it as

$$VoT^{1} = \frac{\lambda^{T1}}{\lambda^{M1}} = \frac{\frac{\partial u}{\partial Z^{b1}}}{\lambda^{M1}} + ((1 - t_{w}) - P^{b})}{L^{b} + 1}$$

$$\tag{13}$$

The numerator of (12) and (13) measure the value of one days work. To see this note that  $\frac{\partial u}{\partial Z^{c1}}$  and  $\frac{\partial u}{\partial Z^{b1}}$  measure the marginal utility of a trip by car or metro respectively. By dividing with  $\lambda^{M1}$  this is converted into monetary terms. One days work also generates income and there is a price on commuting. This is captured by  $((1-t_w)-P^c)$  and  $((1-t_w)-P^b)$  which is the income from one days work consisting of the after tax income,  $(1-t_w)$ , and the price of the commuting trip,  $P^b$  or  $P^c$ . The denominator measure the time used on a days work which consist of the time spend working (which we normalized to one) and commuting time. We thus see that (12) and (13) describes the value of time for car owners. From (12) and (13) we get that a tax on commuting working through either  $P^b$  or  $P^c$  reduce the value of time, which is also pointed out in De Borger and Van Dender (2003). It is also obvious that a reduction of the labor tax,  $t_w$ , will increase the value of time and increased congestion reduce the value of time. The expressions in (12) and (13) for the value of time resemble those found in De Borger and Van Dender (2003), and as they note the value of time is independent of the activity in which it is used.

Using the first order conditions (8) through (11) together with the budget constraint (4) and the time resource constraint (5) we get the ordinary demand functions for car owners

$$L^{1}(P, P^{b}, P^{c}, t_{w}, L^{b}, L^{c}(\overline{Z}), y, \overline{P}, \overline{L})$$

$$\tag{14}$$

$$Z^{1}(P, P^{b}, P^{c}, t_{w}, L^{b}, L^{c}(\overline{Z}), y, \overline{P}, \overline{L})$$

$$\tag{15}$$

$$Z^{b1}(P, P^b, P^c, t_w, L^b, L^c(\overline{Z}), y, \overline{P}, \overline{L})$$

$$(16)$$

$$L^{c1}(P, P^b, P^c, t_w, L^b, L^c(\overline{Z}), y, \overline{P}, \overline{L})$$

$$(17)$$

together with shadow prices  $\lambda^{M1}$  and  $\lambda^{T1}$ . Using these together with

$$L^{w1} = Z^{b1} + Z^{c1} (18)$$

we can write the indirect utility function for households owning a car as

$$V^{1}(P, P^{b}, P^{c}, t_{w}, L^{b}, L^{c}(\overline{Z}), y, \overline{P}, \overline{L})$$

$$= \max_{L^{1}, Z^{1}, Z^{b^{1}}, Z^{c^{1}}} \{U(L^{1}, Z^{1}) + u(Z^{b^{1}}, Z^{c^{1}})$$

$$-\lambda^{M1}(PZ^{1} + P^{b}Z^{b^{1}} + P^{c}Z^{c^{1}} + \overline{P} - (1 - t_{w})L^{w^{1}} - y)$$

$$-\lambda^{T1}(L^{1} + L^{b}Z^{b^{1}} + L^{c}(\overline{Z})Z^{c^{1}} + L^{w^{1}} - \overline{L})\}$$
(19)

which is now given as a function of variables exogenous to the household.

#### 2.1.1.2 The choice for non-car owners

Letting subscript 0 indicate non-ownership of cars, households not owning a car have to solve

$$\max_{L^{0}, Z^{0}, Z^{b0}} U(L^{0}, Z^{0}) + u(Z^{b0}, 0)$$
s.t.
$$PZ^{0} + P^{b}Z^{b0} = (1 - t_{w})L^{w0} + y \qquad (\lambda^{M0})$$

$$L^{0} + L^{b}Z^{b0} + L^{w0} = \overline{L} \qquad (\lambda^{T0})$$
(20)

where  $\lambda^{M0}$  is the marginal utility of income,  $\lambda^{T0}$  is the marginal utility of time as a resource, and labor supply will be given by

$$T^{w0} = Z^{b0} (21)$$

With (20) being a standard maximization problem the first order conditions are given by

$$\frac{\partial U}{\partial I^0} = \lambda^{T0} \tag{22}$$

$$\frac{\partial U}{\partial Z^0} = \lambda^{M0} P \tag{23}$$

$$\frac{\partial u}{\partial z^{b0}} = \lambda^{M0} (P^b - (1 - t_w)) + \lambda^{T0} (L^b + 1)$$
 (24)

together with the budget constraint (4) and the time resource constraint (5).

As before we follow DeSerpa (1971) define the value of time for non-car owners,  $VoT^0$ , as  $\frac{\partial U}{\partial L^0} / \frac{\partial U}{\partial Z^0}$ . By using (22), (23), and (24) we can write this as

$$VoT^{0} = \frac{\lambda^{T0}}{\lambda^{M0}} = \frac{\frac{\partial u}{\partial Z^{b0}}}{\lambda^{M0} + ((1 - t_{w}) - P^{b})}$$

$$L^{b} + 1$$
(25)

As in (12) and (13) the numerator is the value of one days work and the denominator is the time used on one days work. Again we see that a tax on commuting (in this case public transport) raise the cost of travel through  $P^b$  and this reduce the value of time. It is also obvious that a reduction of the labor tax,  $t_w$ , will increase the value of time. From (13) and (25) we also see that if the government implement a policy where the time requirement for public transport,  $L^b$ , is reduced the value of time for both car owners and non-car owners will go up<sup>46</sup>.

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<sup>&</sup>lt;sup>46</sup> In Copenhagen such a policy could be the opening of the new metro line in 2007 and the extension of the metro system with a city ring expected to be build during the next 10 years.

The problem in (20) is structurally identical to (6). As before we get the ordinary demand functions for non-car owners

$$L^{0}(P, P^{b}, t_{w}, L^{b}, y, \overline{L})$$

$$(26)$$

$$Z^{0}(P, P^{b}, t_{\dots}, L^{b}, y, \overline{L}) \tag{27}$$

$$Z^{b0}(P, P^b, t_w, L^b, y, \overline{L})$$
 (28)

together with shadow prices  $\lambda^{M0}$  and  $\lambda^{T0}$ . Again we can write the indirect utility function for households not owning a car as

$$V^{0}(P, P^{b}, t_{w}, L^{b}, y, \overline{L})$$

$$= \max_{L^{0}, Z^{0}, Z^{b0}} \{U(L^{0}, Z^{0}) + u(Z^{b0}, 0)$$

$$-\lambda^{M0}(PZ^{0} + P^{b}Z^{b0} - (1 - t_{w})L^{w0} - y)$$

$$-\lambda^{T0}(L^{0} + L^{b}Z^{b0} + L^{w0} - \overline{L})\}$$
(29)

which is now given as a function of variables exogenous to the household.

### 2.1.2 The choice at the extensive margin

Facing the price structure  $(P, P^b, P^c, \overline{P})$ , wage tax  $t_w$ , having non-labor income y, facing time requirements  $L^b$  and  $L^c$  together with externality level  $\overline{Z}$ , the household chooses between the utility level  $V^0$  and  $V^1$  given in (19) and (29). Since households are utility maximizing, they each choose  $i \in \{0,1\}$  such that

$$V^{i} = \max\{V^{0}, V^{1}\} \tag{30}$$

Using the random utility approach pioneered by Daniel McFadden (McFadden (1974)) we assume that the households behave as if the indirect utility function is composed of an observable deterministic part,  $V^i$ , together with stochastic error term,  $\varepsilon^i$ . We write this as  $V^i + \varepsilon^i$ . The error term captures the unobservable characteristics which make the

household choice seem random to the government. For simplicity and to ensure a closed form solution, we assume that these error terms are independently and identically distributed following a double exponential distribution with scale parameter  $\eta$  which we will normalize to 1. This gives us a logit model for discrete choice<sup>47,48</sup>. We know (Ben-Akiva & Lerman (1985) and Train (2003)) that the probability of choosing not to buy a car,  $\pi^0$ , and the probability of choosing to buy a car,  $\pi^1$ , are given by<sup>49</sup>

$$\pi^{0}(P, P^{b}, P^{c}, t_{w}, L^{b}, L^{c}(\overline{Z}), y, \overline{P}, \overline{L}) = \frac{e^{V^{0}}}{e^{V^{0}} + e^{V^{1}}}$$
(31)

$$\pi^{1}(P, P^{b}, P^{c}, t_{w}, L^{b}, L^{c}(\overline{Z}), y, \overline{P}, \overline{L}) = \frac{e^{V^{1}}}{e^{V^{0}} + e^{V^{1}}}$$
(32)

It is worth noting that the probabilities shown in (31) and (32) depend on all the parameters in the model. This means that even though households not owning a car do not directly affect the total level of congestion, by changing behavior on the intensive margin they still indirectly affect the level of congestion, via their impact on the extensive margin. The expected maximum utility W for a representative household is given by<sup>50</sup>

$$W = \ln(e^{V^0} + e^{V^1}) \tag{33}$$

 $\pi^0 = \Pr(V^0 + \varepsilon^0 > V^1 + \varepsilon^1) = \Pr(\varepsilon^1 < \varepsilon^0 + V^0 - V^1) = \int (e^{-e^{-(\varepsilon^0 + V^0 - V^1)}})e^{-\varepsilon^0}e^{-e^{-\varepsilon^0}}d\varepsilon^0 = \frac{e^{V^0}}{e^{V^0} + e^{V^1}}$  and thus that

$$\pi^{1} = 1 - \pi^{0} = \frac{e^{V^{1}}}{e^{V^{0}} + e^{V^{1}}}$$
.

<sup>&</sup>lt;sup>47</sup> Further details about the logit model, the double exponential distribution and a discussion of the scale parameter  $\eta$  can be found in Ben-Akiva and Lerman (1985) and Train (2003).

<sup>&</sup>lt;sup>48</sup> A discussion of the restrictiveness relating to the choice of the double exponential distribution of the error terms can be found in appendix C of De Borger (2000).

<sup>&</sup>lt;sup>49</sup> From Train (2003) we have that

From Train (2003) we get that the maximum expected utility for a representative household is given by  $E[\max\{V^0 + \varepsilon^0, V^1 + \varepsilon^1\}] = \ln(e^{V^0} + e^{V^1})$ 

which is also known as the log-sum. The demand for goods and commodities for a representative (or average) household as well as the supply of labor can now be written as

$$\hat{Z} = \pi^0 Z^0 + \pi^1 Z^1 \tag{34}$$

$$\hat{Z}^b = \pi^0 Z^{b0} + \pi^1 Z^{b1} \tag{35}$$

$$\hat{Z}^c = \pi^1 Z^{c1} \tag{36}$$

$$\hat{L}^{w} = \pi^{0} L^{w0} + \pi^{1} Z^{w1} \tag{37}$$

which is a weighted average of the demand for the two types of households in the economy. By defining  $L^b(\overline{Z})$  as a function of  $\widehat{Z}^c$  instead of  $\overline{Z}$  we have that (36) plays the role of the total number of trips by car in the economy, i.e.  $\widehat{Z}^c = \overline{Z}$ . As expected we see that the level of  $\overline{Z}$  depend on both the choice on the intensive margin,  $\pi^1$ , and the choice on the extensive margin,  $Z^{c1}$  which is an extension compared to previous models where the level only depend on the choice on the extensive margin. To simplify notation we will make the assumption that  $\widehat{Z}^c = \overline{Z}$  in the coming derivations.

## 2.2 The production sectors and the government

We assume that all production sectors are fully competitive and operate under constant returns to scale. No profits thus exist and the producer prices p,  $p^b$ , and  $p^c$  for commodities, public transport (a 'ticket') and private transport ('fuel') become constant and equal to the marginal cost of production. The government can tax both private and public transport. Letting  $t^b$  and  $t^c$  represent the tax on public and private transport we can write

$$P^b = p^b + t^b \tag{38}$$

$$P^c = p^c + t^c \tag{39}$$

We assume that the fixed fee  $\overline{P}$  for the purchase of a 'car' is paid directly to the government which is just a normalization not affecting the results. The government has to raise revenue G for some unspecified purposes using the taxes defined in (38) and (39), together with the labor tax,  $t_w$ , and the fixed fee,  $\overline{P}$ . Following De Borger (2000) we use (33) to write the social welfare function as

$$W(P, P^{b}, P^{c}, (1-t_{w}), L^{b}, L^{c}(\overline{Z}), y, \overline{P}, \overline{L}) = \ln(e^{V^{0}} + e^{V^{1}})$$
(40)

which the government seeks to maximize. We define the government's revenue function as

$$R(\overline{P}, t_w, t^b, t^c) = \pi^1 \overline{P} + t_w \widehat{L}^w + t^b \widehat{Z}_b + t^c \widehat{Z}_c$$

$$\tag{41}$$

where the first term is the fixed fee collected from car owners, the second term is the total labor tax, and the last two terms represent taxes on metro and car transport respectively. The government's budget constraint is now given by

$$R(\overline{P}, t_w, t^b, t^c) = G \tag{42}$$

Taking a closer look at the formula for expected maximum utility for a representative household (33), we see that the effect of changes in parameters is a weighted sum of changes in the indirect utility functions for households owning a car and households not owning a car. Letting  $\Theta$  represent some policy parameter that is changed, the change in maximum expected utility will be given by

$$\frac{\partial W}{\partial \Theta} = \pi^0 \frac{\partial V^0}{\partial \Theta} + \pi^1 \frac{\partial V^1}{\partial \Theta} \tag{43}$$

where we can interpret the probabilities as fractions of households not owning and owning a car. Since for households being at the border between having and not having a car we have  $V^0 = V^1$ , the change in the probability of car ownership at the margin does not change the overall welfare. Hence the change in probabilities does not enter the expression above.

## 3. Tax reform analysis

In this section we examine how social welfare changes when the government implements a tax reform reducing the purchase tax on cars while increasing the variable tax on the use of cars in a revenue neutral manner.

### 3.1 Helpful derivations

Using (19), (29), and (43) we know that

$$\frac{\partial W}{\partial t^c} = -\pi^1 \lambda^{M1} Z^{c1} \tag{44}$$

$$\frac{\partial W}{\partial L^c} = -\pi^1 \lambda^{T_1} Z^{c_1} \tag{45}$$

$$\frac{\partial W}{\partial \overline{P}} = -\pi^1 \lambda^{M1} \tag{46}$$

The marginal welfare effects (44) and (45) resemble the results from Parry & Bento (2001) except for the probability weighting included here. Note that the effect of the fixed fee derived in (46) is identical to the effect of a lump-sum transfer in the Parry and Bento paper (again except for the probability weighting here). This is a consequence of  $\overline{P}$  only having an income effect on the intensive margin for car owners.

#### 3.2 Feedback effects

It will be useful to know how the demand for car transport changes as a function of the fixed fee. Since we have externalities in the model, we expect feedback effects to be present both on the intensive margin and on the extensive margin in the demand for private transport.

Evaluating the change in demand when the fixed fee changes and the revenue is recycled through  $t^c$  we find by using (36) that

$$\frac{d\widehat{Z}^{c}}{d\overline{P}} = \left(\frac{\partial \pi^{1}}{\partial \overline{P}} + \frac{\partial \pi^{1}}{\partial t^{c}} \frac{dt^{c}}{d\overline{P}} + \frac{\partial \pi^{1}}{\partial L^{c}} L^{\prime} \frac{d\widehat{Z}^{c}}{d\overline{P}}\right) Z^{c1} + \left(\frac{\partial Z^{c1}}{\partial \overline{P}} + \frac{\partial Z^{c1}}{\partial t^{c}} \frac{dt^{c}}{d\overline{P}} + \frac{\partial Z^{c1}}{\partial L^{c}} L^{\prime} \frac{d\widehat{Z}^{c}}{d\overline{P}}\right) \pi^{1}$$

$$(47)$$

which by manipulation gives

$$\frac{d\widehat{Z}^{c}}{d\overline{P}} = \frac{\frac{(\widehat{\partial}\pi^{1}}{\widehat{\partial}P}Z^{c1} + \frac{\widehat{\partial}Z^{c1}}{\widehat{\partial}P}\pi^{1}) + (\frac{\widehat{\partial}\pi^{1}}{\widehat{\partial}P}Z^{c1} + \frac{\widehat{\partial}Z^{c1}}{\widehat{\partial}P}\pi^{1}) \frac{dt^{c}}{d\overline{P}}}{(1 - L'(\frac{\widehat{\partial}Z^{c1}}{\widehat{\partial}P}\pi^{1} + \frac{\widehat{\partial}\pi^{1}}{\widehat{\partial}P}Z^{c1}))} = \frac{(\frac{\widehat{\partial}\pi^{1}}{\widehat{\partial}P} + \frac{\widehat{\partial}\pi^{1}}{\widehat{\partial}P}dt^{c})Z^{c1} + (\frac{\widehat{\partial}Z^{c1}}{\widehat{\partial}P} + \frac{\widehat{\partial}Z^{c1}}{\widehat{\partial}P}dt^{c})Z^{c1})}{(1 - L'(\frac{\widehat{\partial}Z^{c1}}{\widehat{\partial}P}\pi^{1} + \frac{\widehat{\partial}\pi^{1}}{\widehat{\partial}P}Z^{c1}))}$$

$$(48)$$

The numerator captures the effect of the change in the fixed fee had there been no externalities present in the model, since there is a direct response to the increase in  $\overline{P}$  and a response from the revenue recycling given by  $\frac{\partial \overline{Z}^c}{\partial t^c} \frac{dt^c}{d\overline{P}}$ . With externalities present a feedback effect is also present which is captured by the denominator. By assumption L'>0, and since we expect private transport to be a normal good, the increase in  $L^c$  will cause the generalized price to increase. We therefore have that  $\frac{\partial Z^{c1}}{\partial L^c} < 0$  and  $\frac{\partial \pi^1}{\partial L^c} < 0$  thus making the denominator exceeds 1. Since we normally also expect  $L'' = \frac{\partial L'}{\partial \overline{Z}}$  to be larger than zero, the feedback effect becomes larger when the congestion externality increases.

Furthermore, we see that the size of the feedback effect is determined by both the change on the intensive margin,  $\frac{\partial Z^{c1}}{\partial L^c}$ , and the change on the extensive margin,  $\frac{\partial \pi^1}{\partial L^c}$ .

## 3.3 Shifting from fixed to variable tax on cars

We now examine the effect of changing the fixed tax,  $\overline{P}$ , on the purchase of cars and financing this by raising the variable tax,  $t^c$ , on the use of cars. The welfare effect is given by

$$\frac{dW}{d\overline{P}} = \frac{\partial W}{\partial \overline{P}} + \frac{\partial W}{\partial t^c} \frac{dt^c}{d\overline{P}} + \frac{\partial W}{\partial L^c} L' \frac{dZ^c}{d\overline{P}}$$
(49)

The first term on the right hand side is the direct effect on welfare from the change in  $\overline{P}$ . The second term captures the revenue recycling effect that works through  $t^c$ . The last term captures the welfare effect of the change in the level of congestion.

To find  $\frac{dt^c}{d\overline{P}}$ , we now differentiate the government budget constraint (42) with regard to  $\overline{P}$ . Using  $\frac{dR}{d\overline{P}} = 0$  since we consider a budget neutral reform, we get

$$0 = \frac{d\pi^{1}}{d\overline{P}} \overline{P} + \pi^{1} + \frac{dt^{c}}{d\overline{P}} \widehat{Z}^{c} + t^{c} \frac{d\widehat{Z}^{c}}{d\overline{P}} + t_{w} \frac{d\widehat{L}^{w}}{d\overline{P}}$$

$$(50)$$

which by manipulation gives

$$\frac{dt^c}{d\overline{P}} = -\frac{\frac{d\pi^1}{d\overline{P}} \overline{P} + \pi^1 + t_c}{d\overline{P}} \frac{d\widehat{Z}^c}{d\overline{P}} + t_w} \frac{d\widehat{L}^w}{d\overline{P}}$$
(51)

Substituting this together with (44), (45), and (46) into (49) gives

$$\frac{dW}{d\overline{P}} = -\lambda^{M1} + \lambda^{M1} Z^{c1} \frac{\frac{d\pi^{1}}{d\overline{P}} \overline{P} + \pi^{1} + t_{c}}{\widehat{d}\overline{P}} \frac{d\widehat{Z}^{c}}{d\overline{P}} - \lambda^{T1} Z^{c1} L^{\dagger} \frac{d\widehat{Z}^{c}}{d\overline{P}}$$

$$(52)$$

Using the definition given in (36) together with (48) we can rewrite (52) as

$$\frac{dW}{d\overline{P}} = \lambda^{M1} \left( \frac{d\pi^{l}}{d\overline{P}} \frac{\overline{P}}{\pi^{l}} + \frac{d\widehat{Z}^{c}}{d\overline{P}} \frac{t_{c}}{\pi^{l}} + \frac{d\widehat{L}^{w}}{d\overline{P}} \frac{t_{w}}{\pi^{l}} \right) - \frac{\lambda^{T1}}{\lambda^{M1}} Z^{c1} L \cdot \frac{(\frac{\partial \pi^{l}}{\partial P} + \frac{\partial \pi^{l}}{\partial t_{c}} \frac{dt_{c}}{\partial P})Z^{c1} + (\frac{\partial Z^{c1}}{\partial \overline{P}} + \frac{\partial Z^{c1}}{\partial t_{c}} \frac{dt_{c}}{\partial P})\pi^{l}}{(1 - L(\frac{\partial Z^{c1}}{\partial t^{c}} \pi^{l} + \frac{\partial \pi^{l}}{\partial t^{c}} Z^{c1}))}$$

$$(53)$$

which simplifies to

$$\frac{dW}{d\overline{P}} = \lambda^{M1} \left( \frac{d\pi^{1}}{d\overline{P}} \overline{P} + \left[ t_{w} \frac{d\widehat{L}^{w}}{d\overline{P}} + \left\{ -\frac{\frac{\partial\widehat{Z}^{c}}{\partial\overline{P}} Z^{c} + \frac{\partial\widehat{Z}^{c}}{\partial t_{c}} \frac{dt_{c}}{d\overline{P}}}{1 - L^{c} \frac{\partial\widehat{Z}^{c}}{\partial I^{c}}} \right\} \left\{ \frac{\lambda^{T1}}{\lambda^{M1}} \widehat{Z}^{c} L' - t_{c} \right\} \right] \right)$$
(54)

Taking a closer look at this expression, we see that several effects affect the outcome of the proposed tax reform. The first term,  $\frac{d\pi^1}{d\overline{P}}\overline{P}$ , captures revenue effects coming from changes in the number of car owners. This effect is new compared to previous models since they do not capture the changes on the extensive margin. The expression in square brackets is structurally identical to formula 10 in Parry & Bento (2001). It is comprised of two terms. The first term,  $t^w \frac{d\widehat{L}^w}{d\overline{P}}$ , captures revenue effects coming from changes in labor market participation, since changes in labor supply will change the tax revenue collected from labor taxes. The second term in the square bracket is a bit more complex. The first part captures changes in the demand for car transport including both tax interaction effects (the second term in the numerator) and feedback effects (the denominator). We see that the feedback effect can be decomposed into a feedback effect from the intensive margin,  $L'Z^{c1}\frac{\partial \pi^1}{\partial E}$ , and a feedback effect from the extensive margin,  $L'\frac{\partial Z^{c1}}{\partial E}\pi^1$ . This last term is new compared to previous models since they only model intensive responces. The second part of this term describes the difference between the marginal external cost of transport,  $\frac{\lambda^{T1}}{\lambda^{M1}} \hat{Z}^c L'$ , and the variable tax on transport,  $t_c$ . The marginal external cost of transport includes an expression for the value of time savings for car owners,  $\frac{\lambda^{T1}}{\lambda^{M1}}$ , which we discussed in (12) and (13). Multiplying the value of time savings with the marginal

time lost due to congestion,  $\hat{Z}^c L'$ , gives the marginal cost of the externalities. The sign of this last parenthesis depends on the level at which  $t_c$  is set. If it is above the marginal social cost, the term is negative, and if it is set below marginal cost it is positive. In the special Pigouvian case where the tax on transport is equal to the marginal external costs we see that this term cancels out. Assuming that Pigouvian marginal cost pricing is implemented by letting  $t_c = \frac{\lambda^{T1}}{\lambda^{M1}} \hat{Z}^c L'$  we end up with a welfare effect given by

$$\frac{dW}{d\overline{P}} = \lambda^{M1} \left( \frac{d\pi^1}{d\overline{P}} \overline{P} + t_w \frac{d\widehat{L}^w}{d\overline{P}} \right) \tag{55}$$

which only includes revenue effects. That only revenue effects are relevant in this case is intuitive since the externalities from the congestion are internalized through  $t_c$ .

## 4. Numerical example

In this section we use simulation to illustrate how the welfare effects could change when a tax reform shifting taxes from fixed fees to variable taxes is implemented. To illustrate this, we make some assumptions concerning the functional form of the utility function.

To keep things simple we specify the two parts of the utility function given in (1) as the CES functions

$$U(L,Z) = (\beta_L L^{\frac{\sigma-1}{\sigma}} + \beta_T Z^{\frac{\sigma-1}{\sigma}})^{\frac{\sigma}{\sigma-1}}$$
(56)

$$u(Z^{b}, Z^{c}) = (\beta_{Z^{b}}(Z^{b})^{\frac{\sigma^{T}-1}{\sigma^{T}}} + \beta_{Z^{c}}(Z^{c})^{\frac{\sigma^{T}-1}{\sigma^{T}}})^{\frac{\sigma^{T}}{\sigma^{T}-1}}$$
(57)

where  $\sigma$  is the elasticity of substitution between leisure and consumption and where  $\sigma^T$  is the elasticity of substitution between the two transport modes.  $\beta_L$ ,  $\beta_Z$ ,  $\beta_{Z^b}$  and  $\beta_{Z^c}$ 

are share parameters. We also need to specify how the total level of car travel affects the time required for car transport. Letting  $L_{free}^c$  denote the free-flow time requirement for a trip by car and C the denote the road capacity, we choose one of the most commonly used speed-flow travel time functions

$$L^{c}(\overline{Z}) = L_{free}^{c}(1 + 0.15(\overline{Z})^{4})$$
 (58)

which was derived in Bureau of Public Roads (1964) and also used in Parry and Bento  $(2002)^{51}$ . Furthermore, the two types of households face the budget constraints given by (4) and (5), and finally the government faces the constraint given by (42).

#### 4.1 The benchmark case and calibration

The model split in the year 2000 between transport by car and other types of transport is shown in table 1. One problem with our model is that we do not allow non-car owners to commute by car but as we can see in table 1 around 9.7 % of the commuting trips done by non-car owners are by car. One explanation for this could be that people not owning a car have borrowed one or they share a ride with someone owning a car. In the calibration we assume that all commuting trips by non-car owners are done by public transport. For households having a car available around 63% of the commuting trips is done by car. From the Danish Transport Diary Survey we also know that the fraction of households now having access to a car in 2000 is approximately 21 % with 79 % having access to at least one car.

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 $<sup>^{\</sup>rm 51}$  A discussion of speed-flow functions can be found in Small (1992) chapter 3.

	Car owners	Non-car owners	Total
Use other transport	37.3 %	90.3 %	45.5 %
Use private transport	62.7 %	9.7 %	54.5 %

Table 1: Modal split on commuting trips (Source: Danish Transport Diary Survey).

Referring to (IRIS (1993)) and (Pollet (2000)) it is argued in De Borger and Van Dender (2003) that in Belgium around 67% of commuting trips are done by car. We see that for Denmark around 55 % of commuting trips is done by car (according to the Danish Transport Diary Survey). One possible explanation for this could be the 180% marginal tax on car purchase prices in Denmark and the fact that the car ownership ratio in Denmark is one of the lowest in EU (Dargay et al. (2006)).

In our benchmark scenario the modal split can be seen in table 2. The fraction of households owning a car (measured by the choice probabilities) is given by 68.5% and the fraction of households without car is 31.5%. In our model fewer households thus owns a car than the Danish Transport Diary shows. We assume this since we want the total modal split between car and other types of transport (our metro service) to be comparable. Initially we have a model split as shown in table 2. Here 24% of car owners use some other modes of transport than car on commuting trips and households not owning a car only goes by other modes (e.g. metro). The total modal split is comparable to the modal split in table 1 with 56.6% of commuting trips done by car and 43.4% of commuting trips done by some other mode.

	Car owners	Non-car owners	Total
Use other transport	24.0%	100.0%	43.4%
Use private transport	76.0%	0.0%	56.6%

Table 2: Modal split on commuting trips in base scenario.

The model is calibrated so that in the benchmark case the car ownership elasticity with regards to fixed cost of purchasing a car is -0.20 which is numerically higher than the estimates from Dargay (2001) where a long term elasticity of -0.11 is found<sup>52</sup>. Fosgerau et al. (2004) estimate a short term elasticity of -0.19 and a long term elasticity of -0.48 for Denmark. With the high tax on cars in Denmark we would expect the elasticity to be numerically higher in Denmark than in the UK which is also confirmed by the findings in Fosgerau et al. (2004). The car ownership elasticity with regards to variable transport cost (i.e. fuel price) is our benchmark case -0.19 which is numerically higher than those found in Dargay (2002) where the long run fuel cost elasticities is found to be between -0.14and -0.06. Fosgerau et al. (2004) again reports values for Denmark which are numerically higher than those reported by Dargay. They find a short term elasticity of -0.22 and a long term elasticity of -0.55 with regards to user cost. The car transport elasticity with regards to the fuel price in our baseline scenario is -0.17 which is in line with the fuel price elasticity of -0.14 found in Nielsen (2006c) and Dargay (2004) find a long run fuel cost elasticity for car travel to be between -0.18 and -0.10. Fosgerau et al. (2004) find the short term elasticity with regards to running costs of car use to be -0.30 and a long term elasticity to be -0.37. The car transport elasticity with regards to car purchase cost is in

<sup>&</sup>lt;sup>52</sup> In Dargays paper the elasticity depend on the number of cars per household. The elasticity of -0.09 applies when the number of cars per household is 1.2. If the number of cars is 1.0 the elasticity is found to be -0.11 and if the number of cars per household is 1.5 the elasticity is -0.07.

our model -0.08. This is numerically much lower than the findings in Nielsen (2006c) where a long run elasticity of -1.13 is found for the entire country. Dargay (2004) find long run elasticities between -0.62 and -0.43 which is closer to our value than the finding in Nielsen (2006c). Since the values from Dargay (2004) and the value of -1.13 in Nielsen (2006c) is found on a national level it is interesting that Nielsen (2006c) also examines rural and urban households separately. He finds an insignificant effect of the car purchase price on the travel demand in urban areas. This finding would call for a very low elasticity to apply for these and we can thus se our model as being calibrated to fit a situation in for example Copenhagen where a metro is also present. The labor supply elasticity is chosen to be 0.5 which is in line with the reported values in Fuchs et al. (1998).

In the situation where no congestion exists we have specified the speed-flow function (58) such that the use of public transport takes 50% longer than transport by car. In the basic scenario congestion exist and the use of public transport only takes 20% longer than car transport. Furthermore we have specified the tax system so that a very high tax exists on the purchase of cars (in our model 180%) and a fairly high tax exists on the use of cars (in our model 100%). Furthermore car owners supply approximately 7.8 hours of work per day and those who do not own a car supply approximately 5.9 hours of work per day with an average working day of 7.2 hours in the economy. With an official working week of 37 hours the average working time for Denmark is 7.4 hours and since we do expect car owners to work more than those households who do not own a car, we find our model to be well specified with regards to the time allocation. Using (12) and (25) to derive the

value of time for car owners and non-car owners we find that  $VoT^0$  is approximately equal to 48% of the gross wage and  $VoT^1$  is approximately equal to 64% of the gross wage. Expecting the value of time to be close to the after-tax wage rate we find these values to be reasonable since the tax on labor in the model is 50% and since we expect car owners to have a higher value of time than people not owning a car.

#### 4.3 Shifting from fixed to variable taxes on cars

Labeling our base scenario as SC0 we now analyze two tax reforms (SC1 and SC2) involving a reduction of the fixed tax on the purchase of cars in a budget-neutral way. The first reform (SC1) reduces the tax on the purchase of cars from 180% to 170% financing it by increasing the tax on the use of cars. The second reform (SC2) reduces the taxation of cars to 100% again financing it by increasing the tax on the use of cars. The modal split before and after the reforms can be found in table 3.

	SC0	SC1	SC2
Car owners (% of population)	68.5 %	68.6 %	68.8%
Use private transport (car)	76.0 %	75.8 %	74.2%
Use public transport (metro)	24.0 %	24.2 %	25.8 %
Non-car owners (% of population)	31.5 %	31.4 %	31.2%
Use other transport	100.0 %	100.0 %	100.0 %
Use private transport	0.0 %	0.0 %	0.0 %
Total modal split			
Use private transport (car)	56.6 %	56.5 %	55.3%
Use public transport (metro)	43.4 %	43.5 %	44.7%

Table 3: Modal split before and after the tax reforms

We see that more households purchase a car which is expected since the purchase price goes down. We also see that the modal split shifts away from the use of cars to other modes. If the elasticity was higher in the model we could end up in a situation where both the number of car owners and the use of cars increase thus reversing the effect on the modal split. The tax reforms also affect other parts of the economy. The changes in the tax system, the change in labor supply, and the change in welfare is shown in table 4.

	SC1	SC2
Taxes		
Change in car purchase tax (relative to SC0)	-5.56%	-44.44%
Change in tax on car use (relative to SC0)	7.20%	59.20%
Effect on travel time by car (relative to SC0)	-0.40%	-2.40%
Effect on the labor supply		
For non-car owners (relative to SC0)	0.00%	0.00%
For car owners (relative to SC0)	-0.10%	-0.85%
Total effect on labor supply (relative to SC0)	-0.07%	-0.58%
The welfare effect (relative to SC0)	0.005%	0.034%

Table 4: Changes in the tax system, travel time by car, and labor supply.

What we see from table 4 is that the reforms reduce the travel time by car. Furthermore the effect on labor supply is interesting since our model shows that implementing a tax reform like the one proposed here will reduce labor supply. This finding is in line with results found elsewhere since increased taxation of commuting reduce the value of

working and thus discourage the supply of labor. Had the model not incorporated the discrete nature of car ownership decision the conclusion could have been that the reduction in labor supply would be closer to 0.85% but since we have households who change car-ownership status and thus starts supplying more labor since car owners work more., the reduction in labor supply could have been higher.

We also see that the tax reform actually improves welfare. It is however not clear if welfare will continue to increase if the fixed tax is reduced even further. In order to examine this we simulate a series of reforms which gradually reduce the fixed tax from 180% to 20%. The change in welfare when the taxes changes are shown in figure 1, figure 2. In the figures the full line comes from the model and we also show best fit of a high degree polynomial approximation to these results.

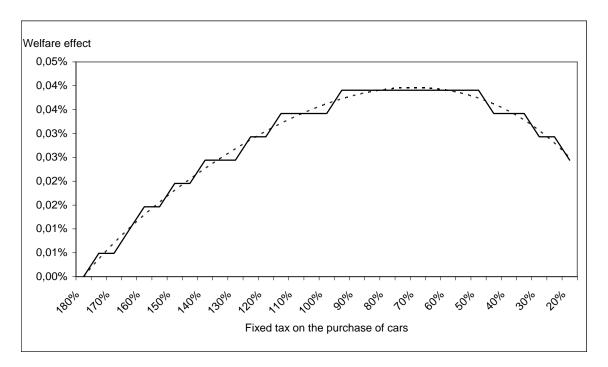


Figure 1: Change in welfare at different levels of the fixed tax relative to the welfare when the tax is 180%.

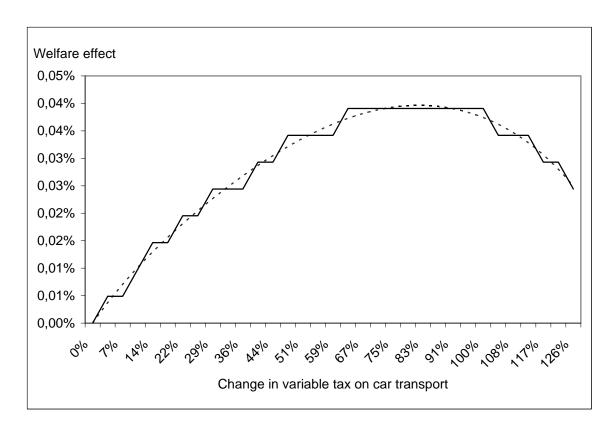


Figure 2: Change in welfare when the tax on car transport is increased relative to the tax in SC0.

Figure 1 shows that as the tax on purchase of cars decrease the welfare increase to a point around 70% at which point we can see in figure 2 that the variable tax have increased with around 85% (to approximately 190% in our model). One could speculate if this external damage cost in this point is below, above or exactly equal to the tax rate imposed. To examine this we specify the model in such a way that the variable tax is given by  $t_c = \frac{\lambda^{T1}}{\lambda^{M1}} \hat{Z}^c L^t$ , keeping the budget balanced, and adjusting the fixed tax accordingly. By doing so we find that a variable tax of 199% is internalizing the externalities and the resulting fixed tax will be set equal to 50%. Using the model to examine how welfare changes around this 'pigouvian point' we conclude that the

implementation of pigouvian taxation is the optimal choice if a tax reform which shifts taxes from purchase of cars to use of cars are to be implemented.

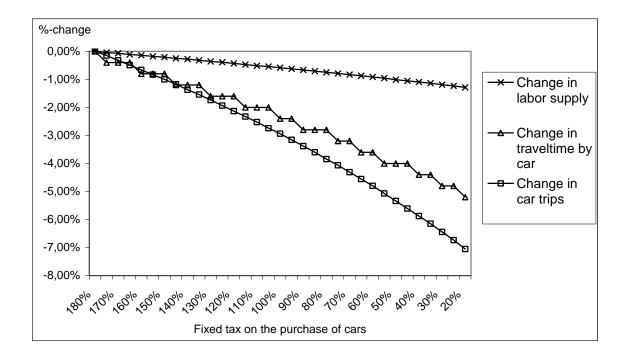


Figure 3: Change in some key parameters relative to SC0 when the fixed tax is reduced.

The effect of such reform on other key variables can be seen in table 3. As in figure 1 we depict the effect of the reduced purchase tax. We see that if we reduce this to 50% (the pigouvian case) the labor supply will fall by approximately 1% and the travel time on car trips will be reduced with around 4.5%. The number of car trips falls with approximately 5%.

# 5. Concluding remarks

This paper shows that the decision on the extensive margin (e.g., of car ownership), is an important element in welfare evaluation of tax reforms in the transport sector. Omission

of decisions on the extensive margin could therefore be critical. Using a simple model for household decisions, taxation, and discrete choice, we show how the feedback effect as well as the welfare effect depends on the ownership decision and on the interaction with the labor supply decision. We illustrated the effects in a small numerical model showing that if we choose to implement a balanced budget tax reform which shifts taxes from car ownership (or purchase) to car use it is optimal to implement pigouvian taxation.

Several extensions to this model can be envisioned. Including only commuting is not satisfactory and leisure trips ought to be included. This extension could possibly be implemented following the ideas of De Borger and Van Dender (2003). One could also wish for more than two types of transport with different levels of fixed taxes to be allowed for. This extension is straight forward to implement in the discrete setup presented here and it would probably not influence the theoretical results. The simulation used here should also be improved by calibrating it toward a specific city with specifically obtained parameters and tailor made speed-flow functions for different types of transport. Interaction between different transport modes could also be wished for since congestion on the streets affects the travel time by bus but probably still leaves a metro system unaffected. All these shortcomings will be left for future research and we still believe that the findings of this paper will benefit future modeling of transport tax reforms.

### References

Becker, G. (1965) A Theory of the Allocation of Time, *The Economic Journal* **75**, 493-517.

Ben-Akiva, M. and Lerman, S. R (1985) *Discrete Choice Analysis: Theory and Application to Travel Demand*, The MIT Press, Cambridge, Massachusetts.

Traffic Assignment Manual, Urban Planning Division, US Department of Commerce, Washington DC.

Dargay, J., Hivert, L. and Legros, D. (2006) The dynamics of car ownership in E.U countries: a comparison based on the european household panel survey, *unpublished* working paper.

De Borger, B. (2000) Optimal two-part tariffs in a model of discrete choice, *Journal of Public Economics* **76**, 127-150.

De Borger, B. (2001) Discrete choice models and optimal two-part tariffs in the presence of externalities: optimal taxation of cars, *Regional Science and Urban Economics* **31**, 471-504.

De Borger, B. & Van Dender, K. (2003) Transport tax reform, commuting, and endogenous values of time, *Journal of Urban Economics* **53**, 510-530.

De Jong, G. C. (1990) An Indirect Utility Model of Car Ownership and Private Car Use, *European Economic Review* **34**, 971-985.

DeSerpa, A. C. (1971) A Theory of the Economics of Time, *The Economic Journal* **81**, 828-846.

DØRS (2006) Dansk Økonomi, forår 2006, Det Økonomiske Råd, www.dors.dk.

Fosgerau, M., Holmblad, M. and Pilegaard, N. (2004) ART – En aggregeret prognosemodel for dansk vejtrafik, *Notat* **5 2004**, Danish Transport Research Institute.

McFadden, D. (1974) The measurement of urban travel demand, *Journal of Public Economics* **3:4**, 303-328.

Nielsen, J. E. (2006a) Taxation, Time Allocation, and Externalities, *Thesis Paper*.

Nielsen, J. E. (2006b) Real estate ownership and the demand for cars in Denmark – A pseudo-panel analysis, *Thesis Paper*.

Nielsen, J. E. (2006c) Demand for car transport in Denmark – Differences between rural and urban households, *Thesis Paper*.

Forsyth, P. J. (1980) The value of time in an economy with taxation, *Journal of Transport Economics and Policy* **14:3**, 337-362.

Fuchs, V. R., Krueger, A. B. & Poterba, J. M. (1998) Economists' Views about Parameters, Values, and Policies: Survey Results in Labor and Public Economics, *Journal of Economic Literature* **36**, 1387-1425.

IRIS (1993) Region de Bruxelles-Capitale, Plan regional des Deplacements - Plan IRIS - Constat et Scenario Tendanciel, Region de Bruxelles-Capitale.

Jara-Díaz, S. R. (2003) On the goods-activities technical relations in the time allocation theory, *Transportation* **30**, 245-260.

Parry, I. W. H. & Bento, A. (2001) Revenue Recycling and the Welfare Effects of Road Pricing, *Scandinavian Journal of Economics* 103:4, 645-671.

Parry, I. W. H. & Bento, A. (2002) Estimating the Welfare Effect of Congestion Taxes: The Critical Importance of Other Distortions within the Transport System, *Journal of Urban Economics* 51, 339-365.

Pollet, I. (2000) Nationale Enquete over Mobiliteit van Huishoudens---Verslag van de Eerste Analyse, Langzaam Verkeer, Leuven.

Small, K. A. & Rosen, H. S. (1981) Applied welfare economics with discrete choice models, *Econometrica* **49**, 105-130.

Small, K. A. (1992) *Urban Transport Economics*, Fundamentals of Pure and Applied Economics Series, Harwood Academic Press, Philadelphia, USA.

Train, K. E. (2003) *Discrete Choice Methods with Simulation*, Cambridge University Press.

Van Dender, K. (2003) Transport Taxes with Multiple Trip Purposes, *Scandinavian Journal of Economics* **105:2**, 295-310.