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Macroeconomic transmission mechanisms in a non-stationary world: Evidence from I(1) and I(2) cointegrated VAR models

by

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Academic advisors: Katarina Juselius and Heino Bohn Nielsen

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Abstract

This thesis consists of four chapters plus introduction and summary. The purpose of the thesis is to identify a set of broad empirical regularities which can be interpreted in light of economic theory. The analyses presented here are thus meant to generate 'refined stylised facts' rather than to provide strict statistical testing of economic models. The results of the different chapters may in many cases be used to suggest modifications of theory. In this sense, the aim of thesis is primarily to generate new empirically relevant hypotheses rather than to formally test existing ones.

The methodology employed throughout is the cointegrated VAR (CVAR) model which is used within different modelling frameworks to study macroeconomic transmission mechanisms. Economic theory is consistently used to guide identification of the statistical models but it is shown that a trade-off between economic identification and statistical significance often arises.

The thesis was originally motivated by a wish to understand price dynamics in the euro area and to assess the ability of the European Central Bank to control inflation. Chapter 1, *A multi-sector model of policy transmission and inflation dynamics in the euro area*, is a general-to-specific study based on the methodology of Juselius (2006). From a set of disequilibria measures derived from I(1) CVAR models of different economic sectors, I find that euro-area inflation has been determined by a range of factors, notably product-market competition and international financial-market conditions, both of which remain outside central-bank control. Overall, inflation appears to have been driven mainly by supply factors rather than by excess demand.

Chapter 1 highlights the importance of interactions of monetary and fiscal policy but also points to the existence of an I(2) component in the public debt-to-GDP ratio. Chapter 2, *Interactions of monetary and fiscal policy: An I(2) cointegrated VAR study of deficit-debt dynamics in the euro area*, thus provides an in-depth analysis of the public sector in the euro area and argues that the special nature of deficit-debt dynamics calls

for an I(2) analysis. In contrast with the conventional prediction, I find evidence of two nominal trends: one which stems from shocks to monetary policy (the short rate), and one which arguably arises from shocks to bond markets' view of fiscal policy (the long rate). This provides further evidence that the nominal anchor has not been provided solely by the monetary authorities.

Together, the findings of the first two chapters suggest that central banks have had little influence on long-term interest rates and consumer-price inflation alike in recent decades. But, while the prices of goods and services have been kept down by globalisation-induced competition, the prices of other assets have surged. In Chapter 3, *Has excess global liquidity fuelled asset prices? An I(2) cointegrated VAR study of bubble dynamics* (with *Julia Giese*), we argue that failure of long-run price homogeneity could be a signal of asset-price bubbles. Results from an I(2) CVAR in globally aggregated variables point to the existence of asset-price bubbles in the sample period, likely induced by an abundance of liquidity, prior to, for example, the recent credit crisis. We find that both house and share prices have had a tendency to rise in response to excessively low policy rates. But, whereas house-prices have been fuelled by excess money supply, this is not the case for goods and share prices.

In today's highly integrated markets, the burst of a bubble in any one country could be transmitted quickly to other regions. Chapter 4, *Asset prices and liquidity spill-overs: A global VAR perspective* (also with *Julia Giese*) develops a global VAR (GVAR) model in order to study the transmission of shocks between the US, the UK, the euro area and Japan. Impulse response analysis suggest that stock markets move largely in sync across regions whereas this is not necessarily the case for housing markets. We show that the stability of the GVAR is sensitive to the choice of cointegration ranks and/or to deviations from long-run price homogenenity.

In sum, the thesis provides a consistent set of evidence against the existence of a clear link between the short- and the long-term interest rate. As a result, monetary policy seems to have had little control over inflation in recent decades, and the apparent success of inflation-targeting central banks has been greatly aided by downward pressure on prices induced by globalisation. The results presented here taken alongside events during the current financial and economic crisis suggest that it may be beneficial for policy makers to focus less on the prices of goods and services and direct more attention towards the prices of assets.

Resumé

Denne afhandling består af fire kapitler samt indledning og sammenfatning. Formålet med afhandlingen er at identificere et sæt af empiriske regulariteter og fortolke disse i lyset af økonomisk teori. Analyserne er således beregnet på at generere en række 'refined stylised facts' snarere end som sådan at teste økonomiske modeller. Resultaterne kan derfor i mange tilfælde benyttes til at foreslå modifikationer af foreliggende teori. Afhandlingen søger på denne måde primært at generere nye empirisk relevante hypoteser fremfor formelt at teste eksisterende teori.

Et gennemgående træk i alle kapitler er anvendelse af den kointegerede VAR (CVAR) model indenfor forskellige modelrammer med henblik at kortlægge en række makroøkonomiske transmissionsmekanismer. Økonomisk teori benyttes konsekvent til at guide identifikation af de statistiske modeller, men en afvejning af økonomisk identifikation og statistisk signifikans viser sig ofte påkrævet.

Afhandlingen var oprindelig motiveret af et ønske om at modellere inflationsdynamik i Euro-området med henblik på at undersøge, i hvilket omfang Den Europæiske Centralbank har kontrol over prisdannelsen. Kapitel 1 er således et 'general-to-specific'-studie baseret på en metodologi foreslået af Juselius (2006). I papiret analyseres en række økonomiske sektorer, og afvigelser fra de identificerede kointegrationsrelationer benyttes som mål for uligevægte i forskellige markeder. Resultaterne viser, at inflationen i de seneste årtier i høj grad har været bestemt af faktorer udenfor centralbankens kontrol, såsom øget konkurrence på verdensmarkedet og betingelserne på de finansielle markeder. Overordnet set, synes inflationen i vid udstrækning drevet af udbuds- snarere end af efterspørgselseffekter.

Multi-sektor analysen i Kapitel 1 understreger dels vigtigheden af interaktion mellem de penge- og finanspolitiske myndigheder og dels eksistensen af en I(2) komponent i offentlig gæld/BNP-variablen. Kapitel 2 giver på denne baggrund en detaljeret analyse af den offentlige sektor i euro-landene og argumenterer for, at dynamikken mellem budgetunderskud og offentlig gæld kræver en I(2) analyse. I modsætning til den teoretiske

forudsigelse viser modellen, at der forefindes to nominelle trende: én, der stammer fra pengepolitik (choks til den korte rente), og én, der synes at have sin oprindelse i obligationsmarkedernes syn på finanspolitisk holdbarhed (choks til den lange rente). Dette indikerer, at det nominelle anker for økonomien ikke udelukkende sættes af centralbanken.

Resultaterne i de to første kapitler peger i retning af, at ECB har haft ringe kontrol over såvel inflation som de lange renter. Mens priser på varer og serviceydelser har været holdt nede af øget konkurrence som følge af globaliseringen, har verdensøkonomien dog oplevet en markant stigning i hus- og aktiepriser. I Kapitel 3 (skrevet i samarbejde med Julia Giese) argumenterer vi for, at afvisning af langsigts-prishomogeneitet kan være et signal om prisbobler. Resultater fra en I(2) CVAR model baseret på globalt aggregerede variable viser tegn på bobler på såvel bolig- som aktiemarkedet i perioden og disse synes at være opstået som følge af overskydende likviditet. Mens huspriserne har haft en tendens til at stige som følge af ekstraordinær høj kreditgivning, så er dette ikke i samme grad tilfældet for vare- og aktiepriser. Både bolig- og aktiemarkedet har dog haft en tendens til at se prisstigninger som en følge af ekspansiv pengepolitik.

Analysen i Kapitel 3 peger på, at aktivprisbobler har været til stede i de seneste årtier, men hvad sker der når en boble brister? De finansielle markeder er i dag tæt forbundne, men samtidig er økonomisk vækst mindre geografisk koncentreret end tidligere. I Kapitel 4 (ligeledes med Julia Giese) konstruerer vi en global VAR (GVAR) model med henblik på at undersøge, i hvor høj grad choks til den amerikanske økonomi påvirker resten af verden. Resultaterne viser, at mens bevægelser på aktiemarkederne i høj grad er synkroniserede på tværs af lande, er dette ikke i samme udstrækning gældende for boligmarkederne. Vi finder desuden, at stabiliteten af GVAR-proceduren er følsom overfor afvigelser fra langsigts-prishomogeneitet samt overfor hvordan kointegrationsrangen sættes.

Samlet set, indeholder afhandlingen en konsekvent afvisning af den konventionelle hypotese om at den korte og den lange rente er nært forbundne. Dette indebærer, at inflationen i høj grad synes at have været udenfor pengepolitisk kontrol i de seneste årtier. Centralbankerne har fået betydelig hjælp til at opfylde deres inflationsmål fra et vedvarende nedadgående pres på vareprisерne som følge af øget konkurrence på verdensplan. Sammenholdt med begivenhederne under den nuværende finansielle og økonomiske krise, henstiller resultaterne her til, at centralbankerne med fordel kan koncentrere sig mindre om vareprisinflation og i større udstrækning end hidtil rette fokus mod prisudviklingen på aktie- og boligmarkederne.

Contents

Preface

This thesis was written in the period August 2004 - August 2009 while I was enrolled as a PhD student at the Department of Economics, University of Copenhagen. I am indebted to my Ph.D. advisors, Katarina Juselius and Heino Bohn Nielsen, for always taking their time for questions and discussions. Heino has provided invaluable guidance on many aspects of both applied and theoretical econometrics. I remain sincerely grateful to Katarina for being a continuous source of inspiration and support from the day I came to her with only a vague idea for a PhD subject, during research ups and downs, and to this day.

From September 2006 - June 2007 I visited Nuffield College, University of Oxford. I am thankful to Bent Nielsen for all his help during my stay and to David Hendry for inviting me to take part in his weekly PhD student meetings; both have helped shape my views and work in important ways. My time at Nuffield spurred a collaboration with Julia Giese, and I am very appreciative of having her as both a co-author and a friend. Financial support from August Christgaus mindefond, Augustinus fonden, British Import Union, Codans medarbejderfond, Etly og Jørgen Stjerngrens fond, Euroclear legatet, Knud Højgaards fond, Nordea Danmark fonden, Oberstløjtnant Max Nørgaard og Hustru Magda Nørgaards legat and Oticon fonden made my stay in Oxford possible and is gratefully acknowledged.

From July 2007 - June 2008 I was on working leave, spending two months in the Systemic Risk Assessment Division, Financial Stability, at the Bank of England, and ten months at Lehman Brothers Global Economics in London. I am grateful to the Macro Shocks team at the Bank and to my former colleagues at Lehman for invaluable exposure to ‘real-world’ economic forecasting, and to Katarina for understanding my call for a non-academic experience amid my studies.

Finally, thanks to my fellow students and friends in Århus, Copenhagen, London and Oxford for making everyday life so much more enjoyable. A special thanks to my parents for always being there, and to my brother, Anders, for sharing experiences in rather dissimilar research areas. And a very special thanks to Dan for being my daily fixed point throughout this process, for always listening - and for LaTeX assistance.

Needless to say, none of the above people bear the least of responsibility for any remaining errors or omissions in the thesis.

Christin Kyrme Tuxen, August 2009

Introduction and summary

This thesis, *Macroeconomic transmission mechanisms in a non-stationary world: Evidence from I(1) and I(2) cointegrated VAR models*, consists of four self-contained manuscripts which broadly reflect the research areas I have been working on during my time as a PhD student. The chapters can be read independently but are related as they reflect that the answer to one research question in turn suggested new issues to be investigated.

The purpose of the thesis is to identify a set of broad empirical regularities which can be interpreted in light of economic theory. The analyses presented here are thus meant to generate 'refined stylised facts' rather than to provide strict statistical testing of economic models. The results of the different chapters may thus in many cases be used to suggest modifications of theory. In this sense, the aim of thesis is primarily to generate new empirically relevant hypotheses rather than to formally test existing ones.

Common to all chapters is applications of the cointegrated VAR (CVAR) model (see Johansen (1996) for an overview) and the use of this as a tool for structuring the information in non-stationary data in an economically meaningful way. The CVAR is incorporated into different frameworks in each chapter. The study of Chapter 1 combines a set of I(1) CVAR models of different economic sectors to form a multi-sector model, Chapters 2 and 3 translate economic models to I(2) CVAR space, and, finally, Chapter 4, combines a set of I(1) CVAR models of different countries to form a global VAR (GVAR) model. In terms of applications, the first two chapters are concerned with macroeconomic transmission mechanisms in the euro area whereas the final two chapters consider the dynamics of global asset markets.

Another general feature is the use of an identification strategy where we start from a theoretical model based on economic theory and re-formulate this to test some of its implications in CVAR space. The proposed long-run relations are then used as a guide for identification. Any restrictions are, as far as possible, imposed on the models only insofar as they comply with Hendry (1980)'s three golden rules of econometrics: "test, test, test". Notwithstanding, it proved reasonable in some cases to impose theoretical restrictions, such as long-run price homogeneity, although rejected based on the sample, on the grounds that they must hold in the (very) long run. We can then observe the implications of deviations from equilibrium on the rest of the system. Although the methodology used throughout the thesis falls predominantly within the 'general-to-specific' category, it turned out useful in some settings to exploit the invariance of the cointegration concept to extensions of the information set; i.e. start from a small set of variables, add new ones gradually, and observe how the model changes.

Finally, a recurring issue is how to bridge theory and data. As pointed out by Hoover, Johansen, and Juselius (2008), the notion of general equilibrium implies that a persistent departure from a partial-equilibrium relation must generate a similarly persistent disequilibria elsewhere in the economy. From a statistical point of view, this means that by considering deviations from equilibrium jointly, we can obtain stationarity at a higher level via cointegration. Economically, deviations from a theoretical relation can arise for (at least) two reasons. A temporary, but possibly persistent, deviation indicates a short-run disequilibrium, for example, the formation of a speculative bubble that will eventually burst. A permanent deviation suggests that theory must be revised, for example, that we should re-consider our specification of any trade-off between inflation and unemployment. It is seldom an easy task to separate the two cases however; the present thesis contains examples of treating deviations from theory both as temporary and as permanent.

Initially, the thesis was motivated by a wish to understand price dynamics in the euro area and to assess the ability of the European Central Bank to control inflation. Chapter 1, *A multi-sector model of policy transmission and inflation dynamics in the euro area*, is a general-to-specific study based on the methodology of Juselius (2006). I use a multi-sector modelling (MSM) approach based on the I(1) CVAR to model the transmission of policy in the euro area, focusing in particular on inflation dynamics. The paper includes a joint analysis of disequilibria in the public sector, the financial market, the external sector and the labour market alike. A set of checks to verify the robustness of the procedure to alternative specifications is proposed, and I show how to distinguish the short-, medium- and long-run dynamics using the MSM framework. The results suggest that prices have exhibited Phillips-curve effects in both the short and the medium run when a non-constant NAIRU is taken into account. In the medium run, unemployment has been driven by the need for euro-area producers to restore external competitiveness; in face of a rigid real wage, improvements in labour productivity appear to have been facilitated by lay-offs. In the long run, the Phillips curve is found to be vertical at a NAIRU level which varies with the real bond rate, and inflation largely seems to have been determined by factors outside central-bank control such as increased product-market competition and financial-market conditions, both of which are found to be exogenous. Notably, supply factors rather than excess demand appear to have been crucial in driving inflation in this period.

The multi-sector analysis in Chapter 1 points to the importance of monetary and fiscal policy interactions but also highlights the existence of an I(2) component in the public debt-to-GDP ratio. Chapter 2, *Interactions of monetary and fiscal policy: An I(2) cointegrated VAR study of deficit-debt dynamics in the euro area*, thus provides an in-depth analysis of the public sector in the euro area and argues that the special nature of deficit-debt dynamics calls for an I(2) analysis. In particular, the focus of the paper is on assessing the dynamic effects of public debt on bond yields. I show that the time-series persistency of deficits and debt over the sample period implies that an I(2) model is required to appropriately characterise the dynamics of government variables. A small economic model of policy interactions is translated into a set of polynomially cointegrating relations and an I(2) CVAR is estimated to test its empirical coherence. With some modifications, I am able to identify relations in the data which can be given broadly similar interpretations as the theoretical ones. In contrast with the standard prediction however, the results point to the existence of two I(2) trends, arising from shocks to the short rate and the long rate, respectively. This suggests that the nominal anchor of the economy has not been provided solely by monetary policy; fiscal policy has played a role as well. Together, the identified cointegration and common-trends structures suggest that a vicious spiral of rising public debt, bond yields and unemployment has been at play during the integration process leading up to the establishment of the European Monetary Union.

The findings of Chapter 1 and 2 suggest that central banks have had little influence on long-term interest rates and consumer-price inflation alike in recent decades. But, although the prices of goods and services have been kept down by globalisation-induced competition, the prices of other assets have surged. In Chapter 3, *Has excess global liquidity fuelled asset prices? An I(2) cointegrated VAR study of bubble dynamics*, which is written jointly with *Julia Giese*, we use the I(2) CVAR model to study the relationship between asset prices and liquidity on a global scale. Starting from a small New-Keynesian model, we propose a set of long-run relations, which allow both the price of liquidity (interest rates) and the quantity (money supply) to potentially affect house and share prices. Among the variables of the model, we find strong evidence of two I(2) trends, arising from twice cumulated shocks to the short and the long rate, respectively. As a result, long-run price homogeneity is rejected and we argue that this could be a first sign of bubbles. Imposing homogeneity on the polynomially cointegrating relations, albeit rejected in-sample, we are able to study the effects of different types of disequilibria and thus the dynamics of asset-price bubbles. We find that a key fundamental of asset prices, output, is excluded from the cointegrating relations for both house and share prices and argue that this may be a second sign of bubbles. Both house and share prices have had a tendency to rise in response to excessively low policy rates, but whereas house-price inflation is fuelled by excess money supply this is not the case for goods and share prices.

The results of Chapter 3 point to the existence of asset-price bubbles on a global scale, likely induced by an abundance of liquidity, prior to the credit crisis. In today's highly integrated markets, the burst of a bubble in any one country could be transmitted quickly to other regions. Chapter 4, *Asset prices and liquidity spill-overs: A global VAR perspective*, is also written jointly with *Julia Giese*. The paper develops a GVAR model to study the transmission of shocks between the US, the UK, the euro area and Japan. We first estimate I(1) CVAR models for the different countries/regions including a set of rest-of-the world variables in each, so-called CVARX* models, to take account of first-round effects of shocks. The set of foreign variables is chosen to reduce dimensionality, yet allowing for the presence of some key international parity relations. We are able to recover a set of economically meaningful relations but the country models all have large roots when the rank is set according to the economic prior. This points to the existence of temporary, yet persistent disequilibria during the sample, likely a result of asset-price bubbles. Lowering the rank when linking the country models ensures that the GVAR is stable and thus that we can conduct impulse response analysis which takes second-round effects of shocks into account. This allows us to assess the dynamic effects of liquidity and asset-price shocks. Shocks analysis shows that stock markets have a tendency to move in sync across regions whereas this is not necessarily the case for housing markets. For simulations of the credit crunch, we argue that the GVAR should be used with care however.

An empirical finding which shows up consistently throughout the thesis is a breakdown of the link between the short- and the long-term interest rates predicted by conventional economic theory. This, combined with the finding that prices have been driven mainly by supply factors, means that central banks have not been able to control the nominal side of the economy in recent years. Hendry and Mizon (2009) note that since inflation is observed to be non-stationary, partly as a result of regime shifts, it is unlikely that a single policy instrument is capable of controlling prices. Indeed, the apparent success of inflation-targeting central banks in past decades has been greatly aided by downward pressure on prices from globalisation. The results presented here taken alongside events during the current financial and economic crisis thus suggest that economists have become too fixated on the prices of goods and services while neglecting the prices of other assets. As a result, it may be beneficial to allow central banks more flexibility in responding to fluctuations in economic activity and asset prices. In today's highly integrated world economy, it is however likely to be the global stance of monetary policy that matters, calling for a large degree of coordination across countries. Finally, as proposed by Colander, Föllmer, Haas, Goldberg, Juselius, Kirman, Lux, and Sloth (2009), the development of 'early-warning schemes' that can detect the formation of bubbles is an important issue for research in the future and would be a vital tool for policy makers. Evidence from CVAR models could provide useful contributions in this respect.

References

Colander, D., H. Föllmer, A. Haas, M. Goldberg, K. Juselius, A. Kirman, T. Lux, and B. Sloth (2009). The financial crisis and the systemic failure of academic economics. *Working Paper, University of Copenhagen. Department of Economics 09-03.*

Hendry, D. F. (1980). Econometrics: Alchemy or science? *Economica 47.*

Hendry, D. F. and G. E. Mizon (2009). Economic policy in a rapidly changing world. *Manuscript.*

Hoover, K. D., S. Johansen, and K. Juselius (2008). Allowing the data to speak freely: The macroeconometrics of the cointegrated vector autoregression. *American Economic Review 98*(2), 251–255.

Johansen, S. (1996). *Estimation and Hypothesis Testing of Cointegration Vectors in Gaussian Vector Autoregressive Models* (2nd ed.). Oxford University Press, Oxford.

Juselius, K. (2006). *The Cointegrated VAR Model - Methodology and Applications.* Oxford University Press, Oxford.

Chapter 1

A multi-sector model of policy transmission and inflation dynamics in the euro area

A multi-sector model of policy transmission and inflation dynamics in the euro area

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Abstract

We use a multi-sector modelling (MSM) approach based on the cointegrated VAR to model the transmission of policy in the euro area, focusing in particular on inflation dynamics. The paper extends the 'general-to-specific' procedure of Juselius (2006) to include a joint analysis of disequilibria in the public sector (fiscal policy), the financial market (monetary policy), the external sector and the labour market alike. We also propose a set of checks to verify the robustness of the procedure to alternative specifications and show how to distinguish the short-, medium- and long-run dynamics by means of the MSM. Our results suggest that prices exhibit Phillips-curve effects in the short and the medium run when a non-constant NAIRU, which varies with the cost of capital, is taken into account. In the medium run, unemployment is driven by the need for euro-area producers to restore external competitiveness. In the long run, the interest-rate augmented Phillips curve is found to be vertical and inflation appears to have been driven mainly by supply-side factors rather than by excess demand. This suggests that central banks have exerted little control over prices in past decades.

Keywords: Time-Series Models, Price Level, Inflation; Deflation, Financial Markets and the Macroeconomy, Comparative or Joint Analysis of Fiscal and Monetary Policy; Stabilization

JEL Classification: C32, E31, E44, E63

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1 Introduction

Based on a 'general-to-specific' modelling approach, we construct a model of policy transmission mechanisms to study inflation dynamics in the euro area. The significance of up-to-date models of policy propagation became pertinent when the European Central Bank (ECB) assumed responsibility of monetary policy for the euro area in 1999. The primary objective of the ECB continues to be that of maintaining price stability, and its policy strategy is based on the recommendation of the modern monetary-economics literature that the best contribution central banks can make to economic prosperity is to keep inflation under control. As a result, smoothing of business cycles is left to the fiscal authorities.

An extensive literature deals with inflation determination and policy transmission in the euro zone, see *inter alia* Coenen and Vega (1999), Coenen and Wieland (2000), Fagan, Henry, and Mestre (2001), Smets and Wouters (2002) and Gerlach and Svensson (2003). Theoretical consistency of models is important for central banks in communicating their policy strategies but is often achieved by trading off empirical relevance. Most modern macro models derive inflation as a simple function of marginal costs but restricting price-determining factors to a few variables is at odds with the existence of a plethora of potential inflationary pressures (Surrey 1989). A range of papers in the recent special issue of the *Economics E-journal: Using Econometrics for Assessing Economic Models*, such as Bjørnstad and Nymoen (2008) and Juselius (2008), indeed show that important relationships commonly employed in the theoretical literature, for example the New-Keynesian Phillips curve (NKPC), obtain little empirical support.

What has then determined euro-zone inflation in past decades? Anecdotal as well as formal empirical evidence (see *inter alia* Pain, Koske, and Sollie 2006) point to a number of special mechanisms having been at play. Evidently, trade liberalisation and competition from low-wage economies have exerted downward pressure on consumer prices and wages in developed countries as producers and workers have had to compete worldwide for consumers and jobs, respectively. Moreover, de-regulation in trade and finance implies that markets are becoming ever more integrated across borders with important consequences for the determination of long-term interest rates and currency movements. Although these mechanisms are by now widely recognised by politicians, central bankers and financial market participants alike [1], few papers incorporate the effects of globalisation within

[1]See IMF (2006), Bordio and Filardo (2007), and a range of ECB speeches (for example Jean-Claude

a broader modelling framework.[2]

In this paper, we study inflation determination by searching for empirically relevant factors against the background of a range of potential price-driving mechanisms. Our starting point is the paper on the ECB's Area-Wide Model (AWM) by Fagan, Henry, and Mestre (2001) (henceforth FHM).[3] The AWM characterises inflation as a function of unit labour costs which, in turn, depend on capacity utilisation. Dynamic homogeneity is imposed on the model, albeit strongly rejected by the data, and thus the long-run Phillips curve is assumed to be vertical. The analysis of the euro-area labour market in Juselius (2003) supports the existence of a long-run relationship between prices and wages but it does however point to a different dynamic structure than that incorporated in the AWM. Specifically, Juselius finds that internal and external competitiveness alike have become increasingly important since the early 1980s and that this led to weaker unions, large-scale lay-offs and productivity adjustment as globalisation gained pace.

The Juselius (2003) study is however based on a small set of variables compared with the AWM. Notably, it has little to say about the relative effectiveness of monetary and fiscal policies in stabilising inflation and unemployment. In contrast, Hendry (2001) takes a multi-sector approach and models UK inflation as a function of excess demand in different markets quantified by equilibrium-correction terms. Using a mix of theoretical assumptions and single-equation cointegration techniques, Hendry concludes that inflation developments are inconsistent with any single-cause explanation. In this paper, we use an extension of the procedure suggested by Juselius (2006) who applied it to Danish data. This approach is based on deriving inflationary pressures from a range of markets using the cointegrated VAR (CVAR) model and is used to construct what we shall refer to as a multi-sector model (MSM) of transmission mechanisms. In allowing for both monetary and fiscal factors the approach taken here is essentially a simultaneous-equation extension of Hendry (2001). Other related studies developing disequilibria measures from different sectors include Juselius (1992) and Metin (1995).

Trichet: *Globalisation, inflation and the ECB monetary policy*, Barcelona, 14 February 2008, and José Manuel González-Páramo: *Globalisation and monetary policy*, Helsinki, 15 March 2007). These issues have also been addressed in numerous articles in the Economist and the Financial Times.

[2]The global VAR (GVAR) approach advocated by H. Pesaran and co-authors, see *inter alia* Dees, Holly, Pesaran, and Smith (2007), is a notable exception.

[3]The New Area-Wide Model (NAWM), a micro-founded open-economy (DSGE) model estimated by Bayesian methods, is under development at the ECB, see Christoffel, Coenen, and Warne (2008). The NAWM emphasises theoretical coherence, as opposed to empirical coherence, to an even larger extent than the original AWM.

Our modelling strategy is the following. We first identify cointegration relations within CVAR models for different markets/sectors of the economy, i.e. the public sector, the financial market, the external sector and the labour market, in order to locate potential inflationary pressures in each. We take deviations from the cointegrating relations as measures of disequilibria and combine these in order to study their joint effects on the variables of interest. This allows us to assess the effects and interactions of monetary and fiscal policy; we focus in particular on the dynamics of inflation. We also propose a set of robustness checks to verify the robustness of the procedure to alternative specifications and show how to distinguish dynamics at different time horizons using the MSM.

The results show that, in the long run, inflation has largely been determined by supply factors outside domestic control rather than by excess demand. We also find evidence that fiscal policy has distorted productivity to some degree and that monetary policy have been effective in steering economic activity in the short run. These results have important consequences regarding the appropriate mandates of monetary and fiscal authorities.

The rest of the paper proceeds as follows. Section 2 presents the MSM procedure and discusses its practical implementation. We present cointegration results for each of the four sectoral CVAR models in Section 4. Section 5 discusses the results of MSM; we focus on price dynamics and compare the explanatory power of the MSM inflation model with that of the AWM. In relation to these results, we naturally also discuss unemployment dynamics and the effectiveness of policy. Section 6 concludes.[4]

2 Methodology: a multi-sector model

We argue that the CVAR methodology provides natural point of departure for a multi-sector modelling approach; we then motivate the MSM procedure and propose a set of robustness checks.

2.1 The cointegrated VAR

Hoover, Johansen, and Juselius (2008) argue that in order to learn about empirically relevant mechanisms we must allow the data to speak freely, albeit guided by a theoretical conception of what we are looking for. In assessing which mechanisms have been important in driving the euro-area economy, a statistical model that validly approximate the

[4]Calculations were conducted using CATS 2.03 in Rats 7.2 (Dennis, Hansen, Johansen, and Juselius 2006), OxMetrics 5.0 (Doornik 2007b), and Autometrics 1.5 (Doornik 2007a).

data-generating process is thus vital. The majority of macroeconomic and financial time-series are found to be well approximated by unit-root processes and inference procedures should take this into account. Accurate modelling of the persistence in the data is a key feature of the CVAR model and the model provides a convenient way of separating the pulling (cointegration) forces from the pushing (common trends) forces in non-stationary data. Indeed, a close correspondence exists between basic economic and CVAR concepts, see Møller (2009). The basic structure of the I(1) CVAR is reviewed below.

We start from the p-dimensional VAR($k = 2$) in its equilibrium-correction model (ECM) representation,

$$\Delta \mathbf{x}_t = \mathbf{\Pi x}_{t-1} + \mathbf{\Psi}_0 \Delta \mathbf{z}_t + \mathbf{\Psi}_1 \Delta \mathbf{z}_{t-1} + \mathbf{\Gamma}_1 \Delta \mathbf{x}_{t-1} + \boldsymbol{\phi} \mathbf{D}_t + \boldsymbol{\varepsilon}_t, \qquad t = 1, 2, ..., T \quad (1)$$

where $\mathbf{x}_t$ is a $p \times 1$ vector of endogenous variables, $\mathbf{z}_t$ a vector of variables assumed weakly exogenous *a priori*, $\mathbf{D}_t$ a vector of deterministic components (some of which may be restricted to the $\boldsymbol{\alpha}$-space) and $\boldsymbol{\varepsilon}_t$ a $p \times 1$ vector of errors for which we assume $\boldsymbol{\varepsilon}_t \sim$ *iid* $N_p(\mathbf{0}, \mathbf{\Omega})$ with $\mathbf{\Omega} > 0$. In the statistical analysis we condition on the initial values, $(\mathbf{x}_{-1}, \mathbf{x}_0)$, and hence these are treated as fixed.

The I(1) CVAR is defined by the reduced-rank restriction,

$$H(r) : \mathbf{\Pi} = \boldsymbol{\alpha\beta}' \quad (2)$$

where both $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are $p \times r$ matrices with $r < p$. Imposing the condition (2) on the model (1), we obtain,

$$\Delta \mathbf{x}_t = \boldsymbol{\alpha\beta}' \mathbf{x}_{t-1} + \mathbf{\Psi}_0 \Delta \mathbf{z}_t + \mathbf{\Psi}_1 \Delta \mathbf{z}_{t-1} + \mathbf{\Gamma}_1 \Delta \mathbf{x}_{t-1} + \boldsymbol{\phi} \mathbf{D}_t + \boldsymbol{\varepsilon}_t \quad (3)$$

Under the additional assumption that the characteristic polynomial has exactly $(p - r)$ unit roots and the remaining roots are outside the unit circle,[5] $\Delta \mathbf{x}_t$ and $\boldsymbol{\beta}' \mathbf{x}_t$ can be made stationary. In this case, r denotes the number of cointegrating relations which may be determined by the LR test proposed by Johansen (1988). The cointegrating relations are given by $\boldsymbol{\beta}' \mathbf{x}_t$, and the $\boldsymbol{\alpha}$-matrix contains information on the short-run adjustment following disequilibria.

Maximum likelihood-inference procedures are derived by Johansen (1988, 1991, 1996). To ensure valid inference, it is vital to check the necessary conditions for the CVAR to

[5]This assumption implies $|\boldsymbol{\alpha}'_{\perp} \mathbf{\Gamma} \boldsymbol{\beta}_{\perp}| \neq \mathbf{0}$ with $\mathbf{\Gamma} = \mathbf{I} - \mathbf{\Gamma}_1$ which requires the variables to be integrated of order at most one; we use $_{\perp}$ to denote the orthogonal complement.

provide an adequate description of the data. Diagnostic testing to ensure parameter stability and that the residuals are well-behaved is therefore crucial. In particular, tests for multivariate normality and absence of autocorrelation should be consulted.

2.2 Motivation for the MSM

The choice of data vectors, $\mathbf{x}_t$ and $\mathbf{z}_t$, is crucial. The literature suggests a number of different channels through which, for example, monetary policy may affect inflation and other key economic variables, see *inter alia* Mishkin (1995), Bernanke and Gertler (1995) and ECB (2004b), but there is little consensus with respect to the relative importance of these. For this reason, we draw on a large information set in order to encompass a range of proposed transmission channels. The CVAR methodology is useful for this purpose as it allows a flexible specification of the data and, at the same time, facilitates formal testing of many economic hypotheses. Although it would be preferable to analyse a large set of variables jointly to characterise the whole economy all at once, this is not feasible given that the VAR is heavily parameterised combined with the fact that we have a relatively short sample. To reduce the dimensionality of the model we adopt an estimation strategy similar to that suggested by Juselius (1992, 2006). We first estimate small-scale CVAR models to identify long-run relations within different sectors of the economy. The cointegration relations identified in each market can be viewed as measures of disequilibria. These measures of deviations from the long-run steady state are then combined in a joint model of the short-run adjustment structure. We denote the latter the MSM and this includes, among other things, an inflation equation.

Applying this type of modelling approach to Danish inflation data, Juselius (1992, 2006) analysed the financial market, the import-export market and the labour market to construct measures of 'excess money supply', 'imported inflation' and excessive wage claims', respectively. She finds that monetary policy alone is insufficient to explain movements in inflation. Besides the above-mentioned sectors, we include also the public sector. This extension is considered important for a number of reasons. First of all, the government plays a vital role in the European welfare states and may therefore have significant effects on domestic demand. Deficit-debt dynamics may also affect the view of financial markets on the sustainability of fiscal policy and thus risk premia and interest rates. In the longer run, fiscal policy may also have supply-side effects, for example, via its impact on the taxation structure and labour market institutions. Secondly, the joint modelling of monetary and fiscal policy issues has received renewed attention in the literature recently, see *inter alia* Kirsanova, Stehn, and Vines (2005) and the references therein. Indeed, the unique feature of the euro zone as a currency union with no direct coordination of national fiscal policies has generated a range of ECB working papers on possible interactions between the monetary and fiscal authorities, see *inter alia* Beetsma and Jensen (2002), Ferrero (2005) and Leith and Thadden (2006). The existence of the Stability and Growth Pact (SGP), and the warnings that the continual violation of its regulations by some countries have given rise to, underline the concern among policy makers that unsound fiscal policies in individual member states pose a latent threat to overall price stability in the euro zone.

Our modelling strategy is illustrated in Figure 1; variables and notation are presented in Table A.1 in Appendix A. For each sector, a set of variables is chosen to represent its dynamics and, based on this, a number of cointegrating relations are identified in the first step of the procedure. For example, in the public-sector model, deficits in excess of the level suggested by a fiscal-policy rule could be indicative of upward pressure on prices. Since the variables entering the CVAR models are potentially non-stationary, in the second step the MSM models the first difference of the variables of interest, notably inflation and unemployment: $\Delta\mathbf{x}_t^{MSM} = (\Delta\mathbf{x}^{fis}, \Delta\mathbf{x}^{mon}, \Delta\mathbf{x}^{ext}, \Delta\mathbf{x}^{lab})_t'$.

Invariance of the cointegration structure with respect to *extensions* of the information set rationalises that identification of the long-run relations is done on the basis of partial information sets in the sector models. Also, small CVAR models are easier to manage, i.e. tests of the statistical assumptions have greater power and facilitate identification of relations that are meaningful both in a statistical and an economic sense. Finally, since the cointegration relations are stationary by definition, the short-run dynamics of the MSM can be estimated from standard inference procedures using an equation-by-equation or a system approach.

The way in which our modelling procedure deals with dimensionality, reducing it by extracting a number of disequilibrium terms from each sector, shares some characteristics with factor analysis. The crucial difference between extracting simple principal components from a set of variables as part of a traditional factor analysis and the MSM approach taken here is that we impose and test economically meaningful restrictions on the cointegrating relations. As a result the 'factors' extracted using our approach arguably have a structural interpretation.

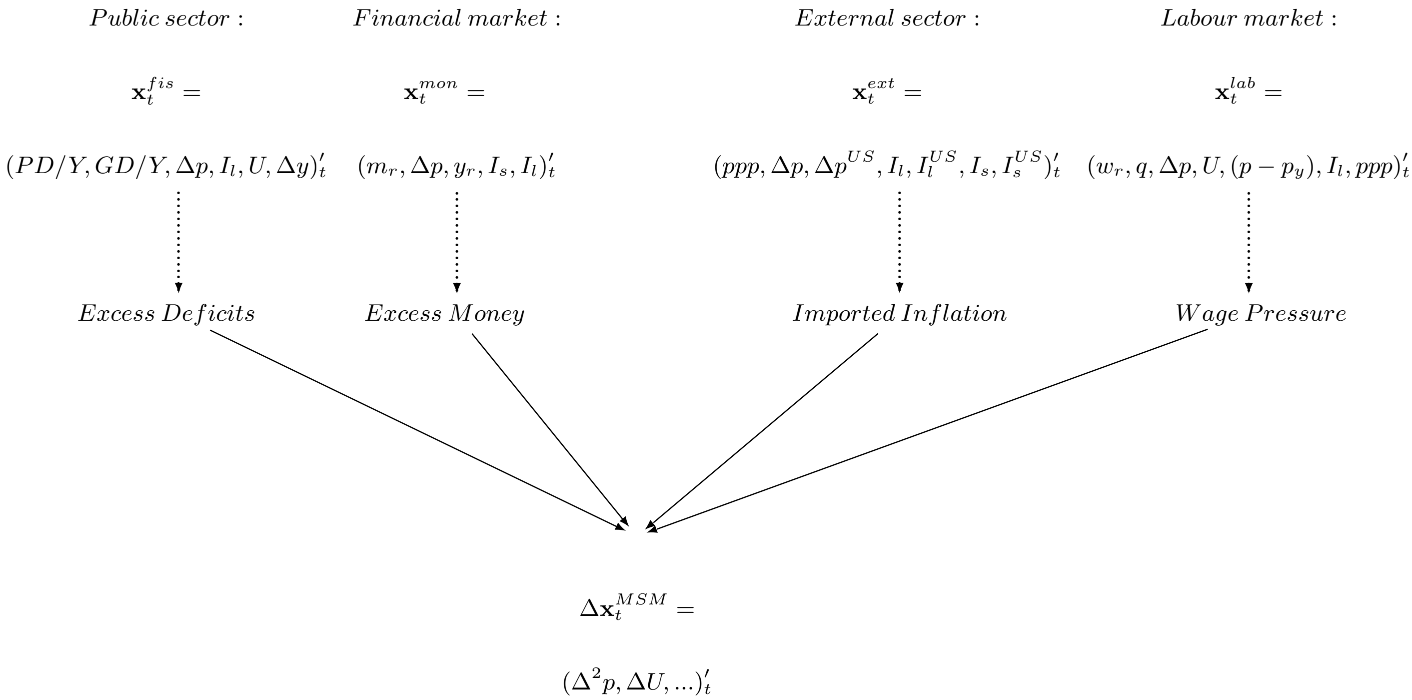

Figure 1: The multi-sector modelling procedure.

2.3 Robustness checks of the MSM

The short-run structure and the common stochastic trends are not invariant to extensions of the information set however. Hence we cannot rely on exogeneity tests within the (partial) sector models but need to determine the exogeneity status of the variables within the MSM based on the full information set. This can be done using the test procedure suggested by Harbo, Johansen, Nielsen, and Rahbek (1998).

Furthermore, the cointegration relations are not invariant to *reductions* of the information set. Although it may seem reasonable from an economic point of view, the sectoral division of the complete set of variables into different, but partly overlapping, sets is arbitrary from a statistical point of view.[6] Splitting up the information set implies that we run the risk of excluding and/or misinterpreting any potentially important cointegration across markets. Similarly, overlapping sets of variables may lead the same cointegration relation to show up in more than one partial model. With r_i the number of cointegration relations identified in sector i, N the number of markets/sectors, and r the cointegra-

[6]In a nutshell, the invariance property with respect to extensions of the information set is only a necessary, but not sufficient, condition for the MSM approach to work: it ensures that we can move from the partial models to the MSM, but not that we can move in the opposite direction.

tion rank of the CVAR model based on the full set of variables, it may thus occur that $\sum_{i=1}^{N} r_i \gtreqless r$.

To account for these potential problems, we propose the following robustness checks of the MSM:

- We add the omitted variables to each sector model one at a time and test if that variable is both weakly exogenous and long-run excludable.
- We estimate an unrestricted CVAR model based on the full information set and check whether the rank equals the sum of the number of cointegration relations from the partial models.

The first check is an evaluation of the specification of the partial models: if the inclusion of an otherwise omitted variable leaves the rank of the partial system unchanged and, in addition, the variable is long-run excludable, it is unlikely to enter an absent cointegration relation.[7] The unrestricted estimates from the full-information CVAR should be interpreted with care given its high dimension and thus low number of degrees of freedom, but as the second check proposes, it may provide tentative evidence on the pushing and pulling forces among the variables, notably the rank.

3 Data

We use quarterly data series from the AWM of the ECB (see FHM) and the OECD for the period 1982:2 to 2002:4. An overview of the data and their sources is given in Table A.1. All series are in logs except fiscal variables, interest rates and the unemployment rate; interest rates have been divided by 400 for comparability with the quarterly inflation rates. Figure A.1 plots the key variables. The sample is set to start in the early 1980s as tests indicate that the transition to a more strict regime of the European Monetary System (EMS) and the demolition of capital restrictions constitute a structural break around that time.

A range of issues arise in aggregating national data of countries as exchange rates have been flexible in part of the sample (see Beyer, Doornik, and Hendry 2001 and Beyer and Juselius 2008). The AWM data set is widely used for area-wide analysis of the euro zone and we abstract from aggregation issues here; in fact, one purpose of this study is to

[7]Adding one variable at a time of course checks only for pairwise cointegration but can of course be generalised such that sets of variables are added and more general cointegration structures checked for.

compare the results of our modelling framework to that of the AWM. Important for our purpose is the extent to which the transition to a monetary union with a common central bank constitutes a structural break. National differences in government policies may also complicate the interpretation of the area-wide stance of fiscal policy. Recursive test procedures do not indicate any major problems with parameter instability but disruptions to intra-euro area PPP convergence need to be explicitly accounted for in the labour-market model (see below).

4 Sector models: cointegrating relations

CVAR models for the four sectors included in Figure 1 have all been analysed for euro-area data elsewhere: the financial market by Coenen and Vega (1999), the external sector by Juselius and MacDonald (2007), the labour market by Juselius (2003) and the public sector by Tuxen (2006). We re-estimate the models based on slightly different model specifications using the AWM data. Tuxen (2006) shows how to drive from an AS-AD model each of the relations identified below. All partial models are based on a VAR with two lags. A number of dummy variables, including centered seasonal dummies, are included in each model to take account of extraordinary events in the sample period. The deterministic specifications vary across the CVAR models as described in the discussion of each sector. Appendix B provides details on misspecification tests, rank test statistics and reports estimates of the cointegrating relations and the corresponding adjustment structure subject to over-identifying restrictions. The identified cointegrating relations are shown in Table 1; Figure 2 plots the deviations from each.

The first robustness check proposed in Section 2 suggests that the partial results are in most cases largely robust to alternative specifications. Expanding the information of each partial model with one omitted variable at a time in most cases results in tests pointing to that variable being both long-run excludable and weakly exogenous. One exception is found in the external model for which the price wedge is borderline rejected as weakly exogenous, a first signal that this variable is to some degree determined by international conditions. In fact, the difference between consumer and producer prices was argued by Juselius (2003) to play an important role as a measure of product-market competition facing producers: a rise in the wedge, $(p-p_y)_t$, represents a rise in the degree of competition.[8]

[8]This interpretation is based on the observation that while consumer prices include the prices of

Label	Acronym	Composition
Public sector:		Test of restrictions: $\chi^2(4) = 3.83(p = 0.43)$
Kirsanova et al. (2005): fiscal-policy rule	$Fecm_{1,t}$	$PD/Y_t = -\underset{[-13.69]}{0.12}\, GD/Y_t + \underset{[11.07]}{0.65}\, U_t$
Kirsanova et al. (2005): government-budget constraint	$Fecm_{2,t}$	$GD/Y_t = \underset{[3.23]}{12.66} PD/Y_t + \underset{[6.57]}{53.94} I_{l,t} - \underset{[-12.71]}{83.79}\, \Delta y_t + \underset{[5.64]}{0.99}$
Phelps(1994)-Phillips(1958) curve I	$Fecm_{3,t}$	$\Delta p_t = -\underset{[-6.23]}{0.06}\, U_t + \underset{[17.19]}{0.73}\, I_{l,t}$
Financial market:		Test of restrictions: $\chi^2(4) = 1.51(p = 0.83)$
Taylor (1993): monetary-policy rule	$Mecm_{1,t}$	$I_{s,t} = \underset{[8.63]}{0.65} \Delta p_t + \underset{[11.17]}{0.10} y_{r,t} - \underset{[-13.77]}{0.07}\, t$
Coenen and Vega (1999): money-demand relation	$Mecm_{2,t}$	$(m_r - y_r)_t = -\underset{[-12.43]}{8.52}\, I_{l,t}$
External sector:		Test of restrictions: $\chi^2(6) = 2.87(p = 0.83)$
Ciccarelli and Mojon (2005): global-inflation attractor	$Eecm_{1,t}$	$ppp_t = -\underset{[-6.32]}{134.32}(I_s - \Delta p)_t - \underset{[-2.69]}{48.73}\, I^{US}_{s,t} + \underset{[9.33]}{305.29} \Delta p^{US}_t - \underset{[-2.53]}{0.66}$
Frydman and Goldberg (2007): imperfect-knowledge economics	$Eecm_{2,t}$	$ppp_t = -\underset{[-14.79]}{72.71}\,(I_l - \Delta p)_t + \underset{[12.11]}{61.01}(I^{US}_l - \Delta p^{US})_t$
Labour market:		Test of restrictions: $\chi^2(7) = 10.20(p = 0.18)$
Phelps(1994)-Phillips(1958) curve II	$Lecm_{1,t}$	$\Delta p_t = -\underset{[-10.10]}{0.23} U_t + \underset{[21.61]}{0.71}\, I_{l,t}$
Juselius (2003): external-competitiveness relation	$Lecm_{2,t}$	$U_t = \underset{[4.68]}{0.31} q_t + \underset{[6.22]}{0.67}(p - p_y)_t - \underset{[-6.06]}{0.05}\, ppp_t - \underset{[-4.63]}{0.13}\, t$, or $(p - p_y)_t = \underset{[9.30]}{0.07} ppp_t + \underset{[8.03]}{1.50} U_t - \underset{[-4.64]}{0.47}\, q_t + \underset{[4.60]}{0.19} t$
Phelps (1994): mark-up relation	$Lecm_{3,t}$	$-(w_r + (p - p_y) - q)_t = \underset{[3.73]}{0.05} ppp_t + \underset{[23.69]}{5.23}\, U_t - \underset{[-6.63]}{0.86}\, conv_t$
Blanchard and Katz (1999): wage-demand relation	$Lecm_4$	$w_{r,t} = -\underset{[-4.63]}{1.28}\, U_t + \underset{[14.15]}{3.27} q_t - \underset{[-9.29]}{0.93}\, t + \underset{[7.41]}{1.27} conv_t$

Table 1: Sector models: composition of the cointegrating relations.

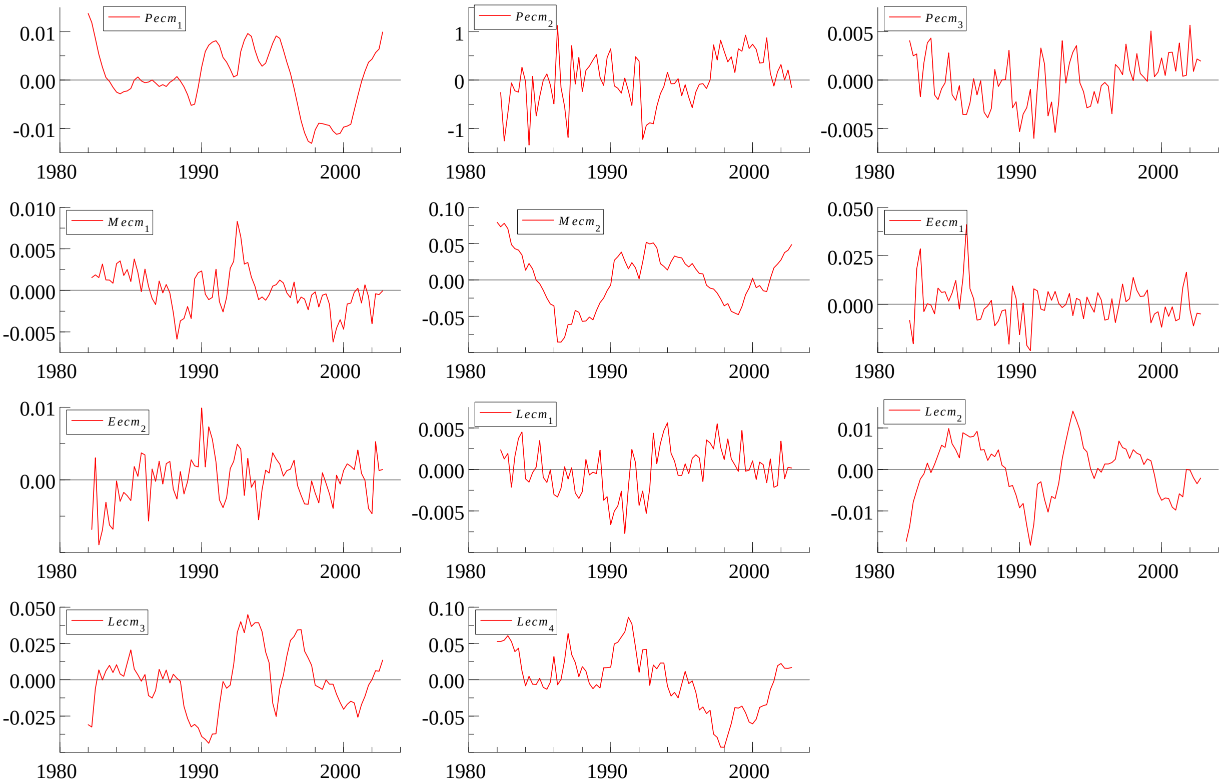

Figure 2: Sector models: deviations from the cointegrating relations (means corrected).

4.1 Public sector

The public sector (fiscal policy) is modelled using the following data vector,

$$\mathbf{x}_t^{fis} = (PD/Y, GD/Y, \Delta p, I_l, U, \Delta y)'_t, \tag{4}$$

where PD/Y_t is the primary deficit-to-GDP ratio (i.e. excluding interest payments), GD/Y_t gross government debt-to-GDP, Δp_t the quarterly inflation rate, $I_{l,t}$ the quarterly long-term interest rate, U_t the unemployment rate, and Δy_t nominal GDP growth. We include a constant restricted to the cointegration space. Details on the model are provided by Table B.1-B.3. After inclusion of a rather large set of dummy variables, the residuals look well-behaved as confirmed by the misspecification tests.[9]

The rank test suggests $r = 3 \vee r = 4$ depending on whether a one- or a five-percent level of significance is used. The companion-form roots remain large irrespective of the choice of rank; this points to I(2) problems. Indeed, the public-sector variables PD/Y_t and GD/Y_t look very persistent from the graphs and the estimated $\mathbf{\Gamma}_1$. We abstract from these issues here, as an I(2) analysis is out of the scope of this paper.[10] We set the rank to three, keeping in mind that this leaves a relatively large root in the model.

After imposing over-identifying restrictions on the model, the first cointegration is identified as a fiscal-policy rule similar to the one proposed by Kirsanova, Stehn, and Vines (2005). This suggests that the primary deficit has reacted to the level of public debt (sustainability motive) as well as to the unemployment rate (stabilisation motive). The second relation resembles that of a public-sector budget constraint where deficits and interest rates add to the net liabilities of the government sector and economic growth eases the burden of a given amount of public debt.[11] Finally, we have a relation which resembles that of a Phillips (1958) curve. However, the usual trade-off between inflation and unemployment needs to be augmented with the real rate of interest to obtain stationarity. This relation resembles that suggested by Phelps (1994) who argues that a rise

both traded and non-traded goods, producer prices (here measured by the GDP deflator) incorporates mainly traded (physical) goods. Traded-goods prices have arguably come under considerable pressure as globalisation has evolved whereas the prices of non-traded goods (e.g. services) have been affected much less.

[9]Dummy specification: $d_t^{fis} = (Dp_{86:2}, Dp_{87:1}, Dp_{89:4}, Dp_{97:2}, Dp_{98:1}, Dp_{00:4}, Dp_{01:2}, Dp_{02:4})'_t$ where $Dp_{QQ:Y}$ denotes a permanent impulse dummy taking the value one at time QQ:Y, zero elsewhere.

[10]See Tuxen (2009) for an analysis of deficit-debt dynamics within an I(2) framework. Multi-cointegration can also be incorporated in an MSM-type approach but this is out of the scope of this paper.

[11]The government budget constraint can be linearised as follows. Simple accounting prescribes,

$$GD_t = (1 + I_{l,t-1})GD_{t-1} + PD_t$$

in the rental cost of capital depresses employment; see also Juselius (2006). We shall refer to this as a Phelps-Phillips curve since Phelps (1994) emphasised the real rate as the core transmission mechanism between the labour market and the wider economy because changes in the cost of capital induces movements in the natural rate of unemployment, i.e. the NAIRU level.

4.2 Financial market

The financial market (monetary policy) is modelled using the data vector,

$$\mathbf{x}_t^{mon} = (m_r, \Delta p, y_r, I_s, I_l)'_t, \tag{5}$$

where $m_{r,t}$ is real money supply, $y_{r,t}$ real GDP, and $I_{s,t}$ the quarterly short-term interest rate. We include a linear trend restricted to the cointegration space. Details on the model are provided by Table B.5-B.7.[12] The rank test suggests $r = 3 \vee 4$ while the Bartlett-corrected test point to $r = 2 \vee 3$. We set the rank to two as a higher choice of rank leaves the cointegrating relations difficult to interpret; this choice is also confirmed by other pieces of information on the rank such as the companion-form roots, the graphs of the cointegrating relations and the adjustment coefficients.

The first cointegration relation resembles a Taylor (1993) rule with the policy rate reacting positively to inflation and the output gap, the latter measured by real output in deviation from trend. The second relation is similar to a money-demand equation as in Coenen and Vega (1999).; while we do not find any significant effects of neither inflation nor the short rate, an increase in the long rate, i.e. the opportunity cost of holding

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Dividing through by Y_t yields,

$$\begin{aligned}\left(\frac{GD}{Y}\right)_t &= (1+I_{l,t-1})\left(\frac{GD}{Y}\right)_{t-1}\underbrace{\frac{Y_{t-1}}{Y_t}}_{\simeq(1-g_{t-1})} + \left(\frac{PD}{Y}\right)_t \\ &\simeq (1+I_{l,t-1}-g_{t-1})\left(\frac{GD}{Y}\right)_{t-1} + \left(\frac{PD}{Y}\right)_t\end{aligned}$$

with $g_{t-1} = \frac{Y_t - Y_{t-1}}{Y_{t-1}}$ denoting GDP growth. Using $g_{t-1} \simeq \Delta y_t$ and a multivariate first-order Taylor approx. e.g. around $\left(\frac{GD}{Y}\right)^* = 0.60$ and $I_l = 0.01 < \Delta y = 0.0125$ (as assumed for the SGP) we obtain,

$$GD/Y_t - \beta_{21}PD/Y_t - \beta_{22}(I_l - \Delta y)_t - \beta_{20} + \nu_{2,t} \sim I(0)$$

where $\beta_{21} > 0$ and $\beta_{22} > 0$.

[12] Dummy specification: $d_t^{mon} = (Dt_{86:2}, Dp_{90:2}, Dp_{92:3})'_t$ where $Dt_{QQ:Y}$ denotes a transitory impulse dummy taking the value one at time QQ:Y, minus one in (one of) the following period(s), and zero elsewhere.

money, has been associated with a decline in real-money balances. Interestingly, there is no evidence of a stationary interest-rate spread nor of a stationary real long-term interest rate, implying both the expectations hypothesis of the term structure in its pure form and the Fisher parity are rejected.

4.3 External sector

The foreign-trade sector is modelled using the data vector,

$$\mathbf{x}_t^{ext} = (ppp, \Delta p, \Delta p^{US}, I_l, I_l^{US}, I_s, I_s^{US})'_t, \tag{6}$$

where ppp_t is the real exchange rate defined as $ppp_t = s_t - p_t - p_t^*$ with s_t the domestic currency price of foreign exchange, Δp_t^{US} US inflation, $I_{s,t}^{US}$ the US short rate, and $I_{l,t}^{US}$ the US long rate. We include a constant restricted to the cointegration space. Details on the model are provided by Table B.9-B.11.[13]

The rank test points to $r = 2$. None of the standard international parity conditions such as the purchasing power parity (PPP) and the uncovered interest rate parity (UIP) are found to be stationary. Similar to Juselius and MacDonald (2007) we instead find linear combinations of deviations from parities to be stationary.

The first relation links the real exchange rate with the euro-area short real rate and its US counterpart, albeit homogeneity is not obtained for the latter. The *sum* of the short-term real rates is stationary which suggests that this represents a global level of rates. The adjustment structure reveals that both the euro-area and US inflation rates have error-corrected strongly to deviations from this relation. This is similar to the finding of Ciccarelli and Mojon (2005) that a global level of inflation exists and acts as an attractor for national inflation. The second relation combines the real exchange rate with the *difference* (spread) between the real long-term interest rates in the euro area and the US. This lends support to the imperfect-knowledge expectations (IKE) model of Frydman and Goldberg (2007) who predict that both the real exchange rate and the real interest-rate differential to be non-stationary but co-moving; see Johansen, Juselius, Frydman, and Goldberg (2008) and Juselius, Frydman, Goldberg, and Johansen (2009) for a discussion of the time-series implications of IKE.

[13] Dummy specification: $d_t^{ext} = (Dp_{84:3}, Dt_{86:2}, Dp_{92:4}, Dp_{93:2})'_t$.

4.4 Labour market

The labour market is modelled using the data vector,

$$\mathbf{x}_t^{lab} = (w_r, q, \Delta p, U, (p - p_y), I_l, ppp)'_t, \tag{7}$$

where $(p - p_y)_t$ is the wedge between consumer and producer prices, $w_{r,t}$ the real wage and q_t labour productivity. We also include as weakly exogenous,

$$\mathbf{z}_t^{lab} = (conv)'_t, \tag{8}$$

which measures the lack of intra-euro zone purchasing power parity (PPP) convergence as suggested by Juselius (2003).[14] We include a trend restricted to the cointegration space. Details on the model are provided by Table B.13-B.15.[15]

The rank test suggests $r = 4$. The first labour-market relation is another version of the real-rate augmented Phillips curve found in the public-sector model.[16] The need for euro-area producers to remain externally competitive in this period is reflected in the second relation where labour productivity and unemployment move together as also found by Juselius (2003) and Juselius and Ordóñez (2009). These authors argue that in face of increased price and wage competition from low-cost producers in developing countries and emerging markets, firms laid off the least productive workers in an attempt to boost productivity as required to match a relatively high and rigid real wage. The third relation sees a positive association of firms' mark-ups[17] and the real exchange rate, suggesting that a real appreciation has made it more difficult for firms in exposed industries to raise prices, another important transmission mechanism emphasised by Phelps (1994). A stationary relationship is however only obtained after controlling for the effect of the business cycle via inclusion of the unemployment rate, indicating that mark-up's are strongly counter-cyclical as is often found to be the case (see *inter alia* Rotemberg and Woodford 1999). The final relation is interpreted as a wage-demand relation and is similar to Blanchard and Katz (1999) in prescribing that the level of consumption wages demanded by workers' unions have been lowered when unemployment has risen and raised in periods of increasing labour productivity.

[14] The nature of $conv_t$ makes it difficult to obtain a well-specified equation for this variable and we set it to be weakly exogenous *a priori* although this variable may clearly feed-back on the system dynamics.

[15] Dummy specification: $d_t^{lab} = (Dp_{84:2}, Dt_{86:2}, Dp_{92:3})'_t$.

[16] The coefficient of unemployment is higher in this model (0.22 vs. 0.07) and the magnitude of the t-statistics suggests that the coefficients of this relation are more precisely estimated compared with the public-sector model.

[17] The mark-up is here defined as -$(w_r + (p - p_y) - q) = p_y - w + q$.

5 MSM: short-, medium- and long-run dynamics

Conditional on the sector-model results, we first show how to estimate the short-run structure for the MSM followed by a discussion of the separation of the short-, medium- and long-run dynamics which the MSM provides.

5.1 Short-run dynamics

The MSM takes the form of a simultaneous equations model where the data vector consists of the stacked set of variables from the four sectors. Conditioning on the cointegration structure identified within the CVAR models, we estimate the following model,

$$\Delta \mathbf{x}_t^{MSM} = \boldsymbol{\alpha}^* \mathbf{ECM}_{t-1} + \boldsymbol{\Gamma}_1^* \Delta \mathbf{x}_{t-1}^{MSM} + \boldsymbol{\Psi}_0^* \Delta \mathbf{z}_t^{MSM} + \boldsymbol{\Psi}_1^* \Delta \mathbf{z}_{t-1}^{MSM} + \boldsymbol{\phi}^* \mathbf{d}_t + \boldsymbol{\varepsilon}_t^*, \quad (9)$$

where $\mathbf{x}_t^{MSM}$ and $\mathbf{z}_t^{MSM}$ are vectors of endogenous and weakly exogenous variables, respectively, $\mathbf{d}_t$ includes various deterministic terms such as a constant, seasonal dummies and impulse dummies, and $\mathbf{ECM}_t$ contains the deviations from the cointegration relations from sector models and we refer to these as equilibrium-correction model (ECM) terms. Details on the composition of $\mathbf{x}_t^{MSM}$, $\mathbf{z}_t^{MSM}$, $\mathbf{d}_t$ and $\mathbf{ECM}_t$ are given below.

Adding up the number of cointegration relations from the partial models yields 11 relations in total. The second robustness check proposed in Section 2 in the form of a tentative assessment of the full-information CVAR, points to a rank of ten however, see Table D.1. As discussed above, $Fecm_{3,t}$ and $Lecm_{1,t}$ have similar interpretations and are indeed highly correlated. We exclude $Fecm_{3,t-1}$ from the second stage of the MSM procedure since the estimated coefficient of U in $Lecm_{1,t}$ is of a more reasonable magnitude, i.e. it suggests that a one-per cent decline in unemployment is associated with an approximate one-per cent increase in the *annual* inflation rate, *ceteris paribus*. Also the parameters in $Lecm_{1,t}$ are more precisely estimated as judged from the t-statistics. This leaves us with effectively ten cointegrating relations.

Moreover, the government budget constraint ($Fecm_{2,t}$) should hold roughly as an accounting identify and hence deviations from this should not be interpreted as disequilibria but simply approximation and/or measurement errors. For this reason we also leave $Fecm_{2,t-1}$ out of the regressor set *a priori*. This results in the following vector of disequilibrium terms,

$$\mathbf{ECM}_t = (Fecm_1, Mecm_1, Mecm_2, Eecm_1, Eecm_2, Lecm_1, Lecm_2, Lecm_3, Lecm_4)'_t. \quad (10)$$

Estimation of (9) provides the short-run adjustment dynamics of the endogenous variables conditional on the identified long-run structure. All variables in (9) are stationary and standard estimation (system or equation-by-equation) and inference techniques can therefore be applied. Because of the large set of regressors, a model-reduction procedure is needed to obtain a parsimonious specification of each variables/equation. The choice of information set, i.e. the right-hand side of (9), used in specifying the general unrestricted model (GUM), which the model search is based is upon, is crucial for the economic sensibility and statistical validity of the terminal models. In order to divide the variables into sets of endogenous and weakly exogenous, $\mathbf{x}_t^{MSM}$ and $\mathbf{z}_t^{MSM}$, we apply the procedure proposed by Harbo et al. (1998) and test the null of weak exogeneity for each variable. This is done by testing, equation-by-equation, whether the union of ECM terms in (10) can be excluded from the set of regressors explaining a given variable. These tests suggest that none of the euro-area variables are weakly exogenous, although the test rejects less strongly for the price wedge and the long-term interest rate. Regarding the US variables, the short rate and long rate can be borderline accepted as weakly exogenous whereas the inflation rate is not, as it reacts strongly to deviations from the first external relation ($Eecm_{1,t}$).

In order to obtain a parsimonious and well-specified model from (9) we use the automatic model-selection algorithm Autometrics to discriminate between the regressors for each equation, see Doornik (2007a).[18] In general, the algorithm performs better the greater the orthogonality of the regressors which suggests that either $\Delta y_{r,t-1}$ or Δq_{t-1}, which are highly correlated, should be removed prior to the search. Standard exclusion tests show that Δq_{t-1}, $\Delta m_{r,t-1}$, $\Delta conv_t$ and $\Delta conv_{t-1}$ can in fact be left out of the model altogether. The set of weakly exogenous variables thus becomes,

$$\mathbf{z}_t^{MSM} = \left(I_s^{US}, I_l^{US}, \Delta p^{US}\right)_t', \tag{11}$$

except that the contemporaneous value of US inflation, Δp_t^{US}, is excluded from the GUM. The set of endogenous variables is,

$$\mathbf{x}_t^{MSM} = (\Delta p, ppp, m_r, p - p_y, w_r, q, I_s, I_l, y_r, U, PD/Y, GD/Y)_t', \tag{12}$$

with the exception that the lagged values Δq_{t-1} and $\Delta m_{r,t-1}$ are excluded.

[18]We take an equation-by-equation approach as the simultaneous-equation version of Autometrics lacks the flexibility required here.

The information set for the GUM is the same for each equation/variable and this is shown in Table C.1. In addition, a constant term and centered seasonal dummies are included. Outlier detection is done by Autometrics and dummies are included where required to obtain a well-specified model. For each variable in $\mathbf{x}_t^{MSM}$, a parsimonious and well-specified equation is located by the algorithm. [19] Information criteria are used in choosing between models in case more than one terminal model are returned by the algorithm. Whenever the Schwarz (SC), Hannan-Quinn (HQ) and Aikaike (AIC) criteria differ with respect to the preferred model, judgement based on economic interpretability is applied in choosing between them. For readability, Table 2 provides the results for each equation, reporting coefficients of the ECM terms only. Table C.2 shows the complete results.

5.2 MSM results

The main advantage of the MSM over the basic structuring of the covariances in the data provided by the $\mathbf{\Pi}$-matrix based on the full information set is that, conditional on our interpretation of the ECM terms as representing deviations from different types of economic equilibrium, we can attach a 'structural' interpretation to the results. Appendix D provides a detailed discussion of the full-information CVAR.

In discussing the empirical results, we first assess the validity of the restrictions imposed on the cointegration structure in the sector models and on the adjustment dynamics in the model-reduction procedure by checking that these do lead to violation of any of the fundamental regularities in the data. This is done through an overview of the main pushing and pulling forces of the MSM, where we check whether these broadly comply with the results of the full-information CVAR in Appendix D regarding weakly exogenous and purely adjusting variables. A separation of the pushing and pulling forces is important as it allows us, in some cases, to assess the direction of causality in a relation between a given set of variables.

In the following, we shall refer to $\boldsymbol{\alpha}$ as capturing 'short-run' adjustment, to $\boldsymbol{\beta}$ as representing 'medium-run' dynamics and to $\mathbf{\Pi} = \boldsymbol{\alpha}\boldsymbol{\beta}'$ as the 'long-run' effects where the interplay of the short- and the medium-run dynamics have settled.[20] In the MSM the

[19] Autometrics was run with the following settings: test form: LR-F; target size: 0.1, outlier detection: 0.05, pre-search reduction: none, backtesting: GUM0, diagnostics p-value: 0.01, search effort: standard.

[20] Note that for convenience in the discussion to follow we refer to the cointegrating relations as describing the medium run despite the fact that these are often denoted long-run or steady-state relations. This choice of wording is only to distinguish the cointegration structure from the notion of the (very)

long-run (static) solutions for each equation are used as a proxy for the latter; indeed, the long-run solution of a dynamic, stochastic process is defined as characterising the hypothetical deterministic situation in which all change has ceased (Hendry 1995). Table C.3 gives the static solutions for each variable.

Following a discussion of the pulling vs. the pushing forces, we consider the inflation equation in details and compare the explanatory power of the MSM information set with that of the AWM. This is naturally followed by a discussion of unemployment dynamics and we end by addressing the effectiveness and interactions of monetary and fiscal policy.

5.2.1 Pulling vs. pushing forces

We consider first the driving forces of the model. Table 2 shows that the long rate and the price wedge have exhibited few and only borderline significant reactions to the ECM terms (t-statistic are below three), consistent with the indications of the Harbo et al. (1998) tests. The bulk of explanatory power for the long rate comes from its own lagged value and the corresponding US rate whereas the price wedge react mainly to the lagged level of the real exchange rate. The long-run static solutions in Table C.3 show that the real exchange rate, the price wedge, real money supply, the long rate and public debt are all largely unrelated to the other euro-area variables in the system when all dynamics have died out and thus seem to determined outside our model. The euro-area long rate has moved roughly with its US counterpart and declined as intra-euro zone PPP convergence improved. The close co-movement of the euro-area and US bond rates point to the existence of one globally determined long-term interest rate. The finding that the real exchange rate and the price wedge are driven largely by factors exogenous to our model are similar to the results of Juselius and Ordóñez (2009) for Spain; this further supports the interpretation of the price wedge as a measure of the exposition of the domestic industry to external competition. Real money sees a borderline significant co-movement with real output in the long run (t-statistic below two); this 'vague' determination of domestic money would be consistent with the hypothesis that 'global liquidity' conditions are becoming ever more important in today's highly integrated financial markets, a hypothesis investigated by Giese and Tuxen (2009). These findings are largely consistent with the tests for zero rows in $\boldsymbol{\alpha}$ on the full-information CVAR in Table D.2.

long run where all dynamics have settled.

	$\Delta^2 p_t$	Δppp_t	$\Delta(p-p_y)_t$	$\Delta w_{r,t}$	Δq_t	$\Delta m_{r,t}$	$\Delta I_{s,t}$	$\Delta I_{l,t}$	$\Delta y_{r,t}$	ΔU_t	$\Delta PD/Y_t$	$\Delta GD/Y_t$
Fiscal-policy rule: $Fecm_{1,t-1} =$ $[PD/Y + 0.12GD/Y - 0.65U]_{t-1}$				-0.24 [-3.45]	-0.48 [-5.04]				-0.65 [-5.69]	0.12 [5.67]	-0.07 [-4.57]	
Monetary-policy rule: $Mecm_{1,t-1} =$ $[I_s - 0.65\Delta p - 0.10y_r + 0.07t]_{t-1}$					-0.90 [-3.71]		-0.34 [-7.08]	-0.06 [-1.91]	-0.66 [-2.85]	0.24 [5.62]	0.13 [3.56]	0.69 [3.67]
Money-demand relation: $Mecm_{2,t-1} =$ $[(m_r - y_r) + 8.52I_l]_{t-1}$	0.03 [3.47]	0.31 [2.24]	0.01 [1.95]		0.03 [1.93]	-0.11 [-7.83]				-0.01 [-3.73]		
Global-inflation attractor: $Eecm_{1,t-1} =$ $\left[ppp + 134.32(I_s - \Delta p) + 48.73I_s^{US} - 305.29\Delta p^{US} + 0.66\right]_{t-1}$	0.07 [2.17]	-3.06 [-3.59]										
Imperfect-knowledge economics: $Eecm_{2,t-1} =$ $\left[ppp_t + 72.71(I_l - \Delta p) - 61.01(I_l^{US} - \Delta p^{US})\right]_{t-1}$		-6.99 [-2.49]					-0.08 [-2.63]		0.64 [3.27]	-0.07 [-2.21]		
Phelps-Phillips curve: $Lecm_{1,t-1} =$ $[\Delta p + 0.23U - 0.71I_l]_{t-1}$	-1.02 [-8.61]	-12.00 [-3.82]			0.64 [2.80]		-0.09 [-2.39]		1.26 [4.00]			
External competitiveness: $Lecm_{2,t-1} =$ $[U - 0.31q - 0.67(p - p_y) + 0.05ppp + 0.13t]_{t-1}$	0.23 [3.88]			0.40 [4.71]	0.28 [3.10]	-0.27 [-3.34]		0.04 [2.41]	0.40 [5.29]	-0.10 [-4.30]		
Mark-up relation: $Lecm_{3,t-1} =$ $[(w_r + (p - p_y) - q) + 0.05ppp + 5.23U + 0.86conv]_{t-1}$				-0.26 [-9.77]	0.07 [2.50]			-0.01 [-2.66]		0.02 [2.63]		
Wage-demand relation: $Lecm_{4,t-1} =$ $[w_r - 3.27q + 1.28U - 1.27conv + 0.89t]_{t-1}$		-0.66 [-4.87]			0.15 [8.50]				0.18 [8.23]			
$[w_r - 3.27q + 1.28U - 1.27conv + 0.89t]_{t-1}$												
	...	...	...	...	...	...	...	...	...	...	...	...
$\hat{\sigma}$	0.0018	0.0337	0.0021	0.0035	0.0024	0.0040	0.0006	0.0007	0.0034	0.0006	0.0006	0.0024

Table 2: MSM: ECM estimates

We consider next the pulling forces of the model. Adjustment back to equilibrium in face of shocks should have taken place by means of corrections in the non-weakly exogenous variables. Focusing on the most significant entries in Table 2, we see that real wages have reacted strongly to deviations from the mark-up relation ($Lecm_{3,t-1}$) such that a decline in the mark-up has been followed by lower real wages. This underlines that external-competition pressures have forced not only producers to adjust mark-ups as suggested by the cointegrating relation itself, unions have had a tendency to adjust wage demands as well. Lack of adjustment of wages towards the wage relation ($Lecm_{4,t-1}$) signals that any adjustment has been very slow and possibly affected by insider-outsider effects. The interpretation of the external-competitiveness relation ($Lecm_{2,t-1}$) is sustained by the short-run adjustment as both unemployment and productivity have witnessed error-correction in face of this type of disequilibria. Labour productivity has improved when real wages have risen above the level dictated by the demand relation, which is also consistent with high and rigid real wages leaving firms to lay-off the least productive workers and produce the same amount of output with less labour. Indeed, the adjustment patterns found in the real-GDP equation are broadly the same as for productivity. While the Phillips curve has played an important role for inflation adjustment (we discuss this in details below), it does not feature in the wage equation. Similarly, wage disequilibria do not appear to have triggered adjustment in prices. The lack of such effects suggests that price-wage spirals have to a large degree been absent in the sample period. These adjustment patterns are again broadly in line with the tests for unit vectors in $\boldsymbol{\alpha}$ in Table D.2.

The long-run static solutions for each variable in Table C.3 support the finding of the above wage- and price-competition effects: real wages have moved negatively with the price wedge and thus possibly been kept down by fiercer competition, albeit with some additional effects from productivity (positive) and unemployment (negative). The long-run equation for labour productivity includes a range of significant variables but the main component is, by far, a positive association with unemployment. These long-run co-movements are similar to the effects found by inspection of the $\boldsymbol{\Pi}$-matrix of the full-information CVAR in Table D.2.

5.2.2 Inflation

We reproduce the $\Delta^2 p_t$-equation below alongside a range of tests for misspecification[21],

$$\begin{aligned}
\widehat{\Delta^2 p_t} &= \underset{[3.47]}{0.03} Mecm_{2,t-1} + \underset{[2.17]}{0.07} Eecm_{1,t-1} - \underset{[-8.61]}{1.02}\, Lecm_{1,t-1} + \underset{[3.88]}{0.23} Lecm_{2,t-1} \\
&\quad + \underset{[4.36]}{0.02} \Delta ppp_{t-1} - \underset{[-1.91]}{0.49}\, \Delta I_{l,t-1} - \underset{[-2.10]}{0.38}\, \Delta U_{t-1} - \underset{[-3.01]}{0.13}\, \Delta GD/Y_{t-1} \\
&\quad + \underset{[2.66]}{0.18} \Delta^2 p^{US}_{t-1} + \underset{[2.51]}{0.36} \Delta I^{US}_{s,t-1} + \underset{[2.92]}{0.01} Dp_{01:2,t} \qquad (13)
\end{aligned}$$

$$\begin{aligned}
F(15,55) &= \underset{[p=0.86]}{0.02}\,; \chi^2(2) = \underset{[p=0.87]}{0.28}\,; F_{RESET}(1,69) = \underset{[p=0.90]}{0.02}\,; \\
F_{AR}(4,66) &= \underset{[p=0.80]}{0.40}\,; F_{ARCH}(4,62) = \underset{[p=0.51]}{0.83}\,; F_{White}(21,48) = \underset{[0.86]}{0.70}.
\end{aligned}$$

The F-test for reduction from the GUM in (9) to the parsimonious model (13) does not reject the null and the diagnostic tests do not indicate problems with the model specification. Figure 3 depicts the fitted values and some residual properties which all seem to suggest that the equation is well-specified. Parameter stability is checked using recursive estimation with an initialisation sample of 30 observations. From Figure 4 the coefficient estimates appear stable for all regressors and the Chow tests cannot reject stability of (13) as a whole.

Equation (13) shows that inflation has error-corrected strongly to the Phelps-Phillips curve such that inflation in excess of the level suggested by $Lecm_{1,t-1}$ has led to adjustment back towards equilibrium.[22] The type of Phillips curve found to play a crucial role for inflation dynamics here is rather different from the NKPC of modern macro models as the former suggests that inflation is driven by cost-push (cost of capital) in addition to the traditional demand-pull (marginal cost) effects. Thus we do not find evidence for the prominent role attached to inflation expectations by the NKPC; the appropriate measure of expected inflation would here be the spread between the long- and the short-term interest rate. This is consistent with the findings of *inter alia* Bårdsen, Jansen, and Nymoen (2004) and Juselius (2008) who alike reject the NKPC specification.

Overall, there is a fair amount of evidence that inflation has to a large extent been determined internationally. Besides the importance of the near-weakly exogenous bond

[21]Conditional on a well-specified GUM, Autometrics ensures that no reduction is made if this leads any one of the diagnostic tests to fail.

[22]The magnitude of the coefficient (close to one in absolute value) may be partly ascribed to the decision to treat p_t as I(2) (rather than $I(1)$). Tests for stationarity point to the inflation rate being non-stationary in all sector models and thus $\Delta p_t \sim I(1)$ and $\Delta^2 p_t \sim I(0)$. Although this appears to be the best statistical approximation given the sample, $\Delta^2 p_t$ may nevertheless suffer from some slight over-differencing, contributing to the coefficient of $Lecm_{1,t-1}$ exceeding one in absolute value.

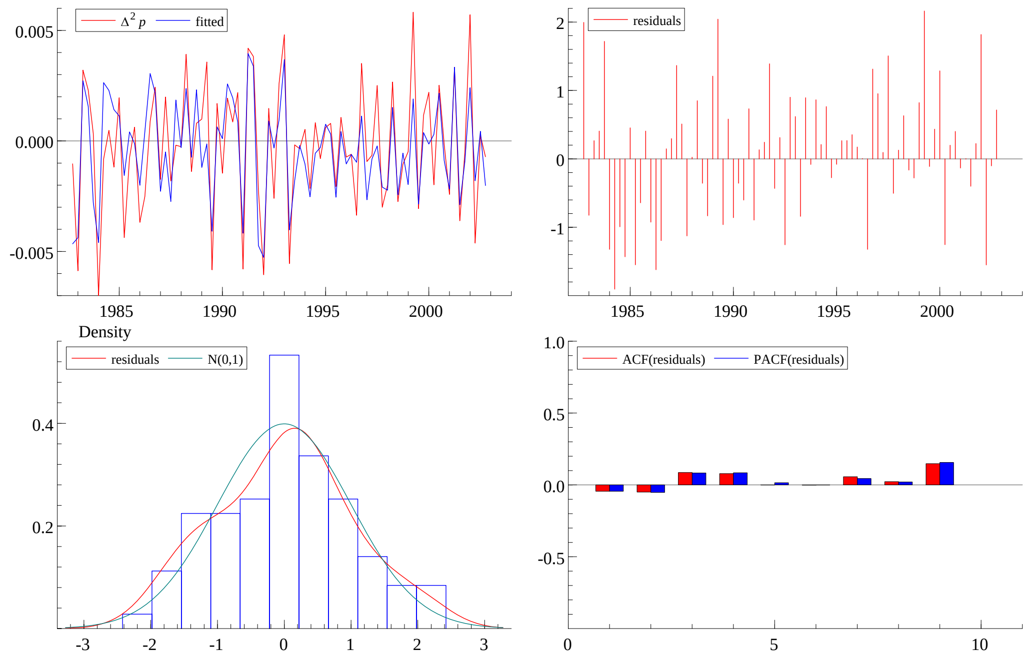

Figure 3: MSM: $\Delta^2 p$-equation: fit, scaled residuals, correlogram and density.

rate in the Phillips curve, deviations from the external-competitiveness relation have also been followed by inflation adjustment. This is likely due to the inclusion of the real exchange rate and the price wedge in $Lecm_{2,t-1}$ as illustrated by the alternative normalisation reported in Table 1; there is a tendency for inflation to decline with an increase in the wedge and with a real appreciation. Both of these variables were found to be among the driving forces of the MSM. The negative effect of the wedge is consistent with the hypothesis that globalisation, by increasing competition, has helped to keep inflationary pressures low. The negative impact on prices of a real appreciation could represent either imported inflation and/or the need for euro-area producers to remain competitive in the global market when the domestic currency strengthens.

Money in excess of the level proposed by the money-demand relation, represented by positive deviations from $Mecm_{2,t-1}$, has had a tendency to lead to higher inflation, consistent with Friedman (1969). Notably, money supply was not found to have been controllable by domestic authorities however; indeed, Ruffer and Stracca (2006) find that 'global liquidity' are driving euro-area market conditions to a large extent. Moreover, $Eecm_{1,t-1}$, which arguably reflects a 'global inflation attractor', has seen euro-area inflation adjusting towards it in an error-correcting fashion.

The lack of price adjustment towards the mark-up relation ($Lecm_{3,t-1}$) and the wage

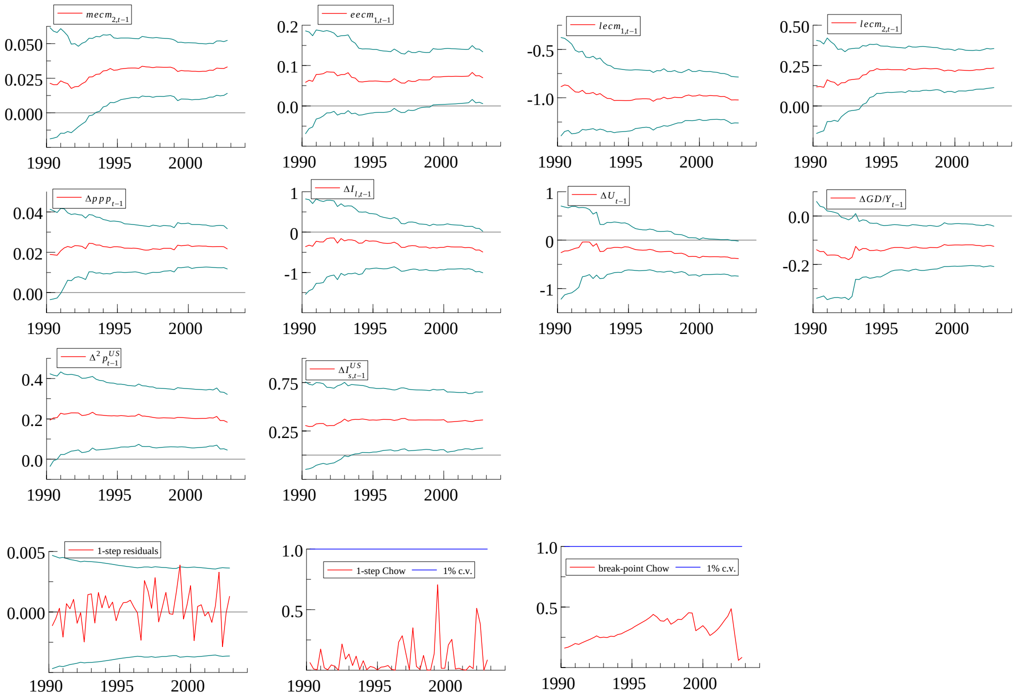

Figure 4: MSM: $\Delta^2 p$-equation: recursive tests (the first three rows show tests on individual regressors; the last row shows joint tests).

relation ($Lecm_{4,t-1}$) is of interest. The absence of deviations from the wage relation in the inflation equation provides further support for the indications above that a wage-price spiral has not been a dominating mechanism in the euro-area labour markets in this period. Further, the absence of the mark-up relation is an indication that firms have increasingly been using pricing-to-market rather than simple mark-up pricing in face of varying degrees of local competition in the world market.

Moreover, it is noteworthy that neither the fiscal-policy rule ($Fecm_{1,t-1}$) nor the monetary-policy rule ($Mecm_{1,t-1}$) enter the inflation equation. The only short-run effect of policy on inflation comes from money supply, which is only very imperfectly controlled by the central bank. Controllability of inflation by means of a monetary-policy rule is based on the assumption that inflation is mainly determined by the demand side. The results from the MSM here are instead consistent with supply-side effects dominating inflation dynamics. As a result, central banks have largely been unable to control inflation in this period. This is in line with the results of Castle and Hendry (2009) who, analysing UK wages and prices, show that an inflation target works only when inflation is excess-demand driven. When price changes are supply-driven the effectiveness of a policy trying to control inflation via interest rates becomes much less effective.

Based on (13), we can solve for the long-run solution for inflation,

$$\Delta p^* = \underset{[1.45]}{0.01} ppp^* - \underset{[-1.84]}{0.25} (p - p_y)^* + \underset{[3.27]}{0.06} m_r^* + \underset{[3.71]}{0.66} I_l^* + \underset{[2.23]}{0.26} I_l^{US\,*} - \underset{[-3.07]}{0.30} \Delta p_l^{US\,*} \tag{14}$$

where an asterisk denotes the long-run solution derived from substituting the definitions of the ECM terms and setting all changes to zero in (13). The static equation shows the long-run impact of (the level) of the individual variables in the system (insignificant terms have been left out). The Wald test strongly rejects the null of all coefficients being equal to zero and tests confirm that it forms a stationary (cointegrating) relation. For all variables in (14), we previously found evidence that they were likely determined primarily by outside factors and thus a causal interpretation might be justified here.

Based on the significance of the right-hand side variables, inflation moves mainly with the long-term interest rate but there are also significant positive contributions from real money and the approximate US real bond rate. The presence of real money in (14) gives some support to the monetarist view that money causes prices to move. However, broad money seems only imperfectly controlled by the central bank. Indeed, financial deregulation and innovations in finance over past decades imply that credit expansion within the banking system is largely out of the hands of monetary policy. The short rate, which is the only instrument under direct control by the monetary authorities, does not enter (14). In contrast with the monetarist prediction, money is also far from being the sole factor determining inflation. Notably, the importance of the long-term interest rates again points to the importance of supply-side factors: while a rise in interest rates may affect demand in the short run, the net effect of a rise in the cost of capital in the long run is a rise in firms' costs; (14) suggests that this has been passed on to consumers.

The static solution mirrors the short-run effects insofar as the real exchange rate and the price wedge are concerned. Inflation is positively associated with a real depreciation which could suggest that higher costs of imported production parts is passed on to consumers. This again points to inflation being largely cost-based. The pricing power of producers rises when the currency weakens which could be another explanation that a depreciation is associated with higher consumer prices both at home and abroad. The price wedge sees a negative co-movement with inflation, highlighting that increasing competition in the global product market has exerted a persistent downward pressure on inflation during the sample. Notably, both real wages and unemployment are missing in (14). The fact that the unemployment rate is absent in the long run, despite the finding

	MSM	AWM
AIC	$\mathbf{-9.77850}$	-9.45379
SC	$\mathbf{-9.48289}$	-9.15818
HQ	$\mathbf{-9.65990}$	-9.33519
$\widehat{\sigma}$	**0.001719**	0.002023
H_0: model encompasses rival	$F(9,62) = \underset{[p=0.3372]}{1.1585}$	$F(9,62) = \underset{[p=0.0003^{**}]}{4.0726}$

Table 3: MSM vs. AWM Δp-equation: information criteria and encompassing tests.

that inflation adjusted towards the Phelps-Phillips curve in the short run, suggests that the trade-off has existed as a 'medium-run' relation only.

Together these observations on $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$ and the static solution for inflation provide a picture of inflation determination at different horizons. The evidence from the MSM on inflation dynamics can be summarised as follows. A rise in the long rate brings about an increase in the NAIRU level, which the unemployment rate adjusts towards, and this in turn prompts a downward adjustment in inflation. In the long run, this trade-off dies out and prices are instead determined by firms' costs as measured by the cost of capital at home (I_l) and abroad ($I_l^{US} - \Delta p^{US}$), liquidity conditions (m_r), the pricing power of firms and the cost of imported parts (ppp), and the level of competition in the tradable sector ($p - p_y$).

5.2.3 Comparison of the AWM and the MSM

To evaluate the explanatory power of the MSM inflation equation, we compare it to the AWM. The AWM is essentially a medium-scale macroeconomic model based on a synthesis of New-Classical and New-Keynesian economics. In the long run, output is determined by technological progress and available factors of production, the Phillips curve is vertical and money is neutral as well as super-neutral. In the short run, output is driven by demand such that wage and price rigidities are generated. The wage-price block of the AWM models inflation and growth in unit labour costs jointly, assuming that firms set prices as a mark-up over unit costs. Unit costs in turn depend on, among other things, capacity utilisation. Wages respond to the level of slack in the labour market. Dynamic homogeneity is imposed and the NAIRU level is taken as exogenous.

Focusing on inflation, we contrast the explanatory power of the information set that the MSM procedure gives rise to with that of the price-wage block of the ECB's AWM; this type of comparison is inspired by that used in Bårdsen, Eitrheim, Jansen, and Nymoen (2005). The Autometrics algorithm is used in locating parsimonious models from the GUMs that the two information sets give rise to. Table C.1 shows the information sets used in specifying the AWM and the MSM GUMs. For the comparison, we re-estimate the MSM inflation equation with Δp_t as the left-hand side variable and include Δp_{t-1} in the GUM since inflation is treated as a stationary variable in the AWM. From Autometrics we obtain the following reduced-form AWM inflation equation,

$$\begin{aligned}\widehat{\Delta p_t}^{AWM} &= \underset{[5.18]}{0.09}\Delta p_{i,t-1} - \underset{[-1.98]}{0.03}\,AWMecm_{2,t-1} - \underset{[-1.69]}{0.07}\,AWMecm_{3,t-1} \\ &\quad +\underset{[5.38]}{0.46}\overline{ulc}_{t-2} - \underset{[-4.15]}{0.34}\,\Delta ws_{t-2} - \underset{[-3.47]}{0.01}\,\Delta p_{r,t-2} - \underset{[-3.78]}{0.34}\,\Delta q_{t-2} + \underset{[9.11]}{0.0058}, \qquad (15)\end{aligned}$$

where the algorithm includes two out of three ECM terms from the price-wage block of the AWM. The bulk of explanatory power in (15) comes from lagged values of the trend in unit labour costs, import-price inflation and the wage share however. The re-estimated MSM inflation equation becomes,

$$\begin{aligned}\widehat{\Delta p_t}^{MSM} &= \underset{[3.96]}{0.03}Mecm_{2,t-1} - \underset{[-3.86]}{0.36}\,Eecm_{2,t-1} - \underset{[-10.3]}{1.17}\,Lecm_{1,t-1} + \underset{[4.41]}{0.18}Lecm_{2,t-1} \\ &\quad +\underset{[15.1]}{0.72}\Delta p_{t-1} + \underset{[5.03]}{0.02}ppp_{t-1} - \underset{[-2.15]}{0.30}\,\Delta PD/Y_{t-1} + \underset{[3.78]}{0.23}\Delta^2 p^{US}_{t-1} + \underset{[4.63]}{0.0023}, \qquad (16)\end{aligned}$$

which is broadly similar to (13) although $Eecm_{2,t-1}$ is included in place of $Eecm_{1,t-1}$.

The information criteria reported in Table 3 all point to the superiority of the MSM in explaining the variation in inflation as do the standard errors of regression. The null that the inflation equation of the MSM encompasses that of the AWM cannot be rejected whereas the complementary test of the AWM encompassing the MSM strongly rejects the null. With the reservation that the AWM serves other purposes than simply fitting the data well, it is remarkable how clearly our model outperforms the AWM when it comes to characterising the dynamics of inflation.

5.2.4 Unemployment

Considerable error-correction of the unemployment rate towards deviations from the external-competitiveness relation ($Lecm_{2,t-1}$) points to the importance of the unemployment-productivity dynamics argued by Juselius (2003) to arise from firms laying off the least productive workers in an attempt to boost productivity in an environment of increased competition and rigid real wages. Although unemployment has not responded to the wage relation, a sign that unions moderated wage demands only slowly, it has however reacted to deviations from the mark-up relation ($Lecm_{3,t-1}$) such that a rise in productivity-adjusted real wages, approximately the inverse of firms' mark-ups, has triggered a fall in employment. The sign of the IKE relation ($Eecm_{2,t-1}$) shows that an 'excessive' real depreciation has had a tendency to be followed by lower unemployment, likely via a boost to external competitiveness. Moreover, monetary and fiscal policy appear to have affected employment. But, while a loosening of monetary policy shows potential in rising employment, expansionary fiscal policy surprisingly has somewhat surprisingly had the opposite effect.

From its long-run static solution we see that the unemployment rate has been negatively related to a real depreciation and to real output but positively to labour productivity, public deficits and debt. These effects are broadly similar to the short-run dynamics. The bulk of explanatory power comes from the interest rates though. In particular, the spreads between the long- and short-term interest rates in both the euro-area and the US which have co-moved with the unemployment rate. The euro-area spread has a negative sign whereas the US one enters with a positive coefficient; a crude calculation of the 'net effect' of the four interest rates suggests that it is positive. One possible explanation is that while the domestic central bank(s) has (have) been able to affect the cyclical component of unemployment by changing the short rate, it is the global (US) long rate that has driven the NAIRU component in accordance with the mechanism discussed in relation to inflation dynamics. A detailed study of the determinants of the 'natural rate' would be required to test this hypothesis in depth.

5.2.5 Fiscal policy

The primary deficit has exhibited a large degree of persistency, likely reflecting that once a fiscal act has been authorised, it bears on the stance of policy for a long period of time. The deficit error-corrects significantly to fiscal disequilibria as measured by $Fecm_{1,t-1}$, a sign that the government sector in the euro area as a whole has indeed followed a fiscal rule, attempting to keep fiscal policy sustainable as prescribed by the SGP while, at the same time, reacting to the business cycle. It is nonetheless surprising that deviations from the fiscal rule have not been accompanied by a reaction in outstanding public debt-to-GDP as expected from a pure accounting point of view. Instead, debt is highly persistent, an indication that the debt-service burden has overshadowed additions from contemporaneous primary deficits.

Although fiscal policy has had little direct effect on prices, it nevertheless seems to have affected activity in an unfavourable direction: expansionary policy has had a tendency to be followed by declines in both output and employment. The composition of the fiscal rule shows that this is not due to pro-cyclical policy and it may thus instead be attributed to crowding-out and higher risk premia. While bond yields seemed largely exogenous in our modelling framework, Tuxen (2009) find evidence of the hypothesis in Juselius (2002) that a vicious circle of rising unemployment and public debt has exerted upward pressure on bond yields in this period, *ceteris paribus*. Hence it is likely that the demand-supporting effects of a rise in the government deficit have been neutralised as the market has become concerned about the sustainability of policy and the implications for future tax burdens and growth; see also ECB (2004a). The finding that fiscal stimulus has had an outright negative effect on the economy is an indication that public expenditure may not have been put to its most productive use in this period.[23] This could reflect the rigid institutional structures dominating many European labour markets with high minimum wages, inflexible hiring/firing regulations and high unemployment benefits dis-encouraging job search for some groups of workers.[24] A disaggregated model of public finances would be required to study more carefully the supply-side effects of public policy, for example using a model similar to that of Henry, de Cos, and Momigliano (2004).

5.2.6 Monetary policy

The reaction of the short rate to $Mecm_{1,t-1}$ supports the appraisal of this as a monetary-policy rule, albeit the short rate shows a large degree of persistency as judged from the significance of the lagged value in this equation. Similarly, the money-demand interpretation of $Mecm_{2,t-1}$ is sustained by the considerable error-correction behaviour of money supply to deviations from this relation. Expansionary monetary policy appears to have had an employment-stimulating effect as a fall in the policy rate and/or excess money supply have led unemployment to decline.

Of the two ECM terms related to monetary policy, only excess money has had a

[23]As the primary deficit excludes the costs of servicing debt this 'unproductive use' cannot solely be attributed to high levels of outstanding debt-to-GDP.

[24]The Lisbon Strategy constitutes an attempt to promote structural reforms, and thereby growth prospects, in the euro area in order to cope with the disincentives created by inflexible institutional arrangements in, for example, the labour market.

direct effect on inflation. This may not seem surprising given the small coefficient of inflation in the monetary-policy rule which indicated that not all central banks within the euro area and/or the ECB have reacted firmly to price pressures at all times. From studying the inflation equation it was clear that the downward pressures on wages and prices from increased competition has been an essential factor in determining inflation. This mechanism seems to have dominated any effect of central bank policy on prices in the medium run. The near-weak exogeneity status of both the long rate and the level of real money supply in fact suggests that domestic monetary authorities committed to price stability would not be able to control inflation simply by means of the short rate.

Together, these results highlight that it be beneficial to allow central banks more flexibility in responding to economic activity rather than adhere to strict inflation targeting.

5.2.7 Policy interactions

The short (policy) rate has not showed significant reactions to deviations from the fiscal rule ($Fecm_{1,t-1}$), but it has had a tendency to rise in response to an increase in the level of debt-to-GDP. This type of reaction may represent the response of central banks to growth in public debt. On the other hand, real-money supply has increased to some extent in response to a rise in the deficit. Although monetary financing of deficits, i.e. using the printing press, is prohibited by the Treaty and has not been a predominant source of funding in this period, there is evidence that some central banks have to a certain extent accommodated government policy by an increase in the quantity of money. On the other hand, it also appears that monetary-policy makers have raised the price of money, i.e. the policy rate, when debt levels have become 'too high'.

A tightening in credit conditions as measured by the monetary rule ($Mecm_{1,t-1}$) has been followed by a rise in the primary deficit and the debt-to-GDP ratio alike. From this, it seems that there has been a tendency for the fiscal stance to loosen, either due to automatic stabilisers or as a result of discretionary policy, following contractive monetary policy and hence to counteract central-bank decisions.

These findings raise the question of whether the EMU is a regime of monetary dominance (as the Maastricht Treaty requires) or fiscal dominance (as our results hint at). The above indications of monetary and fiscal authorities apparently working against each other could be an additional explanation why policies have seen difficulties in contributing to their respective targets. To the extent that governments have counteracted tighter monetary policy by loosening the fiscal stance, for example, to improve chances of re-election, this may help explain the adverse effects of monetary policy. This is consistent with the prediction of the theoretical model developed by Kirsanova et al. (2005) who argue that because the fiscal authorities within the euro area are unlikely to be able to coordinate among each other, the EMU arrangement could potentially result in large welfare costs. An empirical analysis of the differences in fiscal-policy responses across euro-area countries would be required to shed more light on this issue.

6 Conclusion

This paper has used a multi-sector modelling (MSM) approach based on the CVAR to model the transmission of policy in the euro area, with a special view to study inflation dynamics in recent decades. We extended the procedure of Juselius (2006) to include a joint analysis of disequilibria in the public sector, the financial market, the external sector and the labour market alike and proposed a set of robustness checks to study its sensitivity to alternative specifications. Exploiting the information on the pulling vs. the pushing forces that the CVAR provides, we showed how to distinguish the empirical evidence on short-, medium- and long-run dynamics based on the MSM.

Our results suggest that prices have exhibited Phillips-curve effects in the short and the medium run when a non-constant NAIRU, which varies with the cost of capital, is taken into account. In the medium run, unemployment has been driven by the need for euro-area producers to restore external competitiveness; in face of a rigid real wage, improvements in labour productivity appear to have been facilitated by lay-offs, consistent with the findings of Juselius (2003). In the long run, the Phillips curve was found to be vertical, and inflation largely seemed to have been determined by factors outside central-bank control. Prices have been driven by the cost of capital at home and abroad, liquidity conditions, the pricing power of firms and the cost of imported parts, as well as the level of competition in the tradable sector. From a comparison of the information set of the ECB's AWM to that of the MSM, we found that the latter outperforms the former regarding inflation dynamics in this period.

The MSM gave a detailed picture on macroeconomic transmission in general, and, in particular, it allowed us to assess the effects of monetary and fiscal policy as well as their interactions. We found that fiscal policy has had a negligible effect on inflation but that an expansionary policy stance led to negative disruptions in productivity and employment.

This could be a signal of the necessity of structural reforms, for example, of the rigid labour-market institutions in many euro-area countries. Central banks were found to have had little influence on bond yields and inflation as both seemed largely determined by exogenous factors. Notwithstanding, we did find some evidence that expansionary monetary policy affected output and employment positively in the short run.

Overall, inflation appears to have been mainly cost-based in the past decades. Combined with evidence of downward pressure on prices from competitive pressures arising from low-cost production in a range of emerging economies, this suggests that inflation has been determined primarily from the supply side. Under these circumstances, the role of central banks may have to be re-considered: changes in the policy rate will be much less effective for controlling inflation when excess demand is not the primary cause of price movements. Our results suggest that it may be beneficial to allow the ECB more flexibility in responding to fluctuations in economic activity and, as the recent experience suggests, possibly asset prices too. In a globalised world with highly integrated goods and capital markets, it is likely to be the global stance of monetary policy that matters however. A larger degree of coordination of actions among central banks could therefore be crucial for policy effectiveness.

References

Bårdsen, G., Ø. Eitrheim, E. S. Jansen, and R. Nymoen (2005). *The Econometrics of Macroeconomic Modelling*. Oxford University Press, Oxford.

Bårdsen, G., E. S. Jansen, and R. Nymoen (2004). Econometric evaluation of the New-Keynesian Phillips curve. *Oxford Bulletin of Economics and Statistics 66 (Supplement)*, 671–686.

Beetsma, R. M. W. J. and H. Jensen (2002). Monetary and fiscal policy interactions in a micro-founded model of a monetary union. *ECB Working Paper 166*.

Bernanke, B. S. and M. Gertler (1995). Inside the black box: The credit channel of monetary policy transmission. *Journal of Economic Perspectives 9*(4), 27–48.

Beyer, A., J. A. Doornik, and D. F. Hendry (2001). Constructing historical euro-zone data. *The Economic Journal 111*, F102–F121.

Beyer, A. and K. Juselius (2008). Does it matter how to measure aggregates? The case of monetary transmission mechanisms in the euro area. *Working paper, Department of Economics, University of Copenhagen 08-07*.

Bjørnstad, R. and R. Nymoen (2008). The New-Keynesian Phillips curve tested on OECD panel data. *Economics: The Open-Access, Open-Assessment E-Journal 2*(2008-23).

Blanchard, O. and L. F. Katz (1999). Wage dynamics: Reconciling theory and evidence. *The American Economic Review 89*(2), 69–74.

Bordio, C. and A. Filardo (2007). Globalisation and inflation: New cross-country evidence on the global determinants of domestic inflation. *BIS Working Paper 227*.

Castle, J. L. and D. F. Hendry (2009). The long-run determinants of UK wages, 1860-2004. *Journal of Macroeconomics 31*(5-28).

Christoffel, K., G. Coenen, and A. Warne (2008). The new area-wide model of the euro-area: A micro-founded open-economy model for forecasting and policy analysis. *ECB Working Paper 944*.

Ciccarelli, M. and B. Mojon (2005). Global inflation. *ECB Working Paper 537*.

Coenen, G. and J. L. Vega (1999). The demand for M3 in the euro area. *ECB Working Paper 6*.

Coenen, G. and V. Wieland (2000). A small estimated euro area model with rational expectations and nominal rigidities. *ECB Working Paper 30*.

Dees, S., S. Holly, M. H. Pesaran, and L. V. Smith (2007). Long run macroeconomic relations in the global economy. *Economics: The Open-Access, Open-Assessment E-Journal 2007-3*.

Dennis, J. G., H. Hansen, S. Johansen, and K. Juselius (2006). CATS for RATS, version 2. Available from Estima.

Doornik, J. A. (2007a). Autometrics. *Working paper, Department of Economics, University of Oxford*.

Doornik, J. A. (2007b). An introduction to OxMetrics 5. Available from Timberlake Consultants Ltd.

ECB (2004a). EMU and the conduct of fiscal policies. *ECB Monthly Bulletin January*.

ECB (2004b). *The Monetary Policy of the ECB*. European Central Bank, Frankfurt am Main.

Fagan, G., J. Henry, and R. Mestre (2001). An Area-Wide Model (AWM) for the euro area. *ECB Working Paper 42*.

Ferrero, A. (2005). Fiscal and monetary rules for a currency union. *ECB Working Paper 502*.

Friedman, M. (1969). The optimum quantity of money. In *The Optimum Quantity of Money and Other Essays*. Aldine, Chicago.

Frydman, R. and M. Goldberg (2007). *Imperfect Knowledge Expectations: Exchange rates and risk*. Princeton University Press.

Gerlach, S. and L. E. O. Svensson (2003). Money and inflation in the euro area: A case for monetary indicators? *Journal of Monetary Economics 50*, 1649–1972.

Giese, J. V. and C. K. Tuxen (2009). Asset prices and liquidity spill-overs. Evidence from a Global VAR. *Manuscript*.

Harbo, I., S. Johansen, B. Nielsen, and A. Rahbek (1998). Asymptotic inference on cointegrating rank in partial systems. *Journal of Business and Economic Statistics 16*(4), 388–399.

Hendry, D. F. (1995). *Dynamic Econometrics*. Oxford University Press, Oxford.

Hendry, D. F. (2001). Modelling UK inflation, 1875-1991. *Journal of Applied Econometrics 16*, 255–275.

Henry, J., P. H. de Cos, and S. Momigliano (2004). The short-term impact of government budgets on prices. Evidence from macroeconometric models. *ECB Working Paper 396*.

Hoover, K. D., S. Johansen, and K. Juselius (2008). Allowing the data to speak freely: The macroeconometrics of the cointegrated Vector Autoregression. *American Economic Review 98*(2), 251–255.

IMF (2006). How has globalization affected inflation? *World Economic Outlook April 2006*, 97–134.

Johansen, S. (1988). Statistical analysis of cointegration vectors. *Journal of Economic Dynamics and Control 12*, 231–254.

Johansen, S. (1991). Estimation and hypothesis testing of cointegration vectors in Gaussian vector autoregressive models. *Econometrica 59*, 1551–1580.

Johansen, S. (1996). *Estimation and Hypothesis Testing of Cointegration Vectors in Gaussian Vector Autoregressive Models* (2nd ed.). Oxford University Press, Oxford.

Johansen, S. and K. Juselius (2003). Controlling inflation in a cointegrated VAR model with an application to US data. *Working Paper, Department of Economics, University of Copenhagen 01-03*.

Johansen, S., K. Juselius, R. Frydman, and M. Goldberg (2008). A resolution of the purchasing power parity puzzle: Imperfect knowledge and long swings. *Working Paper, Department of Economics, University of Copenhagen 08-31*.

Juselius, K. (1992). Domestic and foreign effects on prices in an open economy: The case of Denmark. *Journal of Economic Modelling 14*(4), 401–428.

Juselius, K. (2002). European integration: Aspects on inflation and unemployment. *Manuscript (available at www.econ.ku.dk/okokj)*.

Juselius, K. (2003). Wage, price and unemployment dynamics and the convergence to purchasing power parity in the euro area. *Working Paper, Institute of Economics, University of Copenhagen*.

Juselius, K. (2006). *The Cointegrated VAR Model - Methodology and Applications*. Oxford University Press, Oxford.

Juselius, K., R. Frydman, M. Goldberg, and S. Johansen (2009). Scenario analysis of economic models based on RE and IKE. *Manuscript*.

Juselius, K. and R. MacDonald (2007). *International Macroeconomics: Recent Developments*, Chapter : International Parity Relationships between Germany and the United States: A Joint Modelling Approach, pp. 79–103. Nova Science Publishers.

Juselius, K. and J. Ordóñez (2009). Balassa-Samuelson and wage, price and unemployment dynamics in the Spanish transition to EMU membership. *Economics: The Open-Access, Open-Assessment E-Journal 4*.

Juselius, M. (2008). Testing the New-Keynesian model on U.S. and euro area data. *Economics: The Open-Access, Open-Assessment E-Journal 2*(2008-24).

Kirsanova, J. T., S. J. Stehn, and D. Vines (2005). Five-equation macroeconomics: A simple view of the interactions of fiscal and monetary policy. *Oxford Review of Economic Policy 21*(4), 532–564.

Leith, C. and L. V. Thadden (2006). Monetary and fiscal policy interactions in a New-Keynesian model with capital accumulation and non-Ricardian consumers. *ECB Working Paper 649*.

Metin, K. (1995). An integrated analysis of Turkish inflation. *Oxford Bulletin of Economics and Statistics 57*, 513–531.

Mishkin, F. S. (1995). Symposium on the monetary transmission mechanism. *Journal of Economic Perspectives 9*(4), 3–10.

Møller, N. F. (2009). Bridging economic theory models and the cointegrated vector autoregressive model. *Economics: The Open-Access, Open-Assessment E-Journal 2*(2008-36).

Pain, N., I. Koske, and M. Sollie (2006). Globalisation and inflation in the OECD economies. *OECD Economics Department Working Papers 524*.

Phelps, E. S. (1994). *Structural Slumps - The Modern Equilibrium Teory of Unemployment, Interest, and Assets.* Harvard University Press, Cambridge, Massachusetts and London, England.

Phillips, A. W. (1958). The relation between unemployment and the rate of change of money wage rates in the United Kingdom. *Econometrica 25*, 1861–1957.

Rotemberg, J. and M. Woodford (1999). The cyclical behavior of prices and costs. In J. B. Taylor and M. Woodford (Eds.), *Handbook of Marcoeconomics*, pp. 1052–1135. Elsevier, Amsterdam.

Ruffer, R. and L. Stracca (2006). What is global excess liquidity, and does it matter? *ECB Working Paper 696*.

Smets, F. and R. Wouters (2002). An estimated stochastic general equilibrium model of the euro area. *ECB Working Paper 171*.

Surrey, M. J. C. (1989). Money, commodity prices and inflation: Some simple tests. *Oxford Bulletin of Economics and Statistics 51*.

Taylor, J. B. (1993). Discretion versus policy rules in practice. *Carnegie-Rochester Conference Series on Public Policy 39*, 195–214.

Tuxen, C. K. (2006). An empirically relevant model of inflation in the euro area. Master's thesis, Department of Economics, University of Copenhagen.

Tuxen, C. K. (2009). Interactions of monetary and fiscal policy: An I(2) cointegrated VAR study of deficit-debt dynamics in the euro area. *Manuscript*.

A Data series

Variable	Notation	Source
Consumer price index (CPI)	p	AWM: PCD
CPI inflation	Δp	-
Public debt-to-GDP	GD/Y	AWM: GDN_YEN
Primary deficit-to-GDP	PD/Y	AWM: (-) GPN_YEN
Nominal money stock	m	OECD: M3
Real money stock	$m_r \equiv m - p$	-
Nominal output (GDP)	y	AWM: YEN
Real GDP	$y_r \equiv y - p$	-
Nominal GDP growth	Δy	-
Short-term interest rate	I_s	AWM: STN
Long-term interest rate	I_l	AWM: LTN
Compensation to employees	w	AWM: WIN
Real wages	$w_r \equiv w - p$	-
Labour productivity	q	AWM: LPROD
Unemployment	U	AWM: URX
GDP deflator / producer price index (PPI)	p_y	AWM: YED
Price wedge (CPI vs. PPI)	$p - p_y$	-
Exchange rate (*vis-à-vis* US)	s	OECD: EUR per USD
Foreign CPI inflation	Δp^{US}	OECD: CPI
Foreign short-term interest rate	I_s^{US}	OECD: 10-year bond rate
Foreign long-term interest rate	I_l^{US}	OECD: three-month interbank rate
Real exchange rate	$ppp \equiv s - p + p^{US}$	-
PPP convergence (intra-euro area)	$conv$	See Tuxen (2006) for details
Wage share	ws	AWM: WRN/(LPROD*PCD)
Commodity price index	p_r	AWM: COMPR*EEN
GDP deflator at factor prices	p_f	AWM: YFD
Import price deflator	p_i	AWM: MTD
Indirect taxes	tax	AWM: (1+TIN/YEN)
Trend unit labour costs	$\overline{ulc}$	AWM: ULT
Trend unemployment	$\overline{U}$	AWM: URT
Output gap	gap	AWM: YGA

Table A.1: Variables and data sources (4th update of the AWM database).

Figure A.1: Data series in levels.

B Sector models

B.1 Public sector

Lag	Test statistic	p
LM tests for no autocorrelation		
1	$\chi^2(36) = 45.78$	0.13
2	$\chi^2(36) = 36.72$	0.44
3	$\chi^2(36) = 38.31$	0.37
4	$\chi^2(36) = 40.48$	0.28
Test for multivariate normality		
	$\chi^2(12) = 18.45$	0.10
LM tests for no ARCH effects		
1	$\chi^2(441) = 447.20$	0.41
2	$\chi^2(882) = 934.46$	0.11
3	$\chi^2(1323) = 1392.17$	0.09
4	$\chi^2(1764) = 1742.58$	0.64

Table B.1: Public sector: misspecification tests.

$p-r$	r	Eigenvalue	Trace	$CV_{95\%}$	p
6	0	0.66	198.32	103.68	0.00
5	1	0.43	110.49	76.81	0.00
4	2	0.26	64.31	53.94	0.00
3	3	0.22	39.60	35.07	0.01
2	4	0.19	19.56	20.16	0.06
1	5	0.03	2.07	9.14	0.76

Table B.2: Public sector: rank test.

$\boldsymbol{\beta}'$	PD/Y_t	GD/Y_t	Δp_t	$I_{l,t}$	U_t	Δy_t	1
β_1'	1.00 [NA]	0.12 [13.69]	0.00 [NA]	0.00 [NA]	−0.65 [−11.07]	0.00 [NA]	0.00 [NA]
β_2'	−12.66 [−3.23]	1.00 [NA]	0.00 [NA]	−53.94 [−6.57]	0.00 [NA]	83.79 [12.71]	−0.99 [−5.64]
β_3'	0.00 [NA]	0.00 [NA]	1.00 [NA]	−0.73 [−17.19]	0.06 [6.23]	0.00 [NA]	0.00 [NA]

$\boldsymbol{\alpha}$	α_1	α_2	α_3
$\Delta PD/Y_t$	−0.05 [−3.00]	−0.00 [−0.99]	−0.02 [−0.61]
$\Delta GD/Y_t$	−0.06 [−1.00]	−0.00 [−1.95]	−0.28 [−2.13]
$\Delta^2 p_t$	−0.04 [−0.58]	−0.00 [−0.17]	−0.72 [−5.13]
$\Delta I_{l,t}$	0.00 [0.07]	0.00 [1.74]	−0.03 [−0.67]
ΔU_t	0.11 [4.32]	0.00 [0.79]	0.09 [1.59]
$\Delta^2 y_t$	−0.20 [−1.79]	−0.02 [−7.96]	0.74 [3.01]

Table B.3: Public sector: cointegration relations and adjustment structure.

$\boldsymbol{\Gamma}_1$	$\Delta PD/Y_{t-1}$	$\Delta GD/Y_{t-1}$	$\Delta^2 p_{t-1}$	$\Delta I_{l,t-1}$	ΔU_{t-1}	$\Delta^2 y_{t-1}$
$\Delta PD/Y_t$	0.90 [17.95]	−0.01 [−0.42]	0.03 [0.94]	0.30 [4.14]	0.12 [1.64]	0.00 [0.13]
$\Delta GD/Y_t$	−0.21 [−1.29]	0.85 [14.06]	0.18 [1.56]	0.55 [2.29]	0.14 [0.58]	0.06 [1.24]
$\Delta^2 p_t$	0.12 [0.71]	−0.15 [−2.32]	−0.00 [−0.01]	−0.33 [−1.29]	0.17 [0.65]	−0.02 [−0.31]
$\Delta I_{l,t}$	−0.11 [−1.79]	−0.02 [−0.75]	0.09 [2.16]	0.46 [5.29]	0.10 [1.10]	−0.02 [−1.10]
ΔU_t	0.04 [0.54]	0.06 [2.34]	0.02 [0.49]	−0.29 [−2.90]	0.42 [4.13]	−0.02 [−1.01]
$\Delta^2 y_t$	0.41 [1.35]	−0.49 [−4.40]	−0.13 [−0.63]	0.84 [1.90]	−0.73 [−1.59]	0.19 [1.96]

Table B.4: Public sector: short-run parameters.

B.2 Financial market

Lag	Test statistic	p
LM tests for no autocorrelation		
1	$\chi^2(25) = 26.65$	0.37
2	$\chi^2(25) = 28.29$	0.29
3	$\chi^2(25) = 29.41$	0.25
4	$\chi^2(25) = 19.80$	0.76
Test for multivariate normality		
	$\chi^2(10) = 13.20$	0.21
LM tests for no ARCH effects		
1	$\chi^2(225) = 222.88$	0.53
2	$\chi^2(450) = 441.61$	0.60
3	$\chi^2(675) = 697.68$	0.26
4	$\chi^2(900) = 967.79$	0.06

Table B.5: Financial market: misspecification tests.

$p-r$	r	Eigenvalue	Trace	$CV_{95\%}$	p
5	0	0.52	146.72	88.55	0.00
4	1	0.30	87.10	63.66	0.00
3	2	0.28	58.58	42.77	0.00
2	3	0.23	31.63	25.73	0.01
1	4	0.12	10.16	12.45	0.12

Table B.6: Financial market: rank test.

$\boldsymbol{\beta}'$	$m_{r,t}$	$y_{r,t}$	Δp_t	$I_{s,t}$	$I_{l,t}$	t
β'_1	0.00 [NA]	−0.10 [−11.17]	−0.65 [−8.63]	1.00 [NA]	0.00 [NA]	0.07 [13.77]
β'_2	1.00 [NA]	−1.00 [NA]	0.00 [NA]	0.00 [NA]	8.52 [12.43]	0.00 [NA]

$\boldsymbol{\alpha}$	α_1	α_2
$\Delta m_{r,t}$	−0.01 [−0.02]	−0.07 [−3.99]
$\Delta y_{r,t}$	−0.95 [−3.41]	−0.05 [−2.99]
$\Delta^2 p_t$	0.36 [2.15]	−0.01 [−0.84]
$\Delta I_{s,t}$	−0.31 [−5.92]	0.00 [0.63]
$\Delta I_{l,t}$	−0.05 [−0.88]	−0.00 [−0.78]

Table B.7: Financial market: cointegration relations and adjustment structure.

$\boldsymbol{\Gamma}_1$	$\Delta m_{r,t-1}$	$\Delta y_{r,t-1}$	$\Delta^2 p_{t-1}$	$\Delta I_{s,t-1}$	$\Delta I_{l,t-1}$
$\Delta m_{r,t}$	0.18 [2.39]	−0.16 [−1.46]	−0.29 [−1.30]	0.71 [1.43]	−0.77 [−1.27]
$\Delta y_{r,t}$	−0.01 [−0.10]	−0.19 [−1.82]	−0.45 [−2.08]	0.51 [1.06]	1.06 [1.81]
$\Delta^2 p_t$	0.06 [1.28]	0.10 [1.55]	−0.13 [−0.99]	−0.14 [−0.49]	0.07 [0.20]
$\Delta I_{s,t}$	0.00 [0.31]	0.00 [0.10]	−0.06 [−1.58]	0.44 [4.96]	0.02 [0.17]
$\Delta I_{l,t}$	0.00 [0.03]	0.03 [1.51]	0.05 [1.24]	−0.05 [−0.56]	0.47 [4.21]

Table B.8: Financial market: short-run parameters.

B.3 External sector

Lag	Test statistic	p
LM tests for no autocorrelation		
1	$\chi^2(49) = 52.56$	0.34
2	$\chi^2(49) = 35.31$	0.93
3	$\chi^2(49) = 69.22$	0.03
4	$\chi^2(49) = 65.01$	0.06
Test for multivariate normality		
	$\chi^2(14) = 16.87$	0.26
LM tests for no ARCH effects		
1	$\chi^2(784) = 841.20$	0.08
2	$\chi^2(1568) = 1719.21$	0.00
3	$\chi^2(2352) = 2268.00$	0.89
4	$\chi^2(3136) = 2268.00$	1.00

Table B.9: External sector: misspecification tests.

$p - r$	r	Eigenvalue	Trace	$CV_{95\%}$	p
7	0	0.55	171.02	134.54	0.00
6	1	0.41	106.55	103.68	0.03
5	2	0.31	63.41	76.81	0.35
4	3	0.18	32.88	53.94	0.81
3	4	0.14	16.90	35.07	0.88
2	5	0.05	4.35	20.16	0.99
1	6	0.01	0.46	9.14	0.99

Table B.10: External sector: rank test.

$\boldsymbol{\beta}'$	ppp_t	Δp_t	Δp_t^{US}	$I_{l,t}$	$I_{l,t}^{US}$	$I_{s,t}$	$I_{s,t}^{US}$	1
β_1	1.00 [NA]	−134.32 [−6.32]	−305.29 [−9.33]	0.00 [NA]	0.00 [NA]	134.32 [6.32]	48.73 [2.69]	0.66 [2.53]
β_2	1.00 [NA]	−72.71 [−14.79]	61.01 [12.11]	72.71 [14.79]	−61.01 [−12.11]	0.00 [NA]	0.00 [NA]	0.00 [NA]

$\boldsymbol{\alpha}$	α_1	α_2
Δppp_t	−0.01 [−2.26]	−0.02 [−0.74]
$\Delta^2 p_t$	0.00 [4.97]	0.01 [4.64]
$\Delta^2 p_t^{US}$	0.00 [6.63]	−0.00 [−0.74]
$\Delta I_{l,t}$	0.00 [0.59]	−0.00 [−0.85]
$\Delta I_{l,t}^{US}$	−0.00 [−0.69]	−0.00 [−2.15]
$\Delta I_{s,t}$	−0.00 [−0.51]	−0.00 [−1.25]
$\Delta I_{s,t}^{US}$	−0.00 [−0.78]	−0.00 [−3.52]

Table B.11: External sector: cointegration relations and adjustment structure.

$\boldsymbol{\Gamma}_1$	Δppp_{t-1}	$\Delta^2 p_{t-1}$	$\Delta^2 p_{t-1}^{US}$	$\Delta I_{l,t-1}$	$\Delta I_{l,t-1}^{US}$	$\Delta I_{s,t-1}$	$\Delta I_{s,t-1}^{US}$
Δppp_t	0.35 [3.30]	−2.98 [−1.49]	−0.13 [−0.09]	−10.01 [−1.85]	10.81 [2.43]	1.97 [0.48]	−3.23 [−0.87]
$\Delta^2 p_t$	0.02 [3.55]	−0.04 [−0.38]	0.07 [0.92]	0.14 [0.45]	0.62 [2.43]	−0.04 [−0.19]	0.17 [0.80]
$\Delta^2 p_t^{US}$	−0.00 [−0.40]	0.35 [2.56]	0.01 [0.11]	0.10 [0.26]	0.34 [1.12]	0.48 [1.73]	0.40 [1.58]
$\Delta I_{l,t}$	−0.00 [−0.27]	0.05 [1.32]	0.00 [0.19]	0.42 [4.14]	0.36 [4.27]	0.08 [1.03]	−0.04 [−0.54]
$\Delta I_{l,t}^{US}$	−0.00 [−0.74]	−0.11 [−2.03]	0.03 [0.77]	0.07 [0.46]	0.36 [2.91]	−0.02 [−0.15]	−0.22 [−2.14]
$\Delta I_{s,t}$	−0.00 [−0.21]	0.04 [0.93]	0.02 [0.55]	−0.03 [−0.21]	−0.19 [−1.86]	0.50 [5.25]	0.25 [2.96]
$\Delta I_{s,t}^{US}$	−0.00 [−0.99]	−0.11 [−1.91]	0.02 [0.48]	0.15 [1.02]	0.04 [0.36]	−0.15 [−1.33]	0.28 [2.72]

Table B.12: External sector: short-run parameters.

B.4 Labour market

Lag	Test statistic	p
LM tests for no autocorrelation		
1	$\chi^2(49) = 49.71$	0.44
2	$\chi^2(49) = 55.99$	0.23
3	$\chi^2(49) = 38.79$	0.85
4	$\chi^2(49) = 54.90$	0.26
Test for multivariate normality		
	$\chi^2(14) = 13.25$	0.51
LM tests for no ARCH effects		
1	$\chi^2(784) = 838.67$	0.09
2	$\chi^2(1568) = 1652.96$	0.07
3	$\chi^2(2352) = 2268.00$	0.89
4	$\chi^2(3136) = 1831.34$	1.00

Table B.13: Labour market: misspecification tests.

$p-r$	r	Eigenvalue	Trace	$CV_{95\%}$	p
7	0	0.65	279.26	166.05	0.00
6	1	0.52	194.70	131.10	0.00
5	2	0.50	134.54	100.13	0.00
4	3	0.30	78.73	73.13	0.02
3	4	0.28	49.57	50.08	0.06
2	5	0.21	23.42	30.91	0.28
1	6	0.06	4.82	15.33	0.82

Table B.14: Labour market: rank test.

$\boldsymbol{\beta}'$	$w_{r,t}$	q_t	Δp_t	U_t	$(p-p_y)_t$	$I_{l,t}$	ppp_t	$conv_t$	t
β_1'	0.00 [NA]	0.00 [NA]	1.00 [NA]	0.23 [10.10]	0.00 [NA]	-0.71 [-21.61]	0.00 [NA]	0.00 [NA]	0.00 [NA]
β_2'	0.00 [NA]	-0.31 [-4.68]	0.00 [NA]	1.00 [NA]	-0.67 [-6.19]	0.00 [NA]	0.05 [6.06]	0.00 [NA]	0.13 [4.63]
β_3'	1.00 [NA]	-1.00 [NA]	0.00 [NA]	5.23 [23.69]	1.00 [NA]	0.00 [NA]	0.05 [3.73]	-0.86 [-6.63]	0.00 [NA]
β_4'	1.00 [NA]	-3.27 [-14.15]	0.00 [NA]	1.28 [4.63]	0.00 [NA]	0.00 [NA]	0.00 [NA]	1.27 [7.41]	0.89 [9.34]

$\boldsymbol{\alpha}$	α_1	α_2	α_3	α_4
$\Delta w_{r,t}$	0.29 [0.96]	0.62 [4.91]	-0.21 [-5.87]	0.01 [0.48]
Δq_t	1.08 [4.52]	0.17 [1.73]	0.04 [1.38]	0.13 [6.86]
$\Delta^2 p_t$	-0.81 [-4.60]	0.07 [0.93]	0.03 [1.30]	-0.00 [-0.27]
ΔU_t	0.01 [0.15]	-0.09 [-2.88]	0.01 [1.58]	0.01 [2.28]
$\Delta(p-p_y)_t$	0.09 [0.41]	-0.10 [-1.12]	0.02 [0.88]	0.00 [0.17]
$\Delta I_{l,t}$	0.15 [2.23]	0.03 [0.93]	0.00 [0.10]	0.01 [2.54]
Δppp_t	-1.86 [-0.54]	-4.64 [-3.28]	1.30 [3.17]	0.13 [0.47]

Table B.15: Labour market: cointegration relations and adjustment structure.

$\mathbf{\Gamma}_1$	$\Delta w_{r,t-1}$	Δq_{t-1}	$\Delta^2 p_{t-1}$	ΔU_{t-1}	$\Delta(p-p_y)_{t-1}$	$\Delta I_{l,t-1}$	Δppp_{t-1}
$\Delta w_{r,t}$	0.03 [0.26]	0.06 [0.68]	−0.41 [−1.87]	−1.42 [−2.32]	0.50 [1.98]	−0.07 [−0.15]	−0.04 [−3.31]
Δq_t	−0.11 [−1.37]	0.05 [0.67]	−0.66 [−3.92]	−1.84 [−3.87]	−0.18 [−0.91]	1.04 [2.82]	0.02 [1.86]
$\Delta^2 p_t$	−0.03 [−0.47]	−0.02 [−0.30]	−0.08 [−0.66]	−0.25 [−0.71]	−0.17 [−1.20]	−0.06 [−0.21]	0.02 [3.42]
ΔU_t	−0.07 [−2.69]	0.02 [0.86]	0.05 [1.07]	0.41 [2.87]	−0.12 [−2.01]	−0.35 [−3.10]	0.00 [1.20]
$\Delta(p-p_y)_t$	0.00 [0.06]	0.02 [0.39]	0.09 [0.61]	0.08 [0.20]	−0.19 [−1.11]	0.22 [0.67]	0.02 [3.23]
$\Delta I_{l,t}$	0.02 [0.88]	0.03 [1.51]	−0.03 [−0.68]	−0.33 [−2.41]	−0.02 [−0.32]	0.36 [3.38]	0.00 [0.67]
Δppp_t	−0.36 [−0.31]	−0.22 [−0.22]	−0.81 [−0.33]	−8.36 [−1.21]	−2.20 [−0.78]	−4.56 [−0.85]	0.32 [2.63]

$(\mathbf{\Psi}_0, \mathbf{\Psi}_1)$	$\Delta conv_t$	$\Delta conv_{t-1}$
$\Delta w_{r,t}$	−0.04 [−0.66]	−0.08 [−1.35]
Δq_t	−0.05 [−1.04]	0.02 [0.45]
$\Delta^2 p_t$	−0.01 [−0.37]	0.00 [0.09]
ΔU_t	0.03 [2.14]	0.01 [0.73]
$\Delta(p-p_y)_t$	0.01 [0.14]	−0.01 [−0.25]
$\Delta I_{l,t}$	−0.01 [−0.55]	−0.01 [−0.58]
Δppp_t	−1.02 [−1.48]	0.50 [0.74]

Table B.16: Labour market: short-run parameters

C Multi-sector model results

MSM	AWM
$\Delta^2 p_{t-1}$ (Δp_{t-1} and Δp_{t-2} in AWM comparison)	$\Delta p_{t-1}, \Delta p_{t-2}$
Δppp_{t-1}	$\Delta ws_{t-1}, \Delta ws_{t-2}$
$\Delta(p - p_y)_{t-1}$	$\Delta\overline{ulc}_{t-1}, \Delta\overline{ulc}_{t-2}$
$\Delta w_{r,t-1}$	$\Delta p_{r,t-1}, \Delta p_{r,t-2}$
Δq_{t-1}	$\Delta p_{f,t-1}, \Delta p_{f,t-2}$
$\Delta I_{s,t-1}$	$\Delta U_{t-1}, \Delta U_{t-2}$
$\Delta I_{l,t-1}$	$\Delta q_{t-1}, \Delta q_{t-2}$
ΔU_{t-1}	$\Delta p_{i,t-1}, \Delta p_{i,t-2}$
$\Delta PD/Y_{t-1}$	$\Delta tax_{t-1}, \Delta tax_{t-2}$
$\Delta GD/Y_{t-1}$	$gap_{t-1}, \Delta gap_{t-1}$
$\Delta^2 p^{US}_{t-1}$	$AWMecm_{1,t-1} = \left[U - \overline{U}\right]_{t-1}$
$\Delta I^{US}_{s,t}, \Delta I^{US}_{s,t-1}$	$AWMecm_{2,t-1} = \left[p_f - \overline{ulc} + ln(1 - 0.41)\right]_{t-1}$
$\Delta I^{US}_{l,t}, \Delta I^{US}_{l,t-1}$	$AWMecm_{3,t-1} = [p - 0.94 p_y - 0.06 p_i]_{t-1}$
$Fecm_{1,t-1}$	
$Mecm_{1,t-1}, Mecm_{2,t-1}$	
$Eecm_{1,t-1}, Eecm_{2,t-1}$	
$Lecm_{1,t-1}, Lecm_{2,t-1}, Lecm_{3,t-1}, Lecm_{4,t-1}$	

Table C.1: Information sets: MSM vs. AWM price-wage block.

	$\Delta^2 p_t$	Δppp_t	$\Delta(p-p_y)_t$	$\Delta w_{r,t}$	Δq_t	$\Delta m_{r,t}$	$\Delta I_{s,t}$	$\Delta I_{l,t}$	$\Delta y_{r,t}$	ΔU_t	$\Delta PD/Y_t$	$\Delta GD/Y_t$
$Fecm_{1,t-1}$				-0.24 $[-3.45]$	-0.48 $[-5.04]$				-0.65 $[-5.69]$	0.12 $[5.67]$	-0.07 $[-4.57]$	
$Mecm_{1,t-1}$					-0.90 $[-3.71]$		-0.34 $[-7.08]$	-0.06 $[-1.91]$	-0.66 $[-2.85]$	0.24 $[5.62]$	0.13 $[3.56]$	0.69 $[3.67]$
$Mecm_{2,t-1}$	0.03 $[3.47]$	0.31 $[2.24]$	0.01 $[1.95]$		0.03 $[1.93]$	-0.11 $[-7.83]$				-0.01 $[-3.73]$		
$Eecm_{1,t-1}$	0.07 $[2.17]$	-3.06 $[-3.59]$										
$Eecm_{2,t-1}$		-6.99 $[-2.49]$					-0.08 $[-2.63]$		0.64 $[3.27]$	-0.07 $[-2.21]$		
$Lecm_{1,t-1}$	-1.02 $[-8.61]$	-12.00 $[-3.82]$			0.64 $[2.80]$		-0.09 $[-2.39]$		1.26 $[4.00]$			
$Lecm_{2,t-1}$	0.23 $[3.88]$			0.40 $[4.71]$	0.28 $[3.10]$	-0.27 $[-3.34]$		0.04 $[2.41]$	0.40 $[5.29]$	-0.10 $[-4.30]$		
$Lecm_{3,t-1}$				-0.26 $[-9.77]$	0.07 $[2.50]$			-0.01 $[-2.66]$		0.02 $[2.63]$		
$Lecm_{4,t-1}$		-0.66 $[-4.87]$			0.15 $[8.50]$				0.18 $[8.23]$			
$\Delta^2 p_{t-1}$					-0.47 $[-2.92]$				-0.51 $[-2.38]$	0.09 $[2.80]$	0.06 $[2.15]$	0.26 $[2.12]$
Δppp_{t-1}	0.02 $[4.36]$	0.23 $[2.25]$	0.02 $[3.60]$	-0.04 $[-4.19]$	-0.02 $[-2.81]$	-0.02 $[-1.49]$			-0.04 $[-4.56]$	0.01 $[3.87]$		-0.02 $[-2.60]$
$\Delta(p-p_y)_{t-1}$		-4.72 $[-2.09]$				0.66 $[3.03]$						0.39 $[3.30]$
$\Delta w_{r,t-1}$		-2.20 $[-2.92]$			-0.16 $[-2.26]$				-0.23 $[-2.44]$			
Δq_{t-1}												
$\Delta I_{s,t-1}$			-0.36 $[-1.84]$		0.48 $[1.68]$		0.33 $[5.27]$					
$\Delta I_{l,t-1}$	-0.49 $[-1.91]$	-10.75 $[-2.37]$						0.37 $[5.13]$			0.34 $[4.61]$	
ΔU_{t-1}	-0.38 $[-2.10]$				-1.75 $[-3.91]$				-2.30 $[-4.60]$	0.49 $[6.66]$	0.09 $[1.61]$	
$\Delta PD/Y_{t-1}$		-6.89 $[-2.52]$				0.91 $[2.95]$					0.81 $[16.9]$	
$\Delta GD/Y_{t-1}$	-0.13 $[-3.01]$				0.25 $[2.41]$		0.05 $[2.74]$					0.67 $[9.34]$
$\Delta^2 p^{US}_{t-1}$	0.18 $[2.66]$		0.15 $[2.64]$		-0.17 $[-2.42]$	-0.24 $[-1.95]$	0.06 $[2.71]$		-0.56 $[-4.45]$			
$\Delta I^{US}_{s,t}$					-0.49 $[-2.24]$		-0.21 $[-3.69]$			0.16 $[2.65]$		
$\Delta I^{US}_{s,t-1}$		-8.12 $[-2.19]$					0.16 $[3.23]$				-0.18 $[-2.99]$	
$\Delta I^{US}_{l,t}$												
$\Delta I^{US}_{l,t-1}$	0.36 $[2.51]$	14.25 $[3.34]$				-0.51 $[-1.55]$		0.36 $[6.49]$		-0.25 $[-4.25]$	0.14 $[2.01]$	
1				0.0035 $[8.72]$	0.0039 $[7.24]$	0.0077 $[16.2]$	-0.0004 $[-4.35]$		0.0065 $[12.4]$	0.0002 $[2.23]$		0.0012 $[3.12]$
$\hat{\sigma}$	0.0018	0.0337	0.0021	0.0035	0.0024	0.0040	0.0006	0.0007	0.0034	0.0006	0.0006	0.0024

Table C.2: MSM: complete short-run structure (dummy estimates not reported).

	Δp^*	ppp^*	$(p-p_y)^*$	w_r^*	q^*	m_r^*	I_s^*	I_l^*	y_r^*	U^*	PD/Y^*	GD/Y^*
Δp^*	$*$				1.26 [3.88]		0.57 [4.50]	0.32 [1.65]	−1.88 [−1.97]		−1.12 [−1.61]	
ppp^*	0.01 [1.45]	$*$			0.03 [4.50]				−0.07 [−2.32]	−0.02 [−2.36]		−0.43 [−1.42]
$(p-p_y)^*$	−0.25 [−1.84]		$*$	−1.90 [−4.29]								
w_r				$*$	0.26 [3.64]							
q^*				0.58 [1.89]	$*$				2.15 [4.26]	0.30 [2.58]		
m_r^*	0.06 [3.27]					$*$						
I_s^*					−0.74 [−2.22]		$*$		2.20 [2.21]	0.89 [2.78]		
I_l^*	0.66 [3.71]							$*$		−0.78 [−2.01]	1.57 [2.15]	
y_r^*					0.28 [2.17]	1.48 [1.65]	0.12 [5.41]		$*$	−0.23 [−1.77]		
U^*				−1.58 [−2.11]	1.38 [7.52]				−3.15 [−3.29]	$*$	1.37 [2.49]	
PD/Y^*				−1.16 [−1.80]	−0.39 [−2.39]				1.08 [1.86]	0.34 [2.17]	$*$	
GD/Y^*				−0.35 [−1.60]	0.08 [1.77]					0.08 [1.79]	−0.32 [−2.51]	$*$
$\Delta p^{US,*}$	−0.30 [−3.07]						−0.17 [−1.62]		−1.14 [−2.13]			
$I_s^{US,*}$	0.26 [2.23]									−0.91 [−3.78]		
$I_l^{US,*}$								0.39 [1.66]	1.67 [2.18]	1.05 [3.72]		
$conv^*$				−0.76 [−2.93]	−0.11 [−2.15]			−0.09 [−2.60]	0.29 [1.65]			
t				0.00 [1.84]	0.00 [1.60]		−0.00 [−5.74]				0.00 [1.84]	
1	0.92 [1.50]			7.67 [11.8]	−2.77 [−3.49]		−1.05 [−5.26]		7.32 [7.09]	2.16 [2.78]		
$\widehat{\sigma}^*$	0.0015	0.2551	0.0198	0.0143	0.0024	0.0203	0.0018	0.0030	0.0061	0.0021	0.0068	0.1613

Table C.3: MSM: long-run static solutions (coefficients with t-statistics below 1.4 not reported).

$p-r$	r	Eigenvalue	Trace	$CV_{95\%}$	p
15	0	0.98	1321	589	0.00
14	1	0.95	1003	523	0.00
13	2	0.87	758	460	0.00
12	3	0.72	594	401	0.00
11	4	0.70	492	346	0.00
10	5	0.64	395	295	0.00
9	6	0.61	312	248	0.00
8	7	0.56	237	205	0.00
7	8	0.41	172	166	0.02
6	9	0.36	129	131	0.07
5	10	0.30	93	100	0.14
4	11	0.25	65	73	0.19
3	12	0.23	42	50	0.24
2	13	0.18	21	31	0.45
1	14	0.05	4	15	0.87

Table D.1: Full-information CVAR: rank test.

D Full-information CVAR model

Although it is in practice difficult to identify the long-run relations in the CVAR based on the full information set, the model stilll provides instructive information on the total number of cointegration relations and on which variables may be weakly exogenous when the sets of variables from the sector models are combined. We consider the following vector of potentially endogenous variables,

$$\mathbf{x}_t^{\text{all}} = (\Delta p, ppp, m_r, p - p_y, w_r, q, I_s, I_l, \Delta y, U, PD/Y, GD/Y, \Delta p^{US}, I_s^{US}, I_l^{US})_t', \tag{17}$$

and as weakly exogenous,

$$\mathbf{z}_t^{\text{all}} = (conv)_t, \tag{18}$$

which give rise to a 15-dimensional VAR with one weakly exogenous variable ($conv_t$).

Table D.2 reports the trace-test statistics and at a five-per cent level these indicate $r = 10$ cointegration relations. The graphs of the cointegrating relations and the significance of the $\boldsymbol{\alpha}$-coefficients (not reported) also support the existence of ten cointegrating relations (not reported). This choice of rank leaves a large root in the model which appears to result primarily from a high degree of persistence in the public deficit and debt ratios. We conclude that the most reasonable choice of rank is $r = 10$. This implies $p - r = 5$ common stochastic trends in the model in addition to the one assumed to originate from cumulated shocks to $conv_t$.

D.1 Short-run adjustment

The tests on $\boldsymbol{\alpha}$ reported in Table D.2 (upper panel) suggest that only the US long rate and possibly the real exchange rate can be considered weakly exogenous (have zero rows in $\boldsymbol{\alpha}$); the joint test results in a p-value of 0.02. A closer look at the magnitude of the test statistics show that shocks to real money, the price wedge, the long rate and the two remaining US variables are not too far from non-rejection as weakly exogenous and thus potentially among the driving forces in the system. The test of whether a given variable is purely adjusting (has a unit column in $\boldsymbol{\alpha}$) is essentially the mirror image of the corresponding weak-exogeneity test. The inflation rate, real wages, labour productivity, nominal GDP growth and government debt all seem to be adjusting to disequilibria. The test also suggests however that, for example, the price wedge and the US inflation rate may be purely adjusting. This apparent inconsistency between the broad conclusions of two types of tests on $\boldsymbol{\alpha}$ for some variables can arise when the unit column coefficient is close to zero; in such cases, the weak exogeneity test is usually the more reliable of the two (Juselius 2006). We conclude that the US interest rates, the real exchange rate, real money, and possibly the price wedge and the euro-area long rate are likely to constitute the main pushing forces given the information set.

D.2 Long-run co-movements

Although we refrain imposing over-identifying restrictions on $\boldsymbol{\beta}$ due to the high dimension of the system, the unrestricted $\boldsymbol{\Pi}$-matrix may nevertheless contain important information regarding the possible long-run relationships between the variables. The rows of $\boldsymbol{\Pi}$ show the 'net effect' of the long- and short-run dynamics and are thus broadly comparable to the long-run static solution for the variable in question. Table D.2 (lower panel) reports the unrestricted $\boldsymbol{\Pi}$-matrix given $r = 10$.

Starting from below, the rows corresponding to the two US interest rates have few significant entries, supporting the finding of weak exogeneity of these two variables. Regarding the euro-area variables, the ppp_t, the $m_{r,t}$ and the $(p-p_y)_t$ rows have few significant entries underlining the possible status of these as exogenous as well. The I_l row shows that the long rate has followed the US rate closely but also that a higher degree of intra euro-area PPP convergence has been associated with a lower long-term interest rate. In the $I_{s,t}$ row, a rise in the short rate has been accompanied by a rise in unemployment as predicted by the conventional monetary transmission mechanism. A short-rate hike has also been associated with higher public deficits and debt. There seems to be little long-run connection between the short (policy) rate and the long rate however. The short rate moves positively with inflation, likely reflecting a policy response.

While the $w_{r,t}$ row sees a relationship between real wages and unemployment (negative) as well as between real wages and labour productivity (positive) as in a standard real-wage relation, it also reveals a strong negative relation between the real wage and PPP convergence. This is a hint that wage claims were gradually moderated as intra euro-area convergence was achieved. The q_t row features a strong positive association of labour productivity on the one side, and wages and unemployment on the other, pointing at vital productivity adjustments in this period.

In the U_t row, unemployment is seen to be positively related to a rise in the short rate (possibly the spread to the corresponding US rate), i.e. to contractive monetary policy (in relative terms). In addition, higher deficits/debt levels, i.e. expansionary fiscal policy, have a tendency to be associated with higher unemployment, likely reflecting a policy response. Crucially, it appears that there is no Phillips-curve trade-off in the long run. Finally, the Δp_t-row suggests that inflation has moved positively with a depreciation of the real exchange rate, the two short rates and the US long rate; a negative association is found with the price wedge and the US inflation rate. The fiscal variables do not seem to have been related to inflation though.

The direction of causality is not clear from the correlation patterns outlined by the estimated $\mathbf{\Pi}$-coefficients. The moving average (MA) representation of the model (1) would allow us to study the long-run impact of shocks to, for example, the short-term interest rate on inflation, and hence to assess whether prices can be controlled by the central bank (Johansen and Juselius 2003). For the system based on all variables in (17) and (18), the long-run impact matrix has only insignificant entries, likely a result of its large dimension which leaves it somewhat imprecisely estimated, and/or the fact that a large root remains in the system. A separation of the pushing and pulling forces through identification of $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ as provided by the MSM is therefore essential.

	$CV_{95\%}$	Δp_t	ppp_t	$m_{r,t}$	$(p-p_y)_t$	$w_{r,t}$	q_t	$I_{s,t}$	$I_{l,t}$	Δy_t	U_t	PB/Y_t	GD/Y_t	Δp_t^{US}	$I_{s,t}^{US}$	$I_{l,t}^{US}$
Zero row	18.31	54.65 [0.00]	21.02 [0.02]	31.51 [0.00]	29.27 [0.00]	61.21 [0.00]	54.33 [0.00]	58.53 [0.00]	42.61 [0.00]	78.51 [0.00]	119.10 [0.00]	69.58 [0.00]	124.72 [0.00]	36.38 [0.00]	30.24 [0.00]	15.38 [0.12]
Unit col	11.07	8.41 [0.14]	13.94 [0.02]	11.95 [0.04]	7.77 [0.17]	3.99 [0.55]	7.10 [0.21]	10.47 [0.06]	14.41 [0.01]	3.58 [0.61]	12.72 [0.03]	12.83 [0.03]	9.21 [0.10]	5.06 [0.41]	10.21 [0.07]	16.16 [0.01]

	Δp_{t-1}	ppp_{t-1}	$m_{r,t-1}$	$(p-p_y)_{t-1}$	$w_{r,t-1}$	q_{t-1}	$I_{s,t-1}$	$I_{l,t-1}$	Δy_{t-1}	U_{t-1}	PB/Y_{t-1}	GD/Y_{t-1}	Δp_{t-1}^{US}	$I_{s,t-1}^{US}$	$I_{l,t-1}^{US}$	$conv_{t-1}$	t
$\Delta^2 p_t$	−0.87 [−8.04]	0.01 [2.60]	0.01 [1.34]	−0.23 [−2.34]	0.00 [0.02]	−0.11 [−1.80]	0.37 [3.20]	0.21 [0.86]	−0.06 [−0.97]	0.07 [0.50]	0.05 [0.89]	0.01 [0.73]	−0.27 [−3.14]	0.35 [2.92]	0.44 [2.52]	0.02 [1.12]	0.05 [2.37]
Δppp_t	0.01 [0.00]	−0.18 [−1.77]	−0.04 [−0.21]	3.02 [1.47]	0.60 [0.74]	−0.23 [−0.17]	7.24 [2.94]	−15.22 [−2.92]	0.21 [0.18]	−1.64 [−0.55]	2.97 [2.50]	0.77 [1.85]	−2.42 [−1.33]	3.34 [1.33]	10.69 [2.90]	1.08 [2.56]	−0.20 [−0.48]
$\Delta m_{r,t}$	−0.22 [−0.87]	−0.04 [−3.27]	−0.04 [−2.09]	0.49 [2.18]	−0.04 [−0.45]	0.19 [1.35]	0.65 [2.41]	−1.52 [−2.64]	−0.11 [−0.86]	−0.38 [−1.18]	0.09 [0.68]	−0.09 [−1.91]	0.14 [0.70]	−0.07 [−0.27]	0.11 [0.27]	−0.10 [−2.15]	−0.01 [−0.16]
$\Delta(p-p_y)_t$	0.06 [0.45]	0.02 [3.16]	0.03 [2.51]	−0.40 [−3.55]	−0.04 [−0.79]	−0.08 [−1.10]	0.05 [0.37]	0.20 [0.71]	−0.03 [−0.48]	0.07 [0.46]	−0.03 [−0.48]	0.04 [1.53]	−0.10 [−0.95]	−0.04 [−0.30]	0.78 [3.85]	0.03 [1.32]	0.03 [1.39]
$\Delta w_{r,t}$	0.03 [0.13]	−0.03 [−2.61]	0.02 [1.05]	0.20 [0.99]	−0.15 [−1.87]	0.26 [1.99]	−0.58 [−2.34]	−0.90 [−1.71]	−0.02 [−0.20]	−0.61 [−2.07]	−0.18 [−1.55]	−0.10 [−2.44]	0.22 [1.23]	−0.39 [−1.54]	−0.21 [−0.58]	−0.28 [−6.52]	−0.07 [−1.75]
Δq_t	0.25 [1.46]	0.02 [2.03]	0.01 [0.36]	−0.07 [−0.45]	0.25 [4.20]	−0.67 [−7.00]	−0.37 [−2.04]	0.15 [0.39]	0.21 [2.32]	1.11 [5.06]	−0.48 [−5.49]	−0.03 [−1.01]	0.09 [0.64]	0.32 [1.69]	−0.31 [−1.12]	−0.19 [−6.15]	0.16 [5.29]
$\Delta I_{s,t}$	0.07 [1.77]	−0.00 [−0.42]	−0.01 [−2.53]	0.01 [0.33]	0.05 [3.62]	−0.00 [−0.19]	−0.26 [−5.77]	0.11 [1.15]	−0.04 [−1.62]	0.29 [5.38]	−0.13 [−6.04]	−0.07 [−9.27]	−0.01 [−0.19]	0.14 [3.11]	−0.12 [−1.77]	−0.05 [−5.77]	0.01 [1.12]
$\Delta I_{l,t}$	0.08 [1.77]	−0.00 [−1.55]	−0.00 [−0.57]	−0.01 [−0.13]	0.01 [0.86]	−0.02 [−0.78]	0.05 [1.06]	−0.35 [−3.56]	0.00 [0.20]	0.05 [0.94]	−0.03 [−1.15]	−0.02 [−2.18]	−0.05 [−1.32]	0.01 [0.15]	0.34 [4.86]	−0.03 [−3.98]	0.01 [1.50]
$\Delta^2 y_t$	0.21 [1.01]	0.01 [1.02]	0.02 [1.18]	0.05 [0.27]	0.22 [2.91]	−0.63 [−5.24]	−0.44 [−1.93]	0.21 [0.43]	−0.76 [−6.79]	0.85 [3.07]	−0.52 [−4.68]	−0.04 [−1.12]	0.09 [0.50]	0.88 [3.75]	−0.95 [−2.75]	−0.24 [−6.00]	0.13 [3.44]
ΔU_t	0.18 [3.88]	0.00 [0.59]	−0.01 [−2.88]	−0.03 [−0.86]	−0.02 [−1.50]	−0.02 [−0.71]	0.26 [5.42]	−0.09 [−0.86]	−0.01 [−0.22]	−0.15 [−2.61]	0.09 [4.00]	0.04 [5.37]	−0.05 [−1.31]	−0.24 [−4.80]	0.15 [2.03]	0.02 [1.99]	0.02 [1.92]
$\Delta PB/Y_t$	−0.19 [−3.41]	0.01 [3.23]	−0.01 [−1.48]	−0.18 [−3.55]	−0.05 [−2.62]	−0.01 [−0.23]	0.28 [4.64]	0.32 [2.52]	−0.00 [−0.07]	0.01 [0.11]	−0.04 [−1.51]	−0.02 [−1.71]	0.15 [3.23]	−0.18 [−2.84]	−0.01 [−0.12]	0.00 [0.31]	0.05 [4.53]
$\Delta GD/Y_t$	−0.06 [−0.42]	−0.00 [−0.65]	−0.09 [−7.10]	−0.11 [−0.86]	−0.08 [−1.49]	0.25 [3.07]	0.48 [3.07]	0.94 [2.82]	−0.30 [−3.92]	−0.06 [−0.32]	0.23 [2.97]	−0.04 [−1.43]	0.06 [0.53]	−0.81 [−5.07]	0.01 [0.05]	0.10 [3.72]	0.02 [0.87]
$\Delta^2 p_t^{US}$	−0.15 [−1.00]	0.00 [0.65]	0.00 [0.35]	−0.00 [−0.03]	0.07 [1.25]	−0.00 [−0.03]	0.11 [0.66]	−0.50 [−1.42]	0.08 [0.97]	−0.16 [−0.82]	0.24 [2.98]	0.05 [1.83]	−0.85 [−6.95]	0.50 [2.95]	0.61 [2.46]	0.02 [0.77]	−0.03 [−1.12]
$\Delta I_{s,t}^{US}$	0.04 [0.59]	−0.00 [−0.26]	0.01 [1.32]	0.00 [0.08]	0.01 [0.59]	−0.04 [−1.20]	0.02 [0.25]	−0.20 [−1.36]	0.07 [2.02]	0.01 [0.17]	0.03 [0.80]	0.03 [2.27]	−0.03 [−0.51]	0.00 [0.01]	0.26 [2.50]	0.01 [1.05]	−0.00 [−0.15]
$\Delta I_{l,t}^{US}$	0.04 [0.67]	−0.00 [−1.12]	0.00 [0.03]	0.10 [1.87]	0.04 [2.08]	−0.10 [−2.93]	0.07 [1.14]	−0.32 [−2.31]	0.04 [1.24]	0.06 [0.75]	−0.01 [−0.19]	0.02 [1.42]	−0.03 [−0.73]	0.11 [1.64]	0.01 [0.07]	−0.02 [−1.94]	0.01 [1.30]

Table D.2: Full-information CVAR: tests on $\boldsymbol{\alpha}$ and $\boldsymbol{\Pi}$-matrix given $r = 10$.

Chapter 2

Interactions of monetary and fiscal policy: An I(2) cointegrated VAR study of deficit-debt dynamics in the euro area

Interactions of monetary and fiscal policy: An I(2) cointegrated VAR study of deficit-debt dynamics in the euro area

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Abstract

This paper studies the interactions of monetary and fiscal policy in euro area using a cointegrated VAR (CVAR) approach. We show that the time-series persistency of deficits and debt over the sample period imply that an I(2) model is required to appropriately characterise the dynamics of the data. We translate a small economic model of policy interactions into a set of polynomially cointegrating relations and estimate an I(2) CVAR to test its empirical coherence. With some modifications we are able to recover a set of economically meaningful relations in the data. In contrast with the theoretical prediction however, we find evidence of two I(2) trends, one arising from shocks to the short rate and another from shocks to the long rate. This suggests that the nominal anchor of the economy has not been provided solely by monetary policy; fiscal policy has played a role as well. Moreover, the identified cointegration and common-trends structures provide evidence in favour of the hypothesis that a vicious spiral of rising public debt, bond yields and unemployment has been at play during the integration process leading up to the EMU.

Keywords: Time-Series Models, Financial Markets and the Macroeconomy, Comparative or Joint Analysis of Fiscal and Monetary Policy; Stabilization

JEL Classification: C32, E44, E63

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1 Introduction

We study interactions of monetary and fiscal policy in the euro area using an I(2) cointegrated VAR (CVAR) model with a special view to investigate the dynamic effects of fiscal policy on bond yields. In the European welfare-states, the public sector constitutes an essential part of the economy, and fiscal policies potentially have significant effects on demand and price pressures via the level and composition of government revenue and expenditure as well as via public deficits and debt. With the introduction of the European Monetary Union (EMU), which combines a centralised monetary policy with decentralised fiscal and structural policies, the latter have become ever more important in dealing with asymmetric shocks.

The notion of 'unpleasant monetarist arithmetic' of Sargent and Wallace (1981) underlines the importance of monetary- and fiscal-policy interactions. Central banks are usually assumed capable of steering long-term interest rates via control over the policy rate. However, government actions may affect a range of economic variables such as aggregate demand, potential output, prices, risk premia, and thus bond yields. An inflation-targeting central bank, such as the ECB, should thus care about the fiscal stance.

A range of theoretical papers address monetary-fiscal interdependency, see *inter alia* Leith and Wren-Lewis (2000), Beetsma and Jensen (2002), Schmitt-Grohe and Uribe (2004) and Leith and Thadden (2006). This strand of literature recommends the use of non-discretionary rules and clear mandates in order to avoid that policy makers pursue, for example, excess output growth at the expense of higher inflation (Barro and Gordon 1983). In a monetary union with a common central bank but no coordination of fiscal policy, government incentives are further distorted as the punishment by financial markets for high debt in one country is shared by all member states, thereby creating a moral-hazard problem. The institutional framework for the EMU laid down by the Treaty and the Stability and Growth Pact (SGP) requires member states to "avoid excessive government deficits" to secure room for manoeuvre in dealing with business-cycle fluctuations in the medium run and to ensure fiscal sustainability in the long run. These guidelines have however been continuously violated by some member states, raising the question whether the EMU is a regime of monetary or fiscal dominance (see also Tuxen 2009).

In this paper, we use the model of Kirsanova, Stehn, and Vines (2005) as the point of departure for an empirical study of euro-area bond markets. These authors develop a five-equation system in order to analyse the interaction of simple rules for both monetary and fiscal policy. Considering the game played between optimising policy makers they conclude that the institutional arrangements within the EMU may lead to social welfare costs. National fiscal policy interests are likely to differ from those of the ECB as they are taken by elected politicians, and because the SGP provides a relatively loose framework for fiscal discipline within the EMU, the authors argue that the fiscal authorities are unlikely to coordinate among themselves. In setting policy, governments may thus fail to acknowledge that the monetary authority will react aggressively to fight inflation in face of upwards pressure on prices arising from fiscal policy. If, in addition, governments discount the future too much and/or aim for excess output, this could lead to a struggle between national governments and the (benevolent) central bank: if governments pursue excessive deficits, the central bank will increase the interest rate more than otherwise needed. This results in a larger accumulation of public debt which damages social welfare.

We translate the 'five-equation model' proposed by Kirsanova et al. (2005) to CVAR space and show that in order to appropriately account for deficit-debt dynamics and the time-series persistency of the fiscal variables, an I(2) model is required. We use the relations proposed by the economic model to guide identification of the polynomially cointegrating relations in the statistical model. This allows us to assess the empirical relevance of the theoretical model and, if required, to suggest possible modifications of it. Related studies using CVAR models to analyse fiscal policy include Reade (2007) and Tuxen (2006, 2009) which both use an I(1) CVAR approach to model interactions of monetary and fiscal policy for the US and the euro area, respectively. Our analysis differs from the above studies in that we use the I(2) CVAR which allows for a rich dynamic structure and a direct assessment of the (nominal) I(2) trend(s).

Here, we focus in particular on testing a central hypothesis of Juselius (2002), who speculates that economic integration in Europe early on produced a vicious circle of rising unemployment, public deficits/debt and interest rates, *ceteris paribus.* Juselius argues that with the introduction of a 'stricter' version of the European Monetary System (EMS), internal exchange rates among European countries became fixed at levels which were not sustainable in the long run, and to even out imbalances in intra-European PPP levels, high-PPP member states (such as Germany and France) had to experience negative inflation rates, and *vice versa* for low-PPP countries (such as Italy and Spain). Downward sticky wages and prices in the high-price countries were a crucial factor behind the rise in unemployment: in face of high wage claims and given a fixed exchange rate within the zone, firms attempted to boost productivity in order to remain externally competitive by producing the same output with less labor (see also Juselius 2003). This led to declining export demand in low-price countries, raising unemployment in these countries as well albeit for different reasons. High unemployment had a dampening effect on union power and thus on wage demand and inflation. Long-term interest rates remained relatively high as did real rates, further dampening European economic activity. One hypothesis of Juselius (2002) is that the large budget deficits required to finance unemployment increased the demand for (unproductive) capital relative to supply, thereby exerting upward pressure on the long end of the yield curve. This arguably depressed employment further and a self-reinforcing mechanism could take off.

Existing evidence on the dynamics of debt and bond yields point to only small effects, if any, of fiscal policy on interest rates in the euro area. Using evidence from seemingly unrelated regression models, Afonso and Strauch (2004) assess the importance attached by capital markets to the credibility of the European fiscal framework. They estimate the impact of fiscal events on swap spreads, which is used as a measure of the risk premia, and find significant, but small, effects of budget deficits on long-term interest rates. In a related study, Heppke-Falk and Hüfner (2004) find no significant impact of expected deficits on swap spreads however. In a recent paper, Haugh, Ollivaud, and Turner (2009) find, using two-stage least squares estimation, that fiscal policies play an important role in explaining bond yield spreads in the euro area. Notably, the effect is amplified by interaction with general risk aversion.

None of above-mentioned empirical studies take account of the interactions of debt and sovereign bond yields with unemployment. In contrast, the I(2) CVAR framework used in this paper allows us to study the dynamics of these key variables jointly. Notably, the Kirsanova et al. (2005) model does not incorporate any effect of fiscal policy on interest rates; in fact, their model include only one interest rate (of unspecified maturity) which is perfectly controlled by the central bank. By allowing for a potential separation of the dynamics of short- and long-term rates, our model reveals that the yield curve is not driven solely by monetary policy; fiscal policy plays an autonomous role.

The rest of the paper is organised as follows. Section 2 presents the Kirsanova et al. (2005) model and Section 3 discusses the fiscal data series. The I(2) CVAR model is introduced in Section 4 and we derive some statistical implications of the economic model in Section 5. Estimation results are discussed in Section 6, and Section 7 concludes.[1]

2 Economic model: 'five-equation macroeconomics'

The majority of modern short-run macroeconomic analysis is based on a New-Keynesian framework consisting of an IS curve, a Phillips curve and a Taylor rule, as proposed by Clarida, Gali, and Gertler (1999). This type of system includes a description of monetary policy but fiscal policy is usually taken as exogenous or left out altogether. Kirsanova et al. (2005) endogenise fiscal policy by adding to the three baseline relations a fiscal rule and an equation describing debt accumulation. We present the main building blocks of this 'five-equation macro model' below.

The model consists of the following relations. First, a dynamic IS curve,

$$y_{r,t}^{gap} = \kappa y_{r,t-1}^{gap} - \sigma(I - \Delta p)_{t-1} + \varphi g_t^r + \psi b_t^r + \varepsilon_t^{IS} \tag{1}$$

where $y_{r,t}^{gap}$ is the real output gap, I_t the interest rate, Δp_t the inflation rate, b_t^r the stock of government debt (either measured in real terms or in ratio to GDP) and g_t^r is the value of primary government expenditure (again in real terms or ratio to GDP); ε_t^{IS} is a demand shock. Besides allowing for output persistency (with coefficient κ), (1) incorporates a negative effect of the real interest rate on output. The fiscal instrument is here taken to be changes in government expenditure, and fiscal policy affects output directly via aggregate demand (with multiplier φ). If Ricardian equivalence (REq) fails there is also an indirect effect via the level of public debt (with multiplier ψ) as a fraction of the debt issued by the government will be treated as net wealth by the private sector.

An accelerationist Phillips curve,

$$\Delta p_t = \Delta p_{t-1} + \lambda y_{r,t-1}^{gap} + \varepsilon_t^{PC} \tag{2}$$

where ε_t^{PC} is an inflation shock. This exhibits the usual trade-off between inflation and capacity utilisation, here measured by the output gap, and, at the same time, incorporates the idea that the dynamics of inflation expectations lead this relationship shift around

[1] All calculations were conducted using CATS 2.01 (Dennis, Hansen, Johansen, and Juselius 2006) in Rats 6.3 and Ox/OxMetrics 5 (Doornik 2007).

over time. Inflation expectations are here assumed to be entirely backward-looking however. This stand in contrast to the New-Keynesian Phillips curve which emphasises that (2) is not a structural relation, and thus that changes in policy may induce changes in expectations formation which could lead the trade-off to break down.

A monetary-policy rule,

$$I_t = \theta^{\Delta p} \Delta p_t + \theta^{y_r} y^{gap}_{r,t} + \varepsilon^{mon}_t \tag{3}$$

such that the central bank adjusts the interest rate in order to stabilise inflation and output; ε^{mon}_t is a monetary-policy shock. The coefficients $\theta^{\Delta p} \simeq 1.5$ and $\theta^y \simeq 0.5$ were proposed by Taylor (1993).

A fiscal-policy rule,

$$g^r_t = -\phi y^{gap}_{r,t-1} - \mu b^r_{t-1} + \varepsilon^{fis}_t \tag{4}$$

where fiscal policy feeds back on the level of government debt and the government aims at stabilising output; ε^{fis}_t is a fiscal-policy shock.

The government-budget constraint tracks the accumulation of debt; from a log-linearisation around the steady-state values of debt and the real interest rate, e.g. $b^r_0 \approx 0.6$ and $(I - \Delta p)_0 \approx 0.01$, we obtain

$$b^r_t \simeq b^r_{t-1} + \overbrace{\underbrace{(I - \Delta p)_0 b^r_{t-1} + (I - \Delta p)_{t-1} b^r_0}_{\text{interest payments}} + \underbrace{g^r_{t-1} - \tau y^{gap}_{r,t-1}}_{\text{primary deficit}}}^{\text{operating deficit}} + \varepsilon^b_t \tag{5}$$

where tax revenues are assumed to vary with the output gap (with elasticity τ); ε^b_t is a public-debt shock. This completes our description of the theoretical model.

3 Data and graphical analysis

The economic model suggests that an empirical analysis should be based on the following set of variables,

$$(y^{gap}_r, \Delta p, I, g^r, b^r)_t \tag{6}$$

where g^r_t may be either real expenditure or expenditure-to-GDP, and similarly for b^r_t. The choice of denominator is irrelevant from a theoretical point of view because the underlying nominal growth rate in the economy should be reflected equally well by p_t or y_t. Here Δp_t is used to represent the nominal anchor.

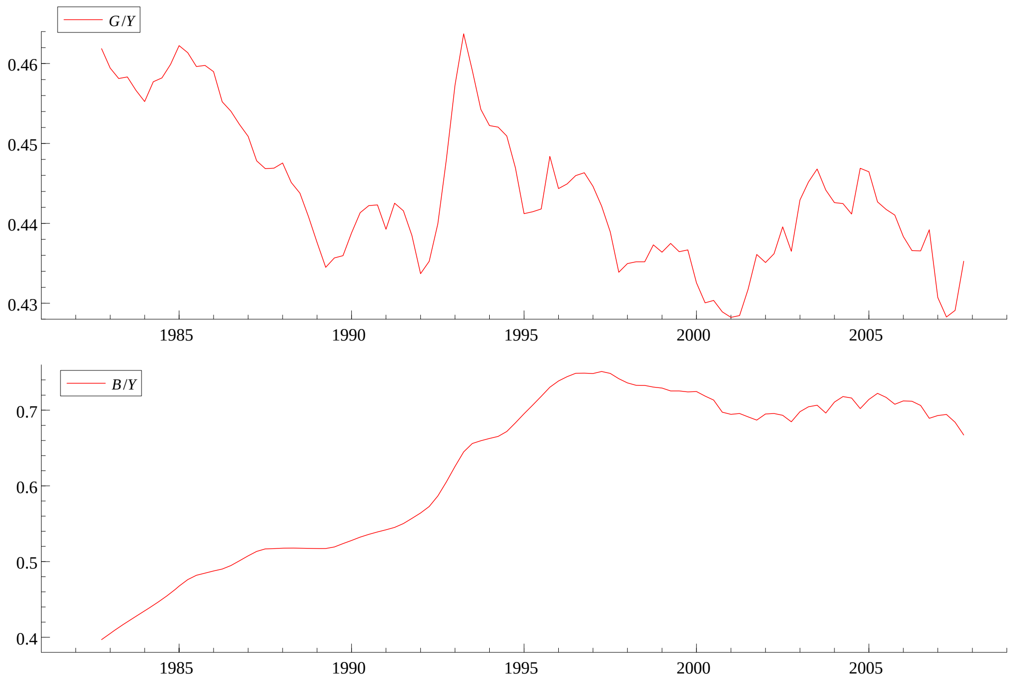

Figure 1: Upper panel: government expenditure. Lower panel: public debt.

Unless otherwise stated, we use quarterly data series from the ECB's Area-Wide Model (AWM) (Fagan, Henry, and Mestre 2001), and consider the period 1982:4 to 2007:4. An overview of the data and their sources is given in Table A.1. The sample is set to start in the early 1980s as tests indicate that the transition to a more strict regime of the European Monetary System (EMS) and the demolition of capital restrictions are likely to constitute a structural break around that time. A range of issues arise in aggregating national data of countries with flexible exchange rates in part of the sample (Beyer, Doornik, and Hendry 2001, Beyer and Juselius 2008). The AWM data set is however widely used for area-wide analysis of the euro zone and we shall abstract from aggregation issues here. National differences in government policies may further complicate the interpretation of the area-wide stance of monetary policy and, let alone, fiscal policy. We provide some individual country estimation results in Appendix C in order to assess the sensitivity of the area-wide results to cross-country differences.

Figure 1 plots some key fiscal variables in ratios to GDP. The ratio of government expenditure-to-GDP, $(G/Y)_t$ is clearly non-stationary and downward-trending overall in the sample period. The evolution of the ratio of government debt-to-GDP, $(B/Y)_t$, is much more smooth, apart from seasonality effects in the final part of the sample, and is

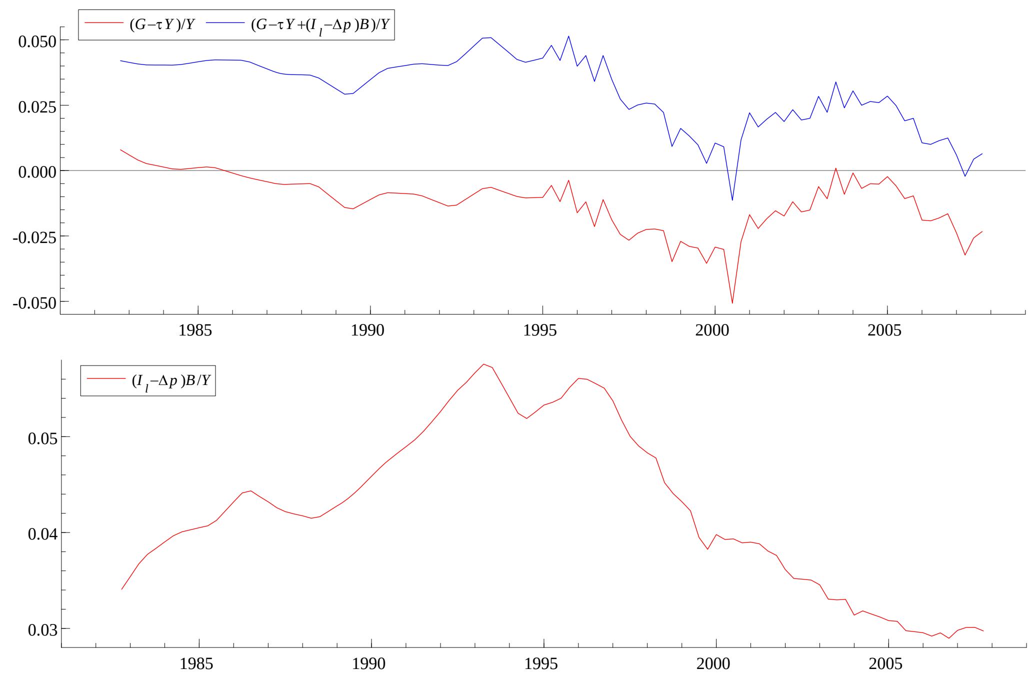

Figure 2: Upper panel: primary deficit vs. operating deficit. Lower panel: interest payments on debt.

trending upwards until around 1995 and is broadly stable thereafter.[2]

Figure 2 plots the primary deficit-to-GDP, $((G - \tau Y)/Y)_t$, alongside the operating deficit-to-GDP, $((G - \tau Y + (I_l - \Delta p)B)/Y)_t$. The difference between the two series, i.e. interest payments on debt, $((I_l - \Delta p)B/Y)_t$, is also depicted.[3] Admission to the EMU requires a budget deficit of less than three per cent of GDP and a public debt less than 60 per cent of GDP. These requirements were made a permanent rule within the monetary union by the SGP in 1997. In the period between the ratification of the Maastricht Treaty in 1993 and the introduction of the euro in 1999, fiscal balances generally improved. In particular, interest payments on debt started to fall markedly, a decline partly brought about by worldwide falling yields.

The different degrees of persistency in government expenditure, G/Y_t, and public debt, B/Y_t, evident from Figure 1 is unsurprising given that expenditure (in excess of revenue), i.e. net borrowing, adds to the level of debt each period as described by (5). Introducing the following definition of the operating deficit (in real terms),

$$d^r_{t-1} \equiv (I - \Delta p)_{t-1} b^r_0 + (I - \Delta p)_0 b^r_{t-1} + g^r_{t-1} - \tau y^{gap}_{r,t-1}, \tag{7}$$

[2]Public debt is 'general-government consolidated gross debt' (ESA95 definition).

[3]The smoothness of the series prior to 1995, after which behaviour becomes more erratic, arise from the fact that some national series were interpolated from an annual to a quarterly frequency.

we can re-write the budget constraint as,

$$b_t^r \simeq b_{t-1}^r + d_t^r + \varepsilon_t^{b_r} \tag{8}$$

Solving for b_t^r by backward substitution we obtain,

$$b_t^r \simeq b_0^r + \sum_{j=1}^{t} d_j^r + \sum_{j=1}^{t} \varepsilon_j^{b_r} \tag{9}$$

which illustrates that the integration order of debt must exceed that of the deficit by one. Assuming debt shocks are i.i.d. (stationary), $\varepsilon_t^{b_r} \sim I(0)$, debt must be integrated of at least order one. However, if $d_t^r \sim I(1)$ then $b_t^r \sim I(2)$. A range of possibilities regarding the time-series properties of the fiscal variables thus exists, and it is an empirical question which is the relevant statistical treatment in a given sample. From a graphical inspection of $-((G - \tau Y)/Y - (I_l - \Delta p)B/Y)_t$ in Figure 2 (upper panel) it seems that the operating surplus is non-stationary within our sample. Figure 1 (lower panel) indeed suggests that $(B/Y)_t \sim I(2)$ is the more likely case here. This in turn implies that the nominal level of debt and/or output must be I(2) (ruling out variables of integration order higher than two). The I(2) CVAR allows variables to be either I(2), I(1) or I(0).

In the empirical analysis, we allow all variables to be potentially I(2) with I(1) obtained as a special case. This decision should not be given a structural/economic interpretation but from a statistical point of view this is likely to provide a better approximation in the given sample. As illustrated by Johansen (2006), treating a highly persistent (near-unit) root as stationary may seriously distort inference. [4]

4 Statistical model: the I(2) CVAR model

The VAR model allows for a flexible description of the regularities in the data and is in its unrestricted form simply a reformulation of the auto-covariances in the data. When $b_t^r \sim I(2)$ and thus $d_t^r = \Delta b_t^r \sim I(1)$ we need to allow for polynomial cointegration in order to model the dynamics of debt and deficits. This ensures that the levels of the I(2)

[4] In the very long run, fiscal sustainability in principle requires that the debt-to-GDP ratio is stationary, i.e. $b_t^r \sim I(0)$; see Baldwin and Wyplosz (2004) for the assumptions and arithmetics of deficits and debt upon which the SGP requirements are based. This allows $(b, y)_t \sim I(1)$ or $I(2)$ but requires that debt and GDP cointegrate such that $(b - y)_t \sim I(0)$. This would then require that the change in debt, $\Delta(b - y)_t \simeq d_t^r + \varepsilon_t^{b_r}$, is integrated of order minus one. This allows the individual components of d_t^r to be I(0) but requires that the primary deficit and interest payments 'move together'. From an empirical point of view, a 'dynamic steady-state relation' for debt which takes a non-stationary real interest rate into account however seems to be a more appropriate sustainability condition.

variables are allowed to cointegrate among each other to become I(1) and further with the first differences to become I(0). To take this into account, we consider an I(2) CVAR, the main structure of which is given below.

We start from the p-dimensional VAR($k = 2$) in acceleration rates,

$$\Delta^2 \mathbf{x}_t = \mathbf{\Pi}\mathbf{x}_{t-1} + \mathbf{\Gamma}\Delta\mathbf{x}_{t-1} + \boldsymbol{\varepsilon}_t, \qquad t = 1, 2, ..., T \tag{10}$$

where $\mathbf{x}_t$ is a $p \times 1$ data vector and $\boldsymbol{\varepsilon}_t$ a $p \times 1$ vector of error terms for which we assume $\boldsymbol{\varepsilon}_t \sim i.i.d.N(\mathbf{0}, \mathbf{\Omega})$ with $\mathbf{\Omega} > \mathbf{0}$. In the statistical analysis, we condition on the initial values, $(\mathbf{x}_{-1}, \mathbf{x}_0)$, and hence these are treated as fixed. The I(2) model is defined by two reduced-rank restrictions (Johansen 1992),

$$\mathbf{\Pi} \quad = \quad \boldsymbol{\alpha}\boldsymbol{\beta}' \tag{11}$$

and

$$\boldsymbol{\alpha}'_{\perp}\mathbf{\Gamma}\boldsymbol{\beta}_{\perp} \quad = \quad \boldsymbol{\xi}\boldsymbol{\eta}' \tag{12}$$

where $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are $p \times r$ with $r < p$, and $\boldsymbol{\xi}$ and $\boldsymbol{\eta}$ are $(p - r) \times s_1$ with $s_1 \leq r - p$; we use $_{\perp}$ to denote the orthogonal complement. We can decompose the $p \times (p - r)$-matrices $\boldsymbol{\alpha}_{\perp}$ and $\boldsymbol{\beta}_{\perp}$ into the I(1) and I(2) directions: $\boldsymbol{\alpha}_{\perp} = [\boldsymbol{\alpha}_{\perp 1}, \boldsymbol{\alpha}_{\perp 2}]$ and $\boldsymbol{\beta}_{\perp} = [\boldsymbol{\beta}_{\perp 1}, \boldsymbol{\beta}_{\perp 2}]$, where $\boldsymbol{\alpha}_{\perp 1}$ and $\boldsymbol{\beta}_{\perp 1}$ are $p \times s_1$ and defined by $\boldsymbol{\alpha}_{\perp 1} = \overline{\boldsymbol{\alpha}}_{\perp}\boldsymbol{\xi}$ and $\boldsymbol{\beta}_{\perp 1} = \overline{\boldsymbol{\beta}}_{\perp}\boldsymbol{\eta}$; $\boldsymbol{\alpha}_{\perp 2}$ and $\boldsymbol{\beta}_{\perp 2}$ are $p \times s_2$ and defined by $\boldsymbol{\alpha}_{\perp 2} = \boldsymbol{\alpha}_{\perp}\boldsymbol{\xi}_{\perp}$ and $\boldsymbol{\beta}_{\perp 2} = \boldsymbol{\beta}_{\perp}\boldsymbol{\eta}_{\perp}$; we use the notation $\overline{\mathbf{v}} = \mathbf{v}(\mathbf{v}'\mathbf{v})^{-1}$.

Imposing the I(2) restrictions, (11) and (12), we can re-write (10) to obtain the parameterisation used in Johansen (1997) for maximum-likelihood estimation,

$$\Delta^2 \mathbf{x}_t = \boldsymbol{\alpha}[\boldsymbol{\rho}'\boldsymbol{\tau}'\mathbf{x}_{t-1} + \boldsymbol{\psi}'\boldsymbol{\Delta}\mathbf{x}_{t-1}] + \boldsymbol{\alpha}_{\perp\Omega}\boldsymbol{\kappa}'\boldsymbol{\tau}'\boldsymbol{\Delta}\mathbf{x}_{t-1} + \boldsymbol{\varepsilon}_t, \tag{13}$$

where the parameters are variation-free. The parameters in (10) can be recovered from (13) by setting $\boldsymbol{\rho}' = (\mathbf{I}_r, \mathbf{0})$ and thus $\boldsymbol{\tau}' = (\boldsymbol{\beta}, \boldsymbol{\beta}_{\perp 1})'$, $\boldsymbol{\psi} = -(\boldsymbol{\alpha}\mathbf{\Omega}^{-1}\boldsymbol{\alpha})^{-1}\boldsymbol{\alpha}\mathbf{\Omega}^{-1}\mathbf{\Gamma}$,
$\boldsymbol{\alpha}_{\perp\mathbf{\Omega}} = -\mathbf{\Omega}\boldsymbol{\alpha}_{\perp}(\boldsymbol{\alpha}'_{\perp}\mathbf{\Omega}\boldsymbol{\alpha}_{\perp})^{-1}$and $\boldsymbol{\kappa} = (\boldsymbol{\alpha}'_{\perp}\mathbf{\Gamma}\overline{\boldsymbol{\beta}}, \boldsymbol{\xi})$. Under the assumption that the characteristic polynomial has exactly $2(p - r) - s_1$ unit roots and the remaining roots are outside the unit circle, $\Delta^2\mathbf{x}_t$, $(\boldsymbol{\rho}'\boldsymbol{\tau}'\mathbf{x} + \boldsymbol{\psi}'\Delta\mathbf{x})_t$ and $\boldsymbol{\tau}'\Delta\mathbf{x}_t$ all have stationary representations. In this case, r denotes the number of multi-cointegrating relations and s_1 the number of I(1) trends. The total number of common trends is $p - r = s_1 + s_2$ with s_2 is the number of I(2) trends; this leaves a total of $s_1 + 2s_2$ unit roots in the model. The reduced ranks (r, s_1) can be determined using the LR test proposed by Nielsen and Rahbek (2007). The multi-cointegrating relations are given by $\boldsymbol{\rho}'\boldsymbol{\tau}'\mathbf{x}_{t-1} + \boldsymbol{\psi}'\boldsymbol{\Delta}\mathbf{x}_{t-1}$, where the

combinations defined by $\boldsymbol{\rho}'\boldsymbol{\tau}'\mathbf{x}_{t-1}$ cointegrate from I(2) to I(1), and $\boldsymbol{\psi}'\boldsymbol{\Delta}\mathbf{x}_{t-1}$ is I(1) and cointegrate with the former to I(0). When $r > s_2$, the multi-cointegrating relations may be split into $r - s_2$ static (directly stationary) long-run relations which cointegrate from I(2) to I(0), and s_2 dynamic long-run relations which need the growth rates to become I(0). The $\boldsymbol{\alpha}$-matrix contains information on short-run adjustment in face of disequilibria. The $(r + s_1)$-dimensional vector $\boldsymbol{\tau}'\boldsymbol{\Delta}\mathbf{x}_{t-1}$ defines combinations of the growth rates which are I(0) and these may be given an interpretation as medium-run steady-state relations.

To facilitate the economic interpretation of estimation results, we use the following parameterisation obtained from re-writing (13) (Paurolo and Rahbek 1999),

$$\Delta^2\mathbf{x}_t = \boldsymbol{\alpha}[\boldsymbol{\beta}'\mathbf{x}_{t-1} + \boldsymbol{\delta}'\Delta\mathbf{x}_{t-1}] + \boldsymbol{\zeta}\boldsymbol{\tau}'\Delta\mathbf{x}_{t-1} + \boldsymbol{\varepsilon}_t, \tag{14}$$

where we have used the projection identity, $\mathbf{I}_p = \overline{\boldsymbol{\tau}}\boldsymbol{\tau}' + \overline{\boldsymbol{\tau}}_\perp\boldsymbol{\tau}'_\perp$, $\boldsymbol{\delta}' = \boldsymbol{\psi}'\overline{\boldsymbol{\tau}}_\perp\boldsymbol{\tau}'_\perp$ and $\boldsymbol{\zeta} = \boldsymbol{\alpha}_{\perp\boldsymbol{\Omega}}\boldsymbol{\kappa}' + \boldsymbol{\alpha}\boldsymbol{\psi}'\overline{\boldsymbol{\tau}}'$. For this model, a data vector $b'\mathbf{x}_t$ is said to be weakly exogenous provided,

$$b'(\boldsymbol{\alpha}, \boldsymbol{\xi}, \widetilde{\boldsymbol{\zeta}}) = 0, \tag{15}$$

where $\widetilde{\boldsymbol{\zeta}}$ contains the first r columns of $\boldsymbol{\zeta}$. The test of $b'\boldsymbol{\alpha} = 0$ amounts to the less restricted hypothesis of 'no long-run levels feed-back'. Equilibrium correction, or lack thereof, is a useful piece of information in understanding the dynamics of the model. Two levels of equilibrium-correcting behaviour can be defined within this model (Juselius 2006). First, the acceleration rates are equilibrium-correcting to the growth rates if,

$$\alpha_{ij}\delta_{ji} < 0, \tag{16}$$

where α_{ij} denotes the (i,j)'th element of $\boldsymbol{\alpha}$ and δ_{ji} the (j,i)'th element of $\boldsymbol{\delta}'$ with $i = 1, ..., p$ and $j = 1, ..., r$. Moreover, the growth rates are equilibrium-correcting to the levels provided that,

$$\delta_{ji}\beta_{ji} > 0. \tag{17}$$

with β_{ji} the (j,i)'th element of $\boldsymbol{\beta}'$.

While inference on $\boldsymbol{\tau}$ can in many cases be based on the χ^2-distribution, see Boswijk (2000) and Johansen (2006), Paruolo (1996) shows that the distribution of the test for restrictions on the multi-cointegration parameter, $\boldsymbol{\delta}$, is not mixed Gaussian. The test of $\boldsymbol{\delta} = \mathbf{0}$ is of particular interest because this implies that $\boldsymbol{\beta}'\mathbf{x}_t \sim I(0)$. Bootstrap methods might be used to simulate the distribution of $\boldsymbol{\delta}$; this is out of the scope of this paper however. Kurita, Nielsen, and Rahbek (2009) give some distributional results for $\boldsymbol{\psi}$ but this parameter does not have an obvious economic interpretation. Moreover, because $\widetilde{\boldsymbol{\beta}}_{\perp 2}$ is not identified, we cannot formally test which variables are affected by the I(2) trend(s). For both $\boldsymbol{\delta}$ and $\widetilde{\boldsymbol{\beta}}_{\perp 2}$, a provisional judgement based on the sign and magnitude of the estimated coefficients may nevertheless be made.

In order to study the common trends, we consider the solution for the levels of the process, $\mathbf{x}_t$. For the I(2) model, the moving-average (MA) representation takes the form,

$$\mathbf{x}_t = \mathbf{C}_2 \boldsymbol{\Sigma}_{j=1}^{t} \boldsymbol{\Sigma}_{i=1}^{j} \boldsymbol{\varepsilon}_i + \ \mathbf{C}_1 \Sigma_{i=1}^{t} \boldsymbol{\varepsilon}_i + \mathbf{C}^*(L)\boldsymbol{\varepsilon}_i + \mathbf{A} + \mathbf{B}t, \tag{18}$$

where $\mathbf{A}$ and $\mathbf{B}$ are functions of the initial values, $\mathbf{C}^*(L)$ is an infinite polynomial in the lag operator L, and $\mathbf{C}_2 = \widetilde{\boldsymbol{\beta}}_{\perp 2}\boldsymbol{\alpha}'_{\perp 2}$ with $\boldsymbol{\alpha}'_{\perp 2}\boldsymbol{\Sigma}_{j=1}^{t}\boldsymbol{\Sigma}_{i=1}^{j}\boldsymbol{\varepsilon}_i$ define the s_2 I(2) trends while $\widetilde{\boldsymbol{\beta}}_{\perp 2} = \boldsymbol{\beta}_{\perp 2}[\boldsymbol{\alpha}'_{\perp 2}(\boldsymbol{\Gamma}\overline{\boldsymbol{\beta}}\overline{\boldsymbol{\alpha}}'\boldsymbol{\Gamma} + \mathbf{I}_p - \boldsymbol{\Gamma}_1)\boldsymbol{\beta}_{\perp 2}]^{-1}$ provide the loadings to these. Similarly, $\boldsymbol{\alpha}'_{\perp 1}\boldsymbol{\Sigma}_{i=1}^{t}\boldsymbol{\varepsilon}_i$ defines the s_1 (separate) I(1) trends. The $\mathbf{C}_1$-matrix cannot be given a simple decomposition but Johansen (2005) derives an analytical expression, $\mathbf{C}_1 = \boldsymbol{\varpi}_0\boldsymbol{\alpha}' + \boldsymbol{\varpi}_1\boldsymbol{\alpha}'_{\perp 1} + \boldsymbol{\varpi}_2\boldsymbol{\alpha}'_{\perp 2}$ where $\boldsymbol{\varpi}_0$, $\boldsymbol{\varpi}_1$ and $\boldsymbol{\varpi}_2$ are complicated functions of the parameters.

For nominal variables such as debt, output and prices, linear trends in the levels is a reasonable starting hypothesis (Rahbek, Kongsted, and Jørgensen 1999). Johansen, Juselius, Frydman, and Goldberg (2009) and Kurita et al. (2009) show how to restrict deterministic shift terms appropriately in an I(2) model with piecewise linear deterministic trends and derive the distribution of $\boldsymbol{\beta}$ and the distributions of $\boldsymbol{\tau}$ and $\boldsymbol{\psi}$, respectively. When including deterministic shifts in the CVAR, two concerns must be accommodated. First of all, we need to consider which components are relevant from an economic point of view. In the I(2) model, all deterministic terms are cumulated both once and twice. It is therefore crucial to ensure that if, say, a trend is appropriate in the levels of the series, then this is properly restricted to ensure that it is not allowed to enter the first and second differences of the model. If left unrestricted, this will cumulate to produce quadratic and cubic trends in the data, respectively, both of which are not economically viable. Secondly, we need to consider which components are needed to ensure similarity in the test procedures; see Nielsen and Rahbek (2000) on the I(1) case. To achieve this, we should allow the same type of deterministic components in all directions of the model, i.e. in the $\boldsymbol{\alpha}, \boldsymbol{\alpha}_{\perp 1}, \boldsymbol{\alpha}_{\perp 2}$ directions alike, in order for tests of stationarity to be conducted against the appropriate alternatives, thereby improving the power of the tests.

We have left out deterministic components in the presentation above but in the empirical analysis we shall consider (14) augmented with a trend and broken linear trends restricted to the $\boldsymbol{\alpha}$-space and unrestricted permanent and transitory impulse dummies,

$$\begin{aligned}\Delta^2\mathbf{x}_t &= \boldsymbol{\alpha}\left[\underbrace{\begin{pmatrix}\boldsymbol{\beta}' & \widetilde{\boldsymbol{\beta}}_0' & \widetilde{\boldsymbol{\beta}}_1'\end{pmatrix}}_{\widetilde{\boldsymbol{\beta}}'}\begin{pmatrix}\mathbf{x}\\ t\\ t\mathbf{Ds}\end{pmatrix}+\underbrace{\begin{pmatrix}\boldsymbol{\delta}' & \widetilde{\boldsymbol{\delta}}_0' & \widetilde{\boldsymbol{\delta}}_1'\end{pmatrix}}_{\widetilde{\boldsymbol{\delta}}'}\begin{pmatrix}\Delta\mathbf{x}\\ 1\\ \mathbf{Ds}\end{pmatrix}\right]_{t-1} \\ &+\boldsymbol{\zeta}\underbrace{\begin{pmatrix}\boldsymbol{\beta}' & \widetilde{\boldsymbol{\beta}}_0' & \widetilde{\boldsymbol{\beta}}_1'\\ \boldsymbol{\beta}_{\perp1}' & \widetilde{\boldsymbol{\beta}}_{\perp1}' & \widetilde{\boldsymbol{\beta}}_{\perp1}'\end{pmatrix}}_{\widetilde{\boldsymbol{\tau}}'}\begin{pmatrix}\Delta\mathbf{x}\\ 1\\ \mathbf{Ds}\end{pmatrix}_{t-1}+\boldsymbol{\phi}_{cs}\mathbf{Dcs}_t+\boldsymbol{\phi}_p\mathbf{Dp}_t+\boldsymbol{\phi}_{tr}\mathbf{Dtr}_t+\boldsymbol{\varepsilon}_t,\end{aligned} \tag{19}$$

where t denotes a linear deterministic trend, $\mathbf{Ds}_t$ is a matrix of shift dummies, 1 a constant term, $\mathbf{Dcs}_t$ a matrix of centered seasonal dummies, $\mathbf{Dp}_t$ a matrix of permanent impulse dummies and $\mathbf{Dtr}_t$ a matrix of transitory impulse dummies; $t\mathbf{Ds}$ thus contains broken linear trends.

5 Transforming the economic model to I(2) CVAR space

We consider first the combined implications of the economic model and the graphical analysis for the specification of the empirical model, and then derive a set of long-run relations to guide identification of the multi-cointegration space. We end by setting up a scenario for the statistical behaviour of the variables based on the theoretical model.

5.1 Specification

In the empirical analysis we shall consider a slightly different data vector compared with (6),

$$\mathbf{x}_t = (b, y, p, U, I_s, I_l)_t' \tag{20}$$

where b_t is log nominal (gross) debt[5], y_t is log nominal GDP, p_t is log consumer prices, U_t is the unemployment rate, and $I_{s,t}$ and $I_{l,t}$ is the short- and long-term interest rate, respectively. Lower-case letters denote that the variable has been log-transformed, and interest rates have been divided by 400 to achieve comparability with the quarterly growth rates. The real output gap can be defined as $(y-p)_t$ in deviation from a linear deterministic

[5]Because we use a gross debt measure it is meaningful to consider its log-transform; this would ne conceptually more problematic for a 'net measure' that could, in principle, turn negative, i.e. net wealth. Using log-transformations of all nominal variables improves the model specification and makes interpretations more straightforward.

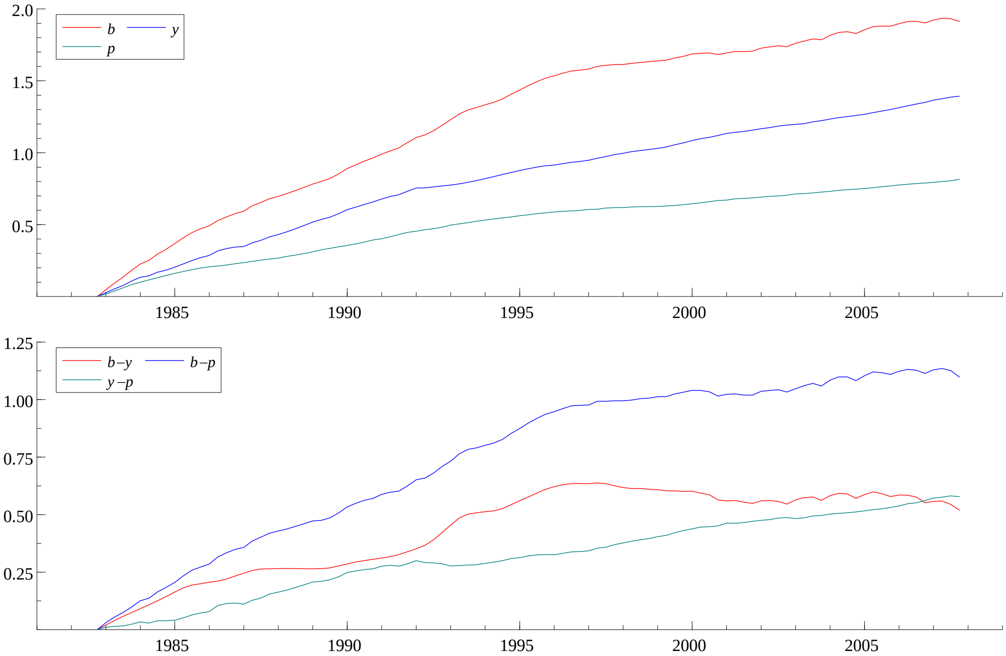

Figure 3: Upper panel: nominal variables in levels. Lower panel: transformations of the nominal variables.

time trend. We allow also another indicator of capacity utilisation, the unemployment rate, U_t, to enter the model and argue below that the fiscal rule and the Phillips curve are more appropriately defined in terms of this variable.

We include a trend in the multi-cointegrating relations and two broken trends to take account of the shifts in the deterministic trends in output and debt, respectively,

$$\widetilde{\mathbf{x}}_t = (\mathbf{x}_t', t, tDs_{92:2}, tDs_{95:3})_t' \tag{21}$$

Figure 3 and 4 show the levels of the nominal variables and the first differences plus some selected transformations of these, respectively. These illustrate the apparent trend breaks in the levels of debt around 1995 and in output around 1992. Corresponding level shifts are seen in the first differences.[6]

Figure 5 shows the unemployment rate and the two interest rates. We differentiate here between the short- and the long-term interest rate in order to allow for a slope in the yield curve. This stands in contrast to the model in Section 2 where no such distinction was made. We assume that the short rate is set by monetary policy while the long rate is

[6]The public-debt series is seasonally adjusted prior to 2000 but not thereafter. We include an additional set of seasonal dummies, $Dsc_iDs_{00:1}$, to take this into account.

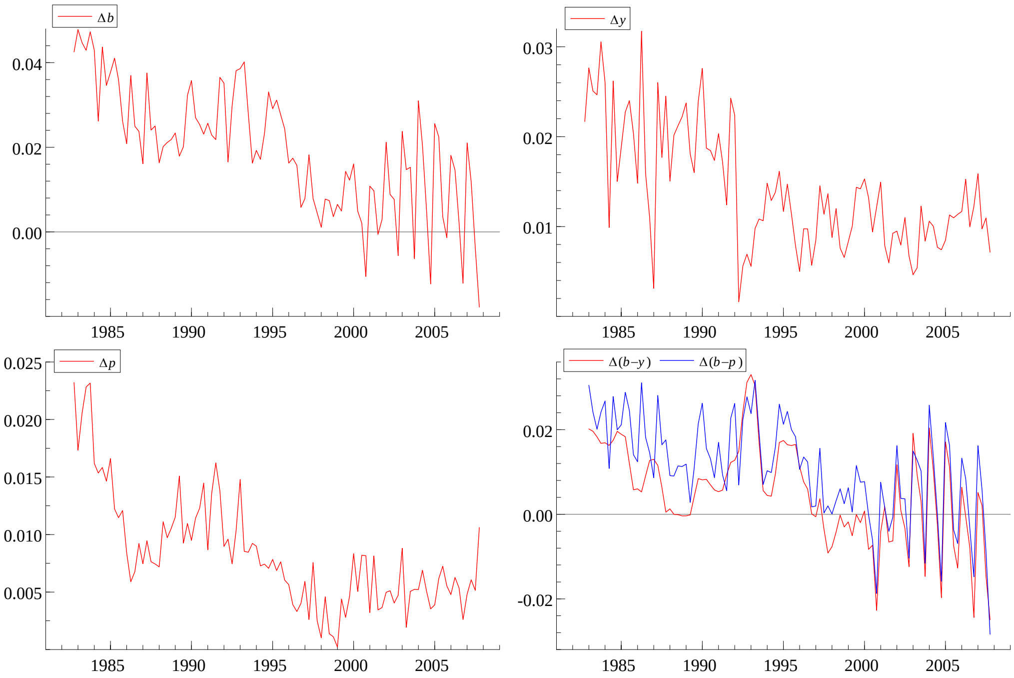

Figure 4: First differences of the nominal variables and of selected transformations.

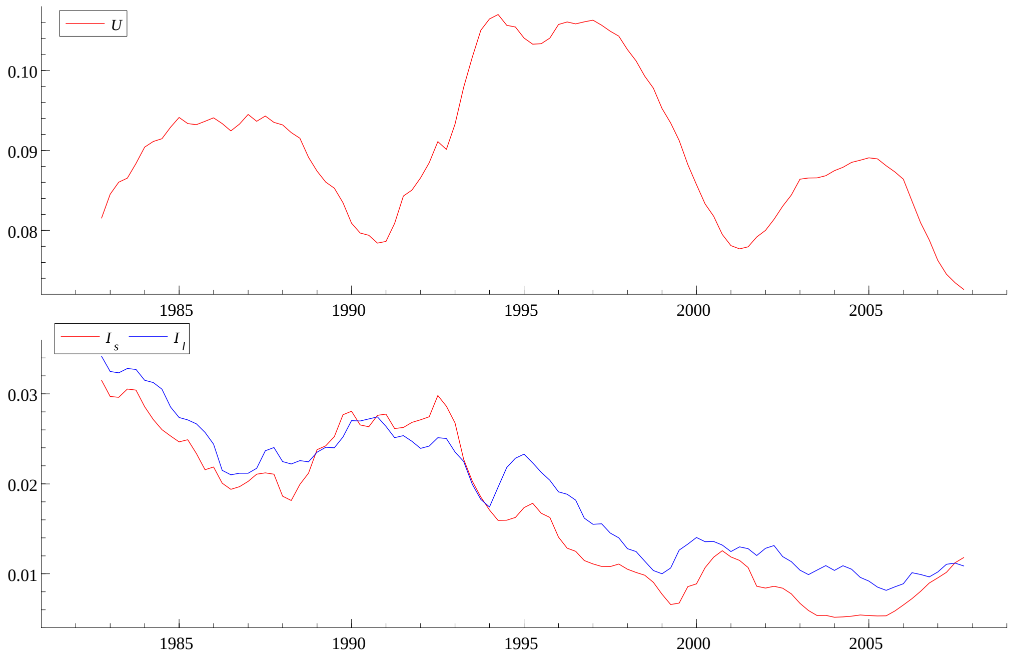

Figure 5: Upper panel: unemployment rate. Lower panel: short- and long-term interest rates.

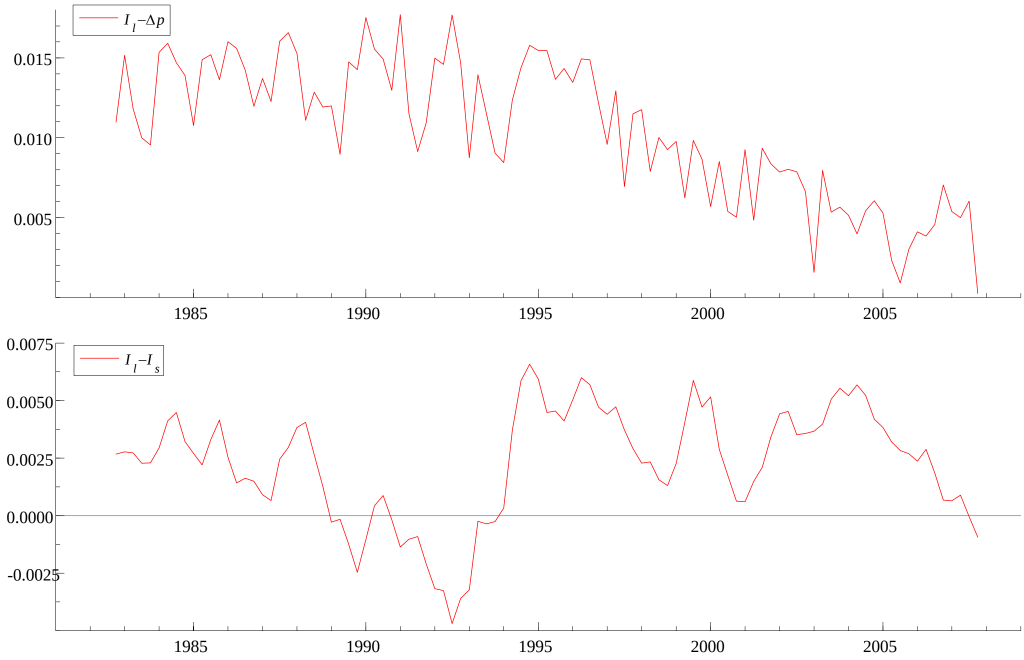

Figure 6: Upper panel: real long-term interest rate. Lower panel: long-short interest spread.

the interest rate paid on debt. A key purpose of our analysis is to see what determines the latter. The expectations hypothesis of the term structure predicts $(I_l - I_s)_t \sim I(0)$ such that, in the long run, the short and the long rate move together (Campbell and Shiller 1987, 1991). Assuming the former is controlled by monetary policy, this means that the central bank is ultimately in charge of bond yields as well. Figure 6 shows that the spread between the two interest rates has seen persistent deviations from any equilibrium mean. The real long-term interest rate, $(I_l - \Delta p)_t$, which the Fisher parity predicts to be stationary, is also depicted; this is broadly constant until the end of the 1990s and trends downward thereafter.

With the choice of data vector (20), we exploit the fact that the first differences of all variables enter the I(2) CVAR model by construction, see (19). Thus, by including levels of the nominal variables, we automatically include also their growth rates. The change in debt, Δb_t, is approximately equal to the operating deficit and we take $\Delta(b - y)_t$, rather than g_t^r as in the theoretical model, to be the instrument of fiscal policy in the empirical analysis. Including g_t^r separately would lead to (near) multi-collinearity. This deviation from the theoretical concepts means that it is not possible to do an exact mapping of the economic model to CVAR space, but we argue that the main features can be incorporated.

We might alternatively consider $\Delta(b-p)_t$ to constitute the policy instrument. As inflation has been subdued in recent decades due to increased competition in a globalised economy (Tuxen 2009) the stochastic trend in y_t is likely to be a better proxy for the 'burden of debt' than that of p_t. We shall therefore use this assumption in the transformation of the economic model but the empirical model allows for both possibilities.

5.2 Identification

Based on the above model specification, we derive some implications of the theoretical relations in terms of the CVAR. These will be used in imposing (over-)identifying restrictions on the model. We first discuss the polynomially cointegrating relations and then turn to equilibrium-correction properties.

We re-write the IS curve in (1) as,

$$\underbrace{(y-p)_t - \widetilde{\beta}_{10}t}_{\text{output gap}} - \beta_{11}(b-y)_t - \beta_{12}(I_l - \Delta p)_{t-1} - \delta_{11}\Delta(b-y)_t + \delta_{10} \sim I(0) \qquad (22)$$

where the output gap is represented as real output, $(y-p)_t$, in deviation from a linear deterministic trend, and the stance of fiscal policy by the change in debt-to-GDP, $\Delta(b-y)_t$. The test of $\beta_{11} = 0$ can be taken as an approximate test of REq. We expect $\widetilde{\beta}_{10} > 0, \beta_{11} \geq 0, \beta_{12} < 0, \delta_{11} > 0$.

We re-specify the Phillips curve (2) in terms of deviations of unemployment from its 'natural rate', i.e. NAIRU level,

$$\Delta p_t = \Delta p_t^e - \vartheta(U - U^{NAIRU})_t + v_t^{PC} \qquad (23)$$

where v_t^{PC} is an inflation shock. Inflation expectations, Δp_t^e, may be forward-looking and the NAIRU level is allowed to be time-varying. Specifically, we look for U_t^{NAIRU} to be a function of the cost of capital, $I_{l,t}$, as suggested by Phelps (1994) and found by Tuxen (2009). Moreover, we propose measuring inflation expectations by the long-short spread, $(I_l - I_s)_t$,

$$\Delta p_t - \beta_{21}U_t - \underbrace{\beta_{22}(I_l - I_s)_t}_{\simeq \Delta p_t^e} - \underbrace{(\beta_{23} - \beta_{22})I_{l,t}}_{\simeq U_t^{NAIRU}} + \delta_{20} \sim I(0) \qquad (24)$$

where we expect $\beta_{21} < 0, \beta_{22} > 0, \beta_{23} > \beta_{22}$.

The monetary-policy rule (3) is re-written in terms of the short rate,

$$I_{s,t} - \delta_{33}\Delta p_t - \beta_{31}\underbrace{(y-p)_t - \widetilde{\beta}_{30}t}_{\text{output gap}} + \delta_{30} \sim I(0) \qquad (25)$$

where the central bank is assumed to adjust the interest rate in order to stabilise inflation and output gap around targets. We expect $\widetilde{\beta}_{30} < 0, \delta_{33} > 0, \beta_{31} > 0$.

The fiscal-policy rule (4) is re-specified in terms of the level of unemployment and the change in the debt-to-GDP ratio. Using the relation between g_t^r and b_t^r in (5) and some approximations we obtain,

$$\overbrace{\underbrace{\Delta(b-y)_t}_{\text{fiscal instrument}} - \beta_{43}(I_l - \Delta p)_t}^{\text{fiscal stance corr. for interest payments}} - \beta_{41}(b-y)_t - \widetilde{\beta}_{40}t - \beta_{42}U_t + \delta_{40} \sim I(0) \tag{26}$$

where the government is assumed to adjust the operating deficit 'corrected for' interest payments, in order to stabilise the level of debt, possibly around a 'target trend rate',[7] and to loosen fiscal policy in face of rising unemployment. We expect $\widetilde{\beta}_{40} > 0, \beta_{41} < 0, \beta_{42} > 0, \beta_{43} > 0$.

Finally, the budget constraint (9) simply prescribes that $(b-y)_t$ is a cumulation of $d_t^r = \Delta(b-y)_t$. However, this accounting identity does not constitute a potential cointegrating relation: any deviations from it inherently represents measurement/approximation errors which should not be given an economic interpretation as disequilibria.

We can summarise the proposed cointegration structure as,

$$\begin{aligned}
&\widetilde{\boldsymbol{\beta}}'\widetilde{\mathbf{x}}_{t-1} + \widetilde{\boldsymbol{\delta}}'\Delta\widetilde{\mathbf{x}}_{t-1} \\
&= \begin{pmatrix} \beta_{11} & 1 & -1-\beta_{11} & 0 & 0 & \beta_{12} & \widetilde{\beta}_{10} & \widetilde{\beta}_{11} & \widetilde{\beta}_{12} \\ 0 & 0 & 0 & \beta_{21} & \beta_{22} & \beta_{23} & 0 & 0 & 0 \\ 0 & \beta_{31} & -\beta_{31} & 0 & 1 & 0 & \widetilde{\beta}_{30} & \widetilde{\beta}_{31} & \widetilde{\beta}_{32} \\ \beta_{41} & -\beta_{41} & 0 & \beta_{42} & 0 & \beta_{43} & \widetilde{\beta}_{40} & \widetilde{\beta}_{41} & \widetilde{\beta}_{42} \end{pmatrix} \begin{pmatrix} b \\ y \\ p \\ U \\ I_s \\ I_l \\ t \\ tDs_{92:2} \\ tDs_{95:3} \end{pmatrix}_t \\
&+ \begin{pmatrix} \boldsymbol{\delta}_{11} & -\boldsymbol{\delta}_{11} & -\boldsymbol{\beta}_{12} & \delta_{14} & \delta_{15} & \delta_{16} & \widetilde{\delta}_{10} & \widetilde{\delta}_{11} & \widetilde{\delta}_{12} \\ \delta_{21} & \delta_{22} & \mathbf{1} & \delta_{24} & \delta_{25} & \delta_{26} & \widetilde{\delta}_{20} & \widetilde{\delta}_{21} & \widetilde{\delta}_{22} \\ \delta_{31} & \delta_{32} & \boldsymbol{\delta}_{33} & \delta_{34} & \delta_{35} & \delta_{36} & \widetilde{\delta}_{30} & \widetilde{\delta}_{31} & \widetilde{\delta}_{32} \\ \mathbf{1} & \mathbf{-1} & -\boldsymbol{\beta}_{43} & \delta_{44} & \delta_{45} & \delta_{46} & \widetilde{\delta}_{40} & \widetilde{\delta}_{41} & \widetilde{\delta}_{42} \end{pmatrix} \begin{pmatrix} \Delta b \\ \Delta y \\ \Delta p \\ \Delta U \\ \Delta I_s \\ \Delta I_l \\ 1 \\ Ds_{92:2} \\ Ds_{95:3} \end{pmatrix}_t
\end{aligned} \tag{27}$$

[7]Sustainability of policy implies that debt-to-GDP cannot keep growing over time but a trend-adjustment may provide a reasonable approximation in-sample.

where the first relation is supposed to represent the IS curve, the second relation the Phillips curve, the third relation the Taylor-type rule, and the final relation the fiscal-policy rule. The $\widetilde{\boldsymbol{\beta}}$-matrix is only identified (on the rank condition) if further restrictions are imposed. Identification can be achieved if at least one of the following restrictions are imposed/removed: some broken trends are excludable and/or non-homogenous relationships between some of the nominal variables are allowed. We shall consider different combinations of these additional restrictions for identification in the empirical analysis.

The adjustment dynamics contained in the $\boldsymbol{\alpha}$- and $\boldsymbol{\delta}$-matrices is an important piece of information regarding the error-correction behaviour of the model, see (16) and (17). In the IS curve, (22) prescribes that Δy_t be negatively related to y_t as $\delta_{12} = -\delta_{11} < 0$ is required for a higher deficit-to-GDP to raise aggregate demand, *ceteris paribus*; further, if $\Delta^2 y_t$ equilibrium-corrects to Δy_t then $\alpha_{21} < 0$. To support the interpretation of the Phillips curve, we look for either inflation or unemployment to equilibrium-correct and thus $\alpha_{32} < 0$ and/or $\alpha_{42} < 0$. If the third relation is indeed a monetary-policy rule, the short-term interest rate should adjust towards it and we expect $\alpha_{53} < 0$. In the fiscal rule, (26) prescribes that $\Delta(b - y)_t$ be negatively related to $(b - y)_t$ and thus $\beta_{41} < 0$ is required for the stance of fiscal policy to react to the level of debt (given $\delta_{41} = 1$).

The reaction of the long-term interest rate to deviations from the fiscal-policy rule is of particular interest in light of our focus on testing the effects of deficits on bond yields. Rejection of $\alpha_{46} = 0$ in favour of $\alpha_{46} > 0$ would be consistent with excess supply of government bonds having put upward pressure on yields in the short term, *ceteris paribus*. Assessing the long-run effects of fiscal policy on yields requires us to study both the cointegration structure and the composition of the I(2) stochastic trends.

5.3 Standard scenario

In the economic model, the theoretical relations are all specified in terms of real variables or variables-to-GDP. This implicitly assumes the existence of one nominal growth rate. In terms of common stochastic trends, it implies the existence of one I(2) trend which loads identically into each of the nominal variables such that long-run price homogeneity

(LPH) holds. We can write this scenario as,

$$\begin{pmatrix} b \\ y \\ p \\ U \\ I_s \\ I_l \end{pmatrix}_t = \begin{pmatrix} \omega_b \\ \omega_y \\ \omega_p \\ 0 \\ 0 \\ 0 \end{pmatrix} \sum_{j=1}^{t}\sum_{i=1}^{j} u_{1i} + \begin{pmatrix} * & * \\ * & * \\ * & * \\ * & * \\ * & * \\ * & * \end{pmatrix} \begin{pmatrix} \sum_{i=1}^{t} u_{1i} \\ \sum_{i=1}^{t} u_{2i} \end{pmatrix} + \begin{pmatrix} * \\ * \\ * \\ 0 \\ 0 \\ 0 \end{pmatrix} t + \text{stat. and det. comp.} \tag{28}$$

where $\Sigma_{j=1}^{t}\Sigma_{i=1}^{j}u_{1i}$ denotes an I(2) trend arising from twice cumulated shocks to a given linear combination of the residuals. In addition to the $s_2 = 1$ I(2) trend prescribed by the theoretical model, we have $s_1 = p - r - s_2 = 1$ (separate) I(1) trend. Assuming the central bank is in control of inflation, the nominal anchor is set by monetary policy. The nominal trend would thus be expected to stem from twice cumulated shocks to the short-term interest rate,

$$\begin{aligned} \boldsymbol{\alpha}'_{\perp 2} &= (0,0,0,0,1,0) \\ &\Longrightarrow \sum_{j}^{t}\sum_{i}^{j} u_{1i} = \boldsymbol{\alpha}'_{\perp 2}\sum_{j}^{t}\sum_{i}^{j}\varepsilon_i = \sum_{j}^{t}\sum_{i}^{j}\varepsilon_{I_s,i} \end{aligned} \tag{29}$$

When LPH holds, the loadings of the single nominal trend are identical for all I(2) variables, i.e. $\omega_b = \omega_y = \omega_p = \omega$, and thus we have,

$$\widetilde{\boldsymbol{\beta}}'_{\perp 2} = (\omega, \omega, \omega, 0, 0, 0) \tag{30}$$

In this case, the nominal trend in each series can be eliminated by subtracting one of the other nominal series, typically the price level, such that the transformed system is defined in terms of real variables (notably interest rates and unemployment are not transformed). The linear combination $(1, -1)$ of each I(2) variable and the nominal denominator ensures cointegration from I(2) to I(1) space and the transformed system can be analysed in an I(1) model. In order to keep track of the nominal growth rate and allow for multi-cointegration involving the nominal growth rate, the first difference of either of the nominal variables, typically the inflation rate, must be included in the model alongside the transformed variables. As long as the restrictions imposed in this nominal-to-real transformation (NRT) are valid, no information is lost (Kongsted 2005). We can write the transformed system based on (28) and (30) as,

$$\begin{pmatrix} b_r \\ y_r \\ \Delta p \\ U \\ I_s \\ I_l \end{pmatrix}_t = \begin{pmatrix} * & * \\ * & * \\ * & * \\ * & * \\ * & * \\ * & * \end{pmatrix} \begin{pmatrix} \sum_{i=1}^{t} u_{1i} \\ \sum_{i=1}^{t} u_{2i} \end{pmatrix} + \begin{pmatrix} * \\ * \\ * \\ 0 \\ 0 \\ 0 \end{pmatrix} t + \text{stat. and det. comp.} \tag{31}$$

In contrast with this standard theoretical scenario, the graphical analysis in Section 3 indicated that LPH does not hold within our sample as both real output and debt-to-GDP appeared to be I(2). This hypothesis can be formally tested within the I(2) model.

6 Empirical results

We first discuss specification of the model and determination of the I(2) ranks. We then proceed to interpret the cointegration structure and discuss the common stochastic trends with a view to investigate the dynamics of unemployment, debt and bond yields.

6.1 Specification tests

Based on the set of variables in (20) and the trend specification in (21) we estimate a VAR(2) model. To take account of extraordinary events we include a set of dummy variables,

$$d_t = (Dtr_{86:2}, Dtr_{87:1})'_t \tag{32}$$

where $Dtr_{YY:Q}$ is a transitory blip dummy which takes a value of one at time YY:Q, minus one in the following period, and zero otherwise.[8] We set the lag length to two based on information criteria and autocorrelation tests. Table 1 shows that the choice $k = 2$ minimises both the Schwarz criterion and the Hannan-Quinn statistic. The LR-tests for lag reduction point to a higher lag length but the test for reduction from $k = 3$ to $k = 2$ rejects 'less strongly'. The LM-tests for autocorrelation suggest that $k = 2$ does not leave any significant autocorrelation at the first or the second lag, and inspection of the coefficient matrix for the lagged acceleration rates for $k = 3$ shows that this has few significant entries. Table 2 shows a set of tests for misspecification. Multivariate normality cannot be rejected with a large margin. At the five- but not the one-per cent level third- and fourth-order autocorrelation and some ARCH effects show up as

[8]For later use, we define $Dp_{YY:Q}$ as a permanent impulse dummy which takes a value of one at time YY:Q, and zero otherwise; Dsc_i with $i = 1, 2, 3$ denote centered seasonal dummies.

Model	no obs	no reg	SC	H-Q	VAR(k)⟶VAR(k-1)	LM(1)	LM(k)
VAR(4)	97	38	-71.26	-74.86	$\chi^2(36) = \underset{[0.00]}{101.60}$	$p = 0.21$	$p = 0.00$
VAR(3)	97	32	-71.91	-74.94	$\chi^2(36) = \underset{[0.00]}{77.90}$	$p = 0.06$	$p = 0.15$
VAR(2)	97	26	$\mathbf{-72.80}$	$\mathbf{-75.27}$	$\chi^2(36) = \underset{[0.00]}{202.97}$	$p = 0.38$	$p = 0.15$
VAR(1)	97	20	-72.41	-74.30	$-$	$p = 0.00$	$p = 0.00$

Table 1: Lag length determination: information criteria and autocorrelation.

Lag	Test statistic	p-value
LM tests for no autocorrelation:		
1	$\chi^2(36) = 50.35$	0.06
2	$\chi^2(36) = 47.00$	0.10
3	$\chi^2(36) = 56.61$	0.02
4	$\chi^2(36) = 60.03$	0.01
Test for multivariate normality:		
	$\chi^2(12) = 8.04$	0.78
LM tests for no ARCH effects:		
1	$\chi^2(441) = 515$	0.01
2	$\chi^2(882) = 977$	0.01
3	$\chi^2(1323) = 1420$	0.03
4	$\chi^2(1764) = 1852$	0.07

Table 2: Misspecification tests

significant. Increasing the lag length does not solve the autocorrelation problem though. We conclude that $k = 2$ provides the best trade-off between preserving degrees of freedom and achieving a well-specified model.

6.2 Determination of the I(2) ranks

We determine the number of polynomially cointegrating relations, r, and the number of I(2) trends, s_2, using the maximum-likelihood procedure of Johansen (1997). Our theoretical prior is $r = 4, s_2 = 1$ as shown in (28). Since the model includes broken linear trends in the levels, the asymptotic distribution derived by Nielsen and Rahbek (2007) for a restricted trend only cannot be used. Instead we simulate the critical values to take account of the specific location of the breaks.[9]

The test procedure starts from the most restricted model ($r = 0, s_2 = 6$) in Table 3 and proceeds row-wise until the first non-rejection. This is found to be ($r = 4, s_2 = 2$) for

[9]Ox code for simulating the distribution of the I(2) rank test was kindly provided by Heino Bohn Nielsen (length of random walk: 2000; number of replications: 20,000).

$p-r$	r	$s_2=6$	$s_2=5$	$s_2=4$	$s_2=3$	$s_2=2$	$s_2=1$	$s_2=0$
6	0	505.21 [359]	420.33 [317]	367.45 [260]	326.77 [246]	291.11 [217]	269.96 [191]	256.20 [170]
5	1		335.09 [271]	287.67 [236]	246.74 [204]	214.93 [176]	196.88 [153]	181.79 [133]
4	2			232.00 [195]	194.87 [165]	158.73 [140]	140.31 [118]	127.27 [100]
3	3				144.53 [130]	112.03 [107]	89.48 [87]	76.63 [71]
2	4					**73.31** [77]	55.33 [59]	44.43 [45]
1	5						29.97 [36]	21.25 [23]

Table 3: Test for I(2) ranks (simulated 95-per cent critical values in brackets).

which the test statistic is insignificant at the five-per cent level. The graphs of the multi-cointegrating relations (not reported) look stationary and significance of the $\boldsymbol{\alpha}$-coefficients (not reported) suggests that adjustment takes place towards at least four cointegrating relations. We conclude that four polynomially cointegrating relations is a reasonable choice.

As an alternative to $(r = 4, s_2 = 2)$ we might consider $(r = 4, s_2 = 1)$ for which the test statistic is clearly below the critical value. The choice between one and two nominal trends is particularly important from an economic point of view as the former is consistent with the standard scenario, (28), whereas the latter is not. The two preferred models are nested and these can be compared directly using the 'maximum-eigenvalue' test proposed by Nielsen (2007). The test statistic is 17.97 and the simulated five-per cent critical value is approximately 23. Hence the test does not reject the reduction down to $(r = 4, s_2 = 2)$. We conclude that two I(2) trends, rather than one, more accurately represent the properties of the data and continue based on this choice. This implies that we have $r - s_2 = 2$ directly stationary relations in the system.

6.3 Data-consistent scenario

The finding of two I(2) trends implies that we need to revise the standard scenario in (28) to be consistent with the data,

$$\begin{pmatrix} b \\ y \\ p \\ U \\ I_s \\ I_l \end{pmatrix}_t = \begin{pmatrix} \omega_{1b} & \omega_{2b} \\ \omega_{1y} & \omega_{2y} \\ \omega_{1p} & \omega_{2p} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \sum_{j=1}^{t} \sum_{i=1}^{j} u_{1i} \\ \sum_{j=1}^{t} \sum_{i=1}^{j} u_{2i} \end{pmatrix} \tag{33}$$

$$+ \begin{pmatrix} * & * \\ * & * \\ * & * \\ * & * \\ * & * \\ * & * \end{pmatrix} \begin{pmatrix} \sum_{i=1}^{t} u_{1i} \\ \sum_{i=1}^{t} u_{2i} \end{pmatrix} + \begin{pmatrix} * \\ * \\ * \\ 0 \\ 0 \\ 0 \end{pmatrix} t + \text{stat. and det. comp.},$$

where we allow two separate I(2) trends to drive the nominal variables. The finding that $s_2 = 2$ implies that there is no simple transformation to I(1) space, such as the standard NRT, which can be made without loss of information. At least two nominal growth rates would be required in an I(1) model to keep track of the I(2) components and to allow for polynomial cointegration. The hypotheses of medium-run price homogeneity, a zero-sum restriction on $\mathbf{\Gamma}$, and of long-run price homogeneity, a zero-sum restriction on $\boldsymbol{\tau}$, are thus both implicitly rejected by the finding of $s_2 > 1$.

6.4 Preliminary tests

Arbitrage arguments prescribe that bond yields should behave approximately as random walks and thus be I(1) (Johansen and Juselius 2006). The unemployment rate, being the ratio of unemployed to the labour force, might similarly be expected to be show I(1) behaviour. Table 4 also shows tests for whether $(U, I_s, I_l)_t$ are individually I(1) by restricting each to have a unit vector in $\boldsymbol{\tau}$. The I(1) hypothesis is rejected in all cases however. The finding that $(I_l, I_s)_t$ are integrated of order higher than one is consistent with the prediction of Juselius, Frydman, Goldberg, and Johansen (2009) that interest rates should exhibit near-I(2) behaviour when agents form expectations based on imperfect knowledge. Using the same notation as for the nominal variables, this in principle implies that either $\omega_{1i} \neq 0$ and/or $\omega_{2i} \neq 0$ for $i = U, I_s, I_l$ in (33).

	Unit vector in $\boldsymbol{\tau}$	Zero row in $\boldsymbol{\alpha}$	Unit vector in $\boldsymbol{\alpha}$
b_t	-	$\chi^2(4) = 41.46$ [0.00]	$\chi^2(2) = 4.13$ [0.13]
y_t	-	$\chi^2(4) = 37.93$ [0.00]	$\chi^2(2) = 0.75$ [0.69]
p_t	-	$\chi^2(4) = 25.79$ [0.00]	$\chi^2(2) = 1.53$ [0.46]
U_t	$\chi^2(5) = 21.88$ [0.0006]	$\chi^2(4) = 2.40$ [0.66]	$\chi^2(2) = 7.05$ [0.03]
$I_{s,t}$	$\chi^2(5) = 19.10$ [0.0018]	$\chi^2(4) = 6.45$ [0.17]	$\chi^2(2) = 0.83$ [0.66]
$I_{l,t}$	$\chi^2(5) = 20.37$ [0.0011]	$\chi^2(4) = 9.88$ [0.04]	$\chi^2(2) = 5.92$ [0.05]

Table 4: Tests of restrictions on $\boldsymbol{\tau}$ and $\boldsymbol{\alpha}$.

Table 4 also reports tests on $\boldsymbol{\alpha}$.[10] The tests for no long-run levels feed-back, i.e. whether any of the variables have a zero row in $\boldsymbol{\alpha}$, show that this hypothesis is not rejected for the unemployment rate and the short rate, and only borderline so at the five-per cent level for the long rate. The joint test of both U_t and $I_{s,t}$ exhibiting no long-run levels feed-back is not rejected with a large margin ($p = 0.33$). The tests for whether any of the variables are purely adjusting within the system, i.e. have a unit vector in $\boldsymbol{\alpha}$, suggest that both the unemployment rate and the long rate may not be adjusting to the long-run relations at all.

6.5 Cointegration structure

We impose the theoretical restrictions on $\widetilde{\boldsymbol{\beta}}$ in (27) and achieve formal identification by excluding $tDs_{92:2}$ in the IS relation, both broken trends in Taylor rule and $tDs_{95:3}$ in the fiscal rule. This set of restrictions is rejected ($\chi^2(9) = 46.68$) which is not surprising given the implicit rejection of long-run homogeneity above (all combinations of exclusion restrictions on the broken trends result in rejection).

Allowing for a non-homogenous relation between b_t and y_t in the fiscal rule, between b_t, y_t and p_t in the IS curve, leaving out $I_{s,t}$ in the Phillips curve and $I_{l,t}$ in the fiscal rule, result in (borderline) non-rejection at the one-per cent level with a test statistic of $\chi^2(8) = 20.60$ ($p = 0.01$). Figure 7 shows graphs of the polynomially cointegrating relations. These suggest that the structure imposed on $\widetilde{\boldsymbol{\beta}}$ is capable of greatly reducing, albeit not completely eliminating, the large degree of persistency in both the levels and first differences of the series. Although the low p-value points to some problems with

[10] For computational reasons, the tests on $\boldsymbol{\alpha}$ are conducted in I(1) space with $r = 4$; this should not have major effects on the conclusions.

non-stationarity, from the graphs we conclude that the slightly modified set of theoretical restrictions are not 'too far from satisfied' by the data. It is surely possible to find a set of restrictions which results in a higher p-value but this can only be achieved by sacrificing economic significance. In order to focus on the most fundamental deviations from the theoretical scenario, we decide here on trading off statistical identification in favour of a structure that comes rather close to the theoretical model, yet allows us to illustrate where theory might have to be modified. We consider also the common trends subject only to reduced-rank restrictions and show that the results on the pushing forces are robust to this choice.

We consider each of the long-run relations in turn below. Table B.1 and Table B.2 provide the estimates of $(\widetilde{\boldsymbol{\beta}}, \boldsymbol{\alpha}, \widetilde{\boldsymbol{\delta}})$ and $(\boldsymbol{\zeta}, \boldsymbol{\phi}_{cs}, \boldsymbol{\phi}_{tr}, \boldsymbol{\phi}_p)$, respectively.[11] In interpreting the magnitude of the estimated coefficients we keep in mind that these might be influenced by the inclusion of the full set of first differences, a lot of which may show up as insignificant, if standard errors on $\widetilde{\boldsymbol{\delta}}$ were available. The results of an I(1) analysis of a transformed set of variables, which allows us to restrict the nominal growth rate(s) to zero, is used as a check in this respect.

The IS curve takes a form similar to (22),

$$\begin{aligned}&\underbrace{(y-p)_t}_{\text{real output}} - \underset{[-28.17]}{2.17}\underbrace{(b-1.68p)_t}_{\text{real debt}} - \underset{[-12.40]}{0.01}\, tDs_{95:3} + \underset{[6.71]}{0.01}t \\ &\quad + \underset{[6.88]}{4.28}\underbrace{(I_l-\Delta p)_t}_{\text{real rate}} - \underset{[N.A.]}{8.83}\,\underbrace{\Delta(b-y)_t}_{\text{deficit-to-GDP}} + ... \sim I(0),\end{aligned} \tag{34}$$

where the real bond rate is negatively related to real output, *ceteris paribus*. Fiscal policy plays a role for the real economy both via the stance of policy, as measured by the deficit-to-GDP ratio, and via the level of real debt. The finding that $\beta_{11} > 0$ implies that the (approximate) REq hypothesis is rejected. This is in line with other studies on the effects of financing decisions of governments in the euro area, see *inter alia* Nickel and Vansteenkiste (2008).

The standard Phillips curve is augmented with the bond rate as suggested by (24),

$$U_t + \underset{[N.A.]}{0.85}\Delta p_t - \underset{[-16.12]}{2.54}\, I_{l,t} + ... \sim I(0), \tag{35}$$

which incorporates the usual trade-off between unemployment and inflation after correcting for the level of the long-term interest rate. This resembles the hypothesis of Phelps

[11] The large t-statistics for the coefficients of the nominal variables in $\widetilde{\boldsymbol{\beta}}$ are a result of the super-super consistency of the estimates, leaving these very precisely estimated and thus very sensitive to deviations from LPH.

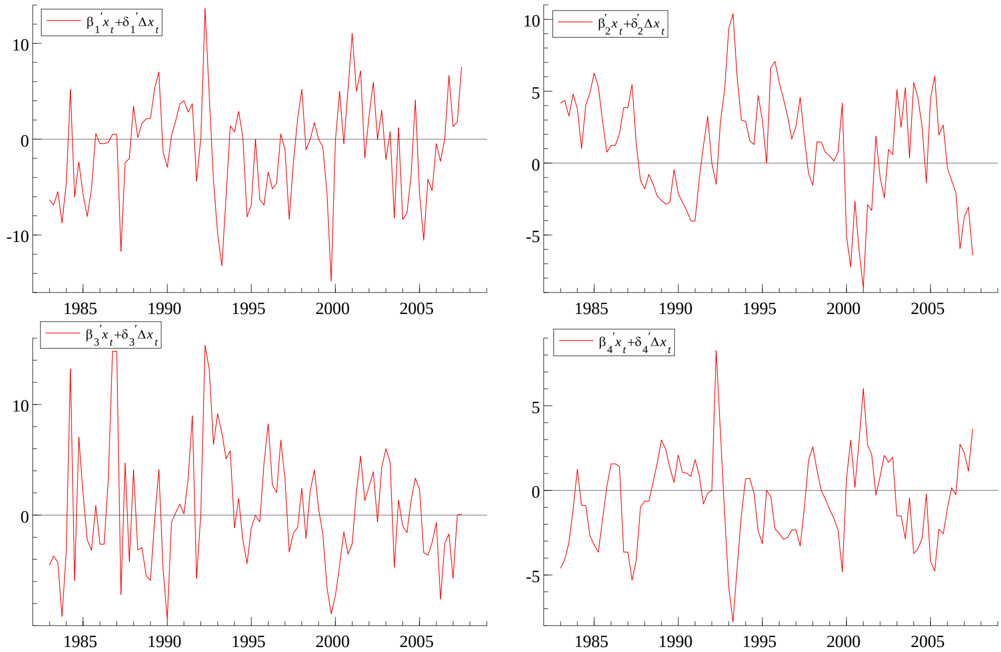

Figure 7: Multi-cointegrating relations corrected for short-run dynamics (over-identifying restrictions on $\boldsymbol{\beta}$ imposed).

(1994) who argues that a rise in the rental cost of capital depresses employment. Phelps emphasised the (real) rate of interest as the core transmission mechanism between the labour market and the wider economy because changes in the cost of capital induce movements in the NAIRU level. Thus (35) points to the existence of a non-constant NAIRU which moves with the long rate. The lack of a role for inflation expectations, as arguably measured by the spread, implies an implicit rejection of the New-Keynesian Phillips curve, consistent with the findings in Bårdsen, Jansen, and Nymoen (2004), Bjørnstad and Nymoen (2008) and Juselius and Ordóñez (2009).

The monetary-policy rule is similar to the Taylor-type rule (25),

$$I_{s,t} - \underset{[-519.11]}{0.10} \underbrace{(y-p)_t}_{\text{real output}} + \underset{[20.78]}{0.00}\, t - \underset{[N.A.]}{0.77} \Delta p_t + \ldots \sim I(0), \tag{36}$$

where the central bank appears to raise the short rate in response to a rise in inflation and/or the output gap. The magnitude of the coefficients differ from those suggested by Taylor (1993); this may be attributed to the fact that the associated $\boldsymbol{\delta}$-row signals that, besides Δp_t, monetary policy also reacts to the two other nominal growth rates, Δb_t and Δy_t.

The final relation takes the form,

$$(b - \underset{[-91.79]}{3.42}\, y)_t - \underset{[-34.03]}{0.01}\, tDs_{92:2} + \underset{[71.95]}{0.04} t - \underset{[-14.14]}{3.19}\, U_t + \underset{[N.A.]}{2.21} \Delta b_t - \underset{[N.A.]}{1.67} \Delta y_t + ... \sim I(0), \quad (37)$$

which we can re-write to illustrate its interpretation as a fiscal-policy rule,

$$\underbrace{\Delta(b-y)_t}_{\text{deficit-to-GDP}} + 0.45\underbrace{(b-3.71y)_t}_{\text{debt-to-GDP}} - 0.01tDs_{92:2} + 0.04t - 3.19U_t + ... \sim I(0), \quad (38)$$

which suggests that a loosening of the fiscal-policy stance is associated with a decline in the trend-adjusted debt-to-GDP ratio and/or a rise in unemployment. Such apparent policy actions may be discretionary or simply due to automatic stabilisers. Compared with (26), the bond rate is excluded which means that the level of the interest rate is not corrected for by the government in setting policy. Hence policy makers seem to steer the operating, rather than the primary, surplus.

Conditional on the cointegration structure, the dynamic adjustment in face of disequilibria is as follows,

$$\begin{pmatrix} \Delta^2 b \\ \Delta^2 y \\ \Delta^2 p \\ \Delta^2 U \\ \Delta^2 I_s \\ \Delta^2 I_l \end{pmatrix}_t = \underbrace{\begin{pmatrix} \underset{[8.44]}{0.27} & \underset{[5.97]}{0.44} & \underset{[-3.92]}{-0.55} & \underset{[6.00]}{0.25} \\ \underset{[5.32]}{0.12} & \underset{[2.15]}{0.11} & \underset{[-2.51]}{-0.24} & \underset{[7.48]}{0.22} \\ \underset{[2.36]}{0.04} & \underset{[-3.17]}{-0.13} & \underset{[0.18]}{0.01} & \underset{[3.59]}{0.08} \\ \underset{[-0.56]}{-0.00} & \underset{[-2.50]}{-0.04} & \underset{[-3.19]}{-0.10} & \underset{[-1.14]}{-0.01} \\ \underset{[1.49]}{0.01} & \underset{[1.65]}{0.03} & \underset{[-2.44]}{-0.07} & \underset{[1.36]}{0.01} \\ \underset{[4.52]}{0.03} & \underset{[2.95]}{0.04} & \underset{[-4.82]}{-0.12} & \underset{[3.30]}{0.03} \end{pmatrix}}_{\boldsymbol{\alpha}} \begin{pmatrix} \widetilde{\beta}_1' \widetilde{\mathbf{x}} + \widetilde{\delta}_1' \Delta\widetilde{\mathbf{x}} \\ \widetilde{\beta}_2' \widetilde{\mathbf{x}} + \widetilde{\delta}_2' \Delta\widetilde{\mathbf{x}} \\ \widetilde{\beta}_3' \widetilde{\mathbf{x}} + \widetilde{\delta}_3' \Delta\widetilde{\mathbf{x}} \\ \widetilde{\beta}_4' \widetilde{\mathbf{x}} + \widetilde{\delta}_4' \Delta\widetilde{\mathbf{x}} \end{pmatrix}_{t-1} + ..., \quad (39)$$

This shows that there is a tendency for output to adjust so as to increase equilibrium errors when GDP growth is away from the level suggested by the IS curve, $\alpha_{21} > 0$. Hence even though Δy_t error-corrects (this is 'built into' the relation) this is not the case for $\Delta^2 y_t$. This may reflect outside factors holding up economic growth, such as the integration of China and a range of other emerging countries into the world economy, arguably leading to an extraordinary rise in output in this period. Both unemployment and prices error-correct in second differences to the Phillips curve, $\alpha_{32} < 0$ and $\alpha_{42} < 0$, as does the short rate to deviations from the monetary-policy rule, $\alpha_{53} < 0$. Within the fiscal-policy rule, debt-to-GDP error-corrects in first differences (again built into the relation) whereas this is not the case for the second differences as public debt has a tendency to error-increase when deficits deviate from the level suggested by the fiscal rule, $\alpha_{14} > 0$. While this may partly represent a (purely mechanical) accounting effect, it may also be taken as further evidence of a vicious circle of rising deficits/debt.

Adjustment towards the medium-run relations is given by $\boldsymbol{\zeta}$ in Table B.2. Significant reactions are seen in one or more variables in face of deviations from all relations, even when tested against one (rather than zero) as is the appropriate null for the I(1) variables due to over-differencing (Juselius 2006). Notably both unemployment and the short rate react to changes in deviations from the fiscal and the monetary rule, respectively. Thus, although these variables exhibited little feed-back on the long-run relations, see Table 4, neither is weakly exogenous on the condition (15).

Regarding policy interactions, the picture is mixed. However, $\zeta_{13} > 0$ and $\zeta_{54} > 0$ are consistent with Tuxen (2009) in suggesting that there has been some tendencies for policy makers to attempt to counteract the actions of the other authority. With respect to short-run effects of policy on GDP and employment, these are also somewhat mixed but here $\alpha_{23} < 0$ and $\zeta_{44} > 0$ are consistent with the results in Tuxen that monetary policy goes some way in affecting output whereas fiscal policy has had adverse effects on economic activity.

Considering disequilibria in a range of economic sectors and thus a larger information set than the one employed here, Tuxen (2009) found that bond yields are mainly driven by factors outside the euro area. Notably downward pressure from US yields seemed to have been a key factor in this period. In comparison, the model of this paper provides estimates of the 'isolated effect' of fiscal policy. The $\boldsymbol{\alpha}$-matrix shows that a range of factors have affected bond yields in the short run. The fact that bond-rate dynamics manifest itself more clearly here, i.e. the long rate is not weakly exogenous, despite a smaller information set, suggests that the I(2) model is a more appropriate framework for modelling debt dynamics.

Specifically, the I_l row in $\boldsymbol{\alpha}$ shows that the bond rate has had a tendency to rise in response to excessively loose fiscal policy. The fiscal rule inherently takes both the level of public debt and the unemployment rate into account, but even after controlling for the level of these variables, both of which may individually affect bond markets, do budget deficits lead long rates to rise in the short run. Deviations from the other multi-cointegrating relation also affect bond yields. Excessively loose monetary policy causes the long rate to increase, possibly reflecting market expectations of the necessity of higher policy rates in the future. Moreover, the long rate seems to rise when output increases

$\boldsymbol{\alpha}'_{\perp 2}$	$\Sigma\Sigma\varepsilon_b$	$\Sigma\Sigma\varepsilon_y$	$\Sigma\Sigma\varepsilon_p$	$\Sigma\Sigma\varepsilon_U$	$\Sigma\Sigma\varepsilon_{I_s}$	$\Sigma\Sigma\varepsilon_{I_l}$
$\alpha'_{\perp 21}$	-0.02 $[-0.50]$	-0.11 $[-1.79]$	0.15 $[2.02]$	-0.34 $[-1.63]$	1.00 $[NA]$	0.00 $[NA]$
$\alpha'_{\perp 22}$	-0.08 $[-2.75]$	-0.09 $[-1.71]$	0.12 $[1.78]$	-0.55 $[-3.05]$	0.00 $[NA]$	1.00 $[NA]$
$\widehat{\sigma}_\varepsilon$	0.0038	0.0026	0.0021	0.0008	0.0008	0.0007

Table 5: I(2) trends (over-identifying restrictions on $\boldsymbol{\beta}$ imposed).

above the level suggested by the IS curve; unemployment in excess of the Phelps-Phillips curve level also puts upward pressure on bond yields, consistent with the vicious-circle hypothesis.

In sum, we have identified a positive long-run co-movement between debt/deficits and unemployment (the fiscal rule) as well as between unemployment and the long rate (the Phelps-Phillips curve). In addition, we found a positive short-run effect of a fiscal loosening on bond yields. To see whether the latter materialises into a long-run effect requires us a study the common trends.

6.6 Common trends

While the cointegration structure shows how the system is pulled back towards equilibrium in face of shocks, the composition of the common stochastic trends reveals which forces are pushing the system away from steady state. The finding of two I(2) trends, as opposed to just one predicted by theory, calls for an investigation of the origin of stochastic trends. We thus consider next the MA representation. The common trends can be interpreted as arising from twice cumulated shocks to a relation between the variables to which shocks enter the rows of $\boldsymbol{\alpha}'_{\perp 2}$. *A priori*, we expected one I(2) trend to stem from shocks to the short rate, see (28). If fiscal policy has played a separate role in driving the long end of the term structure in this period, then the additional I(2) trend which we identified, see (33), might reflect this. We consider here the area-wide I(2) trends whereas Appendix C studies cross-country differences for Germany, France, Italy and the Netherlands. Table 5 shows the composition of the I(2) trends in terms of the model residuals subject to the over-identifying restrictions on $\boldsymbol{\beta}$ from before. We normalise on the largest (in absolute value) significant coefficient in each $\boldsymbol{\alpha}_\perp$-column.

The first I(2) trend stems almost solely from shocks to the short rate, albeit with a

$\boldsymbol{\alpha}'_{\perp 2}$	$\Sigma\Sigma\varepsilon_b$	$\Sigma\Sigma\varepsilon_y$	$\Sigma\Sigma\varepsilon_p$	$\Sigma\Sigma\varepsilon_U$	$\Sigma\Sigma\varepsilon_{I_s}$	$\Sigma\Sigma\varepsilon_{(I_l-I_s)}$
$\alpha'_{\perp 21}$	-0.022 $[-0.693]$	0.003 $[0.064]$	-0.057 $[-0.983]$	-0.228 $[-1.238]$	1.000 $[NA]$	0.000 $[NA]$
$\alpha'_{\perp 22}$	-0.053 $[-1.593]$	-0.058 $[-1.059]$	0.110 $[1.833]$	-0.057 $[-0.298]$	0.000 $[NA]$	1.000 $[NA]$
$\widehat{\sigma}_{\varepsilon}$	0.0039	0.0027	0.0019	0.0009	0.0008	0.0008

Table 6: Alternative parameterisation: I(2) trends (no identifying restrictions on $\boldsymbol{\beta}$ imposed).

borderline significant (negative) contribution from, what is approximately, real output,

$$\alpha'_{\perp 21}\sum_{j=1}^{t}\sum_{i=1}^{j}\boldsymbol{\varepsilon}_i=\sum_{j=1}^{t}\sum_{i=1}^{j}(\varepsilon_{I_s}-\underset{[-1.79]}{0.11}\,\varepsilon_y+\underset{[2.02]}{0.15}\varepsilon_p+...)_i \tag{40}$$

The composition of (40) is in accordance with the identification of a monetary-policy rule as positive shocks to output is associated with positive shocks to short-term interest rates. Shocks to the short rate have thus been one of the pushing force in the system, and we interpret the first I(2) trend as induced by monetary policy.

The second I(2) trend is primarily made up of shocks to the bond rate plus some (negative) contributions from shocks to real public debt and the unemployment rate,

$$\alpha'_{\perp 22}\sum_{j=1}^{t}\sum_{i=1}^{j}\boldsymbol{\varepsilon}_i=\sum_{j=1}^{t}\sum_{i=1}^{j}(\varepsilon_{I_l}-\underset{[-3.05]}{0.55}\,\varepsilon_U-\underset{[-2.75]}{0.08}\,\varepsilon_b+\underset{[1.78]}{0.12}\varepsilon_p+...)_i, \tag{41}$$

The composition of (41) is in accordance with the hypothesis that positive shocks to interest rates and/or unemployment have been associated with positive shocks to the real value of public debt. This, together with the evidence from the multi-cointegration structure, is consistent with a vicious circle of rising unemployment and budget deficits having put upward pressure on bond yields: the Phelps-Phillips curve pointed to a NAIRU level varying with the bond rate, the fiscal rule suggested that deficits/debt co-moved with unemployment, and (41) highlights a positive relationship of bond yields with both public debt and unemployment. Together these findings are compatible with the existence of a self-reinforcing mechanism where market participants drive up long-term rates when unemployment rises in anticipation of an increase in the supply of government bonds. Based on this reasoning, we interpret the second I(2) trend as induced by fiscal policy, corrected for bond-markets reactions.

6.7 Empirically relevant scenario

The clear separation of shocks to the short rate from shocks to the long rate as the two nominal driving forces implies that the expectations hypothesis of the term structure does not hold. Giese (2008) finds that among a set of US zero-coupon rates, two common trends, arising from a level and a slope factor, respectively, are present. This invites a re-parameterisation of our model in terms of the interest-rate spread (slope) alongside one of the two interest rates (level). The spread may here be interpreted as representing bond markets' assessment of fiscal policy 'corrected for the monetary-policy stance'. Table 6 shows the composition of the I(2) trends for the model based on the alternative re-parameterisation,

$$\mathbf{x}_t^{alt} = (b, y, p, U, I_s, (I_l - I_s))'_t \tag{42}$$

To avoid dependence of the common trends on restrictions imposed on $\boldsymbol{\beta}$ in the discussion to follow, we do not impose any restrictions here besides reduced ranks. The interpretation of the two trends is similar to before but $\Sigma\Sigma\varepsilon_U$ and $\Sigma\Sigma\varepsilon_b$ are now insignificant in the fiscal trend. The first and second trends are thus made up of twice cumulated shocks to the short rate and to the spread, respectively.

Substituting for $\boldsymbol{\alpha}'_{\perp 2}$ and $\tilde{\boldsymbol{\beta}}'_{\perp 2}$ in (33), the empirically relevant scenario becomes,

$$\begin{pmatrix} b \\ y \\ p \\ U \\ I_s \\ I_l \end{pmatrix}_t \simeq \underbrace{\begin{pmatrix} \mathbf{1.17} & \mathbf{2.19} \\ \mathbf{1.13} & 0.45 \\ \mathbf{1.71} & \mathbf{1.95} \\ -0.25 & 0.42 \\ 0.30 & 0.16 \\ -0.07 & 0.06 \end{pmatrix}}_{\tilde{\boldsymbol{\beta}}'_{\perp 2}} \begin{pmatrix} \sum_{j=1}^{t}\sum_{i=1}^{j} \varepsilon_{I_s,i} \\ \sum_{j=1}^{t}\sum_{i=1}^{j} \varepsilon_{(I_l - I_s),i} \end{pmatrix} + ... \tag{43}$$

Positive shocks to the short rate have thus affected all nominal variables, notably the price level, positively. This is consistent with the findings in Tuxen (2009) that inflation has largely been cost-determined in this period with higher interest rates leading nominal growth to rise because the cost of capital rises. Positive shocks to the spread have (mainly) affected public debt and prices positively. This is consistent with the vicious-circle hypothesis in that higher risk premia lead interest payments and thus deficits/debt to rise.

Figure 8 plots the common trends. Since the residuals represent unexpected shocks given the estimated model, the common trends inherently reflect the effects of outside

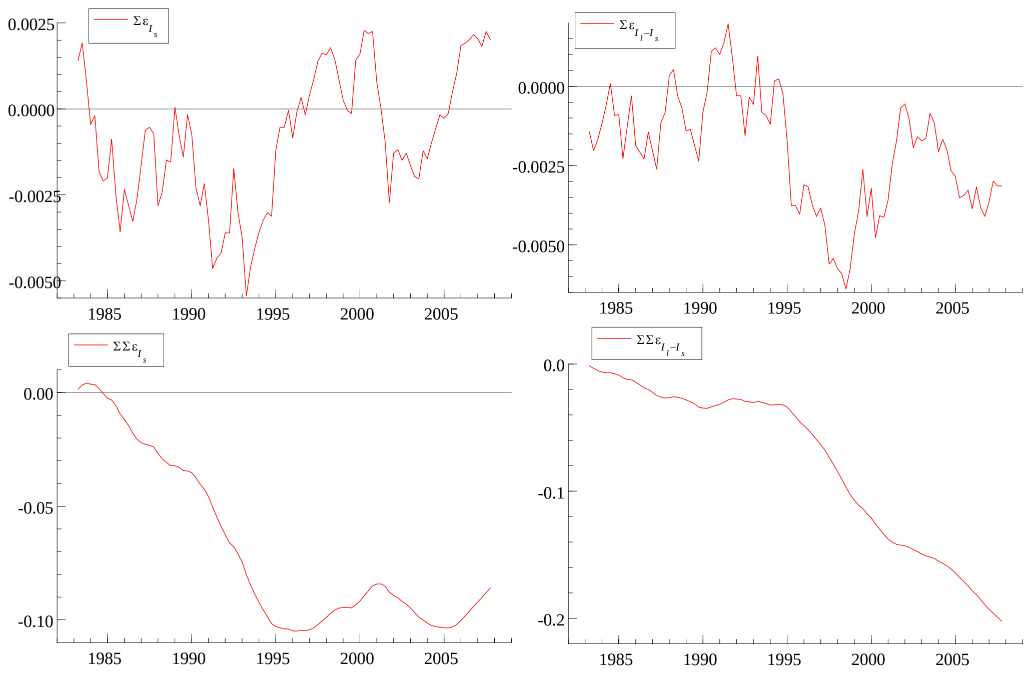

Figure 8: Upper panel: I(1) common trends. Lower panel: I(2) common trends.

factors and thus dynamics not explained by our framework. Shocks to the short rate, essentially representing unexpected shifts in monetary policy, were predominantly negative prior to 1995. Thus, prior to 1995, $\sum \varepsilon_{I_s}$ is mainly negative and $\sum\sum \varepsilon_{I_s}$ sees a downward trend. In contrast, shocks to the spread, essentially reflecting unexpected shifts in bond markets' view of fiscal policy, fluctuated largely around zero prior to 1995 and then become mainly negative. Hence, after 1995, $\sum \varepsilon_{I_l - I_s}$ is primarily negative and $\sum\sum \varepsilon_{I_l-I_s}$ shows a downward trend.

In the first half of the sample, the short rate is systematically lower than the model predicts whereas this is not the case for shocks to the spread. Both shocks to the short and to the long rate were thus primarily negative prior to 1995. This invites the explanation that global factors caused a downward shift in the yield curve. Indeed, interest rates fell from the high levels of the 1970s to much lower levels as inflation came down.

In the second half of the sample, the spread is generally lower than predicted by the model whereas this is not for the case the short rate. As a result, shocks to the long rate must be causing the downward trend in the second I(2) trend. This would be consistent with the ratification of the Treaty, and thus the acceptance of national governments of the requirements regarding fiscal discipline, making bond markets change their expectations of future public deficits and debt levels as national governments were now explicitly committed to ensure fiscal sustainability. The turning point in the two stochastic trends in fact coincide with the time at which the SGP was introduced and thus with the time at which the perceived credit risk of some government bonds likely started to fall.

In sum, the MA representation shows that whereas one I(2) trend comes from monetary policy, as conventional theory predicts, an additional nominal trend arises from the interaction of fiscal policy with bond markets. This implies that a nominal anchor has not been solely provided by monetary policy; fiscal policy and associated bond-market reactions have played a separate role in driving the nominal side of the economy.

7 Conclusion

This paper has used the I(2) CVAR model to study the interactions of fiscal and monetary policy in the euro area. We argued that an I(2) specification was required to take account of the different degrees of time-series persistency of the debt and deficit. By translating the 'five-equation macro model' of Kirsanova et al. (2005) to I(2) CVAR space, we derived a set of potential polynomially cointegrating relations to guide identification of the statistical model. Using area-wide data we were able to recover a set of economically meaningful long-run relations, but crucially the Phillips-curve trade-off had to be modified to allow for a time-varying NAIRU level. Also, in conflict with the standard prediction, we found two stochastic trends driving the nominal side of the economy.

Within the multi-cointegration framework we identified a positive long-run co-movement between debt/deficits and unemployment (the fiscal rule) as well as between unemployment and the long rate (the Phelps-Phillips curve). Central banks seemed to follow a Taylor rule in setting the policy rate with the latter adjusting to deviations from the rule. In contrast, there was a tendency for debt to error-increase following deviations from the fiscal rule. Notably we found a positive short-run effect of a fiscal loosening on the bond rate. The common-trends representation showed that twice cumulated shocks to the short rate made up the first I(2) trend whereas a second I(2) trend consisted of shocks to the bond rate and to public debt. Country models largely confirmed this separation. These results indicate that the nominal anchor of the economy has not solely provided by monetary policy; fiscal policy has played an autonomous role. The determination of long-term interest rates has thus been outside central-bank control.

Together, the polynomially cointegrating relations and the composition of the common trends are consistent with the hypothesis of Juselius (2002) that a vicious spiral of rising debt levels, bond yields and unemployment rates has been at play in the euro area during the integration process leading up to the EMU. Combined with the results in Tuxen (2009), who found adverse effects of fiscal policy on economic activity, the results in this paper point to significant costs associated with the extensive public sector in the euro area: financial markets foresee that an economic downturn will put considerable strain on government finances and thus require higher risk premia. This makes it harder for the policy makers to stimulate economic growth when most needed, *ceteris paribus*.

During the current financial and economic crisis, central banks have cut policy rates to historically low levels and introduced a range of unconventional measures to drive down longer-term rates. At the same time, governments have loosened fiscal policies and flooded the bond market with new issuances. Sovereign spreads (to Germany) have widened markedly in many countries and the ECB's covered-bond purchase programme so far appears to have had limited effect on longer-term yields. Our results suggest that with unemployment rising, fiscal sustainability is likely to take center stage in the near future and bond rates could stay elevated for prolonged period of time.

References

Afonso, A. and R. Strauch (2004). Fiscal policy and interest rate swap spreads: Evidence from the EU. *Journal of International Financial Markets, Institutions and Money 17*(3), 261–276.

Baldwin, R. and C. Wyplosz (2004). *The economics of European integration.* McGraw-Hill.

Bårdsen, G., E. S. Jansen, and R. Nymoen (2004). Econometric evaluation of the New-Keynesian Phillips curve. *Oxford Bulletin of Economics and Statistics 66 (Supplement)*, 671–686.

Barro, R. J. and D. B. Gordon (1983). Rules, discretion and reputation in a model of monetary policy. *Journal of Monetary Economics 12*(1), 101–121.

Beetsma, R. M. W. J. and H. Jensen (2002). Monetary and fiscal policy interactions in a micro-founded model of a monetary union. *ECB Working Paper 166.*

Beyer, A., J. A. Doornik, and D. F. Hendry (2001). Constructing historical euro-zone data. *The Economic Journal 111*, F102–F121.

Beyer, A. and K. Juselius (2008). Does it matter how to measure aggregates? The case of monetary transmission mechanisms in the euro area. *Working paper, Department of Economics, University of Copenhagen 08-07.*

Bjørnstad, R. and R. Nymoen (2008). The New-Keynesian Phillips curve tested on OECD panel data. *Economics: The Open-Access, Open-Assessment E-Journal 2*(2008-23).

Boswijk, H. P. (2000). Mixed normality and ancillarity in I(2) systems. *Econometrics theory 16*, 878–904.

Campbell, J. Y. and R. J. Shiller (1987). Cointegration and tests of present value models. *Journal of Political Economy 95*, 1062–1088.

Campbell, J. Y. and R. J. Shiller (1991). Yield spreads and interest rate movements: A bird's eye view. *Review of Economic Studies 58*, 495–514.

Clarida, R., J. Gali, and M. Gertler (1999). The science of monetary policy: A New-Keynesian perspective. *Journal of Economic Literature 37*, 1661–1707.

Dennis, J. G., H. Hansen, S. Johansen, and K. Juselius (2006). CATS for RATS, version 2. Available from Estima.

Doornik, J. A. (2007). An introduction to OxMetrics 5. Available from Timberlake Consultants Ltd.

Fagan, G., J. Henry, and R. Mestre (2001). An Area-Wide Model (AWM) for the euro area. *ECB Working Paper 42*.

Giese, J. V. (2008). Level, slope, curvature: Characterising the yield curve in a cointegrated VAR model. *Economics: The Open-Access, Open-Assessment E-Journal 2*(2008-28).

Haugh, D., P. Ollivaud, and D. Turner (2009). What drives sovereign risk premiums? An analysis of recent evidence from the euro area. *OECD Economics Department Working Papers 718*.

Heppke-Falk, K. and F. P. Hüfner (2004). Expected budget deficits and interest rate swap spreads: Evidence for France, Germany and Italy. *Deutsche Bundesbank Discussion Paper 40*.

Johansen, S. (1992). A representation of vector autoregressive processes integrated of order 2. *Econometric Theory 8*.

Johansen, S. (1997). Likelihood analysis of the I(2) model. *Scandinavian Journal of Statistics 24*, 433–462.

Johansen, S. (2005). The interpretation of cointegrating coefficients in the cointegrated vector autoregressive model. *Oxford Bulletin of Economics and Statistics 67*, 93–104.

Johansen, S. (2006). *Post Walrasian Macroeconomics. Beyound the Dynamic Stochastic General Equilibrium Model*, Chapter : Confronting the Economic Model with the Data. Cambridge University Press, Cambridge.

Johansen, S. and K. Juselius (2006). *Post Walrasian Macroeconomics. Beyound the Dynamic Stochastic General Equilibrium Model*, Chapter : A European view on empirical macro. Cambridge University Press, Cambridge.

Johansen, S., K. Juselius, R. Frydman, and M. Goldberg (2009). Testing hypotheses in an I(2) model with piecewise linear trends: An analysis of the persistent long swings in the Dmk/dollar rate. *Journal of Econometrics. (forthcoming)*.

Juselius, K. (2002). European integration: Aspects on inflation and unemployment. *Manuscript (available at www.econ.ku.dk/okokj).*

Juselius, K. (2003). Wage, price and unemployment dynamics and the convergence to purchasing power parity in the euro area. *Working Paper, Institute of Economics, University of Copenhagen.*

Juselius, K. (2006). *The Cointegrated VAR Model - Methodology and Applications.* Oxford University Press, Oxford.

Juselius, K., R. Frydman, M. Goldberg, and S. Johansen (2009). Scenario analysis of economic models based on RE and IKE. *Manuscript.*

Juselius, K. and J. Ordóñez (2009). Balassa-Samuelson and wage, price and unemployment dynamics in the Spanish transition to EMU membership. *Economics: The Open-Access, Open-Assessment E-Journal 4.*

Kirsanova, J. T., S. J. Stehn, and D. Vines (2005). Five-equation macroeconomics: A simple view of the interactions of fiscal and monetary policy. *Oxford Review of Economic Policy 21*(4), 532–564.

Kongsted, H. C. (2005). Testing the nominal-to-real transformation. *Journal of Econometrics 124*(2), 205–225.

Kurita, T., H. B. Nielsen, and A. Rahbek (2009). An I(2) cointegration model with piecewise linear trends: Likelihood analysis and application. *Working Paper, Department of Economics, University of Copenhagen 09-13.*

Leith, C. and L. V. Thadden (2006). Monetary and fiscal policy interactions in a New-Keynesian model with capital accumulation and non-Ricardian consumers. *ECB Working Paper 649.*

Leith, C. and S. Wren-Lewis (2000). Interactions between monetary and fiscal policies. *Journal of Monetary Economics 110*, 93–108.

Nickel, C. and I. Vansteenkiste (2008). Fiscal policies, the current account and Ricardian equivalence. *ECB Working Paper 935.*

Nielsen, B. and A. Rahbek (2000). Similarity issues in cointegration models. *Oxford Bulletin of Economics and Statistics 62*, 5–22.

Nielsen, H. B. (2007). A 'maximum-eigenvalue' test for the cointegration rabks in I(2) vector autoregressions. *Economics Letters 94*, 445–451.

Nielsen, H. B. and A. Rahbek (2007). The likelihood ratio test for cointegration ranks in the I(2) model. *Econometric Theory 23*, 615–637.

Paruolo, P. (1996). On the determination of integration indices in I(2) systems. *Journal of Econometrics 72*, 313–356.

Paurolo, P. and A. Rahbek (1999). Weak exogeneity in I(2) VAR systems. *Journal of Econometrics 93*, 281–308.

Phelps, E. S. (1994). *Structural Slumps - The Modern Equilibrium Teory of Unemployment, Interest, and Assets*. Harvard University Press, Cambridge, Massachusetts and London, England.

Rahbek, A., H. C. Kongsted, and C. Jørgensen (1999). Trend-stationarity in the I(2) cointegration model. *Journal of Econometrics 90*, 265–289.

Reade, J. (2007). Econometric modelling in a non-stationary world: Monetary and fiscal policy interactions in the US. *D.Phil. thesis, University of Oxford*.

Sargent, T. J. and N. Wallace (1981). Some unpleasant monetarist arithmetic. *Quarterly Review, Federal Reserve Bank of Minneapolis Fall*.

Schmitt-Grohe, S. and M. Uribe (2004). Optimal fiscal and monetary policy under sticky prices. *Journal of Economic Theory 114*, 198–230.

Taylor, J. B. (1993). Discretion versus policy rules in practice. *Carnegie-Rochester Conference Series on Public Policy 39*, 195–214.

Tuxen, C. K. (2006). An empirically relevant model of inflation in the euro area. Master's thesis, Department of Economics, University of Copenhagen.

Tuxen, C. K. (2009). A multi-sector model of policy transmission and inflation dynamics in the euro area. *Manuscript*.

A Data series

Variable	Notation	AWM variable
Consumer-price index (CPI)	p	PCD
Public (gross) debt	b	GDN
Nominal output (GDP)	y	YEN
Short-term interest rate	I_s	STN
Long-term interest rate	I_l	LTN
Unemployment rate	U	URX

Table A.1: Variables and data sources (8th. update of the AWM database).

B Estimation results

$\widetilde{\boldsymbol{\beta}}'$	b	y	p	U	I_s	I_l	$tDs_{92:2}$	$tDs_{95:3}$	t
β_1'	-2.17 [-28.17]	1.00 [NA]	2.64 [22.93]	0.00 [NA]	0.00 [NA]	4.28 [6.88]	0.00 [NA]	-0.01 [-12.40]	0.01 [6.71]
β_2'	0.00 [NA]	0.00 [NA]	0.00 [NA]	1.00 [NA]	0.00 [NA]	-2.54 [-16.12]	0.00 [NA]	0.00 [NA]	0.00 [NA]
β_3'	0.00 [NA]	-0.10 [-519.11]	0.10 [519.11]	0.00 [NA]	1.00 [NA]	0.00 [NA]	0.00 [NA]	0.00 [NA]	0.00 [20.78]
β_4'	1.00 [NA]	-3.42 [-91.79]	0.00 [NA]	-3.19 [-14.14]	0.00 [NA]	0.00 [NA]	-0.01 [-34.03]	0.00 [NA]	0.04 [71.95]

$\boldsymbol{\alpha}$	α_1	α_2	α_3	α_4
$\Delta^2 b$	0.27 [8.44]	0.44 [5.97]	-0.55 [-3.92]	0.25 [6.00]
$\Delta^2 y$	0.12 [5.32]	0.11 [2.15]	-0.24 [-2.51]	0.22 [7.48]
$\Delta^2 p$	0.04 [2.36]	-0.13 [-3.17]	0.01 [0.18]	0.08 [3.59]
$\Delta^2 U$	-0.00 [-0.56]	-0.04 [-2.50]	-0.10 [-3.19]	-0.01 [-1.14]
$\Delta^2 I_s$	0.01 [1.49]	0.03 [1.65]	-0.07 [-2.44]	0.01 [1.36]
$\Delta^2 I_l$	0.03 [4.52]	0.04 [2.95]	-0.12 [-4.82]	0.03 [3.30]

$\widetilde{\boldsymbol{\delta}}'$	Δb	Δy	Δp	U	I_s	I_l	$tDs_{92:2}$	$Ds_{95:3}$	1
δ_1'	-8.83	1.07	-5.33	-3.73	0.66	-1.47	0.10	0.05	15.18
δ_2'	2.01	-1.29	0.85	2.01	-0.22	0.79	-0.02	0.01	-0.06
δ_3'	-0.22	-2.02	-0.77	2.11	-0.13	0.83	-0.01	-0.00	1.47
δ_4'	2.21	-1.67	0.49	2.89	-0.24	1.13	-0.12	-0.05	33.01

Table B.1: Multi-cointegrating relations and adjustment structure.

$\boldsymbol{\zeta}$	ζ_1	ζ_2	ζ_3	ζ_4
$\Delta^2 b$	-0.03 [-0.54]	-0.06 [-0.29]	1.22 [2.56]	0.19 [2.22]
$\Delta^2 y$	0.02 [0.60]	0.14 [1.06]	0.98 [2.97]	0.20 [3.50]
$\Delta^2 p$	-0.23 [-7.23]	-0.27 [-2.49]	0.24 [0.95]	-0.09 [-2.04]
$\Delta^2 U$	0.00 [0.14]	0.08 [1.88]	-0.24 [-2.33]	0.10 [5.39]
$\Delta^2 I_s$	0.05 [3.88]	0.07 [1.76]	-0.62 [-6.35]	0.06 [3.72]
$\Delta^2 I_l$	-0.00 [-0.14]	0.18 [4.96]	0.02 [0.19]	0.04 [2.89]

$\boldsymbol{\phi}_{cs}$	Dcs_1	Dcs_2	Dcs_3	$Dcs_1 Ds_{00:1}$	$Dcs_2 Ds_{00:1}$	$Dcs_3 Ds_{00:1}$
$\Delta^2 b$	0.00 [0.76]	0.00 [0.65]	0.00 [0.37]	0.03 [11.91]	0.01 [5.52]	0.01 [3.56]
$\Delta^2 y$	0.00 [0.73]	0.00 [1.41]	0.00 [1.33]	0.00 [0.92]	0.00 [1.98]	0.00 [0.96]
$\Delta^2 p$	0.00 [1.14]	0.00 [1.08]	0.00 [1.52]	0.00 [0.91]	0.00 [0.81]	0.00 [0.43]
$\Delta^2 U$	-0.00 [-1.22]	-0.00 [-0.47]	-0.00 [-0.86]	0.00 [0.28]	-0.00 [-1.72]	-0.00 [-0.80]
$\Delta^2 I_s$	-0.00 [-0.26]	0.00 [2.35]	0.00 [1.85]	0.00 [0.05]	0.00 [2.77]	0.00 [0.53]
$\Delta^2 I_l$	0.00 [2.35]	0.00 [1.49]	0.00 [0.06]	-0.00 [-0.71]	0.00 [0.30]	-0.00 [-0.44]

$(\boldsymbol{\phi}_{tr}, \boldsymbol{\phi}_p)$	$Dtr_{86:2}$	$Dtr_{87:1}$	$Dp_{92:2}$	$Dp_{95:3}$
$\Delta^2 b$	0.00 [0.08]	-0.01 [-1.84]	-0.01 [-2.85]	0.01 [1.90]
$\Delta^2 y$	0.01 [2.43]	-0.01 [-3.82]	-0.01 [-4.99]	-0.00 [-0.79]
$\Delta^2 p$	-0.00 [-1.35]	0.00 [0.38]	-0.00 [-0.14]	-0.00 [-0.51]
$\Delta^2 U$	0.00 [0.91]	0.00 [0.95]	0.00 [1.03]	-0.00 [-0.15]
$\Delta^2 I_s$	-0.00 [-1.60]	0.00 [0.21]	0.00 [1.62]	-0.00 [-1.33]
$\Delta^2 I_l$	-0.00 [-3.66]	0.00 [0.65]	0.00 [0.98]	0.00 [0.01]

Table B.2: Medium-run relations and dummy variables.

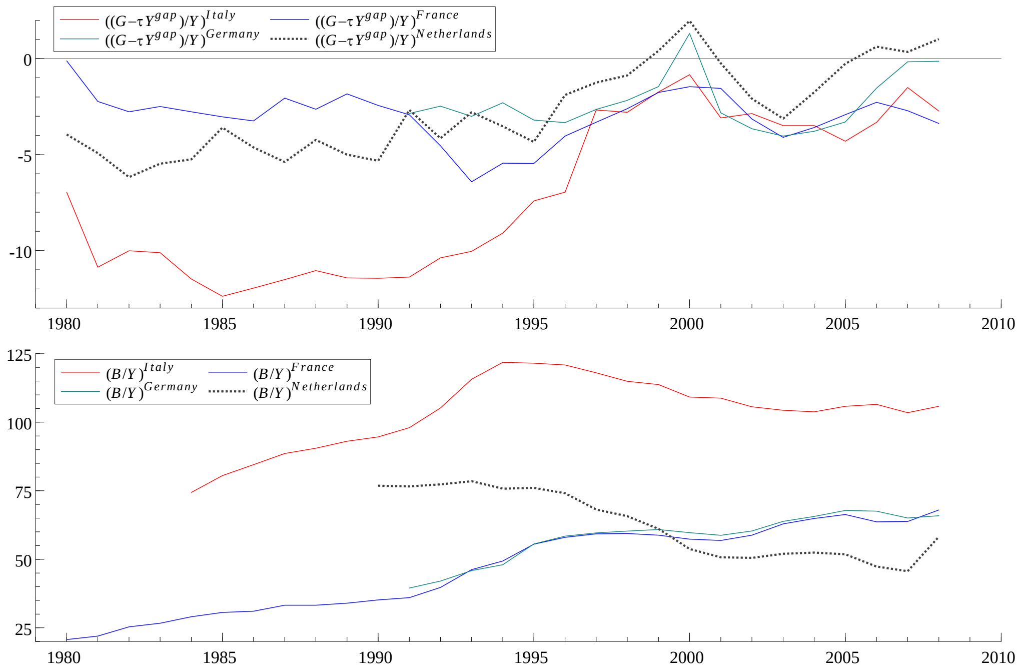

Figure C.1: Upper panel: primary deficit-to-GDP. Lower panel: public debt-to-GDP. Source: ECB.

C Cross-country policy differences

National differences in government policies may complicate the interpretation of an area-wide consideration of fiscal policy. Markets for public goods in the euro-area countries are disintegrated to a much larger extent than is the case for financial products. Monetary-policy actions affect investor decisions abroad almost immediately and lead to intra-day movements in e.g. exchange-rates. In contrast, public services are inherently non-traded goods and the international transmission of domestic fiscal-policy actions is much less direct.

Figure C.1 shows the (annual) level of public debt-to-GDP and the budget balance-to-GDP for Italy, France, Germany and the Netherlands. These major euro-area countries differ considerably regarding degree of indebtedness; for example, highly indebted Italy contrasts with the more disciplined Netherlands. All countries however appear to invoke on a less expansive stance of fiscal policy after 1995. Figure C.2 and C.3 plot the remaining variables of interest for France, Germany, the Netherlands, and Italy alongside their area-wide counterpart; the former three countries represent high-PPP countries whereas Italy falls in the low-PPP category.[12] The graphs reveal important differences in developments

[12] Annual data on public debt for Italy have been interpolated to a quarterly frequency. Data for

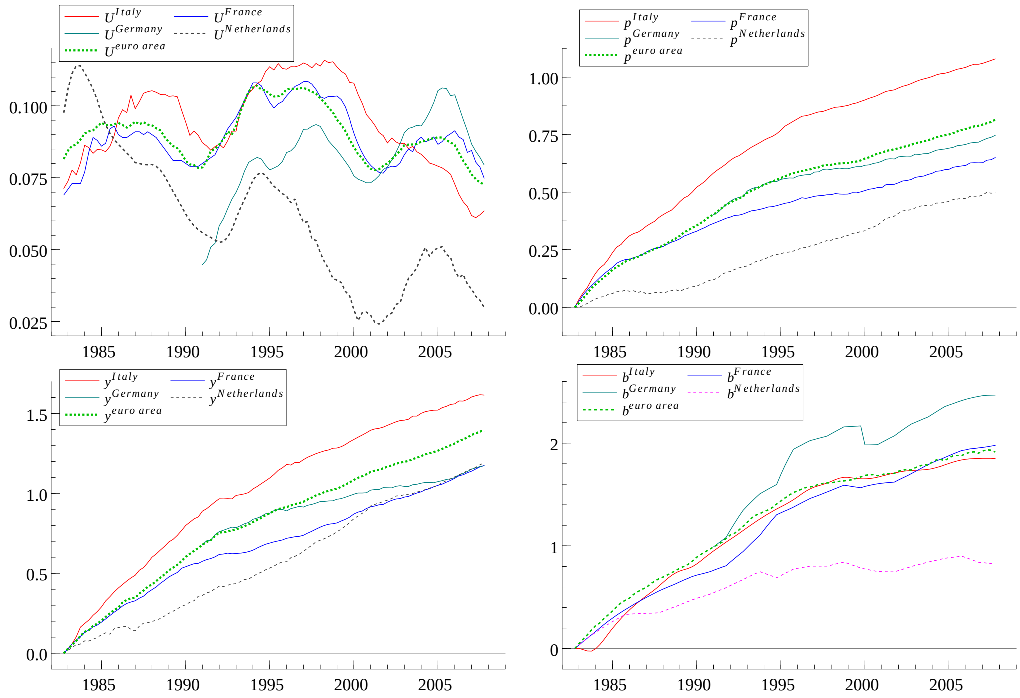

Figure C.2: Clockwise from left: unemployment rate, prices, public debt and output. Source: OECD.

across countries since the start of the sample. In contrast to the rest of the euro zone, the Netherlands has overall experienced a downward trend in unemployment. Italy stands out by having seen prices and output rise much faster than the other euro-area countries. Although starting from a lower level of debt, Germany has witnessed above average growth in public-sector debt following the re-unification.

We set up VAR models for Italy, France, Germany and the Netherlands based on the data vector (20). The sample is again 1982:4 - 2007:4 except in the German case where data are only available from 1991:1 and onwards. We allow the number of lags as well as the broken trend and dummy specifications to differ across countries. The I(2) rank tests generally point to $r \leq 4$ and thus a lower rank than that found for the euro area as a whole. This could be due to the fact that prior to 1999 monetary policy in most European countries was effectively determined by the Bundesbank as the 'hard EMS' implied that the German mark was used as the anchor currency; a monetary-policy rule may therefore not be expected to show up among the cointegrating relations for some countries.

Germany are only available from 1991:1 and onwards, and $(p, b, y)_t^{Germany}$ have been set to coincide with those of the euro-area series in 1991:1. The shift in mean in $b_t^{Germany}$ is due to changes in data methodology and this is accounted for by inclusion of a shift dummy. Sufficient data for Spain were not available for the required time period.

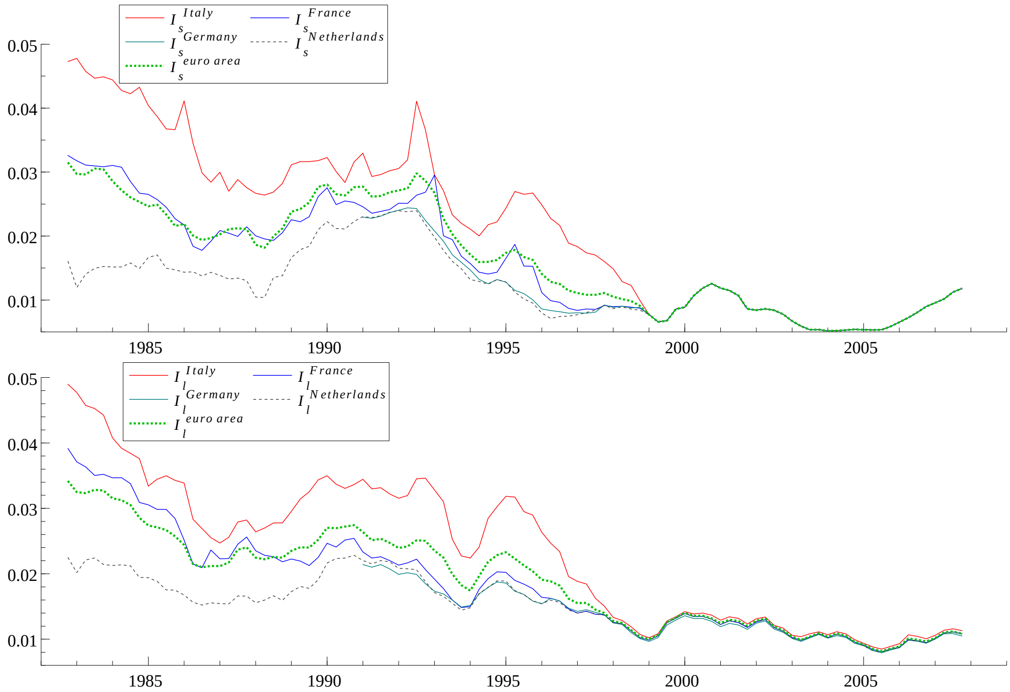

Figure C.3: Upper panel: short-term interest rates. Lower panel: long-term interest rates. Source: OECD.

For the sake of a comparison of the I(2) trends in the different countries, we impose the same set of reduced ranks on all models as was found for the euro area as a whole, i.e. $(r = 4, s_2 = 2)$, despite the fact that the tests suggest otherwise for some countries. Table C.1 and C.2 show the first and second I(2) trends, respectively, for each country with no further restrictions, besides reduced ranks, imposed on $\boldsymbol{\beta}$.

Apart from some minor contributions from debt shocks $\alpha'_{\perp 21} \sum_{j=1}^{t} \sum_{i=1}^{j} \boldsymbol{\varepsilon}_i$ is almost solely made up of shocks to the short rate in all countries except France. Debt shocks enter positively for Italy and negatively in the Dutch trend. The positive relation between short rates and public debt in the Netherlands may reflect that De Nederlandsche Bank has had a tendency to hike policy rates to counteract any fiscal loosening prior to 1999. Unemployment has seen a persistent decline in the Netherlands during the sample and enters the second I(2) trend significantly with a positive coefficient. This may be a signal that the relatively flexible Dutch labour-market institutions have allowed the monetary authorities to hike rates without harming employment. For France, the second I(2) trend consists mainly of shocks to 'real output' (measured as $(y - 0.50p)_t$) and the short rate is insignificant. This may be attributed to the 'franc fort' policy with Banque de France pegging the French franc to the German mark, thereby rendering monetary policy focused on exchange-rate movements. Currency pressures are often associated with the developments in domestic growth which could explain why GDP shocks make up the second nominal trend. The second area-wide I(2) trend closely resembles that of Germany, supporting the view that monetary policy in the euro zone was effectively conducted by the Bundesbank prior to the establishment of the ECB.

In all models, except the Italian, debt shocks enter $\alpha'_{\perp 22} \sum_{j=1}^{t} \sum_{i=1}^{j} \boldsymbol{\varepsilon}_i$ negatively, pointing to a positive relation between public debt and bond yields. For Italy, shocks to all variables enter significantly, and although the long rate shows up with a significant and large coefficient, the coefficient of the unemployment rate is larger in absolute value; we therefore normalise on this. The relationship between public debt and the bond rate is negative and thus the debt dynamic appears to have been rather different in Italy compared with the rest of the euro zone. Italy starts off from a very high level of debt-to-GDP (see Figure C.1) and high interest rates, but during the sample period debt evolves in line with the euro-area average whereas prices and output rise faster than in the rest of the zone. Hence Italy seems to go through a period of fiscal consolidation during the sample. This is likely related to low-PPP Italy 'catching up' with the high-PPP countries in the euro area in terms of productivity in this period; see Juselius and Ordóñez (2009) for an analysis of PPP dynamics for the case of Spain. Despite some differences in the magnitude of the coefficient to public debt, the first I(2) trend for France, Germany and the Netherlands alike are similar to their area-wide counterpart.

$\boldsymbol{\alpha}_{\perp 22}$	$\alpha_{\perp 22}^{\text{euro area}}$	$\alpha_{\perp 22}^{\text{Italy}}$	$\alpha_{\perp 22}^{\text{France}}$	$\alpha_{\perp 22}^{\text{Germany}}$	$\alpha_{\perp 22}^{\text{Netherlands}}$
$\Sigma\Sigma\varepsilon_b$	-0.02 $[-0.69]$	0.24 $[2.39]$	0.32 $[0.77]$	0.02 $[0.88]$	-0.14 $[-2.12]$
$\Sigma\Sigma\varepsilon_y$	0.00 $[0.06]$	-0.08 $[-2.75]$	1.00 $[NA]$	0.02 $[0.93]$	-0.01 $[-0.22]$
$\Sigma\Sigma\varepsilon_p$	-0.06 $[-0.98]$	0.03 $[0.47]$	-0.50 $[-2.54]$	-0.03 $[-0.83]$	0.10 $[1.52]$
$\Sigma\Sigma\varepsilon_U$	-0.23 $[-1.24]$	0.00 $[NA]$	0.09 $[0.246]$	-0.26 $[-1.40]$	0.55 $[3.45]$
$\Sigma\Sigma\varepsilon_{I_s}$	1.00 $[NA]$	1.00 $[NA]$	-0.25 $[-0.56]$	1.00 $[NA]$	1.00 $[NA]$
$\Sigma\Sigma\varepsilon_{I_l}$	0.00 $[NA]$	-0.16 $[-0.86]$	0.00 $[NA]$	0.00 $[NA]$	0.00 $[NA]$

Table C.1: First I(2) trend for different countries (no restrictions imposed on $\boldsymbol{\beta}$).

$\boldsymbol{\alpha}_{\perp 21}$	$\alpha_{\perp 21}^{\text{euro area}}$	$\alpha_{\perp 21}^{\text{Italy}}$	$\alpha_{\perp 21}^{\text{France}}$	$\alpha_{\perp 21}^{\text{Germany}}$	$\alpha_{\perp 21}^{\text{Netherlands}}$
$\Sigma\Sigma\varepsilon_b$	-0.08 $[-2.43]$	-0.51 $[-5.02]$	-0.57 $[-2.64]$	-0.06 $[-1.84]$	-0.16 $[-2.11]$
$\Sigma\Sigma\varepsilon_y$	-0.05 $[-1.07]$	0.19 $[6.06]$	-0.00 $[NA]$	0.03 $[1.05]$	0.10 $[1.66]$
$\Sigma\Sigma\varepsilon_p$	0.05 $[0.95]$	-0.36 $[-5.12]$	-0.06 $[-0.61]$	-0.02 $[-0.57]$	0.15 $[1.99]$
$\Sigma\Sigma\varepsilon_U$	-0.29 $[-1.61]$	1.00 $[NA]$	0.08 $[0.41]$	0.32 $[1.50]$	0.25 $[1.38]$
$\Sigma\Sigma\varepsilon_{I_s}$	0.00 $[NA]$	0.00 $[NA]$	-0.07 $[-0.31]$	0.00 $[NA]$	-0.00 $[NA]$
$\Sigma\Sigma\varepsilon_{I_l}$	1.00 $[NA]$	-0.91 $[-4.71]$	1.00 $[NA]$	1.00 $[NA]$	1.00 $[NA]$

Table C.2: Second I(2) trend for different countries (no restrictions imposed on $\boldsymbol{\beta}$).

Chapter 3

Has excess global liquidity fuelled asset prices? An I(2) cointegrated VAR study of bubble dynamics

Has excess global liquidity fuelled asset prices? An I(2) cointegrated VAR study of bubble dynamics

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Abstract

We use the I(2) cointegrated VAR (CVAR) model to study the relationship between asset prices and liquidity on a global scale. Starting from a small New-Keynesian model, we propose a set of long-run relations which allow both the price of liquidity (interest rates) and the quantity (money supply) to potentially affect house and share prices. We find strong evidence of two I(2) trends which arise from twice cumulated shocks to the short and the long rate, respectively. As a result, long-run price homogeneity is rejected and we argue that this could be a first sign of bubbles. Imposing homogeneity on the polynomially cointegrating relations, albeit rejected in-sample, we are able to study the effects of different types of disequilibria and thus the dynamics of asset-price bubbles. We find that a key fundamental of asset prices, output, is excluded from the cointegrating relations for both house and share prices and argue that this may be a second sign of asset-price bubbles. Both house and share prices have had a tendency to rise in response to excessively low policy rates, but whereas house-price inflation is fuelled by excess money supply this is not the case for goods and share prices.

Keywords: Time-Series Models, Financial Markets and the Macroeconomy, Price Level; Inflation; Deflation, Monetary Policy

JEL Classification: C32, E44, E31, E52

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1 Introduction

This paper uses an I(2) cointegrated VAR (CVAR) model to study the relationship between asset prices and liquidity on a global scale, allowing both the price and quantity of credit to play a role in generating asset-market disequilibria. It is by now generally accepted that the current economic and financial crisis was preceded by bubbles in both the housing and the stock market in many countries. Understanding how the global economy could be led astray despite the apparent success of modern risk-management techniques and central-bank policies, is vital in avoiding a repetition in the future. The notion of 'excess global liquidity' received a lot of attention among policy makers, bankers and the financial press alike even prior to the crisis[1] but academic studies remain few. In fact modern monetary economics does not ascribe a separate role for money or credit in policy-decision making over and above what can be inferred from the level of the interest rate (Woodford 2007).

Until the early 1980s, leading central banks used to monitor money supply, based on the Friedman (1969) idea that "inflation is always and everywhere a monetary phenomenon". To control inflation via money supply, stability of money demand is a prerequisite, and money-targeting regimes were abandoned by policy makers because empirical analysis often found money demand to be unstable. The majority of central banks today adhere to some form of inflation targeting and have adopted dynamic stochastic general equilibrium (DSGE) models, see *inter alia* Woodford (2003), as the core of their forecasting models.[2] These models build on real-business cycle models (Kydland and Prescott 1982) with no role for financial intermediation (Modigliani and Miller 1958). Although the literature on DSGE models with financial frictions is emerging, see *inter alia* Christiano, Motto and Rostagno (2007a, 2007b), even the basic assumptions of DSGE models have been found to be rejected when tested empirically (Franchi and Juselius 2007).

Another strand of literature originating from Fisher (1933) focuses on the importance of financial aspects in macro models. Friedman and Schwartz (1963) emphasised the strong correlation of money supply and output and although the direction of causality is not clear, this suggests that banks' liabilities matter as part of the money creation

[1]See *inter alia* IMF(1999, 2007), the speech by ECB Vice-President Lucas D. Papademos, on May 35, 2006, various publications by Lehman Brothers and Deutsche Bank and a plethora of articles in the Financial Times and the Economist.

[2]The European Central Bank (ECB) continues to follow a 'two-pillar strategy' where both developments in the real economy and in money supply are monitored closely. The ECB thus maintains a 'prominent role of money'.

process. This is the *money view* reflected in the textbook IS-LM model. Bernanke (1983), Bernanke and Blinder (1988) and Bernanke and Gertler (1990) instead insist on the importance of banks' assets (bank loans) as opposed to other sources of funds for borrowers due to financial imperfections. This is the *credit view* reflected in models of the financial accelerator.

In this paper, we use a cointegration framework to construct measures of excess liquidity based on both the price and the quantity of money. In Giese and Tuxen (2007) we showed using an I(1) CVAR that 'excess liquidity' was present from 2001 onwards both on a Taylor-rule and a money-demand deviations measure. The credit expansion in the early years of the new millennium did not appear to have much of an effect on goods prices however, likely because inflation was kept down by global competitive pressures (Tuxen 2009b). In today's closely linked financial markets, excess liquidity may instead have poured into assets such as bonds, shares and housing in a 'search for yield'. This is the question we address in the present paper.

Prior to the onset of the credit crisis the literature on 'global liquidity' mainly focused on identifying spill-overs between countries, see *inter alia* Baks and Kramer (1999), Sousa and Zaghini (2004) and Ruffer and Stracca (2006). Our model differs from these studies in that we focus on the long- and short-run co-movements of liquidity and asset prices on a global scale rather than on cross-country linkages.[3] In a recent study, Ahrend (2008) constructs measures of deviations from Taylor rules for individual OECD countries and find that accommodating monetary policy over the period 2002-05 together with financial innovation contributed to the run-up in asset prices prior to the crisis. Bracke and Fidora (2008), using a structural VAR approach, test the explanatory power of three competing explanations for the build-up in global (current-account) imbalances and find that the 'liquidity glut' beats both the 'savings glut' and the 'investment drought' hypotheses in explaining the variation in imbalances and asset prices.

Here, we argue that the test of whether financial asset prices have followed the same nominal trend as that of output and physical goods can be taken as a test of asset-price bubbles. Using an I(2) CVAR we find that the routinely applied nominal-to-real transformation (NRT) is strongly rejected as two I(2) trends are present within the sample. We argue that this could be a first sign of price bubbles. Starting from a small New-

[3]In a companion paper, Giese and Tuxen (2009), we analyse the transmission of liquidity among countries using the global VAR (GVAR) framework developed by *inter alia* Dees, Holly, Pesaran, and Smith (2007).

Keynesian model, we propose a set of long-run relations and use these as a guide to identification of the polynomially cointegrating relations. A gradual extension of the information set allows us to study the existence of excess liquidity and the effects of this on asset prices in turn. Imposing long-run price homogeneity (LPH) on the model on the grounds that it should hold in the (very) long run, albeit rejected for the sample period, allows us to study the effects of persistent deviations from the equilibrium relations. We argue that the exclusion of a key fundamental, output, in the cointegrating relations for house and share prices may be a second sign of asset-price bubbles. Although our motivation is the recent bubble, the empirical model is based on regularities of global asset-price cycles since the early 1980s.

The literature contains a wide range of suggestions of how to detect asset-price bubbles, see Gürkaynak (2005) for a survey. Diba and Grossman (1988) propose a simple test for the existence of a bubble which amounts to a check of whether stock prices and dividends are cointegrated. Within the CVAR literature, our methodological approach contrasts with that used by Nielsen (2005) and Engsted (2006) who consider I(1) processes with explosive roots for analysing hyperinflation data and stock-market bubbles, respectively. We do not allow for explosive roots but instead make an attempt to account for roots close to, but below, one in models of real-transformed variables by modelling their nominal counterparts in I(2) space. Hence our approach is similar to Taylor (1991) in treating nominal variables as I(2) but we use the I(2) CVAR model (Johansen 1992) rather than the I(1) model (Johansen 1988) employed by Taylor.

The paper is structured as follows. Section 2 outlines a simple theoretical framework and Section 3 discusses the data set and some graphical observations. Section 4 presents the statistical framework while Section 5 discusses some econometric implications of economic theory. Section 6 presents results from estimation of a series of models arising from gradually extending the information set. Section 7 concludes.[4]

[4]All calculations were conducted using CATS 2.01 (Dennis 2006) in Rats 6.3 and Ox/OxMetrics 5 (Doornik 2007).

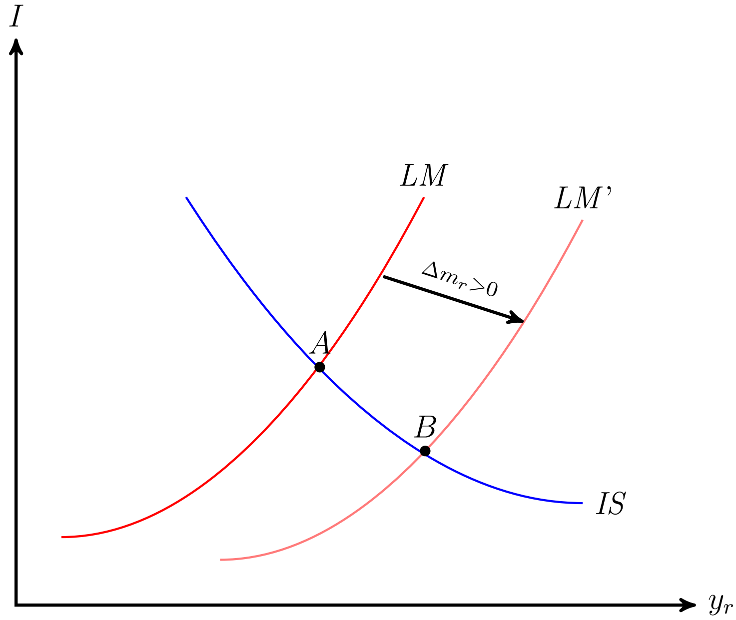

Figure 1: Liquidity glut: downward shift of LM curve.

2 Economic framework

We first discuss the 'excess liquidity' hypothesis in a simple IS-LM framework and then outline a set of equilibrium relations which incorporates this idea.

2.1 The 'excess liquidity' hypothesis

In a low-inflation environment, the textbook IS-LM model may provide a reasonable description of some basic dynamics of the economy.[5] In this framework, see *inter alia* Mankiw (2006), a loosening of monetary policy is represented by an exogenous increase in money supply, $\Delta m_r > 0$. We can consider the effects of this on real output, y_r, and interest rate, I, graphically. Figure 1 shows that this implies a downward shift of the upward-sloping LM curve; the downward-sloping IS curve is not affected. The equilibrium shifts from A to B and the latter will be characterised by a higher level of output and a lower level of the interest rate. The New-Neo-Classical synthesis would then predict that with output above its potential level, prices start to rise, and in the long run the AS curve (not depicted) is vertical. This increase in prices shifts the LM curve back to its starting point and thus monetary policy has no long-run effect on the real economy.

[5]This use of the IS-LM model was put forward by the Economist: *A Working Model. Is the world experiencing excess saving or excess liquidity?*, print edition, 11 August 2005.

It may be argued that prior to 2006, goods prices did not respond to domestic imbalances and hence the global economy only experienced the downward shift of the LM curve, and not the shift back. This may be taken as evidence that global imbalances prior to the recent crisis were generated by a liquidity glut (supply) rather than a savings glut (demand).[6] In case of an increase in the demand for savings, the IS curve would have shifted left, thereby reducing both output and interest rates. However, output has grown at a historically fast pace in the new millennium, rendering the hypothesis of a savings glut less convincing.

The backward shift of the LM curve may not have been needed if the AS curve did in fact shift permanently to the right as might have happened as a result of the integration of China, India and other emerging markets into the global economy. The credit crisis has led to a considerable contraction in credit and an rise in interest-rate spreads (over policy rates). According to this framework the crisis brought about a shift in the LM curve. This signals that the AS curve did not shift as much as required and thus that the global economy was away from its long-run equilibrium path prior to the downturn in the US housing market.

2.2 A simple economic model

As a starting point we consider a simple New-Keynesian model. In resemblance with the majority of modern macroeconomic models of short-to-medium-term fluctuations, our theoretical model has a three-equation system at its core: an IS curve, a Taylor-type rule for monetary policy and a Phillips curve, see Clarida, Galí, and Gertler (1999). We add money and asset prices to this below but leave out fiscal policy.

The demand side is represented by an expectational IS curve,

$$y_{r,t} = \tau_1 E_t y_{r,t+1} - \tau_2 (I_{l,t} - E_t \Delta p_{t+1}) + \varepsilon_t^{IS} \tag{1}$$

where $y_{r,t}$ is the real output gap at time t, $I_{l,t}$ a long-term interest rate and Δp_t the corresponding inflation rate at time t; ε_t^{IS} is a demand shock and E_t denotes expectation as of time t. Thus, (1) implies that demand for physical goods depends positively on expected future output and negatively on the real interest rate.

The supply side is described by a New-Keynesian Phillips curve (NKPC) specified in

[6] The savings-glut hypothesis was proposed by Ben Bernanke: *The global saving glut and the US current account deficit*, lecture to Virginia Association of Economics, 10 March 2005.

terms of the output gap,

$$\Delta p_t = \delta_1 E_t \Delta p_{t+1} + \delta_2 (y_r - y_r^*)_t + \varepsilon_t^{NKPC} \tag{2}$$

where an asterisk denotes a target/potential value and ε_t^{NKPC} a supply (cost) shock. Hence, (2) characterises inflation as a function of expected future inflation and of firms' marginal costs, here measured by the output gap, $(y_r - y_r^*)_t$.

The central bank is assumed to follow a Taylor (1993)-type rule,

$$I_{s,t} = I_s^* + \gamma_1 (\Delta p - \Delta p^*)_t + \gamma_2 (y_r - y_r^*)_t + \varepsilon_t^{Taylor} \tag{3}$$

where $I_{s,t}$ is the short-term (policy) rate and I_s^* the corresponding 'neutral level'; ε_t^{Taylor} is a policy-rate shock. Thus, (3) prescribes that the policy rate is raised whenever inflation (output) rises above target (potential). Excessively loose monetary policy can then be defined as policy rates below the level suggested by (3). According to the expectations hypothesis of the term structure (see *inter alia* Campbell and Shiller 1987), a change in the policy rate, $I_{s,t}$, in (3) causes the long rate, $I_{l,t}$, in (1) to move as well. Notably, control of inflation by means of a Taylor rule implicitly assumes that demand shocks are predominant. If, on the other hand, shocks to output and/or prices are caused by supply factors, rate cuts will mainly lead firms' costs to fall, thereby dampening inflationary pressures.

Adding money The New-Keynesian model assumes no separate role for money (over and above the interest rate) in the conduct of monetary policy. Here we explicitly want to allow the quantity of credit to play a potential role however. The quantity side is introduced in the model by means of a money-demand (LM) relation, (see *inter alia* Romer 1986),

$$m_{r,t} = y_{r,t} - \lambda_1 (I_l - I_s)_t - \lambda_2 E_t \Delta p_{t+1} + \varepsilon_t^{LM} \tag{4}$$

where $m_{r,t}$ denotes real money supply; ε_t^{LM} is a money-supply shock. This proposes that the demand for real balances increases in proportion with the need to conduct transactions measured by the level of real output, decreases with the opportunity cost of holding money measured by the spread between the long- and the short-term interest rate, and declines with the rate of inflation because purchasing power of cash balances is wiped out by higher prices. Excess money can then be defined as money supply above the level suggested by (4).

Adding asset prices Incorporating prices on assets such as housing and shares into the model could add wealth effects in the IS curve, (1), such that booming property and stock markets lead to higher demand. Moreover, portfolio re-balancing may lead a rise in asset prices to increase the demand for money, (4). We propose that a relation describing the determination of asset prices could take the form,

$$q_{r,t} = \kappa_1 \underbrace{y_{r,t}}_{\text{fundamentals}} - \underbrace{\kappa_2[I_t - E_t\Delta p_{t+1} - (I^* - \Delta p^*)_t] + \kappa_3(m_r - y_r)_t}_{\text{excess-liquidity bubble}} + \varepsilon_t^{asset} \tag{5}$$

where $q_{r,t}$ is a vector of real asset prices, i.e. $q_{r,t} = (h_r, s_r)'_t$ with $h_{r,t}$ and $s_{r,t}$ real house and real share prices, respectively; ε_t^{asset} is an asset-price shock. The fundamental value of an asset is given by the net present value of future 'dividends' (Campbell and Shiller 1988), here proxied by real output. In addition, (5) suggests that 'excess liquidity', measured either by interest rates of different maturities, i.e. $I_t = (I_s, I_l)'_t$, in deviation from a 'neutral' level, $(I^* - \Delta p^*)_t$, and/or by the inverse velocity of money, $(m_r - y_r)_t$, may play a separate role in driving asset prices. This component may be interpreted as a bubble component and would inherently exist only as a temporary phenomena as money and output should move together in the (very) long run and the real interest rate should equal its 'neutral' counterpart. Here $\kappa_1 > 0$ and $\kappa_2 = \kappa_3 = 0$ would be consistent with 'no bubbles'.

3 Data aggregation and graphical analysis

We first discuss aggregation of national data and then analyse graphically the constructed global series to anticipate their time-series behaviour.

3.1 Construction of global data series

Based on the theoretical framework above we choose the set of variables to enter the empirical analysis. The economic model was specified in terms of real-transformed variables but because we are interested in assessing potential divergence in nominal growth rates, we consider the following nominal data vector,

$$\mathbf{x}_t = (y, p, I_s, I_l, m, h, s)'_t \tag{6}$$

where y_t, p_t, m_t, h_t, and s_t are the nominal counterparts of the previously defined real variables. We use quarterly time series from 1982:4 to 2006:3 for France, Germany[7], Italy, Japan, United Kingdom (UK) and United States (US) and aggregate these to obtain 'global series' as discussed below. Emerging economies such as China and India are not included due to lack of sufficiently long time series. The start of the sample is set to coincide with the time at which the Fed shifted away from its approach of targeting the quantity of money (M1) and started targeting the federal funds rate in September 1982. The sample ends mid 2006 approximately at the peak of the US housing bubble. Table A.1 in Appendix provides an overview of the national data sources.

Country series are aggregated to a global level using the aggregation method proposed by Beyer, Doornik, and Hendry (2001).[8] For volume series (money and output), aggregation weights of each country are based on the relative share of the variable measured in a common currency (here USD). The time-varying weights are used to aggregate within-country growth rates calculated in national currency. The aggregated growth rates are then cumulated to obtain levels and an anchor value is chosen. For non-volume variables (house- and share-price indices and interest rates), GDP weights are used to aggregate levels. The goods-price index is calculated as the implicit deflator of the GDP series. All variables are log-transformed (denoted by lower-case letters) except interest rates which are divided by 400 to obtain rates which are comparable to the quarterly inflation rate. In contrast to standard aggregation procedures, which often do not account for exchange-rate movements, the Beyer et al. (2001) method ensures that when a variable increases in each country, the aggregate increases as well.

The price of credit in the short end of the maturity spectrum is measured by the three-month interbank rate as this is the rate is used as a reference for pricing a range of financial contracts. For the longer end of the curve, we use the yield on 10-year government bonds. The quantity of credit can be represented by a range of measures such as different monetary aggregates (narrow vs. broad), interbank lending, loans granted to the private sector, indicators from central banks' bank-lending surveys, etc. As an example, IMF (2007) suggests the use of base money plus reserves to account for the accumulation of foreign currency by central banks in emerging markets with large external surpluses. We

[7]Prior to the German re-unification in 1991:1 growth rates in the West-German variables are used to splice the data series and thus to construct historical data for Germany as a whole.

[8]This aggregation has been criticised by Beyer and Juselius (2007) for sensitivity to PPP deviations in the base year but we abstract from this issue here.

leave out current-account issues *per se* and represent non-price credit conditions by broad money supply (M3 or M4). Broad money provides a standardised measure across countries of the amount of liquidity created by both central banks and financial intermediaries in a fractional reserve-banking system.[9]

Central banks set the policy rate or, equivalently, the supply of narrow money. As part of the money-multiplier mechanism banks then determine the supply of broad money conditional on the monetary base. An expansion of the ratio of broad-to-narrow money does not necessarily reflect a loosening of the lending standards applied by banks as money supply could change endogenously as a result of an increase in the demand for money. But, money supply in excess of the level suggested by a money-demand relation could be taken as an indicator of a loosening of (non-price) credit conditions. It is therefore important to control for factors determining money demand in concluding whether money supply is 'excess'. One advantage of the cointegration framework is that it allows us to disentangle such effects.

3.2 Graphical analysis

Figure 2 depicts nominal broad money, output, goods prices, house and share prices (here re-based to start at zero). All series except share prices evolve smoothly, a clear indication that they are likely to be I(2). Over the sample period share prices see very significant growth whereas goods prices in contrast rise by much less. Money and output have largely followed the same path and notably share a kink around the start of the 1990s; since 2000 money grew faster than GDP however. House prices have generally increased by more than goods prices but by less than money, output and share prices alike. The house-price series also exhibits a break in trend around 1990. Whereas money and output immediately invoke on a new lower trend path, house prices see almost a decade of stagnating growth before taking off just prior to the start of the new millennium.

Figure 3 shows some simple transformations of the baseline variables. Real money, $(m-p)_t$, and real GDP, $(y-p)_t$, have followed each other closely up to 2001 where the former appears to shift to a permanently higher level. The behaviour of inverse velocity, $(m-y)_t$, is a mere reflection of these observations: it has been mean-reverting up to around 2001 where an upward shift occurs. Interest rates have been falling over

[9]The ECB has argued that portfolio shifts may have affected M3 dynamics in the period 2000-03. The Fed discontinued the publication of M3 data, arguing that it did not convey any additional information over M2 due to the increasing use of securitisations and rising off-shore positions.

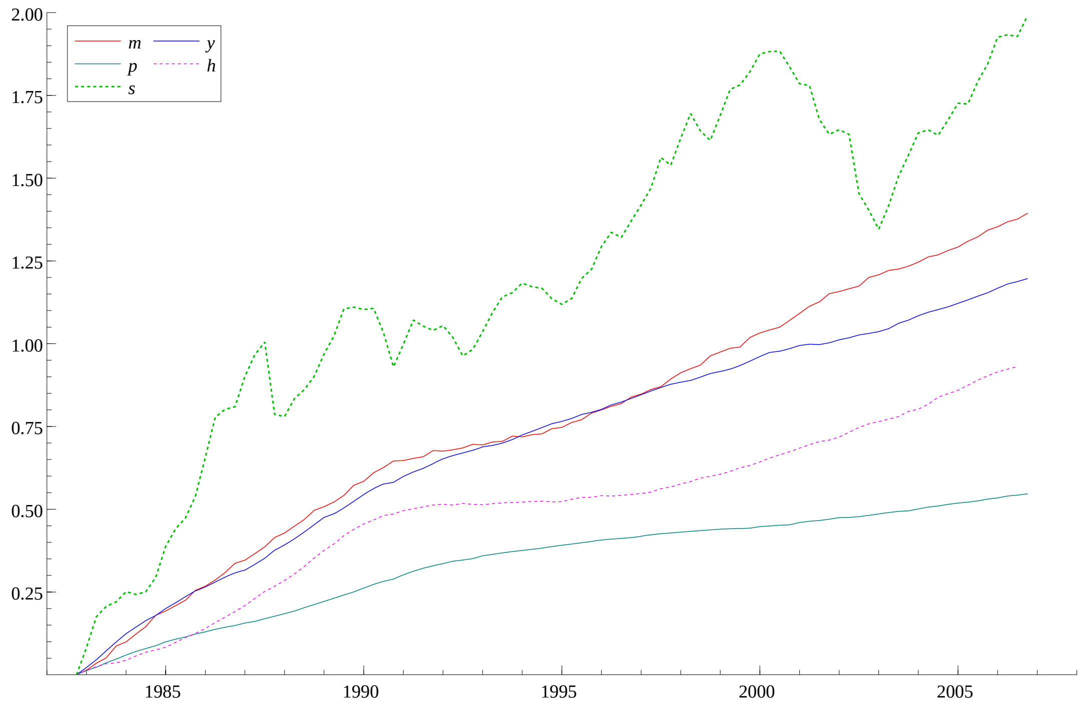

Figure 2: Nominal broad money, output, goods prices, house and share prices.

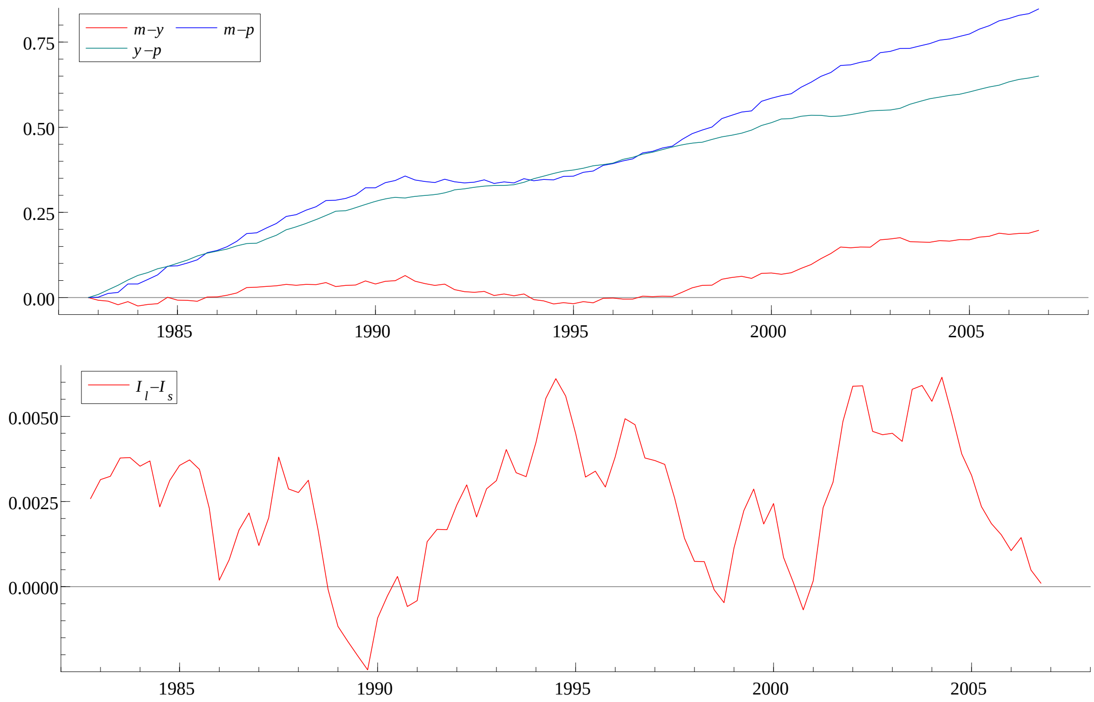

Figure 3: Upper panel: inverse velocity, real money and real output. Lower panel: interest-rate spread.

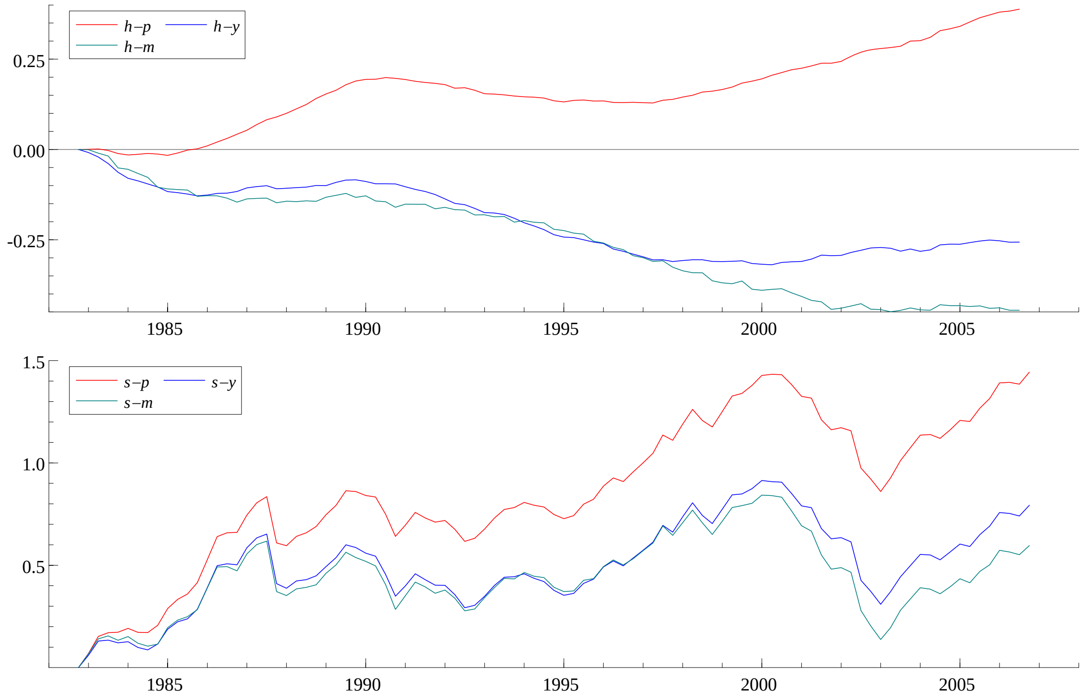

Figure 4: Upper panel: real house prices, house prices-to-GDP and house prices-to-money. Lower panel: real share prices, share prices-to-GDP and share prices-to-money.

the sample period across the maturity spectrum but the long-short spread has remained positive most of the time. Inversions of the term structure are witnessed around the 1990 and the 2001 recessions. Towards the end of the sample when policy rates were raised, bond rates remained at historically low levels and the spread narrowed, giving rise to Greenspan's bond-yield conundrum.[10] This apparent non-stationarity of the spread suggests that the short and the long end of the yield curve are driven by different factors.

Figure 4 shows transformations of the two asset-price series. Real house prices have developed smoothly with housing market peaks in the early 1990s and in 2006. The ratio of house prices to GDP however fell persistently until the late 1990s and increased slightly thereafter. When the price of homes is measured relative to the supply of money, the housing market appears to have been in a slump up to 2001; from 2001-06 house prices then evolved broadly in line with money supply. In contrast, share prices reveal broadly similar time-series behaviour regardless of whether money, output or GDP is used as the denominator. Indeed, all transformations of the share-price series exhibit somewhat erratic movements pointing to bull- followed by bear-market behaviour. Hence

[10] Alan Greenspan, testimony before the Committee on Banking, Housing, and Urban Affairs, U.S. Senate, February 16, 2005.

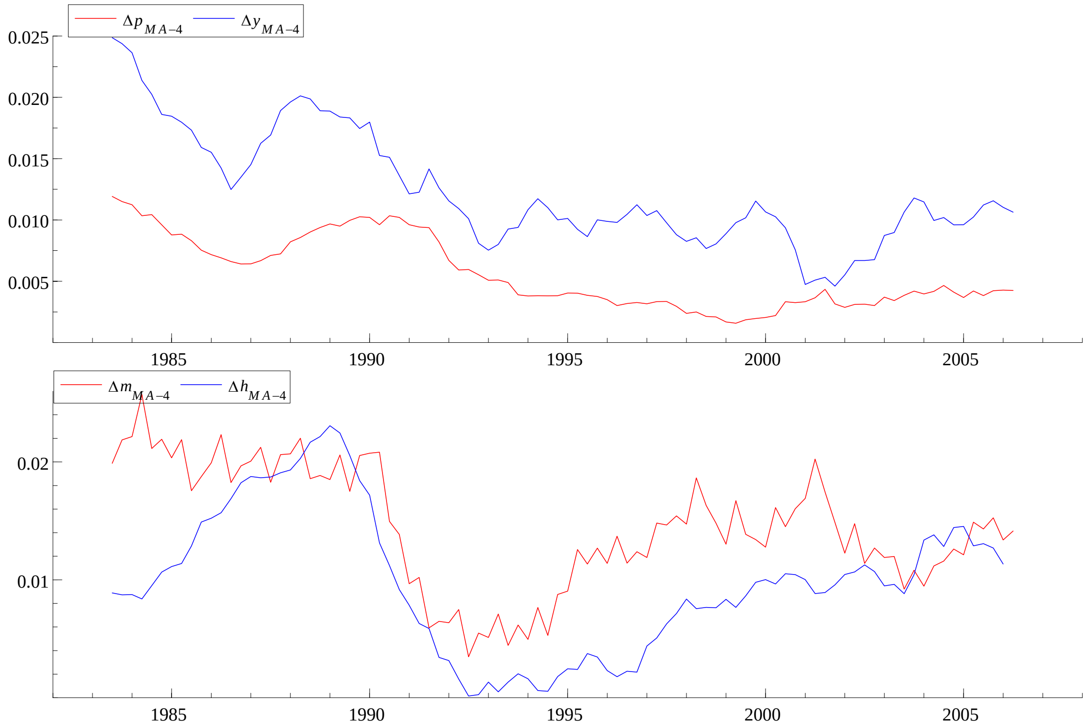

Figure 5: Upper panel: Goods-price and GDP growth. Lower panel: Money and house-price growth. (four-quarter moving averages)

share prices do not appear to be driven by an I(2) trend.

Figure 5 plots moving averages of the growth rates of the four variables which are likely to be I(2). Output growth and goods-price inflation largely share the same overall movements during the sample period. Indeed, growth in real output (not shown) seems to be mean-reverting. Similarly, growth in money supply and house prices have been moving closely together since the mid-1980s, and both series see a marked decline in growth around 1990. Prior to the bursting of the housing bubbles in 1990-91 and 2006 the growth rate of house prices exceeded money growth for a short period of time.

It is evident from the graphical analysis above that some sort of 'shift' occurs in the nominal variables around 1990-91 and 2000-01. These two points in time mark the start of the downturn in the early 1990s and the burst of the dot-com bubble at the start of the new millennium, respectively. In specifying the empirical model we are faced with the choice of modelling these shifts stochastically by the inherent model dynamics or deterministically by including either level shifts and/or trend breaks. Even if the individual series exhibit deterministic shifts in mean or trend, these shifts may cancel in the cointegration space, and tests can reveal whether co-breaking occurs. Juselius (2006) argues that a shift should be modelled stochastically if its effects are fully unanticipated. In contrast, if agent behaviour changes systematically as a result of the change in regime or if the effects of the change are fully anticipated, a deterministic modelling approach may be more appropriate. In Figure 2, the trend break in early 1990s is quite remarkable and this is likely to have been brought about by a series of changes in the global economy such as the breakup of the former Soviet Union, the integration of new countries, notably China and India, into the global economy, and the failure of the European Monetary System. On this background, it seems reasonable *a priori* to allow the trending variables to evolve around a different trend after 1990-01 and we include a broken trend from 1990:4 denoted $tDs_{90:4}$.[11] This leads to the data vector,

$$\widetilde{\mathbf{x}}_t = (\mathbf{x}', t, tDs_{90:4})'_t \tag{7}$$

which we shall analyse in the empirical analysis.

4 The I(2) CVAR model

The VAR allows flexible modelling of the regularities in the data and is in its unrestricted form simply a reformulation of the auto-covariances in the data. The graphical analysis pointed to some of the nominal variables being I(2) and we therefore consider here the CVAR under the I(2) restriction. Tests of the NRT (Kongsted 2005), which can only be conducted within the I(2) model, can then reveal whether a valid transformation to I(1) space can be made. We outline the basic components of the I(2) CVAR framework below.[12]

We start from the p-dimensional VAR($k = 3$) in acceleration rates,

$$\Delta^2\mathbf{x}_t = \mathbf{\Pi}\mathbf{x}_{t-1} + \mathbf{\Gamma}\Delta\mathbf{x}_{t-1} + \mathbf{\Psi}\mathbf{\Delta}^2\mathbf{x}_{t-1} + \boldsymbol{\varepsilon}_t, \qquad t = 1, 2, ..., T \tag{8}$$

where $\mathbf{x}_t$ is a $p \times 1$ data vector and $\boldsymbol{\varepsilon}_t$ a $p \times 1$ vector of error terms for which we assume $\boldsymbol{\varepsilon}_t \sim i.i.d.N(\mathbf{0}, \mathbf{\Omega})$ with $\mathbf{\Omega} > \mathbf{0}$. In the statistical analysis we condition on the initial values, $(\mathbf{x}_{-2}, \mathbf{x}_{-1}, \mathbf{x}_0)$, and hence these are treated as fixed. The I(2) model is defined by

[11] This quarter in fact coincides with the NBER dating of the start of the 1990-01 recession in the US. We experimented with other deterministic terms as well including levels shifts and a trend break in 2001:2. We did not find any of these to improve the specification of the model though.

[12] This section is based on Tuxen (2009a).

two reduced-rank restrictions (Johansen 1992),

$$\mathbf{\Pi} = \boldsymbol{\alpha}\boldsymbol{\beta}' \tag{9}$$

and

$$\boldsymbol{\alpha}_{\perp}'\mathbf{\Gamma}\boldsymbol{\beta}_{\perp} = \boldsymbol{\xi}\boldsymbol{\eta}' \tag{10}$$

where $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are $p \times r$ with $r < p$, and $\boldsymbol{\xi}$ and $\boldsymbol{\eta}$ are $(p-r) \times s_1$ with $s_1 \leq r-p$; we use $_\perp$ to denote the orthogonal complement. We can decompose the $p \times (p-r)$-matrices $\boldsymbol{\alpha}_\perp$ and $\boldsymbol{\beta}_\perp$ into the I(1) and I(2) directions: $\boldsymbol{\alpha}_\perp = [\boldsymbol{\alpha}_{\perp 1}, \boldsymbol{\alpha}_{\perp 2}]$ and $\boldsymbol{\beta}_\perp = [\boldsymbol{\beta}_{\perp 1}, \boldsymbol{\beta}_{\perp 2}]$, where $\boldsymbol{\alpha}_{\perp 1}$ and $\boldsymbol{\beta}_{\perp 1}$ are $p \times s_1$ and defined by $\boldsymbol{\alpha}_{\perp 1} = \overline{\boldsymbol{\alpha}}_\perp \boldsymbol{\xi}$ and $\boldsymbol{\beta}_{\perp 1} = \overline{\boldsymbol{\beta}}_\perp \boldsymbol{\eta}$; $\boldsymbol{\alpha}_{\perp 2}$ and $\boldsymbol{\beta}_{\perp 2}$ are $p \times s_2$ and defined by $\boldsymbol{\alpha}_{\perp 2} = \boldsymbol{\alpha}_\perp \boldsymbol{\xi}_\perp$ and $\boldsymbol{\beta}_{\perp 2} = \boldsymbol{\beta}_\perp \boldsymbol{\eta}_\perp$; we use the notation $\overline{\mathbf{v}} = \mathbf{v}(\mathbf{v}'\mathbf{v})^{-1}$.

Imposing the I(2) restrictions, (9) and (10), we can re-write (8) to obtain the parameterisation used in Johansen (1997) for maximum-likelihood estimation,

$$\Delta^2 \mathbf{x}_t = \boldsymbol{\alpha}[\boldsymbol{\rho}'\boldsymbol{\tau}'\mathbf{x}_{t-1} + \boldsymbol{\psi}'\mathbf{\Delta}\mathbf{x}_{t-1}] + \boldsymbol{\alpha}_{\perp\Omega}\boldsymbol{\kappa}'\boldsymbol{\tau}'\mathbf{\Delta}\mathbf{x}_{t-1} + \mathbf{\Psi}\mathbf{\Delta}^2\mathbf{x}_{t-1} + \boldsymbol{\varepsilon}_t, \tag{11}$$

where the parameters are variation-free. The parameters in (8) can be recovered from (11) by setting $\boldsymbol{\rho}' = (\mathbf{I}_r, \mathbf{0})$ and thus $\boldsymbol{\tau}' = (\boldsymbol{\beta}, \boldsymbol{\beta}_{\perp 1})'$, $\boldsymbol{\psi} = -(\boldsymbol{\alpha}\mathbf{\Omega}^{-1}\boldsymbol{\alpha})^{-1}\boldsymbol{\alpha}\mathbf{\Omega}^{-1}\mathbf{\Gamma}$, $\boldsymbol{\alpha}_{\perp\boldsymbol{\Omega}} = -\boldsymbol{\Omega}\boldsymbol{\alpha}_\perp(\boldsymbol{\alpha}_\perp'\boldsymbol{\Omega}\boldsymbol{\alpha}_\perp)^{-1}$and $\boldsymbol{\kappa} = (\boldsymbol{\alpha}_\perp'\mathbf{\Gamma}\overline{\boldsymbol{\beta}}, \boldsymbol{\xi})$. Under the assumption that the characteristic polynomial has exactly $2(p-r) - s_1$ unit roots and the remaining roots are outside the unit circle, $\Delta^2\mathbf{x}_t$, $(\boldsymbol{\rho}'\boldsymbol{\tau}'\mathbf{x} + \boldsymbol{\psi}'\Delta\mathbf{x})_t$ and $\boldsymbol{\tau}'\Delta\mathbf{x}_t$ all have stationary representations. In this case, r denotes the number of multi-cointegrating relations and s_1 the number of I(1) trends. The total number of common trends is $p - r = s_1 + s_2$ with s_2 is the number of I(2) trends; this leaves a total of $s_1 + 2s_2$ unit roots in the model. The reduced ranks (r, s_1) can be determined using the LR test proposed by Nielsen and Rahbek (2007). The multi-cointegrating relations are then given by $\boldsymbol{\rho}'\boldsymbol{\tau}'\mathbf{x}_{t-1} + \boldsymbol{\psi}'\mathbf{\Delta}\mathbf{x}_{t-1}$, where the combinations defined by $\boldsymbol{\rho}'\boldsymbol{\tau}'\mathbf{x}_{t-1}$ cointegrate from I(2) to I(1), and $\boldsymbol{\psi}'\mathbf{\Delta}\mathbf{x}_{t-1}$ is I(1) and cointegrate with the former to I(0). When $r > s_2$, the multi-cointegrating relations may be split into $r - s_2$ static (directly stationary) long-run relations which cointegrate from I(2) to I(0), and s_2 dynamic long-run relations which need the growth rates to become I(0). The $\boldsymbol{\alpha}$-matrix contains information on short-run adjustment in face of disequilibria. The $(r + s_1)$-dimensional vector $\boldsymbol{\tau}'\mathbf{\Delta}\mathbf{x}_{t-1}$ defines combinations of the growth rates which are I(0) and these may be given an interpretation as medium-run steady-state relations. To facilitate the economic interpretation of estimation results, we use the following parameterisation obtained from re-writing (11) (Paurolo and Rahbek

1999),

$$\Delta^2\mathbf{x}_t = \boldsymbol{\alpha}[\boldsymbol{\beta}'\mathbf{x}_{t-1} + \boldsymbol{\delta}'\Delta\mathbf{x}_{t-1}] + \boldsymbol{\zeta}\boldsymbol{\tau}'\Delta\mathbf{x}_{t-1} + \boldsymbol{\Psi}\boldsymbol{\Delta}^2\mathbf{x}_{t-1} + \boldsymbol{\varepsilon}_t, \tag{12}$$

where we have used the projection identity, $\mathbf{I}_p = \overline{\boldsymbol{\tau}}\boldsymbol{\tau}' + \overline{\boldsymbol{\tau}}_\perp\boldsymbol{\tau}'_\perp$, $\boldsymbol{\delta}' = \boldsymbol{\psi}'\overline{\boldsymbol{\tau}}_\perp\boldsymbol{\tau}'_\perp$ and $\boldsymbol{\zeta} = \boldsymbol{\alpha}_{\perp\Omega}\boldsymbol{\kappa}' + \boldsymbol{\alpha}\boldsymbol{\psi}'\overline{\boldsymbol{\tau}}'$. For this model, a data vector $b'\mathbf{x}_t$ is said to be weakly exogenous provided,

$$b'(\boldsymbol{\alpha}, \boldsymbol{\xi}, \widetilde{\boldsymbol{\zeta}}) = 0, \tag{13}$$

where $\widetilde{\boldsymbol{\zeta}}$ contains the first r columns of $\boldsymbol{\zeta}$. The test of $b'\boldsymbol{\alpha} = 0$ amounts to the less restricted hypothesis of 'no long-run levels feed-back'. Equilibrium correction, or lack thereof, is a useful piece of information in understanding the dynamics of the model. Two levels of equilibrium-correcting behaviour can be defined within this model (Juselius 2006). First, the acceleration rates are equilibrium-correcting to the growth rates if,

$$\alpha_{ij}\delta_{ji} < 0, \tag{14}$$

where α_{ij} denotes the (i,j)'th element of $\boldsymbol{\alpha}$ and δ_{ji} the (j,i)'th element of $\boldsymbol{\delta}'$ with $i = 1, ..., p$ and $j = 1, ..., r$. Moreover, the growth rates are equilibrium-correcting to the levels provided that,

$$\delta_{ji}\beta_{ji} > 0. \tag{15}$$

with β_{ji} the (j,i)'th element of $\boldsymbol{\beta}'$.

While inference on $\boldsymbol{\tau}$ can in many cases be based on the χ^2-distribution, see Boswijk (2000) and Johansen (2006), Paruolo (1996) shows that the distribution of the test for restrictions on the multi-cointegration parameter, $\boldsymbol{\delta}$, is not mixed Gaussian. The test of $\boldsymbol{\delta} = \mathbf{0}$ is of particular interest because this implies that $\boldsymbol{\beta}'\mathbf{x}_t \sim I(0)$. Bootstrap methods might be used to simulate the distribution of $\boldsymbol{\delta}$; this is out of the scope of this paper however. Kurita, Nielsen, and Rahbek (2009) give some distributional results for $\boldsymbol{\psi}$ but this parameter does not have an obvious economic interpretation. Moreover, because $\widetilde{\boldsymbol{\beta}}_{\perp 2}$ is not identified, we cannot formally test which variables are affected by the I(2) trend(s). For both $\boldsymbol{\delta}$ and $\widetilde{\boldsymbol{\beta}}_{\perp 2}$, a provisional judgement based on the sign and magnitude of the estimated coefficients may nevertheless be made.

In order to study the common trends, we consider the solution for the levels of the process, $\mathbf{x}_t$. For the I(2) model, the moving-average (MA) representation takes the form,

$$\mathbf{x}_t = \mathbf{C}_2\boldsymbol{\Sigma}_{j=1}^{t}\boldsymbol{\Sigma}_{i=1}^{j}\boldsymbol{\varepsilon}_i + \mathbf{C}_1\Sigma_{i=1}^{t}\boldsymbol{\varepsilon}_i + \mathbf{C}^*(L)\boldsymbol{\varepsilon}_i + \mathbf{A} + \mathbf{B}t, \tag{16}$$

where $\mathbf{A}$ and $\mathbf{B}$ are functions of the initial values, $\mathbf{C}^*(L)$ is an infinite polynomial in the lag operator L, and $\mathbf{C}_2 = \widetilde{\boldsymbol{\beta}}_{\perp 2}\boldsymbol{\alpha}'_{\perp 2}$ with $\boldsymbol{\alpha}'_{\perp 2}\boldsymbol{\Sigma}_{j=1}^{t}\boldsymbol{\Sigma}_{i=1}^{j}\boldsymbol{\varepsilon}_i$ define the s_2 I(2) trends while $\widetilde{\boldsymbol{\beta}}_{\perp 2} = \boldsymbol{\beta}_{\perp 2}[\boldsymbol{\alpha}'_{\perp 2}(\boldsymbol{\Gamma}\overline{\boldsymbol{\beta}}\overline{\boldsymbol{\alpha}}'\boldsymbol{\Gamma} + \mathbf{I}_p - \boldsymbol{\Gamma}_1)\boldsymbol{\beta}_{\perp 2}]^{-1}$ provides the loadings to these. Similarly, $\boldsymbol{\alpha}'_{\perp 1}\boldsymbol{\Sigma}_{i=1}^{t}\boldsymbol{\varepsilon}_i$ defines the s_1 (separate) I(1) trends. The $\mathbf{C}_1$-matrix cannot be given a simple decomposition; Johansen (2005) derives an analytical expression, $\mathbf{C}_1 = \boldsymbol{\varpi}_0\boldsymbol{\alpha}' + \boldsymbol{\varpi}_1\boldsymbol{\alpha}'_{\perp 1} + \boldsymbol{\varpi}_2\boldsymbol{\alpha}'_{\perp 2}$ where $\boldsymbol{\varpi}_0$, $\boldsymbol{\varpi}_1$ and $\boldsymbol{\varpi}_2$ are complicated functions of the parameters.

For nominal variables such as output, consumer, house and share prices, linear trends in the levels is a reasonable starting hypothesis (Rahbek, Kongsted, and Jørgensen 1999). Johansen, Juselius, Frydman, and Goldberg (2009) and Kurita et al. (2009) show how to restrict deterministic shift terms appropriately in an I(2) model with piecewise linear deterministic trends and derive the distribution of $\boldsymbol{\beta}$ and the distributions of $\boldsymbol{\tau}$ and $\boldsymbol{\psi}$, respectively. When including deterministic shifts in the CVAR, two concerns must be accommodated. First of all, we need to consider which components are relevant from an economic point of view. In the I(2) model, all deterministic terms are cumulated both once and twice. It is therefore crucial to ensure that if, say, a trend is appropriate in the levels of the series, then this is properly restricted to ensure that it is not allowed to enter the first and second differences of the model. If left unrestricted, this will cumulate to produce both quadratic and cubic trends in the data, respectively, both of which are not economically viable. Secondly, we need to consider which components are needed to ensure similarity in the test procedures; see Nielsen and Rahbek (2000) on the I(1) case. To achieve this, we should allow the same type of deterministic components in all directions of the model, i.e. in the $\boldsymbol{\alpha}, \boldsymbol{\alpha}_{\perp 1}, \boldsymbol{\alpha}_{\perp 2}$ directions alike, in order for tests of stationarity to be conducted against the appropriate alternatives, thereby improving the power of the tests.

We left out deterministic components in the presentation above but in the empirical analysis we shall consider (12) augmented with a trend and a broken linear trend restricted to the $\boldsymbol{\alpha}$-space in (7) as well as unrestricted permanent and transitory impulse dummies,

$$
\begin{aligned}
\Delta^2\mathbf{x}_t \;=\; & \boldsymbol{\alpha}\left[\underbrace{\begin{pmatrix} \boldsymbol{\beta}' & \widetilde{\boldsymbol{\beta}}'_0 & \widetilde{\boldsymbol{\beta}}'_1 \end{pmatrix}}_{\widetilde{\boldsymbol{\beta}}'}\begin{pmatrix} \mathbf{x} \\ t \\ t\mathbf{Ds} \end{pmatrix} + \underbrace{\begin{pmatrix} \boldsymbol{\delta}' & \widetilde{\boldsymbol{\delta}}'_0 & \widetilde{\boldsymbol{\delta}}'_1 \end{pmatrix}}_{\widetilde{\boldsymbol{\delta}}'}\begin{pmatrix} \Delta\mathbf{x} \\ 1 \\ \mathbf{Ds} \end{pmatrix}\right]_{t-1} \qquad (17)\\
& + \boldsymbol{\zeta}\begin{pmatrix} \boldsymbol{\beta}' & \widetilde{\boldsymbol{\beta}}'_0 & \widetilde{\boldsymbol{\beta}}'_1 \\ \boldsymbol{\beta}'_{\perp 1} & \widetilde{\boldsymbol{\beta}}'_{\perp 1} & \widetilde{\boldsymbol{\beta}}'_{\perp 1} \end{pmatrix}\begin{pmatrix} \Delta\mathbf{x} \\ 1 \\ \mathbf{Ds} \end{pmatrix}_{t-1} + \boldsymbol{\phi}_{cs}\mathbf{Dcs}_t + \boldsymbol{\phi}_p\mathbf{Dp}_t + \boldsymbol{\phi}_{tr}\mathbf{Dtr}_t + \boldsymbol{\Psi}\boldsymbol{\Delta}^2\mathbf{x}_{t-1} + \boldsymbol{\varepsilon}_t,
\end{aligned}
$$

where t denotes a linear deterministic trend, $\mathbf{Ds}_t$ is a matrix of shift dummies, 1 a constant term, $\mathbf{Dcs}_t$ a matrix of centered seasonal dummies, $\mathbf{Dp}_t$ a matrix of permanent impulse dummies and $\mathbf{Dtr}_t$ a matrix of transitory impulse dummies; $t\mathbf{Ds}$ thus contains broken linear trends.

5 Transforming the economic model to CVAR space

We consider a CVAR scenario that assumes validity of the NRT and is consistent with the economic model of Section 2. We test for I(2) ranks and propose an alternative transformation consistent with the empirical finding of two I(2) trends. Finally, we suggest a data-consistent scenario that allows for failure of LPH. We first discuss our identification strategy however.

5.1 Identification strategy

To facilitate the identification process, we consider three models, called Model I, II and III, in turn. This sequence of models results from a gradual extension of the information that resembles that used in the write-up of the economic model. We start from a small set of variables, $(y, p, I_s, I_l)_t$, and gradually extend the information set to include m_t and (h_t, s_t) in turn as done in Section 2. In this process, we exploit the fact that the cointegrating relations are, in principle, invariant to extensions of the information set. When a new (non-stationary) variable is added, two possibilities therefore exist. The first possibility is that the cointegration rank increases by one which implies that the new variable cointegrates with the previous set of variables and thus adds relevant information. The second possibility is that the rank is unchanged and thus that a new stochastic trend, which may be either I(1) or I(2), is introduced; in this case the new variable adds no relevant long-run information.

From this procedure, we can only identify irreducible cointegrating relations as the cointegrating relations are not invariant to reductions of the information set. For example, having identified an IS curve in Model I, this procedure does not allow us to go back an revise the specification of this relation to assess whether, say, asset prices play a role in the IS curve of Model III. Indeed, empirically it is possible that asset prices could enter with a significant coefficient in spite of the baseline IS relation being stationary by itself and thus that adding asset prices might then lead the p-value to increase. We abstract from this complication here.

Starting from the baseline set of variables and observing whether adding money and the two asset-price series leads the cointegration rank to increase, we can answer the following questions in turn:

- Model I, $(y, p, I_s, I_l)_t$: Have excessively low policy rates fuelled goods-price inflation?
- Model II, $(y, p, I_s, I_l, m)_t$: Has excess money supply fuelled goods-price inflation?
- Model III, $(y, p, I_s, I_l, m, h, s)_t$: Has 'excess liquidity' fuelled asset-price inflation?

5.2 Standard scenario

The theoretical relations were all specified in terms of *real* variables (real output, real money and real asset prices) and one *nominal* growth rate (the inflation rate) plus a long- and a short-term interest rate. Conventional economic theory prescribes the existence of one single trend underlying the growth in all nominal variables. Assuming the central bank is in control of inflation, the nominal anchor is set by monetary policy.

In terms of time-series behaviour, this implies that all nominal variables (output, money, goods and asset prices) share a common I(2) trend, i.e. only one I(2) trend exists and the loadings of the variables to this are identical. The nominal component can then be removed by a simple transformation to I(1) space: the nominal trend in each series is eliminated by subtracting one of the other nominal series, typically the price level such that the transformed system becomes a set of real variables. In this case, the linear combination (1,-1) of each pair of variables ensures cointegration from I(2) to I(1) space. The transformed system can be analysed in an I(1) model and in order to keep track of the nominal growth rate and multi-cointegration, the first difference of either of the nominal variables, typically the inflation rate, should be included in the model. The NRT thus requires the existence of exactly one I(2) trend and LPH. If these conditions are fulfilled no information is lost in the transformation (Kongsted 2005).

We discuss the econometric implications of this standard hypothesis by first setting up a scenario for the time-series behaviour of the variables in terms of the MA representation (16), and then turning to the long-run relations between the variables in terms of the ECM representation. Among the $p = 7$ variables in (6), standard theory predicts $s_2 = 1$ I(2) trend which results in the following specification for the levels,

$$\begin{pmatrix} y \\ p \\ I_s \\ I_l \\ m \\ h \\ s \end{pmatrix}_t = \begin{pmatrix} \omega_y \\ \omega_p \\ 0 \\ 0 \\ \omega_m \\ \omega_h \\ \omega_s \end{pmatrix} \sum_j^t \sum_i^j u_{1i} + \begin{pmatrix} * & * \\ * & * \\ * & * \\ * & * \\ * & * \\ * & * \\ * & * \end{pmatrix} \begin{pmatrix} \sum_i^t u_{1i} \\ \sum_i^t u_{2i} \end{pmatrix}_t + \begin{pmatrix} * \\ * \\ 0 \\ 0 \\ * \\ * \\ * \end{pmatrix} t + \text{ det. and stat.comp.,} \tag{18}$$

where $\sum_j^t \sum_i^j u_{\ell i}$ denotes the ℓ^{th} I(2) trend arising from the twice cumulated residuals; similar notation is used for the I(1) trends. With a cointegration rank of $r = 5$ as suggested by the theoretical model, see also (22) below, we have $s_1 = p - r - s_2 = 1$ (separate) I(1) trends in the model. In (18) all nominal variables share the I(2) trend, $\sum\sum u_{1i,t}$, but for the NRT to hold we require also LPH, i.e. $\omega_y = \omega_p = \omega_m = \omega_h = \omega_s = \omega$. This implies that the I(2) loadings matrix takes the form,

$$\widetilde{\boldsymbol{\beta}}'_{\perp 2} = (\omega, \omega, 0, 0, \omega, \omega, \omega) \tag{19}$$

If monetary policy controls the price level then the cumulated shocks to the policy rate, proxied by the short rate, constitutes the I(2) trend,

$$\begin{aligned} \boldsymbol{\alpha}'_{\perp 2} &= \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix} \\ &\Longrightarrow \sum\sum u_{1,t} = \boldsymbol{\alpha}'_{\perp 2} \sum_j^t \sum_i^j \boldsymbol{\varepsilon}_i = \sum_j^t \sum_i^j \varepsilon_{Is,i} \end{aligned} \tag{20}$$

If the NRT defined by (19) and (18), is valid, we can transform the I(2) model to its simpler I(1) counterpart formulated in terms of the real-transformed variables and, say, the inflation rate,

$$\begin{pmatrix} y_r \\ \Delta p \\ I_s \\ I_l \\ m_r \\ h_r \\ s_r \end{pmatrix}_t = \begin{pmatrix} * & * \\ * & * \\ * & * \\ * & * \\ * & * \\ * & * \\ * & * \end{pmatrix} \begin{pmatrix} \sum_i^t u_{1i} \\ \sum_i^t u_{2i} \end{pmatrix}_t + \begin{pmatrix} * \\ 0 \\ 0 \\ 0 \\ * \\ * \\ * \end{pmatrix} t + \text{ det. and stat.comp.,} \tag{21}$$

where $y_{r,t} = (y-p)_t$ is real output, $m_{r,t} = (m-p)_t$ real money supply, $h_{r,t} = (h-p)_t$ real house prices, and $s_{r,t} = (s-p)_t$ real share prices. All variables in (21) are at most I(1) and can thus be analysed within the I(1) CVAR model, see Johansen (1988, 1991) and Juselius (2006).

The equilibrium relations proposed in Section 2 provide the basis for deriving cointegration relations for the model (21), i.e. imposing identifying restrictions on $\boldsymbol{\beta}$. The IS relation, the NKPC and the Taylor rule can however not be separately identified given the choice of variables in (6). We leave out the NKPC below and price adjustment would is to be captured by the $\boldsymbol{\alpha}$-matrix. With $r = 5$ we have the following structure,

$$\boldsymbol{\alpha}\boldsymbol{\beta}'\mathbf{x}_t^{\text{real}} = \begin{pmatrix} \boldsymbol{\alpha}_{11} & * & * & * & * \\ * & * & * & * & * \\ * & \boldsymbol{\alpha}_{32} & * & * & * \\ * & * & * & * & * \\ * & * & \boldsymbol{\alpha}_{53} & * & * \\ \alpha_{61} & \alpha_{62} & \alpha_{63} & \boldsymbol{\alpha}_{64} & \alpha_{65} \\ \alpha_{71} & \alpha_{72} & \alpha_{73} & \alpha_{74} & \boldsymbol{\alpha}_{75} \end{pmatrix}$$

$$\begin{pmatrix} 1 & -\beta_{11} & 0 & \beta_{11} & 0 & 0 & 0 \\ \beta_{21} & \beta_{22} & 1 & 0 & 0 & 0 & 0 \\ -1 & \beta_{31} & -\beta_{32} & \beta_{32} & 1 & 0 & 0 \\ \beta_{41}-\beta_{44} & -(\beta_{42}+\beta_{43}) & \beta_{42} & \beta_{43} & \beta_{44} & 1 & 0 \\ \beta_{51}-\beta_{54} & -(\beta_{52}+\beta_{53}) & \beta_{52} & \beta_{53} & \beta_{54} & 0 & 1 \end{pmatrix} \begin{pmatrix} y_r \\ \Delta p \\ I_s \\ I_l \\ m_r \\ h_r \\ s_r \end{pmatrix}_t \qquad (22)$$

where, in addition, the trend and the broken linear trend in (7) might enter the cointegration space.

The first relation represents an IS curve where real output is negatively related to the long real rate of interest, $\beta_{11} > 0$. The second relation is a Taylor rule which suggests that the short rate increases when inflation and output (gap) rise above target, $\beta_{21}, \beta_{22} < 0$. The NKPC might be recovered by combining the IS relation and the Taylor rule, possibly weighted by the corresponding α-coefficients to capture adjustment in prices. The third relation is a money-demand relation where inverse velocity is negatively related to the interest-rate spread and the inflation rate, $\beta_{31}, \beta_{32} > 0$.[13]

The last two relations describe demand for housing and shares, respectively, and these are only identified (on the order condition) if further restrictions are imposed, for example, by exclusion/inclusion of the trend and/or broken trend and/or if some of the excess-liquidity components are excludable. These relations reflect the idea that the fundamental value for both asset prices is driven by real output, $\beta_{41}, \beta_{51} < 0$. In addition, the excess-liquidity components suggest that lower long- and/or short-term real interest rates should

[13]The stylised downward shift in the LM curve depicted in Figure 1 could potentially be captured by inclusion of a shift dummy in the money-demand relation here.

be associated with rising asset prices, $\beta_{42}, \beta_{52} > 0$ and $\beta_{43}, \beta_{53} > 0$, as should money in excess of GDP, $\beta_{44}, \beta_{54} < 0$.

In the adjustment structure, error-correction sustaining the interpretation of each equation would be supported by the variable corresponding to the coefficient on which we have normalised to exhibit error-correcting behaviour, $\alpha_{11}, \alpha_{32}, \alpha_{53}, \alpha_{64}, \alpha_{75} < 0$. We focus here on the reaction of asset prices. Output in excess of the IS level could fuel asset prices in the short run, $\alpha_{61}, \alpha_{71} > 0$. If the central bank raises the policy rate above its Taylor-rule level, asset prices are likely to be held back, *ceteris paribus*, $\alpha_{62}, \alpha_{72} < 0$. The standard New-Keynesian model would predict that conditional on this reaction to interest-rate changes, there should be no effects of deviations from the money-demand relation, $\alpha_{63} = \alpha_{73} = 0$. In contrast, our hypothesis is that a fall in money supply below its equilibrium level could spur a decline in asset prices on top of the effects of interest-rate changes, $\alpha_{63}, \alpha_{73} > 0$. Finally, cross-asset effects may be present such that a rise in house prices causes share prices to rise (push channel), and *vice versa*; the opposite effects may also occur (pull channel) and thus $\alpha_{74}, \alpha_{65} \gtreqless 0$.

5.3 Alternative scenario

Before proceeding to tests for I(2) ranks we propose also an alternative hypothesis consistent with the graphical analysis which suggested: $s_2 = 2$, $s_{r,t} \sim I(1)$ and that excess (monetary) liquidity have fuelled house prices rather than goods prices. This alternative hypothesis takes the form,

$$\begin{pmatrix} y \\ p \\ I_s \\ I_l \\ m \\ h \\ s \end{pmatrix}_t = \begin{pmatrix} \omega_{1y} & 0 \\ \omega_{1p} & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & \omega_{2m} \\ 0 & \omega_{2h} \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \sum_j^t \sum_i^j u_{1i} \\ \sum_j^t \sum_i^j u_{2i} \end{pmatrix}_t + \begin{pmatrix} * & * \\ * & * \\ * & * \\ * & * \\ * & * \\ * & * \\ * & * \end{pmatrix} \begin{pmatrix} \sum_i^t u_{1i} \\ \sum_i^t u_{2i} \end{pmatrix}_t + \begin{pmatrix} * \\ * \\ * \\ 0 \\ 0 \\ * \\ * \end{pmatrix} t$$
$$+ \text{ det. and stat. comp.} \tag{23}$$

where y_t and p_t are driven by the first I(2) trend and m_t and h_t by the second; $s_t \sim I(1)$.

If the two I(2) trends are shared proportionally by the relevant sets of variables, i.e. if $\omega_{1p} = \omega_{1y} = \omega_1$ and $\omega_{2m} = \omega_{2h} = \omega_2$, then the loadings of the common trends take the form,

$$\widetilde{\boldsymbol{\beta}}'_{\perp 2} = \begin{pmatrix} \omega_1 & \omega_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \omega_2 & \omega_2 & 0 \end{pmatrix} \tag{24}$$

Excess money measured by $(m-y)_t \sim I(2)$ could thus be a likely cause, or effect, of housing-market bubbles as $(h-p)_t \sim I(2)$.

Under this scenario, an alternative transformation to I(1) space could take the form,

$$\begin{pmatrix} y_r \\ \Delta p \\ \Delta m \\ I_s \\ I_l \\ h-m \\ s \end{pmatrix}_t = \begin{pmatrix} * & * \\ * & * \\ * & * \\ * & * \\ * & * \\ * & * \\ * & * \end{pmatrix} \begin{pmatrix} \sum_i^t u_{1i} \\ \sum_i^t u_{2i} \end{pmatrix}_t + \begin{pmatrix} * \\ 0 \\ 0 \\ 0 \\ 0 \\ * \\ * \end{pmatrix} t + \text{det. and stat.comp.} \tag{25}$$

where all variables are assumed to be at most I(1) and we notably include two nominal growth rates, inflation and nominal-money growth. This transformation instead of the NRT allows an IS curve and a Taylor rule to be among the long-run relations. The money-demand and asset-price relations must however be modified to take account of the transformation.

5.4 Tests for I(2) ranks and LPH

We estimate a VAR($k = 3$) based on the data vector (7). The deterministic specification restrict the trend and the broken trend to the $\boldsymbol{\beta}$-space; correspondingly a constant and a level shift in 1990:4, $Ds_{90:4}$, are included and restricted to the $\boldsymbol{\delta}$-space, see (17), and a permanent blip in 1990:4, $Dp_{90:4}$, in the second differences (unrestricted). To account for extraordinary events we include the following transitory blip dummies,

$$\mathbf{Dtr}_t = (\mathbf{Dtr}_{84:4}, \mathbf{Dtr}_{87:4}, \mathbf{Dtr}_{90:4}, \mathbf{Dtr}_{03:4})'_t \tag{26}$$

Finally, centered seasonal dummies are included. For reporting results, $\boldsymbol{\phi}$ is used to denote all unrestricted deterministic terms.

Table A.2 shows misspecification tests for each of the models I, II and III. With three lags, tests for autocorrelation suggest a reasonably well-specified model in all cases apart from some problems at the second lag in Model II. After inclusion of the transitory dummies in (26), the assumption of multivariate normality cannot be rejected for any of the models, albeit only at the one-per cent level for Model II and III. Some ARCH effects are present but this should not affect the rank test too much, see Dennis, Hansen, and Rahbek (2002) for results on the I(1) case.

Table 1, 2 and 3 report the I(2) rank test statistics for Model I, II and III, respectively,

$p-r$	r	$s_2=4$	$s_2=3$	$s_2=2$	$s_2=1$	$s_2=0$
4	0	226.82 [170]	185.38 [142]	147.45 [119]	127.20 [99]	113.26 [83]
3	1		129.46 [111]	95.10 [89]	77.51 [72]	63.12 [58]
2	2			**60.21** **[64]**	43.72 [48]	32.28 [37]
1	3				22.03 [29]	15.82 [18]

Table 1: Model I: Rank test statistics (95-per cent critical values in brackets).

$p-r$	r	$s_2=5$	$s_2=4$	$s_2=3$	$s_2=2$	$s_2=1$	$s_2=0$
5	0	325.55 [239]	280.52 [205]	241.88 [177]	207.03 [151]	188.67 [130]	174.44 [112]
4	1		202.74 [170]	165.39 [142]	140.52 [119]	125.68 [94]	111.64 [83]
3	2			121.84 [111]	92.96 [89]	76.02 [72]	65.50 [58]
2	3				**61.05** **[64]**	45.28 [48]	34.56 [37]
1	4					21.11 [29]	14.58 [18]

Table 2: Model II: Rank test statistics (95-per cent critical values in brackets).

$p-r$	r	$s_2=7$	$s_2=6$	$s_2=5$	$s_2=4$	$s_2=3$	$s_2=2$	$s_2=1$	$s_2=0$
7	0	532.55 [413]	473.81 [367]	426.29 [290]	387.98 [257]	354.40 [257]	330.58 [228]	315.48 [203]	305.50 [182]
6	1		385.99 [320]	345.89 [280]	308.40 [246]	279.77 [215]	255.28 [188]	241.27 [165]	234.07 [145]
5	2			272.57 [239]	238.36 [205]	210.63 [177]	185.51 [151]	170.83 [130]	164.04 [112]
4	3				183.33 [170]	155.47 [142]	132.16 [119]	118.97 [99]	111.59 [83]
3	4					115.00 [111]	94.78 [89]	81.27 [72]	74.19 [58]
2	5						**62.45** **[64]**	49.00 [48]	41.77 [37]
1	6							22.12 [29]	11.33 [22]

Table 3: Model III: Rank test statistics (95-per cent critical values in brackets).

	Model I	Model II	Model III
Choice of I(2) ranks	$r = 2 \wedge s_2 = 2$	$r = 3 \wedge s_2 = 2$	$r = 5 \wedge s_2 = 2$
Unit vector in τ:			
$I_{s,t} \sim I(1)$	$\chi^2(4) = \underset{[0.0016]}{17.44}$	$\chi^2(4) = \underset{[p=0.0028]}{16.16}$	$\chi^2(4) = \underset{[p=0.0002]}{21.84}$
$I_{l,t} \sim I(1)$	$\chi^2(4) = \underset{[p=0.0070]}{14.10}$	$\chi^2(4) = \underset{[p=0.0601]}{9.04}$	$\chi^2(4) = \underset{[p=0.0489]}{9.54}$

Table 4: Model I, II and III: Choice of ranks and tests for I(1).

alongside five-percent critical values simulated to take account of the trend break.[14] Starting from Model I in the four baseline variables, $(y, p, I_s, I_l)_t$, the test points to $s_2 = 2$ I(2) trends and a rank of $r = 2$. Model II adds m_t to the baseline set of variables which leads the rank to increase to three but the number of I(2) trends is unchanged, $(r = 3, s_2 = 2)$. Model III further adds house and share prices which seems to lead the rank to increase by two, again leaving the number of common trends unchanged. Plots of the first five multi-cointegrating relations (not shown) reveal that these indeed look stationary.

The choice of rank for the sequence of models suggest that all nominal variables are driven by the same two I(2) trends. This implies however that the standard NRT cannot be imposed as more than one nominal growth rate is needed to keep track of the I(2) components. Hence the standard scenario (22) is not a valid description of the data. The finding that $s_2 > 1$ points to significant divergence in nominal growth rates over the sample period and thus LPH does not hold in-sample. This could be a first sign of asset-price bubbles: for example, if financial prices have moved out of sync with goods prices, such that the real returns on financial asset have surged, this could indicate the presence of a bubble. The test of the alternative transformation represented by the scenario (25) is a special case of this hypothesis assigning a special role to money in driving asset prices; this restriction also leads to rejection of the null ($\chi^2(10) = 76.16$, $p = 0.00$) however. The dynamics of the nominal variables has thus been more complicated in our sample than allowed for by these simple transformations.

Table 4 reports tests of whether the two interest rates are I(1), i.e. has unit vectors

[14] Ox code for simulating the distribution of the I(2) rank test was kindly provided by Heino Bohn Nielsen (length of random walk: 2000; number of replications: 20,000).

in $\boldsymbol{\tau}$. The long rate cannot be rejected as I(1) at the one per-cent level in all three models. The I(1) hypothesis is rejected for the short rat in all models, albeit not very strongly so. The finding that the two interest rates are likely to be integrated of order higher than one is consistent with the prediction of Juselius, Frydman, Goldberg, and Johansen (2009) that interest rates should exhibit near-I(2) behaviour when agents form expectations based on imperfect knowledge. Share prices are unlikely to be I(2) based on the graphical analysis in Section 3 and we treat it as I(1), possibly around a trend, hereafter.[15]

5.5 Data-consistent scenario

Given that no economically relevant transformation to I(1) space seems to exist, we stay with the I(2) model and look for polynomial cointegration. A scenario consistent with the tests above must allow for non-identical loadings to the I(2) trends,

$$\widetilde{\boldsymbol{\beta}}'_{\perp 2} = \begin{pmatrix} \omega_{1y} & \omega_{1p} & 0 & 0 & \omega_{1m} & \omega_{1h} & 0 \\ \omega_{2y} & \omega_{2p} & 0 & 0 & \omega_{2m} & \omega_{2h} & 0 \end{pmatrix} \tag{27}$$

Our hypothesis is that shocks to interest rates in either end of the maturity spectrum constitute the two I(2) trends,[16]

$$\begin{aligned} \boldsymbol{\alpha}'_{\perp 2} \quad &= \quad \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix} \\ &\Longrightarrow \quad \begin{pmatrix} \sum_j^t \sum_i^j u_{i,1} \\ \sum_j^t \sum_i^j u_{i,2} \end{pmatrix} = \begin{pmatrix} \boldsymbol{\alpha}'_{\perp 21} \sum_j^t \sum_i^j \boldsymbol{\varepsilon}_i \\ \boldsymbol{\alpha}'_{\perp 22} \sum_j^t \sum_i^j \boldsymbol{\varepsilon}_i \end{pmatrix} = \begin{pmatrix} \sum \sum_i^t \varepsilon_{I_s,i} \\ \sum \sum_i^t \varepsilon_{I_l,i} \end{pmatrix} \end{aligned} \tag{28}$$

[15] The economically relevant hypothesis of whether trend-adjusted share prices are I(1) around a deterministic trend cannot be tested in CATS.

[16] For example, Giese (2008) finds that the US term structure is affected by two types of stochastic trends: one arising from cumulated shocks to the short end of the term structure and one arising from cumulated shocks to the long end of the curve.

In light of the rejection of LPH, we propose a set of multi-cointegrating relations which allow for non-homogeneous relations,

$$\boldsymbol{\beta}'\mathbf{x}_t^{\text{nom}} + \boldsymbol{\delta}'\Delta\mathbf{x}_t = \begin{pmatrix} 1 & \beta_{11} & 0 & \beta_{12} & 0 & 0 & 0 \\ \beta_{21} & \beta_{22} & 1 & 0 & 0 & 0 & 0 \\ \beta_{31} & 0 & -\beta_{33} & \beta_{33} & 1 & 0 & 0 \\ 0 & 0 & \beta_{41} & \beta_{42} & \beta_{43} & 1 & 0 \\ \beta_{51} & 0 & \beta_{52} & \beta_{53} & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} y \\ p \\ I_s \\ I_l \\ m \\ h \\ s \end{pmatrix}_t + \begin{pmatrix} \delta_{11} & \delta_{12} & 0 & 0 & \delta_{13} & \delta_{14} & 0 \\ \delta_{21} & \delta_{22} & 0 & 0 & \delta_{23} & \delta_{24} & 0 \\ \delta_{31} & \delta_{32} & 0 & 0 & \delta_{33} & \delta_{34} & 0 \\ \delta_{41} & \delta_{42} & 0 & 0 & \delta_{43} & \delta_{44} & 0 \\ \delta_{51} & \delta_{52} & 0 & 0 & \delta_{53} & \delta_{54} & 0 \end{pmatrix} \begin{pmatrix} \Delta y \\ \Delta p \\ \Delta I_s \\ \Delta I_l \\ \Delta m \\ \Delta h \\ \Delta s \end{pmatrix}_t \tag{29}$$

where we look for relations similar to those in I(1) space but allowing for lack of homogeneity between the nominal variables. All $\boldsymbol{\delta}$-terms associated with the I(2) variables are set to potentially enter each relation.

We need new and/or additional restrictions compared with (22) in order to achieve formal identification of the cointegration space. If deviations from homogeneity are small we expect $\beta_{11} \simeq -1$, $\beta_{21} \simeq -\beta_{22}$ and $\beta_{31} \simeq -1$. A negative sign of the goods-price inflation term in the second relation, $\delta_{22} < 0$, is key to identify this as a standard Taylor-type rule. In addition, there could be a role for asset prices in the first three relations as rising asset prices may lead aggregate demand to rise due to wealth effects, central banks to hike rates, and/or money demand to contract as a result of portfolio re-balancing; we have left these terms out here. For identification of the asset-price relations, we propose that house prices move with money supply, $\beta_{43} < 0$, as suggested by Figure 5, and with the real either of the real interest rates, $\beta_{41}, \beta_{42} > 0, \delta_{42} < 0$. Share prices are here assumed to vary with output, $\beta_{51} < 0$, but also with the real interest rates, $\beta_{52}, \beta_{53} > 0, \delta_{52} < 0$. In terms of the adjustment structure, we expect the signs of the $\boldsymbol{\alpha}$-coefficients to be similar to those discussed in relation to (22).

6 Empirical results

Based on the identification strategy above, we estimate the sequence of models defined by gradually extending the information set. Assuming that homogeneity must hold in the (very) long run, its rejection within our sample is a signal of temporary, but persistent, deviations from equilibrium. This makes it difficult to achieve empirical and economic identification simultaneously. As a remedy to this problem, we use an estimation strategy where each of the models, Model I, II and III, is in two steps:

1. We start by imposing the theoretical restrictions in (29) except for the homogeneity restrictions.

2. Provided the restrictions cannot be rejected, possibly after incorporating some modifications to the theoretical relations, we then proceed to impose homogeneity on the system, despite its likely rejection within the sample.

This procedure allows us to separate the importance of (lack of) homogeneity from the validity of other economic restrictions. Studying the effects of deviations from the homogenous relations on the model variables then gives a picture of its effect on the model dynamics in our (finite) sample.

6.1 Model I: Baseline variables

The baseline model consists of the four variables that make up the simple New-Keynesian model,

$$\mathbf{x}_t = (y, p, I_s, I_l)'_t \tag{30}$$

The I(2) rank test statistics pointed to $(r = 2, s_2 = 2)$, see Table 1. As suggested by (29) we look for an IS relation and a monetary-policy rule. Table 5 shows that imposing just-identifying restrictions on $\boldsymbol{\beta}$, but allowing y_t and p_t to be non-homogeneous, identifies $\beta_1'\mathbf{x}_t$ as an IS curve and $\beta_2'\mathbf{x}_t$ as a policy rule, although the former is defined in terms of the interest-rate spread rather than the real long rate. The coefficient of p_t in $\beta_1'\mathbf{x}_t$ is not 'too far' from suggesting homogeneity and this over-identifying restriction is not rejected $(\chi^2(1) = 1.07, p = 0.30)$. The coefficient of y_t in $\beta_2'\mathbf{x}_t$ is highly significant, a likely result of the super-super consistency of $\boldsymbol{\beta}$. The coefficient of p_t however seems far from homogeneous with that of y_t, and notably p_t is much less significant. The broken linear-trend coefficients hint at significantly weaker output growth after 1990:4.

$\widetilde{\boldsymbol{\beta}}'$	y	p	I_s	I_l	$t_{90:4}$	t
$\widetilde{\beta}'_1$	1.00 [NA]	−0.74 [−7.40]	−4.98 [−10.58]	4.98 [10.58]	0.00 [4.96]	−0.01 [−9.54]
$\widetilde{\beta}'_2$	−0.27 [−1043.40]	0.07 [4.19]	1.00 [NA]	0.00 [NA]	−0.00 [−16.71]	0.00 [24.99]

Table 5: Model I: CI(2,1) relations (homogeneity not imposed).

$\boldsymbol{\beta}'$	y	p	I_s	I_l	$t_{90:4}$	t
β'_1	1.00 [NA]	−1.00 [NA]	−4.52 [−10.64]	4.52 [10.64]	0.00 [10.69]	−0.01 [−49.43]
β'_2	−0.32 [−2756.31]	0.32 [2756.31]	1.00 [NA]	0.00 [NA]	−0.00 [−9.29]	0.00 [37.07]

$\widetilde{\boldsymbol{\delta}}'$	Δy	Δp	ΔI_s	ΔI_l	$Ds_{90:4}$	1
$\widetilde{\delta}'_1$	0.86	1.13	−0.00	0.00	0.01	−30.26
$\widetilde{\delta}'_2$	−0.23	−1.79	0.47	0.15	−0.01	9.74

$\boldsymbol{\alpha}$	α_1	α_2
$\Delta^2 y$	−0.15 [−2.62]	0.16 [1.17]
$\Delta^2 p$	0.07 [2.80]	0.28 [4.26]
$\Delta^2 I_s$	−0.02 [−1.21]	−0.07 [−2.08]
$\Delta^2 I_l$	−0.06 [−3.69]	−0.11 [−2.83]

Table 6: Model I: Multi-cointegrating relations and adjustment structure (homogeneity imposed).

$\tilde{\boldsymbol{\beta}}'_{\perp 2}$	y	p	I_s	I_l
$\tilde{\beta}'_{\perp 21}$	2.11	0.20	0.61	0.19
$\tilde{\beta}'_{\perp 22}$	−0.63	0.46	−0.35	−0.11

$\boldsymbol{\alpha}'_{\perp 2}$	$\Sigma\Sigma\varepsilon_y$	$\Sigma\Sigma\varepsilon_p$	$\Sigma\Sigma\varepsilon_{I_s}$	$\Sigma\Sigma\varepsilon_{I_l}$
$\alpha'_{\perp 21}$	0.01 [0.21]	0.26 [1.76]	1.00 [NA]	0.00 [NA]
$\alpha'_{\perp 22}$	−0.15 [−2.18]	0.47 [2.75]	0.00 [NA]	1.00 [NA]
$\widehat{\sigma}_\varepsilon$	0.0027	0.0013	0.0007	0.0007

Table 7: Model I: Composition and loadings of the I(2) trends (homogeneity imposed).

Table 6 shows the results of imposing homogeneity between y_t and p_t in both relations. This results in rejection of the restrictions ($\chi^2(2) = 24.74, p = 0.00$) but graphs of the multi-cointegrating relations appear stationary, see Figure A.1, an indication that homogeneity is not completely off.

The IS relation,

$$(y-p)_t + \underset{[10.64]}{4.52}(I_l - I_s)_t + \underset{[N.A.]}{1.13}\Delta p_t + \underset{[10.69]}{0.00}\,t_{90:4} \underset{[-49.43]}{-0.01}\,t + \underset{[N.A.]}{0.01}Ds_{90:4} + ... \sim I(0), \quad (31)$$

has real output negatively related to the spread, possibly reflecting inflation expectations, and to increases in nominal growth rates as measured by Δp_t. The $\boldsymbol{\alpha}$-matrix shows that there is a tendency for output growth to fall and for inflation to rise whenever output exceeds the level suggested by (31), $\alpha_{11} < 0, \alpha_{21} > 0$, supporting the interpretation of this as an IS relation.

The monetary-policy rule,

$$I_{s,t} \underset{[-2756.31]}{-0.32}\,(y-p)_t - \underset{[N.A.]}{1.79}\Delta p_t \underset{[-9.29]}{-0.00}\,t_{90:4} + \underset{[37.07]}{0.00}\,t - \underset{[N.A.]}{0.01}Ds_{90:4} + ... \sim I(0), \quad (32)$$

sees the short rate being positively related to real output and inflation, and the magnitude of the coefficients are close to those suggested by Taylor (1993). The short rate reacts to positive deviations from (32) by decelerating, $\alpha_{32} < 0$, consistent with our interpretation of this as a policy-reaction function. Inflation has a tendency to rise immediately after a rate hike though, $\alpha_{22} > 0$, an instance of the 'price puzzle'. Based on the conditions (14) and (15), the structure in (6) exhibits error correction in the first differences to the levels and in the second differences to the first differences alike.

Table 7 shows the composition and the loadings of the I(2) trends with homogeneity restrictions imposed. The first I(2) trend consists mainly of cumulated shocks to the short rate suggesting that this trend originates from monetary policy. Output appears to be positively affected by this trend; prices are influenced as well, albeit less so as judged from the relative magnitude of the $\tilde{\beta}'_{\perp 21}$ coefficients. The second I(2) trend primarily stems from cumulated shocks to the long rate but there are some contributions from output shocks (negative) and price shocks (positive) as well. This stochastic trend causes movements of opposite signs in output (negative) and prices (positive).

$\widetilde{\boldsymbol{\beta}}'$	y	p	I_s	I_l	m	$t_{90:4}$	t
$\widetilde{\beta}'_1$	1.00 [NA]	−1.58 [−7.82]	−14.40 [−8.49]	14.40 [8.49]	0.00 [NA]	−0.01 [−4.17]	0.00 [0.96]
$\widetilde{\beta}'_2$	−0.31 [−1020.10]	0.06 [3.11]	1.00 [NA]	0.00 [NA]	0.00 [NA]	−0.00 [−17.33]	0.00 [25.09]
$\widetilde{\beta}'_3$	−0.67 [−8.07]	0.00 [NA]	−21.07 [−7.32]	21.07 [7.32]	1.00 [NA]	−0.01 [−19.90]	0.00 [NA]

Table 8: Model II: CI(2,1) relations (homogeneity not imposed).

6.2 Model II: Adding money

Adding money to the baseline model (30), the set of variables becomes,

$$\mathbf{x}_t = (y, p, I_s, I_l, m)'_t \tag{33}$$

For this model we previously found ($r = 3, s_2 = 2$), see Table 2. In addition to the two relations found in Model I, we now look for a money-demand relation. Table 8 shows the results of imposing the theoretical restrictions apart from homogeneity; these are rejected at the five- but not the one-per cent level ($\chi^2(1) = 4.70, p = 0.03$). The coefficient of p_t in the IS curve now exceeds one in absolute value and homogeneity seems far from satisfied in the policy rule. The new money-demand relation also seems a long way from a one-for-one relationship between money and output.

Table 9 reports the results subject to the homogeneity restrictions which are, not surprisingly, rejected ($\chi^2(4) = 34.71, p = 0.00$). Again graphs of the relations look stationary however, see Figure A.2. Compared with Model I, the IS relation is broadly unchanged. The coefficient of real output in the policy rule drops somewhat and the short rate now appears to be reacting to a combination of growth in prices, output and money.

The money-demand relation takes the form,

$$(m-y)_t + \underset{[5.96]}{9.43}(I_l - I_s)_t + \underset{[N.A.]}{14.92}\Delta m_t \underset{[-32.28]}{-0.01}\, t_{90:4} + \underset{[N.A.]}{0.22} Ds_{90:4} + \ldots \sim I(0), \tag{34}$$

where inverse velocity is negatively related to the spread and nominal growth, here measured by the first difference of money. Exclusion of the (regular) trend suggests that, although the individual series might be trending up to the early 1990s, velocity is not. The significance of the broken trend nevertheless shows that m_t and y_t do not co-break in 1990:4. The $\boldsymbol{\alpha}$-matrix reveals that growth in nominal money declines when money supply exceeds the level suggested by (34), $\alpha_{53} < 0$. Also, the short rate has a tendency to fall when money supply increases relative to demand, $\alpha_{33} < 0$. Finally, it appears that excess

$\widetilde{\boldsymbol{\beta}}'$	y	p	I_s	I_l	m	$t_{90:4}$	t
$\widetilde{\beta}'_1$	1.00 [NA]	−1.00 [NA]	−5.53 [−10.40]	5.53 [10.40]	0.00 [NA]	0.00 [9.19]	−0.01 [−47.96]
$\widetilde{\beta}'_2$	−0.17 [−2285.86]	0.17 [2285.86]	1.00 [NA]	0.00 [NA]	0.00 [NA]	−0.00 [−8.36]	0.00 [39.84]
$\widetilde{\beta}'_3$	−1.00 [NA]	0.00 [NA]	−9.43 [−5.96]	9.43 [5.96]	1.00 [NA]	−0.01 [−32.28]	0.00 [NA]

$\widetilde{\boldsymbol{\delta}}'$	Δy	Δp	ΔI_s	ΔI_l	Δm	$Ds_{90:4}$	1
$\widetilde{\delta}'_1$	0.71	0.75	0.04	0.01	1.02	0.01	−30.28
$\widetilde{\delta}'_2$	−0.77	−1.02	0.04	−0.00	−0.39	−0.01	5.23
$\widetilde{\delta}'_3$	2.12	−5.39	1.30	−0.06	14.92	0.22	−0.04

$\boldsymbol{\alpha}$	α_1	α_2	α_3
$\Delta^2 y$	−0.11 [−2.37]	0.39 [1.87]	0.01 [0.94]
$\Delta^2 p$	0.06 [2.77]	0.49 [5.25]	0.02 [3.44]
$\Delta^2 I_s$	0.01 [0.61]	−0.10 [−1.92]	−0.01 [−2.36]
$\Delta^2 I_l$	−0.03 [−2.26]	−0.12 [−2.13]	−0.00 [−1.47]
$\Delta^2 m$	0.07 [1.28]	0.18 [0.70]	−0.08 [−5.46]

Table 9: Model II: Multi-cointegrating relations and adjustment structure (homogeneity imposed).

$\widetilde{\boldsymbol{\beta}}'_{\perp 2}$	y	p	I_s	I_l	m
$\widetilde{\beta}'_{\perp 21}$	0.52	−0.11	0.11	−0.00	1.59
$\widetilde{\beta}'_{\perp 22}$	0.22	0.79	−0.10	0.00	−0.75

$\boldsymbol{\alpha}'_{\perp 2}$	$\Sigma\Sigma\varepsilon_y$	$\Sigma\Sigma\varepsilon_p$	$\Sigma\Sigma\varepsilon_{I_s}$	$\Sigma\Sigma\varepsilon_{I_l}$	$\Sigma\Sigma\varepsilon_m$
$\alpha'_{\perp 21}$	0.10 [1.42]	0.13 [0.94]	1.00 [NA]	0.00 [NA]	−0.04 [−1.27]
$\alpha'_{\perp 22}$	−0.09 [−1.20]	0.33 [2.18]	0.00 [NA]	1.00 [NA]	0.00 [0.03]
$\widehat{\sigma}_\varepsilon$	0.0027	0.0012	0.0007	0.0008	0.0032

Table 10: Model II: Composition and loadings of the I(2) trends (homogeneity imposed).

$\widetilde{\boldsymbol{\beta}}'$	y	p	I_s	I_l	m	h	s	$t_{90:4}$	t
$\widetilde{\beta}_1$	1.00 [NA]	−0.71 [−14.17]	−3.69 [−14.63]	3.69 [14.63]	0.00 [NA]	0.00 [NA]	0.00 [NA]	0.00 [12.69]	−0.01 [−21.14]
$\widetilde{\beta}_2$	−0.61 [−1046.77]	−0.31 [−8.43]	1.00 [NA]	0.00 [NA]	0.00 [NA]	0.00 [NA]	0.00 [NA]	−0.01 [−28.99]	0.01 [37.01]
$\widetilde{\beta}_3$	−0.99 [−22.57]	0.00 [NA]	−6.89 [−8.52]	6.89 [8.52]	1.00 [NA]	0.00 [NA]	0.00 [NA]	−0.00 [−29.24]	0.00 [NA]
$\widetilde{\beta}_4$	0.00 [NA]	0.00 [NA]	0.00 [NA]	0.00 [NA]	−1.09 [−147.82]	1.00 [NA]	0.00 [NA]	0.00 [NA]	0.01 [17.89]
$\widetilde{\beta}_5$	0.00 [NA]	0.00 [NA]	−73.29 [−15.67]	73.29 [15.67]	0.00 [NA]	0.00 [NA]	1.00 [NA]	0.00 [NA]	−0.01 [−14.19]

Table 11: Model III: Multi-cointegrating relations (homogeneity not imposed).

money leads goods prices to rise within this information set, $\alpha_{23} > 0$, but, as we shall see below, this effect disappears when asset prices are introduced. In combination with $\boldsymbol{\beta}$ and $\boldsymbol{\alpha}$, the $\boldsymbol{\delta}$-matrix shows that error correction with respect to both second and first differences occur in all relations.

Table 10 shows that the composition of the two I(2) trends is broadly similar to that of Model I with one trend originating from cumulated shocks to the short rate and one from shocks to the long rate, here with some contributions from shocks to goods prices only in the latter. Shocks to the short rate mainly drive money and output (both positively) whereas shocks to the long rate drive prices (positively) and money (negatively).

6.3 Model III: Adding asset prices

Adding house and share prices to the model (33), the extended set of variables becomes,

$$\mathbf{x}_t = (y, p, I_s, I_l, m, h, s)'_t \tag{35}$$

For this, we concluded on $(r = 5, s_2 = 2)$ above, see Table 3. Since the number of both I(1) and I(2) common stochastic trends is unchanged when adding s_t and h_t, the asset-price variables must be driven by the same stochastic trends as the baseline variables. The graphical analysis in Section 3 found share-prices to be I(1) and as the number of I(1) trends is unchanged here share prices must be driven by the same type of residuals which make up the two I(2) trends, albeit only cumulated once. In addition to the three relations found in Model II, we now look for a demand-for-housing and a demand-for-shares relation as suggested by (29).

Table 11 shows the results of imposing theoretical restrictions on $\boldsymbol{\beta}$. Identification is achieved by setting $\beta_{41} = \beta_{42} = 0$, such that both interest rates are excluded in the housing relation, and $\beta_{51} = 0, \beta_{53} = -\beta_{52}$, such that share prices move with the spread but not with output. The restrictions are not rejected at the ten-per cent level ($\chi^2(5) = 9.28, p = 0.10$). The broken linear trend can be left out in both asset-demand relations and thus money and house prices appear to co-break in 1990:4. The house-price relation may alternatively be specified as an inverse relation between real house prices and the interest-rate spread but this results in a considerably lower p-value ($\chi^2(4) = 9.10, p = 0.06$). When money supply is used as denominator the significance of the spread disappears. Excess money turns out insignificant when included in the share-price relation. The adjustment structure and the common trends are largely invariant to these alternative specifications of the asset-price relations.

Table 12 shows the estimation results after homogeneity has been imposed in all relations; Table A.3 reports the $\boldsymbol{\zeta}$-, $\boldsymbol{\Psi}$- and $\boldsymbol{\phi}$-matrices. As expected, the restrictions are rejected ($\chi^2(9) = 39.28, p = 0.00$) but graphs indicate stationarity, see Figure A.3. Imposing homogeneity in all relations except the policy rule does not lead to rejection of the restrictions ($\chi^2(8) = 9.50, p = 0.30$). The first three relations are similar to those of Model II. Exclusion of the key fundamental driver of asset prices, output, in the long-run relations for both housing and shares could be taken as another signal that bubbles have occurred in this period. Inclusion of money supply in the housing relation and of the interest-rate spread in the share-price relation shows that 'liquidity conditions' have instead played a vital role. In terms of (5), we have $\kappa_1 = 0$ and slightly different specifications for the liquidity-bubble components. On the conditions (14) and (15), error correction in second and first differences only takes place in the house-price relation; in the remaining relations, the variable on which we have normalised does not error-correct.

The house-price relation,

$$(h - m)_t + \underset{[25.10]}{0.00}\, t + ... \sim I(0), \tag{36}$$

consists of a homogeneous relationship between house prices and money, but the growth rates of the I(2) variables all enter with large coefficients in δ_4. Adding the long-short spread to (36) results in a small and insignificant coefficient. Thus, in the long run, house prices seem to move closely with the quantity of money rather than with the interest rates. The $\boldsymbol{\alpha}$-matrix reveals that house prices exhibit only borderline significant adjustment behaviour, $\alpha_{64} < 0$, a hint that persistent deviations from equilibrium in the housing market have occurred. There is a tendency for goods-price inflation to rise and for output growth to decrease in response to a rise in house prices above their steady-state

$\widetilde{\boldsymbol{\beta}}'$	y	p	I_s	I_l	m	h	s	$t_{90:4}$	t
$\widetilde{\beta}_1$	1.00 [NA]	−1.00 [NA]	−4.46 [−14.58]	4.46 [14.58]	0.00 [NA]	0.00 [NA]	0.00 [NA]	0.00 [30.52]	−0.01 [−101.13]
$\widetilde{\beta}_2$	−0.29 [−5984.27]	0.29 [5984.27]	1.00 [NA]	0.00 [NA]	0.00 [NA]	0.00 [NA]	0.00 [NA]	−0.00 [−39.15]	0.00 [110.44]
$\widetilde{\beta}_3$	−1.00 [NA]	0.00 [NA]	−5.80 [−6.77]	5.80 [6.77]	1.00 [NA]	0.00 [NA]	0.00 [NA]	−0.00 [−57.34]	0.00 [NA]
$\widetilde{\beta}_4$	0.00 [NA]	0.00 [NA]	0.00 [NA]	0.00 [NA]	−1.00 [NA]	1.00 [NA]	0.00 [NA]	0.00 [NA]	0.00 [25.10]
$\widetilde{\beta}_5$	0.00 [NA]	0.00 [NA]	−76.78 [−18.38]	76.78 [18.38]	0.00 [NA]	0.00 [NA]	1.00 [NA]	0.00 [NA]	−0.01 [−18.69]

$\widetilde{\boldsymbol{\delta}}'$	Δy	Δp	ΔI_s	ΔI_l	Δm	Δh	Δs	$Ds_{90:4}$	1
$\widetilde{\delta}_1'$	−1.14	−0.90	0.01	0.01	−1.11	−1.00	0.08	−0.03	−30.15
$\widetilde{\delta}_2'$	0.07	−0.01	−0.00	−0.00	0.07	0.04	0.08	0.01	8.84
$\widetilde{\delta}_3'$	−1.20	−1.28	0.02	0.01	−1.10	−1.11	1.28	0.07	0.30
$\widetilde{\delta}_4'$	3.82	3.60	0.00	−0.00	3.84	3.75	0.41	0.18	24.58
$\widetilde{\delta}_5'$	12.11	12.44	−0.09	−0.02	11.73	11.74	−5.11	−0.29	−3.89

$\boldsymbol{\alpha}$	α_1	α_2	α_3	α_4	α_5
$\Delta^2 y$	−0.03 [−0.45]	0.87 [2.46]	0.03 [1.61]	−0.07 [−3.38]	0.00 [0.48]
$\Delta^2 p$	0.14 [5.02]	0.09 [0.60]	0.01 [1.07]	0.03 [3.23]	−0.01 [−4.65]
$\Delta^2 I_s$	−0.00 [−0.20]	−0.08 [−0.99]	−0.00 [−0.78]	0.00 [0.98]	0.00 [1.56]
$\Delta^2 I_l$	−0.04 [−2.31]	−0.20 [−2.17]	−0.01 [−2.02]	0.00 [0.86]	−0.00 [−1.50]
$\Delta^2 m$	0.02 [0.27]	0.54 [1.42]	−0.13 [−7.00]	−0.07 [−3.38]	0.00 [0.40]
$\Delta^2 h$	−0.07 [−1.16]	−0.77 [−2.75]	0.04 [3.10]	−0.03 [−1.72]	0.01 [1.78]
$\Delta^2 s$	−1.76 [−1.81]	−13.70 [−2.84]	−0.04 [−0.16]	0.08 [0.30]	−0.29 [−4.41]

Table 12: Model III: Multi-cointegrating relations and adjustment structure (homogeneity imposed).

level, $\alpha_{24} > 0, \alpha_{14} < 0$. Money growth has a tendency to contract whenever house prices rise more than warranted by the long-run relation, $\alpha_{54} < 0$. This suggests that credit conditions have been tightened to some degree when a housing bubble has materialised, but only when prices have risen above the level suggested by the supply of money, which is itself not necessarily reflecting sustainable fundamentals.

The share-price relation,

$$s_t + \underset{[18.38]}{76.78}(I_l - I_s)_t \underset{[-18.69]}{-0.01}\, t + ... \sim I(0), \tag{37}$$

sees share prices being negatively related to the long-short spread. It is possible to include also real output here but the coefficient is only borderline significant but negative as expected. Share prices adjust significantly to deviations from (37), $\alpha_{75} < 0$. In contrast with house prices, share prices in excess of its steady-state level lead goods-price inflation to decline. Moreover, while share prices did not react to housing disequilibria, there is some evidence of positive spill-overs to house prices when share prices rise above their equilibrium level.

The results on determination of asset prices can be summarised as follows. The acceleration rate of house prices is determined by, among other factors, the following disequilibria,

$$\begin{aligned} \widehat{\Delta^2 h_t} \;\; = \;\; & \underset{[-2.75]}{-0.77}[\underbrace{\hat{\beta}_2^{\prime}\mathbf{x} + \hat{\delta}_2\Delta\mathbf{x}}_{\text{excess rate hikes}}]_{t-1} + \underset{[3.10]}{0.04}[\underbrace{\hat{\beta}_3^{\prime}\mathbf{x} + \hat{\delta}_3\Delta\mathbf{x}}_{\text{excess money supply}}]_{t-1} \\ & - \underset{[-1.72]}{0.03}[\underbrace{\hat{\beta}_4^{\prime}\mathbf{x} + \hat{\delta}_4\Delta\mathbf{x}}_{\text{excess house prices}}]_{t-1} + \underset{[1.78]}{0.01}[\underbrace{\hat{\beta}_5^{\prime}\mathbf{x} + \hat{\delta}_5\Delta\mathbf{x}}_{\text{excess share prices}}]_{t-1} + ... \end{aligned} \tag{38}$$

where all components are I(0). This illustrates that conditional on the effect of policy-rate changes, the share-price relation and adjustment to the house-price relation itself, there is an additional effect on house prices from excess money supply. This suggests that, in addition to the role played by low interest rates (price of credit), the availability of loans (quantity of credit) has played a separate role in driving house prices in this period.

The acceleration rate of share prices is determined by the following disequilibria,

$$\widehat{\Delta^2 s_t} = \underset{[-1.81]}{-1.76}[\underbrace{\hat{\beta}_1^{\prime}\mathbf{x} + \hat{\delta}_1\Delta\mathbf{x}}_{\text{excess output}}]_{t-1} - \underset{[2.84]}{13.70}[\underbrace{\hat{\beta}_2^{\prime}\mathbf{x} + \hat{\delta}_2\Delta\mathbf{x}}_{\text{excess rate hikes}}]_{t-1} - \underset{[-4.41]}{0.29}[\underbrace{\hat{\beta}_5^{\prime}\mathbf{x} + \hat{\delta}_5\Delta\mathbf{x}}_{\text{excess share prices}}]_{t-1} + ... \tag{39}$$

where all components are again I(0). This shows that share prices equilibrium-correct significantly towards the share-price relation but there is also a significant positive effect of

$\tilde{\boldsymbol{\beta}}'_{\perp 2}$	y	p	I_s	I_l	m	h	s
$\tilde{\beta}'_{\perp 21}$	0.05	−0.85	0.26	0.06	1.22	1.22	15.50
$\tilde{\beta}'_{\perp 22}$	0.44	0.81	−0.11	−0.03	−0.04	−0.04	−6.40

$\boldsymbol{\alpha}'_{\perp 2}$	$\Sigma\Sigma\varepsilon_y$	$\Sigma\Sigma\varepsilon_p$	$\Sigma\Sigma\varepsilon_{I_s}$	$\Sigma\Sigma\varepsilon_{I_l}$	$\Sigma\Sigma\varepsilon_m$	$\Sigma\Sigma\varepsilon_h$	$\Sigma\Sigma\varepsilon_s$
$\alpha'_{\perp 21}$	0.12 [1.35]	0.08 [0.54]	1.00 [NA]	0.00 [NA]	−0.00 [−0.12]	−0.03 [−0.51]	0.00 [0.64]
$\alpha'_{\perp 22}$	0.11 [1.49]	0.23 [1.80]	0.00 [NA]	1.00 [NA]	−0.00 [−0.10]	0.08 [1.62]	−0.01 [−2.22]
$\widehat{\sigma}_\varepsilon$	0.0026	0.0010	0.0006	0.0007	0.0028	0.0021	0.0356

Table 13: Model III: Composition and loadings of the I(2) trends (homogeneity imposed).

a loosening in monetary policy. Somewhat surprisingly, output above the level suggested by the IS relation has had a tendency to depress share prices.

With homogeneity rejected, yet imposed, we could expect to see significant deviations from the multi-cointegrating relations depicted in Figure A.3. From the year 2000 and onwards, deviations from the Taylor rule are predominantly negative, an indication that policy rates were kept low relative to the policy-rule level. In contrast with the results from an I(1) CVAR in Giese and Tuxen (2007), there are no clear signs here of money supply growing significantly faster than demand in the new millennium. House prices appear to have moved out of sync with money between 2001 and 2005, but towards the end of the sample the supply of money increased rapidly and thus made up for this development. In comparison, share prices did not witness a similar movement away from fundamentals.

Regarding the driving forces of the system, the short rate no longer reacts significantly to deviations from the multi-cointegrating relations as was the case for Model I and II, i.e. the $I_{s,t}$ row in $\boldsymbol{\alpha}$ is insignificant, and it is therefore a candidate for a weakly exogenous variable as defined by (13). Similarly, the long rate sees only minor reactions to the long-run relations. Adjustment of both interest rates to the medium-run relations, see $\boldsymbol{\zeta}$ in Table A.3, is however highly significant, even when tested against one (rather than zero) which is a more appropriate null due to over-differencing of the I(1) variables (Juselius 2006). It therefore does not seem reasonable to categorise any of the variables as weakly exogenous within this information set.

Table 13 reports the composition and loadings of the I(2) trends. The trends are similar to those found in Model I and II and are thus again made up of cumulated shocks to each of the two interest rates, respectively, but there are some (negative) contributions from shocks to share prices in the second trend.[17] The composition of the I(2) trends is very similar to the structure found by Tuxen (2009a). As a result of the restrictions imposed on $\boldsymbol{\beta}$, money and house prices have identical loadings (positive) to the common trend arising from shock to the short rate. Physical-goods prices are also affected by the first trend but with the opposite sign (negative). In light of this, it is not surprising that LPH was rejected: not only are the loadings of all I(2) variables not close in magnitude they are actually of opposite signs for some pairs. Both output and goods prices appear to be driven by the common trend induced by shocks to the long rate (both positively). This suggests that the alternative scenario (23) is not 'too far' from describing the dynamics of the data, and that the behaviour of goods prices is the main reason that it is nevertheless formally rejected. This seems to be a reflection of the fact that output has risen much faster than prices in this period, see Figure 2, likely a result of globalisation which has arguably led world output to rise while prices have been kept down by increased competition, see also Tuxen (2009b).

7 Conclusion

We have used the I(2) cointegrated VAR (CVAR) model to study the relationship between asset prices and liquidity in recent decades on a global scale. Starting from a small New-Keynesian model we proposed a set of long-run relations which allowed both the price of liquidity (interest rates) and the quantity (money supply) to potentially affect house and share prices. We found strong evidence of two I(2) trends, which seem to arise from twice cumulated shocks to the short and the long rate, respectively. These common trends drive nominal output, goods prices, money and house prices alike but do not load identically into the variables. We argued that the finding of more than one nominal trend, and thus the rejection of LPH, could be a first sign that asset-price bubbles have been present.

A gradual expansion of the information set facilitated identification of the polynomially cointegrating relations in an I(2) CVAR model. In the long run, we found that asset prices co-moved closely with liquidity conditions, i.e. interest rates for shares and money supply for housing, rather than with the theoretical fundamentals proxied by GDP. This

[17] Although share prices were found to be I(1), the residuals of the s_t equation have a relatively large variance, which is the likely reason this I(1) series appears to drive and be driven by the I(2) trends as suggested by its significant coefficient in both $\boldsymbol{\alpha}_{\perp 22}$ and large coefficients in $\tilde{\boldsymbol{\beta}}_2'$.

could be a second sign of price bubbles. By imposing price homogeneity on the long-run relations, despite its rejection, we were able to study the effects of temporary, yet persistent, disequilibria and thereby the dynamics of asset-price bubbles. In the short run, we found that deviations from a Taylor rule did not drive the prices of physical goods but fuelled both house and share prices. In contrast, deviations from a money-demand relation only spurred a rally in the housing market but not in stocks.

If bubbles arise should central banks attempt to pop these? Bernanke and Gertler (2001) propose that asset prices are only relevant for monetary policy makers to the extent that they signal inflationary or deflationary forces. Based on the results presented here, it is not clear however that an inflation-targeting central bank should react to such signals of excess liquidity and/or asset-price bubbles as neither appeared to drive goods-price inflation. Moreover, as central banks control only one of the two shocks driving the nominal variables, the ability of monetary policy to provide a nominal anchor of goods and asset prices is likely to be, at best, imperfect. Notwithstanding, the crisis has highlighted the potential usefulness of providing monetary-policy makers with a mandate and the tools to 'lean against the wind' in face of an asset-price bubble to reduce some of the adverse effects that the eventual burst could have on economic growth.

References

Ahrend, R. (2008). Monetary ease: A factor behind financial crises? some evidence from OECD countries. *Discussion Paper, Economics: The Open-Access, Open-Assessment E-Journal 44*.

Baks, K. and C. Kramer (1999). Global liquidity and asset prices: Measurement, implications, and spillovers. *IMF Working Paper*.

Bernanke, B. (1983). Non-monetary effects of the financial crisis in the propagation of the Great Depression. *American Economic Review 73*(3), 257–276.

Bernanke, B. and A. Blinder (1988). Credit, money and aggregate demand. *American Economic Review 78*(2), 435–439.

Bernanke, B. and M. Gertler (1990). Financial fragility and economic performance. *Quarterly Journal of Economics 105*(1), 87–114.

Bernanke, B. and M. Gertler (2001, May). Should central banks respond to movements in asset prices? *American Economic Review*, 253–257.

Beyer, A., J. A. Doornik, and D. F. Hendry (2001). Constructing historical euro-zone data. *Economic Journal 111*, 308–327.

Beyer, A. and K. Juselius (2007). Does it matter how to measure aggregates? The case of monetary mechanisms in the Euro area. Proceedings: Conference in honour of David F. Hendry, Oxford, August 2007.

Boswijk, H. P. (2000). Mixed normality and ancillarity in I(2) systems. *Econometric Theory 16*, 878–904.

Bracke, T. and M. Fidora (2008). Global liquidity glut or global savings glut? A structural VAR approach. *ECB Working Paper 911*.

Campbell, J. Y. and R. J. Shiller (1987). Cointegration and tests of present value models. *Journal of Political Economy 95*, 1062–1088.

Campbell, J. Y. and R. J. Shiller (1988). The dividend/price ratio and expectations of future dividends and discount factors. *Review of Financial Studies 1*, 195–228.

Christiano, L. J., R. Motto, and M. Rostagno (2007a). Financial factors in business cycles. Working paper, Northwestern University.

Christiano, L. J., R. Motto, and M. Rostagno (2007b). Two reasons why money and credit may be useful in monetary policy. *NBER Working Paper W13502.*

Clarida, R. H., J. Galí, and M. Gertler (1999). The science of monetary policy: A New-Keynesian perspective. *Journal of Economic Literature 37.*

Dees, S., S. Holly, M. H. Pesaran, and L. V. Smith (2007). Long run macroeconomic relations in the global economy. *Economics - The Open-Access, Open-Assessment E-Journal 2007-3.*

Dennis, J. G. (2006). *CATS in RATS - Cointegration Analysis of Time Series, Version 2*. Evanston, Illinois: Estima.

Dennis, J. G., E. Hansen, and A. Rahbek (2002). ARCH innovations and their impact on cointegration rank testing. *Working paper, Centre for Analytical Finance, University of Copenhagen 22.*

Diba, B. and H. Grossman (1988). Explosive rational bubbles in stock prices? *American Economic Review 78*, 520–530.

Doornik, J. A. (2007). An introduction to OxMetrics 5. Available from Timberlake Consultants Ltd.

Engsted, T. (2006). Explosive bubbles in the cointegrated VAR model. *Finance Research Letters 3*, 154162.

Fisher, I. (1933). The debt-deflation of great depressions. *Econometrica 1*(4), 337–357.

Franchi, M. and K. Juselius (2007). Taking a DSGE model to the data meaningfully.

Friedman, M. (1969). The optimum quantity of money. In *The Optimum Quantity of Money and Other Essays.* Aldine, Chicago.

Friedman, M. and A. Schwartz (1963). *A monetary history of the United States, 1867-1960.* Princeton University Press; Princeton, N.J.

Giese, J. V. (2008). Level, slope, curvature: Characterising the yield curve in a cointegrated VAR model. *Economics: The Open-Access, Open-Assessment E-Journal 2*(2008-28).

Giese, J. V. and C. K. Tuxen (2007). Global liquidity and asset prices in a cointegrated VAR. *Manuscript.* Conference in honour of David F. Hendry, Oxford, August 2007.

Giese, J. V. and C. K. Tuxen (2009). Asset Prices and Liquidity Spill-overs: A GVAR Perspective. *Manuscript.*

Gürkaynak, R. S. (2005). Econometric tests of asset price bubbles: Taking stock. *Finance and Economics Discussion Series, Federal Reserve Board, Washington, D.C. 2005-04*.

IMF (1999, October). Safeguarding macroeconomic stability at low inflation. IMF, Washington DC. World Economic Outlook.

IMF (2007, October). Global prospects and policies. IMF, Washington DC. World Economic Outlook.

Johansen, S. (1988). Statistical analysis of cointegration vectors. *Journal of Economic Dynamics and Control 12*, 231–254.

Johansen, S. (1991). Estimation and hypothesis testing of cointegration vectors in Gaussian vector autoregressive models. *Econometrica 59*, 1551–1580.

Johansen, S. (1992). A representation of vector autoregressive processes integrated of order 2. *Econometric Theory 8*.

Johansen, S. (1997). Likelihood analysis of the I(2) model. *Scandinavian Journal of Statistics 24*, 433–462.

Johansen, S. (2005). The interpretation of cointegrating coefficients in the cointegrated vector autoregressive model. *Oxford Bulletin of Economics and Statistics 67*, 93–104.

Johansen, S. (2006). *Post Walrasian Macroeconomics. Beyound the Dynamic Stochastic General Equilibrium Model*, Chapter : Confronting the Economic Model with the Data. Cambridge University Press, Cambridge.

Johansen, S., K. Juselius, R. Frydman, and M. Goldberg (2009). Testing hypotheses in an I(2) model with piecewise linear trends: An analysis of the persistent long swings in the Dmk/dollar rate. *Journal of Econometrics. (forthcoming).*

Juselius, K. (2006). *The Cointegrated VAR Model: Econometric Methodology and Macroeconomic Applications*. Oxford: Oxford University Press.

Juselius, K., R. Frydman, M. Goldberg, and S. Johansen (2009). Scenario analysis of economic models based on RE and IKE. Manuscript.

Kongsted, H. C. (2005). Testing the nominal-to-real transformation. *Journal of Econometrics 124*(2), 205–225.

Kurita, T., H. B. Nielsen, and A. Rahbek (2009). An I(2) cointegration model with piecewise linear trends: Likelihood analysis and application. *Working Paper, Department of Economics, University of Copenhagen 09-13*.

Kydland, F. and E. Prescott (1982). Time to build and aggregate fluctuations. *Econometrica 50*, 1350–1372.

Mankiw, G. (2006). *Macroeconomics* (6 ed.). W H Freeman and Co.

Modigliani, F. and M. Miller (1958). The cost of capital, corporation finance and the theory of investment. *American Economic Review 48*(3), 261–297.

Nielsen, B. (2005). Analysis of co-explosive processes. *Working Paper, Nuffield College, Oxford University W8*. (forthcoming in Econometric Theory).

Nielsen, B. and A. Rahbek (2000). Similarity issues in cointegration models. *Oxford Bulletin of Economics and Statistics 62*, 5–22.

Nielsen, H. B. and A. Rahbek (2007). The likelihood ratio test for cointegration ranks in the I(2) model. *Econometric Theory 23*, 615–637.

Paruolo, P. (1996). On the determination of integration indices in I(2) systems. *Journal of Econometrics 72*, 313–356.

Paurolo, P. and A. Rahbek (1999). Weak exogeneity in I(2) VAR systems. *Journal of Econometrics 93*, 281–308.

Rahbek, A., H. C. Kongsted, and C. Jørgensen (1999). Trend-stationarity in the I(2) cointegration model. *Journal of Econometrics 90*, 265–289.

Romer, D. (1986). A simple general equilibrium version of the Baumol-Tobin model. *Quarterly Journal of Economics 101*(4), 663–686.

Ruffer, R. and L. Stracca (2006). What is global excess liquidity, and does it matter? *ECB Working Paper 696*.

Sousa, J. and A. Zaghini (2004). Monetary policy shocks in the euro area and global liquidity spillovers. *ECB Working Paper 696*.

Taylor, J. B. (1993). Discretion versus policy rules in practice. *Carnegie-Rochester Conference Series on Public Policy 39*, 195–214.

Taylor, M. P. (1991). The hyperinflation model of money demand revisited. *Journal of Money, Credit and Banking 23*(3), 327–351.

Tuxen, C. K. (2009a). Interactions of monetary and fiscal policy: An I(2) cointegrated VAR study of deficit-debt dynamics in the euro area. *Manuscript.*

Tuxen, C. K. (2009b). A multi-sector model of policy transmission and inflation dynamics in the euro area. *Manuscript.*

Woodford, M. (2003). *Interest and Prices: Foundations of a Theory of Monetary Policy.* Princeton University Press.

Woodford, M. (2007). How important is money in the conduct of monetary policy? *NBER Working Paper 13325.*

Appendix

Variable	Description	Source
y	Nominal output (GDP)	OECD EOL
m	Broad money stock	National sources[18]
p	GDP deflator (implicit)	OECD EOL
I_s	Short-term interest rate (three-month deposits)	OECD EOL
I_l	Long-term interest rate (10-year government bonds)	OECD EOL
h	House-price index	BIS[19]
s	Share-price index (key industrial indices)	National sources[20]

Table A.1: Overview of variables and national data sources.

[17]For most countries, M3 is used as the broad money measure. For the UK and Japan M4 and M2 plus cash deposits is used, respectively. US M2 growth was used to extrapolate the US M3 series from 2006:1 and onwards when publication of M3 was discontinued.

[18]BIS calculation based on national sources. Series for the US and UK are quarterly throughout; for France, Italy and Japan semi-annual series were interpolated to create quarterly series, and for Germany annual series were interpolated.

[19]Indices used: France: Paris Stock Exchange SBF 250, Italy: ISE MIB Storico Generale, Japan: TSE Topix, UK: FTSE 100, US: NYSE Composite, Germany: CDAX.

	Model I	Model II	Model III
No autocorrelation:			
LM(1)	$\chi^2(16) = \underset{[p=0.10]}{23.67}$	$\chi^2(25) = \underset{[p=0.13]}{33.10}$	$\chi^2(49) = \underset{[p=0.42]}{50.38}$
LM(2)	$\chi^2(16) = \underset{[p=0.08]}{24.53}$	$\chi^2(25) = \underset{[p=0.00]}{47.80}$	$\chi^2(49) = \underset{[p=0.92]}{35.90}$
LM(3)	$\chi^2(16) = \underset{[p=0.20]}{20.50}$	$\chi^2(25) = \underset{[p=0.49]}{24.53}$	$\chi^2(49) = \underset{[p=0.92]}{36.03}$
LM(4)	$\chi^2(16) = \underset{[p=0.61]}{13.83}$	$\chi^2(25) = \underset{[p=0.05]}{37.67}$	$\chi^2(49) = \underset{[p=0.18]}{57.96}$
Normality	$\chi^2(8) = \underset{[p=0.67]}{5.81}$	$\chi^2(10) = \underset{[p=0.04]}{19.25}$	$\chi^2(14) = \underset{[p=0.03]}{25.48}$
No ARCH effects:			
LM(1)	$\chi^2(100) = \underset{[p=0.23]}{110.22}$	$\chi^2(225) = \underset{[p=0.23]}{240.21}$	$\chi^2(784) = \underset{[p=0.26]}{809.00}$
LM(2)	$\chi^2(200) = \underset{[p=0.08]}{228.59}$	$\chi^2(450) = \underset{[p=0.00]}{501.76}$	$\chi^2(1568) = \underset{[p=0.01]}{1964.95}$
LM(3)	$\chi^2(16) = \underset{[p=0.00]}{375.13}$	$\chi^2(675) = \underset{[p=0.00]}{809.49}$	$\chi^2(2352) = \underset{[p=0.05]}{2469.12}$
LM(4)	$\chi^2(16) = \underset{[p=0.02]}{463.72}$	$\chi^2(900) = \underset{[p=0.00]}{1014.80}$	$\chi^2(3136) = \underset{[p=1.00]}{2604.00}$

Table A.2: Model I, II and III: Misspecification tests.

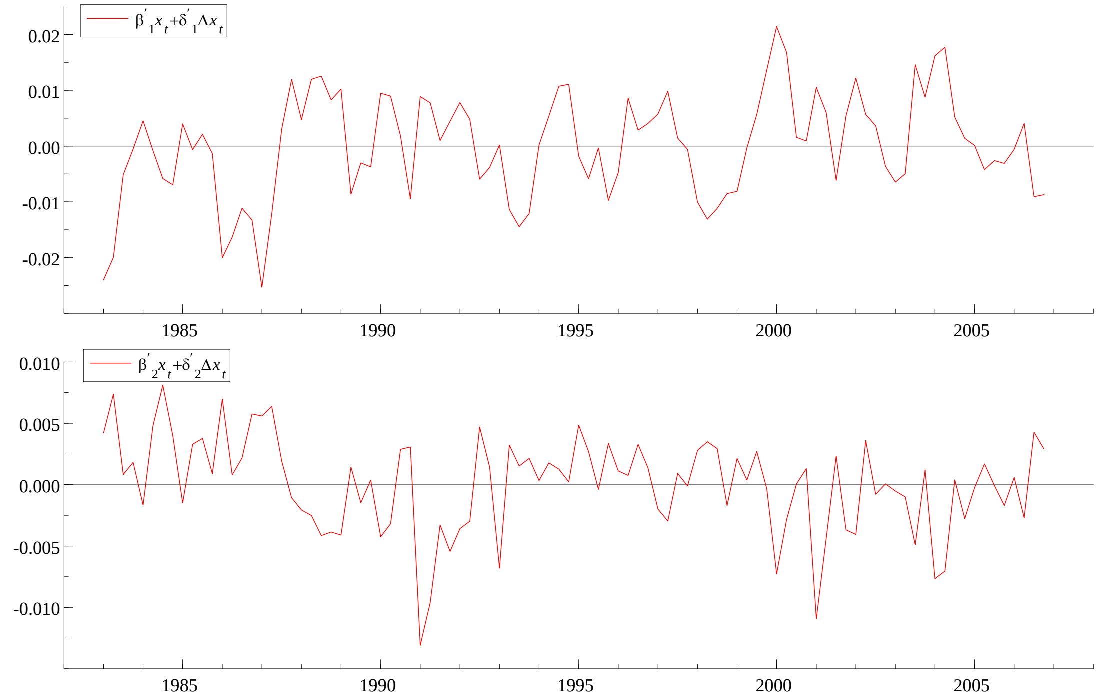

Figure A.1: Model I: Multi-cointegrating relations (means-corrected).

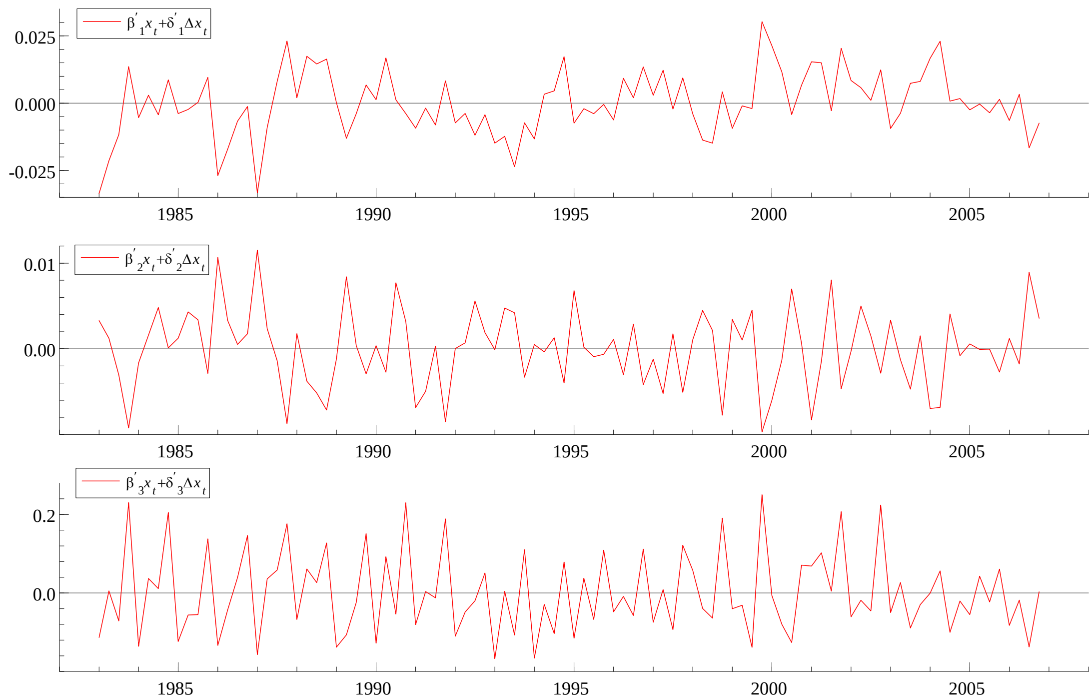

Figure A.2: Model II: Multi-cointegrating relations (means-corrected).

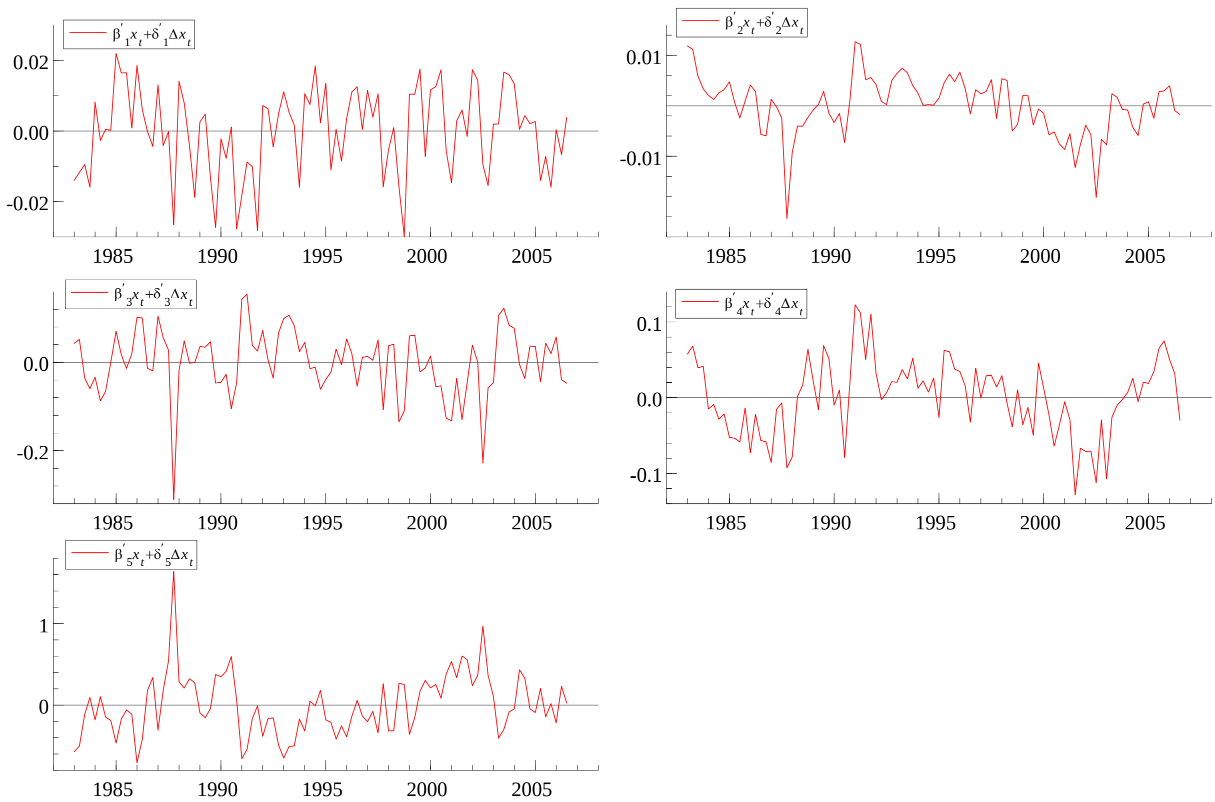

Figure A.3: Model III: Multi-cointegrating relations (means-corrected).

$\boldsymbol{\zeta}$	ζ_1	ζ_2	ζ_3	ζ_4	ζ_5
$\Delta^2 y$	0.46 [1.43]	1.71 [2.56]	0.30 [1.28]	0.22 [1.32]	−0.04 [−1.30]
$\Delta^2 p$	0.79 [6.15]	0.41 [1.53]	0.59 [6.27]	0.35 [5.23]	−0.08 [−6.27]
$\Delta^2 I_s$	−0.20 [−2.80]	−0.69 [−4.57]	−0.08 [−1.42]	−0.02 [−0.57]	0.02 [2.38]
$\Delta^2 I_l$	−0.24 [−2.86]	−0.75 [−4.35]	−0.05 [−0.80]	0.02 [0.55]	0.02 [1.94]
$\Delta^2 m$	−1.10 [−3.18]	−0.36 [−0.49]	−0.84 [−3.34]	0.48 [2.62]	0.13 [3.79]
$\Delta^2 h$	−0.69 [−2.72]	0.62 [1.18]	−0.75 [−4.04]	−0.83 [−6.24]	0.07 [2.85]
$\Delta^2 s$	10.42 [2.38]	22.58 [2.48]	2.30 [0.72]	−0.51 [−0.22]	−0.58 [−1.32]

$\boldsymbol{\Psi}$	$\Delta^2 y_{-1}$	$\Delta^2 p_{-1}$	$\Delta^2 I_{s,-1}$	$\Delta^2 I_{l,-1}$	$\Delta^2 m_{-1}$	$\Delta^2 h_{-1}$	$\Delta^2 s_{-1}$
$\Delta^2 y$	0.02 [0.22]	−0.06 [−0.26]	−1.15 [−2.27]	−0.08 [−0.17]	0.06 [0.82]	0.02 [0.24]	−0.00 [−0.23]
$\Delta^2 p$	0.17 [4.03]	−0.22 [−2.23]	−0.04 [−0.19]	−0.41 [−2.20]	−0.01 [−0.25]	−0.16 [−4.06]	0.00 [0.58]
$\Delta^2 I_s$	−0.04 [−1.60]	−0.03 [−0.50]	0.04 [0.35]	0.14 [1.37]	0.01 [0.49]	0.00 [0.11]	0.00 [2.59]
$\Delta^2 I_l$	0.04 [1.46]	−0.07 [−1.06]	0.29 [2.20]	−0.32 [−2.65]	0.04 [2.12]	−0.03 [−1.10]	0.01 [3.47]
$\Delta^2 m$	0.26 [2.26]	−0.33 [−1.21]	−0.35 [−0.64]	0.44 [0.88]	0.37 [4.39]	−0.26 [−2.46]	0.00 [0.30]
$\Delta^2 h$	−0.02 [−0.27]	−0.14 [−0.70]	−0.45 [−1.12]	0.47 [1.28]	0.04 [0.65]	0.15 [1.97]	−0.01 [−2.47]
$\Delta^2 s$	−0.82 [−0.57]	5.84 [1.72]	1.40 [0.20]	−11.60 [−1.83]	−0.23 [−0.22]	2.79 [2.08]	−0.05 [−0.53]

$\boldsymbol{\phi}$	$Dp_{90:4}$	$Dtr_{84:4}$	$Dtr_{03:4}$	$Dtr_{90:4}$	$Dtr_{87:4}$	$Dsea_1$	$Dsea_2$	$Dsea_3$
$\Delta^2 y$	−0.01 [−1.23]	−0.01 [−2.46]	−0.00 [−1.95]	−0.01 [−2.04]	0.00 [0.61]	−0.00 [−0.30]	0.00 [1.25]	0.00 [0.15]
$\Delta^2 p$	0.00 [0.74]	−0.00 [−2.56]	−0.00 [−2.69]	−0.00 [−1.98]	0.00 [1.17]	−0.00 [−1.49]	−0.00 [−1.34]	−0.00 [−1.06]
$\Delta^2 I_s$	0.00 [1.80]	−0.00 [−4.31]	−0.00 [−0.24]	−0.00 [−1.26]	0.00 [0.98]	0.00 [1.05]	0.00 [1.16]	0.00 [0.62]
$\Delta^2 I_l$	0.00 [1.30]	−0.00 [−2.57]	0.00 [0.11]	−0.00 [−0.72]	0.00 [0.96]	−0.00 [−1.96]	−0.00 [−2.54]	−0.00 [−1.74]
$\Delta^2 m$	−0.00 [−0.37]	0.00 [1.24]	−0.01 [−3.86]	−0.00 [−1.32]	0.00 [1.03]	−0.01 [−5.38]	0.00 [3.34]	−0.01 [−5.49]
$\Delta^2 h$	0.00 [0.25]	−0.00 [−0.08]	0.01 [3.63]	−0.01 [−3.25]	−0.00 [−0.63]	−0.00 [−0.00]	−0.00 [−2.35]	−0.00 [−1.35]
$\Delta^2 s$	0.08 [1.33]	−0.02 [−0.74]	0.01 [0.28]	−0.09 [−1.92]	−0.14 [−4.26]	−0.01 [−0.49]	−0.00 [−0.05]	0.02 [0.82]

Table A.3: Model III: Medium-run relations, short-run dynamics and dummy variables.

Chapter 4

Asset prices and liquidity spill-overs: A global VAR perspective

Asset prices and liquidity spill-overs: A global VAR perspective

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Abstract

This paper develops a global VAR (GVAR) model to study the transmission of shocks between the US, the UK, the euro area and Japan. We first estimate cointegrated VAR models for the different countries/regions including a set of rest-of-the world variables in each to take account of first-round effects of shocks. The set of foreign variables is chosen to reduce dimensionality, yet allowing for the existence of key international parity relations derived from the Dornbusch-Frankel model. We are able to replicate a set of economically meaningful relations but these I(1) country models all have large roots when the rank is set according to the economic prior. This points to the existence of temporary, yet persistent, disequilibria during the sample, likely a result of asset-price bubbles. Lowering the rank when linking the country models ensures that the combined model is stable. The GVAR allows us to assess the dynamic effects of liquidity and asset-price spill-overs between countries, taking second-round effects of shocks into account. Shocks analysis shows that stock markets have a tendency to move in sync across regions whereas this is not always the case for housing markets. For simulations of the credit crunch, we argue that the GVAR should be used with care however.

Keywords: Time-Series Models, Forecasting and Other Model Applications, Financial Markets and the Macroeconomy, International Financial Markets

JEL Classification: C32, C53, E44, G15

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1 Introduction

This paper develops a global VAR (GVAR) model to study the importance of cross-country linkages in the transmission of shocks between the US, the UK, the euro area and Japan. The current financial and economic crisis has highlighted the importance of financial linkages between countries as falling house prices in the US were transmitted rapidly into soaring interbank rates and a worldwide lending freeze. Spill-overs of financial-market developments between countries have thus become a topical issue.

It is by now widely recognised that an abundance of 'global liquidity' created by low interest rates and loose lending standards existed in the years preceding the credit crisis. This in turn appears to have fuelled a boom in stock, bond and housing markets, in particular in the US and the UK. In contrast, goods-price inflation remained subdued. Such anecdotal evidence was supported by the findings of Giese and Tuxen (2008) (henceforth GT). But what happens when liquidity dries up and/or asset prices start to fall? Prior to the crisis there was a general belief that a shock to the US economy would not necessarily have a large effect on the rest of the world. Due to the emergence of a range of new economies (mainly in Asia) and structural changes in the global economy associated with the 'Great Moderation', global decoupling was considered a reasonable scenario in face of a shock to the US (IMF 2007). Recent developments suggest that the credit shock has in fact been highly contagious, and thus the catchphrase "when the US sneezes the world catches a cold" still seems to hold.

In this paper, we use the GVAR framework proposed by *inter alia* Pesaran, Schuermann, and Weiner (2004) to study cross-country linkages in the transmission of liquidity and asset-price shocks. The GVAR has previously been employed to explore, among other things, the effects of various risk scenarios on a bank's loan portfolio (Pesaran, Schuermann, and Weiner 2004), the international linkages of the euro area (Dees, di Mauro, Pesaran, and Smith 2007), and the transmission of oil-price, policy and equity-price shocks (Dees, Holly, Pesaran, and Smith 2007). Notably Dees et al. 2007 (henceforth DHPS) find that financial markets tend to correct disequilibria faster than do goods markets. Here, we argue that impulse response analysis based on a GVAR provides a convenient set-up for assessing the long- and short-run dynamics of liquidity spill-overs and asset prices. A natural evaluation of the GVAR would be its ability to replicate broad developments during the current crisis.

The previous literature on 'global liquidity' spill-overs has focused on the existence of a push as opposed to a pull channel (Baks and Kramer 1999). If an increase in domestic money gives rise to capital-flows to ('push') foreign asset markets, we would expect asset prices to rise (downward pressure on yields) abroad, i.e. a positive correlation between domestic money growth and foreign asset prices. A positive correlation could however also be consistent with economic spill-overs, i.e. 'a rising tide that lifts all boats'. This case is denoted a 'push channel'. Similarly, a negative correlation between domestic money and foreign asset prices would imply a 'pull channel'. Baks and Kramer (1999) construct several G7 measures of monetary liquidity using narrow and broad money and different weighting schemes; from studying contemporaneous correlations, simple regression results and Granger-causality tests these authors find that stock returns increase with global money growth while real interest rates decline. Their evidence thus points to the existence of a push channel.

Another strand of literature uses structural VAR (SVAR) models to study the effects of global liquidity on consumer and asset prices. Sousa and Zaghini (2004) construct an SVAR for the euro area and study the effects of an expansion in global money. An unexpected increase in money abroad is found to lead to a permanent increase in euro-area money supply (push channel) which in turn causes upward pressure on goods prices. Ruffer and Stracca (2006) define a range of excess-liquidity measures based on ratios of broad and narrow money to output. Across a range of countries they find a common factor which they ascribe to global liquidity but they find only "scattered evidence" that this common factor Granger-causes domestic money supply and output in the individual countries. They set up a SVAR based on globally aggregated data and find that the global monetary stance, measured by money supply and the short-term interest rate, appears to be a useful indicator for inflationary pressure at the global level. Other papers on global liquidity taking a SVAR approach include Gouteron and Szpiro (2005), Adalid and Detken (2006) and Belke and Orth (2007).

The main advantage of a GVAR over a SVAR is that the former incorporates second-round effects of shocks and that it allows us to study how a shock in any one country affects the rest of the world. To construct the GVAR, we first estimate cointegrated VAR (CVAR) models for the US, the UK, the euro area (EA) and Japan (JP) including a limited set of country-specific foreign (rest-of-the world, ROW) variables in each; these are called CVARX* models. Cointegration between *domestic* variables are identified based on the economic framework in GT whereas cointegration between *domestic and* *foreign* variables are identified based on the predictions of the Dornbusch-Frankel model under imperfect knowledge expectations (IKE), see Johansen, Juselius, Frydman, and Goldberg (2008), rather than under rational expectations (RE). The country models are then linked up into a global model, the GVAR, in a way consistent with the scheme used for constructing foreign variables. However, when the ranks of the country models are set according to the economic prior, large unrestricted roots are present and the GVAR becomes unstable. This is likely a result of asset-price bubbles which implies that we have to set the rank artificially low to achieve convergent impulse response functions. We evaluate the GVAR by its predictions regarding the credit crunch. But, in light of the difficulties in reconciling economic and statistical identification that our analysis points to, combined with the possibility that the crisis might constitute a structural break, we propose using the GVAR with care in this context.

The paper is structured as follows. Section 2 discusses the Dornbusch model as a basis for identifying potential linkages across countries and derive some statistical implications of the RE hypothesis and contrast these with the predictions of the IKE hypothesis. Section 3 discusses how to incorporate foreign variables in a CVAR model and the data set is presented in Section 4, and cointegration results for each country/region are presented in Section 5. Section 6 derives a modified version of the GVAR and discusses stability conditions. Results from shocks analysis are discussed in Section 7, and Section 8 concludes.[1]

2 Economic framework: the Dornbusch model

The majority of GVAR applications do not impose identifying restrictions on the CVARX* models. Here we emphasise the importance of both economic and statistical identification of the country models in interpreting the dynamics of the data. We look to economic theory as guide for identification. DHPS list a number of relationships linking domestic and foreign variables, [2] but set the cointegration rank in the country models too low for all of these to be identified. GT propose a number of potential relations linking the domestic variables: an IS curve, a Taylor rule, a money-demand relation and a demand-for-housing

[1]All calculations were conducted using CATS 2.01 (Dennis 2006) in Rats 6.3, Ox/OxMetrics 5 (Doornik 2007) and Matlab R2008a.

[2]DHPS consider the following relations: a Balassa-Samuelson modified version of purchasing power parity (PPP), an output-convergence condition, the Fisher parity, a term premium, an equity-price relation and an uncovered interest-rate parity (UIP) condition, where the rate of change in the exchange rate is proposed stationary and thus excluded in the cointegrating relation.

and a demand-for-shares relation. Here we use the relations in GT as the starting point for identifying domestic linkages in each country model. Regarding international relationships, we use the Dornbusch (1976)-Frankel (1979) model of exchange-rate dynamics for deriving potential cross-country linkages.

In the following, we outline the basic structure of the Dornbusch-Frankel model and derive some cointegration implications of the model under RE and contrast these with the predictions of the alternative expectations formation scheme, IKE.

2.1 The sticky-price monetary model

The two-country sticky-price monetary model due to Dornbusch (1976) and Frankel (1979) was proposed as a tool for studying the effects of shocks to domestic money supply; the model does not take current-account and fiscal-policy issues into account. The Dornbusch-Frankel model is essentially a perfect-foresight extension of the Mundell-Fleming, i.e. an open-economy IS-LM model with goods prices assumed to be sticky. Short-run purchasing power parity (PPP) deviations therefore occur. PPP is satisfied in the long run however and asset markets always clear. The overshooting property implies that the model is able to account for part of the large volatility observed in the movements of exchange rates. Modifications of the basic model structure within the New Open-Economy Macro tradition have later focused on introducing explicit micro-foundations and allowing monetary policy to be endogenous. Here we focus on the predictions of the model regarding the co-movement of real interest-rate differentials and the real exchange rate. We present a standard version of the model below, assuming foreign variables to be exogenous.[3]

Monetary equilibrium is characterised by equality of supply and demand of money,

$$m_t - p_t = \phi_1 y_t^r - \phi_2 I_t \tag{1}$$

where m_t, p_t and y_t^r denote domestic (nominal) money supply, price level and real output, respectively; I_t is the domestic (nominal) interest-rate, assumed to measure the opportunity cost of holding money.

Free capital mobility implies UIP,

$$E_t \Delta e_{t+1} = I_t - I_t^* \tag{2}$$

[3]This section draws on Obstfeld and Rogoff (1996) and Johansen, Juselius, Frydman, and Goldberg (2008).

with I_t^* the interest-rate abroad and e_t is the spot exchange rate (domestic-currency price of foreign exchange); E_t denotes the expectation as of time t. Note that this assumes that domestic and foreign bonds are perfect substitutes (no risk premium).

Aggregate demand (domestic relative to foreign) is increasing in the real exchange rate and decreasing in the real interest-rate differential,

$$y_{r,t}^d - y_{r,t}^{d,*} = \overline{y_r} - \overline{y_r^*} + \phi_3(e_t - p_t + p_t^* - \overline{ppp}) - \phi_4\left[(I_t - \overline{\Delta p}) - (I_t^* - \overline{\Delta p}^*)\right] \tag{3}$$

where a bar indicates a steady state value. The real exchange rate is given by $ppp_t \equiv (e - p + p^*)_t$, $\overline{y_r}$ is the natural rate of output, and $\overline{\Delta p}$ and $\overline{\Delta p}^*$ denote the domestic and foreign steady-state rates of inflation, respectively; $\overline{ppp}$ is the PPP level of the real exchange rate. This assumes that domestic and foreign goods are imperfect substitutes; if perfect substitutability exists, then $\phi_3 \longrightarrow \infty$ as a real depreciation would induce an 'infinitely' large shift in demand from foreign towards domestic goods.

Aggregate supply is represented by an inflation-expectations augmented Phillips curve where prices are sticky but adjust proportionally to excess demand,

$$E_t\Delta p_{t+1} = \phi_5[y_r^d - \overline{y_r} - (y_r^{d,*} - \overline{y_r^*})]_t + E_t\Delta(e + p^*)_{t+1} \tag{4}$$

With fixed prices we have $\phi_5 = 0$ whereas $\phi_5 > 0$ implies that prices are flexible to some degree.

2.1.1 Partial equilibrium

Assuming prices are fixed, equilibrium in the goods market is found by setting aggregate demand equal to the natural rate in (3),

$$\begin{aligned} y_{r,t}^d - y_{r,t}^{d,*} &= y_r - y_r^* \Rightarrow \\ \phi_3(e_t + p_t^* - p_t - \overline{ppp}) &= \phi_4\left[(I_t - \overline{\Delta p}) - (I_t^* - \overline{\Delta p}^*)\right] \Rightarrow \\ (I_t - \overline{\Delta p}) - (I_t^* - \overline{\Delta p}^*) &= \varphi(e_t + p_t^* - p_t - \overline{ppp}) \end{aligned} \tag{5}$$

where $\varphi = \phi_3/\phi_4 > 0$ such that a real depreciation of the currency is associated with a higher real interest-rate differential. One possible causal interpretation of this positive correlation is that a real depreciation requires a rise in domestic real rates relative to foreign rates.

2.1.2 General equilibrium

Once we take price adjustment dynamics (aggregate supply) into account, the sign of the relation in (5) is not longer unambiguous. To see this, substitute (3) into (4),

$$E_t\Delta p_{t+1} = \phi_5 \left\{ \phi_3(e_t + p_t^* - p_t - \overline{ppp}) - \phi_4 \left[(I_t - \overline{\Delta p}) - (I_t^* - \overline{\Delta p}^*) \right] \right\} + E_t\Delta(e + p^*)_{t+1} \tag{6}$$

Imposing UIP, see (2), and setting expected inflation rates equal to their steady-state values, $E_t\Delta p_{t+1} = \overline{\Delta p}$ and $E_t\Delta p_{t+1}^* = \overline{\Delta p^*}$, we obtain

$$\overline{\Delta p} = \phi_5 \left\{ \phi_3(e_t + p_t^* - p_t - \overline{ppp}) - \phi_4 \left[(I_t - \overline{\Delta p}) - (I_t^* - \overline{\Delta p}^*) \right] \right\} + I_{l,t} - I_{l,t}^* + \overline{\Delta p^*} \Rightarrow$$

$$\begin{aligned} \phi_5\phi_3(e_t + p_t^* - p_t - \overline{ppp}) &= (\phi_5\phi_4 - 1) \left[(I_t - \overline{\Delta p}) - (I_t^* - \overline{\Delta p}^*) \right] \Rightarrow \\ (e_t + p_t^* - p_t - \overline{ppp}) &= \theta \left[(I_t - \overline{\Delta p}) - (I_t^* - \overline{\Delta p}^*) \right] \end{aligned} \tag{7}$$

where the sign of $\theta = (\phi_5\phi_4 - 1)/\phi_5\phi_3$ is ambiguous as it depends on the relative magnitude of the model parameters, i.e. if $\phi_5\phi_4 < 1$ the predicted correlation is negative, and *vice versa*. The more sticky prices are ($\phi_5 \to 0$) and/or the lower the interest-rate elasticity of demand ($\phi_4 \to 0$), the greater the probability that a rise in the real-interest rate differential is associated with a real appreciation in the long run. A causal interpretation of a negative correlation could be that a rise in the real interest-rate differential, e.g. induced by monetary-policy actions, leads to a stronger currency.

In sum, the dynamics of exchange rates and interest rates is the key transmission mechanism in the Dornbusch model. In relation to the discussion of 'global liquidity' spill-overs above, we note that money does not enter the key relation (7). Money does however have an effect impact via its effect on the price level.

2.2 Cointegration implications of RE vs. IKE

We now contrast the cointegration implications of the Dornbush model under RE with those of IKE. Allowing both a short and a long rate in each region, the data vector of interest for the Dornbush model becomes,

$$\mathbf{x}_t^{Dornbusch} = (ppp, \Delta p, \Delta p^*, I_l, I_l^*, I_s, I_s^*)_t' \tag{8}$$

where $I_{s,t}$ and $I_{l,t}$ denote the domestic short and long rate, respectively, and similarly for the foreign interest rates.

2.2.1 Rational expectations

RE equates individuals' forecasts of the aggregate variables with the model predictions on the aggregate level. As a result, the Dornbusch model implies PPP, UIP and international Fisher parity in steady state. Forecasters are assumed to update their point estimates of the nominal exchange rate such that it adjusts towards PPP at all times,

$$E_t e_{t+1}^{RE} = e_t + \xi(\overline{e} - e_t) \tag{9}$$

where ξ is one minus the stable root of the system (see Johansen et al. 2008). The expected value of the nominal exchange rate next period thus equals the value this period plus adjustment towards the steady-state value. The assumption of PPP adjustment implies that a similar relation holds for domestic goods prices and hence for the real exchange rate, assuming foreign goods prices are fixed.

Under the RE hypothesis, the Dornbusch model predicts that the variables in (8) cointegrate as follows,

$$ppp_t \sim I(0) \tag{10}$$

$$(I_j - \Delta p)_t - (I_j^* - \Delta p^*)_t \sim I(0) \tag{11}$$

where $j \in \{s, l\}$. Here, (10) and 11 imply PPP and international Fisher parity, respectively. The equilibrium condition in (7), which essentially combines these two parities, therefore does not constitute an irreducible cointegrating relation under RE as both the real exchange rate and the relative level of the real interest rates are I(0) by themselves.

2.2.2 Imperfect knowledge expectations

Real exchange rates are however often found to be non-stationary, even over long time spans, and thus (10) does not hold. Johansen et al. (2008) argue that the empirical failure of RE models to resolve the PPP puzzle (see Rogoff 1996), is due to fundamental flaws arising from the use of RE in modelling forecasting behaviour. Instead, the authors propose IKE (see Frydman and Goldberg 2007) as an alternative expectations formation scheme and show that when imperfect knowledge is recognised, the monetary model is able to resolve the PPP puzzle. This is achieved by allowing the forecasts of agents to play a separate role in driving the model, thereby introducing additional persistence in the exchange rate.

IKE postulates that agents produce forecasts according to diverse models of the type (Johansen et al. 2008),

$$E_t^{i,IKE} e_{t+1} = \widehat{\zeta}_t^i \mathbf{x}_t^i + \widehat{\rho}^i e_t \tag{12}$$

where $\mathbf{x}_t^i$ represent the variables that individual i uses in forming his forecasts (these may not coincide with those in $\mathbf{x}_t^{Dornbusch}$); $\widehat{\zeta}_t^i$ and $\widehat{\rho}^i$ are the parameters attached to these variables and to the current value of the nominal exchange rate, respectively. Forecasting models are thus assumed to differ between agents, leaving the aggregate of individual forecasts, $E_t^{IKE} e_{t+1}$, to differ from the model prediction of the aggregate, $E_t^{RE} e_{t+1}$. Moreover, IKE assumes that agents revise their forecasting strategies over time in a way which depends, among other things, on the size of the departure from PPP. Notably, IKE imposes only qualitative restrictions on forecasting behaviour but these are sufficient to generate persistent deviations from PPP.

When agents are assumed to have imperfect knowledge, $E_t^{IKE} e_{t+1}$ influences the steady-state values of the nominal exchange rate and the price level differently. As a result, the steady-state real exchange rate depends on $E_t^{IKE} e_{t+1}$, i.e. $\overline{ppp}_t^{IKE} = \overline{ppp} + \varsigma(E_t^{IKE} e_{t+1} - E_t^{RE} e_{t+1})$ where $\overline{ppp}$ is the RE PPP level and $\varsigma > 0$ depends on the model parameters. When $E_t^{IKE} e_{t+1} \neq E_t^{RE} e_{t+1}$ changes in the nominal exchange rate may imply movement either away from or towards PPP. This further implies that the international Fisher parity does not hold in steady state; a deviation from parity in one place is corresponded by another deviation elsewhere.

In contrast with RE, IKE is consistent with the existence of (7) as a genuine (irreducible) cointegrating relation as both ppp_t and $[(I_j - \Delta p) - (I_j^* - \Delta p^*)]_t$ are non-stationary and possibly near-I(2) under IKE, see Juselius, Frydman, Goldberg, and Johansen (2009). The Dornbusch model indeed pointed to co-movement of these variables, see (7), which suggests a cointegrating relation of the form,

$$ppp_t - \theta \left[(I_j - \Delta p) - \omega(I_j^* - \Delta p^*)\right]_t \sim I(0) \tag{13}$$

where we expect $\omega \simeq 1$ and $\theta \gtreqless 0$. Cointegrating relations of this form were identified by Juselius and MacDonald (2007) for Germany vs. the US, and by Tuxen (2009) for the euro area vs. the US.

In sum, (13) suggests two potential cointegration relations linking the domestic and foreign variables, including the short and long rates, respectively, in addition to the five relations involving only domestic variables proposed by GT. We use these seven relations to guide the identification of each CVARX* model below. For Japan, where the debt-deflation experience left monetary policy focused on quantitative easing for an extended period of time, a Taylor rule may not be expected among the cointegrating relations.

3 Statistical framework

We outline the CVAR methodology and discuss how to set up CVARX* models to account for existence of the cross-country links suggested by the economic framework.

3.1 The CVAR model

The CVAR model provides a convenient way of separating the pulling (cointegration) forces from the pushing (common trends) forces in non-stationary data. We review the basic structure of the I(1) CVAR below.[4]

We start from a p-dimensional VAR($k = 2$) in equilibrium correction model (ECM) form,

$$\Delta \mathbf{x}_t = \mathbf{\Pi x}_{t-1} + \mathbf{\Psi}_0 \Delta \mathbf{w}_t + \mathbf{\Psi}_1 \Delta \mathbf{w}_{t-1} + \mathbf{\Gamma}_1 \Delta \mathbf{x}_{t-1} + \boldsymbol{\phi} \mathbf{d}_t + \boldsymbol{\varepsilon}_t, \qquad t = 1, 2, ..., T \quad (14)$$

where $\mathbf{x}_t$ is a $p \times 1$ vector of endogenous variables, $\mathbf{w}_t$ a vector of variables assumed weakly exogenous *a priori*, $\mathbf{d}_t$ a vector of deterministic components (some of which may be restricted to the $\boldsymbol{\alpha}$-space) and $\boldsymbol{\varepsilon}_t$ a $p \times 1$ vector of errors for which we assume $\boldsymbol{\varepsilon}_t \sim iid$ $N_p(\mathbf{0},\mathbf{\Omega})$ with $\mathbf{\Omega} > 0$. In the statistical analysis, we condition on the initial values, $(\mathbf{x}_{-1}, \mathbf{x}_0)$, and hence these are treated as fixed.

The I(1) CVAR is defined by the reduced-rank restriction,

$$H(r) : \mathbf{\Pi} = \boldsymbol{\alpha\beta}' \quad (15)$$

where both $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are $p \times r$ matrices with $r < p$. Imposing the condition (15) on the model (14) we obtain,

$$\Delta \mathbf{x}_t = \boldsymbol{\alpha\beta}' \mathbf{x}_{t-1} + \mathbf{\Psi}_0 \Delta \mathbf{w}_t + \mathbf{\Psi}_1 \Delta \mathbf{w}_{t-1} + \mathbf{\Gamma}_1 \Delta \mathbf{x}_{t-1} + \boldsymbol{\phi} \mathbf{d}_t + \boldsymbol{\varepsilon}_t \quad (16)$$

Under the additional assumption that the characteristic polynomial has exactly $(p - r)$ unit roots and the remaining roots are outside the unit circle,[5] $\Delta \mathbf{x}_t$ and $\boldsymbol{\beta}' \mathbf{x}_t$ can be

[4]This subsection is partly based on Tuxen (2009).

[5]This assumption implies $|\boldsymbol{\alpha}'_\perp \mathbf{\Gamma} \boldsymbol{\beta}_\perp| \neq \mathbf{0}$ with $\mathbf{\Gamma} = \mathbf{I} - \mathbf{\Gamma}_1$ which requires the variables to be integrated of order at most one; we use $_\perp$ to denote the orthogonal complement.

made stationary. In this case, r is the number of cointegrating relations which can be determined by the LR test proposed by Johansen (1988). The cointegrating relations are given by $\boldsymbol{\beta}'\mathbf{x}_t$, and the $\boldsymbol{\alpha}$-matrix contains information on the short-run adjustment following disequilibria.

While the ECM parameterisation illustrates the equilibrium-correcting forces of the model, the moving average (MA) representation is the key to the common stochastic trends driving the system. This parameterisation is also used for deriving impulse response functions. Under the above assumptions, the level of the process, $\mathbf{x}_t$, has the following solution, (Engle and Granger 1987, Johansen 1996),

$$\mathbf{x}_t = \mathbf{C}\boldsymbol{\Sigma}_{i=1}^{t}(\boldsymbol{\varepsilon}_i + \boldsymbol{\phi}\mathbf{d}_i) + \text{stat. comp.}, \tag{17}$$

where the long-run impact matrix, $\mathbf{C} = \boldsymbol{\beta}_\perp(\boldsymbol{\alpha}'_\perp\boldsymbol{\Gamma}\boldsymbol{\beta}_\perp)^{-1}\boldsymbol{\alpha}'_\perp$, reveals how a shock to one variable lead the rest of the variables in the system to react. The common trends are defined as $\boldsymbol{\alpha}'_\perp\boldsymbol{\Sigma}_{i=0}^{t}\boldsymbol{\varepsilon}_i$ and $\widetilde{\boldsymbol{\beta}} = \boldsymbol{\beta}_\perp(\boldsymbol{\alpha}'_\perp\boldsymbol{\Gamma}\boldsymbol{\beta}_\perp)^{-1}$ provides the loadings to these.

3.2 The CVARX* model

For each of the regions included in the GVAR, a CVAR in the following set of domestic variables is our point of departure,

$$\mathbf{x}_t = (m_r, y_r, \Delta p, I_s, I_l, h_r, s_r)'_t \tag{18}$$

where all variables were introduced above except real house prices, h_r, and real share prices, s_r, and we distinguish between a short-term interest rate, $I_{s,t}$ and a long-term interest rate, $I_{l,t}$.

To capture cross-country links such as the relation (7), we add a set of foreign variables, $\mathbf{x}_t^*$, to (18) to form a CVARX* model The foreign variables are assumed to be weakly exogenous for the domestic variables, i.e. $\mathbf{w}_t = \mathbf{x}_t^*$. The CVARX* models of country i in ECM representation (14) thus takes the form,

$$\begin{aligned} \boldsymbol{\Delta}\mathbf{x}_{i,t} &= \boldsymbol{\mu}_i + \boldsymbol{\alpha}_i\widetilde{\boldsymbol{\beta}}_i'\widetilde{\mathbf{x}}_{i,t-1} + \boldsymbol{\Gamma}_{i1}\boldsymbol{\Delta}\mathbf{x}_{i,t-1} + \boldsymbol{\Upsilon}_{i0}\boldsymbol{\Delta}\mathbf{x}_{i,t}^* + \boldsymbol{\Upsilon}_{i1}\boldsymbol{\Delta}\mathbf{x}_{i,t-1}^* + \boldsymbol{\phi}_i\mathbf{d}_{i,t} + \mathbf{u}_{i,t}, \qquad (19) \\ i &= 0, 1, ..., N \text{ and } t = 1, 2, ..., T \end{aligned}$$

where $\mathbf{x}_{i,t}$ is the $k_i \times 1$ vector of domestic variables of country i, $\mathbf{x}_{i,t}^*$ the $k_i^* \times 1$ vector of country-specific foreign variables, $\widetilde{\mathbf{d}}_{i,t}$ a matrix of (unrestricted) dummy variables and $\boldsymbol{\mu}_i$ an (unrestricted) constant term; $\mathbf{u}_{i,t}$ is an error term assumed to be *i.i.d.* $N_{k_i}(\mathbf{0}, \boldsymbol{\Omega}_i)$.

Finally, $\widetilde{\mathbf{x}}_{i,t}= (\mathbf{x}'_{i,t}, \mathbf{x}^{*\prime}_{i,t}, t)'$ which is $(k_i + k_i^* + 1) \times 1$ and t is a linear deterministic time trend which is restricted to the cointegration space. We again assume fixed values of $(\mathbf{x}_{i,-1}, \mathbf{x}_{i,0})$ and $(\mathbf{x}^*_{i,-2}, \mathbf{x}^*_{i,-1}, \mathbf{x}^*_{i,0})$.

The Dornbusch set-up suggests adding the real exchange rate to the set of endogenous variables in (18). To allow for the key relationship (7) and thereby first-round effects of foreign shocks on the domestic economies, we include the foreign inflation rate, short-term and long-term interest rate,

$$\mathbf{x}^*_{i,t} = (\Delta p^*, I^*_s, I^*_l)'_{i,t} \tag{20}$$

as weakly exogenous. This results in the data vector,

$$\widetilde{\mathbf{x}}_{i,t} = (\mathbf{x}', ppp, \mathbf{x}^{*\prime}, t)'_{i,t} \tag{21}$$

such that we have $p_1 = 8$ endogenous variables and $p_2 = 3$ weakly exogenous variables in each of the CVARX* country models.

4 Data

We use quarterly time series from 1982:4 to 2006:3 for the US, the UK, euro area and Japan.[6] Emerging economies such as China and India are not included due to lack sufficiently long time series. Our GVAR thus consists of a relatively small number of countries compared with other applications. Table B.1 in Appendix B provides an overview of the national data sources. The euro area is here defined as the aggregate of France, Italy and Germany[7] and we use the aggregation method proposed by Beyer, Doornik, and Hendry (2001) to define euro-area aggregates. This method uses time-varying weights and bases aggregation on growth rates. For volume series (money and output) weights are based on the relative share of the variable in question measured in a common currency (here USD). The start of the sample is set to coincide with time at which the Fed shifted away from its approach of targeting the quantity of money (M1) and started targeting the federal funds rate in September 1982. The sample ends mid 2006, approximately at the peak of the US housing bubble. Table B.1 to B.4 plot the domestic and foreign variables of each region. All variables are log-transformed (denoted by lower-case letters) except interest rates which are divided by 400 to make them comparable with the quarterly inflation rates.

[6] This section draws on GT.

[7] Prior to the German re-unification in 1991:1 growth rates in the West-German variables are used to splice the data series and thus to construct historical data for Germany as a whole.

The 'price of credit' in the short end of the maturity spectrum is measured here by the three-month interbank rate as this the interest rate is used as a reference for pricing a range of financial contracts. For the longer end of the curve we use the yield on 10-year government bonds. The 'quantity of credit' can be represented by a range of measures such as different monetary aggregates (narrow vs. broad), interbank lending, loans granted to the private sector, indicators from central banks' bank-lending surveys, etc. As an example, IMF (2007) suggests the use of base money plus reserves to account for the accumulation of foreign currency by central banks in emerging markets with large external surpluses. We leave out current-account issues *per se* and represent non-price credit conditions by broad money supply (M3 or M4). Broad money provides a standardised measure across countries of the amount of liquidity created by both central banks and financial intermediaries in a fractional reserve-banking system.[8]

For each country/region, we aggregate the data for the three other regions in order to obtain ROW series for each. The Dornbusch model proposes that crucial cross-country links are provided by bond markets. Whereas housing markets are arguably little integrated across countries, stock markets are potentially an important additional source of spill-overs between regions. To reduce the dimensionality of the CVARX* models we focus here on bond-market linkages, leaving in mind that we may miss transmission effects that run solely through share prices. Since we include only non-volume variables (interest rates and the inflation rate) in $\mathbf{x}'_{i,t}$, time-varying GDP weights are used for all ROW variables.[9] The exchange rate is taken to be the national currency per USD.[10]

[8]The ECB has argued that portfolio shifts may have affected M3 dynamics in the period 2000-03. The Fed discontinued the publication of M3 data, arguing that it did not convey any additional information over M2 due to the increasing use of securitisations and rising off-shore positions.

[9]Alternatively, money-supply weights or weights reflecting the size of capital flows between regions may be used. To comply with the aggregation method used in GT we here use to GDP weights.

[10]Because the exchange rate measure, e_t, is defined in terms of USD whereas the foreign price deflator, p_t^*, is a ROW measure, the variable $ppp_t = e_t - p_t + p_t^*$ is not strictly a measure of the real exchange rate but it should provide a good approximation. DHPS use the real *effective* exchange rate. Note also that because the dollar is used as the reference currency, for the US, we have $ppp_t = p_t^* - p_t$ which is then simply a measure of relative prices.

5 Country models: CVARX*

Each of the country models is based on the data vector (21). In addition, the models include centered seasonal dummies, an unrestricted constant term, and a set of dummy variables to take account of extraordinary events specific to each country.[11] All models are based on two lags as suggested by information criteria (not reported). Misspecification tests for each country are reported in Appendix C. Multivariate normality is rejected for the US and Japan and rejected at the five- but not the one-per cent level for the UK despite inclusion of a set of permanent impulse dummies for each. Rejection however appears to be mainly due to excess kurtosis (not reported) which the rank test is largely robust to (Dennis, Hansen, and Rahbek 2002). In the following, we first test for the cointegration rank and check the weak-exogeneity assumption for the foreign variables. We then consider the identified long-run relations for each country in turn, focusing on interpretation of relations of the type (13).

5.1 Rank determination

When weakly exogenous variables are present, the test for cointegration rank depends not only on the number of common trends in the full system, but also on the number of common trends generated by the partial system. This is because the weakly exogenous variables may cointegrate, not only with the endogenous variables, but also among themselves. Harbo, Johansen, Nielsen, and Rahbek (1998) (see also Pesaran, Sin, and Smith 2000) derive the asymptotic distribution and provides critical values for the test for cointegration rank under the assumption of weak exogeneity. However, because the number of foreign variables in our country models is small compared with other GVAR applications, we are able to test the rank of the full system. We thus consider the system,

$$\mathbf{x}_{i,t} = (m_r, y_r, \Delta p, h_r, s_r, I_s, I_l, ppp, \Delta p^*, I_s^*, I_l^*)'_{i,t} \tag{22}$$

[11]Dummy specifications,

$$\begin{aligned}
d_t^{US} &= (Dp_{84:2}, Dp_{87:4}, Dp_{99:4}, Dp_{00:2}, Dp_{02:3}, Dp_{04:3})'_t \\
d_t^{UK} &= (Dp_{84:2}, Dp_{85:1}, Dp_{86:3}, Dp_{87:4}, Dp_{92:4}, Dp_{97:3})'_t \\
d_t^{EA} &= (Dp_{87:1}, Dp_{88:3}, Dp_{92:1})'_t \\
d_t^{JP} &= (Dp_{85:4}, Dp_{87:2}, Dp_{89:2}, Dp_{95:2}, Dp_{97:2})'_t
\end{aligned}$$

where $Dp_{YY:Q}$ is a permanent impulse dummy that takes a value of one at time YY:Q and zero elsewhere. We use $Dcs_{yy:q}$ to denote centered seasonal dummies.

	US	UK	EA	JP
$r = 8$	1.0233	0.9920	0.9648	0.9782
$r = 7$	1.0275	1.0156	0.9806	0.9700
$r = 6$	1.0107	0.9764	0.9741	0.9959
$r = 5$	0.9840	0.9552	0.9287	0.9793
$r = 4$	0.9446	0.9467	0.8989	0.9762
$r = 3$	0.9427	0.9368	0.8960	0.8970
$r = 2$	0.9297	0.9528	0.8924	0.9163
$r = 1$	0.9096	0.8985	0.8887	0.8744
$r = 0$	0.7301	0.5907	0.8547	0.8461

Table 1: Modulus of the largest unrestricted companion-form roots for different choices of rank.

where all variables are allowed to be potentially endogenous, i.e. $\mathbf{x}^*_{i,t} = \{\emptyset\}$. The rank test statistics for each country are shown in Appendix C. The ranks suggested by the standard trace tests and by the Bartlett-corrected ones differ markedly for all countries, which is often a sign of I(2) problems (Johansen 2000). The non-corrected tests point to ranks in the range five to eight whereas the Bartlett-corrected ones suggest only two-three stationary relations. This compares with the theoretical prediction of $r = 7$ ($r = 6$ for Japan). Table 1 shows the largest unrestricted companion-form roots for different choices of rank; these likewise suggest that the rank should be set notably lower than proposed by theory as a large root remains in all models for any reasonable choice of rank. In contrast, the $\boldsymbol{\alpha}$-matrices (not reported) shows significant reaction to up to seven relations. The recursively calculated trace test statistics (not reported) grow linearly up to the start of the new millennium for the US and the UK; for the euro area and Japan the test statistics rise linearly more or less throughout sample. Notably, when the sample is set to end at the start of the 00s recursive estimation instead hints at a considerably higher rank.

Overall, there is evidence that the rank could be close to the theoretical prior when disregarding the potential I(2) problems that likely arise due to asset-price bubbles in the most recent period (see also GT). From a longer-term perspective, a 'high' rank thus seems to be the appropriate choice and we base the identification of the country models below on this choice, i.e. $r = 7$ for the US, the UK and the euro area and $r = 6$ for Japan. For the GVAR to be stable, it nevertheless turns out that we have to base the country models on a lower choice of rank, i.e. $r = 3$ for the US and the UK, $r = 4$ for the euro area and $r = 2$ for Japan. We discuss stability of the GVAR further in Section 6.3.

	US	UK	EA	JP
Δp^*	$\chi^2(3) = \underset{[0.00]}{16.24}$	$\chi^2(3) = \underset{[0.01]}{11.46}$	$\chi^2(4) = \underset{[0.08]}{8.42}$	$\chi^2(3) = \underset{[0.01]}{8.66}$
I_s^*	$\chi^2(3) = \underset{[0.64]}{1.71}$	$\chi^2(3) = \underset{[0.53]}{2.22}$	$\chi^2(4) = \underset{[0.09]}{7.91}$	$\chi^2(3) = \underset{[0.14]}{3.89}$
I_l^*	$\chi^2(3) = \underset{[0.21]}{4.50}$	$\chi^2(3) = \underset{[0.35]}{3.25}$	$\chi^2(4) = \underset{[0.00]}{16.20}$	$\chi^2(3) = \underset{[0.01]}{9.04}$

Table 2: Tests for weak exogeneity.

5.2 Testing weak exogeneity

To save degrees of freedom it is preferable to specify the country models as partial models where we condition on the foreign variables. For this, we need weak exogeneity of $\mathbf{x}_{i,t}^*$ with respect to $\mathbf{x}_{i,t}$. Table 5.2 shows tests based on (22) and the low-rank choice (making it easier not to reject weak exogeneity *ceteris paribus*). Weak exogeneity cannot be rejected in most cases but for the US and the UK, foreign inflation is rejected as weakly exogenous, suggesting that these two regions exert significant influence on inflation in the rest of the world. Somewhat surprisingly, the foreign long-term interest rate is found to be weakly exogenous for the US but not for the euro area. Notwithstanding that weak exogeneity is rejected for a few of the variables, we condition on all of the foreign variables in the CVARX* models in order to reduce dimensionality.

5.3 Identification of the country models

Identification of the country models below is based on the 'high-rank' choice. We look the five relations linking the domestic variables suggested by GT plus variants of the relation (13). In the following, we focus in particular on discussing the latter. The complete set of cointegration relations for each region is given in Appendix C. Our identification strategy differs from that of DHPS in that we focus on identifying the full set of theoretical relations for each region, and in that we do not set the variables to be long-run excludable unless suggested by tests. This implies that we trade off statistical significance in favour of economic identification to some extent and thus some models are identified with low p-values.

5.3.1 US

Table C.3 reports the cointegrating relations and the associated adjustment structure subject to over-identifying restrictions. The test for restrictions results in a test statistic of $\chi^2(20) = 37.67$ which rejects at the five- but not the one-per cent level ($p = 0.01$).

The first five relations have interpretations similar to those of GT although some foreign interest rates are included in the house- and share-price relations. The sixth relation relates the real exchange rate to the real short-term interest rate,

$$ppp_t \underset{[-6.95]}{-0.03} (I_s - \Delta p)_t \sim I(0) \tag{23}$$

which notably does not include the foreign short rate. This suggests that a fall in the US real short rate is lowered, for example, as the Fed responds to a recession that takes off in the US, is associated with a strengthening of the dollar in relative terms. This is likely due to the special role played by the dollar as a world reserve currency, which means that market participants have a tendency to substitute to USD in a bear market when risk aversion is high because the US is a viewed as a 'safe haven'. The real exchange rate adjusts significantly in face of deviations from this relation but not in an equilibrium-correcting direction. The sign of the short rate suggests some equilibrium-correction in that variable but this effect is insignificant. Real output has a tendency to rise when the currency depreciates, likely a result of increased competitiveness and thus a rise in exports.

The seventh relation relates the real exchange rate to the real long-term interest rate differential,

$$ppp_t - \underset{[-9.37]}{0.07} \left[(I_l - \Delta p) - (I_l^* - \Delta p^*)\right]_t \sim I(0) \tag{24}$$

which similarly suggests that a higher real long rate relative to the rest of the world is accompanied by a real depreciation. In terms of (13), this implies that $\omega = 1$ and $\theta > 0$, indicating that prices in the US are relatively flexible and/or the interest elasticity of demand is high relative to the rest of the world. The signs of the adjustment coefficients are the now the exact opposite compared with (23): the real exchange rate equilibrium-corrects to deviations as does the long rate.

5.3.2 UK

Table C.6 shows the identified cointegration and adjustment structures. The restrictions are not rejected with a test statistic of $\chi^2(20) = 30.17$ ($p = 0.07$). Again the first five relations are broadly similar to those in GT.

The sixth relation combines the real exchange rate with the real short-term interest rate differential,

$$ppp_t + \underset{[10.47]}{0.24}(I_s - \Delta p)_t - \underset{[-17.60]}{0.44}(I_s^* - \Delta p^*)_t \sim I(0) \tag{25}$$

such that a rise in the (approximate) real interest differential is associated with a real appreciation. The real exchange rate equilibrium-corrects to this relation and there is a tendency for the long rate to rise whenever sterling weakens more than suggested by the fundamentals in (25); real money supply contracts in this case whereas inflation tends to rise somewhat in the short run.

The seventh relation is similar to (25) but includes the long rates,

$$ppp_t + \underset{[18.87]}{0.74} \left[(I_l - \Delta p) - (I_l^* - \Delta p^*) \right]_t \sim I(0) \tag{26}$$

where we now have the additional restriction of homogeneity in the interest differential. In terms of (13), this implies that $\omega = 1$ and $\theta < 0$. In contrast with the US, this points to prices in the UK being relatively sticky and/or the interest elasticity of demand being low relative to the rest of the world. As was the case for the US, the (significant) adjustment coefficients have opposite signs of those associated with (25). The real exchange rate shows error-increasing behaviour in response to deviations from the relation involving the long rate differential whereas it was equilibrium-correcting for the US. This could be a first indication that it is the US which is driving the world long-term interest rate to a large degree.

5.3.3 Euro area

Table C.9 shows the cointegration and adjustment structures. The restrictions result in a test statistic of $\chi^2(21) = 41.63$ ($p = 0.01$). The first five relations are broadly similar to those of GT except that the IS relation sees a negative relationship between real output and the *foreign* real long rate.

The sixth relation combines the real exchange rate and the foreign real short-term interest rate,

$$ppp_t + \underset{[11.30]}{3.50} (I_s^* - \Delta p^*)_t \sim I(0) \tag{27}$$

which notably does not include the domestic short rate. This is in fact conceptually the mirror image of (23): the US short rate has a large weight in the euro-area foreign short rate and because the real exchange rate of (27) has to be inverted to become (at least approximately) comparable with its US counterpart, so has the sign of the relation. This could therefore be interpreted as representing the safe-haven effect from a different angle; the (absolute) value of the coefficient in (27) is however somewhat large compared with that in (23). The real exchange rate does not adjust significantly towards this relation but real share prices and the two domestic interest rates tend to rise when the euro depreciates more than warranted by (27).

The seventh relation also includes only one real interest rate,

$$ppp_t + \underset{[14.32]}{0.94}(I_l - \Delta p) \sim I(0) \tag{28}$$

such that a rise in the domestic real long rate is associated with a strengthening of the euro. The exclusion of the foreign long rate could be a signal that investors in the, to some degree, closed euro-area economy do not view foreign bond as the alternative asset class for investments but rather look to, for example, housing and stock markets within the euro area. Only inflation and the short rate adjusts to deviations from (28).

5.3.4 Japan

Table C.12 reports the identified cointegrating relations and adjustment coefficients. The restrictions are not rejected with a test statistic of $\chi^2(20) = 26.43$ ($p = 0.15$). The rank is set one lower than in the other models as a Taylor rule does not exist for Japan but otherwise the first four relations are similar to GT.

The fifth relation combines the real exchange rate with the real short-term interest rate differential,

$$ppp_t + \underset{[11.73]}{9.01}(I_s - \Delta p)_t - \underset{[-3.82]}{0.21}(I_s^* - \Delta p^*)_t \sim I(0) \tag{29}$$

which is similar to its UK counterpart albeit with the coefficient of $(I_s - \Delta p)_t$ much larger in magnitude, which may be due to the extremely low levels of interest rates in Japan in part of the sample. Indeed, carry-trade strategies may have played a role in making the exchange rate highly sensitive to movements in the interest rate differential: a low interest rate encourages investors to take short positions in yen and use the proceeds to invest (long) in higher-yield currencies; both types of trade are highly liquid which could explain the high sensitivity suggested by the large coefficient in (29). The real exchange rate equilibrium-corrects to (29) as was the case for the UK. There is also a tendency for share prices to decline when the yen depreciates.

The sixth relation is similar to (29) but includes the long rates,

$$ppp_t + \underset{[12.07]}{16.10}(I_l - \Delta p)_t - \underset{[-7.27]}{2.22}(I_l^* - \Delta p^*)_t \sim I(0) \tag{30}$$

which is again similar to its UK counterpart (26), albeit with larger coefficients and lack of homogeneity in the differential term. Mainly the short rate and real output adjust to deviations from (30), both with a negative sign. The fact that the real exchange rate is not reacting here suggests that short-rate differentials is the main driver of the yen, consistent with the importance of carry-trade activities.

5.3.5 Nature of cross-country linkages

We found above that the Dornbusch-IKE relation (13) plays a central role in all countries for both the short and the long interest rates. But, the sign of θ was found to be positive in case of the US and negative for the remaining countries. Goods prices in the UK, the euro area and Japan thus appears to be relatively less flexible and/or the interest elasticity of demand lower relative to the US, see (7). Since the US is used as the reference country for exchange-rate movements, these findings regarding the sign of θ are thus consistent across countries in broad terms. In resemblance with the results in Juselius and MacDonald (2007), the US dollar seems to represent a safe haven, meaning that market participants has a tendency to flee other currencies when risk aversion increases. This is consistent with the fact that the 'carry trade' was predominantly based on short positions in low-yield currencies such as yen or sterling prior to the credit crisis.

6 Global model: GVAR

We now show how to derive the GVAR from the CVARX* models. Modifications of the standard GVAR resulting from our specific specification of the CVARX* models and stability issues are also discussed.

6.1 Building the GVAR from the CVARX* models

For deriving impulse response functions, the CVARX*(2,2) in (19) is re-written in levels,

$$\mathbf{x}_{i,t} = \boldsymbol{\mu}_i + \boldsymbol{\pi}_i t + \boldsymbol{\Phi}_{i1}\mathbf{x}_{i,t-1} + \boldsymbol{\Phi}_{i2}\mathbf{x}_{i,t-2} + \boldsymbol{\Psi}_{i0}\mathbf{x}^*_{i,t} + \boldsymbol{\Psi}_{i1}\mathbf{x}^*_{i,t-1} + \boldsymbol{\Psi}_{i2}\mathbf{x}^*_{i,t-2} + \boldsymbol{\phi}_i\mathbf{d}_{i,t} + \mathbf{u}_{i,t}, \quad (31)$$

where

$$\begin{aligned}
\boldsymbol{\Phi}_{i2} &= -\boldsymbol{\Gamma}_{i1} \\
\boldsymbol{\Phi}_{i1} &= \mathbf{I}_{k_i} + \boldsymbol{\Pi}_i + \boldsymbol{\Gamma}_{i1} \\
\boldsymbol{\Psi}_{i0} &= \boldsymbol{\Upsilon}_{i0} \\
\boldsymbol{\Psi}_{i2} &= -\boldsymbol{\Upsilon}_{i1} \\
\boldsymbol{\Psi}_{i1} &= \boldsymbol{\Pi}^*_i - \boldsymbol{\Upsilon}_{i0} + \boldsymbol{\Upsilon}_{i1}
\end{aligned} \quad (32)$$

with the $k_i \times (k_i + k_i^* + 1)$ matrix $\widetilde{\mathbf{\Pi}}_i = \boldsymbol{\alpha}_i \widetilde{\boldsymbol{\beta}}'_i$ decomposed as,

$$\widetilde{\mathbf{\Pi}}_i = (\mathbf{\Pi}_i, \mathbf{\Pi}_i^*, \boldsymbol{\pi}_i) \tag{33}$$

such that $\mathbf{\Pi}_i$ contains the first k_i columns and $\mathbf{\Pi}_i^*$ columns $k_i + 1$ to $k_i + k_i^*$ of $\mathbf{\Pi}_i$, and $\boldsymbol{\pi}_i$ the last column corresponding to the restricted linear trend.

The data vector of country i can be written as,

$$\mathbf{z}_{i,t} = \mathbf{W}_i \mathbf{x}_t, \tag{34}$$

where $\mathbf{z}_{i,t} = (\mathbf{x}'_{i,t}, \mathbf{x}^{*\prime}_{i,t})'$ is $(k_i + k_i^*) \times 1$ and $\mathbf{x}_t = (\mathbf{x}'_{0t}, \mathbf{x}'_{1t}, ..., \mathbf{x}'_{Nt},)'$ is a k-dimensional data vector with the domestic variables from the N countries stacked on top of each other and $k = \sum_{i=0}^{N} k_i$ is the total number of variables in the system (the domestic variables are picked out by 'weights' of one); $\mathbf{W}_i$ is the weighting matrix of country i which is $(k_i + k_i^*) \times k$. Using (34), we can re-write the country models (31) in terms of $\mathbf{z}_{i,t}$,

$$\mathbf{A}_{i0}\mathbf{z}_{i,t} = \boldsymbol{\mu}_i + \boldsymbol{\pi}_i t + \mathbf{A}_{i1}\mathbf{z}_{i,t-1} + \mathbf{A}_{i2}\mathbf{z}_{i,t-2} + \boldsymbol{\phi}_i \mathbf{d}_{i,t} + \mathbf{u}_{i,t}, \tag{35}$$

where the $\mathbf{A}_{ij}$-matrices are of dimension $k_i \times (k_i + k_i^*)$, $j = 0, 1, 2$, and

$$\begin{aligned} \mathbf{A}_{i0} &= (\mathbf{I}_{k_i}, -\mathbf{\Psi}_{i0}) \\ \mathbf{A}_{i1} &= (\mathbf{\Phi}_{i1}, \mathbf{\Psi}_{i1}) \\ \mathbf{A}_{i2} &= (\mathbf{\Phi}_{i2}, \mathbf{\Psi}_{i2}) \end{aligned} \tag{36}$$

and we require $rank(\mathbf{A}_{i0}) = k_i$ (full row rank).

In order to link the different country models to one single global model, the GVAR, it is convenient to stack the country models. For this purpose, define a $\sum_{i=0}^{N}(k_i + k_i^*) \times k$ weight matrix, $\mathbf{W} = (\mathbf{W}'_0, \mathbf{W}'_1, ..., \mathbf{W}'_N)'$, and likewise for $\boldsymbol{\mu}$, $\boldsymbol{\pi}$ and $\boldsymbol{\phi}$. In addition, define a new matrix, $\mathbf{A}_j$, of dimension $k \times \sum_{i=0}^{N}(k_i + k_i^*)$ with the $\mathbf{A}_{ij}$ terms on the diagonal and zeros elsewhere,

$$\mathbf{A}_j = \begin{pmatrix} \mathbf{A}_{0j} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{A}_{1j} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{A}_{Nj} \end{pmatrix} \tag{37}$$

Using (37), we stack the N country models (35) to obtain in terms of $\mathbf{z}_t$,

$$\mathbf{A}_0\mathbf{z}_t = \boldsymbol{\mu} + \boldsymbol{\pi} t + \mathbf{A}_1\mathbf{z}_{t-1} + \mathbf{A}_2\mathbf{z}_{t-2} + \boldsymbol{\phi}\mathbf{d}_t + \mathbf{u}_t \tag{38}$$

Substituting the expression (34) and using the definition of $\mathbf{W}$, we can re-write in terms of $\mathbf{x}_t$,

$$\mathbf{A}_0\mathbf{W}\mathbf{x}_t = \boldsymbol{\mu} + \boldsymbol{\pi}t + \mathbf{A}_1\mathbf{W}\mathbf{x}_{t-1} + \mathbf{A}_2\mathbf{W}\mathbf{x}_{t-2} + \boldsymbol{\phi}\mathbf{d}_t + \mathbf{u}_t \tag{39}$$

Now isolate $\mathbf{x}_t$ to arrive at the model of interest,

$$\mathbf{x}_t = \mathbf{f}_0 + \mathbf{f}_1\mathbf{t} + \mathbf{F}_1\mathbf{x}_{t-1} + \mathbf{F}_2\mathbf{x}_{t-2} + \boldsymbol{\phi}\mathbf{d}_t + \boldsymbol{\varepsilon}_t \tag{40}$$

where $\boldsymbol{\varepsilon}_t = (\mathbf{A}_0\mathbf{W})^{-1}\mathbf{u}_t$, $\mathbf{f}_0 = (\mathbf{A}_0\mathbf{W})^{-1}\boldsymbol{\mu}$, and the other terms are defined in a similar way. Notably $\mathbf{A}_0\mathbf{W}$ must be invertible for (40) to be valid.

In order to compute impulse response functions, we consider the MA representation of the model (40), see (17) and DHPS,

$$\mathbf{x}_t = +\sum_{j=0}^{\infty} \mathbf{K}_j \boldsymbol{\varepsilon}_{t-j} + \text{det. and stat. comp.}, \tag{41}$$

where $\mathbf{K}_j = \mathbf{F}_1\mathbf{K}_{j-1} + \mathbf{F}_2\mathbf{K}_{j-2}, j = 1, 2, ...$ with $\mathbf{K}_0 = \mathbf{I}_k$, and $\mathbf{K}_j = 0$ for $j < 0$. The properties of $\mathbf{K}_j$, and thus of $\mathbf{F}_1$ and $\mathbf{F}_2$, determine how fast effect of shocks die out. In line with other applications of the GVAR, we consider generalised impulse response functions (GIRFs) for shocks analysis (Pesaran and Shin 1998). These differ from the standard orthogonalised impulse responses based on, for example, Cholesky decompositions, in that the historical distribution of errors is used to integrate out the effects of other shocks when any one element of $\mathbf{x}_t$ is shocked. From (41) we can derive the GIRF for a one-standard deviation shock to the ℓ^{th} element of $\mathbf{x}_T$ on its $\jmath^{\text{th}}$ element as, see DHPS,

$$GIRF(\mathbf{x}_{\jmath,t}; u_{\ell t}, h) = \frac{e_\jmath' \mathbf{K}_h (\mathbf{A}_0\mathbf{W})^{-1} \boldsymbol{\Sigma}_u e_\ell}{\sqrt{e_\ell' \boldsymbol{\Sigma}_u e_\ell}}, h = 0, 1, 2, ..., H; \ell, \jmath = 1, 2, ..., k, \tag{42}$$

where h is time horizon index for which GIRFs are to be computed, H is the total number of periods to be considered, and $\boldsymbol{\Sigma}_u$ is the covariance matrix of $\mathbf{u}_t$; e_ℓ and $e_\jmath$ are $k \times 1$ selection (unit) vectors with an entry of one at the ℓ^{th} and $\jmath^{\text{th}}$ elements, respectively, and zeros elsewhere.

GIRFs take account of the historical correlation of shocks as gauged by the off-diagonal elements of the residual covariance matrix $\boldsymbol{\Sigma}_u$. The initial shock is thus based on both by the weighting matrix, $\mathbf{W}$, and the coefficient matrix containing the contemporaneous effects of weakly exogenous variables, $\mathbf{A}_0$. The GVAR-GIRF framework also allows us to consider also the effects of shocks that do not originate from a particular country as such but which is common to all countries. The effects of such a 'global shock', i.e. a similar shock to the same variable in all countries simultaneously, can be studied by setting the entries of e_ℓ corresponding to the variable to be shocked equal to the weight of the variable in question.

Pesaran and Shin (1998) argue that GIRFs have the advantage that unlike conventional impulse response analysis they do not require orthogonalisation of shocks and are thus invariant to the ordering of variables. However, this feature is essentially achieved by changing the ordering of the variables for each type of shock such that the shocked variable is always put first in the causal order used for identification. As discussed by Johansen (2004), this approach can make it difficult to interpret the type of shock being analysed because a range of variables is in fact shocked at the same time as a result of the role played by the residual covariance matrix in integrating out the effects of shocks to other variables. A causal interpretation of the results may thus be difficult and inspection of the composition of the shock on impact is pertinent in understanding the type of shock under scrutiny.

Figure 1 illustrates the GVAR framework where the cointegration structures for each country are linked in a way consistent with the aggregation scheme used in constructing $\mathbf{x}^*_{i,t}$. The GVAR is thus effectively a two-step procedure: the CVARX* models take first-round effects of shocks into account via $\mathbf{x}^*_{i,t}$, and the GIRFs account for second-round effects via the feed-back effects incorporated in $\mathbf{K}_j$ and $(\mathbf{A}_0\mathbf{W})^{-1}$ in (42).

6.2 Modifications of the GVAR

In the country models of Section 5, we made a set of modifications to the standard specification of the CVARX* models employed by most GVAR applications:

- Construction of foreign variables: we use time-varying (GDP) weights to aggregate inflation and interest rates to a ROW measure as discussed in Section 4.

- Choice of foreign variables: we include only a subset of the type of variables from the domestic set, $\mathbf{x}_{i,t}$, as potential foreign variables, $\mathbf{x}^*_{i,t} = (\Delta p^*, I_s, I_l)'_{i,t}$, as discussed in Section 3.

The first modification can be incorporated by simplify allowing the weight matrix be time-varying, i.e. $\mathbf{W}_t = (\mathbf{W}'_0, \mathbf{W}'_1, ..., \mathbf{W}'_N)'_t$. To simply calculation of the impulse response functions, we use the weight matrix at time T in period $T+h$ for $h > 0$, i.e. $\mathbf{W}_{t+h} = \mathbf{W}_T$.

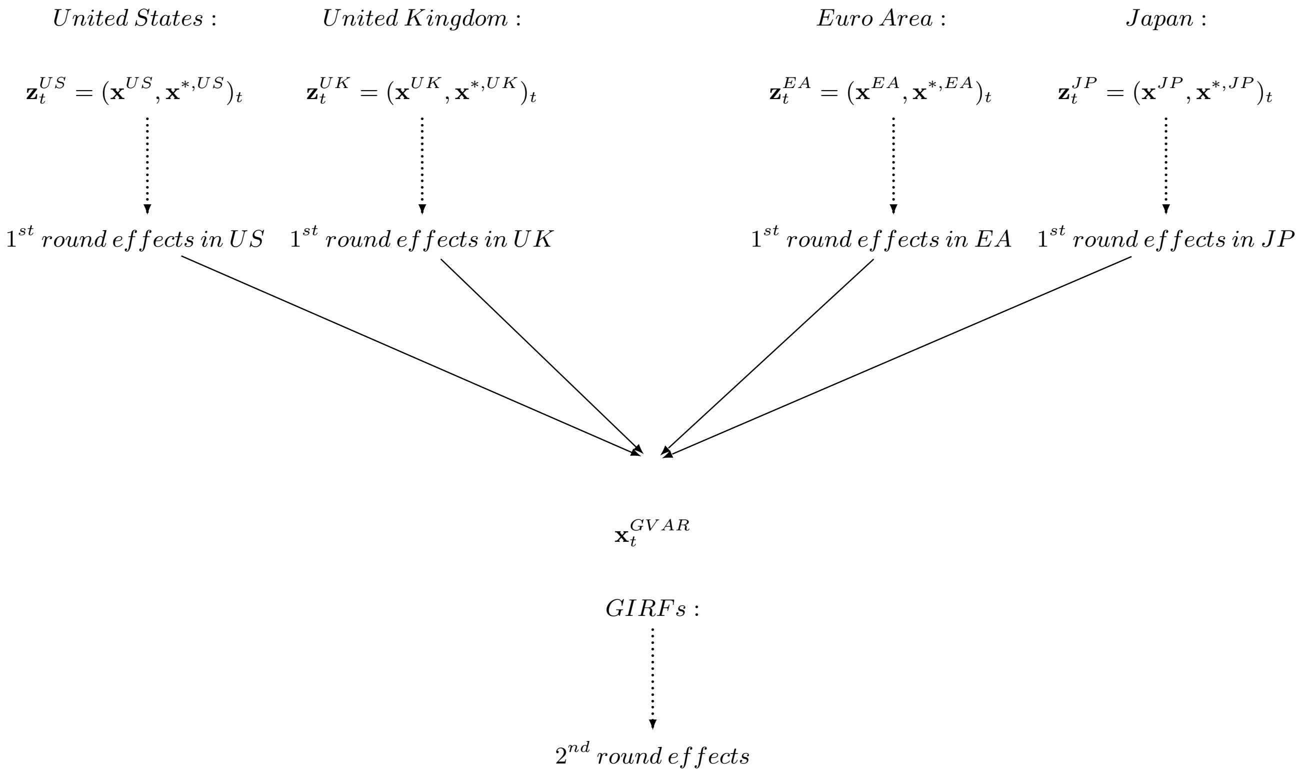

Figure 1: Global VAR procedure for shocks analysis.

The second modification introduces a number of zeros in the $\mathbf{\Psi}_{ij}$ matrices and thus in $\mathbf{A}_j$. In order to derive (40), we noted that $\mathbf{A}_0\mathbf{W}$ must be invertible. A necessary condition for this is that $\mathbf{A}_0\mathbf{W}$ is a square matrix (of dimension k). Hence a variable in $\mathbf{x}_{i,t}$ can only be excluded if the corresponding variable is excludable from $\mathbf{x}^*_{i,t}$ for all $i = 1, ..., N$ as $\mathbf{x}^*_{i,t}$ is constructed on the basis of $\mathbf{x}_t$, see (34). This means that the number of endogenous variables in each country model must be the same, i.e. $k_i = k_j$ with $i \neq j$. In contrast, the number of foreign variables could be allowed to vary between different country models, i.e. $k^*_i \neq k_i$ and $k^*_i \neq k^*_j$ with $i \neq j$ is allowed. We allowed for the former in specification of the CVARX*s; the latter is not relevant here as none of the foreign variables are found to be long-run excludable.

6.3 Stability of the GVAR

Pesaran et al. (2004) discuss stability of the GVAR with one lag and notes that this depends crucially on the eigenvalues of the autoregressive parameter matrix. In our model with two lags, the dynamic properties depends on the companion-form roots of the system. Re-write (40) in companion form,

$$\begin{aligned} \mathbf{x}_t &= \mathbf{F}_1\mathbf{x}_{t-1}+\mathbf{F}_2\mathbf{x}_{t-2}+\boldsymbol{\varepsilon}_t \Rightarrow \\ \begin{pmatrix} \mathbf{x}_t \\ \mathbf{x}_{t-1} \end{pmatrix} &= \underbrace{\begin{pmatrix} \mathbf{F}_1 & \mathbf{F}_2 \\ \mathbf{I} & \mathbf{0} \end{pmatrix}}_{=\mathbf{F}} \begin{pmatrix} \mathbf{x}_{t-1} \\ \mathbf{x}_{t-2} \end{pmatrix} + \begin{pmatrix} \boldsymbol{\varepsilon}_{t-1} \\ \boldsymbol{\varepsilon}_{t-2} \end{pmatrix} \end{aligned}$$

where the $2k$ roots of $\mathbf{F}$ can be found by solving the eigenvalue problem, $\boldsymbol{\rho}\mathbf{v} = \mathbf{F}\mathbf{v}$. Unit roots imposed in the CVARX*s will impose unit roots in the GVAR and these will show up as unit eigenvalues of $\mathbf{F}$. If the cointegration ranks of the country models have been set to appropriately account for the number of unit roots, then the CVARX* models are, by construction, individually stable and equilibrium-correcting forces will eventually bring the domestic economy back to steady state after a shock has hit. When the individual country models are combined to form the GVAR and second-round effects are incorporated, there are however no built-in conditions to guarantee that the joint model will be stable. Since coefficient estimates from different country models are combined within the $\mathbf{A}_i$-matrices, the $\mathbf{F}$-matrix will most likely have eigenvalues that differ from the joint set of the CVARX*s, and eigenvalues close to and/or above one is a possibility. Explosive roots imply that the GIRFs will not converge, and if complex eigenvalues are present oscillations in the projections could occur.

Table 1 in Section 5 showed that for $r = 7$ both the US and the UK model have explosive roots. This finding is consistent with the fact that these two countries towards the end of the sample were among the major deficit countries in the world economy and thus involved in the build-up of global current account imbalances with countries such as China, Germany and Japan as surplus counterparts. For the euro area and Japan, the roots are below, albeit close to, one when the rank is set equal to the theoretical suggestion. Table 3 reports the largest companion-form roots of the CVARX* models subject to the high-rank choice and the over-identifying restrictions presented in Section 5. For all models, except the Japanese one, explosive roots are present.

In light of the large country-model roots, it is not surprising that some eigenvalues of $\mathbf{F}$ for the GVAR fall outside the unit circle. In contrast, all eigenvalues remain firmly below one when the GVAR is based on the low-rank choice and hence GIRFs converge; some roots have complex parts which could lead to (dampened) oscillations however. Indeed, the approach taken by DHPS is in line with this choice: they do not use the theoretical prior to guide the cointegration rank but rather base their choice on the test procedure

US	UK	EA	JP
$r = 7$	$r = 7$	$r = 7$	$r = 6$
1.0191	1.0012*	1.0051	0.9910

Table 3: Modulus of the largest unrestricted companion-form roots with over-identifying restrictions imposed. An * denotes a double root.

in Pesaran, Sin, and Smith (2000) and with a view to ensure that persistence profiles are well-behaved.

Despite the problems with the roots, we continued the analysis in Section 5 based on the I(1) CVAR; indeed, Nielsen (2000) showed that the I(1) model can be used to estimate long-run relations even in the presence of explosive roots. We did encounter some signs of I(2) problems however which is not surprising given that GT found that an I(2) model was required for analysing a similar set of variables on a global level. The nominal-to-real transformation (Kongsted 2005) implicit in the formulation of the data vector (22) is routinely applied in the GVAR literature, but the validity of this requires that at most two I(2) trends, likely originating from twice cumulated shocks to monetary policy at home and abroad, are present and that these load identically into all nominal variables (except the interest rates). Considering the nominal counterpart of (22) for each country, the I(2) rank test procedure (Johansen 1995, 1997) shows that for all countries the number of I(2) trends exceed the RE prediction of two. Thus, analysing (21) in I(1) space we will loose some information compared with an I(2) analysis as the transformation will not reduce the order of integration of the variables to I(1) or I(0); thus there will be an I(2) component left in the model.[12] In GT, we argued that imposing long-run price homogeneity on the model, despite its rejection, may nevertheless be useful for studying the causes and effects of disequilibria.[13]

In sum, the GVAR cannot be implemented based on I(1) CVAR models without ignoring potentially important signals in the data about disequilibria. Below we conduct impulse response analysis based on the low-rank choice (and no restrictions on the $\boldsymbol{\beta}_i$'s) to ensure convergence of the GIRFs, noting that this trades off economic identification in

[12]It is feasible to base a GVAR on sets of polynomially cointegrating relation estimated from CVARX* models in I(2) space, i.e. the I(2) model is a restricted version of its I(1) counterpart. Another alternative could be to treat the real-transformed series as I(1) with explosive roots as proposed by Nielsen (2005). Both of these modifications are out of the scope of this paper however.

[13]The $\boldsymbol{\alpha}$-coefficients of the country models are potentially of interest as they allow us to assess, for example, whether money supply in excess of the level suggested by money demand lead asset prices to rise. Here, our focus is on spill-overs across countries however and we shall not discuss the domestic effects in details.

favour of a well-behaved statistical model.

7 Shocks analysis

We use the GVAR to analyse the propagation of shocks between regions in order to assess whether push- or pull-channel effects are predominant following liquidity shocks. A natural use of the GVAR is to consult it on its predictions regarding a credit crunch. For this purpose, we are however faced with the complication that the credit crunch might constitute a structural break which the CVARX* may not be invariant to. Indeed, impulse response analysis assumes super-exogeneity of the counterfactual shocks generating the impulses (Ericsson, Hendry, and Mizon 1998) and this assumption is not straightforward to test for the GVAR. We evaluate the GVAR on its ability to replicate observed developments during the current crisis. We consider in turn: a negative shock to US, to global money supply, and to US house prices.[14] In reporting results, the following legends are used: blue line: model projection; green line: five-per cent lower band; red line: five-per cent upper band. The 90-per cent error bands are calculated using the bootstrap procedure described in Appendix A. It generally takes at least 12 years ($H = 48$ periods) from the initial shock has hit until all variables have converged to their new long-run levels.

7.1 Negative shock to US money supply

Figure C.1 shows that a negative one-standard error shock to US money supply is associated with an upward jump in both the US short-term interest rate; the long rate also rises, albeit by less.[15] House prices start to decline which is not surprising given the strong co-movement with money supply found previously. Output and share prices rise (not significantly so however). This could suggest that the shock may alternatively represent a central-bank action to cool a booming economy, i.e. the Fed attempting to 'lean against the wind', rather than an actual credit crunch. Long-run effects must be considered to decide whether the credit-crunch or monetary-policy interpretation is more reasonable here.

In the long run, both money supply and house prices are at lower levels than before the shock and the dollar has depreciated in real terms. These effects seem reasonable

[14]Matlab code for calculating GIRFs based on the GVAR and their bootstrapped confidence bands is available from the authors on request.

[15]The effects of this type of shock are broadly similar to those of a positive shock to the US short rate except that for the latter US house prices fall by less and the dollar is broadly unchanged in the long run.

given that the US market was in a major disequilibrium at the end of the sample. The exchange-rate projection seems to represent the safe-haven effect found in (24) with a rise in the US long rate associated with a booming world economy and thus low risk aversion which leads investors to flee the dollar.

The shock induces a 'push effect' in the UK as money supply contracts here as well, following a small rise on impact. The shock is associated with falling output, interest rates and house prices in the UK; in contrast, share prices broadly move in sync with the US. A 'pull effect' seems present for the euro area where money supply expands following an initial dip. Output and house prices rise as do interest rates, leading the euro to appreciate. In Japan, the effects of the shock are small and generally insignificant; the stock market is lifted in the long run however.

Overall, it seems reasonable to interpret the shock as an approximate credit-crunch shock. In accordance with what has been observed during the crisis, the US and the UK have moved closely together. Both push and pull effects can be observed following movements in US money supply. Unsurprisingly stock-market developments are largely synchronised whereas housing markets seem to move closely with domestic money supply. Interest-rate reactions differ across regions, leading to some real-exchange rate shifts consistent with the Dornbusch relationships identified in Section 5. The significant effects are broadly in line with what has been observed during the crisis so far.

7.2 Negative shock to global money supply

Figure C.2 shows that a negative one-standard error shock to global money supply , constructed as a weighted shock to m_r in all countries simultaneously, induces reactions that are similar to a negative shock to US money supply. US output now sees a significant rise and the effect on house prices in the long run is insignificant. In Japan, both money supply and house prices decline significantly but so do short rates; the latter may reflect the lack of success of quantitative easing in raising inflation (and interest) rates during the time of the 'liquidity trap'. The reactions of housing markets are smaller compared with before. House prices in the euro area still rise despite the contraction in money supply, suggesting that this market has been less sensitive to credit conditions.

7.3 Negative shock to US house prices

Figure C.3 shows that a negative one-standard error shock to US house prices is associated with drops in US share prices, money supply and output alike.[16] After rising a little on impact, interest rates start to fall as well, likely a policy response to deteriorating growth prospects. The short-run effects turn into permanently lower levels for the majority of the variables, albeit the dollar is broadly unchanged in real terms in the long run.

In the UK, a pull effect seems present for this type of shock following small initial dips: house prices rise as do money supply, output, inflation and interest rates. Share-price movements are again synchronised across countries and the UK sees a fall in the stock market despite the otherwise prospering economy. The euro area experiences only a small drop in house prices, consistent with a short-term push effect, possibly a result of the rise in money supply despite falling output. Interest rates decline but relative to the US, euro-area rates are up, leading the euro to appreciate in real terms. Japan is again largely unaffected in the long run, apart from its market for shares which see prices fall in sync with the ROW.

Overall, both push and pull effects can again be observed, but the roles of the UK and the euro area have been swapped compared with the US money supply shock considered above. The predictions of the GIRFs here thus differ markedly from what has been observed during the crisis where the UK has moved largely with the US and the euro area has been hit less hard by the global recession.

7.4 Importance of spill-overs

The bootstrap bands for the GIRFs above suggest that the long-run effects of the shocks considered above are insignificant for the majority of the variables. Both US and global shocks did however have some significant effects on the UK and the euro area in particular. Notably, impulse response analysis of liquidity spill-overs pointed to a push channel for the UK and a pull channel for the euro area insofar as house prices were concerned. The effects on Japanese house prices was insignificant for a money-supply shock originating in the US but a push channel was identified for a global shock. In contrast, liquidity shocks were found to induce pull effects for share prices in all regions.

Indeed, stock markets were found to be highly synchronised across regions whereas

[16]The effects of this type of shock are broadly similar to those of a negative shock to the US share prices and a negative shock to US output.

housing markets did not appear to be so. House prices moved largely with money supply although this relationship was less pronounced for the euro area. Share prices in all countries seemed to move largely with US output, albeit the co-movement with interest rates was less clear from the GIRFs. Goods-price inflation did not react much to any of the shocks considered; in fact, most inflation profiles turn out insignificant in the long run.

In bond markets, long- and short-term interest rates moved in sync within regions but shifts were observed between regions. The US and the euro area see rates move broadly together in face of both money and house-price shocks. In the UK, rates moved with the US immediately following the shocks but the reaction was reversed after a few periods. Our results suggest that a shock following a major disequilibria could see interest rates move in opposite directions during the adjustment process when imbalances are evened out. The induced shifts in real interest-rate differentials implied movements in real exchange rates in accordance with the central relation (13), in particular for sterling and the euro.

8 Conclusion

This paper has evaluated the potential of the GVAR in modelling cross-country linkages via liquidity and asset prices. We made a set of modifications to the standard GVAR procedure such as using time-varying, rather than constant, weights, and limiting the set of foreign variables on economic grounds, as opposed to simply using the mirror image of the domestic variables. Moreover, we allowed the theoretical prior to systematically guide the choice of cointegration rank as well as identification of the country CVARX* models, rather than relying solely on the indications of statistical tests and leaving the long-run relations unrestricted. We showed however that this more economically appealing approach than the one usually taken in GVAR studies could lead the combined model to become statistically unstable if persistent disequilibria, such as asset-price bubbles likely present in our case, have occurred within the sample.

In the first stage of the procedure, we considered CVARX* models of the US, the UK, euro area and Japan separately. We found that the cointegrating relation linking real-interest rate differentials with PPP deviations derived from the Dornbusch model under IKE played a central role in all countries in capturing first-round effects of spill-overs. Notably, the US dollar appeared to possess a safe-haven status, meaning that market participants have had a tendency to flee other currencies in favour of the dollar when risk aversion increases. This is consistent with the fact that the 'carry trade' has predominantly been based on short positions in low-yield currencies such as yen or sterling.

The I(1) country models however suffered from large unrestricted roots when the rank was set equal to the theoretical prior. This pointed to a crucial trade-off between economic and statistical significance: in the presence of bubbles it will be difficult to achieve both simultaneously without fundamental modifications of the theory. Combining the country models to a GVAR in the second step of the procedure thus led to instability. In order to conduct impulse response analysis and take second-round effects of shocks into account, we therefore had to set the rank considerably lower for each CVARX* to ensure stability of the GVAR. Shocks analysis suggested that while stock markets have moved largely in sync across regions, this was not always the case for housing markets. 'Push channel' effects were observed for house prices in response to a liquidity shock except in the euro area. For share prices, 'pull effects' were identified across regions.

From the outset, the GVAR seems a convenient framework for analysing feed-back effects of shocks within the global economy and thus to assess how the credit crunch might play out in different countries. We found that a negative GIRF shock to money supply was able to replicate some key features of the credit crunch, such as significant spill-overs to the UK, whereas an analogous interpretation of a negative shock to US house prices was less clear. Still, in light of the difficulties in reconciling economic and statistical identification that our analysis points to, combined with the possibility that the crisis might constitute a structural break, we propose using the GVAR with care in this context.

References

Adalid, R. and C. Detken (2006). Excessive liquidity and asset price boom/bust cycles. *Manuscript.*

Baks, K. and C. Kramer (1999). Global liquidity and asset prices: Measurement, implications, and spillovers. IMF Working Paper No 168.

Belke, A. and W. Orth (2007). Global excess liquidity and house prices: A VAR analysis for OECD countries. *Ruhr Economic Papers* (37).

Benkwitz, A., H. Lütkepohl, and J. Wolters (2001). Comparison of bootstrap confidence intervals for impulse reponse of German monetary systems. *Macroeconomic Dynamics 5*, 81–100.

Beyer, A., J. A. Doornik, and D. F. Hendry (2001). Constructing historical euro-zone data. *Economic Journal 111*, 308–327.

Dees, S., F. di Mauro, M. H. Pesaran, and L. V. Smith (2007). Exploring the international linkages of the euro area: A Global VAR analysis. *Journal of Applied Econometrics 22*, 1 – 38.

Dees, S., S. Holly, M. H. Pesaran, and L. V. Smith (2007). Long run macroeconomic relations in the global economy. *Economics - The Open-Access, Open-Assessment E-Journal 2007-3.*

Dennis, J. G. (2006). *CATS in RATS - Cointegration Analysis of Time Series, Version 2*. Evanston, Illinois: Estima.

Dennis, J. G., E. Hansen, and A. Rahbek (2002). ARCH innovations and their impact on cointegration rank testing. *Working paper, Centre for Analytical Finance, University of Copenhagen 22.*

Doornik, J. A. (2007). An introduction to OxMetrics 5. Available from Timberlake Consultants Ltd.

Dornbusch, R. (1976). Expectations and exchange-rate dynamics. *Journal of Political Economy*, 1161 – 1176.

Engle, R. F. and C. W. J. Granger (1987). Co-integration and error correction: Representation, estimation and testing. *Econometrica 55*, 251–276.

Ericsson, N. R., D. F. Hendry, and G. E. Mizon (1998). Exogeneity, cointegration, and economic policy analysis. *International Finance Discussion Papers, Board of Governors of the Federal Reserve System 616*.

Frankel, J. (1979). On the mark: A theory of floating exchange rates based on real interest differentials. *American Economic Review 69*, 610–623.

Frydman, R. and M. Goldberg (2007). *Imperfect Knowledge Expectations*. Princeton, New Jersey: Princeton University Press.

Giese, J. V. and C. K. Tuxen (2008). Has excess global liquidity fuelled asset prices? Evidence from I(1) and I(2) cointegrated VAR models. *Manuscript*.

Gouteron, S. and D. Szpiro (2005). Excess monetary liquidity and asset prices. *Working Paper, Banque de France*.

Harbo, I., S. Johansen, B. Nielsen, and A. Rahbek (1998). Asymptotic inference on cointegration rank in partial systems. *Journal of Business and Economic Statistics 16*(4), 388–399.

IMF (2007, October). Global prospects and policies. IMF, Washington DC. World Economic Outlook.

Johansen, S. (1988). Statistical analysis of cointegration vectors. *Journal of Economic Dynamics and Control 12*, 231–254.

Johansen, S. (1995). A statistical analysis of cointegration for I(2) variables. *Econometric Theory 11*(1), 25–29.

Johansen, S. (1996). *Likelihood-based Inference in Cointegrated Vector Auto-regressive Models*. Oxford: Oxford University Press.

Johansen, S. (1997). Likelihood analysis of the I(2) model. *Scandinavian Journal of Statistics 24*(4), 25–29.

Johansen, S. (2000). A Bartlett correction factor for tests on the cointegrating relations. *Econometric Theory 16*(5), 740–778.

Johansen, S. (2004). Comment. *Journal of Business and Economic Statistics 22*(2), 169–172.

Johansen, S., K. Juselius, R. Frydman, and M. Goldberg (2008). A resolution of the purchasing power parity puzzle: Imperfect knowledge and long swings. *Working Paper, Department of Economics, University of Copenhagen 08-31*.

Juselius, K., R. Frydman, M. Goldberg, and S. Johansen (2009). A scenario analysis of economic models based on RE and IKE. Manuscript.

Juselius, K. and R. MacDonald (2007). *International Macroeconomics: Recent Developments*, Chapter : International Parity Relationships between Germany and the United States: A Joint Modelling Approach, pp. 79–103. Nova Science Publishers.

Kongsted, H. C. (2005). Testing the nominal-to-real transformation. *Journal of Econometrics 124*(2), 205–225.

Nielsen, B. (2000). The asymptotic distribution of likelihood ratio test statistics for cointegration in unstable autoregressive processes. *Nuffield College, Discussion Paper W24*.

Nielsen, B. (2005). Analysis of co-explosive processes. *Working Paper, Nuffield College, Oxford University W8*. (forthcoming in Econometric Theory).

Obstfeld, M. and K. Rogoff (1996). *Foundations of International Macroeconomics*. The MIT Press.

Pesaran, H., Y. Sin, and R. J. Smith (2000). Structural analysis of Vector Error Correction Models with exogenous I(1) variables. *Journal of Econometrics 97*, 293–343.

Pesaran, M. H., T. Schuermann, and S. M. Weiner (2004). Modeling regional interdependencies using a global error-correcting macroeconometric model. *Journal of Business and Economic Statistics 22*(2), 129–162.

Pesaran, M. H. and Y. Shin (1998). Generalized impulse response analysis in linear multivariate models. *Economics Letters 58*, 17–29.

Rogoff (1996). The purchasing power parity puzzle. *Journal of Economic Literature 34*, 647–668.

Ruffer, R. and L. Stracca (2006). What is global excess liquidity, and does it matter? *ECB Working Paper 696*.

Sousa, J. and A. Zaghini (2004). Monetary policy shocks in the euro area and global liquidity spillovers. *ECB Working Paper 309*.

Swensen, A. R. (2006). Bootstrap algorithms for testing and determining the cointegration rank in VAR models. *Econometrica 74*(6), 1699–1714.

Tuxen, C. K. (2009). A multi-sector model of policy transmission and inflation dynamics in the euro area. *Manuscript*.

A Bootstrapping GIRF bands

We calculate confidence bands for the GIRFs using the following non-parametric bootstrap procedure along the lines of the suggestions in Benkwitz, Lütkepohl, and Wolters (2001):

1. Set initial values in bootstrap sample equal to those in original sample.
2. Draw with replacement from the set of residuals.
3. Construct bootstrap data series recursively using the original estimates of the VAR parameters and the re-sampled residuals (weakly exogenous variables are not re-sampled).
4. Re-estimate the short-run parameters based on the bootstrap data (using OLS equation-by-equation) with the (super-consistent) $\boldsymbol{\beta}_i$-parameters fixed at the original estimates from the CVARX* models.
5. Construct GIRFs based on the bootstrap estimates using fixed out-of-sample aggregation weights.
6. Repeat step 2-5, say, 2000 times and find the 5- and 95-per cent quantiles of the bootstrapped GIRF distribution in order to construct 90-per cent error bands.

We allow bands to be non-symmetric and bias-correct (based on the bootstrap) the central projection (based on the sample).[17]

[17] Explosive and/or highly oscillating bootstrap replications are removed prior to calculation of the quantiles; see also Swensen (2006).

B Data series

Variable	Description	Source
y	Nominal output (GDP)	OECD EOL
m	Broad money stock	National sources*
p	GDP deflator (implicit)	OECD EOL
I_s	Short-term interest rate (three-month deposits)	OECD EOL
I_l	Long-term interest rate (10-year government bonds)	OECD EOL
h	House-price index	BIS**
s	Share-price index (key industrial indices)	National sources***

Table B.1: Overview of variables and national data sources.

*For most countries, M3 is used as the broad money measure. For the UK and Japan, M4 and M2 plus cash deposits is used, respectively. US M2 growth was used to extrapolate the US M3 series from 2006:1 and onwards when publication of M3 was discontinued.

**BIS calculation based on national sources. Series for the US and UK are quarterly throughout; for France, Italy and Japan semi-annual series were interpolated to create quarterly series, and for Germany annual series were interpolated.

***Indices used: France: Paris Stock Exchange SBF 250, Italy: ISE MIB Storico Generale, Japan: TSE Topix, UK: FTSE 100, US: NYSE Composite, Germany: CDAX.

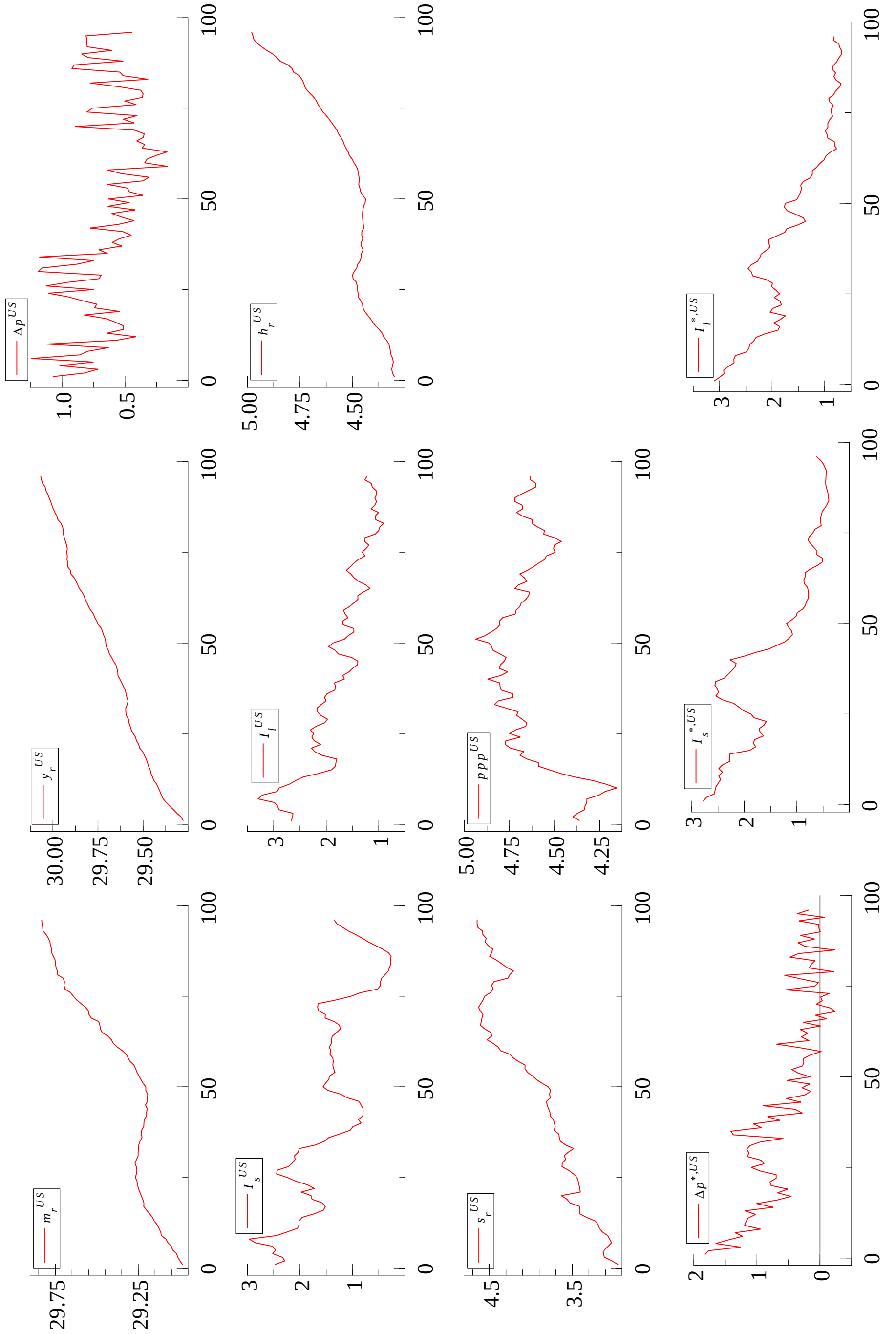

Figure B.1: US variables.

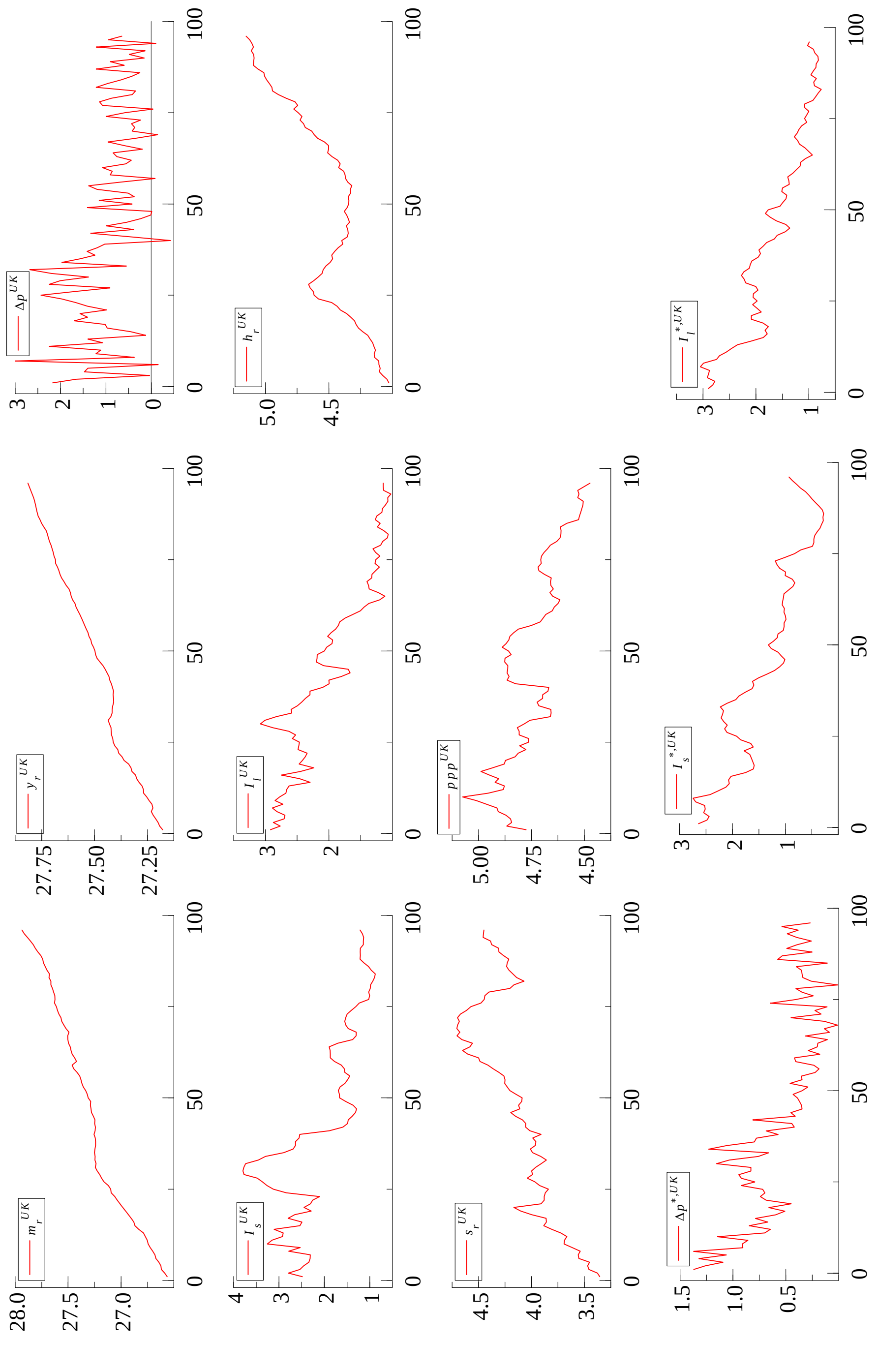

Figure B.2: UK variables.

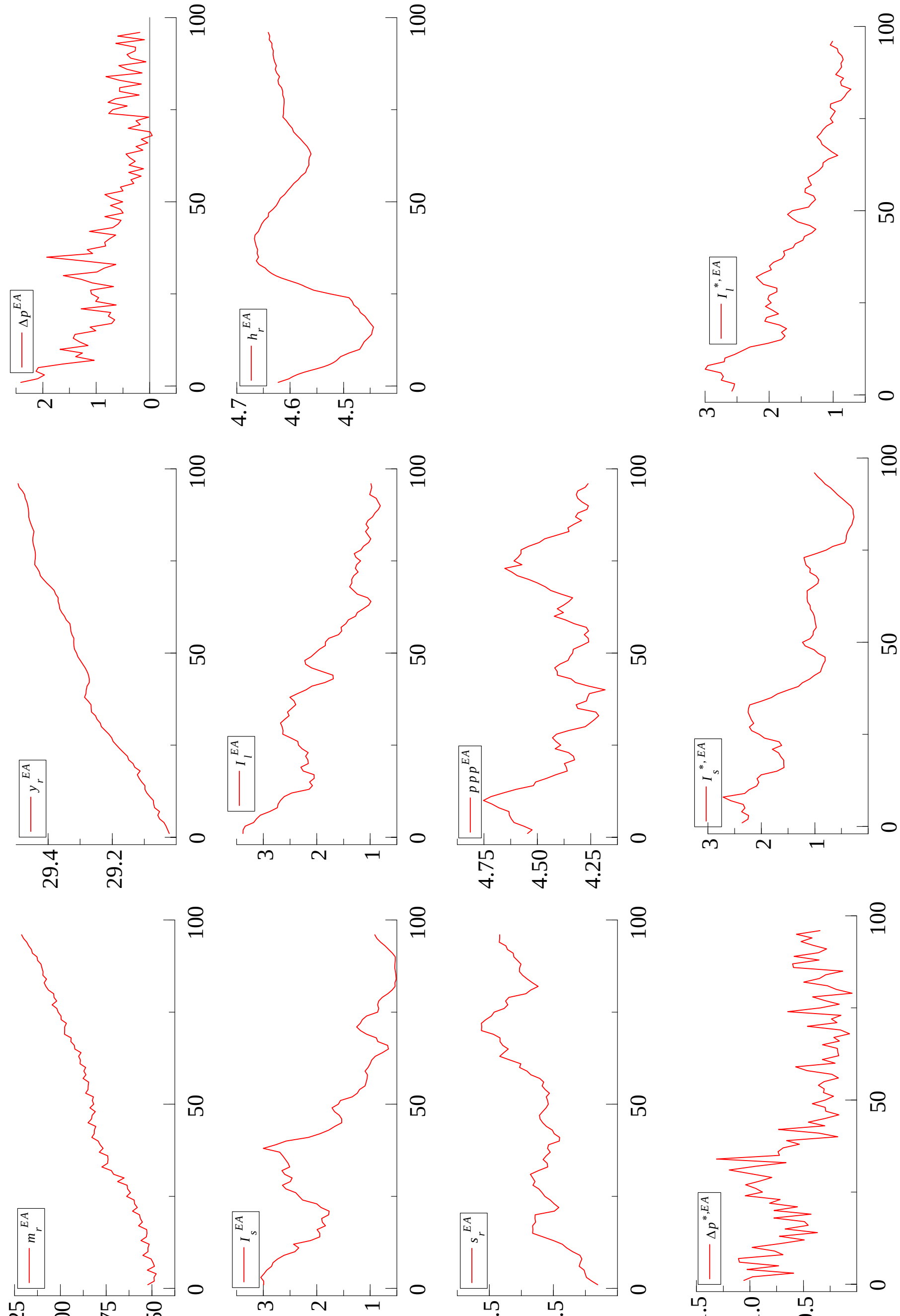

Figure B.3: EA variables.

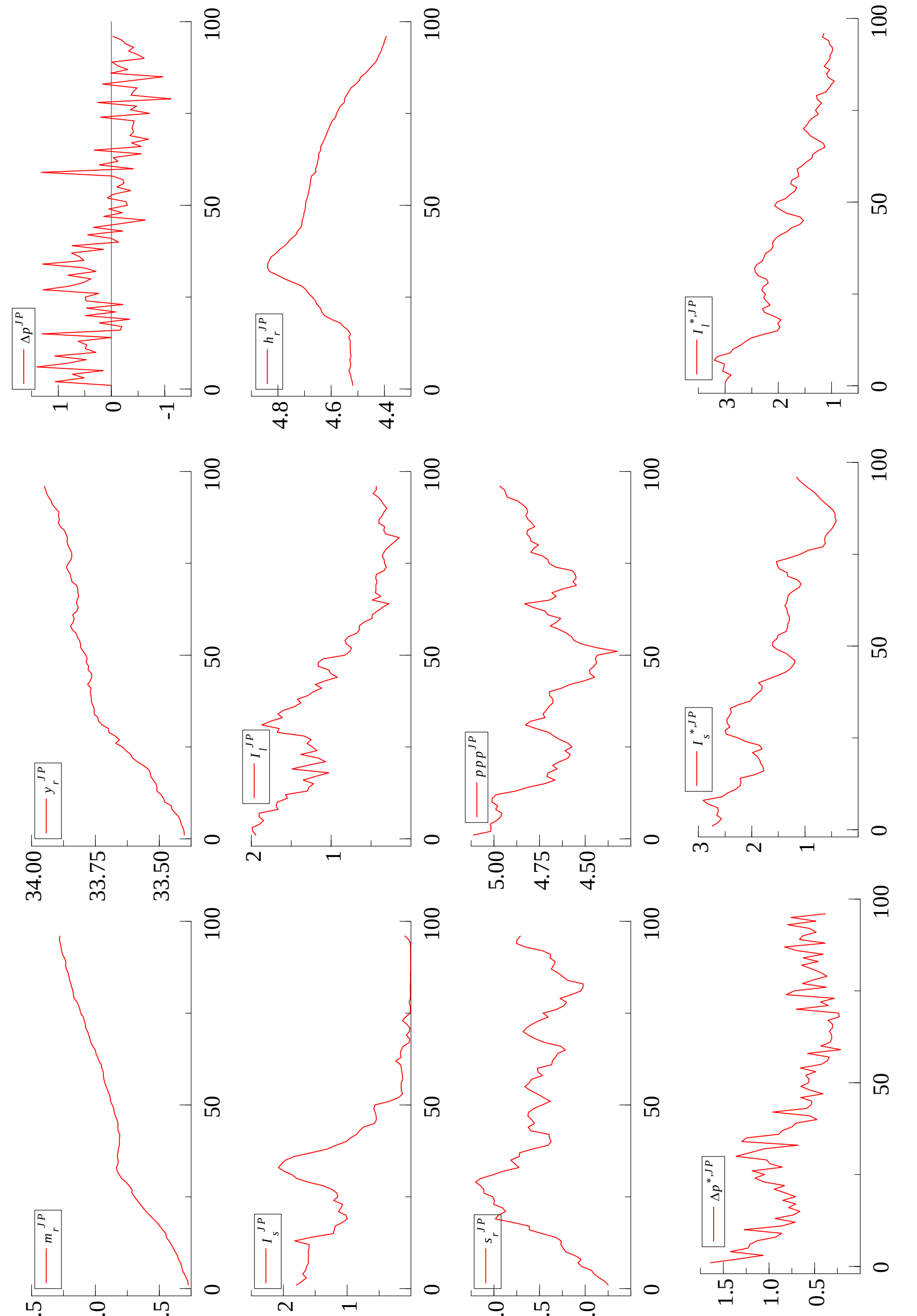

Figure B.4: JP variables.

C CVARX* results

C.1 US

Lag	Test statistic	p
LM-tests for no autocorrelation		
1	$\chi^2(64) = 72.70$	0.21
2	$\chi^2(64) = 76.43$	0.14
3	$\chi^2(64) = 85.25$	0.04
4	$\chi^2(64) = 55.07$	0.78
Test for multivariate normality		
	$\chi^2(16) = 42.15$	0.00
LM-tests for no ARCH effects		
1	$\chi^2(1296) = 1305.84$	0.42
2	$\chi^2(2592) = 2686.73$	0.10
3	$\chi^2(3888) = 3384.00$	1.00
4	$\chi^2(5184) = 3384.00$	1.00

Table C.1: US: misspecification tests.

$p-r$	r	Eigenvalue	Trace	Trace*	$CV_{95\%}$	p	p^*
11	0	0.70	556.33	418.05	321.94	0.00	0.00
10	1	0.65	442.35	299.04	273.04	0.00	0.00
9	2	0.59	344.41	225.38	228.15	0.00	0.07
8	3	0.49	261.34	166.86	187.25	0.00	0.34
7	4	0.42	198.67	117.83	150.35	0.00	0.74
6	5	0.36	147.70	87.61	117.45	0.00	0.77
5	6	0.31	106.44	65.93	88.55	0.00	0.67
4	7	0.29	71.19	48.70	63.66	0.01	0.48
3	8	0.21	38.99	28.94	42.77	0.12	0.57
2	9	0.12	17.23	14.91	25.73	0.41	0.59
1	10	0.05	5.25	4.90	12.45	0.57	0.62

Table C.2: US: rank test statistics.

$\boldsymbol{\beta}'$	m_r	y_r	Δp	I_s	I_l	h_r	s_r	ppp	Δp^*	I_s^*	I_l^*	t
β_1'	0.00 [NA]	1.00 [NA]	−0.06 [−10.34]	0.00 [NA]	0.06 [10.34]	0.00 [NA]	0.00 [NA]	0.00 [NA]	0.00 [NA]	0.00 [NA]	0.00 [NA]	−0.01 [−43.33]
β_2'	0.00 [NA]	−9.28 [−6.12]	−1.66 [−16.14]	1.00 [NA]	0.00 [NA]	0.00 [NA]	0.00 [NA]	0.00 [NA]	0.00 [NA]	0.00 [NA]	0.00 [NA]	0.08 [6.67]
β_3'	1.00 [NA]	−1.00 [NA]	0.00 [NA]	−0.11 [−4.94]	0.11 [4.94]	0.00 [NA]	0.00 [NA]	0.00 [NA]	0.00 [NA]	0.00 [NA]	0.00 [NA]	0.00 [NA]
β_4'	−1.00 [NA]	0.00 [NA]	0.00 [NA]	0.16 [3.86]	−1.48 [−17.77]	1.00 [NA]	0.00 [NA]	0.00 [NA]	0.00 [NA]	−0.71 [−14.39]	1.48 [17.77]	−0.01 [−6.05]
β_5'	0.00 [NA]	−12.98 [−15.43]	0.00 [NA]	0.43 [13.56]	0.00 [NA]	0.00 [NA]	1.00 [NA]	0.00 [NA]	0.00 [NA]	−0.43 [−13.56]	0.00 [NA]	0.08 [12.10]
β_6'	0.00 [NA]	0.00 [NA]	0.03 [6.95]	−0.03 [−6.95]	0.00 [NA]	0.00 [NA]	0.00 [NA]	1.00 [NA]	0.00 [NA]	0.00 [NA]	0.00 [NA]	0.00 [NA]
β_7'	0.00 [NA]	0.00 [NA]	0.07 [9.37]	0.00 [NA]	−0.07 [−9.37]	0.00 [NA]	0.00 [NA]	1.00 [NA]	−0.07 [−9.37]	0.00 [NA]	0.07 [9.37]	0.00 [NA]

$\boldsymbol{\alpha}$	α_1	α_2	α_3	α_4	α_5	α_6	α_7
Δm_r	0.22 [4.73]	−0.01 [−3.35]	−0.01 [−0.60]	0.01 [2.02]	0.03 [7.68]	0.05 [1.14]	−0.08 [−1.65]
Δy_r	−0.14 [−4.06]	−0.00 [−0.43]	−0.00 [−0.22]	−0.01 [−1.66]	0.02 [5.05]	0.15 [4.28]	−0.14 [−4.05]
$\Delta^2 p$	5.50 [3.91]	0.36 [4.48]	1.62 [4.10]	0.59 [3.78]	−0.28 [−2.25]	1.18 [0.83]	−0.87 [−0.60]
ΔI_s	−0.35 [−0.36]	−0.22 [−4.00]	−0.58 [−2.13]	0.16 [1.50]	0.07 [0.79]	0.61 [0.62]	−0.79 [−0.79]
ΔI_l	0.40 [0.59]	−0.13 [−3.29]	0.37 [1.96]	0.10 [1.29]	0.24 [3.98]	−1.73 [−2.54]	1.77 [2.56]
Δh_r	0.17 [3.20]	−0.01 [−2.18]	0.04 [2.50]	0.00 [0.74]	0.01 [3.20]	0.05 [0.94]	−0.05 [−0.93]
Δs_r	−0.76 [−2.09]	0.06 [2.99]	−0.00 [−0.02]	−0.05 [−1.30]	−0.02 [−0.73]	0.71 [1.94]	−0.59 [−1.58]
Δppp	1.36 [4.18]	−0.04 [−2.36]	−0.17 [−1.84]	0.19 [5.19]	−0.05 [−1.61]	0.99 [2.98]	−1.26 [−3.76]

Table C.3: US: cointegrating relations and adjustment structure.

C.2 UK

Lag	Test statistic	p
LM-tests for no autocorrelation		
1	$\chi^2(64) = 61.72$	0.56
2	$\chi^2(64) = 105.15$	0.00
3	$\chi^2(64) = 88.03$	0.02
4	$\chi^2(64) = 84.43$	0.04
Test for multivariate normality		
	$\chi^2(16) = 33.67$	0.01
LM-tests for no ARCH effects		
1	$\chi^2(1296) = 1276.45$	0.65
2	$\chi^2(2592) = 2640.83$	0.25
3	$\chi^2(3888) = 3384.00$	1.00
4	$\chi^2(5184) = 3384.00$	1.00

Table C.4: UK: misspecification tests.

$p-r$	r	Eigenvalue	Trace	Trace*	$CV_{95\%}$	p	p^*
11	0	0.77	518.03	403.12	321.94	0.00	0.00
10	1	0.59	381.35	292.71	273.04	0.00	0.00
9	2	0.52	297.84	226.89	228.15	0.00	0.06
8	3	0.52	227.94	173.36	187.25	0.00	0.21
7	4	0.42	158.92	124.83	150.35	0.01	0.54
6	5	0.32	107.32	87.61	117.45	0.19	0.77
5	6	0.23	70.77	57.91	88.55	0.48	0.90
4	7	0.17	46.60	38.62	63.66	0.57	0.89
3	8	0.15	29.04	24.95	42.77	0.57	0.79
2	9	0.09	13.34	11.70	25.73	0.71	0.83
1	10	0.05	4.67	4.53	12.45	0.65	0.67

Table C.5: UK: rank test statistics.

$\boldsymbol{\beta}'$	m_r	y_r	Δp	I_s	I_l	h_r	s_r	ppp	Δp^*	I_s^*	I_l^*	t
β_1'	0.00 [NA]	1.00 [NA]	−0.12 [−18.54]	0.00 [NA]	0.12 [18.54]	0.00 [NA]	0.00 [NA]	0.00 [NA]	0.00 [NA]	0.00 [NA]	0.00 [NA]	−0.01 [−42.15]
β_2'	0.00 [NA]	−9.82 [−7.25]	−6.46 [−16.14]	1.00 [NA]	0.00 [NA]	0.00 [NA]	0.00 [NA]	0.00 [NA]	0.00 [NA]	0.00 [NA]	0.00 [NA]	0.00 [NA]
β_3'	1.00 [NA]	−1.00 [NA]	0.00 [NA]	−0.22 [−13.34]	0.22 [13.34]	0.00 [NA]	0.00 [NA]	0.00 [NA]	0.00 [NA]	0.00 [NA]	0.00 [NA]	−0.01 [−15.26]
β_4'	−1.00 [NA]	0.00 [NA]	−2.41 [−4.40]	2.41 [4.40]	0.00 [NA]	1.00 [NA]	0.00 [NA]	0.00 [NA]	0.00 [NA]	8.57 [12.64]	0.00 [NA]	0.24 [14.23]
β_5'	0.00 [NA]	0.00 [NA]	0.00 [NA]	0.00 [NA]	7.65 [18.04]	0.00 [NA]	1.00 [NA]	0.00 [NA]	0.00 [NA]	0.00 [NA]	0.00 [NA]	0.16 [15.00]
β_6'	0.00 [NA]	0.00 [NA]	−0.24 [−10.47]	0.24 [10.47]	0.00 [NA]	0.00 [NA]	0.00 [NA]	1.00 [NA]	0.44 [17.60]	−0.44 [−17.60]	0.00 [NA]	0.00 [NA]
β_7'	0.00 [NA]	0.00 [NA]	−0.74 [−18.87]	0.00 [NA]	0.74 [18.87]	0.00 [NA]	0.00 [NA]	1.00 [NA]	0.74 [18.87]	0.00 [NA]	−0.74 [−18.87]	0.00 [NA]

$\boldsymbol{\alpha}$	α_1	α_2	α_3	α_4	α_5	α_6	α_7
Δm_r	0.02 [0.25]	−0.01 [−4.61]	−0.10 [−4.37]	−0.00 [−2.52]	−0.00 [−1.75]	−0.13 [−5.18]	0.09 [5.27]
Δy_r	−0.02 [−0.95]	0.00 [0.34]	−0.02 [−2.43]	−0.00 [−3.91]	−0.00 [−1.47]	−0.02 [−2.47]	0.01 [2.37]
$\Delta^2 p$	19.11 [4.61]	−0.08 [−1.15]	4.84 [3.70]	0.12 [3.00]	−0.26 [−2.96]	5.43 [4.04]	−2.60 [−2.78]
ΔI_s	2.59 [2.89]	−0.06 [−3.54]	0.18 [0.65]	−0.01 [−1.24]	−0.05 [−2.58]	−0.75 [−2.58]	0.37 [1.82]
ΔI_l	2.09 [2.76]	−0.02 [−1.50]	1.43 [5.97]	0.03 [4.54]	−0.09 [−5.37]	1.06 [4.30]	−0.70 [−4.07]
Δh_r	0.50 [3.78]	−0.00 [−1.13]	−0.07 [−1.77]	−0.01 [−4.63]	−0.01 [−2.10]	−0.10 [−2.36]	−0.02 [−0.65]
Δs_r	−0.09 [−0.25]	−0.01 [−2.18]	−0.41 [−3.39]	−0.01 [−3.54]	−0.00 [−0.22]	−0.44 [−3.60]	0.28 [3.24]
Δppp	0.02 [0.10]	−0.01 [−1.99]	−0.16 [−2.46]	−0.00 [−1.39]	−0.01 [−2.13]	−0.29 [−4.43]	0.17 [3.70]

Table C.6: UK: cointegrating relations and adjustment structure.

C.3 Euro area

Lag	Test statistic	p
LM-tests for no autocorrelation		
1	$\chi^2(64) = 77.78$	0.12
2	$\chi^2(64) = 105.15$	0.12
3	$\chi^2(64) = 88.03$	0.41
4	$\chi^2(64) = 84.43$	0.33
Test for multivariate normality		
	$\chi^2(16) = 33.67$	0.11
LM-tests for no ARCH effects		
1	$\chi^2(1296) = 1368.51$	0.08
2	$\chi^2(2592) = 2674.19$	0.13
3	$\chi^2(3888) = 3440.03$	1.00
4	$\chi^2(5184) = 3384.00$	1.00

Table C.7: Euro area: misspecification tests.

$p-r$	r	Eigenvalue	Trace	Trace*	$CV_{95\%}$	p	p^*
11	0	0.67	492.73	364.95	321.94	0.00	0.00
10	1	0.62	388.29	276.66	273.04	0.00	0.03
9	2	0.55	297.89	212.94	228.15	0.00	0.21
8	3	0.47	221.94	159.72	187.25	0.00	0.52
7	4	0.41	161.86	117.51	150.35	0.01	0.74
6	5	0.33	112.61	80.30	117.45	0.10	0.92
5	6	0.29	75.40	46.04	88.55	0.31	1.00
4	7	0.18	42.97	26.55	63.66	0.74	1.00
3	8	0.10	24.49	15.78	42.77	0.81	1.00
2	9	0.08	14.93	5.58	25.73	0.59	1.00
1	10	0.07	6.81	2.74	12.45	0.38	0.89

Table C.8: Euro area: rank test statistics.

$\boldsymbol{\beta}'$	m_r	y_r	Δp	I_s	I_l	h_r	s_r	ppp	Δp^*	I_s^*	I_l^*	t
β_1'	0.00 [NA]	1.00 [NA]	0.00 [NA]	0.00 [NA]	0.00 [NA]	0.00 [NA]	0.00 [NA]	0.00 [NA]	−0.15 [−7.35]	0.00 [NA]	0.15 [7.35]	−0.00 [−17.04]
β_2'	0.00 [NA]	−4.25 [−6.55]	−2.36 [−32.01]	1.00 [NA]	0.00 [NA]	0.00 [NA]	0.00 [NA]	0.00 [NA]	0.00 [NA]	0.00 [NA]	0.00 [NA]	0.00 [NA]
β_3'	1.00 [NA]	−1.00 [NA]	0.00 [NA]	−0.17 [−12.11]	0.17 [12.11]	0.00 [NA]	0.00 [NA]	0.00 [NA]	0.00 [NA]	0.00 [NA]	0.00 [NA]	−0.00 [−13.27]
β_4'	−1.00 [NA]	0.00 [NA]	0.18 [22.12]	0.00 [NA]	−0.18 [−22.12]	1.00 [NA]	0.00 [NA]	0.00 [NA]	−0.19 [−8.66]	0.00 [NA]	0.19 [8.66]	0.01 [18.64]
β_5'	0.00 [NA]	−1.00 [NA]	−1.73 [−21.41]	0.00 [NA]	1.73 [21.41]	0.00 [NA]	1.00 [NA]	0.00 [NA]	−1.96 [−9.35]	0.00 [NA]	1.96 [9.35]	0.00 [0.19]
β_6'	0.00 [NA]	0.00 [NA]	0.00 [NA]	0.00 [NA]	0.00 [NA]	0.00 [NA]	0.00 [NA]	1.00 [NA]	−3.50 [−11.30]	3.50 [11.30]	0.00 [NA]	0.00 [NA]
β_7'	0.00 [NA]	0.00 [NA]	−0.94 [−14.32]	0.00 [NA]	0.94 [14.32]	0.00 [NA]	0.00 [NA]	1.00 [NA]	0.00 [NA]	0.00 [NA]	0.00 [NA]	0.00 [NA]

$\boldsymbol{\alpha}$	α_1	α_2	α_3	α_4	α_5	α_6	α_7
Δm_r	−0.38 [−3.41]	0.01 [1.78]	−0.09 [−2.01]	0.21 [3.42]	0.01 [0.70]	0.00 [0.22]	0.01 [0.71]
Δy_r	−0.09 [−2.51]	−0.00 [−1.74]	0.02 [1.23]	−0.01 [−0.50]	0.01 [2.71]	0.00 [1.27]	−0.00 [−0.82]
$\Delta^2 p$	−4.66 [−1.98]	0.27 [2.05]	2.26 [2.30]	1.79 [1.39]	−0.16 [−0.82]	0.00 [0.02]	1.05 [4.51]
ΔI_s	1.00 [1.43]	−0.08 [−1.98]	1.20 [4.10]	−0.70 [−1.85]	0.04 [0.78]	0.03 [2.05]	−0.25 [−3.65]
ΔI_l	0.23 [0.31]	0.10 [2.37]	0.70 [2.27]	0.72 [1.80]	−0.08 [−1.36]	0.04 [2.75]	−0.07 [−1.04]
Δh_r	0.11 [2.88]	−0.01 [−4.34]	−0.10 [−6.19]	−0.16 [−7.77]	−0.00 [−0.99]	0.00 [1.36]	−0.01 [−1.78]
Δs_r	2.64 [3.97]	0.16 [4.42]	1.19 [4.26]	−0.25 [−0.67]	−0.28 [−5.17]	0.06 [4.91]	0.08 [1.25]
Δppp	0.06 [0.18]	0.08 [3.95]	0.40 [2.76]	0.51 [2.70]	−0.02 [−0.69]	0.01 [1.06]	−0.04 [−1.29]

Table C.9: Euro area: cointegrating relations and adjustment structure.

C.4 Japan

Lag	Test statistic	p
LM-tests for no autocorrelation		
1	$\chi^2(64) = 72.70$	0.21
2	$\chi^2(64) = 76.43$	0.14
3	$\chi^2(64) = 85.25$	0.04
4	$\chi^2(64) = 55.07$	0.78
Test for multivariate normality		
	$\chi^2(16) = 42.15$	0.00
LM-tests for no ARCH effects		
1	$\chi^2(1296) = 1305.84$	0.42
2	$\chi^2(2592) = 2686.73$	0.10
3	$\chi^2(3888) = 3384.00$	1.00
4	$\chi^2(5184) = 3384.00$	1.00

Table C.10: Japan: misspecification tests.

$p-r$	r	Eigenvalue	Trace	Trace*	$CV_{95\%}$	p	p^*
11	0	0.71	550.14	391.93	321.94	0.00	0.00
10	1	0.61	433.00	310.77	273.04	0.00	0.00
9	2	0.56	343.39	231.66	228.15	0.00	0.03
8	3	0.51	266.72	176.61	187.25	0.00	0.16
7	4	0.49	200.29	132.36	150.35	0.00	0.33
6	5	0.38	137.61	83.62	117.45	0.00	0.87
5	6	0.32	92.71	43.31	88.55	0.02	1.00
4	7	0.21	56.32	26.45	63.66	0.18	1.00
3	8	0.14	33.60	4.93	42.77	0.31	1.00
2	9	0.11	19.86	11.95	25.73	0.24	0.81
1	10	0.09	8.68	6.07	12.45	0.21	0.46

Table C.11: Japan: rank test statistics.

$\boldsymbol{\beta}'$	m_r	y_r	Δp	I_s	I_l	h_r	s_r	ppp	Δp^*	I_s^*	I_l^*	t
β_1'	0.00 [NA]	1.00 [NA]	−33.97 [−10.58]	0.00 [NA]	46.43 [13.39]	0.00 [NA]	0.00 [NA]	0.00 [NA]	0.00 [NA]	0.00 [NA]	0.00 [NA]	0.29 [10.41]
β_2'	1.00 [NA]	−1.00 [NA]	0.00 [NA]	−0.70 [−14.31]	0.70 [14.31]	0.00 [NA]	0.00 [NA]	0.00 [NA]	0.00 [NA]	0.00 [NA]	0.00 [NA]	−0.00 [−17.50]
β_3'	−1.00 [NA]	0.00 [NA]	2.24 [10.99]	−2.24 [−10.99]	0.00 [NA]	1.00 [NA]	0.00 [NA]	0.00 [NA]	0.00 [NA]	0.00 [NA]	0.00 [NA]	0.01 [36.57]
β_4'	−3.45 [−6.90]	−1.00 [NA]	−4.34 [−10.03]	4.34 [10.03]	0.90 [6.36]	0.00 [NA]	1.00 [NA]	0.58 [3.94]	0.00 [NA]	0.00 [NA]	−0.90 [−6.36]	0.02 [4.00]
β_5'	0.00 [NA]	0.00 [NA]	−9.01 [−11.73]	9.01 [11.73]	0.00 [NA]	0.00 [NA]	0.00 [NA]	1.00 [NA]	0.21 [3.82]	−0.21 [−3.82]	0.00 [NA]	0.00 [NA]
β_6'	0.00 [NA]	0.00 [NA]	−16.10 [−12.07]	0.00 [NA]	16.10 [12.07]	0.00 [NA]	0.00 [NA]	1.00 [NA]	2.22 [7.27]	0.00 [NA]	−2.22 [−7.27]	0.00 [NA]

$\boldsymbol{\alpha}$	α_1	α_2	α_3	α_4	α_5	α_6
Δm_r	−0.00 [−5.82]	0.01 [0.35]	−0.03 [−1.95]	0.00 [1.23]	−0.01 [−2.39]	0.00 [2.54]
Δy_r	0.00 [0.91]	0.15 [3.70]	−0.02 [−0.90]	0.01 [1.18]	0.00 [0.30]	−0.01 [−4.51]
$\Delta^2 p$	−0.02 [−1.25]	−0.57 [−0.30]	−0.35 [−0.30]	−0.48 [−2.14]	0.13 [0.43]	0.13 [1.80]
ΔI_s	−0.00 [−0.86]	1.51 [5.41]	0.12 [0.71]	0.08 [2.47]	0.08 [1.72]	−0.05 [−4.51]
ΔI_l	−0.00 [−0.19]	−0.59 [−0.95]	−0.57 [−1.51]	−0.16 [−2.23]	−0.10 [−1.01]	0.02 [0.75]
Δh_r	0.00 [0.28]	−0.02 [−0.72]	−0.02 [−1.33]	0.01 [2.03]	−0.01 [−2.46]	0.00 [0.12]
Δs_r	−0.00 [−0.53]	−1.17 [−2.73]	−1.05 [−3.95]	−0.17 [−3.36]	−0.25 [−3.78]	0.05 [2.98]
Δppp	0.01 [4.09]	−0.63 [−2.97]	−0.65 [−4.91]	−0.10 [−3.92]	−0.16 [−4.93]	0.01 [1.19]

Table C.12: Japan: cointegrating relations and adjustment structure.

D GIRF results

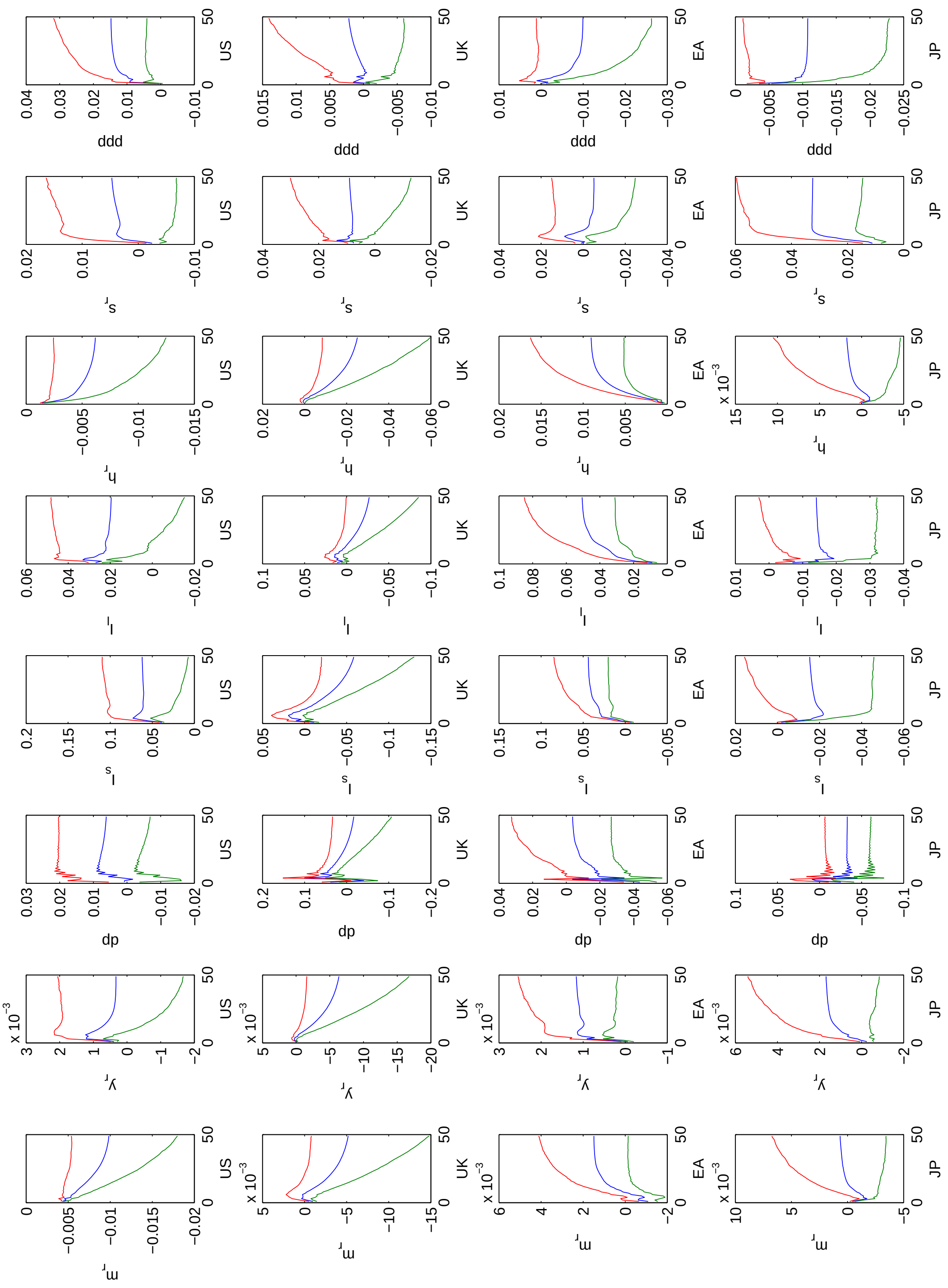

Figure C.1: GIRF: One-standard deviation negative shock to US money supply (90-per cent error bands).

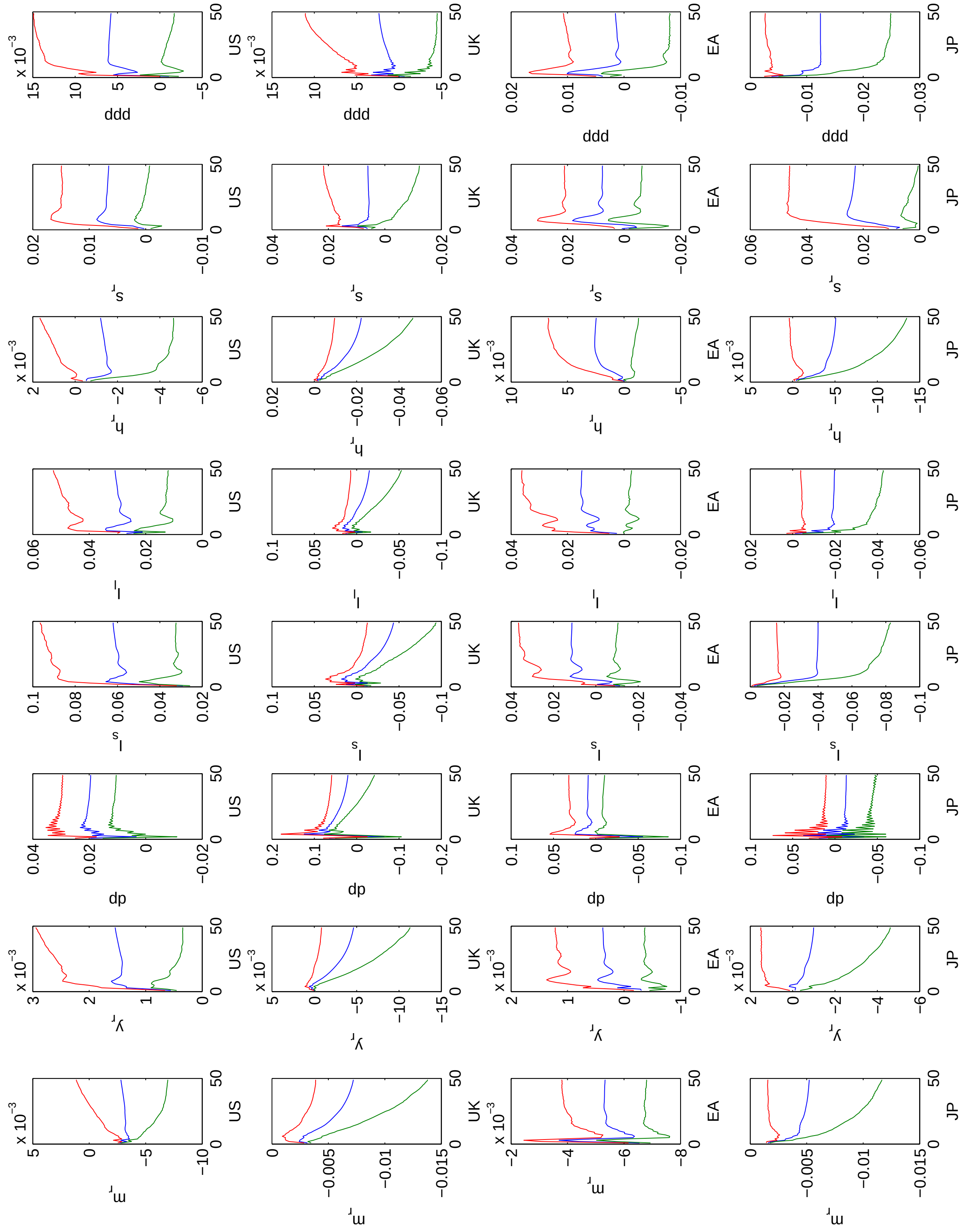

Figure C.2: GIRF: One-standard deviation negative shock to global money supply (90-per cent error bands).

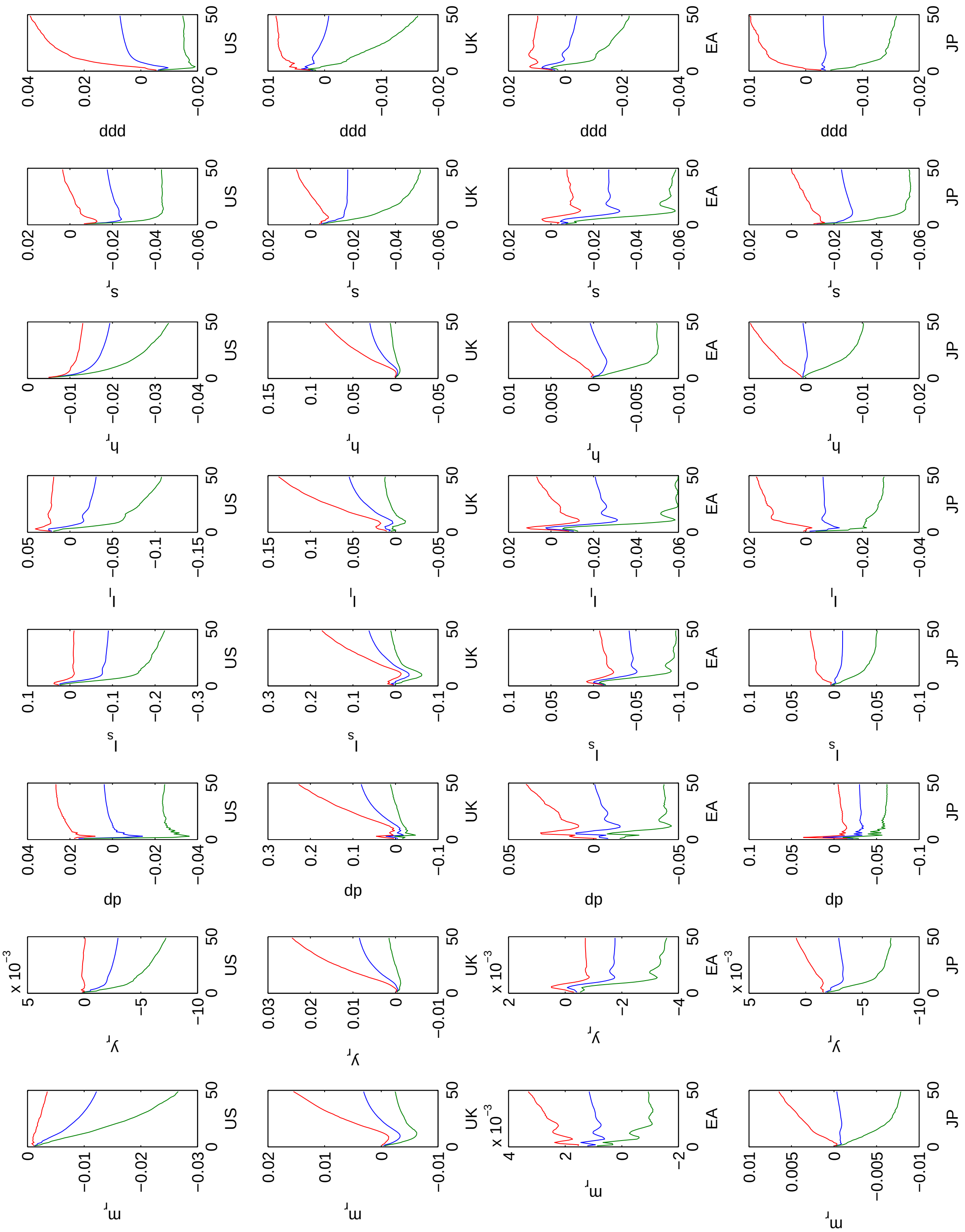

Figure C.3: GIRF: One-standard deviation negative shock to US house prices (90-per cent error bands).

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