FACULTY OF SOCIAL SCIENCES
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Macroeconomics under the Microscope

Heterogeneity and Uncertainty

PhD dissertation

Emil Holst Partsch

Supervisors: Emiliano Santoro and Jeppe Druedahl

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Contents

Foreword

I first discovered the power of the economic method while writing my bachelor thesis, a game theoretical exercise on inter-country climate negotiations. For the first time, I felt that the toolbox I had given during my undergraduate studies was useful for investigating real-world issues. I was, however, still rather discontent with the aggregation assumptions typically made in undergraduate courses. Thus, at the initiation of my master's degree, I felt the need to investigate how other fields thought about this issue. Hence, I took two great courses in the physics department given by professor Kim Sneppen and associate professor Namiko Mitarai respectively. Here I discovered statistical physics, where systems are modeled explicitly as many small interacting objects subject to random processes. It was also here I learned the power of numerical methods. On my own, I discovered that economics has a long tradition of dynamic models with explicit distributions, specifically the heterogeneous agent literature – which interestingly also required a great deal of numerical skills. As it turned out, there were already people in the economics department at Copenhagen University with such an interest. Hence, I wrote my master's thesis on heterogeneous agent macro-models with associate professor Jeppe Druedahl as my supervisor, who would later become one of my Ph.D. supervisors. Despite COVID lockdowns making the already lonely process of a Ph.D. even more lonely and full of obstacles, practical as well as mental, the Ph.D. process taught me a great deal about macroeconomics, mainly thanks to my supervisors, professor Emiliano Santoro and associate professor Jeppe Druedahl, and my co-authors Ivan Petrella and (at the time, fellow Ph.D. student) Luca Neri. Though the supply of Ph.D. courses was limited, two courses, in particular, helped shape and tie together the final product. The first was an incredibly eye-opening course given by professor Petr Sedlacek on firm heterogeneity in macroeconomics, a topic I had not touched much upon prior but ended up being invaluable as a large part of my thesis ended up dealing with firm heterogeneity. Assistant professor Ludwig Straub on HANK models gave the second course. I

took this course towards the end of my Ph.D., so the chapters of my thesis were already rather settled. Despite that, the course was incredibly inspiring and helped me clarify how to tie my research together.

I want to thank the Danish central bank and especially the former head of research, Federico Ravenna, for believing in me and choosing to fund my Ph.D. The central bank provided an excellent research environment of like-minded people, significantly adding to the research environment available at the university. Regarding the university, I was lucky that at the last stage of my Ph.D. years, the macro research environment grew significantly due to the establishment of the international HANK group under Søren Hove Ravn. This thesis has greatly benefitted from discussions with my fellow Ph.D. students from this group, Jacob Marott Sundram and Nicolai Waldström. In addition, as I went further into empirical analysis than initially planned, I greatly benefitted from intense discussions with office colleagues and great econometricians Christian Philip Hoeck and Rasmus Bisgaard Larsen. Finally, I want to thank my family and those close to me for listening to my rants and their patience and support. I also want to thank my father for igniting my interest in social science at an early age and for his invaluable insights into the, at times, very unpredictable research process.

Emil Holst Partsch

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English summary

This Ph.D. thesis consists of four self-contained chapters, which all investigate macroeconomic questions from a micro perspective: through structural models with either full distributional heterogeneity or approximations thereof, and in one case, through microdata. In addition, all projects are bound together by incorporating risk and/or uncertainty as drivers of outcomes.

In the *first* chapter, co-authored with Emiliano Santoro and Ivan Petrella, we quantify the direct and indirect effects of monetary policy transmission in a two-sector Heterogeneous Agent New Keynesian (HANK) with durable and nondurable goods. With an empirically plausible share of liquidity-constrained consumers, we find that transitory income (indirect) effects drive the brunt of the consumption response of both goods and are key in generating positive co-movement when prices are asymmetrically sticky between sectors. Direct effects, however, regain strength relative to one-sector HANK models, as durables are quite interest rate sensitive. We show that results are robust to the realistic cases of deficit financing and sticky wages.

In the *second chapter*, co-authored with Emiliano Santoro and Ivan Petrella, we build tractable two-sector HANK models where households may infrequently participate in financial markets. Both durables and nondurables are available for household consumption. In a model with two household types – households constrained in bonds and those not – having access to durables implies a de facto risk-sharing condition. Thus, the amplification of household-specific and sectoral nondurable consumption in response to monetary shocks depends on preference heterogeneity over nondurables. This is unlike similar one-sector models, where fiscal redistribution from unconstrained to constrained households plays a prominent role. When introducing a third agent, hand-to-mouth consumers with no access to durables or financial assets, fiscal redistribution amplifies the conditional volatility of GDP, which is opposite to one-sector

economies.

In the *third* chapter, co-authored with Jeppe Druedahl, we solve and evaluate global solutions of heterogeneous agent models with non-linear aggregate dynamics. Specifically, we investigate models where a small number of states can summarize aggregate dynamics. We let the perceived law-of-motion that agents use for now- and forecasting be arbitrarily non-linear. We use either a neural net or radial basis function interpolation to deliver precise global solutions. Radial basis function interpolation is much faster and more stable in terms of convergence. We solve our benchmark model with an aggregate non-linearity and period-by-period market clearing in less than 15 minutes.

The *fourth* and final chapter, co-authored with Luca Neri, documents the dynamic effects of uncertainty shocks on skilled and unskilled labor using Danish registry data. In particular, we use Denmark as a small open economy subject to several aggregate uncertainty shocks. Identification relies on differential industry exposure to these shocks. Our results highlight that the labor displacement effects ascribed to uncertainty shocks affect unskilled labor more than skilled. We build a dynamic partial equilibrium heterogeneous firm with skilled and unskilled labor inputs and heterogeneous labor adjustment costs to rationalize our findings.

For convenience, the full abstracts of the four projects are listed below.

1. Durable Goods and Monetary Transmission in a HANK Economy

with Emiliano Santoro and Ivan Petrella

We quantify the direct and indirect effects of monetary policy transmission in a two-sector Heterogeneous Agent New Keynesian (HANK) setting where consumers may purchase both durable and nondurable goods. Given a realistic share of liquidity constrained households, (indirect) transitory income effects represent the most significant transmission channel of monetary shocks and are key in generating positive comovement between durable and nondurable expenditure. Moreover, (direct) interest rate effects regain strength relative to one-sector HANK models through their marked impact on durable spending. Our results are robust to realistic extensions involving deficit financing and sticky wages.

2. Long-lived Durables in T(H)ANK Economies

with Emiliano Santoro and Ivan Petrella

We devise tractable heterogeneous-agent New Keynesian economies where households may infrequently participate in financial markets. Both nondurable and durable goods are available for consumption. To the extent that durables feature slow depreciation rates, both their stock and their shadow value display negligible variation in the face of temporary monetary shocks. In light of this, the marginal utility of nondurable consumption for households having access to durable purchases is approximately the same, thus realizing, *de facto*, a risk-sharing condition. As a result, when all agents may access durable purchases, regardless of their financial status, the amplification of both household-specific and sectoral nondurable consumption in the face of monetary shocks may only depend on preference heterogeneity over nondurables. By contrast, factors typically key in shaping monetary transmission in benchmark one-sector economies- primarily fiscal redistribution from financially unconstrained to constrained households- only affect households' durable expenditure, with their effects intimately connected with the degree of sectoral price stickiness. When introducing hand-to-mouth consumers with no access to durable purchases and financial assets, fiscal redistribution tends to amplify the conditional volatility of GDP, unlike in one-sector economies featuring only nondurables.

3. Global Solutions of Heterogeneous Agent Models with Non-linear Aggregate Dynamics

with Jeppe Druedahl

In this paper, we present and evaluate global solution methods for heterogeneous agent models with non-linear aggregate dynamics. We consider models with weak approximate aggregation, where the aggregate dynamics can be summarized with a small number of states. We allow the perceived law-of-motion the agents use for now- and forecasting to be non-linear, and do not impose any parametric restrictions on it. Specifically, we derive the perceived law-of-motion using either a *neural net* or *radial basis function interpolation*. Both methods deliver precise global solutions, but radial basis function interpolation is faster because of a slow training step for the neural net, and more stable in terms of ensuring convergence. We are able to globally solve our benchmark model with an aggregate non-linearity and period-by-period market clearing in less than 15 minutes on a desktop computer.

4. Firm Uncertainty and Labor Composition Dynamics

with Luca Neri

We document the effects of uncertainty shocks on *skilled* and *unskilled* employment at the firm level using Danish registry data. To investigate the potential effects of uncertainty on net-hiring, we use that industries are differentially exposed to several aggregate shocks. We take advantage of this fact to identify industry-specific uncertainty shocks. We show that, while unskilled net-hiring is negatively affected by uncertainty shocks on impact, skilled net-hiring is not. Our dynamic approach shows that skilled labor falls with a lag. Unskilled labor shows similar dynamics, with the effect of uncertainty being strongest after impact. Our results highlight that labor displacement effects ascribed to uncertainty shocks affect unskilled labor relatively and absolutely more than skilled and are persistent. We contextualize our empirical findings within a heterogeneous firm model with skilled and unskilled labor inputs and heterogeneous labor adjustment costs.

Danish summary

Denne Ph.D.-afhandling indeholder fire selvstændige kapitler, der alle undersøger makroøkonomiske spørgsmål fra et mikro-perspektiv: gennem strukturelle modeller med enten fuld fordelingsmæssig heterogenitet eller approksimeringer heraf, og i et tilfælde ved brug af mikrodata. Derudover er alle projekter bundet sammen ved at inkorporere risiko og/eller usikkerhed som essentiel for udfald.

I første kapitel, skrevet med Emiliano Santoro og Ivan Petrella, kvantificerer vi "direkte" og "indirekte" effekter af pengepolitisike stød i en to-sektor heterogen agent Ny-Keynesiansk (HANK) model, hvor husholdninger kan forbruge både varige og ikke-varige forbrugsgoder. Vi finder at transitoriske indkomst (indirekte) effekter driver størstedelen af forbrugsresponsen i begge goder til et pengepolitisk stød, og at dette er essentielt for at skabe ko-variation når prisstivheder er asymmetriske mellem sektorer. Dog genoprettes noget af styrken i direkte effekter (fra repræsentativ agent modeller) relativt til en sektor HANK modeller, da varige forbrugsgoder er tilstrækkeligt sensitive over for renter. Vi viser at vores resultater er robuste over for realistiske modeludvidelser med underskudsfinansiering og stive nominelle lønninger.

I andet kapitel, skrevet med Emiliano Santoro og Ivan Petrella, bygger vi analytiske to-sektor HANK modeller, hvor husholdninger ind i mellem kan deltage i finansielle markeder. Både varige og ikke-varige forbrugsgoder er tilgængelige for husholdninger. I en model med to typer af husholdninger – kreditbegrænsede og kredit-ubegrænsede – betyder adgangen til varige forbrugsgoder at der er en de fakto risikodelingsbetingelse. Som følge heraf vil amplificering af husholdningsspecifik og sektormæssig ikke-varigt forbrugsgode forbrug i respons til pengepolitiske stød afhænge af præference heterogenitet over ikke-varige forbrugsgoder. Dette er modsat lignende et-sektor økonomier, hvor skattemæssig omfordeling fra kreditbegrænsede til ubegrænsede husholdninger spiller en stor rolle. Når man introducerer en tredje type husholdning, "hånd-til-mund" forbrugere, uden adgang til varige forbrugsgoder eller finansielle aktiver, vil skattemæssig omfordeling amplificere betinget BNP volatilitet. Dette resultat er modsat lignende et-sektor økonomier.

I tredje kapitel, skrevet med Jeppe Druedahl, løser og evaluerer vi globale løsninger af heterogen agent modeller med aggregerede ikke-lineariteter. Specifikt undersøger vi modeller hvor et lille antal states ("tilstande") kan opsummere aggregeret dynamik. Vi lader den opfattede bevægelseslov ("PLM") som agenter burger til at now- og forecasts være arbitrært ikke-lineær. Vi burger enten neurale net eller "radial basis function" (RBF) interpolation til at give os præcise globale løsninger. RBF interpolation er meget hurtigere og mere stabilt i forhold til konvergens end neurale net. Med RBF interpolation kan vi løse vores benchmark model med aggregeret ikke-linearitet og periode-for-periode markeds clearing på mindre end 15 minutter.

Det fjerde og sidste kapitel, skrevet med Luca Neri, dokumenterer dynamiske effekter af usikkerhedsstød på faglært og ufaglært arbejdskraft ved brug af dansk register data. Vi bruger at Danmark er en lille åben økonomi der er udsat for en række aggregerede usikkerhedsstød. Vores identifikationsstrategi beror på at forskellige industrier er forskellig eksponeret over for disse stød. Vores resultater viser at de negative beskæftigelseseffekter der tilskrives til usikkerhedsstød rammer ufaglærte arbejdere hårdest. Vi bygger en partiel ligevægts heterogen virksomheds model med faglærte og ufaglærte arbejdere som produktionsinput og arbejdskraftsjusteringsomkostninger til at rationalisere vores empiriske resultater.

Chapter 1

Durable Goods and Monetary Transmission in a HANK Economy

Durable Goods and Monetary Transmission in a HANK Economy

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August 31, 2022

Abstract

We quantify the direct (interest-rate) and indirect (general-equilibrium) effects of monetary policy transmission in a two-sector Heterogeneous Agent New Keynesian (HANK) setting where consumers may purchase both durable and nondurable goods. Given a realistic share of liquidity-constrained households, indirect effects represent the most significant transmission channel of monetary shocks, and are key in generating positive comovement between durable and nondurable expenditure. Moreover, direct effects (re)gain strength relative to standard one-sector HANK models through their marked impact on durable spending. Our results are robust to realistic extensions involving deficit government financing and sticky wages.

Keywords: Heterogeneous agents, durable goods, monetary policy.

JEL codes: E21, E31, E40, E44, E52.

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1 Introduction

A flourishing literature has recently emerged with the aim of incorporating rich household heterogeneity into the workhorse New Keynesian model, yielding learnings that profoundly change our understanding of the transmission of monetary policy. A main insight of these economies is that, in the presence of transitory income shocks, general-equilibrium effects drive the brunt of the response to monetary policy shocks (Kaplan et al., 2018; Auclert, 2019). This stands in stark contrast with the predictions of Representative Agent New Keynesian (RANK) economies, where nearly the entire response is driven by intertemporal substitution. Such Heterogeneous Agent New Keynesian (HANK) models typically feature one sector and one type of (perishable) consumption goods. However, it is well known that durable spending is much more interest-rate sensitive than nondurable spending (see, e.g., Mankiw, 1985), along with typically being more volatile over the business cycle. Since durables differ in important ways from nondurables, being both a consumption good and a store of value involved in household-portfolio choices, it seems legitimate to ask whether the lessons learned from one-sector HANK frameworks carry over to richer models of consumption. More specifically: what are the main propagation channels of monetary shocks on durable and nondurable expenditure in the presence of meaningful wealth heterogeneity? And, further, how does the answer to such question help us refine our understanding of monetary policy?

To address these questions, we devise and calibrate a two-sector HANK model based on US data. We retain the building blocks of standard two-sector New Keynesian models with asymmetric price stickiness between sectors, in the vein of Barsky et al. (2007) and Monacelli (2009), augmented to reflect uninsurable idiosyncratic risk on the household side.[1] We then decompose the response of consumption on both durables and nondurables to a contractionary monetary policy shock into a *direct* (or interest-rate) effect—as captured by intertemporal substitution—and an *indirect* effect, which operates through the general-equilibrium increase in labor demand. In turn, we further decompose the latter into the response ascribable to changes in the *relative price* of durables to nondurables, and to *pure income* effects.

[1]Relative to the HA literature that deals with durable expenditure at the household level, we focus on durable adjustment along the intensive margin, rather than on the extensive margin (in this respect, see Berger and Vavra, 2015; Harmenberg and Öberg, 2021; McKay and Wieland, 2021, among others).

We show that, even in the presence of a realistic wealth distribution, durables are more interest-rate sensitive than nondurables, and that interest-rate effects account for a non-negligible share of the total response of consumer durables. On the other hand, the brunt of the response of both durables and nondurables is driven by indirect effects, in line with the quantitative insights from one-sector HANK economies à la Kaplan et al. (2018). Decomposing the response of durable consumption further, based on liquid-wealth holdings, allows us to ascribe the importance of indirect effects to the hand-to-mouth behavior of liquidity-constrained households, while durables' interest-rate sensitivity is largely driven by savers reallocating resources from their stock of durables to bond holdings, in the face of a monetary contraction. On a side note, we show that general-equilibrium effects are key in overcoming the relative-price force that induces consumers to substitute durables for nondurables and *vice-versa*—depending on the relative degree of sectoral price stickiness—thus allowing us to address the comovement puzzle that typically characterizes otherwise standard two-sector RANK models with asymmetric sectoral price stickiness (Barsky et al., 2007).

Our results are robust to realistic extensions, such as deficit financing. We also augment our model to feature sticky wages, as it has become standard in the HANK literature (see, e.g., Auclert et al., 2020b). In this case, indirect effects are responsible for an even larger share of the response of both types of consumption, being relative-price effects relatively muted. While the interest rate appears as less of a driver of the response of durables, as compared with the case of flexible wages, it is still true that durables are more interest-rate sensitive than nondurables. In addition, interest-rate effects—as mainly channeled through savers' portfolio reallocation—capture a non-negligible fraction of the response of durables.

Related literature We relate to a burgeoning literature on monetary policy transmission in New Keynesian models with rich wealth distributions. Our work is inspired by the seminal work of Kaplan et al. (2018), who investigates the effects of monetary policy in a rich calibrated one-sector HANK model. In this respect, we also relate to Alves et al. (2020), who expands on the same one-sector framework, emphasizing the role of fiscal adjustments outside the steady state. Another relevant contribution is Auclert (2019), who reports that redistribution triggered by monetary policy is key in amplifying its effect in the aggregate. Relative to these papers, we extend the analysis to a two-sector HANK setting, and show how indirect effects dominate the response of both durables and non-durables, the former being more interest-rate sensitive than the latter. In this respect, we also relate to McKay and Wieland (2022), who build a model that features durable adjustment along the extensive margin, exploiting its sensitivity to the (contemporaneous) user cost to address the forward guidance puzzle. We abstract from this channel, while casting an otherwise standard two-sector NK model in a HA setting, so as to retain closer comparability with a long-standing tradition that studies monetary transmission in multi-sector economies.

Our paper also relates to a large literature tackling the *comovement puzzle* that typically characterizes standard two-sector New Keynesian models with asymmetric price rigidity. Remedies that have been put forward to address this puzzle can essentially be divided into three categories: *i*) opting for non-separable preferences between a composite of sectoral consumption goods and labor supply (see, e.g., Katayama and Kim, 2013) and (Dey and Tsai, 2012); *ii*) adopting sticky prices of the production inputs, such as Carlstrom and Fuerst (2010)—who assume sticky wages—or Sudo (2012) and Petrella et al. (2019), who both allow for input-output interactions; *iii*) embedding financial frictions in the vein of Tsai (2016)—who emphasizes the importance of working capital—or Monacelli (2009), who emphasizes the importance of the collateral constraints applying to households. All these modeling devices influence the extent of the fall in the relative price of durables (the latter are typically assumed to display prices that are more flexible than those of non-durables), in the face of a monetary tightening, thus helping to address the problem. Our framework takes a different route, and reproduces sectoral comovement not by weakening the relative-price channel, but by highlighting the importance of transitory income movements in two-sector HANK economies.

Structure The paper is structured as follows: Section 2 details the baseline two-sector HANK model. Section 3 details the calibration and the solution of the deterministic steady state. In Section 4 we generate the responses to a monetary policy shock and decompose them into direct, relative-price, and pure income effects. In Sections 4.3 and 4.4 we extend the baseline model to account for deficit financing and sticky wages, respectively. Section 5 concludes.

2 A two-sector HANK model with durables

The economy is populated by households with preferences over durable and nondurable consumption, as well as labor hours that are supplied to intermediate-goods firms operating in a regime of monopolistic competition. The latter, in turn, sell their products to final-goods firms. The government pursues monetary policy, while balancing its budget on a period-by-period basis. The remainder of this section details the key blocks of the model, as well as how equilibrium obtains.

2.1 Households

We assume a continuum of households, indexed by $s \in [0,1]$. Consumer preferences are defined over nondurable consumption, $C_{n,t}(s)$, durable goods, $D_t(s)$, as well as over labor hours, $\mathcal{N}_t(s)$. Households' intertemporal utility reads as

$$\mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[\frac{\left(C_{n,t}^{\theta} D_t^{1-\theta}\right)^{1-\sigma}}{1-\sigma} - \psi_N \frac{\mathcal{N}_t^{1+\varphi}}{1+\varphi} \right] \right\}. \tag{1}$$

We define the durable flow as $C_{d,t}(s) = D_{t+1}(s) - (1-\delta)D_t(s)$. Household s's budget constraint (deflated by the price of nondurables) is given by

$$\begin{gathered} C_{n,t}(s) + Q_t C_{d,t}(s) + B_{t+1}(s) = \\ \left(1 + r(B_t(s))_t\right) B_t(s) + w_{n,t} N_t \exp\left\{e_t(s)\right\} + Div_t \overline{Div}(s) - \tau_t \bar{\tau}(s) - \frac{\alpha}{2} \left(\frac{C_{d,t}(s)}{D_t(s)}\right)^2 D_t(s), \end{gathered} \tag{2}$$

where $B_{t+1}(s)$ denotes bond holdings, Q_t is the price of durables relative to that of nondurables, $w_{n,t}$ is the real wage rate, α scales the adjustment cost on durables, $\delta \in [0,1]$ is the depreciation rate and $e_t(s)$ is an idiosyncratic productivity shock with mean normalized to one. Furthermore, $r(B_t(s))_t$ is the real return on bonds when $B_t(s) > 0$, while it equals the real rate plus a borrowing wedge, κ, when $B_t(s) < 0$ (see Kaplan et al., 2018). Households pay taxes, τ_t, and receive dividends, Div_t, from the ownership of firms, according to the incidence rules $\bar{\tau}(s)$ and $\overline{Div}(s)$, which are set so that taxes and dividends are distributed according to productivity. Note that the nominal wage is equalized across sectors, as we assume perfect labor mobility. Finally, households face a borrowing constraint:

$$B_t(s) \geqslant -\psi Y, \tag{3}$$

where Y is steady-state total output, and ψ is a scaling parameter. We assume that all households supply labor according to the solution given by the RA representation of the model (see, e.g., Debortoli and Galí, 2021), that is:

$$w_{n,t} = \psi_N N_t^{\varphi} \frac{1}{\theta} \left(C_{n,t}^{\theta} D_t^{1-\theta}\right)^{\sigma} \left(\frac{C_{n,t}}{D_t}\right)^{1-\theta}, \tag{4}$$

where $C_{n,t} \equiv \int_0^1 C_{n,t}(s)ds$ and $N_t = \mathcal{N}_t(s)$ for all s.[2]

2.2 Production

Final-goods producers There are two sectors, indexed by $j = \{n, d\}$. Two sectoral representative final-goods producers aggregate a continuum of intermediate goods $y_{j,t}(i)$ indexed by $i \in [0,1]$, with prices $p_{j,t}(i)$ such that

$$Y_{j,t} = \left(\int_0^1 y(i)_{jt}^{\frac{\epsilon_j - 1}{\epsilon_j}} di\right)^{\frac{\epsilon_j}{\epsilon_j - 1}}, \tag{5}$$

where ϵ_j is the elasticity of substitution across goods of type j. Given $Y_{j,t}$, profit maximization for the jth final goods producer implies a demand for intermediate good i in the same sector:

$$y(i)_{j,t} = y\left(p(i)_{j,t}; P_{j,t}, Y_{j,t}\right) = \left(\frac{p(i)_{j,t}}{P_{j,t}}\right)^{-\epsilon} Y_{j,t}, \tag{6}$$

where $P_{j,t}$ denotes the equilibrium price of the final good:

[2]We take a RA stand about labor supply, as it is well known that assuming frictionless endogenous labor supply at the household level in HANK economies runs counter the empirical evidence (see, e.g., Auclert et al., 2020a). We relax our assumption in subsection 4.4, where we introduce sticky wages (see, e.g., Hagedorn et al., 2019; Auclert et al., 2020b).

$$P_{j,t} = \left(\int_0^1 p(i)_{jt}^{1-\epsilon_j} di \right)^{\frac{1}{1-\epsilon_j}}. \tag{7}$$

Intermediate-goods producers Intermediate-goods producers in either sector employ a linear production technology:

$$Y_{j,t}(i) = A_{j,t} N_{j,t}(i), \tag{8}$$

where $A_{j,t} \equiv \exp \{a_{j,t}\}$ represents total factor productivity, assumed to be common to all firms in sector j. Price setting in each sector is subject to virtual Rotemberg adjustment costs $\mathcal{C}_j(\cdot) = \frac{\xi_j}{2} \left(\frac{P_{j,t}(i)}{P_{j,t-1}(i)} - 1 \right)^2 Y_{j,t}$ (with $\xi_j > 0$) as in, e.g., Hagedorn et al. (2019). Each firm's value function in real terms reads as

$$V_{j,t}^{IGF} \left(p(i)_{j,t-1} \right) \equiv \max_{p(i)_{j,t}} \frac{p(i)_{j,t}}{P_{j,t}} y \left(p(i)_{j,t}; P_{j,t}, Y_{j,t} \right) - w_{j,t} N_{j,t} - \frac{\xi_j}{2} \left(\frac{p(i)_{j,t}}{p(i)_{j,t-1}} - 1 \right)^2 Y_{j,t} + \beta V_{j,t+1}^{IGF} \left(p(i)_{j,t} \right). \tag{9}$$

The problem yields the usual New Keynesian Phillips curve(s):

$$(1 - \epsilon_j) + \epsilon_j w_{j,t} / A_{j,t} - \xi_j \left(\Pi_{j,t} - 1 \right) \Pi_{j,t} + \beta \xi_j \left(\Pi_{j,t+1} - 1 \right) \Pi_{j,t+1} \frac{Y_{j,t+1}}{Y_{j,t}} = 0, \tag{10}$$

while total real dividends (deflated by $P_{n,t}$) are

$$Div_t = \sum_j Div_{j,t} = Y_{n,t} - w_{n,t} N_{n,t} + Q_t \left(Y_{d,t} - w_{d,t} N_{d,t} \right). \tag{11}$$

2.3 Policy

Monetary policy Monetary policy sets the nominal rate according to a Taylor rule that features a shock u_t^r

$$i_t = \phi_{\tilde{\pi}} \tilde{\pi}_t + u_t^r, \tag{12}$$

where $\tilde{\pi}$ is the net (aggregate) rate of inflation, with $\tilde{\Pi}_t \equiv \Pi_{n,t}^{1-\gamma} \Pi_{d,t}^{\gamma}, \gamma \in [0, 1]$.

Fiscal policy The fiscal authority issues one-period nominal bonds, B^g, maintaining this constant in fulfillment of the steady-state bond-to-output ratio, and adjusts the level of lump-sum taxes, τ_t, to balance its budget period-by-period:

$$\tau_t = r_t B^g. \tag{13}$$

2.4 Equilibrium

Market clearing Bonds market clearing obtains as

$$B_t = \int_0^1 B_t(s) ds = B_g. \tag{14}$$

Aggregate labor hours are given by

$$N_t = \sum_j \int_0^1 N_{j,t}(i) di = \sum_j Y_{j,t} / A_{j,t}, \tag{15}$$

and are assumed to be distributed uniformly among household types, i.e. $N_t(s) = N_t$ for all $s \in (0, 1)$. The sectoral resource constraints are

$$Y_{d,t} = C_{d,t}, \tag{16}$$

and

$$Y_{n,t} = C_{n,t} + \chi_t + \kappa \int \max(-B_t(s), 0) ds, \tag{17}$$

where the last two terms respectively represent household adjustment of durables and borrowing costs. It follows from equations (16) and (17) that the market for aggregate goods clears by

$$Y_t = Q_t Y_{d,t} + Y_{n,t} = Q_t C_{d,t} + C_{n,t} + \chi_t + \kappa \int \max(-B_t(s), 0) ds. \tag{18}$$

Equilibrium definition An equilibrium in this economy is defined as paths for individual household decisions, $\{C_{n,t}(s), D_t(s), B_t(s)\}_{t\geqslant 0}$, inflation rates and relative prices, $\{\Pi_{n,t}, \Pi_{d,t}, Q_t\}_{t\geqslant 0}$, real wages, $\{w_{n,t}, w_{d,t}\}_{t\geqslant 0}$, sectoral output and employment, $\{Y_{n,t}, Y_{d,t}, N_{n,t}, N_{d,t}\}_{t\geqslant 0}$, dividends, $\{Div_t\}_{t\geqslant 0}$, interest rates, $\{i_t, r_t\}_{t\geqslant 0}$, government supply of bonds and taxes, $\{B^g_t, \tau_t\}_{t\geqslant 0}$, such that:

1. Households maximize their objective functions, given the $\{Q_t, r_t, w_{n,t}, N_t, Div_t, \tau_t, \}_{t\geqslant 0}$ sequences;
2. Firms in each sector maximize their profits, taking as given the $\{w_{n,t}, w_{d,t}\}_{t\geqslant 0}$ sequences;
3. Given the $\{C_{n,t}, D_t\}_{t\geqslant 0}$ sequences, the real-wage sequences, $\{w_{n,t}\}_{t\geqslant 0}$ and $\{w_{d,t}\}_{t\geqslant 0}$, are consistent with the wage schedule, (4), conditional on perfect sectoral mobility, as captured by $Q_t w_{d,t} = w_{n,t}$;
4. The government budget constraint, (13), is satisfied;
5. Bonds, labor, nondurable and durable goods markets clear;
6. Distributions fulfill consistency requirements.

3 Calibration

An overview of our (quarterly) calibration is presented in Table 1. We calibrate the discount factor, β, so the steady-state annual real risk-free rate is 3 percent. The coefficient of relative risk aversion, σ, and the inverse Frisch elasticity of labor supply, φ, are set to 1. The utility weight on nondurables, θ, is set to 0.7 to match a steady-state nondurable to total consumption ratio of 0.60, which is in the middle of the range provided in Beraja and Wolf (2021). Durables' depreciation, δ, is set to 0.068, as in McKay and Wieland (2021).

The idiosyncratic income parameters, σ_e and ρ_e, are set to 0.1928 and 0.9777, respectively, following McKay et al. (2016) and Auclert (2019). On the supply side, we set ϵ_n and ϵ_d to 0.6, as in Monacelli (2009). As for the policy parameters, the steady-state government debt-to-output ratio is set to 0.26, as in Kaplan et al. (2018). The reaction parameter in the Taylor rule, ϕ_π, is set to 1.5. The weight on durables in the monetary authority's inflation index, γ, is set to the steady-state share of durable consumption to total consumption. Finally, we implement the simulated method of moments (SMM), using α and ξ_n, ξ_d to target: *i*) the relative volatility of durable to nondurable consumption, calculated using log quantities of HP-filtered data; *ii*) the stickiness of durable and nondurable prices.[3] We target Calvo probabilities (i.e, the sectoral probabilities of not being able to adjust prices in a given quarter) of 0.75 and 0.25 for nondurable and durable prices, respectively, given that Nakamura and Steinsson (2008) report median price durations between 8 and 11 months (with one of the most prominent durables, *transportation goods*, denoting a price duration of 2.7 months; see their Table II). As there is no clear-cut sorting of durables and nondurables in the micro price-setting literature, we take these Calvo probabilities as being within plausible ranges. Thus, we impose durables to be more price-flexible than nondurables, as is standard in the business-cycle literature.[4]

Finally, note that, based on this calibration, the unconditional correlation between durable and nondurable consumption amounts to 0.495 (conditional on our baseline monetary policy shock, and measured over 10 quarters), which is very close to the same moment computed with NIPA HP-filtered data (0.422).

3.1 Deterministic steady state

Let a generic variable x_t be denoted by x in the steady state. When solving for the steady state, we use a multi-dimensional root finder to guess on β, Q, N_d, to target: *i*) bonds market clearing; *ii*) durable goods market clearing; *iii*) total employment ($N = 1$). Given bonds and durable goods markets clearing, the nondurable goods market clears by

[3]The relative volatility of durables to nondurables is computed as the *on-impact* relative response to a 0.25% monetary policy shock with persistence set to 0.5, as in Kaplan et al. (2018).

[4]We rely on the mapping between Calvo probabilities and Rotemberg adjustment costs, $\xi_j = \theta_j^{Calvo}(\epsilon_j - 1)/((1-\theta_j^{Calvo})(1-\beta\theta_j^{Calvo}))$, where θ_j^{Calvo} is the probability of not being able to adjust prices in sector j. From this, we obtain $\theta_n^{Calvo} = 0.62$ and $\theta_d^{Calvo} = 0.40$ (corresponding to median price durations of 7 and 5 months, respectively), and a relative on-impact volatility of C_d to C_n of 3.560. This value is in line with the evidence in, e.g., Erceg and Levin (2006) and Sterk and Tenreyro (2018).

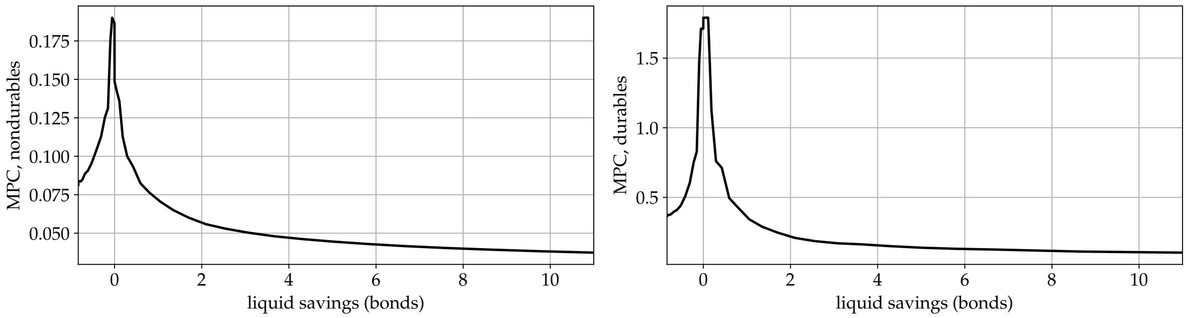
Table 1: Baseline model calibration

Parameter	Value	Target/Source
Household parameters		
β	0.9652	Steady-state adjustment
σ	1	Std. business-cycle literature value
φ	1	Std. business-cycle literature value
θ	0.7	$\frac{C_n}{C_n+C_d}$; Beraja and Wolf (2021)
α	0.119	SMM target volatility of $C_d/C_n = 3.572$; BEA, NIPA accounts
δ	0.068	BEA Fixed Asset, McKay and Wieland (2021)
ψ_N	0.764	Steady-state adjustment
ψ	0.833	Average quarterly steady-state wage
κ	0.0465	Steady-state share of households with $B(s) = 0$; Kaplan et al. (2018)
ρ_e	0.9777	McKay et al. (2016) and Auclert (2019)
σ_e	0.1928	McKay et al. (2016) and Auclert (2019)
Supply-side paramaters		
r	0.03/4	Debortoli and Galí (2021)
ϵ_n, ϵ_d	6	Monacelli (2009)
ξ_n	20.21	SMM target Calvo probability of 0.75; Nakamura and Steinsson (2008)
ξ_d	5.43	SMM target Calvo probability of 0.25; Nakamura and Steinsson (2008)
A_n	1.0	Steady-state adjustment
A_d	2.16	Steady-state adjustment
Policy parameters		
B^g/Y	0.26	Liquid assets/GDP; Kaplan et al. (2018)
ϕ_π	1.5	Taylor (1993)
γ	0.40	Steady-state $C_d/(C_n + C_d)$

Walras' law. The household solution is obtained using the endogenous grid method algorithm of Auclert et al. (2021) in two dimensions; see Appendix A. The steady-state distribution is retrieved by relying on the deterministic histogram method of Young (2010). Given guesses for β, Q, N_d, we can solve for equilibrium quantities, as described in Appendix B.

We obtain a steady-state skewness of the durable stock over nondurable consumption of 0.867, which is remarkably in line with micro evidence in Bertola et al. (2005) (see Figure 7 in Appendix F for a density plot), especially if we consider that the present framework does not feature any adjustment along the extensive margin.In addition, Figure 1 reports the steady-state marginal propensities to consume (MPCs) as functions of liquid assets. Both MPCs peak roughly where bond holdings are nil due to the debt cost, as captured by the borrowing wedge, κ. Notice that households with zero liquidity but median holdings of durables can use the durable stock as a consumption-smoothing device (subject to an adjustment cost). As such, durables assume the dual role of a consumption good and of an (illiquid) asset, at the eyes of ”wealthy hand-to-mouth” households (see Kaplan et al., 2018). Despite this feature, the MPC is still relatively large for households who are constrained in the access to liquid savings.

Figure 1: Marginal propensities to consume as a function of liquid savings



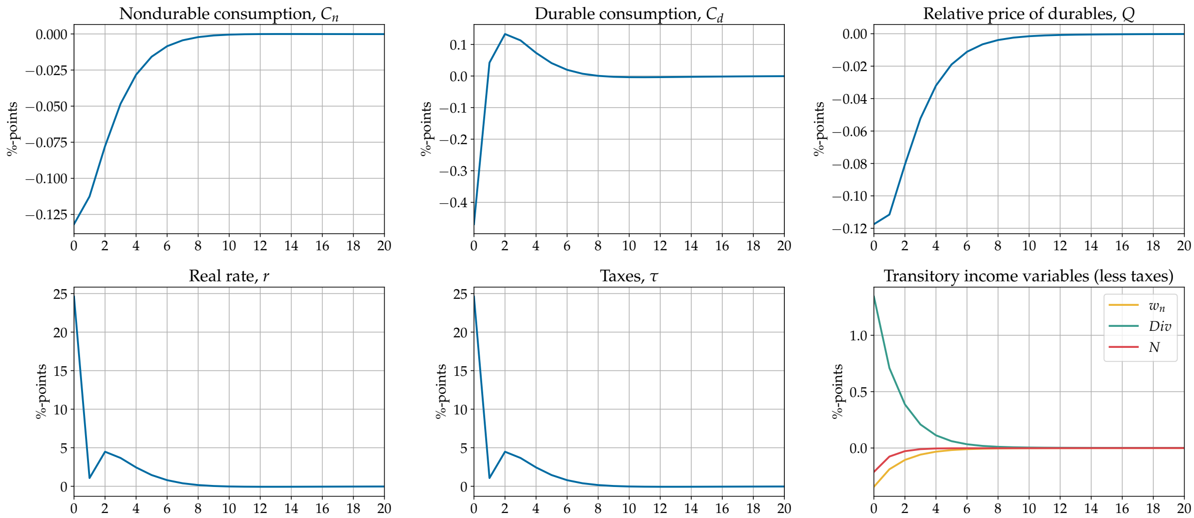
Note: To plot MPCs in two dimensions, we fix the idiosyncratic income shock, $e(s)$, as well as the stock of durables, $D(s)$, at their median steady-state value.

4 Monetary transmission

We now study monetary transmission, with a special focus on how the response of different types of consumption, both at the aggregate and at the household level, can be decomposed into direct and (different) indirect effects. We then test the robustness of our main insights to accounting for realistic extensions to the original framework.

4.1 Impulse responses to a monetary policy shock

Figure 2: Impulse responses to a contractionary monetary policy shock



Note: We consider a 0.25% monetary-policy innovation occurring at $t = 0$.

To obtain impulse responses, we solve the model to the first order, around the deterministic steady state, using the sequence-space method, as described in Auclert et al. (2021).[5] We consider a monetary policy shock at time $t = 0$. As in Kaplan et al. (2018), we set the quarterly innovation to 0.25%, while the shock-persistence parameter, ρ_{r*}, is set to 0.5.

The results are presented in Figure 2. We may notice how the monetary shock pushes both types of consumption down, with durable expenditure featuring a hump-shaped recovery, as it has typically been shown in both theoretical and empirical settings (see, e.g., Beraja and Wolf, 2021). Also the drop in the relative price is consistent with what expected on *a priori* grounds, given that durables feature relatively more flexible prices. The main scope of the subsequent analysis is to study the determinants of the contraction in both types of consumption goods, as well as their relative strength.

[5]For the sequence-space formulation of the model, we refer the reader to Appendix C.

4.2 Consumption decomposition

Following Kaplan et al. (2018), we can decompose the consumption responses at some time $t = 0$ into *direct* (i.e., interest-rate) and *indirect* (i.e., general-equilibrium) effects, by total differentiation of the impulse-response paths of $\{C_{j,t}\}_{t \geqslant 0}$, for $j = \{n, d\}$:

$$dC_{j,0} = \underbrace{\sum_{t=0}^{\infty} \frac{\partial C_{j,0}}{\partial r_t} dr_t}_{\text{direct effect}} + \underbrace{\sum_{t=0}^{\infty} \left(\underbrace{\frac{\partial C_{n,0}}{\partial Q_t} dQ_t}_{\text{relative-price effect}} + \underbrace{\frac{\partial C_{j,0}}{\partial N_t} dN_t + \frac{\partial C_{j,0}}{\partial w_{n,t}} dw_{n,t} + \frac{\partial C_{j,0}}{\partial Div_t} dDiv_t + \frac{\partial C_{j,0}}{\partial \tau_t} d\tau_t}_{\text{pure income effects}} \right)}_{\text{indirect effects}}. \tag{19}$$

Each effect is computed by moving only the variable with respect to which the partial differential is taken. For example, the direct effect is a partial-equilibrium one where all variables other than the real rate are kept fixed. we are in a two-sector setting, indirect effects can be grouped into two terms: a *relative-price* effect— which captures both income and substitution effects—and terms that exclusively correspond to *income-related* effects. Numerically, we calculate the partial-equilibrium household paths by varying only the relevant inputs, while keeping the remaining terms fixed. For example, in the case of the direct effect on nondurable consumption, we need to compute

$$\sum_{t=0}^{\infty} \frac{\partial C_{n,0}}{\partial r_t} dr_t = \sum_{t=0}^{\infty} \left(\int \frac{\partial C_{n,0} \left(e_t(s), B_t(s), D_t(s); \{r_t, Q, w_n, N, Div, \tau\}_{t>0} \right)}{\partial r_t} ds \right) dr_t. \tag{20}$$

In practice, this is accomplished by varying one input at a time, given the general-equilibrium path computed through household Jacobians, which are calculated when tackling the sequence-space solution of the impulse-response functions (see Auclert et al., 2021).

Figure 3: Consumption response decomposition

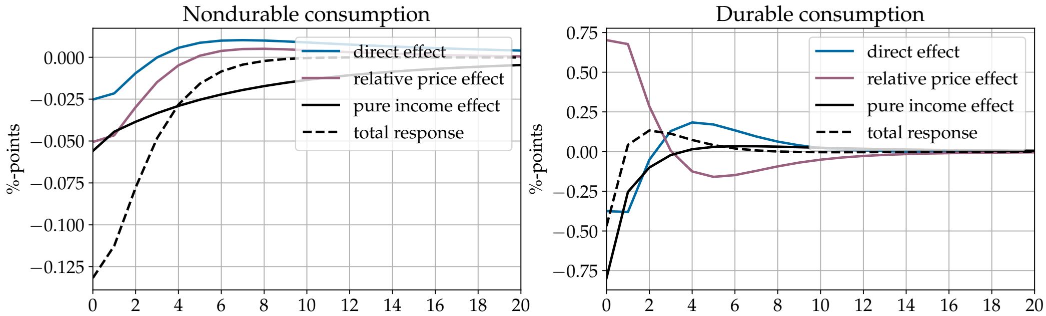


Figure 3 reports our baseline consumption-response decomposition. This shows how both the direct and the pure income effects push both types of consumption down. On the other hand, the fall in the relative price would *per se* lead to substitute nondurable for durable consumption, potentially yielding an empirically counterfactual negative comovement. In fact, summing the relative-price to the direct effect would still imply negative comovement between durables and nondurables, as the intratemporal substitution motive—which is driven by the drop in Q_t—is way more powerful than the intertemporal substitution motive, as is typically the case in standard two-sector RANK models.[6] In fact, indirect effects prove key in generating positive consumption comovement.

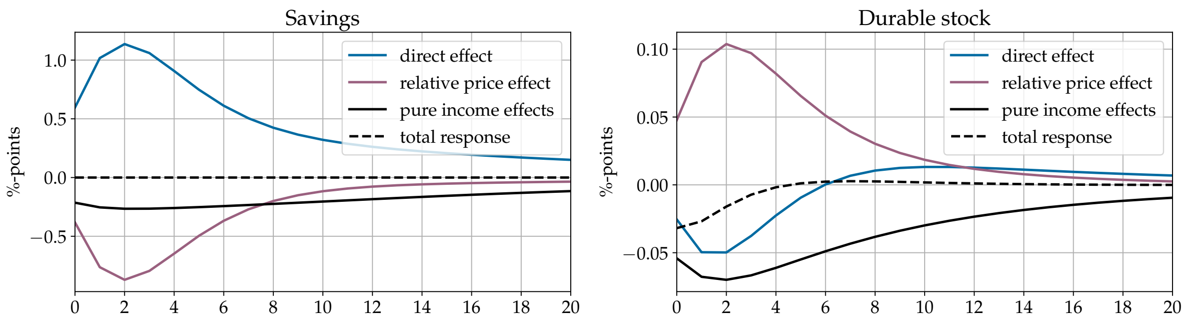
Quantitatively, the on-impact interest-rate effect amounts to -0.025 percentage points (pp) for nondurables, while indirect effects amount to -0.050 pp. As for durable consumption, the corresponding figures are -0.37 pp and -0.70 pp, respectively. Over a year, the contribution of the direct effect is 15% for nondurables[7] and 374% for durables, while the contributions of the indirect effects amount to 47% and 647% for each type of consumption, respectively. Thus, the contribution of indirect effects to the fall in consumption of both goods is roughly twice as large as that of the direct effect. Moreover, durables are much more interest-rate sensitive than nondurables. As for total consumption, direct and indirect effects contribute by 94% and 176% of the drop, respectively.

[6]In this respect, it should be noted that, on impact, Q_t is very effective at contributing to the fall in nondurable consumption. This is because the drop in Q_t represents a particularly powerful negative income effect for liquidity-constrained households, as we will see more clearly when performing a wealth-based decomposition.

[7]To establish a term of comparison, in Kaplan et al. (2018), the relative contribution of the direct effect to (nondurable) consumption is around 20% over a year.

We can further decompose the indirect effects into a variety of sub-components. We do this in Appendix F, Figure 8. Here, we see that the brunt of the negative effect from the income components arises from labor income variables, N and w_n. Taxes matter for a smaller share of the total negative push. The main reason for this is that taxes are progressively distributed according to productivity, so that low-income households—who are more sensitive to transitory income shocks—are partially insulated from this force. From Kaplan et al. (2018), it is well known that the exact assumptions about how the government budget constraint adjusts outside the steady state matter when budgets are balanced period-by-period. In Section 4.3 we show that our set of results still holds in the presence of deficit financing. Moreover, one should recall that dividends are expansionary in the present scenario, as is typically the case in New Keynesian economies featuring rigid prices. In light of this, we argue that positive-comovement forces would be even stronger in a similar model where dividends are procyclical or neutralized. To test such conjecture, we introduce sticky wages in Section 4.4.

Figure 4: Portfolio-based response decomposition

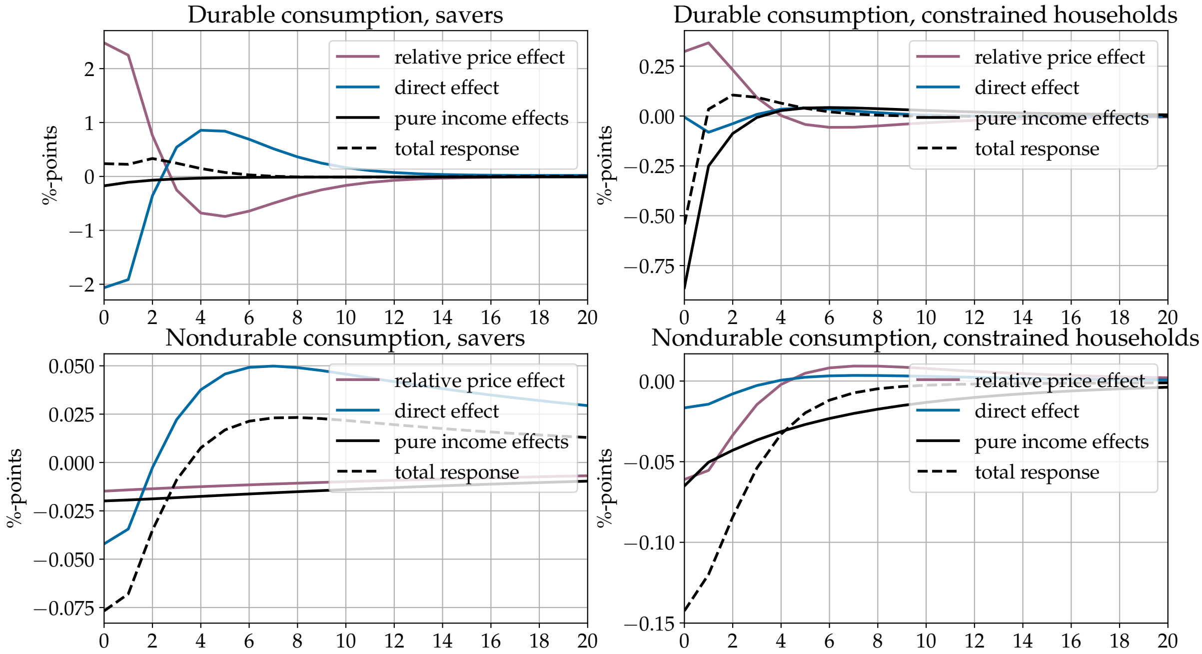


A portfolio-based decomposition In the present setting, a particularly useful perspective to examine this issue consists of considering that, together with liquid assets, durables are implicitly involved in a portfolio allocation choice. Thus, we report a response decomposition of the *portfolio* featuring bonds and the stock of durables (see Figure 4). Diverging effects of an increase in the interest rate on the holdings of the two assets are to be ascribed to the relative behavior of their rates of return: as the "spread" between these increases, households are progressively induced to tilt their portfolio towards bonds. An opposite force emanates from the relative price of durables, whose contraction would *per se* induce a substitution from bonds towards durables. Pure income effects, instead, are contractionary with respect to both forms of assets. In sum, whereas the overall impact is nil for bonds—by virtue of market clearing—the stock of durables contracts due to the decisive impact of indirect effects.

A wealth-based decomposition We go even deeper in examining the role of different transmission channels, decomposing the consumption responses of liquidity-constrained and unconstrained households. Specifically, the first row of Figure 5 reports the durable consumption decompositions for households at the bottom 50% steady-state bond holdings (i.e., most of the liquidity-constrained households), and for households at the steady-state top 1% of bond holdings (i.e., savers). We see that savers are quite interest-rate sensitive, given their motive to re-balance their portfolio of assets, moving away from durables and towards bonds. Instead, liquidity-constrained households respond very little to interest-rate changes, as expected in light of their hand-to-mouth behavior. Savers' response with respect to durable consumption is also more sensitive to the change in the relative price, to the point that this overcomes the contractionary force exerted by intertemporal substitution. On the other hand, the impact of the relative price on liquidity-constrained households' durable consumption is relatively muted, both because these agents are limited in portfolio (re-)allocation, and because the fall in Q_t represents a substantial negative income effect. In this respect, notice that the fall in the relative price is rather effective at compressing financially constrained households' nondurable consumption: on impact, this effect is comparable with that emanating from different determinants of disposable income, which are by far the major driver of the conditional response of both durables and nondurables, for this class of consumers. By contrast, savers' nondurable consumption response is poorly shaped by both relative-price and pure income effects.[8]The emerging picture is such that negative comovement between durable and nondurable consumption appears as a distinctive trait of savers' consumption response in the face of a monetary disturbance, whereas constrained agents' feature positive conditional comovement.

[8]Setting relative risk aversion to one implies that the relative price does not affect unconstrained households' nondurable consumption through substitution effects.

Figure 5: Durable consumption response decomposition by steady-state wealth percentiles



Note: Liquidity-constrained households are defined as households at the bottom 50% steady-state bond holdings. Savers are defined as households at the steady-state top 1% bond holdings. The effects are calculated using the (initially) truncated distributions relative to a simulation of the relevant truncated distribution conditional on all input variables being at their steady-state values.

4.3 Deficit financing

It is well known that the specific assumptions about how the government budget constraint adjusts outside the steady state matter in HANK economies, especially when governments balance their budget period-by-period. As seen in Section 4.2, part of the co-movement through indirect effects is driven by the increase in taxes. Thus, to neutralize movements in taxes we replace equation (13) with (21), as in Auclert et al. (2020b):

$$\begin{aligned} (1 + r_t)\, B^g_{t-1} &= \tau_t + B^g_t, \\ \tau_t &= \tau + \phi_\tau \left(B^g_{t-1} - B^g \right), \end{aligned} \tag{21}$$

where τ and B^g denote steady-state taxes and government bonds, respectively, while ϕ_τ determines how fast deficits are closed. Note that such formulation does not affect the steady state. Outside the steady state, we find taxes in each period conditional on the government budget constraint holding; see Appendix D.

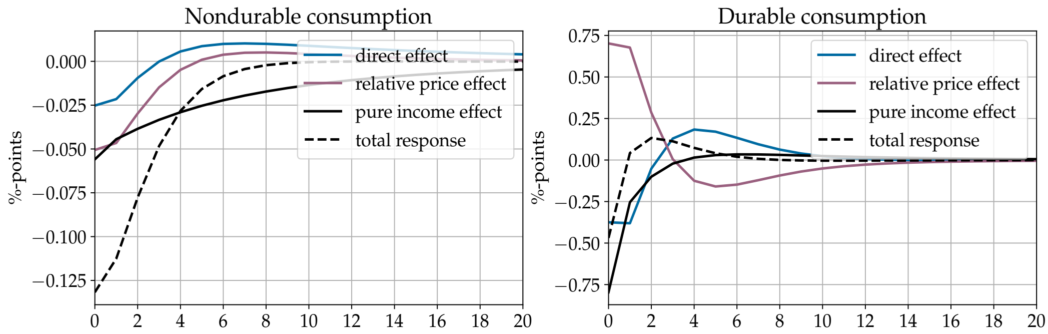
Re-calibration We set ϕ_τ to 0.1, as in Auclert et al. (2020b). Note that, under deficit financing, we need to re-perform our SMM calibration exercise for the scaling parameter in the adjustment cost of durables and the price adjustment costs, so as to target the volatility of durables to nondurables. Doing so results in $\alpha = 0.137$, while the Calvo probability amounts to 0.62 for nondurables and 0.40 for durables, thus mapping into $\xi_n = 19.90$ and $\xi_d = 8.37$, respectively. The discount factor, β, is now 0.965, the borrowing wedge, κ, is 0.0454, while the total factor productivities of nondurable and durable production are 1.0 and 2.15, respectively. Finally, the scaling parameter for labor disutility, ψ_N, is 0.765. The resulting volatility of durables-to-nondurables is 3.563, while steady-state ratio between nondurable to total consumption is 0.60 (implying γ=0.4), in line with the baseline calibration.

Consumption decomposition The second row of Figure 6 contains a consumption decomposition of the effects induced by a monetary tightening in the presence of fiscal deficits, in line with the analogous decomposition for the baseline model in Section 4.2 (which has been reproduced in the first row of the figure, to enhance comparability). Even with fiscal deficit financing, income effects still drive the brunt of the contractionary response of both types of consumption goods. For a more detailed view, we refer the readero to Figure 9 in Appendix F. As expected, taxes barely move in the presence of deficit financing.

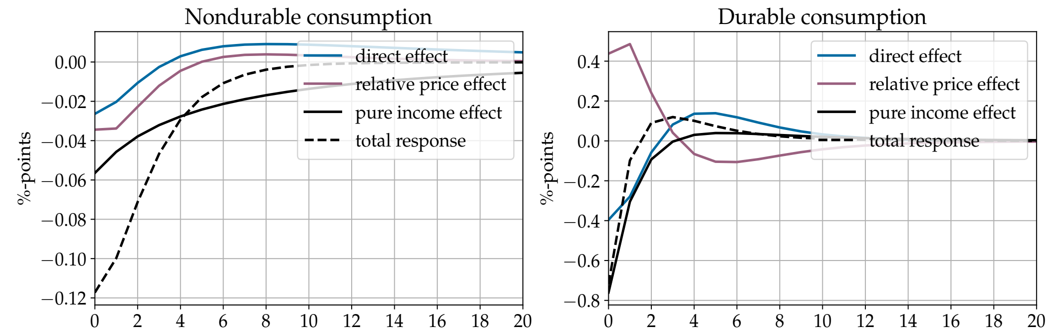
4.4 Sticky wages

It is well known that sticky wages alleviate the issue of countercyclical profits. In addition, they also reinforce durable and nondurable consumption comovement, as sticky wages dampen relative-price changes (see Carlstrom and Fuerst, 2010). Thus, given these reasonable properties, we investigate the consumption responses of our HANK model, extended to accommodate sticky wages.

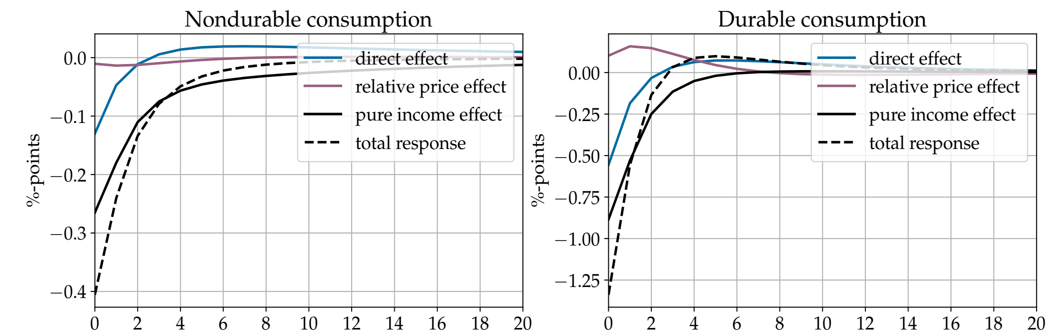
Figure 6: Consumption response decomposition, robustness to different model alterations



(a) Baseline model



(b) Deficit financing



(c) Sticky wages

Labor union We replace the wage schedule equation, (4), with a wage Phillips curve as in Erceg et al. (2000), Erceg and Levin (2006) and Hagedorn et al. (2019). Specifically, each household provides differentiated labor services, which are transformed into aggregate effective labor, N_t, by perfectly competitive labor packers, using the technology

$$N_t = \left(\int_0^1 \exp\{e(s)_t\} \left(\mathcal{N}(s)_t\right)^{\frac{\epsilon_w - 1}{\epsilon_w}} ds \right)^{\frac{\epsilon_w}{\epsilon_w - 1}}. \tag{22}$$

A union sells labor services at the nominal wage W_t (equalized across production sectors) to the labor recruiter, who minimizes costs given the aggregate demand for labor, implying

$$\mathcal{N}(s)_t = \mathcal{N}\left(W(s)_t; W_t, N_t\right) = \left(\frac{W(s)_t}{W_t}\right)^{-\epsilon_w} N_t \tag{23}$$

for the sth household, and where the equilibrium nominal wage amounts to

$$W_t = \left(\int_0^1 \exp\{e(s)_t\} W(s)_t^{1-\epsilon_w} ds \right)^{\frac{1}{1-\epsilon_w}}. \tag{24}$$

The union sets the nominal wage for one effective labor unit, $\hat{W}_t$, such that $\hat{W}_t = W_t$ subject to virtual Rotemberg adjustment costs:

$$\mathcal{C}_w(\cdot) = \exp\left\{e(s)_t\right\} \frac{\xi_w}{2} \left(\frac{W_{it}}{W_{it-1}} - 1\right)^2 N_t, \tag{25}$$

assuming steady-state $\Pi_w = 1$. The union's wage-setting problem maximizes

$$V_t^w\left(\hat{W}_{t-1}\right) \equiv \max_{\hat{W}_t} \int \frac{\exp\left\{e(s)_t\right\}(1-\tau_t)\hat{W}_t}{P_{n,t}} \mathcal{N}\left(\hat{W}_t; W_t, N_t\right) - \frac{v\left(\mathcal{N}\left(\hat{W}_t; W_t, N_t\right)\right)}{U'_{C_n}\left(C_{n,t}, D_t\right)} \Bigg) ds$$
$$- \int \exp\left\{e(s)_t\right\} \frac{\xi_w}{2} \left(\frac{\hat{W}_t}{\hat{W}_{t-1}} - 1\right)^2 N_t ds + \beta V_{t+1}^w\left(\hat{W}_t\right).$$

This problem yields a wage Phillips curve:[9]

[9]See Hagedorn et al. (2019).

$$(1-\epsilon_w)\, w_{n,t} + \epsilon_w \frac{U'_N(N_t)}{U'_{C_n}(C_{n,t}, D_t)} - \xi_w (\Pi_{w,t} - 1)\, \Pi_{w,t} + \beta \xi_w (\Pi_{w,t+1} - 1)\, \Pi_{w,t+1} \frac{N_{t+1}}{N_t} = 0, \quad (26)$$

where the aggregation assumptions are as in Hagedorn et al. (2019), so that one obtains the RA outcome as heterogeneity is turned off.

The steady state is solved as described in Section B: however, instead of varying ψ_N such that the wage schedule (4) holds in the steady state, we vary it to ensure that the steady-state wage Phillips curve holds. For the dynamic solution, we refer the reader to Appendix E.

Re-calibration Given this extended structure, we need to re-calibrate some parameters. We set the labor unions' market power so that $\epsilon_w = \epsilon_n = \epsilon_d = 0.6$. We set $\xi_n = 54.42$ and $\xi_d = 2.20$, such that the corresponding Calvo probabilities for prices are right on target (i.e, 0.75 and 0.25, respectively). As for wage stickiness, we set $\xi_w = 54.42$ to target a Calvo probability of 0.75, yielding an implied duration of wage contracts of one year, in line with the estimates of Smets and Wouters (2003) and Levin et al. (2005). We re-calibrate the parameter scaling the adjustment of durables, α, to 1.522, so as to target the relative (on-impact) volatility of C_d to C_n. The model can now hit that target of 3.572 exactly. The borrowing wedge, κ, is re-calibrated to 0.0368 to hit the share of 30% liquidity-constrained households in the steady state. The discount factor, β, is now 0.9634. The scaling of labor disutility, ψ_N, is 0.633. Finally, the implied steady-state total factor productivity in each sector are $A_n = 1$ and $A_d = 2.58$, while steady-state nondurable-to-total consumption, $C_n/(C_n + C_d)$, equals 0.61 (so that $\gamma = 0.39$).

Consumption decomposition The last row of Figure 6 reports a consumption decomposition for the model with sticky wages. In this case, pure income effects make up an even larger part of the consumption response. This is because prices inherit some stickiness from wages, causing relative-price movements to be smaller, thus alleviating dividend countercyclicality.[10] It should be stressed, however, that durables are still quite interest-rate sensitive: over a year, the relative contribution of the interest rate to the drop in sectoral consumption is 21% for nondurables and 37% for durables, while the corresponding

[10]For a detailed account, see Appendix F, Figure 10.

figures for the indirect effects are 73% and 89%, respectively. As for total consumption, we have a contribution of 26% and 75% from direct and indirect effects, respectively. Finally, it is important to stress that the main takeaways from the wealth-based decomposition in Section 4.2 carry over to the present setting. That is, liquidity constrained households (savers) are those whose durable consumption is mainly sensitive to interest-rate effects (see Figure 11 in Appendix F).

5 Concluding remarks

We introduce durable goods into an otherwise standard New Keynesian model with heterogeneous households, showing that pure income effects dominate the consumption response of both nondurables and durables to a monetary policy shock. In the simplest setting with sticky prices and flexible wages, indirect effects are enough to undo negative comovement arising from changes in the relative price of the two goods. The fact that indirect effects dominate the response of consumption is akin to what the HANK literature based on one-sector economies generally indicates, and reinforces the demand for further understanding of how such effects generate in their respective markets/institutions.

We also find that durables are more interest-rate sensitive than nondurables and, most importantly, that interest-rate effects make up a non-negligible part of the total response of durables to monetary policy shocks, even in the presence of sticky wages. This indicates that changes in conventional policy instruments have a grip that is typically downplayed by analyses that limit their focus to one-sector HANK economies. Considering the dominant impact of durables on business-cycle volatility, and specifically on the transmission of monetary shocks, it is essential to consider the lessons learned in this paper for understanding and designing monetary policy.

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Appendix

A Endogenous grid method with non-separable utility in durable and nondurable consumption

A.1 Model setup

Households face the following optimization problem:

$$\begin{aligned} V_t(z_t, b_t, d_t) &= \max_{c_t, d_{t+1}, b_{t+1}} u(c_t, d_t) + \beta \mathbb{E}_t V_{t+1}(z_{t+1}, b_{t+1}, d_{t+1}) \\ \text{s.t. } c_t + b_{t+1} + Q_t(d_{t+1} - (1-\delta) d_t) &= z_t + (1 + r_t) b_t - \Psi(d_{t+1}, d_t) \\ b_t \geqslant \underline{b}, &\quad d_t \geqslant 0, \end{aligned} \tag{27}$$

where z_t denotes idiosyncratic income, b_t is wealth, d_t denotes durables and Q_t is the price of durables relative to that of nondurables. In the general equilibrium setting, $z_t = \exp\{e_t\}[w_{n,t} N_t - \tau_t + Div_t]$. The rest, except for utility and the cost function $\Psi(\cdot)$ is standard. The utility and the adjustment cost functions are

$$\begin{aligned} u(c_t, d_t) = \tfrac{\psi(c_t, d_t)^{1-\sigma}}{1-\sigma} \quad &\text{and} \quad \psi(c_t, d_t) = c_t^{\theta} d_t^{1-\theta}, \\ \Psi(d_{t+1}, d_t) &= \tfrac{\alpha}{2} \left(\tfrac{d_{t+1} - (1-\delta) d_t}{d_t} \right)^2 d_t. \end{aligned} \tag{28}$$

A.2 First-order and envelope conditions

Re-write the Bellman equation by substituting out consumption using the budget constraint

$$\begin{aligned} V_t(z_t, b_t, d_t) = \max\nolimits_{b_{t+1}, d_{t+1}} \quad & u\left(z_t + (1 + r_t) b_t - Q_t(d_{t+1} - (1-\delta) d_t) - \Psi(d_{t+1}, d_t) - b_{t+1}, d_t\right) \\ & + \mu_t d_{t+1} + \lambda_t (b_{t+1} - \underline{b}) + \beta \mathbb{E} V_{t+1}(z_{t+1}, b_{t+1}, d_{t+1}), \end{aligned} \tag{29}$$

where μ_t and λ_t are the multipliers for the non-negativity constraint on durables and the unsecured credit-borrowing constraint, respectively.

The first-order conditions with respect to d_{t+1} and b_{t+1} yield

$$\begin{aligned} \partial_{c_t} u(c_t, d_t) \left(Q_t + \partial_{d_{t+1}} \Psi\left(d_{t+1}, d_t\right)\right) &= \mu_t + \partial_{d_{t+1}} \beta \mathbb{E} V_{t+1}\left(z_{t+1}, b_{t+1}, d_{t+1}\right), \\ \partial_{c_t} u(c_t, d_t) &= \lambda_t + \partial_{b_{t+1}} \beta \mathbb{E} V_{t+1}\left(z_{t+1}, b_{t+1}, d_{t+1}\right). \end{aligned} \tag{30}$$

The envelope conditions are

$$\begin{aligned} &\partial_{b_t} V_t\left(z_t, b_t, d_t\right) = \left(1 + r_t\right) \partial_c u(c_t, d_t), \\ &\partial_{d_t} V_t\left(z_t, b_t, d_t\right) = \partial_{d_t} u(c_t, d_t) + \partial_{c_t} u(c_t, d_t)\left[Q(1-\delta) - \partial_{d_t} \Psi\left(d_{t+1}, d_t\right)\right]. \end{aligned} \tag{31}$$

For later use, it is convenient to define the post-decision value function as

$$W_t\left(z_t, b_{t+1}, d_{t+1}\right) \equiv \beta \mathbb{E}_t V_{t+1}\left(z_t, b_{t+1}, d_{t+1}\right). \tag{32}$$

A.3 Main equations of the algorithm

First, we combine the equations in (30) to obtain

$$\frac{\mu_t + \partial_d \beta \mathbb{E} V_{t+1}\left(z_{t+1}, b_{t+1}, d_{t+1}\right)}{\lambda_t + \partial_b \beta \mathbb{E} V_{t+1}\left(z_{t+1}, b_{t+1}, d_{t+1}\right)} = Q_t + \alpha\left(\frac{d_{t+1}}{d_t} - (1-\delta)\right). \tag{33}$$

From the F.O.C. wrt. b_{t+1} in (30) we can pin down nondurable consumption:

$$\begin{aligned} \tfrac{\partial u(c_t, d_t)}{\partial c_t} &= \lambda_t + \partial_{a_{t+1}} \beta \mathbb{E} V_{t+1}\left(z_{t+1}, b_{t+1}, d_{t+1}\right) \\ \Rightarrow \theta c_t^{\theta-1} d_t^{1-\theta}\left[c_t^{\theta} d_t^{1-\theta}\right]^{-\sigma} &= \lambda_t + \partial_{b_{t+1}} \beta \mathbb{E} V_{t+1}\left(z_{t+1}, b_{t+1}, d_{t+1}\right) \\ \Rightarrow c_t &= \left[\tfrac{1}{\theta}\left(\lambda_t + \partial_{b_{t+1}} \beta \mathbb{E} V_{t+1}\left(z_{t+1}, b_{t+1}, d_{t+1}\right)\right) d_t^{(\theta-1)(1-\sigma)}\right]^{\frac{1}{\theta(1-\sigma)-1}}. \end{aligned} \tag{34}$$

A.4 Algorithm

The algorithm is based on the two-asset algorithm described in Auclert et al. (2021). For a generic variable x_t, denote today's grid by x and tomorrow's grid by $x^{'}$. Thus, according to the EGM algorithm:

1. When seeking for steady-state policies, initialize the guess on $\partial_b V\left(z,b,d\right), \partial_d V\left(z,b,d\right)$. Otherwise, start backward induction by using steady-state $\partial_b V\left(z,b,d\right), \partial_d V\left(z,b,d\right)$ (used when calculating household Jacobians).

2. Let the productivity-shock transmission matrix be notated by Π. The value functions have a common $z' \to z$ so the post-decision functions are:

$$\begin{aligned} W_b\left(z,b',d'\right) &= \beta\Pi V_b\left(z',b',d'\right), \\ W_d\left(z,b',d'\right) &= \beta\Pi V_d\left(z',b',d'\right). \end{aligned} \tag{35}$$

3. Find $d'(z,b',d)$ for the *unconstrained* case using (33):

$$\frac{W_d\left(z,b',d'\right)}{W_b\left(z,b',d'\right)} = Q + \alpha\left(\frac{d'}{d} - (1-\delta)\right). \tag{36}$$

4. Use $d'(z,b',d)$ to map $W_b\left(z,b',d'\right)$ into $W_b\left(z,b',d\right)$ by interpolation. Then compute consumption by using (34):

$$c(z,b',d) = \left(W_b\left(z,b',d\right)d^{\theta-1}\cdot d^{(1-\theta)\sigma}\right)^{\frac{1}{\theta(1-\sigma)-1}}. \tag{37}$$

5. Now it is possible to find total assets by inserting $d'(z,b',d)$ and $c(z,b',d)$ into the budget constraint:

$$b\left(z,b',d\right) = \frac{c\left(z,b',d\right) + Q\left(d'\left(z,b',d\right) - (1-\delta)\,d\right) + b' + \Psi\left(d'(z,b',d),d\right) - z}{1+r}. \tag{38}$$

6. Invert $b\left(z,b',d\right)$ to obtain $b'\left(z,b,d\right)$ by interpolation. Use the same interpolation weights to obtain $d'(z,b,d)$.

7. Find $d'(z,\underline{b},d)$ for the *constrained* case using (33). For scaling, define $\kappa \equiv \lambda/W_b\left(z,\underline{b},d'\right)$.

Then (33) becomes

$$\frac{1}{1+\kappa}\frac{W_d\left(z,\underline{b},d'\right)}{W_b\left(z,\underline{b},d'\right)} = Q + \alpha\left(\frac{d'}{d} - (1-\delta)\right). \tag{39}$$

8. Use (39) to solve for $d'(z,\kappa,d)$, that is over a grid of κ values. Then compute consumption as

$$c(z,\kappa,d) = \left((1+\kappa)W_b\left(z,\kappa,d\right)d^{\theta-1}\cdot d^{(1-\theta)\sigma}\right)^{\frac{1}{\theta(1-\sigma)-1}}. \tag{40}$$

9. Using $d'(e,\kappa,d)$, $c(e,\kappa,d)$ and the budget constraint obtain

$$b(z,\kappa,d) = \frac{c(z,\kappa,d) + Q\left(d'(z,\kappa,d) - (1-\delta)\,d\right) + \underline{b} + \Psi\left(d'(z,\kappa,d),d\right) - z}{1+r}. \tag{41}$$

10. Invert $b(z,\kappa,d)$ by interpolation to obtain $\kappa(z,b,d)$. The same interpolation weights can be used to map $d'(z,\kappa,d)$ into $d'(z,b,d)$. By definition, $b'(z,b,d) = \underline{b}$.

11. Combine the constrained and the unconstrained solutions of $b'(z,b,d)$ and $d'(z,b,d)$. Then compute consumption from the budget constraint:

$$c(z,b,d) = z + (1+r)\,b - Q\left(d'(z,b,d) - (1-\delta)\,d\right) - \Psi\left(d',d\right) - b'(z,b,d). \tag{42}$$

12. Update $\partial_b V\left(z,b,d\right)$ and $\partial_d V\left(z,b,d\right)$ using the envelope conditions from (31):

$$\begin{aligned}\partial_b V\left(z,b,d\right) &= (1+r)\,\partial_c u(c,d),\\ \partial_d V\left(z,b,d\right) &= \partial_d u(c,d) - \partial_c u(c,d)\left[Q(1-\delta) + \partial_d \Psi\left(d',d\right)\right].\end{aligned} \tag{43}$$

13. For the steady-state solutions: Return to step 2 and follow the same steps until the change in $\partial_b V\left(z,b,d\right)$ and $\partial_d V\left(z,b,d\right)$ between iterations is ≈ 0. Otherwise, solve paths by backward iteration (used to obtain household Jacobians given some shock to a given household input variable).

Finally, to obtain aggregates we need to simulate the distribution of households. We use the histogram method as developed in Young (2010). In the steady state, we simulate forwards until the change in the distribution between iterations is ≈ 0. Outside the steady state, one can simply simulate forward given a path length.

B Deterministic steady state

The distribution is obtained by relying on the deterministic histogram method of Young (2010). Given guesses for β, Q, N_d, we can solve for equilibrium quantities as follows:

1. We set $P_n = 1$ as the numeraire, so that $\Pi_n = 1$;
2. We get that $\Pi_d = 1$, as $\Pi_d = \Pi_n$ in the steady state;
3. Given a calibration target for Y_d (which is set to 0.5), we pin down $A_d = Y_d/N_d$;[11]
4. We obtain $w_d = A_d \cdot \frac{\epsilon_d - 1}{\epsilon_d}$ from the durable-goods sector Phillips curve;
5. The latter then yields real wage in the nondurable-goods sector as $w_n = Q \cdot w_d$, as the nominal wage is equalized across sectors;
6. From the nondurable-goods sector Phillips curve we can pin down $A_n = w_n \cdot \frac{\epsilon_n}{\epsilon_n - 1}$;
7. We set $Y_n = 1 - Q \cdot Y_d$, such that total output, $Y = 1$;
8. We then obtain employment in the nondurable-goods sector as $N_n = Y_n/A_n$;
9. We get dividends from (11), $Div(Y_n, Y_d, Q, w_n, w_d)$;
10. Taxes are pinned down as $\tau = r \cdot B^g$.

As we pin down all variables from aggregate relationships, it is then possible to solve the household problem to obtain C_n, C_d, B, and check root-finding target residuals. Thus, after root-finding, we set ψ_N given w_n, C_n, C_d and the parameters, such that the wage schedule, (4), holds in the steady state.

[11] $Y_d = 0.5$ is a reasonable choice—given that $Y_d = C_d$—as C_d makes up a empirically plausible share of total consumption; cf. the calibration target for $C_n/(C_n + C_d)$.

C Sequence space formulation for impulse responses

In sequence space, the model can be summarized by the equation system

$$H\left(N_{n,t}, N_{d,t}, \Pi_{n,t}, Q_t, w_{n,t}, u_t^r\right) = \begin{pmatrix} \text{Wage schedule} \\ \text{NKPC durables} \\ \text{NKPC nondurables} \\ \text{Bonds market} \\ \text{Goods market durables} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{44}$$

Denoting aggregate solution variables with $\mathcal{B}, \mathcal{C}_n$,$\mathcal{C}_d$,$\mathcal{D}$, the system can be reported as

$$H\left(N_{n,t}, N_{d,t}, \Pi_{n,t}, Q_t, w_{n,t}, u_t^r\right) =$$
$$\begin{pmatrix} w_{n,t} - \psi_N N_t^{\varphi} \frac{1}{\theta} \left(\mathcal{C}_{n,t}^{\theta} \mathcal{D}_t^{1-\theta}\right)^{\sigma} \left(\frac{\mathcal{C}_{n,t}}{\mathcal{D}_t}\right)^{1-\theta} \\ (1-\epsilon_d) + \epsilon_d w_{d,t}/A_{n,t} - \xi_d \left(\Pi_{d,t} - 1\right) \Pi_{d,t} + \beta \xi_d \left(\Pi_{d,t+1} - 1\right) \Pi_{d,t+1} \frac{Y_{d,t+1}}{Y_{d,t}} \\ (1-\epsilon_n) + \epsilon_n w_{n,t}/A_{d,t} - \xi_n \left(\Pi_{n,t} - 1\right) \Pi_{n,t} + \beta \xi_n \left(\Pi_{n,t+1} - 1\right) \Pi_{n,t+1} \frac{Y_{n,t+1}}{Y_{n,t}} \\ \mathcal{B}_t - B^g \\ Y_{d,t} - \mathcal{C}_{d,t} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{45}$$

where we have

$$\Pi_{d,t} = \frac{Q_t}{Q_{t-1}} \Pi_{n,t} \tag{46}$$

$$Y_{n,t} = A_{n,t} N_{n,t} \tag{47}$$

$$Y_{d,t} = A_{d,t} N_{d,t} \tag{48}$$

$$N_t = N_{n,t} + N_{d,t} \tag{49}$$

$$w_{d,t} = Q_t^{-1} w_{n,t} \tag{50}$$

$$Div_t = Y_{n,t} - w_{n.t} N_{n,t} + Q_t \left[Y_{d,t} - w_{d,t} N_{d,t} \right] \tag{51}$$

$$\tilde{\Pi}_t = \Pi_{n,t}^{1-\gamma} \Pi_{d,t}^{\gamma} \tag{52}$$

$$i_t = u_t^r + \phi_{\tilde{\pi}} \tilde{\pi}_t \tag{53}$$

$$r_t = \frac{1 + i_{t-1}}{1 + \pi_{n,t}} - 1 \tag{54}$$

$$\tau_t = r_t B_g \tag{55}$$

and where the market for nondurable goods clears by Walras' law.

D Sequence space formulation with deficit financing

All targets and variables stay the same as in Appendix C. The only difference is that we replace (55) with

$$\tau_t = \tau + \phi_\tau \left(B_{t-1}^g - B^g \right), \tag{56}$$

where it has to hold that

$$\left(1 + r_t\right) B_{t-1}^g = \tau_t + B_t^g. \tag{57}$$

Thus, we use a root-finder to solve for the path of B_t^g consistent with (57), nested in the sequence space formulation. For further details, see Appendix C.5 in Auclert et al. (2021). The model can the be solved in sequence space, as described in Appendix C.

E Sequence space formulation with sticky wages

In sequence space, the model with the wage Phillips curve can be summarized by the equation system

$$H\left(N_{n,t}, N_{d,t}, \Pi_{n,t}, Q_t, w_{n,t}, u_t^r\right) = \begin{pmatrix} \text{Wage Phillips curve} \\ \text{Phillips curve durables} \\ \text{Phillips curve nondurables} \\ \text{Bonds market clearing} \\ \text{Durable goods market clearing} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{58}$$

Using caligraphic variables $\mathcal{B}, \mathcal{C}_n$,$\mathcal{C}_d$,$\mathcal{D}$ to denote the aggregated household solution variables counterparts, the system reads as

$$\begin{gathered} H\left(N_{n,t}, N_{d,t}, \Pi_{n,t}, Q_t, w_{n,t}, u_t^r\right) = \\ \begin{pmatrix} (1-\epsilon_w) w_{n,t} + \epsilon_w \frac{U'_{\mathcal{N}}(N_t)}{U'_{C_n}(\mathcal{C}_{n,t}, \mathcal{D}_t)} - \xi_w \left(\Pi_{w,t} - 1\right) \Pi_{w,t} + \beta \xi_w \left(\Pi_{w,t+1} - 1\right) \Pi_{w,t+1} \frac{N_{t+1}}{N_t} \\ (1-\epsilon_d) + \epsilon_d w_{d,t}/A_{n,t} - \xi_d \left(\Pi_{d,t} - 1\right) \Pi_{d,t} + \beta \xi_d \left(\Pi_{d,t+1} - 1\right) \Pi_{d,t+1} \frac{Y_{d,t+1}}{Y_{d,t}} \\ (1-\epsilon_n) + \epsilon_n w_{n,t}/A_{d,t} - \xi_n \left(\Pi_{n,t} - 1\right) \Pi_{n,t} + \beta \xi_n \left(\Pi_{n,t+1} - 1\right) \Pi_{n,t+1} \frac{Y_{n,t+1}}{Y_{n,t}} \\ \mathcal{B}_t - B^g \\ Y_{d,t} - \mathcal{C}_{d,t} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \end{gathered} \tag{59}$$

where we have

$$\Pi_{d,t} = \frac{Q_t}{Q_{t-1}} \Pi_{n,t} \tag{60}$$

$$\Pi_{w,t} = \frac{w_{n,t}}{w_{n,t-1}} \cdot \Pi_{n,t} \tag{61}$$

$$Y_{n,t} = A_{n,t} N_{n,t} \tag{62}$$

$$Y_{d,t} = A_{d,t} N_{d,t} \tag{63}$$

$$N_t = N_{n,t} + N_{d,t} \tag{64}$$

$$w_{d,t} = Q_t^{-1} w_{n,t} \tag{65}$$

$$Div_t = Y_{n,t} - w_{n.t} N_{n,t} + Q_t \left[Y_{d,t} - w_{d,t} N_{d,t} \right] \tag{66}$$

$$\tilde{\Pi}_t = \Pi_{n,t}^{1-\gamma} \Pi_{d,t}^{\gamma} \tag{67}$$

$$i_t = u_t^r + \phi_{\tilde{\pi}} \tilde{\pi}_t \tag{68}$$

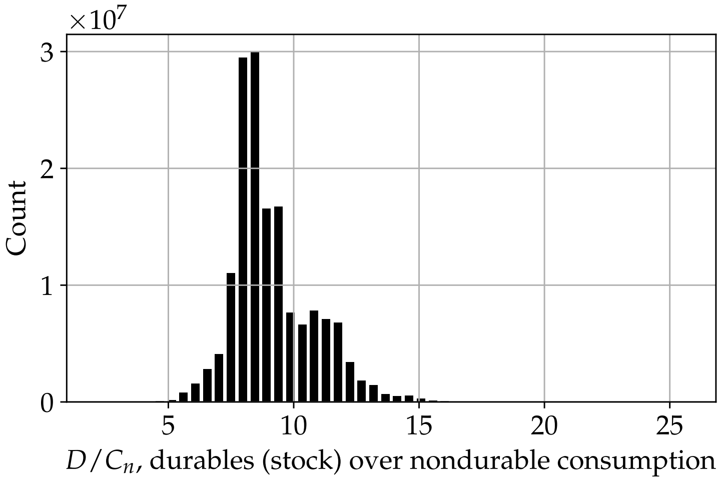
$$r_t = \frac{1 + i_{t-1}}{1 + \pi_{n,t}} - 1 \tag{69}$$

$$\tau_t = r_t B_g \tag{70}$$

and where the nondurable goods market clears by Walras' law.

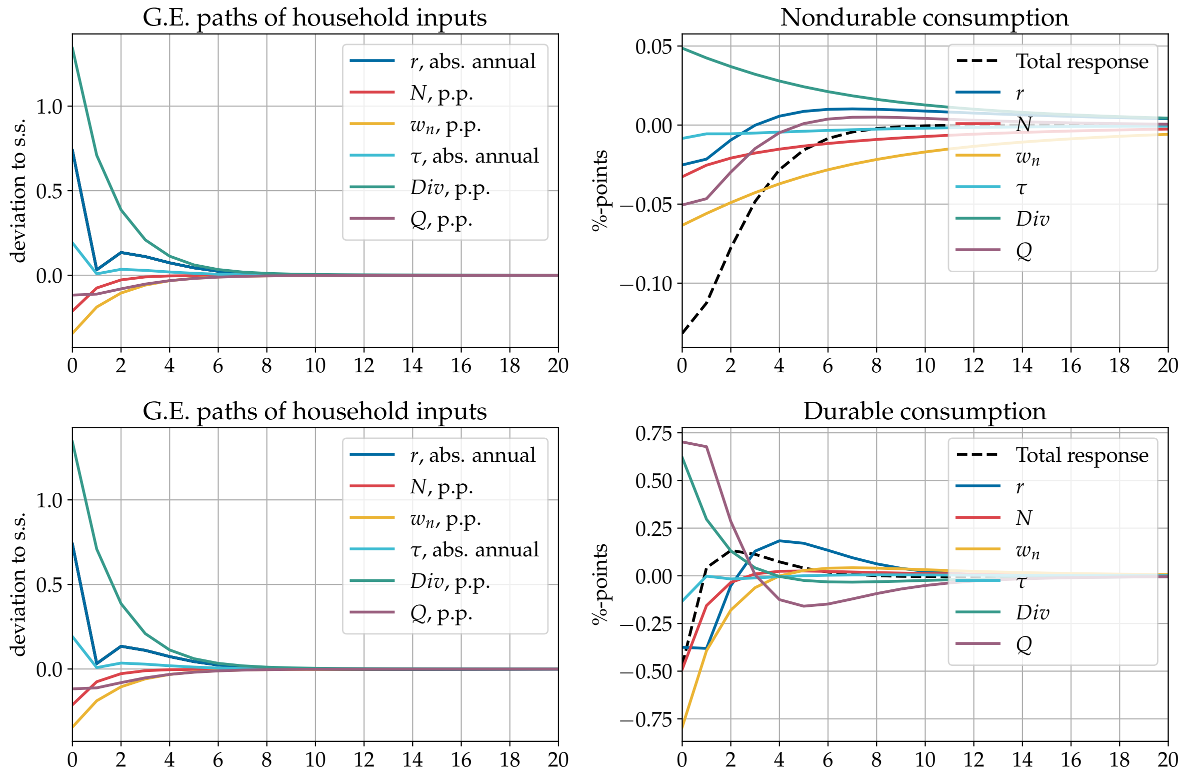
F Additional figures

Figure 7: Steady-state histogram of durables (stock) to nondurable consumption



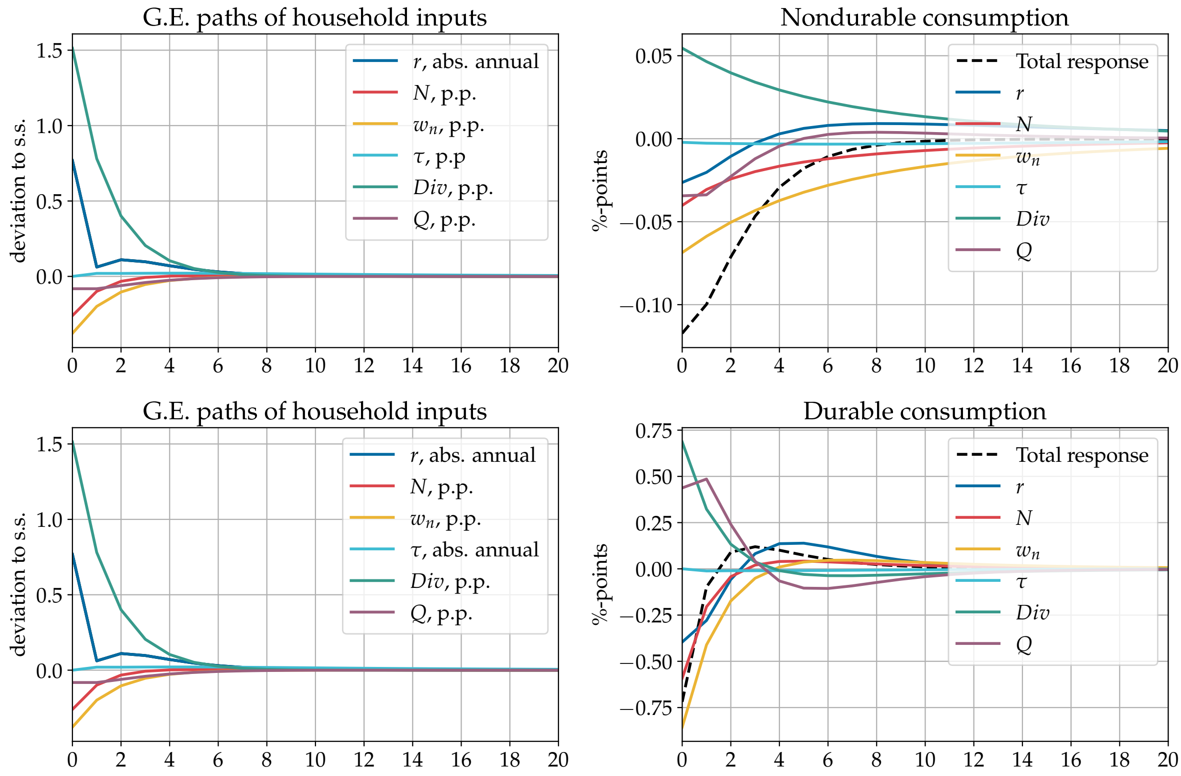
Note: To generate the histogram, we Monte Carlo simulate the steady-state household distribution using 2D linear interpolation over policy functions. We simulate 80.000 households for 2.000 periods and discard the first 1.000 periods. We use 50 bins for plotting.

Figure 8: Detailed consumption response decomposition



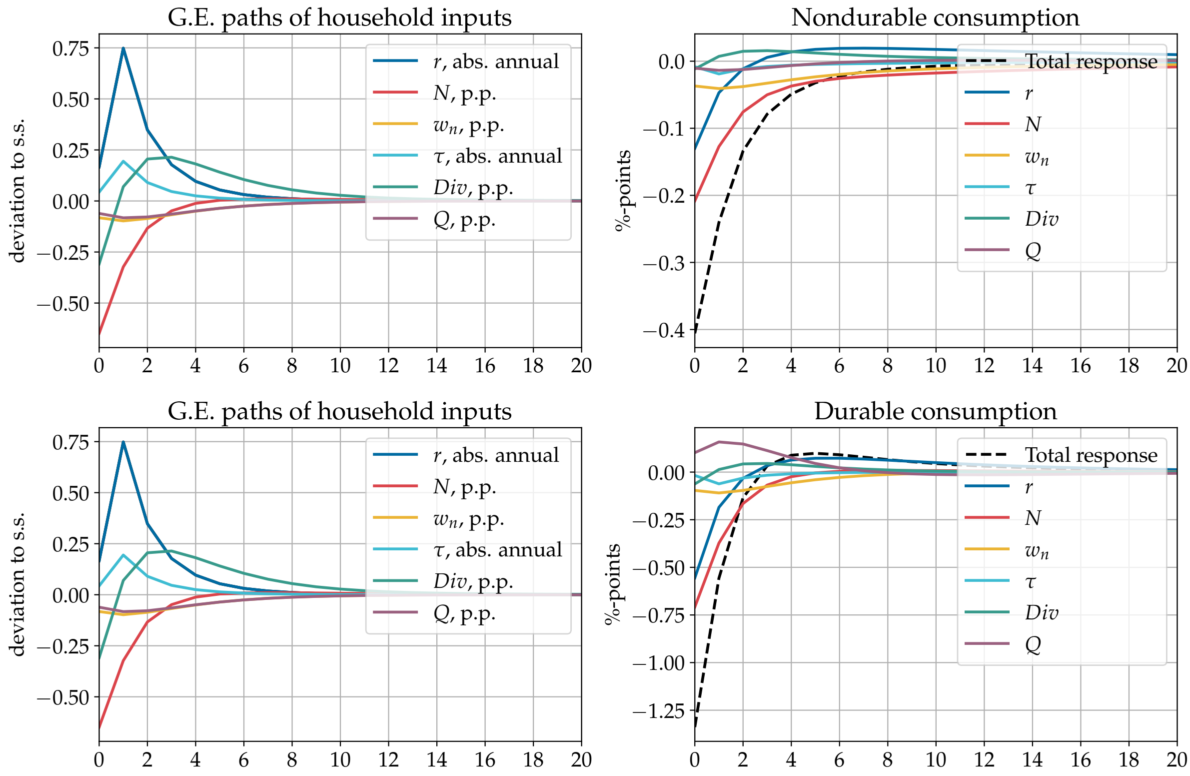
Note: Absolute annual deviations are calculated for visualization purposes.

Figure 9: Detailed consumption response decomposition under deficit financing



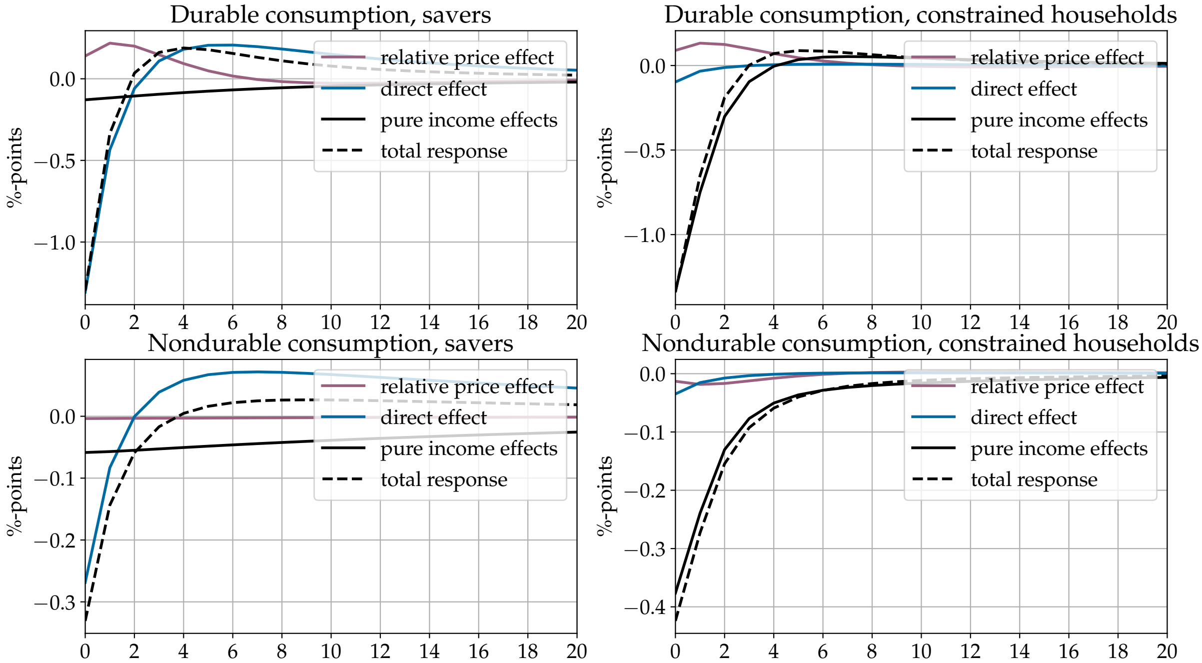
Note: Absolute annual deviations are calculated for visualization purposes.

Figure 10: Detailed consumption response decomposition with sticky wages



Note: Absolute annual deviations are calculated for visualization purposes.

Figure 11: Consumption response decomposition by steady-state wealth percentiles in the model with sticky wages



Note: Constrained households are defined as households at the bottom 50% steady-state bond holdings. Savers are defined as households at the steady-state top 1% bond holdings. The effects are calculated along the paths of interest using initially truncated distributions, relative to a simulation of the relevant truncated distribution conditional on all input variables being at their steady-state values.

Chapter 2

Long-lived Durables in T(H)ANK Economies

Long-lived Durables in T(H)ANK Economies

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Abstract

We devise tractable heterogeneous-agent New Keynesian economies where households may infrequently participate in financial markets. Both nondurable and durable goods are available for consumption. To the extent that durables feature slow depreciation rates, both their stock and their shadow value display negligible variation in the face of temporary monetary shocks. In light of this, the marginal utility of nondurable consumption for households having access to durable purchases is approximately the same, thus realizing, *de facto*, a risk-sharing condition. As a result, when all agents may access durable purchases, regardless of their financial status, the amplification of both household-specific and sectoral nondurable consumption in the face of monetary shocks may only depend on preference heterogeneity over nondurables. By contrast, factors typically key in shaping monetary transmission in benchmark one-sector economies- primarily fiscal redistribution from financially unconstrained to constrained households- only affect households' durable expenditure, with their effects intimately connected with the degree of sectoral price stickiness. When introducing hand-to-mouth consumers with no access to durable purchases and financial assets, fiscal redistribution tends to amplify the conditional volatility of GDP, unlike in one-sector economies featuring only nondurables.

Keywords: Heterogeneous agents, durable goods, monetary policy.

JEL codes: E21, E31, E40, E44, E52.

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1 Introduction

In recent years, the macroeconomic literature has established various lines of enquiry on the connection between incomplete markets and household heterogeneity, with the aim of understanding both the aggregate and the distributive outcomes of shocks to the economy (Coibion et al., 2017; Kaplan et al., 2018; Kogan et al., 2020, among others). Concurrently, a certain interest has emerged in developing analytically tractable models that can capture the salient features of heterogeneous-agent (HA) economies (see, e.g., Bilbiie, 2018, 2020; Ravn and Sterk, 2021). That said, much of our understanding of the trasmission of monetary policy—and most of the analytical literature employing HA New Keynesian (HANK, hereafter) models is no exception—comes from one-sector economies where only nondurable goods are available for consumption. Yet, it is well known that most of the fluctuations in aggregate consumption reflect the volatility of durable consumption (both at the household and at the aggregate level; see Attanasio, 1999; Stock and Watson, 1999). Moreover, consumer spending on durables is far more sensitive to changes in the interest rate than is expenditure on nondurables and services (Mankiw, 1985).

In this paper, we focus on the role of long-lived consumer durables for monetary transmission. To this end, we devise tractable HA economies where agents may infrequently participate to financial markets. Thus, conditional on their financial status, they may also feature durables with low depreciation rates, along with nondurables, in their consumption baskets. A well-established property of RANK economies with long-lived durable goods is that these preserve a quasi-constant shadow value (along with their stock), conditional on temporary shocks (Barsky et al., 2007). In light of this, the marginal utility of nondurable consumption for an agent that buys durables mirrors changes in the relative price of durables. A direct implication of this feature in HA settings is that an endogenous *risk-sharing* condition emerges among all consumers having access to durable purchases. We show this to be the case both in a two-agent New Keynesian (TANK) setting where limited participation to the *financial* market applies deterministically—so that households are inviariantly sorted into savers and hand-to-mouth (HtM) consumers—and in a setting characterized by idiosyncratic uncertainty, where consumers may switch between the two financial states: we refer to this as the 2-state HANK model. Even if HtM households cannot access a saving technology— at least from time to time— they can still smooth their nondurable consumption profile through durable purchases. Due to this property, in the face of monetary shocks both savers' and HtM households' nondurable consumption levels remain at the (symmetric) steady state—when the relative price of durables does not vary—or display similar deviations from the steady state—net of a factor that depends on agent-specific degree of relative risk aversion—when sectors exhibit asymmetric price stickiness (and the relative price varies in response to monetary shocks). In this second scenario, we derive an Euler equation for aggregate nondurable consumption where the elasticity of (expected) aggregate consumption growth to the real rate of interest depends on the share of HtM households, as well as on both households' degree of relative risk aversion. This factor loading indexes the so-called *HtM* channel in both our 2-agent/state economies.

A key departure of the TANK and the 2-state HANK economies from their respective one-sector benchmarks is that fiscal redistribution—in the form of subsidies to HtM households' income to be financed from firm profits—is irrelevant to both household-specific and sectoral nondurable consumption. In fact, only preference heterogeneity may activate the HtM channel. This is because long-lived durables insulate HtM households' nondurable consumption from the adverse effects of profits going down as demand (and, thus, the real wage) expands, for whatever reason, and in either sector. By contrast, while transfers are also neutral to the sectoral production of durables, household-specific durable expenditure is shaped by fiscal redistribution. Specifically, increasing the transfer is always beneficial to constrained households' durable expenditure under any degree of sectoral price stickiness and, thus, the behavior of sectoral profits.

A notable feature of the aggregate nondurable Euler in the 2-state HANK model, as compared with its TANK counterpart, is that discounting (compounding) of news about future expenditure may emerge, but only to the extent that HtM households are more (less) risk averse than savers. In fact, even if they acknowledge that in some states of the world, they might find themselves financially constrained, households are still able to access a durable-goods saving technology. Therefore, as far as only nondurable consumption is concerned, the *self-insurance* channel still emerges from the interaction between aggregate and idiosyncratic uncertainty, and is complementary to the *HtM* channel—as in the 2-state HANK model of Bilbiie (2018, 2020)—though the way these operate only hinges on households displaying preference heterogeneity over nondurable consumption. In light of this, different types of household-specific consumption behave exactly in the same way they do in the TANK economy. This finding challenges the conventional emphasis on the interplay between idiosyncratic uncertainty and HtM behavior as a key driver of aggregate *nondurable* consumption (Bilbiie, 2008), especially in connection with the self-insurance channel, which is regarded as a powerful intertemporal propagator of the HtM channel. Seen in this perspective, the ability to buy durable goods, along with their role as a store of value, brings the 2-state HANK model closer to a setting with complete markets (to the extent that preference heterogeneity is considered of second-order importance).

In light of this property, we introduce a third class of households that are limited in access to both financial markets and durables—a category that we label *pure HtM*—assuming they may switch to/from a third state embodying these restrictions. Within this setting, we retrieve a key property: fiscal redistribution—in the form of a fraction of savers' sectoral dividends being rebated to both types of HtM households—*amplifies* the conditional volatility of GDP. We show how such prediction ultimately depends on the behavior of durables volatility with respect to transfers. Thus, even in a more realistic setting with pure HtM households, a main takeaway is that considering a one-sector economy may be misleading in that fiscal redistribution *attenuates* the response of GDP to monetary shocks. Not only do durables induce higher overall volatility— even if produced by a relatively small sector in the economy— but the way they affect monetary transmission, both in the aggregate and at the household level, has important implications about its interaction with fiscal policy.

Related literature Our paper relates to a broad literature employing saver-spender models to investigate the transmission of monetary policy (see Campbell and Mankiw, 1989; Mankiw and Zeldes, 1991) and fiscal policy (see Galí et al., 2007). Inspired by this tradition, Bilbiie (2008) devises a one-sector TANK model where profits and their redistribution through fiscal policy take center stage. While building up on this, our settings represent non-trivial two-sector extensions, where the *propagation* of monetary policy may change profoundly.

In this respect, we relate to Barsky et al. (2007) and other contributions employing RANK models with durables to investigate the transmission of monetary policy (e.g. Erceg and Levin, 2006; Monacelli, 2009; Sudo, 2012; Tsai, 2016; Petrella et al., 2019) in that we report how profit redistribution and other structural characteristics interact with sectoral price stickiness, and may ultimately affect monetary transmission as observed in RA economies.

On the HANK front, Bilbiie (2018, 2020) surveys both the analytical and the quantitative literature. As for the first strand, he extensively traces out the main differences between his framework and other approaches in generating amplification of demand shocks. Thus, we refer to this for a detailed mapping of the available contributions. As for the second strand of the literature, our paper closely relates to McKay and Wieland (2022), who show how embedding durables into an otherwise standard HANK economy is key to attenuating the forward guidance puzzle due to higher interest rate sensitivity of the demand for durables. Furthermore, in a companion paper we devise a calibrated two-sector setting featuring durable and nondurable production, showing how transitory income effects represent the bulk of monetary propagation to both types of consumption goods, as compared with intertemporal substitutability (see Holst Partsch et al., 2022).

Finally, we relate to some contributions examining households' adjustment of the durable-nondurable consumption mix in the face of transitory income shocks. In this respect, Parker (1999) suggests that constrained households cut back more on goods that exhibit high intertemporal substitution because the utility cost of fluctuations in these is lower than goods that are less substitutable over time. Browning and Crossley (2000) formally shows this effect is equivalent to that characterizing the adjustment of luxury-goods expenditure in Hamermesh (1982).[1] While our main focus is on the transmission of monetary policy shocks, a main point of tangency with these studies is that, in our economies, durables act as an "inefficient" saving technology that bears the burden of the adjustment, for they show a quasi-constant shadow value and, thus, close-to-infinite intertemporal substitutability.

Structure The rest of the paper is organized as follows. In Section 2 we outline the baseline structure of our modular economy. Section 3 discusses the role of long-lived durables, thus reporting the behavior of both household-specific and sectoral consumption in the benchmark TANK economy. Section 4 introduces a 2-state HANK economy where consumers can switch between different states (financially constrained vs. unconstrained), while still holding a stock of durables. Section 5 focuses on aggregate amplification, ex-

[1] Browning and Crossley (2009) complements this accelerator effect with irreversibility in durable purchases.

tending the 2-state HANK economy to a 3-state economy, where the additional state contemplates the presence of agents with access to neither a saving technology nor durable purchases. Section 6 concludes.

2 Modeling strategy

We will proceed modularly, starting from a TANK model with savers and HtM households that buy both nondurables and long-lived durables. After discussing the main implications of this in connection with Bilbiie (2008, 2018, 2020), we will extend the model into a HANK economy with two states (i.e., savers vs. HtM households). The final step consists of elaborating an analytical HANK model featuring a 3-state structure, with potential transition among savers, *wealthy HtM* (who can smooth through durables but no financial markets), and *pure HtM* households (who hold neither durables nor financial assets).

The core of the model is a standard cashless dynamic general equilibrium economy augmented for limited asset market participation (LAMP). In line with Bilbiie (2008, 2018, 2020), we assume that a fraction of the households are excluded from asset markets, while others trade in complete markets for state-contingent securities (including a market for shares in firms). The main point of departure from conventional LAMP economies lies in differentiating consumption goods into nondurables and (long-lived) durables.

2.1 TANK economy

There is a continuum of households and two sectors of production, each of them populated by a single perfectly competitive final-good producer and a continuum of monopolistically competitive intermediate-goods producers setting prices on a staggered basis. There is also a government pursuing a redistributive fiscal policy and a nominal interest-rate monetary policy. A continuum of households is envisaged over the support $[0, 1]$, all having a similar utility function. A λ_S share is represented by households who can trade in all markets for state-contingent securities. We will interchangeably refer to these as asset holders or savers.

2.1.1 Households

Each asset holder chooses consumption, asset holdings, and leisure, solving a standard intertemporal problem featuring an additively separable CRRA time utility:

$$\max_{C_{S,t},B_{S,t},X_{S,t},N_{S,t}} E_t \left\{ \sum_{i=0}^{\infty} \beta^i \left(\frac{C_{S,t}^{1-\sigma_S}}{1-\sigma_S} + \eta_S \frac{X_{S,t}^{1-\chi_S}}{1-\chi_S} - \varpi_S \frac{N_{S,t}^{1+\phi_S}}{1+\phi_S} \right) \right\}$$

s.t.

$$B_{S,t}+\Omega_{S,t}V_t \leqslant (1+r_{t-1})\, B_{S,t-1}+\Omega_{S,t-1}\,(V_t+P_{C,t}D_t)+W_tN_{S,t}-P_{C,t}C_{S,t}-P_{X,t}\left[X_{S,t}-(1-\delta)\,X_{S,t-1}\right],$$

where $\beta \in (0,1)$ is the discount factor, $\eta_S > 0$ and $\varpi_S > 0$ indicate how durable consumption and leisure are valued relative to nondurable consumption, $\phi_S > 0$ is the inverse of the labor supply elasticity, while $\sigma_S \geq 1$ and $\chi_S \geq 1$ index the curvature of the utility in nondurables and durables, respectively. $C_{S,t}$, $X_{S,t}$, $N_{S,t}$ are nondurable consumption, the stock of durables and hours worked by saver (time endowment is normalized to unity). $P_{C,t}$ (taken as the numeraire) and $P_{X,t}$ are the nominal prices of nondurable and durable goods, respectively. There are two financial assets: a government-issued riskless bond paying a nominal return r_t (> 0), denoted by $B_{S,t}$, and shares in monopolistically competitive firms, denoted by $\Omega_{S,t}.V_t$ is average market value at time t shares in intermediate good firms, while $D_t = D_{C,t} + D_{X,t}^C$ are total dividend payoffs aggregated over the two sectors in terms of nondurable prices, with $D_{C,t}$ denoting profits from the nondurable goods sector and $D_{X,t}^C$ indicating profits from the durable goods sector (deflated by $P_{C,t}$).

Maximizing utility subject to this constraint gives the first-order necessary and sufficient conditions at each date and in each state:

$$1 = \beta E_t \left\{ \frac{C_{S,t+1}^{-\sigma_S}}{C_{S,t}^{-\sigma_S}} \frac{1+r_t}{1+\pi_{C,t+1}} \right\}, \tag{1}$$

$$\frac{V_t}{P_{C,t}} = \beta E_t \left\{ \frac{C_{S,t+1}^{-\sigma_S}}{C_{S,t}^{-\sigma_S}} \left(\frac{V_{t+1}}{P_{C,t+1}} + D_{t+1} \right) \right\}, \tag{2}$$

$$Q_t C_{S,t}^{-\sigma_S} = \eta_S X_{S,t}^{-\chi_S} + \beta(1-\delta) E_t \left\{ Q_{t+1} C_{S,t+1}^{-\sigma_S} \right\}, \tag{3}$$

$$\varpi_S N_{S,t}^{\phi_S} = C_{S,t}^{-\sigma_S} \frac{W_t}{P_{C,t}}, \tag{4}$$

where $Q_t \equiv P_{X,t}/P_{C,t}$, and $(1+\pi_{C,t+1}) \equiv \frac{P_{C,t+1}}{P_{C,t}}$.

The rest of the households (labeled non-assetholders or HtM households, and indexed by H) have no financial assets and solve

$$\max_{C_{H,t},X_{H,t},N_{H,t}} E_t \left\{ \sum_{i=0}^{\infty} \beta^i \left(\frac{C_{H,t}^{1-\sigma_H}}{1-\sigma_H} + \eta_H \frac{X_{H,t}^{1-\chi_H}}{1-\chi_H} - \varpi_H \frac{N_{H,t}^{1+\phi_H}}{1+\phi_H} \right) \right\}$$

s.t.

$$C_{H,t} + Q_t \left[X_{H,t} - (1-\delta) X_{H,t-1} \right] = \frac{W_t}{P_{C,t}} N_{H,t} + T_t^H,$$

where T_t^H denotes fiscal transfers. The first order conditions are

$$\varpi_H N_{H,t}^{\phi_H} = C_{H,t}^{-\sigma_H} \frac{W_t}{P_{C,t}}, \quad (5)$$

$$Q_t C_{H,t}^{-\sigma_H} = \eta_H X_{H,t}^{-\chi_H} + \beta(1-\delta) E_t \left\{ Q_{t+1} C_{H,t+1}^{-\sigma_H} \right\}. \quad (6)$$

2.1.2 Firms

In each sector $j = \{C, X\}$, the final good is produced by a representative firm using a CES production function (with elasticity of substitution ε^j) to aggregate a continuum of intermediate goods indexed by i : $Y_{j,t} = \left(\int_0^1 Y_{j,t}(i)^{(\varepsilon_j - 1)/\varepsilon_j} di \right)^{\varepsilon_j/(\varepsilon_j-1)}$. Final-good producers behave competitively, maximizing profits $P_{j,t}Y_{j,t} - \int_0^1 P_{j,t}(i) Y_{j,t}(i) di$ each period: for the j^{th} sector, $P_{j,t}$ is the overall price index of the final good and $P_{j,t}(i)$ is the price of intermediate good i. For $j = \{C, X\}$, the demand for each intermediate input is $Y_{j,t}(i) = (P_{j,t}(i)/P_{j,t})^{-\varepsilon_j} Y_{j,t}$ and the price index is $P_{j,t}^{1-\varepsilon_j} = \int_0^1 P_{j,t}(i)^{1-\varepsilon_j} di$. Each intermediate good is produced by a monopolistically competitive firm indexed by i, using a linear technology $Y_{j,t}(i) = N_{j,t}(i)$, by bearing a nominal marginal cost W_t. The profit function in real terms is thus given by: $D_{j,t}(i) = \left(1+\tau_j^S\right) \left[P_{j,t}(i)/P_{j,t}\right] Y_{j,t}(i) - \left(W_t/P_{j,t}\right) N_{j,t}(i) - T_{j,t}^F$, where $1+\tau_j^S$ denotes a production subsidy, while $T_{j,t}^F$ stands for a lump-sum profit tax. We assume the subsidy to be set to eliminate the markup distortion in the steady state: the pricing condition under flexible prices, $P_{j,t}^*(i)/P_{j,t} = 1 = \varepsilon_j \left(W_{j,t}^*/P_{j,t}\right) \left[\left(1+\tau_j^S\right)(\varepsilon_j - 1)\right]^{-1}$, allows us to pint down this value at $\tau_j^S = (\varepsilon_j - 1)^{-1}$. Financing the total cost of this subsidy by the profit tax ($T_{j,t}^F = \tau_j^S Y_{j,t}$) leads to aggregate sectoral profits $D_{j,t} = Y_{j,t} - \left(W_t/P_{j,t}\right) N_{j,t}$, which are zero in the steady state, thus allowing for full insurance in both nondurable and durable consumption—i.e. $C_S = C_H = C$ and $X_S = X_H = X$—and implying $Q = 1$. Loglinearizing around this undistorted steady state, with $d_{j,t} \equiv \ln(D_{j,t}/Y_j)$, leads to $d_{j,t} = -(w_t - p_{j,t})$.[2]

Next, we allow for monetary policy to affect the real allocation in this simple cashless model, introducing sticky prices in the vein of Calvo (1983) and Yun (1996). Intermediate-good firms in each sector $j = \{C, X\}$ adjust their prices infrequently, with θ_j being both the history-independent probability of keeping the price constant and the fraction of firms that keep their prices unchanged. Asset holders (who, in equilibrium, will hold all the shares in firms) maximize the value of the firm, i.e. the discounted sum of future nominal profits, choosing the price $P_{j,t}(i)$ and using $\Lambda_{t,t+i}$, the relevant stochastic discount factor (pricing kernel) for nominal payoffs:
$\max E_t \sum_{s=0}^{\infty} \left(\theta^s \Lambda_{t,t+s} \left[\left(1+\tau_j^S\right) P_{j,t}(i) Y_{j,t,t+s}(i) - MC_{t+i} \quad Y_{j,t,t+s}(i) - T_{j,t+s}^F\right]\right.$, subject to the demand equation, and where $\Lambda_{t,t+1}$ is S's the marginal rate of intertemporal substitution between time t and $t+1$. In equilibrium, each producer that chooses a new price $P_{j,t}(i)$ in period t will choose the same price and the same level output, so that the sectoral price index is $P_{j,t}^{1-\varepsilon_j} = (1-\theta_j)\left(P_{j,t}^*\right)^{1-\varepsilon_j} + \theta_j P_{j,t-1}^{1-\varepsilon_j}$.

2.1.3 Government

The government conducts fiscal and monetary policy. Along with the tax and the subsidy applied to sectoral production, the former consists of a redistribution scheme that taxes S's dividends at τ^D and rebates the proceedings to H, so that $T_t^H = \frac{\tau^D}{\lambda_H} D_t$. As stressed by Bilbiie (2020), this is key to determining the extent of indirect monetary policy transmission in his HA economies.

Monetary policy is conducted by means of a standard interest-rate rule that sets the nominal rate of interest in reaction to aggregate inflation and featuring a non-systematic component:

$$\frac{R_t}{R} = (1+\pi_t)^{\phi_\pi} \exp(\nu_t), \tag{7}$$

where R is the steady-state (gross) nominal interest rate, ϕ_π denotes the degree to which

[2]Notice that, due to the subsidy leading to an undistorted steady state, $d_{X,t} = d_{X,t}^C$.

the nominal interest rate responds to aggregate inflation, $\pi_t = \alpha \pi_{C,t} + (1 - \alpha) \pi_{X,t}$ (with $\alpha \in [0, 1]$), whose steady state has been implicitly assumed to be zero, as that for the sectoral inflation rates, and where $\nu_t = \rho_\nu \nu_{t-1} + \varepsilon_t^\nu$, ε_t^ν being i.i.d., zero-meaned and with variance σ_ν^2.

2.1.4 Equilibrium and market clearing

A rational expectations equilibrium is a sequence of processes for all prices and quantities introduced above, such that the optimality conditions hold for all agents and all markets clear at any given time t. Specifically, labor market clearing requires that labor demand and total labor supply to be equal, $N_t = \lambda_H N_{H,t} + \lambda_S N_{S,t} = \sum_{j=\{C,X\}} N_{j,t}$. With uniform steady-state hours, this implies the log-linear relationship $n_t = \lambda_H n_{H,t} + \lambda_S n_{S,t}$.

State-contingent assets are in zero net supply (markets are complete and agents trading in them are identical), whereas equity market clearing implies that shareholdings of each asset holder are

$$\Omega_{S,t+1} = \Omega_{S,t} = \Omega = \frac{1}{\lambda_S}. \tag{8}$$

Finally, by Walras' Law, the goods markets also clear, so that $C_t \equiv \lambda_H C_{H,t} + \lambda_S C_{S,t}$ and $X_t \equiv \lambda_H X_{H,t} + \lambda_S X_{S,t}$: in light of full consumption risk-sharing in the steady state, once log-linearized these respectively translate into $c_t = \lambda_H c_{H,t} + \lambda_S c_{S,t}$ and $x_t = \lambda_H x_{H,t} + \lambda_S x_{S,t}$.

2.2 Loglinear economy

The TANK economy can be summarized by the following log-linear relationships:

Savers:

$c_{S,t} = E_t c_{S,t+1} - \frac{1}{\sigma_S}\left(r_t - E_t \pi_{t+1}\right)$

$q_t - \sigma_S c_{S,t} = -\left[1-\beta(1-\delta)\right]\chi_S x_{S,t} + \beta(1-\delta)\left(E_t q_{t+1} - \sigma_S E_t c_{S,t+1}\right)$

$\phi_S n_{S,t} = \omega_t - \sigma_S c_{S,t}$

$c_{S,t} + \frac{Y_X}{Y_C} e_{S,t} = \frac{Y}{Y_C}\left(\omega_t + n_{S,t}\right) + \frac{1-\tau^D}{\lambda_S} d_{C,t} + \frac{1-\tau^D}{\lambda_S}\frac{Y_X}{Y_C} d_{X,t}$

$e_{S,t} = q_t + \frac{1}{\delta} x_{S,t} - \frac{1-\delta}{\delta} x_{S,t-1}$

Hand-to-mouth:

$q_t - \sigma_H c_{H,t} = -\left[1-\beta(1-\delta)\right]\chi_H x_{H,t} + \beta(1-\delta)\left(E_t q_{t+1} - \sigma_H E_t c_{H,t+1}\right)$

$\phi_H n_{H,t} = \omega_t - \sigma_H c_{H,t}$

$c_{H,t} + \frac{Y_X}{Y_C} e_{H,t} = \frac{Y}{Y_C}\left(\omega_t + n_{H,t}\right) + \frac{\tau^D}{\lambda_H} d_{C,t} + \frac{\tau^D}{\lambda_H}\frac{Y_X}{Y_C} d_{X,t}$

$e_{H,t} = q_t + \frac{1}{\delta} x_{H,t} - \frac{1-\delta}{\delta} x_{H,t-1}$

Production and pricing:

$y_{j,t} = n_{j,t},\ j = \{C, X\}$

$d_{j,t} = -\left(w_t - p_{j,t}\right),\ j = \{C, X\}$

$\pi_{j,t} = \beta E_t \pi_{j,t+1} + \psi_j rmc_{j,t},\ \ \psi_j \equiv (1-\theta_j)(1-\beta\theta_j)/\theta_j,\ j = \{C, X\}$

$rmc_{j,t} = w_t - p_{j,t},\quad j = \{C, X\}$

$q_t = q_{t-1} + \pi_{X,t} - \pi_{C,t}$

Market clearing:

$n_t = \frac{Y_X}{Y} n_{X,t} + \frac{Y_C}{Y} n_{C,t} = \lambda_H n_{H,t} + \lambda_S n_{S,t}$

$y_{C,t} = c_t = \lambda_H c_{H,t} + \lambda_S c_{S,t}$

$y_{X,t} = \frac{1}{\delta} x_t - \frac{1-\delta}{\delta} x_{t-1}$

$x_t = \lambda_H x_{H,t} + \lambda_S x_{S,t}$

Monetary Policy:

$r_t = \phi_\pi \pi_t + \nu_t$

$\pi_t = \alpha \pi_{C,t} + (1-\alpha)\pi_{X,t}$

$\nu_t = \rho_\nu \nu_{t-1} + \varepsilon_t^\nu$

where ω_t denotes the real wage expressed in units of nondurables, in percentage deviation from its steady state, i.e. $\omega_t \equiv w_t - p_{C,t}$.

3 The role of durability

Consider households' Euler equations for durables. These may be solved forward to yield an expression for the households-specific shadow value of durables:

$$Q_t C_{z,t}^{-\sigma z} = \eta_z E_t \left\{ \sum_{i=0}^{\infty} \beta^i (1-\delta)^i X_{z,t+i}^{-\chi_z} \right\} \equiv \Lambda_{z,t}, \quad z = \{S, H\}. \tag{9}$$

As noted by Barsky et al. (2007), for long-lived durables $\Lambda_{z,t}$ will be largely time-invariant to shocks with short-lived effects. This means that the intertemporal elasticity of substitution in durables demand is close to infinite. Thus, for $\beta(1-\delta)$ close to one, short-term movements in $X_{z,t}$—as those generated by a temporary shock to fiscal spending or monetary policy—will affect the right side of the equation above relatively little, so that $Q_t C_{z,t}^{-\sigma_H} \approx \Lambda_z$. According to this, movements in the relative price of durables is forced to mirror those in either household's shadow value of income, thus reflecting the emergence of an endogenous risk-sharing condition. In a log-linear setting, this implies

$$\sigma_H c_{H,t} = q_t = \sigma_S c_{S,t}. \tag{10}$$

Thus, extending households' consumption possibilities to long-lived durables implies positive comovement between their consumption of nondurables, with the extent of such comovement depending on the relative curvature of the utility functions over nondurables. Allowing for non-separability between durables and nondurables in households' utility would not fundamentally alter this property, given that the stock-flow ratio is high for long-lived durables, and $X_{z,t}$ moves slowly enough to be regarded as nearly constant, in the face of temporary shocks (Barsky et al., 2007).[3] Combining (10) with $c_t =$

[3]Notably, allowing for an adjustment cost of durables would further inhibit changes in their stock, thus exerting little effect on the shadow value from the perspective of either type of household.

$\lambda_H c_{H,t} + \lambda_S c_{S,t}$ returns[4]

$$c_t = \left[1 - \lambda_H \left(1 - \gamma\right)\right] c_{S,t}, \text{ where } \gamma \equiv \frac{\sigma_S}{\sigma_H}, \tag{11}$$

so that we can combine the latter with savers' Euler for nondurables, as well as nondurables' market clearing (i.e., $y_{C,t} = c_t$), obtaining:

$$y_{C,t} = E_t y_{C,t+1} - \chi^{-1} \left(r_t - E_t \pi_{C,t+1}\right), \tag{12}$$

where $\chi \equiv \frac{\sigma_S}{1-\lambda_H(1-\gamma)}$. Therefore, in this TANK setting, intertemporal substitution over nondurable consumption depends on household heterogeneity in terms of the coefficients of relative risk aversion and, conditional on such heterogeneity, on the size of constrained agents with respect to the total population. Notably, unlike Bilbiie (2008), there is no room for an inversion of the elasticity of the of aggregate nondurable demand to the real interest rate. Yet, increasing the wedge between the curvature of S's nondurable consumption utility and that of H may still amplify the impact of $r_t - E_t \pi_{C,t+1}$ on $\Delta E_t y_{C,t+1}$.

3.1 Monetary transmission

We can now examine equilibrium behavior in both sectors, as well as household-specific expenditure in either type of good, conditional on monetary shocks. To this end, we take an economy with symmetric price stickiness as the most straightforward extension of the one-sector framework to elicit the distinctive role of durability in monetary transmission. Thus, in line with Barsky et al. (2007), we alternatively consider the case of purely flexible prices of durables and nondurables. Based on this plan, we first detail the behavior of sectoral purchases, to then discuss the determinants of household-specific consumption of durables and nondurables. The analytics for each scenario are reported in Appendix A.

When the goods produced by both sectors display symmetric stickiness, $q_t = 0$, so that also household-specific and aggregate nondurable consumption remain at their steady-state values, in light of (10). Thus, combining S's Euler for nondurables and the Taylor rule, together with households' labor supply, returns

[4] Note also that, after combining the two labor supply schedules with $c_{H,t} = \gamma c_{S,t}$, the following restriction holds: $\phi_S n_{S,t} = \phi_H n_{H,t}$.

$$y_{X,t} = \frac{Y}{Y_X} \frac{1}{\zeta\psi(\rho_\nu - \phi_\pi)} \varepsilon_t^\nu, \text{ where } \zeta \equiv \phi_S \left[1 - \lambda_H \left(1 - \vartheta\right)\right]^{-1} \text{ and } \vartheta \equiv \phi_S/\phi_H. \tag{13}$$

As in the one-sector RANK model discussed by Barsky et al. (2007), movements in aggregate production are accounted for entirely by production, with nondurables remaining essentially unchanged. This is because HtM households can still smooth nondurable purchases through durables and, due to the combination of equally sticky sectoral prices and slow depreciation, they end up not adjusting their nondurable consumption in the face of monetary shocks, relative to the steady state.

With this picture in mind, obtaining equilibrium household-specific durable expenditure is straightforward. For illustrative purposes, we report this as a function of sectoral durable production, while neutralizing preference heterogeneity in terms of labor supply (so that $\zeta = \phi$):

$$\begin{aligned} e_{S,t} &= \left(1 + \phi \tfrac{\tau^D - \lambda_H}{\lambda_S}\right) y_{X,t}, \\ e_{H,t} &= \left(1 + \phi \tfrac{\lambda_H - \tau^D}{\lambda_H}\right) y_{X,t}, \end{aligned} \tag{14}$$

implying that the magnitude of τ^D relative to λ_H amplifies/attenuates the response of savers relative to that of HtM households. The effect is similar to that highlighted by Bilbiie (2020) in his one-sector economy. In fact, the TANK economy features an externality imposed by H on S through an income effect of real wages (which, for S, also count as marginal costs), though, in the present scenario, this only manifests itself through durable expenditure: fiscal redistribution amounts to impose an incentive on H to internalize the expansionary effects of a monetary tightening on firm profits, as long as we have a finite elasticity of labor supply.[5] As a result, inequality in durable expenditure, as measured by $e_{S,t} - e_{H,t}$, is procyclical whenever $\tau^D > \lambda_H$.

[5]Otherwise, for $\phi = 0$—i.e., when agents are perfectly insured through the real wage—the model retains a neutrality feature, as typically displayed by RANK economies.

3.1.1 Asymmetric price stickiness

We now consider asymmetric price stickiness, in the form of one sector at a time displaying pure price flexibility. Table 1 summarizes household-specific nondurable and durable expenditure elasticity relative to the respective sectoral production. In the analysis of Bilbiie (2020), such elasticity is key to examine the cyclical behavior of aggregate nondurable consumption. We take a similar standpoint, retaining preference heterogeneity, though our main focus is on how household-specific durable and nondurable expenditures are affected by fiscal redistribution and labor market characteristics (for given λ_H).

Table 1: Comovement with sectoral production

Sectoral price stickiness	$c_{S,t}$	$c_{H,t}$	$e_{S,t}$	$e_{H,t}$
Flexible p_X, sticky p_C	$\frac{1}{1-\lambda_H(1-\gamma)}$	$\frac{\gamma}{1-\lambda_H(1-\gamma)}$	$\left[1-\left(\frac{Y}{Y_C}-\frac{1-\tau^D}{\lambda_S}\right)\sigma_S\right]\frac{1}{1-\lambda_H(1-\gamma)}$	$\frac{1}{\lambda_H}-\frac{\lambda_S}{\lambda_H}\left[1-\left(\frac{Y}{Y_C}-\frac{1-\tau^D}{1-\lambda}\right)\sigma_S\right]\frac{1}{1-\lambda_H(1-\gamma)}$
Sticky p_X, flexible p_C	$\frac{1}{1-\lambda_H(1-\gamma)}$	$\frac{\gamma}{1-\lambda_H(1-\gamma)}$	$\left[1-\sigma_S\left(\frac{1-\tau^D}{\lambda_S}\frac{Y_X}{Y_C}-\frac{1}{\phi_S}\frac{Y}{Y_C}\right)\right]\frac{Y_C}{Y_C\zeta+\chi Y}\frac{1}{1-\lambda_H(1-\gamma)}$	$\frac{1}{\lambda_H}-\frac{\lambda_S}{\lambda_H}\left[1-\sigma_S\left(\frac{1-\tau^D}{\lambda_S}\frac{Y_X}{Y_C}-\frac{1}{\phi_S}\frac{Y}{Y_C}\right)\right]\frac{Y_C}{Y_C\zeta+\chi Y}\frac{1}{1-\lambda_H(1-\gamma)}$

Notes: The first column details two scenarios of asymmetric sectoral price stickiness, in which one sector at a time displays full price flexibility. For each of these, the following two columns report the elasticity of S's and H's nondurable expenditure to sectoral nondurable production, while the last two columns report the elasticity of S's and H's durable expenditure to sectoral durable production.

Even in this case, fiscal redistribution and labor market characteristics are irrelevant to households' nondurable consumption response to monetary shocks due to the presence of long-lived durables.[6] Instead, both $c_{S,t}$'s and $c_{H,t}$'s degree of comovement with aggregate nondurable expenditure hinges on the magnitude of σ_S relative to σ_H and, conditional on these being different, on how households split between savers and HtM. Whenever the curvature of H's nondurable utility exceeds that of S, i.e. $\gamma < 1$, $c_{S,t}$ ($c_{H,t}$) moves more (less) than one-for-one with $y_{C,t}$. In light of this, nondurable consumption inequality, as captured by $c_{S,t} - c_{H,t}$, is procyclical when $\gamma < 1$. As for the population shares, instead, increasing λ_H inflates (deflates) the elasticity of household-specific nondurable consumption to its sectoral aggregate, for $\gamma < 1$ (< 1), as is expected on *a priori* grounds.

On the other hand, fiscal redistribution and labor market characteristics do matter

[6]In fact, this is also the case for sectoral and aggregate production, with consumer heterogeneity only playing a role through preferences, as indexed by χ and ζ, depending how sectoral price stickiness is set (cfr. Appendix A). Things will change in this respect when pure HtM consumers—whose expenditure is only directed towards nondurables—will be introduced in Section 5.

for the behavior of household-specific durable consumption (and, thus, the behavior of cyclical inequality). In fact, both $e_{S,t}$'s and $e_{H,t}$'s degree of comovement with aggregate durable expenditure hinges on τ^D and ϕ.[7] Before seeing how such features combine, it is important to recall how sectoral production behaves in response to monetary shocks. In fact, $y_{C,t}$ *i*) contracts in the face of a monetary tightening, when durables feature flexible prices, while *ii*) it expands when it is up to nondurables to display no price stickiness (assuming that the shock is persistent enough). As for $y_{X,t}$, this necessarily comoves negatively with $y_{C,t}$ (this property may be relaxed once adding pure HtM households that consume no durables, as we do in Section 5). With this picture in mind, fiscal policy is always redistributive towards H's durable expenditure, conditional on the sign of $y_{X,t}$'s response to the monetary shock (albeit it is still neutral to sectoral production). Whenever, $y_{X,t}$ contracts (expands), increasing τ^D attenuates (amplifies) the response of $e_{H,t}$. At the same time, whenever labor hours vary—and this is not the case when durables feature flexible prices, in which case $n_{S,t} = n_{H,t} = 0$—increasing the elasticity of labor supply works in the same direction as τ^D, as the slope of the labor supply schedule drops, and a given demand increase corresponds to a more muted contraction of sectoral profits.

Therefore, regardless of how price stickiness is set across sectors, even though H has no access to financial assets, risk-sharing in nondurable consumption emerges, *de facto* allowing constrained households to smooth their nondurable consumption profile through durable purchases. Long-lived durables insulate H's nondurable consumption from the effects of profits going down as real wages expand for whatever reason. On the other hand, H's (and, thus, S's) durable expenditure rests on the cyclicality of sectoral profits with respect to aggregate durable production. In this respect, take the case of flexible prices in the durable sector first: following a monetary contraction, the real wage in units of nondurables (i.e., ω_t) contracts while households' labor supply remains at the steady state—explaining why ϕ plays no role, in this context—and also the real wage in units of durables remains unaffected (so that $d_{X,t} = 0$ too). At the same time, $d_{C,t}$ expands: thus, as τ^D increases, H (S) has more (less) resources to buy durables, for given $y_{X,t}$. In the case of flexible prices in the nondurable sector (and relatively inertial monetary shocks), instead, a monetary contraction compresses the real wage in units of durables (i.e., $\omega_t - q_t$), so that $d_{X,t}$ expands, while leaving ω_t (and, thus, $d_{C,t}$) unaffected. At the same time, households'

[7] While extending the analysis to a two-sector economy, we will mainly focus on these two determinants, taking as given other household-specific or sector-specific traits, such as the curvature of nondurable consumption utility and the relative size of each sector.

labor supply drops. For given $y_{X,t}$, while the first effect increases (restricts) H's (S's) resources to buy durables, as τ^D increases, the second effect restricts either household's durable purchase opportunities, though less so as ϕ_z increases, for $z = \{S, H\}$.

In the next section, we extend the TANK economy into a HANK model to allow agents to switch between different financial states (financially constrained vs. unconstrained) and study how compounding/discounting of news about future nondurable expenditure interacts with the *HtM channel* we have outlined for this type of economy.

4 A 2-state HANK model with long-lived durables

TANK economies miss a key channel in that unconstrained agents do not face the possibility of becoming constrained in the future, and vice versa. In this section, we extend the model from the previous section to allow for this possibility. In fact, Bilbiie (2018, 2020) discusses how extending the TANK setting in this direction is chiefly important to explain short-lived shocks and policies. As in these contributions, we can envisage the problem as featuring a unit mass of households infrequently participate in financial markets: when they do, they can adjust their portfolio frictionlessly and receive dividends from firms in either sector. When they do not participate, they only receive the return on their bond holdings from the previous period. Denote the two states as S and H, respectively. The exogenous change of state follows a Markov chain: the probability to stay type S is ϱ_{SS}, while households have a probability ϱ_{HH} to stay type H (with transition probabilities ϱ_{SH} and ϱ_{HS}, respectively). We focus on stationary equilibria whereby the mass of H is, by standard analysis, $\lambda_H = \frac{\varrho_{SH}}{\varrho_{SH}+\varrho_{HS}}$, with $\varrho_{SS} \geq \varrho_{SH}$, implying that the probability to stay a saver is larger than the probability to become one. We retain preference heterogeneity across states.

We follow Bilbiie et al. (2022) in that we make some assumptions that allow for analytical tractability. First, households are members of a family, whose intertemporal utility is maximized by the head, given limits to risk-sharing. Households are located on two *islands*, only depending on their financial-market participation status: one island is for savers, S, and one for HtM households, H. The family head can transfer resources within islands, but only some resources can be transferred between islands.

In light of these assumptions, there is full insurance within type, in the face of idiosyncratic risk, but limited insurance across types. At the beginning of the period, the family head pools resources within the island. The aggregate shocks are revealed, and the family head determines the consumption/saving choice for each household in each island. Then households learn their next-period status and have to move to the corresponding island accordingly, taking bonds and their stock of durables with them. Different financial assets thus have different liquidity: only one of the two financial assets (bonds) can be used to self-insure before idiosyncratic uncertainty is revealed—i.e., is liquid—while stocks are illiquid, and cannot be used to self-insure.

The problem for the family head reads as:

$$\max_{C_{S,t},C_{H,t},X_{S,t},X_{H,t},N_{S,t},N_{H,t},\Omega_{S,t},Z_{S,t},Z_{H,t}} E_t \left\{ \sum_{i=0}^{\infty} \beta^i \left[\lambda_S \left(\frac{C_{S,t+i}^{1-\sigma_S}}{1-\sigma_S} + \eta_S \frac{X_{S,t+i}^{1-\chi_S}}{1-\chi_S} - \varpi_S \frac{N_{S,t+i}^{1+\phi_S}}{1+\phi_S} \right) \right. \right.$$
$$\left. \left. + \lambda_H \left(\frac{C_{H,t+i}^{1-\sigma_H}}{1-\sigma_H} + \eta_H \frac{X_{H,t+i}^{1-\chi_H}}{1-\chi_H} - \varpi_H \frac{N_{H,t+i}^{1+\phi_H}}{1+\phi_H} \right) \right] \right\}$$

s.t.

$$\begin{aligned}
C_{S,t} + Q_t \left[X_{S,t} - (1-\delta) \widetilde{X}_{S,t-1} \right] + \Omega_{S,t} V_t + Z_{S,t} &= \frac{1+r_{t-1}}{1+\pi_{C,t}} B_{S,t-1} + \Omega_{S,t-1} (V_t + D_t) + \frac{W_t}{P_{C,t}} N_{S,t}, \\
C_{H,t} + Q_t \left[X_{H,t} - (1-\delta) \widetilde{X}_{H,t-1} \right] + Z_{H,t} &= \frac{1+r_{t-1}}{1+\pi_{C,t}} B_{H,t-1} + \frac{W_t}{P_{C,t}} N_{H,t} + T_t^H, \\
\lambda_S \widetilde{X}_{S,t} &= \lambda_S \varrho_{SS} X_{S,t} + \lambda_H \varrho_{HS} X_{H,t}, \\
\lambda_H \widetilde{X}_{H,t} &= \lambda_S \varrho_{SH} X_{S,t} + \lambda_H \varrho_{HH} X_{H,t}, \\
\lambda_S B_{S,t} &= \lambda_S \varrho_{SS} Z_{S,t} + \lambda_H \varrho_{HS} Z_{H,t}, \\
\lambda_H B_{H,t} &= \lambda_S \varrho_{SH} Z_{S,t} + \lambda_H \varrho_{HH} Z_{H,t},
\end{aligned}$$

where, for $z \in \{H, S\}$, $\widetilde{X}_{z,t}$ denotes the end-of-period stock of durables, while $Z_{z,t}$ ($B_{z,t}$) denotes the end-of-period (beginning-of-period) stock of bonds. The zero-liquidity limit à la Krusell et al. (2011) implies that, even though S's demand for bonds is well-defined, the supply is zero, so there are no bonds held in equilibrium. Under this assumption, the only equilibrium condition from this part of the model is the Euler equation for S's bonds:

$$C_{S,t}^{-\sigma_S} = \beta E_t \left\{ \frac{1+r_t}{1+\pi_{C,t+1}} \left[\varrho_{SS} C_{S,t+1}^{-\sigma_S} + \varrho_{SH} C_{H,t+1}^{-\sigma_H} \right] \right\}. \tag{15}$$

As for H, being her constraint binding, the zero-liquidity assumption implies HtM behavior. Furthermore, the 2-state HANK model differs from its TANK counterpart with respect to the following durable Euler equations:

$$Q_t C_{S,t}^{-\sigma_S} = \eta_S X_{S,t}^{-\chi_S} + \beta(1-\delta) E_t \left\{ \varrho_{SS} Q_{t+1} C_{S,t+1}^{-\sigma_S} + \varrho_{SH} Q_{t+1} C_{H,t+1}^{-\sigma_H} \right\}, \tag{16}$$

$$Q_t C_{H,t}^{-\sigma_H} = \eta_H X_{H,t}^{-\chi_H} + \beta(1-\delta) E_t \left\{ \varrho_{HH} Q_{t+1} C_{H,t+1}^{-\sigma_H} + \varrho_{HS} Q_{t+1} C_{S,t+1}^{-\sigma_S} \right\}. \tag{17}$$

We can take the two durable Euler equations and write them in compact form as

$$\mathbf{Y}_t = \mathbf{A} E_t \mathbf{Y}_{t+1} + \mathbf{B} \mathbf{X}_t, \tag{18}$$

with

$$\mathbf{Y}_t = \begin{bmatrix} Q_t C_{S,t}^{-\sigma_S} \\ Q_t C_{H,t}^{-\sigma_H} \end{bmatrix} and \quad \mathbf{X}_t = \begin{bmatrix} X_{S,t}^{-\chi_S} \\ X_{H,t}^{-\chi_H} \end{bmatrix}.$$

Conditional on the two eigenvalues of $\mathbf{A}$ lying within the unit circle, (18) can be iterated forward:[8]

$$\mathbf{Y}_t = \sum_{i=0}^{\infty} \mathbf{A}^i \mathbf{B} E_t \mathbf{X}_{t+i}.$$

Long-lived durables imply $\sum_{i=0}^{\infty} \mathbf{A}^i \mathbf{B} E_t \mathbf{X}_{t+i} \approx (\mathbf{I} - \mathbf{A})^{-1} \mathbf{B}\mathbf{X}$, so that the stationarity condition can be satisfied and both arguments of $\mathbf{Y}_t$ are approximately constant, implying the relative price of durables is forced to mirror households' shadow value of income. An immediate implication is that, as in the TANK setting, nondurable consumption invariantly remains at the steady state, in both states, in the benchmark scenario with symmetric price stickiness.

More generally, log-linearizing the self-insurance equation (15) around the symmetric

[8] The two eigenvalues are $\beta(1-\delta)$ and $\beta(1-\delta)(\varrho_{HH} + \varrho_{SS} - 1)$, so that the condition invariantly holds.

steady state, we obtain

$$c_{S,t} = \mu E_t c_{S,t+1} - \sigma_S^{-1} \left(r_t - E_t \pi_{t+1} \right), \text{where } \mu \equiv \varrho_{SS} + \gamma \varrho_{SH}. \tag{19}$$

Combining the latter with $c_t = [1 - \lambda_H (1-\gamma)] c_{S,t}$ and $c_t = y_{C,t}$:

$$y_{C,t} = \mu E_t y_{C,t+1} - \chi^{-1} \left(r_t - E_t \pi_{t+1} \right). \tag{20}$$

With idiosyncratic uncertainty (i.e., $\varrho_{SS} < 1$), the Euler for nondurables features by discounting/compounding of news about future nondurable expenditure—as captured by the factor loading μ—depending on $\gamma \lesseqgtr 1$.[9] In either of the two cases, even if they acknowledge that in some state of the world, they might find themselves financially constrained, households can still exploit durable goods as a store of value so that the marginal utility from nondurable consumption is equalized across states. In light of this, different forms of household-specific consumption behave exactly in the same way they do in the TANK economy (cfr. Table 1), so that fiscal redistribution and labor market characteristics operate along the same direction across different scenarios, and do not interact with idiosyncratic risk.[10] As a result, μ may only affect the elasticity of *sectoral* production to the monetary shock. In fact, Table 2 shows that μ amplifies the response of both $y_{C,t}$ and $y_{X,t}$, in either direction, when assuming asymmetric sectoral price rigidity (while playing no role under symmetric sectoral price stickiness as, again, $y_{C,t} = 0$ in this case).

Table 2: Sectoral production in the HANK economy (asymmetric price rigidity).

Sectoral price stickiness	$y_{C,t}$	$y_{X,t}$
Flexible p_X, sticky p_C	$-\frac{1-\beta\rho_\nu}{(1-\beta\rho_\nu)(1-\rho_\nu\mu)+\psi_C(\phi_\pi-\rho_\nu)}\chi^{-1}\nu_t$	$\frac{Y_C}{Y_X}\frac{1-\beta\rho_\nu}{(1-\beta\rho_\nu)(1-\rho_\nu\mu)+\psi_C(\phi_\pi-\rho_\nu)}\chi^{-1}\nu_t$
Sticky p_X, flexible p_C	$-\frac{1-\beta\rho_\nu}{(1-\beta\rho_\nu)(1-\rho_\nu\mu)-\phi_\pi\psi_X}\chi^{-1}\nu_t$	$\frac{Y_C\zeta+\chi Y}{Y_X}\frac{1-\beta\rho_\nu}{(1-\beta\rho_\nu)(1-\rho_\nu\mu)-\phi_\pi\psi_X}\chi^{-1}\nu_t$

Notably, when $\gamma < 1$, the impact of monetary policy shocks on either form of sectoral consumption is attenuated, both with respect to the direct impact of the real rate of interest on $y_{C,t}$, and through discounting of future news about nondurable spending. This is a manifestation of the *self-insurance channel* in this economy, though the way this operates and interacts with the HtM channel is, again, different from Bilbiie (2020), due to the presence of long-lived durables. When good news about future aggregate nondurable

[9]Assuming homogeneous preferences, instead, implies that the Euler corresponds to that obtained in a RANK economy with no heterogeneity.

[10]In light of this, also cyclical inequality behaves in line with the TANK economy.

production arrives, households recognize that, in some states of the world, they will be constrained in the access to financial assets while displaying lower intertemporal substitution in nondurable consumption. In light of this, even being able to purchase long-lived durables and, through these smooth nondurable purchases, households recognize they will not be able to make the most of the increase in $E_t y_{C,t+1}$.

As explained by Bilbiie (2020), the interaction between aggregate and idiosyncratic uncertainty represents the motive to self-insure, and more so as ϱ_{ss} drops, so that the HtM spell, as captured by λ_H, lasts longer. Unlike Bilbiie (2020), though, self-insurance operates with respect to nondurable consumption, *de facto*, only to the extent households display preference heterogeneity. The next section is partly aimed at addressing this shortcoming.

5 A 3-state HANK economy

We have seen how, in its 2-state version, the HANK model with long-lived durables impairs the propagation stemming from the interaction between idiosyncratic uncertainty and HtM behavior, which is typically regarded as a key driver of aggregate *nondurable* consumption. For this reason, we now devise a 3-state HANK economy where preference heterogeneity is switched off, and where household members can find themselves in any of the following three states: a state in which they can smooth consumption both over durables and nondurables, one with access to durable purchases but no financial assets—in which case they are considered *wealthy HtM*—and one without access to either durables or financial assets—in which case they are considered *pure HtM*.

We retain most of the assumptions from the previous section while assuming that households in the family infrequently participate in durable purchases too. Thus, we envisage a third island that we label K. Upon learning that they will move to this island, households drop their stock of durables, which are redistributed to S and H. The exogenous change of state follows a Markov chain: the probability to stay type S is p_{SS}, to stay type H is p_{HH}, and to stay type K is p_{KK} (with transition probabilities p_{fl} with $f, l = \{K, S, H\}$ and $f \neq l$). Even in this case, we focus on stationary equilibria.[11] The

[11]The transition matrix is set so that the Markov chain is ergodic. The steady-state solution of the transition probabilities is reported in Appendix B.1.

head of family's optimization problem now reads as

$$\max_{C_{S,t},C_{H,t},C_{K,t},X_{S,t},X_{H,t},N_{S,t},N_{H,t},N_{K,t},\Omega_{S,t},Z_{S,t},Z_{H,t}} E_t \left\{ \sum_{i=0}^{\infty} \beta^i \left[\lambda_S \left(\frac{C_{S,t+i}^{1-\sigma_S}}{1-\sigma_S} + \eta_S \frac{X_{S,t+i}^{1-\chi_S}}{1-\chi_S} - \varpi_S \frac{N_{S,t+i}^{1+\phi_S}}{1+\phi_S} \right) \right. \right.$$
$$+\lambda_H \left(\frac{C_{H,t+i}^{1-\sigma_H}}{1-\sigma_H} + \eta_H \frac{X_{H,t+i}^{1-\chi_H}}{1-\chi_H} - \varpi_H \frac{N_{H,t+i}^{1+\phi_H}}{1+\phi_H} \right)$$
$$\left. \left. +\lambda_K \left(\frac{C_{K,t+i}^{1-\sigma_K}}{1-\sigma_K} - \varpi_K \frac{N_{K,t+i}^{1+\phi_K}}{1+\phi_K} \right) \right] \right\}$$

s.t.

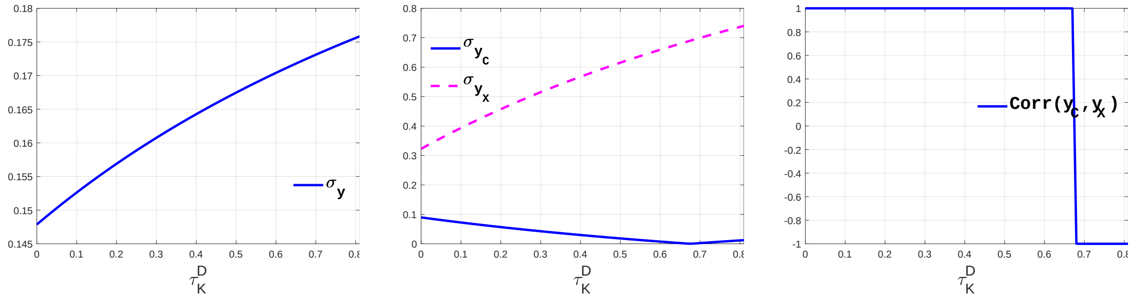
$$\begin{aligned}
C_{S,t} + Q_t \left[X_{S,t} - (1-\delta) \widetilde{X}_{S,t-1} \right] + \Omega_{S,t} V_t + Z_{S,t} &= \frac{1+r_{t-1}}{1+\pi_{C,t}} B_{S,t-1} + \Omega_{S,t-1} (V_t + D_t) + \frac{W_t}{P_{C,t}} N_{S,t}, \\
C_{H,t} + Q_t \left[X_{H,t} - (1-\delta) \widetilde{X}_{H,t-1} \right] + Z_{H,t} &= \frac{1+r_{t-1}}{1+\pi_{C,t}} B_{H,t-1} + \frac{W_t}{P_{C,t}} N_{H,t} + T_t^H, \\
C_{K,t} + Z_{K,t} &= \frac{1+r_{t-1}}{1+\pi_{C,t}} B_{K,t-1} + \frac{W_t}{P_{C,t}} N_{K,t} + T_t^K \\
\lambda_S \widetilde{X}_{S,t} &= (\varrho_{SS}\lambda_S + \varrho_{SK}\lambda_K) X_{S,t} + \varrho_{HS}\lambda_H X_{H,t}, \\
\lambda_H \widetilde{X}_{H,t} &= \varrho_{SH}\lambda_S X_{S,t} + (\varrho_{HH}\lambda_H + \varrho_{HK}\lambda_K) X_{H,t}, \\
\lambda_S B_{S,t} &= \varrho_{SS}\lambda_S Z_{S,t} + \varrho_{HS}\lambda_H Z_{H,t} + \varrho_{KS}\lambda_K Z_{K,t}, \\
\lambda_H B_{H,t} &= \varrho_{SH}\lambda_S Z_{S,t} + \varrho_{HH}\lambda_H Z_{H,t} + \varrho_{KH}\lambda_K Z_{K,t}, \\
\lambda_K B_{K,t} &= \varrho_{SK}\lambda_S Z_{S,t} + \varrho_{HK}\lambda_H Z_{H,t} + \varrho_{KK}\lambda_K Z_{K,t}.
\end{aligned}$$

Under the assumptions above, the only equilibrium condition for the bond market is the Euler equation of S:

$$C_{S,t}^{-\sigma_S} = \beta E_t \left\{ \frac{1+r_t}{1+\pi_{C,t+1}} \left[\varrho_{SS} C_{S,t+1}^{-\sigma_S} + \varrho_{SH} C_{H,t+1}^{-\sigma_H} + \varrho_{SK} C_{K,t+1}^{-\sigma_K} \right] \right\}. \tag{21}$$

As for the demand of durables, this is determined by the combination of the following

Figure 1: Volatility and fiscal redistribution



Notes: Symmetric transition probabilities, $\sigma = 1$, $\phi = 2$, $Y_C = \alpha = 0.75$, $Y_X = 1 - Y_C$, $\beta = 0.97$, $\theta_X = \theta_C = 0.6$,$\delta = 0.025$, $\phi_\pi = 1.5$.

Euler equations for S and H:

$$Q_t C_{S,t}^{-\sigma_S} = \eta_S X_{S,t}^{-\chi_S} + \beta(1-\delta) E_t \left\{ \frac{\varrho_{SS}\lambda_S + \varrho_{SK}\lambda_K}{\lambda_S} Q_{t+1} C_{S,t+1}^{-\sigma_S} + \varrho_{SH} Q_{t+1} C_{H,t+1}^{-\sigma_H} \right\}, \quad (22)$$

$$Q_t C_{H,t}^{-\sigma_H} = \eta_H X_{H,t}^{-\chi_H} + \beta(1-\delta) E_t \left\{ \frac{\varrho_{HH}\lambda_H + \varrho_{HK}\lambda_K}{\lambda_H} Q_{t+1} C_{H,t+1}^{-\sigma_H} + \varrho_{HS} Q_{t+1} C_{S,t+1}^{-\sigma_S} \right\} \quad (23)$$

Having examined the distinctive impact of long-lived durability in a two-agent economy with S and H households, we will now explore the role of K in determining sectoral and aggregate amplification of monetary shocks in the 3-state economy. In doing so, we abstract from preference heterogeneity while retaining potential asymmetry in sectoral price stickiness and size. The complete log-linear economy and the analytical derivations are reported in Appendix B.2.

We primarily focus on our benchmark setting featuring symmetric price stickiness. Figure 1 reports the conditional volatility of GDP, along with that of nondurable and durable production, as well as their correlation, all as functions of τ_K^D, which is the subsidy rate applied to K. A first element to highlight is that fiscal transfers are no longer purely redistributive. In fact, aggregate volatility increases in τ_K^D, displaying a pattern that is broadly in line with the volatility of durable production. This squares with the common view that durables, as well as their pricing, dictate the aggregate behavior of two-sector RANK economies, regardless of the pricing and demand structure of nondurables (Barsky et al., 2007). This is still the case in our HANK environment. Let us see why.

A good starting point is to notice that sectoral comovement is perfect over most of the range of values of τ_K^D (see Figure 1). This can be explained by inspecting the equilibrium sectoral levels of production, whose derivation is reported in Appendix B.3:

$$y_{C,t} = \frac{Y\phi}{Y_C\phi + Y\sigma}\left(\frac{(1+\phi)\lambda_K}{\phi} - \tau_K^D\right)\omega_t, \tag{24}$$

$$y_{X,t} = \frac{Y}{(1-\lambda_K)\,Y_X}\left(\frac{\lambda_S + \lambda_H}{\phi} + \tau_K^D\right)\omega_t, \tag{25}$$

where $\omega_t = \frac{1}{\psi(\rho_\nu - \phi_\pi)}\varepsilon_t^\nu$. Looking at $y_{C,t}$, it is immediate to infer that its volatility drops with τ_K^D up until this is lower than $\frac{(1+\phi)\lambda_K}{\phi}$, to then revert, along with the correlation between $y_{X,t}$ and $y_{C,t}$. On the other hand, the volatility of $y_{X,t}$ increases in τ_K^D. In fact, a key feature emerging from this comparative-statics analysis is that, unlike one-sector models, fiscal redistribution amplifies GDP volatility due to the presence of long-lived durables. To see more closely why this is the case, assume there is an unexpected increase in the real wage (which is now common between sectors) for whatever reason. The attenuation of fluctuations in nondurable production is to be ascribed to K—whom, in the present setting, is the only household adjusting nondurable purchases—internalizing the downward pressure on dividends, which becomes more intense as τ_K^D increases. The inversion then stems from the fact that, as this effect becomes conspicuous—and this is more easily accomplished when labor supply is relatively inelastic—any increase in the real wage compresses nondurable expenditure. As for the volatility of durables, instead, increasing τ_K^D makes more resources available for S's and H's durable consumption while their nondurable consumption remains at the steady state. Thus, increasing fiscal redistribution augments the passthrough of shocks to the real wage. Being durable production the dominant source of GDP volatility and its volatility more sensitive to fiscal redistribution, the overall impact of τ_K^D on σ_Y is necessarily expansionary. After gaining a broader picture, we will elaborate further on this aspect by analyzing the cases involving asymmetric sectoral price stickiness.

5.1 Asymmetric price stickiness

Much like the analysis of Bilbiie (2020), it is possible to characterize the elasticity of pure HtM households' nondurable consumption to aggregate nondurable consumption whenever price stickiness is asymmetric between sectors. To see this, it is possible to show that

$$c_{K,t} = \frac{\mu_K}{\lambda_K} y_{C,t}, \tag{26}$$

where the derivation of μ_K is performed in Appendix B.4. Thus, starting from S's Euler for nondurables (which is the only one holding in equilibrium), we may characterize the behavior of aggregate nondurable consumption in log-linear terms:[12]

$$y_{C,t} = \frac{\lambda_K \left(\varrho_{SS} + \varrho_{SH}\right)\left(1-\mu_K\right) + \varrho_{SK}\mu_K \left(\lambda_H + \lambda_S\right)}{\left(1-\mu_K\right)\lambda_K} E_t y_{C,t+1} - \frac{1-\lambda_K}{\sigma\left(1-\mu_K\right)}\left(r_t - E_t \pi_{C,t+1}\right) \tag{27}$$

Notably, $\mu_K < (>) \lambda_K$ ensures discounting (compounding) of news about the future while attenuating (amplifying) the elasticity of $y_{C,t}$ to the real interest rate.[13] Therefore, the (intratemporal) HtM channel is *complemented* by the (intertemporal) self-insurance channel: bad (good) news about future nondurable production reduce (boost) today's demand for nondurables, implying less (more) need for self-insurance against the K state. Thus, given that $y_{X,t}$ and $y_{C,t}$ display close-to-perfect negative correlation when either sector features purely flexible prices, the volatilities of the two sectoral productions are also characterized by the same determinants. In light of this, we can simply focus on the behavior of μ_K.

Figure 2 in Appendix B.4 reports μ_K as a function of τ_K^D and ϕ. Starting from the scenario featuring flexible prices of durable goods, $c_{K,t}$ reacts more than one-to-one to changes in nondurable production under relatively low fiscal redistribution (and large

[12]Except, again, for the case in which $\theta_X = \theta_C$.

[13]According to the same conditions, procyclical (countercyclical) nondurable consumption inequality emerges.

λ_K), as well as in the presence of a relatively high ϕ, all else equal.[14] To see why this is the case recall that, under $\theta_X = 0$ and $\theta_C > 0$, $d_{X,t} = 0$, so that K's income equals $\frac{Y}{Y_C}(n_{K,t} + \omega_t) + \frac{\tau_K^D}{\lambda_K} d_{C,t}$. Assume a monetary tightening, which causes a contraction in $y_{C,t}$ and ω_t. Raising ϕ attenuates the ensuing increase in $n_{K,t}$, thus acting as a further drag on K's labor income. At the same time, as $d_{C,t} = -\omega_t$, dividends accruing from the nondurables sector necessarily expand—attenuating the impact of the contractionary monetary stance on $c_{K,t}$—but less so as τ_K^D drops and/or λ_K increases, all else equal, as in this case pure HtM consumers progressively internalize less the positive income effect from fiscal redistribution.

Opposite conclusions are drawn when nondurable goods feature flexible prices. In this case, $\mu_K > \lambda_K$ tends to hold more easily under relatively large fiscal redistribution and/or under a relatively small λ_K. Recall that K's income equals $\frac{Y}{Y_C} n_{K,t} + \frac{\tau_K^D}{\lambda_K} \frac{Y_X}{Y_C} d_{X,t}$, in this case. A monetary tightening now induces an expansion in $y_{C,t}$, as nondurable goods become relatively cheaper. At the same time, given that $d_{X,t} = -(w_t - p_{X,t})$, dividends from the durable sector expand. Thus, increasing τ_K^D and/or reducing λ_K enhances such expansion. As for ϕ, instead, raising it amounts to limit the drop in K's labor supply, rendering it increasingly inelastic and attenuating the drag on $c_{K,t}$.

What does this comparative-statics analysis imply for the volatility of GDP and the role of fiscal redistribution? Figure 3 in Appendix B.4 shows that aggregate volatility broadly increases in τ_K^D, regardless of how sectoral volatilities behave in connection with sectoral price stickiness.[15] This can be explained by appealing to a statistical argument. We can formalize the impact of fiscal redistribution on GDP volatility as

$$\frac{\partial \sigma_y^2}{\partial \tau_K^D} = \left(\frac{Y_C}{Y}\right)^2 \frac{\partial \sigma_{y_C}^2}{\partial \tau_K^D} + \left(\frac{Y_X}{Y}\right)^2 \frac{\partial \sigma_{y_X}^2}{\partial \tau_K^D} + 2 \frac{Y_C}{Y} \frac{Y_X}{Y} \frac{\partial \mathrm{Cov}\left[y_{C,t}, y_{X,t}\right]}{\partial \tau_K^D}.$$

As $\mathrm{Corr}\left[y_{C,t}, y_{X,t}\right] = \mathrm{Cov}\left[y_{C,t}, y_{X,t}\right] / (\sigma_{y_C} \sigma_{y_X}) \approx -1$ and nondurables volatility is rather flat with respect to fiscal redistrribution (i.e., $\partial \sigma_{y_C}^2 / \partial \tau_K^D \approx 0$), we can show that

[14]In fact, in Appendix B.4 we show how to retrieve the multiplier in Bilbiie (2020), when abstracting from durability.

[15]It can be shown that this is the case even for economies with no polarization in the sectoral probabilities of price adjustment, along with non-symmetric transition probabilities.

$$\frac{\partial \sigma_y^2}{\partial \tau_K^D} > 0 \Leftrightarrow \frac{Y_X}{Y}\sigma_{y_X} \lesseqgtr \frac{Y_X}{Y}\sigma_{y_C} \quad \text{if } \frac{\partial \sigma_{y_X}}{\partial \tau_K^D} \lesseqgtr 0,$$

Thus, aggregate volatility increases in τ_K^D as long as durables have higher (lower) weighed volatility, as compared with nondurables, and their very same volatility increases (decreases) in τ_K^D. This condition is practically always verified in the scenarios we consider.[16]

To sum up, it is important to recall what happens when all agents may purchase durables. In this case, neutrality in fiscal redistribution realizes so that neither durable nor nondurable expenditure are affected by τ_K^D, in the aggregate. This is because long-lived durables allow for risk-sharing between households/states. When this structure is decoupled, and not all agents have access to durables, fiscal redistribution leads pure HtM households to internalize (through the transfer) some of the income effects of profits. Such effect is *i*) *negative* in the case of symmetric sectoral stickiness or when nondurables have sticky prices, while it is *ii*) *positive* when durables have sticky prices. Thus, in *i*) K's demand for nondurables does not increase by as much for a given increase in the transfer—and the demand externality on savers and wealthy HtM households is attenuated, as in Bilbiie (2020)—while in *ii*) we observe an amplification of their (and, thus, sectoral) nondurable expenditure by reverse logic. This is necessarily reflected in the behavior of durable expenditure, which is only accessible by savers and wealthy HtM households: as the transfer increases, more resources are available for durable spending—and so its volatility increases in τ_K^D—when we have equal stickiness or sticky prices for nondurables, while the opposite happens with sticky prices of durables. Either way, at conventional calibrations, the volatility of nondurables is substantially flat with respect to the transfer, so the behavior of durables volatility dominates. Even in the unlikely event of sticky prices of durables, GDP volatility increases in the transfer because the decline in the volatility of durable expenditure reinforces its covariance with nondurable expenditure, an effect that overcomes the drop in sectoral volatilities.

[16]Notice that, in the scenario with sticky prices for durables, GDP volatility initially drops, being the weighted volatility of nondurables lower than its counterpart for durables. This is only the case, though, for a low degree of fiscal redistribution.

6 Concluding remarks

Long-lived durables are key to the transmission of monetary policy—not just because they are more interest-rate sensitive than nondurables—but also because they represent a store of value through which households may shape their nondurable consumption profile, even when they have no access to financial markets. We highlight this property within modular two-sector New Keynesian economies where part of the households are financially constrained. The amplification/attenuation of both household-specific and sectoral nondurable consumption in TANK and HANK economies where all households can buy durables only hinges on preference heterogeneity, whereas durable consumption at the household level also depends on other structural determinants, primarily the degree of fiscal redistribution from financially unconstrained to constrained households. When contemplating the presence of households that access neither financial assets nor durable purchases, we highlight that GDP volatility increases in fiscal transfers, unlike in one-sector TANK or HANK economies featuring only nondurables. Such prediction ultimately depends on the behavior of durables volatility with respect to fiscal redistribution. These results call for further research on monetary policy's direct and indirect transmission in multi-sector settings with heterogeneous agents.

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Appendix

A TANK economy

Benchmark economy under symmetric sectoral price stickiness

In this case:

$$q_t = y_{C,t} = 0. \tag{28}$$

Thus, by combining the S's Euler for nondurables and the Taylor rule we obtain

$$\pi_t = \frac{1}{\phi_\pi} E_t \pi_{t+1} - \frac{1}{\phi_\pi} \nu_t. \tag{29}$$

So that, assuming $\phi_\pi > 1$ is sufficient to iterate the equation forward and pin down the rate of inflation:

$$\pi_t = -\frac{1}{\phi_\pi} E_t \sum_{s=0}^{\infty} \left(\frac{1}{\phi_\pi}\right)^s \nu_{t+s} = \frac{1}{\rho_\nu - \phi_\pi} \varepsilon_t^\nu. \tag{30}$$

As $\sigma_S c_{S,t} = 0$, labor supply implies

$$\phi_S n_{S,t} = \omega_t. \tag{31}$$

Since $\phi_S n_{S,t} = \zeta n_t$,aggregate inflation is dictated by

$$\pi_t = \beta E_t \pi_{t+1} + \zeta \psi n_t. \tag{32}$$

In light of $E_t \pi_{t+1} = 0$, $n_t = \frac{1}{\zeta\psi} \frac{1}{\rho_\nu - \phi_\pi} \varepsilon_t^\nu$ and

$$y_{X,t} = \frac{Y}{Y_X} y_t = \frac{Y}{Y_X} \frac{1}{\zeta\psi} \frac{1}{\rho_\nu - \phi_\pi} \varepsilon_t^\nu. \tag{33}$$

As usual, for household-specific durable consumption, we employ the usual equilibrium relationships, plugging them into the budget constraints.

Flexible prices of durables

From S's labor supply:

$$\phi_S n_{S,t} = w_t - p_{X,t} + q_t - \sigma_S c_{S,t}, \tag{34}$$

where $w_t - p_{X,t} = 0$ due to the assumption of flexible prices in the durables sector, and $q_t - \sigma_S c_{S,t}$ is approximately null due to the assumption of long-lived durability. Analogous observations for H lead us to conclude that $n_{H,t} = n_{S,t} = n_t = y_t = 0$ and $y_{C,t} = -\frac{Y_X}{Y_C} y_{X,t}$, in line with Barsky et al. (2007). Therefore, the following autonomous system obtains under flexible prices in the durable sector:

$$y_{C,t} = E_t y_{C,t+1} - \chi^{-1} \left(r_t - E_t \pi_{C,t+1} \right), \tag{35}$$

$$\pi_{C,t} = \beta E_t \pi_{C,t+1} + \psi_C \chi y_{C,t}, \tag{36}$$

$$r_t = \phi_\pi \pi_{C,t} + \nu_t. \tag{37}$$

Conjecturing a solution of this type:

$$\begin{aligned} y_{C,t} &= a_y \nu_t, \\ \pi_{C,t} &= a_\pi \nu_t, \\ E_t y_{C,t+1} &= a_y \rho_\nu \nu_t, \\ E_t \pi_{C,t+1} &= a_\pi \rho_\nu \nu_t, \end{aligned}$$

we obtain

$$\begin{aligned} a_y &= -\frac{1 - \beta\rho_\nu}{\chi \left(1 - \beta\rho_\nu\right)\left(1 - \rho_\nu\right) + \psi_C \chi \left(\phi_\pi - \rho_\nu\right)}, \\ a_\pi &= -\frac{\psi_C}{\left(1 - \beta\rho_\nu\right)\left(1 - \rho_\nu\right) + \psi_C \left(\phi_\pi - \rho_\nu\right)}, \end{aligned}$$

so that

$$y_{C,t} = -\frac{1-\beta\rho_\nu}{(1-\beta\rho_\nu)(1-\rho_\nu)+\psi_C(\phi_\pi-\rho_\nu)}\frac{1}{\chi}\nu_t, \tag{38}$$

$$y_{X,t} = \frac{Y_C}{Y_X}\frac{1-\beta\rho_\nu}{(1-\beta\rho_\nu)(1-\rho_\nu)+\psi_C(\phi_\pi-\rho_\nu)}\frac{1}{\chi}\nu_t, \tag{39}$$

where $\sigma_H < \sigma_S$ implies higher reactiveness of $y_{C,t}$ and $y_{X,t}$ in either direction (through the fact that χ is a negative function of $\sigma_S - \sigma_H$). As for agent-specific consumption, recall that $c_t = [1-\lambda_H(1-\gamma)]\,c_{S,t}$ and $c_t = \frac{1-\lambda_H(1-\gamma)}{\gamma}c_{H,t}$, implying:

$$c_{S,t} = -\frac{1-\beta\rho_\nu}{(1-\beta\rho_\nu)(1-\rho_\nu)+\psi_C(\phi_\pi-\rho_\nu)}\frac{1}{\sigma_S}\nu_t, \tag{40}$$

$$c_{H,t} = -\frac{1-\beta\rho_\nu}{(1-\beta\rho_\nu)(1-\rho_\nu)+\psi_C(\phi_\pi-\rho_\nu)}\frac{1}{\sigma_H}\nu_t, \tag{41}$$

so that the sign of the response follows from that of $y_{C,t}$. Finally, to obtain household-specific durable expenditure—which expressed in unit of nondurables is defined as $q_t + \frac{1}{\delta}x_{z,t} - \frac{1-\delta}{\delta}x_{z,t-1}$, with $z = \{S,H\}$—we turn to the budget constraints. Thus, recall that $n_{H,t} = n_{S,t} = 0$, along with $d_{X,t} = -(w_t - p_{X,t}) = -\omega_t + q_t = 0$, and $\sigma_H c_{H,t} = q_t = \sigma_S c_{S,t}$, to obtain

$$e_{S,t} = \left[\left(\frac{Y}{Y_C} - \frac{1-\tau^D}{1-\lambda}\right)\sigma_S - 1\right]\frac{Y_C}{Y_X}c_{S,t} \tag{42}$$

and, thus, $e_{H,t}$.

Flexible prices of nondurables

In this case, from S's labor supply:

$$\phi_S n_{S,t} = \omega_t - \sigma_S c_{S,t}, \tag{43}$$

where $\omega_t = 0$ due to the assumption of flexible prices in the nondurables sector and $\sigma_S c_{S,t} = \chi y_{C,t}$. Thus, through $n_t = [1 - \lambda_H (1 - \vartheta)] n_{S,t}$ (where $\vartheta = \frac{\phi_S}{\phi_H}$) we obtain

$$n_t = y_t = -\frac{\chi}{\zeta} y_{C,t}, \tag{44}$$

where $\zeta = \phi_S [1 - \lambda_H (1 - \vartheta)]^{-1}$, so that

$$y_{C,t} = -\frac{Y_X}{Y_C \zeta + \chi Y} y_{X,t}. \tag{45}$$

Conjecturing

$$\begin{aligned} y_{C,t} &= a_y \nu_t, \\ \pi_{X,t} &= a_\pi \nu_t, \\ E_t y_{C,t+1} &= a_y \rho_\nu \nu_t, \\ E_t \pi_{X,t+1} &= a_\pi \rho_\nu \nu_t, \end{aligned}$$

we obtain

$$\begin{aligned} a_y &= -\chi^{-1} \frac{(1 - \beta\rho_\nu)}{(1 - \beta\rho_\nu)(1 - \rho_\nu) - \phi_\pi \psi_X}, \\ a_\pi &= \frac{\psi_X}{(1 - \beta\rho_\nu)(1 - \rho_\nu) - \phi_\pi \psi_X}, \end{aligned}$$

Thus

$$y_{C,t} = -\frac{(1 - \beta\rho_\nu)}{(1 - \beta\rho_\nu)(1 - \rho_\nu) - \phi_\pi \psi_X} \chi^{-1} \nu_t, \tag{46}$$

$$y_{X,t} = \frac{Y_C \zeta + \chi Y}{Y_X} \frac{(1 - \beta\rho_\nu)}{(1 - \beta\rho_\nu)(1 - \rho_\nu) - \phi_\pi \psi_X} \chi^{-1} \nu_t, \tag{47}$$

where the response of $y_{C,t}$ ($y_{X,t}$) to ν_t tends to be positive if the shock is persistent enough and where, again, $\sigma_H < \sigma_S$ implies higher reactiveness of $y_{C,t}$ and $y_{X,t}$ in either direction (through the fact that χ is a negative function of $\sigma_S - \sigma_H$). As for agent specific consumption:

$$c_{S,t} = -\frac{(1-\beta\rho_\nu)}{(1-\beta\rho_\nu)(1-\rho_\nu)-\phi_\pi\psi_X}\frac{1}{\sigma_S}\nu_t, \tag{48}$$

$$c_{H,t} = -\frac{(1-\beta\rho_\nu)}{(1-\beta\rho_\nu)(1-\rho_\nu)-\phi_\pi\psi_X}\frac{1}{\sigma_H}\nu_t, \tag{49}$$

so that the sign of the response follows from that of $y_{C,t}$. Finally, to obtain $e_{S,t}$ and $e_{H,t}$, we turn to the budget constraints, recalling that $d_{C,t} = -\omega_t = 0$, $n_{z,t} = -\frac{\sigma_z}{\phi_z}c_{z,t}$, $d_{X,t} = -(w_t - p_{X,t})$, and $\sigma_H c_{H,t} = q_t = \sigma_S c_{S,t}$:

$$e_{S,t} = \left[\sigma_S\left(\frac{1-\tau^D}{1-\lambda}\frac{Y_X}{Y_C} - \frac{1}{\phi_S}\frac{Y}{Y_C}\right) - 1\right]\frac{Y_C}{Y_X}c_{S,t}, \tag{50}$$

$$e_{H,t} = \frac{1}{\lambda}y_{X,t} - \frac{1-\lambda}{\lambda}e_{S,t}. \tag{51}$$

B 3-state HANK economy

B.1 Transition probabilities in the steady state

Let us consider the transition probabilities across three states/islands $[S, H, K]$ and assume those are governed by the following transition probabilities

$$\mathbf{P} = \begin{bmatrix} \varrho_{SS} & \varrho_{SH} & \varrho_{SK} \\ \varrho_{HS} & \varrho_{HH} & \varrho_{HK} \\ \varrho_{KS} & \varrho_{KH} & \varrho_{KK} \end{bmatrix}. \tag{52}$$

Denote with $\boldsymbol{\lambda} = [\lambda_S, \lambda_H, \lambda_K]$ the share of population within each of the states/islands. The stationary distribution is found by solving the system of equations $\boldsymbol{\lambda}\boldsymbol{P} = \boldsymbol{\lambda}$:

$$\boldsymbol{\lambda} = \begin{bmatrix} \frac{\varrho_{KH}\varrho_{HS}+\varrho_{KS}\varrho_{HS}+\varrho_{HK}\varrho_{KS}}{\varrho_{KH}\varrho_{HS}+\varrho_{KS}\varrho_{HS}+\varrho_{HS}\varrho_{SK}+\varrho_{SH}\varrho_{KH}+\varrho_{SK}\varrho_{KH}+\varrho_{SH}\varrho_{KS}+\varrho_{HK}\varrho_{KS}+\varrho_{HK}\varrho_{SH}+\varrho_{HK}\varrho_{SK}} \\ \frac{\varrho_{SH}\varrho_{KH}+\varrho_{SK}\varrho_{KH}+\varrho_{SH}\varrho_{KS}}{\varrho_{KH}\varrho_{HS}+\varrho_{KS}\varrho_{HS}+\varrho_{HS}\varrho_{SK}+\varrho_{SH}\varrho_{KH}+\varrho_{SK}\varrho_{KH}+\varrho_{SH}\varrho_{KS}+\varrho_{HK}\varrho_{KS}+\varrho_{HK}\varrho_{SH}+\varrho_{HK}\varrho_{SK}} \\ \frac{\varrho_{HS}\varrho_{SK}+\varrho_{HK}\varrho_{SH}+\varrho_{HK}\varrho_{SK}}{\varrho_{KH}\varrho_{HS}+\varrho_{KS}\varrho_{HS}+\varrho_{HS}\varrho_{SK}+\varrho_{SH}\varrho_{KH}+\varrho_{SK}\varrho_{KH}+\varrho_{SH}\varrho_{KS}+\varrho_{HK}\varrho_{KS}+\varrho_{HK}\varrho_{SH}+\varrho_{HK}\varrho_{SK}} \end{bmatrix} \tag{53}$$

B.2 Loglinear economy

The loglinear economy reads as follows:

Savers:

$c_{S,t} = \varrho_{SS} E_t c_{S,t+1} + \varrho_{SH} E_t c_{H,t+1} + \varrho_{SK} E_t c_{K,t+1} - \frac{1}{\sigma}\left(r_t - E_t \pi_{C,t+1}\right)$

$q_t - \sigma c_{S,t} = -\left[1-\beta(1-\delta)\right]\chi x_{S,t}$

$+\beta(1-\delta)\left[\frac{\varrho_{SS}\lambda_S + \varrho_{SK}\lambda_K}{\lambda_S}\left(E_t q_{t+1} - \sigma E_t c_{S,t+1}\right) + \varrho_{SH}\left(E_t q_{t+1} - \sigma E_t c_{H,t+1}\right)\right]$

$\phi n_{S,t} = \omega_t - \sigma c_{S,t}$

$c_{S,t} + \frac{Y_X}{Y_C} e_{S,t} = \frac{Y}{Y_C}\left(\omega_t + n_{S,t}\right) + \frac{1-\tau_K^D - \tau_H^D}{\lambda_S} d_{C,t} + \frac{1-\tau_K^D - \tau_H^D}{\lambda_S}\frac{Y_X}{Y_C} d_{X,t}$

$e_{S,t} = q_t + \frac{1}{\delta} x_{S,t} - \frac{1-\delta}{\delta}\left(\varrho_{SS}\lambda_S x_{S,t-1} + \varrho_{HS}\lambda_H x_{H,t-1}\right)$

Wealthy hand-to-mouth:

$q_t - \sigma c_{H,t} = -\left[1-\beta(1-\delta)\right]\chi x_{H,t}$

$+\beta(1-\delta)\left[\varrho_{HS}\left(E_t q_{t+1} - \sigma E_t c_{S,t+1}\right) + \frac{\varrho_{HH}\lambda_H + \varrho_{HK}\lambda_K}{\lambda_H}\left(E_t q_{t+1} - \sigma E_t c_{H,t+1}\right)\right]$

$\phi n_{H,t} = \omega_t - \sigma c_{H,t}$

$c_{H,t} + \frac{Y_X}{Y_C} e_{H,t} = \frac{Y}{Y_C}\left(\omega_t + n_{H,t}\right) + \frac{\tau_H^D}{\lambda_H} d_{C,t} + \frac{\tau_H^D}{\lambda_H}\frac{Y_X}{Y_C} d_{X,t}$

$e_{H,t} = q_t + \frac{1}{\delta} x_{H,t} - \frac{1-\delta}{\delta}\left(\varrho_{HH}\lambda_H x_{H,t-1} + \varrho_{SH}\lambda_S x_{S,t-1}\right)$

Pure hand-to-mouth:

$\phi n_{K,t} = \omega_t - \sigma c_{K,t}$

$c_{K,t} = \frac{Y}{Y_C}\left(\omega_t + n_{K,t}\right) + \frac{\tau_K^D}{\lambda_K} d_{C,t} + \frac{\tau_K^D}{\lambda_K}\frac{Y_X}{Y_C} d_{X,t}$

Production, pricing and profits:

$y_{j,t} = n_{j,t},\ j = \{C, X\}$

$rmc_{j,t} = w_t - p_{j,t},\ j = \{C, X\}$

$d_{j,t} = -rmc_{j,t},\ j = \{C, X\}$

$\pi_{j,t} = \beta E_t \pi_{j,t+1} + \psi_j rmc_{j,t}, \psi_j \equiv (1-\theta_j)(1-\beta\theta_j)/\theta_j,\ j = \{C, X\}$

$q_t = q_{t-1} + \pi_{X,t} - \pi_{C,t}$

Market clearing:

$n_t = \frac{Y_X}{Y} n_{X,t} + \frac{Y_C}{Y} n_{C,t} = \lambda_H n_{H,t} + \lambda_K n_{K,t} + \lambda_S n_{S,t}$

$y_{C,t} = c_t = \lambda_H c_{H,t} + \lambda_K c_{K,t} + \lambda_S c_{S,t}$

$y_{X,t} = \frac{1}{\delta(1-\lambda_K)} x_t - \frac{1-\delta}{\delta(1-\lambda_K)} x_{t-1}$

Monetary policy:

$r_t = \phi_\pi \pi_t + \nu_t$

$\pi_t = \alpha \pi_{C,t} + (1-\alpha)\pi_{X,t}$

$\nu_t = \rho_\nu \nu_{t-1} + \varepsilon_t^\nu$

B.3 Sectoral dynamics in the benchmark economy

The real wage can be determined as in the TANK economy. Combine the Euler for nondurables and the Taylor rule to obtain

$$\pi_t = \frac{1}{\phi_\pi} E_t \pi_{t+1} - \frac{1}{\phi_\pi} \nu_t, \tag{54}$$

So that, by assuming $\phi_\pi > 1$, is sufficient to iterate the equation forward and pin down the rate of inflation:

$$\pi_t = -\frac{1}{\phi_\pi} E_{t s=0}^{\infty} \left(\frac{1}{\phi_\pi}\right)^s v_{t+s} = \frac{1}{\rho_\nu - \phi_\pi} \varepsilon_t^\nu, \tag{55}$$

Thus, from the NKPC:

$$\omega_t = \frac{1}{\psi \left(\rho_\nu - \phi_\pi\right)} \varepsilon_t^\nu. \tag{56}$$

Take now H's and S's budget constraints, and aggregate them, considering that *i)* $c_{H,t} = c_{S,t} = q_t = 0$ and, thus $\omega_t = w_t - p_{X,t}$, so that $d_{C,t} = d_{X,t} = -\omega_t$; *ii)* $n_{S,t} = n_{H,t} = \frac{1}{\phi}\omega_t$. Thus, multiply both sides of the constraint by $1/(1-\lambda_K)$ to obtain

$$y_{X,t} = \frac{Y}{(1-\lambda_K) Y_X} \left(\frac{\lambda_S + \lambda_H}{\phi} + \tau_K^D\right) \omega_t. \tag{57}$$

Take now K's budget constraint, and combine it with $c_{K,t} = \frac{1}{\lambda_K} y_{C,t}$, $n_{K,t} = \frac{1}{\lambda_K} n_t - \frac{\lambda_S}{\lambda_K} n_{S,t} - \frac{\lambda_H}{\lambda_K} n_{H,t}$, and $n_{S,t} = n_{H,t} = \frac{1}{\phi}\omega_t$:

$$y_{C,t} = \frac{Y}{Y_C} \left(\lambda_K - \tau_K^D - \frac{\lambda_S + \lambda_H}{\phi}\right) \omega_t + \frac{Y}{Y_C} y_t. \tag{58}$$

Consider y_t from the definition of aggregate hours, and then combine this with the labor supply schedule, in each state (recall that $n_{K,t} = \frac{1}{\phi}\omega_t - \frac{\sigma}{\phi} c_{K,t}$):

$$y_t = \frac{1}{\phi}\omega_t - \frac{\sigma}{\phi} y_{C,t}.$$

Thus, combining the latter with (58):

$$y_{C,t} = \frac{Y\phi}{Y_C\phi + Y\sigma}\lambda_K\left(\frac{1+\phi}{\phi} - \frac{\tau_K^D}{\lambda_K}\right)\omega_t.$$

B.4 Amplification under asymmetric price stickiness

We aggregate the labor supply schedules of households in each of the three states to obtain the aggregate wage schedule:

$$\phi n_t = \omega_t - \sigma c_t. \tag{59}$$

Let us now consider the case of *flexible prices for durables*. Combine the pure HtM households' labor supply with her budget constraint, using $d_{j,t} = -w_{j,t}$ and recalling that $\omega_{X,t} = 0$, to obtain

$$\omega_t = \frac{\left(\phi + \sigma\frac{Y}{Y_C}\right)\lambda_K}{\lambda_K + \phi\left(\lambda_K - \tau_K^D\right)}\frac{Y_C}{Y}c_{K,t}. \tag{60}$$

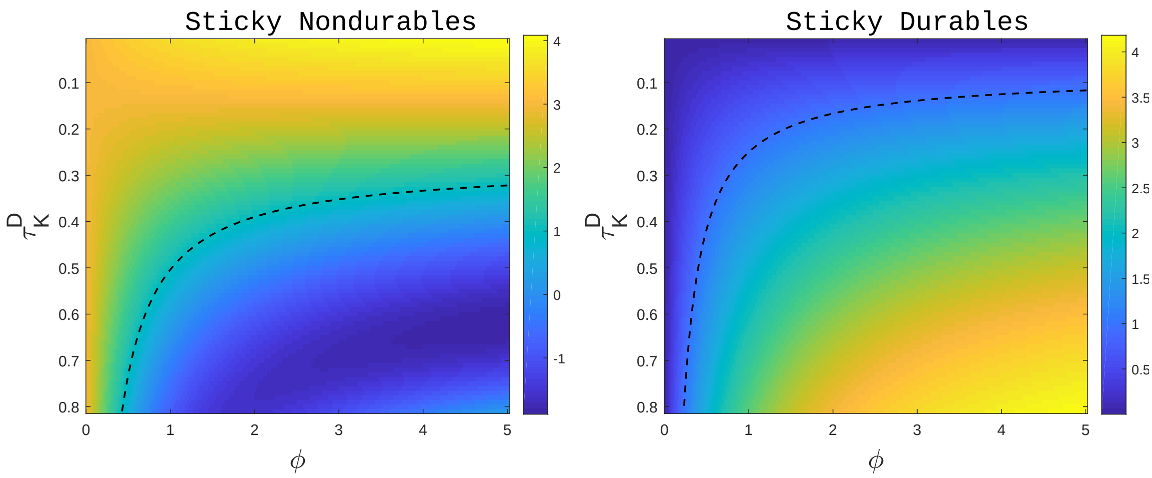
Plugging this into the the aggregate wage equation, and relying on $y_t = n_t$:

$$\phi y_t = \frac{\left(\phi + \sigma\frac{Y}{Y_C}\right)\lambda_K}{\lambda_K + \phi\left(\lambda_K - \tau_K^D\right)}\frac{Y_C}{Y}c_{K,t} - \sigma c_t. \tag{61}$$

This equation is the key to deriving K's consumption as a function of total nondurable production. Furthermore, at this stage it is possible to prove the equivalence with the multiplier in Bilbiie (2020), by simply setting $Y_C = Y$. Recall again that $(w_t - p_{X,t}) = 0$. Thus, by appealing to K's labor supply and $\sigma_H c_{H,t} = q_t = \sigma_S c_{S,t}$,[17] we can show again

[17] As in the case of the 2-state HANK economy, also in this case we can determine a system that is isomorphic to (18). Once again, conditional on the two eigenvalues of $\mathbf{A}$ lying within the unit circle, this system can be iterated forward.

Figure 2: Amplification in nondurable production



Notes: The figure reports μ_K/λ_K as a function of ϕ and τ^D; broken line denotes the unitary value. Calibration: left panel: $\theta_X = 0$ and $\theta_C = 0.6$; right panel: $\theta_X = 0.6$ and $\theta_C = 0$. Common parameters: symmetric transition probabilities, $\sigma = 1$, $Y_C = \alpha = 0.75$, $Y_X = 1 - Y_C$, $\beta = 0.97$, $\delta = 0.025$, $\phi_\pi = 1.5$.

that $n_{H,t} = n_{S,t} = 0$, so that $n_t = \lambda_K n_{K,t}$. In light of this:

$$c_{K,t} = \frac{\left[\lambda_K + \phi\left(\lambda_K - \tau_K^D\right)\right]\left[\lambda_K Y + \phi\left(\lambda_K Y - \tau_K^D Y_C\right)\right]}{\left(\phi + \sigma\frac{Y}{Y_C}\right)\left[\lambda_K Y + \phi\left(\lambda_K Y - \tau_K^D Y_C\right)\right]\frac{Y_C}{Y} - \phi\left[\lambda_K + \phi\left(\lambda_K - \tau_K^D\right)\right]\left[\lambda_K Y_C - \sigma\left(\lambda_K Y - \tau_K^D Y_C\right)\right]} \cdot \frac{\sigma}{\lambda_K} y_{C,t} \tag{62}$$

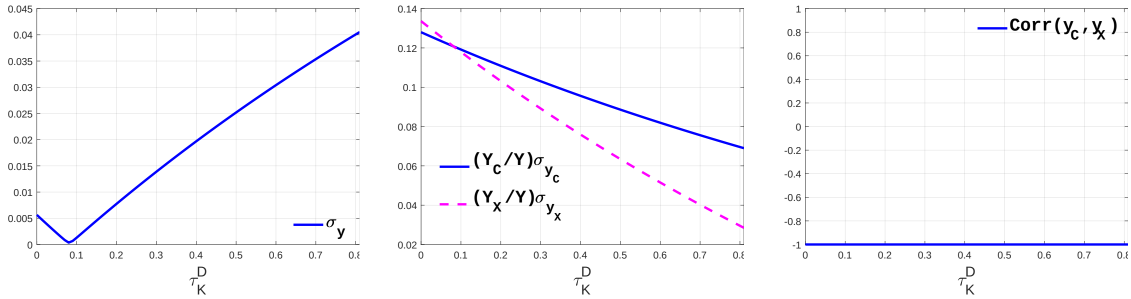
As for the case of *flexible prices for nondurables*, recall that $\omega_t = d_{C,t} = 0$ and, therefore, K's labor supply implies $n_{K,t} = -\frac{\sigma}{\phi} c_{K,t}$. Thus, from K's budget constraint:

$$\left(1 + \frac{\sigma}{\phi}\frac{Y}{Y_C}\right) c_{K,t} = \frac{\tau_K^D}{\lambda_K}\frac{Y_X}{Y_C}\sigma c_{S,t}, \tag{63}$$

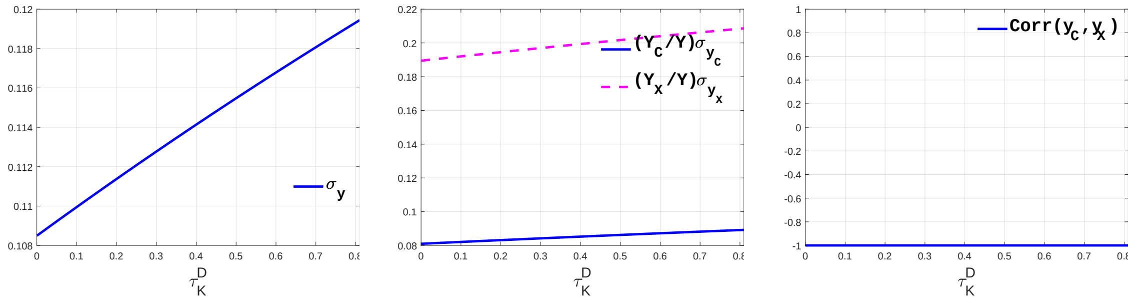
Thus, using $c_{S,t} = \frac{1}{\lambda_H + \lambda_S} c_t - \frac{\lambda_K}{\lambda_H + \lambda_S} c_{K,t}$ and $c_t = y_{C,t}$,, we can prove that

$$c_{K,t} = \frac{\tau_K^D \sigma \phi Y_X}{\phi Y_C\left(\lambda_H + \lambda_S\right) + \sigma Y\left(\lambda_H + \lambda_S\right) + \tau_K^D \sigma \phi Y_X}\frac{1}{\lambda_K} y_{C,t}. \tag{64}$$

Figure 3: Volatility and fiscal redistribution: asymmetric stickiness



(a) Sticky Nondurables



(b) Sticky Durables

Notes: Panel a): $\theta_X = 0$ and $\theta_C = 0.6$; panel b): $\theta_X = 0.6$ and $\theta_C = 0$. Common parameters: symmetric transition probabilities, $\sigma = 1$, $\phi = 2$, $Y_C = \alpha = 0.75$, $Y_X = 1 - Y_C$, $\beta = 0.97$, $\delta = 0.025$, $\phi_\pi = 1.5$.

Chapter 3

Global Solutions of Heterogeneous Agent Models with Non-linear Aggregate Dynamics

Global Solutions of Heterogeneous Agent Models with Non-linear Aggregate Dynamics*

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Abstract

In this paper, we present and evaluate global solution methods for heterogeneous agent models with non-linear aggregate dynamics. We consider models with weak approximate aggregation, where the aggregate dynamics can be summarized with a small number of states. We allow the perceived law-of-motion the agents use for now- and forecasting to be non-linear, and do not impose any parametric restrictions on it. Specifically, we derive the perceived law-of-motion using either a *neural net* or *radial basic function interpolation*. Both methods deliver precise global solutions, but radial basis function interpolation is faster because of a slow training step for the neural net, and more stable in terms of ensuring convergence. We are able to globally solve our benchmark model with an aggregate non-linearity and period-by-period market clearing in less than 15 minutes on a desktop computer.

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1 Introduction

Recently, a burgeoning literature incorporating explicit heterogeneity into business cycle model has come to the forefront of modern macroeconomics. This literature shows that introducing such heterogeneity may radically alter transmission mechanisms and dynamics relative to comparable representative agent economies. The standard approach is to use *local* solution methods. These are generally fast, but can miss central model dynamics due to exogenous non-linearities or strong internal propagation and feedback loops. *Global* solution methods are typically more complicated and slower, but allow us to study a broader range of economic questions. For example, a global solution is necessary to study both the response of precautionary saving and portfolio choice to aggregate risk, and the state dependence of impulse responses form shocks and economic policies.

The first methods for solving heterogeneous agent models globally were presented in Den Haan (1996, 1997) and Krusell and Smith (1997, 1998). The resulting literature was later evaluated in a special issue of *Journal of Economics Dynamics and Control* (Den Haan, 2010b)[1] and surveyed in Algan et al. (2014).[2] A central insight from this line of work was that the dynamics of the macroeconomic aggregates could be modeled in terms of only the mean of the wealth distribution. This is generally referred to as »approximate aggregation«. In this paper, we rely on a *weaker* version of approximate aggregation, where we only assume that the aggregate dynamics can be modeled in terms of a small number of aggregate states. We also deviate from the standard assumption of linear or log-linear aggregate dynamics, allowing the perceived law-of-motion agents use for forecasting to be non-linear. This is important because a full global solution is not needed as soon as aggregate dynamics are approximately linear. If aggregate dynamics are instead linear, one can instead rely on early contributions in Reiter (2002, 2009, 2010) and efficient first order solution methods developed in state-space form (Bayer and Luetticke, 2020) and in sequence-space form (Auclert et al., 2021).[3]

Our proposed solution method is a variant of the standard Krusell-Smith algorithm. The major difference is that we allow the perceived laws-of-motion (PLMs) the agents use

[1] For this paper, the two step approach used in Maliar et al. (2010), and the non-stochastic simulation method developed in Young (2010) (see also Tan (2020)), are in particular relevant.

[2] See Terry (2017) for a survey on methods for solving heterogeneous firm models.

[3] See also Ahn et al. (2018); Boppart et al. (2018); Winberry (2018); Bilal (2021).

for now- and forecasting to be non-linear, and do not impose any parametric restrictions on them. Specifically, we derive the perceived law-of-motion using either a neural net or radial basic function interpolation. We formulate the perceived-laws-of-motion such that it should hold exactly in the limit of no approximation errors.[4] This allows us to reap the full gains of the local flexibility of our function approximation methods. Additionally, we formulate the household problem such that period-by-period market clearing is a pure interpolation problem on already derived policy functions. This provides additional speed-up. We apply our method to a Heterogeneous Agents Neo-Classical (HANC) model with adjustable technology utilization subject to an upper bound and capital adjustment costs.

Our paper is complementary to Fernández-Villaverde et al. (2021), who use the method from Fernández-Villaverde et al. (2020) to solve a Heterogeneous Agent New Keynesian (HANK) model with an occasionally binding zero lower bound using a neural net for the perceived laws-of-motion. In line with their results, assuming linear laws of motion results in higher than acceptable forecast errors. Instead, we propose using radial basis function interpolation for the PLMs to solve the model globally, which is stable, accurate, and fast. Specifically, using our benchmark model, we can find the full global solution in less than 15 minutes on a desktop computer using radial basic function interpolation. This is slightly slower than linear PLMs, but orders of magnitude more precise. Instead, using a neural net to generate PLMs is slower and less precise than using radial basis function interpolation, and in some cases, we face a lack of convergence.

The proposed solution method is easy to implement and Python code for all results in the paper is provided at github.com/JeppeDruedahl/GlobalHA.[5]

Finally, our paper is related to the recent literature on using artificial intelligence algorithms for solving general equilibrium models with heterogeneous agents (Maliar and Maliar, 2020; Gorodnichenko et al., 2021; Hill et al., 2021; Maliar et al., 2021; Valaitis and Villa, 2021; Azinovic et al., 2022; Curry et al., 2022; Han et al., 2022). These methods are designed to also alleviate the cost of solving each agent's dynamic programming problem. This is particularly relevant when there are many aggregate states, as even our weak version of approximate aggregation fails in that case. Our paper focuses on non-linear

[4] This differs from the implementation of the Krusell-Smith method for a Heterogeneous Agent New Keynesian (HANK) model in Bayer et al. (2019), where one of the PLMs is for a variable with an expectation term. Errors should then only be zero *on average*.

[5] We build on top of the Python package GEModelTools, which we have developed for computing local solutions to heterogeneous agent models using the sequence-space method from Auclert et al. (2021).

but lower dimensional problems and can therefore rely on much simpler algorithms.

The paper is structured as follows. Section 2 presents the overall solution method. Section 3 presents our benchmark model. The main results are presented in Section 4 and additional results for a model without capital adjustment costs are presented in Section 5. Section 6 concludes.

2 Solution Method

Setup. To present our solution method in quasi-general terms, before turning to our benchmark model in the following section, we consider a heterogeneous agents model with a single endogenous state and a single continuous choice. The approximate household problem then is

$$\begin{aligned}\tilde{v}(\boldsymbol{Z}_t, \boldsymbol{S}_{t-1}, \boldsymbol{z}_t, a_{t-1}) &= \max_{a_t, c_t} u(c_t) + \beta \mathbb{E}_t \left[\tilde{v}(\boldsymbol{Z}_{t+1}, \boldsymbol{S}_t, \boldsymbol{z}_{t+1}, a_t) \right] \\ &\text{s.t.} \\ \boldsymbol{S}_t, \boldsymbol{P}_t &= \text{PLM}(\boldsymbol{Z}_t, \boldsymbol{S}_{t-1}) \\ a_t + c_t &= m(\boldsymbol{z}_t, a_{t-1}, \boldsymbol{P}_t) \\ \boldsymbol{z}_{t+1} &\sim \Gamma_z(\boldsymbol{z}_t) \\ \boldsymbol{Z}_{t+1} &\sim \Gamma_Z(\boldsymbol{Z}_t) \\ a_t &\geq -b,\end{aligned} \tag{1}$$

where

1. $\boldsymbol{Z}_t$ are exogenous aggregate shocks.
2. $\boldsymbol{S}_{t-1}$ are pre-determined (finite dimensional) aggregate states.
3. $\boldsymbol{P}_t$ are »prices«.
4. PLM($\bullet$) is the *Perceived-Law-of-Motion*.
5. $\boldsymbol{z}_t$ is stochastic and exogenous idiosyncratic states.
6. c_t is consumption providing utility $u(c_t)$ discounted by β.
7. a_t is end-of-period assets (borrowing constraint given by b).

8. $m(\bullet)$ is cash-on-hand with $\frac{\partial m(\bullet)}{\partial a_{t-1}} > 0$.

This problem is approximate because the distribution of households, $\boldsymbol{D}_t$, over the idiosyncratic states $\boldsymbol{z}_t$ and a_{t-1}, are *not* included as a state. In the true model, the distribution matters for determining both current and future aggregate states and prices. In the approximation, this is instead captured by a finite number of aggregate states, $\boldsymbol{S}_t$. These can, however, be functions of the distribution.

The PLM must be specified to let the household update the aggregate states, $\boldsymbol{S}_t$, and infer all the prices, $\boldsymbol{P}_t$, in their budget constraint. The PLM must be consistent with all aggregate model relationship, where expectation terms can be evaluated with e.g. a quadrature method and using that $\boldsymbol{Z}_{t+1} \sim \Gamma_Z(\boldsymbol{Z}_t)$ is known. In general, the PLM is therefore allowed to take the form

$$\text{PLM}(\boldsymbol{Z}_t, \boldsymbol{S}_{t-1}) = \mathbb{E}\left[f(\boldsymbol{Z}_t, \boldsymbol{S}_{t-1}, \boldsymbol{Z}_{t+1}; \Psi) \mid \boldsymbol{Z}_t, \boldsymbol{S}_{t-1}\right], \tag{2}$$

where Ψ are parameters to be determined. In the benchmark model, we specify the PLM without an expectation term, but when searching for market clearing prices we use that an expectation term can be evaluated.

Importantly, the law-of-motion the PLM is approximating is fully *deterministic*, and there is therefore no bias-variance trade-off when estimating it from data on states, $\boldsymbol{S}_t$, shocks, $\boldsymbol{Z}_t$, and prices $\boldsymbol{P}_t$. In other word, there is no fundamental over-fitting risk in letting $f(\bullet)$ have infinitely many degrees of freedom, and it can therefore be modeled with a neural net or radial basis function interpolation.[6]

An *equilibrium condition* is that the PLM is (approximately) self-consistent in terms of the implied dynamics. I.e. as the household observes simulated data, it should not be able to improve forecast errors by updating Ψ.

[6] In the presence of approximation errors some over-fitting is possible. A stochastic simulation might result in differences in outputs, $\boldsymbol{P}_t$ and $\boldsymbol{S}_t$, for (almost) the same inputs, $\boldsymbol{Z}_t$ and $\boldsymbol{S}_{t-1}$. In this case it might therefore be valuable to introduce some smoothing, by restricting $f(\bullet)$ in some way to even this out.

Household solution and simulation. The approximate household problem can be solved with an endogenous-grid-method as

$$\begin{aligned}
q(\boldsymbol{Z}_t, \boldsymbol{S}_{t-1}, z_t, a_t) &= \mathbb{E}\left[v_a(\boldsymbol{Z}_{t+1}, \boldsymbol{S}_t, z_{t+1}, a_t)\right] \\
\tilde{c}(\boldsymbol{Z}_t, \boldsymbol{S}_{t-1}, z_t, a_t) &= (\beta q(\bullet))^{-\frac{1}{\sigma}} \\
\tilde{m}(\boldsymbol{Z}_t, \boldsymbol{S}_{t-1}, z_t, a_t) &= a_t + c(\bullet) \\
c^\star(\boldsymbol{Z}_t, \boldsymbol{S}_{t-1}, z_t, a_{t-1}) &= \text{interp } \tilde{m}(\bullet) \to \tilde{c}(\bullet) \text{ at } m(z_t, a_{t-1}, \boldsymbol{P}_t) \\
a^\star(\boldsymbol{Z}_t, \boldsymbol{S}_{t-1}, z_t, a_{t-1}) &= m(\bullet) - c^\star(\bullet) \\
v_a(\boldsymbol{Z}_t, \boldsymbol{S}_{t-1}, z_t, a_{t-1}) &= \frac{\partial m(\bullet)}{\partial a_{t-1}} c^\star(\bullet)^{-\sigma}.
\end{aligned} \tag{3}$$

Next, define the associated cash-on-hand savings function by

$$\begin{aligned}
a^\star(\boldsymbol{Z}_t, \boldsymbol{S}_{t-1}, z_t, m_t) &= a^\star(\boldsymbol{Z}_t, \boldsymbol{S}_{t-1}, z_t, a_{t-1}) \\
a_{t-1} &= m^{-1,a}(m_t, z_t, \boldsymbol{P}_t).
\end{aligned} \tag{4}$$

The distribution of households can be simulated forward using either Monte Carlo simulation or the non-stochastic histogram method from Young (2010), which we prefer here.

Aggregate simulation. For pre-determined $\boldsymbol{D}_0$, $\boldsymbol{Z}_{-1}$ and $\boldsymbol{S}_{-1}$, and the savings policy function $a^\star(\boldsymbol{Z}_t, \boldsymbol{S}_{t-1}, z_t, m_t)$, the model can be simulated for $t \in \{0, \ldots, T\}$ by the following period-by-period three step procedure

1. Draw $\boldsymbol{Z}_t$ given $\boldsymbol{Z}_{t-1}$;
2. Find $a_t^\star(z_t, m_t) = a^\star(\boldsymbol{Z}_t, \boldsymbol{S}_{t-1}, z_t, m_t)$ by interpolation over $\boldsymbol{Z}_t$ and $\boldsymbol{S}_{t-1}$;
3. Search for $\boldsymbol{P}_t$ so $\int a_t^\star(z_t, m(z_t, a_{t-1}, \boldsymbol{P}_t)) d\boldsymbol{D}_t$ clears the savings market.

A potential computational bottleneck is step 3. A central benefit of the formulation we use here is that the search for market clearing prices only involves interpolating an already known policy function. Typically, a large number of simulation periods is needed, say

5,000, and therefore small computational costs quickly accumulate.[7]

Aggregate solution. The PLM can be found from a fixed-pointed iteration with relaxation as

1. Draw shocks to be used in all simulations
2. Solve and simulate a linearized version of the model
3. Estimate the PLM on the simulated data
4. Given the PLM compute $\check{\boldsymbol{S}}^0$ and $\check{\boldsymbol{P}}^0$ on the grid of $\boldsymbol{Z}_t$ and $\boldsymbol{S}_{t-1}$
5. Set the PLM convergence iteration counter $n = 0$
6. Solve the approximate household problem given $\check{\boldsymbol{S}}^n$ and $\check{\boldsymbol{P}}^n$
7. Simulate the model given household behavior
8. Estimate the PLM on the simulated data
9. Given the PLM compute $\check{\boldsymbol{S}}_{NEW}$ and $\check{\boldsymbol{P}}_{NEW}$ on the grid of $\boldsymbol{Z}_t$ and $\boldsymbol{S}_{t-1}$
10. Stop if $|\check{\boldsymbol{S}}_{NEW} - \check{\boldsymbol{S}}^n|_\infty <$ tol. and $|\check{\boldsymbol{P}}_{NEW} - \check{\boldsymbol{P}}^n|_\infty <$ tol.
11. Update $\check{\boldsymbol{S}}$ and $\check{\boldsymbol{P}}$ by relaxation with $\omega \in (0,1)$

$$
\begin{aligned}
\check{\boldsymbol{S}}^{s+1} &= \omega \check{\boldsymbol{S}}_{NEW} + (1-\omega)\check{\boldsymbol{S}}^s \\
\check{\boldsymbol{P}}^{s+1} &= \omega \check{\boldsymbol{P}}_{NEW} + (1-\omega)\check{\boldsymbol{P}}^s.
\end{aligned}
$$

12. Increment n and return to step 6

The convergence criteria in step 10 and the relaxation in step 11 is done in terms of the evaluated values of the PLM on the grid. This ensures that the relaxation is done on the inputs to the household problem. The standard approach of iterating on the parameters *inside* the PLM is less appealing when the PLM is non-linear in these parameters. It would

[7] In models with additional endogenous states and continuous and discrete choices, clearing the market easily becomes a computational bottleneck. See Bakota (2022) for a method to avoid finding market clearing prices period-by-period.

then not be ensured that a relaxation scheme in terms of averages of current and past parameters would imply smooth changes in the inputs to the household problem.

The potential computational bottlenecks in the algorithm are in steps 6-8. The computational challenges in finding the market clearing prices in step 7 were discussed above. The computational complexity of the household problem in step 6 is increasing in the number of aggregate shocks and states due to the curse of dimensionality, and their combined number must therefore be small. For simplicity, we here consider tensor product grids and multi-linear interpolation.[8] The computational complexity of the estimation in step 8 will depend on the chosen function approximation method for the PLM. It will be negligible, when the parameters in the PLM can be estimated with a simple linear regression, but can increase substantially if training of a neural net is required, as discussed later.

Functional approximation methods. The choice of functional approximation method used for the PLM is central. We consider tree different approaches:

1. Linear regression (short: OLS)
2. Neural net with a single layer (short: NN)
3. Radial basis function interpolation (short: RBF)

To describe these, let $X_{it} \in \mathbf{Z}_t, \mathbf{S_{t-1}}$ denote the i'th input to the PLM for $i \in \{1, \ldots, \#_{ZS}\}$ and let $Y_{jt} \in \boldsymbol{S}_t, \boldsymbol{P}_t$ denote the j'th output for $j \in \{1, \ldots, \#_{SP}\}$.

For *linear regression*, we have

$$Y_{jt} = \Psi_{j0} + \sum_{i=1}^{\#_{ZS}} \Psi_{ji} X_{it}. \tag{5}$$

Estimation of Ψ is straightforward with OLS.[9]

[8] The presence of aggregate non-linearities makes global interpolation methods problematic. A speed-up could, however, probably be achieved with adaptive sparse grids (see e.g. Brumm and Scheidegger, 2017; **?**; Eftekhari and Scheidegger, 2022).

[9] Technically, it is straightforward to include non-linear terms. For a specific model in question, it is, however, not a priori clear, which terms to include.

For the *neural net*, we have

$$Y_{jt} = \Psi_{j0} + \sum_{q=1}^{Q} \Psi_{jq}\phi\left(\Psi_{jq0} + \sum_{i=1}^{\#_{ZS}} \Psi_{jqi} s(X_{it}, \Psi_{js})\right), \tag{6}$$

where $\phi(\bullet)$ is the activation function, $s(\bullet)$ is the scaling function and Q is the number of neurons. We found best behavior with $Q = 5,000$ neurons, and *ReLu* activation functions, $\phi(x) = \max\{0, x\}$, on data scaled to have zero mean and unitary variance.[10] We train the neural net (i.e. find Ψ) using stochastic gradient descend with a Nesterov momentum of 0.90 and mean squared errors as the loss function. We aim for a perfect fit and over-fitting is therefor not an issue. We terminate when no improvement has been achieved for 5 epochs. We use the implementation in *tensorflow* in Python.

For the *radial basis functions*, we have

$$Y_{jt} = \Psi_{j00} + \sum_{i=1}^{\#_{ZS}} \Psi_{j0i} X_{it} + \sum_{\tau=1}^{\mathcal{T}} \Psi_{jk}\phi\left(\sum_{i=1}^{\#_{ZS}} \sqrt{\left(X_{it} - X_{i\tau}^{\text{sim}}\right)^2}\right), \tag{7}$$

where $\phi(\bullet)$ is the kernel, and $X_{i\tau}^{\text{sim}}$is the i'th input to the PLM from the τ'th period in the simulation for $\tau \in \{0, \dots, T\}$. We use a *thin plate function* kernel, $\phi(x) = x^2 \log x$. The parameters are chosen by solving the equation system implied by exactly fitting all the data points from the simulation. We use the implementation in *scipy* in Python.

The choice of function approximation method must be evaluated both in terms of its ability to secure the stability of the solution method, its accuracy at convergence, and the implied solution time. Specifically, we measure the accuracy of the derived aggregate laws of motion using the long-run dynamic forecast errors as suggested by Den Haan (2010a).

A central issue for stability of the solution method is, that the complicated geometry of inputs, $\boldsymbol{Z}_t$ and $\boldsymbol{S}_{t-1}$, change between iterations, even when keeping the shocks used for simulation fixed. Therefore extrapolation beyond the data set where the PLM was estimated is unavoidable. All the three chosen function approximation methods deliver good results in this respect. We have also experimented with global interpolation using Chebyshev polynomials and local interpolation using nearest or natural neighbor interpolation

[10]It is straightforward to use multi-layered neural nets as well. We have found no benefit from doing this.

or Barycentric interpolation from a Delaunay-triangulation. These function approximation methods, however, all suffer from boundary issues, and we have often found stability problems in terms of lack of convergence.

Regarding *implementation complexity*, the three function approximation methods are similar, as they all rely on standard packages.

3 Benchmark model: HANC

In this section, we present a baseline heterogeneous agent neo-classical (HANC) model with aggregate risk. Essentially, it is an extension of Krusell and Smith (1998) to have i) technology utilization costs and ii) capital adjustment costs. These extensions introduce an aggregate non-linearity and a forward looking term, such that we need to search for prices at each period along the simulation.

Firms. A representative firm rents capital, K_{t-1}, and hires labor, L_t, to produce goods, with the production function

$$Y_t = u_t Z_t K_{t-1}^{\alpha} L_t^{1-\alpha}, \tag{8}$$

where Z_t is exogenous technology, and u_t is technology utilization. Technology follows an AR(1):

$$Z_{t+1} - Z_{ss} = \rho_Z (Z_t - Z_{ss}) + \epsilon_{t+1}^Z, \;\; \epsilon_t^Z \sim \mathcal{N}(0, \sigma_Z^2). \tag{9}$$

Changing u_t involves virtual adjustment costs. Capital depreciates with the rate δ. The representative firms objective is given as

$$\max_{L_t, K_{t-1}, u_t} u_t Z_t K_{t-1}^{\alpha} L_t^{1-\alpha} - w_t L_t - r_t^k K_{t-1} - \chi_1 (u_t - \tilde{u}) - \frac{\chi_2}{2} (u_t - \tilde{u})^2$$
$$\text{s.t. } u_t \leq \bar{u}.$$

where r_t^k is rental rate of capital and w_t is the wage rate, χ_1 and χ_2 are parameters determining the size of the linear and convex adjustment costs, and $\tilde{u}$ is zero-cost level of technology utilization. Finally, $\bar{u}$ is an upper bound on u_t introducing an aggregate non-linearity. The problem yields standard pricing equations plus an equation pinning down technology capacity utilization

$$r_t^k = \alpha u_t Z_t (K_{t-1}/L_t)^{\alpha\text{-}\mathbf{1}} \equiv r^k(u_t, Z_t, K_{t-1}, L_t) \tag{10}$$

$$w_t = (1-\alpha) u_t Z_t (K_{t-1}/L_t)^{\alpha} \equiv w(u_t, Z_t, K_{t-1}, L_t) \tag{11}$$

$$u_t = \max\left[\frac{Z_t K_{t-1}^{\alpha} L_t^{1-\alpha} - \chi_1 + \chi_2 \tilde{u}}{\chi_2}, \bar{u}\right] \equiv u(Z_t, K_{t-1}, L_t). \tag{12}$$

The implied (real) interest rate is

$$r_t = r_t^k - q_t\delta = r^k(u_t, Z_t, K_{t-1}, L_t) - \delta \equiv r(u_t, Z_t, K_{t-1}, L_t).$$

Capital producers. Capital producers choose investment, I_t,and take the price of capital, q_t, as given. Investments are subject to quadratic adjustment costs, and capital producers objective is given as

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t I_t \left\{ q_t \left[1 - \frac{\phi}{2}\left(\log \frac{I_t}{I_{t-1}}\right)^2\right] - 1 \right\}.$$

Optimality implies (dropping higher-order terms)

$$q_t \left[1 - \phi \log \frac{I_t}{I_{t-1}}\right] = 1 - \beta \mathbb{E}_t \left[q_{t+1} \phi \log \left(\frac{I_{t+1}}{I_t}\right)\right]. \tag{13}$$

Assuming that capital adjustment costs are virtual, the law of motion for aggregate capital is trivially given by

$$K_t = I_t + (1-\delta) K_{t-1} - \tfrac{\phi}{2}\left(\log \tfrac{I_t}{I_{t-1}}\right)^2 . \tag{14}$$

Implies zero adjustment costs in steady-state.

Households. Households are heterogeneous ex post with respect to their productivity, z_t, and assets, a_{t-1}. The distribution of households is denoted D_t. Each period household exogenously supply z_t units of labor, and choose consumption c_t subject to a no-borrowing constraint. The household problem is

$$
\begin{aligned}
v(Z_t, K_{t-1}, I_{t-1}, z_t, a_{t-1}) &= \max_{a_t, c_t} \frac{c_t^{1-\sigma}}{1-\sigma} + \beta \mathbb{E}_t \left[v(Z_{t+1}, K_t, I_t, z_{t+1}, a_t) \right] \\
\text{s.t.} & \\
L_t &= 1 \\
K_{t-1} &= \int a_{t-1} dD_t \\
r_t, w_t, K_t, I_t &= \text{PLM}(Z_t, K_{t-1}, I_{t-1}) \\
a_t + c_t &= (1 + r_t) a_{t-1} + w_t z_t \\
\log z_{t+1} &= \rho_z \log z_t + \psi_{t+1} \;\; , \psi_{t+1} \sim \mathcal{N}(\mu_\psi, \sigma_\psi), \, \mathbb{E}[z_{t+1}] = 1 \\
Z_{t+1} &= \rho_Z (Z_t - Z_{ss}) + \epsilon_{t+1}^Z \\
D_{t+1} &= \Lambda(Z_t, D_t) \\
a_t &\geq 0.
\end{aligned} \tag{15}
$$

Calibration. The calibration is arbitrary, but implies aggregate and idiosyncratic dynamics within reasonable bounds.

1. **Preferences:** $\sigma = 2$, $\beta = 0.995$
2. **Income:** $\rho_z = 0.96$, $\sigma_\psi = 0.15$
3. **Production and investment:** $\alpha = 0.33$, $\delta = 0.05$, $\varphi = 0.05$
4. **Technology utilization:** $\overline{u} = 1.0$, $\chi_1 = 1$, $\chi_2 = 1$, $\tilde{u} = 0.99$
5. **Aggregate technology:** $\rho_Z = 0.80$, $\sigma_Z = 0.01$

3.1 Global solution

The general solution method can be used cf. Section 2 as follows

1. The shocks are $\boldsymbol{Z}_t = \{Z_t\}$.
2. The aggregate states are $\boldsymbol{S}_t = \{K_t, I_t\}$.
3. The »prices« are $\boldsymbol{P}_t = \{r_t, w_t\}$,

4. The PLM is

$$
\begin{aligned}
K_t &= \text{PLM}_K(Z_t, I_{t-1}, K_{t-1}; \Psi) \\
q_t &= \text{PLM}_q(Z_t, I_{t-1}, K_{t-1}; \Psi) \\
u_t &= u(Z_t, K_{t-1}, 1) \\
w_t &= w(u_t, Z_t, K_{t-1}, 1) \\
r_t^k &= r(u_t, Z_t, K_{t-1}, 1) \\
r_t &= r_t^k - q_t \delta_t.
\end{aligned}
$$

5. The cash-on-hand function is

$$
m(z_t, a_{t-1}, \boldsymbol{P}_t) = (1 + r_t) a_{t-1} + w_t z_t.
$$

6. The market clearing condition is

$$
\int a_t^\star(\boldsymbol{z}_t, m(\boldsymbol{z}_t, a_{t-1}, w_t, r_t)) dD_t = K_t,
$$

where we guess on I_t and get r_t from

$$
\begin{aligned}
q_t &= \frac{1 - \beta \mathbb{E}_t \left[q_{t+1} \phi \log \left(\frac{I_{t+1}}{I_t} \right) \right]}{1 - \phi \log \left(\frac{I_t}{I_{t-1}} \right)} \\
K_{t+1} &= \text{PLM}_K(Z_{t+1}, I_t, K_t; \Psi) \\
I_{t+1} &= K_{t+1} - (1 - \delta) K_t \\
q_{t+1} &= \text{PLM}_q(Z_{t+1}, I_t, K_t; \Psi) \\
u_t &= u(Z_t, K_{t-1}, 1) \\
w_t &= w(u_t, Z_t, K_{t-1}, 1) \\
r_t^k &= r(u_t, Z_t, K_{t-1}, 1) \\
r_t &= r_t^k - q_t \delta_t,
\end{aligned}
$$

and where expectations are evaluated using Gauss-Hermite quadrature on the actual law of motion, $Z_{t+1} = \rho_Z (Z_t - Z_{ss}) + \epsilon_{t+1}^Z$.

Numerical implementation. For the *household problem*, we use $\#_a = 80$ grid points for $a_t \in [0, 100]$ and $\#_z = 3$ grid points for z_t discretized using the Rouwenhorst-method. For the deterministic steady state we solve and simulate the household problem with a tolerance of 10^{-12}. When solving the household problem globally, we lower the tolerance to 10^{-4}.

For the *aggregate states* we use, 10 grid points for Z_t, 20 grid points for K_{t-1}, 15 grid points for I_{t-1} and 3 Gauss-Hermite nodes for Z_{t+1}. The grid spans are chosen relative to the mean values of the PLM states from initial linear simulation.

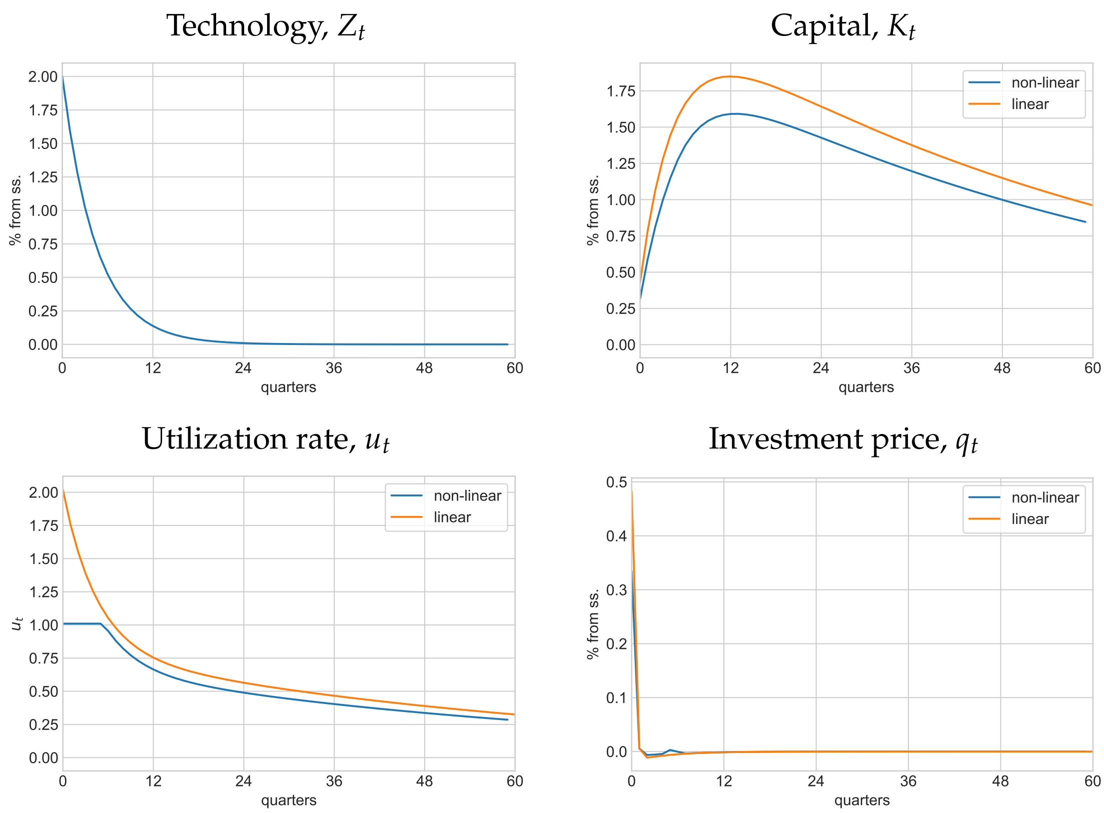
We simulate the model for $5,000$ periods and use a market clearing tolerance of 10^{-6}. For the *PLMs,* we use a relaxation weight of $\omega = 0.65$ and a tolerance of 10^{-4} whenever possible.

Timings were computed on a Windows 10 desktop computer with a i7-4770 3.4 GHz CPU with 4 cores and 32 GB of RAM.

3.2 Perfect foresight IRFs

To get intuition on the aggregate non-linearites present in the model, we first consider a 2 percent shock to technology and compare the linear and non-linear perfect foresight impulse responses computed with the sequence space method in Auclert et al. (2021). In Figure 1, we see that the linearized solution does *not* take the non-linearity of technology utilization into account, and implies a utilization rate way above the allowed upper limit of $\overline{u} = 1$. The positive effect on capital accumulation is consequently overstated relative to the nonlinear impulse response. This shows that a simulation of the model with aggregate risk using linearization will be very inaccurate. Therefore a global solution is needed.

Figure 1: Perfect foresight and linearized impulse responses



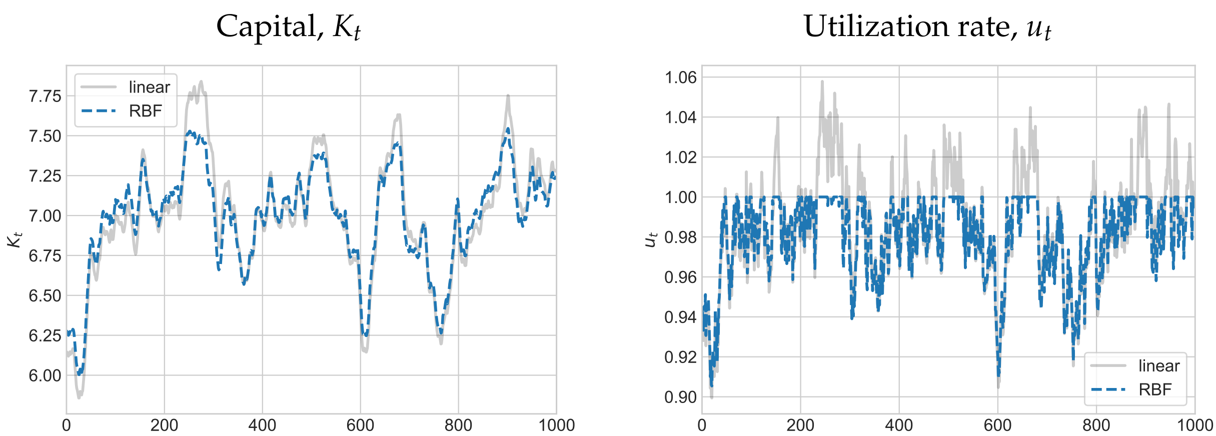
4 Results

In this Section, we present results from using the global solution presented in Section 3 and how the choice of function approximation method in the PLMs affects the accuracy and solution time. This section does not contain results with a neural net for function approximation due to convergence issues. In Section 5, we consider the limit of no capital adjustment costs, $\varphi \to 0$, where all the function approximation methods yields converged global solutions.

4.1 Simulations

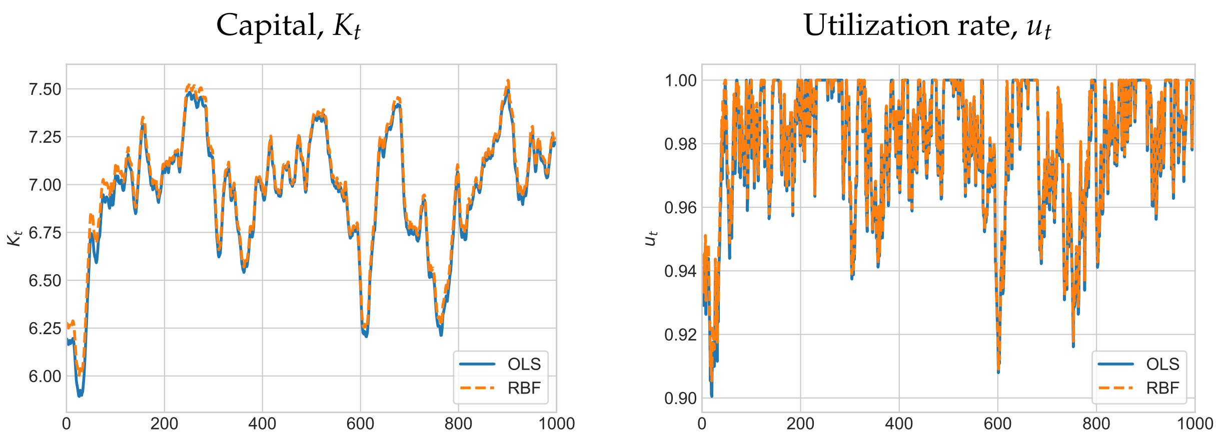
As explained in Section 2, we begin our global solution methods from a linearized simulation given a fixed draw of aggregate shocks. At convergence, our simulation method implies a new simulation with these aggregate shocks. Figure 2 shows the last 1,000 periods from these two simulations, where we use RBF for function approximation in the PLMs. We see that the upper limit of the utilization rate is violated in the linearized simulation, but respected in the global simulation. This naturally affects the dynamics of capital, which is markedly different, but still similar enough to make the linearized model reasonable to start from.

Figure 2: Simulation (in-sample): RBF vs. linear



Next, we consider out-of-sample simulations with new draws of shocks not used in the solution. In Figure 3, we show global simulations with the OLS and RBF function approximation methods for the PLMs. The OLS based simulation is consistently beneath the RBF based one, though the differences are not large.

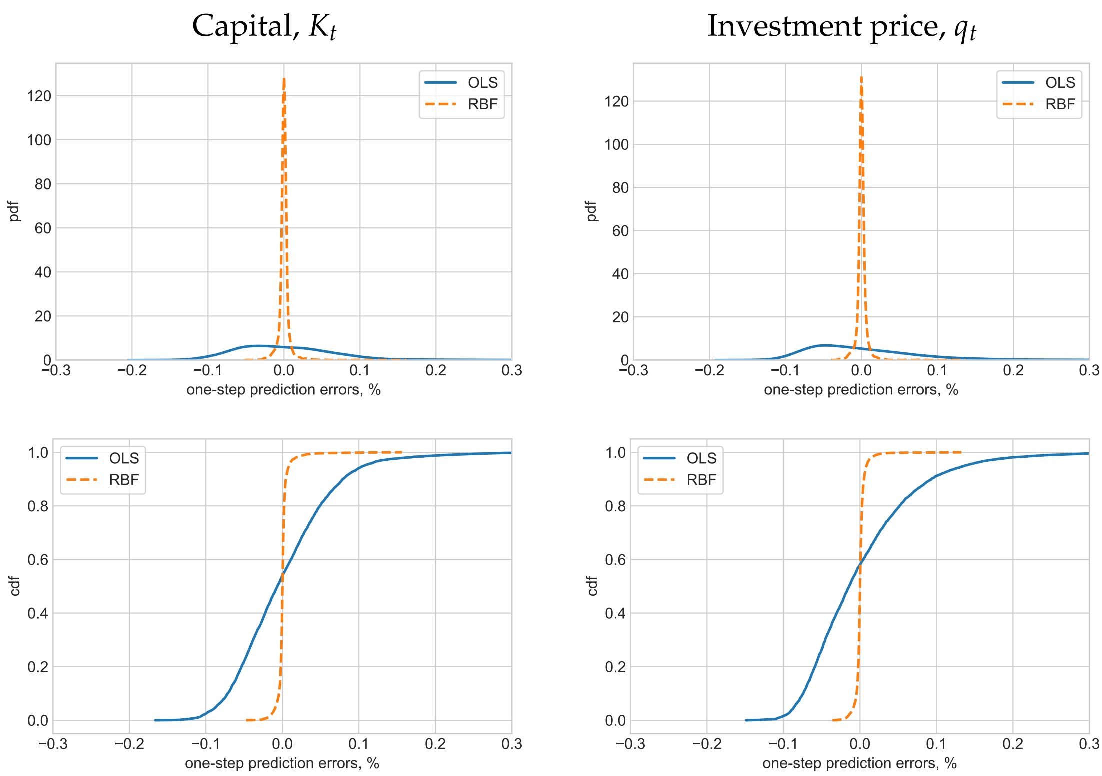
Figure 3: Simulation (out-of-sample): PLM methods



4.2 Accuracy

To choose between the different function approximation methods, we now consider their accuracy. In Figure 4, we first show the one-step prediction errors for capital and the investment price in percent of the true realizations in an out-of-sample simulation. Using RBF delivers much more precise predictions than using OLS. With RBF, the errors are nicely centered around zero. With OLS, the errors are fat-tailed and left-skewed. The skewness is likely due to the upper bound on the technology utilization rate.

Figure 4: One-step ahead PLM errors



A stronger accuracy measure is given by the dynamic Den Haan errors, cf. Den Haan (2010a). We use a standard out-of-sample simulation and a simulation based on the PLMs alone to calculate these. Specifically, the path of capital and the investment price is generated by the PLMs alone as

$$\begin{aligned} K_t^{\text{PLM}} &= \text{PLM}_K(Z_t, K_{t-1}^{PLM}, I_{t-1}^{PLM}) \\ q_t^{\text{PLM}} &= \text{PLM}_K(Z_t, K_{t-1}^{PLM}, I_{t-1}^{PLM}) \\ I_t^{\text{PLM}} &= K_t^{\text{PLM}} - (1-\delta)K_{t-1}^{PLM}. \end{aligned} \tag{16}$$

The dynamic errors are then calculated in percentage terms as

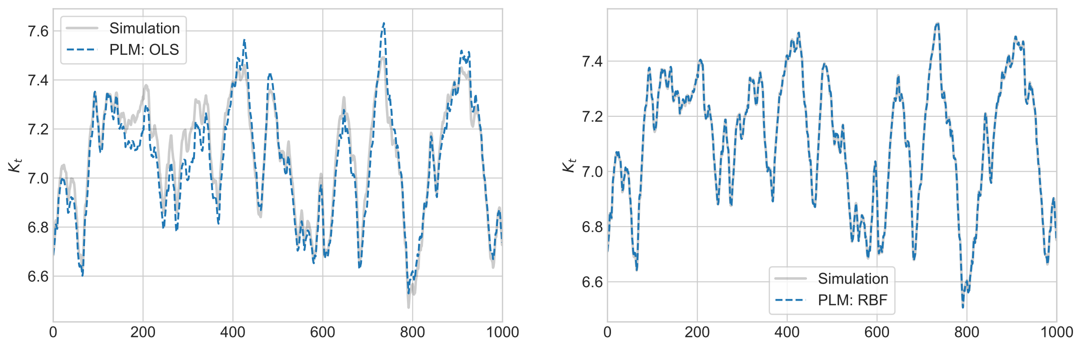
$$\begin{aligned} &100 \times \left|\log K_t^{\text{PLM}} - \log K_t\right| \\ &100 \times \left|\log q_t^{\text{PLM}} - \log q_t\right|. \end{aligned} \tag{17}$$

In Figure 5, we plot the dynamic PLM forecasts against the actual model simulation. We see that with RBF the PLM simulation lies almost exactly on top of the model simulation, while with OLS it is consistently below.

This observation is also confirmed in Table 1. For both the maximum and the average, the dynamic errors are more than an order of magnitude larger with OLS than RBF. This is moreover achieved with a minimal increase in the solution time of about 10 percent due to an increased time to estimate the PLMs. The RBF errors are very small and economically acceptable. Using RBF takes slightly longer due to a more complicated estimation of the PLM. With both methods, the solution time is less than 15 minutes. RBF is therefore clearly preferred.[11]

Figure 5: Dynamic PLM errors

(a) OLS (b) RBF



[11]The low number of required simulations shows the benefits of starting from the linearized solution.

Table 1: HANC: Prediction Errors

	OLS	RBF	NN
	dynamic log prediction errors × 100		
max	3.75	0.26	
mean	0.88	0.04	
median	0.79	0.03	
99th perc.	3.16	0.18	
90th perc.	1.62	0.07	
	timings (secs.)		
total	666.3	722.2	
- solve household problem	356.2	315.4	
- simulate with market clearing	310.0	356.3	
- estimate PLMs	0.0	50.5	
iterations	13	14	

4.3 PLMs

To understand the differences in accuracy, we now investigate how the PLMs actually look like.

In Figures 6 and 7, we plot how the PLMs for capital, K_t, and the investment price, q_t, vary along each input dimension keeping the values of the other inputs fixed at their (deterministic) steady-state values.

We see that RBF interpolation captures non-linearities in the model, especially along the Z_t and the I_{t-1} dimensions, which cannot be captured with linear OLS.

Figure 6: PLM for capital, K_t – 2D slices

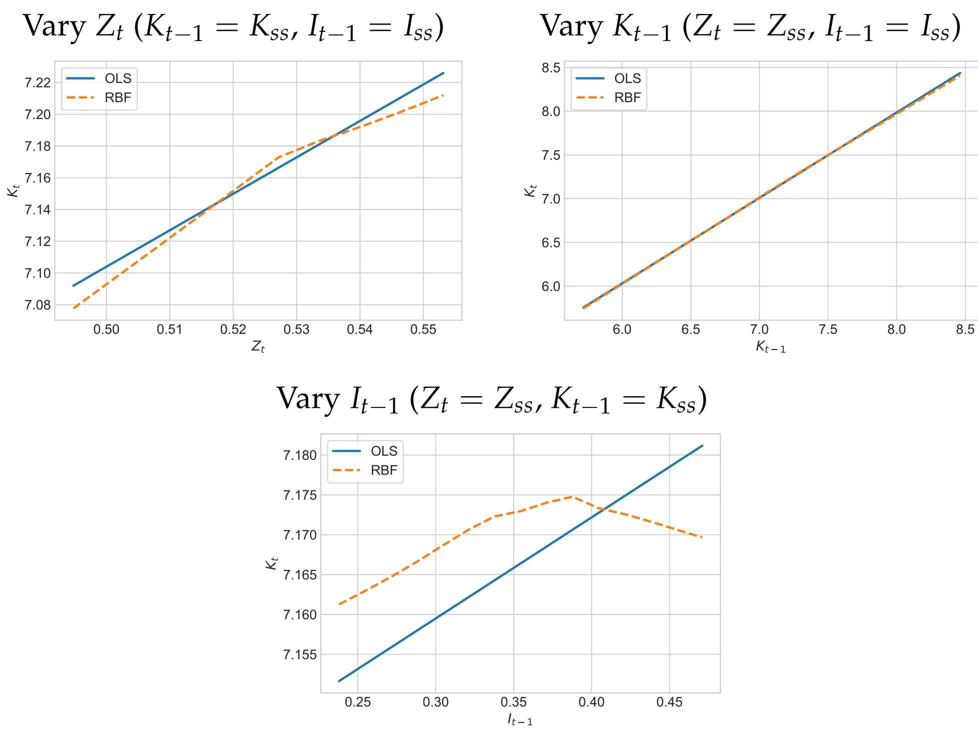
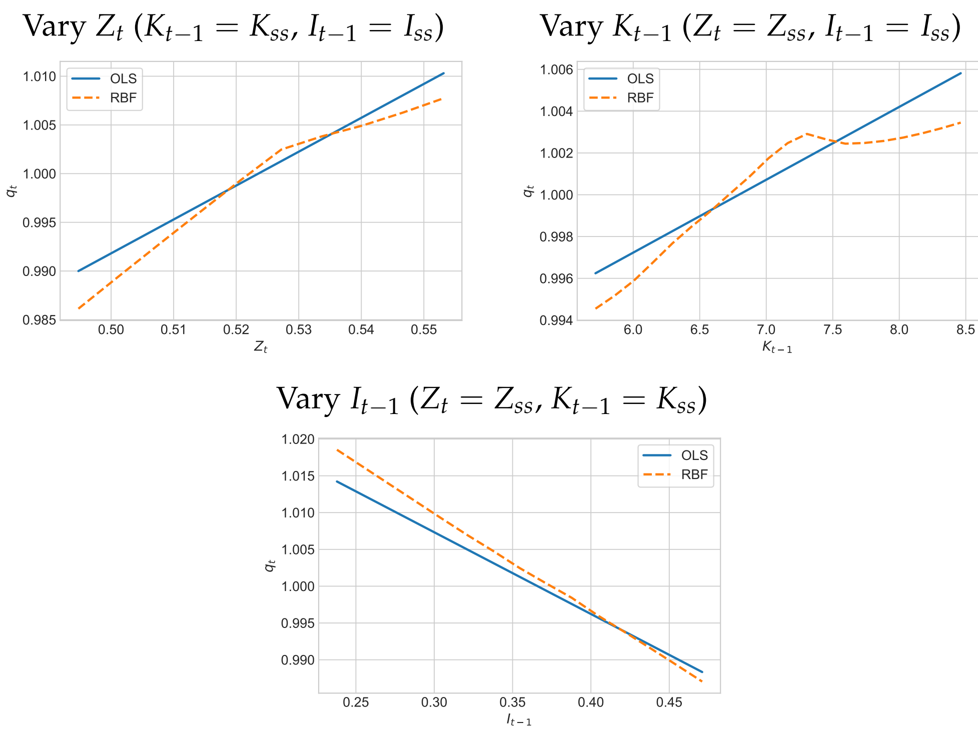


Figure 7: 2PLM for investment price, q_t – 2D slices

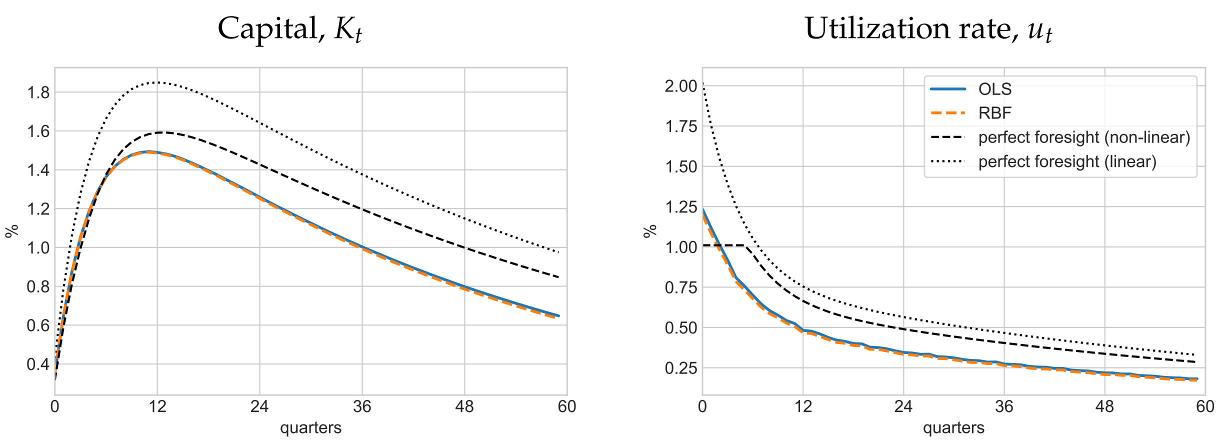


4.4 Global solution IRFs

In this sub-section, we present impulse responses functions (IRFs) from the global solutions and compare them with IRFs from the linear and non-linear perfect foresight solutions. We again consider a 2 percent TFP shock as in Section 3.

To obtain IRFs from the global models, we add the shock to the baseline technology path, Z_t, and simulate forward. We then subtract the baseline simulation (the model simulation without the added shock) and divide it by the initial value. We do this for about 500 evenly spaced different starting points and take the mean to obtain global impulse responses. Results are presented in Figure 8.

Figure 8: Impulse-responses: Global vs. perfect foresight

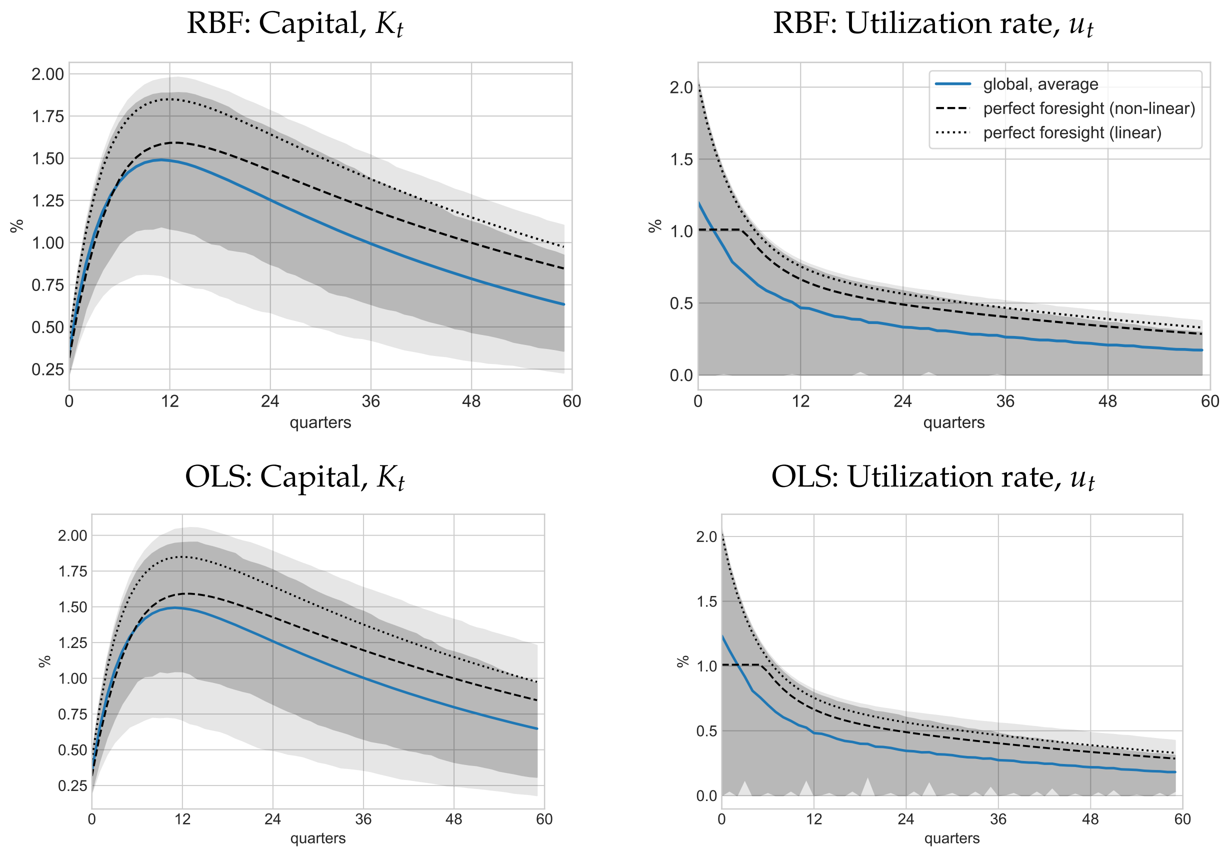


The main takeaway is that the impulses from global non-linear models differ quite a bit from perfect foresight-based solutions. Remember, that the linearized perfect foresight solution is a linearized global solution, and is thus directly comparable with the non-linear global solution. We also note that the OLS and RBF based global solutions are very close. Thus, even if the RBF based PLM is more precise and yields different simulation paths, for shocks of this size, IRFs are pretty similar.

Note that the reason the technology utilization rate, u_t, is never capped off in the global solution IRFs is that the IRFs are means over many IRFs, some of which hit the upper bound and some do not, whereas the perfect foresight IRFs are relative to the deterministic steady-state.

Finally, we also show global IRFs with 90 (light grey) and 70 percentile (dark grey) bands in Figure 9. This shows state dependence; initial values matter for the resulting IRFs. We note minor differences between the OLS and RBF PLM bands; the OLS bands are wider. Thus the OLS based model seems more volatile than the RBF based one.

Figure 9: Impulse-responses: State-dependence



5 Results with no capital adjustment costs

In this section, we consider a simpler variant of the model without capital adjustment costs, $\varphi \to 0$. Then there is only one aggregate state, $S_t = K_t$, and we do not need to search over I_t for market clearing, as the pricing equations give use price, r_t and w_t, directly from the pre-determined stock of capital.

To give an overview of the viability of the different function approximation methods for the PLMs, we again calculate dynamic Den Haan errors and the solution time in Table 2. When using OLS and RBF the results are similar to those obtained in the full model.

The NN -based solution now converges. The implied accuracy is satisfactory and, on average, almost as good as with RBF. The maximum errors are, however, much larger. This is likely because the neural net overestimates curvature along the endpoints of the capital grid, a problem not present in the radial basis function solution, see Appendix Figure A.2. Additionally, the solution time is an order of magnitude longer with NN. This primarily results from a very slow training step, which makes it time-consuming to estimate the PLMs. With NN, the number of required iterations is also higher.

Additional figures are include in Appendix A.

Table 2: HANC: Prediction Errors (no capital adjustment costs)

	OLS	RBF	NN
	dynamic log prediction errors × 100		
max	4.86	0.37	3.17
mean	0.99	0.03	0.08
median	0.84	0.02	0.02
99th perc.	3.52	0.25	1.69
90th perc.	1.99	0.06	0.09
	timings (secs.)		
total	255.7	335.6	1943.0
- solve household problem	230.0	218.9	513.0
- simulate (no market clearing)	25.7	28.0	39.7
- estimate PLMs	0.0	88.7	1390.3
iterations	10	15	22

6 Conclusion

We have presented an extension of the basic Krusell-Smith algorithm to account for non-linear dynamics. Our preferred choice is to model the perceived laws-of-motion (PLMs) with radial basis function interpolation. This delivers stable, accurate, and precise results and is easy to implement. To the best of our knowledge, the proposed global solution method is state-of-the-art for heterogeneous agent models, which can be approximated by a low number of aggregate states, but have non-linear dynamics.

Multiple further lines of inquiry are possible and valuable. The internal robustness of the proposed solution method can be investigated by varying grid sizes, tolerances and simulation lengths. Additional activation functions for the neural net and additional kernels for the radial basis function interpolation can also be investigated. A method related to radial basis function interpolation, gaussian process regression (implemented in e.g. *scikit-learn*) could also be considered. To investigate the convergence issues further, additional models should be considered. Including models with New Keynesian features.

Code speed-up is achievable by using parallelization and graphics cards (especially for the neural net). Or by solving the household problem using a Howard improvement step as in Rendahl (2022) (or a modified policy iteration version hereof).

Finally, the accuracy and speed of radial basis function interpolation make it a good candidate for use in a parameterized expectation based approach.

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Appendix

A Additional Tables and Figures

A.1 No capital adjustment costs

Figure A.1: Simulation (out-of-sample): PLM methods

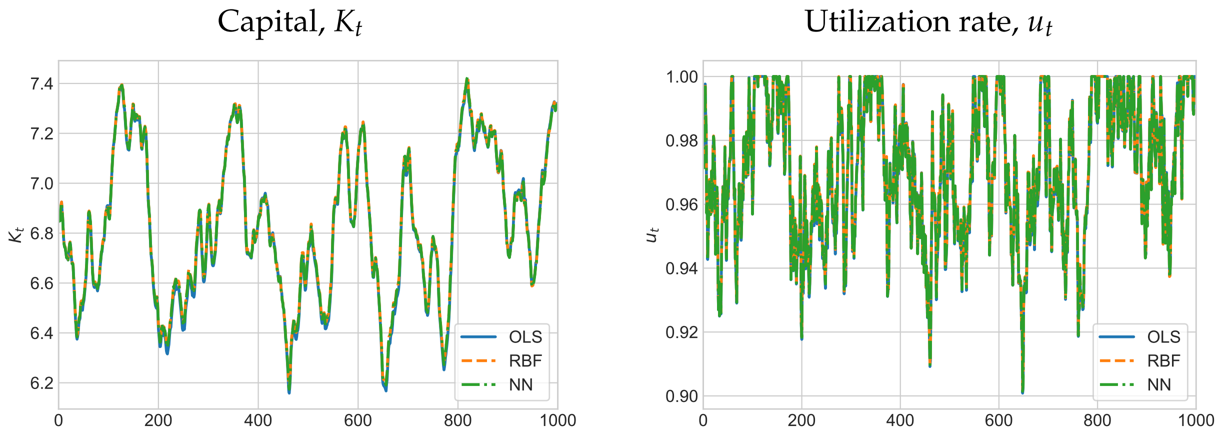


Figure A.2: PLM for capital, K_t – 2D slices

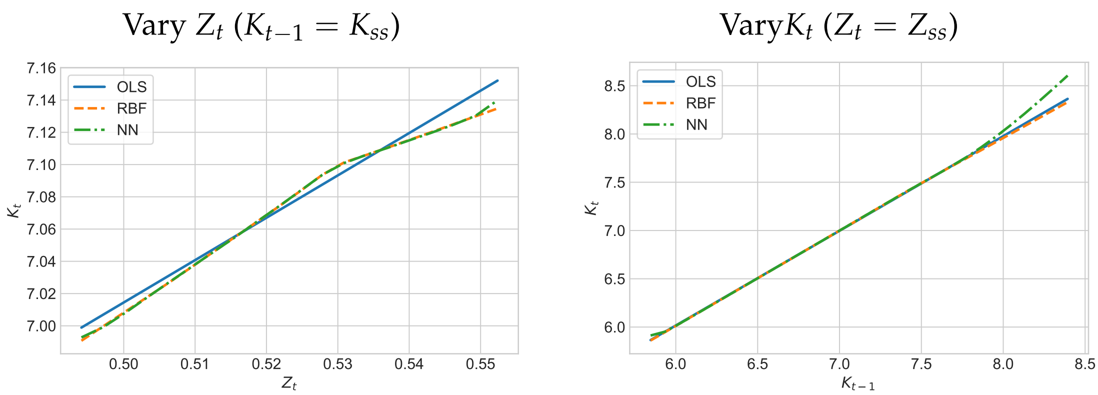


Figure A.3: One-step ahead PLM errors

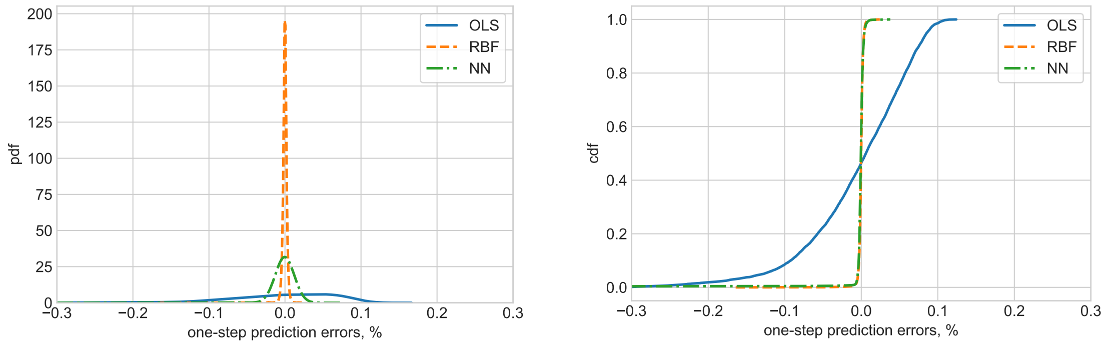
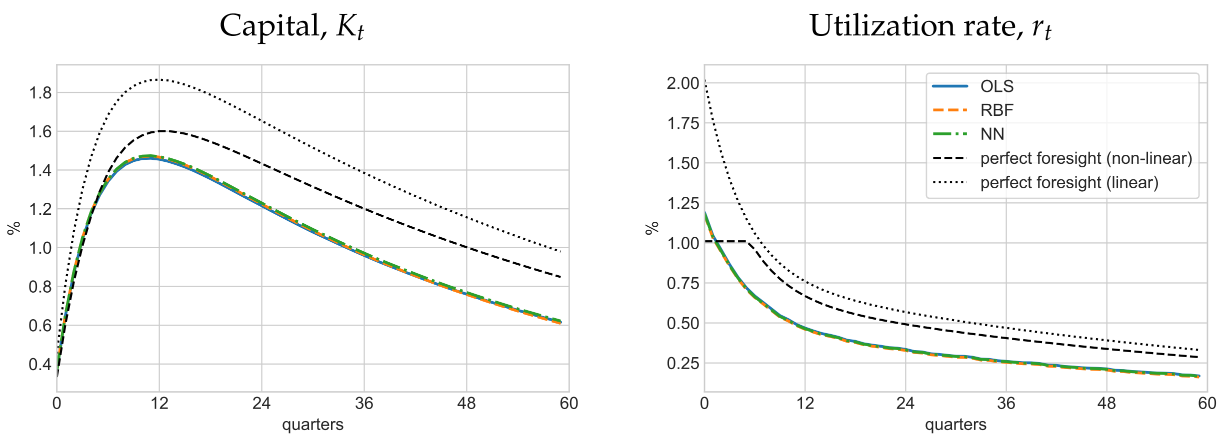


Figure A.4: Impulse-responses: Global vs. perfect foresight



Chapter 4

Firm Uncertainty and Labor Composition Dynamics

Firm Uncertainty and Labor Composition Dynamics*

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August 31, 2022

Abstract

We document the effects of uncertainty shocks on *skilled* and *unskilled* employment at the firm level using Danish registry data. To investigate the potential effects of uncertainty on net-hiring, we use that industries are differentially exposed to several aggregate shocks. We take advantage of this fact to identify industry-specific uncertainty shocks. We show that, while unskilled net-hiring is negatively affected by uncertainty shocks on impact, skilled net-hiring is not. Our dynamic approach shows that skilled labor falls with a lag. Unskilled labor shows similar dynamics, with the effect of uncertainty being strongest after impact. Our results highlight that labor displacement effects ascribed to uncertainty shocks affect unskilled labor relatively and absolutely more than skilled and are persistent. We contextualize our empirical findings within a heterogeneous firm model with skilled and unskilled labor inputs and heterogeneous labor adjustment costs.

Keywords: uncertainty, firm dynamics, employment, panel local projection.
JEL Classification: E23, E24, C55, C26, C23, C01, F31.

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1 Introduction

Increases in uncertainty have been shown to drive recessions, resulting in reductions in investment, hiring, and spending.[1] One of the leading theoretical explanations for the finding of contractionary effects of uncertainty is the *real-options* channel. The idea is that firms see their investment/hiring choices as a series of options. For example, a mining company has a lease to extract undeveloped land with potential natural resources. Embarking on this investment has a set of fixed costs, e.g., building roads and hiring workers. If it becomes uncertain whether the demand for the mined goods is there in the future, management may choose to delay the project to avoid costly mistakes. On the other hand, uncertainty may be expansionary due to e.g. *growth options*, see Bar-Ilan and Strange (1996). If there is a delay from the time an investment decision is taken until the investment yields profit, uncertainty can positively affect investment and hiring. For example, a company experiencing increasing uncertainty about future demand for a product: the project's costs are limited by its associated sunk costs (investment and hiring), but gains (the investment is more profitable than expected) are not bounded in this way. Thus, expected profit goes up with uncertainty incentivizing investment and associated hiring. In light of these potential channels, understanding firm behavior under increasing uncertainty is vital to understand the consequences of uncertainty for aggregates such as hiring and investment. In this paper, we zoom in on firms' labor input response to uncertainty.

Concretely, firms rely on workers of different skill levels to produce their output. As uncertainty increases, firms may be cautious in adjusting their amount of labor, as hiring involves several sunk costs, e.g., posting job ads, screening, buying tools associated with hiring, and training. Unskilled job requirements may be relatively easy to fill, while sunk costs may be higher for skilled positions. Similarly, it may be more costly to fire skilled workers due to e.g., higher dismissal costs, higher levels of firm-specific knowledge, or less flexible labor contracts. In addition, there may be frictions making sudden labor adjustments costly. For example, if ongoing projects require a certain amount of labor, firms may have incentives to preserve their labor stock. Such effects could be stronger for skilled employees if they are for example connected to research and development or administrative tasks of the project (e.g. book keeping). If uncertainty affects heterogeneous

[1] See e.g. Stein and Stone (2013), Baker et al. (2016a), Barrero et al. (2017), Baker et al. (2020) and Leduc and Liu (2020). See also Bloom (2014) for a literature review.

skill groups differently, understanding for whom the real-options channel may or may not be active and to what extent is an essential extension to the existing knowledge on the effects of uncertainty on employment. For example, targeting the correct labor type is essential to design policies to alleviate employment fluctuations during increasing uncertainty, such as the general financial crisis, the recent COVID-19 crisis, and the war in Ukraine. While the link between uncertainty, investment, and employment is relatively well understood, there is less knowledge about how firms' decisions over labor of different skill levels interact with uncertainty. As understanding this illuminates for who, in what direction and to what extent uncertainty affects net-hiring, in this paper, we ask the question "How does uncertainty affect firms net-hiring dynamically across labor of different skill levels?". To do so, we use rich Danish matched employer-employee data, making it possible to investigate the dynamic effects of uncertainty shocks on both skilled and unskilled labor.

To answer our research question, we first partition workers into unskilled and skilled categories using the Danish version of The International Standard Classification of Occupations (ISCO) code, an occupation-based coding. In particular for unskilled workers, we follow the ISCO definition of low-skill workers. These workers perform tasks such as cleaning; digging; lifting and carrying materials by hand; sorting and storing or assembling goods by hand.[2] We let skilled workers be defined by the resulting residual, i.e. workers not in the unskilled category. We construct our measure of uncertainty at the industry level. We take advantage of the fact that Denmark is a small open economy and construct an index measuring industry-specific sensitivities to a range of international aggregate shocks. These shocks are exogenous from the point of view of the firm. Namely, the exogenous factors are volatilities of exchange rates, oil prices, and policy uncertainty indices. The index varies across industries and reflects different exposure to such aggregate factors. We estimate dynamic causal effects of shocks to uncertainty via panel local projection. To our knowledge, we are the first to investigate both effects of uncertainty on different types of labor and to do so in a dynamic panel setting.

Our empirical results show that in the face of increasing uncertainty *i)* net-hiring falls in line with the literature, and *ii)* firms reduce unskilled labor to a larger degree than skilled labor. A one-standard-deviation uncertainty shock reduces average employment

[2] Skill category 1 in ISCO, cf. `https://www.ilo.org/wcmsp5/groups/public/@dgreports/@dcomm/@publ/documents/publication/wcms_172572.pdf`

by 4.64 heads over a three-year horizon. Measured in full-time workers, the effect on unskilled is 2.33 times that of skilled workers. We find that while unskilled labor is affected on impact, skilled labor is affected only with a lag. In general, there is a tendency for the effects of uncertainty to be strongest after impact. In sum, our empirical results point towards displacement effects of uncertainty on unskilled labor that are larger than on skilled labor. Our empirical findings are robust to a battery of controls, firm liquidity, leverage, crises controls, and fixed effects. Furthermore, results are qualitatively consistent across several specifications.

We contextualize our findings within a partial equilibrium firm dynamics model in the spirit of Bloom et al. (2018). The model has two labor input types, skilled and unskilled, a constant elasticity of substitution production technology, and heterogeneous adjustment costs. In the model, sunk costs generate the real-options effect of uncertainty, making uncertainty shocks contractionary, while convex costs generate stickiness in labor adjustment. Sufficiently high convex costs on skilled labor give rise to dynamic responses consistent with the empirical finding that uncertainty does not affect skilled labor on impact. The rationale behind the greater speed of adjustment costs on skilled labor is that firms may have ongoing projects requiring a specific stock of skilled labor to fulfill, e.g. research and development requirements.

After a brief literature review, the rest of the paper is structured as follows: Section 3 details the data and the creation of the uncertainty shock measures. Section 4 investigates heterogeneity in the dynamic responses of different labor types to our uncertainty shocks. Given our empirical findings, Sections 5 presents a heterogeneous firm model with uncertainty and two labor types that can contextualize the stylized facts. Section 6 concludes.

2 Related literature

Theoretically, the idea that uncertainty affects firms' choices of heterogeneous labor is found already in the seminal work of Dixit et al. (1994). They argue that the slow increase in U.S. firms' full-time positions during the 90's recovery was due to uncertainty, i.e., because full-time positions imply higher sunk costs. Bloom (2009) creates an estimated structural model with adjustment cost frictions on a homogeneous labor input and shows that uncertainty reduces employment due to the irreversibility of labor decisions. Bloom et al. (2018) employ a similar mechanism. Alfaro et al. (2018) consider an estimated investment model with adjustment costs only on capital investment where costs of external finance increase with uncertainty. They reconcile such mechanics with their empirical evidence on investment, employment, and cash holding. The theoretical realization that adjustment costs induce misallocation of labor is standard see, e.g., Bentolila and Bertola (1990), and Hopenhayn and Rogerson (1993). That increases in uncertainty theoretically amplifies misallocation of both capital and labor when adjustments of inputs are related to sunk costs is also well known, cf. Bloom (2014). Relative to this literature, we consider a model with both skilled and unskilled labor in firms' production with different adjustment costs, generating differences in skilled and unskilled labor net-flows in response to increases in uncertainty. In addition, we show what that implies for misallocation of both labor types and total labor.

Empirically, our paper especially relates to firm-level studies on uncertainty. Stein and Stone (2013) use the exposure of U.S. firms to variations in energy and currency volatilities as an instrument for uncertainty. They find that firms experiencing rising uncertainty decrease investment, hiring, and advertising. Barrero et al. (2017) investigates the effects of short- vs. long-run uncertainty using 30 and 10-year options at the firm level and finds that hiring tends to respond more in response to short-run uncertainty. In contrast, investment, research, and development respond more to long-run uncertainty. Alfaro et al. (2018) use a similar approach to Stein and Stone (2013) in their empirical investigation, showing that increases in uncertainty significantly decrease investment and cash holding and find a negative coefficient on employment. More recently, Kumar et al. (2022) show quite large and significant contractionary effects of uncertainty on firms hiring and investment using randomized information treatments providing information about the first and second moments of future economic growth. Using macro data, Baker et al. (2016a), Baker et al. (2020) and Leduc and Liu (2020) amongst others have shown that rising uncertainty increases unemployment. Belianska (2020) shows on U.S. aggregate data that increased uncertainty decreases employment and increases the share of skilled to unskilled labor, which is similar to our findings of a stronger response of unskilled labor. Bess et al. (2020) construct a policy uncertainty index for Denmark using newspaper articles and find that rising uncertainty predicts decreases in investments and employment. Relative to this literature, we investigate the effects on skilled and unskilled labor. We are aware of one other study, Belianska (2020), that also investigates effects on skilled vs. unskilled labor. She uses a recursive ordering SVAR approach. Instead, we use a panel data approach allowing us to disentangle first- and second-moment effects, and to control for e.g. financial conditions. Notably, it has been argued that in aggregate data, it is difficult to detach the effects of uncertainty from the endogenous response of other variables, see e.g. Ludvigson et al. (2021). Our results that the effects of uncertainty are stronger for unskilled net-hiring are consistent with the results in Belianska (2020), that the relative employment of skilled to unskilled increases after an uncertainty shock.

In the present paper, we are interested in understanding differences in the effects of uncertainty on employment between workers of different skill types. Relative to the literature, our contribution is to show the connection between firm uncertainty and firms' labor adjustment of different skill types. In addition, we do this in a dynamic setting, which is crucial to our findings. This is possible due to our rich Danish matched employer-employee data. Doing so gives insight into what occupations experience displacement during periods of heightened uncertainty, which is for example helpful when designing wage insurance or hiring subsidy policies to alleviate displacement.

3 Data and identification strategy

The following section details the Danish matched employer-employee data and the identification of uncertainty shocks. The data enables us at the firm level to *i)* obtain detailed information on balance-sheet items and worker flows, and *ii)* generate industry-level uncertainty shocks from TFP shock estimation.

In Section 3.2 we describe the construction of our uncertainty shocks based on differential industry exposure to several aggregate shock series (exchange rates, oil prices and policy uncertainty indexes) through estimated sensitivity parameters.

3.1 Data

We use Danish matched employer-employee data for the years 2000-2016. The frequency of observation is annual. On the firm side, we observe firms' balance sheet items and the total number of workers employed at each firm. We observe each employee connected to a firm (measured in November each year) and her occupational type. We classify workers occupational types according to the Danish version of the International Labour Organisation's International Standard Classification of Occupations (ISCO), named DISCO. We use this to bin workers in *skilled* and *unskilled* occupations. We can calculate the number of employees each year and thus net-hiring from the match. For all specifications, instead of counting heads, we use a measure of full-time equivalent workers; see Appendix A. After cleaning, the unbalanced sample contains $29,317$ observations for $3,950$ firms in five broad industries based on the Danish version of the NACE code (DB07). The industries covered are:

1. Mining, quarrying and manufacturing (unbalanced: 13,493 and balanced: 4,879 observations);
2. Construction (unbalanced: 4,635 and balanced: 1,122 observations)
3. Wholesale trade, retail and repair of vehicles (unbalanced: 24,313 and balanced: 8,738 observations)
4. Information and communication (unbalanced: 2,185 and balanced: 340 observations)
5. Professional, scientific and technical activities (unbalanced: 5,625 and balanced: 918 observations).[3]

The unbalanced sample is used when measuring industry exposure to aggregate volatility shocks in Section 3.2. However, since entry and exit are endogenous to uncertainty, we require a fully balanced panel for running the panel local projections in Section 4.

The total number of firms in our sample is relatively small compared to the size of the Danish economy. This is because we drop firms where balance-sheet data has been partially or completely imputed. We do this as we rely on firm-level balance sheet data both to run production functions and as financial condition controls in our regressions. The retained firms are a good representation of the Danish economy with the caveat that mainly larger firms are retained, and we can safely omit imputed balance sheets without affecting the consistency of our results.[4]

[3]For the documentation regarding the DB07 classification, see `https://www.dst.dk/en/Statistik/dokumentation/nomenklaturer/dansk-branchekode-db07`.

[4]See Appendix B for a discussion regarding sample selection.

We partition workers into skilled and unskilled categories based on their DISCO codes following standard classification. We use the general ISCO definition of unskilled workers as our unskilled definition and the remainder (except for military workers) as skilled. From the ISCO definition:[5]

"Occupations at Skill Level 1 typically involve the performance of simple and routine physical or manual tasks. They may require the use of hand-held tools, such as shovels, or of simple electrical equipment, such as vacuum cleaners. They involve tasks such as cleaning; digging; lifting and carrying materials by hand; sorting, storing or assembling goods by hand (sometimes in the context of mechanized operations); operating non-motorized vehicles; and picking fruit and vegetables."

Note that we exclude managers from the skilled group as they are part of the decision-making process and may undermine our investigation. We create a measure of full-time equivalent workers for the matched workers, see A.3. Table 1 reports the mean and standard deviation of variables of primary importance to the analysis. The balanced sample (Panel a) generally contains larger firms, which is sensible. That is, it is reasonable that firms that survive longer throughout the sample are the larger ones.

Table 1: Summary statistics

Variable	Mean	Standard deviation
Panel a) Unbalanced panel		
Employees	59.60	71.40
Full-time normalized employees	53.07	58.30
Unskilled/total workers	0.13	0.21
Full-time skilled labor	38.11	45.13
Full-time unskilled labor	6.84	16.36
Panel b) Balanced panel		
Employees	104.32	80.30
Full-time normalized employees	97.14	60.32
Unskilled/total workers	0.11	0.17
Full-time skilled labor	60.72	52.26
Full-time unskilled labor	9.78	18.28

Our main empirical specifications include controls that are standard in the literature,

[5]See `https://www.ilo.org/wcmsp5/groups/public/@dgreports/@dcomm/@publ/documents/publication/wcms_172572.pdf`.

such as financial and size controls. We allow for unobserved heterogeneity at the firm level and include time-fixed effects. In addition, we construct a dummy indicating the leverage status of the individual firm. In particular, we use book leverage and weigh short- and long-term debt to account for maturity as in Ottonello and Winberry (2020): (short-term debt$_{i,t}$ $+ 0.5 \cdot$ long-term debt$_{i,t}$)/total assets$_{i,t}$. We then calculate median leverage for each industry in each year and generate a dummy indicating whether firm i in industry j at year t is above or below the industry median.

3.2 Identification strategy

Identification relies on firms' differential exposure to exogenous factors (aggregate shocks). For instance, concerning oil price returns: some firms are positively sensitive to price increases (e.g., oil companies), some are negatively sensitive (e.g., airlines), and others are neutral (e.g., IT service firms). Importantly, firms have differential directional sensitivity to the first-moment of aggregate shocks, while the second moment is non-directional. Thus, we can separate movements in first- and second-moments allowing us to control for first-moment effects. We create an uncertainty index combining a host of exogenous shocks. We exploit the fact that Denmark is a small open economy and pick a set of aggregate series among exchange rates, oil prices, and policy uncertainty measures. In the following, we outline the identification strategy in three steps.

Step 1. In the first step, we collect a set of aggregate shocks, which are exogenous to Danish firms. We obtain a list of exchange rates related to countries that are the main trade partners with Denmark, crude oil prices, and U.S., global and Danish policy uncertainty indices.[6] Note that fluctuations in exchange rates are not only exogenous to the single firm but the Danish economy as a whole, as the Danish krone is pegged to the Euro. The set of currencies are USD, GBP, SEK, NOK, YEN, $SLOT$ and $YUAN$. We obtain daily and annual currency prices from the Danish central bank's statistics database, "Nationalbankens Statistikbank". We download the crude-oil price from FRED (DCOILBRENTEU) and convert it into Danish kroner using the USD to DKK exchange rate. Finally, we ob-

[6]We take the most important trading partners from the weights calculated for the effective DKK exchange rate by the Danish central bank, `https://www.nationalbanken.dk/da/vidensbank/tema/Sider/Effektiv-kronekurs.aspx`. We exclude the exchange rate with the Euro, as the Danish krone is pegged to it.

tain global, US, and Danish policy uncertainty measures from Davis (2016), Baker et al. (2016b) and Bergman and Worm (2021). As mentioned earlier, no firm in the sample is big enough to influence any of the mentioned series.

Step 2. To measure firm-level sensitivity to aggregate shocks, we use value-added. To do so, we first estimate a production function. This is done to clean value-added of the usual determinants; namely capital and labor. In the empirical specification, we also control for time trends (omitted here for notational simplicity). Let $J \in \mathbb{N}$ denote the total number of industries in the sample. Without loss of generality, we assume firms combine inputs of production using a Cobb-Douglas production function, that is

$$e_{i,t} = \beta_0 + \beta_l l_{i,t} + \beta_k k_{i,t} + z_{i,t} + u_{i,t} \quad \text{for all } i \text{ in industry } j = 1, ..., J, \tag{1}$$

where, $e_{i,t}$ indicates the logarithm of the ith-firm's output in year t, $l_{i,t}$ is (log) labor input and $k_{i,t}$ is (log) capital. $z_{i,t}$, the transmitted productivity component (TFP). It is a state variable observed by the firm but unobserved by the econometrician. Finally, $u_{i,t}$ is an error term uncorrelated with input choices. Several methods have been proposed to estimate $z_{i,t}$ in equation (1).[7] We use the method proposed by Wooldridge (2009), since it is robust to numerical issues. Note that this method relies on (the log of) intermediate inputs ($m_{i,t}$) as a proxy to control for unobservables, which is why we need data on intermediate inputs in addition to capital and labor. Empirically, we let $e_{i,t}$ be represented by a value-added measure. Additionally, we drop firms with $e_{i,t}$ above the 95th percentile and add time fixed effects as controls. For each input combination $(k_{i,t}, l_{i,t})$ we construct appropriate deflators.[8] Estimation of $z_{i,t}$ from equation (1) is run on sub-samples comprising one industry at the time from the list of industries in Section 3.1.

Finally, to obtain year-to-year innovations to TFP, we estimate an AR(1) regression as in Bloom et al. (2018):

$$\hat{z}_{i,t} = \rho \hat{z}_{i,t-1} + \mu_i^z + \lambda_t^z + \xi_{i,t} \tag{2}$$

where μ_i^z is a firm-fixed effect (to control for permanent firm level differences, e.g. dif-

[7]Ackerberg et al. (2015) propose methods to estimate production functions using a two-step procedure. Wooldridge (2009) instead proposes a one-step generalized method of moments (GMM) procedure. The method of Ackerberg et al. has been shown to be numerically sensitive to starting guesses for the optimizer (see Rovigatti and Mollisi, 2018).

[8]See Appendix A.

ferences in demand and technology innovation structures) and λ_t^z is a year fixed-effect (to control for cyclical shocks). Thus, year-to-year innovations to TFP are the resulting residuals, $\xi_{i,t}$.

Step 3. In the final step, we measure industry sensitivity to aggregate shocks and use these sensitivities to construct the uncertainty shock.

Let hats denote estimated quantities and $\hat{z}_{i,t}$ denote the estimated TFP process from (1). Let $\xi_{i,t} = z_{i,t} - \mathbb{E}[z_{i,t}|z_{i,t-1}]$ be TFP the innovations and

$$c = USD, GBP, SEK, NOK, YEN, SLOT, YUAN, OIL, GPU, USPU, DKPU$$

denote the selected list of currencies ($USD, ..., YUAN$), prices (OIL) and policy uncertainty indexes ($GPU, USPU, DKPU$) used to construct our uncertainty index.

We measure the sensitivity of earnings to exogenous factors, β_j^c, using

$$\hat{\xi}_{i,t} = \alpha_j + \sum_c \beta_j^c \cdot f_t^c + v_{i,t}, \quad \text{for all } i \text{ in industry } j = 1, ..., J \tag{3}$$

where α_j is the industry-specific intercept, and f_t^c is the log-differenced exogenous return of series c. Since the error term, $v_{i,t}$, is uncorrelated with f_t^c we estimate the sensitivities using OLS industry by industry.

Let σ_t^c be the volatility of factor c using daily data. In sample, $\hat{\sigma}_t^c = \sqrt{\frac{1}{256}\sum_{d=1}^{256}(f_{d_t}^c)^2}$ where $f_{d_t}^c$ is the dth daily return of factor c within year t. Given $\hat{\sigma}_t^c$ and the estimates from regression (3), the uncertainty index for industry j is defined as the weighted sum of the volatilities of the exogenous factors,

$$\hat{\sigma}_{i,t} := \sum_c \hat{w}_j^c \cdot \left|\hat{\beta}_j^c\right| \cdot \hat{\sigma}_{t-1}^c, \quad \text{for all } i \text{ in industry } j = 1, ..., J \tag{4}$$

where w_j^c is a significance weight of factor c on industry j, such that $w_j^c = \left|\mathrm{t}_j^c\right| / \sum_c \left|\mathrm{t}_j^c\right|$ where t_j^c is the usual asymptotic t-statistic associated to sensitivity β_j^c. The weighting scheme is implemented to normalize the processes and address noisy estimates and multicollinearity concerns. On the right-hand side of (4), $\hat{\sigma}_t^c$ is lagged in order to address simultaneity issues between the shock and the outcomes.

Note that the generated index is a *realized* volatility index of exogenous factors. Ideally, one would prefer *implied* volatility, as that contains forward-looking information. However, differences between the two wash out at the annual frequency considered here.[9]

We argue that our index is, in general, a good shock proxy, as commodities and currency options are traded in international (deep) markets, and thus changes in first (and second) moments are unanticipated due to no-arbitrage conditions.[10] Policy uncertainty series can be regarded as news shocks. It is, however, possible that firms know of changes in policy uncertainty before it becomes public information. In Appendix D, we address such concerns by removing the policy uncertainty from our index. We do not observe any substantial difference in the results.

We note that weights differ significantly between industries, validating our identification scheme. For example, in the case of the exchange rate with the U.S. dollar, the $\hat{\beta}_j$'s are 0.250, 0.034, -0.065, -0.059, -0.299 for manufacturing, construction, wholesale, IT and professional, scientific and technical activities. A full overview of all weights, showing that all weights vary significantly between industries, is provided in Appendix C.

First-moment controls Not controlling for first-moments of the shocks may be an issue. For instance, oil prices tend to be high when the volatility of oil prices is high. In order to disentangle movements in the second from the first moments, we create first moment controls as

$$\hat{f}_{j,t} := \sum_c \hat{w}_j^c \cdot \hat{\beta}_j^c \cdot f_t^c, \quad j = 1, ..., J.$$

In addition we control for firm-specific first-moments by using TFP, $\hat{z}_{i,t}$. In sum, our first-moment controls help disentangle correlated movements in the first and second moments.

[9]For example, Alfaro et al. (2018) find similar effects and magnitudes of realized and implied volatility measures on annual data

[10]See e.g. Akram et al. (2008) for foreign exchange markets and Gorton et al. (2013) for commodities markets.

4 Empirical findings

From the real-options theory, one would expect to see that an increase in uncertainty causes a reduction in employment as increased uncertainty implies increases in the variance of future returns to hiring, causing a *wait-and-see* effect on hiring when labor adjustment entails sunk costs. Wait-and-see effects may be dampened if there are costs on the speed of labor adjustment, as is commonly assumed, see Bloom (2009), and Bloom et al. (2018). Such labor smoothing could be related to projects with a long horizon, requiring a certain amount of labor. Different combinations of adjustment costs associated with skilled and unskilled labor may result in different degrees of reductions of each type in response to an increases in uncertainty. On the other hand, growth-options effects may dominate, i.e., the profitability of projects may increase as uncertainty goes up as sunk costs of projects are bounded from below while profits are not. If such projects require additional labor, net-hiring may increase in response to increases in uncertainty. For example, Kraft et al. (2018) have shown that increases in uncertainty can affect research and development positively, referring to growth options as the likely mechanism. In this case, one can hypothesize that growth options may positively affect the net-hiring of skilled workers as research-and-development necessitates specialized workers. We investigate the dynamic effects of uncertainty using local projections to grasp how these potential channels play out in data.

4.1 The effects of an uncertainty shock

In the following, we present specifications and results from local projections on the effect of uncertainty on net-hiring of total, skilled and unskilled labor.

4.1.1 Local projection specifications

We estimate cumulative impulse responses using panel local projections to investigate uncertainty shocks. Our general specification is as follows:

$$\log(y_{i,t+h}) - \log(y_{i,t-1}) = \mu_i^h + \lambda_t^h + \beta^h \Delta\hat{\sigma}_{i,t} + \sum_{k=1}^{K} \boldsymbol{\gamma}_k^{h\prime} \boldsymbol{X}_{i,t-k} + \varepsilon_{i,t+h}^h, \tag{5}$$

where horizons of the projections are indexed by $h = 0, \ldots, 4$, the outcome variable $\log(y_{i,t+h})$ is the full-time equivalent stock of the different labor types, firm's specific fixed effects, μ_i^h, are capturing unobserved heterogeneity to control for firm idiosyncrasies, such as management practices, size, time of origination, production function specification, firm-specific capital-labor complementarities resulting in different adjustment costs of different labor skill levels across firms and so on.[11] Time fixed effects, λ_t^h, control for common trends in the sample. Estimates of β^h are the dynamic responses to the identified uncertainty shock, $\Delta\hat{\sigma}_{j,t} = \log(unc_{j,t}) - \log(unc_{j,t-1})$. We include a sequence of (logged) control variables, with nuisance parameters, γ_k^h. As a set of contemporaneous controls, we use TFP as a firm-specific first-moment control and the industry specific first-moment controls. As lagged controls, we have a set of financial controls, namely maturity-weighted book leverage as described in Section 3, liquid assets, inventories, short-term debt, earnings before interest and taxes (EBIT). In addition, we generate dummies to control for the financial crisis years (2008 and 2009). It takes values one during the core months of the crises in which firms are subject to heightened financial frictions and is interacted with our high-leverage dummy.[12] Recall that the high-leverage dummy controls for firms with leverage greater than the industry median in a given year. Together with time-fixed effects and financial controls, this helps soak up the credit-supply effects of the great recession. Finally, we add lags of the uncertainty shock and the lagged total number of (log) employees as a size control.

To alleviate potential auto-correlation issues, we include lags of the transformed dependent variable and the shock in addition to the set of controls. We include three lags of all variables as impulses stabilize there.[13] Error terms of the local projections, $\varepsilon_{i,t+h}^h$, are moving averages of the forecast errors from t to $t + h$ and are therefore uncorrelated with the regressors. Note that restrictions that the shock is uncorrelated with leads is generally unrestrictive and follows the definition of shocks as unanticipated (Stock and Watson, 2018). Lag exogeneity conditions are satisfied by including lags of the left-hand side variable and the shock. We use Driscoll-Kraay standard errors, which are robust to homoscedasticity assumptions and cross-sectional dependence violations. We control for time-invariant within industry correlations (and all other potential time-invariant groupings) with firm fixed effects.

[11] For the definition of full-time-equivalent worker units, see Appendix A.

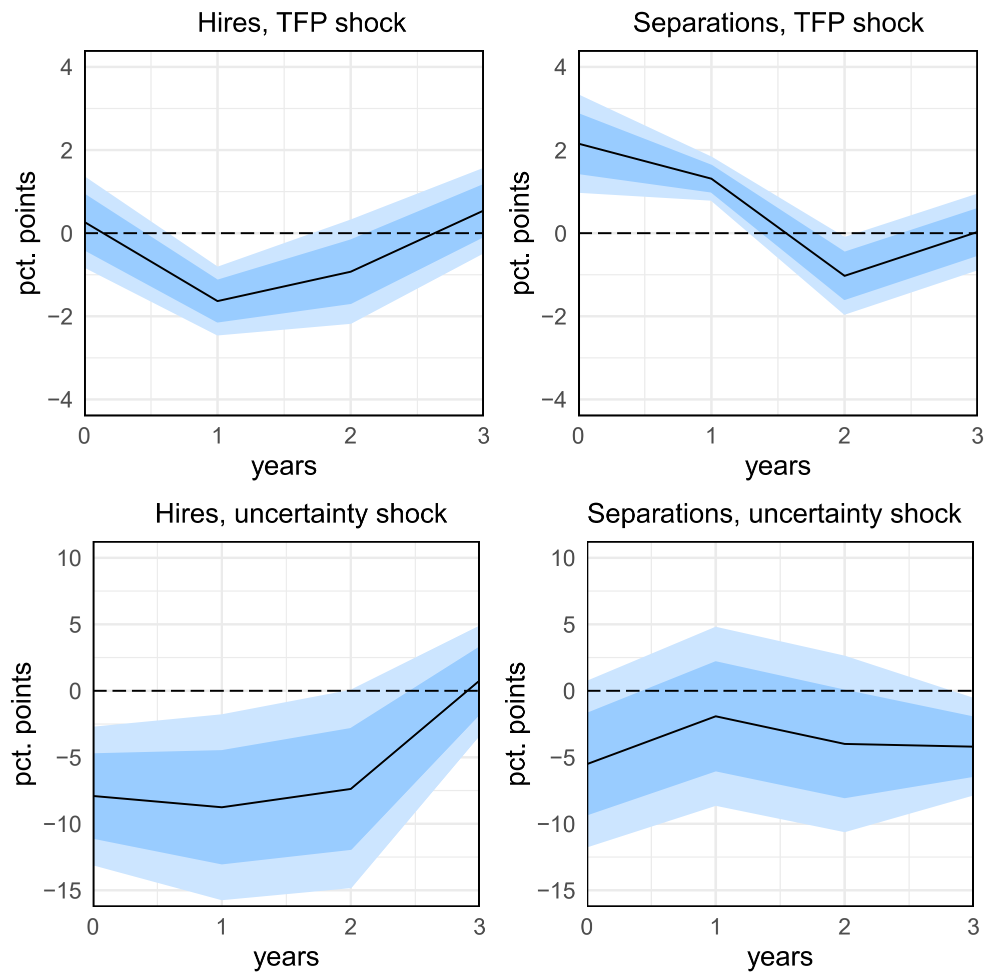
[12] See e.g. Zullig (2020) regarding the period of the financial crisis in Denmark.

[13] Montiel Olea and Plagborg-Møller (2021) recommends against using selection criteria, as such pretesting causes uniformity issues.

4.1.2 First vs. second moment effects

To further validate our identification strategy, we first present local projections where we exchange the outcome variable from being the stock of labor to hiring and separations. We first run local projections with the uncertainty index shock and then run similar local projections using firm-specific negative TFP shocks obtained from the production function estimation exercise in 3. Results are presented in Figure 1.

Figure 1: Gross labor flow responses to 1 standard deviation shocks in TFP and uncertainty



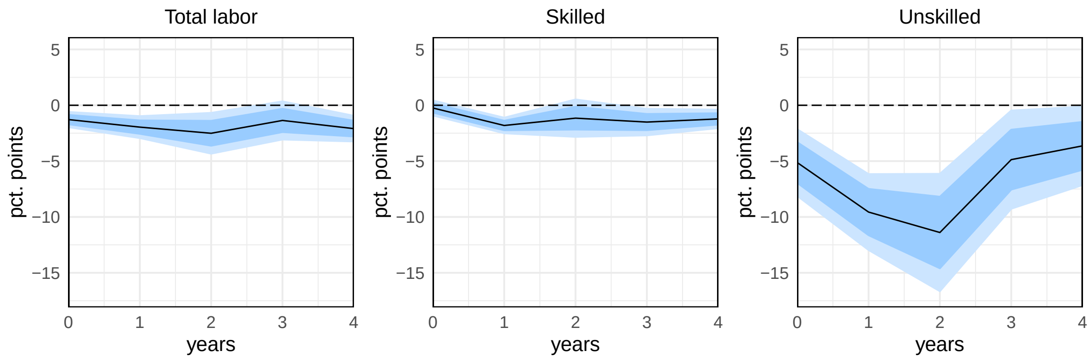
Note: 90% and 68% confidence intervals indicated in light and dark blue shaded areas. Driscoll-Kraay standard errors.

We see that an uncertainty shock tends to decrease both hiring and separations, implying wait-and-see effects in both directions. On the other hand, the negative first-moment shock decreases hiring and increases separations. The wait-and-see predictions would be consistent with a search-and-match model with uncertainty shocks as in Den Haan et al. (2021) extended to have endogenous firing and sunk costs thereof. In addition, we show this in Appendix E.2 with a simple gross flows model with uncertainty shocks. In sum, we provide evidence that our uncertainty index shock is well identified and that we can make inference about effects beyond first-moment.

4.1.3 Main local projection results

Local projection results given the specification in Section 4.1.1 are shown in Figure 2. The local projections confirm the contractionary findings of uncertainty on net-hiring

Figure 2: Net-hiring responses to 1 standard deviation uncertainty shocks

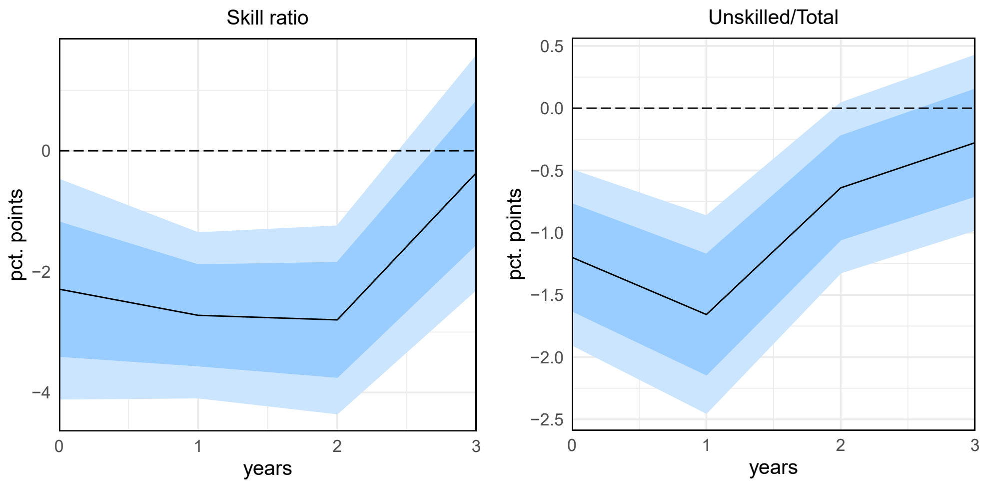


Note: 90% and 68% confidence intervals indicated in light and dark blue shaded areas. Driscoll-Kraay standard errors.

in the literature; i.e., the real-options channel on labor is also at play in Danish data. Notably, total and unskilled labor respond negatively on impact, while skilled workers do not. Dynamics unveil an interesting result: Skilled workers contract, albeit with a lag. Interestingly, we see a similar tendency for unskilled workers - contractionary effects are stronger after impact.

To highlight the compositional effects, we also check what happens if we replace the outcome variable with unskilled/skilled (*skill ratio*) and unskilled/total workers. The results are presented in Figure 3. From these plots, it is clear that there is generally a larger relative negative effect on unskilled labor. This is consistent with the SVAR findings of Belianska (2020). In addition, higher volatility of unskilled labor is consistent with tendenciens over the general business cycle, as suggested by Hagedorn et al. (2016). This

Figure 3: Response in skill ratio to a 1 std. deviation shock to uncertainty



Note: 90% and 68% confidence intervals indicated in light and dark blue shaded areas. Driscoll-Kraay standard errors.

is consistent with the argument in the uncertainty literature that uncertainty shocks are a driver of business cycles (e.g. Bloom (2014)).

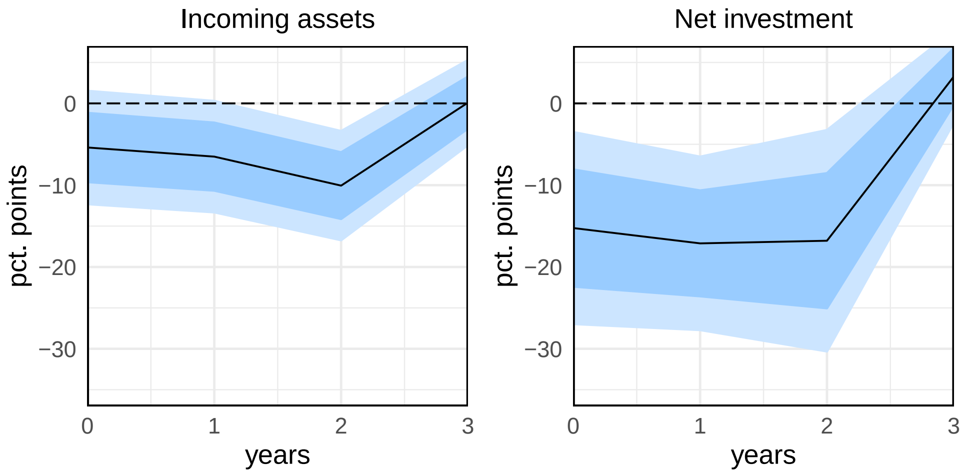
Regarding magnitudes, we note that a shock to uncertainty, $\sigma_{j,t}$, of one standard deviation reduces average full-time workers by 4.64 heads over a three-year horizon, all else being equal.[14] For the average firm, unskilled full-time heads are cumulatively reduced by 2.33 times the reduction of skilled with the effect taking off on impact.[15]

As a sanity check, we document that investment tend to fall after an uncertainty shock, as in line with the literature (cf. e.g., Stein and Stone, 2013; Alfaro et al., 2018). These results are presented in in Figure 4. The specification for the impulse responses follows the baseline uncertainty index specification.

One could posit that the reason skilled labor falls with a lag is due to costs associated with firing of skilled workers.[16] Thus, we check the effect on gross flows for skilled workers in response to an uncertainty shock. This is documented in Appendix D, Figure 19, and confirms that it is the hiring margin that moves, and that, if anything, separations fall.

[14]Calculations are relative to unconditional averages of full-time workers in data and are at 90 % confidence levels.
[15]Given 90 % confidence bands, the lower bound for this difference is 37 pct. points.
[16]"funktionærloven" in Denmark, `https://www.retsinformation.dk/eli/lta/2017/1002`.

Figure 4: Response in investment to a 1 std. deviation shock to uncertainty



Note: 90% and 68% confidence intervals indicated in light and dark blue shaded areas. Driscoll-Kraay standard errors.

In Appendix D, we do various robustness checks. We confirm that including four lags instead of three (Figures 10 and 14), disincluding policy uncertainty measures from our uncertainty index (Figure 11), and taking the square of the shock (Figure 12), do not change results markedly - effects are always larger for unskilled and the ratio of unskilled to skilled is always negatively affected from impact. Figures 13 and 17 document what happens to estimates when one industry at a time is removed as to check whether one industry is driving the results. Orderings between unskilled and skilled stays the same, and the skill ratio still falls after an uncertainty shock, in all cases. Finally, we also check unskilled gross flows on their own, confirming that both hires and separations fall, similarly to the results for total labor in Section 4.1.2.

5 Contextualizing the empirical findings

We build a partial equilibrium heterogeneous firm model to contextualize our empirical findings. Firms produce their output using a combination of skilled and unskilled labor, and adjusting either type of labor entails a combination of fixed and convex adjustment costs. Firms face idiosyncratic productivity shocks with time-varying innovations to the second moment of the process, *i.e.*, uncertainty shocks. We use the model to generate impulse responses to an uncertainty shock.

5.1 Model setup

There exists a continuum of ex-post heterogeneous firms. Firms production function is a constant elasticity of substitution technology with two inputs, skilled and unskilled labor and are subject to idiosyncratic total-factor productivity shocks. Since we are interested firms' behavior concerning labor adjustment, we assume that physical capital is fixed.

In particular, firm i's output is produced as follows

$$Y_{i,t} = z_{i,t} A \left(\left[\gamma \left(A_U u_{i,t} \right)^{\frac{\sigma-1}{\sigma}} + (1-\gamma) \left(A_S s_{i,t} \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \right)^{\alpha},$$

where $A_S > A_U$ are two separate technology constants, A is time invariant total factor productivity common to all firms, $\gamma \in (0,1)$ determines the importance of unskilled labor, σ determines the elasticity of substitution between the two factors and the log of $z_{i,t}$ follows an AR(1) with stochastic standard deviation given by a two-state Markov process where $\sigma_{i,t} \in \{\sigma_{low}, \sigma_{high}\}$ and $\sigma_{high} > \sigma_{low}$. There are decreasing returns to labor, that is $\alpha < 1$. Having decreasing returns in production implies that risk matters as the objective function becomes concave. This is standard in the uncertainty literature, e.g. Bloom et al. (2018).

Firm i's objective is given by

$$\max_{u_{i,t}, s_{i,t}} V_{i,t} \left(\zeta_{i,t}, u_{i,t-1}, s_{i,t-1} \right) = Y_{i,t} \left(\zeta_{i,t}, u_{i,t}, s_{i,t} \right) - w_U u_{i,t} - w_S s_{i,t}$$

$$-\Psi_{i,t}^{U} \left(u_{i,t-1}, s_{i,t} \right) - \Psi_{i,t}^{S} \left(s_{i,t-1}, s_{i,t} \right) + \frac{1}{1+r} \mathbb{E}_t V_{i,t+1} \left(\zeta_{i,t+1}, u_{i,t}, s_{i,t} \right)$$

$$\text{s.t. } u_{i,t} = (1-\delta_u) u_{i,t-1} + h_{i,t}^{u}$$

$$s_{i,t} = (1-\delta_s) s_{i,t-1} + h_{i,t}^{s}$$

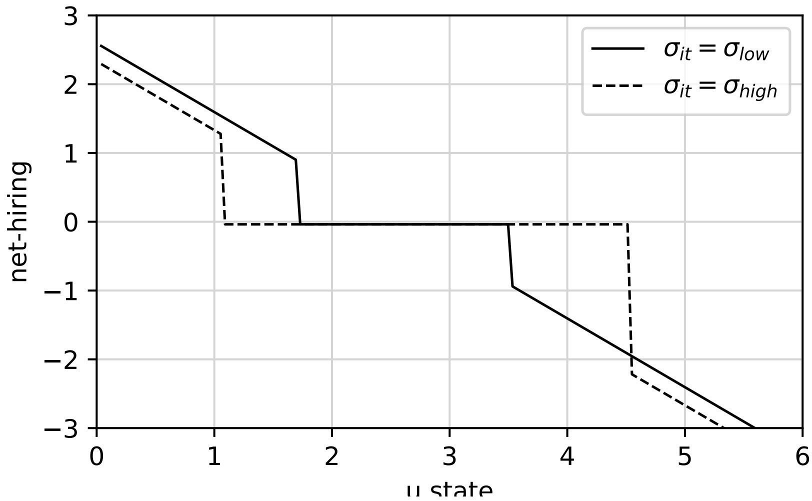
where $h_{i,t}^{s}$ and $h_{i,t}^{u}$ denotes flows into skilled and unskilled labor stocks, $\zeta_{i,t}$ is the firms exogenous state given by the double $(z_{i,t}, \sigma_{i,t})$, w_U, w_S are wages of unskilled and skilled labor, r is the risk-free rate, and $\Psi_{i,t}^{L}(\cdot), \Psi_{i,t}^{H}(\cdot)$ are labor adjustment costs on unskilled and skilled labor. After choosing $l_{i,t}, h_{i,t}$, there is exogenous labor attrition denoted by δ_s, δ_u.

The functional form of the adjustment cost function is

$$\Psi_{i,t}^{X}(\cdot) = \Psi_{i,t}^{X}(\cdot) = \theta_1^X \mathbb{I}\{h_{i,t}^X \neq 0\} + \theta_2^X \frac{1}{2}\left(\frac{h_{i,t}^X}{X_{i,t-1}}\right)^2 X_{i,t-1}$$

where θ_1^X and $\theta_2^X > 0$ are parameters with $X = u, s$. In the case of no adjustment costs $\left(\theta_1^X = \theta_2^X = 0\right)$, the model solution is a repeated static problem where marginal productivities equal wages. That is, labor freely moves around with idiosyncratic TFP shocks. In that case, increases in uncertainty about future $z_{i,t}$ does not matter, only realized volatility will affect outcomes. When instead sunk costs are activated, firms' labor choices follow an S, s policy. In this case, uncertainty about the future will increase the S, s band as having to reverse adjustments becomes more likely.

Figure 5: Net-hiring policy function, fixed adjustment costs only

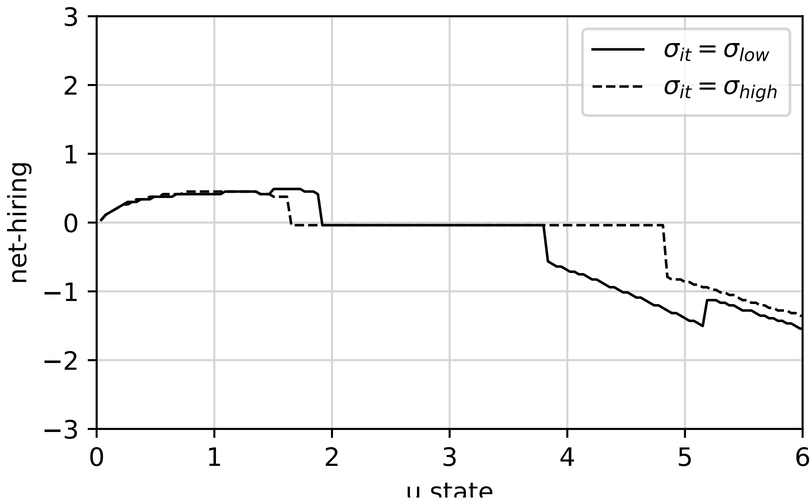


Note: The plot shows the optimal net-hiring policies associated with low and high uncertainty shock states of the single labor version of the model. The value of the idiosyncratic TFP shock is fixed at its mean value.

This is illustrated in Figure 5 for the case of $\gamma = 1$, $A_L = 1$ and $\sigma = \infty$ such that the model is only a function of one labor type ("unskilled") and displays a linear-production function $Y_{i,t} = z_{i,t}u_{i,t}$ for ease of exposition. From Figure 5 we see that in the high uncertainty state, the S, s band widens due to the real-options effect.

In case it is costly to quickly adjust labor $\left(\theta_2^X > 0\right)$, e.g. because it is important to have a certain amount of labor to finish potentially fruitful projects that take time to build, real-options effects may be dampened. We illustrate such effects in Figure 6. From here we

Figure 6: Net-hiring policy function, fixed and convex adjustment costs



Note: The plot shows the optimal net-hiring policies associated with low and high uncertainty shock states of the single labor version of the model. The value of the idiosyncratic TFP shock is fixed at its mean value.

see that the net-hiring effects of uncertainty are dampened, as it is costly not to maintain the same labor stock.

5.2 Model calibration

The (quarterly) model calibration is presented in Table 2. We take the CES elasticity between skilled and unskilled labor from Katz and Murphy (1992). Given the normalizations $w_h = 1, \ A_h = 1$, we also obtain relative factor-augmenting technology and the skilled wage premium from Katz and Murphy. We set the risk-free rate to 5% pro annum as in Alfaro et al. (2018). The weight on unskilled employees in production, γ, is set to match unskilled/total employees in data. Labors share in production, α, is set to a standard value of 1/3. Note that the decreasing returns to scale assumption correspond to having a fixed amount of capital (or any other potential production inputs). We set labor attrition according to estimates in Shimer (2005). A scales the model such that the model is in a region where it is possible to target relative peak responses from local projections. We set the persistence of the AR(1) TFP process as in Khan and Thomas (2008) and the standard deviations, σ_{high}, σ_{low}, and transition probabilities between these, $\pi^{\sigma}_{low,high}$, $\pi^{\sigma}_{high,high}$, as in Bloom et al. (2018). We calibrate the fixed and convex adjustment costs for skilled

Table 2: Model calibration

Parameter	Value	Source/target
σ	1.41	Katz and Murphy (1992)
$1/(1+r)$	0.988	Alfaro et al. (2018)
γ	0.66	Calibrated to match unskilled/total employees ratio in data of 0.13
α	$1/3$	Standard value for labors share of production
A_u	0.0667	Katz and Murphy (1992) (implied)
A_s	1	Normalization
w_u	0.9324	Katz and Murphy (1992) (implied)
w_s	1	Normalization
δ_u, δ_s	0.088	Shimer (2005)
A	8	Scales model to obtain relative peak responses from local projections
θ_1^u	0.294	Calibrated to obtain relative peak responses from local projections
θ_1^s	0.663	Calibrated to obtain relative peak responses from local projections
θ_2^u	21.117	Calibrated to obtain relative peak responses from local projections
θ_2^s	28.035	Calibrated to obtain relative peak responses from local projections
σ_{low}	0.051	Bloom et al. (2018)
σ_{high}	0.210	Bloom et al. (2018)
ρ_z	0.95	Khan and Thomas (2008)
$\pi^{\sigma}_{low,high}$	0.026	Bloom et al. (2018)
$\pi^{\sigma}_{high,high}$	0.943	Bloom et al. (2018)

and unskilled labor to obtain similar dynamics as in the empirics. Our simulated method of moments procedure targets the relative peak responses of unskilled to skilled from local projections. This is calculated as

$$\frac{\text{maximum pct. deviation over the impulse horizon of unskilled labor}}{\text{maximum pct. deviation over the impulse horizon of skilled labor}},$$

both empirically and in the model. From the empirical impulses, this number is 5.280. In the calibrated model, the relative peak is 5.281. From our simulated method of moment exercise, we obtain *i)* the relative peak response between unskilled and skilled labor observed from local projections (cf. Section 4.1), *ii)* no effect on impact of skilled labor and *iii)* hump shapes in responses.

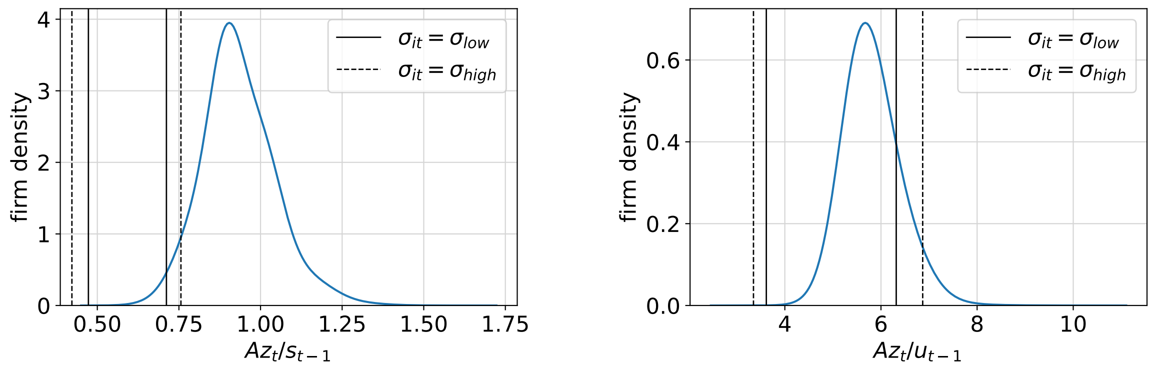
5.3 The effects of an uncertainty shock

This subsection documents the effects of an uncertainty shock on labor allocation dynamics.

5.3.1 Density plots

From policy functions alone, it is unclear whether an increase in uncertainty is strictly contractionary, as the real-options channel alludes to. If average net firing falls by more than average net hiring plus exogenous attrition, increases in uncertainty could theoretically be expansionary. Therefore, inspecting density plots for the full model with (net) hiring and firing threshold indicators is fruitful.

Figure 7: The impact of an increase in uncertainty on net hiring and firing thresholds



(a) Skilled (b) Unskilled

Note: Figures plot the simulated cross-sectional marginal distributions of micro-level labor inputs after productivity shock realizations and before labor adjustment. The marginal distributions are conditional on firms being in the low uncertainty state. The (net) hiring and firing thresholds have been calculated for firms with average idiosyncratic shock values and average skilled/unskilled labor stocks.

We follow Bloom et al. (2018) and plot the steady-state densities of TFP ($A \cdot z_{i,t}$) relative to labor after shocks have been realized but before firms have adjusted. Figure 7 shows firm densities over both unskilled and skilled labor. In each panel, the right solid line shows firm-level net-hiring thresholds, and the left solid line net-firing thresholds in the low uncertainty case. In between the two lines, firms are inactive. Firms to the right of the hiring line will hire, and firms to the left of the firing line will fire. The dashed lines show the same for high uncertainty. An increase in uncertainty increases the mass of firms inside the inaction area due to the real-options effect. As there is exogenous labor attrition, the densities are skewed such that there are almost no firms on the firing threshold. Thus an increase in uncertainty and, therefore a decrease in hiring will typically be contractionary, as labor is depleted due to δ_u and δ_s. Notice the difference between the unskilled and skilled densities: The skilled density has much more mass in the hiring action region due to the large convex cost decreasing the real-options incentive.

5.3.2 Impulse responses

We first simulate the model with 80.000 firms for 2.000 periods and discard the first 500 periods. We then average across firms and time to obtain average steady-state skilled and unskilled labor stocks. As Bloom et al. (2018), we obtain uncertainty shock impulses by setting $\sigma_{i,t} = \sigma_{high}$ for all firms, and letting the model converge back to steady-state. To reduce Monte Carlo error along the path, we simulate 500 independent impulses for 100 periods and average over them. Figure 8 presents the impulse responses. The first panel shows the average path of expected volatility of z, i.e., uncertainty. Both skilled and unskilled labor display hump shapes due to the convex adjustment cost on net-hiring generating a smoothing incentive. As convex costs are sufficiently large for skilled labor, skilled labor barely responds on impact. In contrast, unskilled labor does respond on impact, in line with the empirical evidence in Section 4. Thus, the real-options effect is at play for both labor types, but strongly dampened for skilled labor. Note that total labor responds somewhere in between the two, as in data. In conclusion, augmenting a partial equilibrium firm model in the style of Bloom (2009) and Bloom et al. (2018) with heterogeneous labor inputs and heterogeneous convex net-hiring costs can qualitatively reproduce the stylized empirical facts of Section 4.

Finally, as an external model validity check we plot the impulse responses to changes in misallocation. This is measured by the spread in marginal products, following the definition of Hsieh and Klenow (2009). From Figure 9 we see that relatively, misallocation rises more for unskilled labor as expected. We then do a check in data of the correlation between the inter-quartile range of the TFP residuals obtained in subsection 3.2, similar to Bloom et al. (2018), and misallocation measured as the growth in the inter-quartile range

Figure 8: Impulse responses to an uncertainty shock

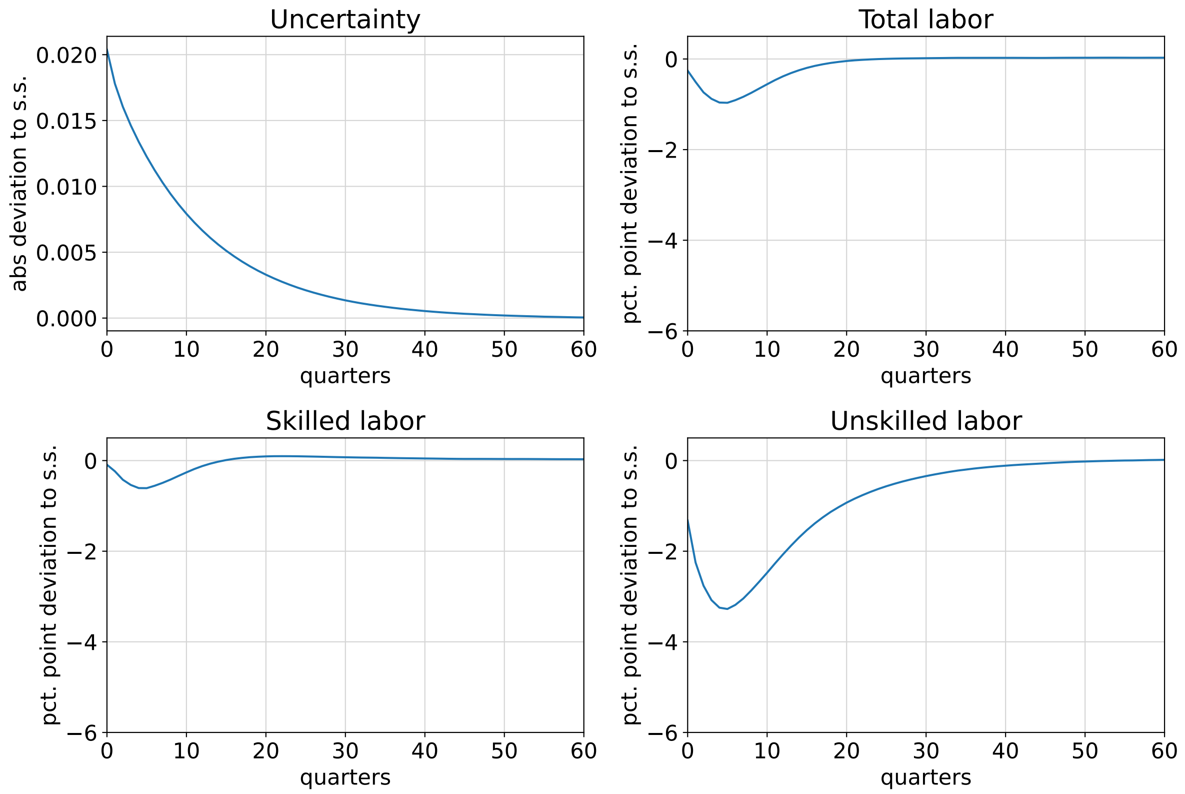


Figure 9: Misallocation impulse responses to an uncertainty shock

over log-linear approximations of marginal products as[17]

$$\operatorname{corr}\left(\text{IQR}\left(\hat{\varepsilon}_{i,t}\right), gIQR\left(\log\left(Y_{i,t}/S_{i,t}\right)\right)\right) = 0.0265$$
$$\operatorname{corr}\left(\text{IQR}\left(\hat{\varepsilon}_{i,t}\right), gIQR\left(\log\left(Y_{i,t}/U_{i,t}\right)\right)\right) = 0.2328$$

[17]We plot the inter-quartile range of TFP shocks against GDP growth in Appendix F as in Bloom et al. (2018) to show that we have a reasonable descriptive measure of micro uncertainty.

These descriptives show that misallocation of unskilled labor is more correlated with uncertainty defined as TFP shock dispersions, consistent with the model prediction.

6 Concluding remarks

Using an identification scheme based on industry sensitivity to exogenous uncertainty shocks, we document that increases in uncertainty decrease net hiring. Our main finding is that unskilled net-hiring is significantly more volatile in response to an uncertainty shock than skilled net-hiring. We find that increasing uncertainty affects the net-hiring of unskilled labor on impact, not skilled labor. Instead, we find that skilled labor drops with a lag. In general, there is a tendency for the effect of uncertainty to be strongest after impact. We find that the displacement effects of uncertainty are stronger for unskilled labor, both in relative and absolute terms. We reconcile our empirical findings with a heterogeneous-firm model with a CES production technology over skilled and unskilled labor and heterogeneous combinations of fixed and convex adjustment costs. Higher convex adjustment costs on skilled labor can produce a model response to an uncertainty shock qualitatively consistent with our empirical findings. We interpret the mechanism with sticky skilled labor as representing firms' need for a particular stock of skilled labor to finish profitable projects with a long-time horizon, dampening the real-options effect of uncertainty.

Our findings point to directions for policy when uncertainty shocks hit the economy: Policymakers may alleviate labor displacement effects of uncertainty by providing e.g. net-hiring subsidies with a focus on unskilled worker hiring. That is, by providing hiring subsidies, wait-and-see effects will be dampened as sunk costs of hiring will be covered to an extent. In addition, as the contractionary effects of uncertainty are stronger after impact, hiring subsidies need to last sufficiently long to prevent the most severe falls in employment from uncertainty shocks.

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Appendix

A Data Appendix

We collect data from various registries made available by Statistics Denmark. Data has the structure of a panel of firms, with yearly frequency of observations, from year 2000 to 2016. The `typewrite` font in this section is reserved for indicating the name of variables directly pulled from the registry.

A.1 Individuals

Data on employees is collected from AKM and IDAN registries.

AKM is used to obtain individuals' occupational classifications (DISCO) into high- and low-skill categories. DISCO is the Danish analogue of the International Standard Classification of Occupations (ISCO). Within the sample, there are two classification codes, DISCO 88 and DISCO 08, with a structural break in 2010. We translate DISCO 88 so that all occupations are in DISCO 08 keys for the entire sample period. Missing information on the category of occupation is either imputed or discarded. The occupational classification of an employee is imputed, if (i) the employee is registered at the same workplace for three consecutive years, (ii) the missing classification corresponds to the middle year, and (iii) both the years previous and after to the missing information are classified with the same occupation code. Otherwise, the entry is discarded.

IDAN has observations recorded in November each year and contains individual-level information on employees related to their workplace. Before 2008, individuals with a yearly salary of less than 10,000 DKK are excluded from the register. For coherency, we drop employees with a yearly salary of less than 10,000 DKK throughout the whole sample. We retain the workplace identifiers and job `TYPE`. The job `TYPE` classifies the type of job into various categories, such as: primary and secondary work in November. We exclude workers for which variable `TYPE` is different from "A" (employer), "H" (primary job in November), “B” (side job in November) or “N” (job in November).[18] It could be that for one worker in a given year there are multiple entries. We use the iterative hierarchical scheme of Hviid and Schroeder (2021) to drop multiple observations of individuals. In practice, if a worker with multiple entries in a given year has one line with job type “A”, that is the one we retain. Otherwise, we repeat the conditional statement in a waterfall manner, through “H”, “B” and “N”. It may be, that the employee has multiple jobs with type “N”. In that case, we select a row at random. Keys on the workplace (`LBNR`, or `ARB_NR` before 2008) are used together with `PNR` and year to match employees to corresponding employers. From the IDAN dataset, we also pull the individuals' contributions to the public pension system while working at the individual plant (`ATP`).

A.2 Firms

Firm-level datasets used are FIDA (match and statistics at plant level) and FIRE (detailed balance sheet). Employees are matched to the FIRE dataset through FIDA. Firms are uniquely identified by the employer's number `CVRNR`. Balance sheet variables are available at the `CVRNR` level.

FIDA contains general information related to firms at the plant level. The dataset matches employees to their corresponding workplace, `LBNR`. This employer-employee match is then used to retrieve the employer's (firm-level) identifier, `CVRNR`.

FIRE contains information of firms on their balance-sheet, financial and other real economic variables. For certain firms, some balance-sheet information is imputed based on their similarity with other firms. Imputation obstructs inference, as we use balance-sheet items as controls and variation for identification. Therefore, we use the journaling code variable (`JKOD`) to exclude firms for which some or all information of the balance-sheet have been imputed. From FIRE we also obtain an outcome variable, `AARSV`, which is the dependent variable of regressions on the total labor force. It measures the total labor stock in full-time equivalent units. `AARSV` bases the calculation of hours worked on total con-

[18]See https://www.dst.dk/da/TilSalg/Forskningsservice/Dokumentation/hoejkvalitetsvariable/beskaeftigelsesoplysninger-der-vedroerer-ida-ansaettelser/type

tributions to the public pension system paid by all the employees in firm i. We construct a similar variable for our matched employees, see A.3. From this table we pull data relevant for production function and TFP estimation. In fact, we use earnings (`XVT`), energy expenses (`KENE`) and the sum of intangible and tangible capital (`IAAT` + `MAAT`) and express them in real terms by constructing industry-specific deflators. Namely, the deflator of variable x for firm i in industry j is defined as the 2010-values index of x in industry j. Finally, we use the total number of workers in full-time equivalent units (`BESK`) as labor-force input in the production function. `BESK` is analogous to `AARSV` but also includes the employer.

A.3 Full-time worker normalization for matched employees

We want to measure the full-time worker equivalent of individual n in firm $i.$ such as to be able to split workers into high- and low-skill full-time worker equivalents. In order to do so, we emulate the construction of the `AARSV` from Statistics Denmark and construct an analogous measure for matched individuals, using their contribution to the public pension system from IDAN (`ATP`). The government sets the total contributions to the public pension system an individual working full-time for a month should pay. We report those total amounts of monthly contributions in the second column of Table 3. The values reported are in DKK.

Table 3: Normalization values in DKK (monthly) for construction of $AARSV$ for matched employee.

Period	Normalization value (τ)
Until 2005	223.65
2006-2008	243.90
2009-2015	270.00
After 2015	284.00

At each point in time, the yearly labor contribution of individual n in full-time equivalent is then defined as

$$AARSV_n \equiv \mathtt{ATP}_n/(12 \cdot \tau).$$

Finally, the $AARSV$ of category worker $d =$ high skilled, low skilled, related to firm i is constructed as

$$AARSV_i^{(d)} = \sum_{\text{worker } n \in \text{ firm } i} AARSV_n \cdot \mathbb{I}\{\text{worker } n \text{ is of category } d\},$$

where $\mathbb{I}\{\bullet\}$ takes values one when the condition is satisfied and zero otherwise.

B Sample Selection (a discussion)

This short section argues why we are not worried about sample selection in the empirical specification. Since 1986 businesses must report standardized accounting information to the tax authorities yearly. Statistics Denmark uses this info together with firm balance-sheet surveys to generate their balance-sheet data sets. However, since then, not all businesses have reported all relevant accounting information. The variable `JKOD` captures the source and the integrity of the reported accounting information. As mentioned in the previous section, some items from the balance sheet are imputed from statistics Denmark. The imputation involves similar companies where all accounting information has been reported.

For the empirical application, we treat firms with partially or entirely imputed balance sheets as missing ($D_i = 0$). Imputed data would bias our estimates, in that they would cause more concentration of firms within sectors. Namely, more firms will have similar, if not equal, balance sheets. In contrast, we are interested in exploiting the dispersion within and differentials across industries. Therefore, formally, we look at the problem of imputed balance sheets from a sample-selection perspective. Data show that the vast majority of firms not reporting all the accounting information are with productivity lower than a cutoff c (namely, $D_i = 0$ for $z < c$). This hints that there is a self-selection mechanism in place: less productive firms show more difficulties in acknowledging items in their balance sheet or have less incentive to report all entries. While empirically, selection is correlated with the productivity level of the individual firm across industries, it appears not to be a deterministic function of productivity solely. Instead, it is the case that other firm-specific characteristics play a role.

We implicitly model selection D_i as a function of productivity z and a random component v, independent of z. Such that, $D_i = 1$ if $z + v \geqslant c$ and $D_i = 0$ if $z + v < c$. That means that firms' data are not missing if productivity and an unobserved random component are larger than the cutoff. Conversely, data are missing if latent utility is smaller than the cutoff. Finally, under the assumption that $\mathbb{E}(\varepsilon|\Delta\sigma, \mu, \lambda, \boldsymbol{X}, v) = 0$, selection is non-consequential for consistency of OLS estimates. In fact, since D is a function of z and v and z is contained in $\boldsymbol{X}$, it follows $\mathbb{E}(\varepsilon|\Delta\sigma, \mu, \lambda, \boldsymbol{X}, D = 1) = 0$.

These assumptions are mild, and we argue they hold in the data. Namely, our empirical specification includes an exhaustive set of controls that may affect selection. We conclude that the non-imputed sample is representative, and our estimates do not suffer from sample selection bias.

C Uncertainty index weights by industry

Table 4: Exchange rate index weights

Industry	Weight USD	weight GBP	weight SEK	weight NOK	weight YEN	weight POL	weight YUAN
Mining, quarrying and manufacturing	0.250	-0.183	0.298	0.017	-0.120	-0.020	0.008
Construction	0.034	-0.058	0.148	0.006	-0.001	-0.032	0.005
Wholesale trade, retail and repair of vehicles	-0.065	0.104	-0.150	-0.004	0.026	0.008	-0.020
Information and communication	-0.059	-0.054	-0.047	0.023	0.028	0.005	0.000
Professional, scientific and technical activities	-0.299	0.123	-0.174	-0.046	0.145	0.040	0.005

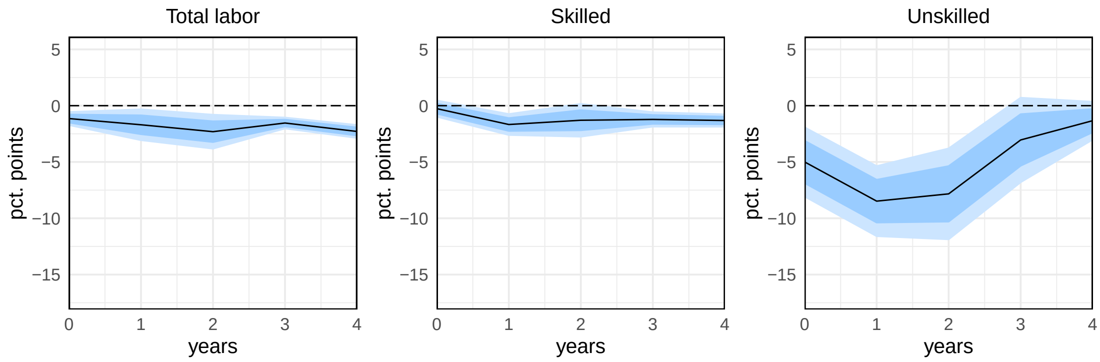
Table 5: Oil and policy uncertainty index weights

Industry	Weight OIL	Weight GPU	Weight DKPU	Weight USPU
Mining, quarrying and manufacturing	-0.002	-0.070	0.036	0.047
Construction	-0.029	-0.010	0.017	-0.010
Wholesale trade, retail and repair of vehicles	0.001	0.049	-0.012	-0.008
Information and communication	0.010	-0.081	-0.031	0.016
Professional, scientific and technical activities	0.007	0.039	-0.032	-0.047

D Local projection robustness

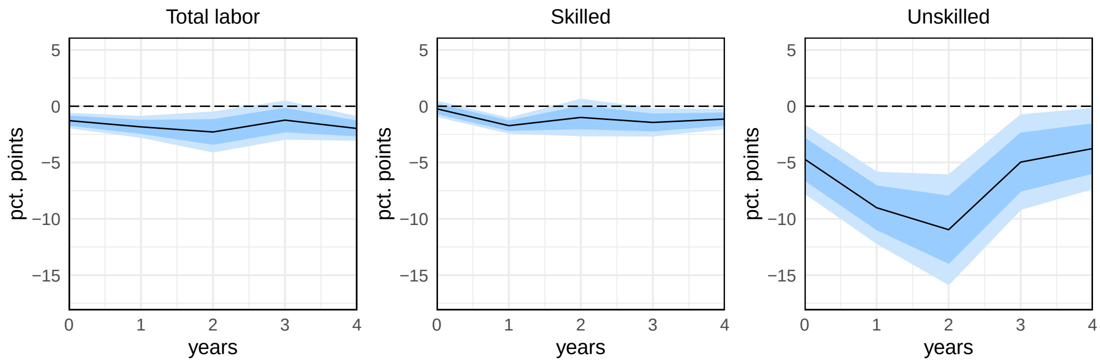
In the following, we provide various robustness checks of our local projections. We include four instead of three lags in Figures 10 and 14. In Figures 11 and 15 we disinclude policy uncertainty indices from the construction of our uncertainty index. In Figures 12 and 16 we use the square of the uncertainty shock. We disinclude industries one by one in Figures 13 and 17. Finally, we check unskilled gross flows on their own in Figure 18. We also note that in absolute terms (full-time workers), the average fall in unskilled workers is always more than skilled over a three-year horizon. For the specification with four lags, the average full-time worker fall is 2.05 times larger for unskilled. For the specification with no policy uncertainty in the uncertainty index, the relative fall in average full-time workers is 2.41 times larger for unskilled. For the specification with the squared shock, the relative fall in average full-time workers is 1.91 times larger for unskilled. Finally, we check skilled gross flows in Figure 19 to see if the reason skilled reacts with a lag is due to firing rising later

Figure 10: Net-hiring, responses to 1 std. dev. uncertainty shock, 4 lags.



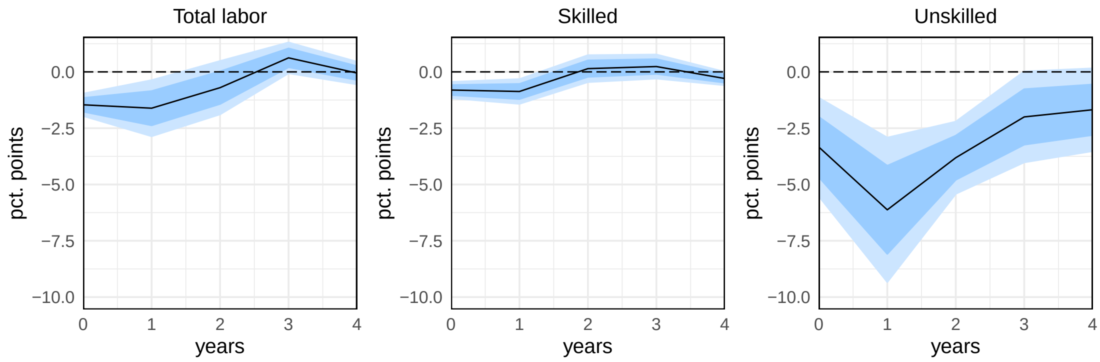
Note: 90% and 68% confidence intervals indicated in light and dark blue shaded areas. Driscoll-Kraay standard errors.

Figure 11: Net-hiring, responses to 1 std. dev. uncertainty shock. No policy uncertainty included in index.



Note: 90% and 68% confidence intervals indicated in light and dark blue shaded areas. Driscoll-Kraay standard errors.

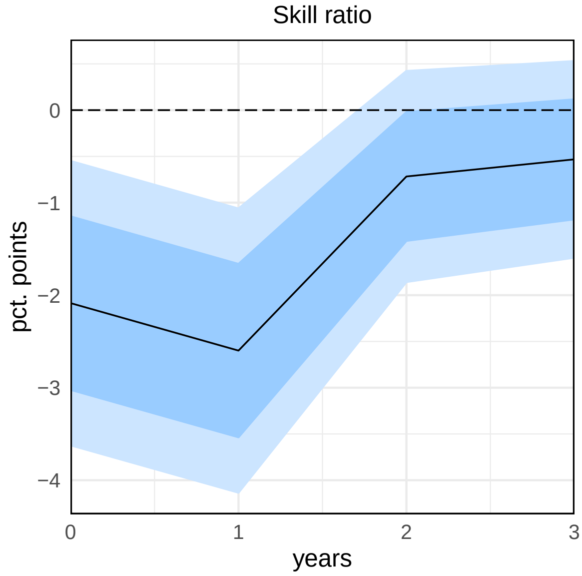
Figure 12: Net-hiring, responses to 1 std. dev. squared uncertainty shock



Note: 90% and 68% confidence intervals indicated in light and dark blue shaded areas. Driscoll-Kraay standard errors.

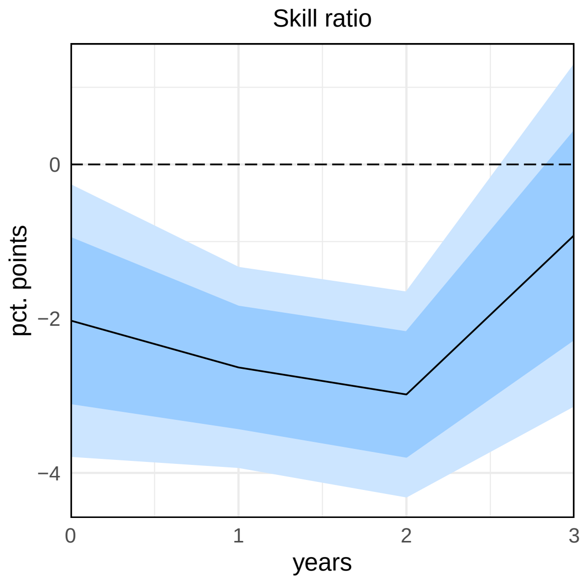
Figure 13: Net-hiring, dropping one industry at a time, responses to a 1 std. dev. squared uncertainty shock

Figure 14: Skill-ratio, responses to 1 std. dev. uncertainty shock, 4 lags.



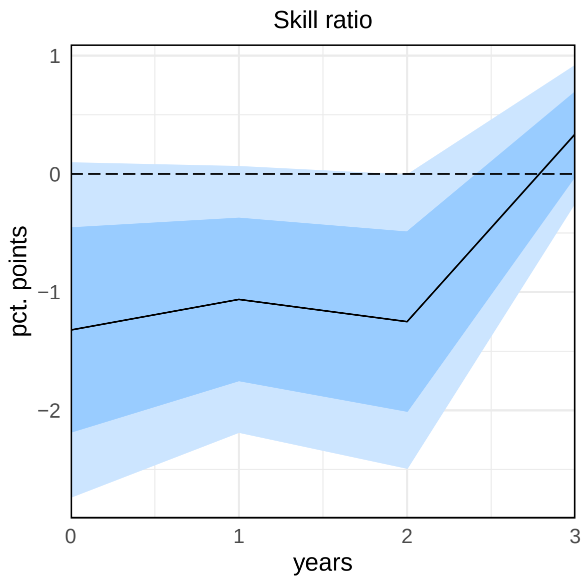
Note: 90% and 68% confidence intervals indicated in light and dark blue shaded areas. Driscoll-Kraay standard errors.

Figure 15: Skill-ratio, responses to 1 std. dev. uncertainty shock. No policy uncertainty included in index.



Note: 90% and 68% confidence intervals indicated in light and dark blue shaded areas. Driscoll-Kraay standard errors.

Figure 16: Skill-ratio, responses to 1 std. dev. in squared uncertainty shock



Note: 90% and 68% confidence intervals indicated in light and dark blue shaded areas. Driscoll-Kraay standard errors.

Figure 17: Skill-ratio, dropping one industry at a time, responses to a 1 std. dev. uncertainty shock

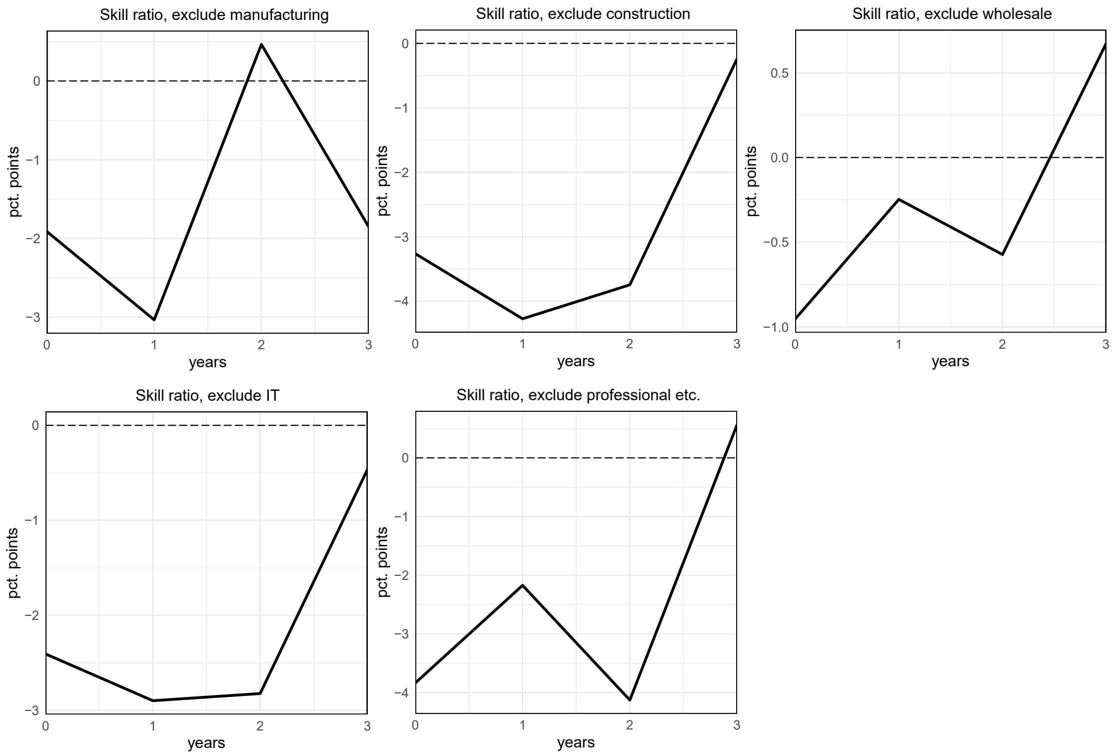
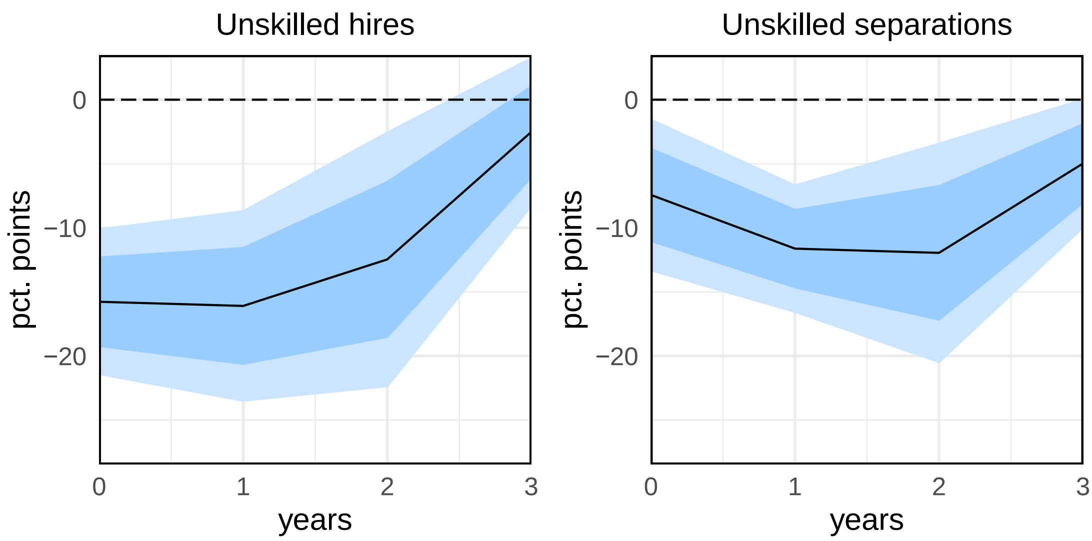
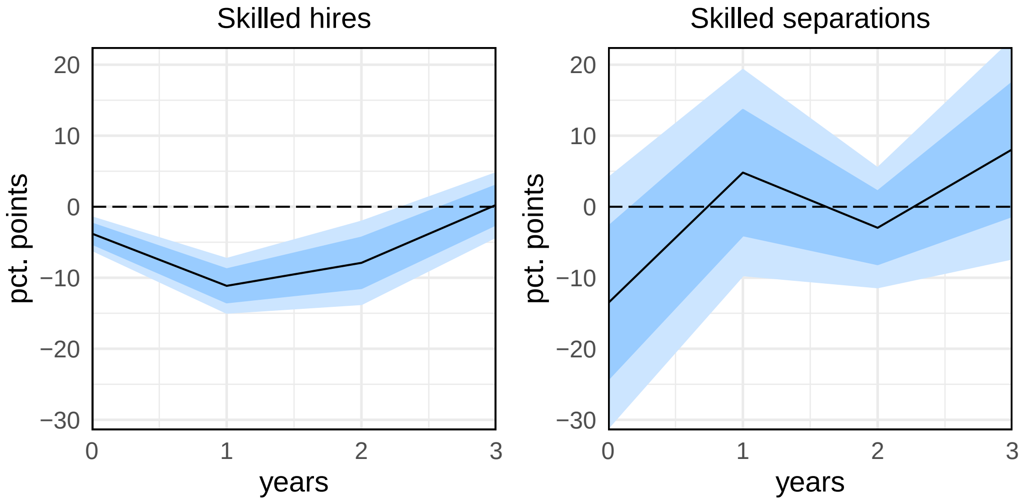


Figure 18: Gross flows unskilled, responses to 1 std. dev. uncertainty shock



Note: 90% and 68% confidence intervals indicated in light and dark blue shaded areas. Driscoll-Kraay standard errors.

Figure 19: Skilled gross-flows in response to a 1 std. deviation shock to uncertainty



Note: 90% and 68% confidence intervals indicated in light and dark blue shaded areas. Driscoll-Kraay standard errors.

E Model appendix

E.1 Numerical solution

The model is solved with value-function iteration over two linear grids for the endogenous variables: a grid over U with 23 equi-logspaced grid points and a grid over S with 38 equi-logspaced grid points. We discretize the firm-specific productivity with two-state Markov process of time-varying conditional volatility into a 5 (productivity level) by 2 grid using the Tauchen method, such that the z grid is the same between the low and high uncertainty states, but the transition matrix changes reflecting higher variance in the high uncertainty state. Value function maximization is done by grid search. In addition, we implement a Howard improvement step to speed up convergence of steady-state policy functions. The model is solved in Python using Numba parallelization.

To obtain average firm steady-state values of skilled and unskilled labor (S, U), the model is simulated using Monte Carlo with linear interpolation of the S, U policy functions for 2,000 periods over a panel of 80,000 firms. We let the first 500 periods be the burn-in period.

Impulse responses are simulated as described in Section 5.3.

E.2 Gross flows model

In the following, we set up a firm model with gross flows due to stochastic labor match qualities.

There exists a continuum of ex-post heterogeneous firms. Firms production function is a constant returns to scale Cobb-Douglas production with two inputs, labor type 1 and labor type 2. In particular, firms output is determined by constant returns to scale Cobb-Douglas function

$$Y_{i,t} = z_{i,t} \cdot l_{1,i,t}^{\alpha_{i,t}} \cdot l_{2,i,t}^{1-\alpha_{i,t}}, \tag{6}$$

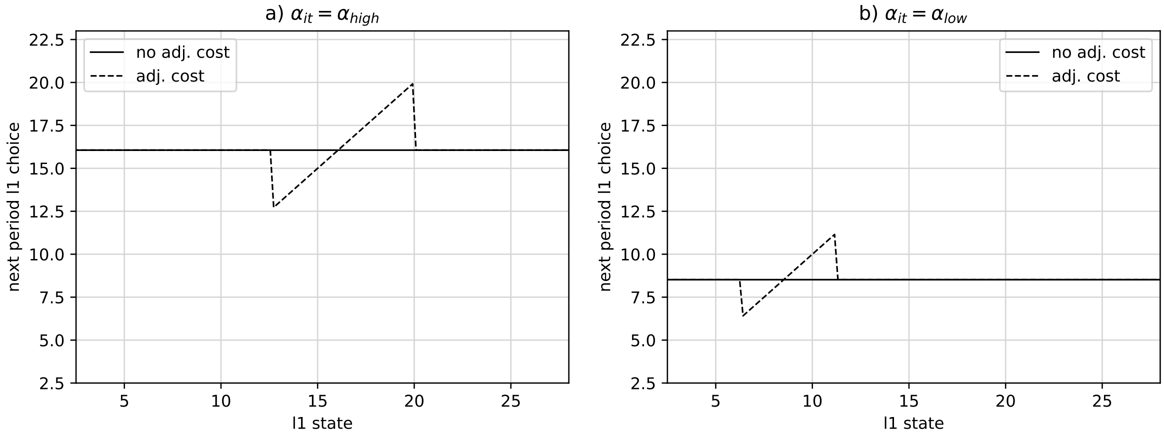
where $z_{i,t}$ follows an AR(1) with a stochastic standard deviation given by a two-state Markov process $\sigma_{i,t} \in \{\sigma_{low}, \sigma_{high}\}$,$\sigma_{high} > \sigma_{low}$. In addition, $\alpha_{i,t}$ follows a two-state Markov process interpreted as labor match quality shocks, $\alpha_{i,t} \in \{\alpha_{low}, \alpha_{high}\}$, $\alpha_{high} > \alpha_{low}$, $\alpha_{low} = 1 - \alpha_{high}$. Firms face an iso-elastic demand curve $Q_{i,t} = BP^{-\varepsilon}$ where, B is a demand-shifter and ε is the demand elasticity. Firm i's objective is given by

$$\begin{aligned} \max_{l_{1,it+1},l_{t+1}} V_{i,t}(\zeta_{i,t}, l_{1,i,t}, l_{2,i,t}) = R_{i,t}(\zeta_{i,t}, l_{1,i,t}, l_{2,i,t}) - w(l_{1,i,t} + l_{2,i,t}) \\ - \textstyle\sum_{j=1}^{2} \Psi_{j,i,t}(l_{j,it}, l_{j,it+1}) + \frac{1}{1+r}\mathbb{E}_t V_{it+1}(\zeta_{it+1}, l_{1,it+1}, l_{2,it+1}), \end{aligned} \tag{7}$$

$\zeta_{i,t}$ is the firms exogenous state given by the triplet $(z_{i,t}, \alpha_{i,t}, \sigma_{i,t})$, adjustment costs $\Psi_{j,i,t}$ are on both and types of labor and identical for each type. In particular, if $l_{k,i,t+1} \neq l_{k,i,t}$ for $k \in (1, 2)$, firms pay a sunk cost $\theta = 1$. The uncertainty process parameters are set in accordance with Bloom et al. (2018). $\alpha = 0.7$ and the transition probability from α_{low} to α_{high} and vice versa is 0.8. We let demand scaling factor $B = 0.35$, $\varepsilon = 0.3$ as in Alfaro et al. (2018) and $1/(1 + r) = 0.988$ also as in Alfaro et al. (2018). The adjustment costs are set to 1. Note that changing parameters within reasonable values (e.g. $1/(1 + r) < 1$) would not change the qualitative implications of the model, and the parameters are only set to illustrate the model mechanisms.

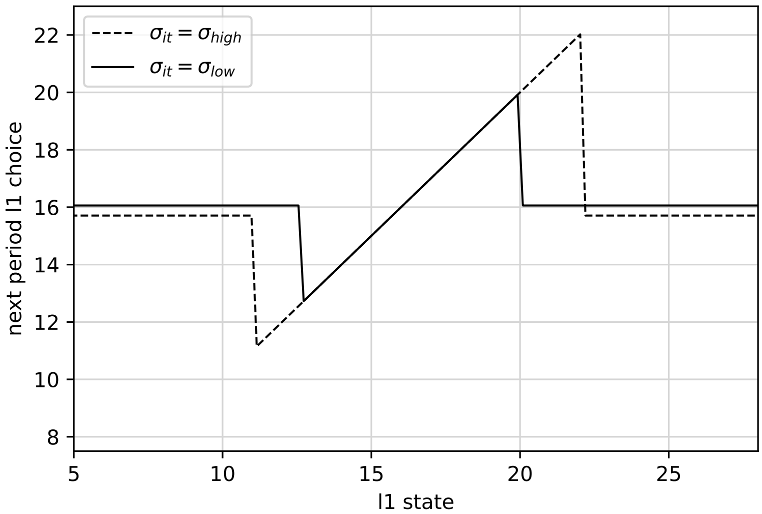
When there are no adjustment costs, labor freely moves around with both TFP and match quality shocks. When sunk adjustment costs are activated instead, we obtain inaction in gross flows. From Figure 20 we see for the no adjustment cost case, that when

Figure 20: Gross flows



$\alpha_{i,t}$ is high, the choice of $l_{1,i,t}$ is higher than when $\alpha_{i,t}$ is low, reflecting changes in match quality. As the solution to $l_{2,i,t}$ is simply the inverse of $l_{1,i,t}$ due to the Cobb-Douglas exponent relations, the stochastic changes to α reflect gross flows for a fixed firm size. We obtain an inaction area when adjustment costs are turned on (the dotted 45 degree lines). That is, firms will choose $l_{1,it+1} = l_{1,i,t}$ given some $\zeta_{i,t}, l_{2,i,t}$. Thus, if there is a shock to $\alpha_{i,t}$ (holding $z_{i,t}$ fixed), firms that would either hire/fire labor type 1 to hire/fire labor type 2, are more likely instead to choose not to hire/fire. Thus, with adjustment costs, gross flows are reduced.

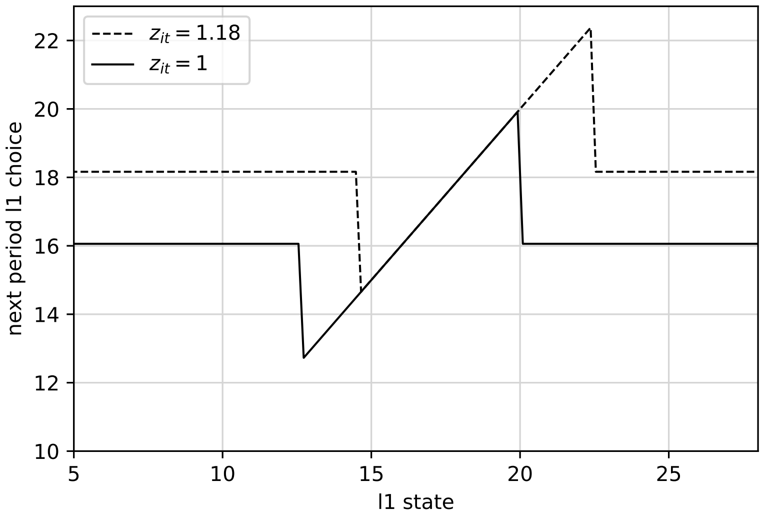
Figure 21: Gross flows, the effects of different levels of uncertainty



In Figure 21, we plot policy functions with adjustment costs comparing high vs. low uncertainty states. The inaction area widens in the high uncertainty state due to "wait-and-see" effects. That is when returns to action have higher variance, firms are more cautious in hiring and firing employees as irreversibilities become more costly. Note that this model is consistent with the prediction that increases in uncertainty decrease firm sizes if there is also exogenous attrition, as in Section 5.

Finally, for a negative TFP shock, it is straightforward to show that we would expect the size of a firm to be reduced; that is, hires decrease and fires increase. This is illustrated in Figure 22. Here we see that for a higher TFP ($z_{i,t}$) value, the next period labor choice is higher than for a low value. The 45-degree lines once again imply inaction due to fixed adjustment costs. We note that this is parsimonious for both labor types as they are symmetrical, only differing in their match quality exponent.

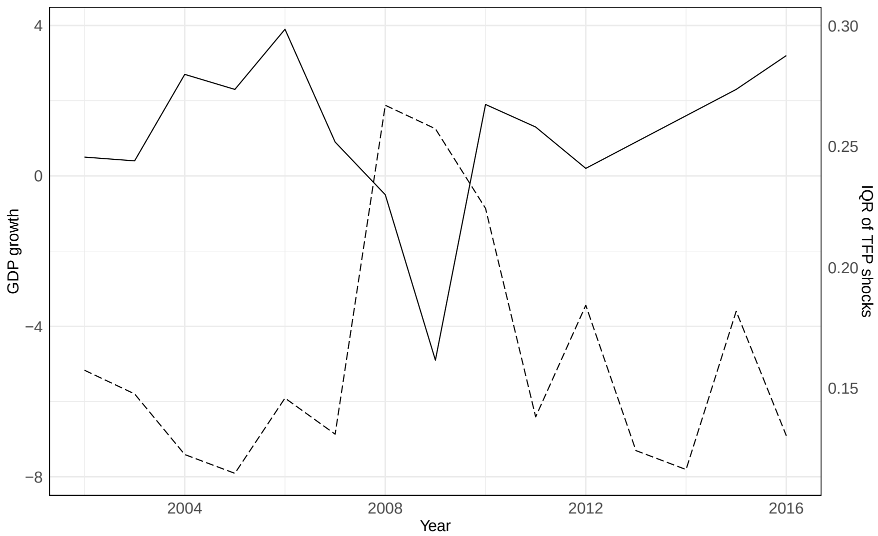
Figure 22: Employment level choices for different TFP values



F Inter-quartile range of TFP shocks vs. growth in output

In Figure 23 we plot the inter-quartile range (IQR) of TFP shocks vs. growth in output for the Danish economy. We find a negative correlation between the dispersion of TFP shocks and the business cycle. The estimated correlation coefficient is -0.56, which is comparable to what Bloom et al. (2018) find.

Figure 23: Interquartile range of "TFP shocks" (dashed line on the right axis) and GDP growth in percent (solid line on the left axis).



Note: GDP growth data was obtained from Statistics Denmark's public website, series NAN1.