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Ph.D. Dissertation

Martin Benedikt Busch

The Formation and Persistence of Perceptions

Statistical Inference and Social Networks

Advisor: Marco Piovesan

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Contents

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Martin Benedikt Busch

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Summary

Our decisions often depend on our subjective perceptions of the surrounding reality and we base our perceptions on the information we have available to us. Consider for instance our assessments of a variety of societal challenges we face, from immigration to inequality. Our perceptions of the severity of these problems depend on the information we extract from the networks around us, consisting of our friends, colleagues, and neighbors, among others. Importantly, our networks might not be representative of the broader population, they might be too small to make valid inferences, or we may not have the ability to process the information efficiently. This results in a gap between subjective perceptions and the objective reality, leading to unintended behavior affecting the individual and the society. To understand the implications of the gap between perception and reality, we need a better understanding of how people form perceptions and the underlying mechanisms creating this gap. We need to investigate the persistence of this gap and consider its long-term consequences. Finally, we may want to test if perceptions are malleable and design interventions to change behavior.

This thesis comprises three self-contained articles that provide new insights on the formation of perceptions and their persistence using economic theory and experiments. The common theme in **Chapter 1** and **2** is that agents in both models act as statisticians. Agents perform statistical inference given their information about others, the amount of information available, and heterogeneity in how they use information, to estimate an unknown but payoff-relevant state. The modeling approach allows us to show and disentangle how sample size and heterogeneity in agents' use of information affects perception and behavior. In both chapters, the sample of data is not representative of the payoff-relevant state as sample selection (**Chapter 1**) or networks (**Chapter 2**) bias the data generating process. In **Chapter 3**, we illustrate how to manipulate perceptions and evaluate its long-term impact on behavior and welfare of subjects embedded in networks. In our laboratory experiment, we show that a simple nudge has long-term payoff consequences, and persistently changes the structure of relationships subjects build with each other over time.

Chapter 1 (*Statistical Inference with Sample Selection in Games – joint with Andreas Bjerre-Nielsen*) investigates sample selection as a source of misperceptions in games where agents know the

utility function and strategies of others. With sample selection, however, agents observe an unrepresentative sample of others' behavior. We analyze equilibrium behavior in binary action games where the decision to take an action depends on the benefit and cost to do so. Whereas agents know their own benefit, cost depends on the estimated share of agents taking one action or another. In equilibrium, the share of agents taking an action must equal the share of agents whose benefit is higher than the estimated cost. We show and disentangle how sample selection, sample size, and heterogeneity in how agents use information about the underlying sample selection process affects equilibrium behavior and outline the welfare implications.

In **Chapter 2** (*Statistical inference and misperceptions in social networks – joint with Andreas Bjerre-Nielsen*), people do not observe others' behavior. Instead, they must form beliefs about others' behavior based on observations of others' social relations using statistical inference. However, a statistical law called the friendship paradox, present in almost all networks, biases observations of others' social relations. Intuitively, the friendship paradox is an over-sampling bias where agents over-sample others with many social relations as network neighbors. We analyze equilibrium behavior under strategic complementarity. Therefore, agents maximize utility if they match their behavior as close as possible to average behavior of others in the network. We show that population behavior depends on the share of agents that either do or do not correct their estimate using information about neighbors' representativeness, captured by network degree. In particular, we show that when all agents in the population are sophisticated (i.e. correct their estimate) the friendship paradox does not affect behavior. We show that the uncertainty from having a finite number of social relations affects behavior and can have a larger impact on behavior than heterogeneity in how agents use network information.

In **Chapter 1** and **2**, agents make a one-time decision in a static environment and the respective game ends. Whereas this is a justifiable assumption to investigate the mechanisms underlying the formation of perceptions, it is a strong assumption, especially when we look at networks. Real world networks are complex as they typically vary in size, structure, and change as time passes. Laboratory experiments present a powerful tool to learn more about networks as the experimental tool allows the researcher to control the network and its features.

In **Chapter 3** (*Nudging Cooperation in Networks – joint with Gorm G. Jensen, Jan O. Haerter, and Marco Piovesan*), we investigate cooperation behavior of agents embedded in networks using a laboratory experiment.[1] In our experiment, subjects build their own network over time. They send costly messages to each other that contain valuable information for the receiver or other subjects in

[1]This Chapter builds on, extends, and repeats text from the master's thesis written by Martin Benedikt Busch at the University of Copenhagen in 2018 (Busch, 2018).

the network. Sending a message is beneficial for the entire network as it increases the probability that subjects find the information they are looking for. However, classical game theory predicts zero cooperation when we measure cooperation by the profit subjects earn. We find subjects do cooperate, generate a profit for themselves and others, and that cooperation prevails for a long period. When we change subjects' perceptions of the network through the provision of initial suggestions of whom to contact, we find subjects send more messages – increasing their own and others' profit. Despite the removal of suggestions, subjects build long-lasting relationships along the suggested contacts.

Summary (in Danish)

Vores beslutninger afhænger ofte af vores subjektive opfattelse af den omkringværende virkelighed, og vi baserer vores virkelighedsopfattelse på den information, som vi har tilgængelig. Dette kunne for eksempel være vurderingen af forskellige samfundsmæssige udfordringer, som vi står over for, fra indvandring til ulighed. Vores opfattelse af alvoren af disse problemer afhænger af den information, som vi får ved at være sammen med vores netværk bestående bl.a. af vores venner, kollegaer og naboer. Men det er vigtigt at være klar over, at vores netværk ikke nødvendigvis er repræsentativt for hele befolkningen, det kan måske være for lille til at lave valid inferens eller vi har måske ikke evnerne til at bearbejde informationen efficient. Disse faktorer resulterer i en forskel mellem den subjektive virkelighedsopfattelse og den objektive virkelighed, hvilket medfører utilsigtet adfærd, som påvirker både individet og samfundet. For at forstå konsekvenserne af forskellen i virkelighedsopfattelsen og den faktiske virkelighed bliver vi nødt til at have en bedre forståelse af, hvordan personer danner deres virkelighedsopfattelse og de underliggende mekanismer, der danner forskellen mellem opfattelsen og realiteten. Vi bliver nødt til at undersøge persistensen af denne forskel og overveje dens langsigtede konsekvenser. Endelig kan vi måske teste, om vores virkelighedsopfattelse er let at påvirke og designe interventioner for at ændre adfærden.

Denne afhandling består af tre selvstændige artikler, som giver nye indsigter i dannelsen af virkelighedsopfattelser og deres persistens ved at bruge økonomisk teori og eksperimenter. I **Kapitel 1** og **2** er det fælles tema, at agenter i begge modeller agerer som statistikkere. Agenterne laver statistisk inferens givet deres information om andre, mængden af tilgængelig information og heterogeniteten i, hvordan de bruger informationen til at estimere en ukendt men payoff-relevant tilstand. Modeltilgangen giver os mulighed for at vise og skelne imellem, hvordan samplestørrelsen og heterogeniteten i agenternes brug af information påvirker virkelighedsopfattelsen og adfærden. I begge kapitler er datasamplet ikke repræsentativt for den payoff-relevante tilstand, fordi sample selektion (**Kapitel 1**) eller netværk (**Kapitel 2**) giver bias i den datagenererende proces. I **Kapitel 3** illustrerer vi, hvordan man manipulerer virkelighedsopfattelsen og evaluerer dens langsigtede virkning på subjekters adfærd og velfærd for de subjekter, som er del af et netværket. I vores laboratorieeksperiment viser vi, at sim-

pel nudging har langsigtede konsekvenser for payoffet og vedvarende ændrer strukturen af de forhold, som subjekterne danner med hinanden over tid.

Kapitel 1 (*Statistical Inference with Sample Selection in Games – joint with Andreas Bjerre-Nielsen*) undersøger, om sampleselektion er en årsag til forkert virkelighedsopfattelse i spil, hvor agenter kender de andre spilleres nyttefunktion og strategier. Dog gør sample selektion, at agenterne observerer et ikke-repræsentativt sample af de andre spilleres adfærd. Vi analyserer ligevægtsadfærden i et binært spil, hvor beslutningen om at tage en handling afhænger af handlingens fordele og omkostninger. Hvor agenterne kender deres egne fordele, afhænger omkostningerne af den estimerede andel af agenter, som tager den ene eller anden handling. I ligevægt skal andelen af agenter, som tager en handling, være lig andelen af agenter, som har større fordele end de estimerede omkostninger. Vi viser og skelner imellem, hvordan sampleselektion, samplestørrelse og heterogenitet i agenternes brug af information om den underliggende sampleselektionsproces påvirker ligevægtsadfærden og skitserer velfærdskonsekvenserne.

I **Kapitel 2** (*Statistical inference and misperceptions in social networks – joint with Andreas Bjerre-Nielsen*) observerer agenterne ikke de andres adfærd. De må i stedet danne overbevisninger om de andres adfærd baseret på observationer af de andres sociale relationer ved at bruge statistisk inferens. Dog er den statistiske lov, friendship paradokset, tilstede i næsten alle netværk og giver bias i observationerne af andres sociale relationer. Intuitivt er friendship paradokset en oversampling bias, hvor agenter oversampler andre med mange sociale relationer som netværksnaboer. Vi analyserer ligevægtsadfærd under strategisk komplementaritet. Agenterne maksimerer derfor deres nytte, hvis deres adfærd er så tæt som muligt på at matche den gennemsnitlige adfærd i netværket. Vi viser, at populationens adfærd afhænger af andelen af agenter, som enten korrigerer eller ikke korrigerer deres estimat ved at bruge information om deres naboers repræsentativitet fanget i netværks-degree'en. Vi viser særligt, at når alle agenter i populationen er sofistikerede (altså korrigerer deres estimat), påvirker friendship paradokset ikke adfærden. Vi viser, at usikkerheden af at have et begrænset antal sociale relationer påvirker adfærden og kan have en større indflydelse på adfærden end heterogenitet i, hvordan agenterne bruger netværksinformationen.

I **Kapitel 1** og **2** tager agenterne en engangsbeslutning i et statisk miljø, hvorefter det respektive spil slutter. Mens dette er en berettiget antagelse til at undersøge de underliggende mekanismer, som danner virkelighedsopfattelsen, er det ellers en stærk antagelse, især når vi ser på netværk. Virkelighedens netværk er komplekse, fordi de typisk varierer i størrelse, strukturer og ændrer sig med tiden. Derfor er laboratorieeksperimenter et stærkt redskab til at lære mere om netværk, fordi de eksperimentale redskaber giver forskeren kontrol over netværkene og dets karakteristika.

I **Kapitel 3** (*Nudging Cooperation in Networks – joint with Gorm G. Jensen, Jan O. Haerter,* *and Marco Piovesan*) undersøger vi samarbejdsadfærden for agenter, som er med i netværk, ved at bruge laboratorieeksperimenter.[2] I vores eksperiment bygger subjekterne deres eget netværk over tid. De sender omkostningsfulde beskeder til hinanden, som indeholder værdifuld information for modtageren eller de andre subjekter i netværket. Det gavner hele netværket at sende en besked, fordi det øger sandsynligheden for at subjekterne finder den information, som de har brug for. Dog forudsiger klassisk game theory, at ingen vil samarbejde, når vi måler samarbejde ud fra den profit, som subjekterne får. Vi finder, at subjekterne samarbejder, genererer profit til dem selv og andre samt at samarbejdet forsætter i lang tid. Når vi ændrer subjekternes opfattelse af netværkene ved initialt at foreslå dem hvem, de skal kontakte, finder vi, at subjekterne sender flere beskeder, hvilket øger deres egen og andres profit. Selv ved at fjerne disse kontaktforslag finder vi, at subjekter bygger langvarige forhold med de foreslåede kontakter.

[2] Dette Kapital bygger på, udvider og gentager dele fra specialeafhandlingen skrevet af Martin Benedikt Bush på Københavns Universitet i 2018 (Busch, 2018).

Chapter I

Statistical Inference with Sample Selection in Games

Statistical Inference with Sample Selection in Games*

Andreas Bjerre-Nielsen† and Martin Benedikt Busch‡

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Abstract

We investigate sample selection as a source of misperceptions in games with complete information. We analyze equilibrium behavior in binary action games where the decision to take the action depends on the estimated share of others taking it. With sample selection, agents observe an unrepresentative sample of others' behavior. We show and disentangle how sample selection, sample size, and heterogeneity in agents' inference procedures affects equilibrium behavior. We outline how our analysis of equilibrium behavior can be useful to analyze the welfare implications of sample selection.

JEL Classification Codes: D80, D90.

Keywords: Sample Selection, Statistical Inference, Complete Information.

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†Department of Economics and SODAS, University of Copenhagen (abn@sodas.ku.dk).
‡Department of Economics and CEBI, University of Copenhagen (mbb@econ.ku.dk).

1 Introduction

People often base decisions on their subjective perception of an unknown, but payoff relevant, objective population distribution. For example, the decision to attend an event in the midst of a pandemic depends on the unknown share of other people attending. In such situations, people must collect a sample of others' decisions to form an estimate as they do not know or do not have access to the entire distribution. More often than not, the collected data is not representative of the population distribution due to sample selection issues (Heckman, 1979). Sample selection issues arise, for instance, when people exclusively sample from their own social network or learn about others' decisions from news media more likely to report exciting rather than non-events.

In this paper, we model statistical decision-making based on selected samples in games with complete information. We ask: 1) How do people form subjective perceptions about the population distribution when the sample is not representative? 2) How do subjective perceptions affect equilibrium behavior and what are its welfare implications? We highlight and disentangle the effect of sample size, heterogeneity in agents' inference procedures, and their respective population share.

We incorporate sample selection into the model of Salant and Cherry (2020). Agents act as statisticians that decide whether or not to take an action by estimating the share of other agents taking it - based on a random sample of other agents' actions. In our model, agents observe a selected sample of other agents' actions. We assume that agents are either naive or sophisticated statisticians when forming their estimate. A naive agent neglects the sample selection issue, that is, thinks that her sample of data is representative of the population distribution.[1] A sophisticated agent uses information about the sample selection process available to both agents to correct for the selection issue, for instance information accumulated from a similar situation in the past. Both agents perform maximum likelihood estimation (MLE) to infer the population share taking one or the other action as any share is equally likely ex ante.

Agents sample data from the equilibrium distribution of actions, use their sample to form an estimate, and respond optimally to maximize utility given their estimate. As a consequence, both sample size and sample selection affect the equilibrium outcome. A small (finite) sample implies that there is uncertainty about the equilibrium distribution with or without sample selection. The uncertainty stems from the fact that agents do not know or do not have access to information about the entire distribution. Sample selection corresponds to the case where agents sample selected parts that are not representative of the distribution instead of random parts. For example, one could imagine that agents

[1] Enke (2020) finds that a substantial share of laboratory subjects are prone to selection neglect when using their sample to estimate a population mean.

are more likely to sample parts of the equilibrium distribution that belong to their social network. To analyze these situations, we use *Sampling Equilibrium with Statistical Inference (SESI)* as the solution concept (Salant and Cherry, 2020).

Our main result highlights the critical role of sophistication, in addition to sample size, to overcome the sample selection issue. We show that the share of agents taking the action with sample selection, i.e. the SESI share, converges to the Nash equilibrium (NE) share if and only if all agents are sophisticated as sample size tends to infinity - independent of whether agents over- or undersample others. Note that NE assumes that agents know the equilibrium distribution. Intuitively, naive and sophisticated agents become "as if" fully informed about the sample selection issue as sample size tends to infinity. However, only a sophisticated agent uses the available information to eliminate the selection issue.

We derive a number of important properties of a SESI with sample selection to establish our main result. First, we show that a SESI with sample selection is unique. Second, a SESI with sample selection deviates persistently from the NE share for any positive share of naive agents - even in the limit. We show that the direction of the deviation depends on sample size, whether agents over- or undersample others taking an action, and the share of sophisticated agents. Third, when all agents are sophisticated, the SESI share with sample selection is strictly smaller than the NE share. Intuitively, the randomness of observations in finite samples alone might lead to inaccurate inferences even if all agents are sophisticated. In other words, with a finite sample of observations sophisticated agents lack the statistical power to completely eliminate the sample selection issue. Fourth, the all naive SESI with sample selection converges to the all sophisticated SESI with sample selection as the share of sophisticated agents increases. The result highlights the important role of sophistication even within any given finite sample size.

We relate to a literature in game theory that models agents as statisticians (Salant and Cherry, 2020; Liang, 2019; Jehiel, 2018). In this literature, agents use statistical inference to estimate a payoff-relevant parameter given their sample of data. With the exception of Jehiel (2018) the data come from an unbiased data generating process. The closest to our paper is the work of Salant and Cherry (2020) who analyze equilibrium behavior when the data generating process is unbiased. We extend their model to allow for sample selection and heterogeneity in how agents use the available information about the biased data generating process. We show how sample selection affects equilibrium behavior and offer a solution to the selection issue.

We relate to a literature in game theory where misperceptions about an unbiased distribution of data stem from agents that use a misspecified causal model to interpret the data (Spiegler, 2016; Schumacher and Thysen, 2017; Spiegler, 2020; Eliaz and Spiegler, 2020). Although our modeling approach is completely different, the insights are quite similar. In this literature, the fact that agents do not fully understand the correlation structures and causal relationships of the underlying data causes persistent misperceptions. In our model, misperceptions persist if naive agents neglect the sample selection issue even with an infinite amount of data. In contrast to this literature, we study sample size in addition to heterogeneity of how agents use available data. We show that even sophisticated agents are prone to misperceptions due to small sample sizes.

We make a methodological contribution to the literature investigating the properties of Bernstein polynomials (see e.g. Davis, 1975; Phillips, 2003). We show that some of the known properties extend to monotone transformations of the Bernstein basis polynomial (see Section 4). This extension is essential to investigate how sample selection affects behavior and welfare in our application to coordination games. The literature uses polynomials in Bernstein form to study, for instance, market equilibria (Salant and Cherry, 2020), voter participation games (Nöldeke and Peña, 2016), evolutionary dynamics in multiplayer games (Peña et al., 2014), and cooperation behavior (Dos Santos and Peña, 2017; Mikkelsen and Bach, 2016). Our extension makes a useful contribution in studying issues of sample selection and cognitive biases, such as selection neglect, in these contexts.

The remainder of this paper proceeds as follows. We introduce the model in Section 2 and present the application to coordination games in Section 3. We present the general framework including our main results concerning behavior and welfare in Section 4. We conclude in Section 5 and the Appendix contains auxiliary results as well as proofs.

2 Model

Consider a unit mass of agents that must choose one out of two available actions, which we denote by B and H.

Utility. An agent receives utility from taking action B as follows:

$$u(\theta, \alpha) = \theta - f(\alpha). \tag{1}$$

First, an agent's utility depends on her individual benefit of taking action B which we denote by $\theta \in U \sim [0, 1]$. We normalize an agent's utility from taking action H to zero. Second, utility depends on the share of other agents taking action B, which we denote by $\alpha \in [0, 1]$. An agent trades off her individual benefit from taking the action with the cost of taking it, which we denote by $f(\alpha)$. We make the following assumptions about the cost function. We assume that $f(\alpha)$ is continuous, convex, and monotonically increasing in α. In other words, the cost of taking the action increases in the share of other agents taking it. We require that $f(0) \geq 0$ and $f(1) \leq 1$ implying that the cost of taking the action is smaller than the maximum individual benefit.

Information structure. The game takes place in an environment of complete information where the structure of the game is common knowledge among all agents. That is, there is no uncertainty about the utility function and the strategies of other agents. In our model, we allow for the possibility that agents only observe a subsample of other agents' actions and that there may be sample selection. In other words, agents receive only an imperfect signal about α, possibly biased as well. Intuitively, with sample selection an agent decides to take an action based on her perceptions but experiences utility according to realized outcomes, as captured in equation (1). We denote the sample selection process, determining the perceived share of agents taking action B, as follows:

$$y(\alpha) = \frac{\beta \cdot \alpha}{\beta \cdot \alpha + (1 - \alpha)}, \tag{2}$$

where $\beta \in (0, +\infty)$ determines the magnitude of the sample selection problem. In our model, $y(\alpha) \in [0, 1]$ is a monotone transformation of the true share of agents taking action B. Note that $y(\alpha) = 1$ whenever $\alpha = 1$ and that $y(\alpha) = 0$ whenever $\alpha = 0$. We define *sample selection* as the case where $\beta \neq 1$ implying that $y(\alpha) \neq \alpha$. There is *no sample selection* when $\beta = 1$ in which case the perceived share $y(\alpha)$ equals the true share α. When $\beta > 1$ we say that agents oversample other's taking action B and when $0 < \beta < 1$ we say that agents undersample others taking the action.

In our model, an agent obtains a sample of size d comprising independent observations of other agents actions drawn from a Bernoulli distribution. Therefore, we can interpret $y(\alpha)$ as the success probability to draw an agent taking action B. For example, when $\beta = 2$ the probability to draw an agent who chooses action B, compared to an agent choosing action H, is twice as likely.

Statistical inference. Agents use their observed sample to estimate α. From the sample they know its size d, the sample mean m of agents taking action B, and the magnitude of the sample selection problem β. Agents use this information in either a naive or sophisticated way to estimate α using maximum likelihood estimation (MLE). We define the MLE estimator as $\hat{\alpha}_r(m)$ where $r \in \{n, s\}$ denotes whether the agent uses a naive (n) or sophisticated (s) updating rule. Both types of agents neglect the randomness of observations in small samples, which the literature refers to as *bounded rationality* (see e.g. Osborne and Rubinstein, 2003). For example, when the sample contains just a single observation, both types of agents believe that they can make valid inferences about α. In addition, a naive agent neglects the sample selection problem. In essence, the naive agent ignores β and estimates that α equals the sample mean m, i.e. $\hat{\alpha}_n(m) = m$. A sophisticated agent does not believe that her sample is representative of the true share due to sample selection, hence, uses the information about β to adjust her estimate as follows:

$$\hat{\alpha}_s(m) = \frac{m}{\beta + m \cdot (1 - \beta)}. \tag{3}$$

In Appendix A.2, we formally derive the estimator and prove that the sophisticated estimator is unbiased and consistent. Note, when there is no sample selection (i.e. $\beta = 1$) the estimator of the naive and sophisticated agent align, i.e. $\hat{\alpha}_s(m) = \hat{\alpha}_n(m) = m$.

3 Example: Sample Selection and Coordination

We illustrate the equilibrium solution concept and our main results on an example where agents must decide whether or not to go to a bar in the midst of a pandemic. In this example, action B implies that an agent decides to go to a bar and action H implies that the agent decides to stay at home. Moreover, θ denotes an agents individual benefit from going to the bar, α denotes the true share of agents going to the bar, $y(\alpha)$ denotes the probability to observe an agent going to the bar, and $f(\alpha)$ denotes the cost of going to the bar. We assume that $f(\alpha) = \alpha^4$. That is, $f(\alpha)$ is continuous, convex, and monotonically increasing. The agent decides to go to the bar based on her perceptions of the share that goes. However, only the true share of agents that go affects the likelihood to catch the virus, hence the utility. In the remainder, we assume that $\beta = 2$ which means that agents oversample others going to the bar. This implies that the perceived cost of going to the bar is higher than the actual cost to do so. In this example, we assume that all agents are either naive or sophisticated. We generalize the example in Section 4.

A Single Observation. Consider a sample size of $d = 1$, which means that the agent has access to a single observation. That is, the agent observes another agent that either goes to the bar or stays at home. When does an agent choose to go to the bar? All agents whose individual benefit lies above (below) the estimated cost of going to the bar strictly prefer to take action B (H) and go (not go) to the bar. The equilibrium share must satisfy the following condition:

$$\alpha = (1 - y(\alpha)) \cdot (1 - f(\hat{\alpha}_r(0))) + y(\alpha) \cdot (1 - f(\hat{\alpha}_r(1))). \tag{4}$$

In the equilibrium, the share of agents that go to the bar must equal the expected share of agents whose individual benefit of going to the bar exceeds the estimated cost to do so. This corresponds to the left and right hand side of equation (4), respectively. Note that it is $1 - f(\hat{\alpha}_r(\cdot))$ as the highest individual benefit θ that an agent can possess is 1, by definition. With probability $(1 - y(\alpha))$ the agent observes someone who stays at home. A naive and sophisticated agent estimates that everybody stays at home as $\hat{a}_r(m) = \hat{a}_s(0) = \hat{a}_n(0) = 0$. Therefore, the agent goes to the bar as $\theta \geq f(0)$ always holds. With probability $y(\alpha)$ the agent observes an agent that goes to the bar. A naive and sophisticated agent estimates that everyone goes to the bar as $\hat{a}_r(m) = \hat{a}_s(1) = \hat{a}_n(1) = 1$. In that case, the agent stays at home since $\theta \geq f(1)$ never holds. We can rewrite equation (4) as follows:

$$1 - \alpha = (1 - y(\alpha)) \cdot f(0) + y(\alpha) \cdot f(1). \tag{5}$$

Without sample selection (i.e. $\beta = 1$), which implies that $y(\alpha) = \alpha$, the unique equilibrium share lies at $\alpha = 1/2$. This is called a *sampling equilibrium with statistical inference (SESI)* as described in Salant and Cherry (2020). With sample selection, however, the unique SESI share lies at $\alpha \approx 0.41$. Intuitively, naive and sophisticated agents arrive at the same conclusion as sophisticated agents lack the statistical power to use the information about the selection issue effectively.

Two Observations. We can apply the same logic as above in the case where an agent has access to a sample of size $d = 2$. The equilibrium share α must satisfy the following condition:

$$1 - \alpha = (1 - y(\alpha))^2 \cdot f(\hat{\alpha}_r(0)) + 2y(\alpha)(1 - y(\alpha)) \cdot f(\hat{\alpha}_r(1/2)) + y(\alpha)^2 \cdot f(\hat{\alpha}_r(1)).$$

With probability $(1 - y(\alpha))^2$ the agent observes two agents who stay at home and estimates that everybody stays at home. With probability $2y(\alpha)(1 - y(\alpha))$ the agent observes one agent who goes to the bar and one agent who stays at home (i.e. $f(\hat{\alpha}_r(1/2))$). The naive agent believes that the share is representative of the population, therefore, estimates that half of the population goes to the bar, i.e. $\hat{\alpha}_n(1/2) = 1/2$. The sophisticated agent adjusts her estimate as follows. The sophisticated agent evaluates the cost at $f(1/3)$ instead of $f(1/2)$ because $\hat{\alpha}_s(1/2) = 1/3$. With probability $y(\alpha)^2$ the agent observes two agents that go to the bar and the naive and sophisticated agent concludes that everybody goes to the bar. Without sample selection, the unique SESI share lies at $\alpha \approx 0.6$ independent of whether agents are naive or sophisticated. With sample selection, the unique SESI share lies at $\alpha \approx 0.51$ when all agents are naive. However, when all agents are sophisticated the unique SESI share lies at $\alpha \approx 0.52$.

Infinitely Many Observations. Consider the case where agents have access to an infinitely large sample of observations. With an infinitely large sample and no sample selection it is "as if" agents know the equilibrium share of agents taking action B. In that case, we can use Nash equilibrium (NE) as the solution concept. In the NE the share of agents going to the bar must satisfy the following condition:

$$\alpha = 1 - f(\hat{\alpha}_r(m)), \tag{6}$$

where $\hat{\alpha}_r(m) = m = \alpha$ in the limit. Intuitively, the share of agents that go to the bar must equal the share of agents whose individual benefit of going to the bar exceeds the estimated cost to do so. There is a unique solution at $\alpha \approx 0.72$. Note that both the naive and sophisticated agent estimates the correct share because sample selection is not a problem. That is, $\approx 72\%$ of agents go to the bar and no agent has an incentive to deviate from her strategy. Now consider the case with sample selection. With infinite estimation precision, the estimator of the naive agent equals $y(\alpha)$ as $\hat{\alpha}_n(m) = m = y(\alpha)$ in the limit. In other words, naive agents are infinitely precise about the bias in the limit. When all agents are naive, there exists a unique limit equilibrium at $\alpha \approx 0.64$.[2] A sophisticated agent, however, adjusts her estimate. The maximum likelihood estimator of a sophisticated agent is unbiased and consistent, therefore, the sophisticated agent correctly estimates the share of agents going to the bar, i.e. $\hat{\alpha}_s(y(\alpha)) = \alpha \approx 0.72$. When all agents are sophisticated, the unique limit equilibrium with sample selection aligns with the NE share.

Illustration of Main Results. Figure 1 provides a visual illustration of the equilibria that we calculated above. Note that we could rewrite the left hand side of each equation as $1 - \alpha$, depicted by the black lines in all panels. The monotonically increasing lines illustrate the right hand sides where the intersection with the black line shows the respective unique equilibrium share. Panel A illustrates the limit equilibria with (dashed) and without (solid) sample selection. Panels B, C, and D show that as sample size increases the unique SESI share converges to the respective limit equilibrium share. Panel B illustrates the solution of Salant and Cherry (2020), where the SESI share without sample selection converges to the NE share. Panel C illustrates that the unique SESI share with sample selection converges to the limit equilibrium share with sample selection when all agents are naive. Intuitively, if all agents are naive and the sample becomes infinitely large, naive agents become infinitely precise about the sample selection bias. Panel D illustrates, however, that the SESI share with sample selection converges to the NE share if and only if all agents are sophisticated. This is the major result of the paper. Intuitively, if sophisticated agents have an infinitely large sample they become, as naive agents, infinitely precise about the underlying sample selection bias. Therefore, sophisticated agents can precisely remove the bias as the estimator is unbiased and consistent. Note that in all panels the SESI share is strictly less than the respective limit equilibrium share for any given finite sample size.

[2] Note that this is not a NE as agents do not know the equilibrium share of agents going to the bar. Instead, agents are infinitely precise about the bias in what we call "the limit equilibrium".

Intuitively, the lack of statistical power causes less agents to go to the bar than in the limit.

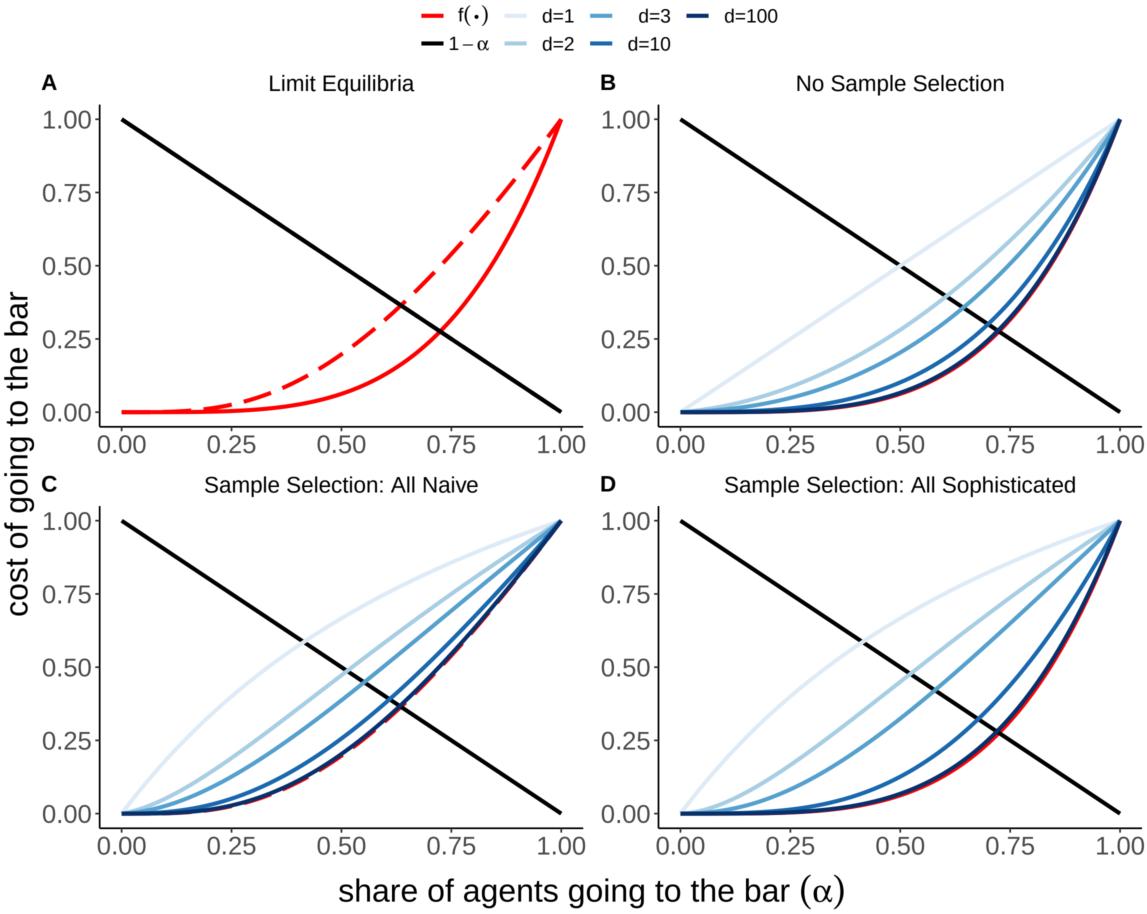

Figure 1: Equilibria with ($\beta = 2$) and without sample selection

The panels illustrate the unique equilibrium shares. The intersection of the black and red lines illustrate the limit equilibria with (dashed) and without (solid) sample selection. The intersection of the black and blue lines denote the SESI shares. Panel **A** illustrates the limit equilibria; Panel **B** replicates the solution of Salant and Cherry (2020); Panel **C** shows the case when all agents are naive; Panel **D** shows the case when all agents are sophisticated. In all panels we assume that $\beta = 2$ when there is sample selection and $\beta = 1$ corresponds to the case without sample selection.

4 General Framework

Consider a population consisting of a mix of naive and sophisticated agents. We denote the share of sophisticated agents by σ and the share of naive agents by $1 - \sigma$. We generalize the example in Section 3 using the theory of Bernstein polynomials, allowing us to represent the equilibrium condition governing any sample size as follows:

$$1 - \alpha = B_d(h_\sigma; y(\alpha)). \tag{7}$$

The right hand side denotes what we call a re-weighted Bernstein polynomial of sample size d. The difference between our re-weighted version of a Bernstein polynomial and the standard version used in the literature (see e.g. Phillips, 2003) is that we allow for sample selection as well as heterogeneity in how agents deal with the sample selection issue. One can rewrite the right hand side as follows:

$$B_d(h_\sigma; y(\alpha)) = \sum_{i=0}^{d} b_{i,d}(y(\alpha)) \cdot \Big(\sigma \cdot f\left(\hat{\alpha}_s(i/d)\right) + (1-\sigma) \cdot f\left(\hat{\alpha}_n(i/d)\right)\Big), \tag{8}$$

where $h_\sigma = \sigma \cdot f(\hat{\alpha}_s(\cdot)) + (1-\sigma) \cdot f(\hat{\alpha}_n(\cdot))$.[3] Note that $\hat{\alpha}_r(i/d)$ denotes the estimator given the observation $i/d = m$ for a given updating rule. Intuitively, one can use a re-weighted Bernstein polynomial to approximate the cost function $f(\cdot)$ using samples of the cost function at $f(\hat{\alpha}_r(m))$. The respective Bernstein basis polynomial weights the samples of the cost function:

$$b_{i,d}(y(\alpha)) = \binom{d}{i} y(\alpha)^i (1-y(\alpha))^{d-i}. \tag{9}$$

Note that a Bernstein basis polynomial equals the probability mass function of a binomial distribution where $\binom{d}{i}$ denotes the binomial coefficient, $y(\alpha)$ the success probability, and d the number of Bernoulli trials. Recall, in Section 3 we could rewrite the left hand side of the equilibrium condition as $1-\alpha$ independent of samples size. This follows directly from the property that the sum of Bernstein basis polynomials of sample size d form a partition of unity by the binomial theorem:

$$\sum_{i=0}^{d} b_{i,d}(y(\alpha)) = \big(y(\alpha)) + (1-y(\alpha)\big)^d = 1.$$

In the two extreme cases where all agents are either naive or sophisticated, equation (7) reduces to:

$$\begin{aligned} 1-\alpha &= B_d(h_1; y(\alpha)) \quad \text{if} \quad \sigma = 1. \\ 1-\alpha &= B_d(h_0; y(\alpha)) \quad \text{if} \quad \sigma = 0. \end{aligned}$$

In the case where all agents are naive (i.e. $\sigma = 0$) and there is no sample selection (i.e. $\beta = 1$) our model reduces to the model of Salant and Cherry (2020).

4.1 Equilibrium Properties

We begin our analysis of the general framework with analyzing the equilibrium properties of a SESI with sample selection.

[3] We use the property that $B_d(h_\sigma; y(\alpha)) = \sigma \cdot B_d(h_1; y(\alpha)) + (1-\sigma) \cdot B_d(h_0; y(\alpha))$ to derive the equation.

Uniqueness. The example in the previous section illustrates that there exists a unique equilibrium corresponding to the case where marginal benefits and cost equal each other for small ($d = 1, 2, 3, \dots$) and large samples ($d \to \infty$). We generalize this uniqueness result below and show it holds that for any sample size, degree of sample selection, and share of sophisticated agents.

Theorem 1. *For any share of sophisticated agents and any sample size, there exists a unique SESI with sample selection.*

Proof. Note that $1 - \alpha$ is a strictly decreasing and continuous function in $\alpha \in [0, 1]$, which does not depend on whether the agent is naive or sophisticated, sample selection, nor sample size. To prove uniqueness, it is sufficient to show that $B_d(h_\sigma; y(\alpha))$ is monotonically increasing in α, which we prove in Fact 1 (see Appendix A.1). As a consequence, $1-\alpha$ and $B_d(h_\sigma; y(\alpha))$ cross only once on the domain $[0, 1]$. □

Theorem 1 extends the uniqueness result of Salant and Cherry (2020), which proves uniqueness for the naive agent without sample selection.

Convergence. The example in the previous section illustrates that the SESI share with sample selection converges to the NE share if and only if all agents are sophisticated. We generalize below how the behavioral assumptions about how agents update affects convergence as sample size tends to infinity.

Theorem 2. *If there is sample selection, the sequence of Bernstein polynomials of sample size d converges uniformly to:*

1. *the misperceived cost function $f(y(\alpha))$ when all agents are naive.*

2. *the true cost function $f(\alpha)$ when all agents are sophisticated.*

3. *the linear combination of the true and mispeceived cost function, i.e. $\sigma \cdot f(\alpha) + (1-\sigma) \cdot f(y(\alpha))$, given a share of sophisticated agents.*

Proof. We provide the proof in Fact 5 of Appendix Section A.1. □

The first result of Theorem 2 states that the SESI share with sample selection, using the naive inference procedure, approximates the limit equilibrium share with sample selection as sample size tends to infinity. Instead, the SESI with sample selection, using the sophisticated inference procedure, approximates the NE share even though there is sample selection. Intuitively, naive and sophisticated agents become fully informed about $y(\alpha)$ as sample size tends to infinity. However, a sophisticated agent uses her knowledge about the sample selection process to cancel the bias. The following reformulation of a Bernstein polynomial provides the intuition:

$$B_d(h_\sigma; y(\alpha)) = E\Big(\sigma \cdot f\left(\hat{\alpha}_s(S_d/d)\right) + (1-\sigma) \cdot f\left(\hat{\alpha}_n(S_d/d)\right)\Big),$$

where S_d counts the number of successes within an agent's sample (i.e. the number of agents taking action B) with success probability $y(\alpha)$. When the agent is naive, the law of large numbers states that the sample average $\hat{\alpha}_n(S_d/d)$ of observed agents taking action B converges in probability to the expected value $y(\alpha)$ as sample size tends to infinity. As $f(\cdot)$ is continuous it should hold that $f(\hat{\alpha}_n(S_d/d))$ converges to $f(y(\alpha))$, because $E(S_d/d) = d \cdot y(\alpha)/d = y(\alpha)$. The sophisticated inference procedure is unbiased and consistent (see Fact 7). Unbiasedness implies that the expectation of the estimator is equal to the true share $E(\hat{\alpha}_s(S_d/d)) = E(\hat{\alpha}_s(m)) = d \cdot \alpha/d = \alpha$. Consistency implies that the sample share converges to the true share as sample size tends to infinity.

4.2 Equilibrium Behavior

In this section, we evaluate how sample selection and the share of sophisticated agents in the population affect equilibrium behavior. We distinguish two cases. First, we analyze the effect of sample selection and sophistication on behavior in isolation. With infinite estimation precision there is no uncertainty about the equilibrium share of agents taking action B. That is, the equilibrium share solely depends on how agents use the information about the sample selection process and the share of sophisticated agents in the population. Second, we allow for uncertainty about equilibrium behavior. Uncertainty comes from the fact that agents only have a finite sample to estimate the equilibrium share.

4.2.1 Behavior with Infinite Estimation Precision

We begin our analysis of equilibrium behavior with a focus on agents who have large (infinite) samples. When the sample is large the right hand side of the equation (7) converges to a linear combination of the true and misperceived cost function (see Theorem 2). In other words, with infinite estimation precision there is no uncertainty about the share of agents taking action B in equilibrium. We can rewrite equation (7) as follows:

$$1 - \alpha = \sigma \cdot f(\alpha) + (1-\sigma) \cdot f(y(\alpha)). \tag{10}$$

When all agents in the population are sophisticated, sample selection has no effect as the estimator of the sophisticated agent is unbiased and consistent. To analyse the effect of sample selection we require that the share of naive agents in the population is positive. i.e. $\sigma \in [0, 1)$.

Theorem 3. *If $f(\alpha)$ is monotonically increasing and continuous on $[0, 1]$ for $\alpha \in [0, 1]$ and there is sample selection then:*

1. *The unique equilibrium share $\alpha^*_{\beta \neq 1}$ decreases as β increases, which holds for any positive share of naive agents in the population and is independent of whether agents oversample or undersample others taking action B.*

2. *The unique equilibrium share increases (decreases) as the share of sophisticated agents increases if there is oversampling (undersampling) for any given β.*

3. *The unique equilibrium share with sample selection converges to the unique NE share as the share of sophisticated agents converges to unity.*

Proof. We provide the proof in Appendix A.3. □

Intuitively, Theorem 3 states that sample selection causes a deviation from the NE for any positive share of naive agents in the population, even though there is no uncertainty about the share of agents taking action B. A sophisticated agent knows about the underlying sample selection problem and perfectly adjusts her estimate. This reduces the sample selection problem as the share of sophisticated agents in the population increases. However, unless all agents in the population are sophisticated, the sample selection issue persists. In other words, the unique equilibrium share with sample selection equals the NE share if and only if all agents are sophisticated and have infinite estimation precision.

4.2.2 Behavior with Finite Estimation Precision

With finite estimation precision sample size affects behavior due to the uncertainty about the number of agents taking action B in equilibrium. Whether there is sample selection or not. As one can infer from the right hand side of equation (7), equilibrium behavior entirely depends on the properties of re-weighted Bernstein polynomials for a given share of sophisticated agents and a given sample size. To derive our finite sample size results we need the additional assumption that the cost function $f(\cdot)$ must be convex. This implies the following properties of a re-weighted Bernstein polynomial of sample size d. We provide the proofs of the respective properties in Appendix A.1.

Property 1. *If $f(y(\alpha))$ and $f(y^{-1}(\alpha))$ are convex on $[0, 1]$ so that the linear combination is convex:*

$$B_d(h_\sigma; y(\alpha)) \geq B_{d+1}(h_\sigma; y(\alpha)),$$

for any share of sophisticated agents and any degree of sample selection. The inequality holds for $\alpha \in [0, 1]$ *and is strict for* $\alpha \in (0, 1)$ *unless* $f(\cdot)$ *is linear.*

Property 1 implies that a re-weighted Bernstein polynomial of sample size d stochastically dominates a re-weighted Bernstein polynomial of sample size $d + 1$. Therefore, an increase in sample size leads to an increase in the equilibrium share. Due to this property the first result of Theorem 3 extends to the case of finite degree requiring convexity, in addition to monotonicity and continuity. In other words, the unique equilibrium share in a SESI with sample selection decreases as β increases for any given sample size.

Property 2. *If* $f(y(\alpha))$ *and* $f(y^{-1}(\alpha))$ *are convex on* $[0, 1]$ *so that the linear combination is convex and there is oversampling:*

$$B_d(h_0; y(\alpha)) \geq B_d(h_0; \alpha),$$

for any $\alpha \in [0, 1]$ *and sample size* $d \geq 1$*, if all agents are naive. Moreover,*

$$B_d(h_0; y(\alpha)) \geq B_d(h_\sigma; y(\alpha)) \geq B_d(h_1; y(\alpha)),$$

for any share of sophisticated agents, any degree of sample selection and $d \geq 2$*. Both inequalities hold for* $\alpha \in [0, 1]$ *and are strict for* $\alpha \in (0, 1)$ *unless* $f(\cdot)$ *is linear. The inverse is true when there is undersampling.*

The first result of Property 2 states that for any given sample size the SESI share with sample selection is smaller (larger) when agents oversample (undersample) others taking action B, compared to the SESI share without sample selection, if all agents are naive. This highlights the fact that sample selection is a persistent issue, independent of sample size. The second result of Property 2 is similar to the second result of Theorem 3, but within a given degree. The re-weighted Bernstein polynomial, assuming that all agents are naive, stochastically dominates the re-weighted Bernstein polynomial assuming that all agents are sophisticated for any sample size $d > 2$ and oversampling.[4] As the share of sophisticated agents increases the re-weighted Bernstein polynomial, assuming that all agents are naive, converges to the re-weighted Bernstein polynomial assuming that all agents are sophisticated.[5]

[4]The inverse is true for undersampling. Note that $d = 1$ implies that $B_1(h_0; y(\alpha)) = B_1(h_1; y(\alpha))$ for any $\alpha \in (0, 1)$. At the extremes (i.e. $\alpha \in \{0, 1\}$) it holds that $B_d(h_0; y(\alpha)) = B_d(h_1; y(\alpha))$ for any given sample size.

[5]See Figure 3 and 4 in Appendix A.4 for a visual representation.

Property 3. *If $f(y(\alpha))$ and $f(y^{-1}(\alpha))$ are convex on $[0, 1]$ so that the linear combination is convex:*

$$B_d(h_0; y(\alpha)) \geq f(y(\alpha)),$$
$$B_d(h_1; y(\alpha)) \geq f(\alpha),$$
$$B_d(h_\sigma; y(\alpha)) \geq \sigma f(\alpha) + (1 - \sigma) f(y(\alpha)),$$

for any share of sophisticated agents and any degree of sample selection. The inequality holds for $\alpha \in [0, 1]$ and is strict for $\alpha \in (0, 1)$ unless $f(\cdot)$ is linear.

Property 3 implies that the SESI share with finite sample size is strictly smaller than the respective limit equilibrium share. That is, the third result of Theorem 3 does not extend to the finite case even if all agents in the population are sophisticated. The result highlights the critical role of sample size as a limiting factor, in addition to sophistication, to overcome the sample selection issue. Intuitively, with a finite sample of observations, sophisticated agents lack the statistical power to completely eliminate the sample selection issue.

5 Concluding Comments

This paper investigates how sample selection affects behavior in games with complete information. The key ingredient to obtain our results is that we model agents as statisticians. In particular, the modeling approach allows us to show and disentangle how sample selection, sample size, and heterogeneity in agents' inference procedures affects behavior. Sample selection corresponds to the case where an agent obtains a selected sample of other agents' actions stemming from a biased data generating process. We show that an agent will never learn the true action distribution unless the agent uses statistical inference to overcome the sample selection issue.

We conclude with two comments about how our results concerning behavior are useful to analyze the effect of sample selection on welfare. We comment on welfare ex-post from the perspective of agents (i.e the consumers of action B) using equilibrium behavior that we pinned down in Section 4.2. The first comment is about welfare in large samples when all agents are either naive or sophisticated as well as a population that consists of a share. The second comment is about welfare implications in small samples.

5.1 Welfare with Infinite Estimation Precision

In the case where all agents are either naive or sophisticated and have a large sample, we can express welfare $W(\cdot)$ as a function of $\alpha \in [0, 1]$ - the share of agents taking action B:

$$W(\alpha) = \int_0^\alpha (1 - x)dx - f(\alpha) \cdot \alpha, \tag{11}$$

where the integral denotes the aggregate benefit θ of agents taking action B and $f(\alpha) \cdot \alpha$ denotes the total cost at any given share of agents taking the action. Naive and sophisticated agents have a different threshold benefit of θ determining whether they take the action or not. However, in the corner cases where all agents are either naive or sophisticated this threshold is the same for all agents, hence, we can evaluate aggregate welfare using equation (11). To determine the welfare maximizing share, we take the derivative of $W(\alpha)$ with respect to α as follows:

$$1 - \alpha = \frac{\partial f(\alpha)}{\partial \alpha} \cdot \alpha + f(\alpha). \tag{12}$$

The left hand side denotes the marginal benefit and the right hand side the marginal cost of an additional agent taking action B.

Theorem 4. *The welfare maximizing share of agents taking action B is strictly smaller than the NE share and is a global maximum.*

Proof. The marginal cost is always greater than or equal to the actual cost by a factor $(\partial f(\alpha)/\partial \alpha) \cdot \alpha$ for any given $\alpha \in [0, 1]$:

$$\frac{\partial f(\alpha)}{\partial \alpha} \cdot \alpha + f(\alpha) \geq f(\alpha),$$

which follows directly from the continuity, monotonicity, and convexity property of the cost function $f(\cdot)$ on its domain $[0, 1]$. We show that the share is a global maximum in Appendix A.3. □

Intuitively, the marginal cost curve always lies above the actual cost curve as the cost increase for all agents who already take action B for each additional agent taking the action. It directly follows that the rent maximizing share of agents taking action B always lies below the NE share.

We can determine the optimal degree of sample selection that implements the welfare optimal share of agents taking action B as follows:

$$\frac{\partial f(\alpha)}{\partial \alpha} \cdot \alpha + f(\alpha) = f(y(\alpha)). \tag{13}$$

Note that equation (13) never holds if all agents are sophisticated as sophisticated agents completely eliminate the sample selection issue with infinite estimation precision.[6] Let us revisit our example from Section 3 and calculate the welfare maximizing degree of sample selection assuming that all agents are naive. First, we calculate the welfare optimal α by solving equation (12) which equates to $\alpha \approx 0.548$. Second, we solve for the degree of sample selection that implements the welfare optimal α. We plug α into equation (13) and solve for the optimal degree of sample selection, that is, $\beta \approx 3.75$. In this example, naivity is good for consumer welfare if agents oversample others actions.

When the population consists of a share of naive and sophisticated agents, there exists a different threshold benefit of θ for each type of agent determining whether the agent takes the action or not. In this case, we can express the welfare function as follows:

$$W(\alpha_r) = \sigma \cdot \Big(\int_0^{\alpha_s} (1-x)dx - f(\alpha_s) \cdot \alpha_s \Big) + (1-\sigma) \cdot \Big(\int_0^{\alpha_n} (1-x)dx - f(\alpha_n) \cdot \alpha_n \Big),$$

where $\alpha_r \in \{\alpha_s, \alpha_n\}$ with the property that $\alpha = \sigma \cdot \alpha_s + (1-\sigma) \cdot \alpha_n$. For example, in the corner case where all agents are naive (i.e. $\alpha_n = \alpha$) we can calculate welfare as in equation (11). From our analysis of equilibrium behavior we know that there are two moderating factors that work in opposite directions (see Theorem 3). First, the unique equilibrium share decreases as the degree of sample selection increases. Second, an increase in the share of sophisticated agents increases the unique equilibrium share when there is oversampling. A challenge for future research is to disentangle the effect of sophistication on aggregate welfare and relate it to the welfare maximizing share.

5.2 Welfare with Finite Estimation Precision

With finite estimation precision one has to consider an additional sorting effect due to the uncertainty stemming from the randomness of observations in small samples - whether there is sample selection or not. As a comparison, note that in the limit there is "perfect" positive sorting. That is, if a naive (sophisticated) agent takes action B then all naive (sophisticated) agents with a higher individual benefit take the action as well. In small samples this is not the case as agents estimates differ due to the randomness of observations. Therefore, agents with a higher individual benefit than an agent who took the action might not necessarily take it as well. We can use the theory of re-weighted Bernstein polynomials to quantify welfare in finite samples as follows:

$$W(\alpha) = \sum_{i=0}^{d} b_{i,d}(y(\alpha)) \cdot \Big(1 - \big[\sigma \cdot f\big(\hat{\alpha}_s(i/d)\big) + (1-\sigma) \cdot f\big(\hat{\alpha}_n(i/d)\big)\big]\Big) - f(\alpha) \cdot \alpha. \qquad (14)$$

[6] In essence, $\frac{\partial f(\alpha)}{\partial \alpha} \cdot \alpha + f(\alpha) = f(\alpha)$ never holds.

To illustrate our point consider a sample size of $d = 1$ in which case naive and sophisticated agents arrive at the same estimate and equation (14) becomes:

$$W(\alpha) = \Big(\big(1 - y(\alpha)\big) \cdot \big(1 - f(0)\big) + y(\alpha) \cdot \big(1 - f(1)\big)\Big) - f(\alpha) \cdot \alpha.$$

There are two possibilities. Either all agents estimate that nobody takes action B or all agents estimate that everybody takes the action. Agents who estimate that nobody takes the action, implying zero cost, always take the action - independent of their individual benefit. Agents who estimate that everybody takes the action, implying maximum cost, never take it. Therefore, an agent with a higher benefit than another agent that already took the action does not necessarily take the action as well. A challenge for future research is to facilitate our results of equilibrium behavior to investigate how sorting affects welfare.

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A Appendix

A.1 Properties of Re-weighted Bernstein Polynomials

Fact 1. *If $f(\cdot)$ is monotonically increasing on* $[0,1]$ *then $B_d\big(h_\sigma; y(\alpha)\big)$ is monotonically increasing in α for $\alpha \in$* $[0,1]$.

Proof. We show that the derivative of $B_d\big(h_\sigma; y(\alpha)\big)$ wrt. α is positive. Using the chain rule we can express this derivative as follows:

$$\frac{\partial B_d\big(h_\sigma; y(\alpha)\big)}{\partial \alpha} = \frac{\partial B_d(h_\sigma; x)}{\partial x}\Big|_{x=y(\alpha)} \cdot \frac{\partial y(\alpha)}{\partial \alpha}$$

From Theorem 7.1.2 in Phillips (2003) we know that $\frac{\partial B_d(h_\sigma;x)}{\partial x}\Big|_{x=y(\alpha)} > 0$ for $\alpha \in [0,1]$. We solve the other part of the derivative and obtain:

$$\frac{\partial y(\alpha)}{\partial \alpha} = \frac{\beta}{(1-\alpha+\alpha\beta)^2},$$

which is positive for $\beta \in (0,\infty)$. Note that if $f(\alpha)$ is monotonically increasing in $\alpha \in [0,1]$ on $[0,1]$, then $f(y(\alpha))$ and $f(y^{-1}(\alpha))$ are monotonically increasing in α on $[0,1]$ as well. □

Fact 2. *If $f(y(\alpha))$ is convex and continuous on* $[0,1]$ *then it holds that $B_d(h_0; y(\alpha)) \geq B_{d+1}(h_0; y(\alpha))$. If $f(y^{-1}(\alpha))$ is convex and continuous on* $[0,1]$ *then it holds that $B_d(h_1; y(\alpha)) \geq B_{d+1}(h_1; y(\alpha))$. If the first two conditions hold then the linear combination of these conditions is convex and continuous on* $[0,1]$ *and $B_d(h_\sigma; y(\alpha)) \geq B_{d+1}(h_\sigma; y(\alpha))$. All three inequalities hold for any $\alpha \in$* $[0,1]$ *and are strict for any $\alpha \in$* $(0,1)$ *unless $f(\cdot)$ is linear.*

Proof. From Theorem 7.1.9 in Phillips (2003) we know that the property holds for any point $\alpha \in [0,1]$, but without the inner transformation of $y(\alpha)$. As we have the map $\alpha = y^{-1}(\alpha)$, which is a one-to-one mapping from $[0,1]$ into $[0,1]$ it follows that the property also holds for the transformation when all agents are naive. When all agents are sophisticated the property must hold as $\hat{\alpha}_s(\cdot)$ maps values from $[0,1]$ into $[0,1]$ as well. Therefore, the property must hold for any linear combination of the two extreme cases. □

Fact 3. *If $f(\alpha)$ is convex and continuous on* $[0,1]$ *then it holds that $B_d(h_0; y(\alpha)) \geq f(y(\alpha))$. If $f(y^{-1}(\alpha))$ is convex and continuous on* $[0,1]$ *then it holds that $B_d(h_1; y(\alpha)) \geq f(\alpha)$. If the first two conditions hold then the linear combination is convex on* $[0,1]$ *and $B_d(h_\sigma; y(\alpha)) \geq \sigma f(\alpha) + (1-\sigma) f(y(\alpha))$. All three inequalities hold for any $\alpha \in$* $[0,1]$ *and are strict for any $\alpha \in$* $(0,1)$ *unless $f(\cdot)$ is linear.*

Proof. One can rewrite a Bernstein polynomial to give it a probabilistic interpretation (see Kowalski (2006), for example). Additionally, we incorporate our estimation strategy as well as the share of naive and sophisticated agents as follows:

$$B_d(h_\sigma; y(\alpha)) = \sigma \cdot E\Big(f\left(\hat{\alpha}_s(S_d/d)\right)\Big) + (1-\sigma) \cdot E\Big(f\left(\hat{\alpha}_n(S_d/d)\right)\Big),$$

where

$$S_d = \sum_{i=1}^{d} A_i.$$

Note that $A_1, A_2, \dots, A_d$ are independent random variables that either take the value $A_i = B$ or $A_i = H$ following the Bernoulli law:

$$P(A_i = B) = y(\alpha) \quad \text{and} \quad P(A_i = H) = 1 - y(\alpha).$$

Thus, we can write the two extreme cases $B_d(h_0; y(\alpha)) \geq f(y(\alpha))$ and $B_d(h_1; y(\alpha)) \geq f(\alpha)$ as follows:

$$\begin{aligned} E\Big(f\left(\hat{\alpha}_n(S_d/d)\right)\Big) &\geq f\Big(E\left(\hat{\alpha}_n(S_d/d)\right)\Big) \quad \text{if} \quad \sigma = 0, \\ E\Big(f\left(\hat{\alpha}_s(S_d/d)\right)\Big) &\geq f\Big(E\left(\hat{\alpha}_s(S_d/d)\right)\Big) \quad \text{if} \quad \sigma = 1, \end{aligned}$$

where both equations hold by Jensen's inequality. Note that $E\left(\hat{\alpha}_n(S_d/d)\right) = y(\alpha)$ as the naive estimator is biased and that $E\left(\hat{\alpha}_s(S_d/d)\right) = \alpha$ as the sophisticated estimator is unbiased (see Appendix A.2). Note that h_σ is a linear combination of the two extreme cases. Therefore, we can write $B_d(h_\sigma; y(\alpha)) \geq \sigma f(\alpha) + (1-\sigma) f(y(\alpha))$ as:

$$E\Big(\sigma \cdot f\left(\hat{\alpha}_s(S_d/d)\right) + (1-\sigma) \cdot f\left(\hat{\alpha}_n(S_d/d)\right)\Big) \geq f\Big(\sigma \cdot E\left(\hat{\alpha}_s(S_d/d)\right) + (1-\sigma) \cdot E\left(\hat{\alpha}_n(S_d/d)\right)\Big),$$

which holds by Jensen's inequality. We can apply the same arguments as above to the right hand side and use that f is a linear operator to obtain the stated result:

$$f\Big(\sigma \cdot \alpha + (1-\sigma) \cdot y(\alpha)\Big) = \sigma f(\alpha) + (1-\sigma) f(y(\alpha)).$$

$\square$

Fact 4. *If $f(y(\alpha))$ is convex and continuous on $[0, 1]$ and there is oversampling ($\beta > 1$) then it holds* *that $B_d(h_0; y(\alpha)) \geq B_d(h_0; \alpha)$. If, additionally, $f(y^{-1}(\alpha))$ is convex and continuous on $[0, 1]$ then it holds that $B_d(h_0; y(\alpha)) \geq B_d(h_1; y(\alpha))$. If both conditions hold then the linear combination is convex and continuous on $[0, 1]$ and $B_d(h_0; y(\alpha)) \geq B_d(h_\sigma; y(\alpha)) \geq B_d(h_1; y(\alpha))$. All three inequalities hold for any $\alpha \in [0, 1]$ and $d \geq 2$. In all three cases, the inequalities are strict for any $\alpha \in (0, 1)$ unless $f(\cdot)$ is linear and the inverse is true when there is undersampling ($\beta < 1$).*

Proof. Phillips (2003) proves in Theorem 7.1.1 that one can write:

$$B_d(z; x) = \sum_{t=0}^{d} \Delta^t f(0) \binom{d}{t} x^t, \tag{15}$$

where $x \in \{y(\alpha), \alpha\}$, $z \in \{h_0, h_1\}$ and $i = 0$.[7] The parameter Δ denotes the forward differences that are positive as long as $f(x)$ is monotonically increasing. Moreover, the forward differences are independent of x as:

$$\Delta^t f(0) = \sum_{k=0}^{t} (-1)^{t-k} \binom{t}{k} \cdot f(\hat{a}_r(k/d)). \tag{16}$$

Let us illustrate this considering a Bernstein polynomial of sample size $d = 2$. First, we need to solve equation (16):

$$\begin{aligned}
&\Delta^0 f(0) = f(\hat{a}_r(0)).\\
&\Delta^1 f(0) = f(\hat{a}_r(1/2)) - f(\hat{a}_r(0)).\\
&\Delta^2 f(0) = f(\hat{a}_r(0)) - 2 \cdot f(\hat{a}_r(1/2)) + f(\hat{a}_r(1)).
\end{aligned}$$

Second, we can insert these equations back into equation (15) which yields the required result:

$$\begin{aligned}
B_2(z; x) &= f(\hat{a}_r(0)) + 2x \cdot \left(f(\hat{a}_r(1/2)) - f(\hat{a}_r(0))\right) + x^2 \cdot \left(f(\hat{a}_r(0)) - 2 \cdot f(\hat{a}_r(1/2)) + f(\hat{a}_r(1))\right)\\
&= (1 - x)^2 \cdot f(\hat{a}_r(0)) + 2x(1 - x) \cdot f(\hat{a}_r(1/2)) + x^2 \cdot f(\hat{a}_r(1)).
\end{aligned}$$

First, we prove that $B_d(h_0; y(\alpha)) > B_d(h_0; \alpha)$ holds when naive agents oversample others taking action B as:

$$\sum_{t=0}^{d} \Delta^t f(0) \binom{d}{t} y(\alpha)^t > \sum_{t=0}^{d} \Delta^t f(0) \binom{d}{t} \alpha^t.$$

[7] Note that i is the running variable in the original formulation of the Bernstein polynomial (see equation (8), for example).

The equation holds whenever $y(\alpha) > \alpha$ which is true for $\beta > 1$. The inverse is true whenever $0 < \beta < 1$. Second, we prove that $B_d(h_0; y(\alpha)) \geq B_d(h_1; y(\alpha))$ holds when naive and sophisticated agents oversample others taking action B. The prove boils down to comparing the forward differences $\Delta^t f(0)$:

$$\sum_{k=0}^{t}(-1)^{t-k}\binom{t}{k} \cdot f(\hat{a}_n(k/d)) \geq \sum_{k=0}^{t}(-1)^{t-k}\binom{t}{k} \cdot f(\hat{a}_s(k/d)).$$

One can see that the inequality holds whenever $f(\hat{a}_n(k/d)) \geq f(\hat{a}_s(k/d))$ which is true for $\beta > 1$. The inverse is true whenever $0 < \beta < 1$. Note that the inequality is strict for $d \geq 2$ as $B_d(h_0; y(\alpha)) = B_d(h_1; y(\alpha))$ for $d = 1$. Third, it directly follows that:

$$B_d(f(0); y(\alpha)) \geq (1-\sigma) \cdot B_d(h_0; y(\alpha)) + \sigma \cdot B_d(h_1; y(\alpha)) \geq B_d(h_1; y(\alpha)),$$

must hold for $\sigma \in [0, 1]$ and $\beta > 1$. Again, the inverse it true for $0 < \beta < 1$ and the inequalities are strict for $d \geq 2$. □

Fact 5. *If $f(\cdot)$ is a continuous function on $[0, 1]$ and d is a positive integer then,*

$$\lim_{d\to\infty} E\Big(\Big|B_d(h_0; y(\alpha)) - f(y(\alpha))\Big|\Big) = 0 \qquad \text{if} \quad \sigma = 0, \tag{17}$$

$$\lim_{d\to\infty} E\Big(\Big|B_d(h_1; y(\alpha)) - f(\alpha)\Big|\Big) = 0 \qquad \text{if} \quad \sigma = 1, \tag{18}$$

$$\lim_{d\to\infty} E\Big(\Big|B_d(h_\sigma; y(\alpha)) - f\big(\sigma \cdot \alpha + (1-\sigma) \cdot y(\alpha)\big)\Big|\Big) = 0 \qquad \text{if} \quad 0 < \sigma < 1. \tag{19}$$

Proof. Kowalski (2006) or Konstantopoulos et al. (2018), for example, prove equation (17), but without the inner transformation of $y(\alpha)$. To prove our case, we can apply the same mapping arguments that we described in Fact 2. To prove equation (18) and (19) note that one can write:

$$B_d(h_\sigma; y(\alpha)) - f\big(\sigma \cdot \alpha + (1-\sigma) \cdot y(\alpha)\big) =$$
$$E\Big(\sigma \cdot f\big(\hat{\alpha}_s(S_d/d)\big) + (1-\sigma) \cdot f\big(\hat{\alpha}_n(S_d/d)\big) - f\Big(\sigma \cdot E\big(\hat{\alpha}_s(S_d/d)\big) + (1-\sigma) \cdot E\big(\hat{\alpha}_n(S_d/d)\big)\Big)\Big).$$

We can use the same arguments as in Fact 3 to derive each of the three equations above. □

A.2 Properties of the sophisticated estimator

Fact 6. *We can write the maximum likelihood estimator (MLE) of a sophisticated agent as follows:*

$$\hat{\alpha}_s(m) = \frac{m}{\beta + m \cdot (1 - \beta)}.$$

Proof. Agents observe a random sample of size d drawn i.i.d. from a Bernoulli distribution with success probability $y(\alpha)$. The agent uses MLE to estimate the most likely parameter that generated the sample. That is, the agent maximizes the following likelihood function:

$$\mathcal{L}(y(\alpha)) = y(\alpha)^{d \cdot m} \cdot (1 - y(\alpha))^{d \cdot (1-m)},$$

where m denotes the sample mean. In other words, the share of agents that take action B in the sample d. We can take the logarithm, maximize $\mathcal{L}(y(\alpha))$ with respect to $y(\alpha)$, and simplify:

$$\begin{aligned}
\log \mathcal{L}(y(\alpha)) &= d \cdot m \cdot \log(y(\alpha)) + d \cdot (1 - m) \cdot \log(1 - y(\alpha)), \\
\frac{\partial \log \mathcal{L}(y(\alpha))}{\partial y(\alpha)} =0 &: \left(dm \cdot \frac{1}{y(\alpha)} - d(1 - m) \cdot \frac{1}{1 - y(\alpha)} \right) \cdot \frac{\partial y(\alpha)}{\partial \alpha} = 0, \\
m &= y(\alpha).
\end{aligned}$$

Note that we can divide by $\partial y(\alpha)/\partial \alpha$ as it is positive (see Fact 1). That is, the maximum of the log-likelihood function is where the sample mean m equals the success probability $y(\alpha)$. Without sample selection (i.e $\beta = 1$), the sample mean equals the population share α of agents that choose action B. With sample selection (i.e. $\beta \neq 1$) the sample mean equals $y(\alpha)$. A sophisticated agent is aware of the sample selection problem. We derive the MLE of the sophisticated agent by inserting the sample selection process from equation (2) and solving for α as follows:

$$\begin{aligned}
m &= y(\alpha), \\
m &= \frac{\beta \cdot \alpha}{\beta \cdot \alpha + (1 - \alpha)}, \\
\hat{\alpha}_s(m) &= \frac{m}{\beta + m \cdot (1 - \beta)}.
\end{aligned}$$

□

Fact 7. *The MLE of the sophisticated agent is unbiased and consistent.*

Proof. The proof of unbiasedness closely follows Bjerre-Nielsen and Busch (2021) and we refer the reader to this article for details. In essence, one must show that the expectation of the sophisticated estimator equals the true share to prove unbiasedness, i.e. $E(\hat{\alpha}_r(m)) = \alpha$. Note that we can write:

$$E(\hat{\alpha}_s(m)) = E\Big(\frac{m}{\beta + m \cdot (1-\beta)}\Big) = \frac{y(\alpha)}{\beta + y(\alpha) \cdot (1-\beta)},$$

as $E(m) = y(\alpha)$. Now we can insert the sample selection process from equation (2) and simplify:

$$\frac{y(\alpha)}{\beta + y(\alpha) \cdot (1-\beta)} = \frac{\frac{\beta \cdot \alpha}{\beta \cdot \alpha + (1-\alpha)}}{\beta + \frac{\beta \cdot \alpha}{\beta \cdot \alpha + (1-\alpha)} \cdot (1-\beta)} = \alpha.$$

To prove consistency of the sophisticated estimator one needs to show that the estimator is identified, α is part of a convex parameter set, and the log-likelihood function is concave. The sophisticated estimator fulfills all three criteria and we we refer to Bjerre-Nielsen and Busch (2021) for details. Furthermore, the sophisticated estimator is the best unbiased estimator, because agents cannot learn anything from the sequence of draws. In our setup, agents play a static game with complete information. That is, agents know either the true share or a share skewed by the sample selection process. □

A.3 Equilibrium Behavior and Welfare

Proof of Theorem 3

Proof. To evaluate how sample selection affects behavior, we take the derivative of the right hand side of equation (10) with respect to β:

$$\frac{d}{d\beta}\Big(\sigma \cdot f(\alpha) + (1-\sigma) \cdot f(y(\alpha))\Big) = (1-\sigma) \cdot \frac{\partial f(y(\alpha))}{y(\alpha)} \cdot \frac{\partial y(\alpha)}{\beta},$$

where

$$\frac{\partial y(\alpha)}{\beta} = \frac{\alpha - \alpha^2}{(\alpha \cdot \beta + (1-\alpha))^2}.$$

The first order condition with respect to β is positive for $\sigma \in [0, 1)$ and $\alpha \in (0, 1)$. That is, as β increases the equilibrium share of agents taking action B decreases given a positive share of naive agents in the population. Note, the left hand side of equation (10) is $1 - \alpha$.

To evaluate how the share of sophisticated agents affects behavior, we take the derivative of the right hand side of equation (10) with respect to σ:

$$\frac{d}{d\sigma}\Big(\sigma \cdot f(\alpha) + (1-\sigma) \cdot f(y(\alpha))\Big) = f(\alpha) - f(y(\alpha)).$$

One can see that when there is oversampling (i.e. $\beta > 1$) the derivative is negative as $f(\alpha) < f(y(\alpha))$. That is, an increase in the share of sophisticated agents leads to an increase in the equilibrium share of agents taking action B. When there is undersampling (i.e. $0 < \beta < 1$) the derivative is positive as $f(\alpha) > f(y(\alpha))$. That is, an increase in the share of sophisticated agents in the population decreases the equilibrium share of agents taking action B. In both cases, an increase in the share of sophisticated agents decreases the gap to the respective limit equilibrium. □

Proof that the welfare maximizing share is a global maximum.

Proof. To show that the welfare maximizing share is a global maximum, we derive the second order derivative of $W(\alpha)$ with respect to α as follows:

$$\frac{\partial^2 W(\alpha)}{\partial \alpha^2} = 0 : -1 - \Big(\frac{\partial^2 f(\alpha)}{\partial \alpha^2} \cdot \alpha + 2 \cdot \frac{\partial f(\alpha)}{\partial \alpha}\Big) < 0, \tag{20}$$

which is negative for any $\alpha \in [0,1]$.[8] Hence, the welfare function is concave. The continuity property of the cost function $f(\cdot)$ on the closed interval $[0,1]$ implies that at least one maximum and minimum exists by the extreme value theorem. There are two options. Either the global maximum is the local maximum in the interior of $[0,1]$ or it lies at its boundaries. Note that $W(0) = 0$ and $W(1) = -1/2$, which is the global minimum. Thus, $W(\alpha)$ has a global maximum that lies in the interior of $[0,1]$. □

[8]Note that the expression in brackets of equation (20) is positive as the cost function $f(\cdot)$ is continuous, monotonically increasing, and convex on its domain $[0,1]$.

A.4 Additional Figures

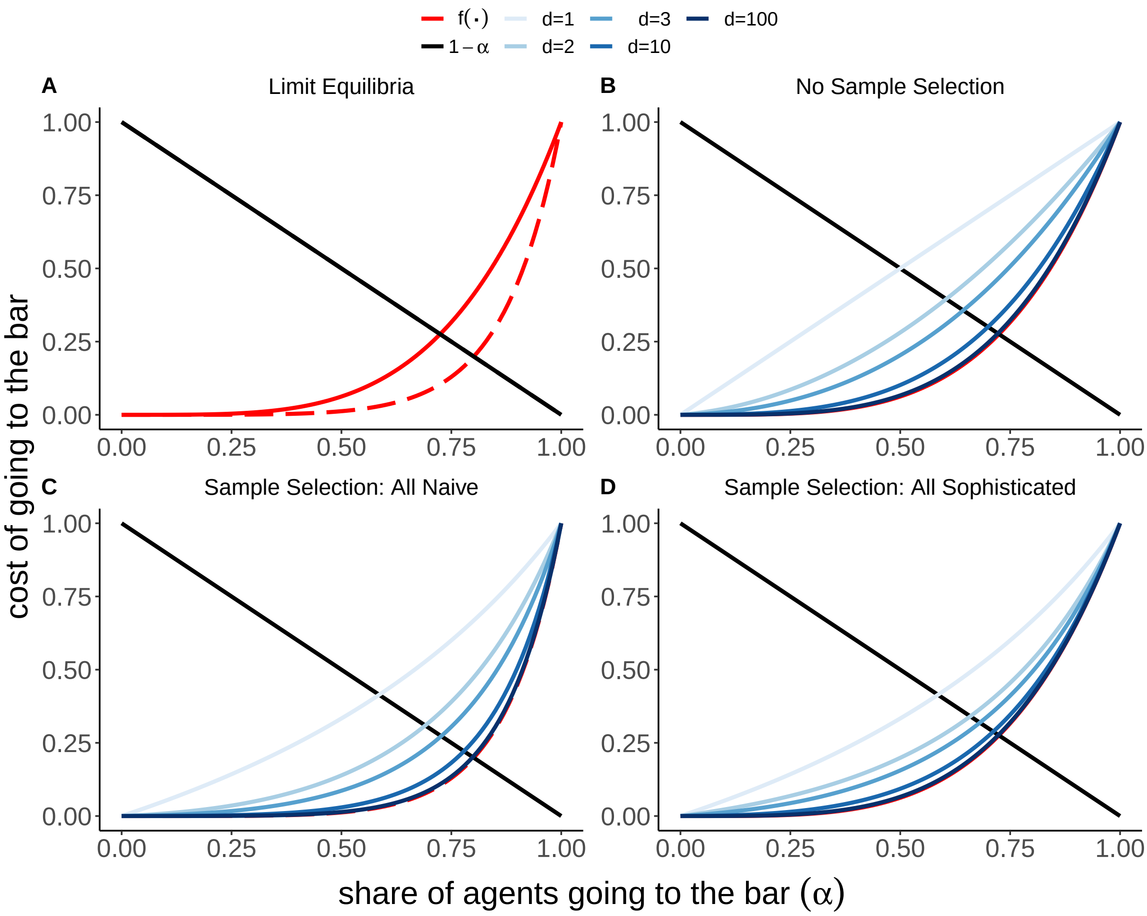

Figure 2: Equilibria with ($\beta = 0.5$) and without sample selection

The panels illustrate the unique equilibrium shares. The intersection of the black and red lines illustrate the limit equilibria with (dashed) and without (solid) sample selection. The intersection of the black and blue lines denote the SESI shares. Panel **A** illustrates the limit equilibria; Panel **B** replicates the solution of Salant and Cherry (2020); Panel **C** shows the case when all agents are naive; Panel **D** shows the case when all agents are sophisticated. In all panels we assume that $\beta = 0.5$ when there is sample selection and $\beta = 1$ corresponds to the case without sample selection.

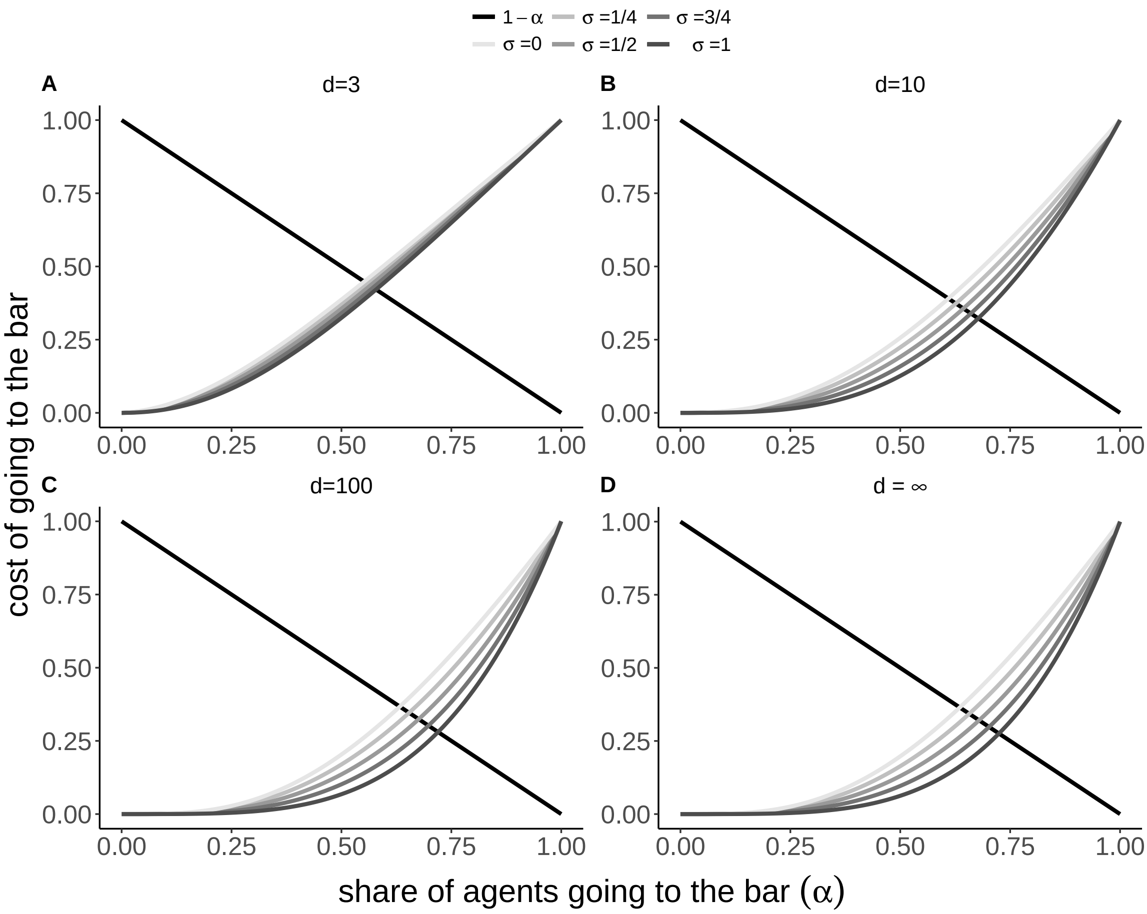

Figure 3: Convergence within a given sample size for $\beta = 2$

The panels illustrate the unique equilibrium shares within a given sample size. The intersection of the lines indicate the unique equilibrium depending on the share of sophisticated agents. Panel **A** illustrates the equilibrium shares for sample size $d = 3$; Panel **B** illustrates the equilibrium shares for sample size $d = 10$; Panel **C** illustrates the equilibrium shares for sample size $d = 100$; Panel **D** illustrates the equilibrium shares for an infinitely large sample. In all panels, we assume that $\beta = 2$.

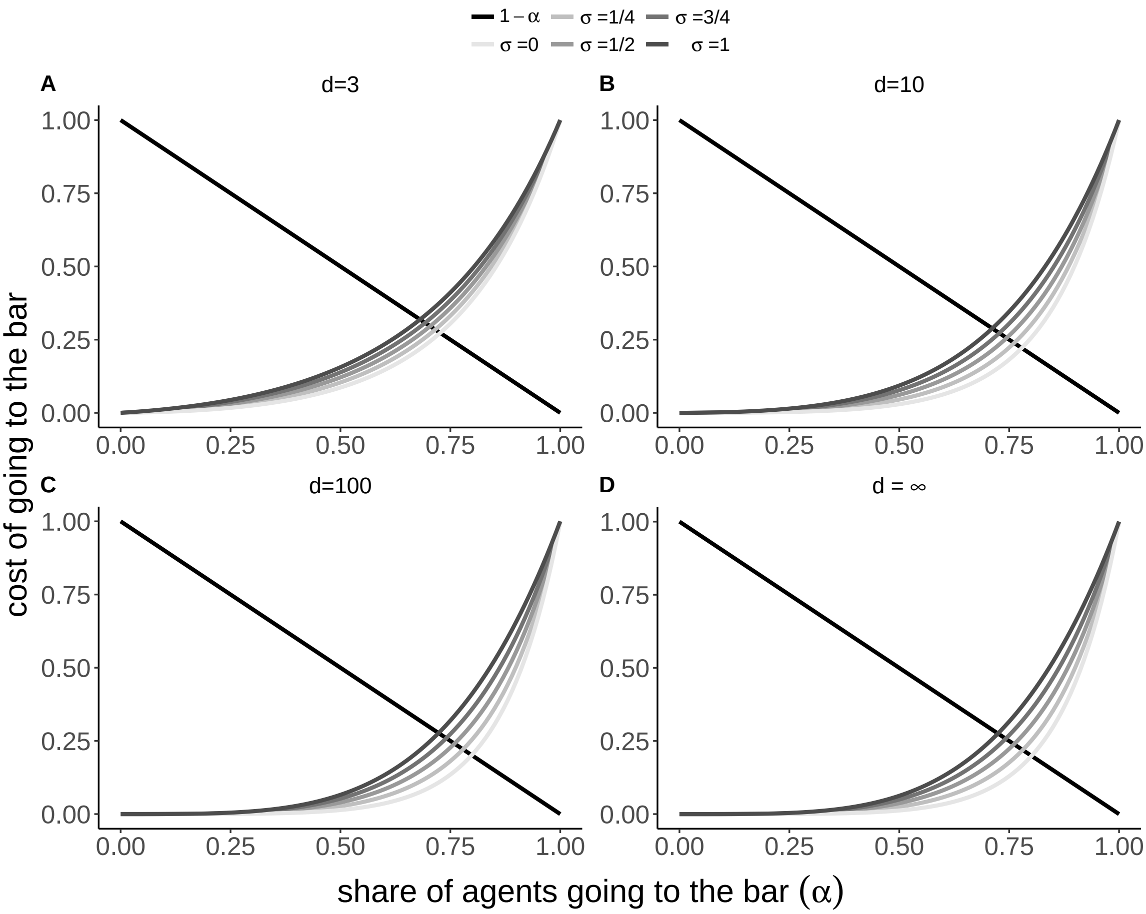

Figure 4: Convergence within a given sample size for $\beta = 0.5$

The panels illustrate the unique equilibrium shares within a given sample size. The intersection of the lines indicate the unique equilibrium depending on the share of sophisticated agents. Panel **A** illustrates the equilibrium shares for sample size $d = 3$; Panel **B** illustrates the equilibrium shares for sample size $d = 10$; Panel **C** illustrates the equilibrium shares for sample size $d = 100$; Panel **D** illustrates the equilibrium shares for an infinitely large sample. In all panels, we assume that $\beta = 0.5$.

Chapter II

Statistical inference and misperceptions in social networks

Statistical inference and misperceptions in social networks*

Andreas Bjerre-Nielsen† and Martin Benedikt Busch‡

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Abstract

We investigate how individuals form beliefs about population behavior using statistical inference based on observations of their social relations, which are biased by the friendship paradox. We analyze equilibrium behavior under strategic complementarity and show that population behavior depends on the share of individuals that either do or do not correct their estimate using information about neighbors' representativeness, which we capture by network degree. We illustrate the role of sample size for estimation precision and outline how our framework is useful for further analysis of behavior by agents embedded in networks.

JEL Classification Codes: D85, D90.

Keywords: Social Networks, Network Information, Friendship Paradox, Statistical inference, Misperceptions, Strategic Complements.

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†Department of Economics and SODAS, University of Copenhagen (abn@sodas.ku.dk).

‡Department of Economics and CEBI, University of Copenhagen (mbb@econ.ku.dk).

1 Introduction

In many real-life situations people base their decisions on what they think other people will do, e.g. to conform to, copy, or oppose others' behaviors. As people rarely observe everyone in a population, they must guess what other people do based on what they actually observe - for example their friends. However, observations based on friends are skewed by a statistical law known as the *friendship paradox* (Feld, 1991), which states that social relations are not representative of the underlying population. Therefore, inferences from friends can cause misperceptions which may lead to unintended behavior.

Consider Jackson (2019)'s example of how the friendship paradox may explain why students misperceive alcohol consumption of others' in school, which, in turn, inflates own and others' alcohol consumption behavior. Before a typical school party, students must decide upon the amount of alcohol to bring for consumption. They attempt to guess the average alcohol consumption of other students by thinking about what their friends would do. Students bring more of their own alcohol when they perceive others' to bring more. However, popular students respond more to their perceptions of others' as they have more social connections and, as a consequence, bring more alcohol on average. Therefore, students may overestimate how much alcohol to bring and consume when forming their perceptions based on friends, which in turn may lead to an inflation of alcohol consumption behavior among all students. Now assume that some students know that popular students are not representative of the entire student population. How can students use this knowledge to estimate correct perceptions of others' consumption behavior and how does it affect their own behavior? Does knowledge affect consumption behavior in the entire school if some students use it and some do not? What are the implications for perceptions and behavior if students can base their estimate only on a limited number of others'?

In this paper, we provide a micro-foundation of how people form beliefs about others' characteristics, given information about their social relations, and show how misperceptions affect own and others' behavior. We make two key assumptions. First, we assume that a share of agents in the population are sophisticated and the rest are naive. Sophisticated agents are aware of the sampling bias due to the friendship paradox, whereas naive agents think that their social relations constitute a representative sample of the population. Second, sophisticated agents use an unbiased and consistent estimation rule to infer population characteristics. In other words, we do not assume that agents know the correct or biased distribution of population characteristics. Our two assumptions come from the empirical literature showing that people use network knowledge (i.e. people are not naive) and usually do not know everyone in a network.[1] Moreover, our setup allows us to illustrate the role of sample size for estimation precision and behavior.

[1] We provide a thorough discussion of the two assumptions and relate our paper to the relevant literature in Section 5.

We consider a large population that consists of two types of people. Popular people (high degree) who have many social connections and less popular people (low degree) who have fewer connections. We model the degree distribution as an unknown state of the world and agents must estimate the population share of popular people based on their sample of network neighbors. We assume that naive and sophisticated agents know the share of popular people among network neighbors and their respective degree. Misperceptions arise due to the friendship paradox, which entails an oversampling of popular people as network neighbors. Jackson (2019) shows how the friendship paradox causes a correlation between an agent's degree and her behavior by embedding naive agents in a game where behavior has strategic complementarity.[2] That is, the friendship paradox affects population behavior through misperceptions of the degree distribution.

We show that when all agents in the population are sophisticated and the number of friends is large, then the friendship paradox has no impact on population perception and behavior, despite agents only seeing a biased sample of network neighbors. When all agents in the population are naive, our results align with Jackson (2019) and the friendship paradox causes an inflation of population behavior. When the population consists of a mix of naive and sophisticated agents, sophisticated agents choose higher equilibrium actions compared to the case where all agents are sophisticated, even though they (correctly) adjust their estimate of the degree distribution. The reason is that equilibrium behavior of naive agents does not change. Naive agents do not think that network information is useful, hence (incorrectly) believe that all other agents are naive. This is common knowledge among sophisticated agents, who increase equilibrium actions as the share of naive agents in the population increases. The contribution here is to provide a micro-foundation of how the friendship paradox may or may not cause systematic misperceptions as well as changes in behavior. Our addition of sophisticated agents is key, because it provides a new channel allowing the researcher to study how the use of network information and which kind (here degree information) determines misperceptions and behavior in mixed populations.

We show that the precision of information, in terms of the number of friends observed, plays a fundamental role in determining perceptions and behavior - independent of the friendship paradox. Intuitively, the estimate of the population degree distribution is imprecise, whether agents are sophisticated or not, when the sample of network neighbors is small. The contribution here is to show that, depending on the sample size, estimation precision defines whether agents can use network information beneficially rather than sophistication. Again, this is only possible because we demand that agents form perceptions based on observations of network neighbors using network information - as agents

[2]He also analyses the effect of the friendship paradox in settings where behaviors are strategic substitutes.

only have access to a finite set of network neighbors in real world settings.

We show that a sophisticated agent, who uses an unbiased and consistent maximum likelihood estimator, can correctly estimate the population degree distribution using neighbor degree information while a naive agent cannot. We derive the maximum likelihood estimator and prove its properties. We relate to the literature that models agents as statisticians (see e.g. Salant and Cherry, 2020; Liang, 2019). In this literature, agents estimate a parameter using data they obtain from an unbiased data generating process and behave rationally with respect to the estimate they get. In our paper, agents obtain data (network neighbors) from a biased data generating process. That is, agents do not learn the true underlying parameter (the true share of high degree agents) even if they have access to an infinitely large sample. The contribution here is to develop a framework where agents do or do not use statistical inference to correct for a naturally occurring sampling bias, present in all real world networks, using information about the data generating process (degree information).

The remainder of this paper proceeds as follows. We introduce the model in Section 2 and derive our main results in Section 3. We derive the maximum likelihood estimator and its properties in Section 4. Section 5 reviews the relevant literature and we conclude in Section 6. The Appendix contains auxiliary results as well as proofs.

2 Model

Let there be a population of agents, denoted by N, which is infinite. Agents are nodes in an undirected network g. Let $ij \in g$ denote that i and j have a link and thus are neighbors in the network. Assume that the set of "local nodes" for an agent be its neighbors and denote this by N_i. We assume that each agent i either has a high degree d_H or a low degree d_L, with $d_H > d_L$ and $d_L > 0$.[3] In other words, it means that high degree agents have more connections than low degree agents. To quantify this, we define $\rho = d_H/d_L - 1$ as the excess ratio. An agent knows the subset of high degree agents among neighbors $\tilde{\delta}_i = \delta \cap N_i$, where we denote the size of the neighborhood by $d_i = N_i$. Let δ denote the share of high degree agents in the population. That is, the share of high degree agents (δ) and the share of low degree agents (1 - δ) constitute the degree distribution. The structure of the network is that edges are formed randomly, without any sorting by degree. To ensure the constraints on the degree distribution we assume that the network was generated by the configuration model (see e.g. Barabási, 2016).[4]

[3]Importantly, we can make this assumption without any loss of generality because one low and one high degree are enough to generate any desired variation in average degree.

[4]The configuration model produces a network as follows. We assign a degree d_i to each agent i by creating d_i open ended links. Then we randomly select two open ended links and connect them until no open ended links are left.

We assume that the data generating process for the share of high degree neighbors $\tilde{\delta}_i$ is affected by the degree of agents in addition to their population share. Hence, unlike a standard setup of estimating δ in a binomial distributed sample, the draw of high and low degree neighbors is not equally likely. We can write the likelihood to draw a high degree neighbor as:

$$E[d_j = d_H | \delta, d_L, d_H] = \frac{d_H \delta}{d_H \delta + d_L(1-\delta)} = \frac{\delta(1+\rho)}{\delta(1+\rho) + (1-\delta)}, \quad \forall i, j \in N : j \in N_i. \tag{1}$$

The implications are as follows. First, the larger the share of high degree agents δ in the population, the more likely it is to draw a high degree agent as a neighbor. Second, the higher the actual degree d_H of high degree agents, the higher the likelihood to draw one as a neighbor. We assume that network neighbors are drawn from an infinite set of agents. The assumption that the population is infinite implies that there is independence between draws. In other words, the likelihood to draw a high degree neighbor is constant across draws.[5]

We impose restrictions on agents' knowledge and perceptions. Each agent observes the share of high degree agents $\tilde{\delta}_i$ among its neighbors and knows the set of degrees $d_i \in \{d_L, d_H\}$. Although each agent possesses the same kind of information, they process it either in a naive or sophisticated manner. We say that agents use an updating rule r_i for forming an expectation about the share of high degree agents based on the set of observed neighbors N_i, where the applied rule is either naive or sophisticated $r \in \{n, s\}$. The crucial difference between types is that the naive agent believes that what she sees ($\tilde{\delta}_i$) is in fact the truth, i.e. she does not incorporate degree information about network neighbors into her estimate. That is, a naive agent thinks that her network neighbors are a random draw from the population. On the other hand, a sophisticated agent uses an unbiased and consistent maximum likelihood estimator, using degree information, to estimate the correct share of high degree agents in the population. We investigate these rules in Section 4, where we formally derive the maximum likelihood estimator and establish its properties.

We denote the share of sophisticated agents in the population by σ. Conversely, $1-\sigma$ denotes the share of naive agents. We assume there is common knowledge, both, of rationality and about the fact that agents observe the share of high degree neighbors. However, we do not assume there is common knowledge about how other agents process signals, i.e. σ. Instead, we assume naive agents believe (incorrectly), both, that all other agents are also naive, i.e. $\sigma = 0$, and that these beliefs are common knowledge among all agents. Conversely, we assume sophisticated agents believe (correctly) that the share equals σ. Moreover, we assume sophisticated agents know of the common knowledge among

[5] Here, the use of the configuration model is crucial, because it allows us to compute network statistics even tough we assume an infinite number of agents.

sophisticated agents about σ as well as about the common knowledge among naive agents.

To determine how an agent's use of information from its local network affect its own and others' behavior, we modify Jackson (2019)'s model framework where the friendship paradox causes misperceptions that affect agents' behavior. The setup consists of agents who choose an action, where actions exhibit strategic complementarity in a simultaneous move game. We define the expected utility function as follows:[6]

$$EU_i(x_i, \theta_i, d_i, r_i, \tilde{\delta}_i) = \theta_i x_i + a x_i d_i E[x_j | r_i, d_i, \tilde{\delta}_i] - \frac{c x_i^2}{2}. \tag{2}$$

The first term denotes an agent's own action, $x_i \in \mathbb{R}_+$ multiplied by her preference $\theta_i \in \Theta$ (where $\theta_i \in \Theta$ is a compact subset of $\mathbb{R}_+$) for the action.[7] We limit our analysis to the case of linear quadratic utility to obtain a closed form solution. We assume that agents' preferences for actions are uncorrelated with degree d_i. That is, agents (correctly) believe that preferences of network neighbors are randomly drawn from the known distribution of preferences. This assumption allows us to focus on misperceptions caused by the friendship paradox where the degree of an agent solely determines her type. We assume that an agents updating rule r is independent of degree. For example, we do not assume that more popular people are more likely to be sophisticated (i.e. think that network information is useful). The second term governs the complementarity in actions, i.e. the extent that own incentives for the action depend on other agents' actions. In other words, agents care about how their action matches with average actions of others in the population. The strength of complementarity depends on agents' own degree d_i and its level $a > 0$. The third term denotes quadratic costs, where c is a positive scalar.

We deviate from Jackson (2019) by replacing expectations in the second term by not assuming that agents have ex-ante knowledge about the degree distribution. That is, we assume that agents only have an uninformative prior about the population degree distribution, which means that the true share of high degree agents (δ) is uniformly distributed on $(0, 1)$. In other words, agents think that every possible share of high degree agents in the population is equally likely, ex ante. In particular, we replace prior beliefs over actions of others with beliefs conditional on observing a set of neighbors N_i and an information updating rule r. The updating rule determines how the agent uses information contained in N_i to estimate the underlying population degree distribution. We express the agents' expectations about other agents' behavior as a function of sufficient statistics rather than the actual sample. This is possible because the individual degree (d_i), which is the sample size, and the observed share of high degree agents ($\tilde{\delta}_i$), together, encompass all the necessary information from the sample

[6] We note that we have omitted a final term from equation (2), compared to Jackson (2019), which governs a global externality which is independent of x_i. We omit this term because we only focus on behavior.

[7] Note that i denotes a generic member of the network.

of network neighbors N_i to obtain the estimated share of high degree agents. Note that $d_i = |N_i|$ determines the precision of the estimate. Intuitively, a larger sample of network neighbors allows agents to improve the precision of the estimate.

3 How network perceptions affect behavior

We proceed with the derivation of our main results, where we compute equilibrium actions for naive and sophisticated agents, depending on their share in the population, under infinite and finite estimation precision. We derive the unique Bayesian Nash equilibrium, where all agents simultaneously choose an action x_i given their beliefs. The equilibrium is a function of an agent's type, because we allow agents to differ with respect to their preference (θ_i) for action x_i, their degree (d_i), updating rule (r_i), and observed share of high degree agents ($\tilde{\delta}_i$). However, for each type of agent, beliefs over the rest of the population are the same. We compute the first order condition for utility wrt. their own action x_i and obtain the following best response function:

$$x_i(\theta_i, d_i, r_i, \tilde{\delta}_i) = \frac{\theta_i}{c} + \frac{ad_i E[x_j | r_i, d_i, \tilde{\delta}_i]}{c}. \tag{3}$$

3.1 Equilibrium expectations and actions under infinite precision

We pin down an expression for individual expectations about other agents' actions. We do this by computing the conditional expectation with respect to the updating rule and information from observing network neighbors. In our computation, we use that a naive agent is defined as someone who essentially ignores network information, for example, because a naive agent thinks that this information is not useful. Therefore, a naive agent does not condition her action on how other agents in the population form their estimates. In other words, this corresponds to the case where a naive agent thinks that all other agents in the population are naive too ($\sigma = 0$). Here, we limit our analysis to the case of asymptotic infinite precision (we relax this assumption in Section 3.3). The assumption allows us to solely focus on misperceptions caused by the friendship paradox through the updating rule r rather than precision. Intuitively, we assume that all agents have a sufficiently large sample of network neighbors to estimate the share of high degree agents in the population - i.e. all agents who use the same rule converge to the same estimate. Therefore, the estimated share of high degree agents only depends on the updating rule r, and not precision, which implies that high and low degree agents have the same conditional expectations. Specifically, we assume that $\lim_{d_L \to \infty}$ where the degree ratio d_H/d_L is fixed. To ensure that our model is independent of the scaling of the minimum sample size,

d_L, we normalize complementarities by defining $a = \alpha/d_L$. We proceed with our first result on the structure of equilibrium expectations.

Lemma 1. *Consider a network generated by the configuration model with two distinct types of degrees where preferences* (θ_i) *and degree* (d_i) *are uncorrelated. For asymptotic infinite low degree and a constant excess ratio* (ρ) *it holds that expectations of other agents' actions are characterized by equation* (4) *for naive agents and equation* (5) *for sophisticated agents.*

$$\lim_{d_L \to \infty} E[x|n, \tilde{\delta}(\delta, \rho)] = \frac{E[\theta]}{c - \alpha \cdot \left[1 + \rho \cdot \tilde{\delta}(\delta, \rho)\right]}, \tag{4}$$

$$\lim_{d_L \to \infty} E[x|s, \tilde{\delta}(\delta, \rho)] = \frac{E[\theta] + \alpha \cdot [1 + \rho \cdot \tilde{\delta}(\delta, \rho)] \cdot (1 - \sigma) \cdot \frac{E[\theta]}{c - \alpha \cdot \left[1 + \rho \cdot \tilde{\delta}(\delta, \rho)\right]}}{c - \alpha \cdot [1 + \rho \cdot \delta] \cdot \sigma}. \tag{5}$$

Proof. We provide the proof of Lemma 1 in Appendix A.2. □

In the case of asymptotic infinite low degree naive agents have the same expectations, irrespective of the share of sophisticated agents σ. However, sophisticated agents adapt their expectations given the share of sophisticated agents. We note that in the case where the measure of naive agents is zero, then the expectations for sophisticated agents is simplified and is equivalent to equation (4) where $\tilde{\delta}(\delta, \rho)$ is substituted for the estimate of a sophisticated agent δ. Our proof of the mixed case relies on the fact that we only need to compute equilibrium expectations from the perspective of a sophisticated agent, because naive equilibrium expectations are independent of the share of sophisticated agents. A sophisticated agent knows that naive agents do not care about the type of other agents in the population. Thus, we can treat $\lim_{d_L \to \infty} E[x|n, \tilde{\delta}(\delta, \rho)]$ as a constant.

We plug the expressions for equilibrium expectations separately into the best response function (3) to derive an expression for equilibrium actions. This allows us to analyze how misperceptions of the degree distribution affect population behavior. We compute the following equations for equilibrium actions:

$$\lim_{d_L \to \infty} x(\theta_i, d_i, n, \tilde{\delta}(\delta, \rho)) = \frac{\theta_i}{c} + \frac{a d_i E[\theta]}{c \cdot (c - \alpha \cdot [1 + \rho \cdot \tilde{\delta}(\delta, \rho)])}, \tag{6}$$

$$\lim_{d_L \to \infty} x(\theta_i, d_i, s, \tilde{\delta}(\delta, \rho)) = \frac{\theta_i}{c} + \frac{a d_i \left(E[\theta] + \alpha \cdot \left[1 + \rho \cdot \tilde{\delta}(\delta, \rho)\right] \cdot (1 - \sigma) \cdot \frac{E[\theta]}{c - \alpha \cdot \left[1 + \rho \cdot \tilde{\delta}(\delta, \rho)\right]}\right)}{c \cdot (c - \alpha \cdot [1 + \rho \cdot \delta] \cdot \sigma)}. \tag{7}$$

We can see that an increase in complementarities a, as well as own degree d_i, increases equilibrium actions. It's exactly these complementarities that provide the channel through which the friendship paradox biases perceptions of population shares. Intuitively, high degree agents benefit more from complementarities, hence choose a higher action in equilibrium which then feeds back to the population and increases overall population behavior. An increase in the cost of taking the action decreases equilibrium actions. Note that we assume that $c > \alpha \cdot [1 + \rho \cdot \tilde{\delta}(\delta, \rho)]$ to ensure that iterative best responses of agents converge and not diverge in equilibrium. The main focus in this section is to analyze how perceptions governed by σ about the share of high degree agents determine equilibrium actions. We compare equilibrium actions to a benchmark case where all agents know the population share of high degree agents δ. Hence, a case where misperceptions are absent. We state our main result in the following theorem that pins down the impact of the friendship paradox on behavior.

Theorem 2. *Consider a network generated by the configuration model with two distinct types of degrees where preferences (θ_i) and degree (d_i) are uncorrelated. For asymptotic infinite low degree and a constant excess ratio it holds that in the unique equilibrium:*

1. *If all agents use naive updating ($\sigma = 0$) then equilibrium actions are higher for all agents compared to the benchmark case without misperceptions.*

2. *If all agents use sophisticated updating ($\sigma = 1$) then equilibrium actions are equal to the benchmark case without misperceptions.*

3. *If a share (σ) of agents in the population use sophisticated updating and a share $(1 - \sigma)$ use naive updating, then*

   (a) *equilibrium actions decrease in σ for sophisticated agents.*

   (b) *equilibrium actions correspond to the case where all agents use naive updating ($\sigma = 0$) for naive agents.*

Proof. We provide the proof of Theorem 2 in Appendix A.2. □

The first and second result of Theorem 2 cover the two extreme cases where all agents in the population are either naive or sophisticated. That is, agents can overcome misperceptions when using degree information of network neighbors. In other words, equilibrium behavior is not affected by the friendship paradox when sophisticated agents use neighbor degree information to estimate the share of high degree agents in the population. However, when agents act naively, the friendship paradox increases equilibrium behavior of all agents, which is consistent with the result of Jackson (2019) for the naive case. In our setting, agents have "as if" perfect knowledge of the degree distribution due to an infinite set of network neighbors, whereas in Jackson (2019) they know the degree distribution, ex ante. Intuitively, a naive agent, who essentially ignores network information, experiences higher activity than there actually is in the population. As a consequence, equilibrium behavior increases given the complementarities. A sophisticated agent, who knows that the activity of her network neighbors is not representative of the population distribution, uses neighbor information to estimate the correct level of population activity.

The third result of Theorem 2 covers the case where the population consists of a mix of naive and sophisticated agents ($0 < \sigma < 1$). Sophisticated agents want to increase (decrease) equilibrium actions as the share of naive agents in the population increases (decreases). Even though sophisticated agents correctly estimate the population share of high degree agents, they choose higher equilibrium actions as compared to the case where all agents in the population use sophisticated updating. Intuitively, sophisticated agents know that naive agents do not form beliefs about how others' update. Hence, naive agents choose an equilibrium action that is inflated by the friendship paradox, where the size of the inflation depends on their share in the population. With strategic complements, sophisticated agents benefit from choosing higher equilibrium actions due to the mistake naive agents make.

3.2 An illustrative example

We now show an example that can help understand the mechanics of the model and illustrate Theorem 2. We assume that the share of high degree agents δ is 40%, and thus the share of low degree agents is 60%. High degree agents have 6 links while low degree agents have 4, where we capture the fact that high degree agents are 1.5 times more popular than low degree agents by the excess ratio ($\rho = \frac{6}{4} - 1 = 0.5$).[8] As high degree agents have more network links, the observed share of network neighbors that are of high degree ($\tilde{\delta}_i = 0.5$) is not representative of the population distribution. Figure 1 displays a finite network representation of these properties.

In this example, we assume $\theta_i = \frac{1}{2}$ for all i which represents an agent's individual preference for action x, and thus $E[\theta_i] = 1/2$. We also assume that $\alpha = 4$ meaning that the level of complementarity $a = \alpha/d_L = 1$, and $c = 6$ denotes the cost of taking action x. Agents don't know δ, hence they need to estimate it using either the naive or sophisticated rule. In the following, we compute equilibrium expectations for each type of agent, depending on their share in the population, and show how this translates into equilibrium actions and expected utility.

Figure 2 illustrates equilibrium expectations of other agents actions x (y-axes) depending on the share of sophisticated agents in the population (x-axes). We calculate equilibrium expectations of naive agents (blue line) about other agents' actions using equation (4) and the benchmark case (red line) assuming that agents know the population degree distribution.[9] The calculations show that equilibrium

[8] See Section 4 for a detailed derivation of ρ.
[9] We provide detailed calculations in the appendix.

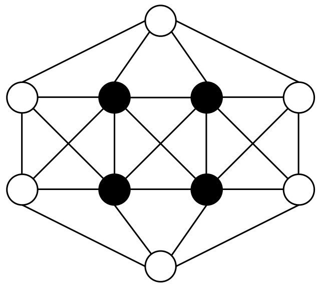

Figure 1: Finite network representation

Figure 1 shows and example network consisting of 10 agents (nodes), where black nodes represent high degree agents ($d_H = 6$) and white nodes represent low degree agents ($d_L = 4$). Each agent observes that 50% ($= \tilde{\delta}$) of network neighbors are of high degree. As in an infinite network, estimation precision does not play a role as, both, high an low degree agents, if sophisticated, correctly estimate δ. Furthermore, the network features a degree-assortativity coefficient of 0 as agents do not sort by degree in an infinite network.

expectations of others' actions increase if all agents are naive (= 0.5) compared to the benchmark case (= 0.417). One can see that as the share of sophisticated agents in the population increases, equilibrium expectations decrease for sophisticated agents (black line). Recall, we assume that a sophisticated agent knows σ and knows that a naive agent does not use network information. Therefore, unless all agents in the population are sophisticated, a sophisticated agent expects higher actions in equilibrium due to the share of naive agents who do not use network information. If all agents are sophisticated, each agent uses an unbiased and consistent estimator to correctly estimate δ. Thus, the calculation of equilibrium expectations aligns with the calculation of the benchmark case.

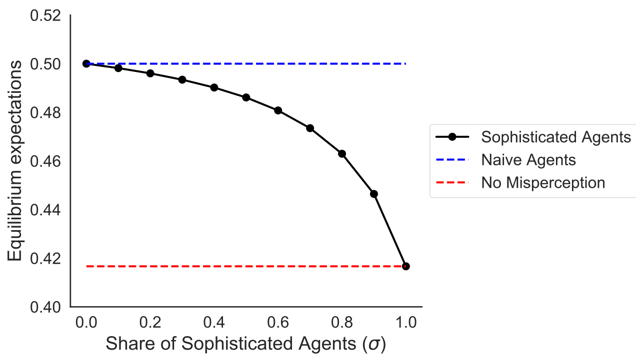

Figure 2: Equilibrium Expectations

Figure 2 shows equilibrium expectations (y-axes) of naive agents (dotted blue line) and sophisticated agents (black line) depending on the share of sophisticated agents in the population (σ). The dotted red line depicts the benchmark case where misperceptions are absent.

Figure 3 shows how equilibrium expectations translate into equilibrium actions and expected utility for each type of agent (high or low degree), using equation (6) and (7) to compute equilibrium actions and equation (2) to compute expected utility. In this example, types only vary by degree $d_i \in \{d_L, d_H\}$ and updating rule $r \in \{n, s\}$ since preferences θ_i for action x are the same for all i. For the benchmark case equilibrium actions equal to 0.361 for the low type and 0.5 for the high type.[10] Utility equates to 0.391 for the low type and 0.75 for the high type. One can see that high degree agents enjoy more interaction, thus have a higher utility than low degree agents.

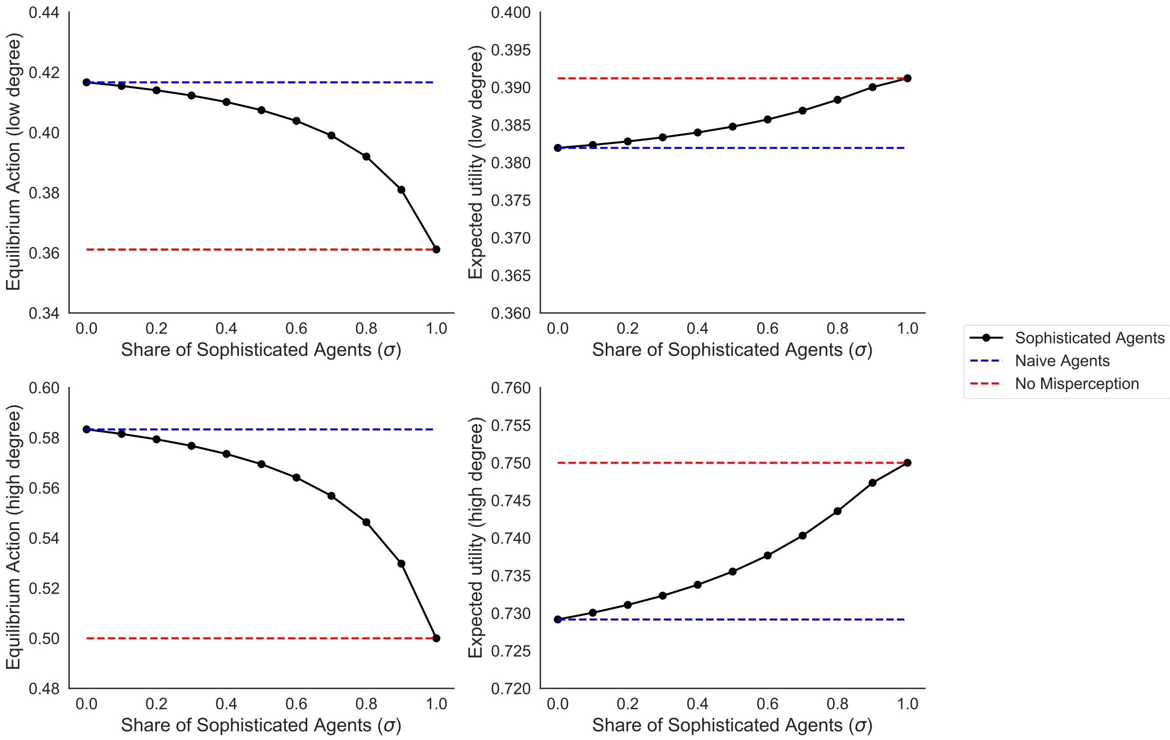

Figure 3: Equilibrium Actions and Utility

The left panels of Figure 3 show equilibrium actions (y-axes) for each type of agent (high and low degree). The right panels show utility (y-axes) for each type of agent (high and low degree). The dotted blue lines depict equilibrium actions and utility for naive agents and the black line depicts equilibrium actions and utility for sophisticated agents depending on the share of sophisticated agents in the population (σ). The dotted red lines depict the benchmark case where misperceptions are absent.

When all agents are naive, each agent estimates that 50% of agents in the population are of high degree, i.e. $\hat{\delta}_i = \tilde{\delta}_i$. We compute equilibrium actions for each type of naive agent where equilibrium actions equal to 0.417 for the low type and 0.583 for the high type. In comparison to the benchmark case, equilibrium actions increase for all types of naive agents. Next, we compute expected utility for both types of naive agents. Here, we compare the utility change directly to the benchmark case. That is, we use the actions naive agents choose under misperceptions about expected actions of others $E[x|n, \tilde{\delta}]$, but use correct expectations $E[x|\delta]$ to calculate the utility change. The result is that expected utility decreases for naive agents compared to the benchmark case for each type of agent.

Equilibrium actions decrease for both types of sophisticated agents as the share of sophisticated

[10] Note that average actions are equal to equilibrium expectations in the benchmark case (i.e. $\frac{4}{10} \cdot 0.5 + \frac{6}{10} \cdot 0.361 = 0.417 = E[x|\delta]$). Under the assumption that people know the population degree distribution there is no need to form expectations and equilibrium actions can be calculated directly.

agents in the population increases. As equilibrium actions start to approach the optimal level (dotted red line), for both types of agents, expected utility increases. If all agents in the population are sophisticated the calculation of equilibrium actions and expected utility align with the benchmark case. In other words, if all agents in the population are sophisticated, the friendship paradox does not affect behavior even though agents do not know the population degree distribution, ex ante. This is the main result of this paper.

3.3 Equilibrium expectations and actions under finite precision

In this section, we relax the assumption of infinite asymptotic precision. This allows us to analyze how estimation precision affects equilibrium expectations and behavior. In particular, we do not assume that agents, naive or sophisticated, have a sufficiently large sample to correctly estimate the share of high degree agents in the population. This entails that high and low degree agents arrive at different estimates, even though they use the same updating rule. Note, network neighbors are still drawn i.i.d (i.e. from an infinite pool of potential network neighbors), but an agent only has a finite sample of network neighbors to estimate the share of high degree agents in the population. In other words, the maximum likelihood estimator of a sophisticated agent is still unbiased but not consistent anymore. In the case where agents have finite degree, equation (8) of Lemma 3 characterizes equilibrium expectations of other agents' actions.

Lemma 3. *Consider a network generated by the configuration model with two distinct types of degrees where preferences (θ_i) and degree (d_i) are uncorrelated. For finite degree and a constant excess ratio (ρ) it holds that expectations of other agents' actions are characterized by equation* (8) *for naive and sophisticated agents.*

$$\xi = -\left(\frac{\alpha}{c}\Pi - I_L\right)^{-1} \frac{E[\theta_i]}{c} \cdot J_L. \tag{8}$$

Proof. We provide the proof of Lemma 3 in Appendix A.2. □

In the equilibrium associated with the finite degree setting each type of agent, as defined by $(r_i, d_i, \tilde{\delta}_i)$, has a different expectation about the equilibrium actions of others. That is, ξ is the vector of beliefs for each type about other agents actions. The solution is mainly affected by Π which denotes the beliefs for each type about the likelihood of observing other agents' types as a function of σ, ρ, and δ. I_L is the identity matrix and J_L a vector of one's, both of size L, where L denotes the number of unique types. The difference between the finite and infinite case is that in the finite case there is also variation in beliefs which stems from the observed share of high degree agents $\tilde{\delta}_i$, which is constant in the infinite case.

Figure 4 visualizes Lemma 3. We visualize equilibrium expectations (y-axis) depending on the true share of high degree agents in the population (x-axis) for a fixed set of parameters (i.e. $\rho = 2$, $\alpha = 1.2$, $E[\theta] = 1$, $c = 3.7$, and $\sigma \in [0, 0.5, 1]$). The figure shows that as the sample of network neighbors increases (i.e. estimation precision increases) equilibrium expectations of the finite case (blue lines) converge to the infinite case (red line). To isolate how estimation precision affects equilibrium expectations, we require a constant excess ratio ρ. As low degree converges towards infinity high degree increases proportionally. In other words, we analyze the effect of estimation precision by holding misperceptions due to the friendship paradox constant. For example, consider the case where low degree is equal to $d_L = 2$ or $d_L = 4$. A constant excess ratio of $\rho = 2$ implies that high degree is equal to $d_H = 6$ and $d_H = 12$, respectively.

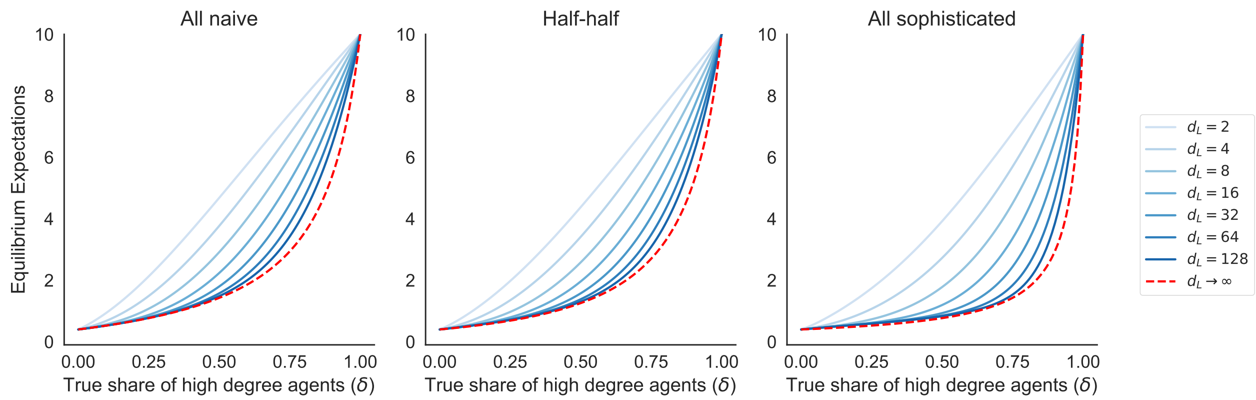

Figure 4: Equilibria with finite degree: convergence and comparison

Figure 4 illustrates the convergence from finite (blue lines) to infinite degree (red line) for a fixed excess ratio of $\rho = 2$. Equilibrium expectations are depicted on the y-axis and the true share of high degree agents δ on the x-axis. Moreover, we fix expectations of others preferences for action x to $E[\theta] = 1$, we normalize complementarities by $\alpha = 1.2$, and choose the cost to take action x to be $c = 3.7$. We compute equilibrium expectations for three different shares of sophisticated agents in the population $\sigma \in [0, 0.5, 1]$. The left panel denotes the case where $\sigma = 0$, the middle panel denotes the case where $\sigma = 0.5$, and the right panel denotes the case where $\sigma = 1$.

When inspecting Figure 4 one sees that as the degree/sample of agents increases then the expectations about others' action in equilibrium decreases. This pattern holds irrespective of estimation rule and for all levels of degree. In fact, it holds not only for the illustrated set of parameters but for any of the approximately five million different feasible parameter combination that we checked computationally.[11] The numerical examination of how the precision of signals affects agents equilibrium actions

[11]We constructed a grid of parameter combinations spanned by $\rho \in \{1, 2, \ldots, 20\}$, $\alpha \in \{0.1, 0.2, \ldots, 12\}$, and $c \in \{1, 10, 20, \ldots, 250\}$ such that the constraint $c > \alpha \cdot [1 + \rho]$ holds. In total, there were 44,458 combinations of the parameters. We verified the condition held for any value of $\delta \in \{0.00, 0.01, \cdots, 1.00\}$, which implies 4,490,258 ($= 44458 \cdot 101$) combinations in total.

led us to articulate the conjecture below.

Conjecture 4. *For any combination of low and high degree, if both of these values are doubled then equilibrium perceptions and actions are lower when keeping other type characteristics constant.*

In some situations, the effect of sophistication may actually play a minor role compared to the effect of precision. This situation is evident in Figure 4: If one compares the case of all sophisticated agents with low precision (e.g. $d_L = 2$) with the case of all naive agents with infinite precision, it is clear that naive agents have lower equilibrium beliefs and, as a consequence, lower actions. That is, even if agents perfectly use network information, estimation precision defines whether they can use network information beneficially or not. As estimation precision increases, equilibrium expectations, hence equilibrium actions, decrease.

As one can see in Figure 4, equilibrium expectations converge to the case of infinite precision as the sample of network neighbors increases. This pattern holds for all feasible parameter combinations and, therefore, underlines the stability of the equilibrium.

4 Forming perceptions from network neighbors

In this section, we show how agents endogenously form perceptions about the population degree distribution by observing a sample of network neighbors - biased by the friendship paradox. The distribution consists of the share of high degree agents (δ) and the share of low degree agents ($1 - \delta$). Based on the observed share of high degree agents among network neighbors ($\tilde{\delta}_i$) and degree information ($d_i \in \{d_L, d_H\}$), agents must provide an estimate ($\hat{\delta}_i$) of the population share of high degree agents. We derive the maximum likelihood estimator for naive and sophisticated agents for the population share of high degree agents δ and show under which assumptions the estimator is unbiased and/or consistent. A sophisticated agent who knows that the degree of agents, in addition to their share, affects the data generating process estimates the population share of high degree agents as follows.

Proposition 5. *The maximum likelihood estimator under neighbor degree information is given by:*

$$\hat{\delta}_i = \frac{\frac{1}{d_H}\tilde{\delta}_i}{\frac{1}{d_H}\tilde{\delta}_i + \frac{1}{d_L}(1-\tilde{\delta}_i)} = \frac{1}{1+(1+\rho)v_i}. \tag{9}$$

Proof. We provide the proof of Proposition 5 in Appendix A.1. □

Note that $v_i = \frac{1-\tilde{\delta}_i}{\tilde{\delta}_i}$ denotes the ratio of observed low and high degree. The maximum likelihood estimate $\hat{\delta}_i$ shows that a sophisticated agent discounts the observed share of high (low) degree agents with their respective degrees. That is, a sophisticated agent realises that high degree agents are over-represented in her neighborhood and adjusts for this. The estimate of a naive agent, who thinks that her neighbors are a random draw from the population distribution, is simply that of a binomial distribution, i.e. $\hat{\delta}_i = \tilde{\delta}_i$. Thus, a naive agent reports the observed share of high degree agents among network neighbors as her estimate of the degree distribution. Note that when the degree of high degree agents equals the degree of low degree agents, i.e. under the assumption that $d_H = d_L$, the estimator for naive and sophisticated agents coincide.

To illustrate the mechanics of the estimator, consider again the illustrative example (section 3.2), where 60% ($1 - \delta = 0.6$) of agents are of low degree ($d_H = 4$) and 40% ($\delta = 0.4$) of agents are high degree ($d_H = 6$). In this example, each agent observes that 50% ($\tilde{\delta}_i = 0.5$) of network neighbors are of high and low degree. Using the estimator (see equation (9)), a sophisticated agent correctly estimates that the true share of high degree agents equals 40%.

We move on to investigating properties of the maximum likelihood estimator. First, we show that the estimator is unbiased and the best unbiased estimator. Second, we show that the estimator is consistent. That is, we show that the estimator converges in probability to the true share δ when the sample of neighbors tends towards infinity.

Theorem 6. *The maximum likelihood estimator under neighbor degree information is the best unbiased estimator.*

Corollary 7. *The maximum likelihood estimator under neighbor degree information is consistent.*

Proof. We provide the proof of Theorem 6 and Corollary 7 in Appendix A.1. □

Figure 5 illustrates these two properties. We visualize how the difference of the naive and sophisticated estimator depends on the share of high degree agents in the population and the excess ratio. The maximum of each curve depicts the point where the combination of δ and the ρ creates maximal misperception. In other words, this is the point where the use of neighbor degree information becomes most valuable to a sophisticated agent. For example, let's consider an excess ratio of 1 which means that high degree agents are twice as popular as low degree agents. Here, the difference between the sophisticated and naive estimate reaches its maximum at $\approx$ 17% points with a true share of high degree agents of $\approx$ 42%. Intuitively, naive agents perceive the share of high degree agents as being around 17% higher than the truth whereas sophisticated agents do not. As the excess ratio increases, misperceptions increase monotonically for a fixed share of high degree agents. Interestingly, as the degree ratio increases, the maximum shifts to the left which means that a small share of high degree agents is sufficient to cause substantive misperceptions. For example, when high degree agents are 100 times

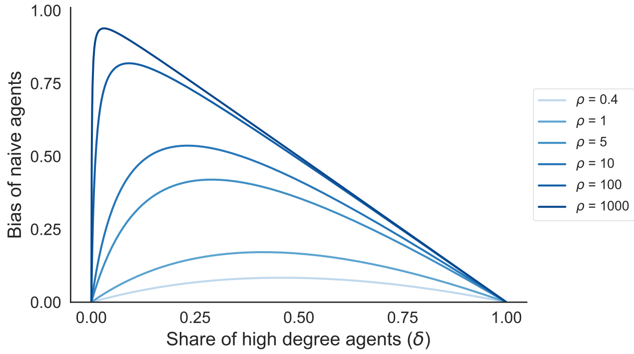

Figure 5: Naive vs. sophisticated estimate

Figure 5 illustrates the difference between the naive and sophisticated estimator (y-axis) depending on the true share of high degree agents (δ) in the population (x-axis). The excess ratio denotes the difference in popularity between high and low degree agents.

more popular then low degree agents only $\approx$ 10% of high degree agents create a difference between the naive and sophisticated estimator of $\approx$ 82% points. We stress that a naive agent misperceives the true share of high degree agents even though the sample of network neighbors is infinite. In other words, the maximum likelihood estimator of a naive agent is not consistent.

5 Related Literature

Recent research demonstrates that network structure can generate misperceptions in people's belief about what is "normal" behavior in the population (Lerman et al., 2016; Jackson, 2019; Lee et al., 2019; Stewart et al., 2019). However, at the core of these results lie two crucial assumptions. First, people do not know the entire structure of the network, consistent with recent evidence from the field (Breza et al., 2018). Once people do not know the entire structure of the network, they must form beliefs based on the people they know - e.g. their network neighbors. However, existing work assumes that people are naive which is the second crucial underlying assumption. In essence, people falsely believe that their neighbors constitute a representative sample of the population. The kind of naivity depends on the setting - in Lerman et al. (2016) and Jackson (2019) agents misperceive the degree distribution, while in Lee et al. (2019) and Stewart et al. (2019) they ignore the underlying sorting in the network. One direct implication of the assumption of naivity is that people do not fully use available network information, for example, about network neighbors to update beliefs about population behavior. In other words, naivity implies that people think that their sample of neighbors is truly random, and sufficiently large, hence there is no need to use additional information.

An emerging literature investigates what people know about the structure of their network (Breza et al., 2018; Banerjee et al., 2019). In particular, this literature finds that people are able to identify central (popular) people in the network. In addition, Banerjee et al. (2019) analyzes how information diffuses through a network and shows that peoples' knowledge about the underlying network facilitates the process of network diffusion. In other words, people are able to use network information beneficially. However, these studies cannot answer the question whether the access to network information changes peoples' behavior or perception, because they don't observe behavior or perceptions in the absence of network information. In the laboratory, the researcher can exogenously vary the information people receive about others to evaluate the effect of information on outcomes and whether people use the information in a naive or sophisticated way. The experimental literature consistently finds that network information does matter for outcomes in a variety of domains, like belief formation (Grimm and Mengel, 2020), cooperation behavior (Gallo and Yan, 2015), equilibrium selection (Charness et al., 2014), and coordination (Kearns et al., 2006).

Complete and incomplete information are two standard assumptions in network games (Jackson and Zenou, 2015). Complete information usually refers to the case where people are able to base their decisions on knowledge about the entire structure of the network. For example, in Lipnowski and Sadler (2019) people know the entire structure of the network, however, use some information exclusively about network neighbors. In particular, Lipnowski and Sadler (2019) assume that people know the (correct) strategies taken by their network neighbors but not by others. Thus, related to our setup, agents make sophisticated inferences about others based on knowledge of network neighbors which has consequences for the entire population.

Incomplete information refers to a case where people need to base their decisions solely on their own degree. However, there exist variations in the literature which depend on the setting. For example, (Gallo and Yan, 2015) combine it with information about cooperativeness of others, whereas Grimm and Mengel (2020) add knowledge about the degree distribution, and Kearns et al. (2006) provides subjects knowledge about neighbor degree in addition to the incomplete information background. In summary, the literature shows that people are not naive about network information, but instead use it in a sophisticated way. Moreover, Grimm and Mengel (2020) directly show that how people form beliefs is inconsistent with a model of naive learning that assumes people do not incorporate network information into their updating process.[12]

Our paper closely relates to the literature that models misperceptions about population characteristics or behavior. Jackson (2019) analyzes behavior of a finite set of agents that have to choose an action based on expected actions of others under incomplete information. In addition to her own degree, the

[12]Grimm and Mengel (2020) show that behavior is also inconsistent with Bayesian belief updating, since participants use a more heuristical (sophisticated) approach to incorporate network information.

agent either knows the population degree distribution or the distribution of expected neighbor degrees. Agents are naive if they misperceive the distribution of expected neighbor degrees as a proxy for the population degree distribution. Jackson (2019) shows that when agents form expectations of others' actions based on expected actions of network neighbors, equilibrium behavior of all agents is higher as compared to the case where agents form expectations of others' actions knowing the population degree distribution. We do not assume that agents know the population degree distribution nor the distribution of expected neighbor degrees. Instead, we micro-found how agents form beliefs about the population degree distribution using network information. Hence, we explicitly condition on the fact that the agent is linked to her network neighbors. We show under which behavioral assumptions agents can overcome misperceptions and analyze the impact on population behavior. Frick et al. (2019a) studies a setting where agents misperceive the distribution of types in the population due to assortativity neglect.[13] In their setting, agents either correctly perceive the type distribution in the population or misperceive the distribution among neighbors as representative of the population distribution. The crucial difference to our paper is that we do not model type assortativity. We focus on misperceptions exclusively caused by degree heterogeneity, where an agents' type equals her degree, without any sorting by degree (i.e. degree assortativity). Frick et al. (2019b) study misperceptions in a social learning environment and show that even slight misperceptions of the type distribution in the population (i.e. others characteristics) can generate long run misperceptions about an underlying true state of the world. In their setup, agents always know the type distribution in the population. However, they either know the correct type distribution or a misspecified version of it. This differs from our approach where we explicitly show how agents estimate the population distribution using network information. That is, we show under which assumptions about agents and their information misspecified distributions arise instead of assuming them. Furthermore, we do not model social learning environments. Instead, we focus on belief formation in a static game played on a network.

Our paper relates to an analogous debate in the finance literature about whether irrational investors pose a threat to efficient market prices in equilibrium or not (Shleifer, 2000).[14] For example, proponents of the efficient market hypothesis argue that trading mistakes of irrational investors do not affect equilibrium prices as rational investors are able to capitalize on them. In contrast, opponents of the efficient market hypothesis argue that irrational investors do affect equilibrium prices because a few rational investors in the market are not able to mitigate all the trading mistakes irrational investors make. Our results reflect the view of the opponents of the efficient market hypothesis in the sense that

[13] Assortativity captures the idea that, for example, high income earners interact disproportionally more with other high income earners than low income earners and vice versa.

[14] See Zwiebel (2002) for a short review.

a few naive agents are able to affect equilibrium perceptions and behavior due to the assumption that naive agents think that all other agents in the population are naive.

6 Conclusion

We show that a sophisticated agent can consistently overcome misperceptions using network information and that their own and others' behavior depends on the mix of naive and sophisticated agents in the population. The existing literature assumes that people have either perfect prior information about the degree distribution or misperceive the distribution to equal the expected distribution of degree among network neighbors. As a consequence, it is assumed that agents ignore network information. This is surprising considering the growing literature, experimental and empirical, that demonstrates that people know quite a bit about their network and use this information to make decisions.

We know from the existing literature in economics that people in the real world are not sophisticated in many respects. However, we also know that people are not naive either. This illustrates that both assumptions about agents are problematic and that the truth probably lies somewhere in between, and probably also depends on the specific application. Second, we assume that the agent knows the degree of her network neighbors which is a critical information assumption in itself. The current literature does not offer a decisive answer on what information people actually use about the network or their network neighbors. We think that this is an interesting avenue for future research.

We finish with a remark about possible extensions. First, one could analyze a more general utility function, e.g. by going beyond linear quadratic and/or incorporating externalities as in Jackson (2019). Moreover, it is possible to introduce a correlation between individual preferences and degree. Such a correlation would lead to another type of misperception known as the "Majority Illusion" (Lerman et al., 2016), and this could further exacerbate behavioral biases. Future work could extend our setup to estimate the share of the type distribution including both degree and preferences. Second, one may allow neighbors to share beliefs among one another by combining it with models of learning in networks. Lastly, one could think of other mechanisms that cause misperceptions in networks other than the friendship paradox. One obvious candidate is homophily (McPherson et al., 2001). That is, people disproportionally form network connections with each other based on their characteristics. One could think of extending the model to allow for homophily by introducing a bias in the way people form links with each other. A sophisticated agent, who knows that her sample of network neighbors is not representative of population characteristics due to homophily, could use this information to correct her estimate of population characteristics.

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A Appendix: auxiliary results and proofs

A.1 Processing information about neighbors

Proof of Proposition 5

Proof. The sample of neighbors is drawn without replacement from the set of agents which is infinite. This implies that even a finite sample drawn without replacement has the property that there is independence between the elements drawn. The agent knows d_H and d_L. The distribution of neighboring agents is Bernoulli distributed where the likelihood to draw a high degree neighbor equals $E[d_j = d_H|\delta, d_L, d_H]$. Therefore, we can express the log-likelihood function, given a sample of network neighbors N_i, as follows:

$$\log \mathcal{L}(\delta|N_i, d_H, d_L) = n_{i,H} \cdot \log(E[d_j = d_H|\delta, d_L, d_H]) + n_{i,L} \cdot \log(1 - E[d_j = d_H|\delta, d_L, d_H]),$$

where $n_{i,H}$ ($n_{i,L}$) denotes the number of high (low) degree agents among network neighbors for agent i. The first-order condition for maximizing the above function can be expressed as follows:

$$\left(n_{i,H}\frac{1}{E[d_j = d_H|\delta, d_L, d_H]} - n_{i,L}\frac{1}{1 - E[d_j = d_H|\delta, d_L, d_H]}\right) \cdot \frac{\partial E[d_j = d_H|\delta, d_L, d_H]}{\partial \delta} = 0,$$

$$n_{i,H}\frac{1}{E[d_j = d_H|\delta, d_L, d_H]} - n_{i,L}\frac{1}{1 - E[d_j = d_H|\delta, d_L, d_H]} = 0,$$

$$n_{i,H}(1 - E[d_j = d_H|\delta, d_L, d_H]) - n_{i,L}(E[d_j = d_H|\delta, d_L, d_H]) = 0,$$

$$E[d_j = d_H|\delta, d_L, d_H] = \frac{n_{i,H}}{n_{i,H} + n_{i,L}}.$$

We now substitute $E[d_j = d_H|\delta, d_L, d_H]$ with its value and simplify:

$$\frac{d_H\delta}{d_H\delta + d_L(1-\delta)} = \frac{n_{i,H}}{n_{i,H} + n_{i,L}},$$

$$d_H\delta \cdot (n_{i,H} + n_{i,L}) = n_{i,H} \cdot (d_H\delta + d_L(1-\delta)),$$

$$\delta = \frac{d_L n_{i,H}}{d_L n_{i,H} + d_H n_{i,L}} = \frac{\frac{1}{d_H}n_{i,H}}{\frac{1}{d_H}n_{i,H} + \frac{1}{d_L}n_{i,L}}.$$

We have now derived the optimal updating rule: $\hat{\delta}_i = \frac{\frac{1}{d_H} n_{i,H}}{\frac{1}{d_H} n_{i,H} + \frac{1}{d_L} n_{i,L}}$. Note that we can denote the number of high (low) degree neighbors $n_{i,H}$ $(n_{i,L})$ as the observed share of high $(\tilde{\delta}_i)$ and low degree neighbors $(1 - \tilde{\delta}_i)$. Furthermore, we denote the excess ratio by $\rho = (d_H/d_L - 1)$ and the ratio of observed low and high degree agents by $v_i = \frac{(1-\tilde{\delta}_i)}{\tilde{\delta}_i}$. Thus we can write the estimator as:

$$\hat{\delta}_i = \frac{1}{1 + (1+\rho) \cdot v_i}.$$

□

Lemma 8. *The maximum likelihood estimator under neighbor degree information is unbiased.*

Proof. To prove that our estimator is unbiased we need to show that the expectation of the estimator equals the true share of high degree agents - i.e. we need to show that $E(\hat{\delta}) = \delta$. The first step is to insert the estimator into the expectation and apply common expectation rules:

$$E\left[\frac{n_{i,H}}{n_{i,H} + (n_{i,L}) \cdot \frac{d_H}{d_L}}\right] = \frac{E\left[n_{i,H}\right]}{E\left[n_{i,H}\right] + E\left[n_{i,L}\right] \cdot \frac{d_H}{d_L}}.$$

Under the assumption that the samples are drawn i.i.d from the true distribution we know that $E(n_{i,H}) = E(n_H)$ and $E(n_{i,L}) = E(n_L)$. Furthermore, we use that $E[n_H] = E[d_j = d_H | \delta, d_L, d_H]$, where

$$E[d_j = d_H | \delta, d_L, d_H] = \frac{\delta d_H}{\delta d_H + (1-\delta) d_L}.$$

Thus:

$$\frac{E\left[n_H\right]}{E\left[n_H\right] + E\left[n_L\right] \cdot \frac{d_H}{d_L}} = \frac{E[d_j = d_H | \delta, d_L, d_H]}{E[d_j = d_H | \delta, d_L, d_H] + (1 - E[d_j = d_H | \delta, d_L, d_H]) \cdot \frac{d_H}{d_L}}.$$

We can insert $E[d_j = d_H | \delta, d_L, d_H]$ into the equation and simplify:

$$\frac{\frac{\delta d_H}{\delta d_H+(1-\delta)d_L}}{\frac{\delta d_H}{\delta d_H+(1-\delta)d_L}+\left(\frac{(1-\delta)d_L}{\delta d_H+(1-\delta)d_L}\right)\frac{d_H}{d_L}} = \frac{\delta d_H}{\left(\delta d_H+(1-\delta)d_L\right)\left(\frac{\delta d_H}{\delta d_H+(1-\delta)d_L}+\left(\frac{(1-\delta)d_L}{\delta d_H+(1-\delta)d_L}\right)\frac{d_H}{d_L}\right)}$$

$$= \frac{\delta d_H}{\left(\delta d_H+(1-\delta)d_L\right)\left(\frac{\delta d_H}{\delta d_H+(1-\delta)d_L}+\frac{(1-\delta)d_H}{\delta d_H+(1-\delta)d_L}\right)}$$

$$= \frac{\delta d_H}{\left(\delta d_H+(1-\delta)d_L\right)\left(\frac{\delta d_H+(1-\delta)d_H}{\delta d_H+(1-\delta)d_L}\right)}$$

$$= \frac{\delta d_H}{\delta d_H+(1-\delta)d_H}$$

$$= \frac{\delta d_H}{d_H}$$

$$= \delta$$

□

Lemma 9. *The statistic $n_{i,H}$ is complete.*

Proof. A statistic is complete if $E_\delta(g(n_{i,H}) = 0$ for all δ which implies that $P_\delta(g(n_{i,H} = 0) = 1$ for all δ and some measurable function $g(\cdot)$. The proof for a statistic that has a binomial distribution with parameters (n_i, δ) can be found in Example 6.2.22 in Casella and Berger (2001). In our case, n_i denotes a random sample of agents where we index each agent by i. Each agent's likelihood of being high degree follows a Bernoulli distribution with probability $E[d_j = d_H|\delta, d_L, d_H]$. Recall that $n_{i,H}$ is the number of high degree agents observed in i's sample. It follows that $n_{i,H}$ is a statistic of the random sample n_i which has a binomial distribution with parameters $(n_i, E[d_j = d_H|\delta, d_L, d_H])$. As the parameter space for $E[d_j = d_H|\delta, d_L, d_H] \in (0, 1)$ is a monotone transformation of the parameter space of $\delta \in (0, 1)$ (i.e. a mapping from the domain $(0, 1)$ into $(0, 1)$, $n_{i,H}$ is a complete statistic. □

Lemma 10. *The statistic $n_{i,H}$ is a sufficient statistic for the maximum likelihood estimator under neighbor degree information.*

Proof. We show that the statistic is sufficient for the underlying parameter $E[d_j = d_H|\delta, d_L, d_H]$. A sufficient statistic is a sample statistic that conveys exactly the same information about the data generating process that created the data as the entire data itself. That is, once we know the sample statistic our inferences are the same as those that one can obtain by conditioning on the entire data. Agent $n_{i,j}$ can either be high degree (H) or low degree (L) where $i \in [1,..,n]$ and $j \in [H, L]$ and we denote an agent's sample consisting of high and low degree nodes by n_i. Whether a node is high degree is Bernoulli distributed with parameter $E[d_j = d_H|\delta, d_L, d_H]$. That is, $P(n_{i,j} = n_{i,H}) = E[d_j = d_H|\delta, d_L, d_H]$ and $P(n_{i,j} = n_{i,L}) = 1 - E[d_j = d_H|\delta, d_L, d_H]$. This can be formulated as follows:

$$P(n_i|n_{i,H}, E[d_j = d_H|\delta, d_L, d_H]) = \frac{P(n_i, n_{i,H}|E[d_j = d_H|\delta, d_L, d_H])}{P(n_{i,H}|E[d_j = d_H|\delta, d_L, d_H])}, \tag{10}$$

where the numerator denotes the joint distribution and the denominator denotes the distribution of the sufficient statistic. If the statistic is sufficient, the ratio will not depend on $E[d_j = d_H|\delta, d_L, d_H]$. We can rewrite the numerator of equation (10) as:

$$P(n_i, n_{i,H}|E[d_j = d_H|\delta, d_L, d_H]) = \prod_{i=1}^{n} E[d_j = d_H|\delta, d_L, d_H])^{n_{i,H}}(1 - E[d_j = d_H|\delta, d_L, d_H]))^{(n_i - n_{i,H})}.$$

where we used that the sample n_i consists of $n_{i,H}$ high degree agents with the property that each draw is independent. We know that our statistic is binomial distributed - i.e. $n_{i,H} \sim Bin(n_i, E[d_j = d_H|\delta, d_L, d_H])$. So we can rewrite the denominator of equation (10) as follows:

$$P(n_{i,H}|E[d_j = d_H|\delta, d_L, d_H]) = \binom{n_i}{n_{i,H}} E[d_j = d_H|\delta, d_L, d_H])^{n_{i,H}}(1 - E[d_j = d_H|\delta, d_L, d_H]))^{(n_i - n_{i,H})},$$

where $\binom{n_i}{n_{i,H}} = \frac{n_i!}{(n_{i,H} - n_i)!\, n_i!}$. Now we can insert the expressions for the numerator and denominator back into equation (10) and simplify:

$$\begin{aligned} P(n_i|n_{i,H}, E[d_j = d_H|\delta, d_L, d_H]) &= \frac{E[d_j = d_H|\delta, d_L, d_H])^{n_{i,H}}(1 - E[d_j = d_H|\delta, d_L, d_H]))^{(n_i - n_{i,H})}}{\binom{n_i}{n_{i,H}} E[d_j = d_H|\delta, d_L, d_H])^{n_{i,H}}(1 - E[d_j = d_H|\delta, d_L, d_H]))^{(n_i - n_{i,H})}} \\ &= \frac{1}{\binom{n_i}{n_{i,H}}}. \end{aligned}$$

which is independent of $E[d_j = d_H|\delta, d_L, d_H])$. Thus, $n_{i,H}$ is a sufficient statistic for $E[d_j = d_H|\delta, d_L, d_H])$. This shows that, for example, knowing the sequence in which the draws of high and low degree agents occurs does not give any further information about $E[d_j = d_H|\delta, d_L, d_H])$. What we are then doing in this paper is to go a step further and see what inferences we can obtain from this about δ - i.e. what we are doing in Proposition 5. For example, the inference we obtain when we assume that a naive agent thinks that $E[d_j = d_H|\delta, d_L, d_H]) = \delta$ in comparison to a sophisticated agent who knows that $E[d_j = d_H|\delta, d_L, d_H]) \neq \delta$. □

Proof of Theorem 6

Proof. We need prove that our estimator $\hat{\hat{\delta}}$, where the agent has neighbor degree information, is the best unbiased estimator of the unknown share of high degree agents δ. That is, there exists no other estimator that, given the provided information about the network, is better at estimating δ. The Lehmann–Scheffé theorem (Lehmann and Scheffé, 1950) states that any unbiased estimator of the unknown share of high degree agents that depends on the data only through a complete and sufficient statistic is the best unbiased estimator of that quantity. These conditions are both satisfied by respectively Lemma 8 for unbiasedness and Lemma 9, 10 for completeness and sufficiency. □

Proof of Corollary 7

Proof. Here we prove consistency of the maximum likelihood estimator $\hat{\hat{\delta}}$. That is, $\hat{\hat{\delta}}$ converges in probability to δ as the sample of network neighbors grows infinitely large. To prove consistency we apply Theorem 2.7 of Newey and McFadden (1994). We show that our estimator is identified, δ is part of a convex parameter set, and the log-likelihood function $\log \mathcal{L}(\delta|N_i, d_H, d_L)$ is concave. First, identification follows directly because our design imposes a unique maximum at δ. Second, the parameter space is the open range $(0, 1)$, which is obviously convex. This follows directly from the fact that for any $x_1, x_2 \in (0, 1)$ it holds that $\lambda x_1 + (1 - \lambda)x_2 \in (\min(x_1, x_2), \max(x_1, x_2))$ and thus $\lambda x_1 + (1 - \lambda)x_2 \in (0, 1)$ as $(min(x_1, x_2), max(x_1, x_2)) \subset (0, 1)$. Third, $\log \mathcal{L}(\delta|N_i, d_H, d_L)$ is concave on its domain $\delta \in (0, 1)$ as long as $\frac{\partial \log \mathcal{L}(\delta|N_i,d_H,d_L)}{\partial\delta}$ is a strictly decreasing function. We already derived the first order condition in Proposition 5 above which is:

$$\left(\underbrace{n_{i,H}\frac{1}{E[d_j = d_H|\delta, d_L, d_H]}}_{A} - \underbrace{n_{i,L}\frac{1}{1 - E[d_j = d_H|\delta, d_L, d_H]}}_{B}\right) \cdot \underbrace{\frac{\partial E[d_j = d_H|\delta, d_L, d_H]}{\partial\delta}}_{C}$$

Note that C is increasing in δ, which directly implies that A (B) is decreasing (increasing) in δ. That is, $A + B$ is decreasing in δ and still decreasing when multiplied by C. □

A.2 How network perceptions affect behavior

Proof of Lemma 1

Proof. We derive the BNE by taking the first order condition for utility (see equation (2)) wrt. own action x_i. We obtain the following best response function:

$$x_i(\theta_i, d_i, r_i, N_i) = \frac{\theta_i}{c} + \frac{a d_i E[x_j | r_i, N_i]}{c}, \qquad d_i = |N_i|. \tag{11}$$

To pin down an expression for individual expectations about other agents' actions we compute the conditional expectation wrt. to the updating rule and the set of network neighbors:

$$E[x_j | r_i, N_i] = \frac{E[\theta_i]}{c} + \frac{a}{c} \cdot \underbrace{E[d_j \cdot E[x_k | r_j, N_j] | r_i, N_i]}_{A}, \tag{12}$$

where A denotes the expectation of population degree times higher order beliefs of others actions x_k, all conditional on the updating rule and neighbor information. We can write $E[x_k | r_j, N_j] = E[x_k | r_j, d_j, \tilde{\delta}_j]$, because d_j and the observed share of high degree agents in the population $\tilde{\delta}_j$ constitute a sufficient statistic for N_j. In other words, d_j and $\tilde{\delta}_j$ encompass all information that can be obtained from the sample of network neighbors N_j. That is, others actions x_k depend on how others estimate others (including j) given the updating rule, the observed share of high degree agents, and degree. We can express A as follows:

$$A = \int \underbrace{d_j \cdot E[x_k | r_j, d_j, \tilde{\delta}_j]}_{B_r} \cdot \underbrace{f(r_j, d_j, \tilde{\delta}_j | r_i, d_i, \tilde{\delta}_i)}_{C} \, \mathrm{d}r_j \mathrm{d}d_j \mathrm{d}\tilde{\delta}_j, \tag{13}$$

where B_r denotes the conditional expectation of others' actions given the updating rule, degree, and the observed share of high degree agents. C denotes the marginal density function for beliefs of other agents about the share of high degree agents, their degree, and their updating rule. We can insert the updating rule and write:

$$A = \int B_r \cdot f(r_j = s, d_j, \tilde{\delta}_j | r_i, d_i, \tilde{\delta}_i) \, \mathrm{d}d_j \mathrm{d}\tilde{\delta}_j + \int B_r \cdot f(r_j = n, d_j, \tilde{\delta}_j | r_i, d_i, \tilde{\delta}_i) \, \mathrm{d}d_j \mathrm{d}\tilde{\delta}_j,$$

where this general notation allows us to distinguish different shares of naive and sophisticated agents in the population. We let σ denote the probability that $r_j = s_j$ and $1 - \sigma$ the probability that $r_j = n_j$, where σ is common knowledge among sophisticated agents. For sophisticated agents it holds that $f(r_j = s_j, d_j \tilde{\delta}_j | r_i, d_i, \tilde{\delta}_i) = \sigma \cdot f(d_j, \tilde{\delta}_j | r_i, d_i, \tilde{\delta}_i)$ and that $f(r_j = n_j, d_j, \tilde{\delta}_j | r_i, d_i, \tilde{\delta}_i) = (1 - \sigma) \cdot f(d_j, \tilde{\delta}_j | r_i, d_i, \tilde{\delta}_i)$. However, for a naive agent it holds that $f(r_j = s_j, d_j, \tilde{\delta}_j | r_i, d_i, \tilde{\delta}_i) = 0$, so that $f(r_j = n_j, d_j, \tilde{\delta}_j | r_i, d_i, \tilde{\delta}_i) = f(d_j, \tilde{\delta}_j | r_i, d_i, \tilde{\delta}_i)$. Recall, a naive agent is defined as someone who essentially ignores network information, for example, because a naive agent thinks that this information is not useful. Therefore, we assume that a naive agent also beliefs that the rest of the population thinks network information is not useful. We can write this as:

$$A = \int B_s \cdot \sigma \cdot f(d_j, \tilde{\delta}_j | r_i, d_i, \tilde{\delta}_i) \, \mathrm{d}d_j \mathrm{d}\tilde{\delta}_j + \int B_n \cdot (1 - \sigma) \cdot f(d_j, \tilde{\delta}_j | r_i, d_i, \tilde{\delta}_i) \, \mathrm{d}d_j \mathrm{d}\tilde{\delta}_j.$$

In the following, the proof proceeds in two steps. First, we solve the case where the agent has access to an infinite sample of network neighbors. Second, we relax the infinity assumption and solve equilibrium expectations when the agents has access to a finite set of network neighbors.

Infinite degree. We make one additional assumption to simplify the analysis. We assume that $\lim_{d_L \to \infty}$ where the degree ratio d_H/d_L is fixed. Intuitively, this assumption ensures that agents have a sufficiently large sample of network neighbors to estimate the share of high degree agents in the population. To ensure that our model does not break down due to the infinity assumption we normalize complementarities $a = \alpha/d_L$. For an asymptotically large sample of neighbors, we have that all agents who use the same rule converge to the same estimate, irrespective of degree. Hence, $f(r_j, d_j, \tilde{\delta}_j | r_i, d_i, \tilde{\delta}_i) = f(r_j, d_j, \tilde{\delta}_j | r_i, \tilde{\delta}_i)$. Therefore, the estimated share of high degree agents only depends on the updating rule r_i, but not precision. This allows us to use the results of Theorem 6 and Corollary 7 and replace the observed share of high degree agents $\tilde{\delta}(\delta, \rho)$ with its estimate δ (which is true share of high degree agents), if the agent is sophisticated. Naive agents do not adjust their estimate, so the estimate for the share of high degree agents is simply the observed share. This implies that for both high and low degree agents it holds that:

$$A = B_s \cdot \sigma + B_n \cdot (1 - \sigma) \quad \text{,where} \tag{14}$$

$$B_s = \big(d_L + (d_H - d_L) \cdot \delta\big) \cdot E[x_k | s, \delta]. \tag{15}$$

$$B_n = \big(d_L + (d_H - d_L) \cdot \tilde{\delta}(\delta, \rho)\big) \cdot E[x_k | n, \tilde{\delta}(\delta, \rho)]. \tag{16}$$

This implies that high and low degree agents have the same conditional expectations. We can insert the excess ratio ($\rho = \frac{d_H}{d_L} - 1$) and write equation (15) and (16) as follows:

$$B_s = d_L \cdot \big(1 + \rho \cdot \delta\big) \cdot E[x_k | s, \delta]. \tag{17}$$

$$B_n = d_L \cdot \big(1 + \rho \cdot \tilde{\delta}(\delta, \rho)\big) \cdot E[x_k | n, \tilde{\delta}(\delta, \rho)]. \tag{18}$$

We can therefore rewrite equation (12) (dropping the subscripts, because expectations are the same for all agents) as an equation with two unknowns ($E[x|s, \delta]$ and $E[x|n, \tilde{\delta}(\delta, \rho)]$), irrespective of degree, but dependent on the updating rule:

$$E[x|r_j, \tilde{\delta}(\delta, \rho)] = \frac{E[\theta]}{c} + \frac{ad_L}{c}\big((1 + \rho \cdot \delta) \cdot \sigma E[x|s, \delta] + (1 + \rho \cdot \tilde{\delta}(\delta, \rho)) \cdot (1 - \sigma)E[x|n, \tilde{\delta}(\delta, \rho)]\big). \tag{19}$$

First, let us compute equilibrium expectations for a naive agent. Recall, a naive agent ignores network information, therefore does not condition her action on how other agents in the population form their estimates. In other words, this corresponds to the case where a naive agent thinks that all other agents in the population are naive too (i.e. $\sigma = 0$). We can write this, substituting in the constant $\alpha = ad_L$, as follows:

$$E[x|n, \tilde{\delta}(\delta, \rho)] = \frac{E[\theta]}{c} + \frac{\alpha}{c} \cdot \big(1 + \rho \cdot \tilde{\delta}(\delta, \rho)\big) \cdot E[x|n, \tilde{\delta}(\delta, \rho)].$$
$$E[x|n, \tilde{\delta}(\delta, \rho)] = \frac{E[\theta]}{c - \alpha \cdot \big[1 + \rho \cdot \tilde{\delta}(\delta, \rho)\big]}.$$

Second, lets compute equilibrium expectations for the mixed case - i.e. sophisticated and naive agents are part of the population. Note, we only need to compute equilibrium expectations from the perspective of a sophisticated agent, because naive equilibrium expectations are independent of the share of sophisticated agents (σ) in the population. We can solve equation (19) for $E[x|s, \delta]$ as follows:

$$E[x|s, \delta] = \frac{E[\theta] + \alpha \cdot [1 + \rho \cdot \tilde{\delta}(\delta, \rho)] \cdot (1 - \sigma) \cdot E[x|n, \tilde{\delta}(\delta, \rho)]}{c - \alpha \cdot [1 + \rho \cdot \delta] \cdot \sigma},$$

and obtain an expression for equilibrium expectations of sophisticated agents, dependent on the share of naive agents in the population. A sophisticated agent knows that naive agents do not care about the type of other agents in the population. Thus, we can treat $E[x|n, \tilde{\delta}(\delta, \rho)]$ as a constant. Inserting the expression for $E[x|n, \tilde{\delta}(\delta, \rho)]$ results in the following equation for equilibrium expectations of sophisticated agents:

$$E[x|s, \delta] = \frac{E[\theta]}{c - \alpha \cdot [1 + \rho \cdot \delta] \cdot \sigma} + \frac{\alpha \cdot E[\theta] \cdot [1 + \rho \cdot \tilde{\delta}(\delta, \rho)] \cdot (1 - \sigma)}{(c - \alpha \cdot [1 + \rho \cdot \delta] \cdot \sigma) \cdot (c - \alpha \cdot [1 + \rho \cdot \tilde{\delta}(\delta, \rho)])}. \tag{20}$$

In case all agents in the population are sophisticated ($\sigma = 1$), we have that:

$$E[x|s, \delta] = \frac{E[\theta]}{c - \alpha \cdot [1 + \rho \cdot \delta]}.$$

Finite degree. In the case where we have finite degree, equation (13) can be rewritten using discrete probabilities. The rewritten expression is the weighted sum of other types expectations where the weights are beliefs about the likelihood of a given type:

$$A = \sum_{r_j \in n,s} \sum_{d_j \in d_L, d_H} \sum_{\tilde{\delta}_j \in \tilde{D}_j} d_j \cdot E[x_k | r_j, d_j, \tilde{\delta}_j] \cdot f(r_j, d_j, \tilde{\delta}_j | r_i, d_i, \tilde{\delta}_i), \quad \tilde{D}_j = \left\{ \frac{0}{d_j}, \frac{1}{d_j}, .., \frac{d_j}{d_j} \right\}. \tag{21}$$

For each feasible type, i.e. $r_i, d_i, \tilde{\delta}_i$, this entails that we can simplify the expression for own expectations about the equilibrium from equation (12). This expression is a linear function of others' expectations about other agents' actions. The number of unique types is $L = 2 \cdot (d_L + 1 + d_H + 1)$ as there is one for each rule times the number of feasible draws that can be obtained ($0, .., d_L$ for low degree and $0, .., d_H$ for high degree).[15] When taken together, equations (12) and (21) span a system of L equations governing the agents belief. Thus, each equation in the system expresses the belief for each type as a linear function of the other L types with L coefficients that measure the types' beliefs about other agents types' likelihood. We can express the system of equations as the following matrix equation:

$$\left(\frac{\alpha}{c} \Pi - I_L \right) \xi' = -\frac{E[\theta_i]}{c} \cdot J_L, \tag{22}$$

where Π denotes the beliefs for each agents type $(r_i, d_i, \tilde{\delta}_i)$ about the likelihood of observing other agents' types. They come from the density function in equation (21). Note that Π is a function of σ, ρ, and δ. That is, we can compute a belief matrix Π for each possible share of sophisticated agents in the population, for example. ξ is the vector of beliefs for each type about other agents actions, I_L is the identity matrix of size L, and J_L is vector of ones with size L. The L equations can be solved with standard linear algebra by inverting the matrix of coefficients for the system of equations:

$$\xi = -\left(\frac{\alpha}{c} \Pi - I_L \right)^{-1} \frac{E[\theta_i]}{c} \cdot J_L.$$

□

Proof of Theorem 2

Proof. First, Theorem 2 states that if all agents use naive updating then $x(\theta_i, d_i, n, \tilde{\delta}(\delta, \rho)) > x(\theta_i, d_i, \delta)$ for all θ_i and d_i. Note that $x(\theta_i, d_i, \delta)$ denotes the benchmark case absent of misperceptions. Second, if all agents are sophisticated then $x(\theta_i, d_i, s, \tilde{\delta}(\delta, \rho)) = x(\theta_i, d_i, \delta)$ for all θ_i and d_i. Thus, $E[x | n, \tilde{\delta}(\delta, \rho)] > E[x | s, \tilde{\delta}(\delta, \rho)] = E[x | \delta]$. Third, if a share (σ) of agents in the population use sophisticated updating and a share $(1 - \sigma)$ use naive updating, then equilibrium actions are decreasing

[15]This assumes that $d_L \neq d_H$.

in σ for sophisticated agents. Note that equilibrium actions of naive agents are not affected by the share of sophisticated agents in the population.

To prove the first two parts of Theorem 2 we need to show that the maximum likelihood estimator of a sophisticated agent is unbiased ($E[s, \tilde{\delta}(\delta, \rho)] = \delta$) and that the estimate of the sophisticated agent is smaller than the estimate of the naive agent ($\delta < \tilde{\delta}(\delta, \rho)$). Note that we already proved that the estimator of a sophisticated agents is unbiased (see Lemma 8). For the second part of the proof we need to show that $\delta < \tilde{\delta}(\delta, \rho)$:

$$
\begin{aligned}
\delta &< \frac{\delta d_H}{\delta d_H + (1-\delta) d_L} = \frac{\delta}{\delta + (1-\delta)\cdot(1+\rho)^{-1}}, \\
\delta \cdot (\delta + (1-\delta)\cdot(1+\rho)^{-1}) &< \delta, \\
(1-\delta)\cdot(1+\rho)^{-1} &< (1-\delta), \\
0 &< \rho.
\end{aligned}
$$

which holds by definition. For the third part we need show that equilibrium actions of sophisticated agents decrease in σ. This is equivalent to showing that equilibrium expectations of sophisticated agents decrease in σ. Therefore, we need to show that the first derivative of equation (20) is negative for each value of σ:

$$
\frac{d}{d\sigma} E[x|s,\delta] = \frac{d}{d\sigma}\left(\frac{E[\theta]}{c - \alpha \cdot [1 + \rho \cdot \delta] \cdot \sigma}\right) + \frac{d}{d\sigma}\left(\frac{\alpha \cdot E[\theta] \cdot [1 + \rho \cdot \tilde{\delta}(\delta, \rho)] \cdot (1-\sigma)}{(c - \alpha \cdot [1 + \rho \cdot \delta] \cdot \sigma) \cdot (c - \alpha \cdot [1 + \rho \cdot \tilde{\delta}(\delta, \rho)])}\right)
$$

$$
= \frac{c \cdot \alpha \cdot E[\theta] \cdot \rho \cdot \delta - c \cdot \alpha \cdot E[\theta] \cdot \rho \cdot \tilde{\delta}(\delta, \rho)}{(c - \alpha \cdot (1 + \rho \cdot \delta) \cdot \sigma)^2 \cdot (c - \alpha \cdot (1 + \rho \cdot \tilde{\delta}(\delta, \rho))}.
$$

where one can see that the nominator is negative because $\delta < \tilde{\delta}(\delta, \rho)$. The denominator is positive due to the assumption that $c - \alpha \cdot (1 + \rho \cdot \tilde{\delta}(\delta, \rho) > 0$. Hence, $\frac{d}{d\sigma} E[x|s,\delta] < 0$. It directly follows that equilibrium actions are decreasing in σ since we only multiply the nominator and denominator with positive constants that are independent of σ. Lastly, we show that equilibrium expectations with a mix of sophisticated and naive agents in the population are bounded by the two extreme cases (all agents are either naive or sophisticated). We need to show that the following inequality holds, where we denote $\tilde{\delta}(\delta, \rho)$ by $\tilde{\delta}$ to save some notation:

$$
\frac{E[\theta]}{c - \alpha \cdot (1 + \rho \cdot \delta)} \leq \frac{E[\theta]}{c - \alpha \cdot (1 + \rho \cdot \tilde{\delta}) \cdot \sigma} + \frac{\alpha \cdot E[\theta] \cdot (1 + \rho \cdot \tilde{\delta}) \cdot (1-\sigma)}{(c - \alpha \cdot (1 + \rho \cdot \delta) \cdot \sigma) \cdot (c - \alpha \cdot (1 + \rho \cdot \tilde{\delta}))} \leq \frac{E[\theta]}{c - \alpha \cdot (1 + \rho \cdot \tilde{\delta})}
$$

where $c > \alpha \cdot (1 + \rho \cdot \tilde{\delta})$ and $0 \leq \sigma \leq 1$. We can multiply by $c - \alpha \cdot (1 + \rho \cdot \tilde{\delta})$, $c - \alpha \cdot (1 + \rho \cdot \delta)$, and divide by $E[\theta]$:

$$c - \alpha \cdot (1 + \rho \cdot \tilde{\delta}) \leq \frac{(c - \alpha \cdot (1 + \rho \cdot \delta)) \cdot (c - \alpha \cdot (1 + \rho \cdot \tilde{\delta})) + (c - \alpha \cdot (1 + \rho \cdot \delta)) \cdot \alpha \cdot (1 + \rho \cdot \tilde{\delta}) \cdot (1 - \sigma)}{c - \alpha \cdot (1 + \rho \cdot \delta) \cdot \sigma}$$
$$\leq c - \alpha \cdot (1 + \rho \cdot \delta)$$

$$c - \alpha \cdot (1 + \rho \cdot \tilde{\delta}) \leq \frac{(c - \alpha \cdot (1 + \rho \cdot \delta)) \cdot (c - \alpha \cdot (1 + \rho \cdot \tilde{\delta}) \cdot \sigma)}{c - \alpha \cdot (1 + \rho \cdot \delta) \cdot \sigma}$$
$$\leq c - \alpha \cdot (1 + \rho \cdot \delta)$$

$$(c - \alpha \cdot (1 + \rho \cdot \tilde{\delta})) \cdot (c - \alpha \cdot (1 + \rho \cdot \delta) \cdot \sigma) \leq (c - \alpha \cdot (1 + \rho \cdot \delta)) \cdot (c - \alpha \cdot (1 + \rho \cdot \tilde{\delta}) \cdot \sigma)$$
$$\leq (c - \alpha \cdot (1 + \rho \cdot \delta)) \cdot (c - \alpha \cdot (1 + \rho \cdot \delta) \cdot \sigma)$$

We first prove that the inequality on the left hand side holds:

$$(c - \alpha \cdot (1 + \rho \cdot \tilde{\delta})) \cdot (c - \alpha \cdot (1 + \rho \cdot \delta) \cdot \sigma) \leq (c - \alpha \cdot (1 + \rho \cdot \delta)) \cdot (c - \alpha \cdot (1 + \rho \cdot \tilde{\delta}) \cdot \sigma)$$
$$-\alpha \cdot (1 + \rho \cdot \delta) \cdot \sigma \cdot c - \alpha \cdot (1 + \rho \cdot \tilde{\delta}) \cdot c \leq -\alpha \cdot (1 + \rho \cdot \tilde{\delta}) \cdot \sigma \cdot c - \alpha \cdot (1 + \rho \cdot \delta) \cdot c$$
$$\alpha \cdot (1 + \rho \cdot \delta) \cdot (c - \sigma \cdot c) \leq \alpha \cdot (1 + \rho \cdot \tilde{\delta}) \cdot (c - \sigma \cdot c)$$
$$(1 + \rho \cdot \delta) \leq (1 + \rho \cdot \tilde{\delta})$$
$$\delta \leq \tilde{\delta}$$

which we just showed above. For the right hand side we need to prove that:

$$(c - \alpha \cdot (1 + \rho \cdot \delta)) \cdot (c - \alpha \cdot (1 + \rho \cdot \tilde{\delta}) \cdot \sigma) \leq (c - \alpha \cdot (1 + \rho \cdot \delta)) \cdot (c - \alpha \cdot (1 + \rho \cdot \delta) \cdot \sigma)$$
$$c - \alpha \cdot (1 + \rho \cdot \tilde{\delta}) \cdot \sigma \leq c - \alpha \cdot (1 + \rho \cdot \delta) \cdot \sigma$$
$$(1 + \rho \cdot \delta) \leq (1 + \rho \cdot \tilde{\delta})$$
$$\delta \leq \tilde{\delta}$$

which we showed above. □

A.3 An illustrative example: additional calculation steps

The assumption underlying the results of Theorem 2 is that the population of agents is infinite, which allows us to isolate misperceptions caused by the friendship paradox and abstract from the following two effects. First, we can abstract from estimation precision, where one could argue that high degree agents might be able to make a better estimation than low degree agents due to a larger sample of network neighbors. This is not the case in an infinite network. Second, there is no degree (dis-)assortativity in an infinite network which can arguably amplify or mitigate the effect of misperceptions caused by the friendship paradox. We would not be able to distinguish to what extent misperceptions are caused by the friendship paradox compared to (dis-)assortativity.

Recall that the example network consists of 10 agents (nodes), where 4 out of 10 agents have high degree (black nodes) and 6 out of 10 agents have low degree (white nodes). That is, the degree distribution consists of a share of high degree agents ($\delta = 40\%$) and the share of low degree agents ($1 - \delta = 60\%$). Each agent observes that 50% ($= \tilde{\delta}$) of network neighbors are of high degree ($d_H = 6$) and the other 50% ($= \tilde{\delta}$) are of low degree ($d_L = 4$). Furthermore, we defined $\theta_i = \frac{1}{2}$ for all i, $E[\theta] = 1/2$, $\alpha = 4$, and $c = 6$.

Benchmark Case with No Misperceptions

Equilibrium actions for each type of agent $x(\theta_i, d_L) = x(\frac{1}{2}, 4)$ and $x(\theta_i, d_H) = x(\frac{1}{2}, 6)$ are defined as follows:

$$x(\frac{1}{2}, 4) = \frac{\theta_i}{c} + \frac{ad_i E[\theta]}{c \cdot (c - \alpha \cdot [1 + \rho \cdot \delta)])} = \frac{0.5}{6} + \frac{1 \cdot 4 \cdot 1/2}{6 \cdot (6 - 4 \cdot [1 + 1/2 \cdot 4/10])} = \frac{13}{36} \approx 0.361.$$

$$x(\frac{1}{2}, 6) = \frac{\theta_i}{c} + \frac{ad_i E[\theta]}{c \cdot (c - \alpha \cdot [1 + \rho \cdot \delta)])} = \frac{0.5}{6} + \frac{1 \cdot 6 \cdot 1/2}{6 \cdot (6 - 4 \cdot [1 + 1/2 \cdot 4/10])} = \frac{18}{36} = 0.5.$$

That is, average actions are equal to $\frac{4}{10} \cdot \frac{18}{36} + \frac{6}{10} \cdot \frac{13}{36} = \frac{15}{36}$ or one can compute average actions as follows:

$$E[x|\delta] = \frac{E[\theta]}{c - \alpha \cdot [1 + \rho \cdot \delta]} = \frac{1/2}{6 - (4 \cdot [1 + 1/2 \cdot 4/10])} = \frac{15}{36} \approx 0.417,$$

where this formulation becomes useful once we do not assume that agents know the degree distribution. We compute utility for both types of agents $U(x, \theta_i, d_L) = U(x, \frac{1}{2}, 4)$ and $U(x, \theta_i, d_H) = U(x, \frac{1}{2}, 6)$:

$$U(x, \frac{1}{2}, 4) = \theta_i x + axd_i E[x|\delta] - \frac{cx^2}{2} = \frac{1}{2} \cdot \frac{13}{36} + 1 \cdot \frac{13}{36} \cdot 4 \cdot \frac{15}{36} - \frac{6 \cdot (\frac{13}{36})^2}{2} = 0.3912,$$

$$U(x, \frac{1}{2}, 6) = \theta_i x + axd_i E[x|\delta] - \frac{cx^2}{2} = \frac{1}{2} \cdot \frac{18}{36} + 1 \cdot \frac{18}{36} \cdot 6 \cdot \frac{15}{36} - \frac{6 \cdot (\frac{18}{36})^2}{2} = 0.75,$$

where high degree agents enjoy more interaction, thus have a higher utility than low degree agents.

All Agents are Naive or Sophisticated

In the following, we do not assume that agents know the population degree distribution. Instead, they must estimate it using information about their sample of network neighbors. When all agents are naive, each agent estimates that 50% of agents in the population are of high degree $\hat{\delta} = \tilde{\delta}$. First, we calculate equilibrium expectations of naive agents about other agents' actions as follows:

$$E[x|n, \tilde{\delta}] = \frac{E[\theta]}{c - \alpha \cdot \left[1 + \rho \cdot \tilde{\delta}\right]} = \frac{1/2}{6 - (4 \cdot [1 + 1/2 \cdot 1/2])} = \frac{18}{36} = 0.5.$$

where equilibrium expectations of others' actions increase if all agents are naive compared to the benchmark case (i.e. $E[x|n, \tilde{\delta}] = 0.5 > E[x|\delta] = 0.417$). Next, we compute equilibrium actions for each type of the naive agent $x(\theta_i, d_L, r) = x(\frac{1}{2}, 4, n)$ and $x(\theta_i, d_H, r) = x(\frac{1}{2}, 6, n)$:

$$x(\frac{1}{2}, 4, n) = \frac{\theta_i}{c} + \frac{ad_i E[\theta]}{c \cdot (c - \alpha \cdot [1 + \rho \cdot \tilde{\delta})])} = \frac{0.5}{6} + \frac{1 \cdot 4 \cdot 1/2}{6 \cdot (6 - 4 \cdot [1 + 1/2 \cdot 1/2])} = \frac{15}{36} \approx 0.417,$$

$$x(\frac{1}{2}, 6, n) = \frac{\theta_i}{c} + \frac{ad_i E[\theta]}{c \cdot (c - \alpha \cdot [1 + \rho \cdot \tilde{\delta})])} = \frac{0.5}{6} + \frac{1 \cdot 6 \cdot 1/2}{6 \cdot (6 - 4 \cdot [1 + 1/2 \cdot 1/2])} = \frac{21}{36} \approx 0.583,$$

where equilibrium actions increase for all types of naive agents compared to the case where misperceptions are absent (i.e. $x(\frac{1}{2}, 4, n) \approx 0.417 > x(\frac{1}{2}, 4) \approx 0.361$ and $x(\frac{1}{2}, 6, n) \approx 0.583 > x(\frac{1}{2}, 6) = 0.5$).

Next, we compute expected utility for both types of naive agents. Here we compare the utility change directly to the case with no misperceptions. That is, we use the actions naive agents choose under misperceptions about expected actions of others $E[x|n, \tilde{\delta}]$, but use correct expectations $E[x|\delta]$ to calculate the utility change:

$$EU(x, \frac{1}{2}, 4, n) = \theta_i x + axd_i E[x|\delta] - \frac{cx^2}{2} = \frac{1}{2} \cdot \frac{15}{36} + 1 \cdot \frac{15}{36} \cdot 4 \cdot \frac{15}{36} - \frac{6 \cdot (\frac{15}{36})^2}{2} \approx 0.3819,$$

$$EU(x, \frac{1}{2}, 6, n) = \theta_i x + axd_i E[x|\delta] - \frac{cx^2}{2} = \frac{1}{2} \cdot \frac{21}{36} + 1 \cdot \frac{21}{36} \cdot 6 \cdot \frac{15}{36} - \frac{6 \cdot (\frac{21}{36})^2}{2} \approx 0.7291,$$

where expected utility decreases for naive agents compared to the benchmark for each type of agent (i.e. $EU(x, \frac{1}{2}, 4, n) \approx 0.3819 < U(x, \frac{1}{2}, 4) \approx 0.3912$ and $EU(x, \frac{1}{2}, 6, n) \approx 0.7291 < U(x, \frac{1}{2}, 6) = 0.75$). If all agents in the population are sophisticated, we can compute their estimate using equation

(9):

$$\hat{\delta}_i = \frac{1}{1 + (1+\rho)v_i} = \frac{1}{1 + (1+\frac{1}{2}) \cdot 1} = \frac{4}{10},$$

where, in this example, each agent (high or low degree) has perfect precision to correctly estimate the share of high degree agents in the population, if sophisticated (i.e. $\hat{\delta}_i = \hat{\delta} = \frac{4}{10}$ for all i). We can see that the sophisticated estimate aligns with the true share of high degree agents δ. Thus, if all agents in the population are sophisticated, the friendship paradox does not influence behavior even though agents do not know the population degree distribution ex ante.

Chapter III

Nudging Cooperation in Networks

Nudging Cooperation in Networks*

Gorm G. Jensen[†]

Martin Benedikt Busch[‡]

Marco Piovesan[§]

Jan O. Haerter[¶]

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Abstract

We investigate cooperation behavior of humans embedded in networks. In our laboratory experiment, subjects build their own network. They send costly messages to each other that contain valuable information for the receiver or other subjects in the network. Sending a message is beneficial for the entire network as it increases the probability that subjects find the information they are looking for. However, classical game theory predicts zero cooperation when we measure cooperation by the profit subjects earn. We find that subjects cooperate, generate a profit for themselves and others', and that cooperation persists. We change subjects' perceptions of the network by providing initial suggestions of whom to contact. We find that subjects send more messages - increasing their own and others' profit. Despite the removal of suggestions, subjects build long-lasting relationships with the suggested contacts.

JEL Classification Codes: D85, C92.

Keywords: Social Networks, Cooperation, Nudge, Social Capital.

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[†]Niels Bohr Institute, University of Copenhagen (gruner.jensen@nbi.ku.dk).

[‡]Deparment of Economics and CEBI, University of Copenhagen (mbb@econ.ku.dk).

[§]Deparment of Economics and CEBI, University of Copenhagen (marco.piovesan@econ.ku.dk).

[¶]Niels Bohr Institute, University of Copenhagen (haerter@nbi.ku.dk).

1 Introduction

Why do people cooperate, trust strangers and build solid networks based on reciprocal help? This is one of the most fundamental puzzles in social sciences. Cooperation, reciprocity and trust are the key ingredients of the emergence of the so-called "social capital" that can foster economic and social development (Nowak and May, 1992; Nowak and Sigmund, 2005; Nowak, 2006; Axelrod and Hamilton, 1981). For instance, through cooperation, reciprocity and trust, individuals can create and share information with one another to reach mutual help. The creation of preferential communication networks allows individuals to locate and access the exact information they need (Trusina et al., 2005; Sneppen et al., 2005) and sustain an information flow that is beneficial for the entire network (Rosvall and Sneppen, 2003, 2006, 2009). This ability to exchange information is not trivial. In particular, cooperation is difficult to achieve in large networks that involve the coordination of many individuals at the same time. The presence of time constraints (Fehl et al., 2011; Rand et al., 2011; Wang et al., 2012; Haerter et al., 2012; Bednarik et al., 2014; Bendtsen et al., 2016) together with the limited capacity of humans to focus on only a finite number of others (Dunbar, 1992; Hill and Dunbar, 2003; Miritello et al., 2013) make it hard to sustain an informal (non-binding) network of cooperation.

In this paper, we study how humans interact, exchange information and learn to trust each other. In a controlled and incentivized experimental setting, we observe subjects that freely build their own communication network to find the information they need. We ask two questions: 1) Are subjects able to cooperate and create an effective communication network even in a complex environment? Our data show that cooperation can emerge spontaneously and remain stable over time. Subjects create their own preferential communication networks that help them to achieve higher payoffs. 2) Can a subtle and non-binding nudge foster cooperation and help to sustain it over time? The provision of a weak suggestion about who and how many subjects to contact at the beginning of the experiment can foster communication and the stability of their network. Interestingly, the effect of the nudge survives long after it is removed.

In our experiment, we create a complex environment that makes cooperation particularly challenging. Subjects interact in a large network comprising 25 individuals for multiple rounds. In each period, subjects can freely choose which other subjects to contact in the network, hence are not forced to cooperate or not with a particular other subject. Subjects are anonymous, i.e. there is no direct face-to-face communication, and cannot build a global reputation. Subjects have the possibility to free ride and exploit the network without direct punishment opportunities by others. In essence, we create an environment where classical game theory would predict zero cooperation.

Subjects in our experiment can send costly messages to each other that contain valuable information for the receiver or other subjects in the network. Subjects can send two types of costly messages. First, they can send inquiries that reveal their distinct information to the receiver of the inquiry. Second, they can send replies that distribute the received information to others in the network. When subjects send a message, they increase the amount of information in the network - beneficial for the entire network. In particular, we define a cooperative act as any kind of message sent during the game. We measure cooperation by the profit subjects earn during the experiment in experimental currency unit (ECU). We derive theoretical conditions, comparable to the data, that illustrate when subjects should engage in the game and when the overall network is in a sustainable state. We illustrate that a subject's decision to engage in the game depends on two factors. First, the belief a subject holds about how many inquiries others send. Second, her beliefs about the willingness of others to reply to inquiries. When taking a network perspective, we show that a state under which cooperation is sustainable (i.e. subjects make a positive profit) depends on the amount of information present in the network, measured by the number of inquiries sent, and subjects willingness to reply to inquiries.

Our results are the following. First, we show that subjects cooperate even in our complex environment. Subjects can build trust and sustain cooperation even if the incentives to do so are small, ex ante. To stimulate cooperation, we suggest six contacts to each subject. The suggestions are visible to subjects during the initial five rounds of the game. Second, we show that this simple nudge is effective in increasing payoffs and the number of messages sent. Payoffs increase even though the higher number of messages sent implies higher cost. Subjects generate more profit because there is more information in the network. We observe a higher level of cooperation and reciprocity with almost no decline. Cooperation is sustainable because subjects are willing to "invest" in others'. Third, we decompose the number of inquiries sent in baseline and nudging sessions into inquiries sent within and outside the suggested network. We show that subjects in our baseline sessions send inquiries even when others' are not (fully) reciprocating. Subjects in nudging sessions are sending even more inquiries: Some inquiries following our suggestion, some to create and explore new networks.

This paper relates to the literature developing since Axelrod's seminal work (Axelrod and Hamilton, 1981), largely based on rational agents striving to maximize gain. Since then, it has become increasingly clear that one cannot categorize humans as pure "Homo Economicus" as they are also "Homo Sociologicus" (Fehr and Gintis, 2007). Whereas the former type, one that is purely self-interested, could prevail in a pure market setting, research shows that it is a limited model for social contexts. Indeed, even under elimination of aiding mechanisms - such as reputation or punishment – altruism prevails as a distinguishing feature of human versus other animal societies (Güth et al., 1982; Fehr and Fischbacher, 2003). Under the temptation of short-term benefits and the complexity of decision-making in communication networks, it seems that the benefit of more immediate gains is more attractive than the investment in long-term links within a network context. Yet, mutual, that is reciprocated, communication and strong links (Krackhardt et al., 2003; Melamed and Simpson, 2016) facilitate the flow of information in email networks (Newman et al., 2002), in the world-wide-web (Albert et al., 1999), or Wikipedia (Zlatić et al., 2006) - hinting at robustness of the finding of reciprocity for network links. However, the literature also suggests weak ties to promote organizational success (Friedkin, 1982), as weak ties often allow for information flow across large societal distances (Granovetter, 1977; Hansen, 1999; Levin and Cross, 2004).

With modern communication increasingly taking place virtually, research addressing the evolution and navigability of corresponding social networks is more relevant than ever (Monge and Contractor, 2001). As opposed to formal, mandated, communication networks, where links are pre-imposed, such as in bureaucratic contexts, informal, emergent, communication networks can be more useful to accomplish creative or innovative tasks (Monge and Contractor, 2001). However, self-interested individuals may then drive communication and cooperation may fail. A theoretical mechanism leading to emergent network structure can be optimization, where people make rational choices to garner personal benefits (Monge and Contractor, 2001). However, the literature suggests that people "satisfice", that is, they do not - and possibly cannot - explore all options before settling for a good solution (Monge and Contractor, 2001). Further distorting the mathematical optimization problem, humans typically lack "time consistency," and thereby favor short-term gains that come with long-term losses (Gintis, 2000). In our setting, we acknowledge that subjects suffer from optimization problems. Instead, we nudge subjects towards the optimal solution. At longer timescales, emergent communication networks relate to the theory of social capital (Coleman, 1988). The theory suggests that humans first build social capital by making investments and subsequently exploit social capital to achieve a benefit as a return.

The remainder of this paper proceeds as follows. We introduce the experimental design in Section 2 and derive the theoretical conditions in Section 3. Section 4 presents the results. We conclude in section 5. The appendix contains auxiliary results and the material used for the laboratory experiment.

2 Experimental Design

We conducted a computerized experiment with 100 players shared in two baseline sessions (B1 and B2) and two nudging sessions (N1 and N2). We endowed players with 100 Experimental Currency Units (ECU; 1 ECU = 3 DKK) and, during the experiments, players made decisions that affected their earnings. Players earned 461.87 DKK on average (min: 71 DKK; max: 767 DKK) for approximately 180 minutes. We distributed written instructions about our interactive game to all players, and we checked their comprehension with on-screen control questions (see Appendix B). Our sessions have a different number of rounds (from 51 to 86) since we defined the game to last for exactly 90 minutes and the speed of each round depends on players' behaviour. Players' ID numbers remain fixed throughout the session to allow each player to identify other players but keeping anonymity.[1] What varies at the beginning of each round is the unique Question and Expertise that each player receives from the computer, represented by neutral letters (e.g. O and P). Each player's Question uniquely matches another player's Expertise.[2] In each round, players have to find out which of the other 24 players has the Expertise for their Question. If they succeed they gain 10 ECU.

Each round has two stages. In stage 1, players can send inquiries to reveal their question and expertise to the receiver. For instance, the inquiry of a player that has Question B and Expertise C says: "I am an expert in C. I have a question about B. Can you help me?" Players can send up to 24 inquiries, but each inquiry has a cost of 1 ECU. Stage 1 ends with the simultaneous delivery of all inquiries. In stage 2, players can reply to the inquiries they received: each reply costs 1 ECU and players cannot send false information. There are three types of costly replies depending on what information each player offers: 1) "I'm sorry, but I don't know anyone who is an expert in B"; 2) "Yes! I am the expert you are looking for"; 3) "The expert you are looking for is player X" if the player has received an Inquiry from player X, revealing that player X has expertise B. Players can avoid sending any inquiry or reply if they want to avoid the cost. Stage 2 ends with the simultaneous delivery of all replies. Nudging sessions differ from baseline sessions for a visual suggestion we gave to players in the first five rounds: six stars appearing next to six players' ID that suggest sending an inquiry to those players. The six suggested ID's form a communication network with three properties:

1. the network is perfectly bi-directional (player i is suggested to player j, if and only if j is suggested to i), i.e., the adjacency matrix $\mathcal{A}$ is symmetric, $\mathcal{A}_{ij} = \mathcal{A}_{ji}$.

2. each player is at most two steps away from any other player, i.e., $\left(\mathcal{A} + \mathcal{A}^2\right)_{ij} > 0 \ \forall \ i, j \in \{1, \dots, N\}$.

3. the graph is symmetric in the sense that all nodes are topologically identical.

We informed subjects that "if every player follows our suggestion and helps other players find their expert, then every player will find her expert in every round". This follows from the second property. Note that players are still free to contact (or not) any other players in the game (see Suggestion in Appendix B).

[1] In practice, to a player sitting at computer m, the player sitting at computer n will appear with ID $n - m \bmod 25$.

[2] First we make a random permutation p of the 25 players and a random permutation q of the 25 first alphabet letters. We then assign to the player p the question q and the expertise $q + 1 \bmod 25$.

3 Theoretical Framework

The game derives its complexity from its network aspect and the trade-offs made by players as they choose whom to communicate with. Classical game theory may not lead to straightforward results, as the size of the strategy space makes it difficult to even suggest the existence of a Nash equilibrium. As each player can send inquiries to any other within stage 1 of each round, $2^{n_r N(N-1)}$ combinations of inquiries are possible during a game comprising n_r rounds and N players. Replies are conditional on inquiries, but further increase the options.

To capture the basic game mechanics, we make the following two assumptions that simplify the strategy space. First, we assume that every player only sends informed replies. An informed reply is a reply where player i sends a reply to an inquiry by player j containing the information on the requested expert. We distinguish informed replies from uninformed replies of the form: "I'm sorry, but I don't know anyone who is an expert in X". Second, we abstract from network structure. That is, we assume that players can choose how many inquiries they want to send, but not whom they send them to. In essence, we model the first round where players do not have a game history with each other, therefore, do not have any incentive to prefer one player over another. The assumption implies that a selfish player does not have a rational incentive to reply to inquiries in the second stage. We evaluate the assumptions in Section 4. Without the incentive to reply, players do not have an incentive to send inquiries in the first stage. In this simplified version, standard rational agent theory predicts a single Nash equilibrium: Selfish players should remain inactive during the game and gain zero ECU. This is a typical example of the tragedy of the commons as replies increase group profit, but players have no incentive to send them.

Humans often do not behave like selfish rational agents, even in idealized lab experiments. Therefore, we consider an additional modification of the game. We define a reply rate $r \in [0, 1]$ capturing the probability that a player sends an informed reply. For example, if the reply rate equals unity all players always send informed replies. If the reply rate is zero, players never send informed replies. For intermediate values, players will send some informed replies but not others. For example, this could be due to inattentiveness or cognitive constraints.

In the following, we distinguish two perspectives. First, the individual perspective where we show under which conditions a player should send at least one inquiry given other players' inquiry sending behavior and reply rate. Second, the network perspective where we illustrate under which conditions the network is in a sustainable state. That is, we compute the expected network profit, given the assumptions, for a given number of inquiries and reply rate. We illustrate the two perspectives using two representative players that we call Alice and Bob, where Alice is the expert for Bob's question.

3.1 Individual Perspective

When should Bob send at least one inquiry? In other words, should Bob engage in playing the game at all? The answer depends on Bob's underlying beliefs about other player's behavior. First, it depends on whether Bob expects Alice to send inquiries and how many. Second, it depends on Bob's expectation about other player's likelihood to reply to inquiries. In the following, we derive belief thresholds under which Bob should engage in playing the game or not.

In a given round, consider the probability $P_{win}(m, n)$ that Bob finds Alice, where m denotes the number of inquiries Bob sends and n denotes the number of inquiries Alice sends. We assume that inquiries sent by Bob and Alice are sent to random receivers. Assume that Bob sends $m = 0$ inquiries. The probability that Bob finds Alice equates to $P_{win}(0, n) = n/24$. For example, if Alice sends $n = 1$ inquiry then this inquiry will reach Bob with probability $P_{win}(0, 1) = 1/24$ as there are 23 other players in the network. If Alice sends an inquiry to every other player in the network (that is, $n = 24$) then Bob will find Alice with certainty, that is, $P_{win}(0, 24) = 1$. We can denote the expected profit $\Pi_i(m, n)$ of Bob as follows:

$$\Pi_i(m, n) = 10 \cdot P_{win}(m, n) - m.$$

The reward for finding Alice equates to 10 ECU, by definition. When Bob sends $m = 0$ inquiries his expected profit equates to $\Pi_i(m, n) = \Pi_i(0, n) = (10 \cdot n)/24$.

When should Bob send at least $m = 1$ inquiry? To answer this, we need to calculate $\Pi_i(1, n)$ and compare it to $\Pi_i(0, n)$. Let us assume that the reply rate r equals unity in the following, so that players always reply if they can (we relax this assumption later). Note that once Bob sends an inquiry, there are two additional scenarios under which he can find Alice. First, Bob sends an inquiry directly to Alice. Second, Bob sends an inquiry to a player that Alice inquires too. To calculate the Probability $P_{win}(1, n)$ that Bob finds Alice, we can ask under which conditions Bob does not find Alice. This depends on the following two probabilities P_A and P_B:

1. $P_A = (24 - n)/24$: None of Alice's n inquiries are sent to Bob.
2. $P_B = (24 - (n + 1))/24$: Bob sends an inquiry neither to Alice nor any of the n players who Alice sends an inquiry to.

For example, if Alice sends one inquiry then the probability that the inquiry of Alice goes to any of the other 23 players equates to $P_A = 23/24$. In the other extreme, if Alice sends an inquiry to every player then Bob finds Alice with certainty, i.e. $P_A = 0$. Now consider the case where Bob and Alice send one inquiry each. The probability that Bob's inquiry neither reaches Alice nor any of the other players Alice inquires equals $P_B = 22/24$. In the other extreme, if Alice sends $n = 23$ inquiries, then Bob finds Alice with certainty, i.e. $P_B = 0$. We can calculate the probability $P_{win}(1, n)$ that Bob finds Alice when Bob sends $m = 1$ inquiry as follows:

$$P_{win}(1, n) = 1 - P_A \cdot P_B = \frac{24 + 47 \cdot n - n^2}{24^2}. \tag{1}$$

Bob's expected profit equates to:

$$\Pi(1, n) = 10 \cdot P_{win}(1, n) - 1 = \frac{240 - 24^2 + 470 \cdot n - 10 \cdot n^2}{24^2}.$$

Thus, Bob should send an inquiry whenever $\Pi(1, n) \geq \Pi(0, n)$. We can solve this equation for n and get the result that this equation holds whenever $1.57 \leq n \leq 21.43$. Whenever Bob expects Alice to send over 1.57 inquiries and less than 21.43 inquiries, Bob should send at least one inquiry. Note that we derived this threshold under the assumption that the reply rate r equals unity. We can account for the reply rate being less than unity considering the probability P_C as follows:

P_C: If Bob does send an inquiry directly to Alice or one of the n players Alice inquired, Alice or one of the other players sends a reply with probability $r < 1$.

The probability P_C extends P_B as it is now possible that Bob does not find Alice even though he sends an inquiry directly to Alice, for example. We can denote the probability P_C by:

$$P_C = P_B + (1 - P_B) \cdot (1 - r) = 1 - r \cdot \frac{n + 1}{24}.$$

One can see that when the reply rate equals unity we are back to the case we calculated above and $P_C = P_B$. However, if the reply rate is less than unity the probability P_C that Bob does not find Alice is greater than P_B as players do not reply with certainty. When the reply rate equals zero the probability P_C that Bob does not find Alice equals unity. Intuitively, if nobody replies Bob cannot find Alice by sending inquiries himself. We can substitute P_C for P_B in equation (1) above and recalculate the interval stating when Bob should send at least one inquiry. For example, for a reply rate of $r = 0.75$ ($r = 0.5$) the interval becomes $2.59 \leq n \leq 20.41$ ($5.09 \leq n \leq 17.91$). Whenever Bob expects Alice to send over 2.59 (5.09) and less than 21.4 (17.91) inquiries, Bob should send at least one inquiry. Note that the interval is shrinking, as a lower reply rate implies that Alice needs to send more (less) inquiries to satisfy the lower (upper) bound of the interval. In other words, if Bob believes that other players do not always reply even though they know who the requested expert is, Alice needs to send relatively more inquiries so that it is worthwhile for Bob to send at least one inquiry. Furthermore, we can calculate that when Bob beliefs that the reply rate is less than ≈ 0.37 he should never send an inquiry independent of how few or many inquiries Alice sends.[3]

3.2 Network Perspective

How does the profit players can expect to make depend on the number of inquiries present in the network and the willingness of players to reply to inquiries? Figure 1 shows the expected profit as a function of the number of inquiries sent per player for random networks. We calculate the expected profit as follows.[4] First, we calculate the probability that Bob will find Alice by counting how many players know this and average this number over all combinations of possible experts. Second, we can calculate how many replies Bob will send, on average. Once we know the average number of messages sent, it is straightforward to calculate the expected profit. The orange star depicts the expected profit of the suggested network.[5]

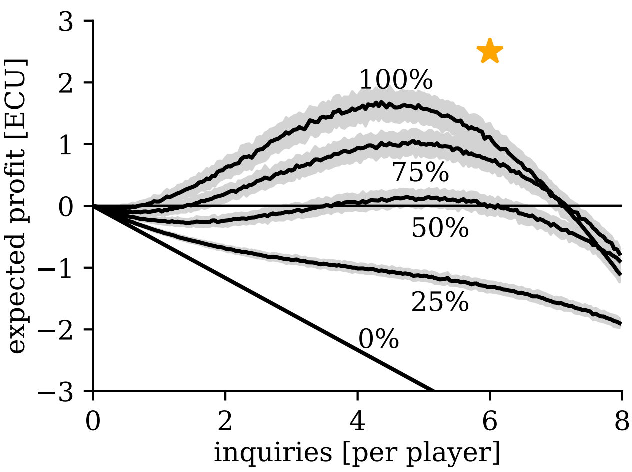

Figure 1: Expected Group Profit in ECU

We generate an ensemble of 1000 random networks for each number of total inquiries ranging from 0 to $8 \cdot 24$. Then we calculate the expected profit for each of the networks in the ensemble. The curves represent the mean of the ensembles, and the shaded areas represent the mean plus/minus one standard deviation. We calculate the expected profit by averaging over all possible question-expertise configurations for each network within an ensemble. We sum the profit of each player and take the average in each round to calculate the average group profit. We measure profit in Experimental Currency Unit (ECU, 1 ECU = 3 DKK).

One can see that when the reply rate equals zero then the expected profit is always negative. Players only find their experts when the expert inquires them through a direct inquiry. Therefore, expected profit linearly decreases in the number of inquiries sent. For example, if players send five inquiries, on

[3]Note that the strategy space, for a given reply rate, is simple enough to explore other Nash equilibria. In Appendix A.4, we show that for sufficiently high reply rates there exist non-trivial "active" Nash equilibrium states in addition to the "no-activity" Nash equilibrium.

[4]See Appendix A.3 for the derivation.

[5]See Appendix A.5 for the derivation.

average, then the expected profit per round equals ≈ -3 ECU. When the reply rate is greater than zero one needs to take the probability into account that Bob finds Alice through a reply sent by a player both Bob and Alice inquired. This non-linear effect creates the curvature of the expected profit curves in Figure 1. One can see that for a reply rate of less than $\approx$ 50% the network does not generate a positive profit under the assumptions we made. The highest profit occurs when players send $\approx$ 4.5 inquiries, on average, given a reply rate of unity. Finally, the expected profit would be significantly higher, compared to the curve that features a reply rate of 100%, if all players were following our suggestion perfectly - as we induce an efficient network structure. We suggest players to each other so that every player finds her expert in every round. That is, the vertical difference between the expected profit curve featuring a reply rate of 100% and our suggested network (orange star) denotes the effect of network structure.

4 Results

Figure 2 displays the average payoffs accumulated during the rounds played by 25 subjects in each of the four sessions (B1 and B2, in blue; N1 and N2, in orange). The payoff structure of our game suggests that subjects should remain inactive (gaining 0 ECU) unless they hold relatively strong beliefs about the number of inquiries their expert sends and others willingness to reply (see Section 3). In the following, we compare outcomes at round 51 as B2 is the session with the lowest number of rounds played among all sessions. In baseline sessions, our subjects do cooperate and play the game. They send, on average, three inquiries in round 1 (B1: 2.88; B2: 3.12) and 2.65 inquiries per round across 51 rounds (B1: 2.53; B2: 2.76). In B2 subjects earned, on average, 22.72 ECU (min: -35, max: 107) after round 51 and 80% of them can make a positive profit. In B1 subjects earned 18.12 ECU after the same number of rounds (min -45, max: 107) and 72% of them made a positive profit.

In nudging sessions, we suggest subjects to send six inquiries per round. However, subjects send, on average, four inquiries in round 1 (N1: 3.96; N2: 4.16) that is lower than what we suggested but significantly more than in baseline sessions (Wilcoxon Rank Sum test, p-value = 0.00323). Across all 51 rounds, on average, subjects sent 3.53 inquiries (N1: 3.22; N2: 3.85). Comparisons of all sessions show that all session means differ significantly from each other at the 1% significance level (Pairwise Wilcoxon Rank-Sum test, p-value < 0.001). This inquiry increase of $\approx$ 33.52% sent per subject almost doubles the cumulative profit. After 51 rounds, on average, subjects earned 39.12 ECU in N1 (min: -32, max 96) and 84% of them made a positive profit. In N2, subjects earned 40.36 ECU (min: -73; max: 105) and 88% of subjects made a positive profit.[6]

[6]Across all rounds of a respective session (B1: 86, B2: 51, N1: 73, N2: 57) subjects make a profit of 30.8 ECU in B1

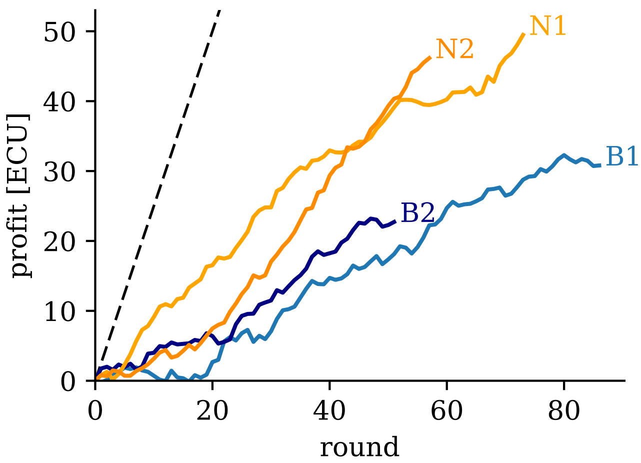

Figure 2: Average Profit (ECU) cumulated during each session (B1, B2, N1 and N2)

We sum the profit of each player and take the average in each round to calculate the average individual profit. We measure profit in Experimental Currency Unit (ECU, 1 ECU = 3 DKK). The number of rounds varies between sessions (86 rounds in B1, 51 in B2, 73 in N1 and 57 in N2). The dotted black line shows the expected profit if all subjects would follow our suggestion. See Appendix A.5 for the derivation of the suggested network.

Figures 3 shows the average number of inquiries sent per round during each of the four sessions. We can say that, on average, a $\approx$ 36.5% investment increase (inquiries and replies sent) in nudging sessions leads to a $\approx$ 94.6% increase in profit at round 51.[7] That is, the increase in information present in nudging sessions outweighs the additional cost of sending more messages. To which extent does the profit increase reflect a pure investment increase, and what is the effect of network structure? Note that our suggested network requires subjects to send more inquiries (i.e. invest more in the network) as well as an efficient network structure. We disentangle the effect of network structure from investment on individual profit and find that investment entirely drives profit differences between baseline and nudging sessions rather than network structure. See Appendix A.2 for a detailed discussion.

We examine to which extent subjects replied to the inquiries they received in stage 1 to determine the reply rate that we used to derive the theoretical benchmarks. As the number of replies any subject can send varies from round to round, we derive a quantity called *conditional informed reply rate*. We normalize the number of replies sent by the number of replies a subject could have sent. For each session this rate becomes B1:0.82, B2:0.87, N1:0.85, and N2:0.81, hence, common to all sessions most subjects replied reliably when they could. The *conditional informed reply rate* is similar in baseline and nudging sessions, that is, the profit increase in N1 and N2 compared to B1 and B2 cannot be explained by changes in subjects' willingness to reply. To learn more about replying behavior,

(min: -39; max: 139), 22.72 in B2 (min: -35; max: 107), 49.48 in N1 (min: -40; max: 123), and 46.16 in N2 (min: -54; max: 128). In total, 72% of subjects make a positive profit in B1, 80% in B2, and 84% in both N1 and N2.

[7] B1: (Cost, Reward, Profit) per round = (71.5, 80.4, 8.9); B2: (82.4, 93.5, 11.1); N1: (94.2, 113.3, 19.2); N2: (116, 136, 19.9).

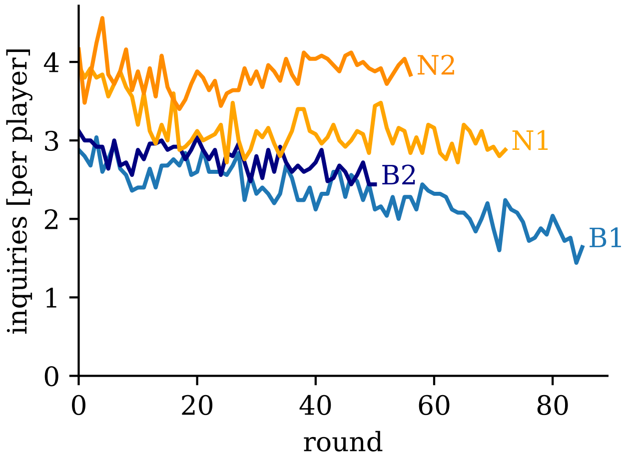

Figure 3: Average Investment for Sending Inquiries (in ECU) per session

We sum the number of inquiries of each player and take the average in each round to calculate the average investment. The number of rounds varies between sessions (86 rounds in B1, 51 in B2, 73 in N1 and 57 in N2). Note that inquiry cost corresponds to the number of inquiries sent per player in a round.

we asked subjects whom they replied to in a post experimental questionnaire (see Appendix A.6). The majority of subjects (58% in baseline sessions and 60% in nudging sessions) claimed they did not distinguish between communication partners they considered friends (defined as reply givers) and those considered acquaintances (defined as communication partners not expected to give replies) when sending replies. However, subjects valued replies they received themself as a sign of friendship. In particular, we asked subjects to score received replies on a scale from 0 to 10 where 10 indicates that subjects value received replies as a sign of friendship. The median subject in baseline (nudging) gives replies a score of 9 (9) - hence not significantly different from each other (p-value of a two sample wilcoxon rank sum test is 0.678).

To derive the theoretical benchmarks, we made the assumption that subjects never send *uninformed replies* of the form "I'm sorry, but I don't know anyone who is an expert in X". This is a relatively good approximation of the experimental findings. We can see that subjects only sent about 3% of the possible *uninformed replies*.[8] However, there are informed replies that cannot help the receiver win the round. If Alice is the expert that Bob is looking for, and they both send inquiries to each other, then it will be redundant for Alice to send a reply of the form "Yes, I'm the expert you are looking for". Subjects should avoid sending redundant replies like this if they were striving to optimize the group profit. The data shows that players sent these replies with a frequency of $\approx$ 75% whether or not they already revealed this by sending an inquiry to that same recipient in stage 1.[9] This does not affect group profit much as a player gets the opportunity to send such a reply only $\approx$ 2 times within 51

[8]The percentage is similar for each session (B1; B2; N1; N2) = (1%; 6%; 2%; 4%).
[9]Subjects sent replies of this form with a frequency of (B1; B2; N1; N2) = (75%; 77.4%; 80.9%; 67.7%).

rounds, on average. Therefore, if subjects did not use this type of reply, group profits would increase by approximately 5.44% in each session, on average.[10]

There is a large variation in how many inquiries subjects send and how many they receive (see Figure 7 in Appendix B). These two measures show a strong positive correlation, which is a sign that subjects prefer reciprocity. The sample Pearson correlation coefficients (Inquiries sent – Inquiries received) for each session are; B1: 0.73, B2: 0.75, N1: 0.51, and N2: 0.60.[11] We claim this because we expect a positive slope if subjects are more likely to contact subjects from whom they have received inquiries in a previous round. Interestingly, inquiry returns roughly mirror inquiry sending behaviour for higher sending rates, but even those sending few inquiries receive about 1.55 inquiries per round. For example, players who do not send any inquiry still receive inquiries from others' who explore the network. In the post experimental questionnaire, we asked subjects to score, on a scale from 0 to 10, whether they perceive received inquiries as a sign of friendship. The median subject in baseline (nudging) sessions gives inquiries a score of 5 (6) which is significantly different (p-value of a two sample wilcoxon rank sum test is 0.008). The network in nudging sessions is more profitable, hence, each inquiry generates a larger profit for subjects, on average. Therefore, subjects might perceive received inquiries to contain a stronger friendship signal.

We investigate to which extent subjects followed our suggestions over time to see whether our suggestions influence network formation even a long time after removal. The bottom panels of Figure 4 decompose the number of inquiries subjects sent in N1 and N2 into "suggested connections" (light + dark orange area) and "others" (light + dark gray area). In the initial five rounds, while we show the suggestions to subjects, almost every inquiry (96% in N1 and 91% N2) is sent to a suggested contact (light + dark orange area). Note, if subjects would have perfectly followed the suggestions, then they would have sent six inquiries each - one to each of the six suggested other subjects. After five rounds, when the suggestions are no longer visible to subjects, we see a sharp drop in the percentage of inquiries sent to suggested players. In rounds 6 to 51, the share of inquiries sent to suggested subjects remains fairly constant at 44% plus/minus 5% in N1 and 53% plus/minus 4% in N2. We would expect that, by chance, 25% of the inquiries would be sent along the suggested links even if we provided no suggestion. This follows because we suggest 6 out of 24 connections to each subject. However, we see that the actual share of inquiries sent to suggested players is significantly higher. We conclude that our initial suggestions influence network formation long after we removed them.

When we disentangle the reward generated from suggested subjects to the reward generated from

[10] In Appendix A.3, we show how it affects our measure of expected profit once we relax the assumption of informed replies.

[11] For the total number of messages sent (i.e. inquiries and replies) the sample pearson correlation coefficients for each session are: B1: 0.84, B2: 0.9, N1: 0.73, and N2: 0.85. See Figure 8 in Appendix B.

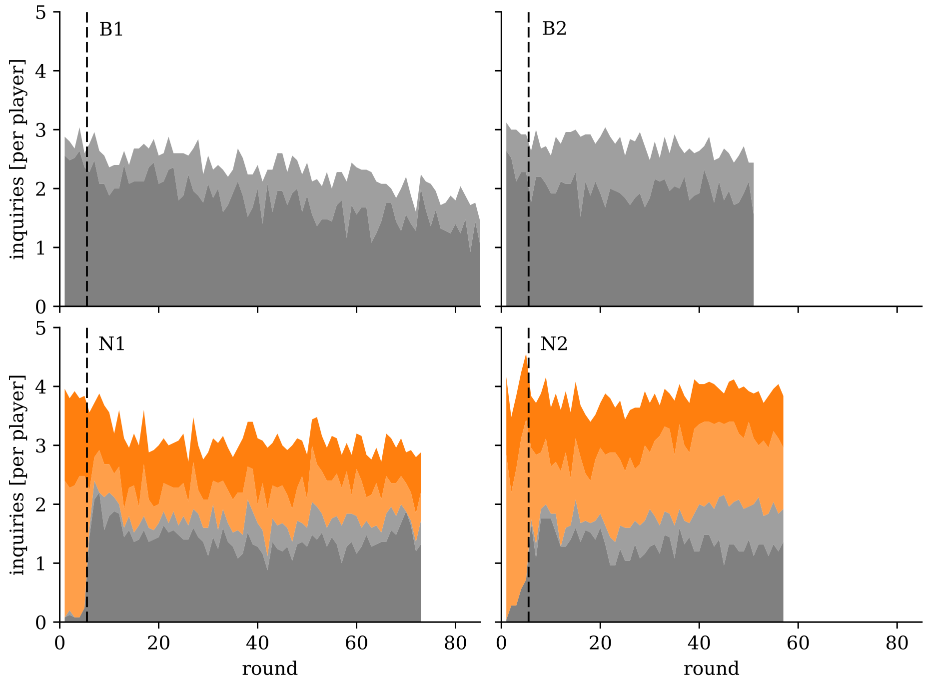

Figure 4: Decomposition of Inquires Sent

The figure shows the number of inquiries sent split by the number of suggested links (light and dark orange) and non-suggested links (light and dark gray) in baseline and nudging sessions as a function of the number of rounds played. The lighter areas (both orange and gray) depict bi-directional links, whereas the darker areas (both orange and gray) depict unidirectional links. The dotted black line depicts the center between the last round where suggestions are active (round 5) and the first round they are removed (round 6).

network links outside the suggested network, we find that for the first 51 rounds the share of reward generated within the suggested network is 48% in N1 and 56% in N2.[12] However, we find that suggested links are not more efficient than links subjects form with others' outside the suggested network. We calculate the relative return on investment (RRI) made by subjects in nudging sessions split into suggested and non-suggested links within 51 rounds. We calculate the RRI by summing the reward and dividing it by the number of inquiries (number of messages) sent as a measure of how much subjects invest in the respective group. In N1, the reward-inquiry ratio (reward-message ratio) equals 1.34 (1.14) among suggested links and 1.47 (1.27) among non-suggested links. In N2, the reward-inquiry ratio (reward-message ratio) equals 1.38 (1.14) among suggested links and 1.44 (1.21) among non-suggested links. It seems surprising that the RRI is lower inside than outside the suggested network since we specifically designed the suggested network to optimize the chance of winning.

One can explain this because suggested connections are bi-directional, that is, pairs of subjects simultaneously sending inquiries to each other. Inquiries sent along bi-directional links are less efficient because they represent redundant information sharing. We observe that there are more bi-directional

[12] A subject receives a reward (10 ECU) if a message informs a subject about who their expert is. If a player receives more than one of such messages in a single round then we divide the reward into equal fractions between subjects.

links then we would expect at random in far most of the rounds across all sessions (see Figure 9 in Appendix B). Therefore, we decompose suggested (non-suggested) links further into suggested (non-suggested) bi-directional links and suggested (non-suggested) unidirectional links. We can see that among suggested links in session N1 (N2) 61% (67%) are bi-directional in the first five rounds, on average. Whereas among non-suggested links only 11% are bi-directional in session N1 in the first five rounds, on average. Not a single non-suggested link is bi-directional in N2 within the first five rounds. After round five, the percentage of bi-directional links among suggested links drops to 43% (58%) in N1 (N2), on average. Among non-suggested links the percentage of bi-directional links increases to 19% (26%).

Finally, we can decompose the links in baseline session B1 and B2 in bi-directional and unidirectional links. We find that the majority of links 88% (82%) in B1 (B2) are uni-directional within the first five rounds, on average. This number decreases slightly to 76% (75%) in B1 (B2) after round five, on average. Hence, most links in baseline sessions where we do not provide a suggestion to subjects are unidirectional. One explanation for the slight decrease in the share of unidirectional links after round five is that subjects develop relationships which leads to the formation of bi-directional links. In summary, the share of bi-directional links among non-suggested ones is similar for baseline and nudging sessions. However, there is a large difference between sessions with respect to the number of bi-directional links. In nudging sessions the share of bi-directional links is not only larger, the large share of bi-directional links prevails at a fairly constant level until the end of the experiment.

5 Conclusion

We use the laboratory to create an environment with little incentives to cooperate and show that subjects cooperate and build their own communication network nevertheless. A subtle and non-binding nudge fosters cooperation and helps to sustain higher levels of cooperation, and thereby profit, even long after its removal. We conclude that the initial conditions are crucial in guiding a community into a productive and profitable state, leading to an increased information flow, and long-lasting relations between players. Across all sessions, even within almost every round, subjects form a higher than expected number of bi-directional relationships with others - a characteristic deeply rooted in the nature of human behaviour.

Our experiments mimic virtual online communication, which is becoming more and more common in a global economy largely based on accumulation and transfer of knowledge and expertise. Our interactive game highlights the complexity arising when the transaction success made with one individual depends on previous transactions made with others. Navigating the strategy space of such a game is complex, and individuals in our experiments likely acted more like intuition, rather than "solving" for the optimal solution during the course of the experiment. We here motivate, that, what at first might seem as "friendly" behavior, namely the pronounced willingness to send replies, could indeed be seen a fairly calculated trade of information: Offering a reply is a display of one's access to information and invites future inquiries from the transaction partner - increasing one's chance to profit at a later stage.

Our results could have powerful practical implications. When building professional teams, such as in business or academia, initial "buddy programs", kindly suggesting professional partnerships between employees would tie informal contracts between them. Thus, allowing the group to perform as an "information processing unit" long after the initial suggestion.

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A Appendix

A.1 The Suggested Network Algorithm

We split the 25 players into five groups of five. Each player can now be indexed with two integers $g, i \in \{1, 2, 3, 4, 5\}$, where g specifies which group the player is in and i is an index within the group. For a player with index (g, i) we add the following six connections:

1. Link the groups together as a ring: $(g, i + 1)$ and $(g, i - 1)$.

2. Inter group links: $(g + 1, 2i + 1)$, $(g + 2, 3i + 3)$, $(g + 3, 2i + 4)$, and $(g + 4, 3i + 2)$.

The indices should all be interpreted as modulo 5. That is, if an index is greater than 5 it loops around, for example $4 + 2 = 1 \mod 5$. It is clear that all players are treated symmetrically. It can be easily checked that the links are indeed bi-directional. Further, the network has diameter two, i.e. any two players are either directly connected or they have a common connection to whom they are both connected.

A.2 Graph Structure and Sending Behavior

To explain the difference between baseline and nudging sessions, we study the effect of network structure and sending behavior on group profit. Recall that the suggested network induces an efficient network structure, but simultaneously suggests subjects to send more inquiries. We use a measure of expected profit, which removes the effect of the inquiry network structure by re-shuffling of network links.

The expected profit is the mean profit that a player receives given the inquiry graph of a single round under the assumption that everybody sends informed replies with a reply rate equal to unity. We compute the average profit obtained by a player over all possible question/expertise configurations. The expected profit helps distinguish between the effects of the inquiry volume and network structure, because it allows us to directly compare the “efficiency” of the actual inquiry networks with randomised ensembles of the same inquiry volume.

The simulation works as follows. We preserve the number of inquiries sent in each round by each player, but the receivers are chosen at random with uniform probability. That is, this measure removes correlations due to relationships that players might form with each other over time. Ideally, the network structure could help increase expected profit, as redundancy could be reduced relative to inquiries sent without coordination.

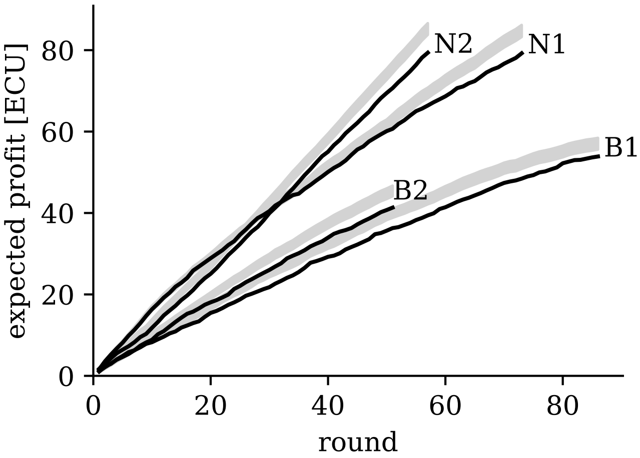


Figure 5: Expected Group Profit in ECU per Round

The figure shows the cumulated expected group profit averaged over all subjects within each session as a function of rounds (solid lines). The grey area shows the simulation outcome from 100 randomised game histories. The simulation preserves the number of inquiries sent by each subject in each round, but receivers are chosen at random with uniform probability. Each shaded area shows the mean plus/minus one standard deviation.

The contrary is the case for our experimental networks. The expected profit in any of the actual inquiry networks is consistently lower when compared to the ensemble where receivers are chosen at random with uniform probability. Whereas there are systematic differences in expected profits for the actual histories and the randomisation, these differences are small compared to those between sessions. We conclude that it is predominantly the amount of sending behaviour driving profit differences, not the network structure itself.

A.3 Expected profit with random inquiry receivers

Assume a directed inquiry network I_{ij} describing a single round of the game, where $I_{ij} = 1$ if player i sends an inquiry to player j, and $I_{ij} = 0$, otherwise. We denote inquiry in-degree of player i by $I_i^{in} = \sum_j I_{ji}$ and inquiry out-degree of player i by $I_i^{out} = \sum_j I_{ij}$. If player j is player i's expert, then the number of players who can send informed replies to player i is:

$$x_{ij} = I_{ij} + \sum_k I_{ik} \cdot I_{jk}.$$

The first term equals one when j can reply directly to player i and 0 otherwise. The second term adds one for every player k that receives an inquiry from player i and j. Player i wins the round if she receives a direct inquiry from player j, or by receiving at least one informed reply. Assume that there is a constant reply rate. Then the probability that player i does not receive any informed replies is given by $(1-r)^{x_{ij}}$. The probability that player i finds her expert is given by:

$$P_{win}(i,j) = 1 - (1-r)^{x_{ij}} \cdot (1 - I_{ji}).$$

For example, player i is certain to find her expert (i.e. $P_{win}(i,j) = 1$) if $I_{ji} = 1$ which means that her expert sends a direct inquiry to her so that $(1-r)^{x_{ij}} \cdot (1 - I_{ji}) = 0$. On the other hand, $P_{win}(i,j)$ solely depends on $(1-r)^{x_{ij}}$ if player i's expert does not sent a direct inquiry (i.e. $I_{ji} = 0$). For example, if the reply rate equals zero player i does not learn who her expert is with certainty (i.e. $P_{win}(i,j) = 0$). In the other extreme, when the reply rate equals unity player i learns who her expert is with certainty given that $x_{ij} > 0$. All players have the same probability of being player i's expert, so the average probability of player i winning is:

$$P(w_i) = \sum_{j \neq i} \frac{P_{win}(i,j)}{24}.$$

The last thing we have to calculate is how many replies player i must send, on average. We distinguish two cases. In the first case, we calculate expected replies under the assumption that players send informed replies. We can write the expected number of replies $E(R_i)$ as follows:

$$E(R_i) = \frac{I_i^{in2}}{24}.$$

For each inquiry player i receives, there is a $I_i^{in}/24$ chance that player i knows who the expert is. However, for bi-directional inquiry links, there is a $1/24$ chance that player i is herself the expert that player j is looking for. In this case, player i knows that her reply does not contain any new information. Therefore, it would be more efficient for the group profit if player i refrains from replying. This would change the number of expected replies as follows:

$$E(R_i) = \left(I_i^{in} - I_i^{rec}\right) \cdot \frac{I_i^{in}}{24} + I_i^{rec} \cdot \frac{I_i^{in} - 1}{24} = \frac{I_i^{in2} - I_i^{rec}}{24}.$$

The first term counts the number of non-reciprocal connections player i has with others' times the probability that player i knows who the expert is. That is, the first term denotes the chance of knowing who the expert is given that the connections are non-reciprocal. The second term counts the number of reciprocal links a player has with others' times the probability that the player knows who the expert is. Player i will reply to $r < 1$ of those, so that we can denote the expected profit of player i by:

$$\Pi_i = 10 \cdot P_{win}(i, j) - \left(I_i^{out} + r \cdot E(R_i)\right),$$

where the first term denotes the expected reward and the second term the expected cost consisting of the inquiry cost I_i^{out} and the expected reply cost $E(R_i)$. In essence, we can calculate the probability that player i will win given that player j is her expert by counting how many players know this. We can average this number over each of the possible experts. In a similar fashion, we can calculate how many replies player i will be able to send on average. When we know the chance of winning and the average number of messages sent, it is straightforward to calculate expected profit.

A.4 Alternative Group Dynamics

Let us consider a different type of idealised game dynamics. We will investigate the expected profit of a player who sends m inquiries in a round when all other players send n inquiries each. The expected profit is the difference between the expected reward and expected cost (inquiry and reply cost). The expected reward is 10 ECU times the probability of winning the round and the cost equates to 1 ECU for each message (inquiries and replies) sent:

$$\Pi_i(m, n) = 10 \cdot P_{win}(m, n) - m - E(R_i).$$

First, we assume that all receivers are chosen independently with uniform probability. Second, all players send informed replies and a reply rate of unity. Given these assumptions we can calculate the probability of winning:

$$P_{win}(m, n) = 1 - \frac{24 - m - n}{24} \prod_{k=1}^{n} \frac{25 - m - k}{25 - k}.$$

The average number of informed replies that player i sends is given by:

$$E(R_i) = \frac{n}{24} + 23 \cdot \left(\frac{n}{24}\right)^2,$$

where the first term counts the expected number of direct replies to players that seek your expertise. The second term counts the number of replies where you can refer players expertises to each other. Note that replies stemming from the first term do not contain any new information if a player already sent an inquiry. This happens with a probability of $m/24$, which changes the average number of replies as follows:

$$E(R_i) = \frac{n}{24} \cdot \left(1 - \frac{m}{24}\right) + 23 \cdot \left(\frac{n}{24}\right)^2.$$

Figure 6 visualises expected profits under the assumptions of informed replies. One can see that the group can produce more profit, on average, if all players maintain a high level of sending activity (i.e. if they send a lot of inquiries). However, players are trying to optimise their individual profit, rather

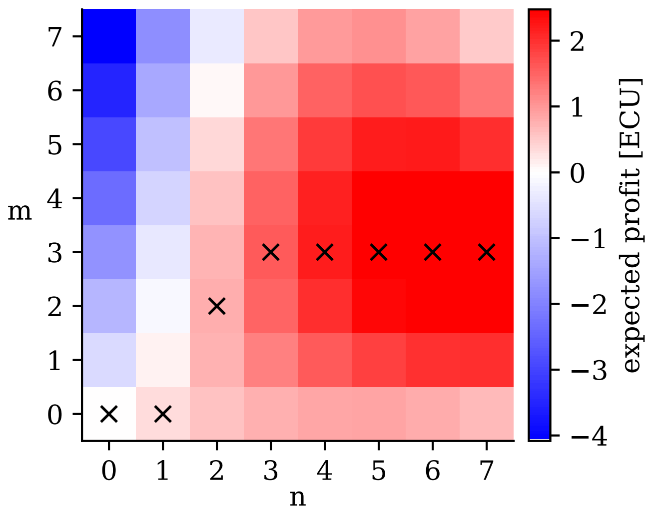


Figure 6: Expected profit of a player when all others' send n inquiries

We sum the profit of each player and take the average in each round to calculate the average group profit. We measure profit in Experimental Currency Unit (ECU, 1 ECU = 3 DKK). The calculation of expected payoff relies on two assumptions. First, all inquiry receivers are chosen at random from a uniform distribution. Second, all players send informed replies. The black x's indicate the optimal strategy for the players sending m inquiries, given all other players sending n inquiries.

than that of the group as a whole. What number of inquiries, m^*, will optimise a players individual profit given that the other players all send n inquiries? In an environment where all other players send $n = 0$, $n = 2$ or $n = 3$ inquiries, all players can optimise their individual profits by conforming to the group and sending the same number of inquiries (i.e. $m^* = n$). These states can be interpreted as Nash equilibria of this restricted version of the game. In contrast, states where all players send $n > 3$ inquiries each can be considered unstable, because the players have an incentive to reduce their sending activity. For $n = 4$ or $n = 5$ it is optimal to only send $m^* = 3$ inquiries, so one could expect the overall activity to drop to this level. A similar logic applies for $n = 1$, where every player would have an incentive to stop sending inquiries all together. Bringing the system into the completely silent state where all communication has perished.

The above considerations indicate that the game has a lower threshold for the overall inquiry activity, below which it is no longer meaningful for the players to keep playing. The value of the threshold might very well be different in the actual game, given that we have ignored the effects of non-random network structures. We also assume that all players except one send the same number of inquiries. However, we can make the qualitative prediction that the inquiry activity should start to consistently decay once it gets below some threshold.

A.5 Expected Profit of the Suggested Network

We provide a suggestion to subjects to create a situation where it is entirely up to themselves to maintain an efficient network. Our suggestion maximizes the expected profits for each subject, while ensuring an equal distribution of profits among subjects. The network we suggest is bi-directional (i.e. all links are reciprocal) and yield the same expected profits for each player (i.e. perfect equality) as opposed to a network that maximizes expected profits (e.g. perfect inequality in a star network). The network connects all 25 players such that each player receives the same expected profit independent of her position in the network. The local network (first degree neighborhood) of each position exhibits the same properties, so that each position receives the same profit in expectation.

To calculate the expected profit $\Pi_i(m, n)$ for each player in the suggested network within a given round, we make the following two assumptions. First, players perfectly follow our suggestion in stage 1 of each round which implies that each player sends the same number of inquiries, $\Pi_i(m, n) = \Pi_i(m, m) = \Pi_i(m)$. Second, we assume a reply rate of unity and that players only send informed replies in stage 2 of each round. Note, the two assumptions ensure that every player finds her expert in every round. Given that a player gains 10 ECU if she finds her expert, the expected reward per player equates to 10 ECU per round.

We calculate the expected cost of sending inquiries and replies for a given round. Calculating the expected cost of sending inquiries is straightforward as 25 players send six inquiries each round according to our suggestions. Each inquiry costs 1 ECU, hence the expected cost of sending inquiries equates to six ECU per player. We calculate the expected reply cost $E(R_i(m))$ as follows. For each inquiry a player receives, the probability $P_R(m)$ that the requested expert is one of the players she is linked to equates to:

$$P_R(m) = \frac{m}{N-1},$$

using our definition of informed replies where N denotes the total number of players. However, for bi-directional inquiry links, there is a 1/24 chance that player i is the expert herself, in which case the reply does not contain any new information. We can account for this and rewrite the probability as follows:

$$P_R(m) = \frac{m-1}{N-1}.$$

Note that it is $m - 1$ as a player cannot send a reply to someone who is already linked to her. Remember when the link is bi-directional, the knowledge about question and expertise pairs is already transferred through inquiries and part of the expected cost of sending inquiries. This being the case, we only have to take the probability into account that one has to refer her direct links to each other. Putting everything together, the expected reply cost for each player at any given round is:

$$E(R_i(m)) = P_R(m) \cdot m.$$

The final step is to insert the parameters specific to the game. The experiment features $N = 25$ players which send six inquiries to each of the six suggested players. This yields expected reply costs of $E(R_i(m)) = 1.5$ ECU per player and round, using the assumption of informed replies. The expected profit $\Pi_i(m)$ of each player, therefore, equates to 2.5 ECU per round. Lastly, we allow for the possibility that a reply sent along a bi-directional link might not contain any new information. In this case, expected reply cost equate to $E(R_i(m)) = 1.25$ ECU per player which yields an expected profit of $\Pi_i(m) = 2.75$ ECU per player and round.

A.6 Questionnaire Results

In the final part of the experiment, subjects must answer a questionnaire (see Questionnaire in Appendix B). which allows us to gain further insights into differences between baseline (B1, B2) and nudging (N1, N2) sessions. Characteristics like age and gender are not able to explain the differences between baseline and nudging sessions. Subjects have a mean (median) age of 25.27 (25) in baseline sessions and 24.84 (23) in nudging sessions. In total, 48 males and 52 females participated in our experiment with a gender ratio (female/male) of 1.174 in baseline sessions and 1 in nudging sessions.

Personality. The questionnaire contains a set of 30 questions to predict certain personality characteristics of subjects which allows us to examine whether personality can explain differences between baseline and nudging sessions. We selected three personality measures (Honest-Humility, Agreeableness versus Anger, and Extraversion) from the HEXACO-60 inventory that assesses the six dimensions of personality structure of the HEXACO model (Ashton and Lee, 2009). We find no significant differences in personality between subjects in baseline and nudging sessions, suggesting that differences in personality are not driving our results. For Agreeableness versus Anger, we find that the average subject in baseline (nudging) scores, on a scale from 1 to 5, 3.16 (3.15) which is not significantly different (p-value of a two sample t-test is 0.94). For Extraversion, we find that the average subject in baseline (nudging) scores 3.33 (3.5) which is not significantly different (p-value of a Welch two sample t-test is 0.165).[13] For Honest-Humility we find that the average subject in baseline (nudging) scores 3.408 (3.398) which is not significantly different (p-value of a two sample t-test is 0.933).

Game Experience. This part of the questionnaire asks subjects about their experiences with the others throughout the game as well as about features of the game itself. The average subject in baseline (nudging) has 5.15 (5.39) acquaintances and 4.29 (4.57) friends. 56% of subjects reported that their group of friends never changed throughout the game (60% in baseline and 52% in nudging), 29% reported that their group of friends changed every 10 rounds (24% in baseline and 34% in nudging), 13% reported that their group of friends changed faster than every 10 rounds (12% in baseline and 14% in nudging).[14] The average (median) subject reports that it took at least $\approx$ 13 (13) rounds to find friends in baseline and $\approx$ 13 (11) rounds in nudging.[15] In baseline (nudging), 26% (26%) of subjects indicated that they found friends in less than 10 rounds, 44% (40%) needed between 10 and 19 rounds to find friends, 18% (28%) reported that they needed more than 20 rounds to find friends, 8% (6%) indicated that they never found friends, and 4% (0%) are missing.

We asked subjects to score, on a scale from 0 to 10, whether they perceive received inquiries or replies from others as a sign of friendship. The median subject in baseline (nudging) gives inquiries a score of 5 (6) which is significantly different (p-value of a two sample wilcoxon rank sum test is 0.008).[16] The median subject in baseline (nudging) gives replies a score of 9 (9) which is not significantly different from each other (p-value of a two sample wilcoxon rank sum test is 0.678). The results show that subjects in baseline and nudging value replies more than inquiries as a sign of friendship. The increase in the value of inquiries in nudging sessions potentially reflects that the networks in nudging sessions generate more profit then the networks in baseline sessions (see Section 4). We further ask subjects if they prefer to send inquiries to friends, acquaintances, or neither of both. 75% of subjects indicate that they prefer friends (68% in baseline and 82% in nudging), indicating that inquiries carry a strong friendship signal. 73% out of the 75% of subjects who prefer friends also indicate that they prefer some friends over others when sending an inquiry. Replies, however, tell a different story. 59% of subjects (58% in baseline and 60% in nudging) indicate that they have no preference over sending

[13]Here we select the Welch two sample t-test, because an f-test shows that the two sample variances are significantly different from each other (p-value of 0.02).

[14]4% of questionnaire answers (two subjects) are missing in baseline sessions due to a data storage error.

[15]To calculate the mean and median we exclude subjects who reported "never" and treat reports of > 25 as 25.

[16]We choose a non-parametric test because the underlying data in baseline and nudging sessions is not normally distributed. For individual sessions (B1; B2; N1; N2) subjects gave inquiries a mean value of (4.5; 4; 5.5; 5.6) and a median value of (4; 5; 5; 6).

replies. That is, the majority of subjects reply independently of whether they consider the individual they are about to help as a friend or an acquaintance. Out of the 33% (30% in baseline and 36% in nudging) who prioritize friends as the receiver of a reply, 61% report that they have no preferences among their friends.

General Questions. We asked subjects whether the game is good at mimicking actual communication, e.g. email communication in an actual business, on a scale from 1 to 10. The average (median) subject gives our game a score of 5.6 (6). Furthermore, our game features a timer that ends a round if the last three subjects need more than 15 seconds to make their decision to prevent the game from stalling. We were worried that this might influence the decisions of subjects so we asked them whether the timer influenced their decisions in the game. 90% of subjects indicated that the timer did not (52%) or just marginally (38%) affect their decisions, confirming that the timer is not responsible for the outcome of the game. Lastly, we asked subjects to indicate whether they were surprised that the game ended since we set a time limit of 90 minutes instead of letting them play a fixed number of rounds. 90% of subjects indicated that they were not surprised (52%) or a little surprised (38%), showing that there might be an endgame effect in the last rounds of our game. However, the data does not confirm such an endgame effect.

B Additional Figures and Laboratory Material

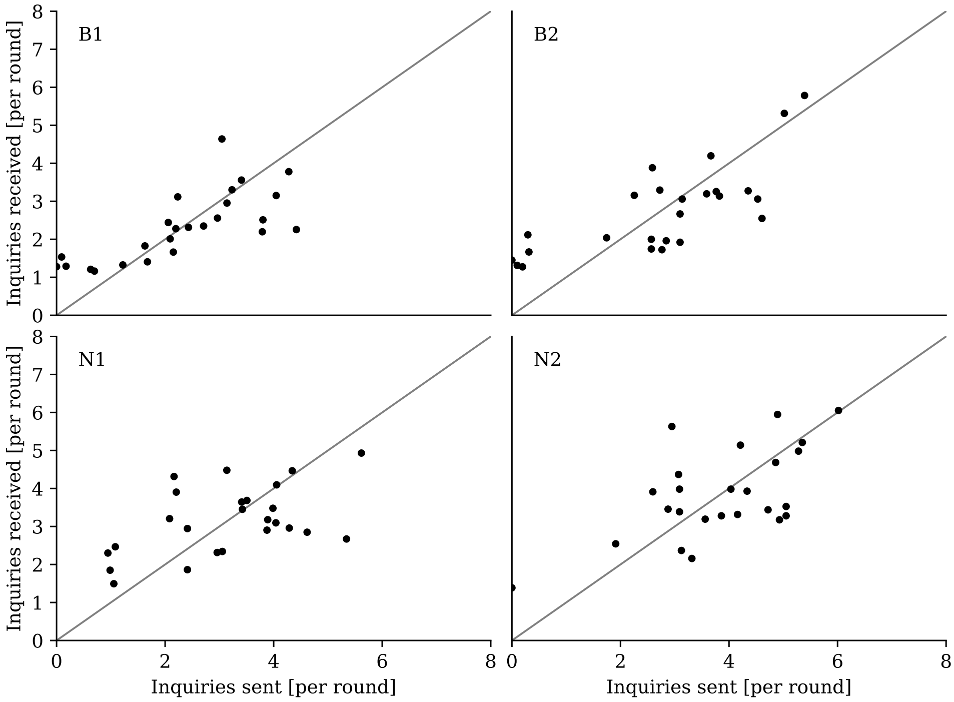


Figure 7: Reciprocal Inquiry Sending Behavior

The scatter plots show the number of inquiries received as a function of the number of inquiries sent, normalized by the number of rounds played in the respective session. The number of rounds varies between sessions (86 rounds in B1, 51 in B2, 73 in N1 and 57 in N2). Each dot represents one out of 25 players for baseline sessions (B1 and B2) and nudging sessions (N1 and N2). The black line denotes the 45 degree line.

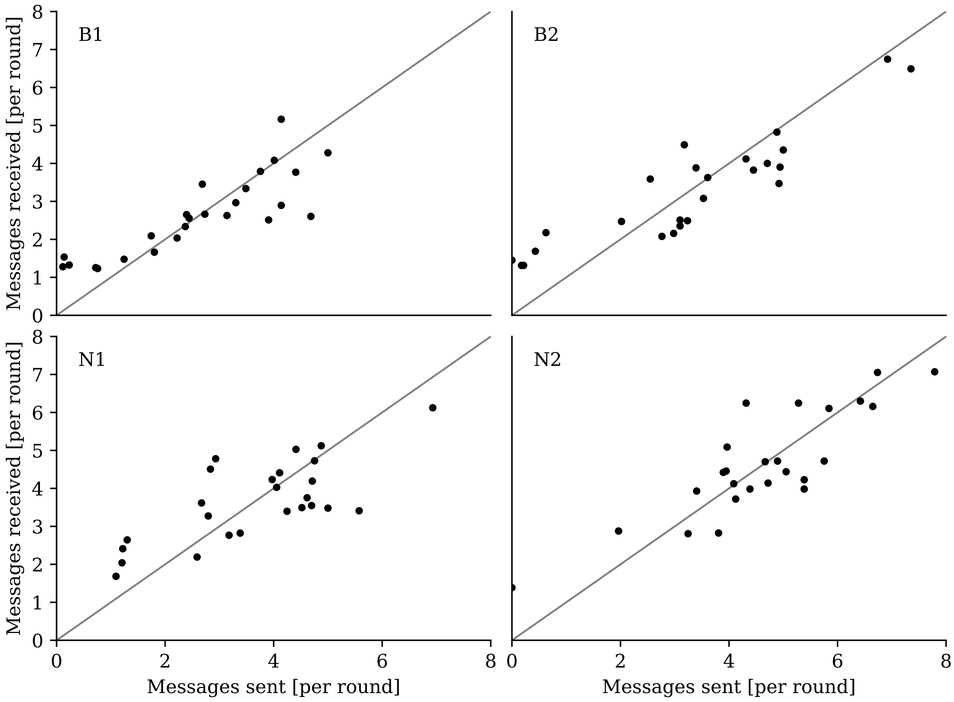


Figure 8: Reciprocal Message Sending Behavior

The scatter plots show the number of messages received as a function of the number of messages sent, normalized by the number of rounds played in the respective session. The number of rounds varies between sessions (86 rounds in B1, 51 in B2, 73 in N1 and 57 in N2). Each dot represents one out of 25 players for baseline sessions (B1 and B2) and nudging sessions (N1 and N2). The black line denotes the 45 degree line.

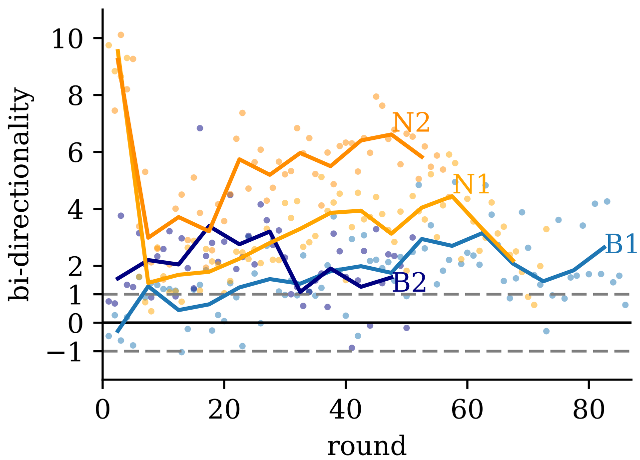


Figure 9: Statistical over-representation of bi-directional links

We generate an ensemble of 100 random inquiry networks, for each round in each session, preserving the in- and out-degrees of the actual (data) networks. We measure the mean and standard deviation of bi-directional links in the ensembles - again for each round in each session. The y-axis shows the standard deviation of our normalized bi-directionality measure. We construct this measure by subtracting the ensemble mean from the actual number of bi-directional links and divide by the standard deviation. The black line shows the mean plus/minus one standard deviation. Dots represent individual rounds and lines show 5 round averages.

I copy part of the following laboratory material from Busch (2018) as the two baseline sessions were part of my Master's Thesis. However, the data for the nudging sessions were collected and analyzed during my Ph.D. studies.

Game Instructions (Note that these Instructions resemble the Tutorial)

Please read through this tutorial carefully to understand the rules of this experiment. Note that at the end of the tutorial you must answer some questions. You can proceed with the experiment only after having correctly answered all the questions. It is important that you understand the “Game” since it is likely that you earn more money when you understand the rules.

Payment

In this experiment, you can earn ECU (Experimental Currency Unit) that will be converted in DKK (Danish krone) at the end of the experiment. The exchange rate is 1:3, which means that for each ECU you have after the experiment, you will receive 3 DKK.

At the end of the experiment you will be payed what your balance is plus 50 DKK as a show-up fee. For example, if your balance is 150 ECU at the end of the game you receive 500 DKK (3x150 + 50).

If you leave before the Experiment is finished you will not receive any money.

Objective

This experiment consists of a series of rounds. In the beginning of each round (stage 1) you will be assigned a "**Question**" and an "**Expertise**".

Your assigned "**Question**" is indicated by a letter (e.g. A, B, C ....). For example, if you are assigned with question A, your goal is to find the player who has the corresponding expertise (In this case a player with expertise A).

Your "**Expertise**" is indicated by a different letter (e.g. if your question is A it is not possible that your expertise is A). For example, if you are assigned with expertise B, some other player assigned with question B is searching for you.

During each round, your goal is to find out which of the other players is the "**Expert**" for your "**Question**". If you succeed, you will earn 10 ECU.

How to play

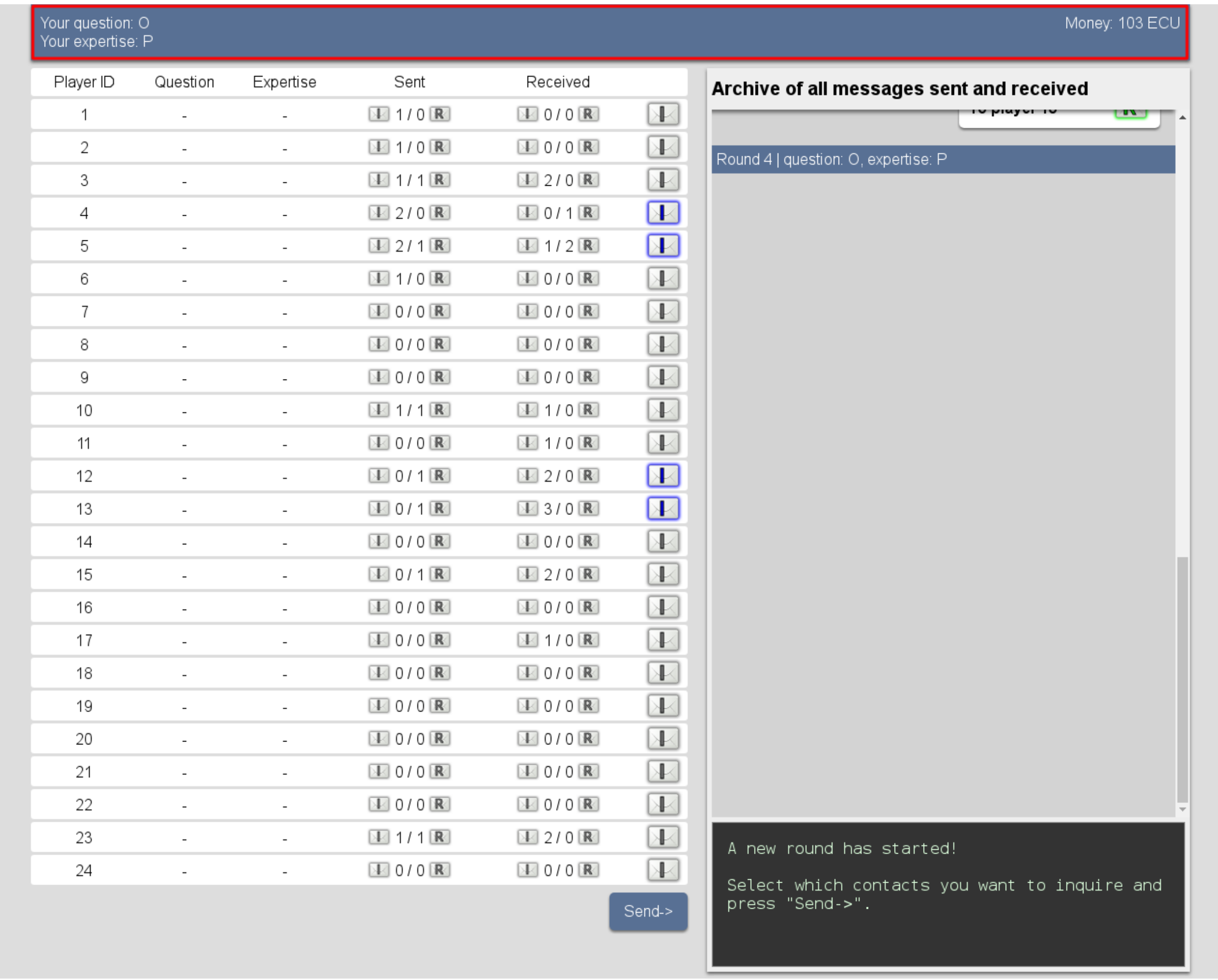
You communicate with the other players by sending and receiving messages. It costs 1 ECU to send a message.

Each round has 2 stages:

- In the first stage, you can send "**Inquiries**" which reveal your question and expertise to the receiver, and ask for help finding your expert. If multiple players send inquiries to you, you might learn that one is the expert another player is searching for.
- In the second stage, you can "**Reply**" to the inquiries you received from the first stage. If you reply to an inquiry, and you know who the expert is, you help the receiver fulfill her task.

In each stage, you may send messages to as many players as you want (but it costs 1 ECU to send each message). The messages will be delivered simultaneously at the end of each stage when all players are finished sending messages.

In the following, you will be guided through an example, which teaches you the game interface and its functions.



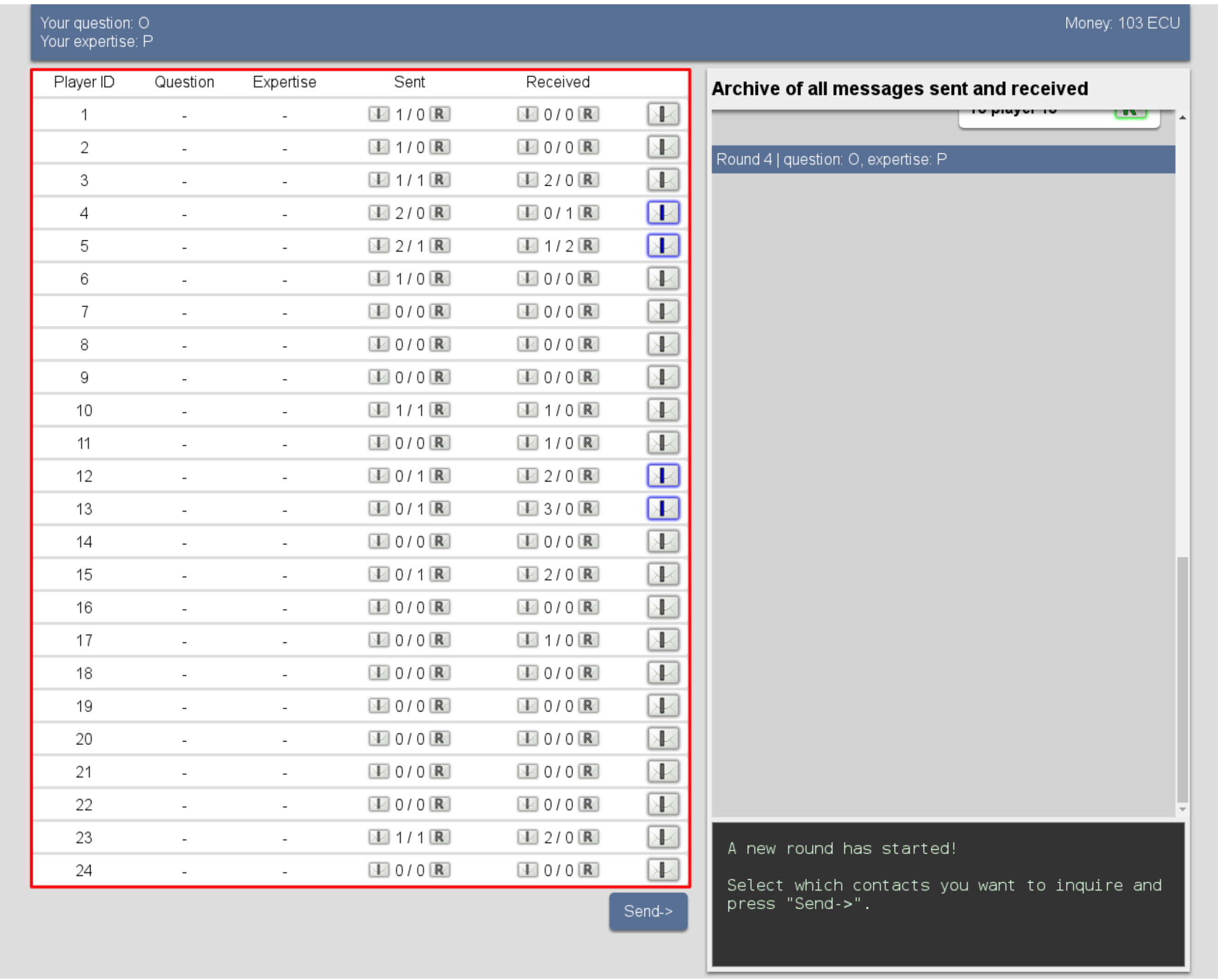
At the top of the screen you can find information about your current state in the game.

- To the left you will find your own "**Question**" and "**Expertise**".
- To the right you can see how much ECU you currently have. When the experiment begins you have 100 ECU.

In this example, your current "**Question**" is "**O**" and your "**Expertise**" is "**P**". This means that during this round you have to find the player with expertise "**O**" to earn 10 ECU. The player with question "**P**" has to find you to earn 10 ECU for himself.

In the example you have 103 ECU, in the beginning of the fourth round. This means that during the first three rounds, you have won 3 ECU more by finding your expert, than you have spent sending messages.

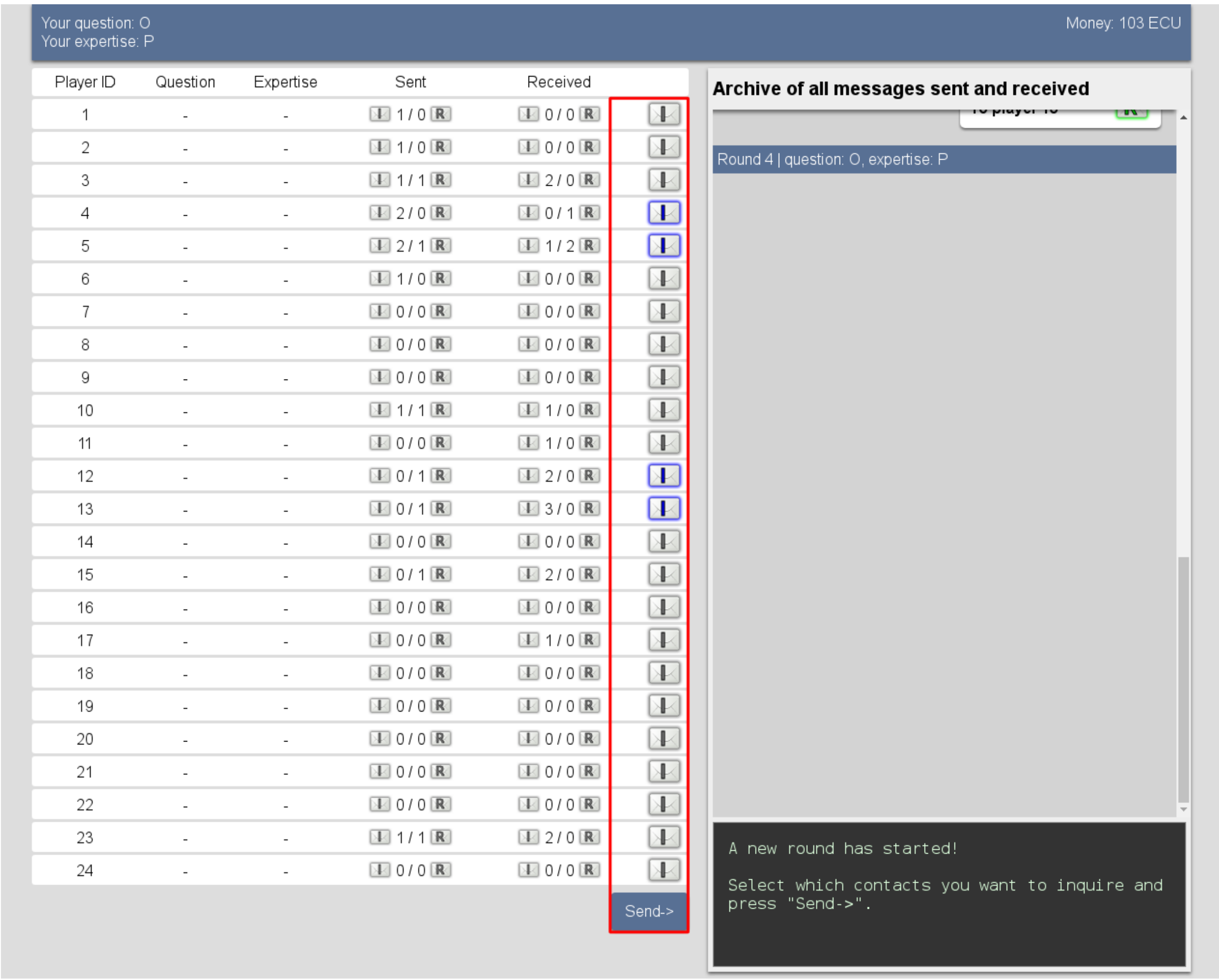
Notice that the panel on top of the screen is **blue** whenever you are expected to make an action, and **grey** when you are waiting for the other players. A *bell*-sound will be played whenever a new stage begins, i.e. when the top-panel changes color from **grey** to **blue**.



Player ID	Question	Expertise	Sent	Received
1	-	-	1 / 0	0 / 0
2	-	-	1 / 0	0 / 0
3	-	-	1 / 1	2 / 0
4	-	-	2 / 0	0 / 1
5	-	-	2 / 1	1 / 2
6	-	-	1 / 0	0 / 0
7	-	-	0 / 0	0 / 0
8	-	-	0 / 0	0 / 0
9	-	-	0 / 0	0 / 0
10	-	-	1 / 1	1 / 0
11	-	-	0 / 0	1 / 0
12	-	-	0 / 1	2 / 0
13	-	-	0 / 1	3 / 0
14	-	-	0 / 0	0 / 0
15	-	-	0 / 1	2 / 0
16	-	-	0 / 0	0 / 0
17	-	-	0 / 0	1 / 0
18	-	-	0 / 0	0 / 0
19	-	-	0 / 0	0 / 0
20	-	-	0 / 0	0 / 0
21	-	-	0 / 0	0 / 0
22	-	-	0 / 0	0 / 0
23	-	-	1 / 1	2 / 0
24	-	-	0 / 0	0 / 0

To the left you will find the ID of all other players (from 1- 24) and information about their "**Question**" and "**Expertise**" in the current round. Note that the number of other players in the session today might be different from this example.

Important: Question and expertise will change at the beginning of each round but the player ID will remain fixed throughout the entire experiment. This means that your ID and the other players ID will not change (e.g. player 17 will be player 17 through the entire experiment). The only thing that changes between rounds is your and other players "**Question**" and "**Expertise**".



In stage 1 you can send "**Inquiries**" to the other players.

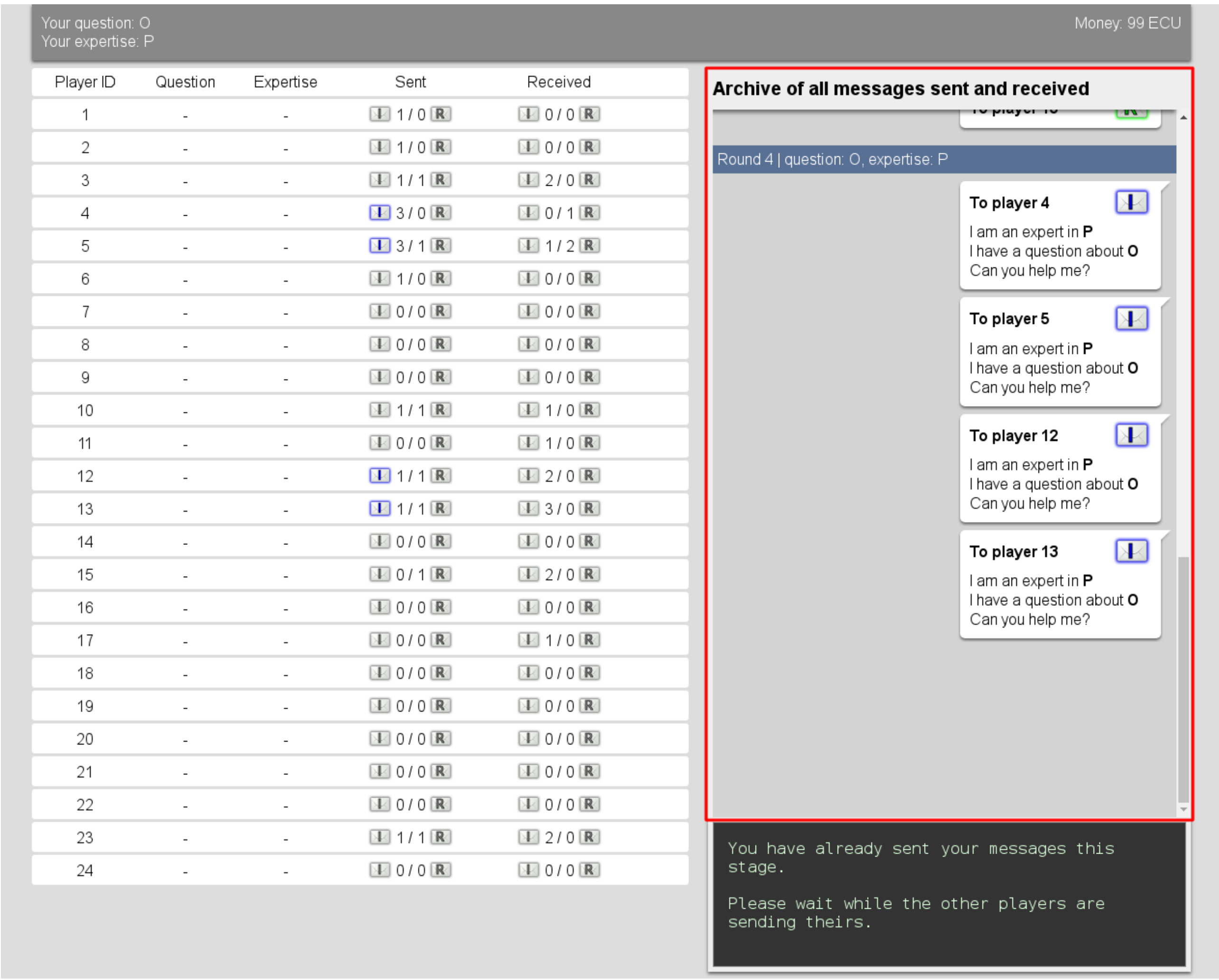
To select who you will send "**Inquiries**" to you click the inquiry-icon  in the right end of each player label.

When clicked, the color of the icon will change from grey to blue  indicating that the player is selected. In the example, you have selected players 4,5,12 and 13 (Note that it is possible to undo your choice i.e. you can press on a blue icon and it turns grey again).

Sending an "**Inquiry**" costs 1 ECU. Remember, you must pay this cost for each inquiry you send. For instance, if you send your inquiry to 4 different players you must pay 4 ECU.

Once you have made your choice you must press the "**Send->**"-button to send your inquiry to the selected players (i.e. player with a blue icon ).

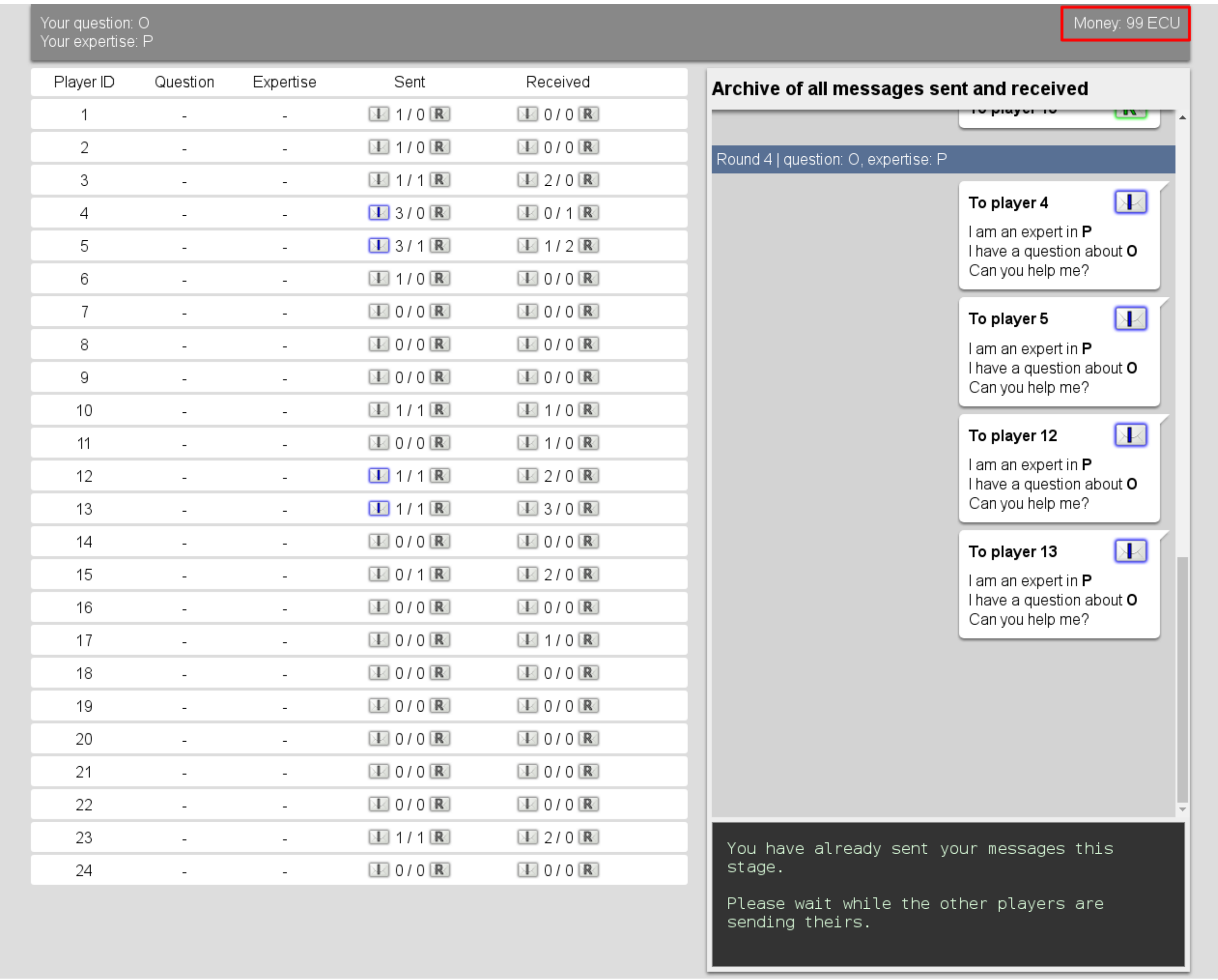
When you have sent your inquiries, you have to wait for the other players to send theirs too. All inquiries will be delivered simultaneously after all players have clicked "**Send->**".



Here you can see the messages you sent and received during the entire experiment.

In the example, in the current round (round 4) you have sent "**Inquiries**" to players 4,5,12, and 13, and you are now waiting for the other players to send their messages, before the game can continue. Notice that the top panel is grey because you are waiting for the other players.

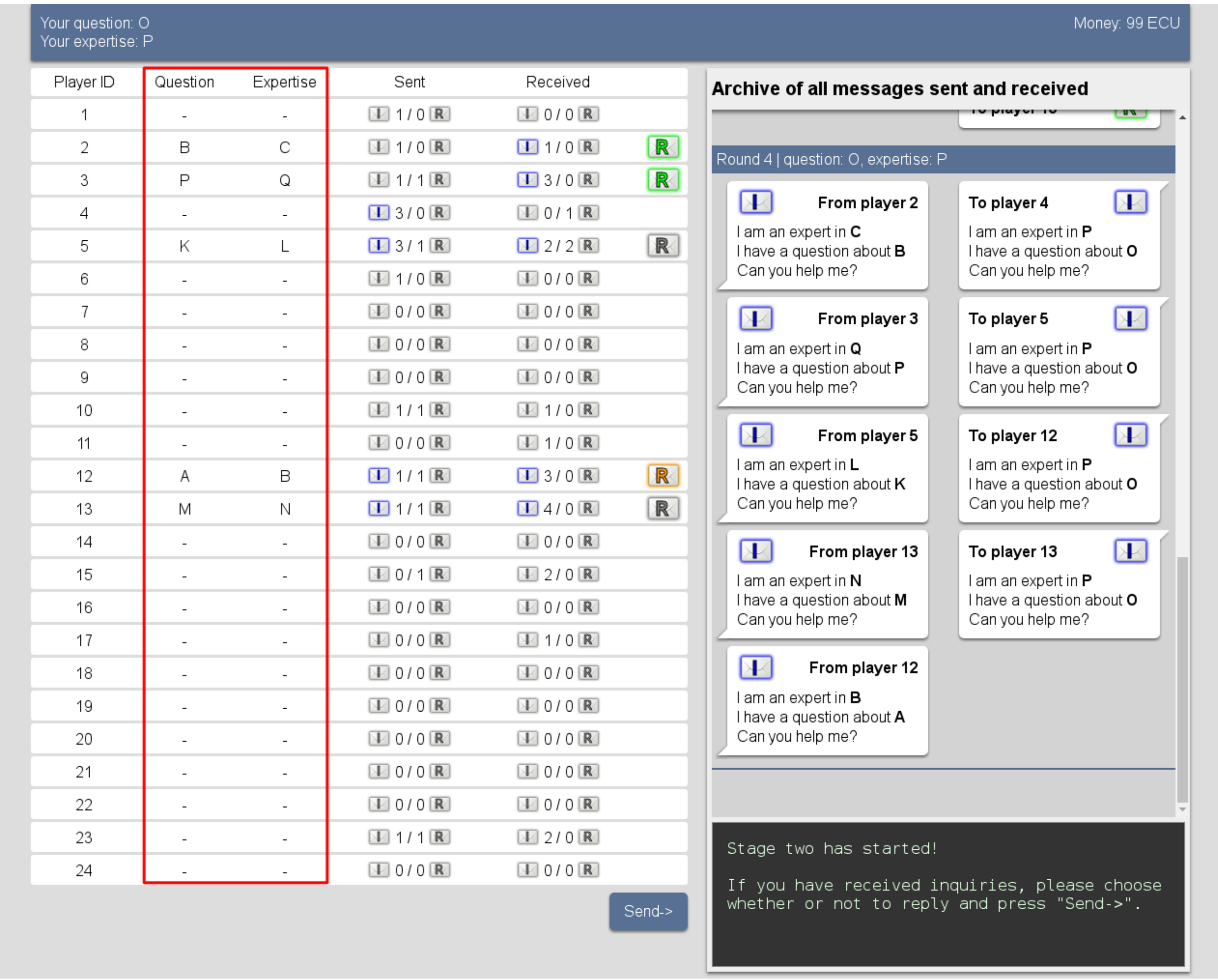
To see the messages you sent and received in the previous rounds you can scroll in the "Archive of all messages sent and received".



Player ID	Question	Expertise	Sent	Received
1	-	-	1 / 0	0 / 0
2	-	-	1 / 0	0 / 0
3	-	-	1 / 1	2 / 0
4	-	-	3 / 0	0 / 1
5	-	-	3 / 1	1 / 2
6	-	-	1 / 0	0 / 0
7	-	-	0 / 0	0 / 0
8	-	-	0 / 0	0 / 0
9	-	-	0 / 0	0 / 0
10	-	-	1 / 1	1 / 0
11	-	-	0 / 0	1 / 0
12	-	-	1 / 1	2 / 0
13	-	-	1 / 1	3 / 0
14	-	-	0 / 0	0 / 0
15	-	-	0 / 1	2 / 0
16	-	-	0 / 0	0 / 0
17	-	-	0 / 0	1 / 0
18	-	-	0 / 0	0 / 0
19	-	-	0 / 0	0 / 0
20	-	-	0 / 0	0 / 0
21	-	-	0 / 0	0 / 0
22	-	-	0 / 0	0 / 0
23	-	-	1 / 1	2 / 0
24	-	-	0 / 0	0 / 0

In the example you spent 4 ECU for sending 4 inquiries. The balance in the upper part of the screen is now 99 ECU (103 – 4 ECU).

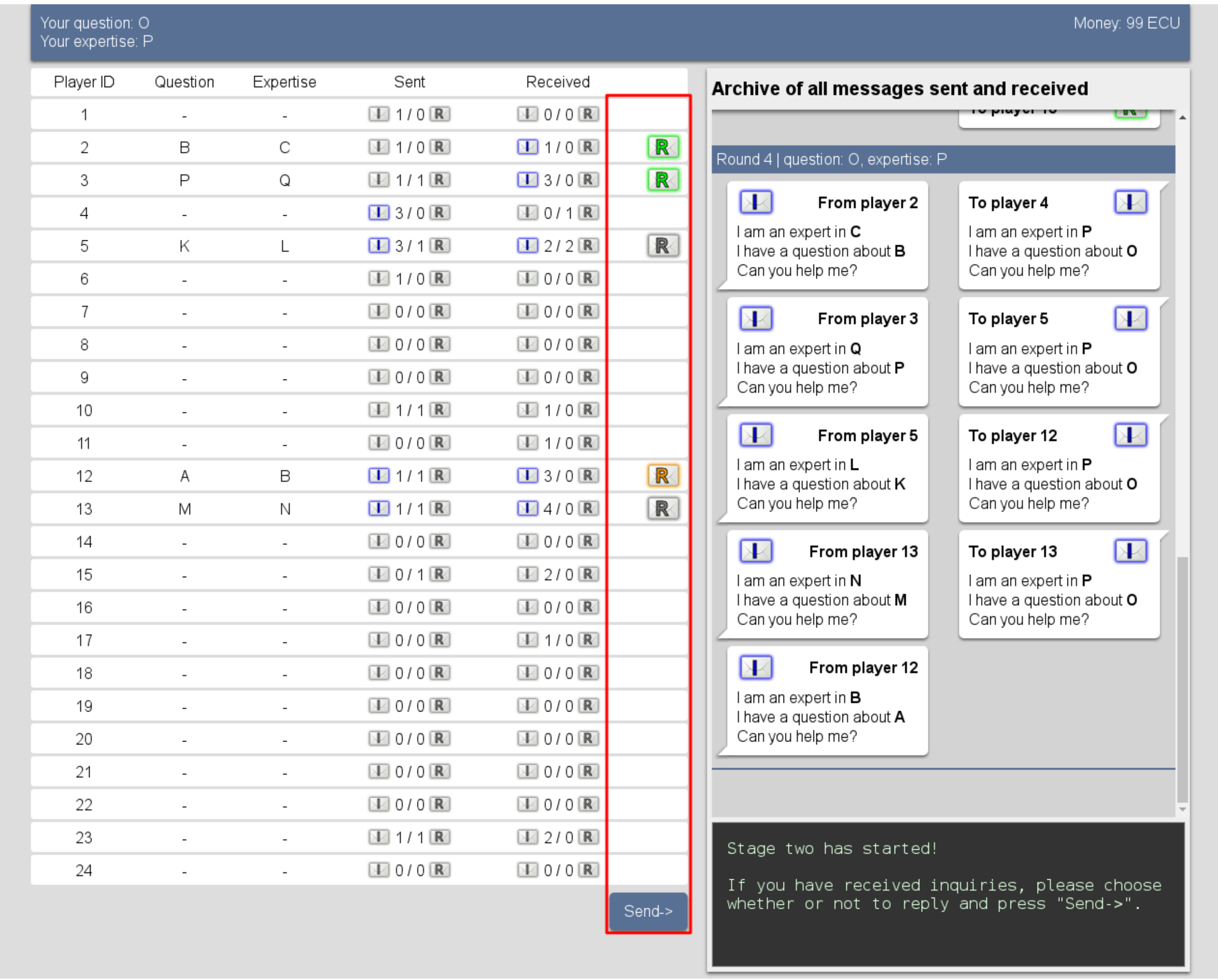
Note that it is not possible to send messages, if their combined price is higher than your current balance.



When the last player has clicked send, you will receive the "Inquiries" that other players have sent to you.

When you receive an inquiry from another Player, her "**Expertise**" and her "**Question**" will appear in the player table.

In the example, you received inquiries from players 2, 3, 5, 12, and 13. Now you know, for example, that in the current round player 2 has a question "**B**" and is an expert in "**C**".



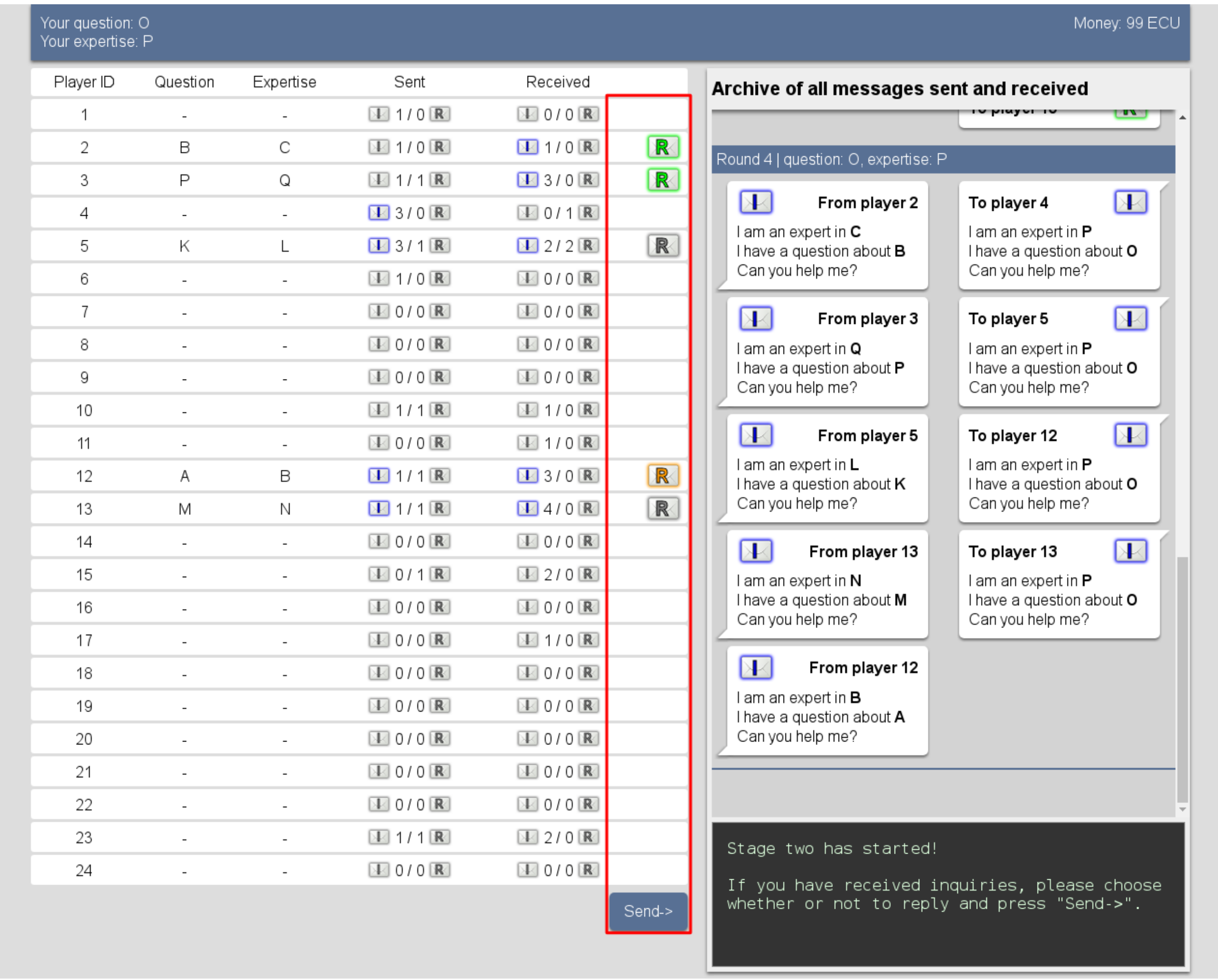
Stage 2 starts immediately after the inquiries sent in stage 1 are delivered. Note that there are now reply icons to the right of the players table. An icon will appear only next to the players from whom you received inquiries. In the example, that are players 2, 3, 5, 12, and 13.

In this stage, if you want, you can reply to the other players' inquiries. Sending a reply can help the receiver find her expert. You may reply to any inquiry you have received. Each reply costs 1 ECU.

You can send two types of (automatically generated) replies:

1. If you know who has an "**Expertise**" matching the inquirer's "**Question**", your reply will say: The expert you are looking for is "Experts player ID", and the reply symbol will be green .
2. If you do not know it, it will say: I'm sorry, but I don't know anyone who's an expert for this "**Question**", and the reply symbol will be orange .

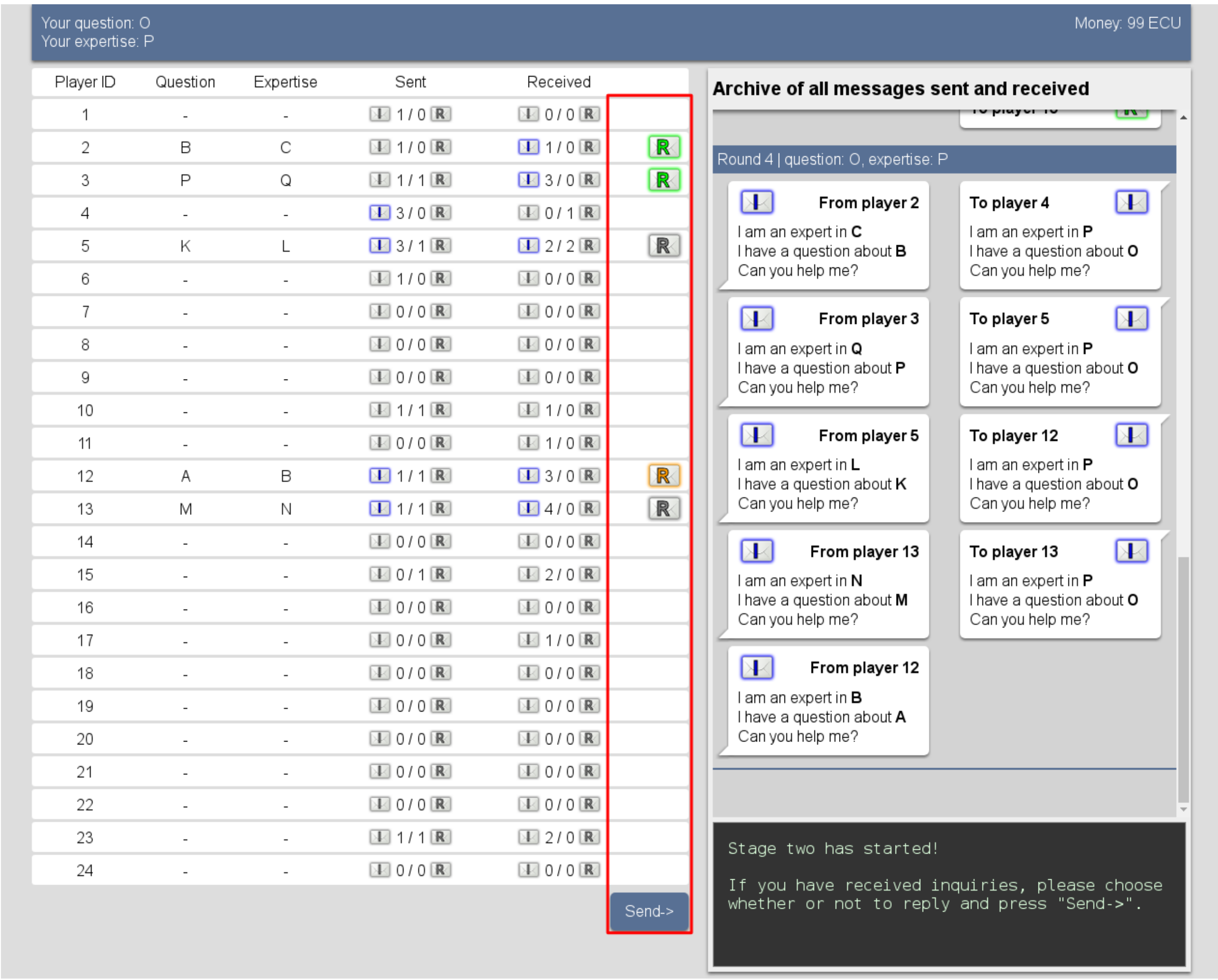
You select which players to reply to by pressing grey reply icons in the right end of the player-labels. A player is selected when the icon changes color. Then press "**Send->**" to actually send the replies.



In this example, you are about to send 3 replies (to players 2, 3, and 12) which will cost 3 ECU in total (3 x 1 ECU). You can send as many or as few replies as you want.

- The reply to player 2 is green, because player 2 has a question about "**B**" you know that player 12 is the expert in "**B**". The reply text will be: "The expert you are looking for is player 12".
- The reply to player 3 is also green, because player 3 has a question about "**P**" and you are an expert in "**P**". The reply text will be: "Yes, I happen to be the expert you are looking for".
- The reply to player 12 is orange because player 12 has a question about "**A**", but you don't know who is an expert in "**A**". The reply text will be: "I'm sorry, but I don't know anyone who's an expert in **A**".

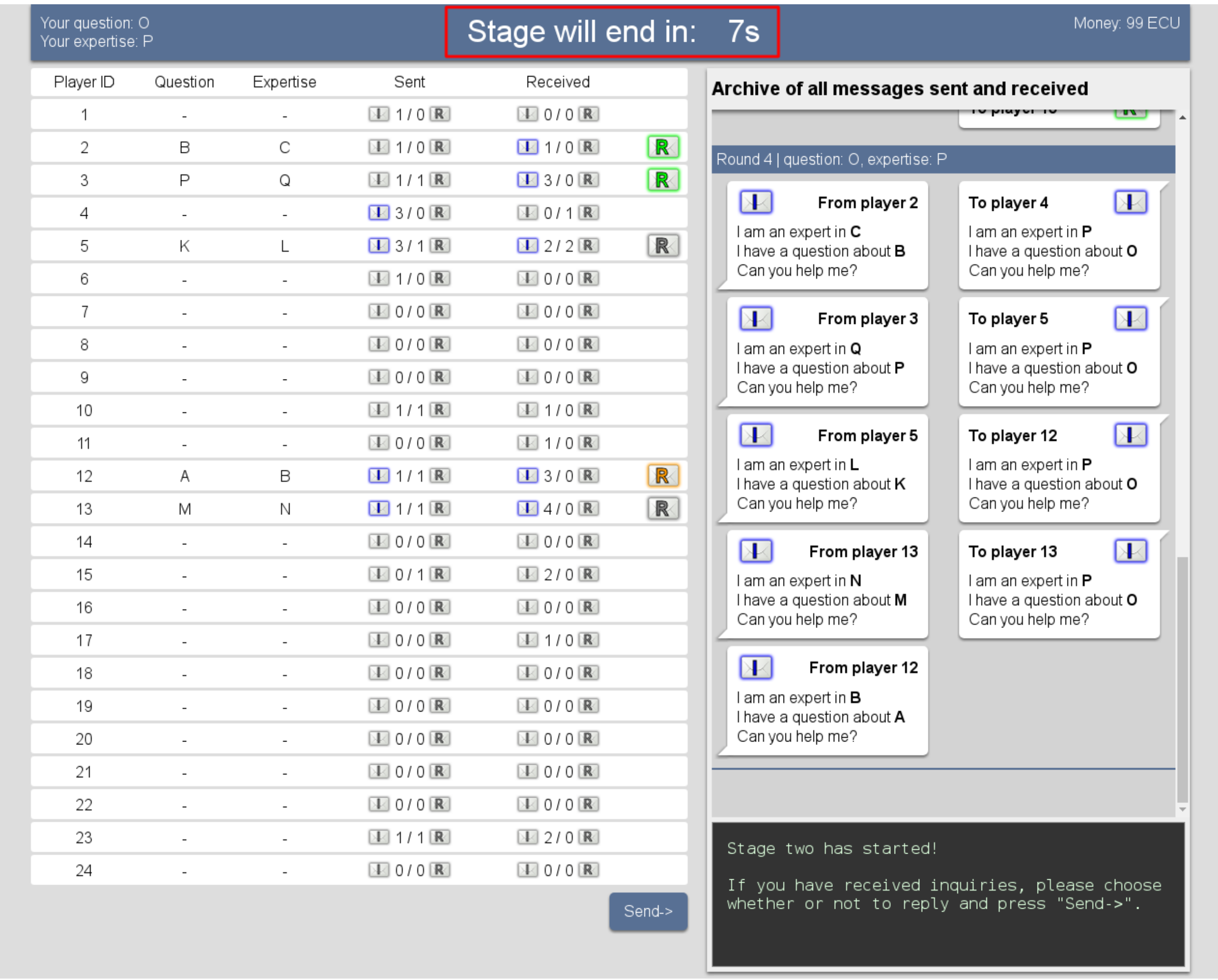
In the example, the reply icons  of player 5 and 13 stay grey, because you choose not to reply to them.



Note that you cannot send false information.

For instance, you cannot send an orange reply (type 2) to player 2. Since you know who the expert of player 2 is, your only option is to either send a green reply (type 1) or you choose not to reply.

In addition, it is not possible to send a green reply (type 1) to player 12. Since you do not know who his expert is, your only option is to either send an orange reply (type 2) or you choose not to reply.



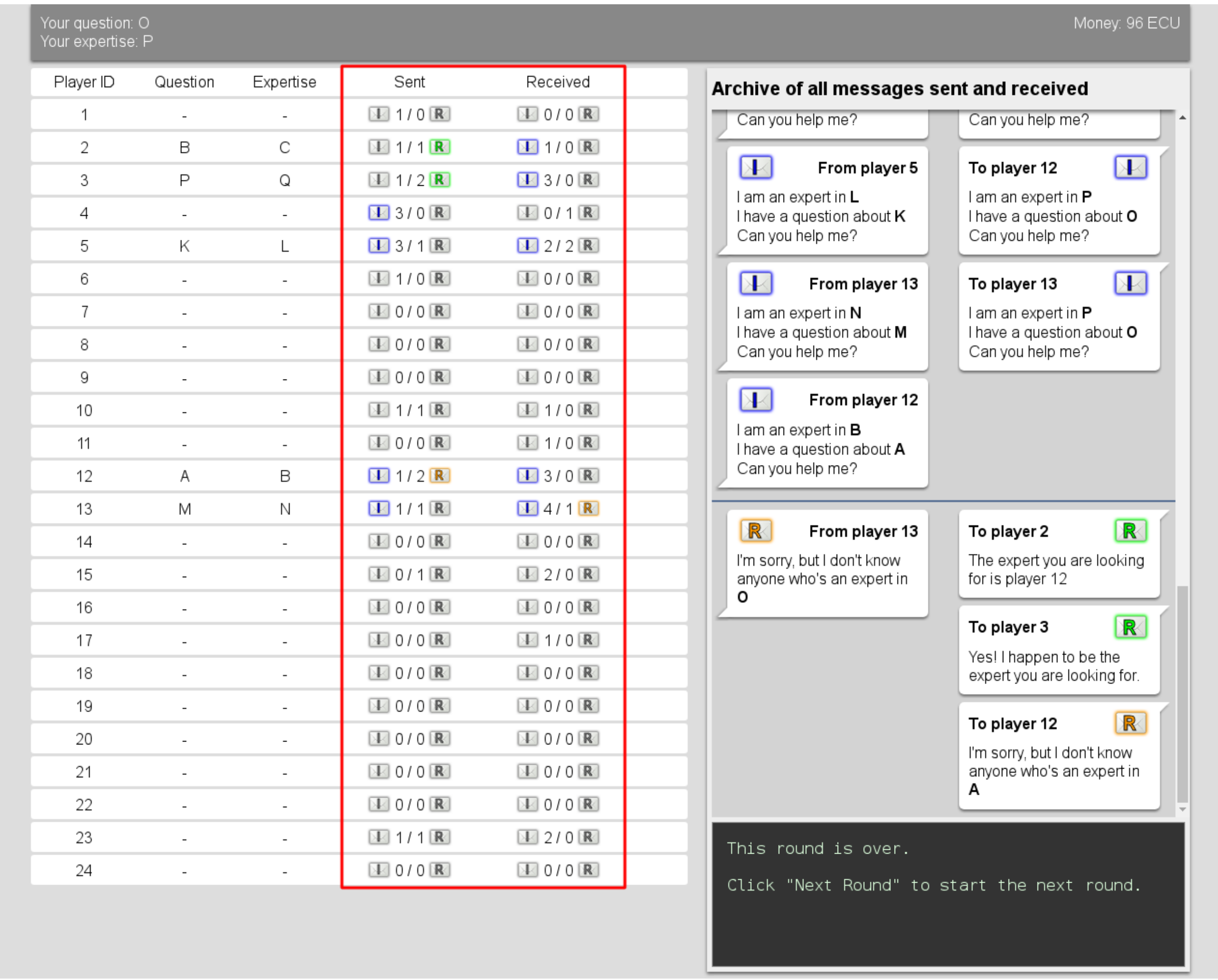
Each stage lasts until the last player has clicked the "**Send->**" button. To prevent the experiment from stalling, the game implements a "soft timer".

The "soft timer" works in the following way:

When all players except for 3 have sent their messages, this timer will appear at the top of the screen of the last three players. The timer will start at 15 seconds, and count down to 0 seconds.

If you are one of the last three players and you don't click the "**Send->**" button before the timer reaches zero seconds, the game will automatically click the button for you. An automatic click works exactly like a manual click, so if the player has selected players to send messages, these messages will be sent.

In the example, you are one of the last three players who have not yet clicked the "**Send->**" button in the current stage, so the timer has started. There are currently 7 seconds until the game will automatically click the send button. If that happens, you will send green replies to the players 2 and 3, an orange reply to player 12, and no replies to player 5 and 13.



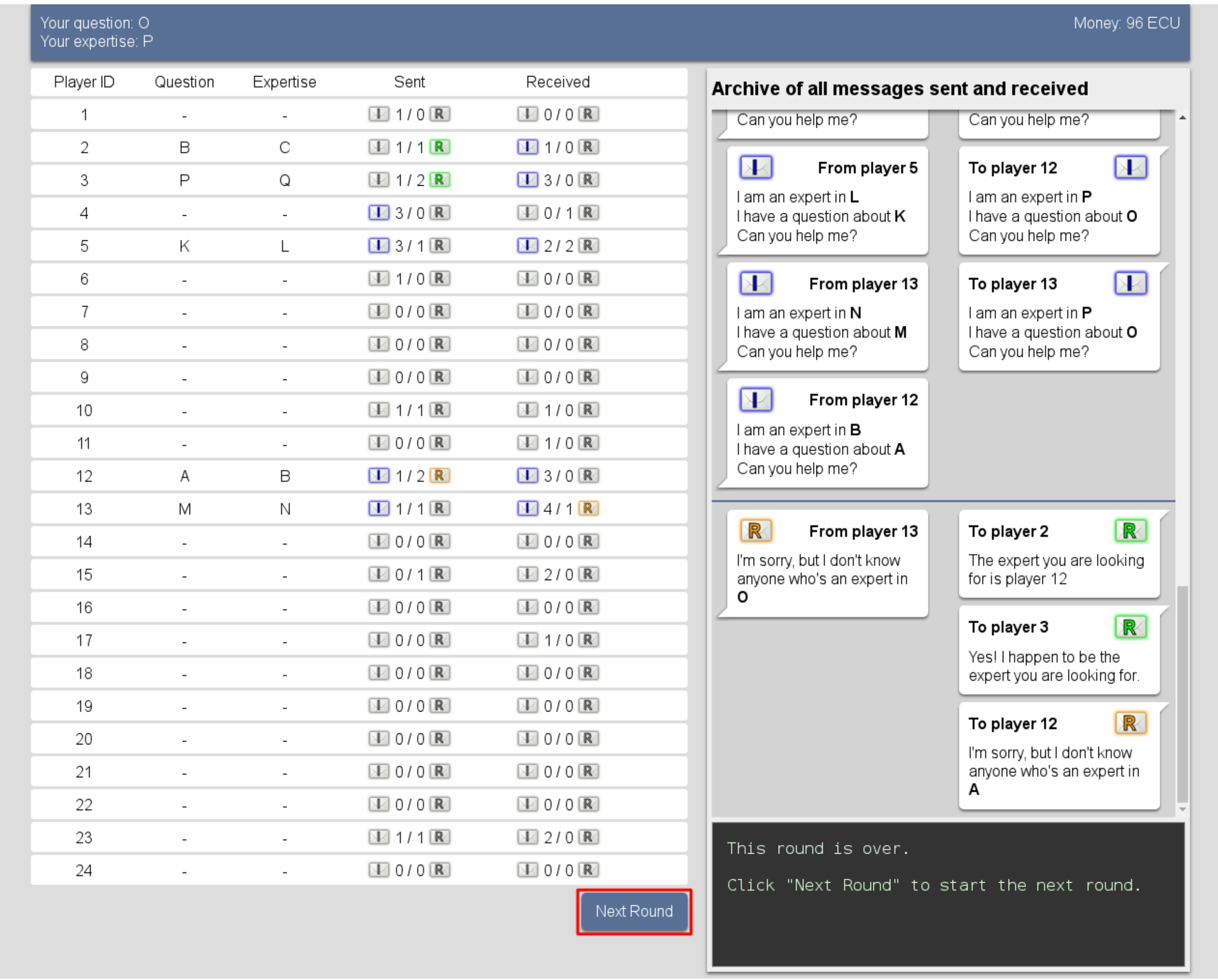
You can use the **message-counters** to get a fast overview of the messages you sent and received in the game.

The color of the small message icons (inquiry: and reply:) show whether you have sent or received such a message in the current round.

- **Sent Column:** In the example you can see, that in the current round you have sent inquiries to the players 4, 5, 12, and 13, and you have sent green replies to the players 2 and 3, and an orange reply to player 12.
- **Received Column:** You can also see that you received inquiries from the players 2, 3, 5, 12, and 13, and you received an orange reply from player 13.

The numbers show how many messages you have sent and received, of the corresponding type, during the entire experiment.

- E.g. in the example your entire correspondence with player 5 consists of you sending 3 inquiries (including one in the current round) and one reply, and you receiving 2 inquiries (including one in the current round) and 2 replies.

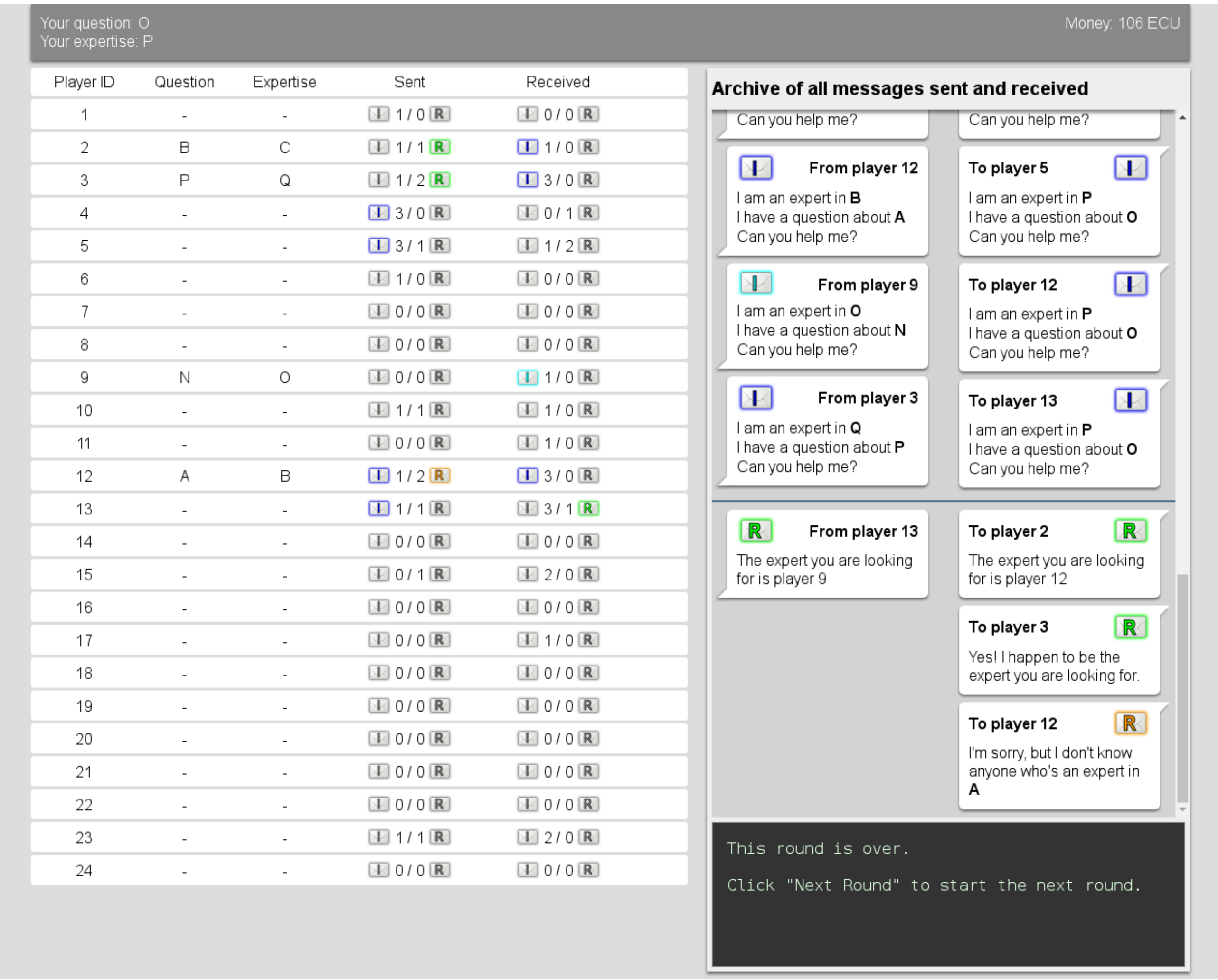


When the last player has sent her or his replies by clicking the "**send->**" button, all replies are delivered and the round is finished.

In the example, you did not find the player which is an expert in "**O**" (your Question). Therefore, you did not earn 10 ECU this round.

When you are finished reading the replies you have received you can start the next round by clicking the "**Next Round**" button.

The experiment consists of many rounds. After 90 minutes, the last round will be allowed to finish, but no new round will be started. You will notice this automatically as the "Next Round" button will not appear on the screen.

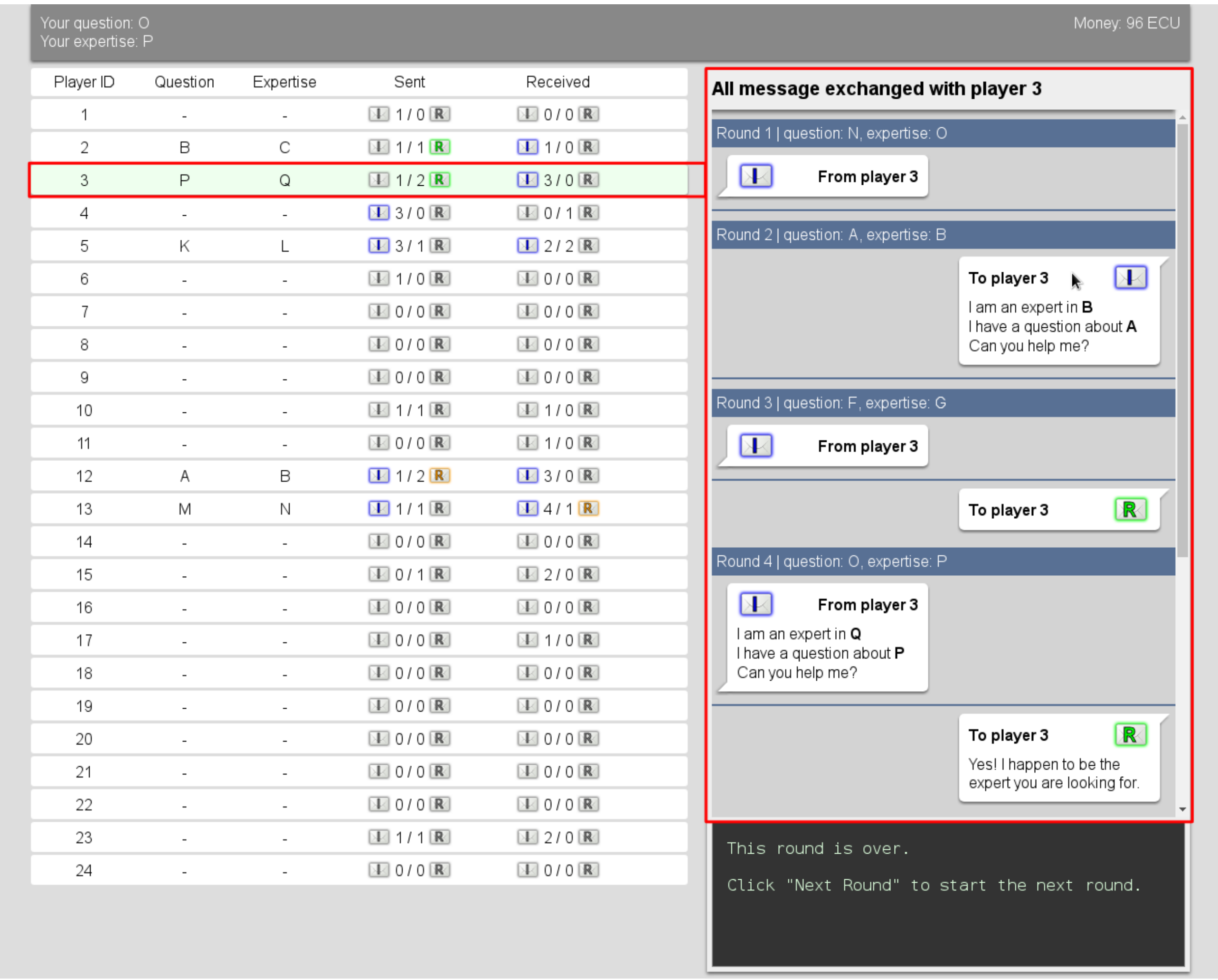


This example shows an altered version of how the round could have played out.

In contrast to the previous example, you have here received an inquiry from player 9. Player 9 is an expert in "**O**" and you have a question about "**O**". The inquiry from player 9 is marked with a turquoise symbol, to indicate that it came from your expert.

In this example, the reply you received from player 13 is green, and is telling you that player 9 is the expert you were looking for. Player 13 must have gotten an inquiry from player 9, to be able to send you this reply.

NOTICE that in this round you have found your expert and won 10 ECU already when receiving the inquiry from player 9 in stage 1. When you received the reply from player 13, you did not win 10 ECU again, because you already knew who your expert was. It is never possible to learn who your expert is and win 10 ECU more than one time each round.



Player ID	Question	Expertise	Sent	Received
1	-	-	1 / 0	0 / 0
2	B	C	1 / 1	1 / 0
3	P	Q	1 / 2	3 / 0
4	-	-	3 / 0	0 / 1
5	K	L	3 / 1	2 / 2
6	-	-	1 / 0	0 / 0
7	-	-	0 / 0	0 / 0
8	-	-	0 / 0	0 / 0
9	-	-	0 / 0	0 / 0
10	-	-	1 / 1	1 / 0
11	-	-	0 / 0	1 / 0
12	A	B	1 / 2	3 / 0
13	M	N	1 / 1	4 / 1
14	-	-	0 / 0	0 / 0
15	-	-	0 / 1	2 / 0
16	-	-	0 / 0	0 / 0
17	-	-	0 / 0	1 / 0
18	-	-	0 / 0	0 / 0
19	-	-	0 / 0	0 / 0
20	-	-	0 / 0	0 / 0
21	-	-	0 / 0	0 / 0
22	-	-	0 / 0	0 / 0
23	-	-	1 / 1	2 / 0
24	-	-	0 / 0	0 / 0

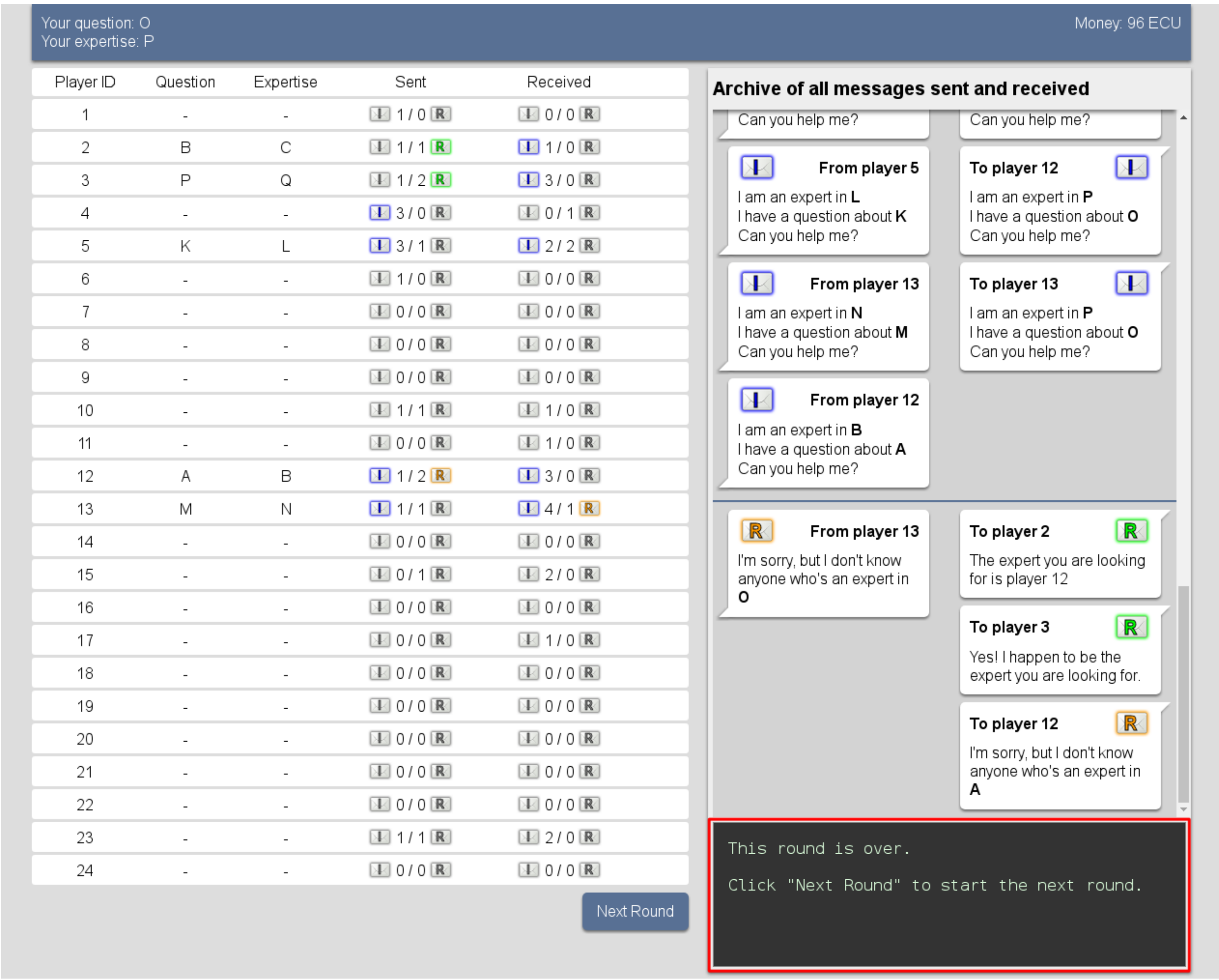
Additional Features

If you click one of the player labels, the archive of messages will show the messages you have sent to-, and received from this player only. You can use this if you want to have a closer look at your correspondence with this particular player.

The archive of messages will return to showing all messages when you click anything that is not a player-label (including the background).

In the example, you have clicked the label of player 3.

Notice that messages you sent and received in previous rounds only display who the message was to or from, and an icon indicating what kind of message it was. To see the content of a message, hold the cursor above it (as with the inquiry sent to player 3 in round 2 in the example).



Additional Features

In the bottom right corner of the screen you can see a dialogue window. If you are ever in doubt about what to do during the game, you are encouraged to look here for instructions of what to do next.

If you have any questions please raise your hand and one of the lab-assistants will assist you in private. Do not talk, communicate or ask questions to the other participants. Thank you!

Important!

For the first 5 rounds, six stars will appear next to six specific players. These stars suggest you six other players to contact. We believe this suggestion is beneficial for you and other players. Following this suggestion can help you find your expert and thus collect more ECU.

Note that:

- Each player receives personalized suggestions. This means that all the players have different suggestions of players to contact.

- The suggested contacts are reciprocal. That means that you are a suggested contact for the same players who are suggested contacts for you.

- If every player follows our suggestion and helps other players find their expert, then every player will find his expert in every round.

- You are free to follow our suggestion or contact any of the other players.

- Stars will appear only for 5 rounds. Note that in these five rounds stars will always suggest the same other players.

The Expert Game: Instructions for Lab-Assistants

General Information:

- The experiment is run online and not using zTree. Therefore, the procedure might be different than usual. Specifically, running the experiment online gives the experimenter/lab assistants less control over and knowledge of the current state of the experiment. Please read these instructions carefully before the start of a session.

- The experiment consists of 3 parts: please note the time of each of the 3 parts for every Session in the respective sheet of the **"Payment.xlsx"** file:

    1. The Expert Game Tutorial/Instructions (roughly 45 minutes)
    2. The Expert Game (90 minutes sharp)
    3. The post experimental Questionnaire (roughly 30 minutes)

General preparation of the lab:

- Start the 28 Client computers and the Experimenter computer.
    - Log in Experimenter computer: username **ibt.ku.dk\okolabadmin** and password **XXXXXXX**
    - Log in Client computers: username **ibt.ku.dk\okolabguest** and password **XXXXXXX**
- Check that there is enough paper in the printer.
- First check if there are written Instructions stored from a previous Session. Print the remaining amount of Instructions, so that you end up with a total of 25. Place them down at each computer and familiarize yourself with the Instructions **(Instructions are the same for control and treatment sessions)**.
- *[Treatment Specific: Print the document called **"Suggestion_Screen"** 25 times. Do **NOT** place them down at each computer.]*
- Print the provided **"Seat-ID/Player-ID"** table (1 printout for you).
- Make sure that there are enough receipts.
- Make sure there is a sufficient amount of money to pay all participants.

- Get the speaker from the IT-support (Building 26, ground floor - room 26.0.37) some days in advance and test if it works. Attach the provided speaker to the Admin Computer and again make sure it works properly **(Turn Volume to 50 on the computer)**.
- Place a sheet of paper with a pen at each seat.

Setting up the game (before participants arrive):

Note: When attempting to log into any of the websites below, the browser may warn you “Your connection is not private”. This happens because the game is set up to use encrypted HTTPS connections, but does not have a validated CA-certificate. You will have to bypass this warning by clicking on “Advanced” and then “Proceed to [address]”.

1. Start the game server on the Experimenter computer by executing “**expertgame2.exe**” saved in the experiment folder in the subfolder “expertgame2”. Click on “allow access” when the pop-up message regarding the connection appears.
2. Start the **Admin Interface** on the Experimenter computer by opening Google Chrome and going to https://localhost:8080/admin**.** Login using the password **XXXXXX**.
3. In the **Admin Interface,** you can configure the game and see players´ current payoff. It is only possible to **add players**, not **remove** them. In addition, players cannot be added after the game has started. Therefore, you will have to count the number of participants who showed up first, before you can log in the Client computers and configure the game according to the number of participants (more information on this later).
4. Open the game on all 28 Client computers by clicking on the Chrome-shortcut named **“expertgame.link”** on the desktop. The game should open in full-screen mode. **Do not login.**
5. Set up the game: Select "**Add Player**" in the “**Action Panel**” and click "**send**" 23 times. The minimum Number of players is 23, the maximum is 25. Verify that the correct number of players has been added to the **“Player Table”**. *[Not in Treatment]*
6. *[Treatment Specific: Set up the game: Select "**Add Player**" in the “**Action Panel**” and click "**send**" 25 times. In the treatment sessions, we **MUST** play with 25 players. Verify that the correct number of players has been added to the **“Player Table”**.]*
7. Configure the game by setting **“Auto-start next round”** to **100** and **“Auto-start timer”** to **3** players and **15** seconds in the **“Settings Panel”**. **Important:** Click on the background outside of the **“Settings Panel”** to implement the changes.
8. Login 23 Client computers (Computer number 1-23) by typing in the respective computer number into the player-ID field and click **“Join Game”**. *[Not in Treatment]*

9. *[Treatment Specific: Login 25 Client computers (Computer number 1-25) by typing in the respective computer number into the player-ID field and click **"Join Game"**.]*
10. After having logged in, click **"Start Tutorial"**. The first page of the tutorial should now fill the screen.
11. Place the keyboards on the side or behind the computer, so that it is not accessible for participants during the **"Tutorial"** and the **"Game"**. They only need it for the **"Questionnaire"**. **Important:** make sure that none of the keys is continuously pressed by placing the keyboard behind the computer, because this will cause problems with the software.

<u>Once all participants have arrived outside the lab:</u>

- Meet participants outside the lab, bringing the participation list and the red coins.
- Ask participants for identification to check whether they registered correctly for the experiment. In case there are **ANY** violations (e.g. a participant is recognized to have participated twice because of a fake account) the participant will be excluded from the experiment.
- Welcome participants and explain the general rules of the lab.
    - Stress that communication during the experiment is forbidden. Otherwise, participants will be asked to leave without receiving any payment.
    - Ask participants to turn off their mobile phones now.
    - No food and drinks are not allowed inside the lab.
    - Tell participants that the experiment is longer than usual and ask them to go to the restroom before entering the lab if they need to.
    - Explain the meaning of the red coins.
    - Ask participants to leave their bags at the left side after entering the lab and then to sit down at their seat and wait for further instructions.
- While one lab-assistant welcomes the participants, the other assistant counts the participants and prepares the corresponding number of red coins.
    - In case there are too many registered participants see section "How to deal with unregistered participants and overbooking" at the end of this script.
- Ask participants to wait outside while you finish preparing the lab: *[Not in Treatment]*
    - If there are 25 participants: Add the remaining 2 by selecting **"Add Player"** in the **"Action Panel"** and click **"send"** 2 more times. *[Not in Treatment]*

    - Login the client computers, by typing the respective computer number into the player-ID field and click **"Join Game"**. *[Not in Treatment]*
    - After having logged in, click **"Start Tutorial".** The first page of the tutorial should now fill the screen. *[Not in Treatment]*
    - Place the keyboards on the side or behind the computer, so that it is not accessible for participants during the **"Tutorial"** and the **"Game"**. They only need it for the **"Questionnaire"**. *[Not in Treatment]*
- Once the computers are ready, go back outside. Openly shuffle the red coins and let participants draw a red coin while entering the lab.

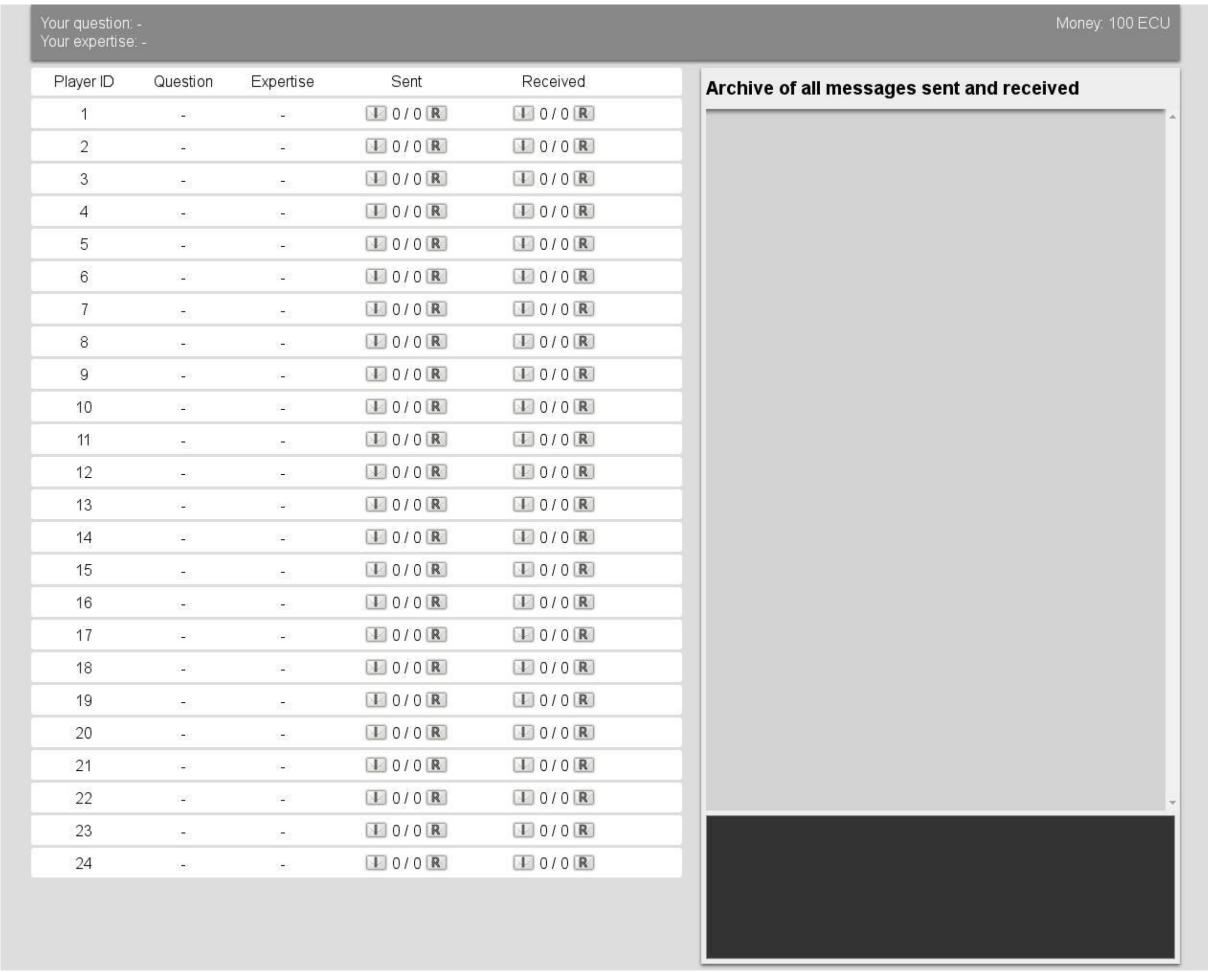
<u>Running the Tutorial:</u>

- When all participants are seated, one lab assistant provides participants with further instructions:
    - Tell them to not click anywhere/press any key other than told.
    - Mention that the experiment consists of three parts: The Tutorial/Instructions, the Game and the Questionnaire.
    - Do **not** say that it is called the "Expert Game" and do **not** mention the duration of the Game.
    - Tell them that the printed instructions are identical to the ones´ they see on the screen. **Emphasize** that they should follow the onscreen instructions and that they can consult the paper instructions just in case (but there is no need to do so).
    - Tell them that they do not need to use the keyboard to answer the control questions.
    - Tell them to wait silently once they finished the Tutorial, which they will notice because they cannot perform any more actions. They will then receive further instructions.
    - If they have questions, they can raise their hand and a lab assistant will come and help them in person. Emphasize that participants are **NOT** allowed to communicate with other participants to answer the control questions.
- Ask the participants if they have any general questions regarding the procedure in an experiment.
- If not, tell them to start the tutorial by following the instructions on the screen.
- If necessary, help participants with the control questions (see solutions at the end of the script).

- **Important:** In case there is one or more participant(s) who take(s) a lot of time ensure that the other participants move the mouse from time to time so that the computer does not shut down.

Running the Game:

- Latest time to start the game (10 o'clock|experiment starts at 9 o'clock)
- Unfortunately, there is no feature that tells you in which stage participants are. Therefore, you will have to check by walking around if all participants have finished answering the control questions. If they have their screen will look like this:



- Let students know that everyone finished the Tutorial part of the experiment and that the Game part will start now.
- *[Treatment Specific: Select "**Suggest Contacts**" in the "**Action Panel**" and click "**send**". A pop-up message will appear which gives participants additional information about today's experiment. After participants read the additional information, they must press the **"Ok"** button. Verify that*

each participant has returned to the screen above (this time including six stars next to some players).]

- *[Treatment Specific: In the meantime, one of the lab-assistants places one printouts of the **"Suggestion_Screen"** down at each computer.]*
- To start the game, select **"Start Game"** in the **Action Panel** and click **"Send"**. The students will now see a blue **"Next Round"** button and you will hear a sound. Ask them to start the Game by clicking **"Next Round"**.
- The game will be played for 90 minutes. Remember to set a timer when you start the game.
- *[Treatment Specific: After **5** rounds select "**Unsuggest Contacts**" in the "**Action Panel**" and click "**send**"].*
- After 90 minutes have passed, end the Game by setting **"Auto-start next game"** to **0** in the **Settings Panel**. This will allow the current round to finish, but the next round will not be started. Let participants know that the game has finished.
- Check how many rounds have been played by looking at one of the computer screens.
- After the Game is over, select **"Start Questionnaire"** in the admin **"Action Panel"**, and click **"Send"**. Tell participants to take the keyboards, because they need it to answer the Questionnaire.
- While the participants fill out the Questionnaire (roughly 30 minutes), prepare the payment. Participants´ payoffs in ECU will be automatically computed and will appear on the Administrator Interface (Column **Reward** in the **Player Table)**. You will have to calculate the profits in DKK by (manually) copying the ECU payoffs into the column "Payoff in ECU" in the **"Payment" file**. The Payment file automatically multiplies participants earnings into DKK (3 x ECU = DKK) and adds a show up fee of 50 DKK. Print out the file, bring it with you to the payment room, and pay out participants.

<u>Payment:</u>

- One lab assistant goes into the payment room while the other assistant asks the participants to remain seated after finishing the Questionnaire. Once everyone is finished, the lab assistant distributes receipts, which participants fill out except from the amount they are going to be paid.
- Once the payment is ready, the lab assistant calls the participants one by one (starting with computer 1) and asks them to go to the payment room.
- Ask them to bring their belongings, the red coins and the receipts with them and leave the rest (Instructions, notes, pens etc.) at their seat.

- The lab-assistant in the payment room then pays the participants. After that, participants can leave.

<u>After the experiment:</u>

- Copy the data files created by the online program and stored in **"expertgame2/files/data_[todays date]"** into the folder "**Data**" in the experiment folder. In case the game-server was started multiple times on the same day, "_x[i]" (where [i] is an increasing counter) is appended to the directory names, to ensure that no data is overwritten.
- To exit the game on the Client computers, press **Alt + F4.**
- Store the written Instructions (if untouched) for the next Session. If there is something written on the instruction, label them with the respective seat number.
- Label the sheet of paper, with the respective seat number if there is something written on it.
- Shut down the computers and clean everything up for the next day.
- Lock the rest of the money in the safe.

<u>How to deal with unregistered participants and overbooking:</u>

1. Print the registration list from ORSEE. You must use the registration list to check if all participants, who showed up are also registered. Note that we are not required to give a show up fee if participants are not registered for the session.
2. Prepare the stack of red coins.
3. The maximum number of participants in the experiment is 25. In case more than 25 registered participants show up follow these steps:
    a. In case someone must be excluded, ask if anybody voluntarily wants to receive a show-up fee of 50 DKK and not participate.
    b. If nobody volunteers, you must randomly choose one or more of the participants. In that case, please include and amount of "X"-coins (corresponding to the number of participants that have to be sent home) in the stack of coins. The participant(s) who receives an "X" will be given a show-up fee and will be excluded from the experiment.
    c. Have some receipts ready in case you must give a show-up fee before the experiment starts.

4. If you must send a participant home and pay him/her a show-up fee, one lab assistant should take that participant into the payment room, ask the participant to sign a receipt and pay.

How to handle special cases:

Note that the following cases are very unlikely and should ideally not happen. However, the following points describe how to deal with such cases. For both cases, the lab assistant is required to note the time the participant (identified by its seat number) leaves and returns to his seat. The information should be added in the provided **"Seat ID/Player-ID"** table.

1. Participant leaves during the experiment: In that case, the participant receives no payment. Then one of the lab assistants must take the seat of that participant and clicks **"only"** the send button for the rest of the Game. Inform the other participants that one player is missing and that this player will not send any messages for the rest of the experiment.
2. Participants leave for the toilette: In that case, the game continues automatically. So, no further action is required by the lab assistant.

Troubleshooting in case participants experience problems:

1. Check whether one key on the keyboard is continuously pressed. In that case, get the keyboard from the place it is stored and check that no key is pressed.
2. If a participant has problems to submit the control questions of the tutorial. First, try to submit it for him again. In case this does not work, press **"Alt + F4"** and look the player in again by clicking on the Chrome-shortcut named **"expertgame.link"** on the desktop. The game should open in full-screen mode. **Login the player with his respective seat number.** Click through the tutorial for him, answer the control questions for the participant until he experienced problems, and click submit.

Solution manual for control questions:

Screen Task:

- What is your Expertise? Solution: **V**
- In which "Round" is the Experiment? Solution: **6**
- In which "Stage" is this "Round"? Solution: **2**
- How many players sent an "Inquiry" to you this round? Solution: **5**
- Did you receive at least one "Inquiry" from player 23 in a previous round? Solution: **Yes**
- Did you send an "Inquiry" to player 2 this round? Solution: **No**
- Did you send an "Inquiry" to player 2 in a previous round? Solution: **Yes**
- Have you already found your "Expert" in the current round? Solution: **No**
- Can you send a green reply of type 1 to player 13? Solution: **No**
- How many green replies of type 1 are you able to send in the current stage? Solution: **2**

Calculation Task:

- In stage 1, you sent 3 "Inquires". How much do they cost in total? Solution: **3 ECU**
- In stage 2, you receive 4 "Inquires". Now you decide to reply to 2 out of the 4 "Inquires" you received. How much do these two replies cost? Solution: **2 ECU**
- At the end of the round, you learn that you did not find your Expert. What is your balance? Solution: **95 ECU**
- Using the exchange rate 1 ECU = 3 DKK, how much did your balance decrease during this round in Danish kroner? Solution: **15 DKK**
- Assume instead that you did learn who your expert was, because someone sent you a green reply. What would your balance be then? Solution: **105 ECU**
- How much did your balance increase during this round in Danish kroner? Solution: **15 DKK**

Note: Participants answered the Questionnaire on screen.

Questionnaire (Paper Version)

Please answer all of the following questions, and then click "Submit questionnaire".

Part 1

Please provide the following information about yourself:

What is your sex? ○ Female or ○ Male

What is your age? ☐ years

Below you will find a series of statements about you. Please read each statement and decide how much you agree or disagree with that statement. Then indicate your response using the following scale:

5 = strongly agree
4 = agree
3 = neutral (neither agree nor disagree)
2 = disagree
1 = strongly disagree

Please answer every statement, even if you are not completely sure of your response.

I rarely hold a grudge, even against people who have badly wronged me.
○ 1 ○ 2 ○ 3 ○ 4 ○ 5

I feel reasonably satisfied with myself overall.
○ 1 ○ 2 ○ 3 ○ 4 ○ 5

I wouldn't use flattery to get a raise or promotion at work, even if I thought it would succeed.
○ 1 ○ 2 ○ 3 ○ 4 ○ 5

People sometimes tell me that I am too critical of others.
○ 1 ○ 2 ○ 3 ○ 4 ○ 5

I rarely express my opinions in group meetings.
○ 1 ○ 2 ○ 3 ○ 4 ○ 5

If I knew that I could never get caught, I would be willing to steal a million dollars.
○ 1 ○ 2 ○ 3 ○ 4 ○ 5

People sometimes tell me that I'm too stubborn.
○ 1 ○ 2 ○ 3 ○ 4 ○ 5

I prefer jobs that involve active social interaction to those that involve working alone.
○ 1 ○ 2 ○ 3 ○ 4 ○ 5

Having a lot of money is not especially important to me.
○ 1 ○ 2 ○ 3 ○ 4 ○ 5

People think of me as someone who has a quick temper.
○ 1 ○ 2 ○ 3 ○ 4 ○ 5

On most days, I feel cheerful and optimistic.
○ 1 ○ 2 ○ 3 ○ 4 ○ 5

I think that I am entitled to more respect than the average person is.
○ 1 ○ 2 ○ 3 ○ 4 ○ 5

My attitude toward people who have treated me badly is “forgive and forget”.
○ 1 ○ 2 ○ 3 ○ 4 ○ 5

I feel that I am an unpopular person.
○ 1 ○ 2 ○ 3 ○ 4 ○ 5

If I want something from someone, I will laugh at that person's worst jokes.
○ 1 ○ 2 ○ 3 ○ 4 ○ 5

I tend to be lenient in judging other people.
○ 1 ○ 2 ○ 3 ○ 4 ○ 5

In social situations, I’m usually the one who makes the first move.
○ 1 ○ 2 ○ 3 ○ 4 ○ 5

I would never accept a bribe, even if it were very large.
○ 1 ○ 2 ○ 3 ○ 4 ○ 5

I am usually quite flexible in my opinions when people disagree with me.
○ 1 ○ 2 ○ 3 ○ 4 ○ 5

The first thing that I always do in a new place is to make friends.
○ 1 ○ 2 ○ 3 ○ 4 ○ 5

I would get a lot of pleasure from owning expensive luxury goods.
○ 1 ○ 2 ○ 3 ○ 4 ○ 5

Most people tend to get angry more quickly than I do.
○ 1 ○ 2 ○ 3 ○ 4 ○ 5

Most people are more upbeat and dynamic than I generally am.
○ 1 ○ 2 ○ 3 ○ 4 ○ 5

I want people to know that I am an important person of high status.
○ 1 ○ 2 ○ 3 ○ 4 ○ 5

Even when people make a lot of mistakes, I rarely say anything negative.
○ 1 ○ 2 ○ 3 ○ 4 ○ 5

I sometimes feel that I am a worthless person.
○ 1 ○ 2 ○ 3 ○ 4 ○ 5

I wouldn’t pretend to like someone just to get that person to do favors for me.
○ 1 ○ 2 ○ 3 ○ 4 ○ 5

When people tell me that I'm wrong, my first reaction is to argue with them.
○ 1 ○ 2 ○ 3 ○ 4 ○ 5

When I'm in a group of people, I'm often the one who speaks on behalf of the group.
○ 1 ○ 2 ○ 3 ○ 4 ○ 5

I would be tempted to buy stolen property if I were financially tight.
○ 1 ○ 2 ○ 3 ○ 4 ○ 5

Part 2

We are now very much interested in your experience with the game. Please give your answers as precisely as possible as they are important for scientific research. Where numbers are required, make your best guess.

For assistance with any of the questions, please ask!

1. Interaction with other players

How well did the other players collaborate with you in the game?
Give each player a score by setting the slider on a scale from bad to good:

Player-ID	Score	
1	bad	good
2	bad	good
3	bad	good
4	bad	good
5	bad	good
6	bad	good
7	bad	good
8	bad	good
9	bad	good
10	bad	good
11	bad	good
12	bad	good
13	bad	good
14	bad	good
15	bad	good
16	bad	good
17	bad	good
18	bad	good
19	bad	good
20	bad	good
21	bad	good
22	bad	good
23	bad	good
24	bad	good

2. Friends and Acquaintances

Let's call those players **"friends"**, from which you generally expected to receive replies, when you sent them an Inquiry. Other players that you also communicated with, but did not expect replies from, are called **"acquaintances"**.

How many **"friends"** did you have approximately? ☐

How many **"acquaintances"** did you have approximately? ☐

Once your group of **"friends"** had formed, did it change over the course of the remaining rounds? Choose the statement which fits best:

○ It stayed the same ○ It changed every 10 rounds or so ○ It changed faster than every 10 rounds

After how many rounds did you feel you had friends in the game?
(please chose the approximate number, if you can't remember the total number of rounds, please ask the lab assistants.)

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	>25	never
○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○	○

3. Message types

Why did you send messages of type **"Inquiry (I)"**? (write anything that comes to your mind)

☐

Why did you send messages of type **"Reply (R)"**? (write anything that comes to your mind)

☐

When receiving any of the two message types of the game, i.e. **"Inquiry (I)"** or **"Reply (R)"**, please give a score for how much you valued each as a sign of friendship. Use a scale where "0" means "no value", and "10" means "highest value".

Inquiry (I): ☐
Reply (R): ☐

When sending Inquiries (stage 1 of each round), who did you prioritize? Choose the one that fits better:
○ Friends ○ Aquaintances ○ Neither

If the answer to the previous question was **“friends”**, did you have a priority list also among your “friends” or did you treat all your “friends” equally? Choose the most appropriate.
○ All equal ○ Some preference ○ Strong preference

When sending Replies (stage 2 of each round), who did you prioritize? Choose the one that fits better:
○ Friends ○ Aquaintances ○ Neither

If the answer to the previous question was **“friends”**, did you have a priority list also among your “friends” or did you treat all your “friends” equally? Choose the most appropriate.
○ Friends ○ Aquaintances ○ Neither

4. General Questions

Thinking of this game as an abstract description of professional communication, e.g. by email in a business, how closely do you find it to mimic actual communication? (check one, 1=bad, 10=excellent)

○ 1 ○ 2 ○ 3 ○ 4 ○ 5 ○ 6 ○ 7 ○ 8 ○ 9 ○ 10

In which aspects did you find that the game had similarities with real communication?

(write at least one thing that comes to your mind)

In which aspects did you find the game did not represent real communication?

(write at least one thing that comes to your mind)

Were you surprised when the experiment ended, i.e. did you expect it to go longer?
○ Quite Surprised ○ A little surprised ○ I knew it was almost over

How much did you find that the timer influenced your decisions during the Game?
○ A lot ○ A little ○ Not at all

If you selected **“A lot”** or **“A little”**. How did the timer influence your decisions?

(write at least one thing that comes to your mind)

How would you describe the strategy you played in the game? Did it change in the course of the experiment, and in what way?