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PhD Thesis

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Essays in Macroeconomics: Expectations, House Prices, and Inflation

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Contents

Acknowledgements

Summary

This thesis consists of three self-contained chapters. The first two chapters focus on the implications of boundedly rational expectations in New Keynesian (NK) macroeconomic models. Specifically, Chapter 1 investigates whether so-called "trend-chasing" expectations can help account for the run-up in US house prices during the boom period from 2000 to 2006. Similarly, Chapter 2, which is co-authored by Kevin J. Lansing, analyzes whether so-called "anchored" inflation expectations can help explain US inflation dynamics since the outbreak of the Great Recession, which standard rational expectations models have difficulties accounting for. Inflation dynamics is also the focus of the final chapter. Specifically, Chapter 3, which is co-authored by Søren Hove Ravn, provides empirical evidence that inflation declines in response to expansionary fiscal policy shocks. Moreover, the decline in inflation is accompanied by an increase in total factor productivity and consumption. We show that the introduction of variable technology utilization can enable an otherwise standard New Keynesian model to reproduce these empirical findings. The following contains a more detailed description of each chapter.

Chapter 1. House Price Booms under Bounded Rationality.
This chapter investigates the causes of the US housing boom from 2000 to 2006. Prior to the outbreak of the financial crisis in 2007, the US economy experienced a large house price boom, a deterioration of the current account, and remarkably low interest rates. The literature has identified at least four factors as important sources of the housing boom: Loose monetary policy (Taylor, 2008), a relaxation of borrowing constraints (Mian and Sufi, 2010, Boz and Mendoza, 2014), a global saving glut (Bernanke, 2005) and deviations of house prices from fundamentals (Shiller, 2007). However, none of these factors can account for house price dynamics in standard NK models with rational expectations. This paper introduces bounded rationality into an otherwise standard two-country NK model. Agents use simple strategies to forecast future variables and switch between strategies based on their relative performance in the recent past. In this model, accommodative monetary policy, relaxed borrowing constraints and a foreign saving glut can jointly account for the dynamics of house prices, the current account, the real interest rate and inflation during the boom period from 2000 to 2006. The shocks produce a run-up in house prices which gradually convinces agents to adopt trend-chasing forecast strategies in

the housing market, leading to self-fulfilling expectations of a boom.

Chapter 2. Inflation Puzzles in the New Keynesian Model: The Implications of Anchored Expectations (joint with Kevin J. Lansing, Federal Reserve Bank of San Francisco)

Since the outbreak of the Great Recession, US inflation dynamics have been associated with two "puzzles". First, the absence of a persistent decline in inflation in the wake of the Great Recession was dubbed the "missing disinflation puzzle" (Coibion and Gorodnichenko, 2015). Subsequently, since 2012, inflation has remained persistently below the Fed's target of 2% despite a simultaneous recovery of the output gap, giving birth to the so-called "missing inflation puzzle". These developments are puzzling from the perspective of standard NK models with rational expectations since these tend to produce large and persistent declines in inflation in response to the Great Recession, followed by a recovery (Auroba and Schorfheide, 2016).

In this paper we introduce bounded rationality into an otherwise standard NK model. Agents are assumed to behave as econometricians, using time-series models to forecast inflation and the output gap similar to that of Stock and Watson (2007). The agent's perceived optimal forecast rules are defined by the Kalman filter. We show that the model has a unique equilibrium, where the values of the two Kalman gain parameters are pinned down by the observed autocorrelation of inflation and output gap changes. This methodology can be applied directly to U.S. data. We show that if agents perpetually update their estimates of the Kalman gains using a moving window of recent data, the identified Kalman gain for inflation exhibits a downward drift during the so-called "Great Moderation" period. A low Kalman gain implies a low weight on recent inflation in the agent's forecast rule. This helps anchor inflation near the central bank's target rate when the output gap falls sharply during the Great Recession. In the longer term, however, the recession leads to a downward revision of the agent's inflation forecast, which generates a moderate – but highly persistent – decline in inflation. Thus, the model can help account for both the "missing disinflation" in the immediate wake of the recession as well as the "missing inflation" in recent years. Forecasts with the model suggest that inflation will undershoot the Fed's target rate for several years after the output gap has fully recovered. Consequently, the model predicts that monetary policy will remain accomodative and contribute to a positive output gap after 2017.

Chapter 3. The Inflation Response to Government Spending Shocks: A Fiscal Price Puzzle? (joint with Søren Hove Ravn, University of Copenhagen)

The final chapter identifies a "fiscal price puzzle" in US data: Based on a Structural Vector Autoregression (SVAR) model, we provide empirical evidence that inflation declines in response to an expansionary fiscal policy shock. Moreover, a fiscal expansion generates an increase in consumption and total factor productivity. These results are highly robust across a wide set of model specifications, price indeces etc. The results, however, are clearly at odds with standard NK models with exogenous productivity, which typically produce a decline in consumption and an increase in prices in response to a positive government spending shock. We show that the introduction of variable technology utilization – along the lines of Bianchi et. al. (2017) – can reconcile an otherwise standard NK model with our empirical findings. Intuitively, variable technology utilization allows firms to accomodate an increase in demand by adopting new technology into the production process. If this mechanism is sufficiently strong, it dominates the upward pressure on marginal costs stemming from higher wages, inducing a net decline in marginal costs. As a result, inflation declines, and the central bank lowers the real interest rate in accordance with the Taylor principle, which paves the way for an increase in consumption.

Resumé (Danish summary)

Denne afhandling består af tre selvstændige kapitler. De første to kapitler omhandler såkaldt "bounded rationality" i Nykeynesianske (NK) økonomiske modeller. Nykeynesianske modeller med rationelle forventninger har haft svært ved at forklare en række centrale udviklingstræk i amerikansk økonomi både før, under og efter Den Store Recession. Kapitel 1 undersøger, hvorvidt såkaldte "trend-følgende" forventninger kan medvirke til at forklare de kraftige boligprisstigninger i USA fra 2000 til 2006. Tilsvarende undersøger kapitel 2, som er skrevet i samarbejde med Kevin J. Lansing, hvorvidt fastforankrede inflationsforventninger kan medvirke til at forklare udviklingen i amerikansk inflation siden Den Store Recession – en udvikling, som almindelige Nykeynesianske modeller har haft vanskeligt ved at forklare. Inflationsudviklingen er ligeledes i fokus i det tredje og sidste kapitel. I kapitlet, som er medforfattet af Søren Hove Ravn, fremlægger vi empirisk evidens for, at inflationen falder efter et positivt stød til det offentlige forbrug. Endvidere er faldet i inflationen ledsaget af en stigning i totalfaktorproduktiviteten og i det private forbrug. Vi viser, at en NK model kan reproducere disse empiriske resultater, hvis standardmodellen udvides med såkaldt "variabel teknologiudnyttelse". Det følgende indeholder en mere detaljeret gennemgang af hvert enkelt kapitel i afhandlingen.

Kapitel 1. House Price Booms under Bounded Rationality.
Dette kapitel analyserer årsagerne til det amerikanske boligboom fra 2000 til 2006. Inden finanskrisen ramte i 2007, var amerikansk økonomi præget af kraftige boligprisstigninger, stigende betalingsbalanceunderskud og særdeles lave renter. Literaturen har identificeret fire centrale årsager til boligboomet: Lempelig pengepolitik (Taylor, 2008), lempelig kreditgivning (Mian og Sufi, 2010, Boz og Mendoza, 2014), et globalt opsparingsoverskud (Bernanke, 2005) samt afvigelser i boligpriserne fra deres fundamentale niveau (Shiller, 2007). Ingen af disse faktorer kan imidlertid forklare boligprisstigningerne i standard NK modeller med rationelle forventninger. I dette kapitel udvides en standard NK model for to lande med "bounded rationality". Det antages, at modellens agenter anvender simple forventningsstrategier til at fremskrive modellens variable, og at agenterne løbende skifter strategier på baggrund af de enkelte strategiers historiske præcision. I denne model kan kombinationen af lempelig pengepolitik - og kreditgivning samt et globalt opsparingsoverskud forklare udviklingen i boligpriserne, betalingsbalancen, realrenten og inflationen i USA fra 2000 til 2006.

Stødene forårsager en stigning i boligpriserne, som gradvist får agenterne til at benytte såkaldte "trend-følgende" forventningsstrategier til at fremskrive boligpriserne. Ultimativt bliver disse trend-følgende forventninger selvopfyldende og medvirker til et kraftigt boligprisboom.

Kapitel 2. Inflation Puzzles in the New Keynesian Model: The Implications of Anchored Expectations (i samarbejde med Kevin J. Lansing, Federal Reserve Bank of San Francisco)

Siden Den Store Recession, har den amerikanske inflationsudvikling været karakteriseret af to "gåder": I kølvandet på recessionen observerede man ikke et persistent fald i inflation, hvilket blev døbt "den manglende disinflation" (Coibion og Gorodnichenko, 2015). Efterfølgende, siden 2012, har inflationen imidlertid ligget persistent under den amerikanske centralbanks målsætning på 2% til trods for en sideløbende normalisering af outputgabet. Denne udvikling er tilsvarende blevet døbt "den manglende inflation". Inflationsudviklingen er vanskelig at forklare indenfor rammerne af en almindelig NK model med rationelle forventninger, da sådanne modeller som hovedregel genererer kraftige og persistente fald i inflationen, når økonomien rammes af en kraftig recession. Dette fald efterfølges typisk af en gradvis stigning (Auroba og Schorfheide, 2016). I dette kapitel udvider vi en standard NK model med "bounded rationality". Agenterne antages at benytte tidsseriemodeller a la Stock og Watson (2007) til at fremskrive inflationen og outputgabet. Agenternes optimale forventningsstrategier er defineret ved Kalman filteret. Vi viser, at modellen har en unik ligevægt, hvor værdierne af de to "Kalman gain"-parametre er bestemt af de observerede autokorrelationskoefficienter for inflations- og outputgab-ændringer. Denne metode kan anvendes direkte på amerikanske data. Vi viser, at hvis agenterne løbende benytter et udsnit af den seneste historiske data til at opdatere deres estimerede Kalman gain-parametre, vil deres estimerede Kalman gain-parameter for inflationen falde gradvist igennem den såkaldte "Store Moderation" (ca. 1984-2007). En lav Kalman gain-parameter indebærer en lav vægt på den senest observerede inflation i agentens fremskrivningsregel. Dette bidrager til at fastforankre inflationen, når outputgabet falder kraftigt under Den Store Recession. På længere sigt medfører recessionen imidlertid et fald i agenternes inflationsforventninger, hvilket skaber et moderat – men stærkt persistent – fald i inflationen. Således kan modellen bidrage til at forklare "den manglende disinflation" i kølvandet på Den Store Recession og "den manglende inflation" i de senere år. Ifølge modelfremskrivninger vil inflationen ligge under den amerikanske centralbanks infla-

tionsmålsætning i adskillige år efter, at outputgabet er lukket. Derfor forudsiger modellen, at pengepolitikken vil forblive lempelig og bidrage til et positivt outputgab efter 2017.

Kapitel 3. The Inflation Response to Government Spending Shocks: A Fiscal Price Puzzle? (i samarbejde med Søren Hove Ravn, Københavns Universitet)

Kapitlet identificerer en "gåde" i amerikansk data: Med afsæt i en strukturel VAR (Vektor Autoregressiv) model fremlægger vi empirisk evidens for, at inflationen falder som følge af et ekspansivt finanspolitisk stød. Endvidere genererer ekspansiv finanspolitik en stigning i privatforbruget og i totalfaktorproduktiviteten (TFP). Disse resultater er særdeles robuste på tværs af en række forskellige modelspecifikationer, prisindekser mv. Resultaterne modsiger imidlertid standard NK modeller, som typisk producerer et fald i privatforbruget og en stigning i priserne efter et positivt finanspolitisk stød. Vi viser, at en modeludvidelse, der følger Bianchi et. al. (2017), med såkaldt "variabel teknologiudnyttelse" muliggør, at en ellers almindelig NK model kan reproducere vores empiriske resultater. Variabel teknologiudnyttelse indebærer, at virksomheder kan imødekomme en stigning i efterspørgslen ved at optage ny teknologi i produktionsprocessen. Hvis denne mekanisme er tilstrækkeligt kraftig, vil en stigning i teknologiudnyttelsen dominere stigningen i lønningerne og generere et fald i virksomhedens marginalomkostninger. Som følge heraf vil priserne falde, og centralbanken vil, jvf. Taylor-princippet, sænke realrenten, hvilket skaber en stigning i privatforbruget.

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Chapter 1

House Price Booms
under Bounded Rationality

House Price Booms under Bounded Rationality[◇]

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Abstract

This paper introduces bounded rationality into an otherwise standard two-country DSGE model. Agents use simple strategies to forecast future variables and switch between strategies based on their relative performance in the recent past. In this model, accommodative monetary policy, relaxed borrowing constraints and a foreign saving glut can jointly account for the dynamics of house prices, the current account, the real interest rate and inflation during the U.S. housing boom from 2000-2006. The shocks produce a run-up in house prices which gradually convinces agents to adopt trend-chasing strategies in the housing market, leading to self-fulfilling expectations of a boom

Introduction

Prior to the outbreak of the financial crisis in 2007, the US economy experienced a large house price boom, a deterioration of the current account, and remarkably low interest rates. The literature has identified at least four factors as important sources of the housing boom: Loose monetary policy (Taylor, 2008), a relaxation of borrowing constraints (Mian and Sufi, 2010, Boz and Mendoza, 2014), a global saving glut (Bernanke, 2005) and deviations of house prices from fundamentals (Shiller, 2007). However, none of these factors can account for house price dynamics in standard Dynamic Stochastic General Equilibrium (DSGE) models with rational expectations. This paper extends a standard DSGE model with boundedly rational agents. In the model, agents can choose between simple strategies (heuristics) to forecast future variables and switch between strategies based on their relative forecast performance in the recent past. The available strategies cover two types of expectations observed in survey data: Trend-chasing expectations and anchored expectations. In this model, monetary policy shocks, credit shocks, and saving glut shocks can largely account for the dynamics of house prices, the current account, interest rates, and inflation from 2000-2006. Moreover, the model establishes a strong negative correlation between house prices and the current account as observed in the data.

Standard DSGE models with rational expectations have difficulties producing large boom-bust cycles in house prices as observed in the U.S. and many European economies over the past decade (Gelain and Lansing, 2014). Typically, these models rely on large and persistent shocks to agents housing preferences to bridge the gap between the model and the data (see, for example, Ferrero, 2015, Justiniano et al., 2013, Gete, 2013). Preference shocks are not unproblematic. Most importantly, they cannot produce a boom in the price-to-rent ratio as observed in the data, since a preference shift towards housing increases both house prices and rents. Many observers have stressed that changes in fundamentals cannot account for house price dynamics (Case and Shiller (2003), Case et al. (2012)). According to Glaeser and Nathanson (2014) "it seems silly now to believe that housing price changes are orderly and driven entirely by obvious changes in fundamentals operating though a standard model" (pp. 40).

A growing body of literature has provided evidence that shifts in expectations were a key contributor to the housing boom (Cheng et al. (2014), Foote et al. (2012), Garriga et al. (2012), among others). According to Piazzesi and Schneider (2009) "starting in 2004, more and more households became optimistic after having watched house prices increase for several years". Some recent papers have emphasized that trend-chasing behavior is key to understanding asset price fluctuations. Greenwood and Shleifer (2014) analyze investor expectations of future stock market returns from six data sources between 1963 and 2011. The authors conclude that survey measures of investor expectations are "reflections of widely shared beliefs about future market returns, which tend to be extrapolative in nature". Frankel and Froot (1990,1991) find evidence of short-term trend-chasing expectations of investors in the U.S. spot exchange market. Similarly, trend-chasing behavior in the U.S. housing market is well documented in Case

et al. (2012), who conducted an extensive questionnaire survey of homebuyers expectations in 1988 and annually from 2003 through 2012. The authors find that 1-year expectations of future house price changes are "fairly well described as attenuated versions of lagged actual 1-year price changes" (p. 282). Gete (2015) inputs the survey data expectations of Case et al. (2012) exogenously in a standard DSGE model, and finds that the model can account for U.S. house price dynamics. He concludes that "DSGE models of housing markets may be failing to explain housing dynamics because they fail to match housing price expectations".

Building on these notions, this paper extends a standard DSGE model with boundedly rational agents. In the model, agents use simple strategies (heuristics) to forecast future variables and switch between strategies based on their recent forecasting performance. This so-called heuristics switching framework was first introduced by Brock and Hommes (1997). The idea that agents are boundedly rational and that individual behavior under uncertainty can best be described by simple "heuristics" dates back to the work of Herbert Simon (1957) and Kahneman and Tversky (1973, 1974). Most heuristics switching models have so far been concerned with financial market applications, but recently they have been applied to the New-Keynesian framework as well (see, for instance, De Grauwe (2011)). For every forward-looking variable in the model, agents can choose between trend-chasing expectations and anchored expectations, respectively. The first strategy is based on the survey evidence in Case et. al. (2012), while the second strategy is based on the large body of empirical literature, which has documented inflation expectations anchoring following the adoption of inflation targeting regimes in many developed countries (see Bernanke et. al. (2001) for a comprehensive survey).

The structural DSGE model in the paper resembles Ferrero (2015). In a two-country rational expectations DSGE model with borrowing constraints, the author seeks to explain three key developments in the U.S. economy prior to the financial crisis: The house price boom, the deterioration of the current account and the low interest rates. One of the key contributions of the paper is to explain the strong negative correlation between house prices and the current account observed both within the U.S. and across countries. The punchline of the paper is a dichotomy between, on the one hand, the factors that explain house prices and the current account deficit, and on the other hand, those that explain the low real interest rate. Ferrero (2015) finds that the house price boom and the deterioration of the current account mainly were driven by domestic housing preference shocks. On the other hand, low real interest rates resulting from overexpansionary monetary policy coupled with foreign exchange rate pegs had practically no effect on house prices. Under bounded rationality this dichotomy breaks down. Low interest rates, either stemming from monetary policy or saving glut shocks, can almost fully account for the house price boom. Low interest rates increase house prices, which eventually convinces agents to follow trend-chasing strategies in the housing market, leading to self-fulfilling expectations of a boom. When overexpansionary monetary policy and savings glut shocks are combined with a relaxation of borrowing constraints, the boundedly rational model can largely account for the house price boom, the deterioration of the current account,

the low real interest rate and inflation observed in the data from 2000-2006. The key to its success is the endogenous switching between forecast strategies, which establishes a strong link between past developments in the underlying DSGE model and the formation of expectations. This mechanism enables the model to reproduce well documented behavior in the U.S. economy during the boom: Trend-chasing behavior in the housing market and the simultaneous anchoring of inflation expectations. Interestingly, with the exception of house prices, the boundedly rational model and the rational expectations model produce practically identical results. This finding supports the view that non-rational expectations are crucial to understanding the housing market specifically. This is closely related to the finding in Adam et. al. (2011) that a standard open economy asset pricing model can account for house price developments over the years 2001-2008 if agents have rational expectations about all variables except house prices. Moreover, it may indicate that housing preference shocks in rational expectations models can be interpreted as a stand-in for non-rational expectations in the housing market.

The remaining proceeds as follows. Section 1 presents the model. Section 2 describes the formation of expectations. Section 3 discusses the calibration. Section 4 discusses the quantitative implications of loose monetary policy and foreign exchange rate pegs. Section 5 adresses the effects of a global savings glut. Section 6 discusses the importance of relaxed credit constraints. Section 7 discusses the monetary policy implications under bounded and strict rationality, respectively. The results are subject to a robustness check in section 8. Finally, section 9 concludes.

1 Model

The model is a two-country model with a collateral constraint which largely resembles the model in Ferrero (2015). Two countries of equal size form the world economy. In each country, a representative household consists of a continuum of measure one of workers. Households consume a composite of goods produced domestically and abroad as well as housing. Housing is assumed to be fixed. Domestic consumers are assumed to be relatively less patient than foreign consumers ($\beta < \beta^*$), thus facing a binding collateral constraint in the vicinity of the steady state. Final goods producers package differentiated intermediate goods inputs under perfect competition, while intermediate goods producers are monopolistically competitive. The only production input in the intermediate goods sector is labor, which is an aggregate of intermediate labor inputs provided by a representative labor agency. Both prices and wages are set on a staggered basis. The law of one price holds, but home bias in consumption means that purchasing power parity is violated. The two countries can trade a one-period nominal risk free bond. The economy is subject to three types of shocks: A relaxation of the collateral constraint of domestic consumers; an expansionary domestic monetary policy shock; and a shock to the intertemporal preferences of foreign consumers.

Domestic consumers

The representative household gains utility from consumption, X_t, and disutility from the labor supply of each household member, $L_t(j)$. The household maximizes the expected discounted utility function:

$$U_t = E_t \left\{ \sum_{k=0}^{\infty} \beta^k \left[\frac{X_{t+k}^{1-\sigma}}{1-\sigma} - \frac{1}{1+\eta} \int_0^1 L_{t+k}(j)^{1+\eta} \, dj \right] \right\} \tag{1}$$

where $\sigma > 0$ is the coefficient of relative risk aversion, $\eta > 0$ is the inverse Frisch elasticity of labor supply.

Aggregate consumption is a composite of non-durable consumption, C_t and housing, H_t:

$$X_t = \left[(1-\alpha) C_t^{\frac{\phi-1}{\phi}} + \alpha H_t^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{\phi-1}} \tag{2}$$

where $\alpha \in (0,1)$ is the share of housing in aggregate consumption and $\phi > 0$ is the constant elasticity of substitution between non-durable consumption and housing. Housing is assumed to be a non-tradable good, while non-durable consumption are tradable goods, consisting of domestically and foreign produced goods:

$$C_t = \left[(1-\triangle_F)^{\frac{1}{\iota_C}} (C_{h,t})^{\frac{\iota_C-1}{\iota_C}} + (\triangle_F)^{\frac{1}{\iota_C}} (C_{f,t})^{\frac{\iota_C-1}{\iota_C}} \right]^{\frac{\iota_C}{\iota_C-1}} \tag{3}$$

where $C_{h,t}$ and $C_{f,t}$ are domestically and foreign produced goods, respectively. $\iota_C > 0$ is the elasticity of substitution between domestic and foreign goods, and $\triangle_F \in (0,1)$ is the share of foreign goods in total non-durable consumption. It is assumed that the consumption of tradable goods exhibit home bias, i.e. $\triangle_F < 0.5$.

Households face the following budget constraint in nominal terms:

$$P_t C_t + Q_t H_t + R_{t-1} B_{t-1} \le B_t + \int_0^1 W_t(j) L_t(j) dj + Q_t H_{t-1} + DIV_t$$

where Q_t is the nominal house price, $W_t(j)$ is the nominal wage of the jth household member, and DIV_t are nominal dividends from intermediate goods firms, which are owned by households. B_t is an internationally traded risk-free bond, denoted in domestic currency, and R_t is the gross nominal interest rate.

In real terms (units of non-durable goods) the budget constraint reads:

$$C_t + q_t H_t + R_{t-1} \frac{b_{t-1}}{\Pi_t} \le b_t + \int_0^1 w_t(j) L_t(j) dj + q_t H_{t-1} + div_t \tag{4}$$

where q_t is the real house price, b_t is real debt, $\Pi_t = P_t / P_{t-1}$ is the gross inflation rate of consumer prices, $w_t(j)$ is the real wage of worker j, and div_t are real dividends.

Domestic consumers face the following collateral constraint (in real terms), which depends on the expected real value of the housing stock:

$$\frac{R_t}{\Pi_{t+1}} b_t \leq \chi_t E_t \left(q_{t+1} H_t \right) \tag{5}$$

where χ_t is the loan-to-value-ratio, which is assumed to follow a first order autoregressive process in log-deviations from steady state:

$$\psi_t = log \left(\chi_t / \chi \right) = \rho_\chi \psi_{t-1} + \varepsilon_{\chi t}$$

The first order conditions of the domestic household read:

$$\lambda_t = MU_t^C, \tag{6}$$

$$\lambda_t q_t = MU_t^H + \beta E_t \left[\lambda_{t+1} q_{t+1} \right] + \lambda_t^{cc} \chi_t E_t \left(\Pi_{t+1} q_{t+1} \right), \tag{7}$$

$$\lambda_t = \beta E_t \left[\lambda_{t+1} \frac{R_t}{\Pi_{t+1}} \right] + R_t \lambda_t^{cc}, \tag{8}$$

$$C_{h,t} = \left(1 - \triangle_F \right) \left(\frac{P_{h,t}}{P_t} \right)^{-\iota_C} C_t, \tag{9}$$

$$C_{f,t} = \triangle_F \left(\frac{\varepsilon_t P_{f,t}^*}{P_t} \right)^{-\iota_C} C_t, \tag{10}$$

$$P_t = \left[\left(1 - \triangle_F \right) \left(P_{h,t} \right)^{1-\iota_C} + \triangle_F \left(\varepsilon_t P_{f,t}^* \right)^{1-\iota_C} \right]^{\frac{1}{1-\iota_C}} \tag{11}$$

where λ_t and λ_t^{cc} are the Lagrange multipliers on the budget constraint and the collateral constraint, respectively, $P_{f,t}^*$ is the price of foreign goods, denoted in foreign currency, and ε_t is the nominal exchange rate (home currency/foreign currency). Note that the law of one price holds, i.e. $\varepsilon_t P_{i,t}^* = P_{i,t}$ for $i = \{h, f\}$. However, due to home bias in consumption, purchasing power parity fails, i.e. $P_t \neq \varepsilon_t P_t^*$, where P_t^* is the foreign price index.

Foreign consumers have the same utility function and face a similar budget constraint as domestic agents. However, due to relative patience, i.e. $\beta < \beta^*$, they are net-lenders in international financial markets and do not face a collateral constraint.

Labor agencies

It is assumed that labor agencies, operating under perfect competition, combine differentiated labor inputs from households, $L_t(j)$, according to the following composite:

$$L_t = \left(\int_0^1 L_t(j)^{\frac{\epsilon_w - 1}{\epsilon_w}} dj \right)^{\frac{\epsilon_w}{\epsilon_w - 1}} \tag{12}$$

where ϵ_w is the elasticity of substitution between labor inputs.

The demand for labor input j is given by:

$$L_t(j) = \left(\frac{W_t(j)}{W_t}\right)^{-\epsilon_w} L_t \tag{13}$$

and the aggregate wage index follows from the zero profit condition:

$$W_t = \left(\int_0^1 W_t(j)^{1-\epsilon_w} dj\right)^{\frac{1}{1-\epsilon_w}} \tag{14}$$

Wage setting

Households are monopolistic suppliers of labor. Wages are set on a staggered basis where θ_w is the probability if not being able to reset the wage in the following period. A household which is able to reset its wage in period t maximizes the following expression with respect to the reset wage, $W_t^{reset}(j)$:

$$E_t \left\{ \sum_{k=0}^{\infty} (\theta_w \beta)^k \left[MU_{t+k}^C W_t^{reset}(j) L_{t+k}(j) - \frac{1}{1+\eta} L_{t+k}(j)^{1+\eta} \right] \right\} \tag{15}$$

subject to the demand function (13). The associated wage Phillips curve is given in the appendix.

Firms

Final goods producers combine intermediate goods according to:

$$Y_t = \left(\int_0^1 Y_t(i)^{\frac{\epsilon-1}{\epsilon}} di\right)^{\frac{\epsilon}{\epsilon-1}} \tag{16}$$

Demands for intermediate good i is given by:

$$Y_t(i) = \left(\frac{P_{h,t}(i)}{P_{h,t}}\right)^{-\epsilon} Y_t \tag{17}$$

while the price index for goods produced in the home country follows from the zero profit condition:

$$P_{h,t} = \left(\int_0^1 P_{h,t}(i)^{1-\epsilon} di\right)^{\frac{1}{1-\epsilon}} \tag{18}$$

Intermediate goods producers use labor as the only production input:

$$Y_t(i) = AL_t(i) \tag{19}$$

where A is a constant productivity factor.

Prices are set on a staggered basis, where θ is the probability that the firm cannot reset its price in the following period. Firms reset their price to maximize their expected discounted future profits for as long as the reset price is expected to remain in place. This implies that firms maximize the following expression with respect to the reset price, $P_t^{reset}(i)$:

$$E_t \left\{ \sum_{k=0}^{\infty} (\theta\beta)^k MU_{t+k}^C \left[P_{h,t}^{reset}(i)Y_{h,t+k} - W_{t+k}L_{t+k}(i) \right] \right\} \tag{20}$$

subject to (19) and the demand function (17). The associated price Phillips-curve is given in the appendix.

Finally, the housing stock is assumed to be fixed[1]:

$$H_t = H \tag{21}$$

Market clearing

International debt market equilibrium requires that the net supply of bonds is zero:

$$B_t + B_t^* = 0 \tag{22}$$

while goods market clearing requires:

$$Y_t = C_{h,t} + C_{h,t}^* \tag{23}$$

Monetary policy

Monetary authorities are assumed to follow a Taylor-type rule:

$$R_t = R_{t-1}^{\mu_R} \left(\bar{R} \left(\frac{\Pi_t}{\bar{\bar{\Pi}}_t} \right)^{\mu_\pi} \left(\frac{Y_t}{\bar{\bar{Y}}} \right)^{\mu_Y} \right)^{(1-\mu_Y)} e^{\mu_{R,t}} \tag{24}$$

where $\mu_{R,t}$ is an innovation to the monetary policy rule.

Equilibrium and steady state

An imperfectly competitive equilibrium of the two-country economy requires that:

i) The representative households maximizes utility subject to the budget constraint and the collateral constraint of domestic consumers, taking prices as given. Also, households set the wages on behalf of its members, taking demand for their specific labor variety as given.

ii) Intermediate goods producers set their price in order to maximize the present discounted value of profits, taking the demand for their specific goods variety as given. Final goods producers minimize costs given the final output price.

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[1]As discussed in Ferrero (2015), housing is assumed to represent land in the model (pp. 270)

iii) Labor and housing markets clear in each country, while goods and financial markets clear internationally.

The assumption $\beta < \beta^*$ gives rise to an asymmetric steady state where the collateral constraint pins down the net foreign assets position. Following Ferrero (2015), the relative productivity level and the housing stock are normalized such that relative prices are equalized across countries and the assymmetry is limited to quantities. The appendix reports the steady state conditions and the log-linearized version of the model.

2 Formation of expectations

The literature on questionnaire survey expectations have generally identified two types of expectations: fundamentalists and chartists expectations (Hommes (2006)). Fundamentalist expectations are stabilizing strategies, such as anchored or mean-reverting expectations. On the other hand, chartist expectations are destabilizing strategies, such as adaptive or trend-following strategies. A growing body of survey litterature have documented the prevalence of trend-chasing behavior in different asset markets, including the housing market (Case et. al. (2012)), the stock market (Vissing-Jorgensen (2004), Greenwood and Shleifer (2014)), and the spot exchange market (Frankel and Froot (1990,1991), Allen and Taylor (1990,1992)). Meanwhile, an extensive body of empirical literature has documented the anchoring of inflation expectations following the adoption of inflation targeting regimes in most OECD countries (see, for instance, Laubach and Posen (1997) or Bernanke et. al. (2001) for a comprehensive survey).

In the following, I will make the simplifying assumption that aggregate expectations are formed as a weighted average of trend-chasing expectations and anchored expectations, respectively, with weights depending on the relative performance of each forecast in the recent past.[2] This seems to be a reasonable simplifying assumption mainly for two reasons. First, anchored and trend-following expectations represent two extremes in terms of volatility: Anchored expectations are highly stabilizing, while trend-chasing expectations are highly destabilizing. This implies that less extreme expectations observed in survey evidence, such as adaptive or mean-reverting expectations, can be expressed as weighted averages of anchored and trend-chasing expectations, respectively. In other words, agents can implicitly choose more moderate aggregate forecasts by appropriately choosing the weights on these two extremes. Second, the assumption enables the model to reproduce two important and well documented features of the U.S. boom: Trend-chasing behavior in the housing market and the simultaneous anchoring of inflation expectations. Section 10 considers an extended model which explicitly allows for adaptive and mean-reverting forecasts.

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[2]Similarly, in an asset pricing model, Barberis et. al. (2015) assumes trend-chasing and rational agents, respectively.

Anchored expectations

Assume that $\hat{x}_t$ denotes the percentage deviation from steady state of the variable x_t. Then anchored expectations simply imply that x_t is expected to equal its steady state value in the future:

$$E_{anch,t}\hat{x}_{t+1} = 0 \tag{25}$$

Agents can choose to apply this forecast to all forward-looking variables in the model, i.e. $\hat{x}_t = \left\{ \triangle\hat{\varepsilon}_t, \hat{q}_t, \hat{q}_t^*, \hat{X}_t, \hat{X}_t^*, \hat{C}_t, \hat{C}_t^*, \pi_t, \pi_t^*, \pi_{h,t}, \pi_{f,t}^*, \pi_{w,t}, \pi_{w,t}^* \right\}$. Note that the inflation forecast, $E_{anch,t}\pi_{t+1} = 0$, corresponds to inflation expectations anchoring when the inflation target of the central bank is normalized to zero.

Trend-chasing expectations

A large body of survey literature finds evidence of trend-chasing behavior in various markets. For instance, based on questionnaire survey evidence on homebuyers expectations of future house prices, Case et. al. (2012) find that "1-year expectations [of future percentage house price changes] are fairly well described as attenuated versions of lagged actual 1-year price changes". Motivated by these findings, the trend-chasing strategy is defined as:

$$E_{trend,t}\left[\hat{x}_{t+1} - \hat{x}_{t-1} \right] = \beta_{trend}\left[\hat{x}_{t-1} - \hat{x}_{t-3} \right] \tag{26}$$

where $0 < \beta_{trend} < 1$. This formulation implies that the expected 6-month change in $\hat{x}_t$, i.e. $E_{trend,t}\left[\hat{x}_{t+1} - \hat{x}_{t-1} \right]$, is a function of the last observed 6-month change, i.e. $\hat{x}_{t-1} - \hat{x}_{t-3}$.[3] The assumption $0 < \beta_{trend} < 1$ is in line with similar survey studies of expected stock market returns (Barberis et. al. (2015)) and expectations in the spot exchange market (Frankel and Froot (1990,1991)). One interpretation of this result is that agents underreact to information (Case et. al. (2012)); another interpretation is that expectations are given by a moving average of past observations, implying that the weight on the most recent observation is smaller than one (Barberis et. al. (2013)). (26) can be applied to all forward-looking variables in the model. The specific value of the trend-chasing parameter, β_{trend}, is discussed in the calibration section.

Selection mechanism and aggregate expectations

Agents continuously evelute the performance of each forecast and switch between forecasts based on their performance in the recent past. I assume that agents prefer strategies with the lowest mean squared forecast errors. The performance of the two strategies are given by:

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[3]To avoid that $\hat{x}_t$ and $E_t\hat{x}_{t+1}$ are determined simultaneously, we assume that agents use lagged information in their forecasts.

$$U^x_{trend,t} = -\sum_{k=1}^{\infty} \omega_k \left[\hat{x}_{t-k} - E_{trend,t-k-1}\hat{x}_{t-k} \right]^2 \tag{27}$$

$$U^x_{anch,t} = -\sum_{k=1}^{\infty} \omega_k \left[\hat{x}_{t-k} - E_{anch,t-k-1}\hat{x}_{t-k} \right]^2 \tag{28}$$

where $U^x_{trend,t}$ and $U^x_{anch,t}$ are the forecast performances of the trend-chasing strategy and the anchoring strategy, respectively. ω_k are geometrically declining weights, implying that agents attach more weight to recent observations.[4]

The share of agents that choose each rule follows Brock and Hommes (1997):

$$\alpha^x_{trend,t} = \frac{exp\left(\gamma U^x_{trend,t}\right)}{exp\left(\gamma U^x_{trend,t}\right) + exp\left(\gamma U^x_{anch,t}\right)} \tag{29}$$

and

$$\alpha^x_{anch,t} = \frac{exp\left(\gamma U^x_{anch,t}\right)}{exp\left(\gamma U^x_{trend,t}\right) + exp\left(\gamma U^x_{anch,t}\right)} = 1 - \alpha^x_{trend,t} \tag{30}$$

where $\alpha^x_{trend,t}$ and $\alpha^x_{anch,t}$ are the share of agents using the trend-chasing strategy and the anchoring strategy, respectively. The parameter γ is the so-called 'intensity of choice', which governs the sensitivity of agent's choices to the relative forecast performance of each strategy. In the limit when $\gamma \to \infty$, all agents choose the best performing strategy, i.e. the strategy with the lowest mean squared forecast error. On the other hand, when $\gamma = 0$, agents do not discriminate between strategies at all, implying that an equal share of agents use the trend-chasing strategy and the anchored startegy, respectively.

The aggregate forecast of any future variable, $\hat{x}_{t+1}$, is given by the weighted average of the trend-chasing and the anchoring forecast:

$$E_t\hat{x}_{t+1} = \alpha^x_{trend,t}\left(E_{trend,t}\hat{x}_{t+1}\right) + \alpha^x_{anch,t}\left(E_{anch,t}\hat{x}_{t+1}\right) \tag{31}$$

Solution under bounded rationality

The log-linearized version of the DSGE model from section 1 can be written in matrix notation as:

$$AZ_t = BE_t\left(Z_{t+1}\right) + CZ_{t-1} + V_t$$

where A, B, and C are appropriately defined matrices, Z_t denotes the state vector, which contains the variables of the model, while V_t contains the shocks. The solution for Z_t is given by:

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[4]The weights are given by $\omega_k = (1-\rho)\rho^k$, where $0 < \rho < 1$.

$$Z_t = A^{-1} \left[B E_t \left(Z_{t+1} \right) + C Z_{t-1} + V_t \right] \tag{32}$$

The solution exists if the matrix A is non-singular. The system describes the solution for the endogenous variables, Z_t, given the expectations, $E_t \left(Z_{t+1} \right)$. Under rational expectations, the equilibrium is unique and stable if the Blanchard-Kahn conditions are satisfied. These conditions rule out exponential growth of the expectations. Under bounded rationality, expectations $E_t \left(Z_{t+1} \right)$ are specified in (25)-(31) and can be substituted into (32). This implies that (32) becomes a non-linear backward-looking dynamic system. Starting from the steady state, and given the shock sequence V_t, we can solve for Z_t in every period by forward iteration. Expectations, as specified in (25)-(31), can become explosive, implying that the Blanchard-Kahn conditions are not necessarily satisfied. The stability of the system depends crucially on the parametrization of the model (particularly, the size of the trend-chasing parameter, β_{trend}, in (26)) and of the size and persistence of the stochastic shocks. While I focus on stable solutions in the paper, the appendix provides some examples of explosive solutions.

3   Calibration

Structural parameters

The calibration of the structural model is identical to that of Ferrero (2015) with the exception of the discount factors, β and β^*, and the housing share parameter, α. The latter denotes the share of housing in total consumption, which is usually calibrated to match an expenditure share in the data. For instance, Monacelli (2009) calibrates α to match a housing investment share of aggregate expenditures of 20 pct. In Ferrero (2015), however, housing is fixed, which implies that housing expenditures are zero.[5] Consequently, I use a different approach. Since realistic collateral effects are crucial to the quantitative analysis in this paper (and since housing serves as collateral for foreign debt), I choose α to match a debt-to-gdp ratio of 15 pct.[6] This corresponds to the Net International Investment Position of the United States as a share of GDP in 2000.

The foreign discount factor, β^*, is set to 0.9908. This implies an annual real interest rate of 3.7 pct., which corresponds to the actual U.S. short-term real interest rate in 2000q1, which is the first quarter in the simulations[7] As discussed in Iacoviello and Neri (2010), the value of the borrower's discount factor, β, has to be sufficiently low to ensure that the collateral constraint stays binding at all times. For instance, Ferrero (2015) works with a relatively low value, $\beta = 0.93$. Under bounded rationality, however, the model can maintain a binding collateral constraint for considerably larger values of the discount factor. Consequently, I use $\beta = 0.96$,

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[5]In the absence of housing investment, Ferrero interprets the value of the housing stock, qH, as housing expenditures and calibrates α such that $\frac{C}{C+qH} = 0.83$.

[6]This condition is satisfied when $\alpha = 0.0082$, which is somewhat larger than the corresponding value in Ferrero (2015).

[7]Ferrero works with $\beta^* = 0.99$, implying a slightly higher annual real interest rate of 4 pct.

which is more in line with empirical estimates (Iacoviello (2005)).

The remaining calibration is identical to Ferrero (2015). The inverse Frisch elasticity of labor supply, η, is set to 2. The elasticity of substitution between goods and labor varieties are both calibrated to match a steady state markup of 15 pct. in the goods and labor market ($\epsilon = \epsilon_w = 7.67$). The price and wage stickiness parameters, θ and θ_w are set to 0.75, which implies a duration of price and wage contracts of four quarters. The share of foreign goods in total tradable consumption, $\triangle_F$, is set to 0.3, while the elasticity of substitution between domestic and foreign goods is set to 2. Moreover, it is assumed that the elasticity of substitution between consumption and housing, ϕ, equals 1. The relative housing stock, H/H^*, is calibrated such that the relative steady state house price, q/q^*, equals unity. Similarly, the productivity ratio, A/A^*, is calibrated such that the relative price of tradable goods in steady state, P_h/P_f^*, equals unity. This ensures that the assymmetric steady state is limited to quantities, while prices are the same in the two countries. The parameters in the Taylor rule takes standard values. The interest smoothing parameter, μ_R, is set to 0.7. The response to inflation, μ_π, is set to 1.5, while the response to output, μ_Y, is set to 0.5.

Behavioral parameters

A key parameter in the behavioral system (25)-(31) is the intensity of choice, γ. This parameter governs the frequency at which agents switch between different strategies, depending on their relative past forecast performance. At one extreme, $\gamma = 0$, agents do not discriminate between rules at all. At the other extreme, $\gamma \to \infty$, all agents will choose the most optimal strategy. Anufriev et. al. (2013) estimates the intensity of choice on experimental data to be between 0.14 and 4.5. The estimated value is heavily influenced by the autocorrelation of the past performance of each forecast. Consequently, white noise data produces estimates between 0.14-0.29, while autocorrelated data produces estimates between 0.38 and 4.5. As a baseline, I fix the intensity of choice to unity, i.e. $\gamma = 1$. In section 8, I show that the qualitative results are robust to changes in γ, though high values may imply unstable inflation dynamics.

The parameter β_{trend} in the trend-chasing forecast (26) governs the degree of trend-chasing. Case et. al. (2012) find estimates between 0.18-0.30 for annual house price expectations across U.S. counties.[8] Barberis et. al. (2015), using survey data on expected annual stock market returns, estimate β_{trend} to around 0.6-0.7.[9] Frankel and Froot (1990,1991) find small but significantly positive estimates for expectations of the U.S. spot exchange rate for short-term horizons (0.13 for 1 week and 0.05 for 1 month, respectively). Since my aim is to analyze the U.S. house price boom, I choose $\beta_{trend} = 0.3$ as a baseline, which is based on the estimates in Case et. al. (2012). In section 8, I test the sensitivity of the results to different values of β_{trend}.

The memory parameter, ρ, which determines the weights on past observations in the selection mechanism, ω_k, is set equal to 0.5, while the window of past periods used to evaluate the

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[8]For the entire sample the authors find $\beta_{ext} = 0.23$.

[9]In Barberis et. al. (2015), the expected future stock return is given by a weighted average of past stock returns, where β_{trend} denotes the weight on the most recent observation.

forecast performance of the rules, z, is assumed to be 20 quarters. These values are standard in the literature (see De Grauwe (2011), Bofinger et. al. (2013)), and the results are highly robust to different assumptions (this is demonstrated in the appendix).

In steady state, both forecasts (25) and (26) and the forecast errors in (27)-(28) equal zero. It follows from (29)-(30) that there is an equal steady state share of anchoring and trend-chasing agents, respectively, i.e. $\alpha^x_{trend} = \alpha^x_{anch} = 0.5$. Section 8 tests the sensitivity of the results to different steady state distributions.

4 Loose monetary policy

Some observers, most notably Taylor (2007, 2008), have argued that loose monetary policy was one of the main drivers of the U.S. house price boom. According to Taylor, the Federal Reserve kept nominal interest rates too low for too long after the 2001 recession. Low interest rates may in turn have contributed to the boom in house prices. Ferrero (2015) documents that the nominal interest rate set by the Federal Reserve between 2000-2006 was indeed more accomodative than prescribed by the Taylor rule (24) (pp. 284). However, the author finds that the quantitative contribution of monetary policy shocks to the house price boom was negligible. This section considers the effects of accomodative monetary policy under bounded rationality.

Figure 1 shows the effects of expansionary monetary policy shocks, identified as deviations of the Federal Funds Rate from the interest rate prescribed by the log-linearized version of (24) from 2000 to 2006. The blue lines show the responses of the boundedly rational model, while the red lines show the responses of a similar model with rational expectations. The responses of the two models are practically identical. In both models, the shocks lower the nominal interest rate by around 1.5 pct. points from 2000-2002, which induces a decline in the real interest rate of almost 2 percentage points. This corresponds to around half of the decrease in the data from 2000-2002. Also, in both models, low real interest rates generates a boom in consumption of around 3-4 pct. Finally, the inflation rate gradually increases to around 2 pct. points above target in 2006, which is well in line with the data.

The only notable difference between the two models is the behavior of house prices and the current account. In the underlying DSGE model, low interest rates facilitate borrowing and increase housing demand. This leads to an increase in real house prices of almost 10 pct. and a deterioration of the current account of roughly 0.5 pct. points as a share of GDP in the boundedly rational model. On the other hand, the responses of house prices and debt are muted under rational expectations. Expectations in the boundedly rational model are formed as a weighted average of anchored expectations (equation (25)) and trend-chasing expectations (equation (26)), with weights depending on the forecast performance of each strategy in the

Figure 1: The effects of expansionary monetary policy shocks under flexible exchange rates
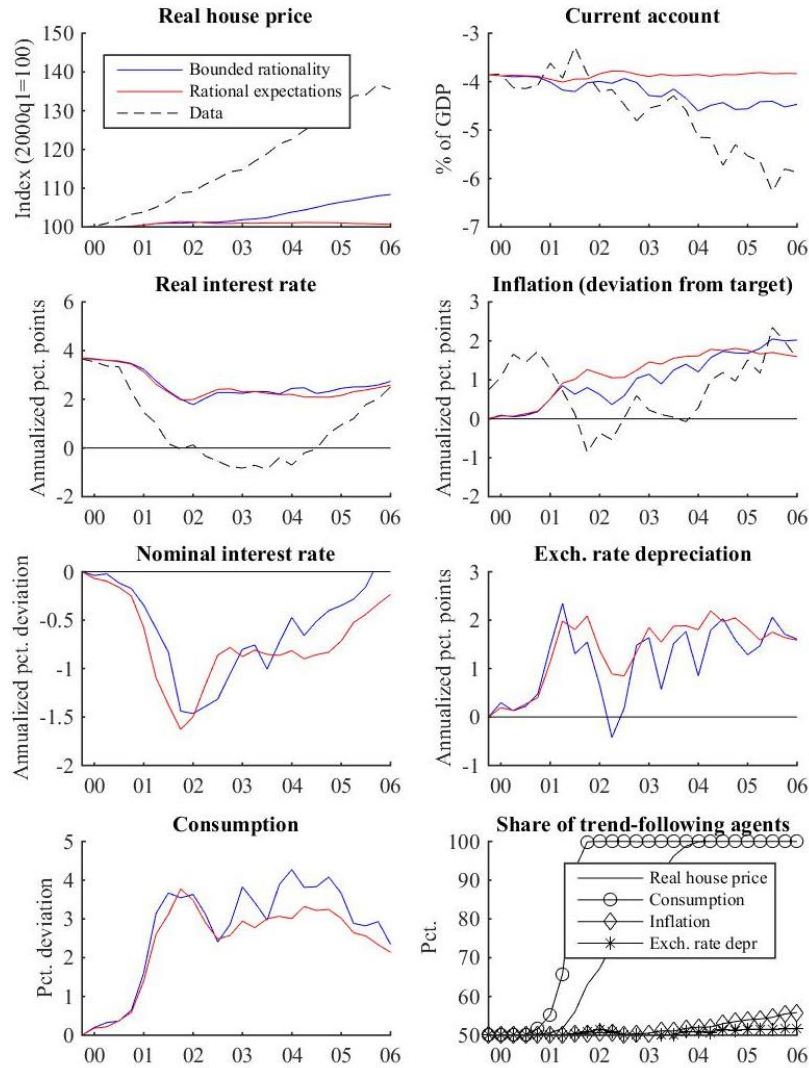
Note (1): The data series for inflation is the annualized quarterly CPI inflation rate subtracted by an assumed annual target of 2 pct.
Note (2): The current account in the model is normalized to equal its data value in 2000q1.
Source: Bureau of Economic Analysis, Federal Housing Finance Agency, Board of Governors of the Federal Reserve, The Federal Reserve Bank of Philadelphia and model simulations

recent past. Figure 1 depicts the development in the distribution of trend-chasing agents over time. Initially, there is an equal distribution of trend-chasing agents and anchoring agents. However, when the economy is subject to shocks, agents will begin to switch between strategies in order to minimize their past squared forecast errors. Accomodative monetary policy generates a persistent boom in consumption, which implies that agents quickly switch to the trend-chasing forecast of future consumption. Similarly, low interest rates increase house prices, which gradually convinces agents to adopt a trend-chasing strategy to forecast house prices. At the same time, agents' expectations of future inflation remain anchored. Clearly, expansionary monetary policy increases the inflation rate. However, the effects are numerically small.

In other words, inflation expectations remain anchored because the inflation rate does not deviate notably from the central bank's target rate.[10] Similarly, exchange rate expectations remain anchored, since the monetary policy shocks only produce a moderate depreciation of the exchange rate.[11]

In the rational expectations model, the quantitative contributions of monetary policy shocks to the house price boom are extremely small.[12] This is essentially an inherent feature of most DSGE models with rational expectations. Expansionary monetary policy shocks are assumed to be entirely stochastic (as in (24)).[13] Under rational expectations, this implies that monetary policy shocks have negligible effects on long run expectations. Absent any effects on long run expectations, monetary policy has negligible effects on contemporary house prices. Importantly, however, even if the monetary policy shocks were assumed to be persistent, they could never generate a housing boom since forward-looking agents would fully anticipate the long-lasting nature of policy deviations. Thus, inflation expectations would no longer remain anchored in response to policy shocks, and a sharp increase in inflation expectations would hinder the central bank from lowering its interest rate in the first place. Thus, even highly persistent monetary policy shocks would not be able to generate a boom in house prices under rational expectations.

4.1 Foreign Exchange Rate Pegs

Expansionary monetary policy shocks could only account for around half of the decline in the real interest rate observed in the data when the exchange rate is flexible (see Figure 1). A key hypothesis in Ferrero (2015) is that the low interest rates were partly a result of foreign currency pegs to the dollar, which transmitted low U.S. interest rates to the rest of the world. As documented in Ferrero (2015, pp. 287), the growing trade deficit of the U.S. during the boom years (2000-2006) was largely financed by China and the OPEC countries, many of which had pegged their currencies to the U.S. dollar. This section examines whether foreign pegs can help account for the decline in the real interest rate in the data and to what extend this could have contributed to the run-up in house prices. In the following I assume that the foreign central bank operates a fixed exchange rate regime. This implies that the foreign Taylor rule is replaced by the following expression:

$$\triangle \hat{\varepsilon}_t = 0 \tag{33}$$

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[10]Note however, that the share of agents who use a trend-chasing inflation forecast increases slightly from 2004-2006. Thus even small deviations from the central bank's target implies a loss of credibility if they are sufficiently long-lasting.

[11]Anchored quarterly exchange rate expectations are consistent with the evidence in Frankel and Froot (1990,1991), which suggests that the extrapolative parameter for the expected quarterly exchange rate depreciation is close to zero.

[12]Ferrero (2015) reaches the same conclusion (p. 264)

[13]This assumption is strong since the Federal Funds rate was consistently lower than the interest rate prescribed by a Taylor rule during the entire boom period from 2000 to 2006 (as shown in Ferrero (2015)). Consequently, the Taylor residuals from (24) are highly autocorrelated during this period.

Figure 2 reconsiders the effects of expansionary monetary policy shocks when the foreign central bank operates a fixed exchange rate regime. By comparing the developments in Figure 1 and Figure 2, it is clear that the domestic real interest rate declines considerably more under a peg both under bounded rationality and rational expectations. Moreover, monetary policy can account for up to half of the boom in house prices under bounded rationality when the exchange rate is fixed. Under rational expectations, however, the response of house prices continues to be relatively muted. Intuitively, expansionary monetary policy will imply a depreciation of the domestic currency under flexible exchange rates. This increases foreign demand, which puts an upward pressure on the domestic price level. The resulting feedback effect from the Taylor rule raises the interest rate. This feedback effect, however, is absent under a full peg, which implies that the interest rate declines more. Under bounded rationality, the lower real interest rate amplifies the house price boom substantially. This is partly because agents switch to the trend-chasing house price forecast at an earlier stage than in the flexible exchange rate scenario considered in Figure 1. Under rational expectations, however, expectations continue to be anchored because the monetary policy shocks are entirely stochastic.

There is no consensus in the literature on the contribution of monetary policy to the U.S. housing boom. Based on FAVAR/VAR models, Del Negro and Otrok (2007) and Jarociski and

Figure 2: The effects of expansionary monetary policy shocks under a foreign exchange rate peg
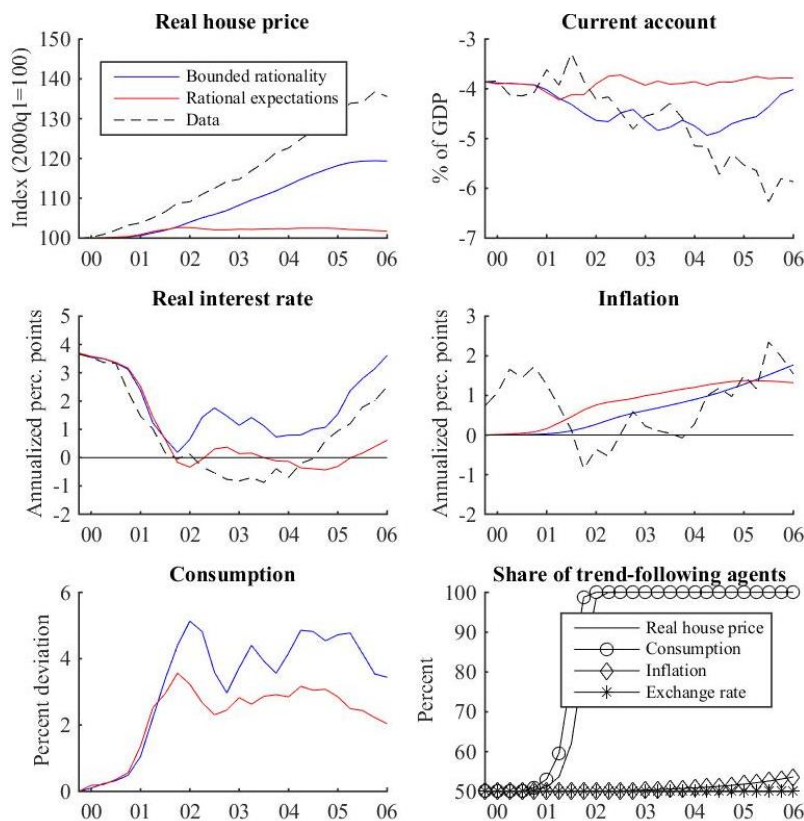
Smets (2008) find that only a small portion of the run up in house prices can be attributed to the stance of U.S. monetary policy. On the other hand, Taylor (2007), based on a single equation estimation, finds that monetary policy can account for roughly one third of the housing boom. The latter matches well with the results in the boundedly rational model where monetary policy shocks can account for between a quarter and half of the house price boom, depending on the exchange rate regime (figure 1 and 2). In an estimated DSGE model with rational expectations, Iacoviello and Neri (2010) find that monetary policy shocks can account for around 15 pct. of the house price boom from 1998-2005. During the same period, however, housing preference shocks can account for 67 pct. of house price dynamics. Thus monetary policy shocks and preference shocks can jointly explain more than 80 pct. of the boom in their model. If preference shocks can be interpreted as a stand-in for non-rational expectations, these findings are well in line with the results in Figure 2.

5   Global Savings Glut

As evident in Figure 2, the boundedly rational model still has some difficulties accounting for the low real interest rate from 2002-2006. A prominent theory about the decline in interest rates in the 2000's is Bernanke's (2005) so-called Global Savings Glut hypothesis. According to Bernanke, increased capital inflows to the United States from countries in which desired saving exceeded desired investment (such as China) brought down U.S. mortgage-related interest rates. Bernanke et. al. (2011) uncovers the size of these capital flows and their effects on U.S. mortage rates. Under the assumption of fixed exchange rates, the authors find that exogenous capital inflows from savings glut countries brought down U.S. mortgage-related interest rates by around 160 bp from 2003-2007. Justiniano et. al. (2014) reach a similar result in an open economy DSGE model. Motivated by these findings, this section considers the effects of an exogenous increase in foreign savings, modelled as a temporary increase in the foreign discount factor, β^*. The baseline corresponds to the scenario considered in Section 2 where the economy is subject to monetary policy shocks under fixed exchange rates. Following Bernanke et. al. (2011), I consider a shock that can produce a decline in the domestic nominal interest rate of roughly 160 basis points in both the rational and the boundedly rational model, respectively.
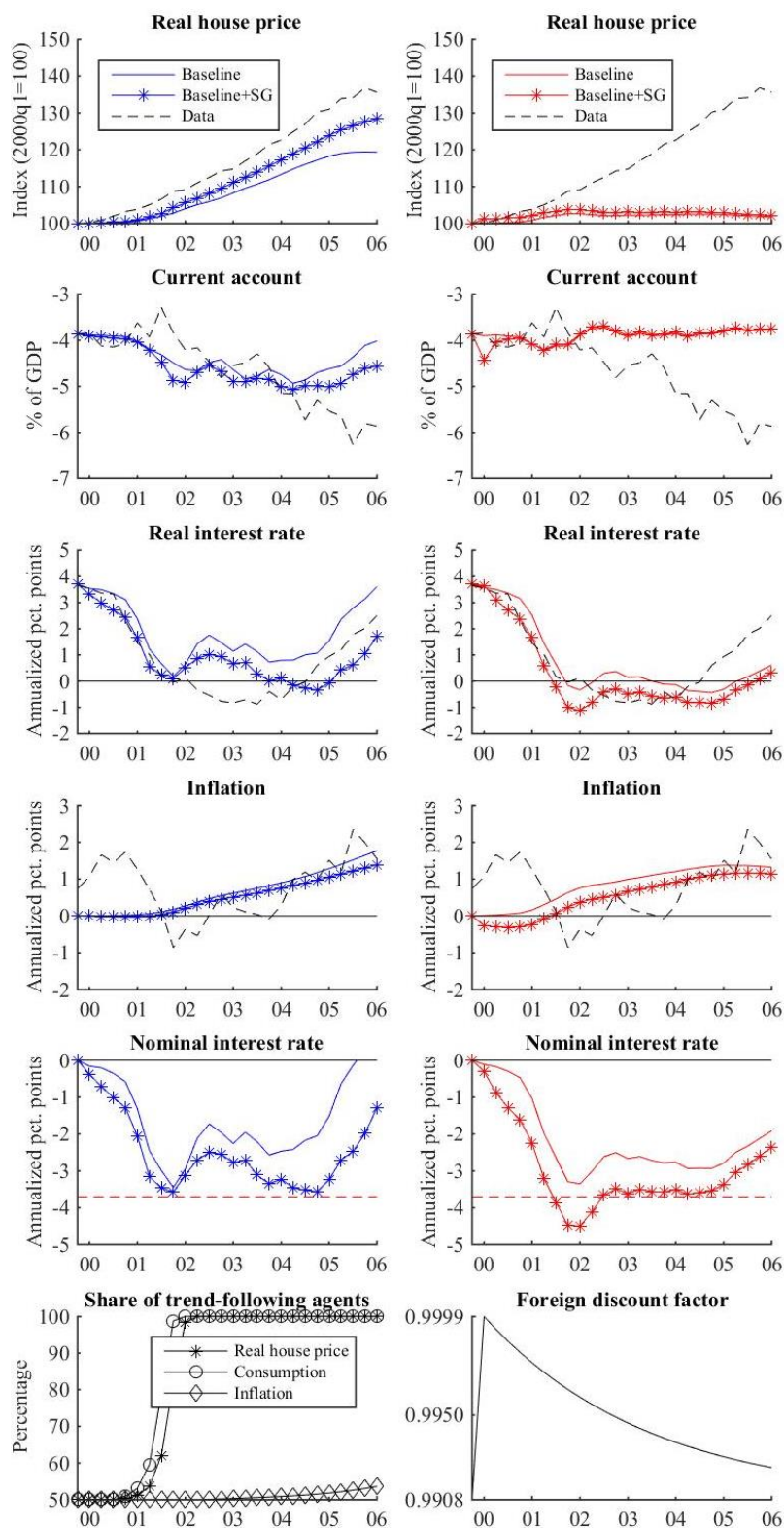
Figure 3 shows the effects of the shock. The left panel shows the impulse responses in the boundedly rational model, while the right panel shows the impulse responses under rational expectations. The foreign discount factor increases to almost 1 in 2000 and then gradually reverts back to its long-run value, $\beta^* = 0.9908$.[14] This induces a decline in the nominal interest rate of around 1-2 pct. points in both models.[15] Qualitatively, the saving glut shock has the exact same implications as a foreign peg. Similarly to a peg, an increase in foreign

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[14]In order to produce a decline in the nominal interest rate which is roughly in line with the results in Bernanke et. al. (2011), the AR(1) coefficient of the discount factor shock, ρ_{β^*}, is set to 0.75.

[15]Note that the zero lower bound on the nominal interest rate is briefly violated in the rational expectations model. This is due to the assumption of a relatively high domestic discount factor.

Figure 3: The effects of an increase in the foreign discount factor

Note (1): The left panel plots the impulse responses under bounded rationality, while the right panel shows the impulse responses under rational expectations.

savings lowers foreign demand, leading to lower domestic GDP and inflation. Lower inflation enables the central bank to lower its interest rate more than in the baseline scenario. A lower real interest rate amplifies the house price boom and the deterioration of the current account. This effect is considerably larger under bounded rationality than under rational expectations. Under bounded rationality, trend-chasing behavior in the housing market amplifies the effects of low interest rates on house prices. Under rational expectations, however, the shock to the discount factor is not sufficiently persistent to affect long term house price expectations and generate any notable run-up in house prices. A highly persistent shock to the discount factor could in principle produce a housing boom. However, since the shock is deflationary this would imply a decline in the nominal interest rate of much larger magnitude than in Bernanke et. al. (2011).

6 The Effects of Relaxed Borrowing Constraints

A large body of literature has linked the U.S. housing boom to relaxed lending standards (see, for example, Demyanyk and Van Hemert (2011), Dokko et. al. (2011)). Duca et. al. (2011) find that the LTV ratio for first-time home buyers from the American Housing Survey increased by roughly 10 percentage points during the boom, from around 90 pct. in 2000 to almost 100 pct. in 2006 (Ferrero (2015), pp. 275). This section considers the effects of relaxing the borrowing constraint of domestic consumers. The shock process is chosen to roughly match the evidence in Duca et. al. (2011). Following Ferrero (2015), the autoregressive coefficient of the LTV shock, ρ_χ, is set to 0.98.

Figure 4 shows the effects of a relaxation of lending standards. The "Baseline+SG"-scenario corresponds to the experiment considered in Section 5 where the economy is subject to monetary policy shocks and foreign intertemporal shocks under a fixed exchange rate regime. The "Baseline+SG+LTV"-scenario includes the shock to the LTV ratio. The relaxation of borrowing constraints allows domestic households to borrow more and increase their consumption of both non-durable goods and housing. In both models, this contributes to the deterioration of the current account. Moreover, the shock drives up the real value of the housing stock as this serves as collateral for debt (see equation (7)). Interestingly, house prices increase more under rational expectations than under bounded rationality. The high persistence of the LTV shock maximizes the effect on rational long-run house price expectations. However, the collateral constraint ceases to bind under rational expectations. To maintain a binding collateral constraint under rational expectations, the model requires a significantly lower value of the domestic discount factor.[16] Moreover, this enables the rational model to generate a larger boom in house prices. I find that for $\beta = 0.90$, the rational model can maintain a binding collateral

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[16]When agents are forward-looking, a highly persistent relaxation of borrowing constraints implies a large decline in the shadow value of borrowing, particularly if agents are relatively patient, i.e. if they have a high discount factor. Therefore, under rational expectations, the discount factor has to be relatively low for the collateral constraint to stay binding.

Figure 4: The effects of an increase in the LTV ratio of domestic consumers
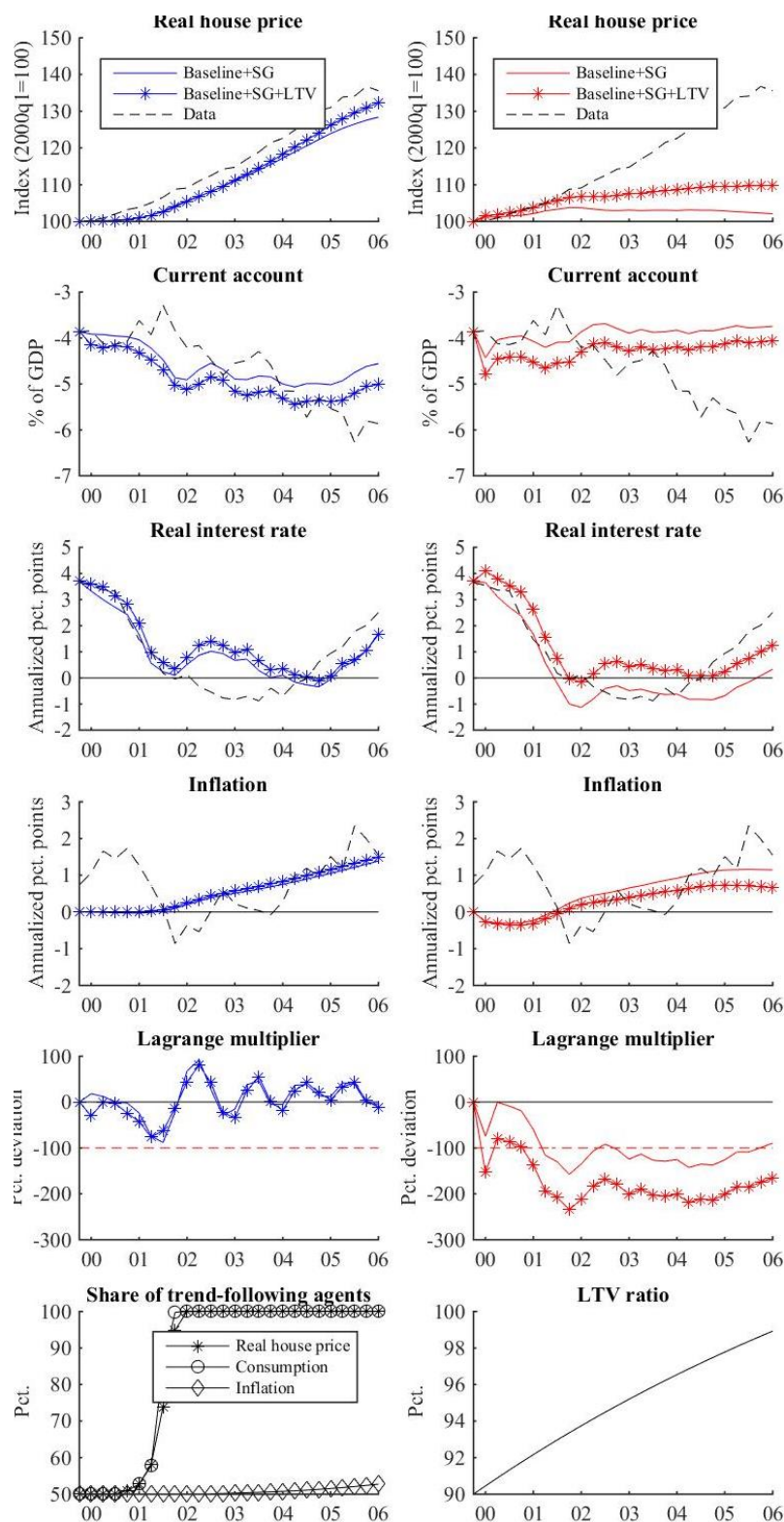
Note (1): The left panel plots the impulse responses under bounded rationality, while the right panel shows the impulse responses under rational expectations.

constraint and reproduce around half of the boom. However, such a low value of the discount factor is not supported by empirical evidence.

Summary

Figure 4 shows that the the boundedly rational model can largely account for the dynamics of house prices, the current account, the real interest rate and inflation from 2000 to 2006. In the model, house prices are largely driven by low interest rates, amplified by trend-chasing expectations. Interestingly, with the exception of house prices, the boundedly rational model and the rational expectations model produce almost identical results. Thus, non-rational expectations may be an important driver of the housing market specifically, while being less important for understanding other parts of the economy. Intuitively, it is the absence of stabilizing feedback mechanisms in the underlying DSGE model, such as foreign tradability, flexible supply or monetary policy response, which means that expectations can end up becoming a self-fullfilling driver of house prices.

An attractive feature of the boundedly rational model is that it establishes a relatively strong negative correlation between house prices and the current account, which is also observed in the data. The correlation coefficient between real house prices and the current account (as a share of GDP) from 2000-2006 is -0.74 in the model, while it is -0.94 in the data. In the model, house prices can become temporarily explosive which is necessary to establish a negative correlation with the current account. The rational expectations model, on the other hand, relies on housing preference shocks that are fitted to match actual house price dynamics to establish this negative correlation.[17]

7   Monetary Policy Implications

The recent financial crisis has re-ignited a long-standing debate as to whether monetary policy should respond to asset price movements or not (see Bernanke and Gertler (2001) for an early contribution). A simple way of evaluating this hypothesis is to modify the domestic Taylor rule to allow for a positive response to house prices:

$$i_t = \mu_R i_{t-1} + (1 - \mu_R) \left[\mu_\pi \pi_t + \mu_Y \hat{Y}_t + \mu_q \hat{q}_t \right] + \mu_{R,t} \tag{34}$$

where $\mu_q > 0$ is the coefficient on the domestic real house price. Figure 5 shows the effects of such a modified rule. μ_q is calibrated such that house prices do not increase by more than 10 pct. between 2000 and 2006 in the boundedly rational model (this is satisfied for $\mu_q = 0.3$). The solid lines without stars show the baseline simulations under

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[17]While this argument holds for the within country evidence of the U.S., temporarily explosive house prices are not necessary to replicate the cross-country evidence presented in Ferrero (2015, pp. 262). This can be replicated in both models.

Figure 5: Monetary policy implications: An augmented Taylor rule
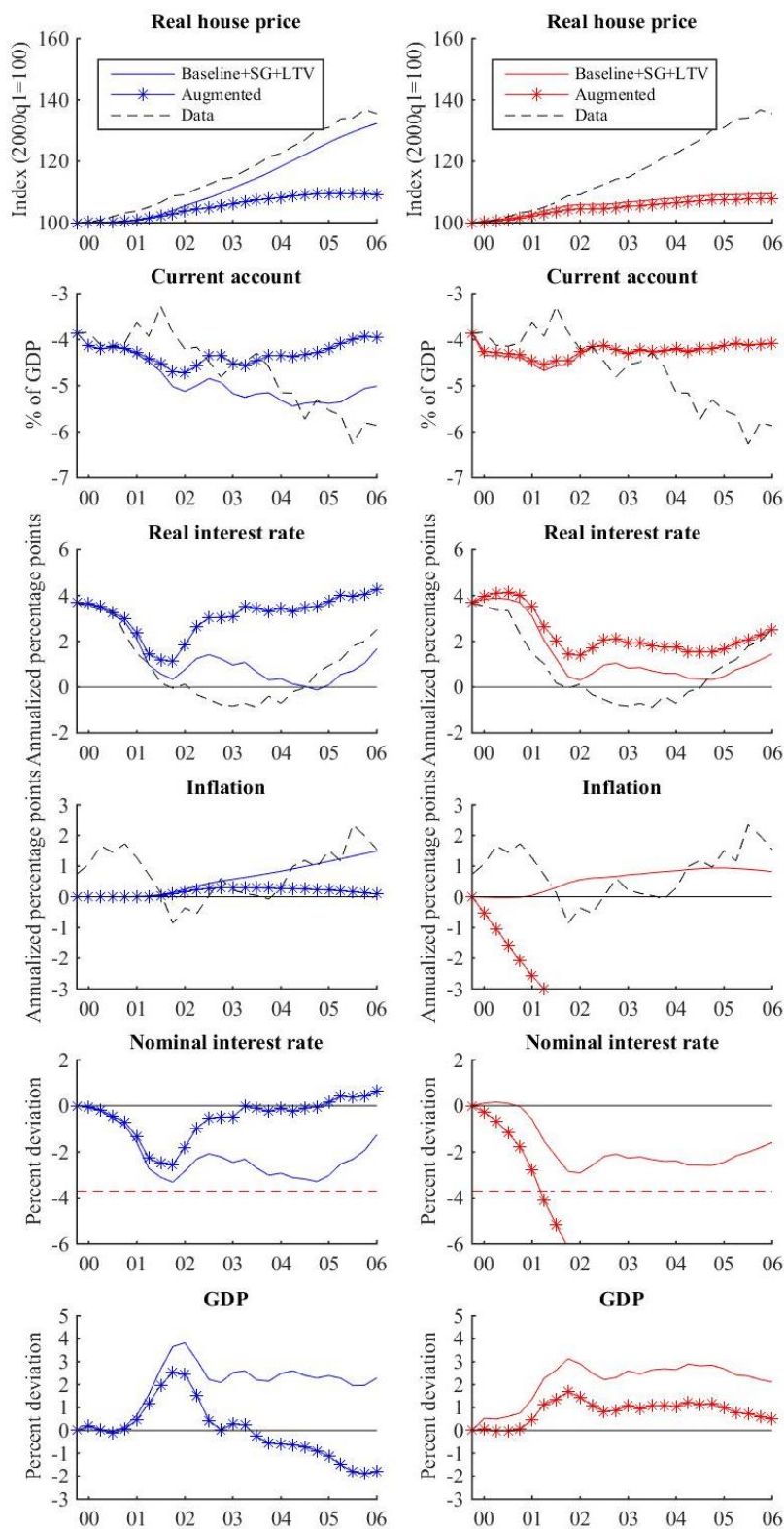
Note (1): The left panel plots the impulse response functions under bounded rationality, while the right panel shows the impulse responses under rational expectations.

bounded rationality (blue) and rational expectations (red) considered in the previous section. The lines with the stars show the paths of the economy when the central bank responds to house prices in the two models. In the boundedly rational model the central bank can stabilize both inflation and GDP and largely prevent the house price boom and the current account deterioration by responding to house prices. This is not surprising since expansionary monetary policy is the main driver of the boom under bounded rationality. Thus, the central bank can meet all of its objectives by increasing its interest rate early on. Under rational expectations, however, a positive response to house prices leads to strong deflation and a violation of the zero lower bound on the nominal interest rate. Intuitively, highly persistent shocks to the LTV-ratio increase rational agents' long run house price expectations. If the central bank aims at stabilizing house prices, this in turn implies that forward-looking agents anticipate higher long run interest rates and lower long run inflation. This decline in long-run inflation expectations sets off a strong deflationary pressure, which forces the central bank to lower its interest rate on impact.

8 Robustness

This section tests the sensitivity of the results in Section 6 to different values of key parameters as well as the steady state conditions and the number of forecast strategies.

8.1 Intensity of choice

The intensity of choice governs the rationality of agents, i.e. the share of agents that choose the better performing rule. When $\gamma = 0$ agents do not discriminate between rules. On the other hand, when $\gamma \to \infty$ all agents choose the most optimal rule. Anufriev et. al. (2013) estimates this parameter on experimental data to lie in the interval [0.38,4.50] when past performances are autocorrelated. Figure 6 tests the sensitivity of the results to changes in the intensity of choice within this interval. Clearly, the results are quite robust to changes in this parameter. However, for high values ($\gamma = 4.50$) inflation may become explosive. As demonstrated in the following, the stability of the model does not only depend on parameter values, but also on the steady-state distribution of trend-chasing agents and anchoring agents, respectively.

8.2 Volatility of trend chasing expectations

The extrapolative parameter, β_{trend}, controls the degree of trend-chasing in the trend-chasing forecast (26). Case et. al. (2012) estimates the parameter on 1-year house price expectations to lie in the interval $\beta_{trend} = [0.18, 0.30]$, while Barberis et. al. (2015) find estimates as high as 0.7 for expected stock returns. Figure 7 tests the robustness of the results to values of the parameter within the interval $\beta_{trend} = [0.18; 0.40]$. Not surprisingly, higher values of the trend-chasing parameter, β_{trend}, produce a larger house price boom.

Figure 6: Varying the intensity of choice
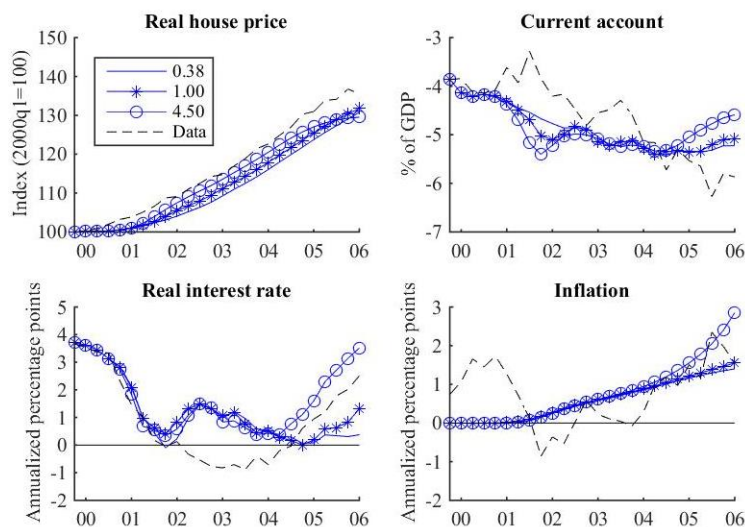
Figure 7: Varying the volatility of trend-chasing expectations (β_{trend})
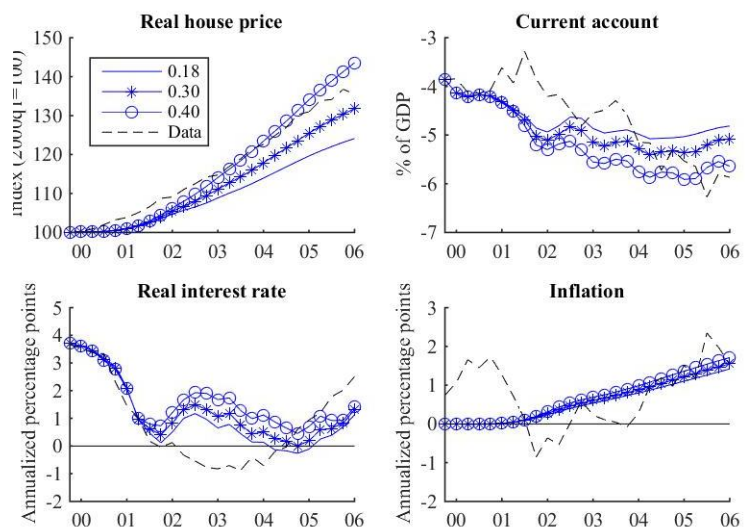
8.3   Share of trend-chasing agents

Figure 8 shows that the results are insensitive to different steady state conditions, unless the steady state share of inflation trend-chasers is very high (75 pct.). In that case inflation may turn explosive.

Figure 8: Varying the steady state share of trend-chasers
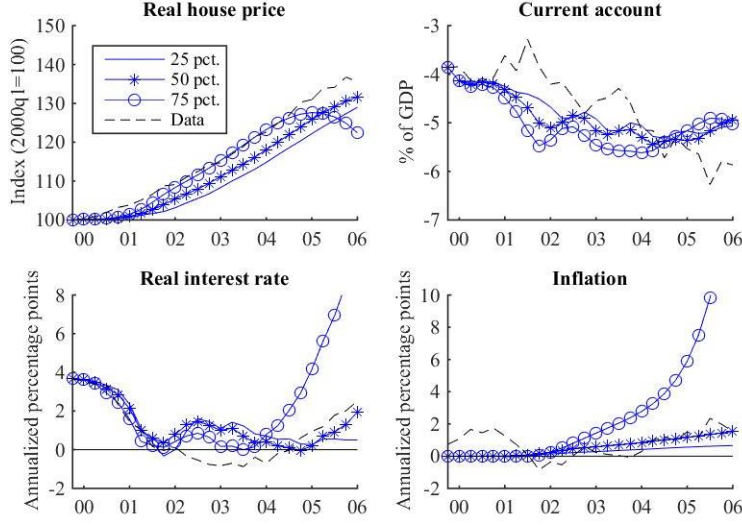
8.4 Number of forecasting strategies

This section considers an extended model which explicitly allows for adaptive and mean-reverting strategies. The adaptive rule is simply

$$E_{adap,t}\left[\hat{x}_{t+1}\right] = \hat{x}_{t-1}$$

while the mean-reverting strategy is given by

$$E_{mean,t}\hat{x}_{t+1} = \beta_{mean}\hat{x}_{t-1}$$

where β_{mean} measures the degree of mean-reversion which I set to 0.75. I assume that the steady state share of agents using an anchoring strategy is 40 pct. (while 20 pct. initially use each of the remaning three strategies). This implies that aggregate expectations in steady state are similar to the baseline model. Figure 9 shows the results. The extended model produces practically identical dynamics as the baseline model in Figure 4. The only notable difference is that agents switch from the trend-following consumption forecast to the adaptive and the mean-reverting consumption forecasts. After the initial increase in consumption in 2000-2002, the central bank stabilizes consumption at a level around 5 pct. above trend. Since the adaptive forecast strategy implies a constant level, it quickly becomes the most accurate ex-post forecasting strategy of consumption. Thus, agents adjust their expectations accordingly.

Figure 9: Number of strategies
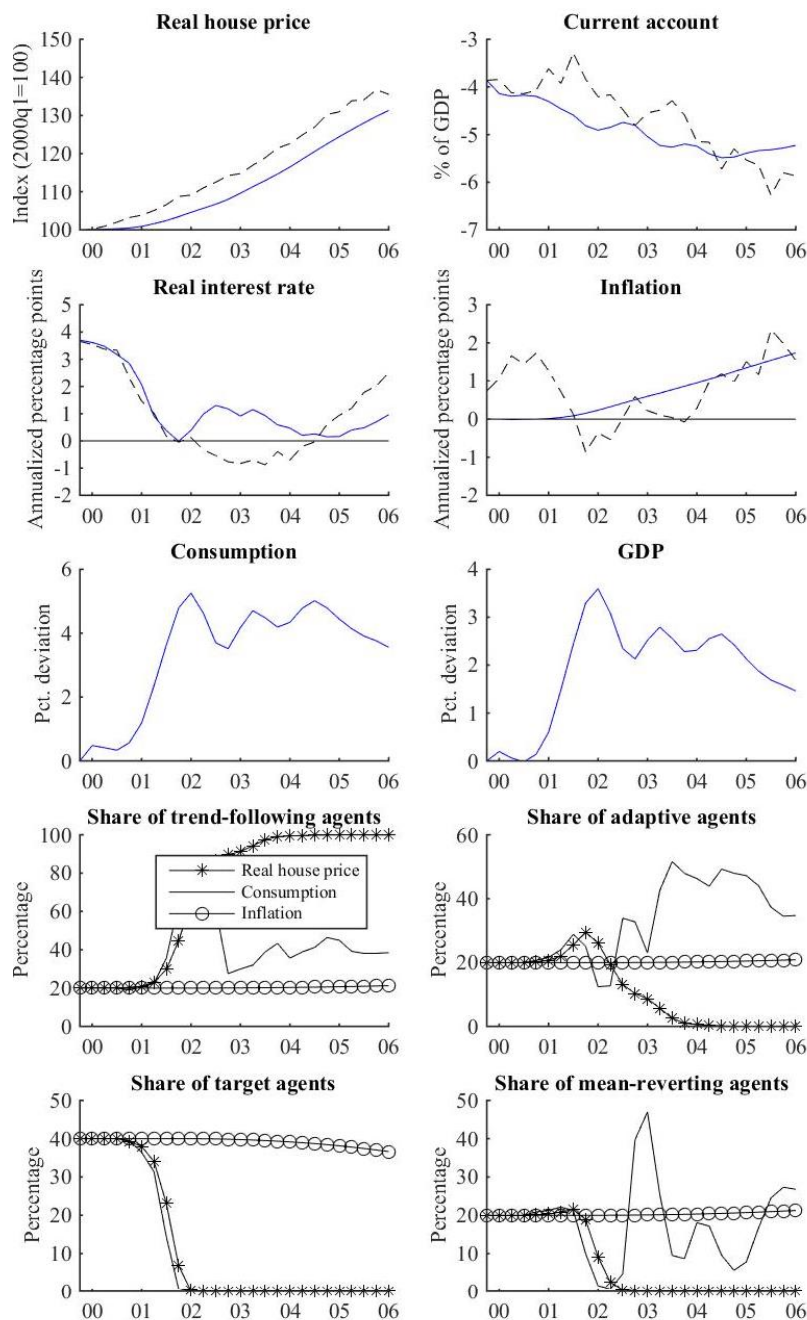
9 Conclusion

The literature has identified at least four important contributors to the U.S. housing boom prior to the financial crisis: Loose monetary policy, a relaxation of borrowing constraints, a global savings glut, and deviations of house prices from fundamentals. However, none of these factors can account for house price dynamics in standard DSGE models with rational expecta-

tions. Instead, these models rely on housing preference shocks to bridge the gap between the model and the data. This paper extends a standard DSGE model with boundedly rational agents. In the model, agents choose between simple strategies to forecast future variables and switch between strategies based on their forecast performance in the recent past. The available forecast strategies are based on behavior observed in survey data. In this model, overexpansionary monetary policy, relaxed borrowing constraints, and a foreign saving glut can almost fully account for the dynamics of house prices, the current account, interest rates, and inflation during the boom period from 2000-2006. Low interest rates, relaxed credit constraints, and rising house prices convince agents to gradually adopt trend-chasing strategies in the housing market, leading to self-fulfilling expectations of a house price boom. Monetary policy can explain between a quarter and half of the house price boom under bounded rationality. In comparison, monetary policy has practically no effect on house prices under rational expectations. In the boundedly rational model, a modified Taylor rule with a positive response to house prices implies an early normalization of the interest rate, which stabilizes inflation and GDP, and largely prevents the house price boom and the deterioration of the current account.

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Appendix

Robustness

Memory

Figure A1 tests the sensitivity of the results to different values of the parameter, ρ, which governs the memory of agents. A high value of ρ implies that agents attach a relatively large weight to observations in the distant past. Clearly, the results are quite insensitive to changes in ρ.

Window

The sensitivity of the results to changes in the number of past observations in the fitness criterion, k, is illustrated in figure A2. The results are practically identical for different values of k.

Elasticity of substitution (consumption/housing)

Figure A3 shows that the results are highly robust to a different elasticity of substitution between consumption and housing.
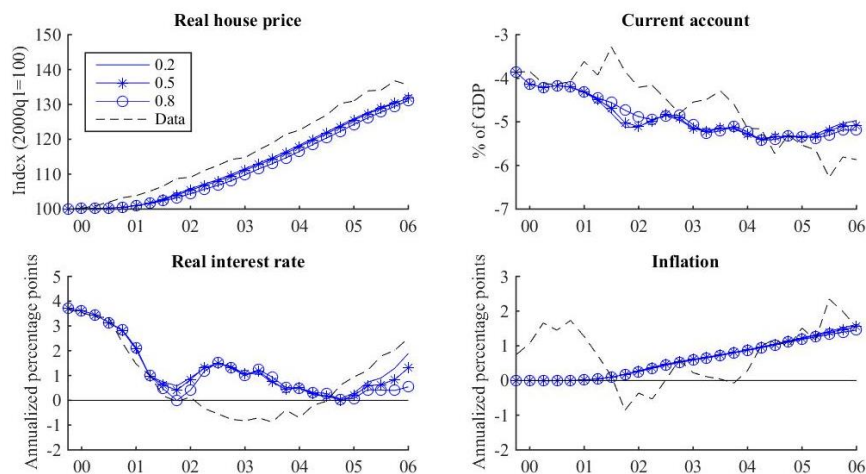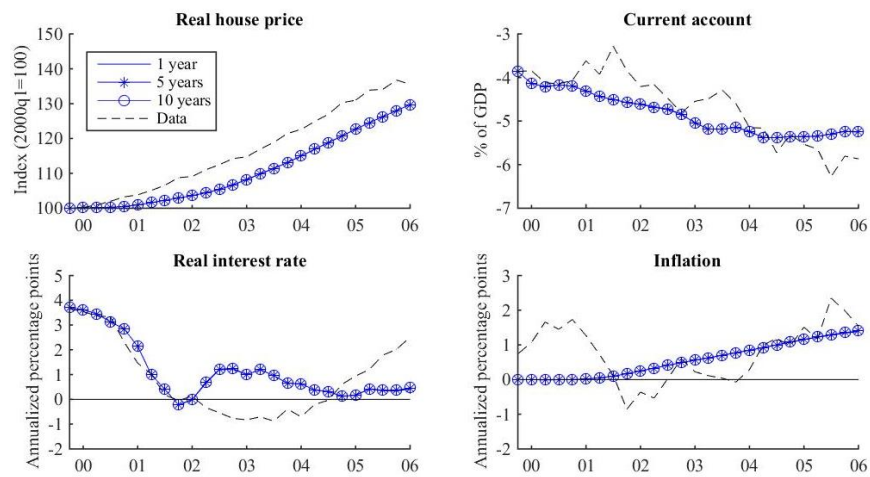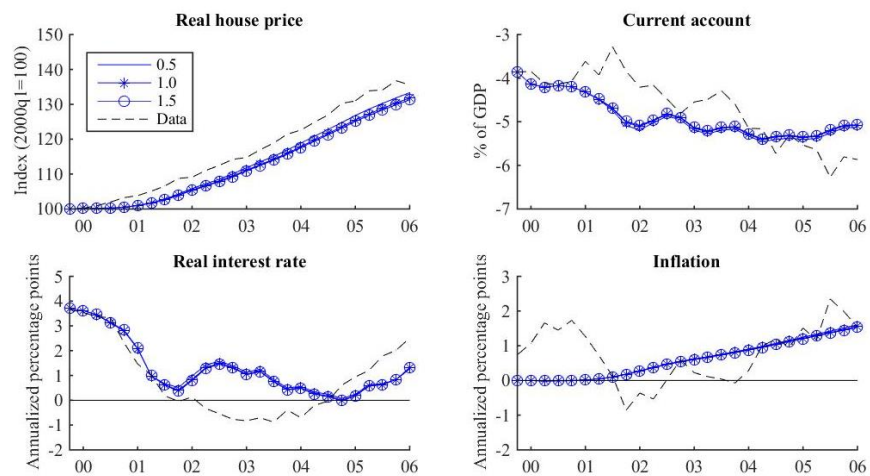
Figure A1: Memory parameter

Figure A2: Window

Figure A3: Elasticity of substitution between durable and non-durable goods

Steady state

$$X = \left[(1 - \alpha) C^{\frac{\phi - 1}{\phi}} + \alpha H^{\frac{\phi - 1}{\phi}} \right]^{\frac{\phi}{\phi - 1}}$$

$$X^* = \left[(1 - \alpha) C^{*\frac{\phi - 1}{\phi}} + \alpha H^{*\frac{\phi - 1}{\phi}} \right]^{\frac{\phi}{\phi - 1}}$$

$$Y = (1 - \triangle_F) C + \triangle_F C^*$$

$$Y^* = (1 - \triangle_F) C^* + \triangle_F C$$

$$Y^\eta = A^{1+\eta} \frac{\epsilon - 1}{\epsilon} (1 - \alpha) X^{\frac{1}{\phi} - \sigma} C^{-\frac{1}{\phi}}$$

$$Y^{*\eta} = A^{*1+\eta} \frac{\epsilon - 1}{\epsilon} (1 - \alpha) X^{*\frac{1}{\phi} - \sigma} C^{*-\frac{1}{\phi}}$$

$$q = \frac{\alpha}{1 - \alpha} \frac{1}{1 - \beta - \chi (\beta^* - \beta)} \left(\frac{C}{H} \right)^{\frac{1}{\phi}}$$

$$q^* = \frac{\alpha}{1 - \alpha} \frac{1}{1 - \beta^*} \left(\frac{C^*}{H^*} \right)^{\frac{1}{\phi}}$$

$$C + b \left(\frac{1}{\beta^*} - 1 \right) = Y$$

$$b = \beta^* \chi q H$$

Log-linearized model

Domestic consumers

$$\hat{X}_t = (1 - \alpha) \left(\frac{C}{X} \right)^{\frac{\phi - 1}{\phi}} \hat{C}_t$$

$$\pi_{w,t} = \beta \pi_{w,t+1} + \kappa_w \left[\eta \hat{L}_t - \left(\frac{1}{\phi} - \sigma \right) \hat{X}_t + \frac{1}{\phi} \hat{C}_t - \hat{w}_t \right]$$

where $\kappa_w = \frac{(1-\theta_w)(1-\beta\theta_w)}{\theta_w(1+\epsilon_w\eta)}$

$$\pi_{w,t} = \hat{w}_t - \hat{w}_{t-1}$$

$$\hat{C}_t = \frac{Y}{C}\left(\hat{Y}_t + \hat{p_{h,t}}\right) + \frac{b}{C}\left[\hat{b}_t - \frac{1}{\beta^*}\left(\hat{b_{t-1}} + \hat{R_{t-1}} - \pi_t\right)\right]$$

$$\hat{q}_t = \frac{1}{\phi}\hat{C}_t + \beta\left[\left(\frac{1}{\phi}-\sigma\right)\left(\hat{X}_{t+1} - \hat{X}_t\right) - \frac{1}{\phi}\hat{C}_{t+1} + \hat{q}_{t+1}\right] + \chi(\beta^*-\beta)\left(\lambda_t^{\hat{cc}} + \hat{\chi}_t + \pi_{t+1} + \hat{q}_{t+1} - \left(\frac{1}{\phi}-\sigma\right)\hat{X}_t\right)$$

$$\frac{1}{\phi}\hat{C}_t = -\hat{R}_t + \left(\frac{1}{\phi}-\sigma\right)\hat{X}_t + \frac{\beta}{\beta^*}\left[\frac{1}{\phi}\hat{C}_{t+1} - \left(\frac{1}{\phi}-\sigma\right)\hat{X}_{t+1} + \pi_{t+1} + \lambda_t^{\hat{cc}}\right] - \lambda_t^{\hat{cc}}$$

$$\hat{C}_{h,t} = -\iota_C\hat{p}_{h,t} + \hat{C}_t$$

$$\hat{C}_{f,t} = -\iota_C\left(\hat{\Gamma}_t + \hat{p}_{h,t}\right) + \hat{C}_t$$

$$\hat{p}_{h,t} = -\triangle_F\hat{\Gamma}_t$$

$$\hat{b}_t = \hat{\chi}_t + \hat{q}_{t+1} - \left(\hat{R}_t - \pi_{t+1}\right)$$

Foreign consumers

$$\hat{X}_t^* = (1-\alpha)\left(\frac{C^*}{X^*}\right)^{\frac{\phi-1}{\phi}}\hat{C}_t^*$$

$$\pi_{wt}^* = \beta^*\pi_{wt+1}^* + \kappa_w^*\left[\eta\hat{L}_t^* - \left(\frac{1}{\phi}-\sigma\right)\hat{X}_t^* + \frac{1}{\phi}\hat{C}_t^* - \hat{w}_t^*\right]$$

where $\kappa_w^* = \frac{(1-\theta_w)(1-\beta^*\theta_w)}{\theta_w(1+\epsilon_w\eta)}$

$$\pi_{wt}^* = \hat{w}_t^* - \hat{w}_{t-1}^*$$

$$\hat{q}_t^* = \frac{1}{\phi}\hat{C}_t^* + \beta^*\left[\left(\frac{1}{\phi}-\sigma\right)\left(\hat{X}_{t+1}^* - \hat{X}_t^*\right) - \frac{1}{\phi}\hat{C}_{t+1}^* + \hat{q}_{t+1}^*\right]$$

$$\frac{1}{\phi}\hat{C}_t^* = \frac{1}{\phi}\hat{C_{t+1}^*} - \left(\frac{1}{\phi}-\sigma\right)\left(\hat{X_{t+1}^*} - \hat{X}_t^*\right) - \left(\hat{R}_t - \pi_{t+1}^* - \triangle\hat{\varepsilon}_{t+1}\right)$$

$$\frac{1}{\phi}\hat{C}_t^* = \frac{1}{\phi}\hat{C_{t+1}^*} - \left(\frac{1}{\phi} - \sigma\right)\left(\hat{X_{t+1}^*} - \hat{X}_t^*\right) - \left(\hat{R}_t^* - \pi_{t+1}^*\right)$$

$$\hat{C_{h,t}^*} = -\iota_C\left(-\hat{\Gamma}_t + \hat{p_{f,t}^*}\right) + \hat{C}^*$$

$$\hat{C_{f,t}^*} = -\iota_C\hat{p_{f,t}^*} + \hat{C}^*$$

$$\hat{p_{f,t}^*} = \triangle_F\hat{\Gamma}_t$$

Firms

$$\hat{Y}_t = \hat{L}_t$$

$$\hat{Y}_t^* = \hat{L}_t^*$$

$$\pi_{h,t} = \beta E_t\pi_{h,t+1} + \kappa\hat{mc}_t$$

where $\kappa = \frac{(1-\theta)(1-\beta\theta)}{\theta}$

$$\pi_{f,t}^* = \beta^* E_t\pi_{f,t+1}^* + \kappa^*\hat{mc}_t^*$$

where $\kappa^* = \frac{(1-\theta)(1-\beta^*\theta)}{\theta}$

$$\hat{mc}_t = \hat{w}_t - \hat{p_{h,t}}$$

$$\hat{mc}_t^* = \hat{w}_t^* - \hat{p_{f,t}^*}$$

Market equilibria

$$\hat{Y}_t = \frac{C_h}{Y}\hat{C_{h,t}} + \frac{C_h^*}{Y}\hat{C_{h,t}^*}$$

$$\hat{Y}_t^* = \frac{C_f^*}{Y^*}\hat{C_{f,t}^*} + \frac{C_f}{Y^*}\hat{C_{f,t}}$$

Taylor rules

$$\hat{R}_t = \mu_R \hat{R}_{t-1} + (1 - \mu_R) \left[\mu_\pi \pi_t + \mu_Y \hat{Y}_t \right] + \mu_{R,t}$$

$$\hat{R}_t^* = \mu_R \hat{R}_{t-1}^* + (1 - \mu_R) \left[\mu_\pi \pi_t^* + \mu_Y \hat{Y}_t^* + \mu_\varepsilon \triangle \hat{\varepsilon}_t \right] + \mu_{R,t}^*$$

Terms of trade

$$\hat{\Gamma}_t - \hat{\Gamma}_{t-1} = \triangle \hat{\varepsilon}_t + \pi_{f,t}^* - \pi_{h,t}$$

CPI inflation

$$\pi_t = \pi_{h,t} - \triangle \hat{p}_{h,t}$$

$$\pi_t^* = \pi_{f,t}^* - \triangle \hat{p}_{f,t}^*$$

Real exchange rate

$$\triangle s_t = \triangle \hat{\varepsilon}_t + \pi_t^* - \pi_t$$

Current account

$$CA_t = \frac{b}{Y} \left(\hat{b}_t - \hat{b}_{t-1} \right)$$

where CA_t is the current account normalized by steady state output.

Chapter 2

Inflation Puzzles in the New Keynesian Model: The Implications of Anchored Expectations

Inflation Puzzles in the New Keynesian Model: The Implications of Anchored Expectations [*]

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Abstract

In this paper we introduce bounded rationality into an otherwise standard New Keynesian model. Agents are assumed to behave as econometricians, using time-series models to forecast inflation and the output gap similar to that of Stock and Watson (2007). The agent's perceived optimal forecast rules are defined by the Kalman filter. In a unique equilibrium, the values of the two Kalman gain parameters are pinned down by the observed autocorrelation of inflation and output gap changes. This methodology can be applied directly to U.S. data. We show that if agents perpetually update their estimates of the Kalman gains using a moving window of recent data, the identified Kalman gain for inflation exhibits a downward drift during the so-called "Great Moderation" period. A low Kalman gain implies a low weight on recent inflation in the agent's forecast rule. This helps anchor inflation near the central bank's target rate when the output gap falls sharply during the Great Recession. In the longer term, however, the recession leads to a downward revision of the agent's inflation forecast, which generates a moderate – but highly persistent – decline in inflation. Thus, the model can help account for both the "missing disinflation" in the immediate wake of the recession as well as the "missing inflation" in recent years. Forecasts with the model suggest that inflation will undershoot the central bank's target rate for several years after the output gap has fully recovered. Consequently, the model predicts that monetary policy will remain accomodative and contribute to a positive output gap in the future.

Keywords: *Inflation Expectations, Phillips Curve, Bounded Rationality*

JEL Classification: E31, E37

1 Introduction

Since the outbreak of the Great Recession, inflation dynamics in the U.S. and the eurozone have sparked a debate over the New Keynesian Phillips Curve (NKPC) (e.g. Hall, 2011). From the perspective of the NKPC, inflation rates did not decline as much as expected in the aftermath of the recession. The absence of a persistent decline in inflation was dubbed the "missing disinflation puzzle" (Coibion and Gorodnichenko, 2015). Subsequently, inflation rates have not recovered as fast as expected. Since mid-2012, PCE inflation has been persistently below the Fed's target of 2%, which has given birth to the so-called "missing inflation puzzle." In this paper, we introduce a form of boundedly-rational expectations into an otherwise standard New Keynesian model. Agents are assumed to behave as econometricians, using time-series models for inflation and the output gap similar to that of Stock and Watson (2007). The agent's perceived optimal forecast rules are defined by the Kalman filter.[1] We show that the model has a unique equilibrium where the perceived optimal values of the two Kalman gain parameters are pinned down by the observed autocorrelations of inflation changes and output gap changes. By computing the values of the two autocorrelation coefficients, the agent can identify the two "signal-to-noise ratios" in the inflation and output gap data.

The model's methodology for identifying the two signal-to-noise ratios can be applied directly to U.S. data. We show that if the agent perpetually updates estimates of the two signal-to-noise ratios using moving windows of recent data, the identified signal-to-noise ratio for inflation will exhibit a downward drift during the so-called "Great Moderation" period from 1984-2007. A lower signal-to-noise ratio implies a lower weight on recent inflation in the agent's forecast rule, which is consistent with the idea of "anchored" inflation expectations (Williams, 2006). Anchored expectations in the NKPC imply that inflation is less sensitive to changes in the output gap. Consequently, when the output gap drops sharply in 2008, the initial response of inflation is muted. However, the recession gradually leads to a moderate downward revision of agent's inflation expectations, which generates a highly persistent decline in inflation in the longer term. Thus, the model can help account for both the "missing disinflation" in the immediate wake of the recession as well as the "missing inflation" since 2012. Model forecasts suggest that inflation will undershoot the central bank's target rate for several years after the output gap has fully recovered. Thus, according to the model, monetary policy will remain accomodative and contribute to a positive output gap in the future.

Standard New Keynesian models with fully rational expectations tend to produce two counterfactual predictions. First, they generate large and persistent declines in inflation in response to the Great Recession.[2] Second, they predict that the recovery of inflation and the

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[1] The paper builds on Lansing (2009), who introduces bounded rationality into the NKPC with an exogenous output gap. We adopt the same type of boundedly rational expectations, but develop a fully-articulated New Keynesian model.

[2] See, for instance, Auroba and Schorfheide (2016) or Christiano et. al. (2015)

output gap closely mirrors the exogenous shock process. Thus, as soon as the shock stops operating, the output gap will be closed and inflation will be back at the central bank's target rate. However, as shown in Figure 1, actual U.S. inflation has behaved markedly different. First, there was no persistent disinflation in the aftermath of the recession. PCE inflation dropped sharply in 2008.q4 when energy prices collapsed, but then almost fully recovered within two quarters. Since mid-2012, however, PCE inflation has been persistently below the Fed's target of 2%. In the context of a standard NKPC this decline is surprising considering the simultaneous recovery of the output gap and the unemployment rate. Figure 1 shows that inflation expectations did not respond much to the Great Recession. After a moderate decline in 2008, 1-year inflation expectations from the Michigan Survey of Consumers recovered between 2009 and 2012.[3] Since 2012, however, these expectations have declined. Similarly, 10-year expectations from the Survey of Professional Forecasters (SPF) began to decline in 2012. 1-year expectations from the SPF did not fully recover after the initial decline in 2008, but have converged to a level which is below its pre-recession trend.

A growing empirical literature has tried to resolve the inflation puzzles that arise in the New Keynesian model. According to Coibion and Gorodnichenko (2015), the missing disinflation is explained by a rise in household inflation expectations from 2009 to 2011, which reflected the increase in oil prices over this time period. In Bobeica and Jarocinski (2017), the inflation puzzles disappear in a vector autoregression which accounts for both domestic and global variables. Closely related to our work is a recent paper by Ball and Mazumder (2011), who argue that the Great Recession provides new evidence against the NKPC with rational expectations.[4] Acccording to these authors, a backward-looking Phillips Curve with a time-varying slope can match U.S. inflation during the Great Recession. Moreover, they find strong evidence of expectations anchoring during the Great Moderation. According to Bernanke (2010), well-anchored inflation expectations made the risk of deflation in the wake of the Great Recession insignificant. In our model, anchored expectations are equivalent to a low perceived signal-to-noise ratio, which in turn implies a low weight on recent inflation in the agent's forecast rule. We argue that inflation forecasts that are based on the unobserved components model of Stock and Watson (2007) may be a good proxy for real world inflation expectations. Since their influential paper, the unobserved components model has become a popular tool among economists to forecast inflation. Several recent papers, including Arouba and Schorfheide (2016), use the unobserved components model to generate inflation forecasts. Moreover, we show that model-based inflation expectations track well with survey-based inflation expectations since the late 1970's.[5]

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[3]Coibion and Gorodnichenko (2015) show that household's inflation expectations followed oil prices closely over this period.

[4]According to these authors, the NKPC fits the data poorly, because it predicts that the output gap has a negative effect on the expected change in inflation.

[5]Other papers reach the same conclusion. For instance, Edge et. al. (2007) find that an unobserved components model using real-time data describes economists long-run productivity growth forecasts extremely
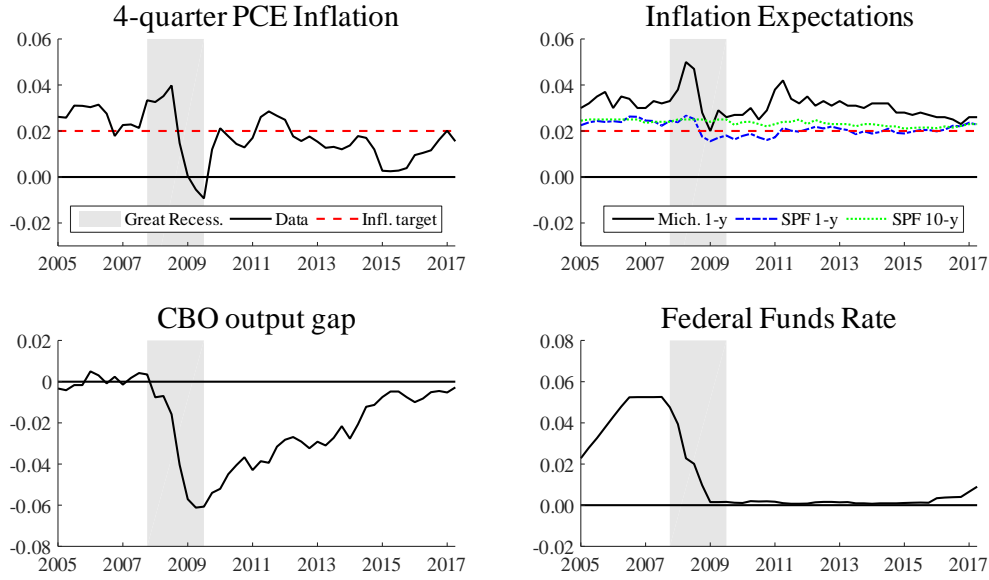
Figure 1: Key Macroeconomic Variables, 2005.q1-2017.q2

In the New Keynesian literature, a series of recent papers have argued that the missing disinflation puzzle can be resolved by extending the standard NK model with various types of financial frictions. For instance, in a model with a working capital channel, Christiano et. al. (2015) shows that a fall in productivity growth and a rise in the costs of working capital can account for the small drop in inflation during the recession. In Del Negro et. al. (2015), the missing disinflation dissappears if the NKPC is sufficiently flat.

Other recent papers deviate from the rational expectations assumption. In the context of an adaptive learning model, Evans et. al. (2017) argue that the U.S. is stuck in a distinct stagnation steady state characterized by pessimistic expectations and a binding ZLB. Lansing (2017), on the other hand, shows that a model with endogenous switching between two local rational expectations equilibria and a time-varying natural rate of interest can produce highly negative output gaps and a binding ZLB, reminiscent of the U.S. Great Recession. While both of these papers are closely related to our work, we focus on the implications of "anchored" inflation expectations.

The remainder of the paper is organized as follows. Section 2 describes the standard New Keynesian model. Section 3 derives the unique solution under rational expectations. In Section 4, we define the concept of a "consistent expectations" (CE) equilibrium and prove the uniqueness of such an equilibrium. In Section 5 we apply the methodology of the CE model directly to U.S. data and reassess the inflation puzzles that arise under rational expectations. Section 6 concludes.

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well.

2 The New Keynesian Model

The starting point for the analysis is a standard New Keynesian model. The model consists of three main elements: A New Keynesian Phillips Curve (NKPC), an IS equations and a Taylor-type rule for monetary policy. Throughout the paper, model variables are expressed in terms of log-deviations from steady state. We use the notation $x_t = \ln(X_t/X^*)$ where X^* is the steady state value of a variable, X_t.

The NKPC can be derived from Calvo's (1983) model of sticky prices. It links inflation to expected inflation and the output gap:

$$\pi_t = \beta \widetilde{E}_t \pi_{t+1} + \kappa y_t + u_t, \qquad \beta \in [0,1), \ \kappa > 0, \ u_t \sim N\left(0, \sigma_u^2\right), \qquad (1)$$

where π_t is the deviation of the inflation rate from the central bank's target.[6] β is the representative agent's subjective time discount factor, y_t is the output gap and u_t is an *iid* cost-push shock. The symbol $\widetilde{E}_t$ represents the agent's subjective expectation conditioned on information available at time t. Under rational expectations, $\widetilde{E}_t$ corresponds to the mathematical expectations operator, E_t.

The IS curve links the output gap to the expected future output gap and the real interest rate:

$$y_t = \widetilde{E}_t y_{t+1} - \alpha \left(R_t - \widetilde{E}_t \pi_{t+1} \right) + v_t, \qquad \alpha > 0, \ v_t \sim N\left(0, \sigma_v^2\right), \qquad (2)$$

where R_t is the log deviation of the gross nominal interest rate, α is the inverse of the coefficient of relative risk aversion and v_t is an *iid* demand shock that is uncorrelated with the cost-push shock.

Monetary policy is characterized by a Taylor-type rule, where the central bank responds to forecasts of inflation and the output gap.

$$R_t = \mu_\pi \widetilde{E}_t \pi_{t+1} + \mu_y \widetilde{E}_t y_{t+1}, \qquad \mu_\pi > 0, \ \mu_y > 0 \qquad (3)$$

where μ_π and μ_y are the Taylor coefficients on the central bank's forecasts of the inflation gap and the output gap, respectively. We assume that the Taylor principle is satisfied, i.e. $\mu_\pi > 1$.

3 Rational Expectations

Under rational expectations, the inflation rate and the output gap are uniquely pinned down by the shocks. The unique rational expectations solution is given by:

$$\pi_t^{re} = \kappa v_t + u_t \qquad (4)$$

---

[6] This holds under the assumption that price adjustments are costless at the steady state rate. Thus we abstract from changes in the functional form of the NKPC that arise when the Calvo pricing equation is log-linearized around a non-zero inflation rate, as shown by Ascari (2004) and Sahuc (2006).

$$y_t^{re} = v_t \tag{5}$$

$$R_t^{re} = 0 \tag{6}$$

(4)-(5) imply that the one-period ahead rational forecasts of inflation and the output gap are always zero:

$$E_t \pi_{t+1}^{re} = 0 \tag{7}$$

$$E_t y_{t+1}^{re} = 0 \tag{8}$$

where we have replaced $\widetilde{E}_t$ with E_t. Moreover, from (4)-(6) we obtain the following unconditional moments:

$$Var\left(\pi_t^{re}\right) = \kappa^2 \sigma_v^2 + \sigma_u^2$$

$$Var\left(y_t^{re}\right) = \sigma_v^2$$

$$Var\left(R_t^{re}\right) = 0$$

$$Corr\left(\pi_t^{re}, \pi_{t-1}^{re}\right) = 0$$

$$Corr\left(y_t^{re}, y_{t-1}^{re}\right) = 0$$

$$Corr\left(R_t^{re}, R_{t-1}^{re}\right) = 0$$

These expressions show that, under rational expectations, the variables inherit their stochastic properties solely from the white-noise shocks. Also, they exhibit no first order autocorrelation. The latter conflicts sharply with U.S. data. From 1984 to 2007 the first order autocorrelation of quarterly PCE inflation, the CBO output gap and the Federal Funds rate were, respectively, 0.46, 0.93 and 0.97. The literature has proposed numerous ways to overcome the problem of weak persistence in New Keynesian models.[7] A straightforward solution would be to model the exogenous shocks, v_t and u_t, as AR(1) processes. However, in the following, we wish to illustrate how the introduction of bounded rationality can generate enough endogenous persistence to match the moments in U.S. data using IID shocks.[8]

4   Consistent Expectations

We introduce bounded rationality by assuming that the representative agent behaves as an econometrician, using time series models to forecast inflation and the output gap. Specifically, the agent is assumed to employ an unobserved components model which allows for both permanent and temporary shocks – along the lines of Stock and Watson (2007). The agent's

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[7]These include introducing a smoothing term in the Taylor rule and habit formation in consumption (see, for instance, Smets and Wouters, 2007)

[8]Later, we extent the rational version of the model with a persistent demand shock which generates the inflation puzzles decribed in the introduction.

perceived law of motion for inflation is given by:

$$
\begin{bmatrix} \pi_t \\ \overline{\pi}_t \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \pi_{t-1} \\ \overline{\pi}_{t-1} \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \zeta_t \\ \eta_t \end{bmatrix}, \qquad
\begin{array}{l} \zeta_t \sim N\left(0, \sigma_\zeta^2\right), \\ \eta_t \sim N\left(0, \sigma_\eta^2\right), \\ Cov\left(\zeta_t, \eta_t\right) = 0, \end{array}
\tag{9}
$$

where $\overline{\pi}_t$ is the unobservable inflation trend, ζ_t is a transitory shock that pushes π_t away from trend, and η_t is permanent shock (uncorrelated with ζ_t) that shifts the trend over time. The specification implies that the subjective forecast $\widetilde{E}_t \pi_{t+1}$ equals the Kalman filter estimate of $\overline{\pi}_t$. Some technical points are worth noting. First, although the perceived law of motion (9) allows for permanent deviations from steady state, the equilibrium inflation process (to be defined below) remains stationary around the steady state inflation rate. Moreover, we abstract from "long-horizon expectations" that arise in the NKPC when forward-looking agents employ subjective forecasts of future inflation, as discussed by Preston (2005). The perceived law of motion (9) implies $\widetilde{E}_t \pi_{t+j} = \widetilde{E}_t \pi_{t+1}$ for all future horizons $j = 2, 3, 4...$ Under consistent expectations, equation (1) can therefore be viewed as a log-linear approximation of a more-complicated NKPC that explicitly incorporates long-horizon inflation expectations.

The representative agent uses a similar time series model to forecast the output gap:

$$
\begin{bmatrix} y_t \\ \overline{y}_t \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} y_{t-1} \\ \overline{y}_{t-1} \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \chi_t \\ \varphi_t \end{bmatrix}, \qquad
\begin{array}{l} \chi_t \sim N\left(0, \sigma_\chi^2\right), \\ \varphi_t \sim N\left(0, \sigma_\varphi^2\right), \\ Cov\left(\chi_t, \varphi_t\right) = 0, \end{array}
\tag{10}
$$

where $\overline{y}_t$ is the perceived long-run output gap , χ_t is a transitory shock and φ_t is permanent shock (uncorrelated with χ_t). Agents do not need to believe that the output gap literally has a unit root but that such a specification is a local approximation that is convenient for forecasting.[9]

As originally shown by Muth (1960), the perceived laws of motion (9) and (10) imply the following error-correction forecasting rules for inflation and the output gap, respectively:

$$
\begin{aligned}
\widetilde{E}_t \pi_{t+1} &= \widetilde{E}_{t-1} \pi_t + \lambda_\pi \left(\pi_t - \widetilde{E}_{t-1}\pi_t\right), \qquad 0 < \lambda_\pi \le 1, \\[2mm]
&= \lambda_\pi \left[\pi_t + (1 - \lambda_\pi)\, \pi_{t-1} + (1 - \lambda_\pi)^2 \, \pi_{t-2} + ...\right],
\end{aligned}
\tag{11}
$$

and

$$
\begin{aligned}
\widetilde{E}_t y_{t+1} &= \widetilde{E}_{t-1} y_t + \lambda_y \left(y_t - \widetilde{E}_{t-1}y_t\right), \qquad 0 < \lambda_y \le 1, \\[2mm]
&= \lambda_y \left[y_t + (1 - \lambda_y)\, y_{t-1} + (1 - \lambda_y)^2 \, y_{t-2} + ...\right],
\end{aligned}
\tag{12}
$$

---

[9] The CBO output gap was negative from 2008-2017, so any mean reversion to zero is obviously very slow. The unobserved components model captures this feature with a unit root.

where $\pi_t - \widetilde{E}_{t-1}\pi_t$ and $y_t - \widetilde{E}_{t-1}y_t$ are the forecast errors in period t. We assume that the agent's subjective forecast makes use of the contemporaneous realizations π_t and y_t. This setup avoids the introduction of an extra lag of variables that might be viewed as artificially influencing the resulting dynamics.[10] Equations (11) and (12) imply that the agent's forecasts at time t are exponentially-weighted moving averages of the current and past observed inflation rates and output gaps, respectively.

The agent's perceived optimal choices of the weights, λ_π and λ_y, in equations (11) and (12) are determined by the Kalman filter, where the objective is to minimize the mean squared forecast errors $E\left(\pi_{t+1} - \widetilde{E}_t\pi_{t+1}\right)^2$ and $E\left(y_{t+1} - \widetilde{E}_t y_{t+1}\right)^2$. In steady-state, the unique solution for the perceived optimal gain parameter for inflation is

$$\lambda_\pi = \frac{-\phi_\pi + \sqrt{\phi_\pi^2 + 4\phi_\pi}}{2}, \tag{13}$$

where $\phi_\pi = \sigma_\eta^2/\sigma_\zeta^2$ is the perceived signal-to-noise ratio for inflation.[11] As $\phi_\pi \to \infty$, the gain parameter approaches 1. From the agent's perspective, the shocks themselves ζ_t and η_t are unobservable but the shock variances σ_η^2 and σ_ζ^2 can be inferred from the moments of inflation changes $\Delta\pi_t$, which are observable. Similarly, the unique solution for the perceived optimal gain parameter for the output gap is:

$$\lambda_y = \frac{-\phi_y + \sqrt{\phi_y^2 + 4\phi_y}}{2}, \tag{14}$$

where $\phi_y = \sigma_\varphi^2/\sigma_\chi^2$ is the perceived signal-to-noise ratio for the output gap.

Proposition 1. *If the representative agent's perceived laws of motion are given by equations (9) and (10), respectively, then the perceived optimal value of the Kalman gain parameter λ_π is uniquely pinned down by the autocorrelation of observed inflation changes, $Corr\left(\Delta\pi_t, \Delta\pi_{t-1}\right)$, while the perceived optimal value of the Kalman gain parameter λ_y is uniquely pinned down by the autocorrelation of observed output gap changes, $Corr\left(\Delta y_t, \Delta y_{t-1}\right)$.*

Proof: From (9), we have $\Delta\pi_t = \eta_t + \zeta_t - \zeta_{t-1}$. Since η_t and ζ_t are perceived to be independent, we have $Cov\left(\Delta\pi_t, \Delta\pi_{t-1}\right) = -\sigma_\zeta^2$ and $Var\left(\Delta\pi_t\right) = \sigma_\eta^2 + 2\sigma_\zeta^2$. Combining these two expressions and solving for the signal-to-noise ratio yields

$$\phi_\pi = \frac{-1}{Corr\left(\Delta\pi_t, \Delta\pi_{t-1}\right)} - 2, \tag{15}$$

where $\phi = \sigma_\eta^2/\sigma_\zeta^2$ and $Corr\left(\Delta\pi_t, \Delta\pi_{t-1}\right) = Cov\left(\Delta\pi_t, \Delta\pi_{t-1}\right)/Var\left(\Delta\pi_t\right)$. The above expres-

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[10] A lagged information assumption is often used in learning models to avoid simultaneity in the determination of the actual and expected values of the forecast variable. In the continuous time limit, the distinction between contemporaneous and lagged information disappears.

[11] For details of the derivation of (13) and (14), see Nerlove (1967, pp. 141-143). His results are expressed as a formula for $1 - \lambda$.

sion shows that $Corr\left(\Delta\pi_t, \Delta\pi_{t-1}\right)$ uniquely pins down ϕ_π which, in turn, uniquely pins down λ_π from equation (13). Similarly, we can prove that the Kalman gain parameter, λ_y, is pinned down by the autocorrelation of changes in the output gap:

$$\phi_y \;=\; \frac{-1}{Corr\left(\Delta y_t, \Delta y_{t-1}\right)} \;-\; 2, \tag{16}$$

■

The model (1)-(3) and the forecast rules (11)-(12) can be written in the following matrix form which defines the actual law of motion of the economy:[12]

$$\mathbf{Z}_t = \mathbf{A}\mathbf{Z}_{t-1} + \mathbf{B}\mathbf{U}_t, \tag{17}$$

where $\mathbf{Z}_t = \begin{bmatrix} \pi_t & y_t & R_t & \widetilde{E}_t\pi_{t+1} & \widetilde{E}_t y_{t+1} \end{bmatrix}'$ and $\mathbf{U}_t = \begin{bmatrix} u_t & v_t \end{bmatrix}'$ The variance-covariance matrix $\mathbf{V}$ of the left-side variables in (17) can be computed using the formula:

$$vec\left(\mathbf{V}\right) = [\mathbf{I} - \mathbf{A} \otimes \mathbf{A}]^{-1} \, vec\left(\mathbf{B}\mathbf{\Omega}\mathbf{B}'\right), \tag{18}$$

where $\mathbf{\Omega}$ is the variance-covariance matrix of the fundamental shocks u_t and v_t. Using (18) we can compute the autocorrelation coefficients of $\Delta\pi_t$ and Δy_t. These coefficients pin down the perceived signal-to-noise ratios ϕ_π and ϕ_y in (15) and (16), which – in turn – uniquely pin down the optimal Kalman gains, λ_π and λ_y, in (13) and (14).

4.1 Defining the Consistent Expectations Equilibrium

This section defines the concept of a "consistent expectations equilibrium" along the lines of Hommes and Sorger (1998) and Lansing (2009).[13]

Definition 1. *A consistent expectations equilibrium is defined as the law of motion (17), and associated Kalman gain parameters, λ_π and λ_y, such that λ_π and λ_y are the fixed points of the multidimensional nonlinear maps $\lambda_\pi = T_\pi\left(\lambda_\pi, \lambda_y\right)$ and $\lambda_y = T_y\left(\lambda_\pi, \lambda_y\right)$, where*

$$T_\pi\left(\lambda_\pi, \lambda_y\right) \;=\; \frac{-\phi_\pi\left(\lambda_\pi, \lambda_y\right) + \sqrt{\phi_\pi\left(\lambda_\pi, \lambda_y\right)^2 + 4\phi_\pi\left(\lambda_\pi, \lambda_y\right)}}{2}, \tag{19}$$

$$\phi_\pi\left(\lambda_\pi, \lambda_y\right) \;=\; \frac{-1}{Corr\left(\Delta\pi_t, \Delta\pi_{t-1}\right)} - 2$$

and

---

[12] The derivations can be found in Appendix A.1

[13] This equilibrium concept is closely related to the "restricted perpections equilibrium" of Evans and Honkapohja (2001) and the "behavioral learning equilibrium" of Hommes and Zhu (2013).

$$T_y\left(\lambda_\pi, \lambda_y\right) = \frac{-\phi_y\left(\lambda_\pi, \lambda_y\right) + \sqrt{\phi_y\left(\lambda_\pi, \lambda_y\right)^2 + 4\phi_y\left(\lambda_\pi, \lambda_y\right)}}{2}, \tag{20}$$

$$\phi_y\left(\lambda_\pi, \lambda_y\right) = \frac{-1}{Corr\left(\Delta y_t, \Delta y_{t-1}\right)} - 2$$

with the unconditional correlations, $Corr\left(\Delta \pi_t, \Delta \pi_{t-1}\right)$ and $Corr\left(\Delta y_t, \Delta y_{t-1}\right)$, computed from the actual law of motion (17), using equation (18).

4.2 Numerical Solution for the Equilibrium

The complexity of the nonlinear maps (19) and (20) necessitates a numerical solution for the equilibrium. To accomplish this, the model is calibrated using a set of baseline parameter values that are standard in the literature. Table 1 reports the baseline calibration.

Table 1: Baseline calibration

Parameter	Value	Description
β	0.995	Discount factor
κ	0.01	Output gap coefficient in NKPC
α	0.5	Interest rate coefficient in IS equation
μ_π	2	Policy response to inflation forecast
μ_y	0.2	Policy response to output gap forecast
σ_u	0.001	Std. dev. of cost push shock
σ_v	0.001	Std. dev. of aggregate demand shock

We choose the discount factor $\beta = 0.995$, which corresponds to an annual real interest rate of 2%. $\kappa = 0.01$ corresponds to a relatively flat NKPC, which is motivated by empirical evidence.[14] The coefficient on the interest rate in the IS equation, $\alpha = 0.5$, corresponds to a coefficient of relative risk aversion of $1/\alpha = 2$, which is common in the literature. The Taylor rule coefficients, $\mu_\pi = 2$ and $\mu_y = 0.2$, are close to the estimates in Smets and Wouters (2007). The standard deviations of the fundamental shocks, σ_u and σ_v, are chosen so that the standard deviations of inflation and the output gap in the consistent expectations model are reasonably close to those observed in US data for the period 1984.Q1 to 2007.Q4. We also examine the sensitivity of the results to alternative parameter values.

Given the parameter values in Table 1, we can solve numerically for the equilibrium. An equilibrium $\left(\lambda_\pi^*, \lambda_y^*\right)$ requires the following two conditions to be satisfied:

$$f_\pi\left(\lambda_\pi^*, \lambda_y^*\right) = \lambda_\pi^* - \frac{-\phi_\pi\left(\lambda_\pi^*, \lambda_y^*\right) + \sqrt{\phi_\pi\left(\lambda_\pi^*, \lambda_y^*\right)^2 + 4\phi_\pi\left(\lambda_\pi^*, \lambda_y^*\right)}}{2} = 0 \tag{21}$$

and

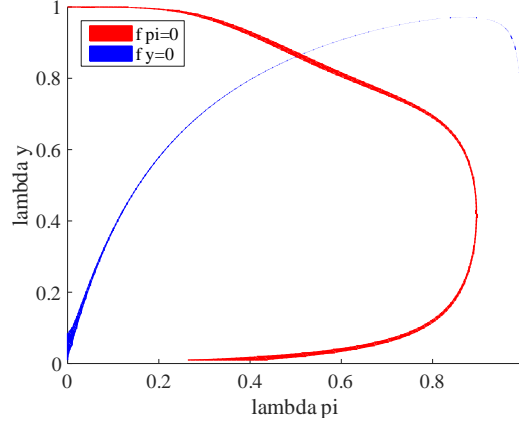[14]See, for instance, Mavroeidis et. al. (2014).

Figure 2: Solving for the Consistent Expectations Equilibrium

$$f_y\left(\lambda_\pi^*, \lambda_y^*\right) = \lambda_y^* - \frac{-\phi_y\left(\lambda_\pi^*, \lambda_y^*\right) + \sqrt{\phi_y\left(\lambda_\pi^*, \lambda_y^*\right)^2 + 4\phi_y\left(\lambda_\pi^*, \lambda_y^*\right)}}{2} = 0 \qquad (22)$$

Figure 2 plots these conditions in (λ_π, λ_y)-space. The figure shows that a unique fixed point occurs at $\left(\lambda_\pi^*, \lambda_y^*\right) = (0.5003, 0.8600)$. This point corresponds to $Corr\left(\Delta\pi_t, \Delta\pi_{t-1}\right) = -0.3999$ and $Corr\left(\Delta y_t, \Delta y_{t-1}\right) = -0.1373$.

Table 2 shows how the equilibrium changes with parameter values. The values of λ_π^* and λ_y^* increase with the values of β and κ, but decrease with the value of μ_y. The intuition behind these effects are straightforward. Roughly speaking, parameter changes that increase the persistence in the model have the effect of increasing the perceived signal-to-noise ratios, ϕ_π and ϕ_y, and hence λ_π^* and λ_y^*. From the agent's perspective, inflation and the output gap are comprised of persistent signal components, $\overline{\pi}_t$ and $\overline{y}_t$, respectively, and transitory noise components, ζ_t and χ_t, respectively. If a parameter shift causes the observed inflation rate or output gap to become more persistent, then the agent's inferred value of the signal-to-noise ratio, ϕ_π or ϕ_y, will increase, resulting in an increase in the corresponding Kalman gain parameter. The table shows that consistent expectations generates a substantial degree of endogenous persistence. Specifically, the autocorrelation coefficients, $Corr\left(\pi_t, \pi_{t-1}\right)$, $Corr\left(y_t, y_{t-1}\right)$ and $Corr\left(R_t, R_{t-1}\right)$ are all close to one in the baseline calibration, whereas the rational model implies zero autocorrelation for all variables.

Table 2: Sensitivity Analysis

Result	Baseline	$\beta = 0.9975$	$\kappa = 0.03$	$\mu_y = 0.1$	$\sigma_v^2/\sigma_u^2 = 2$
ϕ_π^*	0.50	0.53	0.62	0.61	1.02
λ_π^*	0.50	0.51	0.54	0.53	0.62
ϕ_y^*	5.28	5.62	5.78	12.84	1.23
λ_y^*	0.86	0.87	0.87	0.93	0.83
$Corr\,(\pi_t, \pi_{t-1})$	0.79	0.80	0.58	0.67	0.80
$Corr\,(\Delta\pi_t, \Delta\pi_{t-1})$	-0.40	-0.39	-0.38	-0.38	-0.33
$Corr\,(y_t, y_{t-1})$	0.95	0.95	0.86	0.90	0.92
$Corr\,(\Delta y_t, \Delta y_{t-1})$	-0.14	-0.13	-0.13	-0.07	-0.17
$Corr\,(R_t, R_{t-1})$	0.87	0.87	0.74	0.78	0.82

Notes: Baseline parameter values are reported in Table 1. Changes in σ_v^2/σ_u^2 are accomplished by adjusting σ_v^2 while maintaining $\sigma_u^2 = (0.001)^2$. Autocorrelation coefficients in the rational model are always zero.

4.3   Real-Time Learning

In the previous section, the equilibrium Kalman gains, λ_π^* and λ_y^*, were computed using the population autocorrelation coefficients, $Corr\,(\Delta\pi_t, \Delta\pi_{t-1})$ and $Corr\,(\Delta y_t, \Delta y_{t-1})$. This procedure implies that the Kalman gains are fixed over time. However, in a real-time learning environment, agents will only have knowledge of the sample autocorrelations, which, in turn, are influenced by the Kalman gains. To investigate the convergence properties of the equilibrium, we now assume that agents employ sample autocorrelations to compute the Kalman gains. The learning algorithm is summarized in Appendix A.2. We run a series of 10,000 period simulations, each generating a unique path of $(\lambda_{\pi,t}^*, \lambda_{y,t}^*)$. In each simulation we set the Kalman gains equal to their theoretical equilibrium values, i.e. $(\lambda_\pi^*, \lambda_y^*) = (0.5003, 0.8600)$, for the first 500 periods. Figure 3 shows the evolution of the Kalman gains. The end-of-simulation values are clustered in the range of the theoretical equilibrium values. However, sampling variation in the shocks, u_t and v_t, influence the estimated autocorrelation coefficients and therefore produce sizable differences in the end-of-simulation Kalman gains. For instance, the full-sample (10,000 period) autocorrelations of inflation changes range between -0.4441 and -0.3164, and the corresponding end-of-simulation Kalman gains are between 0.4298 and 0.6435. Over the 10 simulations shown in Figure 3, the average autocorrelation coefficients are -0.4187 for inflation, and -0.1450 for the output gap, which are close to the theoretical values of $Corr\,(\Delta\pi_t, \Delta\pi_{t-1}) = -0.3999$ and $Corr\,(\Delta y_t, \Delta y_{t-1}) = -0.1373$, respectively.
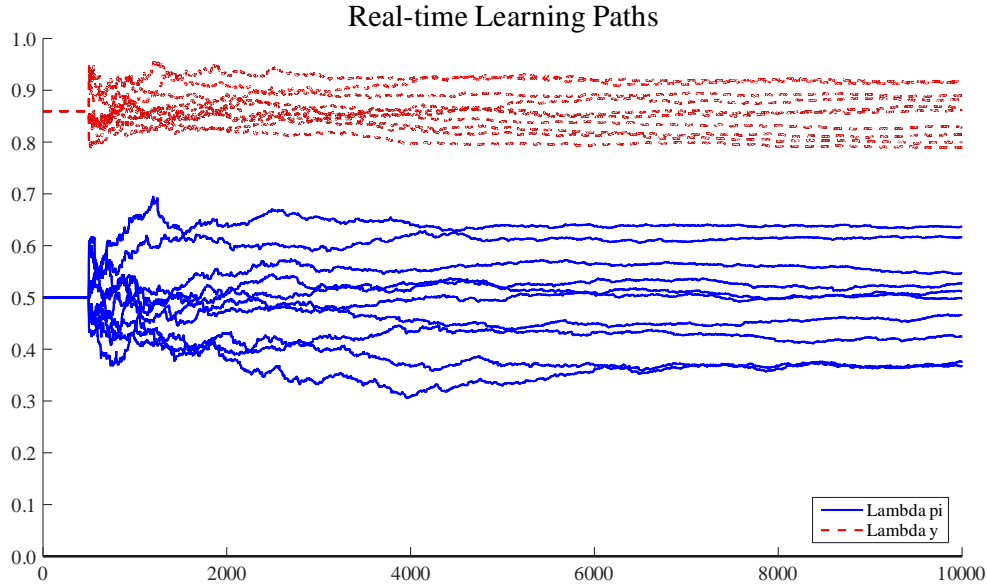
Figure 3: Convergence Properties of Kalman Gain Parameters under Real-time Learning

5 Applying the Model's Methodology to U.S. Data

5.1 Endogenous Volatility and Persistence under Consistent Expectations

This section compares different measures of volatility and persistence in U.S. data with the corresponding moments generated by the consistent expectations (CE) model. We consider three different specifications of the model. In the constant gains model, the Kalman gains are computed using the population autocorrelation coefficients. In the variable gains model, the Kalman gains are computed using rolling windows of recent data.[15] We assume that agents use either 10-year windows (40 quarters) or 20-year windows (80 quarters) of past observations. Table 3 shows the results. First, under rational expectations, the standard deviations of inflation and the output gap are simply given by the standard deviations of the supply and demand shocks, respectively.[16] Consistent expectations generates endogenous volatility. Particularly, the variable gains version of the model is more volatile than the constant gains model, especially when the moving window of past observations is relatively short. Moreover, consistent expectations adds a substantial degree of endogenous persistence into the model. Under rational expectations, none of the variables exhibit first order autocorrelation (as shown

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[15]The learning algorithm is described in Appendix A.2

[16]Note that the standard deviations of the shocks are chosen so that the CE model can roughly match the standard deviations in the data. Thus, the rational model can by construction not match the volatility observed in the data.

analytically in Section 3).[17] In the CE model, the first order autocorrelation coefficients are positive and generally match well with the corresponding moments in the data. For instance, the autocorrelation of the interest rate matches very well with the data despite the absence of a smoothing term in the Taylor rule.

Table 3: Moments

Statistic	U.S. Data 1984.q1-2007.q4		RE	Model simulations CE Constant gains	CE Variable gains	
	CPI	PCE			$T_s = 80$	$T_s = 40$
$Std.\ Dev.\ (4\pi_t)$	1.72	1.23	0.40	1.36	1.64	2.16
$Corr\ (\pi_t, \pi_{t-1})$	0.09	0.46	0.00	0.79	0.74	0.72
$Corr\ (\Delta\pi_t, \Delta\pi_{t-1})$	-0.55	-0.42	-0.50	-0.40	-0.33	-0.30
CBO output gap						
$Std.\ Dev.\ (y_t)$		1.25	0.10	1.55	1.75	2.21
$Corr\ (y_t, y_{t-1})$		0.93	0.00	0.95	0.90	0.91
$Corr\ (\Delta y_t, \Delta y_{t-1})$		0.19	-0.50	-0.14	-0.09	-0.04
Fed. Funds Rate						
$Std.\ Dev.\ (4R_t)$		2.38	0.00	1.44	1.91	2.76
$Corr\ (R_t, R_{t-1})$		0.97	0.00	0.87	0.77	0.67
$Corr\ (\Delta R_t, \Delta R_{t-1})$		0.51	0.00	-0.56	-0.21	-0.25

Notes: T_s = length of rolling sample (in quarters) for computing the perceived signal-to-noise ratios ϕ_π and ϕ_y. Parameter values are reported in Table 1. Standard deviations are denoted in percent.

5.2 Inflation Puzzles under Rational Expectations

Numerous papers have demonstrated that standard NK models with rational expectations have large difficulties accounting for inflation dynamics during and after the Great Recession. In the following we will briefly illustrate the resulting inflation "puzzles" in the context of the rational expectations version of our model.

We adopt a standard approach in the New Keynesian literature and assume that the Great Recession was caused by a highly persistent adverse demand shock. For this purpose, we momentarily replace the IID demand shock, v_t, in the IS-curve (2) with an AR(1) process, v_t:[18]

$$y_t = \widetilde{E}_t y_{t+1} - \alpha \left(R_t - \widetilde{E}_t \pi_{t+1} \right) + v_t, \qquad \alpha > 0, \tag{23}$$

where v_t follows the first order autoregressive process:

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[17]Note that several extensions could help reconcile the rational model with the data, for instance, the introduction of a smoothing term in the Taylor rule

[18]See, for instance, Auroba and Schorfheide (2016). The shock can be interpreted as a shock to the natural rate of interest.

$$v_t = \rho_v v_{t-1} + \varepsilon_t, \qquad 0 < \rho_v < 1, \varepsilon_t \sim N\left(0, \sigma_\varepsilon^2\right). \tag{24}$$

We use $\rho_v = 0.96$, which is standard in the literature[19]. Moreover, we replace the Taylor rule (3) with the ZLB condition:

$$R_t = \max\left\{-\log(R\overline{\Pi}), \mu_\pi \widetilde{E}_t \pi_{t+1} + \mu_y \widetilde{E}_t y_{t+1}\right\} \qquad \alpha > 0, \tag{25}$$

Where R is the steady state gross nominal interest rate and $\overline{\Pi}$ is the central bank's gross inflation target. We assume a net inflation target of 2% annually, i.e. $4\log(\overline{\Pi}) = 0.02$. To keep the exercise as simple as possible we assume that the rational agent employs forecast rules that are derived under the assumption that the ZLB will never bind.[20] Under this assumption, the (near) rational forecasts are given by:[21]

$$E_t \pi_{t+1}^{re} = \frac{\kappa \rho_v}{\left(1 - \beta\rho_v + \frac{\kappa\alpha\rho_v(\mu_\pi - 1)}{1 - \rho_v + \alpha\mu_y\rho_v}\right)\left(1 - \rho_v + \alpha\mu_y\rho_v\right)} v_t, \tag{26}$$

$$E_t y_{t+1}^{re} = \frac{\rho_v}{\left(1 + \frac{1}{1 - \beta\rho_v}\frac{\kappa\alpha\rho_v(\mu_\pi - 1)}{1 - \rho_v + \alpha\mu_y\rho_v}\right)\left(1 - \rho_v + \alpha\mu_y\rho_v\right)} v_t. \tag{27}$$

Thus, the rational model consists of the NKPC (1), the IS curve (23), the shock process (24), the ZLB condition (25) and the two forecast rules (26) and (27).

To replicate the path of the output gap during the recession, we use the CBO output gap as forcing variable from 2008.q1 to 2017.q2. Then we backtrack the sequence of innovations, ε_t, needed to match the output gap data. Specifically, taking y_t as given and treating ε_t as endogenous, we solve the non-linear system (1) and (23)-(27) every period. Figure 4 shows the results. The shock process, v_t, implies that the model matches the CBO output gap from Figure 1 by construction. On impact the recession induces a sharp drop in inflation of around 4 percentage points. This decline is almost fully driven by an equivalent drop in inflation expectations. Following the contraction from 2008 to 2009, the model predicts that inflation will gradually recover. However, inflation is negative for approximately 5 years. These inflation dynamics match poorly with the data. While inflation declined sharply in 2008.q4, it was only negative for two quarters. Moreover, as noted by Coibion and Gorodnichenko (2015), these fluctuations were largely explained by the collapse and subsequent recovery of oil prices. Furthermore, survey-based measures of expectations did not respond much to the recession. However, since mid-2012, inflation has persistently declined. Moreover, this decline was accompagnied by a decline in several survey-based measures of inflation expectations,

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[19]A high autocorrelation coefficient is necessary under rational expectations to match the persistence of the output gap in the data

[20]This assumption implies that expectations are not fully rational. Alternatively, we could use the Occbin Toolkit (Guerrieri and Iacoviello, 2014) to solve endogenously for the expected duration of the ZLB episode. This would imply less stable expectations at the ZLB.

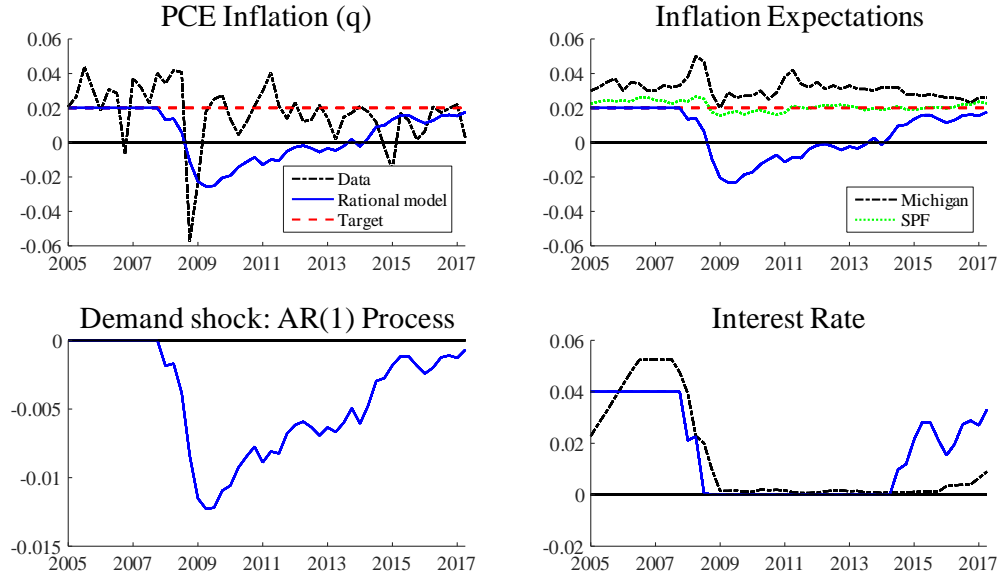[21]It is easy to check that these forecasts collapse to (7) and (8) when $\rho_v = 0$

Figure 4: Inflation Puzzles under Rational Expectations

including 1-year expectations from the Michigan Survey of Consumers. Thus, while the output gap recovered, inflation declined. From the perspective of the rational expectations model, these dynamics are puzzling.

5.3   Inflation Dynamics under Consistent Expectations

In this section we apply the methodology of the CE model directly to U.S. data and reassess the inflation puzzles that arise under rational expectations.

5.3.1   Inflation Expectations During The Great Moderation

Figure 5 shows what happens if the methodology of the consistent expectations model is applied directly to U.S. inflation data. Specifically, we assume that the agent continously updates the estimated signal-to-noise ratio and the associated inflation Kalman gain using a 20-year rolling window of past observations starting in 1978.q1. Thus, the autocorrelation of inflation changes is computed directly from a rolling window of recent data. The signal-to-noise ratio and the associated Kalman gain for inflation are computed from the learning-versions of equations (15) and (13), respectively, while the resulting model-implied inflation expectations are computed from the learning-version of the forecast rule (11).[22]

Figure 5 shows that the estimated signal-to-noise ratio for inflation exhibits a downward drift during the Great Moderation. This development in consistent with the idea of inflation

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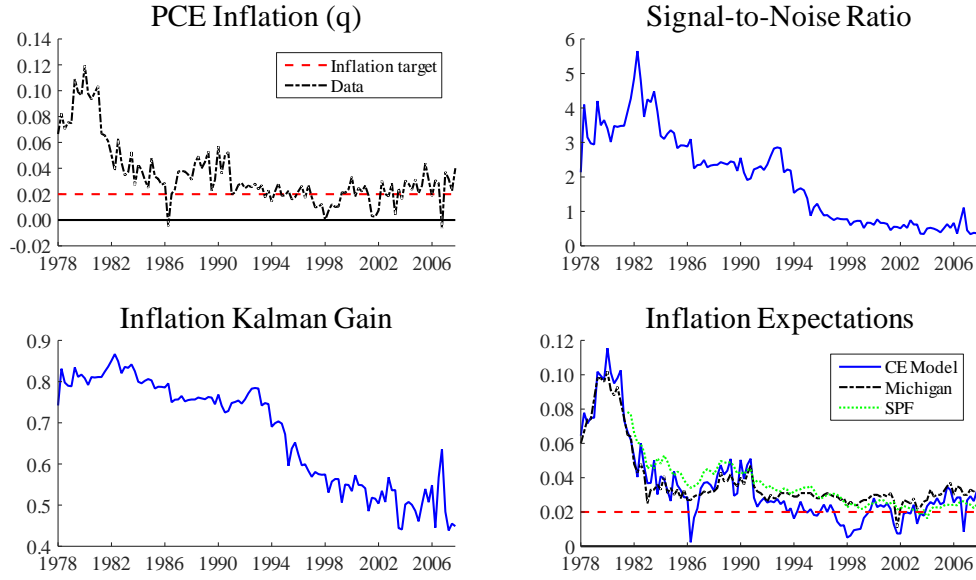
[22]The learning algorithm is described in Appendix A.2

Figure 5: Inflation Expectations and the Great Moderation

expectations becoming more "anchored". The decline in the signal-to-noise ratio implies a lower Kalman gain in the agent's forecast rule (11). The resulting model-based inflation expectations track well with survey-based measures of expectations from the Michigan Survey of Consumers and the Survey of Professional Forecasters over the period.[23] Thus, survey expectations appear to be well described as a moving average of current and past observed inflation rates (as implied by the agent's perceived optimal forecast rule (11)).

5.3.2 Reassessment of Inflation Puzzles

The historical decline in the model-implied signal-to-noise-ratio depicted in Figure 5 is associated with a lower weight on recent inflation in the agent's forecast rule (11). A lower value for λ_π implies that inflation is less sensitive to changes in the output gap. This can be seen by inserting the forecast rule (11) into the NKPC (1), and taking the derivative with respect to y_t:

$$\pi_t = \beta \left[\widetilde{E}_{t-1}\pi_t \;+\; \lambda_\pi \left(\pi_t - \widetilde{E}_{t-1}\pi_t \right) \right] + \kappa y_t + u_t$$

$$\Rightarrow \pi_t = \frac{1}{1 - \beta\lambda_\pi} \left[\left(1 \;-\; \lambda_\pi \right) \beta \widetilde{E}_{t-1}\pi_t + \kappa y_t + u_t \right]$$

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[23]Note, that the survey-based expectations are 1-year expectations, while the model-based expectations are annualized quarterly expectations. This may explain why model-based expectations are slightly more volatile than survey-based measures
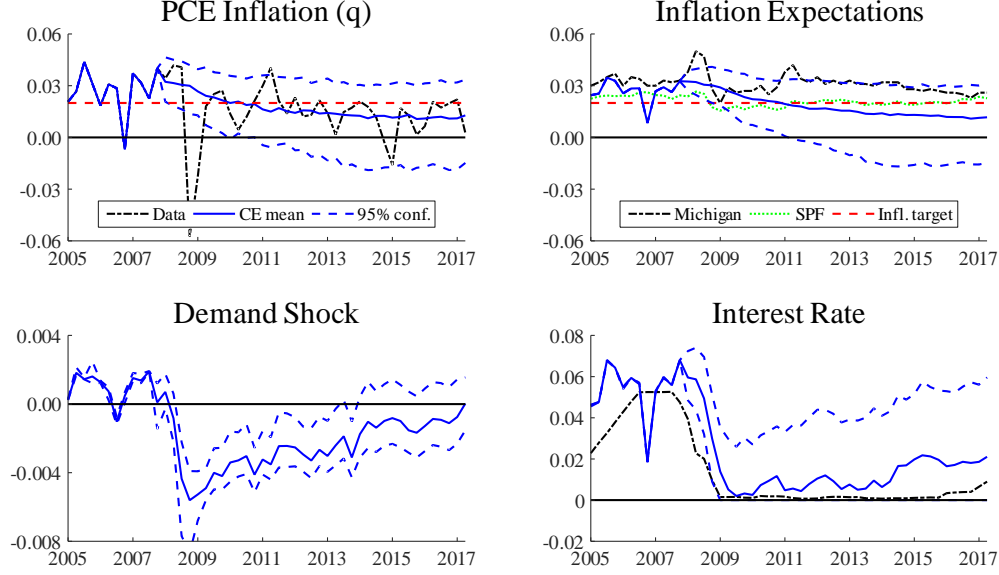
Figure 6: Solving Inflation Puzzles with Consistent Expectations Model

$$\Rightarrow \frac{\partial \pi_t}{\partial y_t} = \frac{\kappa}{1 - \beta \lambda_\pi} \tag{28}$$

which shows that $\partial \pi_t / \partial y_t$ declines as λ_π goes towards zero. In this section we use the CE model to reasses the inflation puzzles that arise under rational expectations. We continue to assume that agents use a rolling window of recent data to compute the signal-to-noise ratios. To make the exercise as realistic as possible, we assume that prior to the contraction starting in 2008.q1, agents use a 20-year window of actual U.S. inflation data. After 2008.q1, inflation is endogenously determined in the model and model-generated inflation data will begin to enter the rolling sample window.[24] Similar to the exercise under rational expectations, we use the CBO output gap as forcing variable from 2008.q1 to 2017.q2 and solve for the sequence of demand shocks, v_t, needed to match the data. Specifically, taking y_t as given, we compute the response of inflation and inflation expectations directly from the NKPC (1) and the learning-version of the forecast rule (11). We then solve for $\widetilde{E}_t y_{t+1}$ and R_t using the learning-version of (12) and the ZLB condition (25). Finally, we use the IS-curve (2) to solve for the demand shock, v_t.

Figure 6 shows the implied model-based forecasts of PCE inflation, inflation expectations and the Federal Funds Rate from 2008.q1 to 2017.q2. The figure also plots the sequence of demand shocks needed to replicate the CBO output gap from Figure 1.[25] The inflation forecast

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[24]Similarly, we assume that agents use a 20-year moving window of output gap data to compute the Kalman gain for the output gap.

[25]The adverse demand shocks peak at around 5 standard deviations on average, which tracks well with output
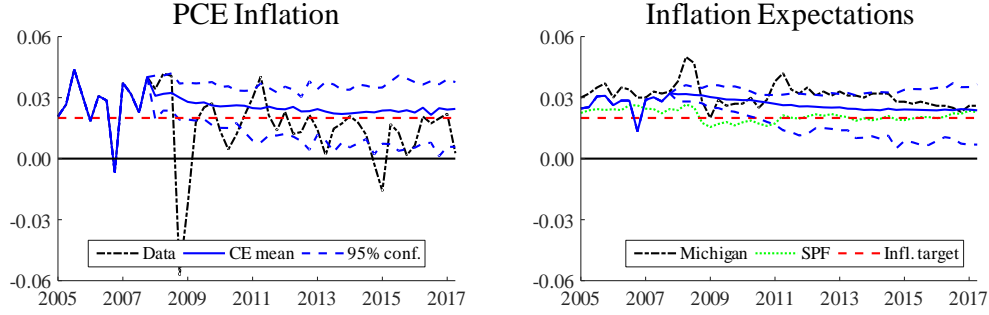
Figure 7: Robustness: 10-year moving window

tracks PCE inflation very accurately. On impact, the response of inflation is muted. This is because expectations are "anchored" prior to the Great Recession due to the historical decline in the estimated signal-to-noise ratio for inflation (as shown in Figure 5). Model-based inflation expectations track well with expectations from the Michigan Survey of Consumers and the Survey of Professional Forecasters. Similar to the survey data, model-based expectations are remarkably stable. Unlike in the rational expectations model, expectations do not drop sharply on impact and then mean-revert. Instead, they gradually decline over the period since they are computed as a moving average of current and past observed inflation rates. On average, model-implied expectations decline slightly more than in survey data, but survey-expectations are generally within the confidence bands of the model.[26] Thus, from the perspective of the CE model, inflation dynamics are associated with neither "missing disinflation" in the wake of the recession, nor "missing inflation" since 2012.[27] If anything, it's puzzling that inflation has not declined to lower levels in recent years given the depth and duration of the downturn. The model-implied interest rate tracks the Federal Funds Rate quite closely over the period. It is close to zero for approximately 5 years – almost as long as in the data.

Figure 7 repeats the exercise under the assumption that agent's use a shorter (10-year) moving window to compute the Kalman gain for PCE inflation. The model continues to track inflation and inflation expectations closely over the period.

5.3.3 The Slope of the NKPC

The response of inflation to changes in the output gap depends crucially on the slope parameter, κ, in the NKPC (which is clear from (28)). A flat Phillips Curve reduces the forecasted

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gap data

[26]These are bootstrapped 95% confidence intervals

[27]The model also resolves the so-called "Forward Guidance Puzzle" (Del Negro et. al, 2012). Since agents' expectations of inflation and the output gap are weighted averages of current and past inflation rates and output gaps, respectively, they are not influenced by central bank announcements about the path of future nominal interest rates. This is related to Gabaix (2017), who also allows for bounded rationality to resolve the "Forward Guidance Puzzles". The puzzle can be resolved under RE by introducing a discount factor in the IS equation that multiplies the expected output term. See McKay et. al. (2017).
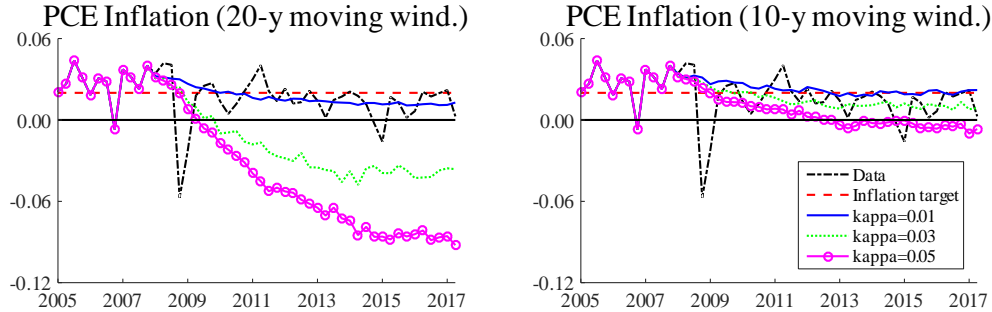
Figure 8: Robustness: Slope of the NKPC

fall in inflation when the output gap drops sharply during the Great Recession. Recent empirical papers supports the assumption of a relatively flat Phillips Curve.[28] Figure 8 shows how sensitive the inflation forecasts are to changes in κ. In this case, the robustness of the results depend crucially on the the length of the moving window. This is because each sample window implies a different value of the inflation Kalman gain. Clearly, when the agent uses a 10-year moving window of PCE inflation data, expectations are more firmly "anchored" prior to the recession compared to the case with a 20-year moving window of data. In the latter case, a relatively steep NKPC ($\kappa = 0.05$) implies that inflation declines to around -10% in 2015. This is clearly at odds with the data. It is important to note, however, that inflation would decline even more under rational expectations.[29]

5.3.4 Inflation Forecast

For how long will inflation continue to undershoot the Fed's target of 2%? This question can be adressed within the context of the CE model. Given the path of inflation and the output gap in the data up to 2017.q2, we solve for the future path of the economy implied by the variable gain model. Specifically, given the data values for inflation and the output gap in 2017.q2 and the implied solution for the remaining variables, we solve the model in each future period by forward iteration, using the learning-version of the recursive law of motion (17). The model-based forecasts for PCE inflation with a 20-year moving window are shown in Figure 9. According to the model, the median inflation rate will undershoot the central bank's target for several years to come. Consequently, monetary policy will remain accomodative and contribute to a positive output gap in the future. The economy is projected to return to its long run equilibrium around 2021. Obviously, the duration of the undershooting episode depends crucially on the initial values of inflation and the output gap (given by the data values in 2017.q2). Moreover, it depends on which price index we consider and the assumed length of the moving window, since each specification implies different values of the Kalman gains for

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[28]See, for instance, Ball and Mazumder (2011)

[29]With rational expectations and $\kappa = 0.05$, the inflation rate declines to roughly -11% already in 2009 and remains negative until 2017.
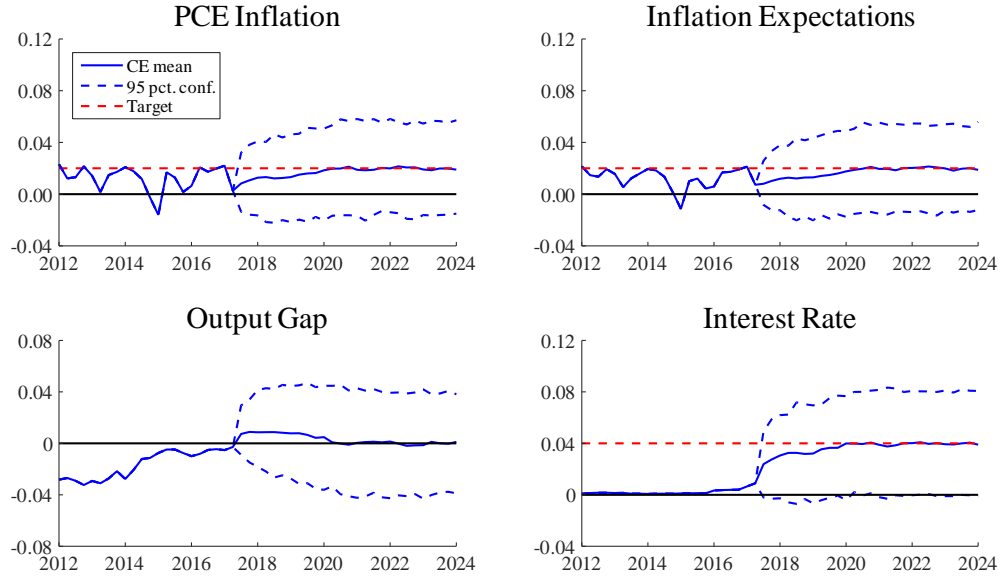
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Figure 9: CE Model Forecasts 2017-2024

inflation and the output gap. Figure 10 plots the model-implied forecasts of inflation and the
output gap for alternative specifications (PCE inflation with a 10-year moving window, CPI
inflation with a 10-year moving window, and CPI inflation with a 20-year moving window,
respectively). Across these specifications, inflation undershoots the central bank's target for
roughly 2 to 6 years into the future and the output gap remains positive for an equivalent
period of time.

6   Concluding Remarks

Since the outbreak of the Great Recession in 2007.q4, U.S. inflation has been associated with
two "puzzles". First, the absence of persistent disinflation in the wake of highly negative output
gaps was dubbed the "missing disinflation puzzle". Subsequently, unexpectedly low inflation
rates gave birth to the so-called "missing inflation puzzle". These are puzzles judging by the
predictions of a standard New Keynesian Phillips Curve with fully rational expectations. In
this paper we introduced a form of boundedly-rational expectations into an otherwise standard
New Keynesian model. In the model, agents use time-series models to forecast inflation and the
output gap. The time series models allow for both permanent and transitory shocks – similar
to that of Stock and Watson (2007). The perceived optimal forecast rules are defined by the
Kalman filter. In a unique "consistent expectations" equilibrium, the values of the Kalman
gains for inflation and the output gap are pinned down by the observed autocorrelation of
inflation and output gap changes, respectively. We showed that if agents continously update
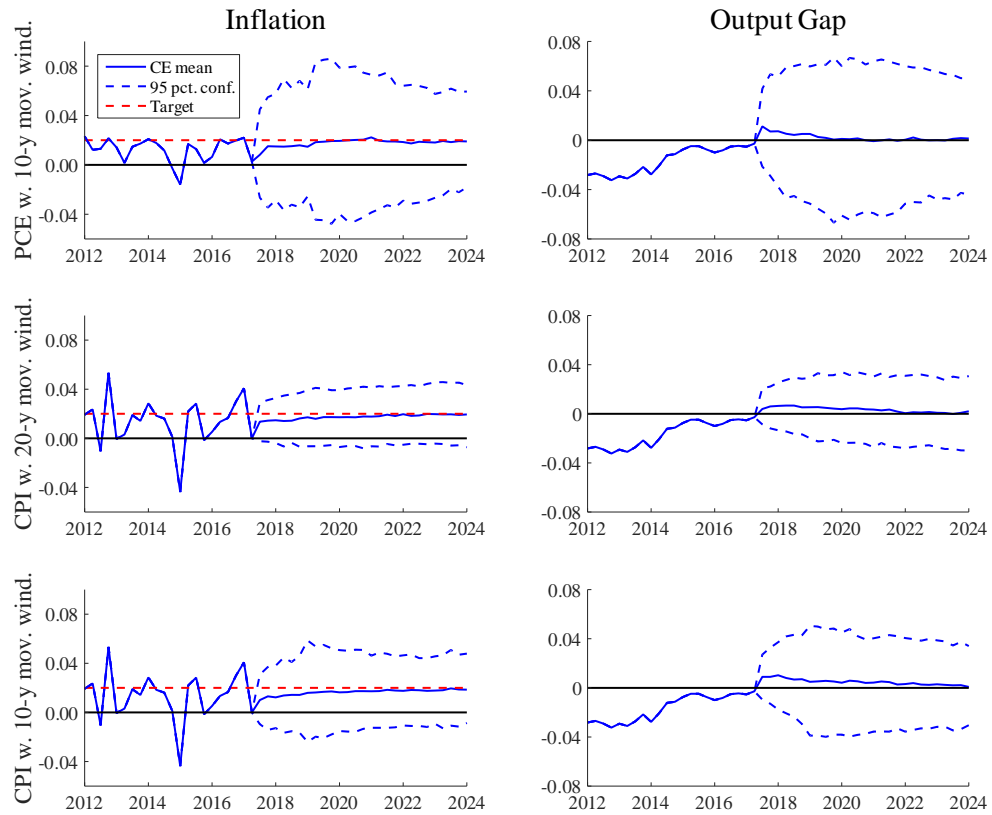
Figure 10: CE Model Forecasts 2017-2024: Robustness

their estimate of the Kalman gain using a moving window of recent data, the identified Kalman gain for inflation exhibits a downward drift during the so-called Great Moderation period (Figure 5). A low Kalman gain implies a low weight on recent inflation in the agent's forecast rule, which is consistent with the idea of "well-anchored" expectations prior to the outbreak of the Great Recession. This helps anchor inflation near the central bank's target rate when the output gap falls sharply during the contraction. The recession, however, gradually generates a moderate – but highly persistent – decline in inflation expectations, which lowers the inflation rate in the longer term. Thus, the model can help account for both the "missing deflation" in the aftermath of the recession as well as the "missing inflation" since 2012. Forecasts with the model suggest that inflation will undershoot the central bank's target rate for several years to come. Consequently, the model predicts that monetary policy will remain accomodative and contribute to a positive output gap in the future.

A Appendix

A.1 Actual Law of Motion under Consistent Expectations

The model is given by the following equations

$$\pi_t = \beta \widetilde{E}_t \pi_{t+1} + \kappa y_t + u_t \tag{A.1}$$

$$y_t = \widetilde{E}_t y_{t+1} - \alpha \left(R_t - \widetilde{E}_t \pi_{t+1} \right) + v_t \tag{A.2}$$

$$R_t = \mu_\pi \widetilde{E}_t \pi_{t+1} + \mu_y \widetilde{E}_t y_{t+1} \tag{A.3}$$

$$\widetilde{E}_t \pi_{t+1} = \widetilde{E}_{t-1} \pi_t + \lambda_\pi \left(\pi_t - \widetilde{E}_{t-1} \pi_t \right) \tag{A.4}$$

$$\widetilde{E}_t y_{t+1} = \widetilde{E}_{t-1} y_t + \lambda_y \left(y_t - \widetilde{E}_{t-1} y_t \right) \tag{A.5}$$

which we wish to write on the form:

$$\mathbf{Z}_t = \mathbf{A}\mathbf{Z}_{t-1} + \mathbf{B}\mathbf{U}_t$$

where $\mathbf{Z}_t = \begin{bmatrix} \pi_t & y_t & R_t & \widetilde{E}_t \pi_{t+1} & \widetilde{E}_t y_{t+1} \end{bmatrix}'$ and $\mathbf{U}_t = \begin{bmatrix} u_t & v_t \end{bmatrix}'$

Insert (A.4) into (A.1) and (A.3)-(A.5) into (A.2) to obtain:

$$\pi_t = \frac{1}{1 - \beta\lambda_\pi} \left\{ \beta \left(1 - \lambda_\pi \right) \widetilde{E}_{t-1} \pi_t + \kappa y_t + u_t \right\}$$

$$y_t = \frac{1}{1 - \lambda_y + \alpha\mu_y\lambda_y} \{ (1 - \lambda_y) \widetilde{E}_{t-1} y_t - \alpha[(\mu_\pi - \mu_\pi\lambda_\pi - (1 - \lambda_\pi)) \widetilde{E}_{t-1} \pi_t$$

$$+ \mu_y \left(1 - \lambda_y \right) \widetilde{E}_{t-1} y_t - (1 - \mu_\pi) \lambda_\pi \pi_t] + v_t \}$$

Combine these two expressions to derive the following equations for π_t and y_t:

$$\pi_t = \frac{1}{1 - \beta\lambda_\pi - \frac{\kappa\alpha(1-\mu_\pi)\lambda_\pi}{1-\lambda_y+\alpha\mu_y\lambda_y}} \{ \beta + \frac{\kappa\alpha \left(1 - \mu_\pi \right)}{1 - \lambda_y + \alpha\mu_y\lambda_y} \left(1 - \lambda_\pi \right) \widetilde{E}_{t-1} \pi_t \tag{A.6}$$

$$+ \frac{\left(1 - \alpha\mu_y \right) \kappa \left(1 - \lambda_y \right)}{1 - \lambda_y + \alpha\mu_y\lambda_y} \widetilde{E}_{t-1} y_t + \frac{\kappa}{1 - \lambda_y + \alpha\mu_y\lambda_y} v_t + u_t \}$$

and

$$y_t = \frac{1}{1 - \lambda_y + \alpha\mu_y\lambda_y - \frac{\alpha(1-\mu_\pi)\lambda_\pi\kappa}{1-\beta\lambda_\pi}} \left\{ \left(1 - \alpha\mu_y\right)\left(1 - \lambda_y\right)\widetilde{E}_{t-1}y_t \right. \tag{A.7}$$

$$\left. + \frac{\alpha\left(1 - \mu_\pi\right)\left(1 - \lambda_\pi\right)}{1 - \beta\lambda_\pi}\widetilde{E}_{t-1}\pi_t + \frac{\alpha\left(1 - \mu_\pi\right)\lambda_\pi}{1 - \beta\lambda_\pi}u_t + v_t \right\}$$

These can then be inserted into the forecast rules to obtain:

$$\widetilde{E}_t\pi_{t+1} = \frac{1 - \lambda_\pi}{1 - \beta\lambda_\pi - \frac{\kappa\alpha(1-\mu_\pi)\lambda_\pi}{1-\lambda_y+\alpha\mu_y\lambda_y}}\widetilde{E}_{t-1}\pi_t \tag{A.8}$$

$$+ \frac{\lambda_\pi}{1 - \beta\lambda_\pi - \frac{\kappa\alpha(1-\mu_\pi)\lambda_\pi}{1-\lambda_y+\alpha\mu_y\lambda_y}}\left(\frac{\left(1 - \alpha\mu_y\right)\kappa\left(1 - \lambda_y\right)}{1 - \lambda_y + \alpha\mu_y\lambda_y}\widetilde{E}_{t-1}y_t + \frac{\kappa}{1 - \lambda_y + \alpha\mu_y\lambda_y}v_t + u_t\right)$$

and

$$\widetilde{E}_ty_{t+1} = \left(1 - \lambda_y\right)\left(\frac{1 - \frac{\alpha(1-\mu_\pi)\lambda_\pi\kappa}{1-\beta\lambda_\pi}}{1 - \lambda_y + \alpha\mu_y\lambda_y - \frac{\alpha(1-\mu_\pi)\lambda_\pi\kappa}{1-\beta\lambda_\pi}}\right)\widetilde{E}_{t-1}y_t \tag{A.9}$$

$$+ \frac{\lambda_y}{1 - \lambda_y + \alpha\mu_y\lambda_y - \frac{\alpha(1-\mu_\pi)\lambda_\pi\kappa}{1-\beta\lambda_\pi}}\left[\frac{\alpha\left(1 - \mu_\pi\right)\left(1 - \lambda_\pi\right)}{1 - \beta\lambda_\pi}\widetilde{E}_{t-1}\pi_t + \frac{\alpha\left(1 - \mu_\pi\right)\lambda_\pi}{1 - \beta\lambda_\pi}u_t + v_t\right]$$

Finally, the forecasts (A.8) and (A.9) can be inserted directly into the Taylor rule (A.3) to obtain a long and complicated expression for the interest rate, R_t. The equations (A.6)- (A.9) and the implied expression for R_t constitutes the actual law of motion under consistent expectations.

A.2   Learning Algorithm

Real-time learning is discussed in Section 4 of the text. The learning algorithm is described by the following system of nonlinear equations

$$\pi_t = \beta\widetilde{E}_t\pi_{t+1} + \kappa y_t + u_t$$

$$y_t = \widetilde{E}_ty_{t+1} - \alpha\left(R_t - \widetilde{E}_t\pi_{t+1}\right) + v_t,$$

$$R_t = \mu_\pi\widetilde{E}_t\pi_{t+1} + \mu_y\widetilde{E}_ty_{t+1},$$

$$\widetilde{E}_t\pi_{t+1} = \widetilde{E}_{t-1}\pi_t + \lambda_{\pi,t-1}\left(\pi_t - \widetilde{E}_{t-1}\pi_t\right), \tag{A.10}$$

$$\widetilde{E}_ty_{t+1} = \widetilde{E}_{t-1}y_t + \lambda_{y,t-1}\left(y_t - \widetilde{E}_{t-1}y_t\right), \tag{A.11}$$

$$Ave_{\pi,t} = \left[\frac{t}{t+1}\right] Ave_{\pi,t-1} + \left[\frac{1}{t+1}\right] \Delta\pi_t, \tag{A.12}$$

$$n_{\pi,t} = n_{\pi,t-1} + \left[\frac{t}{t+1}\right] (\Delta\pi_t - Ave_{\pi,t-1})^2, \tag{A.13}$$

$$m_{\pi,t} = m_{\pi,t-1} + (\Delta\pi_t - Ave_{\pi,t-1})\left[\Delta\pi_{t-1} - \frac{\Delta\pi_t}{(t+1)^2} - \frac{(t^2+3t+1) Ave_{\pi,t-1}}{(t+1)^2}\right], \tag{A.14}$$

$$Ave_{y,t} = \left[\frac{t}{t+1}\right] Ave_{y,t-1} + \left[\frac{1}{t+1}\right] \Delta y_t, \tag{A.15}$$

$$n_{y,t} = n_{y,t-1} + \left[\frac{t}{t+1}\right] (\Delta y_t - Ave_{y,t-1})^2 \tag{A.16}$$

$$m_{y,t} = m_{y,t-1} + (\Delta y_t - Ave_{y,t-1})\left[\Delta y_{t-1} - \frac{\Delta y_t}{(t+1)^2} - \frac{(t^2+3t+1) Ave_{y,t-1}}{(t+1)^2}\right], \tag{A.17}$$

$$\phi_{\pi,t} = -\frac{n_{\pi,t}}{m_{\pi,t}} - 2, \tag{A.18}$$

$$\lambda_{\pi,t} = \frac{-\phi_{\pi,t} + \sqrt{\phi_{\pi,t}^2 + 4\phi_{\pi,t}}}{2}, \tag{A.19}$$

$$\phi_{y,t} = -\frac{n_{y,t}}{m_{y,t}} - 2, \tag{A.20}$$

$$\lambda_{y,t} = \frac{-\phi_{y,t} + \sqrt{\phi_{y,t}^2 + 4\phi_{y,t}}}{2}. \tag{A.21}$$

Equations (A.10) and (A.11) are the forecast rules when the Kalman gains are evolving over time. The first five equations can be written on recursive form similar to the law of motion derived in Appendix A.1. Equations (A.12)-(A.14) and (A.15)-(A.17) are used to recursively estimate the autocorrelation of inflation and output gap changes, respectively, using all past data.[30] Equations (A.18) and (A.20) are the full-sample estimate of the signal-to-noise ratios. Equations (A.19) and (A.21) are the Kalman gain formulas.

To obtain the "variable-gain" version of the model that is discussed in Section 5 of the text, equations (A.12) through (A.17) are modified to compute the autocorrelation of inflation and output gap changes over a rolling sample period rather than over the full sample period. Both the real-time learning algorithm and the variable-gain model employ a "projection facility," which sets $\phi_{i,t} = \phi_{i,t-1}$ for $i = \{\pi, y\}$ whenever the sample autocorrelations of $\Delta\pi_t$ and Δy_t yield the result that $\phi_{i,t} < 0$.

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[30]These formulas are adapted from Hommes and Sorger (1998, pp. 320-321).

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Chapter 3

The Inflation Response to Government Spending Shocks: A Fiscal Price Puzzle?

The Inflation Response to Government Spending Shocks: A Fiscal Price Puzzle?*

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Abstract

Based on a Structural Vector Autoregression (SVAR) analysis, this paper provides empirical evidence that prices decline significantly and persistently in response to a positive government spending shock. This result stands out across a wide variety of specifications of our empirical model and for different price indices. The decline in prices is accompanied by an increase in output and private consumption, as found in most of the existing literature, as well as an increase in Total Factor Productivity. These findings are hard to reconcile with standard New Keynesian models with exogenous productivity, which typically generate higher prices and a drop in consumption following a fiscal expansion. We show that the introduction of variable technology utilization, along the lines of Bianchi *et al.* (2017), can enable an otherwise standard New Keynesian model to match our empirical findings. Intuitively, variable technology utilization allows firms to accommodate an increase in demand by adopting new technology into the production process. The resulting increase in productivity leads to a decline in prices and an increase in consumption. Our paper thus contributes to an emerging literature studying how endogenous movements in productivity affect the business cycle.

Keywords: Government Spending Shocks, Fiscal Policy, Business-cycle Comovement, DSGE Modelling, Endogenous Productivity.

JEL classification: E13, E31, E32, E62.

1 Introduction

The macroeconomic effects of changes in government spending have received widespread attention in the economics profession, not least since the onset of The Great Recession in 2007. Following the tradition of Blanchard and Perotti (2002), a large literature has employed Structural Vector Autoregressive (SVAR) models to characterize the empirical effects of government spending shocks on GDP, private consumption, and a range of other macroeconomic variables (e.g., Ramey, 2011a). However, the response of inflation to government spending shocks has typically received limited attention in the empirical literature. Nonetheless, a common perception is that increases in government spending are inflationary. Indeed, this idea plays an important role in the transmission of fiscal policy shocks across several theoretical models, including the textbook New Keynesian model. A prominent example is the effectiveness of government spending shocks when the nominal interest rate is stuck at the zero lower bound. The finding of a large fiscal multiplier under these circumstances relies entirely on the ability of higher government spending to generate inflation and thus reduce the real interest rate (e.g., Christiano *et al.*, 2011).[1] In this paper, we study the effects of government spending shocks on inflation in the U.S. economy. Our main finding is that prices *decline* significantly and persistently in response to increases in government spending. This result stands out across a variety of specifications of our empirical model, as well as across different price indices. Importantly, the drop in inflation coexists with the increase in output and private consumption found in most of the existing literature (e.g., Blanchard and Perotti, 2002; and Galí *et al.*, 2007), as well as an increase in Total Factor Productivity (TFP).

The existing evidence on the response of prices to government spending shocks is rather mixed, as Table 1 makes clear. Some previous studies have also reported a decline in prices in response to a fiscal expansion. Examples include Fatas and Mihov (2001b) and Mountford and Uhlig (2009). However, Edelberg *et al.* (1999) and Caldara and Kamps (2007) report that inflation increases, whereas Fatas and Mihov (2001a) find an insignificant response of prices. Perotti (2004) studies the response of inflation in the US and 4 other OECD countries across different specifications and subsamples. While the evidence he reports is somewhat mixed, he concludes that there is little evidence of an *increase* in inflation after a government spending shock, consistent with our results.[2] Several prominent studies of fiscal policy do not consider the response of prices. None of the authors who do find evidence of a decline in inflation attempt to provide a structural explanation for it.[3]

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[1] Another example emanates from open-economy models: The fiscal multiplier is typically found to be smaller in countries with floating exchange rates (as compared to countries with a currency peg), as they will experience a tightening of monetary policy to combat the rise in inflation assumed to follow an increase in government spending (e.g., Corsetti *et al.*, 2013).

[2] As seen in Table 1, some studies report the response of the price level, and others that of the inflation rate, but this cannot explain the different findings in the literature. While we use the price level in all our estimations, none of our findings depend on this choice.

[3] Canova and Pappa (2007) offer a discussion of potential explanations for a decline in prices after a fiscal expansion, of which they find some evidence, but they stop short of building a theoretical model.

Table 1: Survey of Empirical Estimates of Inflation Response	
Fiscal Policy Study	**Response of Prices/Inflation**
Edelberg *et al.* (1999)	*Prices increase*
Fatas and Mihov (2001a)	*Prices are insignificant*
Fatas and Mihov (2001b)	*Prices decline*
Blanchard and Perotti (2002)	*N/A*
Canzoneri *et al.* (2002)	*Inflation declines*
Burnside *et al.* (2004)	*N/A*
Perotti (2004)	*Mixed response of inflation*
Canova and Pappa (2007)	*Mixed response of prices*
Galí *et al.* (2007)	*N/A*
Caldara and Kamps (2008)	*Inflation increases*
Mountford and Uhlig (2009)	*Prices decline*
Ramey (2011a)	*N/A*
Ravn *et al.* (2012)	*N/A*
Ben Zeev and Pappa (2017)	*Inflation increases*

Notes: In Edelberg *et al.* (1999), the GDP deflator increases, while the CPI index first increases and then declines. In Fatas and Mihov (2001b) and Canzoneri *et al.* (2002), the decline in inflation is barely significant. All studies use U.S. data, though Perotti (2004) and Canova and Pappa (2007) also report evidence from other OECD countries and from Euro Area countries, respectively.

Our empirical findings seem hard to reconcile with traditional accounts of the transmission mechanism of fiscal policy. To provide a structural interpretation of our empirical findings, we propose a version of the New Keynesian model featuring time-varying adoption of the available technology into the production process, as in recent work by Anzoategui *et al.* (2017) and Bianchi *et al.* (2017). In our model, firms decide on the extent to which they adopt or utilize new technologies as these become available. In response to an increase in government spending, firms find it optimal to raise the utilization rate of technology in order to satisfy the increase in aggregate demand, despite the costs associated with changes in the utilization rate. Higher technology utilization increases measured productivity, in line with the empirical evidence we present. Provided this mechanism is sufficiently powerful, it dominates the upward pressure on marginal costs arising from higher wages, ensuring that marginal costs decline in equilibrium. This paves the way for firms to reduce their prices, generating the desired decline in inflation. In turn, this induces the central bank to reduce the nominal interest rate, in line with what we observe in our SVAR evidence. This leads to a drop in the real interest rate, facilitating an increase in consumption.

The theoretical model is deliberately simple in order to allow for an analytical solution.

As in the basic New Keynesian model, consolidating a standard consumption Euler equation with a version of the Taylor rule for monetary policy results in a negative relationship between consumption and inflation. In our model, an increase in government spending shifts the economy *down* along this consolidated Euler equation, resulting in a *decline* in inflation and an *increase* in consumption, in line with the data. We provide an analytical characterization of the parameter requirements for our model to generate these findings, and show that a range of parameters always exists for which this is the case. We finally show that a calibrated version of our model can account for the dynamic effects of government spending shocks in the data for reasonable parameter values.

Incidentally, the textbook version of the New Keynesian model does feature a negative comovement between inflation and private consumption conditional on a shock to government spending, but of the opposite sign than what we find in the data: inflation *increases* and consumption *declines* after a positive government spending shock. The response of consumption has received substantial attention in the theoretical literature, with several authors proposing mechanisms to obtain an increase in consumption. However, most of these seem to hold little promise for producing a decline in inflation. For example, the introduction of rule-of-thumb households by Galí *et al.* (2007) drives up aggregate demand but has no direct effects on the supply side. Allowing for non-separable utility in consumption and leisure, as in Monacelli and Perotti (2008) and Bilbiie (2011), induces consumption and labor supply to increase in tandem, provided consumption and leisure are substitutes. However, as shown by Bilbiie (2011), the demand-side effects still dominate, leading to a rise in inflation.

Correspondingly, while theoretical models do exist which may potentially be able to generate a decline in inflation after a fiscal expansion, these are generally not consistent with a contemporaneous increase in consumption. In the New Keynesian model, there are essentially three ways to bring about a drop in inflation in response to a government spending shock: a drop in the markup, a drop in the wage rate, or an increase in productivity. A countercyclical markup is the hallmark of the so-called deep habits model of Ravn *et al.* (2006), who show that this mechanism can even generate an increase in consumption after a government spending shock in their flexible-price model. However, Jacob (2015) demonstrates that the deep habits model performs quite differently in sticky-price environments: while it may indeed generate a decline in inflation, this occurs alongside a decline in consumption.[4] A drop in the wage rate may be obtained in the presence of a sufficiently strong increase in labor supply in response to the reduction in permanent income associated with higher government spending (Baxter and King, 1993). Besides requiring an implausibly large Frisch elasticity of labor supply, however, a declining wage makes it very unlikely to observe an increase in consumption, as shown, e.g., by Monacelli and Perotti (2008).[5] Altogether, these considerations lead us to focus on endogenous

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[4]In a nutshell, price stickiness erodes the ability of firms to reduce their markup as desired under deep habits, thus impeding the increase in real wages necessary to drive up consumption.

[5]An alternative way to obtain a decline in the wage rate is to allow for a sufficiently strong reaction of monetary policy to output, as shown by Linnemann and Schabert (2003). Aside from the fact that an increase in the nominal interest rate is in contrast to our empirical evidence, this approach has the disadvantage of

changes in the level of productivity as a more promising avenue for matching the empirical evidence.

We contribute to an emerging literature studying endogenous changes in productivity over the business cycle. We build directly on the work of Bianchi *et al.* (2017), who propose an endogenous growth model capturing both business cycle fluctuations and long-term growth. They find that technology utilization dropped substantially during the Great Recession, explaining most of the observed drop in TFP. While the paper of Bianchi *et al.* (2017) features endogenous technological progress through vertical innovations, as in Aghion and Howitt (1992), other authors have employed horizontal innovations featuring increasing returns to specialization as in Romer (1990) and Comin and Gertler (2006). A prominent recent example is the paper by Anzoategui *et al.* (2017). They also report that most of the observed decline in productivity during the Great Recession is attributable to endogenous factors, primarily a decline in the intensity of technology adoption. Moran and Queralto (2017) use a similar model to study the link between monetary policy shocks and endogenous movements in technology. They first report that a monetary expansion leads to an increase in R&D investment and TFP in the data, and then study how this may strengthen the transmission of monetary policy shocks onto output in their model. Other recent contributions to this literature include Queralto (2013), Kung and Schmid (2015), and Guerron-Quintana and Jinnai (2017), but none of these papers have studied the connection between endogenous productivity and fiscal policy.

The rest of the paper is structured as follows. We present our empirical exercises and results in Section 2. Our model of variable technology adoption is outlined in Section 3, while Section 4 is devoted to studying its properties analytically. We present numerical model simulations in Section 5. Finally, Section 6 concludes.

2   Fiscal Policy and the Price Level: Empirical Evidence

In this section, we set up a Structural VAR model for the U.S. economy to investigate the effects of government spending shocks on key macroeconomic variables. As a baseline, we follow the tradition of Blanchard and Perotti (2002) and identify government spending shocks through a Cholesky decomposition with government spending ordered first. Next, to account for anticipated changes in fiscal policy, we use the forecast errors of government spending computed by Auerbach and Gorodnichenko (2012) to identify government spending shocks. Finally, we conduct a series of robustness checks.

2.1   Baseline VAR Evidence

We estimate the following quarterly VAR model on U.S. data:

$$\mathbf{X}_t = a_0 + a_1 t + a_2 t^2 + B^{-1} A(L) \mathbf{X}_{t-1} + B^{-1} e_t, \tag{1}$$

leading to an even larger drop in consumption.

where $\mathbf{X}_t$ is the vector of endogenous variables, e_t is a vector of IID structural shocks with unit variance, $A(L)$ comprises the coefficients on the lagged endogenous variables, L is the lag operator and B comprises the coefficients on the contemporaneous endogenous variables. We include linear and quadratic time trends, as in Blanchard and Perotti (2002). We follow most of the literature and use 4 lags as our baseline. The baseline model contains the following variables:

$$\mathbf{X}_t = \left[\begin{array}{ccccccc} G_t & Y_t & C_t & T_t & P_t & R_t & A_t \end{array} \right]',$$

where G_t is real government expenditure and investment, Y_t is real GDP, C_t is real private consumption, T_t is real tax revenue (tax receipts less current transfers, interest payments and subsidies), P_t is the Personal Consumption Expenditures (PCE) price index, R_t is the nominal interest rate on 3-month treasury bills, and A_t is Total Factor Productivity (TFP).[6] All variables except R_t are in logs, and the variables G_t, Y_t, C_t and T_t are measured in real per-capita terms. T_t is converted into real terms using the GDP deflator. We use the TFP measure of Fernald (2014).[7] Our data sample covers the period 1960:Q1-2017:Q2. Appendix A contains a detailed description of the data.

Following the approach of Blanchard and Perotti (2002), we assume that the structural shocks to government spending can be recovered from the estimated residuals $B^{-1}e_t$ in (1) by imposing that the matrix B is lower triangular. This implies that government spending does not respond to any other variable within-quarter, but affects other variables within the same quarter. Intuitively, the assumption is motivated by decision lags in fiscal policy. By the time policymakers realize that a shock has hit the economy and implement an appropriate policy response, at least one quarter would have passed. The ordering of the remaining variables is such that real variables (with the exception of TFP) are determined before nominal ones, and that monetary policymakers are assumed to be able to observe and react to changes in output and prices within-period. Our findings are robust to different orderings of the variables.

Figure 1 shows the impulse response functions to a positive government spending shock normalized to 1 percent, along with bootstrapped 68 percent confidence bands. All responses are denoted in percent, except for the interest rate, where the response is reported in basis points. Following a fiscal expansion, output and consumption increase persistently, in line with most of the empirical literature.[8] Prices display a strongly significant and very persistent decline. This response is particularly notable given the mixed evidence available in the existing literature discussed above. The price level drops by around 0.3 percent at the peak. The implied annualized inflation rate drops by slightly more than 25 basis points at its trough 2 quarters after the shock. TFP also increases significantly, in line with the evidence reported

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[6] All results are robust if we use non-durable consumption instead of total consumption.

[7] We use the non-utilization-adjusted TFP measure as our baseline. All results are robust to using the utilization-based measure instead.

[8] The implied government spending multiplier on output can be found by multiplying the reported output response by the inverse of the sample average of the ratio of government spending to output, which is 0.245. This implies an impact multiplier of 1.02, well within the range of available estimates reported in the survey of Ramey (2011b) of $0.8 - 1.5$.

by Afonso and Sousa (2012) for four OECD countries including the US. Finally, the short-term nominal interest drops by around 20 basis points, and tax revenues decline.

2.2 Controlling for Anticipated Shocks

A common criticism of the Cholesky identification strategy employed above is that changes in fiscal policy are–at least to some extent–anticipated by economic agents, as discussed by Ramey (2011a), among others. In this case, it is not possible to recover a structural shock to fiscal policy using the identification strategy of Blanchard and Perotti (2002). To account for this, we consider an alternative identification scheme which controls for fiscal foresight. Following Auerbach and Gorodnichenko (2012), we identify an unanticipated government spending shock as an innovation to the forecast error of the growth rate of government spending. The vector of endogenous variables becomes:

$$\mathbf{X}_t = \left[\begin{array}{cccccccc} FE_t & G_t & Y_t & C_t & T_t & P_t & R_t & A_t \end{array} \right]',$$

where FE_t is the implied forecast error of the survey-based forecasts of the growth rate of government spending, which we obtain from Auerbach and Gorodnichenko (2012). In this specification, we interpret an unanticipated government spending shock as an innovation to the forecast error, FE_t, which is ordered first in the system. The data sample covers the period 1966:Q3-2010:Q3, for which the forecast errors of Auerbach and Gorodnichenko (2012) are available. Figure 2 shows the effects of an unanticipated shock to government spending under the alternative identification scheme. After controlling for fiscal foresight, the results are qualitatively similar to those presented above, with all variables except tax revenues moving in the same direction as before. Notably, a fiscal expansion continues to generate an increase in consumption and productivity and a decline in prices.

2.3 Robustness

We consider a series of alternative specifications of our VAR model to check the robustness of our results. These include a) using alternative price indices, b) including commodity prices in the VAR, c) using an alternative productivity measure, d) excluding TFP from the baseline VAR. We use the Cholesky specification as the baseline for the robustness checks.

Figure 3 shows the impulse responses when the PCE price index is replaced by, respectively, the GDP deflator, the CPI index, and the core version of the PCE index. Qualitatively, the results are very similar across the different price indices. It is worth noting, however, that the core PCE index appears to react more strongly to a government spending shock. This indicates that our main finding is not driven by movements in food and energy prices, which is excluded from this measure.

Figure 4 displays the results from the next set of specifications. First, we include a measure of commodity prices in the VAR model. Sims (1992) showed that prices increase on impact
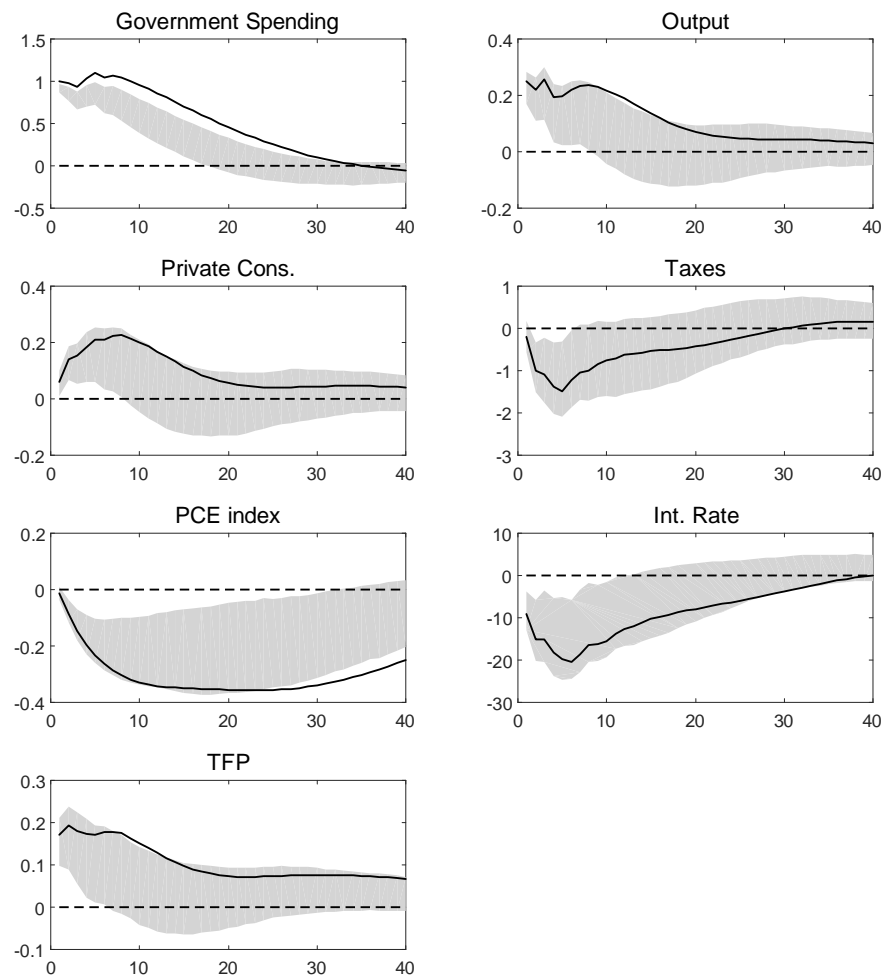
Figure 1: *The dynamic effects of a shock to government spending. Baseline model with Cholesky identification scheme.*
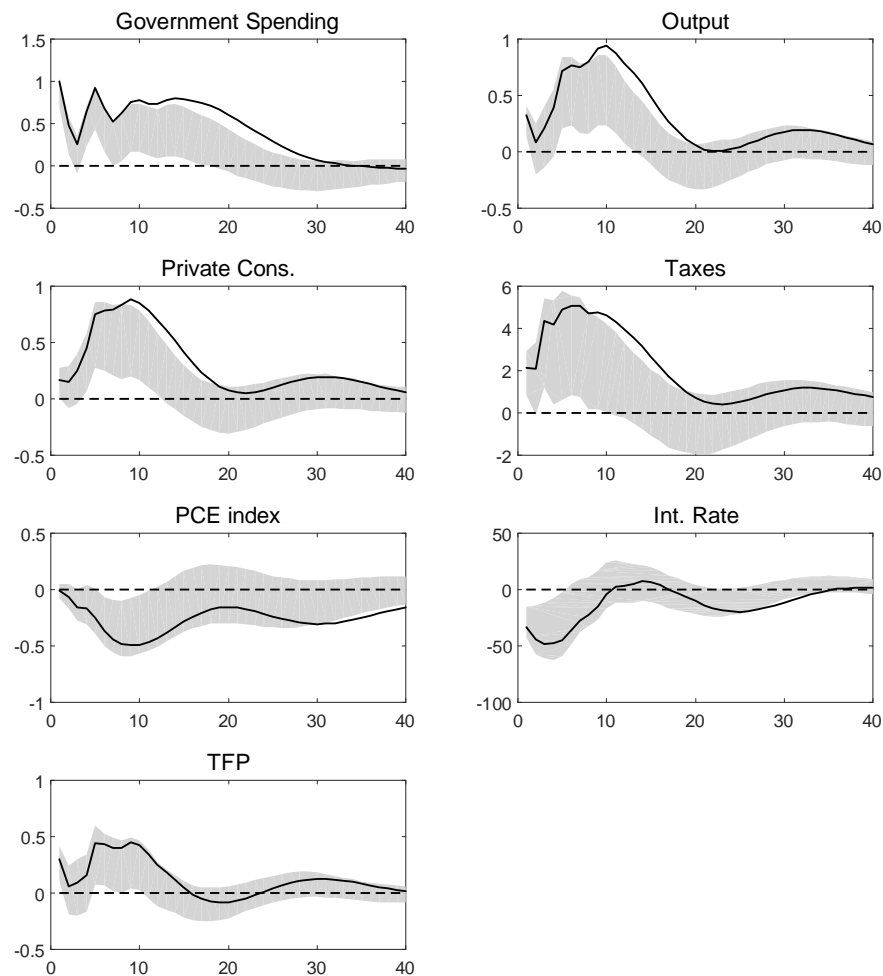
Figure 2: *The dynamic effects of a shock to government spending. Estimates obtained using the identification scheme based on forecast errors.*
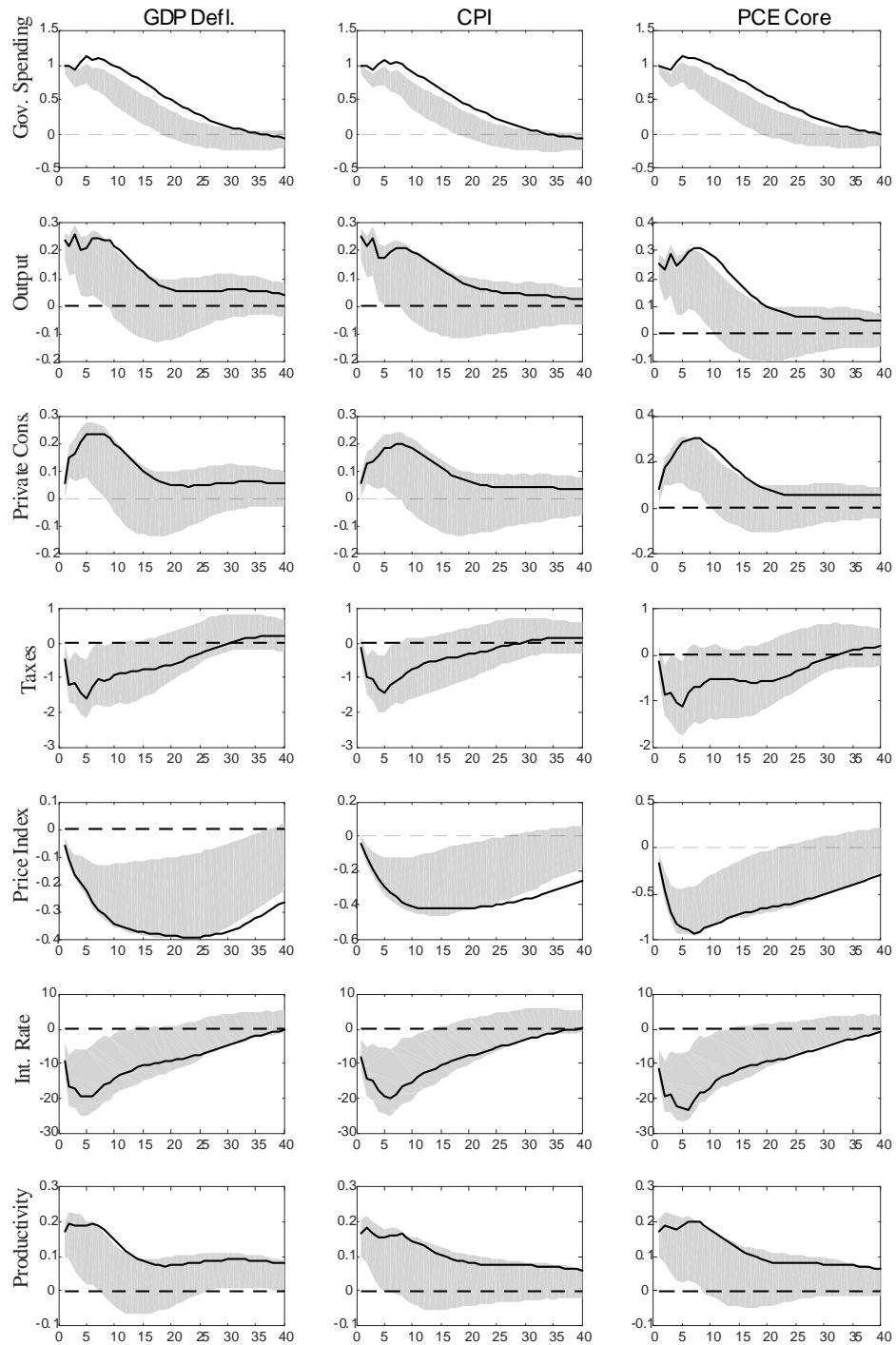
Figure 3: *The dynamic effects of a shock to government spending. Robustness checks: Different price indices: GDP deflator (first column), CPI index (second column), PCE core price index (third column). Estimates obtained using the Cholesky identification scheme.*

in response to a tightening of monetary policy; the so-called "price puzzle", but that this counterintuitive response could be alleviated by including commodity prices in the VAR model. Intuitively, commodity prices may contain signals of future price changes observed by central bankers, but not by an econometrician excluding commodity prices from her model. While this argument appears less appealing in the case of fiscal policy, we check the robustness of our results when commodity prices are included. The first column of Figure 4 confirms that our results are indeed robust.[9] Next, we show that an alternative measure of productivity, the log of real output per hour in the nonfarm sector, responds similarly to TFP. Lastly, we address the potential concern that any of our initial results could be driven by the inclusion of TFP in the VAR model. All results are confirmed when TFP is excluded from the baseline model.

We present an additional battery of robustness checks in Appendix A.1. These include e) alternative ordering of variables, f) changing the lag length, g) excluding the quadratic time trend. Finally, we also perform a complete set of robustness checks performed on the forecast error specification of the VAR model from Section 2.2. The qualitative findings presented above are not altered by these changes. The rest of this paper is dedicated to providing a structural interpretation of our results.

3   The Model

We consider a version of the baseline New Keynesian model without capital, as in Galí (2015). A representative household works, saves, consumes, and owns the firms in the economy. The production side consists of an intermediate goods sector operating under imperfect competition and subject to price rigidities, and a perfectly competitive final goods sector. A central bank conducts monetary policy, and a fiscal authority makes decisions about changes in government spending. A key feature of the model is the presence of variable utilization of the available technology level, as in Bianchi *et al.* (2017). In their model, endogenous variations in total factor productivity (TFP) can arise due to variable technology adoption or R&D investments in "knowledge capital". They find that the endogenous components account for the bulk of fluctuations in TFP growth. Moreover, at business cycle frequencies, endogenous changes in TFP are driven almost exclusively by variable technology adoption, with R&D investments playing an important role only at medium- to long-term frequencies. Given our focus on the business-cycle effects of changes in fiscal policy, we therefore abstract from the accumulation of knowledge capital.

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[9] The commodity price itself displays a decline in response to a government spending shock (not shown).
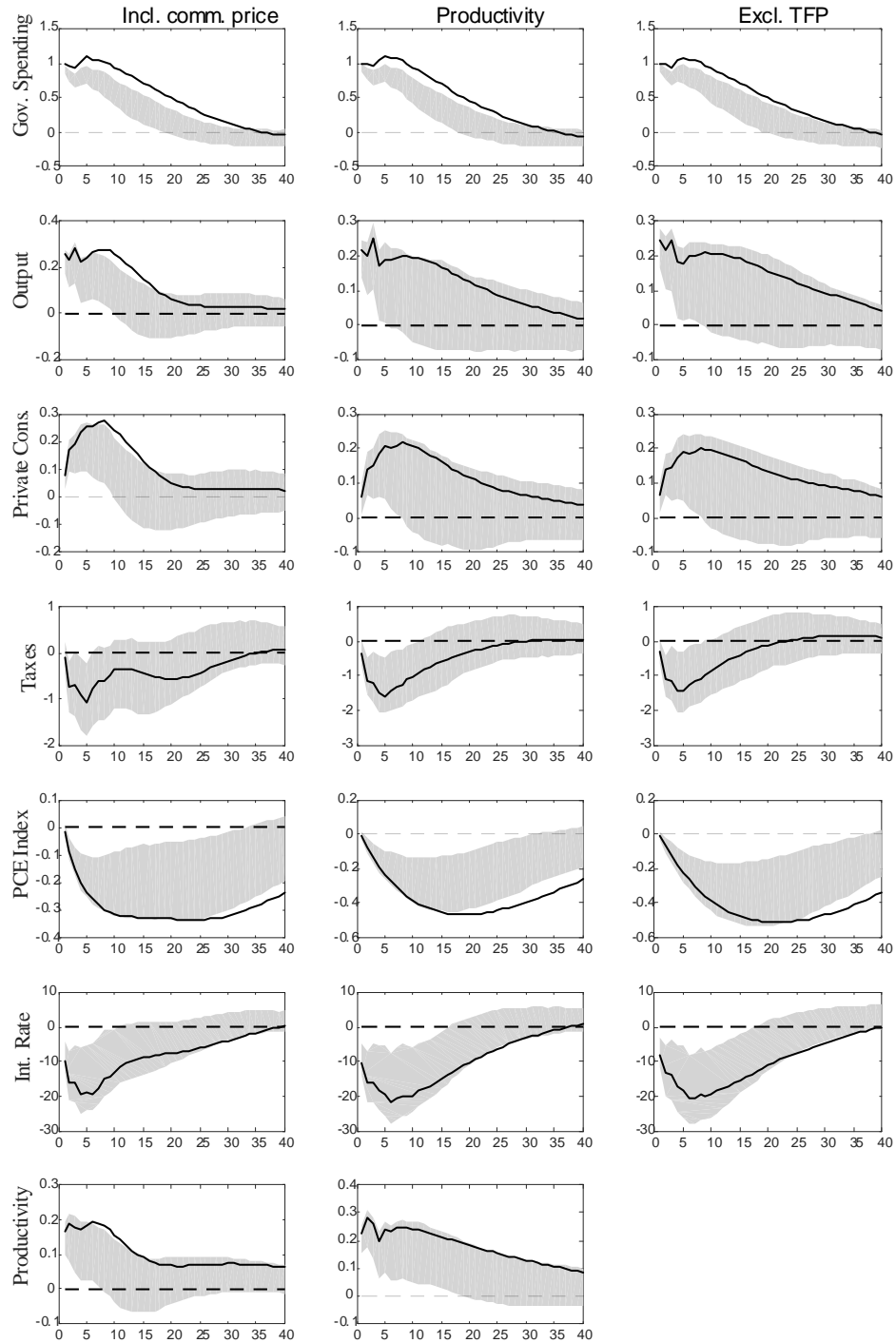
Figure 4: *The dynamic effects of a shock to government spending. Robustness checks: Including commodity prices (first column), alternative productivity measure (second column), excluding TFP from baseline model (third column). Estimates obtained using the Cholesky identification scheme.*

3.1 The Household

The utility function of the representative household takes the following form:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{\psi N_t^{1+\varphi}}{1+\varphi} \right],$$

where C_t and N_t denote non-durable consumption and labor. $\beta \in (0,1)$ is the discount factor, and σ, ψ, and φ are positive parameters. Utility maximization is subject to the following budget constraint:

$$C_t + \frac{R_{t-1}b_{t-1}}{\pi_t} = w_t N_t + b_t + d_t - t_t,$$

where $\pi_t \equiv \frac{P_t}{P_{t-1}}$ denotes the rate of inflation in the price of consumption goods P_t, b_t is one-period debt at the nominal interest rate R_t, w_t is the real wage, d_t is real profits from firms, and t_t is a lump-sum tax. The household chooses C_t, N_t, and b_t, and the associated first-order conditions can be stated as:

$$\psi N_t^\varphi = w_t C_t^{-\sigma}, \tag{2}$$

$$C_t^{-\sigma} = \beta \mathrm{E}_t \frac{R_t C_{t+1}^{-\sigma}}{\pi_{t+1}}. \tag{3}$$

3.2 Final Goods Producers

There is a perfectly competitive sector of final goods producers, who purchase goods from different intermediate goods producers, bundle them together, and sell them to the household or the government. Final goods producers have the following production function:

$$Y_t = \left(\int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} \, di \right)^{\frac{\varepsilon}{\varepsilon-1}}, \quad \varepsilon > 1,$$

where Y_t is aggregate production of the final good, and $Y_t(i)$ denotes the amount produced by individual firm i in the intermediate goods sector. The cost-minimization problem of the representative final goods firm gives rise to the following demand for intermediate good i:

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon} Y_t, \tag{4}$$

where $P_t(i)$ is the price of good i, and where ε thus represents the elasticity of substitution between different intermediate goods.

3.3 Intermediate Goods Producers

There is monopolistic competition in the intermediate goods sector. Individual firm i produces
according to the following production function:

$$Y_{it} = V_{it} N_{it}^{1-\alpha}, \tag{5}$$

where $\alpha \geq 0$, so as to allow for decreasing or constant returns to scale in labor. N_{it} is the
amount of labor hired by firm i, and V_{it} is the level of *utilized* technology. In turn, this is given
by:

$$V_{it} = u_{it} A_t, \tag{6}$$

where u_{it} denotes the firm-specific utilization rate, and A_t is the economy-wide and exogenous
level of technology, which grows deterministically at the rate $\lambda_A \geq 0$:

$$A_t = (1 + \lambda_A) A_{t-1}. \tag{7}$$

We let each firm decide on the rate at which it wishes to utilize the available technology in
society. As in Bianchi *et al.* (2017), technology utilization may be interpreted as a measure of
the capacity of the firm to adopt new knowledge or inventions into the production setup. As
new inventions arrive, each firm needs to exert an effort to internalize this new technology. By
endogenizing the rate of technology adoption, we allow firms to choose when to make this effort,
subject to an adjustment cost whenever u_{it} differs from its steady-state level, denoted u. We
thus assume that it is costly for a firm to fully adopt new inventions into their production process
as they arrive, for example because employees must be trained in using the new technology.
We let the function $z(u_{it})$ denote the adjustment costs associated with the choice of u_{it}. As in
Bianchi *et al.* (2017), this function satisfies $z(u) = 0$, i.e., adjustment costs are zero in steady
state. We also require $z'(u) > 0$ and $z''(\cdot) > 0$. Further, in line with the literature on variable
utilization of capital (e.g., Christiano *et al.*, 2005), we assume that $u = 1$. As we shall see, this
choice pins down $z'(1)$. The curvature parameter $z''(\cdot)$ emerges as a measure of how quickly
adjustment costs rise with changes in the rate of technology utilization.[10]

Each firm chooses labor inputs N_{it} and technology utilization u_{it} so as to minimize its costs
subject to (5). This gives rise to the following first-order conditions:

$$w_t = (1 - \alpha) mc_{it} \frac{Y_{it}}{N_{it}}, \tag{8}$$

$$z'(u_{it}) = mc_{it} \frac{Y_{it}}{u_{it}}, \tag{9}$$

where mc_{it} is the multiplier associated with (5), and represents the real marginal cost of pro-

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[10] The only characteristic of the function z affecting the steady state is $z'(1)$. Moreover, as in Christiano *et
al.* (2005), only the ratio $\frac{z''(\cdot)}{z'(1)}$ affects the dynamics of our model outside steady state.

duction. (8) sets the real wage equal to the marginal product of labor, while (9) says that the marginal cost of higher utilization, given by the increase in adjustment costs $z'(u_{it})$, must equal the marginal product of a higher utilization rate. The utilization rate of technology affects the marginal cost in two ways: On one hand, a higher rate of utilization allows the firm to increase production for given inputs of labor, effectively working like an increase in productivity. On the other hand, higher utilization is costly. As we show later, the former effect will typically dominate, so that a higher utilization rate reduces the marginal cost. In response to a government spending shock, this effect may even be strong enough to overcome the increase in the wage rate, thus paving the way for an equilibrium decline in the marginal cost and, as a consequence, inflation.

When setting their price, intermediate goods firms are subject to a nominal rigidity in the form of quadratic price adjustment costs, as in Rotemberg (1982). Adjustment costs Υ_{it} are scaled by nominal output, and take the following form:

$$\Upsilon_{it} = \frac{\gamma}{2} \left(\frac{P_{it}}{P_{it-1}} - 1 \right)^2 P_t Y_t,$$

where $\gamma > 0$ measures how costly it is to change prices. Firm i sets its price so as to maximize profits, and this problem can be written in real terms as:

$$\max_{P_{it}} \mathrm{E}_0 \sum_{t=0}^{\infty} q_{t,t+1} \left[\left(\frac{P_{it}}{P_t} - mc_{it} \right) Y_{it} - z(u_{it}) - \Upsilon_{it} \right],$$

subject to the demand function (4). Here, $q_{t,t+1} \equiv \beta \frac{\mathrm{E}_t \lambda_{t+1}}{\lambda_t}$ is the stochastic discount factor of the household, with λ_t denoting the marginal utility of consumption. Upon deriving the first-order condition, we impose a symmetric equilibrium in which all firms charge the same price, allowing us to state the optimality condition as:

$$1 - \varepsilon + \varepsilon mc_t = \gamma(\pi_t - 1)\pi_t - \gamma \mathrm{E}_t \frac{q_{t,t+2}}{q_{t,t+1}} (\pi_{t+1} - 1) \frac{Y_{t+1}}{Y_t} \pi_{t+1}. \tag{10}$$

This condition can be written on log-linearized form as a New Keynesian Phillips Curve.

3.4 Monetary and Fiscal Policy

Fiscal policy is assumed to follow a balanced-budget rule:

$$g_t = t_t, \tag{11}$$

where government spending, g_t, satisfies:

$$\log g_t = (1 - \rho_G) g + \rho_G \log g_{t-1} + \varepsilon_t^G, \tag{12}$$

with the innovation ε_t^G following an i.i.d. normal process with standard deviation σ_G, and where g denotes government spending in steady state, while $\rho_G \geq 0$. The monetary policy rule is specified as follows:

$$\frac{i_t}{i} = \left(\frac{\pi_t}{\pi}\right)^{\phi_\pi}\left(\frac{Y_t}{Y}\right)^{\phi_y},\tag{13}$$

with $\phi_\pi > 1$ and $\phi_y \geq 0$, and where π and Y denote the steady-state levels of inflation and output.

3.5   Market Clearing

Bonds are in zero net supply:

$$b_t = 0.\tag{14}$$

The labor market clears when:

$$\int_0^1 N_{it}di = N_t.\tag{15}$$

Finally, goods market clearing requires:

$$Y_t - z\left(u_{it}\right) - \Upsilon_{it} = C_t + g_t.\tag{16}$$

When solving the model, we detrend all variables to eliminate the trend growth in the level of technology. Considering only symmetric equilibria in which all firms make the same decisions allow us to discard subscript i's. We then log-linearize the equilibrium conditions around the non-stochastic steady state of the model, which is described in Appendix B.1. The log-linearized equilibrium conditions are presented in Appendix B.2.

4   Analytics of the Model

To build intuition on the ability of the model to reproduce our empirical findings–in particular, a decline in inflation and an increase in consumption in response to expansionary fiscal policy– we find it useful to offer some analytical insights. To do this, we make the following simplifying assumptions, all of which are regularly encountered in the existing business cycle literature: We assume constant returns to scale in production ($\alpha = 0$), log utility in consumption ($\sigma = 1$), unitary (inverse) Frisch elasticity of labor supply ($\varphi = 1$), no monetary policy reaction to the output gap ($\phi_y = 0$), and a constant level of technology ($A_t = 1, \forall t$). Under these conditions, the log-linearized version of the model can be reduced to two equations in consumption and inflation (plus an exogenous process for government spending), as we show in Appendix B.3. In fact, letting $\widehat{x}_t$ denote the (log) deviation of a generic variable x_t from its steady-state value
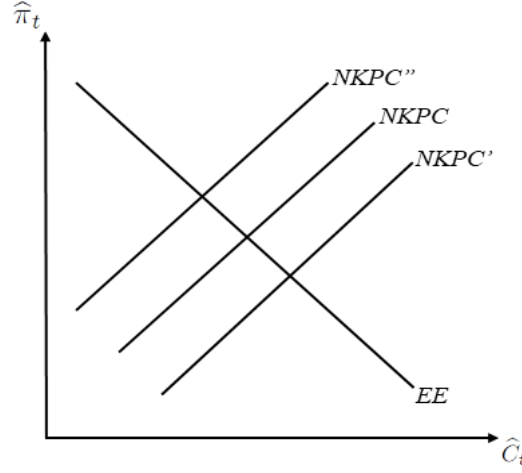
Figure 5: *The Effects of a positive government spending shock. The NKPC'-curve refers to our baseline model, while the NKPC"-curve refers to the basic New Keynesian model without variable technology utilization.*

x, these two equations can be stated as (see Appendix B.3 for details):

$$- \widehat{C}_t \quad = \quad \mathrm{E}_t \left(-\widehat{C}_{t+1} + \phi_\pi \widehat{\pi}_t - \widehat{\pi}_{t+1} \right), \tag{EE}$$

$$\widehat{\pi}_t \quad = \quad \beta \mathrm{E}_t \widehat{\pi}_{t+1} + a \widehat{C}_t - b \widehat{g}_t, \tag{NKPC}$$

where a, b are functions of the deep parameters of the model. We provide necessary and sufficient conditions below for a and b to be strictly positive below. (EE) simply combines the household's Euler equation with the monetary policy rule, while (NKPC) emerges by substitution of the remaining equilibrium conditions into the New Keynesian Phillips Curve. In Figure 5, we provide a graphical representation of the model (EE)-(NKPC) in $(\widehat{C}_t, \widehat{\pi}_t)$-space. (EE) can be represented by a downward-sloping line (this can be seen most clearly in the case of non-persistent shocks, in which case $\mathrm{E}_t \widehat{C}_{t+1} = \mathrm{E}_t \widehat{\pi}_{t+1} = 0$), whereas (NKPC) implies an upward-sloping relationship between the two variables. A positive shock to government spending ($\widehat{g}_t > 0$) shifts the NKPC-curve *down*, leaving the EE curve unaffected. As shown by the curve labelled *NKPC'* in Figure 5, an increase in government spending thus leads to a drop in inflation and an increase in consumption under these conditions, in line with the empirical evidence of Section 2.

We proceed by deriving a closed-form solution of the model, as well as an analytical characterization of the conditions for a unique and determinate solution. We do this under the simplifying assumption that shocks to government spending have no persistence ($\rho_G = 0$). As we show in Appendix B.3 and B.4, the following statements are warranted:

Proposition 1 *The model has a determinate solution (and the parameter a is strictly positive) if and only if the curvature of the cost function associated with changes in the utilization rate of technology is above the following threshold:*

$$z''\left(\cdot\right) > z'\left(1\right)\frac{mc - \frac{g}{Y}}{2 - \frac{g}{Y}}. \tag{17}$$

Proposition 2 *If the model has a unique and determinate solution, it features a decline in inflation along with an increase in consumption on impact in response to a positive shock to government spending (and a strictly positive value of the parameter b) if and only if the curvature of the cost function is below the following threshold:*

$$z''\left(\cdot\right) < z'\left(1\right). \tag{18}$$

Proof. See Appendix B.3 and B.4. ∎

Note that the steady-state value of mc is given by $mc = \frac{\varepsilon-1}{\varepsilon} < 1$. This means that there always exists a range of values for $z''\left(\cdot\right)$ for which both (17) and (18) are satisfied. Our baseline calibration of the next section implies $mc = \frac{5}{6}$, $\frac{g}{Y} = 0.245$, and $z'\left(1\right) = 0.21$. These values produce an admissible range of $z''\left(\cdot\right) \in [0.07, 0.21]$. For all values within this range, the model has a determinate equilibrium featuring, on impact, a drop in inflation and an increase in consumption.

We can explain these requirements as follows: (18) requires that the curvature $z''\left(\cdot\right)$ cannot be *too large*. If $z''\left(\cdot\right)$ is very high, changes to the utilization rate are very costly, so firms will be hesitant to make such changes. In the limiting case of $z''\left(\cdot\right) \to \infty$, firms will choose to never adjust the utilization rate, which will therefore remain constant, exactly as in a model without an endogenous utilization rate. Indeed, we show in Appendix B.5 that for $z''\left(\cdot\right) \to \infty$, the analytical solution to our model collapses to that of a basic New Keynesian model, and that the latter always implies an increase in inflation—driven by the upward movement in the wage rate—along with a decline in consumption when $\widehat{g}_t$ increases. Graphically, this implies that the NKPC-curve is shifted *up*, as illustrated by the curve labelled *NKPC"* in Figure 5.[11] To overturn this, and ensure a positive value of b and a downward shift in the NKPC-curve in Figure 5, it is crucial that the utilization rate is sufficiently responsive, which in turn requires a limited cost of adjusting it.

Conversely, (17) provides a lower bound on the adjustment cost, effectively entailing that the rate of technology utilization cannot be too responsive. If this condition is not met, the model does not have a determinate solution. Intuitively, if the costs associated with changing the utilization rate are sufficiently small, the optimal utilization rate may tend to infinity in response to an expansionary shock. Thus, the adjustment cost function needs to display a

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[11]The basic New Keynesian model—subject to the same parameter restrictions as our model—features an Euler equation identical to (EE), and a rewritten New Keynesian Phillips Curve of the same form as (NKPC), but where the parameter in front of $\widehat{g}_t$ is strictly negative. See Appendix B.5 for details.

certain degree of curvature for the costs to increase sufficiently with the utilization rate and
contain the movements in the latter. In terms of the graphical representation in Figure 5, (17)
ensures an upward-sloping NKPC curve, which is necessary for the model to have a determinate
equilibrium.[12]

The analysis above establishes some general conditions under which our model is able to
generate impact multipliers in line with the empirical evidence from Section 2. Effectively,
our mechanism works much like an increase in the level of technology—in fact, it produces an
increase in *measured* TFP (V_t), as we shall see below. The decline in marginal costs induces
firms to reduce their prices, thus generating a decline in inflation. Simultaneously, the increase
in technology utilization boosts the household's permanent income, all else equal, through
increases in both the marginal product of labor and firm profits. Again, this resembles the
effects of a technology shock. In addition, the central bank engineers a decline in the nominal
and real interest rate, which further stimulates consumption. These insights carry through to
the next section, where we lift some of the simplifying assumptions made in this section and
resort to numerical analyses of the quantitative implications of our model.[13]

5 Dynamic Effects of a Government Spending Shock

In this section, we use model simulations to study the effects of a government spending shock
beyond the quarter in which the shock hits the economy. To this end, we assign realistic values
to all parameters of the model, and study the implied impulse-responses. We also offer a set of
robustness checks regarding certain key parameters.

5.1 Calibration

The baseline calibration of the model is as follows: We set $\beta = 0.99$, implying an annualized
real interest rate of 4% in steady state. We set the coefficient of relative risk aversion to
$\sigma = 2$, in line with microeconometric estimates (see, e.g., Attanasio and Weber, 1995). As in
Christiano *et al.* (2005), we maintain the assumption from the previous section of an (inverse)
Frisch elasticity of labor supply of unity; $\varphi = 1$. This value represents a compromise between
microeconometric studies–where 1 can be regarded as an upper bound; see Chetty *et al.* (2011)–

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[12]Interestingly, this type of constraint does not arise in models featuring variable utilization of the capital
stock. In those models, adjustment costs will be tied to the rental rate of capital in equilibrium: capital
producers will never find it optimal to raise the utilization rate to a level at which the associated adjustment
costs outweigh the rental rate earned on utilized capital (see, e.g., Smets and Wouters, 2007). In our setup,
instead, the utilization decision regards a production "factor" which is intrinsically free to use; the level of
technology in society. The only cost of utilizing technology comes from the adjustment costs, motivating the
presence of a lower bound on these.

[13]A final insight can be obtained from the simple model above: If we were to introduce a monetary policy
shock into the model, the shock would appear in (EE), but not in (NKPC). A contractionary monetary policy
shock would shift the EE-curve down along the NKPC-curve, generating a decline in inflation. In other words,
our model is not able to account for the "price puzzle" of monetary policy (Sims, 1992). Intuitively, a government
spending shock affects inflation directly, whereas a monetary policy shock only affects inflation indirectly through
changes in consumption/saving decisions.

and macroeconomic models, where values above 1 are not uncommon (see, e.g., Hall, 2009). The weight on disutility of labor hours in the utility function, ψ, is calibrated so that the household works 1/4 of her time endowment. On the production side, we follow most of the literature and set $\varepsilon = 6$, implying a steady-state markup of 20 percent. We maintain the assumption of constant returns to scale ($\alpha = 0$) in our baseline analysis, and then study the case of decreasing returns to scale in Section 5.3.3. The adjustment cost associated with price changes is calibrated so that a given price is changed, on average, every 3 quarters, consistent with microeconometric evidence reported by Nakamura and Steinsson (2008). Given the other parameters, this implies a value of $\gamma = 29.41$.

Regarding the policy-related parameters, we follow most of the literature in setting the steady-state inflation rate to zero. The policy response parameters are set in accordance with the original proposal of Taylor (1993), i.e., we set $\phi_\pi = 1.5$ and $\phi_y = 0.125$ (0.5 divided by 4). To calibrate the persistence of government spending shocks, we fit an AR(1)-process to the impulse-response of government spending reported in Figure 1. This yields a value of $\rho_G = 0.955$, slightly higher than that used in Galí *et al.* (2007), who set this parameter to 0.9.[14] The ratio of government spending to output in the model matches the sample average in the data for the period 1960-2017, which equals $\frac{g}{Y} = 0.245$.

Finally, we need to specify and parametrize the functional form of the adjustment cost associated with changes in the technology utilization rate. We assume that adjustment costs are given by:

$$z\left(u_t\right) = \chi_1 \left(u_t - u\right) + \frac{\chi_2}{2} \left(u_t - u\right)^2, \tag{19}$$

where $\chi_1, \chi_2 > 0$, and where $u = 1$ again denotes the steady-state level of u_t. This implies that $z'\left(u_t\right) = \chi_1 + \chi_2 \left(u_t - u\right)$. As already described, we calibrate the value of $z'\left(1\right) = \chi_1$ to ensure that the rate of utilization equals 1 in steady state. This returns a value of $\chi_1 = 0.21$. The curvature parameter $z''\left(\cdot\right) = \chi_2$ is harder to pin down. Conditional on our baseline calibration of all other parameters, the "admissible range" of values for this parameter established analytically in Section 4 changes slightly: For any value of $\chi_2 \in [0.03, 0.15]$, we obtain responses of inflation and consumption in line with the data. In the simulations below, we pick a baseline value of $\chi_2 = 0.05$, while our robustness checks shed more light on the quantitative importance of this parameter.[15]

5.2 Impulse-Response Analysis

Given our baseline calibration, Figure 6 displays the impulse-responses of the model to a government spending shock equal to 1 percent of steady-state output (solid blue lines). As the figure illustrates, the shock leads to an increase in the rate of technology utilization. This is

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[14]All main results reported below are confirmed if we set $\rho_G = 0.9$ instead.

[15]In comparison, Bianchi *et al.* (2017) estimate a value of $z''\left(\cdot\right) = 0.0033$ in their model, but do not report their calibrated value of $z'\left(1\right)$. As noted above, only the ratio between these two affects the dynamics of our model.
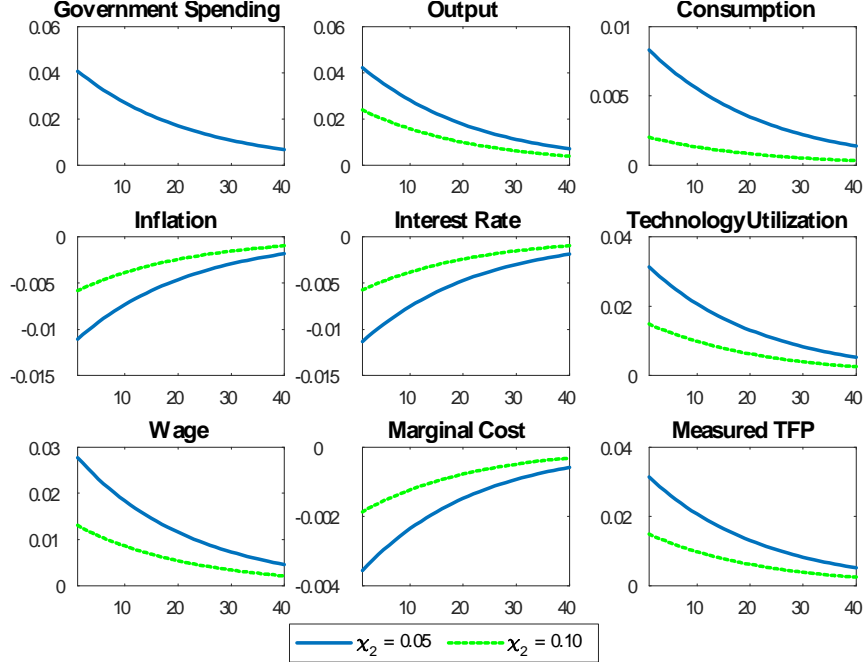
Figure 6: *Impulse-responses of key variables to a positive government spending shock equal to 1 percent of steady-state output. Solid blue lines: baseline model with* $\chi_2 = 0.05$. *Dotted green lines: alternative model with* $\chi_2 = 0.1$.

sufficient to generate a decline in marginal cost, despite the increase in the wage rate. As a consequence, inflation drops. In line with the intuition traced out in the previous section, consumption increases, thus amplifying the increase in output. As we discuss below, the increase in output now enters the central bank's reaction function, which–all else equal–weakens the inverse relationship between consumption and inflation. However, the nominal interest rate is reduced due to the drop in inflation, reducing also the real interest rate, which stimulates consumption. The negative response of the nominal interest rate is in line with the empirical evidence from Section 2.[16] Furthermore, and also in line with the data, we observe an increase in "Measured TFP" as given by the *utilized* technology level, V_t. In the absence of exogenous technology shocks, this variable moves one-for-one with the utilization rate.

5.3   Sensitivity Analysis

Having established that a realistically parametrized version of our model is able to reproduce the empirical evidence presented in Section 2, this subsection explores the robustness of our findings with respect to some of our key parameter values and modeling choices.

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[16]Additionally, the positive response of the wage rate, as well as that of labor hours (not reported), while not included in our VAR model, are in line with existing empirical evidence. Among others, Galí *et al.* (2007) and Andres *et al.* (2015) report that wages and hours both rise in response to an increase in government spending.

5.3.1 Movements in the Technology Utilization Rate

Given the uncertainty surrounding the cost of changing the rate of technology utilization, it is worth pointing out that we do not require dramatic changes in the utilization rate to obtain a decline in inflation: Under our baseline calibration, the utilization rate increases by around 3 percent; somewhat less than the increase in output. This is similar to the behavior of the utilization rate of capital in Christiano *et al.* (2005), which moves slightly less than 1-for-1 with output in the data and in their model. To shed some additional light on the robustness of our findings, the dotted green lines in Figure 6 show the corresponding impulse-responses after changing the value of χ_2 from 0.05 to 0.1. In this case, the utilization rate increases only by around 1.5 percent on impact, which is again somewhat less than the increase in output. In this case, inflation and consumption still behave in accordance with the empirical evidence, but now display much smaller changes. This shows that even relatively small movements in the utilization rate are sufficient to obtain the desired impulse-responses. While data on technology utilization is not readily available, Bianchi *et al.* (2017) argue that their model-implied rate of technology utilization is closely correlated with data on the software expenditures of firms; one potential measure of technology adoption. We have verified that this correlation also emerges conditional on a government spending shock: When we include software expenditures in our baseline VAR model of Section 2, we observe a significant increase in this variable after an increase in government spending (see details in Appendix A.1).

5.3.2 The Role of Monetary Policy

The stance of monetary policy plays a key role in the transmission of fiscal policy. At the heart of the negative relationship between inflation and consumption implied by (EE) are movements in the real interest rate: Consumption is bound to increase if and only if the central bank engineers a decline in the real interest rate upon observing a drop in inflation. This, in turn, requires a sufficiently strong reaction of the nominal policy rate to a given change in inflation. We now characterize the requirements that monetary policy must meet in order for our model to match, at least from a qualitative viewpoint, the empirical evidence.

Figure 7 shows the behavior of our model as a function of the parameters in the monetary policy rule (13), keeping all other parameters at their baseline calibration. For low values of ϕ_π and ϕ_y, as illustrated by the blue area, the model does not have a unique and stable equilibrium given our baseline calibration.[17] As also shown analytically in Appendix B.4, a version of the Taylor principle of standard New Keynesian models holds up in our model: To ensure a unique and stable solution, monetary policy must be sufficiently responsive. When this condition is satisfied, the ratio $\frac{\phi_\pi}{\phi_y}$ must be sufficiently high to ensure that the model produces the desired responses. For relatively high values of ϕ_π, the decline in inflation associated with an increase in government spending leads to a reduction in the nominal and real interest rate, and thus an

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[17]For a given combination of ϕ_π and ϕ_y, there may exist different combinations of the other parameters of the model (in particular of χ_2) for which a unique, stable solution is restored, cf. the discussion in Section 4.
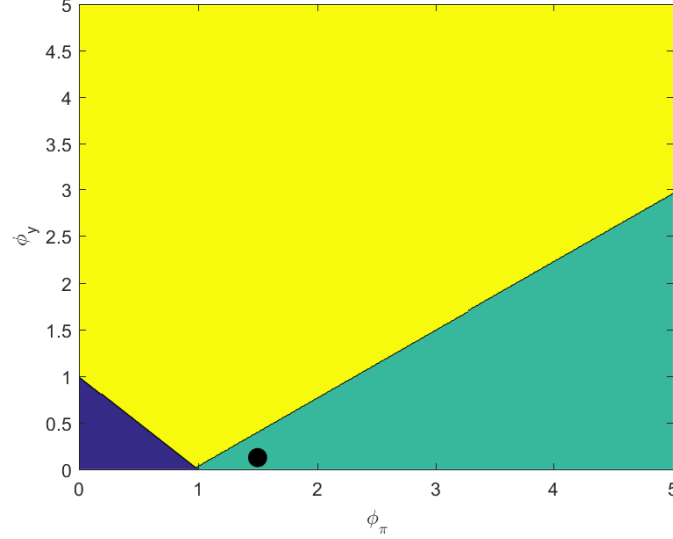
Figure 7: *Model outcomes for different combinations of monetary policy parameters. Blue area: No unique and determinate solution. Yellow area: Model fails to generate a decline in inflation and/or an increase in consumption on impact. Green area: Model generates a decline in inflation and an increase in consumption on impact. The black dot indicates our baseline calibration.*

increase in consumption. Given the empirical responses of private consumption and the nominal interest rate documented in Section 2, this case appears to be the most realistic. Instead, when ϕ_y is relatively high, the increase in the output gap induces the central bank to raise the policy rate (or reduce it by less), so that the real interest rate increases, stimulating savings instead of consumption. In the figure, the green area indicates combinations of policy parameters for which the model produces an increase in consumption and a decline in inflation on impact, while the yellow area indicates combinations where either of these does not obtain (i.e., consumption fails to increase). The black dot denotes our baseline calibration; falling well within the green area.

5.3.3 Decreasing Returns to Scale

So far in our analysis, we have assumed a constant-returns-to-scale technology in the intermediate goods sector. This assumption facilitates a decline in inflation. If instead there are decreasing returns to scale ($\alpha > 0$), a given increase in production requires a larger increase in labor inputs, thus driving up marginal costs, which–all else equal–makes it harder for the technology utilization rate to bring about a decline in marginal costs in equilibrium. It is therefore important to verify that our proposed mechanism can reproduce the empirical evidence even in the case of decreasing returns to scale. To this end, the dotted green lines in Figure 8 show
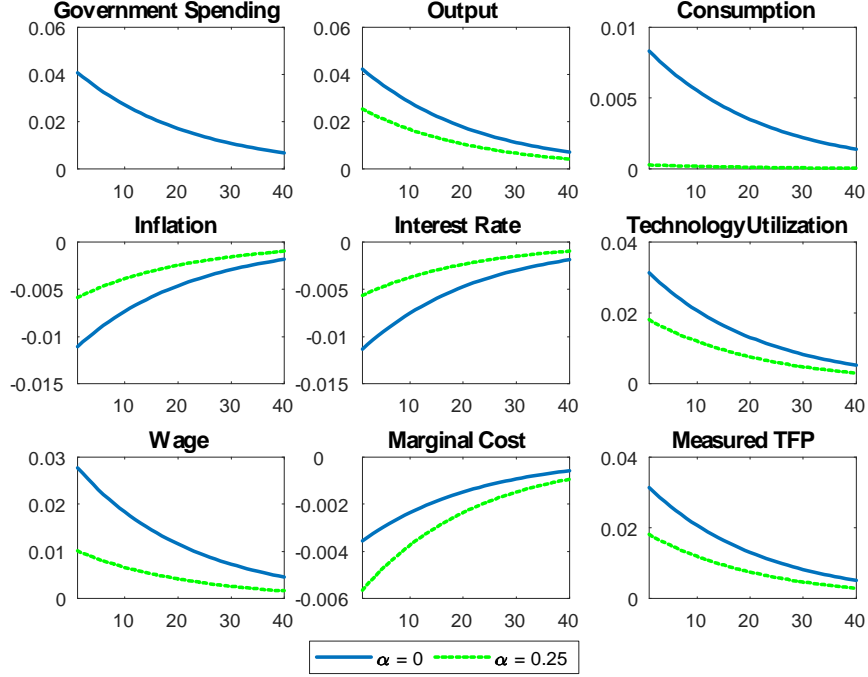
Figure 8: *Impulse-responses of key variables to a positive government spending shock equal to 1 percent of steady-state output. Solid blue lines: baseline model with constant returns to scale ($\alpha = 0$). Dotted green lines: alternative model with decreasing returns to scale ($\alpha = 0.25$).*

impulse-responses from our model under the assumption that $\alpha = 0.25$, as in Galí (2015).[18] The solid blue lines display our benchmark model for comparison. As can be seen, our main result survives, as the model is still able to generate a drop in inflation alongside a (very small) increase in consumption. However, from a quantitative viewpoint, the movements in these variables are substantially smaller than those reported in Figure 6, reflecting that our mechanism of variable technology adoption has less quantitative bite in this case.

6   Conclusion

We have presented empirical evidence that inflation tends to drop in response to increases in government spending in the U.S. economy. This result is robust across a range of different empirical specifications, as well as across price indices. It emerges alongside the increase in output and private consumption documented in previous studies, as well as an increase in TFP. To explain these findings, we have proposed a model of variable technology utilization in the spirit of Bianchi *et al.* (2017) and the related literature on the role of endogenous changes in productivity over the business cycle. We have documented that a realistically calibrated

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[18]This moves the "admissible range" of values for χ_2, which has therefore been changed to 0.025 in the simulations reported in Figure 8. In addition, the calibrated parameters ψ, γ, and χ_1 are automatically adjusted so as to ensure that our calibration targets are maintained.

version of this model can account for the response of a set of key macroeconomic variables to a government spending shock.

Our findings challenge some widely held beliefs about the transmission mechanism of fiscal policy. As we have argued in this paper, the empirical support for a crucial building block of many theoretical accounts of fiscal policy–the assumption that increases in government spending are inflationary–is underwhelming at best. In future work, we plan to extend the study to other countries than the US.

From a modeling viewpoint, we envisage several avenues for further research. In future work, we are planning to augment the model with physical capital to verify that our results survive in this more realistic setting. We also plan to estimate the parameters of the model using impulse-response matching. Finally, it would be interesting to study how variable technology utilization affects the transmission mechanism of additional types of macroeconomic shocks.

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Appendices

A The Data

All data used in the baseline specification – with the exception of total factor productivity (TFP) –
are taken from Federal Reserve Economic Data (FRED). The series are described in detail below with
series names in FRED indicated in brackets:

G_t: Government consumption expenditure and gross investment (GCECE1, seasonally adjusted,
Chained 2009 $).

Y_t: Real GDP (GDPC1, seasonally adjusted, Chained 2009 $).

C_t: Real Personal Consumption Expenditures (PCECC96).

T_t: Government current tax receipts (W054RC1Q027SBEA, seasonally adjusted) - Government
current transfer receipts (A084RC1Q027SBEA, seasonally adjusted) - Government interest payments
(A180RC1Q027SBEA, seasonally adjusted) - Government subsidies (GDISUBS, seasonally adjusted).
We convert from nominal to real terms using the GDP deflator (see below).

P_t: Personal Consumption Expenditures Price Index (PCECTPI, seasonally adjusted, 2009=100).

R_t: Nominal interest rate on 3-month Treasury Bills (TB3MS).

A_t: Raw Total Factor Productivity series constructed by the Federal Reserve Bank of San Francisco
based on the methodology of Fernald (2014).[19]

The first four series are converted to per capita terms using the Census Bureau Civilian Population
(All Ages) estimates, which we collect from the FRED database (POP). We take logs to all variables
except the interest rate, R_t.

In addition, we use the following series from the FRED database for the robustness checks:

CPI index: Consumer Price Index for All Urban Consumers: All Items (CPIAUCSL, seasonally
adjusted, 2009=100).

PCE Core index: Personal Consumption Expenditures Excluding Food and Energy Price Index
(PCEPILFE, seasonally adjusted, 2009=100).

GDP deflator index: Gross Domestic Product: Implicit Price Deflator (GDPDEF, seasonally adjusted, 2009=100).

Commodity price index: Producer Price Index for All Commodities (PPIACO, not seasonally adjusted, 2009=100).

Productivity: Real Output per Hour of All Persons in the Nonfarm Business Sector (OPHNFB,
seasonally adjusted, 2009=100).

A.1 Additional Robustness Checks

We present an additional set of robustness checks here:[20]

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[19]The data can be collected from https://www.frbsf.org/economic-research/indicators-data/total-factor-productivity-tfp/

[20]The alternative ordering used in some of the robustness checks below is the following: $\mathbf{X}_t =
\begin{bmatrix} FE_t & G_t & Y_t & C_t & T_t & P_t & R_t & T_t \end{bmatrix}'$. We have experimented with other alternative orderings, in particular regarding the placement of the price index. This did not lead to any changes in our results.
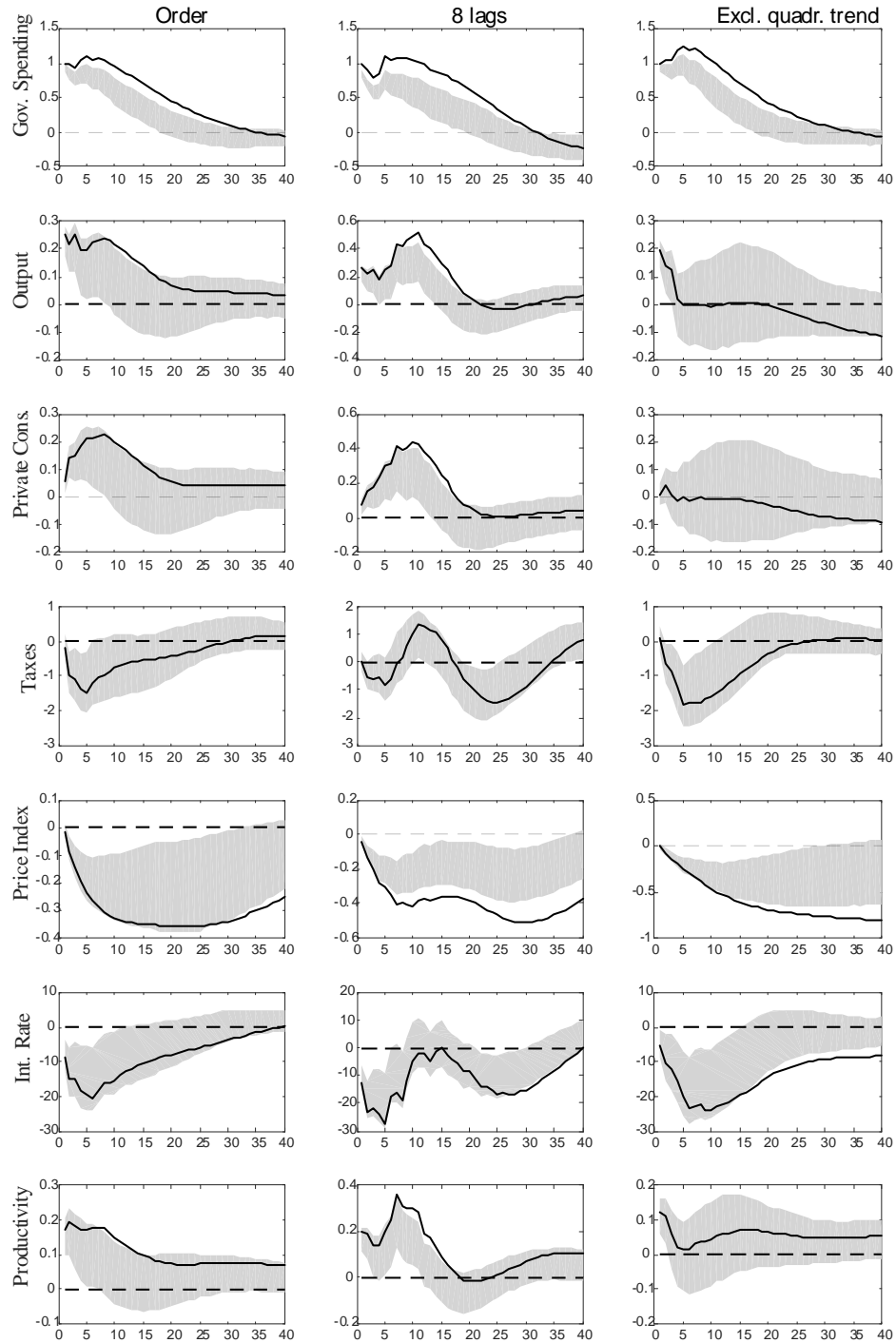
Figure A.1: *The dynamic effects of a shock to government spending. Robustness checks: Alternative ordering of variables (first column), model with 8 lags instead of 4 (second column), model excluding the quadratic trend (third column). Estimates obtained using the Cholesky identification scheme.*
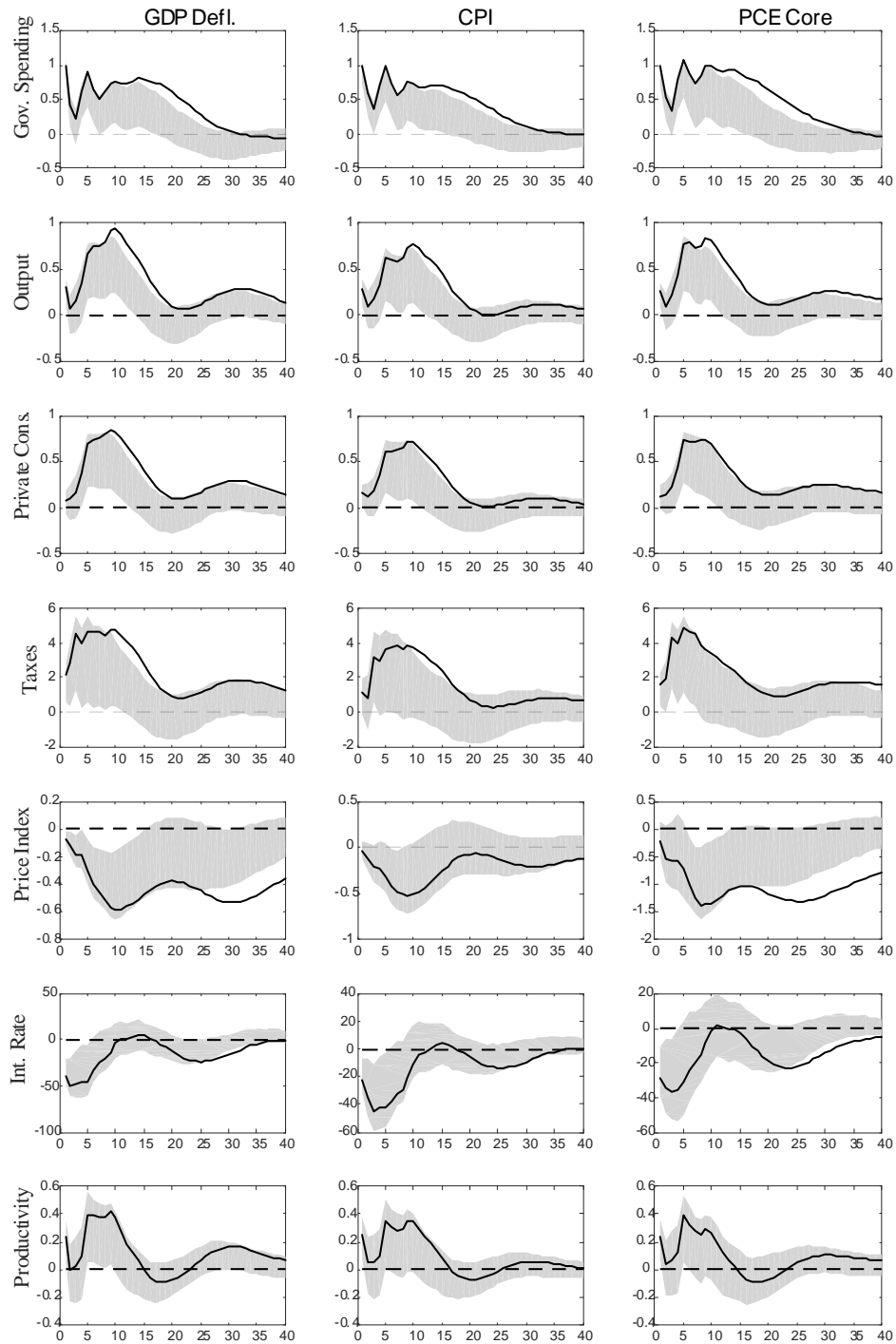
Figure A.2: *The dynamic effects of a shock to government spending. Robustness checks: Different price indices: GDP deflator (first column), CPI index (second column), PCE core price index (third column). Estimates obtained using the identification scheme based on forecast errors.*
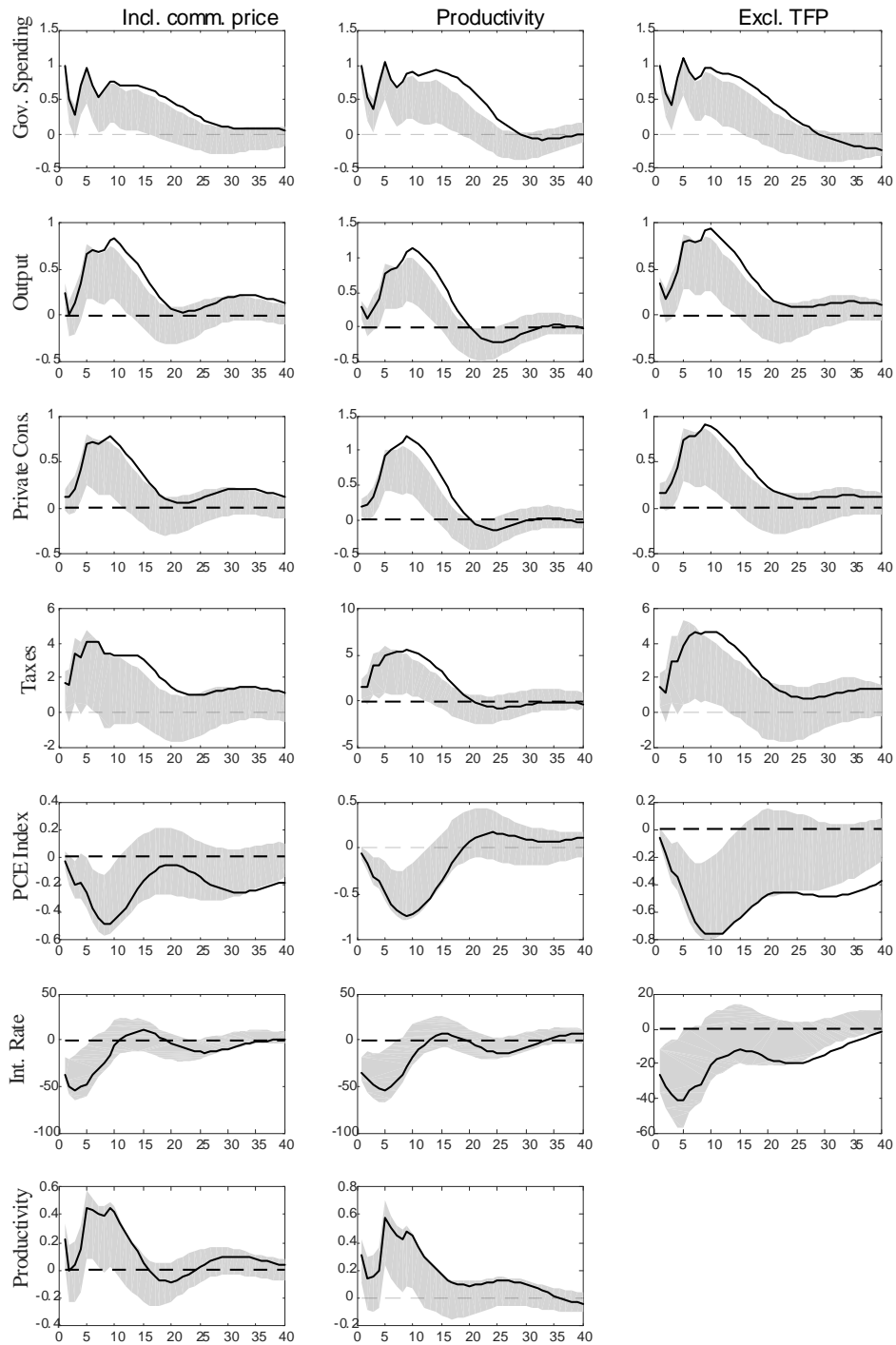
Figure A.3: *The dynamic effects of a shock to government spending. Robustness checks: Robustness checks: Including commodity prices (first column), alternative productivity measure (second column), excluding TFP from baseline model (third column). Estimates obtained using the identification scheme based on forecast errors.*
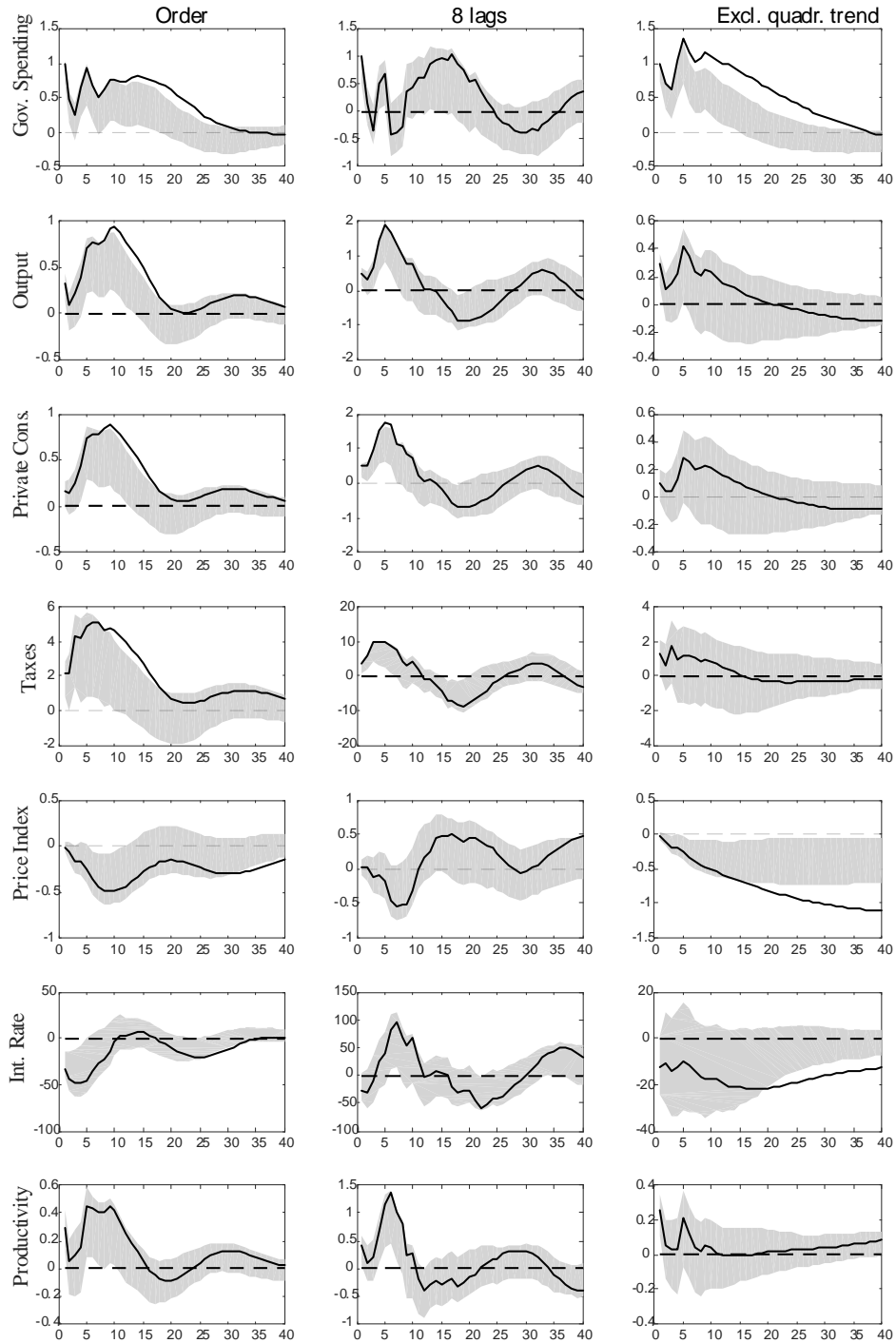
Figure A.4: *The dynamic effects of a shock to government spending. Robustness checks: Alternative ordering of variables (first column), model with 8 lags instead of 4 (second column), model excluding the quadratic trend (third column). Estimates obtained using the identification scheme based on forecast errors.*

Figure A.5: *The dynamic effects of a shock to government spending. Model augmented with software expenditure. Estimates obtained using the Cholesky identification scheme.*

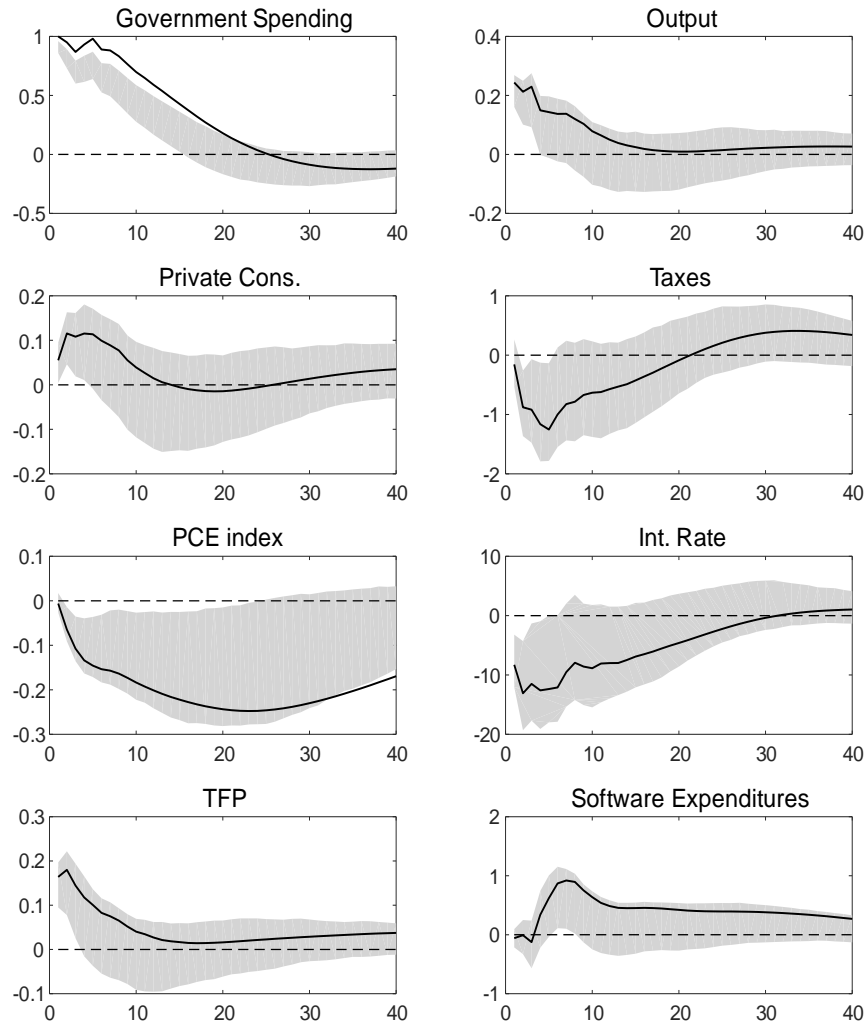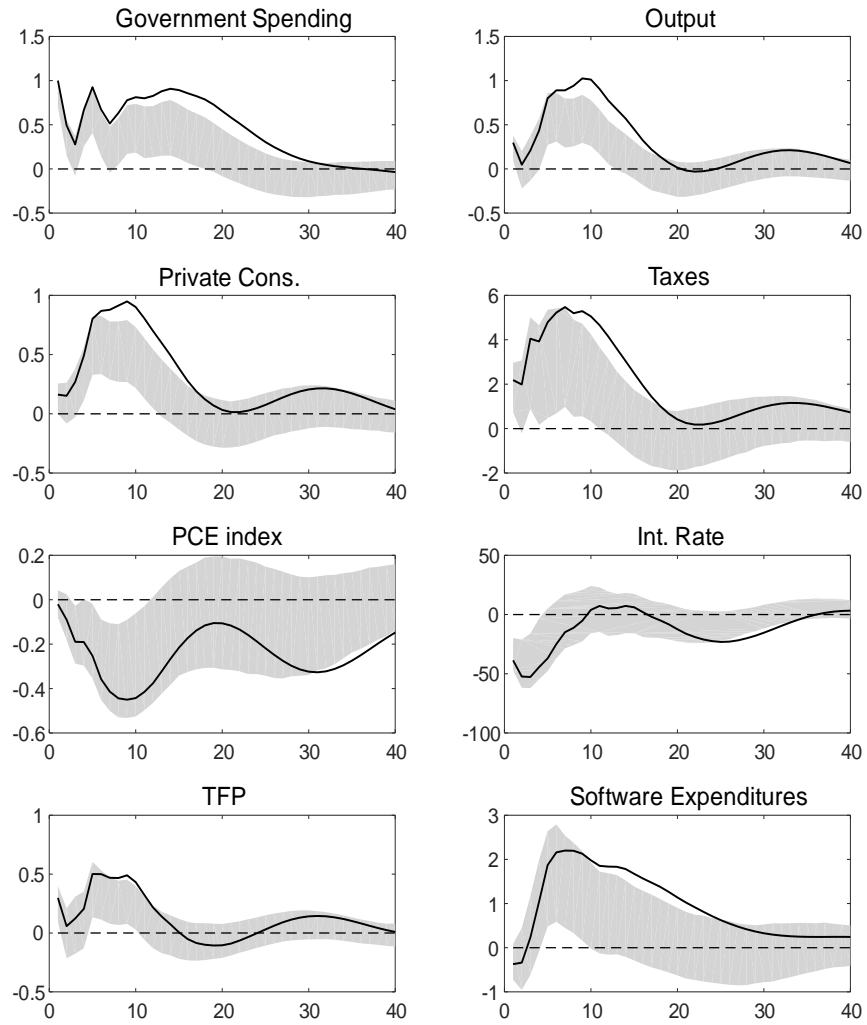Figure A.6: *The dynamic effects of a shock to government spending. Model augmented with software expenditure. Estimates obtained using the identification scheme based on forecast errors.*

B The Model

This appendix presents the details of our model of variable technology utilization. We impose the functional form of $z\left(u_t\right)$ proposed in (19) throughout the appendix.

B.1 The Steady State

As usual, the steady-state interest rate is pinned down by the inverse of the household's discount factor; $R = 1/\beta$. From the optimal price setting of intermediate goods firms (10), we obtain $mc = \frac{\varepsilon-1}{\varepsilon}$. From the goods market clearing condition (16), we get:

$$\frac{C}{Y} = 1 - \frac{g}{Y},$$

where $\frac{g}{Y}$ is determined exogenously. Steady-state production is pinned down from (5):

$$Y = uAN^{1-\alpha},$$

where A is exogenous, u is fixed at 1 in steady state, at N is fixed so that households work 25 percent of their time endowment. Combining labor supply (2) and labor demand (8), and using the production function, we can find the value of ψ that ensures this:

$$\psi N^\varphi = C_t^{-\sigma}\left(1-\alpha\right)mc\frac{Y}{N} \Leftrightarrow$$
$$\psi N^\varphi = C^{-\sigma}\left(1-\alpha\right)mcuAN^{-\alpha} \Leftrightarrow$$
$$\psi = \frac{\left(1-\alpha\right)mcuA}{N^{\varphi+\alpha}C^\sigma}.$$

Finally, to ensure that the utilization rate equals 1 in steady state, we rewrite (9) to get:

$$z'\left(1\right) = mc\frac{Y}{u} \Leftrightarrow$$
$$\chi_1 = mc\frac{Y}{u},$$

which pins down the required value of χ_1. This completes the characterization of the steady state.

B.2 Log-linearized Model

Before simulating the model, we log-linearize it around the non-stochastic steady state. Letting $\widehat{x}_t$ denote the log deviation of a generic variable x_t from its steady-state value x, we obtain the following set of log-linearized equilibrium conditions:

$$\varphi\widehat{N}_t = -\sigma\widehat{C}_t + \widehat{w}_t, \tag{B.1}$$

$$-\sigma\widehat{C}_t = \mathrm{E}_t\left(-\sigma\widehat{C}_{t+1} + \widehat{R}_t - \widehat{\pi}_{t+1}\right), \tag{B.2}$$

$$\widehat{Y}_t = \widehat{u}_t + \widehat{A}_t + \left(1-\alpha\right)\widehat{N}_t, \tag{B.3}$$

$$\frac{C}{Y}\widehat{C}_t = \widehat{Y}_t - mc\widehat{u}_t - \frac{g}{Y}\widehat{g}_t, \tag{B.4}$$

$$\frac{\chi_2}{\chi_1}\widehat{u}_t = \widehat{mc}_t + \widehat{Y}_t - \widehat{u}_t, \tag{B.5}$$

$$\widehat{mc}_t = \widehat{w}_t - \widehat{u}_t - \widehat{A}_t + \alpha\widehat{N}_t, \tag{B.6}$$

$$\widehat{\pi}_t = \beta\mathrm{E}_t\widehat{\pi}_{t+1} + \frac{\varepsilon-1}{\gamma}\widehat{mc}_t, \tag{B.7}$$

$$\widehat{R}_t = \phi_\pi\widehat{\pi}_t + \phi_y\widehat{y}_t, \tag{B.8}$$

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$$\widehat{g}_t = \rho_G \widehat{g}_{t-1} + \varepsilon_t^G, \tag{B.9}$$

$$\widehat{A}_t = \rho_A \widehat{A}_{t-1} + \varepsilon_t^A. \tag{B.10}$$

We thus have a system of 10 equations in 10 variables: $\widehat{Y}_t, \widehat{C}_t, \widehat{g}_t, \widehat{\pi}_t, \widehat{mc}_t, \widehat{u}_t^A, \widehat{w}_t, \widehat{A}_t, \widehat{N}_t, \widehat{R}_t$.

B.3   Analytical Solution

As described in the main text, we derive the analytical solution to the model under the following simplifying assumptions: No technology shocks ($\widehat{A}_t = 0$), constant returns to scale in production ($\alpha = 0$), log utility in consumption ($\sigma = 1$), unitary (inverse) Frisch elasticity of labor supply ($\varphi = 1$), and no monetary policy reaction to the output gap ($\phi_y = 0$). Under these assumptions, it is straightforward to verify that (B.2) and (B.8) can be combined to obtain the Euler equation presented in Section 4:

$$-\widehat{C}_t = \mathrm{E}_t\left(-\widehat{C}_{t+1} + \phi_\pi \widehat{\pi}_t - \widehat{\pi}_{t+1}\right). \tag{B.11}$$

To arrive at the New Keynesian Phillips Curve studied in Section 4, we begin by combining (B.1) and (B.3) to obtain:

$$\widehat{Y}_t = \widehat{u}_t - \widehat{C}_t + \widehat{w}_t.$$

This expression can be inserted twice, into (B.4) and (B.5), to obtain:

$$\frac{C}{Y}\widehat{C}_t = \widehat{u}_t - \widehat{C}_t + \widehat{w}_t - mc\widehat{u}_t - \frac{g}{Y}\widehat{g}_t \Leftrightarrow$$

$$\widehat{w}_t = \left(1 + \frac{C}{Y}\right)\widehat{C}_t - (1 - mc)\widehat{u}_t + \frac{g}{Y}\widehat{g}_t, \tag{B.12}$$

and

$$\frac{\chi_2}{\chi_1}\widehat{u}_t = \widehat{mc}_t + \widehat{u}_t - \widehat{C}_t + \widehat{w}_t - \widehat{u}_t \Leftrightarrow$$

$$\frac{\chi_2}{\chi_1}\widehat{u}_t = \widehat{w}_t - \widehat{u}_t - \widehat{C}_t + \widehat{w}_t \Leftrightarrow$$

$$2\widehat{w}_t = \left(\frac{\chi_2}{\chi_1} + 1\right)\widehat{u}_t + \widehat{C}_t,$$

where the second-to-last line uses (B.6). We can combine these two expressions:

$$\left(1 + \frac{C}{Y}\right)\widehat{C}_t - (1 - mc)\widehat{u}_t + \frac{g}{Y}\widehat{g}_t = \frac{\left(\frac{\chi_2}{\chi_1} + 1\right)\widehat{u}_t + \widehat{C}_t}{2} \Leftrightarrow$$

$$\widehat{u}_t = \frac{\left(1 + \frac{2C}{Y}\right)}{\frac{\chi_2}{\chi_1} + 1 + 2(1 - mc)}\widehat{C}_t + \frac{\frac{2g}{Y}}{\frac{\chi_2}{\chi_1} + 1 + 2(1 - mc)}\widehat{g}_t. \tag{B.13}$$

We are now ready to insert into the original New Keynesian Phillips Curve (B.7), using first (B.6):

$$\widehat{\pi}_t = \beta\mathrm{E}_t\widehat{\pi}_{t+1} + \frac{\varepsilon - 1}{\gamma}\widehat{mc}_t \Leftrightarrow$$

$$\widehat{\pi}_t = \beta\mathrm{E}_t\widehat{\pi}_{t+1} + \frac{\varepsilon - 1}{\gamma}\left(\widehat{w}_t - \widehat{u}_t\right),$$

and then inserting from (B.12):

$$\widehat{\pi}_t = \beta\mathrm{E}_t\widehat{\pi}_{t+1} + \frac{\varepsilon - 1}{\gamma}\left(\left(1 + \frac{C}{Y}\right)\widehat{C}_t - (2 - mc)\widehat{u}_t + \frac{g}{Y}\widehat{g}_t\right),$$

where we can insert from (B.13) to get:

$$
\begin{aligned}
\widehat{\pi}_t &= \beta \mathrm{E}_t \widehat{\pi}_{t+1} + \frac{\varepsilon - 1}{\gamma} \left(\left(1 + \frac{C}{Y} \right) \widehat{C}_t + \frac{g}{Y} \widehat{g}_t \right) \\
&\quad - \frac{\varepsilon - 1}{\gamma} \left(2 - mc \right) \left[\frac{\left(1 + \frac{2C}{Y} \right)}{\frac{\chi_2}{\chi_1} + 1 + 2\left(1 - mc \right)} \widehat{C}_t + \frac{\frac{2g}{Y}}{\frac{\chi_2}{\chi_1} + 1 + 2\left(1 - mc \right)} \widehat{g}_t \right],
\end{aligned}
$$

which can be rewritten as:

$$
\begin{aligned}
\widehat{\pi}_t &= \beta \mathrm{E}_t \widehat{\pi}_{t+1} + \frac{\varepsilon - 1}{\gamma} \left(\frac{\frac{\chi_2}{\chi_1} + 1 + \left(\frac{\chi_2}{\chi_1} - 1 \right) \frac{C}{Y} - mc}{\frac{\chi_2}{\chi_1} + 3 - 2mc} \widehat{C}_t + \frac{\frac{g}{Y} \left(\frac{\chi_2}{\chi_1} - 1 \right)}{\frac{\chi_2}{\chi_1} + 3 - 2mc} \widehat{g}_t \right) \Leftrightarrow \\
\widehat{\pi}_t &= \beta \mathrm{E}_t \widehat{\pi}_{t+1} + a \widehat{C}_t - b \widehat{g}_t,
\end{aligned} \tag{B.14}
$$

after defining

$$
a = \frac{\varepsilon - 1}{\gamma} \frac{\frac{\chi_2}{\chi_1} + 1 + \left(\frac{\chi_2}{\chi_1} - 1 \right) \frac{C}{Y} - mc}{\frac{\chi_2}{\chi_1} + 3 - 2mc}, \tag{B.15}
$$

$$
b = \frac{\varepsilon - 1}{\gamma} \frac{\frac{g}{Y} \left(1 - \frac{\chi_2}{\chi_1} \right)}{\frac{\chi_2}{\chi_1} + 3 - 2mc}. \tag{B.16}
$$

The system (B.11)-(B.14) can be combined with (B.9) to obtain 3 equations in $\widehat{\pi}_t$, $\widehat{C}_t$, and $\widehat{g}_t$. We can solve this system analytically using the method of undetermined coefficients. For expositional simplicity, we assume that the shock to government spending has no persistence ($\rho_G = 0$). We conjecture that the solutions for $\widehat{\pi}_t$ and $\widehat{C}_t$ take the form:

$$
\begin{aligned}
\widehat{C}_t &= \Psi \widehat{g}_t, \\
\widehat{\pi}_t &= \Phi \widehat{g}_t,
\end{aligned}
$$

where the coefficients Ψ and Φ are yet to be determined. Inserting these conjectured solutions into (B.11) and (B.14), we obtain:

$$
\begin{aligned}
-\widehat{C}_t &= \mathrm{E}_t \left(-\widehat{C}_{t+1} + \phi_\pi \widehat{\pi}_t - \widehat{\pi}_{t+1} \right) \Leftrightarrow \\
-\Psi \widehat{g}_t &= \mathrm{E}_t \left(-\Psi \widehat{g}_{t+1} + \phi_\pi \Phi \widehat{g}_t - \Phi \widehat{g}_{t+1} \right) \Leftrightarrow \\
\Psi &= -\phi_\pi \Phi,
\end{aligned}
$$

where we have used that $\mathrm{E}_t \widehat{g}_{t+1} = 0$ when shocks have no persistence. Further, we get:

$$
\begin{aligned}
\widehat{\pi}_t &= \beta \mathrm{E}_t \widehat{\pi}_{t+1} + \frac{\varepsilon - 1}{\gamma} \left(\frac{\frac{\chi_2}{\chi_1} + 1 + \left(\frac{\chi_2}{\chi_1} - 1 \right) \frac{C}{Y} - mc}{\frac{\chi_2}{\chi_1} + 3 - 2mc} \widehat{C}_t + \frac{\frac{g}{Y} \left(\frac{\chi_2}{\chi_1} - 1 \right)}{\frac{\chi_2}{\chi_1} + 3 - 2mc} \widehat{g}_t \right) \Leftrightarrow \\
\Phi \widehat{g}_t &= \beta \mathrm{E}_t \Phi \widehat{g}_{t+1} + \frac{\varepsilon - 1}{\gamma} \left(\frac{\frac{\chi_2}{\chi_1} + 1 + \left(\frac{\chi_2}{\chi_1} - 1 \right) \frac{C}{Y} - mc}{\frac{\chi_2}{\chi_1} + 3 - 2mc} \Psi \widehat{g}_t + \frac{\frac{g}{Y} \left(\frac{\chi_2}{\chi_1} - 1 \right)}{\frac{\chi_2}{\chi_1} + 3 - 2mc} \widehat{g}_t \right) \Leftrightarrow \\
\Phi &= \frac{\varepsilon - 1}{\gamma} \left(\frac{\frac{\chi_2}{\chi_1} + 1 + \left(\frac{\chi_2}{\chi_1} - 1 \right) \frac{C}{Y} - mc}{\frac{\chi_2}{\chi_1} + 3 - 2mc} \Psi + \frac{\frac{g}{Y} \left(\frac{\chi_2}{\chi_1} - 1 \right)}{\frac{\chi_2}{\chi_1} + 3 - 2mc} \right).
\end{aligned}
$$

Combining these two expressions yields:

$$\Phi = \frac{\varepsilon - 1}{\gamma} \left(-\frac{\frac{\chi_2}{\chi_1} + 1 + \left(\frac{\chi_2}{\chi_1} - 1\right)\frac{C}{Y} - mc}{\frac{\chi_2}{\chi_1} + 3 - 2mc} \phi_\pi \Phi + \frac{\frac{g}{Y}\left(\frac{\chi_2}{\chi_1} - 1\right)}{\frac{\chi_2}{\chi_1} + 3 - 2mc} \right) \Leftrightarrow$$

$$\Phi \left[1 + \frac{\varepsilon - 1}{\gamma} \phi_\pi \frac{\frac{\chi_2}{\chi_1} + 1 + \left(\frac{\chi_2}{\chi_1} - 1\right)\frac{C}{Y} - mc}{\frac{\chi_2}{\chi_1} + 3 - 2mc} \right] = \frac{\varepsilon - 1}{\gamma} \frac{\frac{g}{Y}\left(\frac{\chi_2}{\chi_1} - 1\right)}{\frac{\chi_2}{\chi_1} + 3 - 2mc} \Leftrightarrow$$

$$\Phi \frac{\frac{\varepsilon-1}{\gamma}\phi_\pi \left[\frac{\chi_2}{\chi_1} + 1 + \left(\frac{\chi_2}{\chi_1} - 1\right)\frac{C}{Y} - mc\right] + \frac{\chi_2}{\chi_1} + 3 - 2mc}{\frac{\chi_2}{\chi_1} + 3 - 2mc} = \frac{\varepsilon - 1}{\gamma} \frac{\frac{g}{Y}\left(\frac{\chi_2}{\chi_1} - 1\right)}{\frac{\chi_2}{\chi_1} + 3 - 2mc} \Leftrightarrow$$

$$\Phi = \frac{\frac{\varepsilon-1}{\gamma}\frac{g}{Y}\left(\frac{\chi_2}{\chi_1} - 1\right)}{\frac{\varepsilon-1}{\gamma}\phi_\pi \left[\frac{\chi_2}{\chi_1} + 1 + \left(\frac{\chi_2}{\chi_1} - 1\right)\frac{C}{Y} - mc\right] + \frac{\chi_2}{\chi_1} + 3 - 2mc},$$

and then:

$$\Psi = -\phi_\pi \Phi \Leftrightarrow$$

$$\Psi = -\frac{\phi_\pi \frac{\varepsilon-1}{\gamma}\frac{g}{Y}\left(\frac{\chi_2}{\chi_1} - 1\right)}{\frac{\varepsilon-1}{\gamma}\phi_\pi \left[\frac{\chi_2}{\chi_1} + 1 + \left(\frac{\chi_2}{\chi_1} - 1\right)\frac{C}{Y} - mc\right] + \frac{\chi_2}{\chi_1} + 3 - 2mc},$$

so that the solution is:

$$\widehat{C}_t = -\frac{\phi_\pi \frac{\varepsilon-1}{\gamma}\frac{g}{Y}\left(\frac{\chi_2}{\chi_1} - 1\right)}{\frac{\varepsilon-1}{\gamma}\phi_\pi \left[\frac{\chi_2}{\chi_1} + 1 + \left(\frac{\chi_2}{\chi_1} - 1\right)\frac{C}{Y} - mc\right] + \frac{\chi_2}{\chi_1} + 3 - 2mc}\widehat{g}_t, \qquad \text{(B.17)}$$

$$\widehat{\pi}_t = \frac{\frac{\varepsilon-1}{\gamma}\frac{g}{Y}\left(\frac{\chi_2}{\chi_1} - 1\right)}{\frac{\varepsilon-1}{\gamma}\phi_\pi \left[\frac{\chi_2}{\chi_1} + 1 + \left(\frac{\chi_2}{\chi_1} - 1\right)\frac{C}{Y} - mc\right] + \frac{\chi_2}{\chi_1} + 3 - 2mc}\widehat{g}_t. \qquad \text{(B.18)}$$

This confirms the form of our conjectured solution, and provides us with closed-form expressions of how consumption and inflation react to a government spending shock on impact. To establish the sign of these coefficients, we first note that the denominator is positive whenever:

$$\frac{\varepsilon - 1}{\gamma}\phi_\pi \left[\frac{\chi_2}{\chi_1} + 1 + \left(\frac{\chi_2}{\chi_1} - 1\right)\frac{C}{Y} - mc\right] + \frac{\chi_2}{\chi_1} + 3 - 2mc > 0 \Leftrightarrow$$

$$\frac{\chi_2}{\chi_1}\left[1 + \frac{\varepsilon - 1}{\gamma}\phi_\pi\left(1 + \frac{C}{Y}\right)\right] > mc\left(2 + \phi_\pi\frac{\varepsilon - 1}{\gamma}\right) - 3 - \frac{\varepsilon - 1}{\gamma}\phi_\pi\left(1 - \frac{C}{Y}\right) \Leftrightarrow$$

$$\chi_2 > \chi_1 \frac{mc\left(2 + \phi_\pi\frac{\varepsilon - 1}{\gamma}\right) - 3 - \frac{\varepsilon - 1}{\gamma}\phi_\pi\left(1 - \frac{C}{Y}\right)}{1 + \frac{\varepsilon - 1}{\gamma}\phi_\pi\left(1 + \frac{C}{Y}\right)}. \qquad \text{(B.19)}$$

This is a lower bound on χ_2. We show below that this condition is always satisfied when the model has a unique and determinate solution. We therefore obtain a decline in inflation and an increase in

consumption if and only if the numerators in both expressions are negative:

$$
\begin{aligned}
\phi_\pi \frac{\varepsilon - 1}{\gamma} \frac{g}{Y} \left(\frac{\chi_2}{\chi_1} - 1 \right) \quad &< \quad 0 \Leftrightarrow \\
\left(\frac{\chi_2}{\chi_1} - 1 \right) \quad &< \quad 0 \Leftrightarrow \\
\chi_2 \quad &< \quad \chi_1.
\end{aligned}
\tag{B.20}
$$

This is the condition stated in Proposition 2 in the main text. However, to complete the proof, the next subsection derives the conditions for the model to have a unique and stable equilibrium.

B.4 Equilibrium Determinacy and Uniqueness

The system (B.11)-(B.14) has two non-predetermined variables. This implies that a necessary and sufficient condition for the model to have a unique and determinate equilibrium is that both eigenvalues of the characteristic polynomial should be inside the unit circle. To write up the characteristic polynomial, we first restate the system on matrix form. After some algebra, we arrive at the following expression:

$$
\begin{bmatrix} \widehat{C}_t \\ \widehat{\pi}_t \end{bmatrix} = \Omega \begin{bmatrix} \Upsilon & (1 - \beta\phi_\pi)\Upsilon \\ \Gamma & \Gamma + \beta\Upsilon \end{bmatrix} \begin{bmatrix} \mathrm{E}_t\widehat{C}_{t+1} \\ \mathrm{E}_t\widehat{\pi}_{t+1} \end{bmatrix} + \Omega\Xi \begin{bmatrix} -\phi_\pi \\ 1 \end{bmatrix} \widehat{g}_t \Leftrightarrow
$$

$$
\begin{bmatrix} \widehat{C}_t \\ \widehat{\pi}_t \end{bmatrix} = A_0 \begin{bmatrix} \mathrm{E}_t\widehat{C}_{t+1} \\ \mathrm{E}_t\widehat{\pi}_{t+1} \end{bmatrix} + B_0\widehat{g}_t, \quad A_0 \equiv \Omega \begin{bmatrix} \Upsilon & (1 - \beta\phi_\pi)\Upsilon \\ \Gamma & \Gamma + \beta\Upsilon \end{bmatrix}, \quad B_0 \equiv \Omega\Xi \begin{bmatrix} \phi_\pi \\ 1 \end{bmatrix},
$$

where we have defined:

$$
\Omega \equiv \frac{1}{\frac{\chi_2}{\chi_1} + 3 - 2mc + \frac{\phi_\pi(\varepsilon-1)}{\gamma}\left[\frac{\chi_2}{\chi_1} + 1 + \left(\frac{\chi_2}{\chi_1} - 1\right)\frac{C}{Y} - mc\right]},
$$

$$
\Gamma \equiv \frac{\varepsilon - 1}{\gamma}\left[\frac{\chi_2}{\chi_1} + 1 + \left(\frac{\chi_2}{\chi_1} - 1\right)\frac{C}{Y} - mc\right],
$$

$$
\Xi \equiv \frac{\varepsilon - 1}{\gamma}\left(\frac{\chi_2}{\chi_1} - 1\right)\frac{g}{Y},
$$

$$
\Upsilon \equiv \left(\frac{\chi_2}{\chi_1} + 3 - 2mc\right).
$$

The characteristic polynomial is then:

$$
|A_0 - \lambda I| = 0 \Leftrightarrow
$$

$$
\left| \Omega \begin{bmatrix} \Upsilon & (1 - \beta\phi_\pi)\Upsilon \\ \Gamma & \Gamma + \beta\Upsilon \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right| = 0 \Leftrightarrow
$$

$$
0 = \left| \begin{bmatrix} \frac{\left(\frac{\chi_2}{\chi_1} + 3 - 2mc\right)}{\frac{\chi_2}{\chi_1} + 3 - 2mc + \frac{\phi_\pi(\varepsilon-1)}{\gamma}\left[\frac{\chi_2}{\chi_1} + 1 + \left(\frac{\chi_2}{\chi_1} - 1\right)\frac{C}{Y} - mc\right]} - \lambda & \frac{(1 - \beta\phi_\pi)\left(\frac{\chi_2}{\chi_1} + 3 - 2mc\right)}{\frac{\chi_2}{\chi_1} + 3 - 2mc + \frac{\phi_\pi(\varepsilon-1)}{\gamma}\left[\frac{\chi_2}{\chi_1} + 1 + \left(\frac{\chi_2}{\chi_1} - 1\right)\frac{C}{Y} - mc\right]} \\ \frac{\frac{\varepsilon-1}{\gamma}\left[\frac{\chi_2}{\chi_1} + 1 + \left(\frac{\chi_2}{\chi_1} - 1\right)\frac{C}{Y} - mc\right]}{\frac{\chi_2}{\chi_1} + 3 - 2mc + \frac{\phi_\pi(\varepsilon-1)}{\gamma}\left[\frac{\chi_2}{\chi_1} + 1 + \left(\frac{\chi_2}{\chi_1} - 1\right)\frac{C}{Y} - mc\right]} & \frac{\frac{\varepsilon-1}{\gamma}\left[\frac{\chi_2}{\chi_1} + 1 + \left(\frac{\chi_2}{\chi_1} - 1\right)\frac{C}{Y} - mc\right] + \beta\left(\frac{\chi_2}{\chi_1} + 3 - 2mc\right)}{\frac{\chi_2}{\chi_1} + 3 - 2mc + \frac{\phi_\pi(\varepsilon-1)}{\gamma}\left[\frac{\chi_2}{\chi_1} + 1 + \left(\frac{\chi_2}{\chi_1} - 1\right)\frac{C}{Y} - mc\right]} - \lambda \end{bmatrix} \right|.
$$

After some tedious algebra, we are able to restate the implied second-order polynomial as:

$$
0 = \lambda^2 + a_1\lambda + a_0,
$$

where we have defined:

$$a_1 \equiv -\frac{(1+\beta)\left(\frac{\chi_2}{\chi_1}+3-2mc\right)+\frac{\varepsilon-1}{\gamma}\left[\frac{\chi_2}{\chi_1}+1+\left(\frac{\chi_2}{\chi_1}-1\right)\frac{C}{Y}-mc\right]}{\frac{\chi_2}{\chi_1}+3-2mc+\frac{\phi_\pi(\varepsilon-1)}{\gamma}\left[\frac{\chi_2}{\chi_1}+1+\left(\frac{\chi_2}{\chi_1}-1\right)\frac{C}{Y}-mc\right]},$$

$$a_0 \equiv \frac{\beta\left(\frac{\chi_2}{\chi_1}+3-2mc\right)}{\frac{\chi_2}{\chi_1}+3-2mc+\frac{\phi_\pi(\varepsilon-1)}{\gamma}\left[\frac{\chi_2}{\chi_1}+1+\left(\frac{\chi_2}{\chi_1}-1\right)\frac{C}{Y}-mc\right]}.$$

We know from, e.g., LaSalle (1986) that both eigenvalues are inside the unit circle if and only if both of the following conditions are satisfied:

$$|a_0| < 1, \tag{B.21}$$
$$|a_1| < 1+a_0. \tag{B.22}$$

We can check these in turn. The first condition yields:

$$\left|\frac{\beta\left(\frac{\chi_2}{\chi_1}+3-2mc\right)}{\left(\frac{\chi_2}{\chi_1}+3-2mc\right)+\frac{\phi_\pi(\varepsilon-1)}{\gamma}\left[\frac{\chi_2}{\chi_1}+1+\left(\frac{\chi_2}{\chi_1}-1\right)\frac{C}{Y}-mc\right]}\right| < 1.$$

Since $\beta < 1$ and the bracket in the numerator is always positive, the denominator will be larger than the numerator (and thus, the inequality satisfied) as long as the second term in the denominator is non-negative:

$$\frac{\phi_\pi(\varepsilon-1)}{\gamma}\left[\frac{\chi_2}{\chi_1}+1+\left(\frac{\chi_2}{\chi_1}-1\right)\frac{C}{Y}-mc\right] > 0 \Leftrightarrow$$
$$\frac{\chi_2}{\chi_1}+1+\left(\frac{\chi_2}{\chi_1}-1\right)\left(1-\frac{g}{Y}\right)-mc > 0 \Leftrightarrow$$
$$\frac{\chi_2}{\chi_1}\left(1+\left(1-\frac{g}{Y}\right)\right) > mc+\left(1-\frac{g}{Y}\right)-1 \Leftrightarrow$$
$$\frac{\chi_2}{\chi_1} > \frac{mc-\frac{g}{Y}}{2-\frac{g}{Y}}. \tag{B.23}$$

This is the condition stated in Proposition 1 in the main text, providing another lower bound on χ_2. We can verify that this is the relevant, binding lower bound on χ_2 by showing that this expression is strictly larger than the one implied by (B.19):

$$\chi_1\frac{mc-\frac{g}{Y}}{2-\frac{g}{Y}} > \chi_1\frac{mc\left(2+\phi_\pi\frac{\varepsilon-1}{\gamma}\right)-3-\frac{\varepsilon-1}{\gamma}\phi_\pi\left(1-\frac{C}{Y}\right)}{1+\frac{\varepsilon-1}{\gamma}\phi_\pi\left(1+\frac{C}{Y}\right)} \Leftrightarrow$$
$$\left(mc-\frac{g}{Y}\right)\left[1+\frac{\varepsilon-1}{\gamma}\phi_\pi\left(1+\frac{C}{Y}\right)\right] > \left(2-\frac{g}{Y}\right)\left[mc\left(2+\phi_\pi\frac{\varepsilon-1}{\gamma}\right)-3-\frac{\varepsilon-1}{\gamma}\phi_\pi\left(1-\frac{C}{Y}\right)\right] \Leftrightarrow$$
$$6 > 2\frac{g}{Y}(2-mc)+3mc,$$

where the last step follows from some simple but tedious algebra. The right-hand side is maximized when $\frac{g}{Y}$ reaches its upper bound of 1 and mc reaches its upper bound of 1 (when $\varepsilon \to \infty$). In this case, the right-hand side approaches 5. We can thus conclude that this condition is always satisfied, so that the binding lower bound on χ_2 is given from (B.23).

Consider now the second necessary and sufficient condition for a unique and determinate equilibrium, (B.22), which yields:

$$|a_1| < 1+a_0 \Leftrightarrow$$

$$\left| -\frac{(1+\beta)\left(\frac{\chi_2}{\chi_1}+3-2mc\right)+\frac{\varepsilon-1}{\gamma}\left[\frac{\chi_2}{\chi_1}+1+\left(\frac{\chi_2}{\chi_1}-1\right)\frac{C}{Y}-mc\right]}{\frac{\chi_2}{\chi_1}+3-2mc+\frac{\phi_\pi(\varepsilon-1)}{\gamma}\left[\frac{\chi_2}{\chi_1}+1+\left(\frac{\chi_2}{\chi_1}-1\right)\frac{C}{Y}-mc\right]} \right| \quad <$$

$$1+\frac{\beta\left(\frac{\chi_2}{\chi_1}+3-2mc\right)}{\frac{\chi_2}{\chi_1}+3-2mc+\frac{\phi_\pi(\varepsilon-1)}{\gamma}\left[\frac{\chi_2}{\chi_1}+1+\left(\frac{\chi_2}{\chi_1}-1\right)\frac{C}{Y}-mc\right]} \quad \Leftrightarrow$$

$$\left| -\frac{(1+\beta)\left(\frac{\chi_2}{\chi_1}+3-2mc\right)+\frac{\varepsilon-1}{\gamma}\left[\frac{\chi_2}{\chi_1}+1+\left(\frac{\chi_2}{\chi_1}-1\right)\frac{C}{Y}-mc\right]}{\left(\frac{\chi_2}{\chi_1}+3-2mc\right)+\frac{\phi_\pi(\varepsilon-1)}{\gamma}\left[\frac{\chi_2}{\chi_1}+1+\left(\frac{\chi_2}{\chi_1}-1\right)\frac{C}{Y}-mc\right]} \right| <$$

$$\frac{\beta\left(\frac{\chi_2}{\chi_1}+3-2mc\right)+\frac{\chi_2}{\chi_1}+3-2mc+\frac{\phi_\pi(\varepsilon-1)}{\gamma}\left[\frac{\chi_2}{\chi_1}+1+\left(\frac{\chi_2}{\chi_1}-1\right)\frac{C}{Y}-mc\right]}{\left(\frac{\chi_2}{\chi_1}+3-2mc\right)+\frac{\phi_\pi(\varepsilon-1)}{\gamma}\left[\frac{\chi_2}{\chi_1}+1+\left(\frac{\chi_2}{\chi_1}-1\right)\frac{C}{Y}-mc\right]}.$$

We saw above that the last term in the denominator is positive, and we have established that also the first term is positive, so we can cancel out the denominators:

$$\left| -\left[(1+\beta)\left(\frac{\chi_2}{\chi_1}+3-2mc\right)+\frac{\varepsilon-1}{\gamma}\left(\frac{\chi_2}{\chi_1}+1+\left(\frac{\chi_2}{\chi_1}-1\right)\frac{C}{Y}-mc\right)\right]\right| <$$

$$(1+\beta)\left(\frac{\chi_2}{\chi_1}+3-2mc\right)+\frac{\phi_\pi(\varepsilon-1)}{\gamma}\left[\frac{\chi_2}{\chi_1}+1+\left(\frac{\chi_2}{\chi_1}-1\right)\frac{C}{Y}-mc\right].$$

Using the same insights, we conclude that all terms on the left-hand side must be positive, so taking absolute values yields:

$$(1+\beta)\left(\frac{\chi_2}{\chi_1}+3-2mc\right)+\frac{\varepsilon-1}{\gamma}\left(\frac{\chi_2}{\chi_1}+1+\left(\frac{\chi_2}{\chi_1}-1\right)\frac{C}{Y}-mc\right) \quad <$$

$$(1+\beta)\left(\frac{\chi_2}{\chi_1}+3-2mc\right)+\frac{\phi_\pi(\varepsilon-1)}{\gamma}\left[\frac{\chi_2}{\chi_1}+1+\left(\frac{\chi_2}{\chi_1}-1\right)\frac{C}{Y}-mc\right] \quad \Leftrightarrow$$

$$\frac{\varepsilon-1}{\gamma}\left[\frac{\chi_2}{\chi_1}+1+\left(\frac{\chi_2}{\chi_1}-1\right)\frac{C}{Y}-mc\right] < \frac{\phi_\pi(\varepsilon-1)}{\gamma}\left[\frac{\chi_2}{\chi_1}+1+\left(\frac{\chi_2}{\chi_1}-1\right)\frac{C}{Y}-mc\right] \Leftrightarrow$$

$$\phi_\pi > 1, \tag{B.24}$$

which is just the well-known Taylor-principle (in the absence of a monetary policy reaction to output, as assumed above). This condition is satisfied by assumption, as we have assumed $\phi_\pi > 1$ already in the main text.

To sum up, we have established that the model has a unique and determinate solution if and only if conditions (B.23) and (B.24) are satisfied, and that when this is the case, the solution features an increase in consumption and a decline in inflation if and only if condition (B.20) holds. This completes the proof of Propositions 1 and 2 in the main text∎

As a final note, recall our graphical representation of the model (B.11)-(B.14) in Section 4. Given the definition of the parameters a and b in (B.15) and (B.16), it is easy to verify that the condition for the parameter a to be positive, and thus for the rewritten New Keynesian Phillips Curve (B.14) to be upward-sloping, is identical to the condition in (B.23). Likewise, it can be easily verified that the parameter b is positive, so that a government spending shock shifts this curve down, if and only if the condition given by (B.20) is satisfied. This confirms the validity of the graphical representation of our model.

B.5   Detour: The Basic New Keynesian Model

In this subsection, we derive the solution to a model version *without* variable technology utilization. Incidentally, in this case the model collapses to the basic New Keynesian model, as presented, e.g., in Galí (2015), augmented with government spending. For comparison, we make the same assumptions

as in the simplified version of our baseline model: No technology shocks ($\widehat{A}_t = 0$), constant returns to scale in production ($\alpha = 0$), log utility in consumption ($\sigma = 1$), unitary (inverse) Frisch elasticity of labor supply ($\varphi = 1$), no monetary policy reaction to the output gap ($\phi_y = 0$), and no persistence in fiscal policy shocks ($\rho_G = 0$). Under these assumptions, the basic New Keynesian model is given by the following set of equations:

$$\widehat{N}_t = -\widehat{C}_t + \widehat{w}_t,$$

$$-\widehat{C}_t = \mathrm{E}_t\left(-\widehat{C}_{t+1} + \widehat{R}_t - \widehat{\pi}_{t+1}\right),$$

$$\widehat{Y}_t = \widehat{N}_t,$$

$$\frac{C}{Y}\widehat{C}_t = \widehat{Y}_t - \frac{g}{Y}\widehat{g}_t,$$

$$\widehat{mc}_t = \widehat{w}_t,$$

$$\widehat{\pi}_t = \beta\mathrm{E}_t\widehat{\pi}_{t+1} + \frac{\varepsilon-1}{\gamma}\widehat{mc}_t,$$

$$\widehat{R}_t = \phi_\pi\widehat{\pi}_t,$$

plus an exogenous process for $\widehat{g}_t$. We can combine these equations to obtain:[21]

$$\widehat{\pi}_t = \beta\mathrm{E}_t\widehat{\pi}_{t+1} + \frac{\varepsilon-1}{\gamma}\left(\widehat{C}_t\left(1+\frac{C}{Y}\right) + \frac{g}{Y}\widehat{g}_t\right),$$

$$-\widehat{C}_t = \mathrm{E}_t\left(-\widehat{C}_{t+1} + \phi_\pi\widehat{\pi}_t - \widehat{\pi}_{t+1}\right).$$

From these expressions, it follows directly–as argued in Section 4–that this model version also implies a downward-sloping Euler equation in $(\widehat{C}_t, \widehat{\pi}_t)$-space, and an upward-sloping NKPC-curve. Importantly, a positive shock to government spending shifts the NKPC-curve *up*, unlike our model of variable technology utilization, see Figure 5. Following the same steps as in the preceding subsections, we can derive the solution to this model, which is given by:

$$\widehat{C}_t = -\frac{\frac{\varepsilon-1}{\gamma}\frac{g}{Y}\phi_\pi}{1+\frac{\varepsilon-1}{\gamma}\phi_\pi\left(2-\frac{g}{Y}\right)}\widehat{g}_t, \tag{B.25}$$

$$\widehat{\pi}_t = \frac{\frac{\varepsilon-1}{\gamma}\frac{g}{Y}}{1+\frac{\varepsilon-1}{\gamma}\phi_\pi\left(2-\frac{g}{Y}\right)}\widehat{g}_t. \tag{B.26}$$

Both the numerator and denominator of both expressions are necessarily positive. An increase in $\widehat{g}_t$ thus leads to an increase in inflation and a decline in consumption in this model, in contrast to our baseline model studied above, but in line with the claims made in Section 4.

Finally, we can verify that the solution to our baseline model collapses to that of the simple New Keynesian model when the adjustment costs associated with changes in technology utilization become sufficiently high. This can be seen by rewriting the solution given by (B.17) and (B.18) as:

$$\widehat{C}_t = -\frac{\phi_\pi\frac{\varepsilon-1}{\gamma}\frac{g}{Y}\left(1-\frac{\chi_1}{\chi_2}\right)}{\frac{\varepsilon-1}{\gamma}\phi_\pi\left[1+\frac{\chi_1}{\chi_2}+\left(1-\frac{\chi_1}{\chi_2}\right)\frac{C}{Y}-\frac{\chi_1}{\chi_2}mc\right]+1+(3-2mc)\frac{\chi_1}{\chi_2}}\widehat{g}_t,$$

$$\widehat{\pi}_t = \frac{\frac{\varepsilon-1}{\gamma}\frac{g}{Y}\left(1-\frac{\chi_1}{\chi_2}\right)}{\frac{\varepsilon-1}{\gamma}\phi_\pi\left[1+\frac{\chi_1}{\chi_2}+\left(1-\frac{\chi_1}{\chi_2}\right)\frac{C}{Y}-\frac{\chi_1}{\chi_2}mc\right]+1+(3-2mc)\frac{\chi_1}{\chi_2}}\widehat{g}_t,$$

and letting $\chi_2 \to \infty$, in which case these expressions collapse to those presented in (B.25) and (B.26).

---

[21]Having carefully described the analytical solution to our baseline model above, we do not present any intermediate steps in this subsection, but simply state the results.