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Ph.D. Thesis

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Attention and perception in decision-making and interactions

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Endnu en til Ane.

Now pay attention, please.

Q, *Goldfinger*

Acknowledgements

> I realize it's conventional and usual to praise one's fellow workers on these occasions. But why not? Ours is a collaborative medium; we all need each other.
>
> Cary Grant

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Chapter 3, *You've got to accentuate the positive: Theory and applications of warm glow attention*

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Chapter 6, *Manipulating perception: The effect of product similarity on valuations and markets*

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Chapter 7, *Change we can perceive: The economics of dynamic inattention*

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Contents

Summary 1

1.1 English

Ever since the pioneering work of Kahneman and Tversky (1979), there has been a growing acknowledgment that individuals do not behave like the rational *homo economicus*. Ample evidence exists of behaviour inconsistent with the canonical paradigm of individuals maximizing an objective function by integrating all available information with flawless calculation. Furthermore, such departures from rationality are persistent and have significant effects on economic choice and interactions.

In this thesis, I examine the effect of limitations and biases in individuals' attention and perception. The impact of attention has only recently begun to be recognized as a major factor in economic choice, and the research presented here contributes to this growing literature.

We are constantly bombarded with vast amounts of information and humans are far from perfect information processing machines. Furthermore, processing power is effortful and a limited resource. Attention and perception are thus vital topics. To understand human decision making it is necessary to know, not only what information is available, but how individuals receive information, how they select it for attention, and how it is prioritized for processing.

There are physical limitations to perception and these may be exacerbated by busy lifestyles and too little time in which to make considered decisions. As anyone who has seen an optical illusion can attest, perception can be manipulated, so it is of much interest to explore how and when other economic agents will do this in order to exploit those with perceptual limitations.

There are also physical limitations to the amount of information humans can attend to, and so the available information must be heavily filtered. Frequently attention is drawn involuntarily to aspects of the environment which are particularly salient or stand out, even though these aspects may be trivial for decision making. It is common for attention to be distracted by irrelevant things.

Attention and perception are thus crucial concepts to understand in order to accurately model decision making.

A wide variety of approaches and methods are used in the studies presented here, including standard lab experiments, eye-tracking and theoretical models. I study both existing theories of attention as well as presenting new models of economic decision making, and explore the impact of attention and perception on individual choice and the consequences for markets and interactions.

It is of great importance to move beyond individual choice to examine how individuals with limited or biased attention and perception interact in institutions such as markets. The consequences of behavioural models of decision making for interactions, either with other boundedly rational individuals or with fully rational, profit maximizing firms, are often non-intuitive. Such non-intuitive consequences are found in this thesis. Studying interactions, especially with profit maximizing firms, is of especial relevance, both to gain insight into real life situations and to develop welfare enhancing interventions. It is easy to imagine that individuals with boundedly rational decision making will be exploited by firms. However, this is not necessarily the case, and care must be taken to examine the nuances of economic interactions.

Although there is a common thread running through them, each chapter is self-contained. There are two main parts to the thesis. Chapters 2 and 3 form the first part, which concentrates on attention. Attention in this sense refers to individuals focusing more on one part of their environment or one aspect of their choice set and less to the rest. Those attributes which receive more attention loom larger in individuals' thoughts, and are weighted more highly in decision making.

Chapters 4, 5 and 6 form the second part of the thesis and consider perception. Perception determines how individuals see the world, and thus their understanding and beliefs about their economic environment. Individuals make decisions based on their comprehension of their environment, and so understanding how individuals perceive the world is vital in understanding their actions.

Finally, Chapter 7 features both attention and perception, as individuals sometimes fail to attend to changes in their environment, and thus their perception of it is erroneous.

To examine each chapter more closely, chapter 2, *Looking for salience: Eye-tracking and preference reversals*, presents the results of an eye-tracking experiment testing a prediction of salience theory taken from Bordalo, Gennaioli, and Shleifer (2013b). In binary vertically differentiated choice, individuals will tend to choose the low price, low quality good when the overall price level is low, but the high price, high quality good when the overall price level is high, despite the price of the quality premium being held constant. Since willingness-to-pay for the quality premium should not be influenced by the price level, this represents a preference reversal.

Salience theory proposes that the mechanism behind such preference reversals is attention. With a low price level, prices vary a lot relative to their mean. This makes price the salient attribute, drawing attention to it and causing it to be weighted more heavily in decision making. On the other hand, when the price level is high, it is quality that varies more relative to its mean, making it the salient attribute, which draws attention to it, which leads to it being more heavily weighted in choice compared to the case of the low price level.

This situation is operationalized in an experiment using everyday consumer goods and with subjects' eye-movements tracked, with the goal of studying the attentional mechanism of salience theory. Preference reversals were observed, so the behavioural predictions of salience theory were confirmed. However, the eye-tracking hypotheses were all rejected, with the possible implications that eye-tracking is not an appropriate tool to examine salience, or that the underlying mechanism of salience theory is not attentional in nature.

In chapter 3, *You've got to accentuate the positive: Theory and applications of warm glow attention*, a theoretical framework similar to Bordalo, Gennaioli, and Shleifer (2012b) and Kőszegi and Szeidl (2013) is presented. However, unlike in those studies, individuals' attention is not determined by the choice set, but instead is allocated in the best way possible for the individuals. People thus create a "warm glow" by using their attention to see their situations in the best possible light.

It is demonstrated that this is a tractable and powerful theoretical framework by presenting several wide-ranging applications. It is shown that warm glow attention gives rise to an endowment effect, with a novel prediction that the effect is attenuated by the similarity of the endowed good and the alternative good. With Bertrand competition, it is shown that firms can exploit warm glow attention to escape the Bertrand trap and earn positive profits.

It also leads to people overestimating their own ability, reproducing the common observation that far more than 50% of a population consider themselves to be of above average ability. On the other hand, they will also tend to underestimate the abilities of an opponent. Warm glow attention leads to overruns of large projects, as individuals are overly optimistic about their completion dates.

In all of the above cases, the key ingredient required is uncertainty, either about one's own ability, that of an opponent, or about the project's probability of success.

Warm glow attention causes individuals to focus more on states of the world with a good outcome, and so they overweight them when forming expectations. Some possibility of being of high ability, or of a project being completed quickly, is thus required for this to be possible.

Finally, it is shown that warm glow attention causes established firms to be slow to adopt new technologies.

Turning now to the second part of the thesis, which concentrates on perception, Rubinstein (1988) presents an axiomatic treatment of individual choice when people are unable to perceive the difference between sufficiently similar consumption bundles. This is adapted in chapter 4, *If it's all the same to you: Blurred consumer perception and market structure*, to a vertically differentiated market in which individuals can only perceive the difference between goods if they are sufficiently differentiated. There is an incumbent and an entrant firm, and the effect of perceptual limitations is demonstrated to be vastly different depending on the cost structure of firms.

With fixed costs of quality, the incumbent benefits from perceptual limitations and the market becomes more concentrated. With marginal costs of quality, however, it can be beneficial for the entrant to imitate the product of the incumbent and the market can become less concentrated. If firms incur costs to enter the market, it can be that with marginal costs of quality, neither firm opts to produce, which is not the case with fixed costs.

Chapter 5, *Perception and quality choice in vertically differentiated markets*, presents a similar model, but with firms choosing quality simultaneously. It is shown that firms must differentiate themselves to a greater extent, and face a coordination problem in selecting equilibrium. Perception is endogenized, and it is shown that firms will opt to improve their consumers' perception to the greatest extent possible.

Perceptual limitations are taken to the lab in Chapter 6, *Manipulating perception: The effect of product similarity on valuations and markets.* In an experiment it is tested whether it is possible to manipulate the ease of perceiving the difference between two experimental goods by altering the similarity of their visual representation. In individual choice, subjects' perceptual limitations cause their valuations of two experimental goods to become more similar when it is harder for them to tell them apart visually.

These experimental goods were also traded in a market in a second experiment. Although buyers made more mistakes when perceiving the difference between goods is hard, sellers are unable to exploit buyers' perceptual limitations. There is some evidence that buyers use a different method of constructing their valuations in the market than in individual choice, and that this is beneficial to them.

Lastly, Chapter 7, *Change we can perceive: The economics of dynamic inattention*, combines both attention and perception. A theoretical framework is constructed in which individuals neglect to attend to infrequent changes in their environment. Thus their perception of their environment is no longer correct, since they are unaware of the change. Three applications of this theory are given.

In portfolio design, investors will tend to either over- or under-invest in an asset whose fundamental value changes gradually over time. If consumers are inattentive to changes in the aggregate price level, then inflation leads to a real terms decrease in their willingness-to-pay for goods, so firms reduce the size of their goods in response. This leads to the well-known phenomenon of product shrinkage. Finally, if individuals fail to attend to slow-developing symptoms of a disease, then medical screening programs can be beneficial, even if costly.

The above studies give an indication of the wide-ranging and diverse nature of the topics of attention and perception in economics. The research that has been done on them so far represents only the tip of the iceberg of what could, and should, be done in future.

1.2 Dansk

Siden Kahneman and Tversky (1979)s banebrydende værk har der været en voksende anerkendelse af, at individer ikke opfører sig som rationelle *homo economicus*. Der eksisterer rigelige beviser på menneskelig opførsel, som er uforenlig med det kanoniske paradigme om mennesker som individer der altid maksimerer deres udbyttefunktion via en fejlfri integration over alle tilgængelige informationer. Sådanne afvigelser fra rationalitet er ydermere vedvarende og har markante effekter på økonomiske valg og samspil.

I denne afhandling undersøger jeg effekten af begrænsninger og skævheder i individers opmærksomhed og perception. Betydningen af opmærksomhed indenfor økonomi er kun for nylig begyndt at blive anerkendt som en betydelig faktor i økonomiske valg, og forskningen som her fremlægges bidrager til den voksende litteratur. Vi bliver hele tiden bombarderet med umådelige mængder af information, og mennesker er langt fra perfekte informationsbehandlende maskiner. Processering af information er anstrengende og en begrænset ressource. Opmærksomhed og perception er dermed vitale emner: For at forstå menneskers beslutningstagning er det nødvendigt at vide ikke bare hvilken informationen, som er tilgængelig, men også hvordan individer modtager information, hvordan informationen udvælges, og hvordan den prioriteres til behandling.

Der er fysiske begrænsninger for perception, og de kan forværres af travl livsstil og for lidt tid til rådighed til at tage velovervejede beslutninger. Som enhver, der har set en optisk illusion, kan bevidne, så kan opfattelse blive manipuleret; og det er derfor meget interessant at undersøge hvordan og hvornår økonomiske agenter gør netop dette for at udnytte andres perceptionelle begrænsninger.

Der findes også fysiske begrænsninger i mængden af information, som mennesker er i stand til at holde deres opmærksomhed på, og den tilgængelige information bliver derfor stærkt filtreret. Opmærksomhed bliver hyppigt ufrivilligt fokuseret på aspekter i omgivelser og miljø, som er særligt fremtrædende eller som skiller sig ud, selv om disse aspekter kan være trivielle for beslutningstagning. Det er således almindelig for opmærksomhed at blive distraheret af irrelevante ting.

Opmærksomhed og perception er dermed afgørende koncepter, som man må forstå for at modellere beslutningstagning nøjagtigt.

Et bredt udvalg af tilgange og metoder er brugt i de studier, der her bliver fremlagt, inklusiv standart laboratorieeksperimenter, sporing af øjenbevægelser og teoretiske modeller. Jeg studerer både eksisterende teorier for opmærksomhed og fremlægger også nye modeller om økonomisk beslutningstagning, og undersøger opmærksomhed og perceptions indvirkning på individuelle valg og konsekvenser for markeder og samspil.

Det er vigtigt at gå videre end de individuelle valg, for også at undersøge hvordan individer med begrænset eller forudindtaget opmærksomhed eller perception interagerer i institutioner, såsom markeder. Konsekvenserne af adfærdsmodeller for beslutningstagning i samspil, enten med andre individer med afgrænset rationalitet eller med fuldt rationelle og overskudsmaksimerende firmaer, er tit ikke-intuitive. Sådanne ikke-intuitivt konsekvenser præsenteres i denne afhandling. At studere samspil, især med overskudsmaksimerende firmaer, er specielt relevant, både for at få indblik i virkelige situationer og for at udvikle indgreb der styrker velfærd. Det er nemt at forestille sig, at individer med begrænsede rationelle beslutningstagningsevner bliver udnyttet af firmaer. Imidlertid er det ikke nødvendigvis sådan, og nuancerne af økonomisk samspil må omhyggeligt undersøges.

Selvom der er en rød tråd igennem afhandlingen, så er hvert kapitel bygget op som en selvstændig, afgrænset præsentation. Afhandlingen består af to større dele. Kapitlerne 2 og 3 danner første del, som fokuserer på om opmærksomhed. Opmærksomhed betyder her, at individer fokuserer mere på én del af miljøet eller ét aspekt af deres valgmuligheder og mindre på resten. Disse attributter, som får mere opmærksomhed, dominerer da individers tankegang, og de vægtes mere i beslutningstagning.

Kapitlerne 4, 5 og 6 danner afhandlingens anden del, og omhandler perception. Perception bestemmer, hvordan individer ser verden, og dermed deres opfattelse af og overbevisning om deres økonomiske miljø. Individer tager beslutninger på baggrund af deres forståelse af deres miljø; og at forstå, hvordan individer opfatter verden, er vitalt, hvis man vil forstå deres handlinger.

Afslutningsvis handler kapitel 7 om både opmærksomhed og perception, da individer nogen gange undlader at være opmærksomme på forandringer i deres miljø, og deres perception derfor vil være fejlagtig.

En nærmere beskrivelse af de enkelte kapitler følger. Kapitel 2, *Looking for salience: Eye-tracking and preference reversals* fremlægger resultaterne fra et eksperiment af sporing af øjenbevægelser, som tester en forudsigelse fra saliens-teori, fra Bordalo et al. (2013b). I binært lodret differentierede valg plejer individer at vælge den laveste pris og laveste kvalitet vare hvis det generelle prisniveau er lavt, men den højeste pris og højeste kvalitet vare hvis det generelle prisniveau er højt; på trods af at den absolutte pris for kvalitetforskellen holdes konstant. Siden betalingsvillighed for kvalitetforskel ikke burde påvirkes af prisniveauet er der dermed dannet en invertering af præference.

Saliens-teori foreslår, at mekanismen bag sådanne præference-inverteringer er opmærksomhed. Med et lavt prisniveau varierer priser meget i forhold til deres gennemsnit. Det betyder, at pris er den fremtrædende attribut, der tiltrækker opmærksomhed, og dermed får den højere vægt i beslutningstagning. Omvendt, når prisniveauet er højt, er det kvalitet, som varierer mest i forhold til dens gennemsnit, og det betyder, at det er den fremtrædende attribut - den tiltrækker opmærksomhed, og den får højere vægt i beslutningstagning i forhold til situationen, hvor prisniveauet er lavt.

Sådanne situationer blev operationaliseret i et eksperiment, der brugte hverdags forbrugsvarer, og hvor der benyttedes sporing af deltagernes øjenbevægelser, med det formål af at studere saliens-teoriens opmærksomhedsmekanisme. Præference-invertering blev observeret; og det betyder således, at saliens-teoris adfærdsmæssige forudsigelser blev bekræftet. Imidlertid blev øjensporingshypotesen afvist, med den mulige implikation at øjensporing ikke er et passende redskab til at undersøge saliens, eller alternativt at saliens-teoris underliggende mekanisme ikke har noget at gøre med opmærksomhed at gøre.

I kapitel 3, *You've got to accentuate the positive: Theory and applications of warm glow attention*, fremlægges der en teoretisk ramme, som ligner Bordalo et al. (2012b) og Kőszegi and Szeidl (2013). I modsætning til disse studier så er individers opmærksomhed imidlertid ikke er dannet af valgmuligheder, men er i stedet allokeret på den bedste muligt måde for individerne. Mennesker skaber dermed en "varm glød" ved at bruge deres opmærksomhed til at se deres situation i det bedste mulige lys.

Det demonstreres, at denne teori udgør en fleksibel og stærk ramme, idet diverse vidtrækkende anvendelser præsenteres. Det påvises, at varm -glød-opmærksomhed giver anledning til en overvurderings-effekt (endowment effect), med en hidtil ukendte forudsigelse: at effekten er svækket, når den overvurderede og den alternative vare ligner hinanden. Med Betrand-konkurrence kan det påvises, at firmaer kan udnytte varm-glød-opmærksomhed til at undslippe Bertrand-fælden, og dermed opnå overskud.

Denne teori medfører også, at mennesker overvurderer deres evner; og den reproducerer dermed den almindelige observation, at langt mere end 50% af en befolkning betragter deres evner som over gennemsnittet. Tilsvarende kommer de også til at undervurdere en modstanders evner. Varm-glød-opmærksomhed fører til, at større projekter overskrider deres deadlines, fordi individer er for optimistiske om datoen for færdiggørelse.

Fælles for alle disse situationer er, at usikkerhed spiller en central rolle; enten med hensyn til ens egne evner, ens modstanders evner eller sandsynligheden for et projekts succes. Varm-glød-opmærksomhed får individer til at fokusere mere på tilstande med et forventet godt udfald, og disse tilstande får da mere vægt, når der dannes forventninger. Det er derfor nødvendigt, at der eksisterer en vis sandsynlighed for et have høje evner, eller for at et projekt kan afsluttes hurtigt, for at denne effekt optræder.

Til sidst påvises det, at varm-glød-opmærksomhed fører til, at firmaer er langsomme med at indføre nye teknologier.

Den anden del af afhandlingen handler om perception. Rubinstein (1988) fremlægger en aksiomatisk behandling af individuelt valg i situation hvor mennesker ikke kan opfatte (perceptere) forskellen mellem tilstrækkeligt ens varer. Den tilpasses i kapitel 4, *If it's all the same to you: Blurred consumer perception and market structure*, til et lodret differentieret marked med individer som kun kan opfatte forskellen mellem to goder, hvis disse er tilstrækkeligt differentieret. Der er et etableret og et indtrædende firma, og effekten af perceptionsbegrænsninger er vidt forskellig, afhængig af firmaernes omkostningsstruktur.

Når kvalitet har faste omkostninger, så drager det etablerede firma fordel af perceptionsbegrænsninger, og markedet bliver mere koncentreret. Når kvalitet har marginale omkostninger, så kan det indtrædende firma drage fordel af at efterligne det etablerede firmas produkt, og markedet kan blive mindre koncentreret. Hvis der er omkostninger ved at komme ind på markedet, så kan det ske, at intet firma vil producere, hvis kvalitet har marginale omkostninger, hvilket ikke sker, når kvalitet har faste omkostninger.

Kapitel 5, *Perception and quality choice in vertically differentiated markets*, præsenterer en lignende model, men med firmaer som vælger kvalitet samtidigt. Det påvises, at firmaer må differentiere sig i højere grad og må løse et koordinerings problem, når de vælger en ligevægt. Perception endogeniseres, og det påvises, at firmaer vælger at forbedre forbrugernes opfattelse af dem i den højest mulige grad.

Begrænsninger i perception overføres til laboratoriet i kapitel 6, *Manipulating perception: The effect of product similarity on valuations and markets.* I et eksperiment testes det, om det er muligt at manipulere med den lethed, hvormed man opfatter forskellen mellem to eksperimentielle goder ved at ændre ligheden i deres visuelle repræsentation. Ved individuelle valg fører deltagernes opfattelsesbegrænsninger til, at deres vurderinger af to eksperimentelle goder bliver mere ens ,når de to goder er sværere at skelne visuelt fra hinanden.

Disse eksperimentelle goder blev også handlet i et marked i et andet eksperiment. Selvom købere begik flere fejl, når det var svært at opfatte forskellen mellem goder, så var sælgere ikke i stand til at udnytte køberes perceptions-begrænsninger. Dette giver et vist bevis for, at købere bruger en anden metode til at danne deres vurderinger i markeder end i individuelle valg, og at dette er gavnligt for dem.

Til sidst kombinerer kapitel 7, *Change we can perceive: The economics of dynamic inattention*, opmærksomhed og perception. En teoretisk ramme konstrueres med individer, der forsømmer at bemærke langsomme og/eller sjældne forandringer i deres miljø. Dermed er deres opfattelse af miljøet ikke længere korrekt, idet de ikke erkender forandringen. Tre anvendelser af teorien præsenteres.

I portfolio-design vil investorer som oftest enten over- eller under-investere i et aktiv, hvis værdi som forandrer sig langsomt. Hvis forbrugere ikke er opmærksomme på forandringer i det samlede prisniveau, så vil inflation føre til et fald i betalingsvillighed for varer, og firmaer reducerer derfor størrelserne på deres goder. Det fører til det velkendte fænomen, at produkter "krymper" over tid. Til sidst vises, at i situationer, hvor individer ikke er opmærksom på symptomer der udvikler sig langsomt, er screeningsprogrammer gavnlige, selv hvis de er omkostningsfulde.

Disse studier giver en række eksempler på den omfattende og diverse natur af opmærksomhed og perception indenfor økonomi. Den eksisterende forskning er kun spidsen af isbjerget af den forskning som kunne, og burde, foretages.

Looking for salience: Eye-tracking and preference reversals 2

Andreas Gotfredsen, Carsten S. Nielsen, Alexander C. Sebald, & Edward J.D. Webb

Abstract

The results of an eye-tracking experiment designed to study a violation of revealed preference predicted by salience theory are reported. Specifically, individuals will choose a low quality good with a low price level and a high quality good with a high price level, despite the price of the quality premium being held constant. Subjects exhibited a significant numbers of such preference reversals (16.12% of decisions). However, the eye-tracking hypotheses, designed to test the underlying mechanism of salience theory, are all rejected. The implications for salience theory are discussed. More reversals were observed with hypothetical than incentivized choice (22.11% vs. 10.12% of decisions).

2.1 Introduction

How we interpret our economic environment is just as important as the environment itself. Our decisions are influenced not just by the choices available, but how we allocate our attention to different aspects of those choices. A proper understanding of what factors draw, modulate and otherwise influence attention is essential in understanding decision making. Several theoretical and empirical models have begun to address this topic recently, including salience theory (Bordalo et al., 2012b; Bordalo, Gennaioli, & Shleifer, 2013c, 2013a; Bordalo et al., 2013b; Bordalo, Gennaioli, & Shleifer, 2015b). It proposes that attention is drawn to, and choice is shaped by, the most salient aspect in the choice context. The greater the variation an attribute has amongst the goods an individual considers, the more salient an attribute becomes, leading the individual to focus more on it and weight the attribute more highly in decision making.

This paper studies salience theory experimentally in a simple consumer choice setting in which salience theory predicts a violation of revealed preference. It is also the first study to directly test the attentional mechanism of an economic theory of attention. While subjects are making their decisions, their eye movements are tracked, thus enabling measurement of what aspects of the choice set subjects select to attend to. Such eye-tracking measures have been found to be highly correlated with decision making (Armel, Beaumel, & Rangel, 2008; Krajbich, Armel, & Rangel, 2010). Thus if choice is shaped by salience, then it should be expected that data gathered using eye-tracking should reflect this.

The choices presented to subjects are inspired by the following example from Bordalo et al. (2013c): Imagine a choice between two wines, one French and one Australian. Suppose you like the French wine about 50% more than the Australian, but in a shop the former is priced at US$20 and the latter at US$10. The price of the French wine seems too high, so you purchase the Australian wine.

Some time later you are in a restaurant and are faced with the same choice. This time, however, the French wine is US$60, and the Australian wine is US$50. The French wine is 50% better, which seems like a good deal for only a 20% higher price, and so this time you choose the French wine. In both situations, the price difference is US$10. Hence according to revealed preference, if you were not willing to pay an extra US$10 for the quality premium in the shop where the price level was low, you should also be unwilling to pay the extra US$10 in the restaurant.

This example captures the intuition of salience theory: in the shop, the price difference between the cheaper and the more expensive wine is more salient than the quality difference, which attracts the consumer's attention, encouraging her to opt for the cheaper option. At the restaurant, after the mark-up is applied to both wines, the quality difference is more salient, encouraging the consumer to splurge and purchase the expensive option. The consumer's attention is drawn to the salient attributes, which are then overweighted in decision making. Since willingness-to-pay for the quality premium should be unaffected by the addition of a mark-up, salience has induced a preference reversal.

Eye-tracking is much used in psychology, but is it an appropriate tool with which to measure the underlying mechanism of salience theory? It measures which parts of their environment subjects' visual attention is directed towards, and that this reflects salience is evidenced by how the term is defined. Bordalo et al. (2012b), Bordalo et al. (2013c) and Bordalo, Gennaioli, and Shleifer (2015a) all quote the following passage:

> *"Salience refers to the phenomenon that when one's attention is differentially directed to one portion of the environment rather than to others, the information contained in that portion will receive disproportionate weighting in subsequent judgments"* (Taylor & Thompson, 1982, p.175)

Thus observing a change in allocation of visual attention should also be a measure of a change in salience.

Subjects in this study were presented with a binary choice between two goods, one of high quality and one of low quality, with the high quality good also having a higher price. Each pair of goods was shown twice, once at a low price level, and once at a high price level, with the price difference between the goods held constant.

To investigate a pattern in previous results of the effects of salience being greater in hypothetical than incentivized choice, half of subjects were placed in a hypothetical treatment and half in an incentivized treatment.

A significant number of subjects' choices (around 16%) exhibited salience induced preference reversals. There were fewer reversals with incentivized choice than with hypothetical choice, but the number was still significant.

While there are significant differences in eye tracking measures before and after the uniform price increase, the focus is opposite of what could first be expected. When comparing the high price level with the low, subjects spend relatively more time fixating on prices than on qualities. Moreover, there is little predictive power in the eye tracking measures. Neither the difference in dwell time, nor the difference in saccade behaviour is significantly correlated with the occurrence of preference reversals.

Several theories of attention and salience have been proposed recently. As well as salience theory, there is the closely related focusing theory (Kőszegi & Szeidl, 2013) as well as Dahremöller and Fels (2014) and Cunningham (2013). Further applications of the theory include bargaining (Canidio & Karle, 2015) and health insurance (Fels, 2014).

Few experimental studies looking at the predictions of salience theory exist. Bordalo et al. (2012b) present non-incentivized results of choices between money lotteries gathered using Amazon's Mechanical Turk which support the predictions of salience theory when applied to choice under risk.

Azar (2011a) found no evidence of preference reversal in an incentivized field experiment, but when subjects were presented with the same choice situation as a hypothetical question, preference reversals were observed.[1] Similarly in Azar (2011b)

[1] Note that the hypothesis in that study came not from salience theory itself, but as a test of the theory of *relative thinking* (Azar, 2007).

subjects were presented with a high quality good offered at some price and were asked to state hypothetically what price for a low quality good would make them indifferent. The reference price was high in one treatment and low in another, and the stated indifference prices were such that preferences were implied.

Bordalo, Gennaioli, and Shleifer (2012a) show that salience theory predicts a reverse endowment effect for bads (i.e. individuals are less likely to keep a bad they are endowed with, being more likely to switch to an alternative bad). Dertwinkel-Kalt and Köhler (2014) test this in the lab, but find a strong conventional endowment effect when choices were incentivized. However, when the same situation was presented to a different group of subjects as a hypothetical choice, a reverse endowment effect was found.

Similarly, Brenner, Rottenstreich, Sood, and Bilgin (2007) find reverse endowment effects for bads in hypothetical choice, yet Coursey, Hovis, and Schulze (1987) find a conventional WTA-WTP gap for incentivized valuation of bads.

The results summarized above seem to indicate that salience effects are much stronger in hypothetical than in incentivized choice, and may even disappear altogether with incentivized choice. Thus to see whether this phenomenon is reproducible in the current experimental setup, half of subjects were presented with incentivized choices, and half with hypothetical choices (for details see section 2.3).

The persistent difference between individuals' behaviour in incentivized and hypothetical questions has precedent, for example Cummings, Harrison, and Rutström (1995), Blumenschein, Blomquist, Johannesson, Horn, and Freeman (2008), List and Gallet (2001), and Murphy, Allen, Stevens, and Weatherhead (2005), who all find that willingness-to-pay is higher when stated hypothetically. Kang, Rangel, Camus, and Camerer (2011) also find a difference in a neurological study. Although the same areas of the brain were activated in hypothetical and incentivized choice, the level of activation was higher with the latter than the former.

This experiment is not the first in economics to use eye tracking. Russo and Leclerc (1994) and Reutskaja, Nagel, Camerer, and Rangel (2011) use the method to analyse subjects' search patterns. Ashby, Dickert, and Glöckner (2012) study visual attention as a mediator of a revealed preference violation and Arieli, Ben-Ami, and Rubinstein (2011) use saccades as a measure of individuals' decision making processes. Devetag, Di Guida, and Polonio (2015) and Stewart, Gächter, Noguchi, and Mullett (2015) study how much attention subjects give to their opponents' payoffs in simple games.

Section 2.2 presents a brief derivation of how salience theory predicts preference reversals and states the hypotheses to be tested. Section 2.3 then details the experimental procedures and section 2.4 gives the results. Section 2.5 discusses the findings and section 2.6 concludes.

2.2 Salience theory and preference reversals

Following (Bordalo et al., 2013c), we consider the simple case where the generic good k is characterized by its quality-price vector $(q_k, p_k) \in \mathbb{R}^2_+ > 0$. Without salience distortions, the consumer values k according to the linear utility function, $u_k = q_k - p_k$, which attaches equal weights to quality and price. A salient thinker departs from this utility function by giving disproportionate weight to attributes that "stand out", or are salient given the context of the choice set C.[2]

Which attribute is salient for good k depends on the salience function $\sigma : \mathbb{R}^2 \to \mathbb{R}_+$. Denote by $(\bar{q}, \bar{p})$ the reference good with attributes $\bar{q} := \sum_k q_k/N$ and $\bar{p} := \sum_k p_k/N$, where N is the number of goods in C, i.e. $\bar{q}$ and $\bar{p}$ are the mean values of quality and price. The salience of quality for a generic good k is then given by $\sigma(q_k, \bar{q})$, while the salience of price for k is given by $\sigma(p_k, \bar{p})$.

[2]The simplifying assumption is made that C is also the set of goods considered by the individual.

There are some well-established features of human perception that the salience function must reflect. Firstly, humans' perceptive apparatus is attuned to increase in contrast. This is captured by having the salience function satisfy *ordering*: whenever an interval $[x, y]$ is contained in a lager interval $[x', y']$, then $\sigma(x, y) < \sigma(x', y')$.

Secondly, changes in stimuli are perceived with *diminishing sensitivity* (Weber's law): for all $x, y > 0$ and any $\epsilon > 0$, $\sigma(x + \epsilon, y + \epsilon) < \sigma(x, y)$. As in Bordalo et al. (2013c), it is assumed that the salience function is homogenous of degree zero, so that $\sigma(\alpha x, \alpha y) = \sigma(x, y)$ for all $\alpha > 0$, which is sufficient to ensure diminishing sensitivity.

Given the choice set C, quality is then salient for good k when $\sigma(q_k, \bar{q}) > \sigma(p_k, \bar{p})$, price is salient for good k when $\sigma(q_k, \bar{q}) < \sigma(p_k, \bar{p})$, and price and quality are equally salient when $\sigma(q_k, \bar{q}) = \sigma(p_k, \bar{p})$.

Salience causes the individual to overweight salient attributes, and her valuation of k is now given by

$$u_k^S = \begin{cases} \frac{2}{1+\delta} \cdot q_k - \frac{2\delta}{1+\delta} \cdot p_k & \text{if } \sigma(q_k, \bar{q}) > \sigma(p_k, \bar{p}), \\ q_k - p_k & \text{if } \sigma(q_k, \bar{q}) = \sigma(p_k, \bar{p}), \\ \frac{2\delta}{\delta+1} \cdot q_k - \frac{2}{\delta+1} \cdot p_k & \text{if } \sigma(q_k, \bar{q}) < \sigma(p_k, \bar{p}) \end{cases} \tag{2.1}$$

where $\delta \in (0, 1]$ is a measure of the consumer's sensitivity to salience.

Suppose C consists of two goods, a high quality good $h = (q_h, p_h)$ and a low quality good $\ell = (q_\ell, p_\ell)$, with $q_h > q_\ell$ and $p_h > p_\ell$.

From homogeneity of degree zero, $\sigma(q_h, \bar{q}) = \sigma\left(\frac{q_h}{\bar{q}}, 1\right)$ and $\sigma(p_h, \bar{p}) = \sigma\left(\frac{p_h}{\bar{p}}, 1\right)$. Then from the ordering property of σ, price is salient for good h if $\frac{q_h}{\bar{q}} < \frac{p_h}{\bar{p}}$ or

$$\frac{q_h}{p_h} < \frac{q_\ell}{p_\ell} \tag{2.2}$$

i.e. if the quality to price ratio is higher for good ℓ than for good h. An analogous argument shows that the same condition also determines whether price is salient for good ℓ.

Now suppose the consumer is offered the same goods with a uniform mark-up $\Delta > 0$, so that prices become $p_h + \Delta$ and $p_\ell + \Delta$. Note that in the standard case, the individual's preferences between h and ℓ are unchanged, since the price of the quality premium remains the same.

The condition for the quality of good h to be salient becomes $\frac{q_h}{p_h+\Delta} > \frac{q_\ell}{p_\ell+\Delta}$, which can be rewritten

$$\frac{q_h}{p_h} + \frac{\Delta\left(q_h + q_\ell\right)}{p_h p_\ell} > \frac{q_\ell}{p_\ell}. \tag{2.3}$$

Again, the same condition determines whether quality is salient for good ℓ.

Inequalities (2.2) and (2.3) differ only by the term $\frac{\Delta(q_h+q_\ell)}{p_h p_\ell}$, which is positive and increasing in Δ. Thus it follows that if inequality (2.2) holds and price is salient for both goods given p_h and p_ℓ, there is some mark-up Δ such that quality is salient for both goods given prices $p_h + \Delta$ and $p_\ell + \Delta$.

While price is salient with a low price level, since it varies more relative to the reference price $\bar{p}$, the uniform mark-up means that with a high price level, prices don't vary as much relative to the reference price. Thus quality becomes salient.

If price is salient given p_h and p_ℓ, but quality is salient given $p_h + \Delta$ and $p_\ell + \Delta$, there is the possibility for a preference reversal. With the low price level and price being salient, the individual chooses ℓ over h if $(p_h - p_\ell) > \delta\left(q_h - q_\ell\right)$. On the other hand, given $p_h+\Delta$ and $p_\ell+\Delta$, the individual chooses h over ℓ if $(q_h - q_\ell) > \delta\left(p_h - p_\ell\right)$. If δ is sufficiently small, both these inequalities are satisfied, and the individual exhibits a preference reversal, choosing the low quality good with a low price level and the high quality good with a high price level. This occurs if

$$\delta < \min \left\{ \frac{q_h - q_\ell}{p_h - p_\ell}, \frac{p_h - p_\ell}{q_h - q_\ell} \right\}. \tag{2.4}$$

The first hypothesis the experiment seeks to test is whether such preference reversals can be observed in the lab.

Hypothesis 2.1. *A significant number of preference reversals will be observed.*

A further behavioural hypothesis is made based on previously observed experimental findings. As was discussed in section 2.1, there has been a pattern in the literature for salience effects to be stronger with hypothetical choice, and weaker or non-existent with incentivized choice. Thus half of subjects were placed in a hypothetical treatment, in which their choices did not affect their remuneration, and half were placed in an incentivized treatment, in which their reward depended on their choices. This enables the testing of the following hypothesis:

Hypothesis 2.2. *More preference reversals will be observed in the hypothetical treatment and fewer or no preference reversals will be observed in the incentivized treatment.*

By observing choices it can be identified whether or not preferences shift in line with salience theory. However, it is not possible to identify whether or not salience is the cause. Whether salience is in fact driving such preference shifts requires some empirical measure of salience. It is argued that eye-tracking provides such measures.

In order to visually process the attributes of a good, the individual moves her eyes. This fact makes it possible to use eye-movements as indicators of preference formation (Fehr & Rangel, 2011). Eye movements are required because visual acuity across the retina rapidly declines with increased distance from the fovea, which is the central and most sensitive part of the retina.

What are perceived as smooth movements of our eyes in fact consist of two very different components: fixations and saccades. Saccades are rapid, ballistic jumps of the eyes that serve to bring specific locations of a scene into the fovea. Processing is suppressed during the period in which the eye is in motion and as a result, preference formation is not updated until the saccade is completed. Inbetween each saccade, there is a fixation: a period of relative stability during which a small area of the scene is projected onto the fovea for detailed processing and the next saccade's trajectory is calibrated.

Eye-tracking records the coordinates of a fixation and the trajectory of saccades. The amount of time a subject spends fixating on a given area is termed *dwell time* and the relative fraction of a trial she spends looking at a given area is termed *relative dwell time*.

The amount of information that is transmitted through the optic nerve exceeds what the brain can process, so the brain has evolved mechanisms that select a subset of information for processing. When attention selects a particular location in a scene, processing of it is enhanced, and processing of non-selected locations are simultaneously suppressed.

Although it is possible for attention to be distributed across the field of vision (Eriksen & Yeh, 1985), it is assumed that in experiments in which individuals are free to move their eyes, attention is focused on whatever is in the fovea, i.e. subjects attend to whatever the eye-tracking device records them as fixating on. For example, although Posner (1980) finds clear evidence that subjects could allocate attention differentially across their field of vision in certain tasks, he states:

> *"It is true that the separation between attention and the fovea that occurs in our experiments is not a normal property of visual perception. It is revealed only under the close experimental control of the laboratory."*

That individuals' computation of the valuations of consumer goods is influenced by attention has been explicitly tested using eye-tracking (Krajbich et al., 2010). They found evidence for a substantial attention bias in the formation of valuations: goods that were fixated on more were more likely to be chosen. Consistent with this prediction, several studies have found it is possible to bias decision making through exogenous manipulations of visual attention (Armel et al., 2008; Milosavljevic, Malmaud, Huth, Koch, & Rangel, 2010; Hare, Malmaud, & Rangel, 2011).

If an individual's attention to salient attributes distorts the valuation of a good, as is proposed by salience theory, then it is natural to expect that in decision making she will fixate her eyes in different ways depending on the price level. In particular, with a low price level, she fixates more on prices compared to with a high price level, where she fixates relatively more on qualities, due to the saliency of prices being reduced. Thus the first eye-tracking hypothesis is

Hypothesis 2.3. *Subjects will spend more time (less time) looking at the quality (prices) of goods when the price level is high compared to when it is low.*

In addition to dwell time, the saccades which individuals make between the various attributes of the choice set can reflect subjects' decision making. Arieli et al. (2011) use the pattern of subjects' saccades when choosing between two binary lotteries to examine whether they tend to calculate the expected value (indicated by saccades predominately being between a prize and the probability of winning it) or make attribute by attribute comparisons (indicated by saccades being predominantly either between two probabilities or two prizes.

Similarly, it is argued here as more salient attributes of the choice set are more important in decision making, individuals will spend more time comparing goods along more salient attributes. Thus subjects are expected to make relatively more saccades from one price to another with a low price level so that price is salient and more saccades from one quality to another with a high price level so that quality is salient. The second eye-tracking hypothesis is then

Hypothesis 2.4. *Subjects will make more saccades from the quality (price) of one good to another when the price level is high compared to when it is low.*

The reason for measuring subjects' visual attention is that it is proposed that they reveal the mechanism behind the decision predictions of salience theory. Thus it is crucial to link the one to the other.

Hypothesis 2.5. *The amount of time subjects spend looking at an attribute and the frequency of saccades within this attribute are reliable predictors of preference reversals.*

2.3 Experiment design

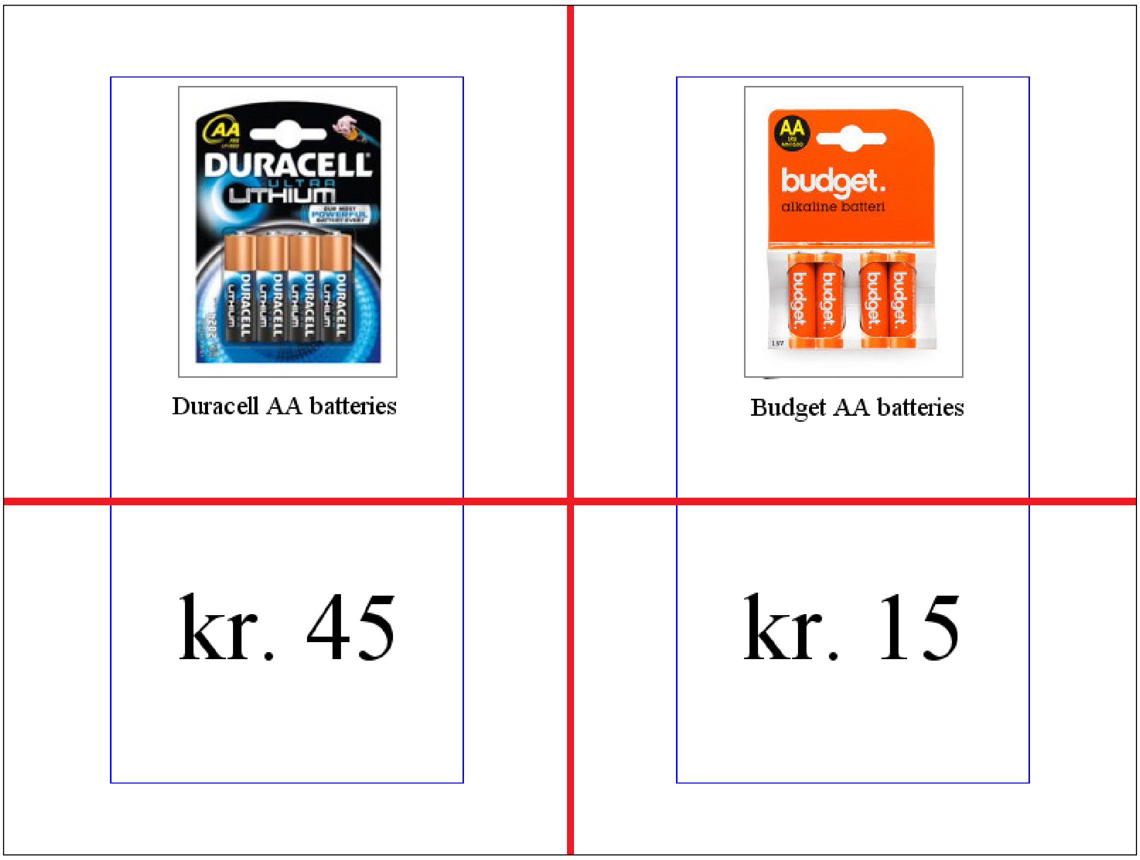
Prior to the main experiment, a calibration exercise was carried out in order to identify suitable goods pairs, to verify that subjects agreed with the experimenters' categorization of high and low quality, and to identify suitable prices. A short description is given in appendix A.2, and full details are available on request.

From an initial list of 50, 40 pairs were selected. Two goods had to be substituted for the main experiment due to them having become unavailable since the pilot. A single subject was sent a slightly different good due to it having become unavailable since the start of the experiment. For full details of the goods used, see table A.7.

Initially, subjects were shown each good individually and were asked to rate how much they liked it on a scale from 1 to 9, where 1 signified that they didn't like it at all and 9 signified that they liked it a lot. For this part of the experiment, subjects' eye movements were not tracked.

Subjects then performed a choice task. They were shown 40 choice sets, each consisting of a high quality good and a low quality good paired by category. The goods were represented by a picture and a brief description and below each good its price was presented. (See Figure 2.1 for an example screen.) Which good was presented on the left and which on the right hand side of the screen was randomized.

FIGURE 2.1: Example of a decision screen. Red lines indicate division of the screen into quadrants. Screens had a resolution of 800x600. Given (0,0) in the bottom left corner of the screen, fixations with an x coordinate of 400 or greater were designated as being on the right hand side of the screen and fixations with a y-coordinate of greater than 250 were considered as being on the upper part of the screen. Conclusions are largely unchanged if the y-axis cut-off is increased to 300 or decreased to 400.



Each set of goods was shown twice, once at a low price level (p_h and p_l), and once at a high price level ($p_h + \Delta$ and $p_\ell + \Delta$). Subjects hence made 80 choices in total, the order of which was randomized. $p_h + \Delta$, the price of the high quality good in the high price level, was chosen to be approximately its retail price or DKK 300, whichever was lower. p_h was chosen to be half this amount, i.e. $\Delta = p_h$. p_ℓ, the remaining degree of freedom, was chosen using data from the calibration exercise (for details see appendix A.2.

Before the appearance of each goods pair subjects were shown a blank screen with a central fixation cross and were instructed to fixate on the cross until the goods appeared. Subjects made their choices using a keyboard. They had as long as they wished to make their choices and abstention was not possible. Response times varied from 0.598s to 18.9s with a median value of 4.23s.

During the choice task, subjects' eye-movements were monitored using a desktop mounted eye tracker (Eyelink 1000, SR Research) at a rate of 1000Hz and an accuracy of $\sim 0.5°$.

There were two treatments, incentivized and hypothetical. For incentivized the treatment, subjects were given an endowment of DKK 300 (approximately $US 45). At the end of the experiment a trial was picked at random, and subjects bought whichever good they had chosen at the stated price, using their endowment. The good was subsequently ordered over the internet by the experimenters and delivered directly to the subject.

For the hypothetical treatment, subjects were informed before making their decisions that they would receive a flat fee of DKK 300 for participation. No good was actually bought. The instructions for both treatments are included in appendix A.4.

38 subjects were recruited using Sona Systems[3] and ORSEE (Greiner, 2015) and participated in the experiment. Eye-tracking data from 2 subjects is excluded due to poor quality. The experiment was implemented using Experiment builder from SR

[3] `www.sona-systems.com`

Research, and the one-on-one sessions were held at the Centre for Visual Cognition, University of Copenhagen.

2.4 Results

Choices

Subjects chose the high quality good 42.7% of the time. The mean number of high quality goods chosen per subject was 34.16 (out of 80) and this is higher with a high price level than with a low price level (19.26 vs. 14.89 out of 40, Wilcoxon signed-rank p-value 0.0003, N=38). The mean number of high quality choices per subject was also higher in the hypothetical than in the incentivized treatment (40.47 vs. 27.84 out of 80, Mann-Whitney U p-value 0.0035, N=38).

A subject is said to have exhibited a *reversal* if, for a given good pair, she chooses the low quality good when the price level is low and the high quality good when the price level is high. It is also possible for the subject to choose the high quality good with low prices and the low quality good with high prices. Such a choice pattern is termed a *B reversal*. Such choices, which could for example be caused by noisy subject choice or income effects, are not the main focus of the analysis. However, it is useful to compare them to the number of reversals.

Table 2.1 gives summary statistics about the number of reversals observed. 16.12% of choices exhibited preference reversals. While this is a significant amount, it could simply be that individuals' choices were noisy in some way. However, this would imply a roughly equal number of reversals and B reversals, whereas only 5.2% of choices exhibited B reversals, significantly fewer than the number of reversals. Hence the observed number of reversals cannot be simply due to noisy subject choice.

Result 2.1. *Subjects exhibited a significant number of preference reversals, and this* *number was significantly higher than the number of B reversals.*

Table 2.1: Mean percentage of subjects' choices exhibiting reversals and B reversals. $N = 38$. *** = significant at 1% level, ** = significant at 5% level, * = significant at 10% level.

	All (p-value for one-sided t-tests of equality to 0 in parentheses)	Hypothetical/ Incentivized (p-values for Mann-Whitney U test in parentheses	Reversals = B reversals (p-value for Wilcoxon signed rank)
Reversals	16.12***	22.10/10.13***	<0.001***
	(<0.001)	(<0.001)	
B reversals	5.20***	4.10/6.32***	
	(<0.001)	(0.004)	
Reversals (ratings based)	9.67***	11.87/7.37**	0.016**
	(<0.001)	(0.032)	
B reversals (ratings based)	5.99***	7.11/4.87*	
	(<0.001)	(0.069)	
Reversals (mean price < DKK 110)	17.89***	23.95/11.84**	<0.001***
	(<0.001)	(0.019)	
B reversals (mean price < DKK 110)	3.55***	2.37/4.74	
	(<0.001)	(0.102)	
Reversals (rating > 5)	21.26***	28.76/13.77***	0.001***
	(<0.001)	(0.009)	
B reversals (rating > 5)	6.95***	4.99/8.90*	
	(<0.001)	(0.081)	

Table 2.1 also shows that subjects in the incentivized treatment had significantly fewer preference reversals than those in the hypothetical treatment. Again, a possible explanation for this is that inentivization reduces random noise. However, a reduction in some symmetric noise term would imply a reduction of approximately the same size in both reversals and B reversals, whereas in fact B reversals significantly increase with incentivization.

Another possibility is that incentivization encourages subjects to frame their endowment as their entire income. This then leads to larger income effects and a greater number of B reversals, dominating any decrease in noisy choices. However, when the analysis is repeated only for the 22 goods pairs whose mean price is less than DKK 110 (approximately the mean price of all goods), there is still a significant reduction in the number of reversals per subject from incentivization, whereas there is no significant reduction (at the 5% level) in the number of B reversals. This pattern is difficult to reconcile with the change being due to a reduction in noise, and it is concluded that

Result 2.2. *Subjects exhibited fewer reversals in the incentivized treatment.*

Table 2.1 shows the results of robustness tests of these results. Subjects rated how much they liked the goods. The difference in the rating of the high and low quality good was non-negative, and therefore (weakly) consistent with the experimenters' categorization of quality in 78.55% of choices. Reclassifying the high (low) quality good as the one given a higher (lower) rating in the pair reduces both types of reversal, due to a fairly high percentage (27.0%) of pairs in which subjects gave both goods the same rating. Nevertheless, conclusions are robust to using this alternative categorization of quality.

It could also be that preference reversals occur simply because subjects did not particularly care for many of the goods on offer, and due to this chose carelessly. However, the pattern is largely unchanged if the analysis is restricted to choices in which subjects gave both goods a rating above 5.

Table 2.2 shows the results of a regression with the existence of a reversal as the dependent variable. It can be seen that the greater the price difference between the high and low quality good, the less the probability of observing a reversal. It also shows that the probability of a reversal is increasing in a subject's response time. If response time is a proxy for the difficulty of a choice, this is consistent with the fact that subjects' preferences between goods cannot differ too much for a reversal to occur, therefore a greater number of reversals should be expected in choice situations which are more difficult.

Dwell time

The decision screen was divided into quadrants, as depicted in figure 2.1. Subjects were considered to be looking at the quality of the goods if a fixation was recorded in the upper half of the screen and at prices if a fixation was recorded in the lower half of the screen. Similarly, they were considered to be looking at the high quality good if a fixation was recorded in the left/right half of the screen, depending on which side it was presented on.

The total sum of all fixation durations in a given area is termed the *dwell time* in that area. The *relative dwell time* is then the proportion of total dwell time spent in a given area. Figure 2.2 shows heat maps of subjects' aggregate dwell time for high price, low price, incentivized and hypothetical choice conditions.

Summary statistics about the relative dwell time on the quality attribute[4] and on the high quality good[5] are shown in table 2.3. Examining the results of the statistical tests in the same table, it can be seen that

Result 2.3. *(i) The relative amount of time subjects spent looking at the quality attribute was lower when the price level was high.*

(ii) The relative amount of time subjects spent looking at the high quality good was higher when the price level was high.

Turning to whether the eye-tracking data can predict preference reversal, two variables are constructed.

[4] I.e. the quality of either the high or low quality good.
[5] I.e. either the quality or price of the high quality good.

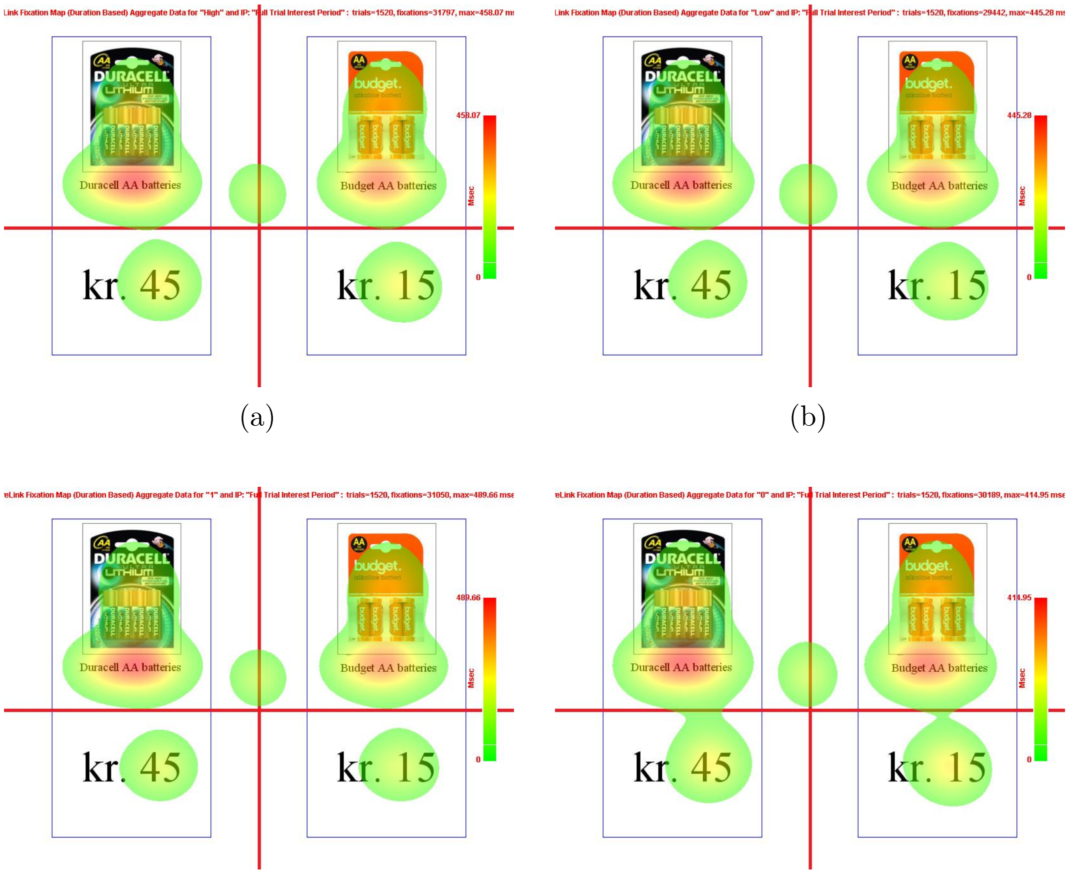
Table 2.2: Logit regressions with subject fixed effects with a dummy indicating the occurance of a reversal as the dependent variable. Lag = No. decision screens between seeing high and low price level, high first = goods pair seen first with high price level. $N = 1440$, *** = significant at 1% level, ** = significant at 5% level, * = significant at 10% level.

	(1)	(2)	(3)	(4)	(5)
Price difference	-0.0216***	-0.0214***	-0.0213***	-0.0212***	-0.0217***
	(0.003)	(0.004)	(0.006)	(0.006)	(0.003)
Price increase	8.36×10^{-3}	8.26×10^{-3}	8.85×10^{-3}	8.76×10^{-3}	8.30×10^{-3}
	(0.121)	(0.126)	(0.121)	(0.125)	(0.124)
Rating difference	8.83×10^{-3}	0.0364×10^{-3}	-7.47×10^{-3}	-7.39×10^{-3}	0.548×10^{-3}
	(0.982)	(0.993)	(0.861)	(0.863)	(0.989)
rt (low price level)	0.179×10^{-3}***	0.184×10^{-3}***	0.188×10^{-3}***	0.190×10^{-3}***	0.174×10^{-3}***
	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)
rt (high price level)	0.150×10^{-3}***	0.144×10^{-3}***	0.167×10^{-3}***	0.165×10^{-3}***	0.158×10^{-3}***
	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)
Lag	4.18×10^{-3}	4.32×10^{-3}	3.22×10^{-3}	3.29×10^{-3}	4.25×10^{-3}
	(0.321)	(0.306)	(0.471)	(0.462)	(0.314)
High first	0.0193	-0.0353	0.600×10^{-3}	-0.0178	0.0912
	(0.910)	(0.842)	(0.997)	(0.924)	(0.606)
Incentivized	-0.159	-0.204	-0.281	-0.301	-0.139
	(0.817)	(0.870)	(0.751)	(0.735)	(0.873)
ΔAT		0.760		0.271	
		(0.287)		(0.718)	
ΔHQ			4.99***	4.98***	
			(<0.001)	(<0.001)	
ΔP					3.50×10^{-3}
					(0.996)
ΔQ					-0.619
					(0.165)

Table 2.3: Means of subjects' dwell time and inter-component saccade categories. P-values for high price vs. low price level are for Wilcoxon signed-rank tests, p-values for incentivized vs. non-incentivized choice are for Mann-Whitney U tests. N=36, *** = significant at 1% level, ** = significant at 5% level, * = significant at 10% level.

	All	High price level	Low price level	P-value (WSR)	Incentivized	Hyopthetical	P-value (MWU)
Mean dwell time on quality attribute	0.809	0.798	0.819	<0.001***	0.821	0.795	0.096*
Mean dwell time on high quality good	0.503	0.512	0.495	0.0104**	0.494	0.514	0.124
Q saccades	0.320	0.317	0.323	0.293	0.325	0.315	0.457
Q saccades (2 step)	0.344	0.342	0.347	0.520	0.351	0.337	0.318
P saccades	0.178	0.186	0.170	<0.001***	0.172	0.186	0.274
P saccades (2 step)	0.188	0.198	0.179	<0.001***	0.182	0.196	0.247

Figure 2.2: Heat maps of subjects aggregated dwell time for (a) high price level, (b) low price level, (c) incentivized choice and (d) hypothetical choice.



(a) (b) (c) (d)

ΔAT : The difference in relative dwell time on the quality attribute between the high and low price levels for a given subject for a given pair of goods.

ΔHQ : The difference in relative dwell time on the high quality good between the high and low price levels for a given subject for a given pair of goods.

Table 2.2 gives the results of regressions with the probability of a preference reversal as the dependent variable with ΔAT and ΔHQ as independent variables. From this it is concluded that

Result 2.4. *(i) The difference in relative dwell time on the quality attribute is not a predictor of reversals.*

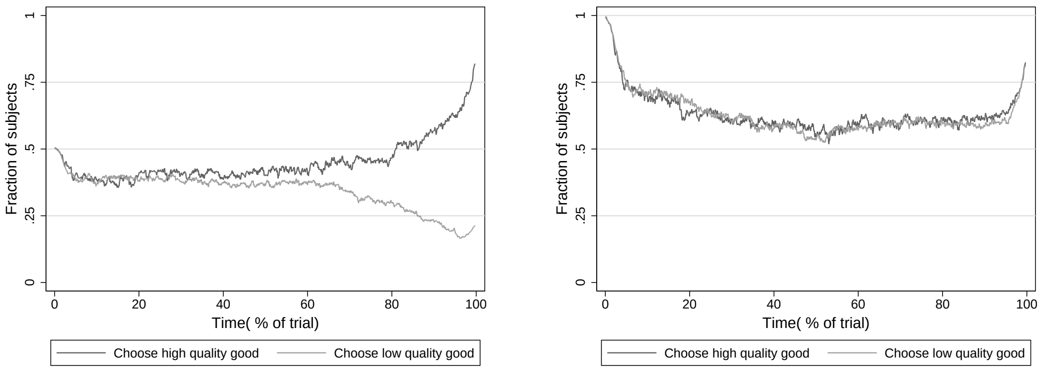
(ii) The difference in relative dwell time on the high quality good between the high and low price levels is a predictor of reversals.

It is a well-established result in drift diffusion studies that when making decisions, individuals' eye-movements tend to be directed in the early stages relatively equally between options, followed by concentrating to a greater and greater extent on the item eventually chosen. (Krajbich et al., 2010; Krajbich, Lu, Camerer, & Rangel, 2012). Since more individuals choose the high quality good with a high price level, this could be consistent with individuals looking more at the high quality good in this condition.

That the results are in line with what should be expected from such studies is confirmed by figure 2.3a. The fraction of individuals at any one time looking at the high quality good is shown for those who went on to chose it, and for those who instead choose the low quality good. Initially, attention is split evenly between the two goods. However, as the trial goes on, it is increasingly more common for subjects to be looking at the item they choose.

Figure 2.3b illustrates the fraction of subjects at a given time looking at the quality attribute. Interestingly, this does not appear to differ over what individuals choose. This is confirmed by comparing subjects' mean dwell time conditional on buying the high quality good (mean 0.813) to the mean dwell time conditional on buying the low quality good (mean 0.806), which does not differ significantly (Wilcoxon signed-rank p-value 0.148, N=36).

Figure 2.3: Mean fraction over all trials of subjects fixating on (a) the high quality good and (b) the quality attribute. All trial lengths normalized to be 100.



(a) (b)

Saccades

Saccades between different areas of the screen are also of interest, as they inform about how subjects compared the different elements of the choice. The first saccade in a trial is discarded, since this represents a transition from the fixation cross. 56.7% of remaining saccades are within a single quadrant, and these are interpreted as exploration of a single component of choice (for example the saccades involved in reading a good's description).

Inter-component saccades are categorized in the following way:-

Quality (Q) saccades: Saccades between qualities

Price (P) saccades: Saccades between prices.

Q and P saccades are of interest since they can be interpreted as measures of salience. If a price difference is salient, this may imply a larger number of saccades between prices, likewise for a quality difference.

Table 2.3 shows summary statistics about the percentages of different categories of saccades which were observed. Following tests, it is found that

Result 2.5. *(i) The amount of Q saccades was the same across price levels, but there were significantly more P saccades with a high price level.*

(ii) There was no significant effect of incentivization on saccades.

These conclusions are robust to classifying as Q saccades sequences which start on one quality, end on the other quality and include a fixation on price inbetween, and similarly allowing P saccades include sequences which start on one price, end on the other price and include a fixation on quality inbetween. (These are referred to as '2 step' in table 2.3.)

To see how well subjects' saccades predict preference reversals, the following measures are created:

ΔQ: The difference in the fraction of inter-component saccades which are Q saccades for a given subject for a given good pair between the high and low price levels.

ΔP: The difference in the fraction of inter-component saccades which are P saccades for a given subject for a given good pair between the high and low price levels.

Table 2.2 shows the results of regressions of probability of a preference reversal against these saccade measures. It can be seen that

Result 2.6. *Differences in saccade behaviour between the high and low price levels are not predictors of preference reversal.*

Robustness checks

Various robustness checks were carried out. Firstly, it could be that the increased dwell time on prices with a high price level is because higher prices are more likely to have three digits as opposed to two digits, and thus take longer to read. The mean change in relative dwell time on the quality attribute was hence compared for four groups:- (1) pairs in which both prices change the number of digits, (2) pairs in which only the high quality good changes the number of digits (3) pairs in which only the low price changes the number of digits and (4) pairs in which both prices have the same number of digits in both price levels. A Kruskal-Wallis test has a p-value of 0.497 (N=36) and so the null hypothesis that the shift in dwell time is unrelated to the number of digits cannot be rejected.

It could be that salience, being an intrinsic, involuntary mechanism, only influences the early part of subjects' gaze. Tables A.1, A.3, and A.4 summarize the analysis for various different ways of defining the early part of trials, both in absolute and relative terms. It can be seen that subjects' attention was still drawn to the price attribute to a greater extent in high price than low price conditions, although the difference is not always significant, and this difference still does not predict preference reversal. The difference in attention received by the high and low quality good across price level is not significant, and it no longer predicts price reversals. This is consistent with models of drift-diffusion, in which there is a gaze cascade with the option eventually chosen receiving more and more attention as the trial goes on (see also figure 2.3a).

Taking this to the extreme and only looking at the very first saccade, it was towards the quality attribute 93.61% of the time with a high price level, and 95.28% of the time with a low price level, a difference which is not significant (Wilcoxon signed-rank p-value 0.203, N=36).

Conversely, it could be argued that, since subjects needed to have both goods and prices in their consideration set, that the early part of a trial is explorative, and that salience theory's influence should only be observed in the latter part of a trial. Tables A.2, A.3, and A.4 show the results of analysis for various definitions of the last part of a trial, and once again there was more attention paid to prices with a high price level. The difference in attention on the high quality good is now a highly significant predictor of a preference reversal, consistent with drift-diffusion.

Again taking this to the extreme and looking at the final fixation, this is on the quality attribute 83.1% of the time with a low price level and 81.6% of the time with a high price level, a difference which is not significant (Wilcoxon signed-rank test p-value 0.252, N=36).

It could also be that the empirical measures of attention are not the best to capture salience. Tables A.5 and A.6 contain the results of an analysis of using alternative measures: instead of the dwell time on an attribute/good the number of fixations is used, and rather than saccades comparing quality and comparing price, saccades which are towards quality, regardless of origin, are used. Results are largely unchanged.

2.5 Discussion

A significant number of preference reversals was observed, demonstrating clear support for the behavioural predictions of salience theory. Preference reversals were also observed within subjects, rather than between subjects as in previous studies, giving evidence of the effect's robustness.

Nor was there an effect of the amount of time between presentations of a goods pair in each price level, as can be seen from the insignificance of the variable *lag* in table 2.2. Indeed, in the 32 cases in which individuals were shown the same goods pair in consecutive trials, reversals were observed 22% of the time, so remembering the choice made the last time the subject saw the pair does not eliminate preference reversals. Subjects simply choose differently given different price levels.

This is an especially relevant finding, since the behavioural hypotheses rely on implicit assumption that the consideration set is identified with the choice set.

However, it is plausible that when subjects were presented with the same goods pairs in close proximity, or even in consecutive trials, that they could, if they so wished, include previously offered prices in their consideration set. That the proximity of presentation has no effect on preference reversals implies that, at least in the current context, individuals do not consider prices outside the choice set.

A possible alternative explanation for the observation of preference reversals is that subjects' decisions were noisy to some degree. However, any symmetric noise term would predict equal numbers of reversals, in which subjects switch from choosing the low quality good with a low price level, to choosing the high quality good with a high price level, and B reversals, in which the opposite choice pattern is observed. Yet there were always significantly more reversals than B reversals, and this is difficult to reconcile with noisy subject choice.

Hypotheses 2.3, 2.4 and 2.5, which deal with the eye-tracking data, are all rejected. The following explanations for this are all considered: that the experiment was an inappropriate operationalization of salience theory, that the allocation of attention described by salience theory is not measurable by using eye-tracking, and that preference reversals are caused by a different mechanism than salience theory.

In order to test any economic theory in the lab, it is necessary to make many auxiliary assumptions in its operationalization. It could be that the hypotheses are rejected due to the choice setting being an inappropriate testing ground for salience theory. Yet the behavioural predictions do hold, which suggests that the binary choices presented to subjects are in fact situations in which the effects of salience theory hold. In addition, it is not the case that no significance was found in the eye-tracking measures, which might be due to an insufficiently powered study. Significant and robust differences were found, but in the opposite direction to that predicted by salience theory.

Figure 2.3a shows that subjects tended to fixate to a greater and greater extent on the good they eventually chose as a trial progressed. This is a well-known, robust result in eye-tracking studies of decision making, so that it is reproduced here is evidence that the eye-tracking data is reliable and was appropriately gathered.

The choice situation subjects were presented with is arguably artificial. Although they were free to move their eyes, their heads were held fixed in place using a chin and forehead rest, an unnatural position. Hence it may be that when making decisions “in the wild”, their visual attention is allocated in line with salience theory.

It is certainly possible to have experiments with improved external validity, using eye-tracking in environments such as supermarkets, for example Pärnamets, Johnasson, Gidlöf, and Wallin (2015) and Gidlöf, Wallin, Dewhurst, and Holmqvist (2013). However, it is impossible to entirely eliminate any bias due to subjects being aware of their eye-movements being monitored. Improving the external validity of the experiment would also mean relinquishing the control, especially of individuals' consideration sets, that the lab provides.

In addition, if the artificiality of the choice situation means that it is an inappropriate testing ground for salience theory, yet visual attention drives decision making in the way salience theory suggests, then the eye-tracking results imply different behavioural results than were observed. Subjects fixated more at prices with a high price level, implying subjects should weight prices more heavily in decision making compared to with a low price level, implying subjects should weight qualities more heavily in decision making. Hence subjects should tend to choose the low quality good more with a high price level and the high quality good more with a low price level, i.e. there should be more B reversals than reversals, whereas precisely the opposite is found.

The purpose of eye-tracking in this study is to measure subjects' allocation of attention. One potential problem with this method is that visual attention may be dissociated from the point of regard. Most of the time, a subject's visual attention is associated with the point of fixation. This is usually referred to as overt attention, due to it being the component of visual attention that can be measured overtly (i.e., by an eye tracker).

However, it is also possible to fix one's gaze at some point, and yet allocate attention to a point or region outside the fovea. This is termed covert attention. The dissociation of attention from ocular fixation poses a challenge for the analysis of attention. When examining the fixations and saccades over the choice set, it is possible to specify what regions were fixated, however, it is not possible to be fully confident that these regions were attended to.

One explanation for the eye-tracking results could be that salience theory captures bottom-up modulation of covert attention (Desimone & Duncan, 1995; Itti & Koch, 2000). Covert attention is shifted towards salient attributes, causing limited cognitive resources to be varied across the visual field (Posner, 1980; Eriksen & Yeh, 1985). When, for example, quality is salient, then the value of quality is more easily perceived than the less salient value of price. Thus quality is weighted more heavily in decision making and the individual is more likely to choose the high quality good than when price is salient, implying preference reversals will occur.

Overt attention is allocated to the attributes least efficiently processed, i.e. towards those which are least salient. Therefore, in the example, although covert attention means quality is weighted more heavily in choice, overt attention captured by eye-tracking, is allocated to a greater extent towards price.

Yet it should be borne in mind that the dissociation of attention from fixation is usually observed in the lab only in specially designed tasks and with subjects specifically instructed not to move their eyes. It is common to assume that when subjects are free to move their eyes, that what they fixate on is a close proxy of what they are processing, as in the passage from Posner (1980) cited in section 2.2. That overt attention is more relevant than covert attention is also argued by Findlay and

Gilchrist (2003):

> *"We argue that spatial selection is best achieved by fixating an item so that it can be processed by the fovea: the processing advantage gained by fixating in this way is substantially greater than the covert attentional advantage."* (Findlay & Gilchrist, 2003, p.35)

Finally, it is possible that salience is not the mechanism behind individuals' pattern of preference reversals. One candidate is that of a Weber's law style effect for prices. Weber's law, a cornerstone of classical psychophysics, states that a given change in some stimulus becomes more difficult to detect as the baseline level of the stimulus increases. Thus the difference in utility terms between two prices becomes more difficult to detect as a mark-up is applied to both. Further support for this possibility comes from the fact that humans' innate sense of numerosity seems to obey a form of Weber's law (Dehaene, Dehaene-Lambertz, & Cohen, 1998).

That it is harder for subjects to determine the value difference between two prices with a higher price level implies both greater attentional resources allocated to the attribute of price and an increased tendency to choose the good which dominates on the attribute of quality, in line with both the behavioural and eye-tracking results. It also implies that subjects should find it harder to make choices with a high price level, which is consistent with decision times being significantly longer with a high price level (mean 4.67s) compared with a low price level (mean 5.10s, Wilcoxon signed-rank test p-value <0.001, $N=36$).

Another possibility consistent with this experiment is that the mechanism behind the behavioural results is cognitive in nature, rather than attentional, as proposed by salience theory. Independently, high prices may simply capture subjects' attention to a greater extent than low prices, leading to the observed eye-tracking results.

Such explanations are, of course, speculative, and it is important to note that this is a single test of salience theory using a single empirical tool to measure salience. Nevertheless, it would be instructive in future research to examine situations in which the behavioural predictions of salience theory and such alternative explanations differ.

It could be questioned whether the eye-tracking results are of any relevance, given that the behavioural predictions are correct. From an instrumentalist standpoint (e.g. Friedman (1953)) they are not: The theory's intended domain of application is choice, so its performance in a domain other than choice, such as the allocation of attention, is irrelevant.

However, as Bardsley et al. (2009) and Hausman (1992) argue, performance in a test domain which differs from the intended domain is important, as it informs about performance in the intended domain. Hausman (1992)'s analogy is test driving a car. Although one may only require a car for short journeys within a city, it is sensible to test it on a motorway as well, as poor performance here may indicate poor performance in the future on the intended route.

Looking at the engine of salience theory is an important indicator of how well it will perform in predicting choice in the future. Understanding the underlying mechanics is even more important given the wide range of applications which already exist.

The effect of salience theory is greater in hypothetical than in incentivized choice, in line with previous findings. Although empirical tests of salience theory are still scarce, there is a pattern emerging, specifically that the effects of salience theory are stronger with hypothetical than with incentivized choice. These results differ slightly from those reported in previous studies (Azar, 2011a; Dertwinkel-Kalt & Köhler, 2014), in that rather than finding no preference reversals with incentivized choice, a significant, but reduced number are found.

This finding raises a methodological issue. Experimental economists' standard practice is to incentivize the choices of their subjects. As incentivized choice is usually the topic of interest, this would appear to be the correct approach. However, in many situations, the rewards for choice are not proximate to the time a decision is made, and so are effectively hypothetical.

It is often the case that rewards in an experiment are proximate, whereas in the environment they are seeking to model, rewards are non-proximate. One example is eliciting preferences over redistributive policies. In the lab, rewards are proximate, since the individual feels the effects of an enacted policy in the size of their experimental payoff within minutes. However, when examining preferences over redistribution in the wild, it can be months or years between a vote, the enactment of policy and its effects coming in to force.

The results here are an indication that whether to incentivize or not is a somewhat more nuanced issue than usually considered by economists, and careful consideration should be made as to whether incentivization with a proximate reward is an appropriate choice, if the environment being modelled has non-proximate rewards.

2.6 Conclusion

The findings of this experiment are mixed. On the one hand, many preference reversals were observed, giving strong support for the behavioural predictions of salience theory. On the other hand, the eye-tracking hypotheses, designed to test the underlying mechanism of salience theory, were firmly rejected. Also, there is a large difference between the strength of the effect in incentivized and hypothetical contexts, the reason for which is not yet clear.

More empirical study of salience theory is hence necessary, especially using eye-tracking and other methods which allow the mechanism behind salience theory to be exposed. It is hoped that this will resolve some of the puzzles presented here. In particular, tests in which the behavioural predictions of salience theory and alternative mechanisms such as a Weber's law style effect differ would be useful. Another fruitful topic of study would be an examination of whether the effect of salience theory with real, but non-proximate rewards differs from its effect with proximate rewards, in the same way as its effect is different with hypothetical choice.

You've got to accentuate the positive: Theory and applications of warm glow attention 3

Edward J.D. Webb

Abstract

A model of warm glow attention is described, in which individuals increase their sense of well-being by diverting attention towards aspects of their environment which are favourable and away from aspects which are unfavourable. It is shown with applications that this can unify many disparate phenomena: it leads to endowment effects, to firms in Bertrand competition spuriously differentiating their goods, to individuals being overconfident about their own abilities, yet underestimating the abilities of an opponent, to the underestimation of the time needed to complete large projects and to firms being slow to adopt new technologies. Novel and testable hypotheses are generated.

3.1 Introduction

As humans, we like to feel good about ourselves. One way we do this is to view the world in a light that suits us, focusing on aspects we like and avoiding aspects we dislike. This paper seeks to explore the economic consequences of such behaviour, presenting a tractable, powerful model with many diverse applications.

When purchasing a good, an individual will regard the features of that product as more important than features possessed only by an alternative. In this way, she feels better about using her product and about her decision to buy it. Yet if at some point prior to purchase she had received the alternative as a gift, her ranking of the attributes of the product would change to make that product seem better.

When staying in the countryside, it is pleasant to focus on the peace and quiet it affords, and when in the city it is pleasant to ignore the noise and focus on the increased leisure opportunities. In their employment, individuals tend to regard the tasks they perform as more important and engaging than in an alternative position. Yet if an individual were to be transferred to a new position, her views on the relative merits of each position would no doubt change.

A particularly striking example of viewing the world in a distorted light to enhance one's own feelings is found in the world of sport. Fans of a team will swear it is the best in the world, despite the evidence of many decades of unmitigated mediocrity.

These examples are formalized in a model of warm glow attention. The various attributes of the options available in an individual's choice set are given some attentional weight, so that attributes with a higher weight receive more attention and are ascribed greater importance in decision making. It is thus strongly linked to the aforementioned salience theory of Bordalo et al. (2012b) and the focusing model of Kőszegi and Szeidl (2013).

The key aspect of the model, and the difference between it and salience theory and focusing theory is that the attentional weights are allocated in such a way that she receives the highest possible payoff. Attention is allocated to benefit the individual as much as possible.

Another feature of the model is persistence. Whilst it is always possible under some circumstances to justify changing one's point of view, it is difficult to believe some feature of a choice is insignificant in one instant, then believe it to be important and valuable the next. Hence any allocation of warm glow attention persists for some time after being made.

After describing the warm glow attention framework, several varied applications are made. It is shown that it leads to an endowment effect, with predictions that differ from both loss aversion and salience theory. In a model of Bertrand competition, it gives firms an incentive to spuriously differentiate their goods. It is shown that warm glow attention causes individuals to be overconfident, as they prefer contemplating being of high ability than of low ability. On the other hand, when facing an opponent, they will tend to underestimate her abilities. Related to overconfidence, it is also shown that warm glow attention can lead to large projects overrunning. Finally, it is shown that if firms are also influenced by warm glow attention, then it leads to them being slow to adopt new technologies.

One of the strengths of warm glow attention is hence that it can explain a wide variety of disparate phenomena. The desire to view the world in a different light to protect one's ego is pervasive, and influences many aspects of life.

As well as unifying different topics, warm glow attention allows new light to be shed on them. Thus several new, testable predictions are developed in the course of this chapter.

The models of attention mentioned previously use a similar mathematical formalism to the one here, with the valuation of choices altered by attentional weights. The principle point of departure is the determination of the weighting. In Bordalo et al. (2012b) and Kőszegi and Szeidl (2013), the weights are reference dependent, in that they are determined by the set of options the individual considers. Here, the weights are determined by the individual, and so will differ, for example, when valuing a good between the perspective of an owner and a non-owner.

Such context dependent effects are no doubt important, and warm glow attention theory is offered as complimentary to them. There are situations in which predictions coincide, and situations in which they differ, and it is an empirical question as to which effect dominates.

Another approach to studying attention in economics is rational inattention (Sims, 2003). However, this involves a rational decision maker allocating attention which is limited due to the limitations or excessive cost of its information gathering technology. As such, it is orthogonal to the current paper, which uses a behavioural methodology.

Warm glow attention is related to the mere ownership effect in psychology (Beggan, 1992; Shu & Peck, 2011). This has often been proposed as an alternative to loss aversion as an explanation of the endowment effect. It proposes that mere ownership of an object leads to individuals associating it with the self. Ego defence mechanisms then create a “warm glow” of ownership and lead to the object being regarded more favourably.

This model is more detailed than the mere ownership effect, as it examines how the warm glow increases the valuations of attributes within a choice object. It is also broader in scope, as its mathematical framework allows it to be applied not only to consumer goods, but also to other choice objects such as risky prospects.

Warm glow attention is also related to Kőszegi (2003), in which patients avoid receiving useful information about their health. The reason is that they wish to feel good about their future health status and to avoid potentially bad news which threatens this feeling. Barigozzi and Levaggi (2010) also show that if individuals gain anticipatory utility from contemplating the future, they may avoid potentially bad news in order to still feel good about the possibility of a favourable future state of the world. Thunström, Nordström, Shogren, Ehmke, and van ’t Veld (2013) present the results of a field experiment in which diners declined free calorific information about their meal. Hence they can continue to feel good about the possibility that the food is low in calories, despite picking a more unhealthy meal.

Section 3.2 gives the exposition of warm glow attention theory. Section 3.3 applies the theory in several different settings, the results of which are discussed in turn. In section 3.3.1, it is shown how it leads to an endowment effect, and in section 3.3.2 two firms in a model of Bertrand competition can spuriously differentiate their goods. In section 3.3.3 an individual has the option of undertaking a task and is overconfident about her ability, and in section 3.3.4 she underestimates the completion date of a large project. Finally, in section 3.3.5 it is shown that firms can be slow to adopt new technologies in a model of Cournot competition. Section 3.4 discusses general aspects of warm glow attention and section 3.5 concludes.

3.2 Theory

A consumption bundle $\boldsymbol{c}$ is a vector of $J \geq 1$ consumption attributes c_j, so that $\boldsymbol{c} = (c_1\ c_2 \ldots c_J)$. It is assumed that the individual's utility function is additively separable in attributes. Let $u_j(\cdot)$ be the utility function for attribute j. The individual's standard utility from consuming $\boldsymbol{c}$ is thus

$$U(\boldsymbol{c}) = \sum_{j=1}^{J} u_j(c_j)\,. \tag{3.1}$$

A warm glow attention vector $\boldsymbol{\eta}$ is a vector of J attentional weights η_j, one for each attribute, so that $\boldsymbol{\eta} = (\eta_1\ \eta_2 \ldots \eta_J)$ with $\sum_{j=1}^{J} \eta_j = 1$. The individual's utility from consuming $\boldsymbol{c}$ given attention vector $\boldsymbol{\eta}$ is modified to become

$$U(\boldsymbol{c}) = \sum_{j=1}^{J} \eta_j u_j(c_j)\,. \tag{3.2}$$

Note that any attention vector with $\eta_i = \eta_j = \frac{1}{J}\ \forall i, j$ corresponds to the standard case.

In salience theory and focusing theory, the weights are determined by the individual's consideration set, with attributes that have greater variation receiving a higher weights are determined by the consumption bundle the individual selects. Attention is directed in such a way to make the individual as well off as possible.

Definition 3.1. *Let $C \neq \varnothing$ be the individual's choice set and let $H \neq \varnothing$ be the set of available warm glow attention vectors. Then conditional on the individual choosing $\boldsymbol{c} \in C$, the attention vector $\boldsymbol{\eta} \in H$ selected satisfies*

$$\sum_{j=1}^{J} \eta_j u_j \left(c_j\right) \geq \sum_{j=1}^{J} \eta_j' u_j \left(c_j\right) \ \forall \ \boldsymbol{\eta}' \in H. \tag{3.3}$$

It may be that there are several $\boldsymbol{\eta} \in H$ satisfying equation (3.3), in which case some tie-breaking rule must be used.

For any given choice of consumption bundle, the warm glow attention vector is determined by the requirement that no alternative allocation of attention can make the individual better off. The individual then chooses the bundle that maximizes utility, given the attention allocation that follows.

Definition 3.2. *Let $\boldsymbol{\eta_c} \in H$ satisfy equation (3.3) given consumption bundle $\boldsymbol{c} \in C$. Then $\boldsymbol{c}$ is a solution to the individual's utility maximization problem (UMP) if*

$$\sum_{j=1}^{J} \eta_{\boldsymbol{c}j} u_j \left(c_j\right) \geq \sum_{j=1}^{J} \eta_{\boldsymbol{c}'j} u_j \left(c_j'\right) \ \forall \boldsymbol{c}' \in C. \tag{3.4}$$

Warm glow attention alters equation (3.1) by allowing the individual to focus to a greater extent on certain attributes than others. The utility gained from attributes with a high weight then contributes more to the overall valuation of a consumption vector than the utility gained from attributes with a low weight. It is important to note that a warm glow attention vector affects the assessment of attributes, rather than individual consumption vectors. Hence for a given attention vector, the individual's valuation of all consumption vectors is affected in the same way.

It should be emphasized that the allocation of attention should be interpreted as an inherent psychological mechanism, and not as a conscious choice by the individual. The individual chooses between alternative consumption vectors, just as in the standard case. Warm glow attention is then an involuntary ego-defense mechanism that shows the choice in the best light possible. In particular, this rules out sophisticated, strategic selection of warm glow attention vectors.

The theory is agnostic as to whether warm glow attention affects the individual's hedonic experience or only the decision making function. Under the former interpretation, warm glow attention causes the food that one eats to taste better and the films that one sees to be more thrilling. This goes against the usual economic approach in which the enjoyment one obtains from a given good is reasonably stable. On the other hand, if warm glow attention works as a mechanism to improve one's mood and feelings, it is plausible to think that it may directly affect the hedonic experience. Consuming something when in a good mood is a far more pleasurable experience than exactly the same consumption when in a bad mood.

Under the alternative interpretation, the hedonic experience is unchanged and equation (3.2) represents the value assigned to a consumption vector for decision making only. Warm glow attention in this case is focused on a higher order consideration of how satisfied one is with one's decision making. It enhances the pleasure of making good decisions.

We are required to make many decisions every day where there is no objectively right choice and the consequences are somewhat unclear. Warm glow attention helps us to regard our decisions as correct, instilling confidence in our decision making and encouraging us to make active decisions in the future.

While the empirical predictions of warm glow attention do not depend on which interpretation is held, there are implications for welfare considerations. Under the decision making interpretation, warm glow attention represents a choice bias, implying scope for beneficial interventions, which is not necessarily the case under the hedonic interpretation. Thus care must be taken when undertaking any welfare calculation. (Though note that warm glow attention is far from unique amongst behavioural theories in this regard.)

Care must also be taken in the definition of attributes, which is subjective, depends on framing and context, and may differ from individual to individual. Consider, for example, an individual deciding whether or not to buy a pizza at some price. Then it is appropriate to model the choice of having two attributes, taste and price. However, if the individual is choosing between several different types of pizza, it may be more appropriate to decompose the taste attribute and consider each topping as an attribute.

How a good is framed can also affect how an individual interprets it in terms of attributes. Section 3.3.2 shows an example of how firms can frame their goods as having different attributes in order to escape the Bertrand trap.

An important property warm glow attention is assumed to have is persistence. Warm glow attention is a way of viewing a choice situation or environment. Thus some consistency over time is required: the individual cannot regard an attribute as hugely important one minute and completely inconsequential the next.

The concept of persistence is formalized.

Definition 3.3. *Let H_t (H_{t-1}) be the set of available warm glow attention vectors at time t ($t-1$) and let $\boldsymbol{\eta}^*_{t-1}$ be the warm glow attention vector selected in period $t-1$. An individual exhibits* persistence *in warm glow attention if*

$$Prob.\left(\boldsymbol{\eta}^*_{t-1} \in H_t\right) > Prob.\left(\boldsymbol{\eta} \in H_t\right)\ \forall\ \boldsymbol{\eta} \in \left\{H_{t-1}/\boldsymbol{\eta}^*_{t-1}\right\}. \tag{3.5}$$

If the individual interprets her environment using some warm glow attention vector, it is more likely that she will continue interpreting it in that way in the future.

However, persistence is by no means permanent. An individual may justify a shift in her focus on attributes by updated information, by a change in the choice set, or simply by the passage of time.

In the applications shown in section 3.3, for ease of exposition a much stronger form of persistence is assumed.

Definition 3.4. *An individual exhibits strict persistence if*

$$H_t = \boldsymbol{\eta}^*_{t-1}. \tag{3.6}$$

A modification to the framework described above is required in order to address choice under risk. Suppose the individual is facing a choice set involving risky prospects. That is, there is a set of I possible states of the world S with typical element s_i and a prospect $\boldsymbol{c}$ specifies the outcome $\boldsymbol{c} = (c_1 \; c_2 \; \dots \; c_I)$ the individual receives in every state of the world.

The standard expected utility maximizing problem (EUMP) is

$$\max_{\boldsymbol{c}} \mathbb{E}U = \sum_{i=1}^{I} p_i U(c_i) \tag{3.7}$$

where p_i is the probability of state s_i occurring.

Following Bordalo et al. (2012b) and Kőszegi and Szeidl (2013), warm glow attention is adapted for risky choice by attaching attention weights to states. Thus the value the individual assigns to some prospect $\boldsymbol{c}$ given some attention vector $\boldsymbol{\eta}$ is

$$\mathbb{E}U = \sum_{i=1}^{I} \eta_i p_i U(c_i) \tag{3.8}$$

and analogous to definitions 3.1 and 3.2

Definition 1′. *Let* $\boldsymbol{c} \neq \varnothing$ *be the individual's choice set and let* $H \neq \varnothing$ *be the set of* *available warm glow attention vectors. Then onditional on the individual choosing* $\boldsymbol{c} \in C$*, the attention vector* $\boldsymbol{\eta} \in H$ *selected satisfies*

$$\sum_{i=1}^{I} \eta_i p_i U(c_i) \geq \sum_{i=1}^{I} \eta_i' p_i U(c_i) \ \ \forall \ \boldsymbol{\eta}' \in H. \tag{3.9}$$

Definition 2′. *Let* $\boldsymbol{\eta_c} \in H$ *satisfy equation (3.9) given* $\mathbf{c} \in C$*. Then* $\mathbf{c}$ *is a solution to the individual's expected utility maximization problem (EUMP) if*

$$\sum_{i=1}^{I} \eta_{\boldsymbol{c}i} p_i U(c_i) \geq \sum_{i=1}^{I} \eta_{\boldsymbol{c}'i} p_i U(c_i') \ \ \forall \ \boldsymbol{c}' \in C. \tag{3.10}$$

The set of attributes is thus identical to the set of states. It is in principle possible to decompose this further so that consumption in a given state has several attributes, however for clarity a state is always modelled as a single attribute.

A further natural extension of the theory, to a continuum of states, is given in section 3.3.3.

3.3 Applications

To illustrate the use and power of the warm glow attention framework, a number of disparate applications are developed. In section 3.3.1, a model of how warm glow attention causes an endowment effect, and how the predictions differ from loss aversion and salience theory. The number of attributes a good has is crucial in section 3.3.2, in which it is shown how firms find it advantageous to spuriously differentiate goods, so that consumers treat each firm's product as a separate attribute, when in fact they are homogeneous.

Uncertainty is introduced in section 3.3.3, where it is shown that if an individual is unsure about her own ability, warm glow attention causes her to be overconfident about her ability, and take on more difficult tasks. On the other hand, when unsure of an opponent's ability, she will underestimate it, and fail to take sufficient defensive action. In a similar vein, in section 3.3.4 it is shown that in projects with many components, uncertainty about the successful completion of each component combined with warm glow attention leads to the project overrunning.

Finally, in section 3.3.5, warm glow attention is applied to firms, rather than individuals. Warm glow attention causes a firm to value its own technology more and so it is reluctant to adopt new technologies, even though they may be superior.

3.3.1 Endowment effects

It is intuitive that when one owns a good, one prefers to pay greater attention to the attributes it has, and less on ones it lacks. This leads naturally to an endowment effect, and this is formally shown here. Thus in this first application, warm glow attention leads to a violation of the choices predicted by standard theory.

Endowment effects have been well documented empirically (see Kujal and Smith (2008) for an overview). Yet usually the explanation for them is loss aversion. In contrast, with warm glow attention it is not that giving up a product is especially painful, but rather that ownership causes an individual to like a product more.

A theoretical model similar to the experimental setup of Morewedge, Shu, Gilbert, and Wilson (2009) is therefore created, to demonstrate the difference between warm glow attention and loss aversion. They compared the WTP of brokers endowed with a mug buying similar mug on behalf of a client to brokers who were not endowed. They found that the bids of brokers who owned the mug were significantly higher than those from brokers who did not own a mug.

When individuals are buying a good on behalf of a client, loss aversion predicts that owning or not owning the product herself should have no difference to her valuation. However, warm glow attention predicts that owning the good should increase her willingness-to-pay, since her valuation of the goods is increased, so that she can feel a warm glow about owning it.

Bordalo et al. (2012a) show that salience theory can also lead to an endowment effect. However, salience predicts a reverse endowment effect for bads, which warm glow attention does not.

The novel prediction is also generated that endowment effects are attenuated by the similarity of goods.

3.3.1.1 Model

Let goods have two attributes, 1 and 2. Let there be two goods in the choice set, $\boldsymbol{x} = (\lambda v \quad (1-\lambda)\, v)$, $\boldsymbol{y} = ((1-\lambda)\, v \quad \lambda v)$, where $v \in \mathbb{R}$ and $\frac{1}{2} \leq \lambda \leq 1$. Thus $\boldsymbol{x}$ contains weakly more of attribute 1 and $\boldsymbol{y}$ contains weakly more of attribute 2. The parameter λ defines how *similar* $\boldsymbol{x}$ and $\boldsymbol{y}$ are, with lower λ meaning they are more similar. When $\lambda = 1$, the goods are completely dissimilar, as they have no attributes in common, and when $\lambda = \frac{1}{2}$, they are identical.

Let there be a unit mass of individuals who value a good with a_1 units of good 1 and a_2 units of good 2 according to the function $u\,(a_1, a_2) = \alpha a_1 + (1-\alpha)\, a_2$, with $0 \leq \alpha \leq 1$. Similar to Knetsch (1989), let half of the individuals be endowed with $\boldsymbol{x}$ and half with $\boldsymbol{y}$. It is assumed that in each group, α is uniformly distributed between 0 and 1. Following the endowment stage, individuals have the opportunity to exchange their endowed good for the alternative. As there are two separate actions, the procedure is modeled as having two periods.

Period 1: Individuals are endowed with either good $\boldsymbol{x}$ or good $\boldsymbol{y}$.

Period 2: Individuals are offered the chance to switch.

Standard case

Regardless of which good they were endowed with, individuals value the goods according to

$$u(\boldsymbol{x}) = \alpha\lambda v + (1-\alpha)(1-\lambda)v \qquad u(\boldsymbol{y}) = \alpha(1-\lambda)v + (1-\alpha)\lambda v. \tag{3.11}$$

Equating these expressions shows that the individual with parameter $\alpha' = \frac{1}{2}$ is indifferent between $\boldsymbol{x}$ and $\boldsymbol{y}$, with those with $\alpha > \alpha'$ preferring $\boldsymbol{x}$ and those with $\alpha < \alpha'$ preferring $\boldsymbol{y}$.

The fraction of consumers who switch from $\boldsymbol{x}$ to $\boldsymbol{y}$ is then $\frac{1}{2}\int_0^{\frac{1}{2}} d\alpha = \frac{1}{4}$, the fraction of consumers who switch from $\boldsymbol{y}$ to $\boldsymbol{x}$ is $\frac{1}{2}\int_{\frac{1}{2}}^{1} d\alpha = \frac{1}{4}$ and the total fraction of switches is $\frac{1}{2}$. There is thus an *endowment effect* if the number of switches is lower than the baseline of $\frac{1}{2}$.

Warm glow attention

There are two attributes, so a warm glow attention vector must contain two weights. The two weights are given the functional form $\frac{1}{1+\gamma}$ and $\frac{\gamma}{1+\gamma}$, with $\gamma \leq 1$. This represents the attribute given weight $\frac{1}{1+\gamma}$ capturing the greatest share of the individual's attention, and the attribute given weight $\frac{\gamma}{1+\gamma}$ the least share. The denominator ensures the weights sum to 1.

In period 1, H_1, the set of available warm glow attention vectors is thus

$$H_1 = \left\{ \frac{1}{1+\gamma}(1\ \gamma),\ \frac{1}{1+\gamma}(\gamma\ 1) \right\}. \tag{3.12}$$

In period 1, individuals cannot yet exchange their endowments. Thus the choice set in period 1 for the group endowed with $\boldsymbol{x}$ ($\boldsymbol{y}$) is $C_1 = \{\boldsymbol{x}\}$ ($C_1 = \{\boldsymbol{y}\}$). Then it follows that

Lemma 3.1. *For the groups endowed with* $\boldsymbol{x}$ *and* $\boldsymbol{y}$ *respectively, the warm glow attention vectors selected,* $\boldsymbol{\eta}_x^*$, $\boldsymbol{\eta}_y^*$, *are*

$$\boldsymbol{\eta}_x^* = \begin{cases} \frac{1}{1+\gamma}(1\ \gamma) & \textit{if } \alpha \geq (1-\lambda) \\ \frac{1}{1+\gamma}(\gamma\ 1) & \textit{if } \alpha < (1-\lambda) \end{cases} \quad (3.13a) \qquad \boldsymbol{\eta}_y^* = \begin{cases} \frac{1}{1+\gamma}(1\ \gamma) & \textit{if } \alpha \geq \lambda \\ \frac{1}{1+\gamma}(\gamma\ 1) & \textit{if } \alpha < \lambda. \end{cases} \quad (3.13b)$$

All proofs are contained in appendix B

Strict persistence is assumed. Thus the set of available warm glow attention vectors in period 2 for the group endowed with $\boldsymbol{x}$ ($\boldsymbol{y}$) is $H_2 = \{\boldsymbol{\eta}_x^*\}$ ($H_2 = \left\{\boldsymbol{\eta}_y^*\right\}$).

Given the allocation of warm glow attention in this stage, it is possible to show that

Proposition 3.1. *The fraction,* F*, of individuals who switch in period 2 is*

$$F = \begin{cases} \frac{\gamma}{1+\gamma} & \textit{if } 1 \geq \lambda \geq \frac{1}{1+\gamma} \\ (1-\lambda) & \textit{if } \frac{1}{1+\gamma} > \lambda \geq \frac{1}{2}. \end{cases} \qquad (3.14)$$

Examining this expression, it follows that

Corollary 3.1. *(i) There is an endowment effect for any* $\frac{1}{2} < \lambda \leq 1$.

(ii) $\frac{\partial F}{\partial \lambda} \leq 0$*, so that the size of the endowment effect is weakly decreasing in the similarity of the goods.*

(iii) F *is independent of* v*, and in particular there is an endowment effect for both goods and bads.*

3.3.1.2 Brokering

Let some individuals act as brokers for other individuals, that is, they must set their clients' WTP for a good $\boldsymbol{y} = ((1-\lambda)\,v \quad \lambda v \quad -p)$. It is again assumed that the brokers are incentivized to elicit their true estimate of their clients' WTP. The brokers themselves are endowed with $\boldsymbol{x} = (\lambda v \quad (1-\lambda)\,v \quad 0)$. Both brokers and clients have utility functions $u(\boldsymbol{x}) = \alpha\lambda v + (1-\alpha)(1-\lambda)\,v - p$ and $u(\boldsymbol{y}) = \alpha(1-\lambda)\,v + (1-\alpha)\,\lambda v - p$ for $\boldsymbol{x}$ and $\boldsymbol{y}$ respectively. There is a unit mass of brokers and a unit mass of clients, with α uniformly distributed between 0 and 1 in both groups. Each broker is matched with a single client, and brokers know the distribution of α, but not the specific α of their client.

It is again assumed that there are two periods.

Period 1: Brokers are endowed with $\boldsymbol{x}$.

Period 2: Brokers state WTP on behalf of their client.

Standard case

A broker wishes to state expected WTP for her client. The WTP of a client with taste parameter α is $wtp = \alpha(1-\lambda)\,v + (1-\alpha)\,\lambda v$ and so expected WTP is $\mathbb{E}wtp \int_0^1 (\alpha(1-\lambda)\,v + (1-\alpha)\,\lambda v)\,d\alpha$ or

$$\mathbb{E}wtp = \frac{1}{2}v. \tag{3.15}$$

Warm glow attention

There are now three attributes in the choice set. Hence the set of available warm glow attention vectors available in period 1 is modified to become

$$H = \begin{Bmatrix} \frac{1}{1+\gamma+\gamma^2}\left(1\ \gamma\ \gamma^2\right), & \frac{1}{1+\gamma+\gamma^2}\left(1\ \gamma^2\ \gamma\right) \\ \\ \frac{1}{1+\gamma+\gamma^2}\left(\gamma\ 1\ \gamma^2\right), & \frac{1}{1+\gamma+\gamma^2}\left(\gamma\ \gamma^2\ 1\right) \\ \\ \frac{1}{1+\gamma+\gamma^2}\left(\gamma^2\ 1\ \gamma\right), & \frac{1}{1+\gamma+\gamma^2}\left(\gamma^2\ \gamma\ 1\right) \end{Bmatrix}. \tag{3.16}$$

In period 1, the brokers' choice set is $H = \{\boldsymbol{x}\}$, from which it can be shown that

Lemma 3.2. *The warm glow attention selected for brokers in period 1 is*

$$\boldsymbol{\eta}_x^* = \begin{cases} \dfrac{1}{1+\gamma+\gamma^2}\left(1\ \gamma\ \gamma^2\right) & \text{if } \alpha \geq (1-\lambda) \\ \\ \dfrac{1}{1+\gamma+\gamma^2}\left(\gamma\ 1\ \gamma^2\right) & \text{if } \alpha < (1-\lambda)\,. \end{cases} \tag{3.17}$$

Strict persistence is once again assumed, so that the set of available warm glow attention vectors in period 2 is $H = \{\boldsymbol{\eta}_x^*\}$. It then follows that

Proposition 3.2. *The mean of brokers' estimates of willingness-to-pay for* $\boldsymbol{y}$ *is*

$$\mathbb{E}\overline{wtp} = \frac{v}{2\gamma^2}\left(2\lambda\left(1-\lambda\right)\left(1-\gamma\right)+\gamma\right). \tag{3.18}$$

Examining this equation, it is clear that

Corollary 3.2. *(i)* $\mathbb{E}\overline{wtp}$ *is greater than the estimate of WTP in the standard case for any* $0 \leq \gamma < 1$.

(ii) $\frac{\partial \mathbb{E}\overline{wtp}}{\partial \lambda} = -\frac{v}{\gamma^2}\left(1\gamma\right)\left(2\lambda-1\right) \leq 0$*, so that the more similar* $\boldsymbol{x}$ *and* $\boldsymbol{y}$*, the greater the mean estimate of WTP.*

3.3.1.3 Discussion

Warm glow attention leads to an endowment effect, a violation of standard theory. Endowment effects are extremely well documented, but are usually in economics accounted for by loss aversion.

However, warm glow attention differs from loss aversion in the results on brokering. When buying a good on behalf of another, loss aversion predicts no difference in whether the broker owns the item herself or not. Yet with warm glow attention, owning the item causes the individual to weight its good attributes more highly. This causes her valuation of the good to increase in general. Her willingness-to-pay on behalf of her client increases, in line with the results of Morewedge et al. (2009).

By valuing a good more highly on behalf of her client, she is able to feel better about the good she herself owns.

There is also a difference in the predictions of warm glow attention and salience, in that for bads, the former predicts a conventional endowment effect, and the latter predicts a reverse endowment effect. The predictions of warm glow attention are fulfilled by Coursey et al. (1987) and Dertwinkel-Kalt and Köhler (2014).

There is also a novel prediction generated by warm glow attention: from corollary 3.2, when goods are more similar, the endowment effect is attenuated. In the extreme case, in which the goods are identical, it disappears entirely. Warm glow attention causes individuals to value the attributes of good she is endowed with more. Thus the attributes the alternative good has in common also have their values increased. Thus the greater the amount of some common attribute the alternative possess, the less the difference in valuation induced by ownership.

An empirical prediction of warm glow attention is thus that is an individual is endowed with a chocolate bar, she is more likely to exchange it for a different flavour chocolate bar than for a mug.

There is a similar effect with brokering, in that the more similar the good owned by the broker to the one she is buying on behalf of her client, the greater the estimate of the client's WTP.

3.3.2 Spurious differentiation

Firms often expend a great deal of effort in branding their goods as distinct from their competitors' products. This happens even with goods such as milk or petrol, which are close to being perfectly homogeneous.

It is shown that the branding of a firm's good as distinct can help firms selling a homogeneous good to escape the Bertrand trap. If each firm's good is framed as having a unique attribute, then once a firm captures a consumer, warm glow attention causes consumers to value its good higher than its rival's. The individual focuses more on the unique attribute of the good they purchase and less on the unique attribute of the good they do not purchase, even though in the standard case the attributes are regarded as identical.

Thus warm glow attention leads to spurious differentiation of firms' products. This differentiation allows the firms to avoid the Bertrand trap and make positive profits.

The model also illustrates the importance of the assumption of persistence. Firms must capture consumers in an initial period of trading, before being able to exploit in a subsequent period the warm glow consumers still feel about the firm's good due to persistence.

Perloff and Salop (1985) introduce spurious differentiation as an addition to actual differentiation, but provide no behavioural mechanism behind why consumers' perception of products is erroneous.

Spiegler (2006) proposes that spurious differentiation arises from consumers using anecdotal reasoning. If the benefit a good provides is uncertain (the example used is a "quack" cure in which individuals may recover naturally rather than as a result of treatment), then consumers call to mind an anecdote in which the good was successful or not successful, and then judge it according to this anecdote. Goods for which the individual recalls a good outcome are hence differentiated from goods for which she recalls a bad outcome, allowing firms to make positive profits.

This model does not rely on such anecdotal reasoning, and so spurious differentiation occurs in riskless choice with goods that consumers are well acquainted with.

3.3.2.1 Model

Let there be two firms, 1 and 2, each costlessly producing a homogeneous good. A unit mass of consumers may purchase at most one unit of the good. If they purchase at price p they gain utility $u = v - p$, $v > 0$, and if they do not purchase their payoff is 0.

There are two periods, $t = 1, 2$, in which trade is possible and firms compete in prices.

Standard case

Firms produce a homogeneous good with identical marginal costs. Hence the standard Bertrand result applies, and in each period prices are $p_1 = p_2 = 0$ and both firms make 0 profit.

Warm glow attention

To illustrate the impact of firms' framing of their goods, consider firstly the case in which there is no differentiation, so that consumers regard both firms' goods to have the same attribute. Thus there are two attributes in individuals' choice sets: the good and price.

For a given individual, a similar functional form is assumed for warm glow attention is used as in section 3.3.1, so H, the set of available warm glow attention vectors in period 1 is

$$H_1 = \left\{ \frac{1}{1+\eta} (1 \ \eta), \frac{1}{1+\eta} (\eta \ 1) \right\}. \tag{3.19}$$

The individual either focuses on the good or on its price.

Individuals are heterogeneous in how much they are influenced by warm glow attention, with η uniformly distributed between $\gamma < 1$ and 1. (Note that $\eta = 1$ is equivalent to standard preferences since both attributes receive equal weighting.)

Persistence means that whatever vector is selected in period 1 is more likely to be in the set of available vectors in period 2 than the other. For clarity and simplicity, strict persistence is assumed so that $H_2 = \boldsymbol{\eta}_1^*$, where $\boldsymbol{\eta}_1^*$ is the warm glow attention vector selected in period 1. The results are qualitatively unchanged if this is relaxed, as long as there is some probability that for an individual in period 2 the set of available warm glow attention vectors consists only of the vector that was selected in period 1.

Since individuals are unsophisticated regarding warm-glow attention, they make no inference about the consequences of their purchase in period 1 on period 2. In each period they simply choose whichever option is optimal in that period. Let $p_i(t)$ be the price set in period t by firm $i \in \{1, 2\}$.

Given $p \leq v$ for both firms, it is clear that $\frac{1}{1+\eta}(1 \ \eta)$ satisfies equation (3.3) regardless of which firm they purchase from. The utility gained from each good $i \in \{1, 2\}$ is $\frac{1}{1+\eta}(v - \eta p_i(1))$ and choosing whichever good has the lowest price is a solution to their UMP.

It follows that in each period, individuals' willingness-to-pay for both goods is higher than in the standard case. However, all consumers still regard the goods as homogeneous. Thus the standard result still applies: $p_1(t) = p_2(t) = 0$, $t = 1, 2$, $\pi_1^T = \pi_2^T = 0$ where $p_i(t)$ is the price firm $i \in \{1, 2\}$ sets in period t and π_i^T is firm i's total profit.

Spurious differentiation

Suppose instead that firm 1's good is represented by the consumption vector $\boldsymbol{c}_1 = (v\ 0\ p_1(t))$ and firm 2's is represented by $\boldsymbol{c}_2 = (0\ v\ p_2(t))$, although the utility from either remains $u = v - p(t)$. Firms frame their good as possessing a unique attribute, which the other firm's good does not.

Since the benefit a consumer obtains from both goods is identical, such differentiation is spurious. The framing is irrelevant in the standard case and the Bertrand result still applies.

Since each good has three attributes, the set of available warm glow attention vectors is modified to become

$$H_1 = \left\{ \frac{1}{1+2\eta} (1\ \eta\ \eta), \frac{1}{1+2\eta} (\eta\ 1\ \eta), \frac{1}{1+2\eta} (\eta\ \eta\ 1) \right\}. \tag{3.20}$$

An individual assigns a high weight to one attribute and the same low weight to the other two attributes. It would also be possible to have an individual assign three different weights, as in section 3.3.1.2. However, the current formulation greatly simplifies the analysis without qualitatively affecting the results.

In period 1 individuals purchase from the lowest price firm, and the warm glow attention vector assigning weight $\frac{1}{1+2\eta}$ to that firm's unique attribute is selected. To verify that this is so, note that if $p_j(t) > p_i(1)$, the payoff from purchasing good j is $u(j) = \frac{1}{1+2\eta}(v - \eta p_j(1))$ but the payoff from purchasing i is $u(i) =$

$\frac{1}{1+2\eta}\left(v-\eta p_i\left(1\right)\right)$.

If the prices are equal, subjects are indifferent. In this case, it is assumed that each firm captures half of the market, and that in each half of the market, η is uniformly distributed between γ and 1.

The firms' equilibrium pricing strategies are found by backwards induction. The first step in this process is to analyse the three possible cases in period 2:-

(i) All consumers have $H_2 = \left\{\frac{1}{1+\eta}\left(1\ \eta\ \eta\right)\right\}$ due to previously purchasing from firm 1.

(ii) All consumers have $H_2 = \left\{\frac{1}{1+\eta}\left(\eta\ 1\ \eta\right)\right\}$ due to previously purchasing from firm 2.

(iii) Half of consumers purchased previously from firm 1 and have have $H_2 = \left\{\frac{1}{1+\eta}\left(1\ \eta\ \eta\right)\right\}$ and half purchased from firm 2 previously and have $H_2 = \left\{\frac{1}{1+\eta}\left(\eta\ 1\ \eta\right)\right\}$.

Lemma 3.3. *(i) If all individuals purchase from firm $h \in \{1,2\}$ in period 1, then the prices set in period 2 by h and $\ell \in \{1,2\}$, $\ell \neq h$ are*

$$p_h\left(2\right) = \left(\frac{1-2\gamma\left(1+\gamma\right)+\sqrt{5+4\gamma}}{2\left(1+\gamma\right)^2}\right)v, \qquad p_\ell\left(2\right) = \left(\frac{2+\gamma-\gamma\sqrt{5+4\gamma}}{2\left(1+\gamma\right)^2}\right)v. \tag{3.21}$$

Period 2 profits are

$$\pi_h^*\left(2\right) = \left(\frac{1-2\gamma\left(1+\gamma\right)+\sqrt{5+4\gamma}}{2\left(1-\gamma\right)\left(1+\gamma\right)^2}\right)\left(\frac{2+\gamma-\gamma\sqrt{5+4\gamma}}{1+\sqrt{5+4\gamma}}\right)v \tag{3.22a}$$

$$\pi_\ell^*\left(2\right) = \left(\frac{2+\gamma-\gamma\sqrt{5+4\gamma}}{2\left(1-\gamma\right)\left(1+\gamma\right)^2}\right)\left(\frac{\sqrt{5+4\gamma}-1-2\gamma}{1+\sqrt{5+4\gamma}}\right)v. \tag{3.22b}$$

(ii) If each firm captures half the market in period 1, prices set in period 2 are

$$p_1(2) = p_2(2) = v(1-\gamma) \tag{3.23}$$

with profits

$$\pi_1(2) = \pi_2(2) = \frac{v}{2}(1-\gamma). \tag{3.24}$$

After some algebra the condition for $\pi_h^* > \pi_\ell^*$ reduces to $-\frac{1}{2} < \gamma < 1$, which holds by assumption, so that capturing the whole market in period 1 yields greater period 2 profit than capturing none of the market.

The total profit of firm $i \in \{1, 2\}$ is then

$$\pi_i^T = \begin{cases} p_i(1) + \pi_h^* & \text{if } 0 \leq p_1(1) < p_j(1) \\ \frac{1}{2}p_i(1) + \frac{v}{2}(1-\gamma) & \text{if } p_i(1) = p_j(1) \\ \pi_\ell^* & \text{if } p_j(1) < p_i(1) \leq v. \end{cases} \tag{3.25}$$

It is clear that the firm never sets $p_i(1) > p_j(1)$, since undercutting its rival ensures higher profits in both periods. Any prices such that $p_i(1) = p_j(1)$ can be sustained as an equilibrium if neither wishes to deviate to set a slightly lower price. Deviating always yields a greater profit in the first period, but does not necessarily in the second. After some algebra, $\pi_h^* = \frac{v}{2}(1-\gamma)$ becomes

$$\gamma^4 - 4\gamma^3 - 4\gamma^2 + 4\gamma + 1 = 0. \tag{3.26}$$

Let $\gamma^* \approx 0.800$ be the unique solution of this equation in the range $0 \leq \gamma \leq 1$. Then for $\gamma < \gamma^*$, firms i always wishes to deviate to a lower price, so they will set $p_1(1) = p_2(2) = 0$. If $\gamma \geq \gamma^*$, any price is sustainable as an equilibrium as long as

$p_i + \pi_h^* \leq \frac{1}{2}p_i + \frac{v}{2}(1-\gamma)$ so that neither firm wishes to undercut its rival. It follows immediately that

Proposition 3.3. *The equilibrium prices set by firm* $i \in \{1,2\}$ *are*

$$p_i(1) = \begin{cases} 0 & \text{if } \gamma < \gamma^* \\ \{p : 0 \leq p \leq v(1-\gamma) - 2\pi_h^*\} & \text{if } \gamma \geq \gamma^* \end{cases} \tag{3.27a}$$

$$p_i(2) = \begin{cases} \left(\dfrac{1 - 2\gamma(1+\gamma) + \sqrt{5+4\gamma}}{2(1+\gamma)^2}\right) v & \text{if } p_i(1) < p_j(1) \\ v(1-\gamma) & \text{if } p_i(1) = p_j(1) \\ \left(\dfrac{2+\gamma-\gamma\sqrt{5+4\gamma}}{2(1+\gamma)^2}\right) v & \text{if } p_i(1) > p_j(1) \end{cases} \tag{3.27b}$$

with equilibrium profits

$$\pi_i^T = \begin{cases} \dfrac{v}{2}(1-\gamma) & \text{if } \gamma < \gamma^* \\ \left\{\pi : \dfrac{v}{2}(1-\gamma) \leq \pi \leq v(1-\gamma) - \pi_h^*\right\} & \text{if } \gamma \geq \gamma^*. \end{cases} \tag{3.28}$$

Firms earn positive profits for any $\gamma < 1$.

3.3.2.2 Discussion

The model shows that warm glow attention gives an incentive for firms to spuriously differentiate their goods. They use branding, advertising and other marketing tools to frame their goods, not necessarily as superior, but possessing unique attributes. This also helps resolve the paradox of a consumer regarding different goods as distinct, but at the same time believing the underlying products to be fairly similar.

In terms of warm glow attention itself, it shows the importance of which attributes goods are framed as having. It also demonstrates the impact of persistence, since when the full set of warm glow attention vectors is available, in period 1 consumers still purchase the cheapest good, as in the standard case. It is only in period 2 that differentiation is introduced via the persistence of the warm glow carrying over into repeat purchases. It is persistence that allows firms to capture customers and exploit them in future periods.

Another aspect the model highlights is that not all consumers need to be strongly influenced by warm glow attention for it to have a significant effect on the outcome of a market. Those with η close to 1 behave approximately as in the standard case, but firms are still able to differentiate themselves due to the influence of those with η close to γ.

There were only two periods in the current model. If there were three or more periods, each period with $t \geq 2$ would look the same, as once each firm captures "their" section of the market they can continue exploiting them in future periods. Thus the longer firms and consumers interact, the greater the increase in profits relative to the baseline.

Both firms entered the market in period 1, with no established firm already present. Given the advantage a firm gains from capturing customers, it would be instructive to examine the effect of warm glow attention on entry into a Bertrand market.

3.3.3 Overconfidence

Individuals are often observed to be overconfident about their own abilities. This was noted by Svenson (1981), who famously reported that a large majority of people regard themselves as being better than average drivers. It is proposed here that warm glow attention can be the driving force behind such phenomena.

In this model, the individual is imperfectly informed about her own ability to take on a task. Undertaking the task is thus a risky prospect and so the model illustrates how warm glow attention influences risky choice. Since the individual is better off when she is of high ability, she focuses more on these states of the world than on states in which she is of low ability.

The result is that she undertakes tasks of greater difficulty than before and to more than half of individuals believing themselves to be of greater than average ability is larger the less they are informed about their ability. This will be contrasted with the results of section 3.3.3.2, in which warm glow attention causes an individual to underestimate the ability of an opponent.

Aside from the extensive documentation of overconfidence in the psychology literature, it has also been demonstrated in economic environments, both experimentally (Camerer & Lovallo, 1999; Clark & Friesen, 2009) and in naturally occurring data (Caliendo & Huang, 2008). It has been examined in a wide variety of settings, including political behaviour (Ortolova & Snowberg, 2015), the principal-agent framework (de la Rosa, 2011) and in the creation of asset market bubbles (Scheinkman & Xiong, 2003).

However, econometric studies generally take overconfidence as a behavioural primitive. This paper contributes by proposing an underlying cause behind overconfidence, which generates novel predictions and suggests under what circumstances economic agents are most likely to be overconfident.

3.3.3.1 Overconfidence in oneself

Suppose an individual has the choice of performing some task, which leads to a reward, $r \geq 0$ at a cost $k \geq 0$, or declining to undertake it. The task's difficulty is represented by the parameter $t \geq 0$. $\frac{\partial r}{\partial t} > 0$ and $\frac{\partial r}{\partial k} > 0$ so that a more difficult task yields a greater reward but at a greater cost.

The individual's ability to do the task is described by the parameter a, $\frac{\partial r}{\partial a} > 0$, so that an individual of greater ability receives a greater reward. (Equivalently, one could assume $\frac{\partial r}{\partial a} = 0$ and $\frac{\partial k}{\partial a} < 0$, i.e. the reward for a given task is constant, but is less costly to obtain for an individual of higher ability.)

The individual observes the difficulty of the task, but is imperfectly informed about her own ability.

Let $r = at$, $k = t^2$, so that the payoff for an individual of ability a taking on a task of difficulty t is $u(a, t) = t(a - t)$, and let the individual's outside option if she declines the task give a payoff of 0.

There is a unit mass of individuals with ability uniformly distributed between $a = 0$ and $a = 1$. Each individual receives a signal of her ability prior to deciding whether or not to perform the task, with $\sigma = a + \zeta$, where ζ is an error term uniformly distributed between $-\varepsilon$ and $+\varepsilon$, $\varepsilon \geq 0$.

Standard case

Let $f_a(\cdot)$ be the probability density of an individual's belief over her ability and let $f_\sigma(\cdot)$ be the distribution of the signal. Then given she receives the signal σ, the individual's expectation of her ability is

$$\mathbb{E}[a|\sigma] = \int_{\max\{0,\sigma-\varepsilon\}}^{\min\{1,\sigma+\varepsilon\}} a f_a(a|\sigma)\, da \tag{3.29}$$

where $f_a(a|\sigma) = \frac{f_\sigma(\sigma|a) f_a(a)}{f_\sigma(\sigma)}$. Let h (ℓ) be the fraction of consumers who believe they are of greater than (less than) average ability, i.e. $\mathbb{E}[a|\sigma] > \mathbb{E}[a]$ ($\mathbb{E}[a|\sigma] < \mathbb{E}[a]$). It is possible to show that

Proposition 3.4. *Given a signal σ the individual undertakes tasks satisfying*

$$t \leq \mathbb{E}[a|\sigma]. \tag{3.30}$$

The expected ratio of h to ℓ is

$$\frac{\mathbb{E}[h]}{\mathbb{E}[\ell]} = 1. \tag{3.31}$$

Warm glow attention

With risky choice, the attributes present in a choice set are identified with the possible states of the world. Here, there is a continuum of states and thus a continuum of attributes.

To adapt warm glow attention to such a continuous setting, a typical allocation of warm glow attention is now a distribution over a continuous S with H the set of available distributions. If $\eta(s)$ the density of some warm glow attention distribution with $\int_S \eta(s)\, ds = 1$, then analogous to equation (3.2), the individual's valuation of some prospect $\boldsymbol{c}$ is

$$\mathbb{E}U = \int_S \eta(s)\, u(c)\, ds. \tag{3.32}$$

In the current model, it is assumed that if the highest (lowest) possible ability and individual can have is $\overline{a}$ ($\underline{a}$), warm glow attention distributions have densities with the functional form

$$f(a,\eta) = \frac{2(1-\eta)\, a + \eta(\overline{a}+\underline{a}) - 2\underline{a}}{\overline{a}-\underline{a}} \tag{3.33}$$

with $\gamma \leq \eta \leq 2-\gamma$, $\gamma \leq 1$. Thus a greater η represents a greater weight being placed on low a and a lesser weight being placed on high a. The set of possible warm glow attention distributions is $H = \{f(a,\eta) : \gamma \leq \eta \leq 2-\gamma\}$.

The value the individual assigns to performing a task of difficulty t given a signal σ is then

$$\mathbb{E}u(t,\eta) = t \int_{\max\{0,\sigma-\varepsilon\}}^{\min\{1,\sigma+\varepsilon\}} (a-t)\, f_a(a|\, \sigma)\, f_\eta(a|\, \eta)\, da. \tag{3.34}$$

The individual's payoff is higher if she is of high ability, so she prefers to focus on those states and neglect states in which she is of low ability. This leads to

Lemma 3.4. *If the individual takes on a task, the warm glow attention distribution selected is* $f_\eta(a, \gamma)$.

When taking on a task the warm glow attention distribution with the lowest possible η is selected, i.e. $\eta = \gamma$.

Denote $f_\eta(a, \gamma)$ as $f_\gamma(a)$ and let $\tilde{\mathbb{E}}$ denote a subjective expectation given warm glow attention. Then the individual's subjective expectation of her ability given a signal σ is $\tilde{\mathbb{E}}[a|\sigma] = \int_{\max\{0,\sigma-\varepsilon\}}^{\min\{1,\sigma+\varepsilon\}} a f_a(a|\sigma) f_\gamma(a)\, da$. It is then possible to show that

Proposition 3.5. *Case (i)* $\varepsilon \leq \frac{1}{2}$

Given a signal σ*, the individual takes on tasks satisfying*

$$t \leq \begin{cases} \frac{1}{6}(4-\gamma)(\sigma+\varepsilon) & \text{if } -\varepsilon \leq \sigma < \varepsilon \\ \\ \frac{1}{3}(\varepsilon(1-\gamma)+3\sigma) & \text{if } \varepsilon \leq \sigma < 1-\varepsilon \\ \\ \frac{1}{6}((4-\gamma)(\sigma-3)+(2+\gamma)) & \text{if } 1-\varepsilon \leq \sigma \leq \varepsilon. \end{cases} \tag{3.35}$$

The expected ratio of h *to* ℓ *is*

$$\frac{\mathbb{E}[h]}{\mathbb{E}[\ell]} = \begin{cases} \frac{3+2\varepsilon(1-\gamma)}{3-2\varepsilon(1-\gamma)} & \text{if } \gamma \geq 4-\frac{3}{2\varepsilon} \\ \\ \frac{4}{9}\varepsilon(4-\gamma)^2-1 & \text{if } \gamma < 4-\frac{3}{2}\varepsilon. \end{cases} \tag{3.36}$$

Case (ii) $\varepsilon > \frac{1}{2}$

Given a signal σ, the individual takes on tasks satisfying

$$t \leq \begin{cases} \frac{1}{6}(4-\gamma)(\sigma+\varepsilon) & \text{if } -\varepsilon \leq \sigma < 1-\varepsilon \\ \frac{1}{6}(4-\gamma) & \text{if } 1-\varepsilon \leq \sigma < \varepsilon \\ \frac{1}{6}((4-\gamma)(\sigma-\varepsilon)+(2+\gamma)) & \text{if } \varepsilon \leq \sigma \leq 1+\varepsilon. \end{cases} \tag{3.37}$$

The expected ratio of h to ℓ is

$$\frac{\mathbb{E}[h]}{\mathbb{E}[\ell]} = \frac{4}{9}\varepsilon(4-\gamma)^2 - 1. \tag{3.38}$$

Examining equations (3.36) and (3.38), it can be seen that

Corollary 3.3. *(i) If $\varepsilon = 0$, $\frac{\mathbb{E}[h]}{\mathbb{E}[\ell]} = 1$.*

(ii) $\frac{\mathbb{E}[h]}{\mathbb{E}[\ell]}$ is decreasing in γ and increasing in ε.

(iii) $\lim_{\varepsilon\to\infty} \frac{\mathbb{E}[h]}{\mathbb{E}[\ell]} = \infty$.

3.3.3.2 Underestimating an opponent

Suppose now there is a unit mass of individuals each facing an opponent of ability a, where the abilities of the opponents are uniformly distributed between 0 and 1. The individuals are imperfectly informed about the ability of their opponent.

Each opponent undertakes a task of difficulty $t \geq 0$ resulting in a transfer of at from the individual to the opponent. The individual can undertake defensive action $\delta \in [0,1]$ which reduces the transfer to $(1-\delta)\,at$ at a cost δ^2.

The loss an individual suffers when facing an opponent of ability a who takes an action of difficulty t after undertaking defensive action δ is then $L(t,a,\delta) = (1-\delta)\,at + \delta^2$

Before deciding on what defensive action to take, each individual perceives a noisy signal $\sigma = a + \zeta$ about the ability of her opponent, with ζ uniformly distributed between $-\varepsilon$ and $+\varepsilon$, $\varepsilon > 0$.

Standard case

The individual's expectation of their opponent's ability is unchanged from equation (3.29). Her expected loss given from undertaking defensive action δ given a task of difficulty t and a signal σ is found from $\mathbb{E}L(t, \sigma, \delta) = \int_{\max\{0,\sigma-\varepsilon\}}^{\min\{1,\sigma+\varepsilon\}} \left((1-\delta)\, at + \delta^2\right) f_a(a|\, \sigma)\, da$. Let h (ℓ) now be the fraction of consumers who believe their opponent to be of higher (lower) than average ability. Then

Proposition 3.6. *Case (i)* $\varepsilon \leq \frac{1}{2}$

Given a signal σ*, the defensive action the individual takes,* δ^**, is*

$$\delta^* = \begin{cases} \left(\dfrac{\sigma+\varepsilon}{4}\right)t & \text{if } -\varepsilon \leq \sigma < \varepsilon \\[2ex] \dfrac{\sigma}{2}t & \text{if } \varepsilon \leq \sigma < 1-\varepsilon \\[2ex] \left(\dfrac{1+\sigma-\varepsilon}{4}\right)t & \text{if } 1-\varepsilon \leq \sigma \leq 1+\varepsilon \end{cases} \tag{3.39}$$

The expected ratio of h *to* ℓ *is 1.*

Case (ii) $\varepsilon > \frac{1}{2}$

Given a signal σ*, the defensive action the individual takes is*

$$\delta^* = \begin{cases} \left(\dfrac{\sigma+\varepsilon}{4}\right)t & \text{if } -\varepsilon \leq \sigma < 1-\varepsilon \\[2ex] \dfrac{1}{4}t & \text{if } 1-\varepsilon \leq \sigma < \varepsilon \\[2ex] \left(\dfrac{1+\sigma-\varepsilon}{4}\right) & \text{if } \varepsilon \leq \sigma \leq 1+\varepsilon \end{cases} \tag{3.40}$$

The expected ratio of h to ℓ is 1.

Warm glow attention

The set of states of the world is identical to that in the previous section. Hence the set of possible warm glow attention distributions is again $H = \{f(a,\eta) : \gamma \leq \eta \leq 2-\eta\}$.

Then given $f(a,\eta)$, the subjective expected loss the individual suffers given a task t and signal σ is

$$\tilde{\mathbb{E}}L(t,\sigma,\delta) = \int_{\max\{0,\sigma-\varepsilon\}}^{\min\{1,\sigma+\varepsilon\}} \left((1-\delta)\,at+\delta^2\right) f_a(a|\,\sigma)\, f_\eta(a,\eta)\, da \tag{3.41}$$

The individual suffers a greater loss if she faces an opponent of higher ability, i.e.

Lemma 3.5. *The warm glow attention vector selected is $f_\eta(a,2-\gamma)$.*

Denote $f_\eta(a,\gamma)$ as $f_{2-\gamma}(a)$. Then the individual's subjective expectation of her loss given a task t and signal σ is
$\tilde{\mathbb{E}}L(t,\sigma,\delta) = \int_{\max\{0,\sigma-\varepsilon\}}^{\min\{1,\sigma+\varepsilon\}} ((1-\delta)\,at+\delta^2)\, f_a(a|\,\sigma)\, f_{2-\gamma}(a,\eta)\, da$ and it follows that

Proposition 3.7. *Case (i) $\varepsilon \leq \frac{1}{2}$*

Given a signal σ, the individual takes defensive action

$$\delta^* = \begin{cases} \frac{1}{12}(2+\gamma)(\sigma+\varepsilon)\,t & \text{if } -\varepsilon \leq \sigma < \varepsilon \\ \frac{1}{6}(3\sigma-(1-\gamma)\,\varepsilon)\,t & \text{if } \varepsilon \leq \sigma < 1-\varepsilon \\ \frac{1}{12}(2+\gamma+(4-\gamma)(\sigma-\varepsilon))\,t & \text{if } 1-\varepsilon \leq \sigma \leq 1+\varepsilon. \end{cases} \tag{3.42}$$

The expected ratio of h to ℓ is

$$\frac{\mathbb{E}[h]}{\mathbb{E}[\ell]} = \begin{cases} \dfrac{3-2\varepsilon(1-\gamma)}{3+2\varepsilon(1\gamma)} & \text{if } \gamma \geq 4-\dfrac{3}{4\varepsilon} \\[2ex] \dfrac{9}{4\varepsilon(4-\gamma)^2-9} & \text{if } \gamma < 4-\dfrac{3}{4\varepsilon}. \end{cases} \tag{3.43}$$

<u>*Case (ii)* $\varepsilon > \frac{1}{2}$</u>

Given a signal σ, the individual takes defensive action

$$\delta^* = \begin{cases} \dfrac{1}{12}(2+\gamma)(\sigma+\varepsilon)\,t & \text{if } -\varepsilon \leq \sigma < 1-\varepsilon \\[2ex] \dfrac{2+\gamma}{12}t & \text{if } 1-\varepsilon \leq \sigma < \varepsilon \\[2ex] \dfrac{1}{12}(2+\gamma+(4-\gamma)(\sigma-\varepsilon))\,t & \text{if } \varepsilon \leq \sigma \leq 1+\varepsilon. \end{cases} \tag{3.44}$$

The expected ratio of h to ℓ is

$$\frac{\mathbb{E}[h]}{\mathbb{E}[\ell]} = \frac{9-2\varepsilon(4-\gamma)(6-\varepsilon(4-\gamma))}{9+2\varepsilon(4-\gamma)(2(1-\gamma)+\varepsilon(4-\gamma))}. \tag{3.45}$$

Examining equations (3.42), (3.43), (3.44) and (3.45), it can be seen that

Corollary 3.4. *(i) Individuals undertake less defensive action than in the standard case.*

(ii) If $\varepsilon = 0$, $\frac{\mathbb{E}[h]}{\mathbb{E}[\ell]} = 1$.

(iii) $\frac{\mathbb{E}[h]}{\mathbb{E}[\ell]}$ is decreasing in γ and increasing in ε.

(iv) $\lim_{\varepsilon\to\infty} \frac{\mathbb{E}[h]}{\mathbb{E}[\ell]} = \infty$.

3.3.3.3 Discussion

The characteristic effect of warm glow attention with risky choice is that the individual focuses on favourable states and neglects unfavourable states. This is intuitively clear in the current example where the uncertainty is with regard to one's own ability: It is much nicer to think of oneself as possessing excellent skills than to contemplate being a dunce.

As was discussed above, overconfidence has been well documented. This paper contributes by proposing a mechanism behind the phenomenon and by generating new predictions.

In Svenson (1981)'s original paper, the percentage of drivers believing themselves to be of higher than average ability reached as high as 90%. Here, provided the feedback individuals receive is noisy enough, this percentage may be arbitrarily large. Indeed in the limit as ε becomes so great that the signal imparts no information, all individuals believe themselves to be of higher than average ability. A further prediction is that overconfidence is more likely to be observed with an individual contemplating a new, unfamiliar task than one they have tried before.

As ε becomes lower and the signal becomes more accurate, the degree of overconfidence becomes lower. The reason is that, although the individual still focuses more on high ability states than low ability ones, the maximum possible ability they have becomes lower. With a reasonably accurate signal, she knows that it is not possible for her to be of much higher ability than she is, and thus cannot focus on such possibilities when estimating her ability. It becomes harder to blame bad feedback on luck rather than on being of low ability.

In the limit, as $\varepsilon \to 0$, i.e. subjects are perfectly informed of their ability, overconfidence is not observed. Thus a novel prediction of the model is that uncertainty is required for an individual to be overconfident. (The uncertainty is modelled here as being with respect to the individual's ability, but it could equivalently be with respect to the difficulty of the task.) When she is certain of her own ability to do the task, there is no "wriggle room" to believe that she is more skilful than she in fact is, so she is not overconfident.

An implication of the model is that the accuracy and frequency of feedback given when undertaking a repeated task is crucial, and should be considered when designing feedback mechanisms. As overconfidence leads to individuals taking on more difficult tasks, this may benefit an employer, so that it is best to avoid giving detailed feedback. On the other hand, too little feedback could also lead to excessive risk taking.

Another aspect the model highlights is that the effects of warm glow attention depend very much on perspective. The space state of abilities and signalling technology is identical in both sections 3.3.3.1 and 3.3.3.2, so that in the standard case, an individual's estimate of ability for a given signal is the same regardless of whether it is her own or her opponent's ability she is estimating.

However, the consequences for an individual of a given state occurring differ between the cases. When it is her own ability she is estimating, her payoff is increasing in ability, whereas if she is estimating the ability of an opponent, her payoff is decreasing in ability. Thus with warm glow attention, she focuses more on states in which she is of high ability, but also on states in which her opponent is of low ability.

A novel prediction of the model then, is that Svenson (1981)'s result is reversed if individuals are estimating the ability of an opponent, rather than themselves.

The comparative statics of the model are similar to those for overconfidence. The greater the amount of noise, the greater the fraction of individuals who believe the opponent is of lower than average ability, with the fraction becoming 1 in the limit as the signal becomes uninformative. Again, it is easier to blame an unfavourable signal on bad luck if it is noisy.

On the other hand, if $\varepsilon = 0$ and the signal is perfectly informative, there is no underestimation. When she knows her opponent's ability with certainty, there is no scope for believing the ability is lower than it is.

There are many diverse applications of overconfidence, mostly taking it as a behavioural primitive. Warm glow attention explains the source of such overconfidence, there is much scope to expand on this model.

3.3.4 Project overruns

A topic related to overconfidence is the frequent overrunning of large projects. There are abundant examples of large scale projects which overran their estimated completion date and cost (Cantarelli & Flyvbjerg, 2013), such as the Channel Tunnel, the Øresund bridge in Denmark and the Second Avenue Subway in New York, yet to be completed despite being first planned in 1929.

It is demonstrated that warm glow attention can cause such overruns. If completion of each stage of a project is uncertain, individuals will focus on states of the world in which a stage is completed quickly and cheaply, and neglect states in which completion is difficult and costly.

Aside from the literature on overconfidence, such overruns are often addressed from a moral hazard perspective, and try to find the optimal procurement contract. (See for example Ganuza (2007), Bajari and Tadelis (2001) and Laffont and Tirole (1987).) As in section 3.3.3, the advantage of studying the problem from a warm glow attention point of view is that it proposes an underlying cause behind overruns. This gives rise to new predictions about where they are likely to occur and possible solutions.

Suppose an individual has undertaken a project giving some reward R which requires $N > 1$ components to complete, with at most one component being completed in a single discrete time period. In a given period, the individual incurs cost c which results in a component being successfully completed with probability p. With probability $1 - p$ the project encounters a problem and no component is completed, but the individual still incurs a cost of c. It is assumed that the individual is either committed to the project, or that the discounted value of R is sufficiently great compared to c that the individual never abandons the project.

The question of interest is the time the individual expects it will take to complete the project. Since the cost of the project is linearly related to the completion time, a time overrun of t implies a cost overrun of ct.

Standard case

The project requires at least N periods to complete. Completion in period $N + t$, $t \geq 0$ requires the success of the final component in that period, i.e. previously there must have been $N - 1$ successful and t unsuccessful periods. Thus Prob. (complete in $N + t$) $= \binom{N+t-1}{t} p^N (1-p)^t$ and if T is the completion time,

$$\mathbb{E}T = \sum_{t=0}^{\infty} \binom{N+t-1}{t} p^N (1-p)^t (N+t) = \frac{N}{p}. \tag{3.46}$$

Warm glow attention

A state of the world in the current model is defined as the completion of the project at some date $N + t$, where $t = 0, 1, 2, \ldots$. As there is no finite horizon by which the project is completed with certainty, there are an infinite number of states and so an infinite number of attributes.

In previous sections, a functional form was assumed for warm glow attention weights of the form $\eta_i = \frac{\gamma^i}{\sum_{i=0}^{J-1} \gamma^i}$, where J is the number of attributes. The same form is here extended to an infinite number of attributes, so that a warm glow attention vector is formed from weights $\eta_i = (1-\gamma)\,\gamma^i$, $i = 0, 1, 2, \ldots$ with a unique i assigned to each state. Note that this still implies that $\sum_{i=1}^{\infty} \eta_i = 1$, even though there are an infinite number of attributes.

The sooner the project is completed, the lower the individual's cost and the sooner she receives the reward. Hence the warm glow attention vector satisfying equation (3.3) assigns weight $(1-\gamma)\,\gamma^t$ to the state in which the project is completed in period $N+t$, $t = 0, 1, 2, \ldots$.

If $\widehat{\mathbb{E}T}$ is estimated completion time with warm glow attention, it then follows that

$$\begin{aligned}\widehat{\mathbb{E}T} &= \sum_{t=0}^{\infty} \gamma^t \binom{N+t-i}{t} p^N (1-p)^t (N+t) \\ &= \frac{N(1-\gamma)\,p^N}{(1-\gamma(1-p))^{N+1}}. \qquad (3.47)\end{aligned}$$

Since each element in the summation above is lower than the element in the analogous summation when calculating $\mathbb{E}T$ (equation (3.46)), it must be that $\widehat{\mathbb{E}T}$ is less than $\mathbb{E}T$. The individual is overoptimistic about the project's completion date.

The ratio of the actual expected completion time to the individual's estimated completion time is

$$\frac{\mathbb{E}T}{\widehat{\mathbb{E}T}} = \frac{1}{1-\gamma}\left(\frac{1-\gamma(1-p)}{p}\right)^{N+1}. \qquad (3.48)$$

This increases in N provided $\frac{1-\gamma(1-p)}{p} > 1$, which may be rearranged to be $(1-p)(1-\gamma) > 0$, which holds. Hence the larger the project, the greater the percentage of overrun will be observed.

3.3.4.1 Discussion

Overrunning projects are often linked to overconfidence. Here, it is shown that both may arise due to warm glow attention. Warm glow attention causes overruns by leading individuals to focus to a greater extent on states of the world in which the project is unproblematic and completed quickly.

It also predicts that the percentage overrun increases in N, the size of the project. To see why this is so, note that the warm glow attention weight given to a state with completion in period $N + t$ is independent of N. As N increases, the actual probability of early completion decreases, and the actual probability of late completion increases. Thus the individual pays more attention to unrealistically early states and less attention to the more realistic late states. A greater percentage of overrun is the result.

If $p = 1$, then equation (3.47) collapses to the standard case, and so the individual is no longer overoptimistic about the completion date. Similar to the results for overconfidence in section 3.3.3, there must be some uncertainty regarding the project the individual to have the scope to convince herself that it will be completed unrealistically early. The prediction is thus that projects with little to no uncertainty will not tend to overrun, even if they are very large.

The model was framed as an individual undertaking a project. There are many relevant examples of individual projects which tend to overrun, such as DIY or writing a PhD thesis. However, the introduction to this section motivated it by considering projects undertaken by large organizations consisting of many people.

So far only individuals have been modelled as being affected by warm glow attention. It is conventional in the behavioural industrial organization literature to consider firms as being fully rational, and only consumers to exhibit biases, heuristics and other departures from rationality.

However, firms consist ultimately of people, so it is plausible that if they are each influenced by warm glow attention, that an organizational structure will not necessarily eliminate it. Project related bonuses are common, so many individuals within the organization will have an incentive structure like the one described above, and so will tend to be overoptimistic about completion. It also implies that even if managers receive unbiased reports on expected completion time, they will tend to underestimate the probability that the reports are true.

The issue of firms being influenced by warm glow attention is further discussed in section 3.3.5.

The results of this section are dependent on the estimate of completion time being made from the perspective of those undertaking the project. If the estimation were made from an independent perspective, the results do not necessarily apply. However, an estimate made by someone not involved with the project is not necessarily correct. Consider, for example, a journalist who will be able to write a large number of articles if a high profile public project overruns. For her, she benefits from states in which the project is late, so warm glow attention will lead to her overestimating its completion time.

3.3.5 Slow technology adoption

The final model developed here illustrates how warm glow attention may affect a firm, rather than an individual. Specifically it shows how a firm may be slow in adapting to technological change.

It is often observed that established that established firms can be slow to adapt to market entrants, which is costly. Nokia is an example of a firm which did not adapt sufficiently quickly to the introduction by Apple of smartphones in the mobile market, and ceded most of its market share as a result[1]. Another example is the

[1] "Nokia slips to seventh in smartphone market" *The Wall Street Journal* November 14th 2012,

music industry, who were slow to react to the dangers and opportunities afforded by online distribution[2].

Here, such behaviour is explained in a model in which an established firm observes the efficiency of an entrant's technology imperfectly. Warm glow attention causes the firm to focus more on states in which its own technology is superior. Hence it chooses not to adopt the new technology and loses market share to the entrant as a result.

There are many studies of technology adoption. Aidrigan and Xu (2014) show empirically that it is hindered by financial frictions and Guadalupe and Kuzmina (2012) show that more technologically innovative firms are more likely to be acquired by multinationals. Milliou and Petrakis (2011) examine the effect of market competition on technology adoption, and find that it occurs faster in Cournot markets, such as the one used here, than in Bertrand markets. Besley and Case (1993) give an overview of technology adoption in developing cultures.

This model contributes to this literature by proposing a behavioural mechanism that may hinder adoption.

Let firm 1 be the incumbent in a Cournot market, using a well-established production method. Firm 2 is an entrant which use a new production method, which may be more or less efficient. The incumbent decides whether or not to adopt it or stay with its own established technology. Each firm has a constant marginal cost which is c_1 for firm 1 and c_2 for firm 2. c_1 is common knowledge, but c_2 is initially firm 2's private information.

The timing of the game is as follows:-

http://www.wsj.com/articles/SB10001424127887324556304578118360942919142 accessed 02/09/15.

[2]"From major to minor", *The Economist*, 10/01/2008, http://www.economist.com/node/10498664 accessed 02/09/15.

Period 1: Firm 1 observes some signal, s of firm 2's marginal cost which informs it that c_2 is uniformly distributed between $s - \varepsilon$ and $s + \varepsilon$, $\varepsilon \geq 0$.

Period 2: Firm 1 decides whether or not to adopt the new technology.

Period 3: Firm 1 observes firm 2's marginal cost.

Period 4: Production takes place.

To abstract from the effects of uncertainty on production, as opposed to the choice of production method, it is assumed that firm 1 observes c_2 after it has had the opportunity to adopt, but before production takes place, regardless of whether it adopts the entrant's technology or not.

After firm 1 observes c_2, each firm then simultaneously chooses some output level, q_1 and q_2 respectively. The inverse demand function is $p\left(Q\right) = 1 - Q$, $Q = q_1 + q_2$.

Standard case

Suppose the incumbent adopts the entrant's production technology so that each have marginal cost c_2. Then the profits of each firm are

$$\pi_1^A = q_1\left(1 - Q - c_2\right) \qquad \pi_2^A = q_2\left(1 - Q - c_2\right) \tag{3.49}$$

and it is a standard result that $q_1^* = q_2^* = \frac{1}{3}\left(1 - c_2\right)$ and $\pi_1^{A*} = \pi_2^{A*} = \frac{1}{9}\left(1 - c_2\right)^2$.

If firm 1 remains with its own technology, firms' profits are

$$\pi_1^{NA} = q_1\left(1 - Q - c_1\right) \qquad \pi_2^{NA} = q_2\left(1 - Q - c_2\right) \tag{3.50}$$

and it is again a standard result that $q_i^* = \frac{1}{3}\left(1 + c_j - 2c_i\right)$ and $\pi_i^{NA*} = \frac{1}{9}\left(1 + c_j - 2c_i\right)^2$, $i, j \in \{1, 2\}$, $i \neq j$.

The expected profits if firm 1 does or does not adopt the new technology are

$$\mathbb{E}\pi_1^A = \int_{s-\varepsilon}^{s+\varepsilon} \frac{(1-c_2)^2}{18\varepsilon} dc_2 = \frac{\varepsilon^2 + 3(1-s)^2}{27} \tag{3.51a}$$

$$\mathbb{E}\pi_1^{NA} = \int_{s-\varepsilon}^{s+\varepsilon} \frac{1+c_2-2c_1}{18\varepsilon} dc_2 = \frac{\varepsilon^2 + 3(1+s-2c_1)^2}{27}. \tag{3.51b}$$

Then combining equations (3.51a) and (3.51b), it is found that the incumbent adopts the entrant's production method if.

$$s < c_1 \tag{3.52}$$

Warm glow attention

The state space is continuous, and so as in section 3.3.3 the allocation of warm glow attention is modelled as a distribution with density $f(c_2, \eta)$. $f(c_2, \eta)$ is given the functional form

$$f(c_2, \eta) = \frac{1}{\varepsilon}((1-\eta)(c_2 - s) + \varepsilon) \tag{3.53}$$

where $\gamma \leq \eta \leq 2 - \gamma$, $\gamma < 1$. Thus low η represents the firm putting a high weight on low values of c_2 and high η represents the firm putting a high weight on high values of c_2.

It was supposed that the firm was well established in the market. Thus when comparing its own technology with other possible technologies without the threat of imminent entry, warm glow attention was allocated so as to paint its own technology in the best light possible. Persistence then implies that it is likely to still do so when the new firm enters. Therefore, it is assumed that due to persistence the set of available warm attention distributions is limited to be

$$H = \left\{ f(c_2, \gamma) = \frac{1}{\varepsilon}((1-\gamma)c_2 + \varepsilon - s(1-\gamma)) \right\}. \tag{3.54}$$

The value the firm then assigns to adopting becomes

$$\mathbb{E}\pi_1^A = \int_{s-\varepsilon}^{s+\varepsilon} \frac{(1-c_2)^2}{18\varepsilon} f(s,\gamma)\, dc_2 = \frac{\varepsilon^2 + 3(1-s)^2 - 2\varepsilon(1-\gamma)(1-s)}{27} \tag{3.55}$$

and the value it assigns to remaining with its own production method is

$$\begin{aligned} \mathbb{E}\pi_1^{NA} &= \int_{s-\varepsilon}^{s+\varepsilon} \frac{(1+c_2-c_1)^2 f(c_2)}{18\varepsilon} dc_2 \\ &= \frac{1}{27}\left(\varepsilon^2 + 3(1+s-2c_1)^2 + 2\varepsilon(1-\gamma)(1+s-2c_1)\right). \end{aligned} \tag{3.56}$$

Combining equations (3.55) and (3.56), it follows directly that

Proposition 3.8. *The condition for the incumbent to adopt the entrant's technology is*

$$s < c_1 - \frac{1}{3}\varepsilon(1-\gamma). \tag{3.57}$$

Comparing inequalities (3.52) and (3.57), it is seen that firm 1 must receive a lower signal of the new cost function to adopt the new technology and warm glow attention leads to slow adoption of the new technology.

3.3.5.1 Discussion

The result of this section is qualitatively similar to section 3.3.3.2. The firm underestimates the ability of a rival due to warm glow attention, causing it to focus to a greater extent on states of the world in which its own technology is superior. Thus it is slow to adopt to new, more efficient production technologies.

A critical part of this section is that it supposes that profit maximization firms exhibit an attentional bias. As was mentioned in section 3.3.4.1, this is unusual in the behavioural economics literature. Rational inattention theory is applied to firms, but this theory is consistent with a conventionally rational firm with limited of costly information gathering technology (Sims, 2003). Here, a true behavioural decision making mechanism is applied to firms.

Warm glow attention was proposed as an inherent psychological effect, and firms obviously do not have a “mind” as such from which psychological effects can spring. However, as Mitt Romney famously observed, corporations are people; firms’ decisions are ultimately made by a collection of individuals. Thus a firm exhibiting the effects of warm glow attention is a shorthand for the individuals taking decisions within the firm exhibiting warm glow attention effects, with an organizational structure which does not serve to eliminate them.

Evidence in favour of warm glow attention influencing firms’ decisions is found in studies of psychological ownership (Shu & Peck, 2011; Kim & Johnson, 2012, 2014, 2015). These show that biases due to ownership, most prominently the endowment effect, are dependent not on the legal fact of ownership, but on whether individuals psychologically feel that they own something. Pierce, Kostova, and Dirks (2001) and van Dyne and Pierce (2004) extend this concept to organizations, showing that employees can associate the firm with the self, and feel that they own it, despite not doing so in a legal sense.

If those making decisions within a firm associate it with the self and feel a sense of psychological ownership, then this implies that they are more likely to make such decisions as if on behalf of themselves, rather than on behalf of the organization. This further implies that personal deviations from conventional rationality, in particular warm glow attention, are likely to be exhibited on a firm level.

This also helps justify the assumption of persistence. If people associate the firm with the self, they should tend to allocate warm glow attention to paint it in the best light possible on a regular basis, leading to persistence when considering changing the technology belonging to the firm for one not owned by the firm.

Given that psychological ownership is a key mediator for firms exhibiting warm glow attention effects, a prediction is those firms in which psychological ownership is stronger should be less nimble in adopting to a changed environment. van Dyne and Pierce (2004) show that psychological ownership in firms is positively related to organizational commitment and job satisfaction, amongst other factors.

This section illustrates one consequence for firms under the assumption that firms are influenced by warm glow attention. Whether they in fact do so remains an open question. Closer investigation of the interaction of individuals influenced by warm glow attention within an organizational structure is required, and this is an avenue for future research.

3.4 General discussion

The idea that people will look at the world in the way that is good for them is intuitive. People protect their egos and feel better about themselves by accentuating the positive and eliminating the negative.

This paper formalizes the idea, and illustrates its potential in economic research. It has been introduced in several diverse settings. In each it has been shown to have large effects, both on individuals and firms. It has also generated many novel predictions.

Warm glow attention is rooted in boosting the feelings of the self, which means it is highly dependent on perspective. Thus, for example, an individuals' evaluation of some good depends very much on whether she owns or intends to purchase it or does not.

Another characteristic is that some of the consequences of warm glow attention, for example overconfidence, require some uncertainty. There must be some possibility of the individual being highly skilful for one to focus on it being so; one cannot make the world seem better by focusing on impossibilities.

The framework is flexible enough to be applied to attentional effects beyond those considered here. One example is that one sometimes cannot avoid paying attention to certain things. If someone talks at length about an unattractive feature of a good one owns or mentions a possible future event with serious negative consequences, it is hard to stop them coming to the front of one's mind, however much one would prefer to ignore them. Warm glow attention can accommodate this effect by a suitable reduction of the available set of attention vectors. A practical application would be firms manipulating the set of available attentional vectors using advertising to draw attention to positive attributes of a good, or negative attributes of consumers' current situation, in order to create demand for a product.

Another strength of the broad scope of the approach is that it unifies various observed phenomena, such as overconfidence and the endowment effect. This allows for a greater understanding of these effects and under which circumstances they are likely to occur.

Not only does warm glow attention bring together many different topics, it provides new insight into them. Several novel predictions have been made, and some of these are summarized in table 3.1. In general these hypotheses are readily testable, and so an empirical examination of the validity of these claims is very much desirable.

The predictions of warm glow attention can be highly dependent on how the set of attributes is defined. This can be a strength, in that it predicts the consequences of consumption being framed in different ways. However, in common with other framing effects, this is sensitive to the somewhat arbitrary definition of the distinct frames. (For example, prospect theory (Kahneman & Tversky, 1979) predicts that individuals behave differently depending on whether something is framed as a loss or a gain relative to some reference point. However, the reference point is exogenous and must be chosen by the modeller.) Hence, as with similar theories based on

Table 3.1: Summary of some novel predictions of warm glow attention

Topic	Prediction
Endowment effect	The more similar two goods, the weaker the endowment effect.
Bertrand competition	Framing homogeneous goods as possessing distinct attributes allows firms to escape the Bertrand trap.
Overconfidence	Some uncertainty about ability is required for an individual to be overconfident about her ability.
	Individuals underestimate the ability of opponents.
Overrunning projects	The larger a project is, as measured by the number of components, the greater the percentage of time it overruns.

framing, care must be taken in applications to define attributes in ways that are reasonable and plausible.

This issue is also present when applying warm glow attention to risky choice. In all models here, it is assumed that individuals treat states as the attributes of a risky prospect. While this is a natural way to approach the problem, there are other alternatives. It could be that the expected utilities of prospects are treated as attributes. Such considerations are no doubt context dependent, and some evidence for this is found in Arieli et al. (2011), who show that individuals integrate the components of money lotteries differently depending on whether expected utility is easy to calculate or not.

The conclusion is that care must also be taken with risky choice to make sure that the definition of attributes as states, expected utility or otherwise is appropriate.

Several of the applications required the assumption of persistence. Persistence is an intuitive property for warm glow attention to have, as it implies that one cannot change the way one views one's environment instantly and at will. However, it is also readily observable that people can be very adept at justifying changes in opinion and attitudes. Therefore closer investigation is needed into the events and circumstances under which individuals can justify changes to the focus of their attention and how radical such changes can be.

An extreme form of persistence was assumed in applications for expositional simplicity. However the conclusions are qualitatively unchanged if warm glow attention only persists with some probability.

There are several ways the warm glow attention framework could be extended. One of the most prominent is in allowing for individuals to be sophisticated. In models of time inconsistency, sophisticated individuals realize that their future selves will have different preferences, and may choose to restrict their available future actions as a result.

A similar approach could be used for warm glow attention. Sophisticated individuals may realize that their future selves will have a different point of view. As an example, imagine deciding whether to leave one's house to socialize with friends. You are tired, and the sofa on which you are sitting is comfortable, so going out is not a particularly attractive prospect. However, you know that once you are there, you will regard it as a more enjoyable experience than remaining at home, so you go out anyway.

So far, warm glow attention has been considered an intrinsic mechanism, out of the individual's control. Another possible extension is to allow individuals some freedom to choose the focus of their attention. This is of possible use in studying dieting behaviour, as the individual may consciously reduce the attentional weight placed on sugar, fat, etc. and increase the weight on healthiness.

3.5 Conclusion

This chapter gives an exposition of a new attentional framework for choice: warm glow attention. This has been shown to cause an endowment effect, to lead to spurious product differentiation, and also to individuals being overconfident and underestimating enemies.

The models presented here are intentionally simple, as they are intended to give an overview of the potential of warm glow attention. As such, each could be developed further.

There are also many other possible applications. One example is politics. If politicians present multidimensional policy platforms, voter's evaluation of them will be influenced by focusing on the policies which are best for them and neglecting the policies they dislike. It could also mean politicians benefit from making promises that fully rational voters regard as incredible. As longer as there is some possibility that a manifesto pledge is enacted, then voters will focus on states of the world in which this is the case, and neglect states in which the promises are broken.

The warm glow attention framework affords many exciting opportunities for future research.

If it's all the same to you: Blurred consumer perception and market structure 4

Edward J.D. Webb

An earlier version of this chapter was published as a working paper (Webb, 2014a), and it builds on a masters thesis (Webb, 2014b).

Abstract

Consumers with bounded perception treat sufficiently similar goods as homogeneous. The effects of bounded perception on a vertically differentiated duopoly with sequential quality choice are examined. With fixed costs of quality, the market becomes more concentrated. With marginal costs of quality, the second mover may profitably imitate the product of its rival, and the market is either more or less concentrated depending on how bounded perception is. When firms incur entry costs, neither firm may opt to produce with marginal costs, whereas at least one firm always produces with fixed costs.

4.1 Introduction

Can we always tell similar goods apart? The quality of a good is a nebulous attribute which is hard to assess at first glance, and so given this limitation to our perception, it is interesting to examine how it influences the selection of goods we are presented with. This is done by looking at firms' product design and the degree of concentration in a market in which consumers are bounded in their ability to distinguish between goods of different quality.

Intuitively, it is not clear whether bounded perception should help or hinder a given firm. On the one hand, it is more difficult for it to distinguish its product. On the other hand, it could produce a "knock off" good and ride on the success of its rival.

It is shown that both intuitions may be correct depending on the market structure, specifically whether the cost of quality is fixed or marginal. A clear illustration is thus given of the importance of studying the interaction between individuals with decision making limitations and other economic agents, as the results are non-straightforward and not necessarily robust to small changes in the market structure.

The model used is a vertically differentiated duopoly, with results of fixed and marginal costs of quality contrasted. With fixed costs, firms must distinguish themselves or fall into the Bertrand trap, leading to greater market concentration. With marginal costs, one firm can "imitate" the other by producing a good of the same perceived quality but with lower marginal costs. The market may then be either more or less concentrated depending on how bounded consumer perception is.

Section 4.2 reviews some of the existing literature on perception. Section 4.3 formalizes bounded perception, and sections 4.4 and 4.5 examine its effect on a model of vertical differentiation with sequential quality choice with respectively fixed and marginal costs of quality. Section 4.6 presents an extension in which firms pay a cost to enter the market. Section 4.7 discusses the findings and finally section 4.8 concludes.

4.2 Literature review

Bounded perception is formalized using Rubinstein (1988)'s concept of a *similarity relation*, which specifies which elements of a set are sufficiently similar to be regarded as identical. Similarity relations are related to earlier work by Luce (1956) on semi-orders and are consistent with much psychophysical research on stimulus detection, particularly the Weber-Fechner law (Falmagne, 2002).

Similarity relations have also been employed to explain anomalies in lottery choice (Aizpurua, Ichiishi, Nieto, & Uriarte, 1993; Leland, 1994; Buschena & Zilberman, 1999) and intertemporal choice (Leland, 2002).

There are fewer articles studying the impact of similarity relations beyond individual choice, but Bachi (2014) uses a Rubinstein similarity relation in a duopoly market in which consumers randomize over firms when prices are sufficiently close together. Chapter 5 uses an identical behavioural mechanism to this article in a vertically differentiated market, but with simultaneous rather than sequential quality choice.

Kalayci and Potters (2011a) investigate experimentally a market in which buyers find it difficult to distinguish between goods. Their experimental design represents goods' value by a sum which is hard to accurately compute in the time allowed for decision making. They find that buyers make a significant number of "mistakes" by not purchasing the good offering the greater surplus.

There has also been some investigation of consumers' perception in the marketing literature. (For example Chandon and Ordabayeva (2009) study how individuals' assessment of a product's volume can be influenced by packaging shape and Kwortnik, Creyer, and Ross (2006) examine the effects of labelling on consumer choice.) However, the studies in this field are mostly targeted at very specific effects with very specific product types.

That consumers are less informed about goods than firms means the situation is somewhat similar to markets with asymmetric information (Akerlof, 1970). However, the key difference is the behaviour of consumers. They act as if fully informed, but with an inherent psychological limitation to their decision making, rather than being aware of the information asymmetry and taking it into account. There is no possibility to learn from past poor transactions, which is an essential feature of a market for lemons.

The information structure also differs in that consumers are perfectly informed about goods which are sufficiently dissimilar, but cannot distinguish at all between similar goods. Even if this structure were approximated by consumers receiving a noisy signal of quality, they do not update prior beliefs in the canonical way. In particular, consumers do not infer anything about the behaviour of firms given that they see two goods of apparently the same quality.

Thus firms face fundamentally different incentives than in a standard asymmetric information situation, and a typical unravelling result is not observed. To emphasize the difference, Baltzer (2012) examines asymmetric information in a market setting very similar to the one considered here, and does find unravelling, in contrast to the findings of this article.

The economic institution utilized is a vertically differentiated product market. Models of vertical differentiation are ideal settings in which to examine perceptual limitations, since firms' profits depend heavily on their ability to distinguish their goods in the eyes of consumers.

Early development of the vertical differentiation framework was undertaken by Gabszewicz and Thisse (1979), Shaked and Sutton (1982) and Hung and Schmitt (1988). There now exists a profusion of theoretical models of vertical differentiation and many empirical applications.

A common behavioural approach when incorporating psychological insights into economics is followed: a standard economic model is taken and an extra parameter is added such that the original model is nested within the new. To do this, a baseline model is required, and the particular model used is most similar to Motta (1993) and Lutz (1997) in the case of 0 entry costs.

4.3 Bounded perception and consumer behaviour

The notion of bounded perception of similar goods will now be formalized. A good $c = (q, -p)$ has two attributes, quality q and price p, with $Q = [0, \infty)$ the set of qualities and $P = [0, \infty)$ the set of prices. The set of goods is then $C = Q \times -P$. $\succsim_u$ is a preference relation on C satisfying the standard assumptions. Let $\sim_s$ be a *Rubinstein similarity relation* (Rubinstein, 1988) on Q. If $q \sim_s q'$, then q and q' are sufficiently similar that an individual regards them as identical. If $q \nsim_s q'$ then an individual regards them as dissimilar.

Together $\succsim_u$ and $\sim_s$ induce a decision preference relation $\succsim_d$ on X in the following way:

(i) If $q \nsim_s q'$ and $c \succsim_u c'$, then $c \succsim_d c'$. (ii) If $q \sim_s q'$ and $p \leq p'$, then $c \succsim_d c'$.

Note that $\succsim_d$ is complete, but not generally transitive. $\succsim_u$ may be thought of as a "true" underlying preference relation and $\succsim_d$ as the relation actually used by in individual in decision making, given the limitations captured by $\sim_s$.

Functional forms are assumed for the various relations. Let $\succsim_u$ be represented by the utility function $u = \alpha q - p$, $\alpha \in \mathbb{R}_+$ and let $\sim_s$ be represented by the parameter $\delta \in [1, \infty)$. δ is termed the *perception threshold*. Let q_h, $q_\ell \in Q$ be the qualities of two goods h and ℓ, with $q_h \geq q_\ell > 0$, then:

(i) If $\frac{q_h}{q_\ell} \geq \delta$, $q_h \nsim_s q_\ell$. (ii) If $\frac{q_h}{q_\ell} < \delta$, $q_h \sim_s q_\ell$.

Furthermore, if $\frac{q_h}{q_\ell} < \delta$ so that q_h and q_ℓ are regarded as similar, the individual is assumed to perceive both goods as having quality $q' = \lambda q_h + (1 - \lambda) q_\ell$, $\lambda \in [0, 1]$.

Thus if two goods are dissimilar enough in quality that the ratio of the high to low quality exceeds the threshold, they are perceived as heterogeneous. On the other hand, if they are similar enough that the ratio of the high to low quality falls below the threshold, they are treated as homogeneous: when expressing a preference between the two, only price information is taken into account and not quality information.

That the perception threshold applies to the ratios of goods' qualities means that as the absolute level of quality increases, so does the absolute difference in quality required for an individual to perceive them as heterogeneous. This is analogous to the classical Weber-Fechner law of psychology, which states that the smallest difference of intensity of some stimulus that an individual can detect (the *just-noticeable difference*, hereafter JND) is greater the greater the absolute level of stimulus.[1]

Note there is a discontinuity in the perception of quality at the perception threshold δ: the consumer perceives quality perfectly when $\frac{q_h}{q_\ell} = \delta$, yet if q_h is reduced or q_ℓ raised by even an infinitesimal amount, the goods are regarded as homogeneous.

The unrealistic nature of this discontinuity may be rationalized by considering it as a simplification of a "smoother", probabilistic mechanism, in which there is some probability of perceiving the goods as homogeneous which becomes greater as qualities become more similar. This is again analogous to psychology, in which in modern research the JND is usually defined as the difference at which stimuli can be

[1]In fact it is equivalent to a common formulation of the law, $\frac{\Delta I}{I} = K$, where I is the intensity if some stimulus, ΔI is the JND and K is a constant. If $\delta = K - 1$, this is exactly the formulation employed here.

distinguished with a certain probability.

The magnitude of the ratio $\frac{q_h}{q_\ell}$ may, given the assumptions made, be interpreted either as a physical or hedonic measure of how far apart the goods are. Thus $\frac{q_h}{q_\ell} = 2$ may be read as "h is of twice as good quality as ℓ, according to some objective measure" or as "the consumer gets twice as much pleasure from h as from ℓ". This property depends on the linearity of the utility function in q, and in general the interpretations do not coincide. The magnitude of the quality ratio is further discussed in section 4.4.

A similarity ratio is not introduced for price, although it is plausible that consumers often act as if very similar prices are identical. Quality is generally a much harder to assess attribute than price, and so the range of prices similar enough to be treated as the same is assumed to be negligible compared to the corresponding range of qualities.[2]

There is hence a tension between a consumer's bounded perception of goods' quality and ability to determine a precise willingness-to-pay (WTP) for them. This is resolved by the assumption that if $\frac{q_h}{q_\ell} < \delta$, both goods h and ℓ are perceived as having quality $q' = \lambda q_h + (1 - \lambda) q_\ell$, $\lambda \in [0, 1]$. The consumer's WTP for both goods is then $\alpha q'$, and although WTP is precisely defined, it is distorted by perceiving the goods as homogeneous, and consumers' decisions whether to participate in the market are non-rational.

It should be noted that although this is a natural assumption, the only necessary restriction on consumer behaviour in section 4.4 is that consumers never purchase the higher priced good when $\frac{q_h}{q_\ell} < \delta$. The assumption will be revisited in section 4.5.

[2]For a treatment of consumers with a similarity relation for prices in a market, see Bachi (2014).

4.4 Fixed costs

Fixed costs of quality will be addressed first, with initially the baseline case of perfect perception derived, before moving on to bounded perception.

4.4.1 Baseline case; $\delta = 1$

Two identical firms produce a good with quality $q \in Q$ which they sell at price $p \in P$. There is a fixed cost of quality $c(q) = \frac{1}{2}q^2$ with all other costs 0. Consumers gain utility $u = \alpha q - p$, $\alpha \in \mathbb{R}_+$, from consuming a single unit of the good. They gain no utility from further units, and the payoff from not consuming is normalized to 0. There is a unit mass of consumers with α uniformly distributed between 0 and 1.

At the start of the game each firm is selected by nature to be the first mover with probability $\frac{1}{2}$. Let firm 1 denote the first mover and firm 2 denote the second mover.[3] The timing of the market is as follows:-

Period 1: Firm 1 chooses quality.

Period 2: Firm 2 observes firm 1's choice and chooses quality.

Period 3: Both firms set prices simultaneously.

The cost of entering the market is 0. A positive entry cost is considered in section 4.6 and the general case with arbitrary entry cost is fully derived in appendix D.

Let $h \in \{1, 2\}$, ($\ell \in \{1, 2\}$) denote the firm producing the high (low) quality good. Then the consumer with taste parameter $\alpha' = \frac{p_h - p_\ell}{q_h - q_\ell}$ is indifferent between purchasing from either firm and the consumer with taste parameter $\alpha'' = \frac{p_\ell}{q_\ell}$ is indifferent between the low quality firm and not consuming. Demands for the high

[3]The first mover is decided by nature, but it could also be that firm 1 is able to move first due to being endowed with some innovation ability that firm 2 lacks. In this case it could be that firm 2's cost of quality is higher, due to its lack of innovation ability, or lower due to learning from observing firm 1 to be more efficient, but this does not qualitatively affect the results.

and low quality firms are thus respectively $1-\alpha'$ and $\alpha'-\alpha''$, implying profits of

$$\pi_h(q_h, q_\ell) = p_h\left(1-\frac{p_h-p_\ell}{q_h-q_\ell}\right)-\frac{1}{2}q_h^2, \qquad \pi_\ell(q_h, q_\ell) = p_\ell\left(\frac{p_h-p_\ell}{q_h-q_\ell}-\frac{p_\ell}{q_\ell}\right)-\frac{1}{2}q_\ell^2. \tag{4.1}$$

From the first order conditions, given in appendix C (equation (C.1)), it is found that equilibrium prices in period 3 are

$$p_h = 2q_h\left(\frac{q_h-q_\ell}{4q_h-q_\ell}\right) \qquad p_\ell = q_\ell\left(\frac{q_h-q_\ell}{4q_h-q_\ell}\right). \tag{4.2}$$

Thus profits for given qualities are

$$\pi_h(q_h, q_\ell) = \frac{4q_h^2(q_h-q_\ell)}{(4q_h-q_\ell)^2}-\frac{1}{2}q_h^2, \quad (4.3a) \qquad \pi_\ell(q_h, q_\ell) = \frac{q_h q_\ell(q_h-q_\ell)}{(4q_h-q_\ell)^2}-\frac{1}{2}q_\ell^2. \quad (4.3b)$$

The first and second order conditions are given in appendix C (see equation (C.2).

Firm 1 takes advantage of its first mover status to produce the high quality good, so $\pi_1(q_1, q_2) = \pi_h(q_1, q_2)$ and $\pi_2 = (q_1, q_2) = \pi_\ell(q_1, q_2)$. Denote the constants solving $\frac{\partial \pi_2(q_1,q_2)}{\partial q_2} = 0$ simultaneously with $\frac{\partial \pi_1\left(q_1, q_2^{BR}(q_1)\right)}{\partial q_1} = 0$ as $r_1 \approx 0.245$ and $r_2 \approx 4.78\times 10^{-2}$.

The profits of each firm in equilibrium are $\pi_1^*(q_1, q_2) \approx 0.0245$ and $\pi_2^*(q_1, q_2) \approx 1.52\times 10^{-3}$. The share of total profit accruing to the incumbent is $\sigma_1 \approx 0.942$. Firm 1 uses its leader status to gain a greater share of the market than firm 2.

Finally, consumer surplus is given by
$CS(q_h, q_\ell) = \int_{\alpha'}^{1}(\alpha q_h - p_h)\,d\alpha + \int_{\alpha''}^{\alpha'}(\alpha q_\ell - p_\ell)\,d\alpha$ which simplifies to

$$CS(q_h, q_\ell) = \frac{q_h^2(4q_h+5q_\ell)}{2(q_h-q_\ell)^2} \tag{4.4}$$

so that in equilibrium $CS^*(q_1, q_2) \approx 0.0421$.

4.4.2 Bounded perception; $\delta > 1$

Now let consumers have bounded perception, i.e. $\delta > 1$. Suppose the firms choose qualities such that $\frac{q_h}{q_\ell} < \delta$, so that consumers perceive them to be homogeneous. Bertrand competition in period 3 drives prices down to marginal cost (i.e. 0) and so firms will make a loss. This leads to a key result:

Lemma 4.1. *With fixed costs of quality, qualities such that $\frac{q_h}{q_\ell} < \delta$ are never observed in equilibrium.*

All proofs are contained in appendix C. Note that this result does not depend on the assumption that when the quality ratio lies below the perception threshold, firms perceive both goods as having quality $q' = \lambda q_h + (1-\lambda) q_\ell$. The only necessary restriction on consumers' behaviour given a quality ratio below the perception threshold is that they never purchase the higher priced good.

By lemma 4.1, it must be that $q_2 \leq \frac{q_1}{\delta}$. If $\operatorname{argmax}_{q_2} \pi_2(q_1, q_2) \leq \frac{q_1}{\delta}$, this must be a best response. If, however, $\operatorname{argmax}_{q_2} \pi_2(q_1, q_2) > \frac{q_1}{\delta}$, then firm 2, given that $\pi_2(q_1, q_2)$ is single-peaked, will choose the highest quality below q_1 such that consumers still perceive the goods as heterogeneous. In summary, firm 2's best response function is

$$q_2^{BR}(q_1) = \min\left\{\operatorname*{argmax}_{q_2} \pi_2(q_1, q_2), \frac{q_1}{\delta}\right\}. \tag{4.5}$$

For small δ then, the outcome is as in the baseline, yet from lemma 4.1, for sufficiently high δ the outcome is changed. Denote the point at which the baseline outcome no longer holds as δ'_f.

Let $\delta > \delta'_f$ so that $q_2^{BR}(q_1) = \frac{q_1}{\delta}$. Substituting into $\pi_1(q_1, q_2)$ and taking the first order condition allows each firm's equilibrium quality to be determined as

$$q_1^* = \begin{cases} r_1 & \text{if } \delta \leq \delta'_f \\ \dfrac{4\delta\left(\delta-1\right)}{\left(4\delta-1\right)^2} & \text{if } \delta > \delta'_f \end{cases}, \qquad q_2^* = \begin{cases} r_2 & \text{if } \delta \leq \delta'_f \\ \dfrac{4\left(\delta-1\right)}{\left(4\delta-1\right)^2} & \text{if } \delta > \delta'_f. \end{cases} \tag{4.6}$$

Further substitution reveals each firm's profit to be

$$\pi_1^*\left(q_1,q_2\right) = \begin{cases} \dfrac{4r_1^2\left(r_1-r_2\right)}{\left(4r_1-r_2\right)^2} - \dfrac{r_1^2}{2} & \text{if } \delta \leq \delta'_f \\ \left(\dfrac{8\delta^2\left(\delta-1\right)^2}{\left(4\delta-1\right)^4}\right) & \text{if } \delta > \delta'_f \end{cases} \tag{4.7a}$$

$$\pi_2^*\left(q_1,q_2\right) = \begin{cases} \dfrac{r_1 r_2\left(r_1-r_2\right)}{\left(4r_1-r_2\right)^2} - \dfrac{r_2^2}{2} & \text{if } \delta \leq \delta'_f \\ \dfrac{4\left(\delta-2\right)\left(\delta-1\right)^2}{\left(4\delta-1\right)^4} & \text{if } \delta > \delta'_f. \end{cases} \tag{4.7b}$$

δ'_f is then found by equating the upper and lower components of $\pi_1^*\left(q_1,q_2\right)$, with $\delta'_f \approx 4.941$.

Firm 1's share of the total profit is

$$\sigma_1 = \begin{cases} \dfrac{8r_1^2\left(r_1-r_2\right) - r_1^2\left(4r_1-r_2\right)^2}{2r_1\left(r_1-r_2\right)\left(4r_1+r_2\right) - \left(4r_1-r_2\right)^2\left(r_1^2+r_2^2\right)} & \text{if } \delta \leq \delta' \\ \dfrac{2\delta^2}{\delta\left(2\delta+1\right)-1} & \text{if } \delta > \delta' \end{cases} \tag{4.8}$$

and consumer surplus is

$$CS^*\left(q_1,q_2\right) = \begin{cases} \dfrac{r_1^2\left(4r_1+5r_2\right)}{2\left(4r_1-r_2\right)^2} & \text{if } \delta \leq \delta'_f \\ \dfrac{2\delta^2\left(4\delta^2+\delta-5\right)}{\left(4\delta-1\right)^4} & \text{if } \delta > \delta'_f. \end{cases} \tag{4.9}$$

These expressions are illustrated in figure 4.1.

The derivatives of each firm's total profit, firm 1's profit share and consumer surplus are

$$\frac{\partial \pi_1^*(q_1,q_2)}{\partial \delta} = \begin{cases} 0 & \text{if } \delta < \delta_f' \\ \frac{16\delta\left(2\delta^2-\delta-1\right)}{\left(4\delta-1\right)^5} & \text{if } \delta > \delta_f' \end{cases} \tag{4.10a}$$

$$\frac{\partial \pi_2^*(q_1,q_2)}{\partial \delta} = \begin{cases} 0 & \text{if } \delta < \delta_f' \\ \frac{-4\left(4\delta^3-29\delta^2+52\delta-27\right)}{\left(4\delta-1\right)^5} & \text{if } \delta > \delta_f' \end{cases} \tag{4.10b}$$

$$\frac{\partial \sigma_1}{\partial \delta} = \begin{cases} 0 & \text{if } \delta < \delta_f' \\ \frac{2\delta\left(\delta-2\right)}{\left(2\delta^2+\delta-1\right)^2} & \text{if } \delta > \delta_f'. \end{cases} \tag{4.10c}$$

$$\frac{\partial CS^*(q_1,q_2)}{\partial \delta} = \begin{cases} 0 & \text{if } \delta < \delta_f' \\ -\frac{2\delta\left(20\delta^2-37\delta-10\right)}{\left(4\delta-1\right)^5} & \text{if } \delta > \delta_f'. \end{cases} \tag{4.10d}$$

From these expressions it is concluded that

Proposition 4.1. *(i) Firm 1's profit is weakly increasing in δ and weakly greater than in the baseline case.*

(ii) Firm 2's profit is weakly decreasing in δ and weakly lower than in the baseline case.

(iii) Firm 1's share of the total profit is weakly increasing in δ and weakly greater than in the baseline case.

(iv) *Consumer surplus is weakly increasing in δ and weakly lower than in the baseline case.*

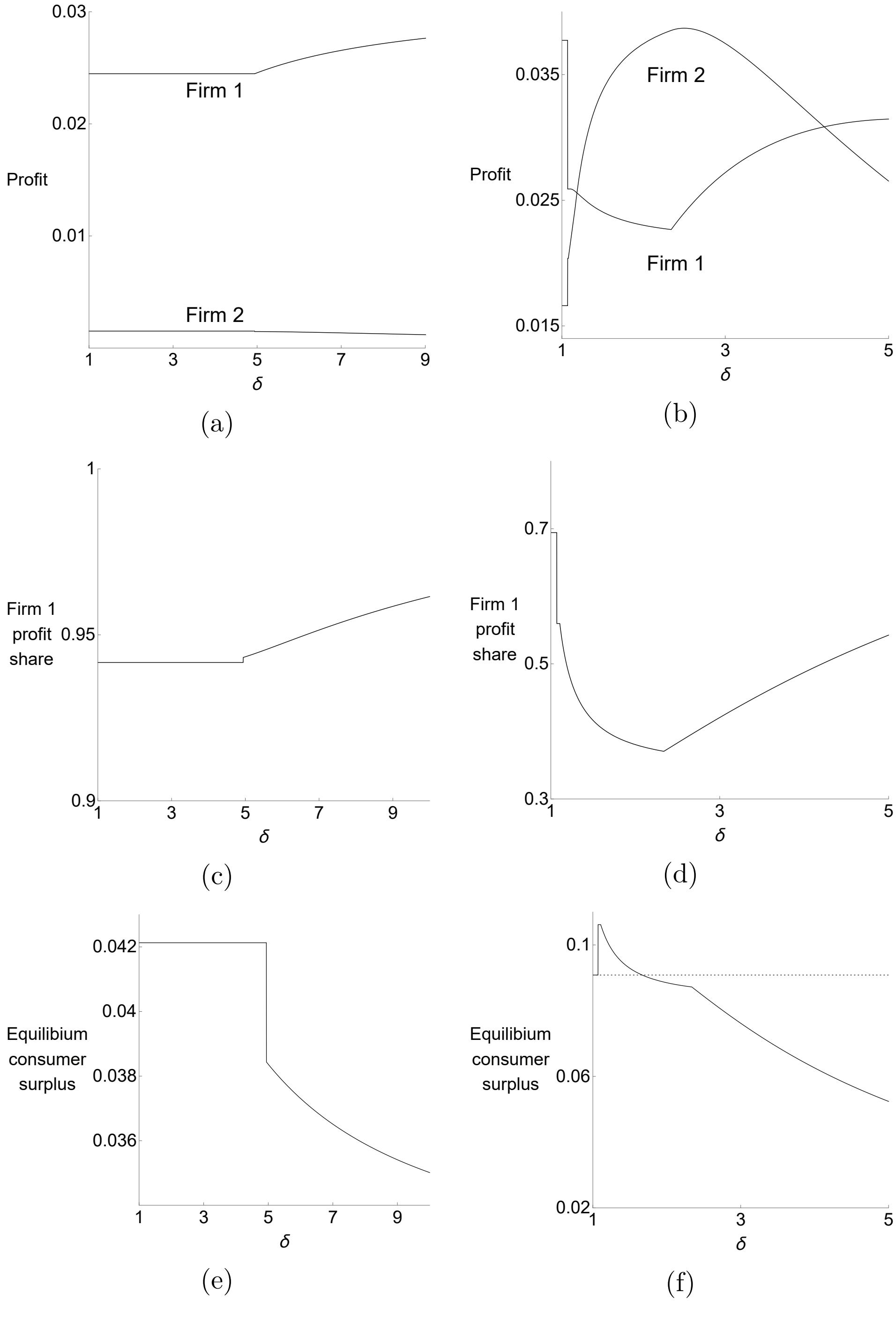
Bounded perception affects the market only when $\delta > \delta'_f \approx 4.941$. However, this number is not general, but rather is determined by the qualities firms produce in the baseline case. Firm 1 chooses a quality almost 5 times higher than firm 2's, and its costs are almost 25 times as great.

A smaller value of δ'_f would result if, for example, q is regarded as excess quality over some minimum feasible level. A more concentrated mass of consumers demanding medium quality, rather than a flat uniform distribution, or a marginal cost of production (independent of quality) would also lead to firms producing qualities closer together in the baseline.

The minimum perception threshold is hence a direct result of the stylized assumptions made for tractability's sake in the baseline model, rather than a property of the bounded perception mechanism itself. This allows it to be reconciled with analogous psychological results on just-noticeable differences in stimuli, which typically take small values.[4]

[4]The ratio $\frac{\Delta I}{I}$, where I is stimulus intensity and ΔI is the JND is termed the Weber fraction, and is equivalent to $\delta - 1$. This has been estimated to be, for example, 0.079 for brightness, 0.048 for loudness and 0.02 for heaviness (Techtsoonian, 1971).

Figure 4.1: Firm profits (a), market concertation (c) and consumer surplus (e) with fixed costs. Firm profits (b), market concentration (d) and consumer surplus (f) with marginal costs.



4.5 Marginal costs of quality

Suppose now each firm incurs marginal costs of quality with the functional form $c(q) = \frac{1}{2}Dq^2$ where D is demand. Fixed costs are 0. The results will be contrasted with the previous section, demonstrating that the effects of bounded perception in a market are highly dependent on its structure. The baseline case of perfect perception is again derived first, before moving on to bounded perception.

4.5.1 Baseline case; $\delta = 1$

Demands for the high and low quality firm are unchanged from the previous section, and so profits are

$$\pi_h(q_h, q_l) = \left(1 - \frac{p_h - p_l}{q_h - q_l}\right)\left(p_h - \frac{1}{2}q_h^2\right) \tag{4.11a}$$

$$\pi_l(q_h, q_l) = \left(\frac{p_h - p_l}{q_h - q_l} - \frac{p_l}{q_l}\right)\left(p_l - \frac{1}{2}q_l^2\right). \tag{4.11b}$$

From the first order conditions (equation (C.3)), equilibrium prices are

$$p_h = q_h \frac{\left(4(q_h - q_l) + 2q_h^2 + q_l^2\right)}{2(4q_h - q_l)}, \tag{4.12a}$$

$$p_l = q_l \frac{\left(2(q_h - q_l) + q_h(q_h + 2q_l)\right)}{2(4q_h - q_l)} \tag{4.12b}$$

and profits for given qualities are

$$\pi_h(q_h, q_l) = q_h^2 \frac{(4 - 2q_h - q_l)^2 (q_h - q_l)}{4(4q_h - q_l)^2} \tag{4.13a}$$

$$\pi_l(q_h, q_l) = q_h q_l \frac{(2 + q_h - q_l)^2 (q_h - q_l)}{4(4q_h - q_l)^2}. \tag{4.13b}$$

To ensure non-negative demand for the high quality good, the restriction $q_\ell \leq q_h \leq 2 - \frac{1}{2}q_\ell$ is imposed.[5] First order conditions are given in the appendix (equations (C.4) and (C.5)). Although the profit functions are not concave, it is possible to show that

Lemma 4.2. *$\pi_h(q_h, q_\ell)$ and $\pi_\ell(q_h, q_\ell)$ have unique local maxima in the range $0 \leq q_\ell \leq q_h$, $q_\ell \leq q_h \leq 2 - \frac{1}{2}q_\ell$.*

Firm 1 again takes advantage of its first mover status to produce the high quality good, so $\pi_1(q_1, q_2) = \pi_h(q_1, q_2)$ and $\pi_2(q_1, q_2) = \pi_\ell(q_1, q_2)$. In the appendix it is shown that firm 1 chooses $q_1^* = s_1 \approx 0.612$ and firm 2 chooses $q_2^* = s_2 \approx 0.309$. The profits of each firm in equilibrium are then $\pi_1^*(q_1, q_2) \approx 0.0377$ and $\pi_2^*(q_1, q_2) \approx 0.0166$ and the proportion of total profit going to the incumbent is $\sigma_1 \approx 0.694$.

Rearranging $CS(q_h, q_\ell) = \int_{\alpha'}^{1}(\alpha q_h - p_h)\,d\alpha + \int_{\alpha''}^{\alpha'}(\alpha q_\ell - p_\ell)\,d\alpha$ gives

$$CS(q_h, q_\ell) = \frac{4q_h^5 - q_h^4(16 - q_\ell) + q_h^3(q_\ell^2 - 4q_\ell + 16) + q_h^2(3q_\ell^3 - 16q_\ell^2 + 20q_\ell)}{8(4q_h - q_\ell)^2} \tag{4.14}$$

so that consumer surplus is $CS^*(q_1, q_2) \approx 0.0908$.

4.5.2 Bounded perception; $\delta > 1$

Suppose consumers have bounded perception. With fixed costs, firm 2 was constrained to choose a quality consumers perceived as distinct (lemma 4.1). This is not the case with marginal costs.

If firms choose qualities such that $\frac{q_h}{q_\ell} < \delta$, consumers regard the goods as homogeneous and hence purchase whichever is cheaper (if any). However, the marginal costs are not identical, in particular the low quality firm enjoys a cost advantage. It is a standard result that in period 3, Bertrand-Nash equilibrium prices

[5] Note that such a restriction also applies with fixed costs, however it is trivially non-binding.

are

$$p_h \in \left(\frac{1}{2}q_h^2, \infty\right) \qquad\qquad p_\ell = \frac{1}{2}q_h^2 \tag{4.15}$$

with the lower quality/cost firm capturing the entire market.

Firm 2, as the second mover always has the option of choosing a quality such that $q_2 < q_1$ and $\frac{q_1}{q_2} < \delta$ and capturing the whole market. Taking such action is referred to as firm 2 *imitating* firm 1. The ability of firm 2 to imitate means that lemma 4.1 does not hold when costs of quality are marginal.

If firm 2 imitates firm 1, then firm 1 captures none of the market. Thus firm 1 is constrained not to choose a quality such that firm 2's best response is imitation.

The demand for firm 2's good when it imitates is $D_2 = 1 - \frac{p_2}{q'}$, where $q' = \lambda q_h + (1-\lambda)\, q_\ell$ is the quality consumers perceive firm 2's (and firm 1's) good to be. So far the only necessary assumption about consumers when $\frac{q_h}{q_\ell} < \delta$ has been that they never purchase the higher priced good. Now however, another simplifying assumption is made to give clarity to the analysis.

As was stated previously, the main focus of this article is the contrast between the cases of fixed and marginal costs of quality. The contrast is largely driven by firm 2's ability to imitate firm 1 with marginal costs, and its profit from imitation is increasing in λ. Therefore, to emphasize the contrast between the two cases and to greatly simplify the analysis, it is assumed that $\lambda = 1$. This implies that if $\frac{q_h}{q_\ell} < \delta$, consumers perceive both goods as being of quality $q' = q_h$. All conclusions are qualitatively unchanged under the oppositely extreme assumption of $\lambda = 0$.

If it imitates, firm 2 minimizes its cost by choosing the lowest quality such that consumers are unable to distinguish it from firm 1's good. However, this is undefined, as firm 2 can choose q_2 arbitrarily close to $\frac{q_1}{\delta}$. Assume that there is some minimum technologically feasible difference in quality ε. Firm 2 will then maximize its profit, conditional on imitating, by choosing $q_2 = \frac{q_1}{\delta} + \varepsilon$.

As ε becomes very close to 0, q_2 is approximately $\frac{q_1}{\delta}$, but with consumers still unable to perceive the difference between q_1 and q_2. Thus in the following section, when it is stated that firm 2 imitates by choosing $q_2 = \frac{q_1}{\delta}$, it should be read as an approximation of choosing $q_2 = \frac{q_1}{\delta} + \varepsilon$ with ε very close to 0.

Conditional on imitating firm 1, firm 2's profit is

$$\pi_I(q_1) = q_1^2 (2 - q_1) \left(\frac{\delta^2 - 1}{4\delta^2} \right). \tag{4.16}$$

Note that this becomes 0 as δ approaches 1, so for sufficiently low δ the market outcome is unchanged from the baseline case. Let the point at which the baseline outcome no longer obtains be δ'_m.

For $\delta > \delta'_m$, firm 2 wishes to imitate firm 1 rather than produce a lower, distinct quality. Firm 1 must avoid this. If it raises its quality, this simply increases firm 2's incentive to imitate. If it reduces its quality, firm 2 finds it less attractive to imitate and more attractive to produce a high quality good.

The threat of being imitated leads firm 1 to cede the advantage of producing the high quality good to its rival.

In the appendix, it is shown that with marginal costs the qualities the firms produce are

$$q_1^* = \begin{cases} s_1 & \text{if } \delta \leq \delta'_m \\ \tilde{s}_1 & \text{if } \delta'_m < \delta \leq \delta''_m \\ \dfrac{4\left(4\mu^2(\delta) - 3\mu(\delta) + 2\right)}{\left(24\mu^3(\delta) - 22\mu^2(\delta) + 5\mu(\delta) + 2\right)} & \text{if } \delta''_m < \delta \leq \delta'''_m \\ \dfrac{-B(\delta) - \sqrt{B^2(\delta) - 4A(\delta)C(\delta)}}{2A(\delta)} & \text{if } \delta > \delta'''_m \end{cases} \tag{4.17a}$$

$$q_2^* = \begin{cases} s_2 & \text{if } \delta \leq \delta'_m \\ \tilde{s}_2 & \text{if } \delta'_m < \delta \leq \delta''_m \\ \mu(\delta)\, q_1^* & \text{if } \delta''_m < \delta \leq \delta'''_m \\ \delta q_1^* & \text{if } \delta > \delta'''_m \end{cases} \tag{4.17b}$$

where $\delta'_m \approx 1.071$, $\delta''_m \approx 1.106$, $\delta'''_m \approx 2.339$ and $\tilde{s}_1 \approx 0.555$, $\tilde{s}_2 \approx 0.906$. $\mu(\delta)$ is defined by the ratio $\left.\frac{q_2^*}{q_1^*}\right|_{\delta'_m < \delta \leq \delta''_m}$ and is given by the unique root of equation (C.6) taking a value greater than 1. $A(\delta)$, $B(\delta)$ and $C(\delta)$ are functions of δ given by equation (C.7).

For δ'_m, the threat of imitation causes firm 1 to allow firm 2 to become the high quality firm[6] and for $\delta'_m < \delta < \delta''_m$, firm 2 strictly prefers to be the high quality firm rather than imitate. When the threshold exceeds δ''_m, however, it prefers to imitate even firm 1's low quality. In response, firm 1 must lower its quality still further until firm 2 (weakly) prefers to enter as the high quality firm rather than imitate.[7]

For $\delta''_m < \delta \leq \delta'''_m$, the equilibrium ratio of high to low quality, $\mu(\delta)$, is above the

[6] I.e. its quality solves $\max_{q_\ell} \pi_\ell\left(q_h^{BR}(q_1), q_1\right)$.
[7] I.e. its quality solves $\pi_h\left(q_h^{BR}(q_1), q_1\right) = \pi_I(q_1)$.

perception threshold. However, for $\delta > \delta'''_m$, $\mu(\delta)$ lies below the threshold. Hence firm 2, if it enters as the high quality firm, must choose a quality higher than it would ideally like in order to distinguish itself.[8]

Substitution then gives firms' profits as

$$\pi_1(q_1,q_2) = \begin{cases} \dfrac{s_1^2(4-2s_1-s_2)^2(s_1-s_2)}{4(4s_1-s_2)^2} & \text{if } \delta < \delta'_m \\[2ex] \dfrac{\tilde{s}_1^2(4-2\tilde{s}_1-\tilde{s}_2)^2(\tilde{s}_1-\tilde{s}_2)}{4(4\tilde{s}_1-\tilde{s}_2)^2} & \text{if } \delta'_m < \delta < \delta''_m \\[2ex] \dfrac{\mu(\delta)(\mu(\delta)-1)q_1^*(2+(\mu(\delta)-1)q_1^*)^2}{4(4\mu(\delta)-1)^2} & \text{if } \delta''_m < \delta \leq \delta'''_m \\[2ex] \dfrac{\delta(\delta-1)q_1^*(2+(\delta-1)q_1^*)^2}{4(4\delta-1)^2} & \text{if } \delta > \delta'''_m \end{cases} \tag{4.18a}$$

$$\pi_2^*(q_1,q_2) = \begin{cases} \dfrac{s_1s_2(2+s_1-s_2)^2(s_1-s_2)}{4(s_1-s_2)^2} & \text{if } \delta \leq \delta''_m \\[2ex] \dfrac{\tilde{s}_1\tilde{s}_2(2+\tilde{s}_1-\tilde{s}_2)^2(\tilde{s}_1-\tilde{s}_2)}{4(\tilde{s}_1-\tilde{s}_2)^2} & \text{if } \delta'_m < \delta \leq \delta''_m \\[2ex] \dfrac{256\mu^4(\delta)(\mu(\delta)-1)^3(4\mu^2(\delta)-3\mu(\delta)+2)}{(24\mu^3(\delta)-22\mu^2(\delta)+5\mu(\delta)+2)^3} & \text{if } \delta''_m < \delta \leq \delta'''_m \\[2ex] q_1^{*2}(2-q_1^*)\left(\dfrac{\delta^2-1}{4\delta^2}\right) & \text{if } \delta > \delta'''_m. \end{cases} \tag{4.18b}$$

[8] I.e. firm 1's quality solves $\pi_h(\delta q_1, q_1) = \pi_I(q_1)$.

Firm 1's share of total profit is

$$\sigma_1 = \begin{cases} \dfrac{s_1 \left(4 - 2s_1 - s_2\right)^2}{4s_1^3 + s_1^2 \left(5s_2 - 16\right) - s_1 \left(s_2^2 + 4s_2 - 16\right) + s_2 \left(2 - s_2\right)^2} & \text{if } \delta \leq \delta'_m \\ \dfrac{\tilde{s}_1 \left(4 - 2\tilde{s}_1 - \tilde{s}_2\right)^2}{4\tilde{s}_1^3 + \tilde{s}_1^2 \left(5\tilde{s}_2 - 16\right) - \tilde{s}_1 \left(\tilde{s}_2^2 + 4\tilde{s}_2 - 16\right) + \tilde{s}_2 \left(2 - \tilde{s}_2\right)^2} & \text{if } \delta'_m < \delta \leq \delta''_m \\ \dfrac{\left(8\mu^2 \left(\delta\right) - 7\mu \left(\delta\right) + 2\right)^2}{64\mu^5 \left(\delta\right) - 64\mu^4 \left(\delta\right) - 48\mu^3 \left(\delta\right) + 81\mu^2 \left(\delta\right) - 28\mu \left(\delta\right) + 4} & \text{if } \delta''_m < \delta \leq \delta'''_m \\ \left(1 + \dfrac{\delta \left(2\delta q_1^* + q_1^* - 4\right)^2}{\left(\delta q_1^* - q_1^* + 2\right)^2}\right)^{-1} & \text{if } \delta \geq \delta'''_m. \end{cases} \tag{4.19}$$

Finally, consumer surplus is given by

$$CS^* \left(q_1, q_2\right) =$$

$$\begin{cases} \dfrac{4s_1^5 - s_1^4 \left(16 - s_2\right) + s_1^3 \left(s_2^2 - 4s_2 + 16\right) + s_1^2 \left(3s_2^3 - 16s_2^2 + 20s_2\right)}{8 \left(4s_1 - s_2\right)^2} & \text{if } \delta \leq \delta'_m \\ \dfrac{4\tilde{s}_2^5 - \tilde{s}_2^4 \left(16 - \tilde{s}_1\right) + \tilde{s}_2^3 \left(\tilde{s}_1^2 - 4\tilde{s}_1 + 16\right) + \tilde{s}_2^2 \left(3\tilde{s}_1^3 - 16\tilde{s}_1^2 + 20\tilde{s}_1\right)}{8 \left(4\tilde{s}_2 - \tilde{s}_1\right)^2} & \text{if } \delta'_m < \delta \leq \delta''_m \\ \dfrac{\left(64\mu^5 \left(\delta\right) + 64\mu^4 \left(\delta\right) - 288\mu^3 \left(\delta\right) + 225\mu^2 \left(\delta\right) - 60\mu \left(\delta\right) + 4\right)}{\left(24\mu^3 \left(\delta\right) - 22\mu^2 \left(\delta\right) + 5\mu \left(\delta\right) + 2\right)^3} \times \ldots & \text{if } \delta''_m < \delta \leq \delta'''_m \\ \cdots \times \left(4\mu^2 \left(\delta\right) - 3\mu \left(\delta\right) + 2\right) & \\ \dfrac{\left(\left(4\delta^5 + \delta^4 + \delta^3 + 3\delta^2\right) q_1^{*3} - \left(16\delta^4 + 4\delta^3 + 16\delta^2\right) q_1^{*2} + \ldots\right.}{8 \left(4\delta - 1\right)^2} & \text{if } \delta > \delta'''_m \\ \cdots + \left(16\delta^3 + 20\delta^2\right) q_1^*\Big) & \end{cases} \tag{4.20}$$

These expressions are shown in figure 4.1.

Derivatives of profits and of firm 1's share of total profit are not explicitly given for reasons of brevity. However, regarding comparative statics it is possible to state that

Proposition 4.2. *(i) Firm 1's profit is weakly lower than in the baseline case.*

(ii) Firm 2's profit is weakly greater than in the baseline case for $\delta \lesssim 7.547$ *and lower than in the baseline case for* $\delta \gtrsim 7.547$.

(iii) Firm 1's share of the total profit is weakly decreasing in δ *for* $\delta < \delta_m'''$ *and increasing in* δ *for* $\delta > \delta_m'''$. *For* $\delta \lesssim 9.453$ *it is lower than in the baseline case and for* $\delta \gtrsim 9.453$ *it is greater than in the baseline case.*

(iv) Consumer surplus is weakly decreasing in δ *for* $\delta > \delta_m'$. *For* $\delta \lesssim 1.667$ *it is greater than in the baseline case and for* $\delta \gtrsim 1.667$ *it is lower than in the baseline case.*

The contrasting conclusions of propositions 4.1 and 4.2 are summarized in table 4.1.

4.6 Entry costs and market creation

So far is has been assumed that there were no costs associated with entering the market. To analyze the impact of relaxing this assumption, suppose firms must now pay an entry cost in order to produce. Let $E \in \left[0, \bar{E}\right)$ represent the fixed cost of entry, identical for both firms, with the upper limit of this cost $\bar{E}$ equal to the monopoly profit which hence ensures the market is viable.[9]

The timing of the game is also altered to reflect the fact that firms must decide whether or not to enter the market, At the start, firm 1 decides whether to be the

[9]From equations (C.2) and (C.4) it is straightforward to show that $\bar{E} = \frac{1}{32}$ with fixed costs and $\bar{E} = \frac{2}{27}$ with marginal costs.

first to enter the market. If it declines, the opportunity to be the first to enter passes to firm 2. If firm 2 declines, it reverts to firm 1, and so the firms alternate in having the opportunity to enter the market first until one does so.

If firm $i \in \{1, 2\}$ enters the market it chooses quality which is observed by firm $j \neq i$. Firm j decides whether or not to also enter the market. If it does so, it incurs entry cost E and chooses its quality. Finally there follows a single price-setting period in which either only firm i or both firms i and j are active.

The introduction of an entry cost allows for the possibility of the first mover deterring entry by the second mover, and it is hence possible to analyse the effect of bounded perception on the number of firms observed in the market at a given entry cost. However, the results closely resemble those already obtained regarding market concentration with 0 entry cost, and so detailed results are relegated to appendix C. Instead, it is examined whether firms ever reach the production stage.

Given that there is a single period in which firms can earn revenue, for a firm to wish to enter the market it must earn enough in that period to cover its entry cost. Thus there is a possibility that in equilibrium firms alternate between refusing to be the first to enter the market, and hence production never takes place. This is referred to as no *market creation*.

Proposition 4.3. *(i) When costs of quality are fixed, a market is always created for any entry cost* $E \in \left[0, \bar{E}\right)$.

(ii) When costs of quality are marginal, there exist entry costs for which market entry does not occur.

The reduction in the profit of the first mover caused by the ability of the second mover to imitate can be so great that neither firm wishes to create the market by being the first mover.

Table 4.1: Summary of results contrasting fixed and marginal costs of quality

	Fixed costs	Marginal costs
Threat of imitation	No	Yes
Firm 1 profit relative to baseline	Greater	Lower
Firm 2 profit relative to baseline	Lower	Greater for $\delta \lesssim 7.547$ Lower for $\delta \gtrsim 7.547$
Firm 1 profit share relative to baseline	Greater	Lower for $\delta \lesssim 9.453$ Greater for $\delta \gtrsim 9.453$
Consumer surplus	Lower	Greater for $\delta \lesssim 1.667$ Lower for $\delta \gtrsim 1.667$
Market creation with entry costs	Market always created	E exist for which no market created

4.7 Discussion

A key insight garnered from sections 4.4 and 4.5 is that bounded perception has very different results depending on the market structure, as summarized in table 4.1. Consumer behaviour for given qualities and prices is fully determined, and so they are not strategic players in the market: it is a game between firms only. Thus the disparate results in the two sections are due to the differing abilities of each firm to exploit bounded perception.

The key difference is whether, when consumers perceive goods as homogeneous, firms are identical when competing in prices, or whether one firm has an advantage in marginal cost. Bertrand competition between identical firms leads to a loss for both, but with differing marginal costs, the low cost firm makes a profit.

Thus with fixed costs, firm 1 as the first mover finds bounded perception an advantage. It picks its quality knowing that firm 2 must position itself so that consumers perceive the goods as distinct, as otherwise it makes a loss. With marginal costs of quality, the low quality/cost firm may make a profit from choosing a good perceived as identical to its rival's. This grants an advantage to firm 2 as the second mover, since it can always choose to be the low quality firm.

It is also important to note that even when the direction of the comparative statics is the same in both cases, the underlying causes are very different. With fixed costs, firm 1 exploits bounded perception to increase its market share. With marginal costs, firm 1 has to ensure firm 2 does not imitate, which it does by choosing a quality so low it is not worth imitating. Even when $\delta > \delta_m'''$ and its market share increases with δ, it is capturing a greater share of a market whose overall value is less.

Turning to consumer surplus, it is decreasing in the perception threshold in both cases with the exception of a jump at δ_m' for marginal costs. It is also lower than in the baseline with the exception of $\delta_m' < \delta \lesssim 1.667$ for marginal costs. Hence for the most part consumers are harmed by perceptual limitations.

With fixed costs, a higher perception threshold benefits firm 1, and its increased power in the market lowers consumer welfare. With marginal costs, on the other hand, with a sufficiently high threshold consumers are harmed by the underprovision of quality by the incumbent in its effort not to be imitated. In both cases, firms' equilibrium qualities are also further apart, implying less intense competition in prices and lower consumer welfare.

A caveat to these results is that they hold strictly in a comparative static setting. Analysis of consumer welfare when consumers do not precisely perceive the quality of goods may be problematic if it is unclear whether welfare should depend on a good's perceived or objective quality. This problem is avoided only due to consumers always being able to perceive goods perfectly in equilibrium.

The perceived quality of goods is context dependent, so if considering the change in welfare for a consumer shifting from one equilibrium to another, it is necessary to take into consideration the new goods being viewed in context with the old. Otherwise paradoxes arise such as an individual being measured as better off when provided with good i rather than j, despite her regarding i and j as perfectly homogeneous.

Allowing for positive costs of firm entry reveals another contrast between the two cases. Firm 1 unsurprisingly always enters the market with fixed costs. With marginal costs, however, there are circumstances in which the prospect of being imitated causes it not to enter.

The stark result of no market being created allows for a welfare result robust to the concerns discussed above. The definitive conclusion is that consumers are left worse off than in the standard case, as they are never presented with the chance to purchase. Such a result justifies patents and intellectual property regulations designed to prevent first mover firms being imitated. However, this result applies only for marginal costs, as restricting entry reduces welfare in the fixed costs case.

For any of the conclusions presented to have relevance, it is necessary for consumers' perceptual limitations to be severe enough. Thus it is important to consider whether any empirical evidence exists that perception influences a market.

Supporting experimental evidence is found in the aforementioned Kalayci and Potters (2011a). However, experimental asset markets often choose stylized parameters for reasons of tractability and closeness to theory, so the external validity of the result should not be taken at face value.

Comparing the market concentration results to the baseline, there is no qualitative difference that could not be observed by altering the parameters of the market. Thus empirical observation of bounded perception may involve comparing two markets with similar cost and demand structures yet demonstrably different consumer ability to compare goods. Another prediction is that, since the perception threshold is defined with respect to the ratio of goods' qualities, at the high-end of a market there should be a greater degree of product differentiation.

A further possibility is to exploit changes in consumer perception over time. Progressive restrictions on advertising and the recently introduced plain packaging law in Australia have made it much more difficult to distinguish between high and low quality cigarettes. Assuming the cigarette market is best characterized by marginal costs and a high δ (exceeding δ_m'''), the prediction is that the interventions lead to a more concentrated market. This has indeed been observed (Clarke & Prentice, 2012).

Although it has no direct bearing on the specific conclusions of this article, some evidence that perception is empirically relevant is that it is easy to observe real world firms taking actions intended to influence consumer perception. This may either try to help perception, for example by adopting a clear colour scheme to make quality discernible at a glance, or to hinder it, for example important nutritional information is often hidden away on the back of food packaging.

Although perception is treated exogenously in the current framework, firms have clear and conflicting incentives to influence perception. The threshold δ may be thought of as the frame in which consumers see goods. It should thus be possible in future research to analyse how firms compete over the frame in which consumers see their goods in a similar way to Spiegler (2014).

Aside from the specific conclusions regarding entry into a vertically differentiated market, conclusions can also be drawn about the impact of individual choice limitations in a market. Consumers are identical in both sections 4.4 and 4.5, and firms differ only in whether the cost of producing a given quality is per-unit or independent of demand.

The interaction between consumers' decision making limitations and firms is vastly different in both cases, however, with the same limitation leading to a natural monopoly for the incumbent in one case and in the other case to a market so unprofitable the incumbent never enters. This highlights the importance of considering decision making limitations not only in the context of individual choice, but also examining the consequences for interactions with other actors.

4.8 Conclusion

The impact on individuals' decision making when their perception is imperfect is a growing area of study in the field of economics. Here, the impact beyond that on the individual is considered. It has been demonstrated that the interactions between perceptual limitations and the cost structure of firms is a complex one, with disparate effects on market outcomes.

There is also potential for extending the simple framework presented here. For example, the perception threshold is entirely exogenous, yet it is plausible that consumers' perception may improve with experience. Another fruitful avenue may be to relax the assumption of a homogeneous threshold, and allow those with higher taste for quality to be better at detecting quality differences.

That consumers are not perfect in their perception of the world is of consequence, and should not be neglected when analysing market structure.

Perception and quality choice in vertically differentiated markets 5

Edward J.D. Webb

An earlier version of this chapter was published as a working paper (Webb, 2014c), and it builds on a masters thesis (Webb, 2014b).

Abstract

Consumers have bounded perception of goods' quality in a vertically differentiated duopoly, treating sufficiently sufficient goods as homogeneous. More extreme product differentiation is observed, and firms must coordinate on equilibrium. Perception is endogenized by allowing firms to influence the ease of product comparison via firms' presentation of their goods. It is shown that firms will act to improve consumer perception to the greatest extent possible.

5.1 Introduction

That consumers' perception of goods may be imperfect is readily observable. Consider Fortnum and Mason's, who in error sold standard quality caviar costing around £40/kg as higher quality caviar costing around £1300/kg. No consumer perceived a difference.[1]

[1]Source: "The fishy caviar toppings of the rich and famous", *The Guardian*, 20/5/13, http://www.theguardian.com/commentisfree/2013/may/20/fishy-caviar-sevruga-fake, accessed

Perception of stimuli is studied in psychophysics, a sub-discipline of psychology (see e.g. Falmagne (2002) and Weber (2004)), where there is a long tradition of studying how dissimilar stimuli must be for a difference to be detectable. As in the previous chapter, this tradition is applied to an economic context: When the qualities of goods are sufficiently similar, consumers do not perceive the difference and treat them as homogeneous. This chapter focuses, however, not on the individual choices of consumers with bounded perception, but rather on the market outcome when such consumers interact with profit maximizing firms.

The actions of firms to influence perception is also studied. The way in which a firm presents a good can have a large impact on the ease with which it can be compared to other goods. It is examined whether competition causes firms to facilitate such comparisons or hinder them.

It is found that firms must differentiate themselves to a greater extent, shifting their qualities further apart than the equilibrium in the baseline case of perfect perception. There is not a unique way of allocating this shift between firms, leading to a continuum of equilibria. The effect on profits compared to the baseline is ambiguous: There are equilibria in which bounded perception leads to greater profits for both firms, one firm and neither firm. This is due to on the one hand firms having to choose excessively high quality (which is costly) or excessively low quality (which consumers do not value) and on the other hand to greater differentiation meaning less intense price competition.

Consumer perception is endogenized by allowing each firm to present their goods to either help or hinder comparison. It is found that firms engage in behaviour to make it as easy as possible for consumers to distinguish between similar goods. The market induces firms to make consumers' decisions as easy as possible, despite potentially greater profits with bounded perception.

9/9/14.

As in the previous chapter, consumers' decision making is founded in Rubinstein (1988)'s notion of a similarity relation.

The specific economic institution considered here is a vertically differentiated product market (Shaked & Sutton, 1982), in which competing firms sell a good with heterogeneous levels of some objectively measurable quality. Consumers with perceptual limitations are introduced to a model similar to that in Motta (1993). This model serves as a tractable baseline against which the effects of perceptual limitations may be judged. The vertical differentiation framework is well suited to yield insights into the effects of bounded perception, as the ability of firms to distinguish the attributes of the goods they produce is instrumental to their ability to make a profit.

The principal difference between this chapter and the preceding one is that choice of quality is simultaneous, allowing quality choice with bounded perception to be studied separately from the strategic concerns of being an incumbent or entrant firm.

Section 5.2 develops and discusses a duopoly model of vertical differentiation when consumers have bounded perception. In section 5.3, perception is endogenized by allowing firms to influence it via the presentation of their goods. Section 5.4 concludes.

5.2 Vertically differentiated duopoly

The behaviour of perceptually bounded consumers is examined when they interact with profit maximizing firms. As the perception threshold is an intrinsic psychological mechanism, consumers' reactions to given qualities and prices are fully determined, and so the following is a game played between firms.

A vertically differentiated market, similar to the model in Motta (1993) will now be developed. As in section 4.3, consumers' ability to perceive the difference between two goods is bounded. Let $\delta \geq 1$ describe consumers' homogeneous perception threshold. Initially the baseline case of $\delta = 1$ (perfect perception) is given, with this then being compared to the case of $\delta > 1$, i.e. consumers with bounded perception.

5.2.1 Baseline case; $\delta = 1$

The model and results from Motta (1993) are presented below with a brief derivation found in section E.1.

Two identical firms, 1 and 2, produce a good with quality $q_i \in Q$ sold at price $p_i \in P$, $i \in \{1, 2\}$. Firms have fixed costs of quality $\frac{1}{2}q_i^2$. Marginal costs of production are normalized to be 0, as this greatly improves tractability whilst leaving results qualitatively unchanged.

Consumers may purchase a single unit of the good, from which their utility is given by $u = \alpha q - p$. The payoff from the outside option of not consuming is normalized to 0. Consumers perceive any $q > 0$ as distinct from the outside option. α represents a consumer's taste for quality, and there is a unit mass of consumers with α uniformly distributed in the interval $[0, 1]$.

The timing of the game is as follows:

Period 1: Firms simultaneously choose qualities, incurring fixed costs of quality.

Period 2: Having observed the chosen qualities, firms set prices, consumers purchase and firms earn revenue.

Let h always denote the firm producing the higher quality good, $h \in \{1, 2\}$ and ℓ the firm producing the lower quality good, $\ell \in \{1, 2\}$, $h \neq \ell$. This implies in equilibrium that $p_h > p_\ell$. The consumer with taste parameter $\alpha' = \frac{p_h - p_\ell}{q_h - q_\ell}$ is indifferent between high and low quality, and the consumer with $\alpha'' = \frac{p_\ell}{q_\ell}$ is indifferent between low quality and not consuming. Demand for firm h is $1-\alpha'$ and demand for firm ℓ is $\alpha'-\alpha''$.

Given qualities q_h and q_ℓ, equilibrium prices in period 2 are

$$p_h^* = 2q_h \left(\frac{q_h - q_\ell}{4q_h - q_\ell}\right), \qquad p_\ell^* = q_\ell \left(\frac{q_h - q_\ell}{4q_h - q_\ell}\right) \tag{5.1}$$

so that firms choose qualities in the first period to maximize

$$\pi_h(q_h, q_\ell) = \frac{4q_h^2(q_h - q_\ell)}{(4q_h - q_\ell)^2} - \frac{1}{2}q_h^2, \tag{5.2a}$$

$$\pi_\ell(q_h, q_\ell) = \frac{q_h q_\ell (q_h - q_\ell)}{(4q_h - q_\ell)^2} - \frac{1}{2}q_\ell^2. \tag{5.2b}$$

Standard equilibrium qualities q_h^* and q_ℓ^* are

$$q_h^* = \mu^{*3}\left(\frac{4\mu^* - 7}{(4\mu^* - 1)^3}\right), \qquad q_\ell^* = \mu^{*2}\left(\frac{4\mu^* - 7}{(4\mu^* - 1)^3}\right) \tag{5.3}$$

where

$$\mu^* = \frac{q_h^*}{q_\ell^*} \tag{5.4}$$

is the ratio of the high quality to the low quality in equilibrium. This ratio is constant and can be shown to be equal to the unique real solution of $4\mu^3 - 23\mu^2 + 12\mu - 8 = 0$, which is approximately $\mu^* \approx 5.251$, implying from equation (5.3) that $q_h^* \approx 0.253$ and $q_\ell^* \approx 0.048$. Equilibrium profits are $\pi_h^* \approx 0.0244$ and $\pi_\ell^* \approx 0.00153$.

5.2.2 Bounded perception; $\delta > 1$

Consumers' ability to perceive the difference between goods' qualities is now bounded in the way described in section 4.3.

Now let all consumers have an identical perception threshold of $\delta > 1$. Given that for $\frac{q_h}{q_\ell} < \delta$ consumers perceive the two goods as homogeneous, both having quality q', they will either purchase the good with the lower price, or not purchase. They will never purchase the good with the higher price.

That consumers never purchase the good with the higher price leads to a key result.

Proposition 5.1. *(i) In equilibrium, firms will never choose qualities such that $\frac{q_i}{q_j} < \delta$, i.e., consumers will always be able to perceive a difference between the firms' equilibrium qualities.*

(ii) If $\delta > \mu^*$, a necessary condition for equilibrium is that $\frac{q_h}{q_\ell} = \delta$.

All proofs are contained in appendix E. It was specified in section 4.3 that, when $\frac{q_h}{q_\ell} < \delta$, consumers perceived both goods as having a quality $q' = \lambda q_h + (1 - \lambda) q_\ell$, $\lambda \in [0, 1]$. However, it should be emphasized that the only necessary restriction for this result on consumers' behaviour when $\frac{q_h}{q_\ell} < \delta$ is that they never buy the higher priced good.

The intuition behind part (i) is straightforward: If firms choose qualities which consumers treat as homogeneous, they end up in the Bertrand trap and make a loss. As they can make 0 by choosing $q = 0$, this will never be observed in equilibrium.

As μ^* is the ratio of q_h to q_ℓ in the baseline case, $\delta > \mu^*$ implies firms will no longer produce the baseline qualities, but choose qualities which are further apart. Part (ii) then states that in equilibrium they will never produce qualities such that $\frac{q_h}{q_\ell} > \delta$. The intuition for this is that if $\frac{q_h}{q_\ell} > \delta$, firms' profits and best response functions are locally as in the baseline case. Thus, as in the baseline case, each firm will wish to choose a quality closer to its rival's.

This is not the case when $\frac{q_h}{q_\ell} = \delta$. Here, it is possible for firms to wish to move closer together in the baseline case, but to be prevented from doing so with $\delta > \mu^*$

by the prospect of consumers perceiving the goods as homogeneous. Hence, it must be a necessary condition for equilibrium that $\frac{q_h}{q_\ell} = \delta$.

When the baseline equilibrium no longer holds, equilibrium qualities may then be found by imposing restrictions on qualities satisfying $\frac{q_h}{q_\ell} = \delta$ such that no firm wishes to deviate. The following six conditions provide an exhaustive list of the possible ways each firm can deviate, and so if all six hold, then the firms must be in equilibrium:-

(i) Firm h does not choose $q_h = 0$, making π_h=0. **(Fh0)**

(ii) Firm ℓ does not choose $q_\ell = 0$, making π_ℓ=0. **(Fℓ0)**

(iii) Firm h does not produce $q_h > \delta q_\ell$ above the perception threshold. **(FhT)**

(iv) Firm ℓ does not produce $q_\ell < \frac{q_h}{\delta}$ below the perception threshold. **(FℓT)**

(v) Firm h does not undercut firm ℓ by producing $q_h < q_\ell$. **(FhU)**

(vi) Firm ℓ does not leapfrog firm h by producing $q_\ell > q_h$. **(FℓL)**

The conditions are explicitly derived in the appendix, and it is shown that Fh0, Fℓ0 and FℓL are redundant. Equilibrium can then be stated.

Proposition 5.2. *In equilibrium, firms choose qualities*

$$q_h^* \begin{cases} = \mu^{*3}\left(\dfrac{4\mu^* - 7}{(4\mu^* - 1)^3}\right) & \text{if } \delta \leq \mu^* \\ \in \left[FhT(\delta), \min\{F\ell T(\delta), FhU(\delta)\}\right] & \text{if } \delta > \mu^* \end{cases} \tag{5.5a}$$

$$q_\ell^* \begin{cases} = \mu^{*2}\left(\dfrac{4\mu^* - 7}{(4\mu^* - 1)^3}\right) & \text{if } \delta \leq \mu^* \\ \in \dfrac{1}{\delta}\left[FhT(\delta), \min\{F\ell T(\delta), FhU(\delta)\}\right] & \text{if } \delta > \mu^* \end{cases} \tag{5.5b}$$

(5.5c)

where

$$FhT(\delta) = \frac{4\delta\left(4\delta^2 - 3\delta + 2\right)}{(4\delta - 1)^3} \tag{5.6a}$$

$$F\ell T(\delta) = \frac{\delta^3(4\delta - 7)}{(4\delta - 1)^3} \tag{5.6b}$$

$$FhU(\delta) = \frac{2\delta^3\left(4\delta^2 - 1\right)}{(4\delta - 1)^2\left(1 + \delta + \delta^2 + \delta^3\right)}. \tag{5.6c}$$

Equilibrium exists for any finite $\delta \geq 1$.

For $\delta \leq \mu^*$, equilibrium is unchanged from the baseline case. For $\delta > \mu^*$ there is a continuum of equilibria, which is illustrated in figure 5.1.

Firms' equilibrium profits are obtained by substituting q_h^* and q_ℓ^* into equation (5.2). Expressions for profits are not given explicitly for reasons of brevity, however it can be shown that

Proposition 5.3. *For $\delta > \mu^*$, firm h (ℓ) makes a greater profit than in the baseline case if inequality (E.9) (inequality (E.10)) is fulfilled, and less profit than in the*

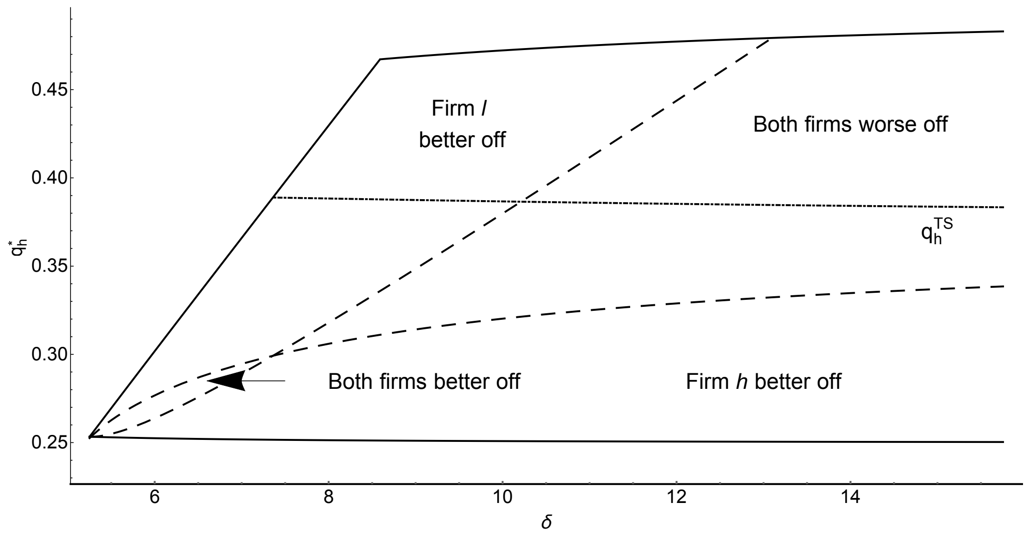


FIGURE 5.1: Solid lines indicate values of q_h which form an equilibrium. Dashed lines indicate whether firms earn more or less than in the baseline. The dot-dashed line shows the q_h at which total surplus is maximized. In the baseline case $q_h^* \approx 0.253$.

baseline case if it is not.

Inequalities (E.9) and (E.10) are not reproduced in the main text due to their length, however they are illustrated in figure 5.1.

It is also possible to compare equilibria for consumers with a given threshold. The comparative statics are given in terms of q_h^*. Note that since $\frac{q_h^*}{q_\ell^*} = \delta$, for fixed δ an increase in q_h^* is equivalent to an increase in q_ℓ^*: It implies the absolute levels of quality in the market are increased.

Proposition 5.4. *In equilibrium, given some $\delta > \mu^*$:-*

(i) Firm h's profit is decreasing in q_h^* and firm ℓ's profit is increasing in q_h^*.

(ii) Consumer surplus is decreasing in q_h^*.

(iii) If $\mu^* < \delta \leq \delta_{TS} \approx 7.359$, total surplus is increasing in q_h^*, where δ_{TS} is defined as the unique positive root of equation (E.12). If $\delta > \delta_{TS}$, total surplus is

maximized at

$$q_h^{TS} = \frac{\delta^2 \left(12\delta^2 - \delta - 2\right)}{2 \left(\delta^2 + 1\right) \left(4\delta - 1\right)^2} \tag{5.7}$$

below which it is increasing in q_h^* *and above which it is increasing in* q_h^*.

q_h^{TS} is illustrated in figure 5.1.

5.2.3 Discussion

Having derived equilibrium for consumers with bounded perception, it can be seen that the perception threshold δ must exceed $\mu^* \approx 5.251$ for there to be any difference to the baseline case. Therefore it is worth examining whether such values of δ could be reasonably expected to be observed, and so whether bounded perception is relevant.

Psychophysical research suggests that the minimum perceivable difference in intensity of some stimulus (equivalent to $\delta - 1$) typically takes values much smaller than the values δ must take to have an impact on quality choice. This *just noticeable difference* has been estimated to be 0.079 for brightness, 0.048 for loudness and 0.02 for heaviness (Techtsoonian, 1971), yet here $\delta - 1 \gtrsim 4.251$ for consumers' bounded perception to be relevant.

However, this is due to the necessity of δ being greater than the standard equilibrium quality ratio μ^*, and μ^* depends heavily on the distribution of consumers in the economy and the assumption of 0 marginal cost. Both a more concentrated distribution of consumers and a positive marginal cost of production would lead to a lower μ^*, and thus a lower perception threshold would be required to influence quality choice.

Another possibility is to regard q as excess quality over some minimum required level q_{min}, so that the ratio of high to low quality is now $\frac{q_h + q_{min}}{q_\ell + q_{min}}$. Thus if $q_{min} = 5$, for example, then the threshold needed for perception to be relevant falls to $\delta - 1 \gtrsim 0.041$, which is in line with the psychophysical results quoted above.

There is now a continuum of equilibria. The reason is that when $\delta > \mu^*$ firms must increase the ratio of high to low quality in order to avoid the Bertrand trap. However, there is no unique way of doing this: a given shift in the ratio of qualities can be split many ways between the firms. For $\delta > \mu^*$, there is a range of qualities such that $\frac{q_h}{q_\ell} = \delta$ and such that in the standard case the only profitable deviation for a firm is to produce a quality closer to its rival's. When consumers have bounded perception, such a deviation now results in a loss, leading to a range of qualities for which there is no profitable deviation.

Firms are presented with a coordination game, in that they must choose qualities such that $\frac{q_h}{q_\ell} = \delta$, but must coordinate on the absolute values that satisfy this ratio. Proposition 5.3 compares the different equilibria and finds that firm h prefers low absolute values of quality and firm ℓ prefers high absolute values of quality. Thus the firms' problem is not one of pure coordination: there is no salient equilibrium which both firms prefer.

To see why higher levels of quality have different effects for each firm, note that if one firm were granted a monopoly, it would choose $q = 0.25$.[2] Duopoly forces one firm to choose above and one firm below the quality it would ideally prefer. Hence higher levels of quality imply firm h is further away from its ideal quality, reducing profit, whereas firm ℓ is closer, increasing profit.

It is possible to compare firms' profits to the baseline as well. Proposition 5.3 shows the effect of bounded perception is ambiguous: compared to the baseline, bounded perception may increase both firms', one firm's or neither firms' profit. This is due to the ratio of high to low quality being greater than in the baseline for $\delta > \mu^*$, which has two contrasting effects. Firstly, price competition in period

[2]This value is easily derived from equation (E.3a) by setting $q_\ell = 0$.

2 is reduced when firms differentiate themselves more. For a given quality choice, the further away the rival firm's quality, the higher the price a firm may charge before consumers switch to the rival (or to not consuming). On the other hand, firms must make suboptimal quality choices. Increasing quality is costly for firm h, and decreasing quality reduces willingness-to-pay for firm ℓ, which tends to reduce profit. This has the opposite effect to reduced price competition, and for each firm, either effect may dominate.

Proposition 5.4 states that consumers prefer high levels of quality. A caveat to this result is that it holds strictly in a comparative static sense. That it is possible to say anything definite regarding consumer surplus is due to the fact that in equilibrium consumers perceive both goods distinctly. Consumers' perceived surplus depends on the context of the choice set they have. Thus when measuring the change in consumer surplus if consumers shift from one equilibrium to another, it is necessary to account for the new goods being viewed in context with the old. It is not clear that a consumer is made better off by consuming a higher quality good if she pays a higher price for it, yet cannot perceive the new good as superior to the old.

The principle result of this section is that the more bounded consumers' perception of quality, the greater the degree of product differentiation. Given that products are differentiated in the baseline, albeit to a lesser degree, the behaviour of firms is qualitatively unchanged. To search for empirical examples of bounded perception, it is necessary to compare markets across sectors or over time.

Comparing different markets, the quality of a product may be assessed more easily in one than the other. This could be due to a fundamental difference in the nature of the quality of different goods, or due to a clear scale by which to measure quality being available in one market but not the other. The prediction of this model is then that products will be differentiated to a greater degree in markets in which product quality is hard to assess than in markets in which it is easy to assess.

It can be reasonably supposed that when a new type of product is introduced, the perception threshold is high due to consumers' inexperience. As they grow more accustomed to it, the threshold is reduced. Thus it should be expected that for new product types, a large degree of product differentiation is observed, followed by less differentiation over time.

Bounded perception does not necessarily improve over time, however. In many markets price comparison websites operate. These facilitate comparing firms on prices, but quality information is often much more difficult to assess. In the airline industry, for example, the advent of flight search websites has led to people to focus far more on the price of a journey than the airline's service level. This has led to success for budget airlines and those with the highest level of service, but problems for more traditional firms who charge more than budget airlines but struggle to differentiate their level of service.

The quality of a good is measured as a single attribute. For the majority of goods, however, there are many different attributes. Quality in this case may be regarded as the single attribute which is the principle source of product differentiation. Another way to consider the single dimension of quality is as an aggregate measure of the quality of many attributes. As such an aggregate measure makes direct comparison more difficult, such an interpretation also makes bounded perception of quality more plausible.

It is also reasonable to assume that the greater the number of attributes that make up this measure, the more difficult the direct comparison of goods is and the higher the perception threshold. Thus the model predicts that in markets with more complex goods, a greater degree of differentiation should be observed then for markets with simple goods.

The current section describes equilibrium in a market with bounded perception taking the perception threshold as exogenous. This allows the threshold to be endogenized in the following section, in which the actions of firms to increase or decrease the ease with which consumers perceive their goods are described.

5.3 Endogenous perception threshold

So far it has been assumed that the perception threshold is exogenous. However, firms can often influence the perception of their goods. This could either have the effect of lowering the threshold, for example if firms coordinated on a clear colour scheme allowing quality to be discerned at a glance, or by the printing of relevant information prominently on a good's packaging, or it could raise the threshold if the low quality firm designs its packaging to ape that of the high quality good, or if information is hidden in the fine print.

5.3.1 Model

To endogenize the perception threshold, suppose each firm may exert presentational effort s to alter the perception threshold, with $s_i \in [\underline{s}, \overline{s}]$, $\underline{s} < 0$, $\overline{s} > 0$, $i \in \{h, l\}$. As methods to influence perception are generally, as in the examples above, rooted in design and presentation, this effort is considered to be costless. Given each firm's presentational effort, the perception threshold becomes $D(s_T, \delta)$, where $s_T = s_1 + s_2$ and $D(s_T, \delta)$ has the following properties:-

(i) $D(s_T, \delta) \geq 1$.

(ii) $D(0, \delta) = \delta$.

(iii) If $D(s_T, \delta) > 1$ and $s'_T > s_T$, then $D(s'_T, \delta) > D(s_T, \delta)$.

(iv) If $D(s_T, \delta) > 1$ and $\delta' > \delta$, then $D(s_T, \delta') > D(s_T, \delta)$.

δ can thus be thought of as a "default" threshold when neither firm exerts any particular effort to manipulate perception. Negative s_T implies improved consumer perception and positive s_T indicates worsened perception relative to the default. The threshold is symmetric with respect to firms' presentational efforts.

Endogenizing bounded perception via presentational effort means effectively that firms can choose the frame in which consumers view their goods, as in Piccione and Spiegler (2012) and Spiegler (2014).

Two extra conditions must now be fulfilled in equilibrium:-

(vii) Firm h does not wish to choose a different presentational effort. **(Fhδ)**

(viii) Firm l does not wish to choose a different presentational effort. **(Fℓδ)**

Let $\underline{D} = D\left(2\underline{s}, \delta\right)$ be the lowest threshold obtainable for a given δ. Then

Proposition 5.5. *In equilibrium, firm choose qualities and presentational efforts*

q_h^*, $\tilde{s}_h^*$ and q_ℓ^*, s_ℓ^* given by

$$q_h^* \begin{cases} = \mu^{*3} \left(\dfrac{4\mu^* - 7}{(4\mu^* - 1)} \right)^3 & \text{if } \underline{D} \leq \mu^* \\ \in \left[FhT\left(\underline{D}\right), \min\left\{ F\ell T\left(\underline{D}\right), FhU\left(\underline{D}\right) \right\} \right] & \text{if } \underline{D} > \mu^* \end{cases} \tag{5.8a}$$

$$s_h^* \begin{cases} \in \left\{ s_h : D\left(s_T, \delta\right) \leq \mu^* \right\} & \text{if } \underline{D} \leq \mu^* \\ = \underline{s} & \text{if } \underline{D} > \mu^* \end{cases} \tag{5.8b}$$

$$q_\ell^* \begin{cases} = \mu^{*2} \left(\dfrac{4\mu^* - 7}{(4\mu^* - 1)} \right)^3 & \text{if } \underline{D} \leq \mu^* \\ \in \dfrac{1}{\delta} \left[FhT\left(\underline{D}\right), \min\left\{ F\ell T\left(\underline{D}\right), FhU\left(\underline{D}\right) \right\} \right] & \text{if } \underline{D} > \mu^* \end{cases} \tag{5.8c}$$

$$s_\ell^* \begin{cases} \in \left\{ s_\ell : D\left(s_T, \delta\right) \leq \mu^* \right\} & \text{if } \underline{D} \leq \mu^* \\ = \underline{s} & \text{if } \underline{D} > \mu^* \end{cases} \tag{5.8d}$$

It is interesting to compare the comparative statics of firms' ability to influence perception (i.e. $\underline{s}$). Consider $\underline{s}' > \underline{s}$ with $\underline{D}' > \mu^*$. Then any q_h^*, q_ℓ^* that formed an equilibrium with $\underline{s}$ will not form an equilibrium with $\underline{s}'$, as it implies a different equilibrium ratio of qualities. However, it is always possible to find some q_i^*, $i \in \{h, \ell\}$ that firm i produces in some equilibrium given both $\underline{s}$ and $\underline{s}'$, even though the quality produced by firm $j \neq i$ will differ. Comparative statics are then performed from the perspective of a firm producing the same quality q_i^* given different $\underline{s}$.

By taking this approach it is possible to show

Proposition 5.6. *(i) Let firm $i \in \{h, \ell\}$ produce quality q_i^* in equilibrium given $\underline{s}$, earning profit $\pi_i^*\left(q_i^*, \underline{s}\right)$. Then given $\underline{s}' < \underline{s}$, conditional on producing the same quality q_i^*, $\pi_i^*\left(q_i^*, \underline{s}'\right) \leq \pi_i^*\left(q_i^*, \underline{s}\right)$.*

(ii) Let firm $i \in \{h, \ell\}$ produce quality q_i^* in equilibrium given $\underline{s}$. Then given $\underline{s}' < \underline{s}$, conditional on the firm producing the same quality q_i^*, consumer surplus is weakly higher.

5.3.2 Discussion

In section 5.2, although bounded perception caused firms to position themselves farther apart than in the baseline and they also has to coordinate on equilibrium, their behaviour was qualitatively unchanged. Here though, a prediction is made that firms will engage in novel behaviour: both low and high quality firms will present their goods so as to make perception of their quality as easy as possible. The effect of a competitive environment on consumers' decision making bias is to reduce it as much as possible.

It is not immediately obvious that firms should always choose to reduce, rather than exacerbate bounded perception, given that one or both firms may with $\delta > \mu^*$ find itself in an equilibrium earning greater profit than in the baseline. Furthermore, provided $\overline{s} \geq -\underline{s}$, a firm can always negate any presentational effort of the other firm to improve perception.

The intuition behind proposition 5.5 is that the equilibrium qualities for $\delta > \mu^*$ found in the previous section (equation (5.5)) form equilibria only due to bounded perception removing a profitable deviation. Without the perception threshold, each firm would prefer to produce a quality closer to its rival's.

When firms are able to influence consumers' perception, the profitable deviation is again present and firms opt to lower the perceptual threshold. Equilibrium then requires that such profitable deviations to produce a quality closer to its rival's does not exist, which is the case only if $s_h = s_\ell = \underline{s}$ and it is not possible to further enhance perception, or the threshold is lowered sufficiently that the baseline equilibrium is reached.

Presentational effort was assumed to be costless. However, it is possible that the costs of such effort may be significant, for example if it involves a large advertisement campaign. Provided the cost is not excessive though, firms will still reduce consumers' perceptual bias to some extent.

Examples of firms presenting their goods in such a way to ease consumer perception of them are readily available. In many countries, for example, dairy firms have coordinated on a colour-coding scheme for different types of milk. Such schemes, which make it easy for consumers to perceive the type of milk at a glance, occur even though they require sustained coordination between firms.

Another example is the carat measure of gold purity. The difference in the purity of different gold items is very difficult to assess. It is made much easier by the fact that jewellers have for many years coordinated on the universal use of an easy to understand scale.

Proposition 5.6 examines the welfare effects of firms' ability to exert presentational effort. For a given equilibrium quality, a firm earns less profit the more firms are able to improve consumer perception. This is due to the resultant lower perception threshold meaning the rival firm choosing a quality closer to its own, and thus in the price setting stage the competition being more fierce. For consumers, on the other hand, the opposite is true, as they benefit from the increased price competition and consumer welfare increases.

A caveat to the welfare results is that, due to the multiplicity of equilibria, an exogenous increase in $\underline{s}$ is not guaranteed to decrease profit or improve consumer welfare.

The result that, rather than trying to exploit their ability to confuse consumers, firms instead try to eliminate consumers" biases is markedly different to, for example, Gabaix and Laibson (2006). In that paper, competitive forces do not act to debias consumers, as such debiased consumers are not then profitable to firms. It also contradicts the experimental findings of Kalayci and Potters (2011a), in which in contrast to here, consumers were homogeneous in their taste for quality.

Aside from the specific results on biased perception, an illustration is thus also given of the complexities of boundedly rational consumers interacting with profit maximizing firms. The outcomes may be vastly different depending on the particular form of bounded rationality being considered, and also on relatively minor changes to the market structure. Thus any potential market intervention should be carefully considered before enactment.

5.4 Conclusion

It is impossible to perceive a small enough difference between two products. This article has taken this simple observation and shown that it can have significant influence on market outcomes, in terms of the qualities and prices observed in equilibria and profit, the effect on which is ambiguous.

There are many extensions that may be made to the simple framework used here. For example, repeated interactions would be a fruitful avenue of research, both if consumers' perception of quality increases with repeated consumption, and if firms have an incentive to reduce quality over time when consumers perceive the good they buy today as the same as the good they bought yesterday.

There is a growing recognition that perceptual issues should not be lightly disregarded in both empirical and theoretical work, and this has been again demonstrated in this study.

Manipulating perception: The effect of product similarity on valuations and markets 6

Andreas Gotfredsen, Carsten S. Nielsen, Alexander C. Sebald, & Edward J.D. Webb

Abstract

The economic impact of perceptual limitations is studied using experimental goods for which, by altering the similarity of their visual representation, the ease of perceiving the difference between them can be manipulated while keeping the underlying value constant. It is shown in the first experiment that perceptual limitations cause subjects' valuations of goods to become more similar when it is harder to tell them apart. In a second experiment, the goods are traded in a market with heterogeneous buyer preferences and seller market power, prices and profits are unchanged by perceptual limitations. Buyers' payoffs are lower, as they make more mistakes and buy fewer goods when perceiving the difference between them is hard. Buyers use a different method of constructing their valuations in the market than in individual choice, and there is weak evidence this is beneficial for buyers.

6.1 Introduction

It is self-evident that out perception of the world is limited. We are constantly bombarded by information, and it is possible to perceive only a small fraction of it.

This cannot but have a large influence on economic decision making and interactions.

This paper examines the impact of individuals' ability to perceive differences between similar goods being limited. As an example of how individuals' ability to perceive differences can have large economic consequences, a British supplier of caviar mistakenly mislabelled its standard variety, costing around £40/kg., as sevruga, which costs much more, around £1280/kg., for several months. No one noticed.[1]

Similarly, a major scandal emerged in several European countries when various processed food products, labelled as containing meats such as beef and chicken, in fact contained horsemeat.[2] However, consumers had apparently been eating such products for some time[3] without perceiving the difference between the adulterated products and the genuine article. It was only after DNA testing by regulatory bodies that the problem emerged.

The above two examples involve consumers interacting with profit maximizing firms. Although there was not necessarily any malice in either case, it is common for policy makers to be wary of firms taking advantage of consumers' perceptual limitations. Thus, for example, there are many laws against watering down alcohol, such as the US requiring beer to be within 0.3% ABV within the stated strength[4] and the EU setting various minimum strengths for spirits.[5]

It is hence also important not only to examine individuals' perceptual limitations in isolation, but also to study the impact of such deviations from the canonical economic model of decision making on strategic interaction. When individuals with perceptual limitations meet profit maximizing firms in a market, it is easy to imagine

[1]Source: http://www.telegraph.co.uk/news/uknews/10067876/Supplier-investigated-after-top-grade-caviar-contained-cheaper-variety.html, accessed 15/12/15

[2]Source: http://www.bbc.com/news/uk-21375594 accessed 15/12/15.

[3]Source: http://www.independent.co.uk/news/uk/home-news/horsemeat-scandal-findus-leak-reveals-horse-in-beef-for-six-months-8486602.html accessed 15/12/15.

[4]Source: http://www.ttb.gov/pdf/ttbp51008_laws_regs_act052007.pdf

[5]Source: http://eur-lex.europa.eu/LexUriServ/LexUriServ.do?uri=OJ:L:2008:039:0016:0054:EN:PDF

that firms will be able to exploit consumers' limitations and benefit from them.

However, this is not necessarily the case. In Akerlof (1970)'s famous model of a market for lemons, consumers are unable to perceive the difference between high and low quality sellers, which results in the high type of firm being harmed. Taneva (2015) even constructs a model in which a buyer finds it optimal to maintain limited perception of a good, to the detriment of the seller, even though she has the option of perceiving its value perfectly.

Recently, there has been a growing acknowledgment that perception and attention are important features of decision making, yet there is still much to learn in this area.

This chapter contributes to the emerging literature on perception and attention in two ways. Firstly, novel experimental goods are introduced that allow the perception of their similarity to remain constant, while the underlying values are held constant. Thus throughout this chapter, when goods are talked of as being more or less similar, it is their visual representations that are being referred to, rather than their underlying values. The goods are also easy for subjects to comprehend and are not over-reliant on their cognitive or mathematical skills. Secondly, the effect is shown of varying subjects' perception of goods in an experimental market with heterogeneous buyer values and sellers with market power.

It is found that in an individual valuation task, subjects' valuations depended on similarity: valuations for more similar goods were closer together. Despite this effect on subjects' willingness-to-pay, in a second experiment in which subjects participated in a market, the behaviour of fully informed sellers did not depend on perceptual limitations. The prices they set and the profits they earned did not differ significantly between goods for which perceiving the difference was easy and goods for which it was hard.

Buyers, however, earn less surplus from more similar goods, partly due to being more reluctant to purchase goods when it is hard to perceive the difference between the choices on offer, and partly due to sometimes mistakenly purchasing goods which offer a low or negative surplus.

There is evidence, on the other hand, that the market mechanism caused buyers to form their valuations in a different way to subjects performing the valuation task, and some indication that this was beneficial to them.

Rubinstein (1988), building on work by Luce (1956) and Fishburn (1970), provides a theoretical underpinning of the current paper. He provides an axiomatic treatment of choice in which individuals are unable to perceive the difference between sufficiently similar components of a choice set, so that the usual economic assumption of transitivity no longer holds.

The experimental goods used in this paper attempt to partially operationalize this notion of similarity. However, it is important to note that Rubinstein treats perception as binary: either individuals are able to perfectly perceive two attributes as distinct or treat them as completely homogeneous. Here, a more continuous definition of similarity is employed, specifically normalized cross-correlation, a measure taken from the vision and image matching literature.

Thus the operationalization of similarity has much in common with the discipline of psychophysics. Commonly psychophysical studies measure how far apart two stimuli (light, heat, sound, etc.) must be in order to detect a difference between them with a given accuracy (Falmagne, 2002). The less similar the stimuli, the more reliably individual can perceive the difference between them.

The theoretical model of similarity has been applied to risky choice and deviations from expected utility theory (Aizpurua et al., 1993; Leland, 1994; Sileo, 1995). Leland (2002) uses the similarity framework to examine anomalies in intertemporal choice.

Chapters 4 and 5 showed applications of the Rubinstein model to consumer choice. Bachi (2014) also studies the effect of consumers not perceiving the difference in utility between sufficiently similar prices.

Kalayci and Potters (2011b) examine a market with many differences to the one presented here (homogeneous buyers, vertical as opposed to horizontal differentiation, different choice variables for sellers who are asymmetric, robot buyers for one treatment, etc.). They introduce experimental goods for which the degree of complexity can be altered while the underlying value is held constant, thus making it more difficult for subjects to perceive the difference in value.

However, their goods introduce complexity by having the value expressed as a sum of increased length. This is different to altering the visual perception of goods, as is done here, and introduces a much greater interdependence with subjects' cognitive and mathematical abilities. It is extremely common for consumers to use visual perception in everyday purchases. Thus the experimental goods used here possess greater external validity for this target domain, since they use a visual representation, and also avoid relying too heavily on subjects' cognitive abilities.

Kalayci and Serra-Garcia (2015) show that complexity in goods' costs drives subjects to choose goods based only on their benefits. However, there is no analogous effect of choosing based only on cost with complex benefits. Spiegler (2015) presents a general framework in which to study the effect of choice complexity on market structure and Crosetto and Gaudeul (2012) show experimentally that consumers prefer choices between simple goods than complex ones.

All the above studies vary perception of their goods by expressing their value as a mathematical formula of varying difficulty of calculation. One study that does use a visual representation, albeit not for valuation or markets, is Ruud, Schunk, and Winter (2014). They use a colour band of varying brightness, with subjects trying to estimate the position of the brightest part. The greater the variation, the easier it is to pinpoint this, which is used to investigate the rounding of subjects' estimates.

The visual perception of goods has often been studied in the marketing literature. For example, Walsh and Mitchell (2005) attempt to measure how susceptible consumers are to treating heterogeneous goods as similar. Chandon and Ordabayeva (2009) and Wansink and van Ittersum (2003) examine how the changing dimensions of a product can influence consumers' perception of its volume while the actual volume is held constant.

Kwortnik et al. (2006) look at the effect of labelling on consumer choice, and similarly Nilufer and Krishna (2011) find that semantic cues from a size label can influence the perceived size of a good. Lamberton and Diehl (2013) study how retailers' physical arrangement of products can influence the similarity of consumers' perceptions of the goods.

There is also a vast amount of research on visual attention in psychology. For an introduction to this literature, see Findlay and Gilchrist (2003).

Section 6.2 describes the experimental goods used. Section 6.3 reports the procedures and results from the first experiment, in which subjects individually state their willingness-to-pay for goods, and section 6.4 details the second experiment, in which subjects trade goods in a market. Section 6.5 discusses the findings of both experiments and section 6.6 concludes.

6.2 Experimental goods

The experiments used pictures, each of which was a 10×10 matrix, with every cell having a value to subjects of between 1 to 9 points. The value of the whole picture was then the sum of values over all cells. The value of a cell was represented by a colour, so that each picture formed a "heat map". There were two treatments, red and blue. In the red treatment, subjects valued red cells more highly than blue, whereas in the blue treatment, subjects valued blue cells more highly than red. An example picture is shown in figure 6.1, along with a scale showing the value of squares in the red treatment. The scale in the blue treatment is the reverse of this, so that the blue square furthest to the left has value 9 and the red square furthest to the right has value 1.

The pictures were presented in pairs. There were 30 picture pairs, with 10 pairs consisting of "block" pictures and 20 pairs consisting of "scrambled" pictures. In each pair, the absolute difference from 500 was the same for both pictures of a pair, with one picture's value being (weakly) less than 500 and one picture being (weakly) greater than 500 (e.g. picture 1 had the value of 473, while picture 2 had the value of 527). Note this implies that in each pair the sum of their values was 1000.

The value difference of picture pairs ranged from 0 to 54, with a step of 6 (see Table 6.1). The pictures used thus ranged in value between 473 and 527. Subjects were informed that all pictures had a value between 460 and 540, and so their responses when stating WTP were restricted to be in this range.

Cell values for the scrambled pictures were randomly generated. Block type picture pairs, however, only differed by a block of cells in the top left-hand corner, with all other cells having a value of 5. The dimensions of the block were the same for both pictures in a pair, with the block on one picture consisting of cells of value

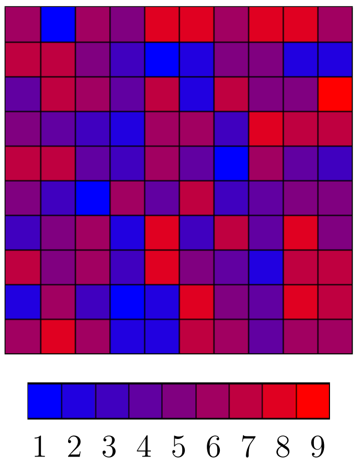


FIGURE 6.1: An example picture and the value scale for the red treatment.

4 and the other of cells of value 6. Examples of both block and scrambled pictures are shown in figure 6.2.

The similarity of the pictures in a given pair is measured by the normalized cross correlation (NCC), a standard measure of similarity in image and vision research (Simpson, Loffler, & Tucha, 2013; Simpson, Falkenberg, & Manahilov, 2003). NCC takes values between -1 (very dissimilar) and 1 (identical). Block pictures all have a value of -1[6] and scrambled pictures were constructed so that for each value difference, one pair had an NCC close to 0 and another pair had an NCC close to 0.9. For full details see table 6.1.

6.3 Experiment 1

The goal of experiment 1 was to study people's ability to perceive differences between goods depending on their similarity and ease of comparison. The task subjects

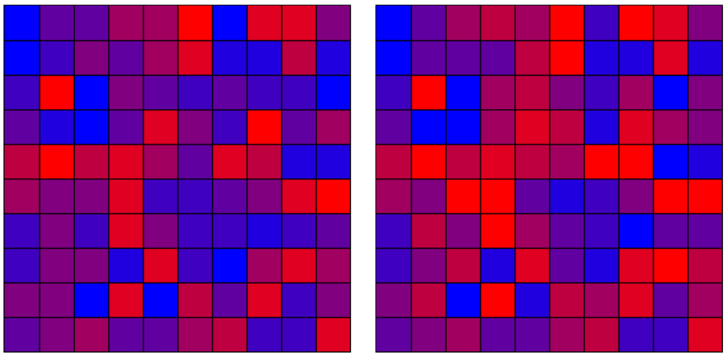
[6]The exception is the (500, 500) pair. The 500 value block pictures consist entirely of cells worth 5, and thus have 0 variance, and since calculating NCC requires dividing by the variance of both pictures, NCC is undefined.

Table 6.1: Picture pair values. 30 picture pairs were generated in total, 2 scrambled pairs and 1 block pair for each picture pair difference.

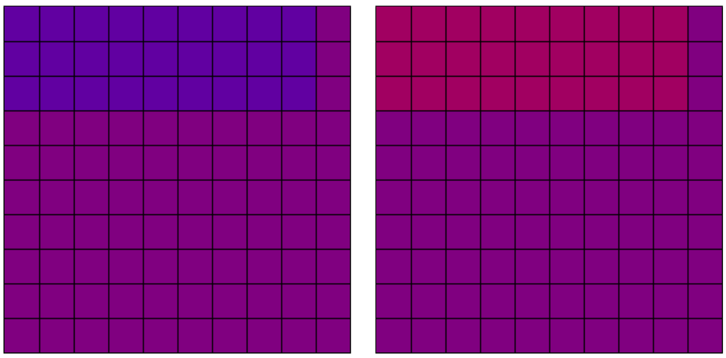
Value difference	0	6	12	18	24	30	36	42	48	54
Picture 1 value										
Red treatment	500	503	506	509	512	515	518	521	524	527
Blue treatment	500	497	494	591	488	485	482	479	476	473
Picture 2 value										
Red treatment	500	497	494	591	488	485	482	479	476	473
Blue treatment	500	503	506	509	512	515	518	521	524	527
NCC (block)	—	-1	-1	-1	-1	-1	-1	-1	-1	-1
NCC (scrambled)	0.00	0.01	0.01	0.00	0.01	0.01	0.00	0.01	0.00	0.01
	0.90	0.91	0.90	0.91	0.91	0.91	0.91	0.91	0.91	0.90

Figure 6.2: Picture pairs with values 473 and 527

(a) Scrambled pictures



(b) Block pictures



performed consisted of stating their willingness-to-pay (WTP) for a series of pairs of the experimental goods described in section 6.2.

6.3.1 Experimental procedures

The valuation task consisted of 30 periods for each subject, one for each of the picture pairs. In each trial, subjects were presented with a picture pair and were asked to indicate the maximum price (in points) that they would be willing to pay for each of the pictures. The Becker-DeGroot-Marschak (BDM) mechanism (Becker, Degroot, & Marschak, 1963) was used to determine the subject's payoff. For each picture, a randomly generated price between 460 and 540 points was drawn. If the randomly generated price was below, or equal to, a subject's willingness to pay for a picture, the subject would buy it, so that the difference between its value and the random price was added to the subjects' earnings. If the randomly generated price was above the subject's indicated willingness to pay, she would not buy the picture. Subjects had 30 seconds to enter their WTP. If they did not submit their WTP in time, they earned 0 for that picture. Figure F.1 shows a sample trial from the valuation task.

To avoid possible order effects, subjects were presented with the picture pairs in random order. It was also randomly determined which picture of a picture pair was shown on the left-hand side and which on the right-hand side of the screen.

After the valuation task, subjects' risk preferences were elicited using a task adapted from Eckel and Grossman (2002). Subjects were presented with a list of gambles and were asked to choose one (see table F.1 for details). For each gamble there was a 50/50 chance of winning a high or low payoff, but as the list goes down the high payoff increases and the low payoff decreases in such a way that the expected value goes up but the guaranteed payoff goes down. Hence choosing a gamble lower on the list implies a subject is less risk averse.

This method has the advantage of being easy to understand and has been found to be correlated with other measures (see Dave, Eckel, Johnson, and Rojas (2010) and Reynaud and Couture (2012)).

Subjects were then presented with six new scrambled picture pairs, two pairs each with the values (497,503), (485,515) and (473,527). In this task, subjects were asked to estimate the values of the pictures, rather than their WTP. In this way, a measure of how skilful they were at distinguishing between the pictures, as opposed to how similar their valuations were, could be obtained. To make estimating the value incentive compatible, subjects' choices were rewarded using a scoring rule: They earned 80 points minus the absolute difference between their estimate and the picture's true value.

Following the experiment, subjects answered a short questionnaire which included demographic information and an additional measure of risk taking behaviour similar to that used in Dohmen et al. (2011).[7]

Before beginning the experiment, subjects were given the instructions to read on paper. To continue to the experiment, the subjects had to answer two control questions to ensure that they had understood the BDM mechanism correctly. Instructions can be found in appendix F.2. At the end of the experiment, the final payoffs were paid out in money using the exchange rate 10 points = 1 DKK. The average payment was DKK 205 (approximately US$30).[8]

The experiment was implemented using zTree (Fischbacher, 2007). 95 subjects were recruited using ORSEE, (Greiner, 2015) and the experiment was carried out at the Laboratory for Experimental Economics (LEE) at the University of Copenhagen. 53 and 42 subjects participated in the red and blue treatment, respectively.

It is expected that as the goods become more similar, that subjects would have greater difficulties perceiving the differences between them. NCC is used as a measure of similarity, and thus it is hypothesized firstly that the difference in WTP between pictures will be greater for block picture pairs than scrambled. Secondly, it is hypothesized that the difference in WTP will be greater for scrambled pairs with low NCC than scrambled pairs with high NCC.

[7]The precise wording of the question was:

> *How do you see yourself: are you generally a person who is fully prepared to take risks or do you try to avoid taking risks? Please tick a box on the scale, where the value 0 means: "not at all willing to take risks" and the value 10 means "very willing to take risks".*

[8]Buyers could theoretically make a loss over the course of the experiment by buying pictures at a price higher than its true value. In the case that a participant made a loss overall they were informed that in this unlikely event they would have to work on a real effort task in order to earn back their show-up fee. No subject ever made an overall loss.

6.3.2 Results

Table 6.2 shows the mean absolute difference in subjects' WTP for block pictures and scrambled pictures with low and high NCC. It can be seen that a greater degree of similarity (as measured by NCC) lowers this difference, although only the difference between low and high NCC scrambled pictures is significant. Thus it is concluded that

Result 6.1. *(i) The absolute difference in subjects' WTP was greater for high NCC than for low NCC scrambled pictures.*

(ii) It is not possible to reject the null hypothesis that the absolute difference in subjects' WTP was the same for both block and low NCC scrambled pictures.

Table 6.2: Mean of subjects' absolute difference in WTP with p-values for tests of difference across picture type. WSR = Wilcoxon signed-rank. *** = significant at 1% level, ** = significant at 5% level, * = significant at 10% level, threshold values for significance adusted for Wilcoxon signed-rank tests using Holm's sequential Bonferroni correction (Holm, 1979). N=95.

	Block	Scrambled	
		Low NCC	HighNCC
Abs. difference in WTP	19.5	18.2	13.8
Kruskal-Wallis	<0.001***		
WSR Test vs. Block	—	0.200	<0.001***
WSR Test vs. low NCC		—	<0.001***

Table 6.3 shows the results of regressions with the absolute difference in WTP as the dependent variable. One model includes subject fixed effects, while the pooled OLS regression includes explanatory variables that are constant over trials for each subject. As should be expected, the underlying value difference of a picture pair is a strong indicator of the difference in willingness-to-pay.

The regressions include dummies for high and low NCC scrambled picture pairs. The dummy for high NCC is significant, but that for low NCC is not, confirming result 6.1. The interaction terms for both dummies with the absolute difference in underlying picture value are negative, however, and not significantly different (F-test p-value 0.556 for fixed effects regression) and so it is concluded that

Result 6.2. *The subjects' willingness-to-pay was less sensitive to the picture value differences for scrambled pictures than for block pictures.*

Otherwise, the most noteworthy result from the pooled OLS regression is that risk preference does not strongly predict WTP. The results also show that there is no significant difference in WTP between subjects in the red and blue treatments.

The pooled regression includes results from the incentivized risk task. As a robustness check, thus was also run with results from the non-incentivized risk measure, with no substantial differences. Analogous checks were carried out for the regressions in section 6.4, with again no material differences found.

Figure 6.3 shows the mean absolute difference in WTP by the picture value difference. It is seen that the slope for scrambled pictures is flatter than for block pictures, which confirms Result 6.2.

6.4 Experiment 2

Experiment 1 documents that by using the experimental goods described in section 6.2, it is possible to manipulate consumers' perception of them whilst holding the underlying values constant. This having been confirmed, it is possible to examine what effect limited perception has on market outcomes.

The goods are now introduced into a market. The market features two sellers and two buyers, with buyers having heterogeneous preferences, giving the firms some

Table 6.3: Regression results with abs. WTP difference as dependent variable. *** indicates significance at 1% level, ** indicates significance at 5% level, * indicates significance at 10% level. N=2850

	(1) Fixed effects	(2) Pooled
Abs. value diff.	0.438***	0.367***
	(<0.000)	(<0.000)
High NCC	0.0191	0.110
	(0.986)	(0.919)
High NCC × Abs. value diff.	-0.211***	-0.220**
	(<0.000)	(<0.000)
Low NCC	3.895***	3.848***
	(<0.000)	(<0.000)
Low NCC × Abs. value diff.	-0.195**	-0.197***
	(<0.000)	(<0.000)
Period	4.70×10^{-3}	-6.04×10^{-3}
	(0.902)	(0.876)
Gamble choice		-0.104
		(0.740)
Gamble choice × Abs. value diff.		0.0105
		(0.451)
Buyer type		-0.0336
		(0.974)
Buyer type × Abs. value diff.		0.0639
		(0.126)
Constant	7.709***	8.270***
	(<0.000)	(<0.000)

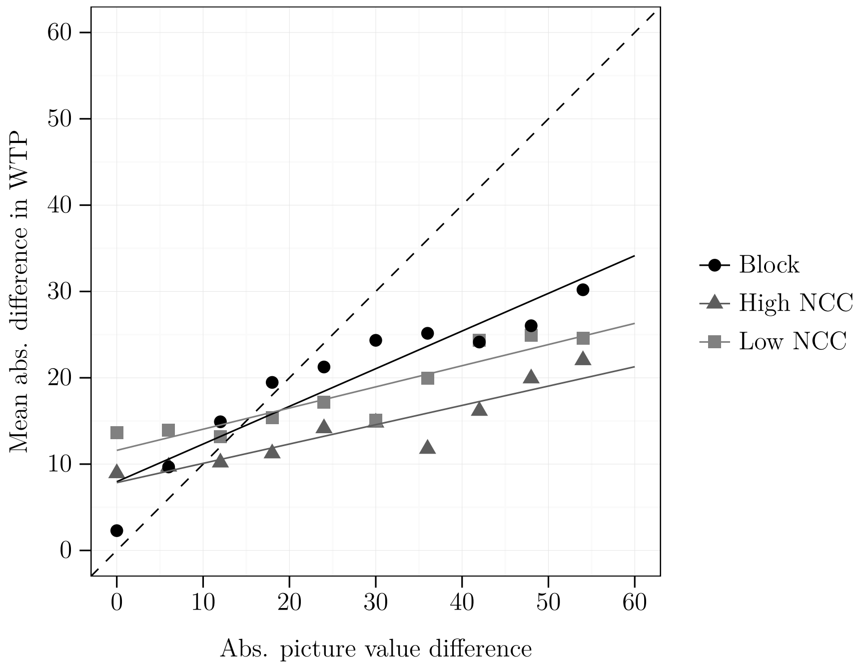


FIGURE 6.3: Willingness-to-pay

market power.

6.4.1 Experimental procedures

Subjects took part in a posted offer market and were assigned a role as either a buyer or a seller. The assigned roles were kept constant throughout the experiment, and buyers were also assigned to be either a red type or a blue type, which again they kept for the entire experiment. Red buyers valued red cells more highly and blue buyers valued blue cells more highly, as in experiment 1.

Each market consisted of one red buyer and one blue buyer. Each seller could sell only one of a pair of goods, with the good pairs used the same as in experiment 1. Thus one seller could sell a good worth (weakly) more to the red type and one could sell a good worth (weakly) more to the blue type.

Each subject traded each good pair once, so that there were 30 market periods in all. In each session approximately half of subjects traded the 10 block pictures first, with the other half trading the 20 scrambled pictures first. Within these sets of block and scrambled pictures the order good pairs were traded in was randomized, but the same for all subjects.

Subjects were randomly rematched to markets every period, and this was common knowledge.

Each trial had two stages. In the first stage, sellers set a price for their picture. Sellers were informed about the true value of their own picture to both buyer types as well as the values of the other seller's picture. Sellers had 60 seconds to set their price (if they exceeded this limit they were prompted to make their choice immediately). In the second stage, buyers could choose between buying a picture from seller 1, or from seller 2, or to abstain from buying. Buyers had 30 seconds in which to make their choice. If they did not make a choice, they bought nothing that round.

If a seller sold a good to a buyer, the seller would earn the price she set minus a fixed cost of 450 points. (Sellers were prevented from setting prices below 450 and thus could not sell at a loss.) If a buyer bought a good, she would earn the true value of the picture purchased minus the price set by the seller. After each trial, sellers were shown a feedback screen informing them of the price they set, the price the other seller set, and the number of goods they sold. Buyers did not receive any feedback about their payoffs between rounds. Figure F.2 shows example screens from stages 1 and 2.

Risk preferences were again measured by presenting subjects with the same gamble choice as in experiment 1. Subjects (both buyers and sellers) were asked to estimate the values of six scrambled pairs and then finally they were presented with a questionnaire, as in experiment 1.

Subjects were given the instructions on paper and had to answer control questions correctly in order to continue. The instructions are reproduced in appendix F.2. At the end of the experiment the final payoffs were paid out in money using the exchange rate 10 points = 1 DKK. The average payment was DKK 230 (approximately US$35).[9]

The experiment was implemented using zTree. 112 subjects were recruited using ORSEE and the experiment took place at LEE at the University of Copenhagen. One subject left part way through a session due to illness, and data from that session following her departure is excluded from the analysis.

6.4.2 Results

Summary statistics for various variables of interest are given in table 6.4.

Beginning with the actions of sellers, the mean prices they set were 490.1 for block pictures, 490.7 for low NCC scrambled pictures and 489.6 for high NCC scrambled pictures. A Kruskal-Wallis test p-value of 0.718 indicates that sellers did not adjust their behaviour across picture type. Similarly, a Kruskal-Wallis test that sellers' mean profit per picture differs over picture type has a p-value of 0.465, indicating no significant difference.

Table 6.5 shows the results of regressions of sellers' price setting behaviour and seller profit. In neither is a significant effect found of picture type.

The above can be summarized as

Result 6.3. *(i) The prices set by sellers did not differ significantly across picture type.*

(ii) Sellers' profits did not differ significantly across picture type.

(iii) Prices and profits were increasing in the value difference between pairs.

[9] As in experiment 1, buyers could theoretically make an overall loss. No one did.

Table 6.4: Results from experiment 2 with p-values for tests of difference across picture type. WSR = Wilcoxon signed-rank. *** = significant at 1% level, ** = significant at 5% level, * = significant at 10% level, threshold values for significance adjusted for Wilcoxon signed-rank tests using Holm's sequential Bonferroni correction. N=56.

	Block	Scrambled	
		Low NCC	HighNCC
Seller price	490.1	490.7	489.6
Kruskal-Wallis	0.718		
WSR Test vs. Block	—	0.403	0.948
WSR Test vs. low NCC		—	0.076
Seller profit per period	33.5	32.6	31.2
Kruskal-Wallis	0.465		
WSR Test vs. Block	—	0.267	0.250
WSR Test vs. low NCC		—	0.168
Buyer purchases per period	0.924	0.851	0.853
Kruskal-Wallis	0.007***		
WSR Test vs. Block	—	0.047*	0.001***
WSR Test vs. low NCC		—	0.770
Buyer mistakes per period	0.174	0.302	0.264
Kruskal-Wallis	<0.001***		
WSR Test vs. Block	—	0.002***	0.003***
WSR Test vs. low NCC		—	0.504
Buyer profit per period	21.7	17.7	19.9
Kruskal-Wallis	0.057*		
WSR Test vs. Block	—	0.017*	0.178
WSR Test vs. low NCC		—	0.048*

Turning to buyers, differences in their behaviour across picture type are found. The mean purchase rate was 92.4% for block pictures, significantly higher than the rate of 85.3% for high NCC scrambled pictures, although it differs from the rate of 85.1% for low NCC scrambled pictures only at the 10% level. There is no significant difference between the purchase rate of high and low NCC pictures.

Table 6.5: Regresion results for seller price and seller profit. *** = significant at 1% level, ** = significant at 5% level, * = significant at 10% level. N=1590

	Seller price		Seller profit	
	(1) Fixed effects	(2) Pooled	(1) Fixed effects	(2) Pooled
Abs. value diff.	0.151***	0.152***	0.167***	0.168***
	(0.005)	(0.005)	(0.021)	(0.020)
Low NCC	0.252	2.665	-0.505	-1.811
	(0.855)	(0.563)	(0.836)	(0.763)
Low NCC× Abs. value diff.	-8.37×10^{-3}	-0.012	-0.035	-0.040
	(0.858)	(0.799)	(0.696)	(0.652)
High NCC	0.322	-1.214	-2.958	-6.342
	(0.827)	(0.773)	(0.346)	(0.370)
High NCC× Abs. value diff.	-0.028	-0.031	5.96×10^{-3}	1.72×10^{-3}
	(0.558)	(0.533)	(0.951)	(0.986)
Period	-0.286***	-0.313***	-0.148*	-0.187**
	(<0.001)	(<0.001)	(0.085)	(0.029)
Order		-6.778***		-5.188***
		(0.004)		(0.002)
Skill		-0.121		-0.505
		(0.750)		(0.106)
Skill× Low NCC		0.066		0.088
		(0.820)		(0.798)
Skill× High NCC		0.020		0.434
		(0.941)		(0.241)
Seller type		2.444		3.399**
		(0.283)		(0.050)
Gamble choice		0.886		0.061
		(0.330)		(0.936)
Gamble choice× Low NCC		-0.698		0.308
		(0.365)		(0.720)
Gamble choice× High NCC		0.532		-0.238
		(0.505)		(0.809)
Constant	490.5***	494.9***	31.6****	40.0****
	(<0.001)	(<0.001)	(<0.001)	(<0.001)

Table 6.6 shows the results of a logit regression with a binary variable indicating that a buyer made a purchase as the dependent variable. There is confirmation that there is a significant difference between the purchase rate of block and scrambled pictures. The interactions between the dummies for scrambled pictures and the absolute value difference are positive and significant, meaning that buyers are more likely to buy scrambled pictures when the pictures' values are far apart.

It is also possible to see whether buyers make "mistakes" when purchasing. A mistake is defined as buying the picture which offers inferior surplus or as not buying when at least one picture offers a positive surplus. The mean mistake rates of 30.2% and 26.4% for high and low NCC scrambled pictures were both significantly higher than the mean mistake rate for block pictures of 17.4%, with no significant difference between the two types of scrambled pictures.

Table 6.6 reports the results of a logit regression with a binary variable indicating whether a buyer made a mistake or not as the dependent variable. The coefficient for the picture value difference is significantly positive, showing that for block type pictures, the probability of a buyer making a mistake was increasing as the pictures become further apart in terms of value. The reverse is seen for scrambled pictures: The interaction between both scrambled dummies and the picture value difference is negative, indicating that the probability of a buyer making a mistake was decreasing in the degree of differentiation. From the regressions it is also seen that the probability of making a mistake fell over the course of the experiment, possibly due to learning.

In terms of buyer surplus, this has a mean of 21.7 points per period for block pictures, compared to 17.7 per period for low NCC and 19.9 per period for high NCC pictures. The results for high NCC pictures do not differ significantly from block pictures, and although the test for a difference between low NCC and block pictures is only significant at the 10% level, note that the p-value of 0.017 is very close to the 5% threshold value of 0.0167 after adjustment for multiple testing using

Table 6.6: Regression results for probability of buyer purchase, probability of a buyer mistake, and buyer surplus. *** = significant at 1% level, ** = significant at 5% level, * = significant at 10% level. N=1590

	Buyer purchase		Buyer mistakes		Buyer surplus	
	(1) Fixed effects	(2) Pooled	(1) Fixed effects	(2) Pooled	(1) Fixed effects	(2) Pooled
Abs. value diff.	-0.016*	-0.015	0.019**	0.019**	0.115**	0.114**
	(0.098)	(0.117)	(0.011)	(0.012)	(0.016)	(0.017)
Low NCC	-1.260**	-3.584***	2.120***	3.600***	-6.154**	-13.460**
	(0.019)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(0.023)
Low NCC	0.017	0.016	-0.045***	-0.042***	0.113*	0.117*
× Abs. value diff.	(0.181)	(0.201)	(<0.001)	(<0.001)	(0.082)	(0.071)
High NCC	-1.874***	-2.315**	1.976***	1.495**	-7.109***	-2.490
	(<0.001)	(0.017)	(<0.001)	(0.047)	(<0.001)	(0.680)
High NCC	0.039***	0.035***	-0.044***	-0.041***	0.197***	0.200***
× Abs. value diff.	(0.003)	(0.005)	(<0.001)	(<0.001)	(0.003)	(0.002)
Period	0.023**	0.019*	-0.035***	-0.032***	0.396***	0.423**
	(0.027)	(0.054)	(<0.001)	(<0.001)	(<0.001)	(<0.001)
Order		-0.062		0.333***		5.525***
		(0.702)		(0.009)		(0.001)
Skill		-0.091**		0.076**		-0.373
		(0.017)		(0.014)		(0.105)
Skill		0.152***		-0.085**		0.555*
× Low NCC		(0.003)		(0.029)		(0.059)
Skill		0.074		0.015		-0.157
× High NCC		(0.134)		(0.698)		(0.568)
Buyer type		0.254		-0.101		0.065
		(0.104)		(0.406)		(0.967)
Gamble choice		0.178*		0.031		0.030
		(0.084)		(0.682)		(0.962)
Gamble choice		0.117		-0.123		-0.204
× Low NCC		(0.374)		(0.201)		(0.783)
Gamble choice		-0.105		0.040		-0.850
× High NCC		(0.412)		(0.683)		(0.272)
Constant	3.318***	2.954***	-2.227***	-3.048***	12.230***	8.941*
	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(<0.001)	(0.097)

Holm's sequential Bonferroni correction (Holm, 1979).

Table 6.6 shows the results of regressions with buyer surplus as the dependent variable. Firstly, it is seen that buyer surplus was increasing in the degree of differentiation. The dummy indicating a low NCC scrambled picture is significantly negative, whereas the dummy for high NCC pictures is only significant in one of three regressions.

To summarize buyers' actions in the market:-

Result 6.4. *(i) Buyers were more likely to purchase block than scrambled pictures, although they were more likely to buy scrambled pictures as the value difference increases. There was no significant difference between low and high NCC scrambled pictures.*

(ii) Subjects were more likely to make mistakes with scrambled than with block pictures, and were less likely to make mistakes with scrambled pictures as the degree of differentiation increases. There was no significant difference between the purchase rates of high and low NCC scrambled pictures.

(iii) Buyers made lower surplus with low NCC pictures than with block pictures, and there was some evidence that their surplus is lower with high NCC pictures compared to block pictures.

It is also possible to compare the actions of buyers in experiment 2 with subjects in experiment 1 to see whether choices observed in experiment 2 are consistent with the results in experiment 1. Given the stated willingness-to-pay of a subject in experiment 1, it is inferred that for a given set of prices, purchasing the picture that maximizes the surplus between WTP and price is the consistent choice (or not purchasing if both prices exceed WTP).

For each subject in experiment 1, the mean numbers of consistent choices of the high and low value goods, as well as the mean number of consistent choices not to buy were calculated. These are compared with the observed outcomes from experiment 2 in table 6.7. In addition, the surplus earned in the market and predicted to be earned given the WTP results are compared.[10] It can be seen that

Result 6.5. *(i) Compared to the predicted purchases from experiment 1, buyers in experiment 2 purchased more high value and fewer low value goods. There is no significant difference in the overall amount of goods purchased.*

(ii) There is weak evidence that buyers made a greater surplus in experiment 2 than is predicted by experiment 1.

Table 6.7: Mean of buyers' purchases and surplus observed in experiment 2 and inferred from experiment 1. P-values for Mann-Whitney U tests in parentheses. *** indicates significance at 1% level, ** indicates significance at 5% level, * indicates significance at 10% level. N=150 for purchases, N=149 for buyer surplus with one outlier removed.

	Buy high value good	Buy low value good	Buy nothing	Buyer Surplus
WTP	17.9***	8.21***	3.89	559.3*
Market	20.8***	5.35***	3.82	596.6*
	(<0.001)	(<0.001)	(0.752)	(0.081)

6.5 Discussion

The results of experiment 1 show that it is possible to manipulate individuals' perception of the experimental goods and that NCC is a useful measure of the goods'

[10] One outlier who earned -216 total surplus in the market is removed. The next lowest surplus is 67 and the mean is 568.

similarity. It is shown that perception has a significant effect on how similarly the goods are treated, with the difference in subjects' WTP for the two goods at its greatest when the pictures are least similar (NCC = -1), and at its lowest when the pictures are most similar (NCC$\approx$0.9).

One possible limitation to the use of the pictures is that there is no significant difference in the result for block pictures and low NCC scrambled pictures (NCC$\approx$0.5). The NCC measure runs only from -1 to 1, so this result seems to indicate that large changes in similarity are required for significant results. This could potentially be improved by using a finer grid than 10x10, or by presenting the pictures to subjects for only a limited amount of time.

In experiment 2, it is examined what impact perception has when individuals interact with better informed sellers. It could be that the sellers use their advantage to exploit buyers with perceptual limitations. On the other hand, it could be that the market institution works to eliminate, at least partially, buyers' limitations.

The type of picture seems to have little effect on sellers' behaviour. There is no significant effect of similarity on the prices sellers set, nor is there an effect on the amount of profit they make. Thus it can be concluded that, at least with the current market structure, sellers do not exploit buyers' perceptual limitations.

One possible explanation for this result is that sellers found the market structure too complicated and confusing, and thus were unable to work out how to make a greater profit from scrambled pictures. Evidence against this explanation is the fact that all sellers were able to answer control questions to verify that they understood their environment. In addition, sellers made a greater profit from pictures with a greater value difference, as should be expected. That they were able to do so is an indication that buyers have some strategic knowledge of the market institution.

The behaviour of buyers, however, is dependent on the similarity of picture pairs. Firstly, buyers are less likely to buy scrambled pictures than block pictures. This finding is consistent with previous research finding that consumers prefer easy to compare offers (in this case meaning block pictures, whose difference is easy to see) to complicated offers (for example Crosetto and Gaudeul (2012)). That the purchasing rates of different types of pictures converge as the value difference between picture pairs increases, leading to even scrambled pictures becoming relatively easy to compare, is also consistent with the interpretation that buyers prefer easy to compare offers.

One possible source of the reluctance to buy scrambled pictures is that buyers realize that they make a greater number of mistakes when faced with scrambled as opposed to block pictures. Again, when the value difference increases, the rates of subjects' mistakes for the different picture types converge, which may indicate why subjects were less reluctant to buy scrambled pictures when their value difference was high.

The combination of fewer purchases and a greater number of mistakes for scrambled pictures means buyers make less from them compared to block pictures. This loss in surplus is thus partly caused by buyers purchasing pictures offering them lower surplus, an aggregate transfer to sellers, but also by refraining from purchasing scrambled pictures to a greater extent than block pictures, a loss to sellers. Hence sellers' net profit does not differ over block and scrambled pictures.

The fact that there are two conflicting effects of increased similarity between pictures may also explain why sellers' price setting behaviour does not differ over picture type.

Unlike experiment 1, there are no real differences found between low NCC and high NCC pictures. A possible indication is thus that individuals employ different strategies due to the different choice mechanisms employed. Further evidence for this is found when comparing the choices observed in the market and those predicted from WTP: In the market, significantly more high value goods are purchased.

The indications thus are that the different choice mechanisms used across the two experiments lead subjects to use a different allocation of attention, and form their valuations in a different manner.

This experiment is a clear demonstration that moving beyond individual choice biases is extremely important. The impact of perceptual limitations on market and strategic interactions can be complex and non-intuitive.

With regards to whether the market institution helps or hurts buyers, it is clear that the market does not eliminate the impact of perceptual limitations entirely, as buyers make less surplus from scrambled than block pictures. However, it is also clear that they are not hurt by the market mechanism either: Sellers do not exploit buyers' perceptual limitations as they make no greater profit from scrambled pictures compared to block pictures. In fact, there is some evidence that buyers are helped by interacting in a market: Buyers' surplus in experiment 2 is significantly greater than predicted by willingness-to-pay stated in experiment 1 at the 10% level. If buyers use a different strategy to construct their valuations when given prices compared to when asked to state WTP, this is certainly not harmful, and is possibly beneficial.

Another advantage to using the goods presented here is that it is possible to compare the relative degrees of similarity of two goods using the empirical measure of normalized cross-correlation. This has shown that NCC is a valid measure of how similar individuals treat the experimental goods.

Comparing the experimental goods used here with the assets expressed with varying degrees of mathematical complexity used by Kalayci and Potters (2011b) and others, it was suggested in the introduction that they possess greater external validity. This certainly is the case when the situation modelled is consumer choice, in which the visual appearance a very large component in individuals' decision making. Visual attention is a much neglected aspect of choice in economics, and should be considered more.

However, it should be acknowledged that for many goods, for example insurance and other financial products, that individuals' cognitive abilities are much more important in choice, in which case it could be argued that a mathematical representation of goods would have greater external validity. The comparison of the relative efficacy and appropriateness of the visual and mathematical representations of goods would be a useful subject for future research.

6.6 Conclusion

This chapter contributes by introducing novel experimental goods. The goods allow their similarity to be varied while holding the underlying value to be held constant. The experimental results verify that subjects are able to understand their use and that the manipulation of perception was successful and that NCC is an appropriate measure of similarity.

It also shows that when individuals find it difficult to perceive similarity the difference between goods, their willingnesses-to-pay for them become closer together. In addition, perceptual limitations increase buyer mistakes in a market, thus lowering their payoff, although there are indications that the market institution aids buyers relative to individual choice.

There is much further work possible using the framework of this experiment. For example, there is a significant effect of time on almost all variables of interest. The goods presented here are unfamiliar to subjects, and the environment can be somewhat confusing, with the value of the traded goods changing each period. Thus the time the markets take to adjust is long, and there is some evidence that the adjustment is still going on at the end of 30 periods.

A future avenue of research might hence be to see the results when a market with perceptually limited buyers has adjusted fully, either by allowing for more periods or by simplifying the environment.

Change we can perceive: The economics of dynamic inattention 7

Edward J.D. Webb

Abstract

A model of dynamic inattention is presented, in which individuals do not pay perfect attention to changes in the economic environment. Three applications are then given. Firstly, in portfolio design, changes in the fundamental value of an asset are neglected, causing over/under-investment in it. Secondly, when consumers are inattentive to changes in the aggregate price level, inflation lowers their willingness to pay for a good, leading to firms shrinking their product's size. Finally, if the symptoms of a disease are slow to develop, individuals do not attend to their growth, and screening programs for the disease are beneficial.

7.1 Introduction

The world around us is constantly changing, and it is impossible for an individual to pay attention to every single change. This article seeks to demonstrate some of the consequences of inattention to such change.

It is often tacitly assumed that economic agents are perfectly attentive to changes in all relevant variables. Yet the incessant and rapid change of economic environments and the sheer number of potential variables an individual would have to keep track of means this cannot always be the case in reality. Individuals will inevitably fail to notice some changes.

The changes that an individual neglects may be either large or small[1], though generally it is assumed that small changes are neglected more often. It could be argued that any change large enough to make a significant difference to agents' outcomes will attract attention. However, psychological research suggests we do often fail to attend to large changes (Simons & Chabris, 1999). Also, even if only small changes are considered, these can have huge knock-on effects. In addition, the cumulative effect of inattention to many small changes is also likely to be significant.

To study the economic effect of unawareness of change, a theory of dynamic inattention, defined as a failure to perfectly attend to changes in the environment over time, is developed. To demonstrate the versatility of the approach, three applications are then given.

Firstly, it is applied to the problem of optimal portfolio construction. With dynamic unawareness, the investor to be unaware of shifts in the expected returns of an asset. Thus if an asset shifts from a state of high average returns to a state of average low returns, she may not attend to the change and as a consequence over-invests in that asset. Similarly, she may not attend to a shift from a state of low average returns to a state of high average returns, meaning she under-invests in the asset.

Secondly, an application is constructed in which consumers are inattentive to changes in the aggregate price level, and the size of the products. However, they are able to track changes in the prices of individual goods. This leads to the well-known phenomenon of goods shrinking over time. Yet the crucial factor which leads to this is not inattentiveness to product size itself, but inattention to the price level.

[1] Although "small" and "large" changes are referred to, in practice this is sometimes proxied in modelling by the speed or frequency of change.

The disparity between individuals' attentiveness to individual price rises and the aggregate price level means individuals' nominal willingness to pay for a given amount of the good remains the same, while their real willingness to pay is reduced by inflation. Thus the firm wishes to sell less of their good to consumers, which it achieves by reducing the size of the product.

Finally it is shown how dynamic inattention causes individuals not to notice slowly developing symptoms of a disease. This means they seek medical help only after a delay. Given that the chances of curing a disease are greater the earlier it is treated, dynamic inattention is welfare reducing. It is also shown how such inattention implies that medical screening programs can have a beneficial effect, even if costly to the individual.

Recently, there have been several studies into the effects of decision makers with biased attention such as the aforementioned Bordalo et al. (2012b, 2013b, 2013c, 2013a, 2015b). A different approach to incorporating attention into economics has been dubbed "rational inattention" (Sims, 2003). This draws on information theory to study how a rational decision maker acts with imperfect information gathering technology. This article is orthogonal to this field, as attention is viewed as an in-built part of an individual's decision making process. The individual is inherently bounded in her attentional abilities and therefore is boundedly rational.

The phenomenon of individuals not attending to a change in their environment (for examine, in Simons and Levin (1998), subjects failed to notice when the person they were conversing with was replaced by someone else) has been well documented in psychology, where it is dubbed *change blindness* (Simons & Levin, 1997). This has been further extended to situations in which individuals are active decision makers and the changes occur to their choice set, which is termed *choice blindness* (Johansson, Hall, & Sikström, 2008; Hall, Johnasson, & Strandberg, 2012; McLaughlin & Somerville, 2013).

An economically interesting illustration of choice blindness is provided by Hall, Johansson, Tärning, Sikström, and Deutgen (2010), who posed as market researchers in a supermarket. They asked shoppers to sample two different types of jam and say which they preferred. They were then told they could take a second taste of their preferred jam, but by sleight of hand were in fact given the alternative. Only around half of the subjects noticed the subterfuge.

This article aims to take such insights in individual choice and see how they affect market outcomes. Behavioural economics has begun to extend its field of vision beyond individual choice to examine the interaction between individuals and profit maximizing firms (see Spiegler (2011) for an overview) and this chapter contributes to the expanding literature on behavioural industrial organization.

Section 7.2 begins with an exposition of the dynamic inattention framework. Applications of the framework to portfolio design, product shrinkage and medical screening programs are then constructed and discussed in sections 7.2.1, 7.2.2 and 7.2.3 respectively. Section 7.3 gives a general discussion of the dynamic inattention approach and section 7.4 concludes.

7.2 Dynamic inattention

Let there be time periods $t = 0, 1, \ldots, T$, where T may or may not be infinite. Let the state of the world be ω with Ω the set of possible states of the world. There is a set of agents I with typical element i. For every i there is a set of possible action A_i and a payoff function $u_i : A \times \Omega \longrightarrow \mathbb{R}$, where $A = \times_{i \in I} A_i$.

For every $\omega, \omega' \in \Omega$ there is a probability function $p_{\omega,\omega'} : A \to [0, 1]$ with $\sum_{\omega' \in \Omega} p_{\omega\omega'} = 1 \; \forall \; \omega \in \Omega$ which gives the probability of transitioning from state ω at time $t - 1$ to state ω' at time t.

In every period there is a *perceived* state of the world $\tilde{\omega} \in \Omega$, where $\tilde{\omega}$ may or may not be the same as the actual state of the world, as well as an initial state $\tilde{\omega}_0 \in \Omega$, with $\sum_{\omega' \in \Omega} q_{\omega\omega'} = 1 \ \forall \ \omega \in \Omega$, which gives the probability that, given a transition from ω at time $t-1$ to ω' at time t, that agents attend to the state of the world at time t, i.e. $q_{\omega\omega'}$ is the probability that $\tilde{\omega}_t = \omega_t$. Otherwise, the perceived state remains the same, i.e. $\tilde{\omega}_t = \tilde{\omega}_{t-1}$ with probability $1 - q_{\omega\omega'}$.

Assumption 7.1. *Individuals are naïve about their perceptual abilities. Thus they act as if the true state of the world is* $\tilde{\omega}$

This setup will now be developed in three applications: portfolio design, product shrinkage and medical screening programs.

7.2.1 Portfolio design

Financial assets fluctuate in value constantly. This may be caused by demand changes, policy changes, both fiscal and monetary, speculation or myriad other causes. However, there is also heterogeneity in the amount of volatility, with some assets' fundamental value changing slowly over time. It is hence proposed that, due to dynamic inattention, investors neglect slowly moving assets, with their attention instead captured by more volatile and frequently fluctuating assets.

That agents in financial markets do not pay perfect attention to all available information is a central theme of rational inattention (Sims, 2003). Peng and Xiong (2006) also show how limited attentional resources can interact with overconfidence to explain various observed features of financial markets at easily captured by standard theory. Della Vigna and Pollet (2007) provide empirical evidence of investor inattention. Demographic changes can reliably predict demand changes can reliably predict demand changes for different sectors at different times. A large birth cohort implies increased demand for toys after a few years, for learning materials after a few more years, and so on until eventually there is an increase in demand for funerary services.

However, investors do not respond to forecast demand shifts due to cohort size until a horizon of approximately five years is reached.

Given that demographic changes are typically slow moving, dynamic inattention would predict their neglect by investors. The link between Della Vigna and Pollet (2007) and the current paper are discussed further in section 7.2.1.2.

For reasons of tractability, the rate of change of an asset's value is proxied by the probability of it shifting from a state of high average returns to a state of low average returns. This may be rationalized by considering that slow shifts from returns being at a high level to a low level also implies fewer shifts from a state of high to a state of low returns.

It is shown that dynamic inattention leads to an investor both over- and under-investing in a risky asset with some probability. However, as the asset becomes more volatile, this probability becomes smaller, as the investor's attention is more likely to be drawn to changes.

7.2.1.1 Model

Let there be two assets, s and r. s is a safe asset giving return m_0 with certainty. r is a risky asset whose return depends on its state s. It may either be in a high state, $\omega = h$, giving a random return m_h with variance $\sigma^2 > 0$ or a low state, $\omega = \ell$, giving a random return m_ℓ with variance $\sigma^2 > 0$, with $\mathbb{E}m_h > \mathbb{E}r_l$. There are a number of periods $t = 0, 1, 2, \ldots$ and in each the individual has an endowment of 1 unit to invest. Let a be the fraction of the endowment invested in the risky asset and let $1 - a$ be the fraction invested in the safe asset.

Let the per-period return to investment be c, from which she gains utility $u\left(c\right)$, where $u\left(\cdot\right)$ satisfies the usual assumptions. She faces no adjustment costs in changing her portfolio, implying that she chooses a in each period to maximize the expected utility from consumption in that period.

In each period, there is some probability $q \in [0,1]$ that the asset changes state, and with probability $1-q$ the asset is in the same state as in the previous period.

Standard case

Given a, in state $\omega = \{h, \ell\}$ the consumption level of the individual is $c = (1-a)\, m_0 + am_\omega$ and the individual maximizes $\mathbb{E}u\left((1-a)\, m_0 + am_\omega\right)$. This has first order condition

$$\frac{\partial \mathbb{E}u\left((1-a)\, m_0 + am_\omega\right)}{\partial c} = \mathbb{E}\left[\frac{\partial u\left((1-a)\, m_0 + am_\omega\right)}{\partial c}\left(m_\omega - m_0\right)\right] \tag{7.1}$$

To ensure that the individual invests in the asset with positive probability, the following assumption is made:

Assumption 7.2. *The expected return on the risky asset exceeds the return on the safe asset in state h, so that*

$$\mathbb{E}m_h > m_0 \tag{7.2}$$

By examining equation (7.1), it is possible to see that when evaluated at $a = 0$ it becomes $\frac{\partial u(m_0)}{\partial c}\left(\mathbb{E}m_s - m_0\right)$, so the assumption ensures that a positive fraction of the endowment is invested in the risky asset in state h.

Denote by a_h^* the amount invested in the risky asset in state h, with a_h^* found from the first order condition (equation (7.1)) with $\omega = h$. Denote by a_ℓ^* the amount invested in the risky asset in state ℓ, with $a_\ell^* = 0$ if $\frac{\partial u(m_0)}{\partial c}\left(\mathbb{E}m_\ell - m_0\right) \leq 0$ and otherwise found from the first order condition (equation (7.1)) with $\omega = \ell$.

Dynamic inattention

Assume that in $t = 0$, the asset is initially in state h. The case of it initially being in state ℓ will be addressed subsequently. Denote the individual's perceived state of the asset by $\tilde{\omega} = \{h, \ell\}$. With dynamic inattention it is necessary to specify the individual's initial perceived state $\tilde{\omega}_0$. It is assumed that $\tilde{\omega}_0 = \omega_0$, so that the individual is aware of the asset's state in period 0.

It is also necessary to specify when the individual is attentive to the current state of the world.

Assumption 7.3. *In each period $t \geq 1$, the individual is attentive to the current state of the world with probability q, so that*

$$\tilde{\omega}_t = \begin{cases} \omega_t & \textit{with probability } q \\ \\ \tilde{\omega}_{t-1} & \textit{with probability } 1-q. \end{cases} \tag{7.3}$$

The probability of being attentive to the current state is identical to the probability that the asset changes state. This functional form for dynamic inattention has the advantage of tractability. It also captures the notion that frequent changes draw attention to a greater extent than infrequent ones.

Thus when q is low, and the asset is likely to remain in its current state for a long time, when a change does come it is less likely that the individual's attention is focused on the state of the asset. On the other hand, when q is high, the asset changes state frequently and it is more important to adjust the portfolio more often. Thus it is also more likely that the individual's attention is focused on the asset's state.

That the probability of being attentive to the state of the world is q in every period means that, after a change of state, conditional on no subsequent changes of state, the probability that the individual does not attend to the change of state decreases over time. No matter how infrequent a change of state is, the individual eventually notices it. The action available to the individual in each state is the fraction to invest in the risky asset for that period.

The individual is naïve regarding her attentional limitations, and so acts as if the asset's state is given by her perception of it. Hence, it is clear that the individual invests a_h^* when $\tilde{\omega}_t = h$ and a_ℓ^* when $\tilde{\omega}_t = \ell$.

Given this strategy for the individual, it is possible that she will over- or under-invest in the risky asset. The individual over-invests if the asset is in state ℓ, implying the optimal investment is a_ℓ^*, but the individual perceives it as being in state h, so that she invests $a_h^* > a_\ell^*$. Similarly, she under-invests if the asset is in state h, implying that the optimal investment is a_h^*, but she perceives it as being in state ℓ, so that she invests $a_\ell^* < a_h^*$. The individual does not attend to the asset shifting from a high to a low returns state, meaning she fails to adjust her portfolio in response, leaving it with too great a stake invested in the risky asset, and vice versa for a shift from a low to a high state.

To find the probability of over/under-investment at some point in time, it is first necessary to find the probability of the asset being in a given state at a given time. Given that it is in state h at time $t = 0$, the probability of it being in state h at time $t > 0$ is

$$\begin{aligned}\text{Prob.}\left(\omega_t = h \middle| \, \omega_0 = h\right) &= \sum_{\tau=0}^{\frac{t}{2}} \binom{t}{2\tau} q^{2\tau} \left(1-q\right)^{t-2\tau} \\ &= \frac{1}{2}\left(1 + \left(1-2q\right)^t\right). \qquad (7.4)\end{aligned}$$

The probability that it is in state ℓ at time $t > 0$ is

$$\text{Prob.}\left(\omega_t = \ell \middle| \omega_0 = h\right) = \sum_{\tau=0}^{\frac{t}{2}} \binom{t}{2\tau+1} q^{2\tau+1} (1-q)^{t-2\tau-1}$$

$$= \frac{1}{2}\left(1 - (1-2q)^t\right). \tag{7.5}$$

Suppose the asset is in state h at time $1 \leq t' < t$. Then the probability that the individual last attended to the asset's state in period t' is $q(1-q)^{t-t'}$. It follows that the probability that the individual last attended to the asset's state in some period prior to the present and that the state was h, denoted $\text{Prob.}\left(\tilde{h}\middle|\,\omega_0 = h\right)$[2], is

$$\text{Prob.}\left(\tilde{h}\middle|\,\omega_0 = h\right) = (1-q)^t + \frac{1}{2}q\sum_{t'=1}^{t-1}(1-q)^{t-t'}\left(1 + (1-2q)^{t'}\right)$$

$$= \frac{1}{2}(1-q)\left(1 + 2(1-q)^t - (1-2q)^t\right). \tag{7.6}$$

Overinvestment occurs when the state of the world is ℓ, whereas the individual perceives its state as h. Thus the probability that the individual over-invests in period t given $\omega_0 = h$, denoted $p_{over}(t)$, is the product of $\text{Prob.}(\omega_t = \ell|\,\omega_0 = h)$ and $\text{Prob.}\left(\tilde{h}\middle|\,\omega_0 = h\right)$, so that

$$p_{over}(t|\,\omega_0 = h) = \frac{1}{4}(1-q)\left(1 + 2(1-q)^t - (1-2q)^t\right)\left(1 - (1-2q)^t\right). \tag{7.7}$$

Suppose that the asset is in state ℓ at time $1 \leq t' < t$. The probability the individual last attended to the state in this period is again $q(1-q)^{t-t'}$. Then the probability that the individual last attended to the asset's state in some period prior

[2]Note that this is not $\text{Prob.}(\tilde{\omega}_t = h)$, as it does not include the possibility that $\tilde{\omega}_t = s_t = h$, i.e. the current state is h, which the individual attends to.

to t and that in this period it was in state ℓ is

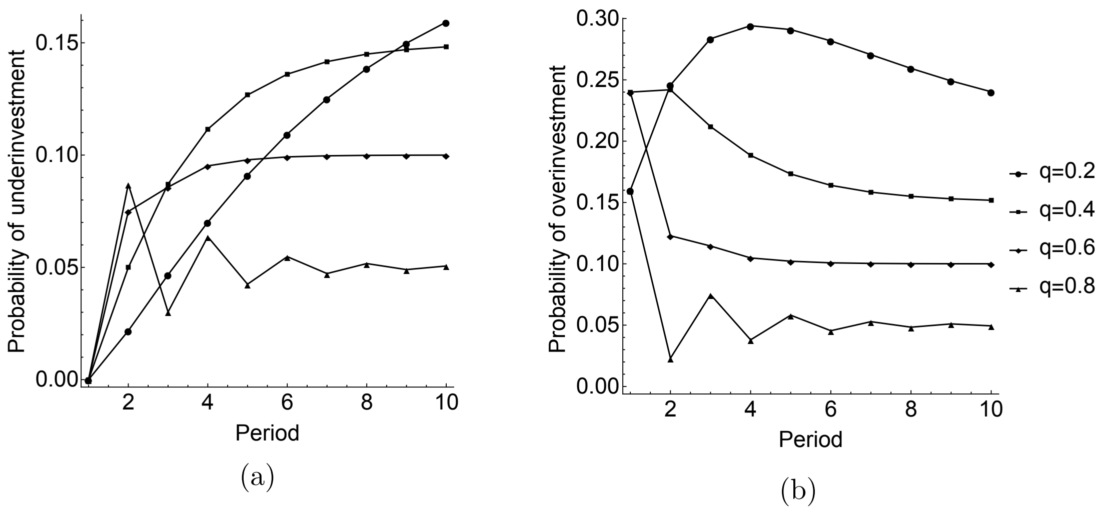
$$\text{Prob.}\left(\tilde{\ell}\middle|\, s_0 = h\right) = \frac{1}{2} q \sum_{t'=1}^{t-1} (1-q)^{t-t'} \left(1 - (1-2q)^{t'}\right)$$
$$= \frac{1}{2}(1-q)\left(1 + (1-2q)^t - 2(1-q)^t\right). \qquad (7.8)$$

Under-investment occurs when the asset's state is h, but the individual perceives it as ℓ, so that the probability of under-investment in period t, $p_{under}(t| < \omega_0 = h)$, is the product of Prob. $(\omega_t | \, \omega_0 = h)$ and Prob. $\left(\tilde{\ell}\middle| \, \omega_0 = h\right)$ and so

$$p_{under}(t| \, \omega_0 = h) = \frac{1}{4}(1-q)\left(1 + (1-2q)^t - 2(1-q)^t\right)\left(1 + (1-2q)^t\right). \qquad (7.9)$$

Illustrations of how the probabilities of over- and under-investment develop over time are shown in figure 7.1.

FIGURE 7.1: Probability of (a) under-investment and (b) over-investment over time for various values of q



Examining equations (7.7) and (7.9), it can be seen that

Proposition 7.1. *(i) If $q = 0$, no over- or under-investment occurs.*

(ii) If $q = 1$, no over- or under-investment occurs.

It is also possible to show that

Proposition 7.2. *If $0 < q < 1$, as t becomes large, the probabilities of over- and under-investment both tend to $\frac{1}{4}(1-q)$.*

If the initial state of the asset is $\omega_0 = \ell$, then by an analogous derivation to the one above, it is possible to show that

$$p_{over}(t|\,\omega_0 = \ell) = \frac{1}{4}(1-q)\left(1 + (1-2q)^t - 2(1-q)^t\right)\left(1 + (1-2q)^t\right) \quad (7.10a)$$

$$p_{under}(t|\,\omega_0 = \ell) = \frac{1}{4}(1-q)\left(1 + 2(1-q)^t - (1-2q)^t\right)\left(1 - (1-2q)^t\right). \quad (7.10b)$$

Comparing this to equations (7.7) and (7.9), it may be observed that

$$p_{over}(t|\,\omega_0 = \ell) = p_{under}(t|\,\omega_0 = h) \quad (7.11a)$$

$$p_{under}(t|\,\omega_0 = \ell) = p_{over}(t|\,\omega_0 = h) \quad (7.11b)$$

and so analogous propositions to 7.1 and 7.2 hold for the case of $\omega_0 = \ell$.

7.2.1.2 Discussion

Proposition 7.1 states that when $q = 0$, there is no under- or over-investment, which should be expected as there is no possibility of the asset changing state. It also states that when $q = 1$, there is also no over- or under-investment. Here, the asset changes state predictably every period, and such regular, frequent change attracts attention with certainty. Thus the two elements needed for dynamic inattention are illustrated: An asset must sometimes change its underlying value and infrequently enough that investors sometimes fail to attend to the change.

Proposition 7.2 shows that in the long run the probability of over- and under-investment both converge to $\frac{1}{4}(1-q)$. As q tends to 1 and the investor is perfectly attentive to change, this tends to the standard case without dynamic inattention. However, comparison with proposition 7.1 shows there is a discontinuity in the long run probabilities of over/under-investment at $q = 0$. In fact, the long run probabilities are decreasing in q, approaching $\frac{1}{4}$ before dropping to 0 as $q = 0$ and the asset never changes state.

For small q the long run probability that the individual has a suboptimal portfolio is approximately 50%, an asset which changes state with very small but positive probability will change state sooner or later, and the less likely the asset is to change state at any given time, the less likely the investor is to attend to the state at that time, and the less likely she is to notice a change. However, it should be noted that the lower the probability of the asset changing state, the longer the expected time becomes before the change required for over- or under-investment to be possible occurs. Thus, the time taken to approach the long run is large.

In this model, there were two possible states of the world, and dynamic inattention took the form of the investor not noticing a change from one state to the other with some probability. The probability of attending to a change of state is identical to the probability of a change of state, so that assets which change more frequently are more likely to be attended to.

This approach captures the intuitive idea that individuals attend to a greater extent to variables which change frequently, and that if a variable changes infrequently, it is more likely that the individual's attention is directed elsewhere when a change occurs. This is in line with the psychological results on change and choice blindness, in which it is unexpected events which go unnoticed (most famously the sudden appearance of someone in a gorilla suit).

However, the size of the change in the asset's expected return does not influence the probability of attending to the change. A change so large that the asset now gives returns of a magnitude not possible in the previous state is attended to with the same probability as a minuscule change. While choice blindness studies show that large differences may go unnoticed, more drastic changes are noticed more frequently (see for example Hall et al. (2010)).

The dynamic inattention framework can easily incorporate such an observation, for example by altering the functional form of paying attention to a change to be $q\frac{m_\ell}{m_h}$.

Della Vigna and Pollet (2007) provide empirical evidence that investors neglect demographic shifts which are slow-moving yet reliably predict demand changes. Yet they explicitly reject neglect of slow-moving variables as an explanation, instead proposing that investors neglect information beyond a horizon of approximately 5 years.

The current model is intended as complimentary to these results. The reason that the horizon is 5 years is that analyst forecasts are not available for horizons beyond 5 years (Della Vigna & Pollet, 2005). The coincidence of timing means that investors attend to changes in variables only when their attention is directed towards the changes by analysts' forecasts is convincing.

However, while Della Vigna and Pollet (2007) explain why investors only become attentive to change once the five year horizon is reached, they give no underlying cause as to why the variables were neglected in the first place. The current model demonstrates that dynamic inattention can give rise to the phenomenon, and gives the theory a psychological base.

The model developed here was intentionally simple in order to clearly illustrate the effects of dynamic inattention on investment decisions. Extending the dynamic inattention approach to more detailed financial models is an avenue for future research.

At present the investor holds an asset purely for consumption. An interesting question is how dynamic inattention would affect the price at which assets are traded between many investors, and in particular how the market institution disseminates information about changes in assets' fundamental values.

It would be interesting to examine alongside this the possibility of heterogeneous inattention amongst individuals. In the current model, the degree of inattention is determined solely by the frequency with which the asset changes state. Different investors being affected by dynamic inattention to differing degrees allows it to be investigated to what extent dynamic inattention is a disadvantage in a market, whether less attentive investors are exploited by more attentive ones and whether investors who are less attentive are driven out of the market.

7.2.2 Product shrinkage

Many will be familiar with the phenomenon of childhood sweets appearing smaller than remembered. Whilst this is partly due to oneself having become larger, the unit size of firms' offerings are often observed to decline over time, as evidenced by this quote from the UK's Office For National Statistics:

> *"Prices overall rose this year. . . , with the main upward contributions coming from ice cream and chocolate bars, where a number of products have reduced in size. This is treated as a price increase in the CPI as consumers get less for their money."* (Office for National Statistics, November 2012)

Consumer organization *Which?* also highlighted the practice in a complaint to the UK's Competition and Markets Authority:

> *"...product pack sizes can shrink without a corresponding decrease in the*

> *price of the product. This technique can mask price rises and make it difficult for consumers to assess the best value product as consumers are not generally informed of the size reduction and may assume that the product they are buying is the same size as it was the last time they bought it.*
>
> *... Consumers may be unaware of the reduction in size of the product and therefore fail to appreciate that they are no longer getting the value they assume."*

That a firm's choice of the unit size of the good(s) it sells impacts on its profit is clear, and often studied in a vertical differentiation framework (Gabszewicz & Thisse, 1979; Shaked & Sutton, 1982).[3] However, firms trade with consumers over many periods. Consumers, especially in an environment such as a supermarket where they are making relatively low value purchases and are faced with a large number of options, cannot pay perfect attention to a good's unit size, or to changes in unit size.

To explain the phenomenon, a model is developed in which consumers exhibiting dynamic inattention interact in a market with a profit maximizing monopoly firm. In particular, in some periods consumers are inattentive to changes in unit size the firm makes and also to the aggregate price level in the economy. The result is that if there is a lag between consumers' perception of the price level and its reality, the firm will produce smaller units of its good. As the probability of a lag increases with time, the expected unit size shrinks over time.

The behaviour of the firm is characterized by one or several small decreases in unit size followed by a big increase when consumers do pay attention to the price level. The fact that consumers may be inattentive towards size decreases means

[3] Usually in a vertical differentiation framework, the variable of interest is product quality. However, as discussed later, this is equivalent to unit size in this model.

there may be one or more periods during which the nominal price of the good is unchanged, despite nominal inflation.

7.2.2.1 Model

There are an infinite number of time periods indexed by $t = 0, 1, 2, \ldots$. The state of the world in period t, ω_t, is characterized by the price level P_t. There is nominal inflation at a constant rate i so that if P_0 is the price level in period 0 the level in period t is $P_t = P_0 (1 + i)^t$. Without loss of generality, let $P_0 = 1$.

Let a monopoly firm sell a good in units of size $v \in [0, \infty)$ at a real price per unit $p \in [0, \infty)$ in time periods $t = 1, 2, 3, \ldots$. There is a constant marginal cost of 1 for producing some amount of the good, so that producing n units of size v has cost nv.

A unit mass of consumers can purchase at most one unit of the good, so that $A_i = \{\text{buy}, \text{not buy}\}\ \forall\ i$, from which they gain utility

$$u = 2\sqrt{v} - p \tag{7.12}$$

If consumers do not purchase, their payoff is 0.

Size can be interpreted as weight, volume, number or any other relevant physical dimension. It is also mathematically equivalent to the quality of the product, as in the canonical model of vertical differentiation. Under this interpretation, a reduction in quality could represent the removal of an ingredient or its replacement with a cheaper substitute. It could also represent the removal of product features or less comprehensive customer support.

Standard case

In the standard case, inflation is irrelevant, and so the firm sets its real price, p_t, equal to consumers' willingness-to-pay (WTP), $2\sqrt{v_t}$, and so its nominal price is

$2P_t\sqrt{v_t}$. Its real per-period profit is then

$$\pi_t = 2\sqrt{v_t} - v_t \tag{7.13}$$

As its unit size in one period does not affect its profit in another period, in all periods it sets v_t to maximize π_t, i.e.

$$v_t^* = 1 \ \forall \ t > 0. \tag{7.14}$$

Total profit is

$$\pi_T = \frac{1}{(1-\delta)} \tag{7.15}$$

where δ is the firm's discount function and in each period the nominal price is simply $2P_t\sqrt{v_t^*} = 2\left(1+i\right)^t$.

Now suppose consumers do not always pay attention to changes in the market environment. This implies that consumers' perception of market variables may differ from their true value. In particular, let $\tilde{v}_t$ and $\tilde{P}_t$ be the perceived unit size and price level respectively. In each period with probability $q_\alpha \in [0,1]$ ($q_\beta \in [0,1]$) consumers do not pay attention to the current period's unit size (price level). If consumers do not pay attention to a given variable, their perception of it is unchanged from the previous period. Thus

$$\text{Prob.}\left(\tilde{v}_t = \tilde{v}_{t-1}\right) = q_\alpha \tag{7.16a}$$

$$\text{Prob.}\left(\tilde{v}_t = v_t\right) = 1 - q_\alpha \tag{7.16b}$$

$$\text{Prob.}\left(\tilde{P}_t = \tilde{P}_{t-1}\right) = q_\beta \tag{7.16c}$$

$$\text{Prob.}\left(\tilde{P}_t = P_t\right) = 1 - q_\beta. \tag{7.16d}$$

Consumers are considered to always pay attention to an individual price, even if they pay no attention to the aggregate price level (i.e. $\text{Prob.}\,(\tilde{p}_t = p_t) = 1)$. Prices of goods are almost always a clearly stated numerical value, facilitating comparison, whereas the aggregate price level is a nebulous variable, subject to measurement error even in official statistics.

The size of a good is also usually more difficult to estimate and even though weight/volume information is often provided, it is not displayed as prominently as price. If "size" is interpreted as lower quality, it may be even more difficult for consumers to detect that one ingredient has been replaced by an inferior one.

It is entirely plausible that a consumer may not notice an individual price increase, for example if she is in a hurry or does not remember the previous price. However, it is considered that the likelihood of failing to notice a price change is sufficiently small compared to the likelihood of failing to notice a change in unit size or aggregate price level that it may be approximated as 0.

The timing of each period is as follows:-

1. The firm chooses unit size, incurring costs.

2. Consumers pay attention to the current unit size with probability q_α and to the current price level with probability q_β. Otherwise their perception of these variables remains unchanged from the previous period.

3. The firm observes consumers' perceptions and sets price.

4. Consumers decide whether or not to purchase and the firm earns revenue.

In period 0, $\tilde{P}_0$ is set to 1. As the firm does not produce prior to period 1, it is assumed that $\tilde{v}_0 = 0$.

As consumers may be unaware of the price level, their WTP in nominal terms no longer necessarily reflects their WTP in real terms. Thus the firm will set its nominal price equal to consumers' nominal WTP, $2\tilde{P}_t\sqrt{\tilde{v}_t}$. Its real per-period profit is then

$$\pi_t = \frac{2\tilde{P}_t\sqrt{\tilde{v}_t}}{P_t} - v_t. \tag{7.17}$$

What happens when the firm chooses some v_t? As q_α is constant, in every period $t+\tau \geq t$ there is some probability that consumers perceive the good as having size v_t, regardless of all future decisions. It follows that setting v_t leads not only to profit in the current period, but expected profit in all future periods, and so v_t will be set to maximize this.

The real expected revenue earned from setting v_t in period t in period $\tau > t$ is

$$2\frac{q_\alpha^\tau (1-q_\alpha)}{P_{t+\tau}}\left(\tilde{P}_{t-1}q_\beta^{\tau+1} + (1-q_\beta)\sum_{t'=0}^{\tau} P_{t+t'}q_\beta^{\tau-t'}\right)\sqrt{v_t} \tag{7.18}$$

and so in period t the firm chooses quality to maximize

$$2\sum_{\tau=0}^{\infty}\frac{(q_\alpha\delta)^\tau (1-q_\alpha)}{P_{t+\tau}}\left(\tilde{P}_{t-1}q_\beta^{\tau+1} + (1-q_\beta)\sum_{t'=0}^{\tau} P_{t+t'}q_\beta^{\tau-t'}\right)\sqrt{v}_t - v_t. \tag{7.19}$$

Thus the firm chooses

$$v^* = \left(\sum_{\tau=0}^{\infty}\frac{(q_\alpha\delta)^\tau (1-q_\alpha)}{P_{t+\tau}}\left(\tilde{P}_{t-1}q_\beta^{\tau+1} + (1-q_\beta)\sum_{t'=0}^{\tau} P_{t+t'}q_\beta^{\tau-t'}\right)\right)^2 \tag{7.20}$$

and then after rearrangement, it follows that:

Proposition 7.3. *Let ℓ be the lag between perceived and actual price level that the firm observes in the beginning of a period*[4]*, i.e. if consumers last paid attention in $\tilde{t}$, $\ell = t-1-\tilde{t}$. Then given a lag of ℓ, the firm will choose unit size*

$$v^*(\ell) = \left(\frac{1-q_\alpha}{1+i-q_\alpha q_\beta \delta}\right)^2\left(\frac{q_\beta}{(1+i)^\ell} + \frac{(1-q_\beta)(1+i)}{(1-q_\alpha\delta)}\right)^2. \tag{7.21}$$

[4] Thus ℓ is the lag it observes when setting unit size.

Given a lag of ℓ in period t, the expected unit size set by the firm in period $t+\tau$ is

$$\mathbb{E}v^*_{t+\tau}(\ell) = q_\beta^\tau v^*(\ell+\tau) + (1-q_\beta)\sum_{m=0}^{\tau-1} q_\beta^m v^*(m). \tag{7.22}$$

The expected profit arising from the firm setting $v^*(\ell)$ in a given period is found by substituting $v^*(\ell)$ into equation (7.19) to be

$$\pi^*(\ell) = \left(\frac{1-q_\alpha}{1+i-q_\alpha q_\beta \delta}\right)^2 \left(\frac{q_\beta}{(1+i)^\ell} + \frac{(1-q_\beta)(1+i)}{(1-q_\alpha\delta)}\right)^2. \tag{7.23}$$

The total expected profit is then

$$\mathbb{E}\pi_T = \sum_{t=1}^{\infty} \delta^{t-1}\left(q_\beta^t \pi^*(t) + (1-q_\beta)\sum_{\ell=0}^{t-1} q_\beta^\ell \pi^*(\ell)\right). \tag{7.24}$$

Expected total profit as a function of q_β is illustrated in figure 7.2.

Examining equations (7.21), and (7.22), it may be seen that

Corollary 7.1. *(i)* $\frac{\partial v^*(\ell)}{\partial \ell} < 0$*, so that the longer ago consumers paid attention to the price level, the lower is the unit size the firm sets.*

(ii) $\frac{\partial \mathbb{E}v_{t+\tau}}{\partial \tau} < 0$*, so that the expected unit size a firm sets decreases over time.*

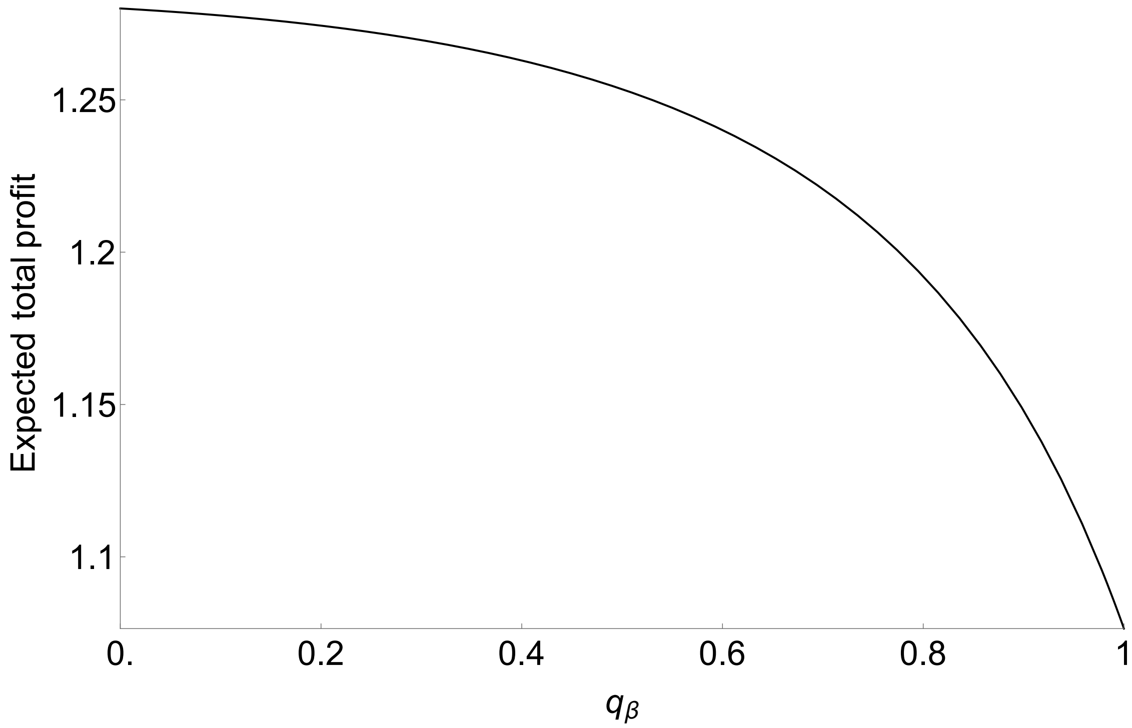
(iii) $v^*(\ell)\big|_{q_\beta=0} = \left(\frac{1-q_\alpha}{1-q_\alpha\delta}\right)^2$ *so that product size is constant if individuals perfectly attend to the price level.*

(iv) $v^*(\ell)\big|_{q_\alpha=0} = 1 - q_\beta\left(1-\frac{1}{(1+i)^{\ell+1}}\right)$ *so that product shrinkage occurs even if individuals attend perfectly to product size.*

Figure 7.3 shows the expected unit size the firm sets over time for various parameter values.

To look further at the firm's choice, suppose it is in period t with consumers last having paid attention to the price level ℓ periods ago. What possible actions will

FIGURE 7.2: Expected total profit. $q_\alpha = \frac{1}{2}$, $\delta = \frac{3}{4}$, $i = 0.02$. In the baseline case, profit is 4.



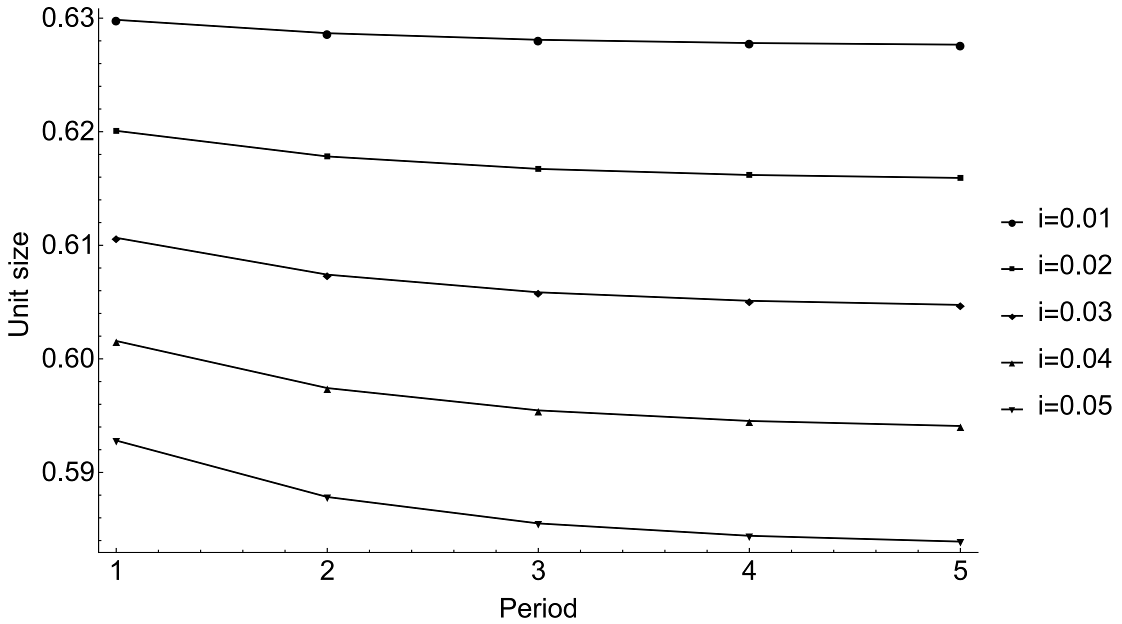
it take in period $t+1$? The answer depends on whether consumers pay attention to the current price level. With probability q_β they do not, in which case the firm reduces size from $v^*(\ell)$ to $v^*(\ell+1)$. On the other hand, with probability $1-q_\beta$ it increases size to $v^*(0)$.

Let $\Delta^-(\ell)$ ($\Delta^+(\ell)$) be the change in size conditional on a reduction (increase). Substitution of the relevant values into equation (7.21) and rearranging

Proposition 7.4. *If the lag is ℓ and the firm subsequently reduces size, the change is*

$$\Delta^-(\ell) = -\frac{iq_\beta(1-q_\alpha)^2}{(1+i-q_\alpha q_\beta\delta)^2(1+i)^\ell}\left(\frac{q_\beta(2+i)}{(1+i)^{\ell+2}} + \frac{2(1-q_\beta)}{(1-q_\alpha\delta)}\right). \tag{7.25}$$

FIGURE 7.3: Expected unit size over time for various values of i. $q_\alpha = \frac{1}{2}$, $q_\beta = \frac{1}{2}$, $\delta = \frac{3}{4}$



If it subsequently increases size, the change is

$$\Delta^{+}(\ell) = q_\beta \left(\frac{1 - q_\alpha}{1 + i - q_\alpha q_\beta \delta} \right)^2 \left(q_\beta \left(1 - \frac{1}{(1+i)^{2\ell}} \right) + \dots \right. \tag{7.26}$$

$$\left. \dots + \frac{2(1 - q_\beta)(1+i)}{(1 - q_\alpha \delta)} \left(1 - \frac{1}{(1+i)^{\ell}} \right) \right).$$

It is also apparent that

Corollary 7.2. *(i)* $|\Delta^{+}(\ell)| \geq |\Delta^{-}(m)| \ \forall \ \ell, m \geq 0$.

(ii) $\frac{\partial \Delta^{-}(\ell)}{\partial \ell} > 0$ *so that the longer ago it was consumers paid attention to the price level, the lower the magnitude of the reduction.*

Turning to the nominal price of the good

Proposition 7.5. *With probability* $(q_\alpha q_\beta)^\tau$ *the firm holds nominal price constant between periods* t *and* $t + \tau$*.*

7.2.2.2 Discussion

Assessing the current price level is a difficult task. Even when done by the teams of professional economists and statisticians who compile official statistics, there is measurement error. It is therefore entirely plausible that consumers are often inattentive to general levels of inflation, even though paradoxically they can gauge changes in the *individual* prices of the goods they consume fairly well.

Similarly, consumers are shown the price of a good prominently, whereas information on volume/weight/quality, etc., may be hard to find. Hence it is also entirely plausible that consumers are inattentive to changes in unit size.

Proposition 7.3 shows that the size of a firm chooses depends only on the lag between the perceived an actual price level. Corollary 7.1 further states that the greater the lag, the greater the reduction in unit size.

The intuition behind the firm reducing its unit size is that consumers wrongly believe that the money in their pockets is worth the same in real terms as it was before. They are hence unwilling to hand over any more money and their nominal WTP for a given good is unchanged. Any price increase would lead to them not buying. This implies that in real terms their WTP decreases. The firm's costs are unchanged, and so it must sell consumers less of the good, which it does by reducing unit size.

The greater the difference between consumers' perception of the price level and its reality, the lower consumers' real WTP. Thus a greater lag leads to a lower unit size set by the firm.

Over time, the probability of there existing a lag between the perceived and actual price levels increases. Likewise the longest lag possible. Thus in expectation the firm's unit size continually decreases over time, as illustrated in figure 7.3.

It is inattentiveness to the price level, rather than to product size itself, that causes shrinkage. Even if $q_\alpha = 0$ and consumers are well aware if products are getting smaller, the phenomenon still occurs. Thus it is debatable whether consumers are necessarily being “tricked” by firms reducing product size.

As can be seen from equation (7.21), if $q_\beta = 0$ (i.e. consumers always pay attention to the price level), unit size is constant. This again illustrates that changes in unit size in the current formulation is due to inattentiveness to inflation, and not due to a firm exploiting inattentiveness to changes in unit size.

If consumers are only inattentive to size changes, then as q_α is constant, the optimal size in a given period is independent of the history of sizes. Thus in every period the firm must solve the same profit maximization problem, and so will choose the same size in every period.

The first part of corollary 7.2 states that the magnitude of any given price increase will be at least as great as that of any given price decrease. The intuition behind this result is that if consumers are attentive to the current price level they are immediately aware of it, regardless of how far their perceptions lagged behind previously. Thus any given increase in quality will encompass all previous decreases, meaning increases must be of greater magnitude. The implication is that firms should be observed to reduce their unit size gradually over time, and that any increase should be large. This could, for example, take the form of “extra-large” or “multipack” versions of the product being introduced.

In the baseline case of perfectly attentive consumers, nominal prices rise steadily over time. However, proposition 7.5 shows that in the current model nominal prices may be held constant over two or more periods. It can also be seen that for such events to be possible, it must be that $q_\alpha > 0$, i.e. there is some probability that consumers are inattentive to changes in unit size.

If consumers are inattentive to the price level so that their real WTP is lowered, the firm wishes to sell them less of the good and reduces size. However, if they are also inattentive to this change, the firm can exploit this and charge the same price as before.

It is possible to compare equations (7.15) and (7.24) numerically and show that the firm makes greater (expected) profit in the baseline case of perfectly attentive consumers. The firm is thus not only unable to exploit the inattentiveness of consumers, it is actively harmed by its presence.

In the case of inattention to price levels, this is perhaps not so surprising as it reduces consumers' WTP in real terms. However, even with $q_\beta = 0$, the firm is still better off in the baseline. The reason for this is that, although the firm can benefit from consumers not noticing a reduction in unit size, it is harmed by consumers not always paying attention to an increase. This is most starkly illustrated in period 1. As $\tilde{v}_0 = 0$, representing the fact that prior to that it was not in the market, it may not attract consumers' attention to its existence. With probability q_α it makes a loss in that period from any positive v_0. Figure 7.2 shows expected total profit as a function of q_β.

In the current model, q_α is entirely exogenous. However, it is clear that firms actually have some ability to influence whether or not consumers pay attention to changes in the composition of their goods. It can, for example, display the information prominently or run an advertising campaign if it wishes to attract consumers' attention. On the other hand, it can make the information obscure and difficult to find if it does not wish to attract attention. A useful future expansion of the model would hence be to allow the firm to influence the probability with which consumers pay attention to unit size.

One limitation of the model is that inflation and the price level are considered entirely exogenous. In reality, the price a firm chooses influences the overall price level. This impact is negligible if the model is considered only as a small sector in a much larger economy. However, if all (or at least many) sectors are under the same influence of inattentive consumers, the aggregate price level will be affected. It would be thus an interesting extension to consider a many-sector economy and endogenize the price level.

So far, only positive values of i have been considered. As inflation is much more commonly observed, there has been no mention of deflation. An intriguing implication of the model is that under deflation the firm's expected unit size should increase over time. This prediction should be taken with a pinch of salt, however, as if unit size becomes too large there may be budget constraints and storage constraints, especially for perishable goods. There are also other factors at work in a deflationary economy, such as consumers' desire to delay purchases.

7.2.3 Medical screening programs

In the final application, a non-monetary effect of dynamic inattention is demonstrated.

Many diseases have gestation periods before severe health consequences are felt. However, they often also have symptoms which are apparent to the sufferer, for example a lump or a persistent cough.

Such symptoms develop slowly. Thus although individuals should seek medical attention quickly, if they are dynamically inattentive, the symptoms remain unnoticed for some time. The individual delays seeking medical attention with negative health consequences.

A model of dynamic inattention to disease symptoms is here constructed, and it is shown how such inattention negatively impacts on an individual's health. It is also shown that disease screening programs can help deduce the impact of dynamic inattention. A beneficial screening program always exists, provided that the cost to individuals attending screening sessions is sufficiently low.

Many economic studies of screening programs have focused on their cost to individuals, both monetary e.g. travel costs and lost earnings, and non-monetary, e.g. the pain of the procedure. These include Aas (2009) and Jonas, Russel, Chou, and Pigone (2010). Picone, Sloan, and Taylor (2004) look at the impact of risk and time preference on whether to attend cancer screenings.

Byrne and Thompson (2001) study a different behavioural bias, specifically the effect of myopia on individual's decision whether of not to attend a screening session. While myopia is undoubtedly an important factor in designing a screening program, it is abstracted from here to highlight the effect of dynamic inattention.

7.2.3.1 Model

Let an individual live in periods $t = 1, 2, 3, \ldots$ and let her discount the future at a rate $\delta \in (0, 1)$. When healthy, she receives some payoff $u \in \mathbb{R}$ in each period.

There is some probability each period $p_d \in (0, 1)$ that the individual contracts a disease. Contracting a disease means that after $T > 1$ periods of gestation the individual's payoff is reduced to $v < u$ each period. Over the gestation time there is a symptom level $s(t)$ that grows at a constant rate $\dot{s} > 0$, so that $s(t) = \dot{s}t$.

During gestation, the individual may seek treatment. If the individual has had the disease for t periods at the time of treatment, it is successful, implying that the individual no longer has the disease, with probability $p_c(t) \in (0, 1)$. $p_c(t)$ is decreasing in t, implying that the chances of curing the disease are lower the later the individual undergoes treatment. Treatment is unsuccessful with probability $1 - p_c(t)$, in which case the loss of payoff occurs at the end of gestation.

It is assumed that the loss of utility provided by the disease is sufficiently severe and the probability of cure sufficiently high that the individual always seeks treatment. Thus it is possible to consider treatment as costless.

Standard case

Let D be the present value of the individual upon contracting the disease. Then her expected utility is

$$\mathbb{E}U = \sum_{t=1}^{\infty} \delta^{t-1} \left((1-p_d)^t u + p_d (1-p_d)^{t-1} D \right) = \frac{(1-p_d) u + p_d D}{1 - \delta (1-p_d)}. \tag{7.27}$$

Suppose the individual employs the strategy of seeking treatment t' periods after contracting the disease, $1 \leq t' \leq T$. Then the expected payoff after contracting the disease is

$$\begin{aligned} D &= \sum_{t=1}^{T} \delta^{t-1} u + p_c(t') \delta^T \mathbb{E}U + (1 - p_c(t')) \sum_{t=T+1}^{\infty} \delta^{t-1} v \\ &= \frac{\left(1-\delta^T\right) u + (1-p_c(t')) \delta^T v}{1-\delta} + \delta^T p_c(t') \mathbb{E}U. \end{aligned} \tag{7.28}$$

Substituting this into equation (7.27) and solving for $\mathbb{E}U$ gives

$$\mathbb{E}U = \frac{\left(1 - \delta + p_d \left(\delta - \delta^T\right)\right) u + p_d (1 - p_c(t')) \delta^T v}{(1-\delta)(1 - \delta(1-p_d) - \delta^T p_d p_c(t'))}. \tag{7.29}$$

From this it is seen that

Lemma 7.1. *In the standard case, the individual seeks treatment as soon as possible.*

In the standard case then, the individual's expected payoff is

$$\mathbb{E}U = \frac{\left(1 - \delta + p_d \left(\delta - \delta^T\right)\right) u + p_d (1 - p_c(1)) \delta^T v}{(1-\delta)(1 - \delta(1-p_d) - \delta^T p_d p_c(1))}. \tag{7.30}$$

Dynamic inattention

Let the probability that the individual attends to the presence or absence of symptoms be $q(\dot{s}) \in (0,1)$, where $q(\dot{s})$ is increasing in $\dot{s}$. Hence the individual's attention is more likely to be drawn to the presence of symptoms the more quickly they develop.

Let $\mathbb{E}p_c$ be the expected probability of the disease being cured. From lemma 7.1, the individual seeks treatment as soon as she first attends to the presence of symptoms, so that

$$\mathbb{E}p_c = q(\dot{s}) \sum_{t=1}^{T} (1 - q(\dot{s}))^{t-1} p_c(t). \tag{7.31}$$

The individual's expected utility becomes

$$\mathbb{E}U = \frac{\left(1 - \delta + p_d\left(\delta - \delta^T\right)\right) u + p_d (1 - \mathbb{E}p_c) \delta^T v}{(1-\delta)(1 - \delta(1 - p_d) - \delta^T p_d \mathbb{E}p_c)} \tag{7.32}$$

from which it follows that

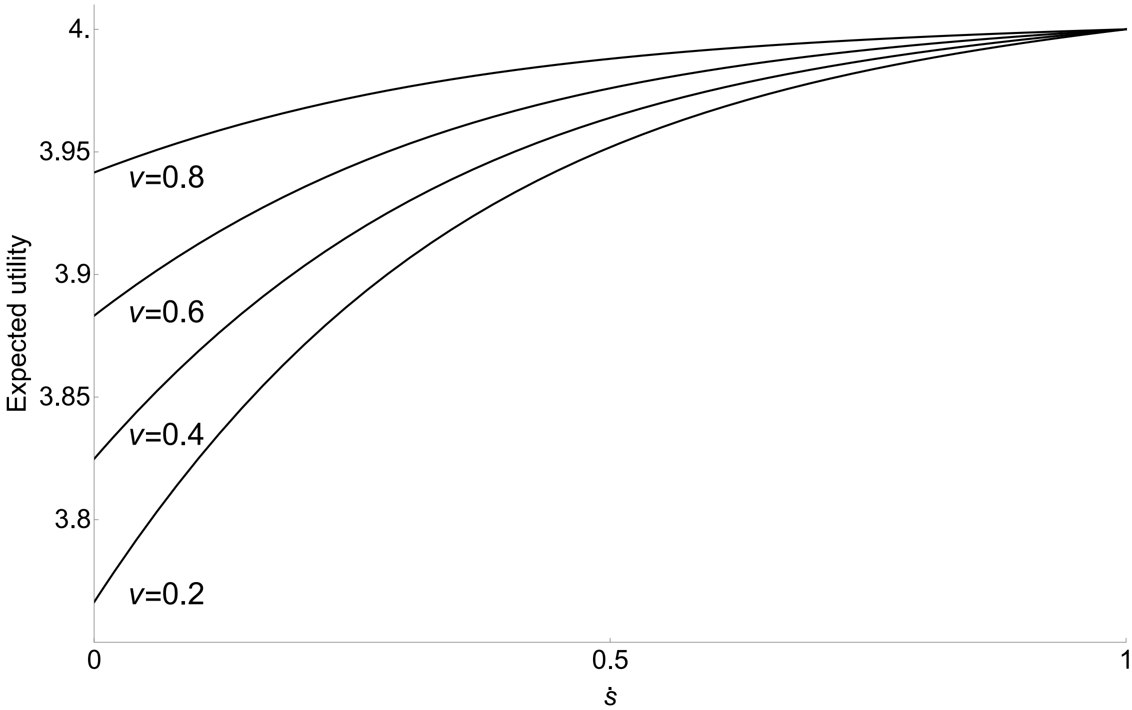
Proposition 7.6. *(i) The probability of cure is lower with dynamic inattention and is increasing in $\dot{s}$.*

(ii) The individual's expected utility is lower with dynamic inattention and is increasing in $\dot{s}$.

Examples of the dependence of expected utility on the speed of symptom development are given in figure 7.4.

That individuals do not immediately attend to symptoms means that a screening program may be welfare enhancing. Screening programs typically involve inviting individuals for a check-up at regular intervals. However, it is more tractable in the current framework to model a screening program as a constant probability each period of being called for a check-up. The expected time between checks is then analogous to the fixed time interval in the deterministic model of screening.

FIGURE 7.4: Dependence of expected utility on speed of symptom development, $T = 5$, $p_d = 0.1$, $\delta = 0.75$, $u = 1$.



One advantage to modelling the screening program in this way is that it allows for individuals' attendance to be imperfect, as they may forget or have other priorities.

Let there be a probability $f \in [0,1]$ each period that the individual attends a screening. If the individual attends a screening, then any disease is detected with perfect accuracy and she undergoes treatment immediately. The cost of attending the screening is $k > 0$ for each visit.

The individual's expected utility becomes

$$\mathbb{E}U = \frac{\left(1 - \delta + p_d\left(\delta - \delta^T\right)\right)u + p_d\left(1 - \mathbb{E}p_c\right)\delta^T v}{\left(1 - \delta\right)\left(1 - \delta\left(1 - p_d\right) - \delta^T p_d \mathbb{E}p_c\right)} - fk. \tag{7.33}$$

The probability of detecting the disease t periods after contracting it is $\left(f + (1-f)\,q\left(\dot{s}\right)\right)\left(1 - f - (1-f)\,q\left(\dot{s}\right)\right)^{t-1}$, which means the expected probability of cure is

$$\mathbb{E}p_c = \sum_{t=1}^{T} \left(f + (1-f)\, q\,(\dot{s})\right) \left(1 - f - (1-f)\, q\,(\dot{s})\right)^{t-1} p_c\,(t)\,. \tag{7.34}$$

From these expressions, it is possible to show that

Proposition 7.7. *There is some screening cost $\hat{k} > 0$ below which there exists a beneficial screening program.*

Examples of how $\hat{k}$ and the optimal screening program depend on the speed of symptom development are given in figures 7.5 and 7.6.

FIGURE 7.5: Dependence of $\hat{k}$ on speed of symptom development, $T = 5$, $p_d = 0.1$, $\delta = 0.75$, $u = 1$.

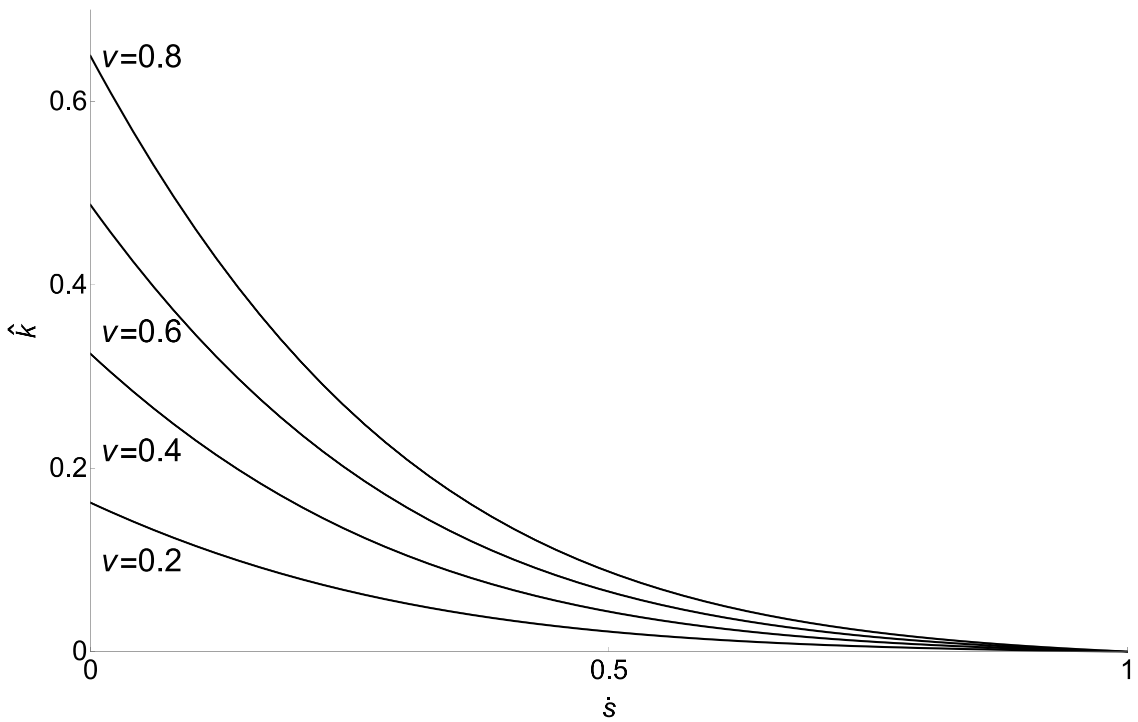
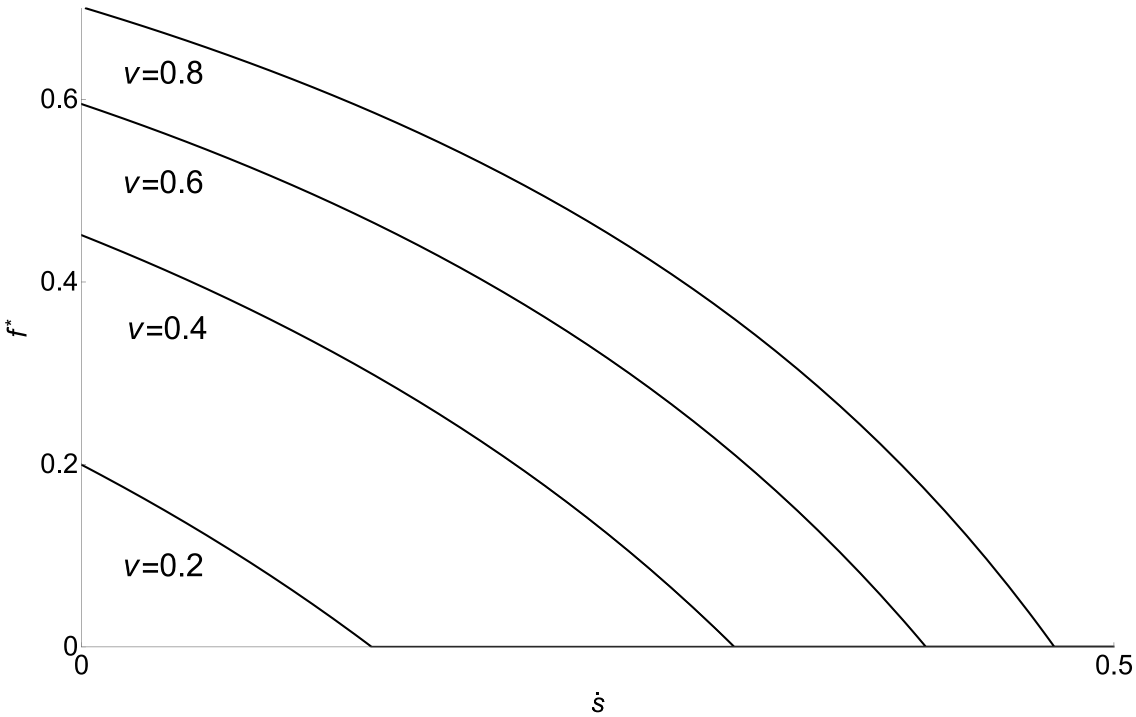


FIGURE 7.6: Dependence of optimal screening frequency on speed of symptom development, $T = 5$, $p_d = 0.1$, $\delta = 0.75$, $u = 1$.



7.2.3.2 Discussion

Proposition 7.6 shows that dynamic inattention can be detrimental to individuals' health. Slow developing symptoms are easy to overlook, and this leads to individuals seeking treatment later than they otherwise would. Since the sooner a disease is treated the better, the delay is bad for their health.

Proposition 7.7 shows that as a result, screening programs, provided they are not too costly, can improve health outcomes. For any degree of dynamic inattention, there exists some cost of screening such that a beneficial screening program exists. However, note that this does not imply that for any given cost of screening that there will be a beneficial program. Indeed, for a sufficiently large cost, no program is beneficial.

The cost of screening can have many components, such as travel costs to the hospital or clinic, or the opportunity cost of the time it requires. Another component may be the cost of a false positive result. A false positive result has potentially very serious consequences to the individual, and so the lower the quality of diagnosis the less likely it is that a beneficial program exists.

A limitation of the model is that the individual is "immortal": Their time horizon is infinite. This may be partly resolved by regarding the discount function δ as a probability of death, but this still means a constant probability of death over an entire lifetime, whereas it in fact increases with age. Thus it should be borne in mind that the benefit of screening for older people is lower than for younger people, as well as the physical cost being higher.

The model also assumes that the designer of a screening program is free to select any frequency of screening. In reality, it can be difficult to ensure perfect attendance. While attendance is individually rational for any program, myopia and forgetfulness provide challenges to the designer of a screening program. It should also be noted that attendance may depend on the degree of sophistication the individual has regarding her degree of dynamic inattention. If she is naïve about her attentional limitations, she may not recognize that participation in the screening program is beneficial.

Screening programs are often considered for diseases which do not exhibit symptoms until too late. Here, it has been demonstrated that they may also be beneficial in combating inattention if clearly visible symptoms do exist, but are slow in developing.

7.3 General discussion

As was previously mentioned, there have recently appeared many models incorporating attention as a factor in decision making. The current chapter contributed to this literature by studying an aspect hitherto neglected. It includes a dynamic inattentional bias, so that decision makers may be unaware of changes in the economic environment over time, and make decisions based on out-of-date information.

It also created a bridge between the economics literature and the psychological literature on change and choice blindness. The studies on choice blindness discussed previously focused on documenting a bias in individual choice. By demonstrating that in a market setting such biases may influence market outcomes, it makes the bias much more relevant for economic study. The applications developed show the wide relevance of the approach, to investment decisions, to consumer choice and in a non-market setting to health outcomes.

However, so far the approach has been purely theoretical in nature. In future, it would be desirable to demonstrate the existence of, and investigate the prevalence of, dynamic inattention "in the wild". Della Vigna and Pollet (2007) give some evidence that investors tend to be unaware of slow shifts in demographics. However, as they discuss, this could have several other sources.

There is also evidence from psychological studies of choice and change blindness that individuals can be inattentive to changes in their surroundings. However, it should be noted that such studies involve somewhat unusual situations, in that the subjects are "tricked" by some sleight of hand into believing that no change has taken place. Nevertheless, the situations are not so different to a firm which alters the formulation of a product yet leaves its packaging unaltered.

It is also necessary to investigate further the sources of dynamic inattention. In the applications developed the functional form of dynamic inattention for a given variable was exogenous. While it is argued, for example, that consumers paying more attention to changes in individual prices than the aggregate price level, is plausible, this is still an ad hoc assumption.

Another avenue for future research is sophistication. Another intertemporal choice bias, myopia, has generated fascinating insights from allowing to individuals to be aware to some extent that their current and future selves possess a bias. However, as yet thus has remained an unexplored area in work on attentional biases.

7.4 Conclusion

A novel way of modelling inattention to dynamic changes in the economic environment was presented. Three applications were then given, firstly to portfolio design where the investor was shown to over- or under-invest in an asset due to changes from states of high to low returns. Secondly, inattention to inflation combined with being attentive to individual price changes caused consumers' nominal WTP for a good to fall, and the firm to shrink the size of its product in response. Finally, individuals neglected slowly developing disease symptoms, and thus delayed seeking medical assistance, giving scope for medical screening programs.

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Derivations, robustness checks, experimental instructions and goods for chapter 2 A

A.1 Preference reversals with multiple attributes

Let each good now have N quality attributes: $h = (q_{h1}, \ldots, q_{hN}, p_h)$, $\ell = (q_{\ell 1}, \ldots, q_{\ell N}, p_\ell)$ with $q_{hk} > q_{\ell k}$ $\forall$ $k = 1, \ldots, N$ and $p_h > p_\ell$. The reference good is then $(\bar{\mathbf{q}}, p) = (\bar{q}_1, \bar{q}_2, \ldots, \bar{q}_N, \bar{p})$ with $\bar{q}_k = \frac{1}{2}(q_{hk} + q_{\ell k})$, $\forall$ $k = 1, \ldots, N$ and $\bar{p} = \frac{1}{2}(p_h + p_\ell)$. Without salience, the individual values good i according to $u = \sum_{k=1}^{N} q_{ik} - p_i$.

Let r_{ij} be the salience rank of quality attribute j for good i, i.e. $r_{ij} < r_{ik}$ iff $\sigma(q_{ij}, \bar{q}_j) > \sigma(q_{ik}, \bar{a}_k)$, with the most salient attribute having rank 1. Then the salience rank attached to attribute q_i when evaluating good j is $\frac{\delta^{r_{ij}}}{\sum_{k=1}^{N+1} \delta^k}$.

Attribute j is more salient than attribute k for good h if $\sigma(q_{hj}, \bar{q}_j) > \sigma(q_{hk}, \bar{q}_k)$, which by homogeneity of degree 0 becomes $\sigma\left(\frac{q_{hj}}{\bar{q}_j}, 1\right) > \sigma\left(\frac{q_{hk}}{\bar{q}_k}, 1\right)$. By ordering this is equivalent to $\frac{q_{hj}}{\bar{q}_j} > \frac{q_{hk}}{\bar{q}_k}$ or

$$\frac{q_{hj}}{q_{\ell j}} > \frac{q_{hk}}{q_{\ell k}}. \tag{A.1}$$

An analogous argument shows this condition also determines whether quality attribute j is more salient than quality attribute k for good ℓ.

Without loss of generality assume $\frac{q_{hj}}{q_{\ell j}} > \frac{q_{hk}}{q_{\ell k}}$ iff $j > k$ and for ease of exposition assume that the ratios of high to low quality are not equal for any two attributes.

By an analogous argument, price is more salient than quality attribute k if $\frac{p_h}{p_\ell} > \frac{q_{hk}}{q_{\ell k}}$. Again for clarity assume that $\frac{p_h}{p_\ell} \neq \frac{q_{hk}}{q_{\ell k}}$ ∀ k.

Let n be such that $\frac{q_{hk}}{q_{\ell k}} > \frac{p_h}{p_\ell}$ iff $k \leq n$ and suppose $n \geq 1$. The individual prefers ℓ to h if

$$\delta^{n+1}\left(p_h - p_\ell\right) > \sum_{i=1}^{n} \delta^k \left(q_{hk} - q_{\ell k}\right) + \sum_{k=n+1}^{N} \delta^{k+1}\left(q_{hk} - q_{\ell k}\right). \tag{A.2}$$

Now suppose prices are subject to a uniform mark-up $\Delta > 0$. Let $\hat{n}$ be such that $\frac{q_{hk}}{q_{\ell k}} > \frac{p_h+\Delta}{p_\ell+\Delta}$ iff $k \leq \hat{n}$. Note that $\hat{n} \leq n$ and for sufficiently high δ, $\hat{n}$ can take any value such that $\hat{n} < n$. h is preferred to ℓ if

$$\delta^{\hat{n}+1}\left(p_h - p_\ell\right) < \sum_{i=1}^{\hat{n}} \delta^i \left(q_{hi} - q_{\ell i}\right) + \sum_{i=\hat{n}+1}^{N} \delta^{i+1}\left(q_{hi} - q_{\ell i}\right). \tag{A.3}$$

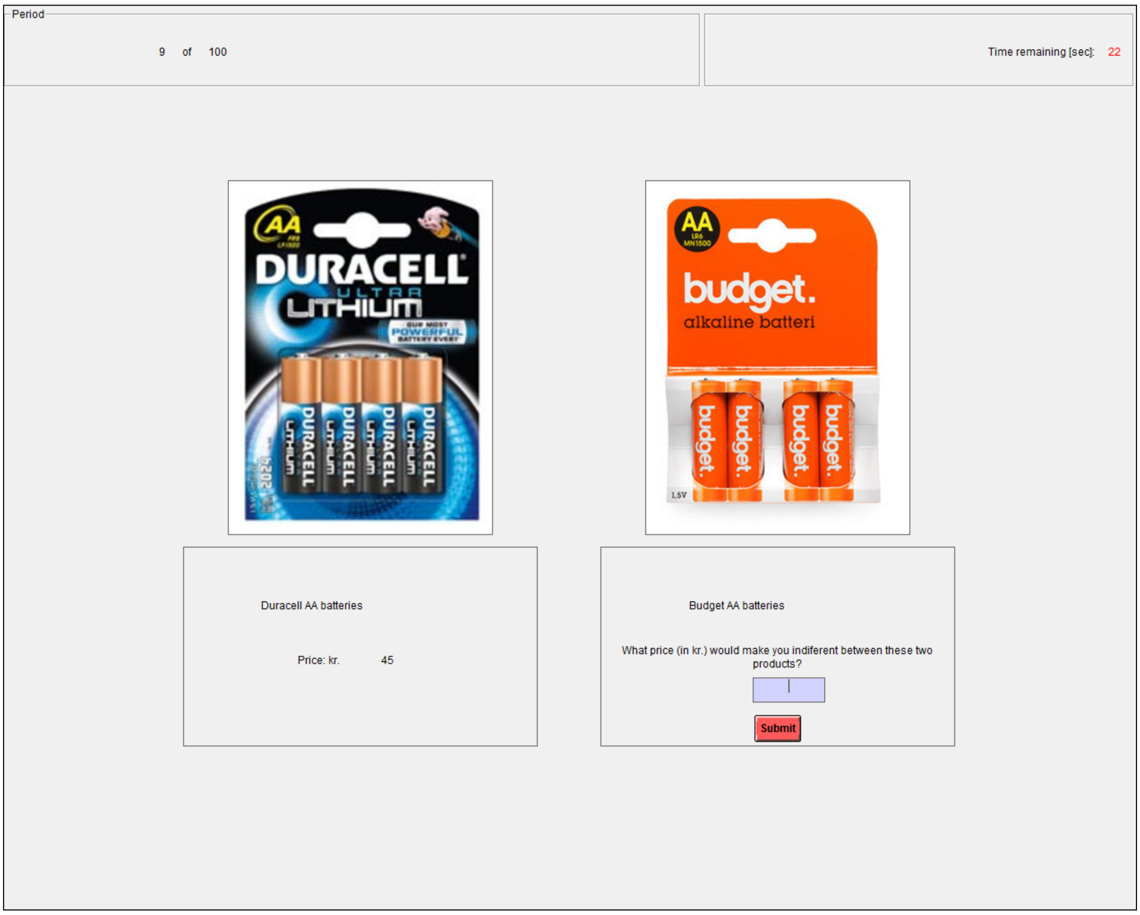
Comparing conditions (A.2) and (A.3), the mark-up decreases the salience of price and increases the salience of quality, meaning preference reversals are possible with multiple quality attributes.

A.2 Calibration exercise

The calibration exercise was designed to identify suitable goods pairs and to find the appropriate price ranges at which preference reversals might be expected to occur.

The experimenters identified 50 pairs of goods, one of which was designated as high and one as low quality. The price of the high quality good was given, and the subjects were asked to state what price for the low quality good would make them indifferent between the two. Note that this could be higher than the reference price if the subject disagreed with the experimenters' categorization of quality. An example screen is shown in figure A.1.

FIGURE A.1: Example of a decision screen from the calibration exercise.



Subjects were shown each pair twice, once with a high reference price ($p_h + \Delta$) and once with a low reference price (p_h). The reference price of the high quality good in the high price condition was its retail price or DKK 300, whichever was lower, and in the low price condition it was halved (i.e. $\Delta = p_h$). All prices were rounded to the nearest DKK 5.

Following this task, subjects were shown all the products again individually and asked to rate how much they liked the good on a scale from 0 to 9, with 9 being high and 0 being low.[1] Previous research has found that liking ratings are very highly correlated with willingness-to-pay (Plassmann, O'Doherty, & Rangel, 2007; Hare,

[1] In the analogous rating task in the main experiment the scale as changed to 1-9 as this was more suitable for the interface.

O'Doherty, Camerer, Schultz, & Rangel, 2008; Hare, Camerer, & Rangel, 2009; Hare, Camerer, Knoepfle, O'Doherty, & Rangel, 2010), so a higher rating for the designated high quality good is taken as a verification of the experimenters' categorization.

26 subjects were recruited using ORSEE (Greiner, 2015) and were paid a flat fee of DKK 150 (approximately US$ 22.5) for their participation. The experiment was programed using z-Tree (Fischbacher, 2007). The session was carried out at the Laboratory for Experimental Economics at the University of Copenhagen and lasted around one hour.

To see how prices were calibrated for the main experiment, using the notation of section 2.2, p_h is the price of the high quality good in the low price condition and $p_h + \Delta = 2p_h$ is the price of the high quality good in the high price condition, both fully determined in the calibration experiment. The remaining degree of freedom, then, is p_ℓ, the price of the low quality good in the low price condition, since once this is set, its price in the high price condition is simply $p_\ell + \Delta$.

Let r_h (r_ℓ) be a subject's response in the calibration experiment with a high (low) reference price. For some p_ℓ, the subject would have purchased ℓ in the low price condition if $p_\ell < r_\ell$, since r_ℓ is defined as the point of indifference between h and ℓ. Similarly, she would have purchased h if $p_\ell + \Delta > r_h$. Thus for a given subject for a given good there is a range

$$r_h - \Delta < p_\ell < r_\ell \tag{A.4}$$

in which her responses imply that she would exhibit preference reversal. Aggregating over all subjects, it is then possible to identify a suitable p_ℓ at which many subjects would exhibit preference reversal.

Out of the 50 pairs, 40 were selected for the main experiment based on whether there was a significant difference in liking ratings between the high and low quality good[2] and on the number of preference reversals that might be expected.

[2]The only stated pair where the high quality good was not rated significantly higher was the chocolate bar. However, it was assumed that when presented with a 70g bar and a 20g bar side-by-side, subjects would prefer the larger bar.

A.3 Robustness checks

Table A.1: Coefficients for regressions with subject fixed effects with a dummy indicating the occurrence of a reversal as dependent variable for the first part of each trial. P-values in parentheses. Control variables: price difference, price increase from low to high price level, rating difference, rt (low price), rt(high price), lag, high first, incentivized. $N = 1440$, *** = significant at 1% level, ** = significant at 5% level, * = significant at 10% level.

	(1)		(2)	
	ΔAT	ΔHQ	ΔP	ΔQ
1st 50% of trial	-0.0854	-0.191	-0.0963	0.0837
	(0.864)	(0.511)	(0.849)	(0.747)
1st 10% of trial	-0.519	-0.129	-1.838	-2.217**
	(0.368)	(0.353)	(0.394)	(0.033)
1st 5% of trial	-0.680	-0.0306	—	—
	(0.366)	(0.806)		
1st 1000ms	-0.158	-0.0805	0.122	0.0315
	(0.723)	(0.638)	(0.791)	(0.887)
1st 500ms	-0.307	-0.157	—	4.75
	(0.647)	(0.277)		(0.134)
1st 200ms	-0.402	-0.0199	—	—
	(0.594)	(0.863)		

Table A.2: Coefficients for regressions with subject fixed effects with a dummy indicating the occurance of a reversal as dependent variable for the last part of each trial. P-values in parentheses. Control variables: price difference, price increase from low to high price level, rating difference, rt (low price), rt(high price), lag, high first, incentivized. $N = 1440$, *** = significant at 1% level, ** = significant at 5% level, * = significant at 10% level.

	(1)		(2)	
	ΔAT	ΔHQ	ΔP	ΔQ
Last 50% of trial	0.279	4.14***	-0.341	-0.306
	(0.532)	(<0.001)	(0.443)	(0.251)
Last 10% of trial	0.337	2.88***	-0.123	-0.0964
	(0.135)	(<0.001)	(0.752)	(0.698)
Last 5% of trial	0.286	2.72***	-0.0371	-0.154
	(0.155)	(<0.001)	(0.947)	(0.592)
Last 1000ms	0.0678	3.70***	-0.0837	-0.0181
	(0.818)	(<0.001)	(0.767)	(0.914)
Last 500ms	0.244	2.99***	-0.912*	0.108
	(0.298)	(<0.001)	(0.056)	(0.826)
Last 200ms	0.369*	2.65***	—	—
	(0.060)	(<0.001)		

Table A.3: Means of eye-tracking measures for high price/low price conditions for the first and last part of each trial. P-values for Wilcoxon signed-rank tests in parentheses. N=36, *** = significant at 1% level, ** = significant at 5% level, * = significant at 10% level.

	Quality attribute	High quality good	Q saccades	P saccades
1st 50% of trial	0.839/0.856*** (<0.001)	0.511/0.503 (0.480)	0.402/0.417 (0.203)	0.162/0.151* (0.084)
1st 10% of trial	0.964/0.973 (0.239)	0.505/0.500 (0.789)	0.494/0.487 (0.730)	0.108/0.113 (0.465)
1st 5% of trial	0.983/0.990*** (0.009)	0.500/0.495 (0.783)	0.524/0.584 (0.294)	0.00980/0.000 (0.317)
1st 1000ms	0.901/0.907 (0.203)	0.510/0.502 (0.499)	0.494/0.487 (0.730)	0.0129/0.0142 (0.676)
1st 500ms	0.966/0.974 (0.218)	0.499/0.498 (0.851)	0.957/0.788 (—)	0.0129/0.0142 (0.676)
1st 200ms	0.994/0.997*** (0.046)	0.495/0.497 (0.869)	0.788/0.957 (—)	0/0 (—)
Last 50% of trial	0.756/0.781*** (<0.001)	0.513/0.487*** (0.006)	0.410/0.452** (0.011)	0.245/0.215*** (0.002)
Last 10% of trial	0.770/0.783* (0.062)	0.513/0.462*** (0.005)	0.330/0.381* (0.182)	0.368/0.316 (0.242)
Last 5% of trial	0.789/0.791 (0.671)	0.507/0.455*** (0.005)	0.267/0.308 (0.180)	0.439/0.427 (0.909)
Last 1000ms	0.751/0.757 (0.414)	0.496/0.466** (0.035)	0422/0.432 (0.322)	0.293/0.268* (0.077)
Last 500ms	0.766/0.771 (0.582)	0.506/0.459*** (0.008)	0.353/0.387 (0.556)	0.387/0.356 (0.499)
Last 200ms	0.792/0.799 (0.561)	0.505/0.454*** (0.009)	0.356/0.350 (0.729)	0.418/0.458 (0.499)

Table A.4: Means of eye-tracking measures for incentivized/hypothetical conditions for the first and last part of each trial. P-values for Mann-Whitney U tests in parentheses. N=36, *** = significant at 1% level, ** = significant at 5% level, * = significant at 10% level.

	Quality attribute	High quality good	Q saccades	P saccades
1st 50% of trial	0.853/0.841 (0.496)	0.509/0.504 (0.788)	0.400/0.420 (0.692)	0.156/0.158 (0.887)
1st 10% of trial	0.971/0.966 (0.437)	0.503/0.502 (0.763)	0.545/0.517 (0.541)	0.0287/0.0491 (0.115)
1st 5% of trial	0.987/0.987 (0.160)	0.503/0.491 (0.669)	0.753/0.636 (0.385)	0.00800/0 (-)
1st 1000ms	0.905/0.903 (0.962)	0.503/0.509 (0.646)	0.488/0.494 (0.843)	0.112/0.109 (0.766)
1st 500ms	0.968/0.972 (0.455)	0.502/0.494 (0.812)	0.509/0.608 (0.386)	6.45×10^{-3}/0.0221 (0.867)
1st 200ms	1.00/0.990** (0.027)	0.498/0.494 (0.887)	0.875/0.591 (1.00)	0/0 (—)
Last 50% of trial	0.787/0.748 (0.132)	0.480/0.524*** (0.010)	0.454/0.405 (0.117)	0.219/0.242 (0.261)
Last 10% of trial	0.797/0.753** (0.048)	0.442/0.538*** (0.006)	0.396/0.306 (0.125)	0.384/0.302** (0.029)
Last 5% of trial	0.810/0.768* (0.068)	0.433/0.535*** (0.002)	0.213/0.353 (0.117)	0.481/0.360 (0.163)
Last 1000ms	0.773/0.732*** (<0.001)	0.448/0.518*** (0.004)	0.456/0.395 (0.159)	0.297/0.266 (0.384)
Last 500ms	0.784/0.752*** (<0.001)	0.436/0.534*** (0.002)	0.400/0.336 (0.217)	0.365/0.377 (0.763)
Last 200ms	0.814/0.775*** (<0.001)	0.432/0.533*** (0.003)	0.460/0.225 (0.508)	0.381/0.496 (0.418)

Table A.5: Means of the proportions of fixation counts and saccades into areas. The proposition of saccades is calculated with respect to the total number of saccades between different interest areas. P-values for high price vs. low price level are for Wilcoxon signed-rank tests, p-values for incentivized vs. non-incentivized choice are for Mann-Whitney U tests. N=36, *** = significant at 1% level, ** = significant at 5% level, * = significant at 10% level.

	All	Highprice	Low price	P-value (WSR)	Incentivized	Hypothetical	P-value (MWU)
Prop. fixations on high quality attribute	0.780	0.773	0.786	<0.001***	0.791	0.767	0.084*
Prop. fixations on high quality good	0.506	0.511	0.500	0.073*	0.498	0.514	0.141
Prop. saccades into quality attribute	0.561	0.556	0.567	0.018**	0.570	0.552	0.223
Prop. saccades into high quality good	0.497	0.501	0.492	<0.001***	0.484	0.511	0.006***

Table A.6: Logit regression with subject fixed effects with a dummy indicating the occurrence of a reversal as the dependent variable. ΔQ count = difference in proportion of fixations on quality attribute between high and low price level. ΔHQ count = difference in proportion of fixations on high quality good between high and low price. ΔQ saccade entry = difference in percentage of inter-component saccades which enter the quality attribute between high and low price level. ΔHQ saccade entry = difference in percentage of inter-component saccades which enter the high quality good between high and low price level. $N = 1440$, *** = significant at 1% level, ** = significant at 5% level, * = significant at 10% level.

	(1)	(2)	(3)	(4)
Price difference	-0.0226***	-0.0218***	-0.0217***	-0.0218***
	(0.003)	(0.004)	(0.002)	(0.005)
Price increase	8.38×10^{-3}	8.31×10^{-3}	8.33×10^{-3}	8.65×10^{-3}
	(0.120)	(0.144)	(0.122)	(0.127)
Rating difference	1.23×10^{-3}	-0.0121	2.30×10^{-3}	3.97×10^{-3}
	(0.976)	(0.776)	(0.954)	(0.925)
rt (low price level)	0.183×10^{-3}***	0.187×10^{-3}***	0.174×10^{-3}***	0.204×10^{-3}***
	(<0.001)	(<0.001	(<0.001)	(<0.001)
rt (high price level)	0.146×10^{-3}***	0.164×10^{-3}***	0.156×10^{-3}***	0.182×10^{-3}***
	(<0.001)	(<0.001)	(<0.001)	(<0.001)
Lag	4.31×10^{-3}	2.41×10^{-3}	4.05×10^{-3}	3.42×10^{-3}
	(0.307)	(0.585)	(0.337)	(0.442)
High first	-0.0177	0.0185	0.0762	0.0536
	(0.920)	(0.916)	(0.663)	(0.767)
Incentivized	-0.194	-0.347	-0.136	-2.06*
	(0.824)	(0.696)	(0.876)	(0.052)
ΔQ count	0.560			
	(0.443)			
ΔHQ count		4.87***		
		(<0.001)		
ΔQ saccade entry			-0.757	
			(0.162)	
ΔHQ saccade entry				5.59×10^{-3}***
				(<0.001)

A.4 Instructions and experimental goods

Text in **bold** was only included in incentivized treatments, text in *italics* was only included in non-incentivized treatments.

In this part of the experiment, you will be making a series of choices about which of two products you wish to buy. **You have been given kr. 300 to spend on a product.** For each choice you are shown a screen like the one below:

[An example screen illustrating a choice between a Blu-ray and DVD was inserted here.]

For each product you are shown a picture, a brief description and its price below it. There is no significance to which product is on the left or the on the right. You have to choose which one you would prefer to purchase, given the prices shown. There is no right or wrong answer, simply choose which one you prefer to buy. You have to buy one or other of the goods. There are 80 choices in total.

You should imagine that you have the opportunity to buy the products at the prices stated, so that there is a trade-off between the product and the cost. For example, suppose you like the left-hand product a little more than the right-hand product, but it is a lot more expensive. Then you would probably choose the right-hand product. On the other hand, suppose the prices of both products are fairly similar, and you like the left-hand product a lot more. Then you would probably choose the left-hand product.

At the end of the experiment, one of the choices you made will be picked at random and enacted for real. You will use the kr. 300 you have been given to pay the price. You will then receive the product and whatever money is left. Note that since you don't know which of the choices will be picked, it is best to treat all of them as if they were for real.

Example: Suppose the choice above was randomly picked to be enacted for real. Then if you chose the Blu-ray, you would receive the Blu-ray. You would pay the price of kr. 150, leaving you with 300 - 150 = kr. 150. On the other hand, if you choe the DVD, you would receive the DVD and pay the price of kr. 120 for it, leaving you with 300 - 120 = kr. 180.

You will be paid a flat fee of kr. 300 for your participation today.

Table A.7: Experimental goods, † indicates replacement after pilot.

Description	High quality Retail price (DKK)	High price (DKK)	Low price (DKK)	Description	Low quality Retail price (DKK)	High price (DKK)	Low price (DKK)	Reversals/ B reversals
Air matress with built in pump	248	250	125	†Simple air matress without foot pump	148	160	35	7/3
Duracell AA batteries	89	90	45	Budget AA batteries	10	60	15	7/2
75cl Jacobsen Saaz Blonde	41	40	20	33cl Red Stripe lager	21	25	5	4/1
Aluminium bike pump with pressure gauge	149	150	75	Bike pump	39	95	20	2/1
Reelight bike light set	199	200	100	Budget bike light set	35	120	20	5/4
Basta bike lock	189	190	95	Budget wire bike lock	49	110	15	7/1
Binoculars, 16 x magnification	319	300	150	Binoculars, 8 x magnification	139	190	40	5/1
Stainless steel toaster	299	300	150	Plastic toaster	79	205	55	10/2
Brunch for two	199	200	100	Coffee for two	149	125	25	11/3
Casio scientific calculator	236	235	120	Canon calculator	73	130	15	3/1

Cont. overleaf

Table A.7: (cont.) Experimental goods, † indicates replacement after pilot.

	High quality				Low quality			
Description	Retail price (DKK)	High price (DKK)	Low price (DKK)	Description	Retail price (DKK)	High price (DKK)	Low price (DKK)	Reversals/ B reversals
Champagne, 75cl bottle	370	300	150	Champagne, 20cl bottle	115	185	35	8/3
Stainless steel cheese slice	200	200	100	Plastic cheese slice	35	115	15	3/2
Milk chocolate bar, 70g	43	45	20	Milk chocolate bar, 20g	15	30	5	14/1
Wooden chopping board 30x45 cm	270	270	135	Plastic chopping board 20x29 cm	49	160	25	7/3
16GB USB stick	90	90	45	8GB USB stick	83	60	15	14/0
5 plug extension lead, 5m length	175	175	90	3 plug extension lead, 1.5 meter length	107	110	25	6/0
Tefal gourmet frying pan (30 cm)	199	200	100	Aida frying pan (20 cm)	50	130	30	5/2
Stainless steel juicer	279	280	140	Plastic juicer	128	205	65	8/0
Kettle, 0.5 litres, 2200 Watt	123	125	60	Kettle, 0.5 litre, 1000 Watt	217	80	15	2/2

Cont. overleaf

Table A.7: (cont.) Experimental goods, † indicates replacement after pilot.

	High quality				Low quality			
Description	Retail price (DKK)	High price (DKK)	Low price (DKK)	Description	Retail price (DKK)	High price (DKK)	Low price (DKK)	Reversals/ B reversals
Advent wireless keyboard	149	150	75	Advent keyboard	99	105	30	8/1
Metal table lamp, height 49cm	119	120	60	Plastic table lamp, height 29cm	15	80	15	4/2
Sandstrøm leather laptop case	299	300	150	Goji fabric laptop case	149	185	35	2/5
Lego hotrod car	250	250	125	Lego blue racecar	40	150	25	5/2
Lemonade (75cl bottle)	35	35	20	Lemonade (20cl bottle)	14	20	5	13/2
4x LED lightbulb	219	220	110	4x Budget lightbulb	116	160	50	6/3
Advent wireless mouse	149	150	75	Advent wired mouse	69	85	10	5/1
Parker chrome ballpoint pen	161	160	80	Plastic ballpoint pen	14	105	25	6/0
Memoryfoam pillow	199	200	100	Fibre filled pillow	25	120	20	2/1
Rucksack, 30 litre capacity	249	250	125	Rucksack, 10 litre capacity	199	155	30	6/4

Cont. overleaf

Table A.7: (cont.) Experimental goods, † indicates replacement after pilot.

	High quality			Low quality				
Description	Retail price (DKK)	High price (DKK)	Low price (DKK)	Description	Retail price (DKK)	High price (DKK)	Low price (DKK)	Reversals/ B reversals
Titanium scissors, 20cm	165	165	85	Scissors, 12.5cm	33	95	15	7/0
Stanley screwdriver set	189	190	95	Stanley mini screwdriver set	60	115	20	6/1
12W 3 piece speaker set	143	145	70	2.3W 2 piece speaker set	127	85	10	7/2
Stapler, 50 page capacity	142	140	70	Stapler, 20 page capacity	58	85	15	4/2
†3 person tent, 210 x 210 cm	278	280	140	2 person tent, 140 x 205 cm	87	215	75	8/3
Stainless steel sandwich toaster	229	230	115	Plastic sandwich toaster	79	140	25	1/5
Duracell torch (200m range)	229	230	115	Duracell torch (20m range)	99	135	20	2/2
Georg Jensen Damask towel, 50x100cm	240	240	120	Towel, 50x90cm	80	165	45	4/4

Cont. overleaf

Table A.7: (cont.) Experimental goods, † indicates replacement after pilot.

	High quality				Low quality			
Description	Retail price (DKK)	High price (DKK)	Low price (DKK)	Description	Retail price (DKK)	High price (DKK)	Low price (DKK)	Reversals/ B reversals
32 GB USB-stick	129	130	65	16 GB USB-stick	79	85	20	12/5
Sigg water bottle	13	135	65	Plastic water bottle	89	95	25	5/1
3 megapixel HD webcam	197	195	100	2 megapixel SD webcam	118	125	30	4/1

Proofs for chapter 3 B

B.1 Proof of lemma 3.1

Let an individual be endowed with $\boldsymbol{x}$. $\boldsymbol{x}$ satisfies equation (3.4) as the choice set has only one element. The individual's valuation of $\boldsymbol{x}$ under $\boldsymbol{\eta}_1 = \frac{1}{1+\gamma}(1\ \gamma)$ is $u(\boldsymbol{x}, \boldsymbol{\eta}_1) = \frac{1}{1+\gamma}(\alpha\lambda v + \gamma(1-\alpha)(1-\lambda)v)$ whereas under $\boldsymbol{\eta}_2 = \frac{1}{1+\gamma}(\gamma\ 1)$ it is $u(\boldsymbol{x}, \boldsymbol{\eta}_2) = \frac{1}{1+\gamma}(\gamma\alpha\lambda v + (1-\alpha)(1-\lambda)v)$. Comparing $u(\boldsymbol{x}, \boldsymbol{\eta}_1)$ and $u(\boldsymbol{x}, \boldsymbol{\eta}_2)$, $\boldsymbol{\eta}_1$ satisfies equation (3.4) if $\alpha \geq 1-\lambda$, whereas $\boldsymbol{\eta}_2$ satisfies it if $\alpha < 1-\lambda$. This directly leads to $\boldsymbol{\eta}_x^*$ being the selected attention vector. An analogous argument holds for individuals endowed with $\boldsymbol{y}$. □

B.2 Proof of proposition 3.1

Let an individual be endowed with $\boldsymbol{x}$ and let $\alpha \geq 1-\lambda$. Given $\boldsymbol{\eta}_x^*$, the individual's valuations of $\boldsymbol{x}$ and $\boldsymbol{y}$ are

$$u(\boldsymbol{x}) = \frac{\alpha\lambda v + \gamma(1-\alpha)(1-\lambda)v}{1+\gamma} \qquad u(\boldsymbol{y}) = \frac{\alpha(1-\lambda)v + \gamma(1-\alpha)\lambda v}{1+\gamma}. \quad \text{(B.1)}$$

Those with low α switch to $\boldsymbol{y}$ and those with high α remain with $\boldsymbol{x}$, so it is necessary to find the critical α at which this change occurs. Equating the above expressions and solving for α shows the individual switches to $\boldsymbol{y}$ if $\alpha < \frac{\gamma}{1+\gamma}$. This critical point is below $1-\lambda$ if $\lambda > \frac{1}{1+\gamma}$.

Let $\lambda \geq \frac{1}{1+\gamma}$. Then all those with α below $\frac{\gamma}{1+\gamma}$ switch to $\boldsymbol{y}$ and the fraction switching is $\frac{1}{2}\int_0^{\frac{\gamma}{1+\gamma}} d\alpha = \frac{\gamma}{2(1+\gamma)}$.

Let $\lambda < \frac{1}{1+\gamma}$. Then all those with $\alpha \geq 1-\lambda$ do not switch. Let $\alpha < 1-\gamma$. Given $\boldsymbol{\eta}_x$, the individual's valuations of $\boldsymbol{x}$ and $\boldsymbol{y}$ are

$$u(\boldsymbol{x}) = \frac{\gamma\alpha\lambda v + (1-\alpha)(1-\lambda)v}{1+\gamma} \qquad u(\boldsymbol{y}) = \frac{\gamma\alpha(1-\lambda)v + (1-\alpha)\lambda v}{1+\gamma}. \tag{B.2}$$

Equating these expressions and solving for α reveals the individual switches if $\alpha < \frac{1}{1+\gamma}$, which is always satisfied given $\alpha < 1-\lambda$ and $\lambda > \frac{1}{1+\gamma}$. Thus the critical α at which individuals switch is $\alpha = 1-\lambda$ and the fraction switching is $\frac{1}{2}\int_0^{1-\lambda} d\alpha = \frac{1}{2}(1-\lambda)$.

By symmetry, the fractions are the same for individuals endowed with $\boldsymbol{y}$, and equation (3.14) follows. □

B.3 Proof of lemma 3.2

The individual's choice set is a single element, so equation 3.4 is satisfied. Note that any attention vector which does not place the lowest weight on p cannot satisfy equation (3.3), since the individual is trivially better off if the lowest weight is placed on p. Thus the possible attention vectors are $\boldsymbol{\eta}_1 = \frac{1}{1+\gamma+\gamma^2}(1\ \gamma\ \gamma^2)$ and $\boldsymbol{\eta}_2 = \frac{1}{1+\gamma+\gamma^2}(\gamma\ 1\ \gamma^2)$. The individual's valuation of $\boldsymbol{x}$ given each vector is then

$$u(\boldsymbol{x}, \boldsymbol{\eta}_1) = \frac{\alpha\lambda v + \gamma(1-\alpha)(1-\lambda)v}{1+\gamma+\gamma^2} \qquad u(\boldsymbol{x}, \boldsymbol{\eta}_2) = \frac{\alpha\gamma\lambda v + (1-\alpha)(1-\lambda)v}{1+\gamma+\gamma^2}. \tag{B.3}$$

Equating these expressions and solving for α shows that $\boldsymbol{\eta}_1$ is selected if $\alpha \geq 1-\lambda$ and $\boldsymbol{\eta}_2$ is selected if $\alpha < 1-\lambda$. Equation (3.17) immediately follows. □

B.4 Proof of proposition 3.2

Let $\alpha \geq 1-\lambda$. Given that $\boldsymbol{\eta}_x^*$, the broker infers that a client with a taste parameter α' values the prospect of buying $\boldsymbol{y}$ at a price p according to $u(\boldsymbol{y}) = \frac{\alpha'(1-\lambda)v+\gamma(1-\alpha')\lambda v-\gamma^2 p}{1+\gamma+\gamma^2}$ Equating to 0 and solving for p shows the broker believes the client's WTP to be $wtp(\alpha') = \frac{\alpha'(1-\lambda)v+\gamma(1-\alpha')\lambda v}{\gamma^2}$ and her estimate of WTP is $\mathbb{E} = \int_0^1 wtp(\alpha') = \left(\frac{1-(1-\gamma)\lambda}{2\gamma^2}\right)v$.

Let $\alpha < 1-\lambda$. By an analogous derivation, the broker's estimate of the client's WTP is $\mathbb{E}wtp = \left(\frac{\gamma+(1-\gamma)\lambda}{2\gamma^2}\right)v$. Mean estimated WTP is then

$$\begin{aligned}\mathbb{E}\overline{wtp} &= \int_0^{1-\lambda}\left(\frac{\gamma+(1-\gamma)\lambda}{2\gamma^2}\right)v d\alpha + \int_{1-\lambda}^{1}\left(\frac{1-(1-\gamma)\lambda}{2\gamma^2}\right)v d\alpha \\ &= \frac{v}{2\gamma^2}\left(2\lambda(1-\lambda)(1-\gamma)+\gamma\right). \qquad \text{(B.4)}\end{aligned}$$

□

B.5 Proof of lemma 3.3

(i) Let all individuals purchase from firm $h \in \{1,2\}$ in period 1, with no demand for firm $\ell \in \{1,2\}$, $\ell \neq h$. An individual's valuations of the goods given an attention parameter of η are

$$u(h) = \frac{1}{1+2\eta}\left(v-\eta p_h(2)\right), \qquad u(\ell) = \frac{1}{1+2\eta}\left(\eta v-\eta p_\ell(2)\right) \qquad \text{(B.5)}$$

Comparing these two expressions, it is can be seen that an individual with attention parameter $\eta' = \frac{v}{v+p_h(2)-p_\ell(2)}$ is indifferent between purchasing from h and ℓ. Those with $\eta < \eta'$ purchase from h and those with $\eta > \eta'$ purchase from ℓ. Profits in period 2 for firms h and ℓ are then

$$\pi_h(2) = p_h(2)\int_{\gamma}^{\eta'} \frac{d\eta}{1-\gamma} = \frac{p_h(2)}{1-\gamma}\left(\frac{v}{v+p_h(2)-p_\ell(2)} - \gamma\right) \tag{B.6a}$$

$$\pi_\ell(2) = p_\ell(2)\int_{\eta'}^{1} \frac{d\eta}{1-\gamma} = \frac{p_\ell(2)\left(p_h(2)-p_\ell(2)\right)}{(1-\gamma)\left(v+p_h(2)-p_\ell(2)\right)}. \tag{B.6b}$$

The first order conditions are

$$\frac{\partial \pi_h(2)}{\partial p_h(2)} = \frac{v\left(v-p_\ell(2)\right) - \gamma\left(v+p_h(2)-p_\ell(2)\right)^2}{(1-\gamma)\left(v+p_h(2)-p_\ell(2)\right)^2} \tag{B.7a}$$

$$\frac{\partial \pi_\ell(2)}{\partial p_\ell(2)} = \frac{p_h^2(2) + p_h(2)\left(v-2p_\ell(2)\right) - p_\ell(2)\left(2v-p_\ell(2)\right)}{(1-\gamma)\left(v+p_h(2)-p_\ell(2)\right)^2} \tag{B.7b}$$

Setting these equal to 0 and solving for prices gives equation (3.21) and substitution into equation (B.6) then gives profits.

(ii) Let each firm capture half the market in period 1 and let $p_i(2) \leq p_j(2)$ in period 2. Since i has the lowest price it captures all consumers who purchased from it in period 1, plus some who purchased from j. The attention parameter of the individual indifferent between j and i is $\eta' = \frac{v}{v+p_j(2)-p_i(2)}$, from which it follows that profits for each firm are

$$\pi_i(2) = \frac{p_i(2)}{2}\left(1+\int_{\eta'}^{1}\frac{d\eta}{1-\gamma}\right) = \frac{p_i(2)}{2}\left(1+\frac{p_j(2)-p_i(2)}{(1-\gamma)\left(v+p_j(2)-p_i(2)\right)}\right) \tag{B.8a}$$

$$\pi_j(2) = \frac{p_j(2)}{2}\int_{\gamma}^{\eta'}\frac{d\eta}{1-\gamma} = \frac{p_j(2)}{2(1-\gamma)}\left(\frac{v}{v+p_j(2)-p_i(2)} - \gamma\right) \tag{B.8b}$$

with first order conditions

$$\frac{\partial \pi_i(2)}{\partial p_i(2)} = \frac{1}{2}\left(1 + \frac{p_i^2(2) + (v + p_j(2))(p_j(2) - 2p_i(2))}{(1-\gamma)(v + p_j(2) - p_i(2))^2}\right) \tag{B.9a}$$

$$\frac{\partial \pi_j(2)}{\partial p_j(2)} = \frac{v(v - p_i(2)) - \gamma(v + p_j(2) - p_i(2))^2}{2(1-\gamma)(v + p_j(2) - p_i(2))^2}. \tag{B.9b}$$

Setting these equal to 0 and solving for prices gives equation (3.23) and then substitution into equation (B.8) gives profits. □

B.6 Proof of proposition 3.4

The probability density of σ is $f_\sigma = \int_{\max\{0,\sigma-\varepsilon\}}^{\min\{1,\sigma+\varepsilon\}} f_a(a)\, f_\sigma(\sigma|\, a)\, da$. Calculating this expression, there are two cases.

Case (i) $\varepsilon \leq \frac{1}{2}$

$$f_\sigma(\sigma) = \begin{cases} \dfrac{\sigma+\varepsilon}{2\varepsilon} & \text{if } -\varepsilon \leq \sigma < \varepsilon \\ 1 & \text{if } \varepsilon \leq \sigma < 1-\varepsilon \\ \dfrac{1-\sigma+\varepsilon}{2\varepsilon} & \text{if } 1-\varepsilon \leq \sigma \leq 1+\varepsilon. \end{cases} \tag{B.10}$$

From this the cumulative distribution function of σ is

$$F_\sigma(\sigma) = \begin{cases} \dfrac{(\sigma+\varepsilon)^2}{4\varepsilon} & \text{if } -\varepsilon \leq \sigma < \varepsilon \\ \sigma & \text{if } \varepsilon \leq \sigma < 1-\varepsilon \\ \dfrac{2(\sigma+\varepsilon) - (\sigma-\varepsilon)^2 - 1}{4\varepsilon} & \text{if } 1-\varepsilon \leq \sigma \leq 1+\varepsilon. \end{cases} \tag{B.11}$$

Given $f(\sigma)$, from $f_a(a|\sigma) = \frac{f_\sigma(\sigma|a) f_a(a)}{f_\sigma(\sigma)}$ it is possible to show

$$f_a(a|\sigma) = \begin{cases} \dfrac{1}{\sigma+\varepsilon} & \text{if } -\varepsilon \leq \sigma < \varepsilon \\ \dfrac{1}{2\varepsilon} & \text{if } \varepsilon \leq \sigma < 1-\varepsilon \\ \dfrac{1}{1-\sigma+\varepsilon} & \text{if } 1-\varepsilon \leq \sigma \leq 1+\varepsilon. \end{cases} \tag{B.12}$$

The individual's expectation of her ability given a signal σ is

$$\mathbb{E}[a|\sigma] = \int_{\max\{0,\sigma-\varepsilon\}}^{\min\{1,\sigma+\varepsilon\}} a f_a(a|\sigma)\, da = \begin{cases} \dfrac{1}{2}(\sigma+\varepsilon) & \text{if } -\varepsilon \leq \sigma < \varepsilon \\ \sigma & \text{if } \varepsilon \leq \sigma < 1-\varepsilon \\ \dfrac{1}{2}(1+\sigma-\varepsilon) & \text{if } 1-\varepsilon \leq \sigma \leq 1+\varepsilon. \end{cases} \tag{B.13}$$

The expected payoff from undertaking a task of difficulty t is found to be

$$\mathbb{E}u = t\int_{\max\{0,\sigma-\varepsilon\}}^{\min\{1,\sigma+\varepsilon\}} (a-t) f_a(a|\sigma)\, da = \begin{cases} \dfrac{t}{2}(\sigma+\varepsilon-2t) & \text{if } -\varepsilon \leq \sigma < \varepsilon \\ t(\sigma-t) & \text{if } \varepsilon \leq \sigma \leq 1-\varepsilon \\ \dfrac{t}{2}(1+\sigma-\varepsilon-2t) & \text{if } 1-\varepsilon < \sigma \leq 1+\varepsilon. \end{cases} \tag{B.14}$$

Equating this to 0 and solving for t then shows that the individual takes on tasks satisfying

$$t \leq \begin{cases} \dfrac{1}{2}(\sigma+\varepsilon) & \text{if } -\varepsilon \leq \sigma < \varepsilon \\ \sigma & \text{if } \varepsilon \leq \sigma < 1-\varepsilon \\ \dfrac{1}{2}(1+\sigma-\varepsilon) & \text{if } 1-\varepsilon \leq \sigma \leq 1+\varepsilon. \end{cases} \tag{B.15}$$

Comparison to equation (B.13) shows this reduces to $t \leq \mathbb{E}\left[a \middle| \sigma\right]$.

Equating equation (B.13) to $\frac{1}{2}$ (i.e. the mean of ability) reveals that the individual believes herself to be of higher than average ability if $\sigma > \frac{1}{2}$. The probability she believes herself to be of lower than average ability is thus $F_\sigma\left(\frac{1}{2}\right)$, and from equation (B.11), $\mathbb{E}\left[\ell\right] = \frac{1}{2}$. Similarly, the probability she believes herself to be of high ability is $1 - F_\sigma\left(\frac{1}{2}\right)$, so that $\mathbb{E}\left[h\right] = \frac{1}{2}$ and $\frac{\mathbb{E}[h]}{\mathbb{E}[\ell]} = 1$.

Case (ii) $\varepsilon > \frac{1}{2}$

The probability density and cumulative density functions of σ are

$$f_\sigma\left(\sigma\right) = \begin{cases} \dfrac{\sigma + \varepsilon}{2\varepsilon} & \text{if } -\varepsilon \leq \sigma < 1 - \varepsilon \\ \dfrac{1}{2\varepsilon} & \text{if } 1 - \varepsilon \leq \sigma < \varepsilon \\ \dfrac{1 - \sigma + \varepsilon}{2\varepsilon} & \text{if } \varepsilon \leq \sigma \leq 1 + \varepsilon \end{cases} \tag{B.16a}$$

$$F_\sigma\left(\sigma\right) = \begin{cases} \dfrac{\left(\sigma + \varepsilon\right)^2}{4\varepsilon} & \text{if } -\varepsilon \leq \sigma < 1 - \varepsilon \\ \dfrac{2\left(\sigma + \varepsilon\right) - 1}{4\varepsilon} & \text{if } 1 - \varepsilon \leq \sigma < \varepsilon \\ \dfrac{2\varepsilon\left(1 + \sigma\right) + \left(1 - \sigma\right)^2 - \varepsilon^2}{4\varepsilon} & \text{if } \varepsilon \leq \sigma \leq 1 + \varepsilon \end{cases} \tag{B.16b}$$

Using $f_a\left(a \middle| \sigma\right) = \frac{f_\sigma(\sigma|a) f_a(a)}{f_\sigma(\sigma)}$ it is possible to show

$$f_a\left(a \middle| \sigma\right) = \begin{cases} \dfrac{1}{\sigma + \varepsilon} & \text{if } -\varepsilon \leq \sigma < 1 - \varepsilon \\ 1 & \text{if } 1 - \varepsilon \leq \sigma < \varepsilon \\ \dfrac{1}{1 - \sigma + \varepsilon} & \text{if } \varepsilon \leq \sigma \leq 1 + \varepsilon \end{cases} \tag{B.17}$$

The individual's expectation of her ability given a signal σ is

$$\mathbb{E}\left[a|\,\sigma\right] = \int_{\max\{0,\sigma-\varepsilon\}}^{\min\{1,\sigma+\varepsilon\}} f_a\left(a|\,\sigma\right) da = \begin{cases} \frac{1}{2}\left(\sigma+\varepsilon\right) & \text{if } -\varepsilon \leq \sigma < 1-\varepsilon \\ \frac{1}{2} & \text{if } 1-\varepsilon \leq \sigma < \varepsilon \\ \frac{1}{2}\left(1+\sigma-\varepsilon\right) & \text{if } \varepsilon \leq \sigma \leq 1+\varepsilon. \end{cases} \tag{B.18}$$

The expected payoff from undertaking a task of difficulty t is

$$\begin{aligned} \mathbb{E}u\left(t\right) &= t\int_{\max\{0,\sigma-\varepsilon\}}^{\min\{1,\sigma+\varepsilon\}} \left(a-t\right) f_a\left(a|\,\sigma\right) da \\ &= \begin{cases} \frac{t}{2}\left(\sigma+\varepsilon-2t\right) & \text{if } -\varepsilon \leq \sigma < 1-\varepsilon \\ t\left(\frac{1}{2}-t\right) & \text{if } 1-\varepsilon \leq \sigma < \varepsilon \\ \frac{t}{2}\left(1+\sigma-\varepsilon-2t\right) & \text{if } \varepsilon \leq \sigma \leq 1+\varepsilon. \end{cases} \end{aligned} \tag{B.19}$$

Equating this to 0 and solving for t shows that the individual takes on tasks satisfying

$$t \leq \begin{cases} \frac{1}{2}\left(\sigma+\varepsilon\right) & \text{if } -\varepsilon \leq \sigma < 1-\varepsilon \\ \frac{1}{2} & \text{if } 1-\varepsilon \leq \sigma < \varepsilon \\ \frac{1}{2}\left(1+\sigma-\varepsilon\right) & \text{if } \varepsilon \leq \sigma \leq 1+\varepsilon. \end{cases} \tag{B.20}$$

Comparison to equation (B.18) shows this reduces to $t \leq \mathbb{E}\left[a|\,\sigma\right]$.

Equating equation (B.18) to $\frac{1}{2}$ shows the individual believes she is of lower than average ability if $\sigma < 1-\varepsilon$ so that $F_\sigma\left(1-\varepsilon\right)$ is the probability of this occurring. Thus from equation (B.16b), $\mathbb{E}\left[\ell\right] = \frac{1}{4\varepsilon}$. She believes she is of high ability with probability $1-F_\sigma\left(\varepsilon\right)$ and so $\mathbb{E}\left[h\right] = \frac{1}{4\varepsilon}$ and $\frac{\mathbb{E}[h]}{\mathbb{E}[\ell]} = 1$. □

B.7 Proof of lemma 3.4

Case (i) $\varepsilon \leq \frac{1}{2}$

An individual's subjective expectation of utility from taking on a task of difficulty t given a signal σ is

$$\tilde{\mathbb{E}}u(t) = t\int_{\max\{0,\sigma-\varepsilon\}}^{\min\{1,\sigma+\varepsilon\}} (a-t)\, f_a(a|\,\sigma)\, f_\eta(a)\, da$$

$$= \begin{cases} \dfrac{t}{6}\left((4-\eta)(\sigma+\varepsilon)-6t\right) & \text{if } -\varepsilon \leq \sigma < \varepsilon \\ \dfrac{t}{3}\left(\varepsilon(1-\eta)+3(\sigma-t)\right) & \text{if } \varepsilon \leq \sigma < 1-\varepsilon \\ \dfrac{t}{6}\left(4-\eta-\varepsilon(2+\eta)+\sigma(2+\varepsilon)-6t\right) & \text{if } 1-\varepsilon \leq \sigma \leq 1+\varepsilon. \end{cases} \tag{B.21}$$

Case (ii) $\varepsilon > \frac{1}{2}$

$$\tilde{\mathbb{E}}u(t) = t\int_{\max\{0,\sigma-\varepsilon\}}^{\max\{1,\sigma+\varepsilon\}} (a-t)\, f_a(a|\,\sigma)\, f_\eta(a)\, da$$

$$= \begin{cases} \dfrac{t}{6}\left((4-\eta)(\sigma+\varepsilon)-6t\right) & \text{if } -\varepsilon \leq \sigma < 1-\varepsilon \\ \dfrac{t}{6}\left(4-\eta-6t\right) & \text{if } 1-\varepsilon \leq \sigma < \varepsilon \\ \dfrac{t}{6}\left(4-\eta-\varepsilon(2+\eta)+\sigma(2+\varepsilon)-6t\right) & \text{if } \varepsilon \leq \sigma \leq 1+\varepsilon. \end{cases} \tag{B.22}$$

From equations (B.21) and (B.22) it is apparent that $\frac{\partial \tilde{\mathbb{E}}u(t)}{\partial \eta} < 0$ and so $\tilde{\mathbb{E}}u(t)$ is maximized at $\min_\eta \{\eta : f_\eta(a,\eta) \in H\} = \gamma$. □

B.8 Proof of proposition 3.5

Case (i) $\varepsilon \leq \frac{1}{2}$

An individual's subjective expectation of utility from taking on a task of difficulty t given a signal σ is found by substituting $\eta = \gamma$ into equation (B.21). Equating this to 0 shows that inequality (3.35) gives the condition for undertaking the task to be a solution to the individual's EUMP. As not undertaking a task gives 0 payoff, this is also the condition for a preferred solution.

The individual's subjective expectation of her ability given a signal σ is

$$\tilde{\mathbb{E}}\left[a|\,\sigma\right] = \int_{\max\{0,\sigma-\varepsilon\}}^{\min\{1,\sigma+\varepsilon\}} a f_a\left(a|\,\sigma\right) f_\gamma\left(a\right) da$$

$$= \begin{cases} \frac{1}{6}\left(4-\gamma\right)\left(\sigma+\varepsilon\right) & \text{if } -\varepsilon \leq \sigma < \varepsilon \\ \sigma + \frac{1}{3}\varepsilon\left(1-\gamma\right) & \text{if } \varepsilon \leq \sigma < 1-\varepsilon \\ \frac{1}{6}\left(4-\gamma+\left(2+\eta\right)\left(\sigma-\varepsilon\right)\right) & \text{if } 1-\varepsilon \leq \sigma \leq 1+\varepsilon. \end{cases} \tag{B.23}$$

Assume $\varepsilon \leq \sigma < 1-\varepsilon$. The individual believes herself to be of below average ability if $\sigma + \frac{1}{3}\varepsilon\left(1-\gamma\right) \leq \frac{1}{2}$ or $\sigma < \frac{1}{2} - \frac{1}{3}\varepsilon\left(1-\gamma\right)$. Then $\mathbb{E}\left[\ell\right]$ is given by $F_\sigma\left(\frac{1}{2} - \frac{1}{3}\varepsilon\left(1-\gamma\right)\right) = \frac{1}{2} - \frac{1}{3}\varepsilon\left(1-\gamma\right)$. $\mathbb{E}\left[h\right]$ is given by $1 - F_\sigma\left(\frac{1}{2} - \frac{1}{3}\varepsilon\left(1-\gamma\right)\right) = \frac{1}{2} + \frac{1}{3}\varepsilon\left(1-\gamma\right)$ and $\frac{\mathbb{E}[h]}{\mathbb{E}[\ell]} = \frac{3+2\varepsilon(1-\gamma)}{3-2\varepsilon(1-\gamma)}$. The assumption holds if $\frac{1}{2} - \frac{1}{3}\varepsilon\left(1-\gamma\right) \geq \varepsilon$, or $\gamma \geq 4 - \frac{3}{2\varepsilon}$.

Let $\gamma < 4 - \frac{3}{2\varepsilon}$ so that all those receiving signals with $\sigma \geq \varepsilon$ believe themselves to be of higher than average ability. The individual believes herself to be of lower than average ability if $\frac{1}{6}\left(4-\gamma\right)\left(\sigma+\varepsilon\right) < \frac{1}{2}$ or $\sigma < \frac{3}{4-\gamma} - \varepsilon$. Then $\mathbb{E}\left[\ell\right]$ is given by $F_\sigma\left(\frac{3}{4-\gamma} - \varepsilon\right) = \frac{9}{4\varepsilon(4-\gamma)^2}$. $\mathbb{E}\left[h\right]$ is given by $1 - F_\sigma\left(\frac{3}{4-\gamma} - \varepsilon\right) = 1 - \frac{9}{4\varepsilon(4-\gamma)^2}$ so that $\frac{\mathbb{E}[h]}{\mathbb{E}[\ell]} = \frac{4}{9}\varepsilon\left(4-\gamma\right)^2 - 1$.

Case (ii) $\varepsilon > \frac{1}{2}$

The individual's subjective expectation of utility when performing a task of difficulty t given a signal σ is found by substituting $\eta = \gamma$ into equation (B.22). Equating this to 0 gives inequality (3.37), which is the condition for undertaking a task to be a solution to the UMP.

An individual's subjective expectation of her ability following a signal σ is

$$\tilde{\mathbb{E}} = \int_{\max\{0,\sigma-\varepsilon\}}^{\min\{1,\sigma+\varepsilon\}} a f_a\left(a|\,\sigma\right) f_\gamma\left(a\right) da$$

$$= \begin{cases} \frac{1}{6}\left(4-\gamma\right)\left(\sigma+\varepsilon\right) & \text{if } -\varepsilon \le \sigma < 1-\varepsilon \\ \frac{1}{6}\left(4-\gamma\right) & \text{if } 1-\varepsilon \le \sigma < \varepsilon \\ \frac{1}{6}\left(\left(2+\gamma\right)\left(\sigma-\varepsilon\right)+\left(4-\gamma\right)\right) & \text{if } \varepsilon \le \sigma \le 1+\varepsilon. \end{cases} \tag{B.24}$$

Note that if $\sigma \geq 1-\varepsilon$, the individual estimates her ability as $\frac{1}{6}\left(4-\gamma\right) \geq \frac{1}{2}$, i.e. she believes she is above average for $\gamma < 1$. She believes she is of lower than average ability if $\frac{1}{6}\left(4-\gamma\right)\left(\sigma+\varepsilon\right) < \frac{1}{2}$, or $\sigma < \frac{3}{4-\gamma} - \varepsilon$, so $\mathbb{E}\left[\ell\right] = F_\sigma\left(\frac{3}{4-\gamma} - \varepsilon\right) = \frac{9}{4\varepsilon(4-\gamma)^2}$. On the other hand, $\mathbb{E}\left[h\right] = 1 - F_\sigma\left(\frac{3}{4-\gamma} - \varepsilon\right) = 1 - \frac{9}{4\varepsilon(4-\gamma)^2}$ and so $\frac{\mathbb{E}[h]}{\mathbb{E}[\ell]} = \frac{4}{9}\varepsilon\left(4-\gamma\right)^2 - 1$.

□

B.9 Proof of proposition 3.6

Case (i) $\varepsilon \leq \frac{1}{2}$

The expected loss of an individual given a task t and a signal σ is

$$\mathbb{E}L(t,\sigma,\delta) = \int_{\max\{0,\sigma-\varepsilon\}}^{\min\{1,\sigma+\varepsilon\}} \left((1-\delta)\,at+\delta^2\right) f_a(a|\,\sigma)\,da \tag{B.25}$$

$$= \begin{cases} \frac{1}{2}(1-\delta)(\sigma+\varepsilon)\,t+\delta^2 & \text{if } -\varepsilon \leq \sigma < \varepsilon \\ (1-\delta)\,\sigma t+\delta^2 & \text{if } \varepsilon \leq \sigma < 1-\varepsilon \\ \frac{1}{2}(1-\delta)(1+\sigma-\varepsilon)\,t+\delta^2 & \text{if } 1-\varepsilon \leq \sigma \leq 1+\varepsilon \end{cases} \tag{B.26}$$

with first order condition

$$\frac{\partial \mathbb{E}L(t,\sigma,\delta)}{\partial\delta} = \begin{cases} -\frac{1}{2}(\sigma+\varepsilon)\,t+2\delta & \text{if } -\varepsilon \leq \sigma < \varepsilon \\ -\sigma t+2\delta & \text{if } \varepsilon \leq \sigma 1-\varepsilon \\ -(1+\sigma-\varepsilon)\,t+2\delta & \text{if } 1-\varepsilon \leq \sigma \leq 1+\varepsilon. \end{cases} \tag{B.27}$$

From this it is easy to see that $\frac{\partial^2 \mathbb{E}L(t,\sigma,\delta)}{\partial\delta^2} > 0$. Equating this expression to 0 and solving for δ then gives equation (3.39).

The state space of ability and signalling mechanism are identical to section 3.3.3.1, so $\frac{\mathbb{E}[h]}{\mathbb{E}[\ell]} = 1$ by the same derivation as in proposition 3.4.

Case (ii) $\varepsilon > \frac{1}{2}$

$$\mathbb{E}L\left(t,\sigma,\delta\right) = \begin{cases} \dfrac{1}{2}\left(1-\delta\right)\left(\sigma+\varepsilon\right)t+\delta^2 & \text{if } -\varepsilon \leq \sigma < 1-\varepsilon \\ \dfrac{1}{2}\left(1-\delta\right)t+\delta^2 & \text{if } 1-\varepsilon \leq \sigma < \varepsilon \\ \dfrac{1}{2}\left(1-\delta\right)\left(1+\sigma-\varepsilon\right)t+\delta^2 & \text{if } \varepsilon \leq \sigma \leq 1+\varepsilon. \end{cases} \tag{B.28}$$

The first order condition is

$$\frac{\partial \mathbb{E}L\left(t,\sigma,\delta\right)}{\partial \delta} = \begin{cases} -\dfrac{1}{2}\left(\sigma+\varepsilon\right)t+2\delta & \text{if } -\varepsilon \leq \sigma < 1-\varepsilon \\ -\dfrac{1}{2}t+2\delta & \text{if } 1-\varepsilon \leq \sigma\varepsilon \\ -\left(1+\sigma-\varepsilon\right)t+2\delta & \text{if } \varepsilon \leq \sigma \leq 1+\varepsilon. \end{cases} \tag{B.29}$$

from which it is clear that $\frac{\partial^2 \mathbb{E}L(t,\sigma,\delta)}{\partial \delta^2} > 0$. Equating this expression to 0 and solving for δ gives equation (3.40).

As both the state space of ability and signaling mechanism are unchanged from section 3.3.3.1, $\frac{\mathbb{E}[h]}{\mathbb{E}[\ell]} = 1$ by the same derivation as in proposition 3.4. □

B.10 Proof of lemma 3.5

Case (i) $\varepsilon \leq \frac{1}{2}$

The subjective expected loss of an individual given task t and signal σ is

$$\tilde{\mathbb{E}}L(t,\sigma,\delta) = \int_{\max\{0,\sigma-\varepsilon\}}^{\min\{1,\sigma+\varepsilon\}} \left((1-\delta)\,at+\delta^2\right) f_a(a|\,\eta)\, f_\eta(a,\eta)\,da$$

$$= \begin{cases} \frac{1}{6}(1-\delta)(4-\eta)(\sigma+\varepsilon)\,t+\delta^2 & \text{if } -\varepsilon \leq \sigma < \varepsilon \\ \frac{1}{3}(1-\delta)(3\sigma+(1-\eta)\,\varepsilon)\,t+\delta^2 & \text{if } \varepsilon \leq \sigma < 1-\varepsilon \\ \frac{1}{6}(1-\delta)(4-\eta+(2-\eta)(\sigma-\varepsilon))\,t+\delta^2 & \text{if } 1-\varepsilon \leq \sigma \leq 1+\varepsilon. \end{cases} \tag{B.30}$$

Case (ii) $\varepsilon > \frac{1}{2}$

$$\tilde{\mathbb{E}}L(t,\sigma,\delta) = \begin{cases} \frac{1}{6}(1-\delta)(4-\eta)(\sigma+\varepsilon)\,t+\delta^2 & \text{if } -\varepsilon \leq \sigma < 1-\varepsilon \\ \frac{1}{6}(1-\delta)(4-\eta)\,t+\delta^2 & \text{if } 1-\varepsilon \leq \sigma < \varepsilon \\ \frac{1}{6}(1-\delta)(4-\eta+(2+\eta)(\sigma-\varepsilon))\,t+\delta^2 & \text{if } \varepsilon \leq \sigma \leq 1+\varepsilon. \end{cases} \tag{B.31}$$

From equations (B.30) and (B.31) it is clear to see that $\frac{\partial \tilde{\mathbb{E}}L(t,\sigma,\delta)}{\partial \eta} < 0$ and so $\tilde{\mathbb{E}}L(t,\sigma,\delta)$ is minimized at $\max_\eta \{\eta : f_\eta(a,\eta) \in H\}$. □

B.11 Proof of proposition 3.7

Case (i) $\varepsilon \leq \frac{1}{2}$

An individual's subjective expectation of her loss given a task t and signal σ is found by substituting $\eta = 2 - \gamma$ into equation (B.30). The first order condition is

$$\frac{\partial \tilde{\mathbb{E}} L(t,\sigma,\delta)}{\partial \delta} = \begin{cases} -\frac{1}{6}(2+\gamma)(\sigma+\varepsilon)t + 2\delta & \text{if } -\varepsilon \leq \sigma < \varepsilon \\ -\frac{1}{3}(3\sigma - (1-\gamma)\varepsilon)t + 2\delta & \text{if } \varepsilon \leq \sigma < 1-\varepsilon \\ -\frac{1}{6}(2+\gamma+(4-\gamma)(\sigma-\varepsilon))t + 2\delta & \text{if } 1-\varepsilon \leq \sigma < 1+\varepsilon. \end{cases} \tag{B.32}$$

From this it is clear that $\frac{\partial \tilde{\mathbb{E}} L(t,\sigma,\delta)}{\partial \delta^2} > 0$. Equating this expression to 0 and solving for δ gives equation (3.42).

An individual's subjective expectation of her opponent's ability given a signal σ is

$$\tilde{\mathbb{E}}[a|\sigma] = \int_{\max\{0,\sigma-\varepsilon\}}^{\min\{1,\sigma+\varepsilon\}} a f_a(a|\sigma) f_{2-\gamma}(a)\, da \tag{B.33}$$

$$= \begin{cases} \frac{1}{6}(2+\gamma)(\sigma+\varepsilon) & \text{if } -\varepsilon \leq \sigma < \varepsilon \\ \frac{1}{3}(3\sigma - \varepsilon(1-\gamma)) & \text{if } \varepsilon \leq \sigma < 1-\varepsilon \\ \frac{1}{6}(2+\gamma+(4-\gamma)(\sigma-\varepsilon)) & \text{if } 1-\varepsilon \leq \sigma \leq 1+\varepsilon \end{cases} \tag{B.34}$$

Assume $\varepsilon \leq \sigma < 1-\varepsilon$. An individual estimates her opponent to be of lower than average ability if $\frac{1}{3}(3\sigma - \varepsilon(1-\gamma)) < \frac{1}{2}$ or $\sigma < \frac{1}{6}(3 + 2\varepsilon(1-\gamma))$. Then $\mathbb{E}[\ell] = F_\sigma\left(\frac{1}{6}(3+2\varepsilon(1-\gamma))\right) = \frac{1}{6}(3+2\varepsilon(1-\gamma))$. Similarly $\mathbb{E}[h] = 1 - F_\sigma\left(\frac{1}{6}(3+2\varepsilon(1-\gamma))\right) = \frac{1}{6}(3-2\varepsilon(1-\gamma))$, so that $\frac{\mathbb{E}[h]}{\mathbb{E}[\ell]} = \frac{3-2\varepsilon(1-\gamma)}{3+2\varepsilon(1-\gamma)}$.

The assumption holds if $\frac{1}{6}(3+2\varepsilon(1-\gamma)) < 1-\varepsilon$ or $\gamma > 4-\frac{3}{2\varepsilon}$. Let $\gamma < 4-\frac{3}{2\varepsilon}$. The individual expects her opponent to be of lower than average ability if $\frac{1}{6}(2+\gamma+(4-\gamma)(\sigma-\varepsilon)) < \frac{1}{2}$ or $\sigma < \frac{1-\gamma}{4-\gamma}+\varepsilon$. Then $\mathbb{E}[\ell] = F_\sigma\left(\frac{1-\gamma}{4-\gamma}+\varepsilon\right) = 1-\frac{9}{4\varepsilon(4-\gamma)^2}$. Similarly, $\mathbb{E}[h] = 1-F_\sigma\left(\frac{1-\gamma}{4-\gamma}+\varepsilon\right) = \frac{9}{4\varepsilon(4-\gamma)^2}$ and $\frac{\mathbb{E}[h]}{\mathbb{E}[\ell]} = \frac{9}{4\varepsilon(4-\gamma)^2-9}$.

Case (ii) $\varepsilon > \frac{1}{2}$

$$\frac{\partial\tilde{\mathbb{E}}L(t,\sigma,\delta)}{\partial\delta} = \begin{cases} -\frac{1}{6}(2+\gamma)(\sigma+\varepsilon)t+2\delta & \text{if } -\varepsilon\leq\sigma<1-\varepsilon \\ -\frac{1}{6}(2+\gamma)t+2\delta & \text{if } 1-\varepsilon\leq\sigma<\varepsilon \\ -\frac{1}{6}(2+\gamma+(4-\gamma)(\sigma-\varepsilon))t+2\delta & \text{if } \varepsilon\leq\sigma<1+\varepsilon. \end{cases} \tag{B.35}$$

From this it can be seen that $\frac{\partial\tilde{\mathbb{E}}L(t,\sigma,\delta)}{\partial\delta^2} > 0$. Equating it to 0 and rearranging gives equation (3.44).

An individual's subjective expectation of her opponent's ability given a task t and signal σ is

$$\tilde{\mathbb{E}}[a|\sigma] = \int_{\max\{0,\sigma-\varepsilon\}}^{\min\{1,\sigma+\varepsilon\}} af_a(a|\sigma)f_{2-\gamma}(a)\,da \tag{B.36}$$

$$= \begin{cases} \frac{1}{6}(2+\gamma)(\sigma+\varepsilon) & \text{if } -\varepsilon\leq\sigma<1-\varepsilon \\ \frac{2+\gamma}{6} & \text{if } 1-\varepsilon\leq\sigma<\varepsilon \\ \frac{1}{6}(2+\gamma+(4-\gamma)(\sigma-\varepsilon)) & \text{if } \varepsilon\leq\sigma\leq1+\varepsilon \end{cases} \tag{B.37}$$

Note that $\frac{2+\gamma}{6} < \frac{1}{2}$ for any $\gamma < 1$, so the individual believes her opponent is of lower than average ability if $\sigma < \varepsilon$.

Assume $\sigma > \varepsilon$. The individual believes her opponent to be of lower than average ability if $\frac{1}{6}(2+\gamma+(4-\gamma)(\sigma-\varepsilon)) < \frac{1}{2}$ or $\sigma < \frac{1-\gamma}{4-\gamma}+\varepsilon$. $\mathbb{E}[\ell]$ is then found to be $F_\sigma\left(\frac{1-\gamma}{4-\gamma}+\varepsilon\right)=1+\frac{1}{2}\varepsilon-\frac{3}{4-\gamma}+\frac{9}{4\varepsilon(4-\gamma)^2}$. Similarly, $\mathbb{E}\left[h\right]$ is found to be $F_\sigma\left(\frac{1-\gamma}{4-\gamma}+\varepsilon\right)=-\frac{3}{4-\gamma}-\frac{1}{2}\varepsilon-\frac{9}{4\varepsilon(4-\gamma)^2}$. Rearrangement of $\frac{\mathbb{E}[h]}{\mathbb{E}[\ell]}$ then gives equation (3.45).

Proofs for chapter 4 C

C.1 Fixed costs of quality

The first order conditions of equation (4.1) are

$$\frac{\partial \pi_h (q_h, q_\ell)}{\partial p_h} = 1 - \frac{2p_h - p_\ell}{q_h - q_\ell} \qquad \frac{\partial \pi_\ell (q_h, q_\ell)}{\partial p_\ell} = \frac{p_h - 2p_\ell}{q_h - q_\ell} - \frac{2p_\ell}{q_\ell} \tag{C.1}$$

from which it is easy to see that the second order conditions are negative.

The first and second order conditions of equation (4.3) are

$$\frac{\partial \pi_h (q_h, q_\ell)}{\partial q_h} = \frac{4q_h (4q_h^2 - 3q_h q_\ell + 2q_\ell^2)}{(4q_h - q_\ell)^3} - q_h \qquad \frac{\partial \pi_\ell (q_h, q_\ell)}{\partial q_\ell} = \frac{(4q_h - 7q_\ell)}{(4q_h - q_\ell)^3} - q_\ell \tag{C.2a}$$

$$\frac{\partial^2 \pi_h (q_h, q_\ell)}{\partial q_h^2} = -\frac{8q_\ell^2 (5q_h - q_\ell)}{(4q_h - q_\ell)^4} - 1 \qquad \frac{\partial^2 \pi_\ell (q_h, q_\ell)}{\partial q_\ell^2} = -\frac{2q_h^2 (q_h + 7q_\ell)}{(4q_h - q_\ell)^4} - 1. \tag{C.2b}$$

C.1.1 Proof of lemma 4.1

If $\frac{q_h}{q_\ell} < \delta$, Bertrand competition with effectively homogeneous goods and identical marginal costs of 0 for each firm occurs. Firms earn no revenue and make a loss for any $q_h, q_\ell > 0$. Then as each firm can make 0 profit from selecting 0 quality, $\frac{q_h}{q_\ell} < \delta$ cannot be an equilibrium. Any $q_h > 0$ is perceivably different to $q_\ell = 0$, so that $q_h = 0$ is not a best response to $q_\ell = 0$. For any $q_h > 0$ it is possible to find some $0 < q_\ell \leq \frac{q_h}{\delta}$ which is strictly positive and perceivably different to q_h. From the first order condition of $\pi_\ell(q_h, q_\ell)$, $\left.\frac{\partial \pi_\ell(q_h,q_\ell)}{\partial q_\ell}\right|_{q_\ell=0} > 0$, so $q_\ell = 0$ is not a best response to any $q_h = 0$. □

C.1.2 Proof of proposition 4.1

(i) Substituting, for example, $\delta = 5$ into equation (4.10a), $\left.\frac{\partial \pi_1^*(q_1,q_2)}{\partial \delta}\right|_{\delta=5} \approx 1.42 \times 10^{-3}$, and
$\left.\frac{\partial \pi_1^*(q_1,q_2)}{\partial \delta}\right|_{\delta>\delta'_f} = 0$ has no solutions for $\delta > 1$.

(ii) Substituting, for example, $\delta = 5$ into equation (4.10b), $\left.\frac{\partial \pi_2^*(q_1,q_2)}{\partial \delta}\right|_{\delta=5} \approx -1.29 \times 10^{-5}$ and $\left.\frac{\partial \pi_2^*(q_1,q_2)}{\partial \delta}\right|_{\delta>\delta'_f} = 0$ has a single solution satisfying $\delta > 1$,
$\delta = \frac{1}{8}\left(25 + \sqrt{193}\right) \approx 4.86 < \delta'_f$.

(iii) Substituting, for example, $\delta = 5$ into equation (4.10c), $\left.\frac{\partial \sigma_1}{\partial \delta}\right|_{\delta=5} \approx 0.103$ and $\left.\frac{\partial \sigma_1}{\partial \delta}\right|_{\delta>\delta'_f} = 0$ has a single solution for $\delta > 1$, $\delta = 2 < \delta'_f$. □

(iv) Substituting, for example, $\delta = 5$ into equation (4.10d), $\left.\frac{\partial CS^*(q_1,q_2)}{\partial \delta}\right|_{\delta=5} \approx -1.23 \times 10^{-3}$ and $\left.\frac{\partial CS^*(q_1,q_2)}{\partial \delta}\right|_{\delta>\delta'_f} = 0$ has a single solution satisfying $\delta > 1$, at $\delta = \frac{1}{40}\left(37 + 3\sqrt{241}\right) \approx 2.089 < \delta'_f$.

C.2 Marginal costs of quality

The first order conditions of equation (4.12) are

$$\frac{\partial \pi_h(q_h, q_\ell)}{\partial q_h} = 1 - \frac{4p_h - 2p_\ell - q_h^2}{2(q_h - q_\ell)} \qquad \frac{\partial \pi_\ell(q_h, q_\ell)}{\partial q_\ell} = \frac{q_\ell(2p_h + q_h q_\ell) - 2p_\ell q_h}{q_\ell(q_h - q_\ell)} \tag{C.3}$$

from which it is easy to see that the second order conditions are negative.

C.2.1 Proof of lemma 4.2

The first order condition of $\pi_h(q_h, q_\ell)$ is

$$\frac{\partial \pi_h(q_h, q_\ell)}{\partial q_h} = \frac{q_h(2q_h + q_\ell - 4)\left(24q_h^3 + 2q_\ell^2(q_\ell - 4) + q_h q_\ell(5q_\ell + 12) - 2q_h^2(11q_\ell + 8)\right)}{4(4q_h - q_\ell)^3} \tag{C.4}$$

so there are at most five stationary points, two of which are $q_h = 0$ and $q_h = 2 - \frac{1}{2}q_\ell$. The polynomial in the rightmost bracket of the numerator has discriminant $\Delta = -71964q_\ell^6 + 415008q_\ell^5 - 627392q_\ell^4 + 121856q_\ell^3 - 94208q_\ell^2$. $\Delta \geq 0$ if $q_\ell = \frac{8}{3}$, $q_\ell \approx 3.005$, both of which lie outside the range, or $q_\ell = 0$, in which case the roots of the polynomial are $q_h = \frac{2}{3}$ and $q_h = 2$. Thus $\Delta < 0$ for $q_\ell < q_h < 2 - \frac{1}{2}q_\ell$ and there is a unique maximum of $\pi_h(q_h, q_\ell)$ in this range.

$\pi_\ell(q_h, q_\ell)$ is continuous in the range $0 \leq q_\ell \leq q_h$, is 0 at $q_\ell = 0$ and $q_\ell = q_h$ and is positive for some q_ℓ within the range. Thus if there is a single stationary point in the interior of the range, it is a maximum. The first order condition of $\pi_\ell(q_h, q_\ell)$ is

$$\frac{\partial \pi_\ell(q_h, q_\ell)}{\partial q_\ell} = \frac{q_h(2 + q_h - q_\ell)\left(4q_h^3 + q_h^2(8 - 19q_\ell) - 2q_\ell^3 + q_h q_\ell(17q_\ell - 14)\right)}{4(4q_h - q_\ell)^3} \tag{C.5}$$

which shows there are at most four stationary points, one of which is $q_\ell = 2 + q_h$. The polynomial in the rightmost bracket of the numerator has discriminant $\Delta = 15633q_h^6 - 4380q_h^5 + 28900q_h^4 - 21952q_h^3$. $\Delta = 0$ if either $q_h = 0$, which implies no q_ℓ such that $0 < q_\ell < q_h$, or if $q_h = \frac{2}{3}$, in which case the sole root of the first order condition is $q_\ell = \frac{1}{3}$. Otherwise $\Delta < 0$ and there is a single root of the polynomial and there is a unique maximum of $\pi_\ell(q_h, q_\ell)$ for $0 < q_\ell < q_h$. □

C.2.2 Derivation of q_h^*, q_ℓ^*

Denote the constants solving $\frac{\partial \pi_\ell(q_h,q_\ell)}{\partial q_\ell} = 0$ simultaneously with $\frac{\partial \pi_h\left(q_h,q_\ell^{BR}(q_h)\right)}{\partial q_h} = 0$ as $k_h \approx 0.567$ and $k_\ell \approx 0.289$. k_ℓ is a best response by firm 2 to $q_1 = k_h$ given that it is constrained to produce $q_2 \leq q_1$, i.e. $q_1 = k_h$, $q_2 = k_\ell$ is an equilibrium only if firm 2 does not wish to deviate to produce some $q_2 \geq q_1$. $\pi_\ell\left(q_h, q_\ell\right)\big|_{(q_h,q_\ell)=(k_h,k_\ell)} \approx 0.0151$, whereas choosing, for example, $q_2 = 0.9$ gives profit $\pi_h\left(q_h, q_\ell\right)\big|_{(q_h,q_\ell)=(0.9,k_h)} \approx 0.0195$, so this is not an equilibrium.

As by lemma 4.2, $\frac{\partial \pi_h\left(q_h,q_\ell^{BR}(q_h)\right)}{\partial q_h} < 0$ for $q_h > k_h$, firm 1 chooses the lowest q_1 such that firm 2 wishes to produce $q_2 < q_1$ rather than deviating to a high quality. Denote the constants solving $\frac{\partial \pi_h(q_h,q_1)}{\partial q_h} = 0$, $\frac{\partial \pi_\ell(q_1,q_\ell)}{\partial q_\ell} = 0$ and $\pi_h\left(q_h, q_1\right) = \pi_\ell\left(q_1, q_\ell\right)$ simultaneously as $s_{2h} \approx 0.939$, $s_1 \approx 0.612$ and $s_2 \approx 0.309$. Equilibrium qualities are then $q_1^* = s_1 \approx 0.612$, $q_2^* = s_2 \approx 0.309$.

Let $\delta > 1$. Define $\delta_m' \approx 1.071$ as the point at which firm 2's profit from imitating $q_1 = s_1$ is equal to its profit in the baseline case. Let $\delta > \delta_m'$. Firm 1 chooses $q_1 < s_1$ so that firm 2 prefers to enter as the high quality firm rather than imitate. Denote the constants solving $\frac{\pi_h(q_h,q_\ell)}{\partial q_h} = 0$ simultaneously with $\frac{\partial \pi_\ell\left(q_h^{BR}(q_\ell),q_\ell\right)}{\partial q_\ell} = 0$ as $\tilde{s}_2 \approx 0.906$ and $\tilde{s}_1 \approx 0.555$. This represents an equilibrium until δ is sufficiently large that it makes a greater profit from imitating $\tilde{s}_1$ than producing $\tilde{s}_2$. Define $\delta_m'' \approx 1.106$ as the solution to $\pi_h\left(q_h, q_\ell\right)\big|_{(q_h,q_\ell)=(\tilde{s}_h,\tilde{s}_\ell)} = \pi_I\left(q_1\right)\big|_{q_1=\tilde{s}_h}$.

Let $\delta > \delta_m''$. Denote by q_{2h} the quality firm 2 chooses conditional on producing such that $q_2 > q_1$ and by q_{2I} the quality firm 2 chooses conditional on imitating. As by lemma 4.2, $\frac{\partial \pi_\ell\left(q_h^{BR}(q_\ell),q_\ell\right)}{\partial q_\ell} < 0$ for $q_\ell < \tilde{s}_1$, firm 1 chooses the highest q_1 such that firm 2 prefers to produce a high, distinct quality rather than imitating, i.e. such that $\pi_I\left(q_1\right) = \max_{q_{2h}} \pi_h\left(q_{2h}, q_1\right)$.

Suppose δ is sufficiently small that $\operatorname{argmax}_{q_{2h}} \pi_h\left(q_{2h}, q_1\right) \geq \delta q_1$. Let $\mu = \frac{q_{2h}}{q_1}$ and substitute $q_{2h} = \mu q_1$ into the first order condition of $\pi_h\left(q_{2h}, q_1\right)$. After rearrangement this gives $q_1 = \frac{4\left(4\mu^2-3\mu+2\right)}{24\mu^3-22\mu^2+5\mu+2}$. Substituting this expression and $q_{2h} = \mu q_1$ into $\pi_I(q_1) = \pi_h(q_{2h}, q_1)$ results in

$$32\delta^2\left(\mu^7 - 3\mu^6\right) + 96\mu^5 + 32\left(5\delta^2 - 6\right)\mu^4 - \left(\delta^2 - 1\right)\left(182\mu^3 - 101\mu^2 + 28\mu - 4\right) = 0 \tag{C.6}$$

Although no analytical solutions exist, numerical approximations are possible to find for given values of δ. Define $\mu(\delta)$ as the unique root of this equation taking a value greater than 1 from which equation (4.17) in the range $\delta''_m < \delta \leq \delta'''_m$ follows.

Define δ'''_m as the solution to $\mu(\delta) = \delta$, with $\delta'''_m \approx 2.339$. Let $\delta > \delta'''_m$. $\operatorname{argmax}_{q_{2h}} \pi_h(q_{2h}, q_1) < \delta q_1$, so firm 2's best response conditional on choosing $q_2 > q_1$ is δq_1. Substituting $q_{2h} = \delta q_1$ into $\pi_I(q_1) = \pi_h(q_{2h}, q_1)$ and rearranging yields $q_1 = \frac{-B(\delta)-\sqrt{B^2(\delta)-4A(\delta)C(\delta)}}{2A(\delta)}$, with

$$A(\delta) = \left(\delta^2 - 1\right)(4\delta - 1)^2 + \delta^4(\delta - 1)(1 + 2\delta)^2 \tag{C.7a}$$

$$B(\delta) = -2\left(\delta^2 - 1\right)(4\delta - 1)^2 - 8\delta^4(\delta - 1)(1 + 2\delta) \tag{C.7b}$$

$$C(\delta) = 16\delta^4(\delta - 1) \tag{C.7c}$$

which completes the derivation of equation (4.17).

C.2.3 Proof of proposition 4.2

(i) For $\delta'_m < \delta \leq \delta''_m$, $\pi_1^*(q_1, q_2) \approx 0.0259 < \pi_1^*(q_1, q_2)|_{\delta=1} \approx 0.0377$. Let $\delta''_m < \delta \leq \delta'''_m$. As $\frac{\partial \pi_I(q_1)}{\partial \delta} > 0$ and $\frac{\partial \pi_2^*(q_1,q_2)}{\partial \delta} = 0$ it must be that $\frac{\partial q_1^*}{\partial \delta} < 0$, which implies by lemma 4.2 that firm 1's profit is decreasing in δ and as $\lim_{\delta \to \delta''^+_m} \pi_1^*(q_1, q_2) \approx 0.0259$, it is lower than in the baseline. Let $\delta > \delta'''_m$. The only solution to $\frac{\partial \pi_1^*(q_1,q_2)}{\partial \delta} = 0$ in the range $\delta > \delta'''_m$ is $\delta \approx 5.240$, and as $\pi_1^*(q_1, q_2)|_{\delta \approx 5.240} \approx 0.315 < \pi_1^*(q_1, q_2)|_{\delta=1} \approx 0.0377$, firm 1's profit is lower than in the baseline.

(ii) For $\delta'_m < \delta \leq \delta''_m$, $\pi_2^*(q_1, q_2) \approx 0.0204 > \pi_2^*(q_1, q_2)\big|_{\delta=1} \approx 0.0166$. Let $\delta''_m < \delta < \delta'''_m$. It was shown above that $\frac{\partial q_1^*}{\partial \delta} < 0$, and as $\frac{\pi_h(q_h, q_\ell)}{\partial q_\ell} < 0$ for $0 \leq q_\ell \leq q_h \leq 2 - \frac{1}{2} q_\ell$ it follows that firm 2's profit is increasing in δ. As $\lim_{\delta \to \delta''^{+}_m} \pi(q_1, q_2) \approx 0.0204$, its profit is greater than in the baseline. Let $\delta > \delta'''_m$. The equation $\pi_2^*(q_1, q_2)\big|_{\delta > \delta'''_m} = \pi_2^*(q_1, q_2)\big|_{\delta=1}$ has a unique solution at $\delta \approx 7.547$.

(iii) For $\delta'_m < \delta \leq \delta''_m$, $\sigma_1 \approx 0.560 < \sigma_1\big|_{\delta=1} \approx 0.694$. Let $\delta''_m < \delta \leq \delta'''_m$. It is shown above that $\frac{\pi_1^*(q_1, q_2)}{\partial \delta} < 0$ and $\frac{\partial \pi_2^*(q_1, q_2)}{\partial \delta} > 0$, from which it follows that $\frac{\partial \sigma_1}{\partial \delta} < 0$. Let $\delta > \delta'''_m$. It was shown above that for $\delta'''_m < \delta \lesssim 5.250$, $\frac{\partial \pi_1^*(q_1, q_2)}{\partial \delta} > 0$ and $\frac{\partial \pi_2^*(q_1, q_2)}{\partial \delta} < 0$, which together imply $\frac{\partial \sigma_1}{\partial \delta} > 0$. As $\frac{\partial \sigma_1}{\partial \delta}\Big|_{\delta > \delta'''_m} = 0$ has no solutions for $\delta > \delta'''_m$, it must be that $\frac{\partial \sigma_1}{\partial \delta} > 0$ for $\delta \gtrsim 5.240$. The equation $\sigma_1\big|_{\delta > \delta'''} = \sigma_1\big|_{\delta=1}$ has the unique solution $\delta \approx 9.453$.

(iv) $CS^*(q_1, q_2)\big|_{\delta'_m < \delta \leq \delta''_m} \approx 0.106$ which is greater than the baseline surplus of $CS^*(q_1, q_2)\big|_{\delta=1} \approx 0.0908$. Let $\delta'' < \delta \leq \delta'''$. By the chain rule, $\frac{\partial CS^*(\mu(\delta))}{\partial \mu(\delta)} \frac{\partial \mu(\delta)}{\partial \delta}$. q_1^* is decreasing in δ. $\frac{\partial q_1^*}{\partial \mu(\delta)} = -\frac{8\left(24\mu^3(\delta)+61\mu^2(\delta)-44\mu(\delta)+4\right)}{\left(24\mu^3(\delta)-22\mu^2(\delta)+5\mu(\delta)+2\right)^2}$, from which it is found that $\frac{\partial q_1^*}{\partial \mu(\delta)} = 0$ has no solutions for $\mu > 1$ and since, for example, $\frac{\partial q_1^*}{\partial \mu(\delta)}\Big|_{\mu(\delta)=2} = -2816$, $\frac{\partial q_1^*}{\partial \mu(\delta)} < 0$ Together with $\frac{\partial q_1^*}{\partial \delta} < 0$ this implies $\frac{\partial \mu(\delta)}{\partial \delta} > 0$ and so a necessary condition for a stationary point of $CS^*(q_1, q_2)$ is $\frac{\partial CS^*(q_1, q_2)}{\partial \mu(\delta)} = 0$. From equation (4.20), $\frac{\partial CS^*(q_1, q_2)}{\partial \mu(\delta)}\Big|_{\delta''_m < \delta \leq \delta'''_m} = 0$ reduces to

$$\begin{aligned} &9217\mu^9(\delta) - 31616\mu^8(\delta) + 51632\mu^7(\delta) - 59920\mu^6(\delta) + 49483\mu^5(\delta) - \\ &\qquad - 27613\mu^4(\delta) + 10868\mu^3(\delta) - 2936\mu^2(\delta) + 416\mu(\delta) - 16 = 0. \end{aligned} \tag{C.8}$$

which has the unique solution satisfying $\mu(\delta) > 1$ of $\mu(\delta) \approx 1.358$. Substituting this value into equation (C.6) shows that this occurs at $\delta \approx 1.0277 < \delta''_m$. Since, for example, $\frac{\partial CS^*(q_1, q_2)}{\partial \delta}\Big|_{\delta=2} \approx -0.0225$, $\frac{\partial CS^*(q_1, q_2)}{\partial \delta}\Big|_{\delta''_m < \delta \leq \delta'''_m} < 0$. Numerically, $\left.\frac{\partial CS^*(q_1,q_2)}{\partial \delta}\right|_{\delta>\delta_m'''} = 0$ has no solutions for $\delta > 1$ and as, for example, $\left.\frac{\partial CS^*(q_1,q_2)}{\partial \delta}\right|_{\delta=5} = -8.77 \times 10^{-3}$, $\left.\frac{\partial CS^*(q_1,q_2)}{\partial \delta}\right|_{\delta>\delta_m'''} < 0$. Numerically, $CS^*\left(q_1, q_2\right)|_{\delta>\delta'} = CS^*\left(q_1, q_2\right)|_{\delta=1}$ has the solution $\delta \approx 1.667$. □

C.3 Entry costs and market creation

C.3.1 Proof of proposition 4.3

(i) From proposition 4.1, firm 2's profit given it enters is weakly decreasing in δ so that the range of entry costs at which firm 1 deters entry is weakly increasing in δ. Also from proposition 4.1, profit given firm 2 enters is weakly increasing in δ, implying that for any $E \in \left[0, \overline{E}\right)$ firm 1's profit is weakly greater with $\delta > 1$ than in the baseline. Thus as the market is always created in the baseline, it is always created with $\delta > 1$.

(ii) With entry costs, firm 1 can potentially deter firm 2 from entering. Such an action is possible if firm 2's maximin profit is negative. By construction, for $\delta > \delta_m''$, firm 2's maximin profit is $\pi_2^*\left(q_1, q_2\right)$, so firm 1 is unable to deter entry for $E < \pi_2^*\left(q_1, q_2\right)$. Firm 1 chooses not to create the market if $E > \pi_1^*\left(q_1, q_2\right)$, so for any δ such that $\pi_2^*\left(q_1, q_2\right) > \pi_1^*\left(q_1, q_2\right)$ there exists a range $\pi_2^*\left(q_1, q_2\right) > E > \pi_1^*\left(q_1, q_2\right)$ such that no market is created. It is hence sufficient to find some δ for which $\pi_2^*\left(q_1, q_2\right) > \pi_1^*\left(q_1, q_2\right)$, and $\pi_2^*\left(q_1, q_2\right)|_{\delta=3} \approx 0.0375$ and $\pi_1^*\left(q_1, q_2\right)|_{\delta=3} \approx 0.0272$. □

Perception and market entry with arbitrary entry cost D

As in section 4.6, let each firm incur a fixed cost of entry $E \in \left[0, \overline{E}\right)$.

D.1 Fixed costs

Suppose firm 1 enters the market. Firm 2 may choose either to choose high quality, $q_{2h} > q_1$, earning profit[1] $\pi_h\left(q_{2h}, q_1\right)$, or low quality, $q_{2\ell} < q_1$, earning profit $\pi_\ell\left(q_1, q_{2\ell}\right)$. From lemma 4.1, it's best responses conditional on entering as the high and low firm are respectively

$$q_{2h}^{BR}\left(q_1\right) = \max\left\{\underset{q_{2h}}{\operatorname{argmax}}\, \pi_h\left(q_{2h}, q_1\right), \delta q_1\right\} \tag{D.1a}$$

$$q_{2\ell}^{BR}\left(q_1\right) = \min\left\{\underset{q_{2\ell}}{\operatorname{argmax}}\, \pi_\ell\left(q_1, q_{2\ell}\right), \frac{q_1}{\delta}\right\}. \tag{D.1b}$$

Equilibrium will now be derived as follows: the condition under which the market is a natural monopoly is found. It is then found when entry deterrence is feasible if the market is not a natural monopoly, followed by comparing firm 1's profit from deterring and allowing entry, making it possible to show when it will deter entry in equilibrium.

[1]For linguistic and notational convenience, *profit* refers to profit net of entry cost and *total profit* refers to profit inclusive of entry cost.

D.1.1 Natural monopoly

The market is a natural monopoly if the entry cost is sufficiently high that firm 1's equilibrium behaviour is unaffected by the presence of firm 2, i.e. if firm 1 chooses the monopoly quality $q^M = \frac{1}{4}$ and firm 2 does not enter the market. Firm 2's best response to q^M is termed the *monopoly best response* (MBR) quality.

Lemma D.1. *The MBR quality is*

$$q_2^{MBR}(\delta) = \begin{cases} \dfrac{1}{4\mu_f^M} & \text{for } \delta \leq \delta'_{fM} \\ \\ \dfrac{1}{4\delta} & \text{for } \delta > \delta'_{fM} \end{cases} \tag{D.2}$$

where μ_f^M is the ratio $\frac{q^M}{q^{MBR}(\delta)}$ and is the unique solution to equation (D.3) which is greater than 1 and $\delta'_{fM} = \mu_f^M$ The constant μ_f^M is approximately $\mu_f^M \approx 5.200$.

Proof. Assume firm 2's best response to q^M is to enter as the low quality firm. This is shown to hold in the derivation of the EPM quality. Let $\mu = \frac{q^M}{q_{2\ell}}$. Substituting $q_1 = q^M$ and $q_{2\ell} = \frac{q^M}{\mu}$ into equation (C.2). and rearranging gives

$$16\mu^4 - 92\mu^3 + 48\mu^2 - 12\mu + 1 = 0 \tag{D.3}$$

Define $\mu_f^M = 5.200$ as the unique root of this equation with $\mu > 1$. Then $q_2^{MBR}(\delta) = \frac{1}{4\mu_f^M} \approx 0.048$. For $\delta > \mu_f^M$, $\frac{q^M}{\delta} > \frac{1}{4\mu_f^M}$ and from $q_{2\ell}^{BR}(q_1)$, $q_2^{MBR}(\delta) = \frac{1}{4\delta}$, which completes the derivation. □

The natural monopoly condition is then be found by requiring firm 2's total profit from entering to be less than 0.

Proposition D.1. *The market is a natural monopoly if* $E \in \left[E^M(\delta), \bar{E}\right)$, *where*

$$E^M(\delta) = \begin{cases} \dfrac{8\mu_f^M - 24\mu_f^{M2} + 8\mu_f^M - 1}{32\mu_f^{M2}\left(4\mu_f^M - 1\right)^2} & \text{for } \delta \leq \delta'_{fM} \\ \\ \dfrac{8\delta^3 - 24\delta^2 + 8\delta - 1}{32\delta^2\left(4\delta - 1\right)^2} & \text{for } \delta > \delta'_{fM}. \end{cases} \tag{D.4}$$

With fixed costs of quality, there is a range of entry costs for which the market is a natural monopoly, and the natural monopoly condition is weakly decreasing in δ.

Proof. Substituting $q^M = \frac{1}{4}$ and equation (D.2) into equation (4.3b) gives equation (D.4). $E^M(\delta)\Big|_{\delta<\delta_f^M} \approx 1.52 \times 10^{-3} < \bar{E}$ and $\frac{\partial}{\partial\delta} E^M(\delta)\Big|_{\delta>\delta_f^M} \geq 0$ can be reduced to $-16\delta^4 + 92\delta^3 - 48\delta^2 + 12\delta - 1 \geq 0$, which has no solutions for $\delta > \delta_f^M$. □

The natural monopoly condition is illustrated in figure D.1.

From lemma 4.1, firm 2 will never choose a quality such that the quality ratio is below the perception threshold, as the Bertrand trap would lead to a loss. In the standard case, its best response to $q^M = \frac{1}{4}$ is to choose a quality such that $\frac{q^M}{q_2^{MBR}} \approx 5.200$. Thus when the threshold excedes this value, it is forced to choose a quality further away from firm 1. This in turn lowers its profit, so that the market is a natural monopoly with lower entry costs than was required without bounded perception.

D.1.2 Entry deterrence

Now let $E < E^M(\delta)$, so that the market is not a natural monopoly. Assume that firm 1 always enters the market (this assumption is shown to hold in proposition D.4). Firm 1 is able to deter entry if it can choose a quality such that firm 2's profit if it enters is not sufficient to cover the cost of entering. Initially, the condition under which it is feasible for entry to be deterred is derived, and whether deterrence is observed in equilibrium is examined subsequently. In period 2, firm 2 may choose either to be the high or low quality firm. Conditional on entering as the high quality firm, firm 2's profit is decreasing in the quality choice of firm 1. Conversely, its profit conditional on entering as the low quality firm is increasing in firm 1's quality choice. The profit of firm 2 is hence minimized if firm 1 chooses its quality such that firm 2 is indifferent between entering as the high or low quality firm. If this minimized profit does not exceed the entry cost, then entry deterrence is feasible.

The quality at which firm 1 minimizes firm 1's profit from entering is termed the *entrant profit minimizing* (EPM) quality. Firm 2's best response to this, conditional on entering as the high (low) quality firm is termed the *high (low) EPM best response* or HEPM (LEPM) quality. Since at the EPM quality firm 2 is indifferent between being the high or low quality firm, these qualities are obtained from $\pi_{2h}\left(q_1, q_{2h}^{BR}(q_1,\delta)\right) = \pi_{2\ell}\left(q_1, q_{2\ell}^{BR}(q_1,\delta)\right)$.

Lemma D.2. *The EPM, HEPM and LEPM qualities are*

$$q_f^D = \begin{cases} b_{f1} & \text{for } \delta \leq \delta'_{fD} \\ \dfrac{\mu_f^{D3}\left(4\mu_f^D - 7\right)}{\left(4\mu_f^D - 1\right)^3} & \text{for } \delta'_f < \delta \leq \delta''_{fD} \\ \dfrac{2\delta^2(\delta-1)(4\delta^2-1)}{(\delta^4-1)(4\delta-1)^2} & \text{for } \delta > \delta''_{fD} \end{cases} \tag{D.5a}$$

$$q_{2h}^{D}(\delta) = \begin{cases} b_{fh} & \textit{for } \delta \leq \delta'_{fD} \\ \delta q_{1}^{D}(\delta) & \textit{for } \delta > \delta'_{fD} \end{cases} \tag{D.5b}$$

$$q_{2\ell}^{D}(\delta) = \begin{cases} b_{f\ell} & \textit{for } \delta \leq \delta'_{fD} \\ \dfrac{q_{1}^{D}(\delta)}{\mu_{f}^{D}} & \textit{for } \delta'_{f} < \delta \leq \delta''_{fD} \\ \dfrac{q_{1}^{D}(\delta)}{\delta} & \textit{for } \delta > \delta''_{fD} \end{cases} \tag{D.5c}$$

where $\delta'_{fD} \approx 1.533$*,* $\delta''_{fD} \approx 3.291$*,* $b_{f1} \approx 0.161$*,* $b_{fH} \approx 0.289$*,* $b_{fL} \approx 0.042$ *and* μ_{f}^{D} *is the ratio* $\left.\frac{q_{1}^{D}(D)}{q_{2\ell}^{D}(\delta)}\right|_{\delta'_{fD} < \delta \leq \delta''_{fD}}$ *and is the unique real root of equation (D.7) greater than 1.*

Proof. For sufficiently low δ, $q_{2h}^{BR}(q_1) = \text{argmax}_{q_{2h}}\, \pi_h(q_{2h}, q_1)$ and $q_{2\ell}^{BR}(q_1) = \text{argmax}_{q_{2\ell}}\, \pi_{2\ell}(q_1, q_{2\ell})$. Denote the constants which solve these equations simultaneously with $\pi_h(q_{2h}, q_1) = \pi_\ell(q_1, q_{2\ell})$ as $b_{f1} \approx 0.161$, $b_{fh} \approx 0.289$, $b_{f\ell} \approx 0.042$. $\frac{b_{fh}}{b_{f1}} \approx 1.792$ and $\frac{b_{f1}}{b_{f2\ell}} \approx 3.862$, so for $\delta > \delta'_f = \frac{b_{fh}}{b_{f1}}$, $q_{2h} = b_{fh}$ is not a best response to b_{f1}. Let $\delta > \delta'_f$ and be sufficiently small that $q_{2\ell}^{BR}(\delta) = \text{argmax}_{2\ell}\, \pi_\ell(q_1, q_{2\ell})$. Let $\mu = \frac{q_1}{q_{2\ell}}$. Inserting $q_1 = \mu q_{2\ell}$ into $\frac{\partial \pi_\ell(q_h, q_\ell)}{\partial q_\ell}$ (equation (C.2)) and rearranging gives $q_{2\ell} = \frac{\mu^2(4\mu - 7)}{(4\mu - 1)^3}$. Inserting $q_{2h} = q_{2h}^{BR}(q_1) = \delta q_1$ and $q_1 = \mu q_{2\ell}$ into $\pi_\ell(q_1, q_{2\ell}) = \pi_h(q_{2h}, q_1)$ and rearranging gives

$$q_{2\ell} = \frac{2\mu}{(\delta^2\mu^2 - 1)} \left(\frac{4\delta^2(\delta - 1)}{(4\delta - 1)^2} - \frac{\mu - 1}{(4\mu - 1)^2} \right). \tag{D.6}$$

Equating the two expressions for $q_{2\ell}$ then yields

$$A(\delta)\,\mu^4 + B(\delta)\,\mu^3 + C(\delta)\,\mu^2 + D(\delta)\,\mu + F(\delta) = 0 \tag{D.7}$$

where

$$A(\delta) = 4\delta^2, \qquad B(\delta) = -\left(\frac{512\delta^2(\delta-1)}{(4\delta-1)^2} + 7\delta^2\right) \tag{D.8a}$$

$$C(\delta) = \frac{384\delta^2(\delta-1)}{(4\delta-1)^2} + 4, \qquad D(\delta) = -\left(\frac{96\delta^2(\delta-1)}{(4\delta-1)^2} + 3\right) \tag{D.8b}$$

$$F(\delta) = \frac{8\delta^2(\delta-1)}{(4\delta-1)^2} + 2. \tag{D.8c}$$

Define μ_f^D as the unique root of this equation taking values greater than 1. The LEPM, EPM and HEPM qualities are found successively from $q_{2\ell} = \frac{\mu^2(4\mu-7)}{(4\mu-1)^3}$, $q_1 = \mu q_{2\ell}$ and $q_{2h} = \delta q_1$. Define $\delta''_f \approx 3.291$ as the solution to $\delta = \mu_f^D$ and let $\delta > \delta''_{fD}$. Inserting $q_{2h} = q_{2h}^{BR}(q_1) = \delta q_1$ and $q_{2\ell} = q_{2\ell}^{BR}(q_1) = \frac{q_1}{\delta}$ into $\pi_\ell(q_1, q_{2\ell}) = \pi_h(q_{2h}, q_1)$ and rearranging gives the EPM quality, the HEPM and LEPM qualities follow.

Note that by construction firm 2 is indifferent between entering with high or low quality in response to $q_1(\delta)$. $q^M > q_1^D(\delta)$, so the assumption in the derivation of the MBR quality that firm 2's best response to q^M is to enter with a lower quality holds. □

Substituting the EPM, HEPM and LEPM qualities into firm 2's profit function, the entry deterrence condition is found.

Proposition D.2. *Firm 1 is able to deter entry by firm 2 if* $E \in \left[E^D(\delta), E^M(\delta)\right)$, *where*

$$E^D(\delta) = \begin{cases} \dfrac{b_{fh}^2\left(8\left(b_{fh}-b_{f1}\right)-\left(4b_{fh}-b_{f1}\right)^2\right)}{2\left(4b_{fh}-b_{f1}\right)^2} & \text{for } \delta \leq \delta'_{fD} \\[2ex] \dfrac{4\delta^2\left(\delta-1\right)\mu_f^{D3}\left(4\mu_f^D-7\right)}{\left(4\delta-1\right)^2\left(4\mu_f^D-1\right)^3}-\dfrac{\delta^2\mu_f^{D6}\left(4\mu_f^D-7\right)^2}{2\left(4\mu_f^D-1\right)^6} & \text{for } \delta'_{fD} < \delta \leq \delta''_{fD} \\[2ex] \dfrac{2\delta^4\left(4\delta^2-1\right)\left(\delta^2-4\right)}{\left(\delta^3+\delta^2+\delta+1\right)^2\left(4\delta-1\right)^4} & \text{for } \delta > \delta''_{fD}. \end{cases} \tag{D.9}$$

When costs of quality are fixed, the entry deterrence condition is weakly decreasing in δ.

Proof. Substituting equation (D.5) into equation (4.3b) gives equation (D.9). For $\delta < \delta'_{fD}$, deterrence is as in the standard case. From the best response functions of firm 2, for a given q_1, $\pi_h\left(q_{2h}^{BR}, q_1\left(q_1\right)\right)$ and $\pi_\ell\left(q_1, q_\ell^{BR}\left(q_1\right)\right)$ are weakly decreasing in δ. As by definition $q_1^D\left(\delta\right)$ minimizes the entrant profit, its dependence on δ implies $\frac{\partial}{\partial\delta} E^D\left(\delta\right)\Big|_{\delta \geq \delta'_{fD}} < 0$. □

The equilibrium deterrence condition is illustrated in figure D.1.

Equation (D.9) gives the condition under which it is feasible for firm 1 to deter entry, but it is as yet unclear when deterrence will be observed in equilibrium. The first stage in determining this is to find the profit of firm 1 from allowing entry. Comparison to its profit from deterring entry will then reveal when deterrence is optimal.

D.1.3 Allowing entry

If firm 1 allows entry, results are as in the standard case, so qualities are given by equation 4.6 and profits are given by equation 4.7.

D.1.4 Equilibrium entry deterrence

The profit from allowing entry is compared to the profit from deterring entry in order to determine when entry deterrence is observed in equilibrium. Let $E \in \left[E^D(\delta), E^M(\delta)\right)$ so that deterrence is feasible. From equation (4.3a), the profit that firm 1 makes if it deters entry is $\pi_{1ED}(q_1) = \frac{q_1}{4}(1 - 2q_1)$. If the cost of entry is sufficiently high, firm 1 enjoys a natural monopoly and chooses its ideal quality of $q_1 = q^M = \frac{1}{4}$. As the cost of entry becomes lower, firm 1 must reduce its quality to deter firm 2 from entering, which also lowers its own profit. It follows that it must be feasible for firm 1 to deter entry at a sufficiently high quality for deterrence to be observed in equilibrium.

The minimum q_1 at which it is optimal for firm 1 to deter entry is termed the *minimum deterrence optimality* (MDO) quality.

Lemma D.3. *The MDO quality and firm 2's best response to it are*

$$q_1^{MDO}(\delta) = \begin{cases} c_{f1} & \text{for } \delta \leq \delta'''_{fE} \\ \frac{1}{4}\left(1 - \frac{\sqrt{1 - 16\delta - 160\delta^2 + 256\delta^3}}{(4\delta - 1)^2}\right) & \text{for } \delta > \delta'''_{fE} \end{cases} \tag{D.10a}$$

$$q_{2\ell}^{MDO}(\delta) = \begin{cases} c_{f2} & \text{for } \delta \leq \delta''_{fE} \\ \frac{q_1^E(\delta)}{\delta} & \text{for } \delta > \delta''_{fE} \end{cases} \tag{D.10b}$$

where $\delta''_{fE} \approx 3.456$, $\delta'''_{fE} = \delta'_f \approx 4.941$, $c_{f1} = \frac{1}{4}\left(1 - \sqrt{1 - 32\,\pi_{1A}\left(q_1^*(\delta), q_2^*(\delta)\right)\big|_{\delta < \delta'_f}}\right) \approx 0.134$ *and* $c_{f2} \approx 0.0386$.

Proof. From $\pi_{1ED}(q_1) = \pi_{1A}(q_1, q_2)\big|_{\delta < \delta'_{fD}}$, $q_1 = c_{f1} = \frac{1}{4}\left(1 - \sqrt{1 - 32\pi_{1A}(q_1, q_2)\big|_{\delta < \delta'_{fD}}}\right)$. Let δ be sufficiently small that $q_{2\ell}^{BR}(\delta) = \operatorname{argmax}_{q_{2\ell}} \pi_\ell(q_1, q_\ell)$ and denote the constant solving $\operatorname{argmax}_{q_{2\ell}} \pi_\ell(c_{f1}, q_{2\ell})$

as $c_{f2} \approx 0.0386$. Let $\delta''_{fE} = \frac{c_{f1}}{c_{f2}} \approx 3.456$. Then for $\delta > \delta''_{fE}$ firm 2's best response to $q_1 = c_{f1}$ is $\frac{q_1}{\delta}$. From $\pi_{1ED}(q_1) = \pi_{1A}(q_1, q_2)|_{\delta > \delta'_{fD}}$, the MDO quality for $\delta > \delta'_{fD}$ is obtained. □

If entry deterrence is feasible at $q > q_1^{MDO}(\delta)$, then it is optimal. If deterrence is only possible if firm 1 lowers its quality to some $q < q_1^{MDO}(\delta)$, then allowing entry is optimal. The lowest quality at which deterrence is feasible is $q_1^D(\delta)$ It follows that, if $q_1^{MDO}(\delta) < q_1^D(\delta)$, then deterrence is only possible at qualities such that it is optimal: the equilibrium deterrence condition is identical to the feasibility condition. Otherwise, the equilibrium deterrence condition is found by requiring firm 2's total profit to be 0 when best responding to $q_1^{MDO}(\delta)$. This leads to

Proposition D.3. *If $E \in \left[E_f^{D*}(\delta), E_f^M(\delta)\right)$, then if firm 1 enters the market it does so by deterring entry, where*

$$E^{D*}(\delta) = \begin{cases} E^D(\delta) & \text{for } \delta \leq \delta'_{fE} \\ \dfrac{c_{f1}c_{f2}(c_{f1} - c_{f2})}{(4c_{f1} - c_{f2})^2} - \dfrac{c_{f2}^2}{2} & \text{for } \delta''_{fE} \geq \delta > \delta'_{fE} \\ \dfrac{(\delta - 1)}{(4\delta - 1)^2} c_{f1} - \dfrac{c_{f1}^2}{2\delta^2} & \text{for } \delta'''_{fE} \geq \delta > \delta''_{fE} \\ \dfrac{(\delta - 1)}{4(4\delta - 1)^2}\left(1 - \dfrac{\sqrt{1 - 16\delta - 160\delta^2 + 256\delta^3}}{(4\delta - 1)^2}\right) - & \\ \qquad - \dfrac{1}{32\delta^2}\left(1 - \dfrac{\sqrt{1 - 16\delta - 160\delta^2 + 256\delta^3}}{(4\delta - 1)^2}\right)^2 & \text{for } \delta > \delta'''_{fE} \end{cases} \tag{D.11}$$

where $\delta'_{fE} \approx 2.883$. The equilibrium deterrence condition is weakly decreasing in δ.

Proof. For $\delta < \delta'_{fD}$, $q_1^D(\delta) \approx 0.161$ and $q_1^{MDO}(\delta) \approx 0.134$, so $E^{D*}(\delta) = E^D(\delta)$. Let δ'_{fE} be the solution to $q_1^D(\delta)\Big|_{\delta'_{fD} < \delta \leq \delta''_{fD}} = q_1^{MDO}(\delta)\Big|_{\delta < \delta''_{fE}}$ with $\delta'_{fE} \approx 2.883$. For

$\delta > \delta'_{fE}$, $E^{D*}(\delta)$ is then found from substituting equation (D.10) into equation (4.3b). By proposition D.2, $E^{D*}(\delta)$ is decreasing in δ for $\delta \leq \delta'_{fE}$ and for $\delta'_{fE} < \delta \leq \delta''_{fE}$ it is not dependent on δ. $\frac{\partial}{\partial\delta} E^{D*}(\delta)\Big|_{\delta''_{fE}<\delta\leq\delta'''_{fE}} > 0$ may be rearranged to become $-4\delta^4 + (7+64)\delta^3 - 48\delta^2 + 12c_{f1}\delta - c_{f1} > 0$ which has no solutions for $\delta > \delta''_{fE}$. $\frac{\partial}{\partial\delta} E^{D*}(\delta)\Big|_{\delta>\delta''_{fD}} > 0$ becomes a lengthy polynomial in δ which is omitted for reasons of space, and has no solutions for $\delta > \delta'''_{fE}$. □

The equilibrium deterrence condition is illustrated in figure D.1.

D.1.5 Equilibrium and incumbent profit

Having derived firms' optimal actions given the assumption that firm 1 enters the market, it can now be stated that

Proposition D.4. *Firm 1 chooses to enter the market in equilibrium.*

Proof. By proposition D.5, firm 1's profit is weakly greater than in the standard case, so it is sufficient to show it enters when $\delta = 1$. By assumption E is less than the monopoly profit, so firm 1 always enters of $E \in \left[E^M(\delta), \bar{E}\right)$. Firm 1's profit must be at least $\pi_{1A}(q_1, q_2)|_{\delta=1} \approx 0.0245$, so it enters unless $E \gtrsim 0.0245$ and the market is a natural monopoly. The market is a natural monopoly for $E \geq E^M(\delta)\Big|_{\delta=1} \approx 0.0015$, so firm 1 enters. □

The results for fixed costs of quality may be summarized in a characterization of equilibrium:

(i) Natural monopoly *If $E \in \left[E^M(\delta), \bar{E}\right)$, firm 1 enters the market and chooses $q_1 = q^M = \frac{1}{4}$. Firm 2 does not enter.*

(ii) Entry deterrence *If $E \in \left[E(\delta), E^M(\delta)\right)$, firm 1 enters the market and chooses the q_1 that satisfies $\pi_\ell\left(q_1, q_{2\ell}^{BR}(q_1)\right) = E$. Firm 2 does not enter.*

(iii) <u>*Duopoly*</u> *If* $E \in \left(0, E^{D*}(\delta)\right)$*, both firms enter, choosing qualities* $q_1 = q_1^*$ *and* $q_2 = q_2^*$*.*

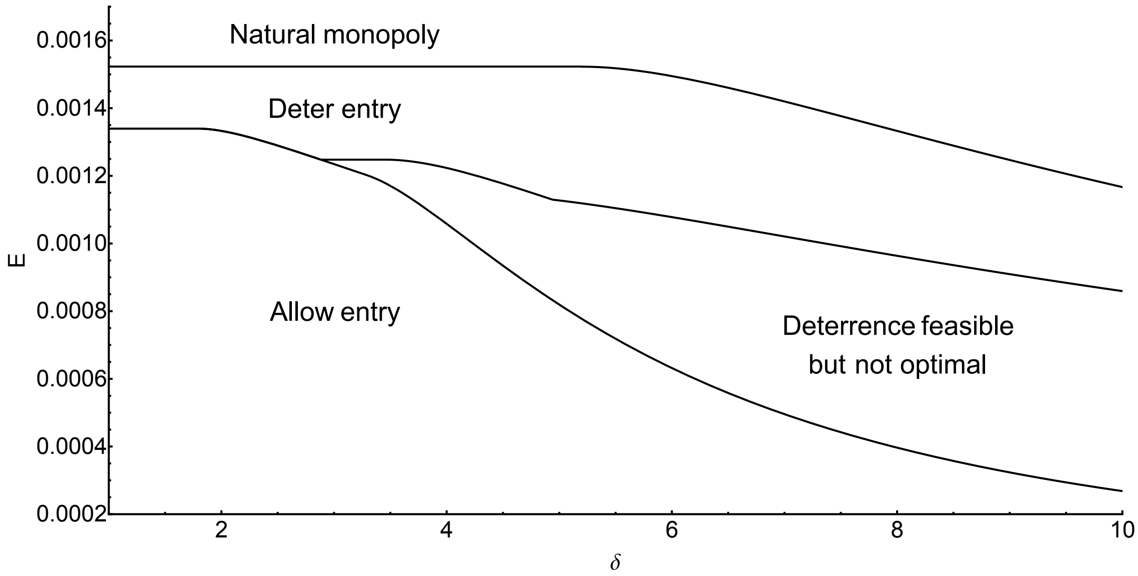


FIGURE D.1: Entry deterrence when quality costs are fixed.

Turning to the effect of bounded perception on profit, it is found that

Proposition D.5. *For fixed costs of quality, incumbent profit is greater than or equal to the standard case when consumers have bounded perception.*

Proof. $\frac{\partial}{\partial \delta} \pi_{1A}(q_1, q_2)|_{\delta > \delta'_{fD}} > 0$ may be reduced to $2\delta^2 - \delta - 1 > 0$ which holds for $\delta > \delta'_f$, so profit from allowing entry is weakly increasing in δ. $\pi_{1ED}(q_1)$ is increasing in q_1 for $q_1 < q^M$. When deterring entry, firm 1's quality solves $\pi_\ell\left(q_1, q_{2\ell}^{BR}(\delta)\right) = E$ so as firm 2's best response is weakly increasing in δ, q_1 is weakly increasing in δ, and so is profit from deterring entry, as $\frac{\partial \pi_{1ED}(q_1)}{\partial q_1} > 0$ for $q_1 < q^M$. Natural monopoly profit is constant. As $E^M(\delta)$ and $E^{D*}(\delta)$ are weakly decreasing in δ, firm 1 may transition from allowing entry to deterring entry and from deterring entry to natural monopoly, both of which increase profit. Incumbent profit is thus weakly increasing in δ and is weakly greater than in the standard case. □

D.2 Marginal costs

When firm 2 imitates it chooses $q_2 = \frac{q_1}{\delta} + \varepsilon \approx \frac{q_1}{\delta}$. To distinguish between firm 2 imitating firm 1 and producing the greatest quality below firm 1 that consumers peceive to be distinct, denote the latter as $\frac{\hat{q}_1}{\delta}$. Then let $\hat{q}_{2\ell}^{BR}(q_1) = \min\left\{\text{argmax}_{q_{2\ell}}\, \pi_\ell(q_1, q_{2\ell}), \frac{\hat{q}_1}{\delta}\right\}$ be firm 2's best response conditional on producing a low quality that consumers perceive as different to q_1.

Firm 2's best responses conditional on entering as the high and low quality firm are then

$$q_{2h}^{BR}(q_1,\delta) = \max\left\{\underset{q_{2h}}{\text{argmax}}\, \pi_{2h}(q_{2h}, q_1), \delta q_1\right\} \tag{D.12a}$$

$$q_{2\ell}^{BR}(q_1,\delta) = \begin{cases} \hat{q}_{2\ell}^{BR}(q_1,\delta) & \text{if } \pi_\ell\left(q_1, \hat{q}_{2\ell}^{BR}(q_1,\delta)\right) \geq \pi_I(q_1) \\ \dfrac{q_1}{\delta} & \text{if } \pi_\ell\left(q_1, \hat{q}_{2\ell}^{BR}(q_1,\delta)\right) < \pi_I(q_1). \end{cases} \tag{D.12b}$$

D.2.1 Natural monopoly

The nature of the entrant's best response having been examined, it can now be found when the market is a natural monopoly. The monopoly quality with marginal costs is $\frac{2}{3}$, and given this it can be shown that

Lemma D.4. *Firm 2's MBR is*

$$q_{2\ell}^{MBR}(\delta) = \begin{cases} \dfrac{1}{3} & \textit{for } \delta \leq \delta'_{mM} \\[2ex] \dfrac{2}{3\delta} & \textit{for } \delta > \delta'_{mM} \end{cases} \tag{D.13}$$

where $\delta'_{mM} = 2\sqrt{\frac{2}{7}} \approx 1.069$.

Proof. Assume firm 2's best response is to enter as the low quality firm (this will be shown to hold in the derivation of the EPM quality). Let δ be sufficiently small that $q_{2\ell}^{BR}(q_1) = \text{argmax}_{q_{2\ell}} \pi_\ell(q_1, q_{2\ell})$. From equation (C.5), $q_{2\ell}^{MBR} = \frac{1}{3}$. Let δ'_{mM} be the solution to $\pi_\ell(q_1, q_{2\ell})\big|_{q_1=\frac{2}{3}, q_{2\ell}=\frac{1}{3}} = \pi_I(q_1)\big|_{q_1=\frac{2}{3}}$ with $\delta'_{mM} = 2\sqrt{\frac{2}{7}} \approx 1.069$. As $\delta'_{mM} < \frac{q^M}{q_{2\ell}^{MBR}}\Big|_{\delta \le \delta'_{mM}} = 2$ this completes the derivation. □

Given these qualities, it can be determined when the market is a natural monopoly.

Proposition D.6. *With marginal costs of quality, the market is a natural monopoly for $E \in \left[E^M(\delta), \bar{E}\right)$, where*

$$E^M(\delta) = \begin{cases} \dfrac{1}{54} & \text{for } \delta \le \delta'_{mM} \\ \dfrac{4}{27}\left(\dfrac{\delta^2 - 1}{\delta^2}\right) & \text{for } \delta > \delta'_{mM}. \end{cases} \tag{D.14}$$

For $\delta \ge \sqrt{2}$ there is no $E \in \left(0, \bar{E}_m\right)$ such that the market is a natural monopoly.

Proof. Equation (D.14) is obtained by substituting equation (D.13) into firm 2's profit function. $E^M\big|_{\delta \le \delta'_{mM}} < \bar{E}$, so natural monopoly is possible. $\frac{\partial}{\partial \delta} E^M(\delta)\Big|_{\delta > \delta'_{mM}} = \frac{8}{27\delta^3} > 0$ and $E^M(\delta) = \bar{E}$ has the solution $\delta = \sqrt{2}$. □

The natural monopoly condition is illustrated in figure D.2.

In contrast to the fixed costs case, bounded perception may be exploited by the entrant, rather than the incumbent, since the entrant can imitate the incumbent's product. Thus a higher perception threshold increases firm 2's profit in response to the monopoly quality, meaning that the range of entry costs for which the market is a natural monopoly shrinks and for a sufficiently high threshold, the market is never a natural monopoly.

D.2.2 Entry deterrence

Let $E < \max\left\{E^M(\delta), \bar{E}\right\}$, so that firm 1 does not enjoy a natural monopoly. As with fixed costs, deterrence is feasible if firm 1 can lower firm 2's profit such that it does not excede the entry costs. It again minimizes firm 2's profit by choosing q_1 such that firm 2 is indifferent between entering as the high or low quality firm, so the EPM, HEPM and LEPM qualities are found from solving $\pi_\ell\left(q_1, q_{2\ell}^{BR}(\delta)\right) = \pi_h\left(q_{2h}^{BR}(\delta), q_1\right)$.

Lemma D.5. *The EPM, HEPM and LEPM qualities are*

$$q_1^D(\delta) = \begin{cases} b_{m1} & \text{for } \delta \leq \delta'_{mD} \\ \dfrac{4\left(4\mu_m^{D2} - 3\mu_m^D + 2\right)}{(24\mu_m^{D3} - 22\mu_m^{D2} + 5\mu_m^D + 2)} & \text{for } \delta''_{mD} \geq \delta > \delta'_{mD} \\ -\dfrac{B(\delta) - \sqrt{B^2(\delta) - 4A(\delta)C(\delta)}}{2A(\delta)} & \text{for } \delta > \delta''_{mD} \end{cases} \tag{D.15a}$$

$$q_{2h}^D(\delta) = \begin{cases} b_{mh} & \text{for } \delta \leq \delta'_{mD} \\ \mu_m^D q_{1D}^D(\delta) & \text{for } \delta''_{mD} \geq \delta > \delta'_{mD} \\ \delta q_1^D(\delta) & \text{for } \delta > \delta''_{mD} \end{cases} \tag{D.15b}$$

$$q_{2\ell}^D(\delta) = \begin{cases} b_{m\ell} & \text{for } \delta < \delta'_{mD} \\ \dfrac{q_1^D(\delta)}{\delta} & \text{for } \delta > \delta'_{mD} \end{cases} \tag{D.15c}$$

where $\delta'_{mD} \approx 1.071$, $\delta''_{mD} \approx 2.336$, $b_{m1} \approx 0.612$, $b_{mh} \approx 0.939$ and $b_{mL} \approx 0.309$. $A(\delta)$, $B(\delta)$ and $C(\delta)$ are functions of δ given by equation (D.17) and μ_m^D is the ratio $\left.\frac{q_{2h}^D(\delta)}{q_1^D(\delta)}\right|_{\delta'_m < \delta \leq \delta''_m}$ and is given by the unique root of equation (D.16) which takes a value greater than 1.

Proof. For sufficiently small δ, $q_{2h}^{BR}(q_1) = \operatorname{argmax}_{q_{2h}} \pi_h(q_{2h}, q_1)$ and $q_{2\ell}^{BR}(q_1) = \operatorname{argmax}_{q_{2\ell}} \pi_\ell(q_1, q_{2\ell})$. Denote the constants solving these equations simultaneously with $\pi_h(q_{2h}, q_1) = \pi_{2\ell}(q_1, q_{2\ell})$ as $b_1 \approx 0.612$, $b_{mh} \approx 0.939$ and $b_{m\ell} \approx 0.309$. Let δ'_{mD} be the solution to $\pi_I(q_1)|_{q_1=b_{m1}} = \pi_\ell(q_1, q_{2\ell})|_{q_1=b_{m1}, q_{2\ell}=b_{m\ell}}$. $\delta'_{mD} \approx 1.071$ and

$\frac{b_{mh}}{b_{m1}} \approx 1.533$, $\frac{b_{m1}}{b_{m\ell}} \approx 1.980$, so for $\delta > \delta'_{mD}$ firm 2's best response to $q_1 = b_{m1}$ is to immitate. Let $\delta > \delta'_{mD}$ but be sufficiently small that $q_{2h}^{BR}(q_1) = \text{argmax}_{q_{2h}} \pi_h(q_{2h}, q_1)$. Let $\mu = \frac{q_{2h}}{q_1}$ and substitute $q_{2h} = \mu q_1$ into equation (C.4), which after rearrangement gives $q_1 = \frac{4\left(4\mu^2-3\mu+2\right)}{24\mu^3-22\mu^2+5\mu+2}$. Substitution of this expression and $q_{2h} = \mu q_1$ into $\pi_I(q_1) = \pi_h(q_{2h}, q_{1l})$ yields

$$\delta^2\left(32\mu^7(\delta) - 96\mu^6(\delta) + 160\mu^4(\delta) - 182\mu^3(\delta) + 101\mu^2(\delta) - 28\mu(\delta) + 4\right) + \dots$$
$$\dots + 96\mu^5(\delta) - 192\mu^4(\delta) + 182\mu^3(\delta) - 101\mu^2(\delta) + 28\mu(\delta) - 4 = 0 \qquad \text{(D.16)}$$

Although no analytical solutions exist, numerical approximations are possible to find for given values of δ. Define μ_m^D as the unique root of this equation taking a value greater than 1, from which the EPM and HEPM qualities are found, with the LEPM quality following from $q_{2\ell} = \frac{q_1}{\delta}$. Define δ''_{mD} as the solution to $\mu_m^D = \delta$, with $\delta''_{mD} \approx 2.336$. Let $\delta > \delta''_{mD}$. Substituting $q_{2h} = \delta q_1$ and $q_{2\ell} = \frac{q_1}{\delta}$ into $\pi_h(q_{2h}, q_1) = \pi_I(q_1)$ and rearranging yields $q_{m1}^D(\delta)\big|_{\delta > \delta''_{mD}} = \frac{-B(\delta)-\sqrt{B^2(\delta)-4A(\delta)C(\delta)}}{2A(\delta)}$, where

$$A(\delta) = \left(\delta^2 - 1\right)(4\delta - 1)^2 + \delta^4(\delta - 1)(1 + 2\delta)^2 \qquad \text{(D.17a)}$$
$$B(\delta) = -2\left(\delta^2 - 1\right)(4\delta - 1)^2 - 8\delta^4(\delta - 1)(1 + 2\delta) \qquad \text{(D.17b)}$$
$$C(\delta) = 16\delta^4(\delta - 1). \qquad \text{(D.17c)}$$

The HEPM and LEPM qualities then follow directly.

$\pi_I(q_1)$ is decreasing in q_1 for $q_1 > \frac{4}{3} > q_m^M$. To show that firm 2's profit is not minimized at $\pi_I(q_1) = \pi_\ell\left(q_1, q_{2\ell}^{BR}(q_1)\right)$, note that as $\frac{\partial \pi_{2\ell}(q_1, q_{2\ell})}{\partial q_1} > 0$, a necessary conditions for this to be the case is that $\pi_\ell\left(q_1, q_{2\ell}^{BR}(q_1)\right)\big|_{q_1 = q_1^M} < \pi_I(q_1)\big|_{q_1 = q_1^D(\delta)}$, which does not hold from $E^M(\delta) > E^D(\delta)$. □

Substituting the EPM, HEPM and LEPM qualities into firm 2's profit gives the entry deterrence condition.

Proposition D.7. *If* $E \in \left[E^D(\delta), \max\left\{E^M(\delta), \bar{E}\right\}\right)$*, it is feasible for firm 1 to deter entry, where*

$$E^D(\delta) = \begin{cases} \dfrac{b_{m1} b_{m\ell} \left(2 + b_{m1} - b_{m\ell}\right)^2 \left(b_{m1} - b_{m\ell}\right)}{4\left(4 b_{m1} - b_{m\ell}\right)^2} & \text{for } \delta \leq \delta'_{mD} \\ \\ \dfrac{1}{4} q_1^{D2}(\delta)\left(2 - q_1^D(\delta)\right)\left(\dfrac{\delta^2 - 1}{\delta^2}\right) & \text{for } \delta > \delta'_{mD}. \end{cases} \tag{D.18}$$

With marginal costs of quality, the entry deterrence condition is weakly increasing in δ *for* $\delta \leq \delta''_{mD}$ *and decreasing* $\delta > \delta''_{mD}$*.*

Proof. Substituting equation (D.15) into equation (4.12a) for $\delta \leq \delta'_{mD}$ and $\pi_I(q_1)$ for $\delta > \delta'_{mD}$ gives equation (D.18). If $q_{2h}^{BR}(q_1) = \operatorname{argmax}_{q_{2h}} \pi_h(q_{2h}, q_1)$ and $q_{2\ell}^{BR}(q_1) = \frac{q_1}{\delta}$, firm 2's maximized profit for a given q_1 must be increasing in δ, so that the entry deterrence condition is increasing in δ for $\delta'_{mD} < \delta \leq \delta''_{mD}$. $\frac{\partial E^D(\delta)}{\partial \delta}$ can be found numerically to be negative at some $\delta > \delta''_{mD}$, $\frac{\partial}{\partial \delta} E^D(\delta)\Big|_{\delta > \delta''_{mD}}$ is continuous and it can be shown numerically that $\frac{\partial}{\partial \delta} E^D(\delta)\Big|_{\delta > \delta''_{mD}} = 0$ has no solutions for $\delta > \delta''_{mD}$. □

The entry deterrence condition is illustrated in figure D.2.

Proposition D.7 determines the feasible actions of firm 1, but does not state when deterrence will be observed in equilibrium. The intial step in addressing this question is to find the incumbent's profit when allowing entry.

D.2.3 Allowing entry

Again, given firm 1 allows entry, results are as in the standard case. Qualities are thus given by equation 4.17 and profits by equation 4.18.

D.2.4 Equilibrium, market creation and incumbent profit

Comparing firm 1's profit from deterring and allowing entry, it is revealed that

Proposition D.8. *With marginal costs of quality, firm 1 always deters entry when it is feasible.*

Proof. From equation (4.13a), if firm 1 deters entry it earns profit $\pi_{1ED}(q_1) = \frac{1}{16} q_1 (2 - q_1)^2$. If $\delta = 1$, $\pi_{1ED}(q_1)|_{q_1 = q_1^D(\delta)} = \pi_{1A}(q_1, q_2)$ and numerically the roots of $\pi_{1ED}(q_1)|_{q_1 = q_1^D(\delta)} - \pi_{1A}(q_1, q_2) = 0$ lie outside the feasible range of quality. □

Unlike with fixed costs, the conditions for equilibrium and feasible deterrence coincide.

Having found firm 1's optimal actions under the assumption that it enters the market, it is possible to revisit that assumption.

Proposition D.9. *If $E \in \left(E^{MC1}(\delta), \bar{E}\right) \cup \left(E^{MC2}(\delta), E^D(\delta)\right)$, firm 1 does not enter the market, where*

$$E^{MC1}(\delta) = \frac{8\delta^2 (\delta^2 - 1)^2}{(5\delta^2 - 4)^3} \tag{D.19}$$

and $E^{MC2}(\delta) = \pi_{1A}(q_1, q_{2\ell})$. There is a range of E satisfying $E \in \left(E^{MC1}(\delta), \bar{E}\right)$ for $\delta > \sqrt{2}$ and a range of E satisfying $E \in \left(E^{MC2}(\delta), E^D(\delta)\right)$ for $\delta'_{MC} < \delta < \delta''_{MC}$, where $\delta'_{MC} \approx 1.184$ and $\delta''_{MC} \approx 4.211$.

Proof. Let $E \in \left[E^D(\delta), \min\left\{E^M(\delta), \bar{E}\right\}\right)$, so that conditional on entering firm 1 sets q_1 to satisfy $\max\left\{\pi_\ell\left(q_1, \hat{q}_{2\ell}^{BR}(q_1)\right), \pi_I(q_1)\right\} = E$, so if $\pi_{1ED}(q_1) < E$ at this quality it will not enter. $\pi_{1ED}(q_1) = \pi_\ell\left(q_1, \hat{q}_{2\ell}^{BR}(q_1)\right)$ has no solutions for $q_1 < q^M$. Let δ be sufficiently high that $\pi_\ell\left(q_1, \hat{q}_{2\ell}^{BR}(q_1)\right) < \pi_I(q_1)$. Solving $\pi_{1ED}(q_1) = \pi_I(q_1)$ gives $q_1^{MC1}(\delta) = \frac{2\delta^2}{5\delta^2 - 4}$. Substitution into $\pi_{1ED}(q_1)$ gives $E^{MC1}(\delta)$. To find when $E^{MC1}(\delta)$ lies within the range $\left(E^D(\delta), \min\left\{E^M(\delta), \bar{E}\right\}\right)$, $q_1^{MC1}(\delta) > q^M$ for $\delta <$

$\sqrt{2}$. $E^{MC1}(\delta) = \bar{E}$ has the solution $\delta = \sqrt{2}$ and $\frac{\partial E^{MC1}(\delta)}{\partial \delta} = -32\delta \frac{(\delta^4 - 3\delta^2 + 2)}{(5\delta^2 - 4)^4}$ so that $\frac{\partial E^{MC1}(\delta)}{\partial \delta} < 0$ for $\delta > \sqrt{2}$. Numerically $E^D(\delta) - E^{MC1}(\delta) = 0$ has no solutions for $\delta > \sqrt{2}$ and so $E^{MC1}(\delta) \in \left(E^D(\delta), \bar{E}\right)$ for $\delta > \sqrt{2}$.

Let $E \in \left(0, E^D(\delta)\right)$ so that conditional on entering firm 1 allows entry. For $\delta \leq \delta'_{mD}$, $\pi_{1A}(q_1, q_2) \approx 0.0379$ and $E^D(\delta) \approx 0.0166$, so firm 1 enters. Let $E^{MC2}(\delta) = \pi_{1A}(q_1, q_2)$ so that firm 1 does not enter for $E \in \left(E^{MC2}(\delta), E^D(\delta)\right)$. Define the unique root of $E^{MC2}(\delta)\Big|_{\delta'_{mD} < \delta \leq \delta''_{mD}} - E^D(\delta)\Big|_{\delta'_{mD} < \delta \leq \delta''_{mD}} = 0$ as $\delta'_{MC} \approx 1.184$ and the unique root of $E^{MC2}(\delta)\Big|_{\delta > \delta''_{mD}} - E^D(\delta)\Big|_{\delta > \delta''_{mD}} = 0$ as $\delta''_{MC} \approx 4.211$. Then $E^{MC2}(\delta) \in \left(0, E^D(\delta)\right)$ for $\delta'_{MC} < \delta < \delta''_{MC}$. □

The regions in which firm 1 does not enter are illustrated in figure D.2. The necessity of avoiding firm 2 imitating its quality means that firm 1's profit may be greatly reduced, and this leads to ranges of entry costs for which it cannot earn enough to make it worthwhile entering the market.

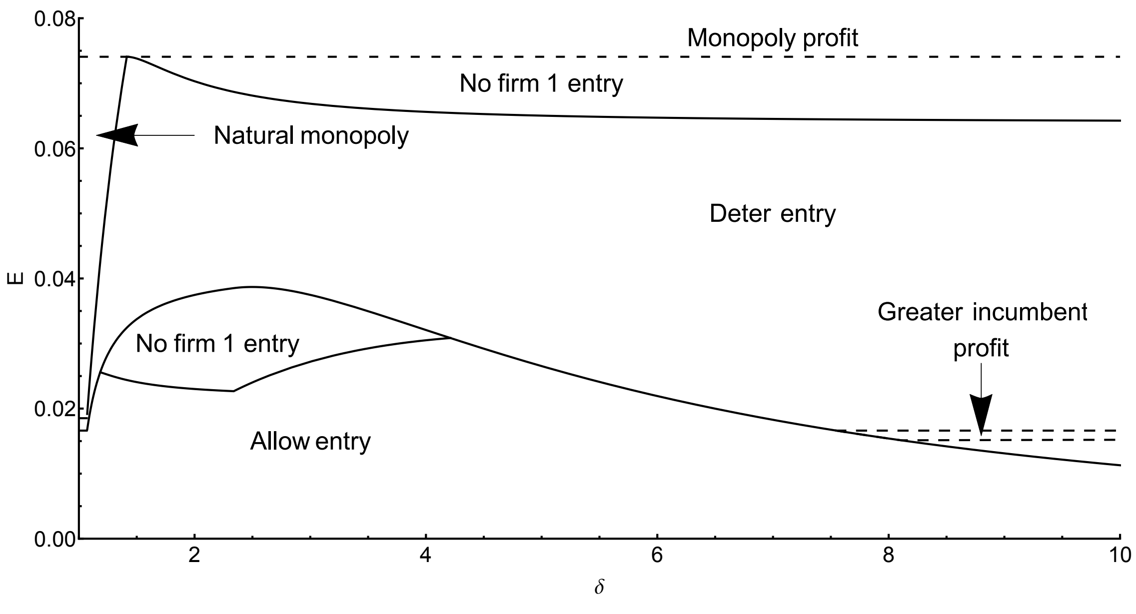


Figure D.2: Entry deterrence when costs of quality are marginal.

The results for marginal costs of quality can be summarized in a characterization of equilibrium:-

(i) *No market created If* $E \in \left(E^{MC1}(\delta), \bar{E}\right) \cup \left(E^{MC2}(\delta), E^{M}(\delta)\right)$ *firm 1 does not enter.*

(ii) *Natural monopoly If* $E \in \left(E^{M}(\delta), \bar{E}\right)$, *then firm 1 enters the market and chooses* $q_1 = \frac{2}{3}$. *Firm 2 does not enter.*

(iii) *Entry deterrence If* $E \in \left(E^{D}(\delta), \max\left\{E^{M}(\delta), E^{MC1}(\delta)\right\}\right]$, *firm 1 enters the market and chooses the* q_1 *that satisfies* $\pi_\ell\left(q_1, q_{2\ell}^{BR}(q_1)\right) = E$. *Firm 2 does not enter.*

(iv) *Allowing entry If* $E \in \left(0, E^{D}(\delta)\right) \setminus \left[E^{MC2}(\delta), E^{D}(\delta)\right)$ *both firms enter, choosing qualities* $q_1 = q_1^*$ *and* $q_2 = q_2^*$.

Turning to the effects of bounded perception, it is found that

Proposition D.10. *With marginal costs of quality, incumbent profit is weakly lower than in the standard case, with the following exception:* $\delta \geq E^{D-1}(E)$ *and* $E \in \left(E^{\pi}(\delta), \pi_{1A}(q_1, q_2)|_{\delta=1}\right)$, *where* $E^{\pi}(\delta)$ *is the unique solution to*

$$\frac{4\delta^2\left(\delta^2-1\right)^2 E}{\left(\delta^2\left(E - 4\,\pi_{1A}(q_1,q_2)|_{\delta=1}\right) - 4\,\pi_{1A}(q_1,q_2)|_{\delta=1}\right)^3} = 1 \tag{D.20}$$

that satisfies $E \in \left(0, \bar{E}\right)$.

Proof. Let $E \in \left(0, E^{D}\big|_{\delta=1}\right)$, so that entry is allowed in the standard case. For $\delta \geq E^{D-1}(E)$, entry is deterred. Firm 1 deters entry by choosing q_1 such that $\pi_I(q_1) = E$, and its profit is equal to that in the standard case if $\pi_{1ED}(q_1) = \pi_{1A}(q_1,q_2)|_{\delta=1}$. Taking the quotient of these two equations and rearranging results in $q_1 = \frac{2\delta^2 E}{E\delta^2 + 4(\delta^2-1)\,\pi_{1A}(q_1,q_2)|_{\delta=1}}$. Substitution into $\pi_{1ED}(q_1) = \pi_{1A}(q_1,q_2)|_{\delta=1}$ yields equation (D.20) from which $E^{\pi}(\delta)$ is found.

For $E \in (0, E^{\pi}(\delta))$, if it deters entry firm 1 makes less than in the standard case, and if it allows entry, from equation (4.17) for $\delta > \delta'_{mD}$ it shares the market by producing the same quality as when it deterred entry when $E = E^{D}(\delta)$, implying a further reduction in profit. If $E \in \left[E^{D}(\delta)\big|_{\delta=1}, E^{M}\big|_{\delta=1} \right)$, for $\delta \leq \delta'_{mD}$ profit is as in the standard case and for $\delta > \delta'_{mD}$ firm 1 either allows entry, implying lower profit than in the standard case, or deters entry by selecting q_1 such that $\pi_I(q_1) = E$. From $\frac{\partial \pi_I(q_1)}{\partial \delta} > 0$ and $\frac{\partial}{\partial q_1} \pi_{1ED}(q_1)\big|_{q_1 < q^M} < 0$, profit is lower than in the standard case. If $E \in \left[E^{M}(\delta), \bar{E} \right)$, profit is as in the standard case for $\delta \leq \delta'_{mM}$ and for $\delta > \delta'_{mM}$ and for $\delta > \delta'_{mM}$ the market is not a natural monopoly, implying lower profit than in the standard case. □

The region in which incumbent profit is higher than in the standard case is illustrated in figure D.2. Generally, the ability to imitate firm 1 leads to lower incumbent profit. However, for some entry costs firm 1 may deter entry with sufficiently severe bounded perception, whereas it would allow entry in the standard case. Thus for some entry costs there is the possibility for incumbent profit to be higher than in the standard case.

Proofs for chapter 5 E

E.1 Derivation of standard equilibrium

Firms profits are

$$\pi_h(q_h, q_\ell) = p_h\left(1 - \frac{p_h - p_\ell}{q_h - q_\ell}\right) - \frac{1}{2}q_h^2, \qquad \pi_\ell(q_h, q_\ell) = p_\ell\left(\frac{p_h - p_\ell}{q_h - q_\ell} - \frac{p_\ell}{q_\ell}\right) - \frac{1}{2}q_\ell^2. \tag{E.1}$$

The first order conditions are

$$\frac{\partial \pi_h(q_h, q_\ell)}{\partial q_h} = 1 - \frac{2p_h - p_\ell}{q_h - q_\ell} \quad \text{(E.2a)} \qquad \frac{\partial \pi_\ell(q_h, q_\ell)}{\partial q_\ell} = \frac{p_h - 2p_\ell}{q_h - q_\ell} - \frac{2p_\ell}{q_\ell} \quad \text{(E.2b)}$$

from which it can be seen that the second order conditions are negative. Solving the first order conditions for p_h and p_ℓ results in the prices in equation (5.1).

The first and second order conditions of equation (5.2) are

$$\frac{\partial \pi_h(q_h, q_\ell)}{\partial q_h} = \frac{4q_h(4q_h^2 - 3q_h q_\ell + 2q_\ell^2)}{(4q_h - q_\ell)^3} - q_h, \quad \frac{\partial^2 \pi_h(q_h, q_\ell)}{\partial q_h^2} = -\frac{q_\ell^2(40q_h + 8q_\ell)}{(4q_h - q_\ell)^4} - 1 \tag{E.3a}$$

$$\frac{\partial \pi_\ell(q_h, q_\ell)}{\partial q_\ell} = \frac{q_h^2(4q_h - 7q_\ell)}{(4q_h - q_\ell)^3} - q_\ell, \qquad \frac{\partial^2 \pi_\ell(q_h, q_\ell)}{\partial q_\ell^2} = -\frac{2q_h^2(8q_h + 7q_\ell)}{(4q_h - q_\ell)^4} - 1. \tag{E.3b}$$

Using the substitution $q_h = \mu q_\ell$, $\mu \geq 1$, the first order conditions may be rearranged to become

$$\frac{4\mu\left(4\mu^2-3\mu+2\right)}{\left(4\mu-1\right)^3}=\mu q_\ell, \qquad \text{(E.4a)} \qquad\qquad \frac{\mu^2\left(4\mu-7\right)}{\left(4\mu-1\right)^3}=q_\ell. \qquad \text{(E.4b)}$$

Taking the ratio of these and rearranging yields $4\mu^3-23\mu^2+12\mu-8=0$, the solution to which is μ^*, and equation (E.4b) then gives the equilibrium qualities in equation (5.3).

E.2 Proof of proposition 5.1

(i) If $\frac{q_h}{q_\ell}<\delta$, Bertrand competition with effectively homogeneous goods and identical marginal costs of 0 for each firm occurs. Firms earn no revenue and make a loss for any $q_h, q_\ell>0$. Then as each firm can make 0 profit from selecting 0 quality, $\frac{q_h}{q_\ell}<\delta$ cannot be an equilibrium. Any $q_h>0$ is perceivably different to $q_\ell=0$, so that $q_h=0$ is not a best response to $q_\ell=0$. For any $q_h>0$ it is possible to find some $0<q_\ell\leq\frac{q_h}{\delta}$ which is strictly positive and perceivably different to q_h. From the first order condition of $\pi_\ell\left(q_h,q_\ell\right)$, $\left.\frac{\partial\pi_\ell(q_h,q_\ell)}{\partial q_\ell}\right|_{q_\ell=0}>0$, so $q_\ell=0$ is not a best response to any $q_h=0$. □

(ii) Let $\delta>\mu^*$. For $\frac{q_h}{q_\ell}>\delta$, firms' profits are given by equation (5.2), and thus their best responses (given firm h (ℓ) is constrained to produce the higher (lower) quality) are determined by the first order conditions (equation (E.3)). The unique solution to $\frac{\partial\pi_h(q_h,q_\ell)}{\partial q_h}=0$, $\frac{\partial\pi_\ell(q_h,q_\ell)}{\partial q_\ell}=0$ is q_h^*, q_ℓ^*, with $\frac{q_h^*}{q_\ell^*}=\mu^*$. Thus if $\delta>\mu^*$, firms cannot be in equilibrium for $\frac{q_h}{q_\ell}>\delta$ and so $\frac{q_h}{q_\ell}=\delta$ is a necessary condition for equilibrium. □

E.3 Proof of proposition 5.2

Firstly it is shown that the baseline equilibrium equilibrium is the unique equilibrium for $\delta \leq \mu^*$. Let $\delta \leq \mu^*$. For any $\frac{q_h}{q_\ell} > \delta$, firms' best response functions are as in the standard case and so the only equilibrium given this is q_h^*, q_ℓ^*. By proposition 5.1, $\frac{q_h}{q_\ell} < \delta$ will not be an equilibrium. If $\frac{q_h}{q_\ell} = \delta$, the only potentially profitable deviation for firm h (ℓ) is to produce a higher (lower) quality, so necessary conditions for equilibrium are $\frac{\partial \pi_h(q_h,q_\ell)}{\partial q_h} \leq 0$ and $\frac{\partial \pi_\ell(q_h,q_\ell)}{\partial q_\ell} \geq 0$. Substituting $q_h = \delta q_\ell$ into the first order conditions, a necessary condition for $\frac{\partial \pi_h(q_h,q_\ell)}{\partial q_h} \leq 0$ and $\frac{\partial \pi_\ell(q_h,q_\ell)}{\partial q_\ell} \geq 0$ is $4\delta^3 - 23\delta^2 + 12\delta - 8 > 0$. The only real root of this polynomial is $\delta = \mu^*$, so for $\delta \leq \mu^*$, $\frac{q_h}{q_\ell} = \delta$ will only be observed in equilibrium if $\delta = \mu^*$ and $q_h = q_h^*$, $q_\ell = q_\ell^*$.

Secondly, equilibrium for $\delta > \mu^*$ is found. By proposition 5.1 it must be that $\frac{q_h}{q_\ell} = \delta$. Substituting $q_\ell = \frac{q_h}{\delta}$ into equation (5.2) gives

$$\pi_h = \frac{4q_h\delta\left(\delta - 1\right)}{\left(4\delta - 1\right)^2} - \frac{1}{2}q_h^2 \tag{E.5}$$

$$\pi_\ell = \frac{q_h\left(\delta - 1\right)}{\left(4\delta - 1\right)^2} - \frac{q_h^2}{2\delta^2}. \tag{E.6}$$

Equilibrium conditions

Condition $Fh0/F\ell 0$

A firm deviates to produce 0 quality if it otherwise makes a loss. Equating equation (E.5) (equation (E.6)) to 0 and rearranging gives condition $Fh0$ ($F\ell 0$) as $q_h \leq Fh0\left(\delta\right) = \frac{8\delta(\delta-1)}{(4\delta-1)^2}$ ($q_h \leq F\ell 0\left(\delta\right) = \frac{2\delta^2(\delta-1)}{(4\delta-1)^2}$).

Condition $FhT/F\ell T$: Firm h/ℓ above/below threshold

For firm h to wish to deviate to produce above the threshold it must be that the first order condition of π_h is positive given $\frac{q_h}{q_\ell} = \delta$. Equating equation (E.3a) to 0 and rearranging gives condition FhT as $q_h \leq FhT(\delta) = \frac{4\delta\left(4\delta^2-3\delta+2\right)}{(4\delta-1)^3}$. Similarly firm ℓ deviates to below the threshold if the first order condition of π_ℓ is negative given $\frac{q_h}{q_\ell} = \delta$. Equating equation (E.11b) to 0 and rearranging gives condition $F\ell T$ as $q_\ell \geq F\ell T(\delta) = \frac{\delta^3(4\delta-7)}{(4\delta-1)^3}$.

Condition FhU: Firm h undercut

Let $h = 1$ so that firm 1 is the high quality firm and firm 2 is the low quality firm. Assume firm 1 undercuts by choosing q_1^{uc} such that $\frac{q_2}{q_1^{uc}} = \delta$, i.e. the highest quality below q_2 such that consumers perceive the goods as distinct. Substituting $q_h = q_2 = \frac{q_1}{\delta}$ and $q_\ell = q_1^{uc} = \frac{q_2}{\delta^2}$ into equation (5.2b) gives $\pi_1 = \frac{(\delta-1)}{\delta(4\delta-1)^2} q_1 - \frac{q_1^2}{2\delta^4}$. Subtracting its profit from deviating from its profit in the candidate equilibium, equating this to 0 and rearranging then allows condition FhU to be given as $q_1 \leq FhU(\delta) = \frac{2\delta^3\left(4\delta^2-1\right)}{(4\delta-1)^2(1+\delta+\delta^2+\delta^3)}$.

The assumption that firm 1's best response conditional on undercutting is $q_1 = \frac{q_2}{\delta}$ is valid only if the first order condition of π_ℓ evaluated at $q_h = q_2 = \frac{q_1}{\delta}$ and $q_\ell = q_1^{uc} = \frac{q_1}{\delta^2}$ is non negative. Substituting the relevant qualities into equation (E.3b) shows the first order condition is $\frac{\partial \pi_1}{\partial q_1} = \frac{\delta^2(4\delta-7)}{(4\delta-1)^3} - \frac{q_1}{\delta^2}$. This is linearly decreasing in q_1, so it is sufficient to show that it is non negative at the maximum q_1 satisfying $q_1 \leq FhU(\delta)$. Substitution reduces the condition to $4\delta^5 - 3\delta^4 - 35\delta^3 + 5\delta^2 + \delta - 2 \geq 0$. Numerically, this is not satisfied for $\delta > \mu^*$ and $FhU(\delta)$ is the condition for no undercutting. An analogous argument holds when firm 2 is the high quality firm.

Condition $F\ell L$: Firm ℓ leapfrog

Let $\ell = 1$ so that firm 1 is the low quality firm and firm 2 the high quality firm. Assume firm 1 leapfrogs to produce q_1^{le} such that $\frac{q_1^{le}}{q_2} = \delta$, i.e. the lowest quality above q_2 that consumers perceive as heterogeneous. Substituting $q_h = q_1^{le} = \delta^2 q_1$ and $q_\ell = q_2 = \delta q_1$ into equation (5.2a) gives $\pi_1 = \frac{4\delta^3(\delta-1)}{(4\delta-1)^2} q_1 - \frac{\delta^4 q_1^2}{2}$. Substituting its profit from deviating from its profit in the candidate equilibrium, equating this to 0 and rearranging then allows condition FℓL to be given as $q_1 \geq F\ell L(\delta) = \frac{2\delta\left(4\delta^2-1\right)}{(4\delta-1)^2(1+\delta+\delta^2+\delta^3)}$.

The assumption that firm 1's best response conditional on leapfrogging is $q_1^{le} = \delta q_2$ is valid only if the first order condition of π_h evaluated at $q_h = q_1^{le} = \delta^2 q_1$ and $q_\ell = q_2 = \delta q_1$ is non positive. Substituting the relevant qualities into equation (E.3a) shows the first order condition is $\frac{\partial \pi_1}{\partial q_1} = \frac{4\delta\left(4\delta^2-3\delta+2\right)}{(4\delta-1)^3} - \delta^2 q_1$. This is decreasing in q_1, so it is sufficient to show it is non positive at the lowest q_1 satisfying $q_1 \geq F\ell L(\delta)$. Substitution reduces the condition to $-8\delta^5+6\delta^4+10\delta^3+5\delta^2-2\delta+4 \leq 0$. Numerically, this is satisfied for $\delta > \mu^*$, so $F\ell L(\delta)$ is the condition for no leapfrogging. An analogous argument holds when firm 2 is the low quality firm.

Condition redundancy

Conditions FhT and FℓL set a lower limit on q_h. Rearranging $F\ell L(\delta) - FhT(\delta) \geq 0$ results in $8\delta^5 + 2\delta^4 - 10\delta^3 + 10\delta^2 + 2\delta + 3 \geq 0$, which is not satisfied for $\delta > \mu^*$, and so FhT is the binding lower limit.

Conditions Fh0, Fℓ0, FhU and FℓT set an upper limit on q_h. $F\ell 0(\delta) - Fh0(\delta) \geq 0$ can be reduced to $\delta \geq 4$ and $Fh0(\delta) - FhU(\delta) \geq 0$ can be reduced to $\delta \geq 2$, so Fh0 and Fℓ0 are not binding for $\delta > \mu^*$. $F\ell T(\delta) - FhU(\delta) = 0$ is reduced to

$$4\delta^4 - 35\delta^3 + 5\delta^2 + 5\delta - 9 = 0. \tag{E.7}$$

Numerically, the unique root of this equation satisfying $\delta > 1$ is $\delta \approx 8.591$. Then $F\ell T(\delta)$ is binding for $\mu^* < \delta \lesssim 8.591$ and $FhU(\delta)$ is binding for $\delta \gtrsim 8.591$.

An equilibrium exists for $\delta > \mu^*$ if there exists some δ satisfying $FhT(\delta) \leq \delta \leq \min\{F\ell T(\delta), FhU(\delta)\}$. $F\ell T(\delta) - FhT(\delta) = 0$ may be rearranged to give to give $4\delta^3 - 23\delta^2 + 12\delta - 8 = 0$, which has a single real solution at $\delta = \mu^*$. Since, for example $F\ell T(6) - FhT(6) \approx 0.049$, $F\ell T(\delta) - FhT(\delta) > 0$ for $\delta > \mu^*$. $FhU(\delta) - FhT(\delta) = 0$ may be rearranged to give $8\delta^5 - 6\delta^4 - 10\delta^3 - 5\delta^2 + 2\delta - 4 = 0$, which numerically has a single real root at $\delta \approx 1.706 < \mu^*$. Since, for example, $FhU(10) - FhT(10) \approx 0.221$, $FhU(\delta) - FhT(\delta) > 0$ for $\delta > \mu^*$ and for $\delta > \mu^*$ there exists some δ satisfying $FhT(\delta) \leq \delta \leq \min\{F\ell T(\delta), FhU(\delta)\}$. □

E.4 Proof of proposition 5.3

By substituting the equilibrium qualities into equation (5.2a), the ratio of equilibrium profit for $\delta > \mu^*$ to profit in the baseline case is

$$\frac{\pi_h^{\delta>\mu*}}{\pi_h^{base}} = \frac{2(4\mu^*-1)^6}{\mu^{*4}(4\mu^*-7)(8-40\mu^*+39\mu^{*2}-4\mu^{*3})}\left(\frac{4\delta(\delta-1)}{(4\delta-1)^2}q_h^* - \frac{q_h^{*2}}{2}\right). \tag{E.8}$$

Equating this to 1 and exploiting the quadratic formula, firm h makes greater profit than in the baseline if

$$\frac{4\delta(\delta-1)}{(4\delta-1)^2} - \sqrt{\frac{16\delta^2(\delta-1)^2}{(4\delta-1)^4} - \frac{\mu^{*4}(4\mu^*-7)(8-40\mu^*+39\mu^{*2}-4\mu^{*3})}{(4\mu^*-1)^6}} \tag{E.9}$$

$$< q_h^* < \frac{4\delta(\delta-1)}{(4\delta-1)^2} + \sqrt{\frac{16\delta^2(\delta-1)^2}{(4\delta-1)^4} - \frac{\mu^{*4}(4\mu^*-7)(8-40\mu^*+39\mu^{*2}-4\mu^{*3})}{(4\mu^*-1)^6}}$$

By the same method it is found that firm ℓ makes greater profit than in the baseline if

$$\frac{\delta^2(\delta-1)}{(4\delta-1)^2} - \sqrt{\frac{\delta^4(\delta-1)^2}{(4\delta-1)^4} - \frac{\delta^2\mu^{*3}(4\mu^*-7)(2-10\mu^*+15\mu^{*2}-4\mu^{*3})}{(4\mu^*-1)^6}} \tag{E.10}$$

$$< q_h^* < \frac{\delta^2(\delta-1)}{(4\delta-1)^2} + \sqrt{\frac{\delta^4(\delta-1)^2}{(4\delta-1)^4} - \frac{\delta^2\mu^{*3}(4\mu^*-7)(2-10\mu^*+15\mu^{*2}-4\mu^{*3})}{(4\mu^*-1)^6}}.$$

□

E.5 Proof of proposition 5.4

(i) The first order conditions of profit given $\frac{q_h}{q_\ell} = \delta$ are

$$\frac{\partial \pi_h^*}{\partial q_h} = \frac{4\delta(\delta-1)}{(4\delta-1)^2} - q_h \tag{E.11a}$$

$$\frac{\partial \pi_\ell^*}{\partial q_\ell} = \frac{(\delta-1)}{(4\delta-1)^2} - \frac{q_h}{\delta}. \tag{E.11b}$$

From equation (E.11a), firm h's profit is maximized at $q_h^* = q_h^{max} = \frac{4\delta(\delta-1)}{(4\delta-1)^2}$. Rearrangement of $FhT(\delta) - q_h^{max} \geq 0$ gives $2\delta + 1 \geq 0$, which holds for all $\delta \geq 1$, so for equilibrium values of q_h and $\delta > \mu^*$, firm h's profit is decreasing in q_h.

From equation (E.11b), firm ℓ's profit is maximized at $q_h^* = q_\ell^{max} = \frac{\delta^2(\delta-1)}{(4\delta-1)^2}$. Rearrangement of $q_\ell^{max} - F\ell T(\delta) \geq 0$ gives $2\delta+1 \geq 0$ which holds for all $\delta \geq 1$. Rearrangement of $q_\ell^{max} - FhU(\delta) \geq 0$ gives $\delta^4 - 8\delta^3 + 2\delta - 1 \geq 0$, which holds for $\delta \gtrsim 7.970$. As $FhU(\delta)$ is only binding for $\delta \gtrsim 8.591$, for equilibrium values of q_h and $\delta > \mu^*$, firm ℓ's profit is decreasing in q_h.

(ii) Aggregate consumer surplus for $\delta > \mu^*$ is

$CS = \int_{\alpha'}^{1} (\alpha q_h - p_h)\, d\alpha + \int_{\alpha''}^{\alpha'} (\alpha q_\ell - p_\ell)\, d\alpha$, where α' (α'') is the taste parameter of the consumer indifferent between q_h and q_ℓ (q_ℓ and not consuming). After integration and the substitution $q_\ell = \frac{q_h}{\delta}$ this becomes $CS = \frac{\delta(4\delta+5)q_h}{2(4\delta-1)^2}$, so that $\frac{\partial CS}{\partial q_h} > 0$.

(iii) Rearranging $\pi_h + \pi_\ell + CS$ gives the total surplus as $TS = \frac{\left(12\delta^2-\delta-2\right)q_h}{2(4\delta-1)^2} - \frac{\left(\delta^2+1\right)q_h^2}{2\delta^2}$. Taking the first order condition and rearranging gives equation (5.7).

$FhU(\delta) - q_h^{TS} \geq 0$ is rearranged to give $4\delta^3 - 11\delta^2 - \delta + 2 \geq 0$ which has the unique root satisfying $\delta \geq 1$ of $\delta \approx 2.775$, above which it holds, thus TS is decreasing for at least some equilibrium qualities of $\delta > \mu^*$. $q_h^{TS} - FhT(\delta) \geq 0$ reduces to $16\delta^4 + 8\delta^3 - 55\delta^2 + 26\delta - 16 \geq 0$, which has a single root satisfying $\delta > 1$ at $\delta \approx 1.439$, above which it holds, and so for $\delta \gtrsim 8.591$ total surplus is increasing in equilibrium values of q_h. $q_h^{TS} - F\ell T(\delta) = 0$ is reduced to

$$8\delta^4 - 62\delta^3 + 24\delta^2 - 7\delta - 2 = 0. \tag{E.12}$$

Let $\delta_{TS} \approx 7.359$ be the unique real root of this equation, so for $\delta \leq \delta_{TS}$ Total surplus is increasing in equilibrium values of q_h and for $\delta > \delta_{TS}$ it is maximized at q_h^{TS}. □

E.6 Proof of proposition 5.5

Consider some q_h, s_h, q_l , s_l such that $\frac{q_h}{q_l} = D(s_T, \delta) > \mu^*$, $s_h > \underline{s}$, $s_l > \underline{s}$ and conditions (i)-(vi) fulfilled. From the derivation of $FhT(\delta)$, $\frac{\partial \pi_h}{\partial q_h} < 0$, so firm h wishes to set $s_h' < s_h$, $q < q_h$. This is the case for all $s_h > \underline{s}$, unless $D(s_T, \delta) \leq \mu^*$. An analogous argument holds for firm l: Consider some $s_h, s_l \in \{(s_h, s_l) : D(s_T, \delta) \leq \mu\}$. Then from the derivation of the standard equilibrium, the only qualities that form an equilibrium are q_h^*, q_l^*. □

E.7 Proof of proposition 5.6

(i) As $\underline{s}' < \underline{s}$ implies $\underline{D}' \leq \underline{D}$ it is sufficient to show that $\frac{\partial \pi_h^*}{\partial D} < 0$ and $\frac{\partial \pi_l^*}{\partial D}$.

Given some threshold D and some equilibrium quality q_h^*, firm h makes profit

$$\pi_h^* = \frac{4D\,(D-1)}{(4D-1)^2} q_h^* - \frac{1}{2} q_h^{*2} \tag{E.13}$$

from which it follows that $\frac{\partial \pi_h^*}{\partial D} = \frac{4(2D+1)}{(4D-1)^3} q_h^* > 0$. Similarly, given D and q_ℓ^* firm ℓ makes profit

$$\pi_\ell^* = \frac{D\,(D-1)}{(4D-1)^2} q_\ell^* - \frac{1}{2} q_\ell^{*2} \tag{E.14}$$

so that $\frac{\partial \pi_\ell^*}{\partial D} = \frac{(2D+1)}{(4D-1)^3} > 0$.

(ii) It is sufficient to show that $\frac{\partial CS}{\partial D} < 0$ for a constant q_h^* and q_ℓ^*. Given some threshold D and firm h producing some equilibrium quality q_h^*, consumer surplus is $CS = \frac{D(4D+5)}{2(4D-1)^2} q_h^*$ so tht $\frac{\partial CS}{\partial D} = -\frac{(28D+5)}{2(4D-1)^3} < 0$. Similarly, given D and firm ℓ producing some equilibrium quality q_ℓ^*, consumer surplus is $CS = \frac{4D+5}{2(4D-1)^2} q_\ell^*$. Thus $\frac{\partial CS}{\partial D} = -\frac{2(4D+11)}{(4D-1)^2} q_\ell^* < 0$. □

Screenshots and instructions for chapter 6 F

F.1 Screenshots

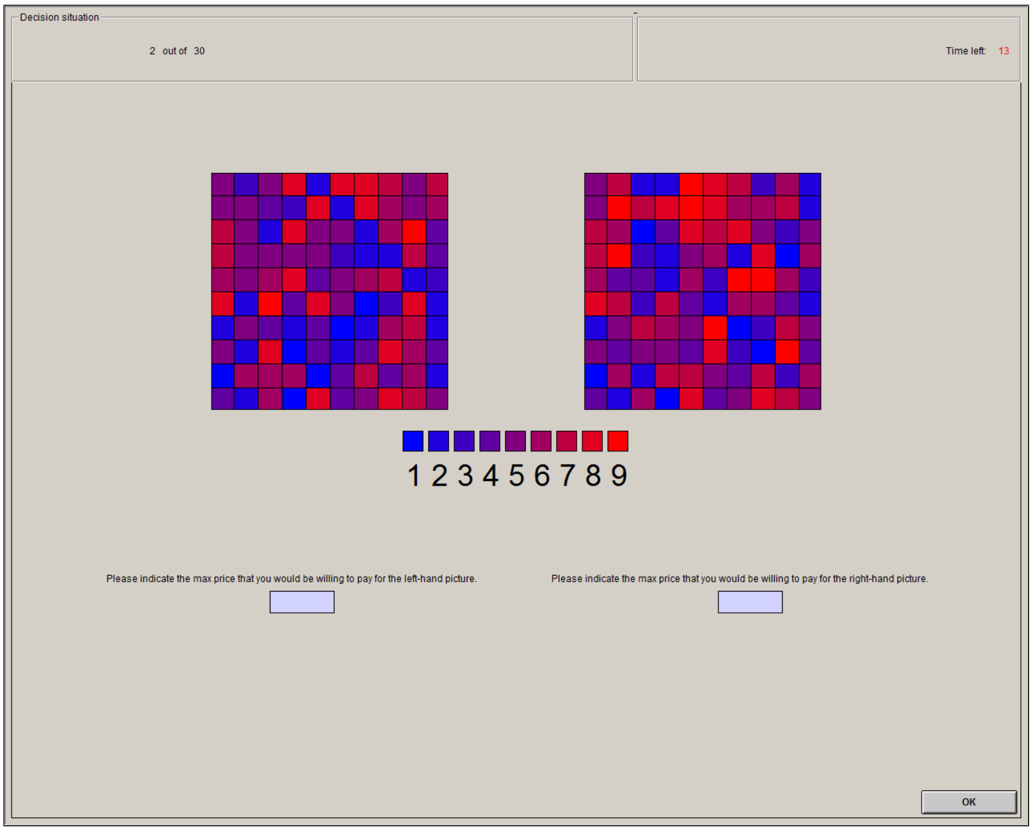
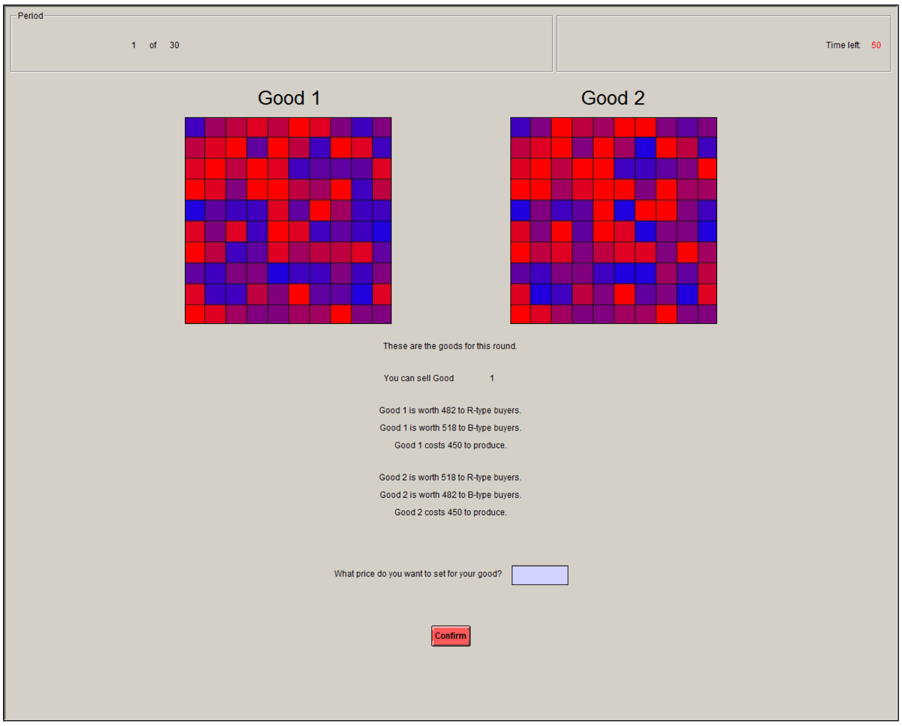


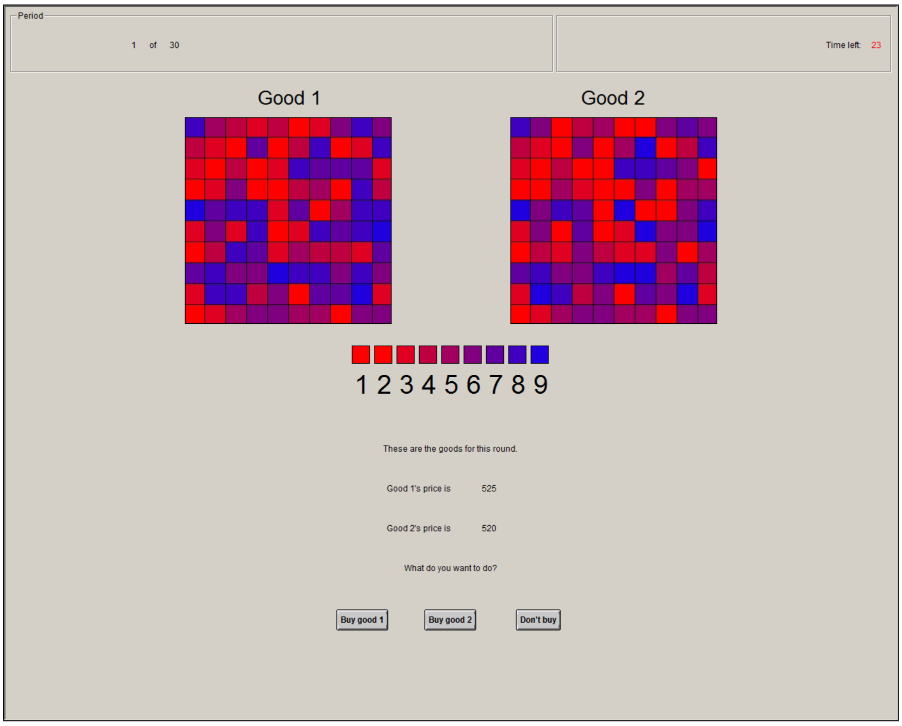
FIGURE F.1: A subject's view of the valuation task (experiment 1)

Figure F.2: Screenshots of stage 1 and 2 (experiment 2)

(a) Stage 1 (seller screen)



(b) Stage 2 (buyer screen)



F.2 Instructions

F.2.1 Experiment 1

Welcome!

This experiment is part of a research project undertaken by researchers from the University of Copenhagen.

It is important that you read the instructions carefully before the experiment starts.

There are three parts to this experiment, of which the first is the longest.

By participating in this experiment, you can earn money. Your payment for all three parts will be added together. In the theoretically possible but extremely unlikely event of you making a loss overall, we have a task to do at the end to make good the loss. (The amount of the loss determines how long you have to work at it.)

Your earnings, plus the show-up fee of kr. 50, will be paid in private at the end of the experiment.

During the experiment you will be confronted with a sequence of decision situations. In each situation you will be asked to make decisions about two pictures, which each have a value between 460 points and 540 points.

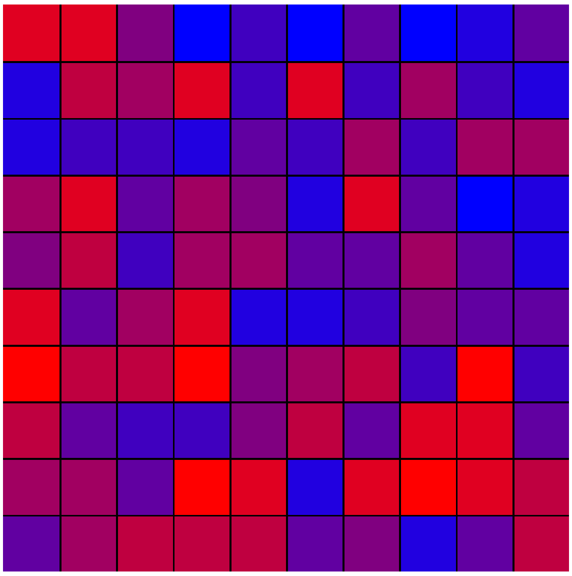
Decision situation

You will be presented with two pictures.

Your task for each is to indicate the max price (in points) that you would be willing to pay for the picture. Notice that the value of the picture implies that your willingness to pay should range between 460 points and 540 points.

Note: There is a time limit of 30 seconds for each decision situation. If you haven't entered anything by the end of 30 seconds, the experiment will move on and you get nothing from that decision stage.

An example of a picture is:



Picture Value

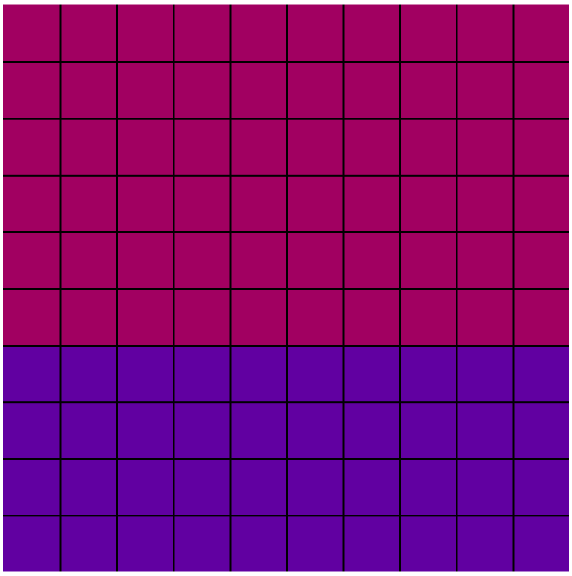
How is the value of a picture determined? Each picture consists of 100 squares. Each square has a colour and each colour has a value. The total value of a picture is the sum of the points over all squares.

Example:

A square like this  is worth 6 and there are 60 of them in the example, giving a total of 360.

A square like this  is worth 4 and there are 40 of them in the example, giving a total of 160.

The value of this picture is then 360 + 160 = 520.



Your Earnings

How is your payoff calculated? The mechanism we use to determine your payoff is explained below. It has one important feature: it is in your best interest to truthfully indicate the max price you would be willing to pay for a picture. You can never do better by indicating a lower or higher price.

The payoff mechanism is as follows:

For each of the shown pictures, we will randomly draw a selling price between 460 and 540 points. The random selling prices are completely independent of the picture's value.

- First, if the random selling price is **lower** than, or equal to your stated willingness to pay, you will **'buy'** the picture. 'Buying' the picture means that the value of the picture is added to your earnings and the random selling price is subtracted.

- Second, if the picture's random selling price is **higher** than your stated willingness to pay, you **do not buy** the picture. Not buying the picture means that nothing is added and nothing is subtracted from your earnings.

- Your earnings (in points) from this experiment will be the sum of your payoffs from all decision situations.

Remember that if you fail to make an enrty within 30 seconds, the experiment will move on and you will earn nothing from that decision stage.

At the end we will convert your earnings in points to money using the following exchange rate: 10 points = 1 DKK.

Detailed payoff examples will be shown on the next page.

Payoff Examples

Example 1

	Left hand picture	Right hand picture
Value	520	495
Your willingness to pay	500	490
Random selling price	465	510

Suppose the underlying values of the pictures, your willingness to pay and the randomly drawn prices were as in the table. You buy the left hand picture, as its randomly drawn price is less than your willingness to pay for it. You don't buy the right hand picture, as its randomly drawn price us higher than your willingness to pay for it. Your payoff is then

Payoff	=	left hand value	-	left hand random selling price
	=	520	-	465
	=	55		

Example 2

Suppose the underlying values of the pictures, your willingness to pay and the random selling prices were as in the table.

	Left hand picture	Right hand picture
Value	525	485
Your willingness to pay	500	480
Random selling price	520	500

For both the left hand and right hand picture, the randomly drawn price is higher than your max willingness to pay for them, so you buy neither of them. Your payoff is simply 0.

Example 3

	Left hand picture	Right hand picture
Value	525	485
Your willingness to pay	525	480
Random selling price	520	470

Suppose the underlying values of the pictures, your willingness to pay and the randomly drawn prices were as in the table.

For both the left hand and right hand pictures, the randomly drawn price is lower than your max willingness to pay for them, so you buy both pictures. Your payoff is then

Payoff	=	left hand value	-	left hand random selling price	+	right hand value	-	right hand random selling price
	=	525	-	520	+	485	-	470
	=	20						

After the experiment has finished you will be able to collect your earnings, plus the show up fee of kr.50, from the experimenter.

In a moment when the experiment starts you will be asked some control questions before the task begins.

The instructions for the other two shorter parts of the experiment will follow later.

F.2.2 Experiment 2

Welcome!

This experiment is part of a research project undertaken by researchers from the University of Copenhagen.

It is important that you read the instructions carefully before the experiment starts.

There are three parts to this experiment, of which the first is the longest.

By participating in this experiment, you can earn money. Your payment for all three parts will be added together. In the theoretically possible but extremely unlikely event of you making a loss overall, we have a task to do at the end to make good the loss. (The amount of the loss determines how long you have to work at it.)

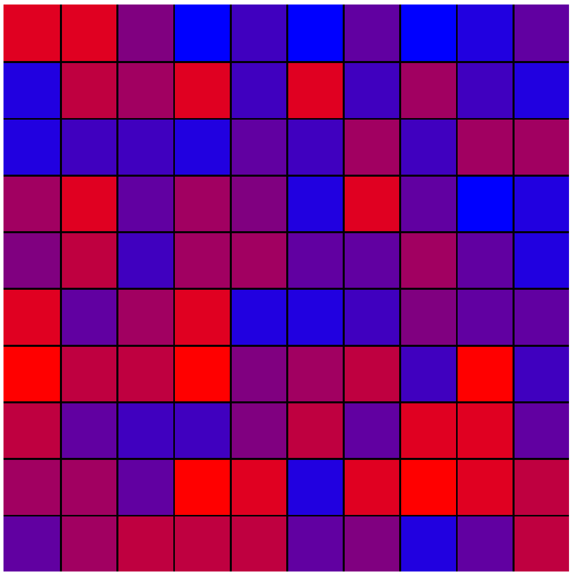
Your earnings, plus the show-up fee of kr. 50, will be paid in private at the end of the experiment.

In this experiment you will be either buying or selling goods in a market. There are two different kinds of roles in this experiment, buyers and sellers. In addition, as will be explained in more detail below, there are two different types of buyers.

In a moment you will randomly be selected into a role and type. That is, we will randomly determine whether you are buyer or seller, and if you are selected to be a buyer, we will randomly determine whether you will be a type R buyer or a type B buyer.

You will keep the same role and type for the whole experiment.

The goods that you will be buying or selling (depending on your role) will be pictures like the one below:

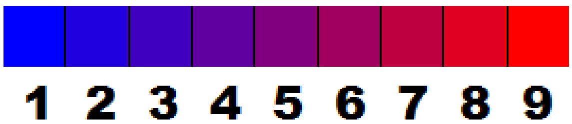


Picture Value

Each picture consists of 100 squares. Each square has a colour and each colour has a value. The value of a picture is the sum of the points over all squares.

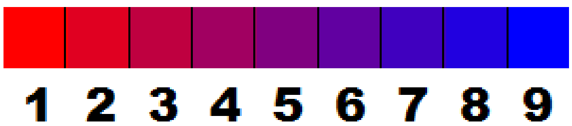
Importantly, however, the different colours have different values for the two different kinds of buyers in the experiment.

R-type buyers value the colours of the different squares according to the following scale:



so they value "redder" colours more than "bluer" colours."

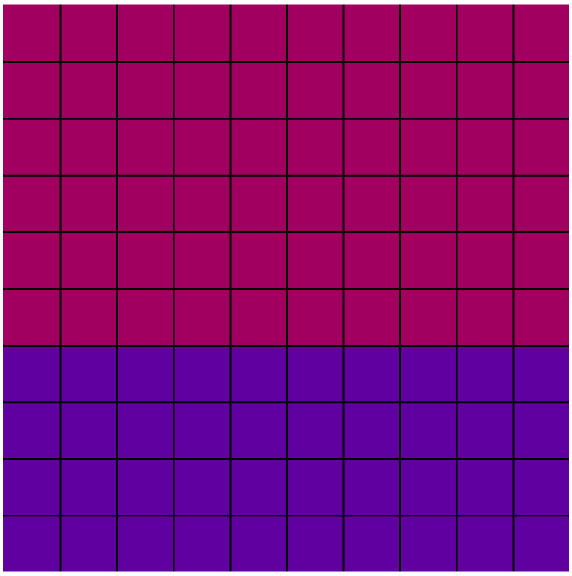
B-type buyers value the colours of the different squares according to the following scale:



so they value "bluer" colours more than "redder" colours.

This implies that the same picture has a different value for R- and B-type buyers in this experiment.

Example



For R-type buyers, a square like this  is worth 4 and there are 40 of them in the example displayed above, giving a total of 160.

For R-type buyers, a square like this  is worth 6 and there are 60 of them in the example, giving a total of 360.

Thus, the value of this picture to R-type buyers is $160 + 360 = 520$.

On the other hand, for B-type buyers, a square like this  is worth 6 and there are 40 of them in the example, giving a total of 240.

For B-type buyers, a square like this  is worth 4 and there are 60 of them in the example, giving a total of 240.

Thus, the value of this picture to B-type buyers is $240 + 240 = 480$.

As mentioned above sellers as well as R- and B-type buyers interact with each other in a market.

Market

There will be 30 market periods in which two sellers and two buyers, one R-type and one B-type, are matched to each other. A single market period works like this:

(1) Sellers are shown the two pictures being traded, told their value to R-type and B-type buyers, and they are informed about which picture of the two they can sell.

(2) Sellers set a price for their picture, without knowing what price the other seller sets. There is a time limit of 60 seconds for this task.

(3) Buyers are shown the pictures (but not told their values) and decide which picture of the two to buy, or not to buy at all. Buyers have 30 seconds to decide. If they haven't entered anything by the end of 30 seconds, the experiment will move on and they get nothing for that period.

After each period we will randomly select a new set of people you interact with in the market, i.e. you will not interact with the same people each time.

Your Earnings

Sellers: For each picture sold a seller earns the price that was paid by the buyer minus a cost of 450.

Seller Earnings = Price – A cost of 450

Buyers: For each picture bought, buyers earn the value of it minus the price they paid.

Buyer Earnings = Value to the buyer – Price

Remember, the 'Value to the buyer' for each picture depends on the type of the buyer.

At the end we will convert your earnings in points to money using the following exchange rate: 10 points = 1 DKK.

In a moment when the experiment starts, you will be assigned your role, and then you will be asked some control questions before the market begins.

The instructions for the other two shorter parts of the experiment will follow later.

F.2.3 Gamble choices

Table F.1: Risk preference elicitation task, gamble list

Choice (50/50 Gamble)	Low payoff	High payoff	Expected return
1	28	28	28
2	24	36	30
3	20	44	32
4	16	52	34
5	12	60	36
6	2	70	36

Proofs for chapter 7 G

G.1 Proof of proposition 7.1

Substituting $q = 0$ into equations (7.7) and (7.9) gives $p_{over}\left(t|\,\omega_0 = h\right) = 0$, $p_{under}\left(t|\,\omega_0 = h\right) = 0$. Substituting in $q = 1$ also gives $p_{over}\left(t|\,\omega_0 = h\right) = 0$, $p_{under}\left(t|\,\omega_0 = h\right) = 0$.

G.2 Proof of proposition 7.2

If $0 < q < 1$, $\lim_{t\to\infty}\left(1-q\right)^t = \lim_{t\to\infty}\left(1-2q\right)^t = 0$, from which it follows that $\lim_{t\to\infty} p_{over}\left(t|\,\omega_0 = h\right) = \frac{1}{4}\left(1-q\right)\left(1+2\cdot 0-0\right)\left(1-0\right) = \frac{1}{4}\left(1-q\right)$ and $\lim_{t\to\infty} p_{under}\left(t|\,\omega_0 = h\right) = \frac{1}{4}\left(1-q\right)\left(1+0-2\cdot 0\right)\left(1+0\right) = \frac{1}{4}\left(1-q\right)$.

G.3 Proof of proposition 7.3

Equation (7.21) follows directly from preceding argument. In period τ the set of possible lags is $\{\ell+\tau\}\bigcup\{m : 0 \leq m < \tau\}$. With probability q_β^τ consumers do not pay attention to price level in any $t \leq t' < \tau$, in which case the lag at the beginning of period τ is $\ell+\tau$ and the firm sets $v^*\left(\ell+\tau\right)$. Otherwise the lag is m and the firm sets $v^*\left(m\right)$ if consumers were attentive to price level m periods ago but were not since, which happens with probability $\left(1-q_\beta\right)q_\beta^m$. Equation (7.22) is then trivial to calculate.

G.4 Proof of proposition 7.4

$$\begin{aligned}\Delta^{-}(\ell+1) &= v^{*}(\ell) - v^{*}(\ell) \\ &= \left(\frac{1-q_\alpha}{1+i-q_\alpha q_\beta \delta}\right)^2 \left(\left(\frac{q_\beta}{(1+i)^{\ell+1}} + \frac{1-q_\beta}{1-q_\alpha \delta}\right)^2 - \left(\frac{q_\beta}{(1+i)^{\ell}} + \frac{1-q_\beta}{1-q_\alpha \delta}\right)^2\right)\end{aligned} \tag{G.1}$$

which after some algebra becomes equation (7.25).

$$\begin{aligned}\Delta^{+}(\ell) &= v^{*}(0) - v^{*}(\ell) \\ &= \left(\frac{1-q_\alpha}{1+i-q_\alpha q_\beta \delta}\right)^2 \left(\left(q_\beta + \frac{(1-q_\beta)(1+i)}{(1-q_\alpha \delta)}\right)^2 - \dots\right. \\ &\quad \left. \dots - \left(\frac{q_\beta}{(1+i)^{\ell}} + \frac{(1-q_\beta)(1+i)}{(1-q_\alpha \delta)}\right)^2\right)\end{aligned} \tag{G.2}$$

which after some algebra becomes equation (7.26).

G.5 Proof of proposition 7.5

The firm sets nominal price equal to consumers' nominal WTP, so for the nominal price to be unchanged between t and $t+\tau$ requires $\tilde{P}_t \tilde{s}_t = \tilde{P}_{t+1} \tilde{s}_{t+1} = \dots = \tilde{P}_{t+\tau} \tilde{s}_{t+\tau}$. Each one of these equalities requires consumers to pay attention to neither price level nor unit size, which happens with probability $q_\alpha q_\beta$, thus all hold with probability $(q_\alpha q_\beta)^\tau$.

G.6 Proof of lemma 7.1

$$\frac{\partial \mathbb{E}U}{\partial p_c(t')} = \frac{\delta^T p_d \left(1-\delta + p_d \left(\delta - \delta^T\right)\right)(u-v)}{(1-\delta)\left(1-\delta + p_d\left(\delta - p_c(t')\,\delta^T\right)\right)^2} \tag{G.3}$$

which is positive. $p_c(t')$ is decreasing in t', so the individual maximizes $\mathbb{E}U$ by setting t' as low as possible and seeking immediate treatment.

G.7 Proof of proposition 7.6

(i) The expected treatment time is $\sum_{t=1}^{\infty} q(\dot{s})(1-q(\dot{s}))^{t-1} t = q(\dot{s})^{-1}$. As $q(\dot{s})$ is increasing in $\dot{s}$, expected treatment time is decreasing in $\dot{s}$. The probability of cure is decreasing in expected treatment time, implying it is increasing in $\dot{s}$.

(ii) $\frac{\partial \mathbb{E}U}{\partial \mathbb{E}p_c(\dot{s})} = \frac{\delta^T p_d \left(1-\delta+p_d\left(\delta-\delta^T\right)\right)(u-v)}{\left(1-\delta+p_d(\delta-\delta^T \mathbb{E}p_c)\right)^2} > 0$ and so as from part (i) $\mathbb{E}p_c$ is increasing in $\dot{s}$, $\mathbb{E}U$ is increasing in $\dot{s}$.

G.8 Proof of proposition 7.7

A sufficient condition for a beneficial screening program to exist is that $\left.\frac{\partial \mathbb{E}U}{\partial f}\right|_{f=0} > 0$. Let $\mathbb{E}W$ be the first term of equation (7.33), i.e. it is expected utility net of screening costs. Then $\frac{\partial \mathbb{E}W}{\partial f} = \frac{\partial \mathbb{E}W}{\partial \mathbb{E}p_c}\frac{\partial \mathbb{E}p_c}{\partial f} - k$. Then if $\left.\frac{\partial \mathbb{E}W}{\partial \mathbb{E}p_c}\frac{\partial \mathbb{E}p_c}{\partial f}\right|_{f=0} > 0$, there is some $\hat{k}$ such that $\left.\frac{\partial \mathbb{E}U}{\partial f}\right|_{f=0} = 0$ and a beneficial program exists for all $k < \hat{k}$. From equation (7.33),

$$\left.\frac{\partial \mathbb{E}W}{\partial \mathbb{E}p_c}\right|_{f=0} = \frac{\delta^T p_d \left(1-\delta+p_d\left(\delta-\delta^T\right)\right)(u-v)}{(1-\delta)\left(1-\delta(1-p_d)-\delta^T p_d \left.\mathbb{E}p_c\right|_{f=0}\right)^2} > 0 \tag{G.4a}$$

$$\left.\frac{\partial \mathbb{E}p_c}{\partial f}\right|_{f=0} = \sum_{t=1}^{T} (1-q(\dot{s}))^{t-1}(1-q(\dot{s})\,t)\,p_c(t). \tag{G.4b}$$

To see that equation (G.4b) is positive, note that the terms in the summation are decreasing in t and negative only if $t > \frac{1}{q(\dot{s})}$. If $p_c(t)$ is constant, the sum becomes $T(1-q(\dot{s}))^T p_c > 0$. As $p_c(t)$ is decreasing in t, it implies a lower relative weight on lower terms in the summations, and thus it is positive.