



# PhD Thesis

Jeppe Druedahl

## Microfounding Consumption

Uncertainty, liquidity and heterogeneity

**Supervisor:** Christian Groth

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# Foreword

Completing this PhD thesis has been a journey on a long, and especially very winding road. My original working title was “Explaining recessions as co-ordination failures”, and I aimed at writing an old style monograph. In the first year and a half I consequently studied New Keynesian Dynamic Stochastic General Equilibrium models. However, I became increasingly dissatisfied with their lack of endogenous propagation and amplification mechanisms. My interests therefore turned toward models of household and firm behavior focusing on the importance of idiosyncratic uncertainty, heterogeneity and non-convexities such as transaction costs. The first product of this interest was my master thesis on the lumpy investment behavior of firms, which especially taught me a great deal about how to solve dynamic programming models in practice.

A real turning point in my studies was a short course taught by Professor Christopher Carroll titled “Microfoundations of Consumption and Saving in an Uncertain World”, which I attended in Oslo in June 2013. This really got me interested in how to microfound the consumption behavior of households using models with a detailed treatment of liquidity issues and heterogeneity – a common theme running through all the papers included in this thesis.

Help from a lot of people have been instrumental for the completion of this thesis. I am thankful for good cooperation with my Jørgensen co-authors, and I wish to thank my supervisor Christian Groth with whom I have had many good discussions; not just related to my own work, but also on the general state of macroeconomics. I likewise want to thank Gianluca Violante for inviting me to visit New York University in the autumn of 2014, where I learned a lot from short intense discussions with him, and through participation in seminars and reading groups. Without the regular “coffee bus” everything would also have been a lot less fun; especially I need to thank Anders Munk-Nielsen, whom I have often shared office with – both for good times with beer, food and board games, and for insightful discussions on economics in general, and computational economics in particular. Good discussions with Thomas Høgholm Jørgensen on all the papers in this thesis have also been very valuable to me.

Finally, I want to thank my family and friends, and especially my girlfriend Julie, for always being there for me.

Jeppe Druedahl  
Copenhagen, August 2015

# English Summary

## Overview

This PhD thesis consists of four self-contained chapters on microfounding the consumption behavior of households in the presence of idiosyncratic income and credit risk, transaction costs, and preference heterogeneity.

The *first* chapter addresses the credit card debt puzzle by constructing a model where it can be resolved by a combination of: (a) specifying credit cards as long-term revolving debt contracts which are partly irrevocable from the lender side, and (b) shocks to the availability of new credit.

The *second* chapter relies on a similar specification of the mortgage contract as being both long-term and in gross debt, to improve our understanding of the life-cycle dynamics of the home ownership rate.

The *third* chapter estimates a non-parametric distribution of preferences using imputed consumption data from the Danish income and wealth registers, and shows that the estimated degree of heterogeneity in impatience and risk aversion is far from enough to explain the observed wealth inequality.

The final and *fourth* chapter shows that adding cyclical variation in idiosyncratic income risk, and in the tightness of the collateral constraint, strongly amplify the model-implied cyclical drop in the share of households who adjust their durable stock during recessions in a standard buffer-stock consumption model extended with a durable good, where trading is subject to transaction costs.

All the four chapters rely on state-of-the-art computational methods to efficiently solve the proposed models, which due to the presence of non-convexities typically are non-standard.

## 1. Precautionary Borrowing and the Credit Card Debt Puzzle

*with Casper Nordal Jørgensen*

This chapter addresses the credit card debt puzzle using a generalization of the buffer-stock consumption model with long-term revolving debt contracts. Closely resembling actual US credit card law, we assume that card issuers can always deny their cardholders access to new debt, but that they cannot demand immediate repayment of the outstanding balance. Hereby, current debt can potentially soften a household's borrowing constraint in future periods and thus provides extra liquidity. We show that for some intermediate values of financial net worth it is indeed optimal for households to simultaneously hold positive gross debt and positive

gross assets even though the interest rate on the debt is much higher than the return rate on the assets. Including a risk of being excluded from new borrowing which is positively correlated with unemployment, we are able to simultaneously explain a substantial share of the observed borrower-saver group and match a high level of liquid net worth.

## **2. The Demand for Housing over the Life-Cycle under Long-Term Gross Debt Contracts**

This chapter generalizes the model of individual demand for housing over the life-cycle in Attanasio, Bottazzi, Low, Nesheim and Wakefield (2012) by formulating the long-term debt contracts in *gross* terms instead of in *net* terms. This more realistic market structure have important implications for the model dynamics because it enables the households to save in financial assets instead of requiring them to save by increasing their mortgage repayments. Moreover the potential for such *precautionary balance sheet expansions* make the households able to self-insure more optimally and thus increase their ex ante expected welfare. Quantitatively the welfare gain is largest when there is no mortgage spread and no forced mortgage repayments. Qualitatively the results are robust to a substantial mortgage spread and forced repayments if just the households are impatient enough. Introducing a combination of proportional and fixed (re)financing costs does likewise not affect the central results.

## **3. Heterogeneous Preferences and Wealth Inequality**

*with Thomas Høgholm Jørgensen*

In this chapter we perform a maximum likelihood estimation of a standard life cycle model allowing for non-parametric heterogeneity in patience and risk aversion using high quality Danish register data. We find substantial preference heterogeneity within educational strata and positive correlation between patience and risk aversion. Across the educational strata, higher educated households are found to be more patient and more risk averse. Although the model fits the average life cycle profiles of consumption and wealth quite well, it cannot explain the observed degree of wealth dispersion over the life cycle. This result suggests that heterogeneity in patience and risk aversion only explains a rather limited part of the observed wealth inequality.

## **4. Business Cycle Fluctuations in the Demand for Durables**

This chapter studies the effect of cyclical variation in both idiosyncratic income risk and in the tightness of the collateral constraint on the demand for consumer

durables in a generalized buffer-stock consumption model. The proposed model includes both transaction costs, an outside option of renting, non-separable utility, and taste shocks. It is shown that empirically plausible fluctuations in income risk and in the down payment ratio, roughly double the model-implied cyclical drop in the share of households who adjust their durable stock during recessions compared to in expansions. This is moreover achieved without any cyclical decrease in the intensive margin in terms of expenditures per purchase, which is contrarily the case when the drop is induced by e.g. lower income growth or higher unemployment.

## Danish Summary

Denne ph.d. afhandling består af fire selvstændige kapitler, der alle omhandler husholdningers forbrugsadfærd i lyset af idiosynkratisk usikkerhed, transaktionskostninger og præference heterogenitet.

Det *første* kapitel omhandler det såkaldte “credit card debt puzzle” [”kreditkortgældsgåden”], og opbygger en model, hvor det kan forstås som en konsekvens af: (a) kreditkortgæld modelleret som en langsigtet løbende gældskontrakt som kun delvist kan ophæves af udlåner, og (b) stød til muligheden for at optage ny gæld.

Det *andet* kapitel anvender en lignende specifikation af realkreditlån som langsigtede og i bruttotermer til at forbedre vores forståelse af ændringerne i andelen af boligejere over livscyklussen.

Det *tredje* kapitel estimerer en ikke-parametrisk fordeling af præference heterogenitet ved brug af imputeret forbrugsdata fra de danske registre over hele befolkningens indkomster og formuer, og viser, at den fundne grad af præference heterogenitet langt fra kan forklare den observerede formueulighed.

Det sidste og *fjerde* kapitel viser, at cyklisk variation i graden af idiosynkratisk indkomstusikkerhed og i lånemulighederne kraftigt forstærker de cykliske udsving i andelen af husholdninger, som køber varige goder, i en standard forbrugsmodel udvidet med et varigt gode og transaktionskostninger.

Alle fire kapitler gør brug af de nyeste numeriske metoder til at løse de opstillede modeller, der grundet ikke-konveksiteter typisk ikke kan løses effektivt med den almindelige værktøjskasse.



# Chapter 1

Precautionary Borrowing and  
the Credit Card Debt Puzzle

# Precautionary Borrowing and the Credit Card Debt Puzzle\*

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August 31, 2015

## Abstract

This paper addresses the credit card debt puzzle using a generalization of the buffer-stock consumption model with long-term revolving debt contracts. Closely resembling actual US credit card law, we assume that card issuers can always deny their cardholders access to new debt, but that they cannot demand immediate repayment of the outstanding balance. Hereby, current debt can potentially soften a household's borrowing constraint in future periods and thus provides extra liquidity. We show that for some intermediate values of financial net worth it is indeed optimal for households to simultaneously hold positive gross debt and positive gross assets even though the interest rate on the debt is much higher than the return rate on the assets. Including a risk of being excluded from new borrowing which is positively correlated with unemployment, we are able to simultaneously explain a substantial share of the observed borrower-saver group and match a high level of liquid net worth.

**Keywords:** Credit Card Debt Puzzle, Precautionary Saving, Consumption.

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# 1 Introduction

Beginning with [Gross and Souleles \(2002\)](#) it has been repeatedly shown that many households persistently have both expensive credit card debt and hold low return liquid assets. This apparent violation of the no-arbitrage condition has been termed the “credit card debt puzzle”, and no resolution has yet been generally accepted (see e.g. the surveys by [Tufano \(2009\)](#) and [Guiso and Sodini \(2013\)](#)).

This paper suggests a new explanation of the puzzle based on *precautionary borrowing*. We begin from the observation that credit card debt is actually a long-term revolving debt contract. Specifically under current US law the card issuer can cancel a credit card at any time, and thus instantly stop the card holder from accumulating additional debt. Contrarily the card issuer *cannot* force the card holder to immediately pay back the remaining balance. Depending on the specific credit card agreement the issuer might be able to increase the minimum payment somewhat, but basically the credit card debt is transformed into an installment loan.<sup>1</sup>

We add such long-term revolving debt contracts, which are partially irrevocable from the lender side, to an otherwise standard buffer-stock consumption model a la [Carroll \(1992, 1997, 2012\)](#). Hereby households gain a motive for *precautionary borrowing* because current debt can potentially relax the borrowing constraint in future periods. For equal (and risk-less) interest rates on debt and assets, the households will therefore always accumulate as much debt as possible maximizing the option value of having a large gross debt. In the more plausible case of a higher interest rate on debt than on assets, there is a trade-off between the benefit of the extra liquidity provided by the debt, and the net cost of the balance sheet expansion.

We further amplify the motive for precautionary borrowing by including credit risk in the model. Specifically we assume that households in any given period might be excluded from new borrowing, and that the risk of this increases under unemployment. The US *Consumer Financial Protection Bureau (CFBP)* shows in its “CARD Act Report” that “over 275 million accounts were closed from July 2008 to December 2012, driving a \$1.7 trillion reduction in total [credit] line” (p. 56, October 2013). It is not clear to which extend this was a demand or supply effect, but anecdotal evidence suggests that the credit card companies unilaterally

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<sup>1</sup> We thank the *National Consumer Law Center* and the *Consumer Financial Protection Bureau* for help in clarifying the rules for us.

changed their lending during the Great Recession, and that the supply effect thus dominated. Consequently, getting a credit card closed seems to be something a rational household should fear. Naturally, households might have an outside option of getting a new credit card at another issuer, but if a household is simultaneously hit by unemployment this might prove impossible.

Based on a careful calibration, we show numerically that there exists a range of intermediate values of net worth for which it is indeed optimal for the households to simultaneously hold positive gross debt and positive gross assets, even though the interest rate on the debt is much higher than the return rate on the assets. This is especially true when we assume that bad income shocks are positively correlated with a high risk of a fall in the availability of new credit. Beyond this, the parametric robustness of our results are rather strong, and we can explain a large part of the observed puzzle group of borrower-savers while matching central moments from the U.S. *Survey of Consumer Finance* (SCF) including a high level of liquid net worth. This indicates that precautionary borrowing is central in understanding the credit card debt puzzle.

We are somewhat cautious in precisely quantifying the importance of precautionary borrowing, because our model for computational reasons does not include illiquid assets (e.g. houses). It is thus not able to match the empirical facts on *total* net worth without muting the precautionary motive completely. Note, however, that [Kaplan and Violante \(2014\)](#) have recently shown that a buffer-stock model with an illiquid asset, subject to transaction costs, can generate a significant share of *wealthy hand-to-mouth* households while still matching total net worth moments. We hypothesize that both poor and wealthy hands-to-mouth households would also rely on precautionary borrowing, and that our results are thus at least qualitatively robust to extending our model in this direction.

The importance of going beyond one-period debt contracts has naturally been noted before. Closest to our paper are [Attanasio, Leicester and Wakefield \(2011\)](#), [Attanasio, Bottazzi, Low, Nesheim and Wakefield \(2012\)](#), [Chen, Michaux and Roussanov \(2013\)](#) and [Halket and Vasudev \(2014\)](#) who all introduce long-term mortgage contracts, and [Alan, Crossley and Low \(2012\)](#) who model the “credit crunch” of 2008 in terms of a drying up of new borrowing (a flow constraint) instead of a recall of existing loans (the typical change in the stock constraint).<sup>2</sup>

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<sup>2</sup> Note that [Alan, Crossley and Low \(2012\)](#) use the term “precautionary borrowing” (borrowing for a rainy day) in a somewhat different fashion than we do because the second asset in their model is a high return risky asset. This e.g. implies that wealthy households also blow up

To the best of our knowledge, [Fulford \(2015\)](#) is the only other paper investigating the importance of multi-period debt contracts for the credit card debt puzzle.<sup>3</sup> Our approach differs from his in a number of important ways. Firstly his model does not include any forced repayment schedule and households are thus (unrealistically) allowed to hold on to once accumulated debt forever. Secondly our formulation of the income process better mimics reality by taking into account *permanent* shocks and *non-zero* income growth which both usually are important in models with a precautionary motive. Consequently our model nests the standard buffer-stock model as a limiting case, while his does not. Thirdly we allow the risk of losing access to the credit market to be positively correlated with unemployment. We show that this is empirically relevant and quantitatively important for explaining a size-able puzzle group. This is especially relevant because introducing permanent shocks strengthens the general precautionary saving motive making the households accumulate a precautionary fund, which diminishes the need for precautionary borrowing and reduces the size of the puzzle group. Fourthly, we are able to explain a large part of the observed puzzle group and simultaneously match a high level of mean liquid net worth, and do so with a more plausible discount factor of 0.90 while [Fulford \(2015\)](#) use a very low discount factor of 0.794.

The paper is structured as follows. Section 2 discusses the related literature. Section 3 presents the model and describes the the solution algorithm briefly. Some stylized facts are presented in section 4 to which the model is calibrated in section 5. Section 6 presents the central results. The welfare gain of the potential for precautionary borrowing is quantified in section 7 and various robustness checks are performed in section 8. Section 9 concludes. Some details are relegated to the appendices A and B.

## 2 Related Literature

### 2.1 Empirical Evidence

[Gross and Souleles \(2002\)](#) showed that in the 1995 *Survey of Consumer Finance* (SCF), and in a monthly sample of credit card holders from 1995-98, almost all

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their balance sheet by taking loans to invest in the risky asset.

<sup>3</sup> We were only made aware of the working paper version of his paper after writing the first draft of the present paper.

households with credit card debt held low return liquid assets (e.g. they had funds in checking or saving accounts). In itself this might not be an arbitrage violation, but could be a pure timing issue if the interview took place just after pay day and just before the credit card bill was due. However, a third of their sample held liquid assets larger than one month's income; without any further explanation this certainly seems to be an arbitrage violation.

Their result has been found to be robust to alternative definitions of the puzzle group<sup>4</sup> and stable across time periods (see [Telyukova and Wright \(2008\)](#), [Telyukova \(2013\)](#), [Bertaut, Haliassos and Reiter \(2009\)](#), [Kaplan, Violante and Weidner \(2014\)](#) and [Fulford \(2015\)](#)). [Telyukova \(2013\)](#) e.g. utilizes certain questions in the SCF to ensure that the households in the puzzle group had credit card debt left over after the last statement was paid, and that they either only occasionally or never repay their balance in full. Recently [Gathergood and Weber \(2014\)](#) has shown that the puzzle is also present in UK data, and that the puzzle group also has many and large expensive installment loans (e.g. car loans).<sup>5</sup>

Across samples and time periods the interest rate differential between the credit card debt and the liquid assets considered has typically been around 8-12 percentage points, and thus economically very significant. Depending on the correction for timing this implies that the net cost of the expanded balance sheets of the puzzle group has been calculated to be in the range of 0.5-1.5 percent of household income.

## 2.2 Other Theoretical Explanations

A number of different rational and behavioral explanations of the credit card debt puzzle has been suggested in the literature. First, [Gross and Souleles \(2002\)](#) informally suggested that a behavioral model of either self/spouse-control or mental accounting might be necessary to explain the puzzle.<sup>6</sup> [Bertaut and Haliassos \(2002\)](#),

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<sup>4</sup> We denote the group of households simultaneously holding both liquid assets and credit card debt as *the puzzle group*.

<sup>5</sup> Looking over the life-cycle the puzzle group is smallest among the young (below 30) and old (above 60). Puzzle households are typically found to be in the middle of the income distribution and have at least average education and financial literacy. Many have sizeable illiquid wealth (e.g. housing and retirement accounts). There is also some evidence of persistence in puzzle status, and in total it thus seems hard to explain the puzzle as a result of simple mistakes or financial illiteracy.

<sup>6</sup> Note that behavioral models with hyperbolic discounting and a present bias such as [Laibson, Repetto and Tobacman \(2003\)](#) can explain that households with credit card debt has illiquid

Haliassos and Reiter (2007) and Bertaut, Haliassos and Reiter (2009) formalized this insight into an accountant-shopper model where a fully rational accountant tries to control an impulsive (i.e. more impatient) fully rational shopper (a different self or a spouse). The shopper can only purchase goods with the credit card which has an upper credit limit, and the accountant thus has a motive to not use all liquid assets to pay off the card balance in order to limit the consumption possibilities of the shopper. Gathergood and Weber (2014) provides some empirical evidence that a large proportion of households in the puzzle group appears to be impulsive spenders and heavy discounters of the future. A fundamental problem with this solution of the puzzle, however, is that it is not clear why the accountant cannot utilize cheaper control mechanisms such as adjusting the credit limit or limiting the shopper's access to credit cards. Furthermore many households with credit cards also have debit cards, which imply that the shopper in practice has direct access to at least some of the household's liquid assets.

Second, beginning with Lehnert and Maki (2007), and continuing with Lopes (2008) and latest Mankart (2014), it has suggested that US bankruptcy laws might make it optimal for households to strategically accumulate credit card debt in order to purchase exemptible assets in the run up to a bankruptcy filing. Even though state level variation in the size of the puzzle group and exemption levels seems to support this explanation, the empirical power seems limited because it is only relevant relatively shortly before a filing.<sup>7</sup> Moreover many households in the puzzle group have both significant financial assets (e.g. bonds and stocks) and non-financial assets (e.g. cars and houses), and generally few households ever file for bankruptcy. Finally it is far from obvious that such a motive for strategic accumulation of exemptible assets can explain the evidence from the UK (see Gathergood and Weber (2014)) which generally has more creditor friendly bankruptcy laws.

A third resolution of the puzzle has been presented by Telyukova (2013) (see also Telyukova and Wright (2008) and Zinman (2007)). She argues that many expenditures (e.g. rents and mortgage payments) can only be paid for by using cash, and that households thus have a classical Hicksian motive for holding liquid assets despite having expensive credit card debt. The strength of this demand for

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assets, but not that they hold fully liquid assets.

<sup>7</sup> Mankart (2014) notes that debt and cash-advances made shortly before the bankruptcy filing (60 or 90 days depending on the time period) are not dischargeable above a rather low threshold.

liquidity is amplified in her model by rather volatile taste shocks for goods that can only be paid for with cash (e.g. many home and auto repairs). It is naturally hard to identify these fundamentally unobserved shocks and their size in the data. A more serious empirical problem is that the use of credit cards has become much more widespread in the last 20 years; in the model this should imply a fall in the size of the puzzle group not seen in the data. Adding a (costly) cash-out option on the credit card to the model, as is now common, could also further reduce the implied size of the puzzle group. In total, this demand for cash might certainly be a contributing factor, but it seems unlikely that it is the central explanation of the credit card debt puzzle. Finally, note that in a model with both a Hicksian motive for holding liquid assets and a precautionary borrowing motive, the two would reinforce each other.

### 3 Model

#### 3.1 Bellman Equation

We consider potentially infinitely lived households characterized by a vector,  $\mathbf{S}_t$ , of the following state variables: end-of-period gross debt ( $D_{t-1}$ ), end-of-period gross assets ( $A_{t-1}$ ), market income ( $Y_t$ ), permanent income ( $P_t$ ), an unemployment indicator,  $u_t \in \{0, 1\}$ , and an indicator for whether the household is currently excluded from new borrowing,  $x_t \in \{0, 1\}$ . In each period the households choose *consumption*,  $C_t$ , and *debt*,  $D_t$ , to maximize expected discounted utility.

Postponing the specification of the exogenous and stochastic income process to section 3.3, the household optimization problem is given in recursive form by

$$V(\mathbf{S}_t) = \max_{D_t, C_t} \frac{C_t^{1-\rho}}{1-\rho} + \beta \cdot \mathbb{E}_t [V(\mathbf{S}_{t+1})] \quad (3.1)$$

s.t.

$$A_t = (1 + r_a) \cdot A_{t-1} + Y_t - C_t \quad (3.2)$$

$$N_t = A_t - D_t \quad (3.3)$$

$$\underbrace{-r_d \cdot D_{t-1}}_{\text{interest}} - \underbrace{\lambda \cdot D_{t-1}}_{\text{installment}} + \underbrace{(D_t - (1 - \lambda) \cdot D_{t-1})}_{\text{new debt}}$$

$$D_t \leq \max \left\{ \underbrace{(1 - \lambda) \cdot D_{t-1}}_{\text{old contract}}, \mathbf{1}_{x_t=0} \cdot \underbrace{(\eta \cdot N_t + \varphi \cdot P_t)}_{\text{new contract}} \right\} \quad (3.4)$$

$$A_t, D_t, C_t \geq 0 \quad (3.5)$$



where  $\rho$  is the risk aversion coefficient,  $\beta$  is the discount factor,  $r_a$  is the (real) interest rate on *assets*,  $r_d$  is the (real) interest rate on *debt* and  $\lambda \in [0, 1]$  is the *minimum payment due rate*. Equation (3.2) is the budget constraint, (3.3) defines end-of-period (financial) net worth, and (3.4) is the borrowing constraint. The model is closed by assuming that the households are required to “die without debt” (i.e.  $N_T \geq 0$  in some infinitely distant terminal period  $T \rightarrow \infty$ ). We only cover the case  $r_d > r_a$ . We denote the optimal debt and consumption functions by  $D^*(\mathbf{S}_t)$  and  $C^*(\mathbf{S}_t)$ .

We assume that  $x_t$  transitions according to a first order Markov process. The (unconditional) risk of losing access to the credit market is given by  $\pi_{x,*}^{lose}$ , and the chance of re-gaining access is given by  $\pi_{x,*}^{gain}$ . Conditional on unemployment we assume that the risk of losing access to the credit market is given by  $\pi_{x,u}^{lose} = \chi_{lose} \cdot \pi_{x,w}^{lose}$ , where  $\pi_{x,w}^{lose}$  is the risk of losing access conditional on employment (in our calibration we choose  $\chi_{lose}$  and let  $\pi_{x,w}^{lose}$  adjust to match the chosen unconditional transition probabilities).

### 3.2 The Borrowing Constraint

Our specification of the debt contract is obviously simplistic, but it serves our purpose, and only add one extra state variable to the standard model. If  $\eta > 0$  asset-rich households are allowed to take on more debt even though there is no formal collateralization. We allow *gearing* in this way to be as general as possible, and we use end-of-period timing and update the effect of income on the borrowing constraint period-by-period following the standard approach in buffer-stock models.<sup>8</sup>

The crucial departure from the canonical buffer-stock model is that we assume that the debt contract is *partially irrevocable from the lender side*. This provides the first term (“old contract”) in the maximum operator in borrowing constraint (3.4), implying that the households can always continue to borrow up to the *remaining principal* of their current debt contract (i.e.  $(1 - \lambda) \cdot D_{t-1}$ ). The second term (“new contract”) is a more standard borrowing constraint and only needs to be satisfied if the households want to take on *new debt* ( $D_t > (1 - \lambda) \cdot D_{t-1}$ ). Hereby current debt can potentially relax the households borrowing constraint in future

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<sup>8</sup> Note than a borrowing constraint such as  $D_t \leq A_t + \alpha \cdot P_t$  would be problematic because it would allow the households to take on infinitely much debt for a given level of consumption. A similar problem would also arise with  $D_t \leq A_{t-1} + \alpha \cdot P_t$  in the time limit if  $r_d = r_a$ .

periods and it thus provides extra liquidity. This implies that it might be optimal for the households to make choices such that both  $D_t > 0$  and  $A_t > 0$ ; i.e to simultaneously be a borrower and a saver.

If there was only one-period debt (i.e.  $\lambda = 1$ ) it would never be optimal for the households to simultaneously have both positive assets and positive debt because the option value of borrowing today would disappear. Consequently it would not be necessary to keep track of assets and debts separately and the model could be written purely in terms of net worth.<sup>9</sup> This would also imply that (3.4) could be rewritten as

$$N_t \geq -\frac{\mathbf{1}_{x_t=0} \cdot \varphi}{1 + \eta} \cdot P_t \quad (3.6)$$

showing that our model nests the canonical buffer-stock consumption model a la Carroll (1992, 1997, 2012) as a limiting case for  $\lambda \rightarrow 1$ .

### 3.3 Income

The income process is given by

$$\begin{aligned} Y_{t+1} &= \tilde{\xi}(u_{t+1}, \xi_{t+1}) \cdot P_{t+1} \\ P_{t+1} &= \Gamma \cdot \psi_{t+1} \cdot P_t \\ \tilde{\xi}(u_{t+1}) &\equiv \begin{cases} \mu & \text{if } u_{t+1} = 1 \\ \frac{\xi_{t+1} - u_* \cdot \mu}{1 - u_*} & \text{if } u_{t+1} = 0 \end{cases} \\ u_{t+1} &= \begin{cases} 1 & \text{with probability } u_* \\ 0 & \text{else} \end{cases} \end{aligned}$$

where  $\xi_t$  and  $\psi_t$  are respectively *transitory* and *permanent* mean-one log-normal income shocks<sup>10</sup> (with finite lower and upper supports), and  $u_*$  is the unemployment rate.<sup>11</sup> Because we have fully permanent shocks, we introduce a small constant mortality rate in the simulation exercise to keep the distribution of income finite.

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<sup>9</sup> If  $N_t \geq 0$  then  $A_t = N_t$  and  $D_t = 0$ , and if  $N_t < 0$  then  $D_t = -N_t$  and  $A_t = 0$ .

<sup>10</sup>Note that the *unconditional* expectation of  $Y_{t+1}$  thus is  $\Gamma \cdot P_t$ .

<sup>11</sup>Throughout the paper we will continue to interpret  $u_t$  as unemployment, but it could also proxy for a range of other large shocks to both income and consumption. This would relax the model's tight link between unemployment and a higher risk of a negative shock to the availability of new borrowing.

### 3.4 Solution Algorithm

As the model has four continuous states, two discrete states and two continuous choices it is not easy to solve, even numerically. We use a novel trick by defining the following helping variables,

$$M_t \equiv (1 + r_a) \cdot A_{t-1} - (r_d + \lambda) \cdot D_{t-1} + Y_t \quad (3.7)$$

$$\bar{D}_t \equiv (1 - \lambda) \cdot D_{t-1} \quad (3.8)$$

$$\bar{N}_t \equiv N_t|_{C_t=0} = M_t - \bar{D}_t \quad (3.9)$$

where  $M_t$  is *market resources*,  $\bar{D}_t$  is the *beginning-of-period debt principal*, and  $\bar{N}_t$  is *beginning-of-period net worth*. Also using the standard trick of normalizing the model by permanent income<sup>12</sup> denoting normalized variables with lower cases, we make  $\bar{n}_t$  a *state variable* instead of  $m_t$  (the standard choice). This speeds up the solution algorithm substantially because a change in  $\bar{d}_t$  then only affects the set of feasible debt choices; we hereby get that if the optimal debt choice is smaller than the current debt principal, then all households with smaller debt principals will make the same choice if it is still feasible, i.e.

$$k < 1 : d^*(\bar{d}_t, \bar{n}_t) = d \leq k \cdot \bar{d}_t \Rightarrow \forall \bar{d} \in [k \cdot \bar{d}_t, \bar{d}_t] : d^*(\bar{d}, \bar{n}_t) = d$$

One further complicating issue in solving the model, is that if  $\eta \neq 0$  then the choice set might be non-convex as illustrated in figure 3.1 using the following characterization of the choice set

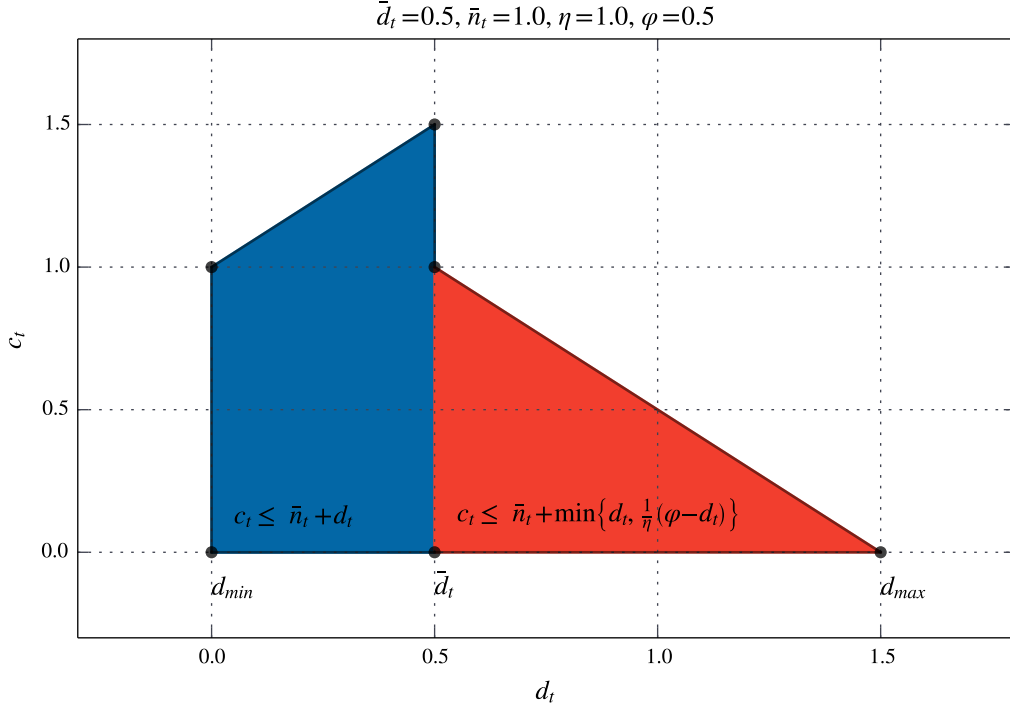
$$d_t \in \left[ \max \{-\bar{n}_t, 0\}, \max \left\{ \bar{d}_t, \eta \cdot \bar{n}_t + \mathbf{1}_{x_t=0} \cdot \varphi \right\} \right] \quad (3.10)$$

$$c_t \in \left[ 0, \bar{c}(x_t, \bar{d}_t, \bar{n}_t, d_t) \right] \quad (3.11)$$

$$\bar{c}(x_t, \bar{d}_t, \bar{n}_t, d_t) \equiv \begin{cases} \bar{n}_t + d_t & \text{if } d_t \leq \bar{d}_t \\ \bar{n}_t + \min \left\{ d_t, \frac{1}{\eta} (\mathbf{1}_{x_t=0} \cdot \varphi - d_t) \right\} & \text{if } d_t > \bar{d}_t \end{cases}$$

---

<sup>12</sup>See appendix B for the normalized model equations and details on the solutions algorithm for the discretized model.

Figure 3.1: Choice Set (*example of non-convexity*)


This possible *non-convexity* of the choice set and the general *non-concavity* of the value function due to the maximum operator in the borrowing constraint (3.4), imply that many of the standard results do not apply directly. Using a recent result from Clausen and Strub (2013) it can, however, be proven<sup>13</sup> that the optimal consumption choice,  $c_t^*(u_t, x_t, \bar{d}_t, \bar{n}_t)$ , conditional on the debt choice, still needs to satisfy the standard Euler-equality, i.e.

$$(c_t^*(\bullet))^{-\rho} = \beta \cdot (1 + r_a) \cdot \mathbb{E}_t \left[ \left( \Gamma \cdot \psi_{t+1} \cdot c_{t+1}^*(\bullet) \right)^{-\rho} \mid d_t = d \right]$$

This makes the Euler-equation a *necessary* condition for an interior solution. *Sufficiency* can then be ensured by numerically checking that the Euler-equation does not have multiple solutions.

Similar to Barillas and Fernández-Villaverde (2007), Hintermaier and Koeniger (2010), Kaplan and Violante (2014), Iskhakov, Jørgensen, Rust and Schjerning (2015) and especially Fella (2014), the endogenous grid points method originally developed by Carroll (2006) can thus be *nested* inside a *value function iteration* algorithm with a grid search for the optimal debt choice further speeding up the

<sup>13</sup>See appendix A.

solution algorithm.<sup>14</sup> The full solution algorithm is presented in appendix B.

## 4 Stylized Facts

Table 4.1: Stylized Facts

	Puzzle	Borrower	Saver	All
Share	27 %	5 %	68 %	100 %
<i>U.S. Dollars 2001</i>		<i>mean / median</i>		
Credit Card Debt	5,766	5,172	317	2,050
	3,800	3,340	0	0
Liquid Assets	7,237	227	17,386	13,734
	3,000	200	3,200	2,800
Liquid Net Worth <sup>1</sup>	1,471	-4,945	17,069	11,684
	-270	-3,200	3,000	1,700
Total After-Tax Income (annual)	52,114	28,032	64,331	59,116
	43,600	25,350	39,950	39,950
Installment Loans <sup>2</sup>	10,957	8,216	5,889	7,386
	6,100	3,600	0	600
Total Net Worth	187,912	36,231	466,463	368,367
	84,650	9,450	104,830	86,480
<i>Relative to quarterly income</i>		<i>mean / median</i>		
Credit Card Debt	0.44	0.74	0.02	0.14
	0.35	0.53	0.00	0.00
Liquid Assets	0.56	0.03	1.08	0.93
	0.28	0.03	0.32	0.28
Liquid Net Worth <sup>1</sup>	0.11	-0.71	1.06	0.79
	-0.02	-0.50	0.30	0.17

*Source:* 2001 SCF, all households with heads of age 25-64. Weighted averages within subgroups.

<sup>1</sup> Defined as liquid assets – credit card debt.

<sup>2</sup> Mortgages are not included.

For comparison between our model and the data, table 4.1 presents the central stylized facts on the credit card debt puzzle using the exact same methodology and 2001 *Survey of Consumer Finance* (SCF) data as Telyukova (2013). The facts are very similar to what other papers has found. *Credit card debt* is measured as

<sup>14</sup>On the precision and speed-up benefits of using EGM see Jørgensen (2013).

the balance due on the credit card left over *after* the last statement was paid, and *liquid assets* includes checking and savings accounts plus idle money in brokerage accounts, but not cash.<sup>15</sup>

All working age households are divided into *three* subgroups. Households are included in the “*puzzle*” group (or interchangeably the “*borrower-saver*” group) if they have more than \$500 in both credit card debt and liquid assets, and report repaying their balance off in full *only sometimes or never*. On the contrary households are denoted as pure “*borrowers*” if they have more than \$500 in credit card debt, but less than \$500 in liquid assets. Finally households with less than \$500 in credit card debt are denoted pure “*savers*”.

Approximately one in four households are measured to be in the puzzle group. For the median puzzle household both gross debt and gross assets equals about one month’s of after-tax income implying zero *liquid net worth* (liquid assets minus credit card debt). The distribution of liquid net worth is, however, somewhat right skewed in the sense that the mean household in the puzzle group has significantly larger gross assets than gross debts. Income wise, the average puzzle household has *less* income than the average income of the total population, but the median puzzle household has *more* income than the median income of the total population. The borrower group has mean credit card debt equal to about two month’s income, and an income level significantly below the average for both the mean and median household. Finally the distribution of gross assets in the saver group is highly right skewed with the mean household holding liquid assets worth more than one quarter’s income, but the median holding less than than one month’s. Including money market funds, and directly held mutual funds, stocks, bonds and T-bills in the measure of liquid assets would amplify this unbalancedness even further.

A novel fact presented in table 4.1 is that the puzzle households also hold many *installment loans*, most of which are *car loans*. The interest rates on such loans are typically significantly lower than on credit cards, and there can be some contractual terms that disincentivize premature repayment. Nonetheless it is an indication that the puzzle households are also using other precautionary borrowing channels than credit cards.

Finally, as also noted by Telyukova (2013), the puzzle households are often rather wealthy measured in *total net worth* (thus also including illiquid assets). This

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<sup>15</sup>See Telyukova (2013) for more details on the data, and a discussion of alternative procedures to quantify the credit card debt puzzle.

is to a large degree explained by *housing equity*. For computational reasons our model does not include an illiquid asset, but as shown in [Kaplan and Violante \(2014\)](#), a buffer stock model with an illiquid asset, and a transaction cost for tapping into this wealth, can imply that households between adjustments act as hand-to-mouth households. In a similar way *hyperbolic discounting* such as in [Laibson, Repetto and Tobacman \(2003\)](#) might further imply that households “*over*”-accumulate illiquid assets in order to strengthen their self-control abilities and better counteract the present bias of their future selves.

## 5 Calibration

Table 5.1: Calibration

Parameter	Value	Note / Source
<b>income</b>		
$\Gamma$ (annual)	1.02	Avg. US GDP per capita growth rate 1947-2014.
$u_*$	0.07	<a href="#">Carroll, Slacalek and Tokuoka (2015)</a> .
$\sigma_\psi^2$	$0.01 \cdot \frac{4}{11}$	<a href="#">Carroll, Slacalek and Tokuoka (2015)</a> .
$\sigma_\xi^2$	$0.01 \cdot 4$	<a href="#">Carroll, Slacalek and Tokuoka (2015)</a> .
$\mu$	0.30	<a href="#">Martin (1996)</a> .
<b>borrowing and saving</b>		
$r_a$ (annual)	-1.48 %	<a href="#">Kaplan and Violante (2014)</a> .
$r_d - r_a$ (annual)	12.36 %	<a href="#">Telyukova (2013)</a> and <a href="#">Edelberg (2006)</a> .
$\varphi$	0.74	<a href="#">Kaplan and Violante (2014)</a> .
$\eta$	0.00	Standard buffer-stock model.
$\lambda$	0.03	Standard credit card contract.
<b>credit risk</b>		
$\pi_{x,*}^{lose}$	2.63 %	<a href="#">Fulford (2015)</a> .
$\pi_{x,*}^{gain}$	6.07 %	<a href="#">Fulford (2015)</a> .
$\chi_{lose}$	4	See text.
<b>preferences</b>		
$\beta$ (annual)	0.90	Matched to empirical moments. See text.
$\rho$	3.00	Matched to empirical moments. See text.

The calibrated parameters are presented in table 5.1. The model is simulated at a *quarterly* frequency, but we discuss discount and interest rates in annualized

terms. In section 8 we present a detailed discussion of how robust the results are to changing each single parameter.

The gross income growth factor  $\Gamma = 1.02$  is chosen to match U.S. trend growth in GDP per capita. The variances of the income shocks and the unemployment rate are all taken from [Carroll, Slacalek and Tokuoka \(2015\)](#) who show that they parsimoniously match central empirical facts from the literature on estimating uncertain income processes. In annual terms the variance of both the permanent and the transitory shock are 0.01.<sup>16</sup> The unemployment replacement rate  $\mu$  is set to 0.30 as documented in [Martin \(1996\)](#); we find the choice of  $\mu = 0.15$  in [Carroll, Slacalek and Tokuoka \(2015\)](#) to be too extreme.

Regarding *borrowing and saving* we first follow [Kaplan and Violante \(2014\)](#), who based on SCF data, set the real interest rate on liquid wealth to  $-1.48$  percent (annually) and find that the borrowing constraint binds at 74 percent of quarterly income. We thus set  $\varphi = 0.74$ , and choose  $\eta = 0$  to stay as close as possible to these results (and the standard parametrization of the buffer-stock model). The interest rate on credit card debt is taken from [Telyukova \(2013\)](#); she finds that the mean nominal interest rate in the borrower-saver group is 14 percent which we then adjust for 2.5 percentage points of inflation and a 0.62 percentage points default risk (see [Edelberg \(2006\)](#)). In total this implies an interest rate spread of 12.4 percent, which is a bit lower than the 13.2 percent spread in [Fulford \(2015\)](#), but larger than the 10.0 percent spread in [Telyukova \(2013\)](#). We set  $\lambda = 0.03$  because many credit card companies use a minimum payment rate of 1 percent on a monthly basis.

For the *credit risk* we set the *unconditional* probabilities equal to the empirical results in [Fulford \(2015\)](#) who utilize a proprietary data set containing a representative sample of 0.1 percent of all individuals with a credit report at the credit-reporting agency *Equifax* from 1999 to 2013. Each quarter the risk of losing access to credit is thus 2.63 percent, while the chance of regaining access is 6.07 percent. Unfortunately Fulford is only able to condition on general covariates such as age, year, credit risk, geographical location and reported number of cards; specifically he is not able to say anything on the relationship between credit risk and income risk or unemployment. To calibrate  $\chi_{lose}$  we therefore instead turn to the *Survey of Consumer Finance* (SCF) 2007-2009 panel where households were asked whether

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<sup>16</sup>For the transitory shock the variance at a quarterly frequency is simply  $4 \times$  annual transitory variance, while [Carroll, Slacalek and Tokuoka \(2015\)](#) show that for the permanent shock the conversion factor should be  $\frac{4}{11}$ .



or not they have a credit card both in 2007 and then again in 2009. This measure of credit card access is inferior to Fulford’s, but we believe that the two measures are rather closely related. We restrict attention to all stable couples between age 25 and 59, with positive income, and who in 2007 had and used a credit card. Table 5.2 shows that 7.7 percent of these household when re-interviewed in 2009 reported *not* having a credit card anymore; we denote this as having “lost access”. Conditional on experiencing any weeks of unemployment the fraction of those who have lost access increases to 15.2 percent. Like Fulford we have no way of determining whether this indicates voluntary choices made by the households, but table 5.3 reports the odds-ratios from logit estimations controlling for both various *background* variables (age, age squared, minority, household size) and *economic* variables (homeownership, log (normal) income, liquid assets, self-employment, education). The effect from unemployment remains significant even when all controls are used though the odds-ratios falls a bit. This is also in line with Crossley and Low (2013) who showed using the 1995 *Canadian Out of Employment Panel* that current unemployment was important for explaining the share of households answering “no” to the question “[i]f you needed it, COULD you borrow money from a friend, family, or a financial institution in order to increase your household expenditures”.

To choose an extract number for  $\chi_{lose}$  we use that the *theoretical* odds-ratio of losing access to new borrowing if treated with some unemployment in the last year (four quarters), and conditional on having had access two years (eight quarters) ago, is given by

$$\text{Odds-Ratio} = \frac{f(1, 1) / f(0, 1)}{f(1, 0) / f(0, 0)} \quad (5.1)$$

where we have defined

$$f(\hat{x}, \hat{u}) \equiv \mathbb{E}(x_8 = \hat{x} \mid x_1 = 0, \exists k \in \{5, 6, 7, 8\} : u_k = \hat{u}) \quad (5.2)$$

which can easily be calculated for  $\hat{x} \in \{0, 1\}$  and  $\hat{u} \in \{0, 1\}$  given the Markov processes of  $x_t$  and  $u_t$ .

Setting  $\chi_{lose} = 4$  we hereby get an odds-ratio of 1.8. This is somewhat below the odds-ratios we find in the data, but due to Fulford’s very low estimate of gaining access, we still have that the risk of losing access conditional on being affected by unemployment is 19.9 percent; if not affected by unemployment the probability is 12.4 percent and in percentage points the increase is thus 7.5 similar to what

we see in table 5.2. We thus stick with the choice of  $\chi_{lose} = 4$  and perform an extensive robustness analysis of these calibrations in section 8.

Finally we calibrate the discount factor  $\beta$  and relative risk aversion  $\rho$  to match central moments from table 4.1. Our first target is that the mean level of liquid net worth across all households should be equal to about *two month's of income*. The exact data counterpart is a bit higher at 79 percent of quarterly income; we choose the lower target because the *median* level is always only a bit below the *mean* level in the model while the median level in the data is only 17 percent of quarterly income and thus substantially below the mean level. Our second target is that the median puzzle household should have approximately zero liquid net worth as we see in the data. Increasing either  $\beta$  or  $\rho$  both increases the liquid net worth of the full population, but  $\beta$  only marginally affects the net worth of the puzzle group whereby we use the second moment to identify  $\rho$ . We obtain  $\beta = 0.90$  and  $\rho = 3$ . The discount factor is including an exogenous quarterly death probability of 1 percent; having mortality is technically necessary to ensure that the cross-sectional distribution of income is finite.<sup>17</sup>

Table 5.2: Lost Access and Unemployment - Raw

	Lost Access <sup>1</sup>	Share of Sample
	<i>percent</i>	
All	7.7	-
No Unemployment over last year <sup>2</sup>	5.6	78.9
Any Unemployment	15.2	21.1
Some Unemployment ( $\geq 1$ month)	15.2	19.3
Deep Unemployment ( $\geq 3$ months)	15.9	14.0

*Source:* SCF panel 2007-2009; Households between age 25 and 59, positive income, had and used a credit card in 2007 (X410=1 and X09205>0). Adjusted for survey weights and multiple imputations.

<sup>1</sup> *Lost Access:* Report not having a credit card in 2009 (P410 = 5).

<sup>2</sup> *Unemployment:* Sum of head and spouse over the last 12 months (P6781 and P6785).

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<sup>17</sup>When a household dies it is replaced with a new household without any debt and assets equal to one week's permanent income, and with the same lagged permanent income as the *mean* of the current population. See e.g. McKay (2015) for a similar approach. The assets of the household is taxed away.

Table 5.3: Lost Access and Unemployment - Logit

	Any Unemployment			Deep Unemployment		
	(1)	(2)	(3)	(1)	(2)	(3)
	<i>odds-ratio (s.e.)</i>					
Unemployment	3.00*** (0.93)	2.98*** (0.94)	2.34** (0.83)	2.82*** (0.95)	2.75*** (0.98)	2.27** (0.89)
Background Controls <sup>1</sup>		✓	✓		✓	✓
Economic Controls <sup>2</sup>			✓			✓
Observations	1,079	1,079	1,079	1,079	1,079	1,079

Source: See table 5.2. \*:  $p < 0.10$ , \*\*:  $p < 0.05$ , \*\*\*:  $p < 0.01$ .

<sup>1</sup> *Background controls*: Age, age squared, minority, household size.

<sup>2</sup> *Economic controls*: Homeownership, log (normal) income, liquid assets, self-employment, education (none, high school, college).

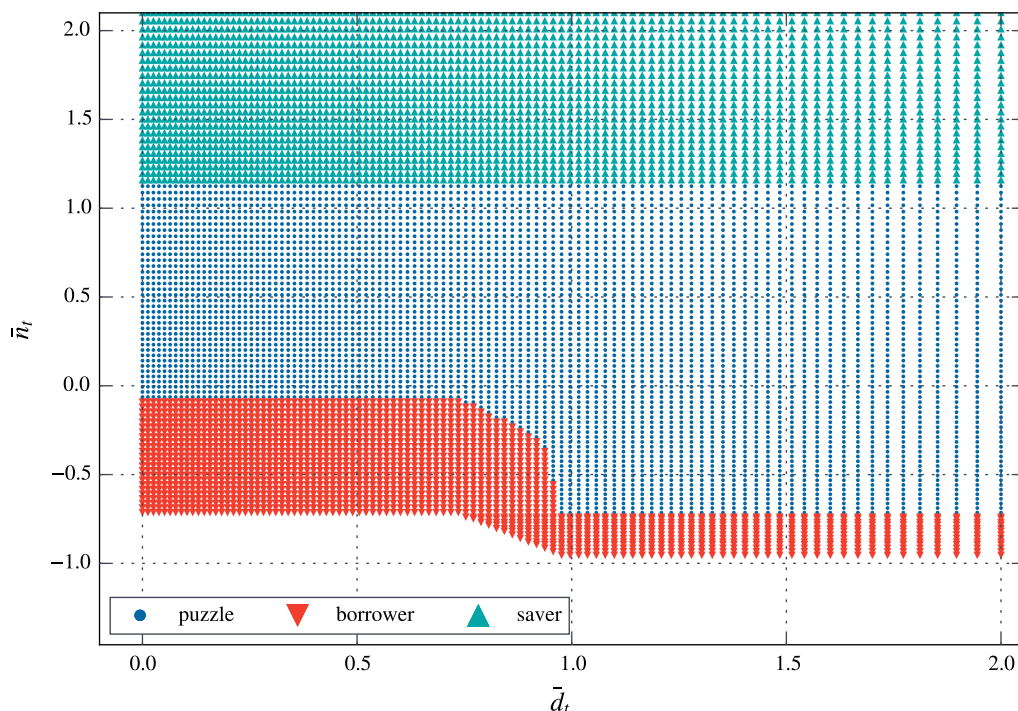
## 6 Results

### 6.1 Policy Functions

Based on the converged policy functions, figure 6.1 shows in which disjoint sets of states the households choose to respectively be a borrower, a saver and a borrower-saver.

The general conclusion is that households always choose to be *savers* if their beginning-of-period net worth ( $\bar{n}_t$ ) is high enough, and *borrowers* if it is low enough. For the wealthy households the option value of holding debt is zero because they have no liquidity problems. In contrast, poor households are already borrowing so much that they either cannot borrow any more, or the option value of more debt is not large enough to cover the net cost of expanding the balance sheet.

The households choose to be in the puzzle group if their beginning-of-period net worth is in between the two extremes mentioned above. If the beginning-of-period debt principal ( $\bar{d}_t$ ) is high, a household can easily accumulate more debt in excess of what it needs to accumulate for consumption purposes. Hence, for a given (low) beginning-of-period net worth it might therefore be optimal for households with a *high* debt principal to be a borrower-saver, while it is optimal to be a borrower for households with a *low* debt principal.

Figure 6.1: **Implied Group Choice** ( $u_t = 0, x_t = 0$ )

## 6.2 Simulation

Given the converged policy functions it is straightforward to simulate the model. Table 6.1 presents the cross-sectional results from a simulation with 100,000 households (after an initial burn-in period).

We see that the model under the chosen parametrization can explain that 16.6 percent of households choose to be borrower-savers. This is a bit below the empirical estimate of 27 percent (see table 4.1) but still a large share. This shows that precautionary borrowing is at least one of the central explanations of the credit card debt puzzle. It is especially important that such a large proportion of the puzzle group can be explained even when the model also implies that the level of mean net worth is above two months income. Furthermore the implied size of the balance sheets of the puzzle households are also rather large; the median puzzle household e.g. has an asset to income ratio of 0.24, while the data counterpart is 0.28.

On the other hand the size of the borrower group is much too small in the simulation, and the model generally has a hard time explaining why some households decide to go so deeply into debt. Consequently it also overshoot the median net

Table 6.1: Results

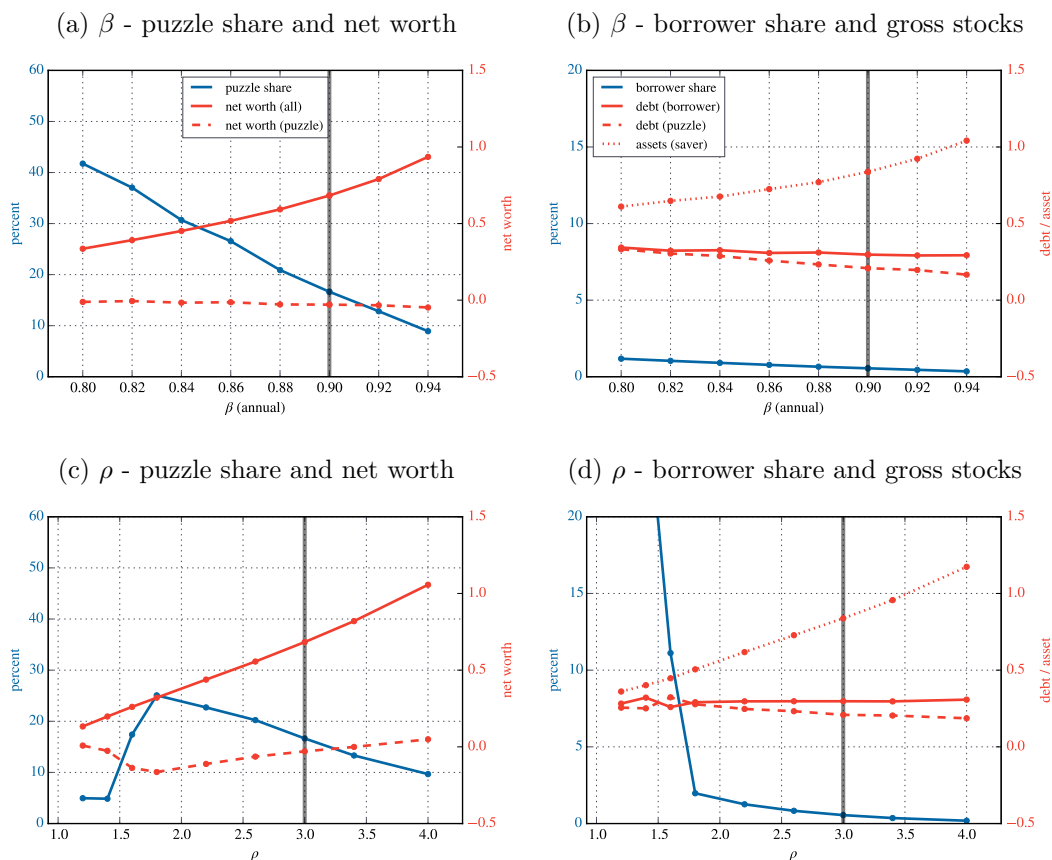
	Puzzle	Borrower	Saver	All
	<i>percent</i>			
Share	16.6	0.5	82.8	100.0
$u_t$ (4 qrt.)	16.6	43.7	4.8	7.0
$x_t$	13.9	79.4	33.2	30.2
	<i>mean / median</i>			
<i>Relative to income</i>				
$d_t$	0.30	0.21	0.00	0.05
	0.24	0.01	0.00	0.00
$a_t$	0.27	0.00	0.84	0.74
	0.24	0.00	0.73	0.61
$n_t$	-0.03	-0.21	0.84	0.68
	-0.00	-0.01	0.73	0.59
$Y_t$ (4 qrt.)	0.88	0.66	1.03	1.00
	0.80	0.60	0.92	0.89
$P_t$ (4 qrt.)	1.03	1.01	1.00	1.00
	0.94	0.92	0.90	0.91

“4. qrt.”: Average of the last four quarters.

Puzzle group definition:  $d_t, a_t > 0.04$ .

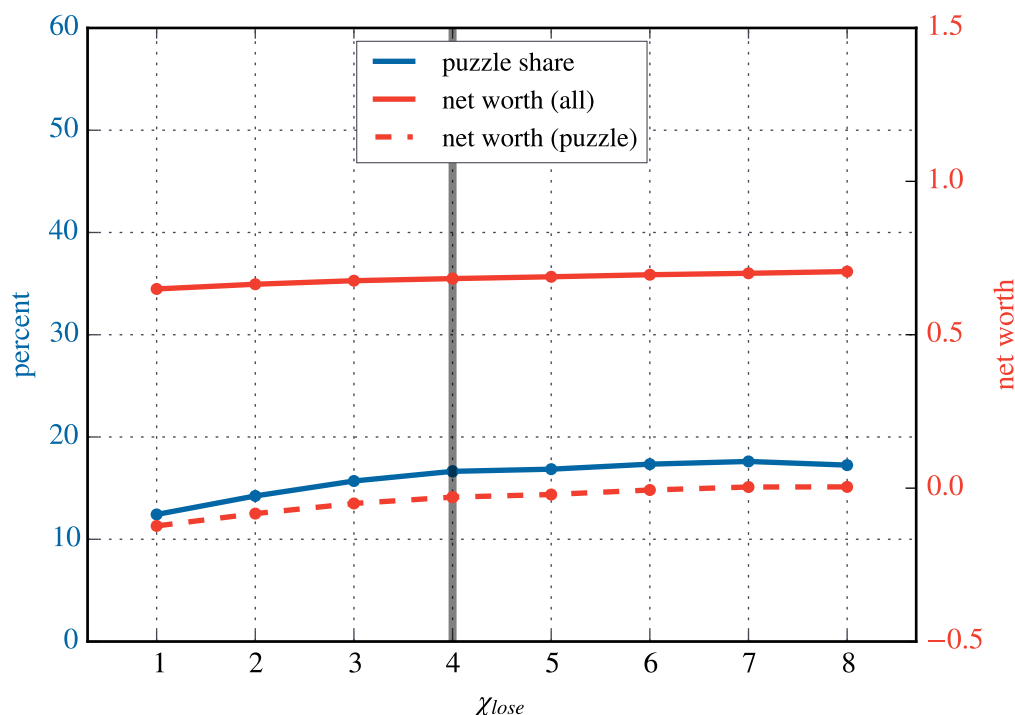
worth of the full population substantially. We do not worry too much about these two shortcomings of the model because introducing heterogeneous impatience and risk aversion would probably be a simple cure. Figure 6.2 thus shows that a large borrower share can be explained if some of the households have a relative risk aversion coefficient below 2. Our calibration implies a somewhat higher degree of risk aversion in order to match the observed net worth of the puzzle households (which are *increasing* in  $\rho$ , while the share of borrowers are *decreasing*). Such heterogeneous risk preferences or heterogeneous impatience would also improve the model’s ability to match a lower median net worth among savers, which it currently overshoots. In general, both lower  $\rho$  and  $\beta$  increases the puzzle group, but make it harder for the model to match the observed level of net worth.

Figure 6.2: Alternative Preferences



Note: Vertical gray line represents baseline parameter value.

Figure 6.3 shows that the assumed positive correlation between unemployment and losing access to the credit markets is rather important for the quantitative results. Removing the extra risk of losing access when unemployed (setting  $\chi_{lose} = 1$  for unchanged  $\pi_{x,*}^{lose}$ ) reduces the puzzle group by a fourth. Increasing  $\chi_{lose}$  above the calibration value of 4 also increases the size of the puzzle group somewhat further.

Figure 6.3: The importance of  $\chi_{lose} > 1$ 

Note: Vertical gray line represents baseline parameter value.

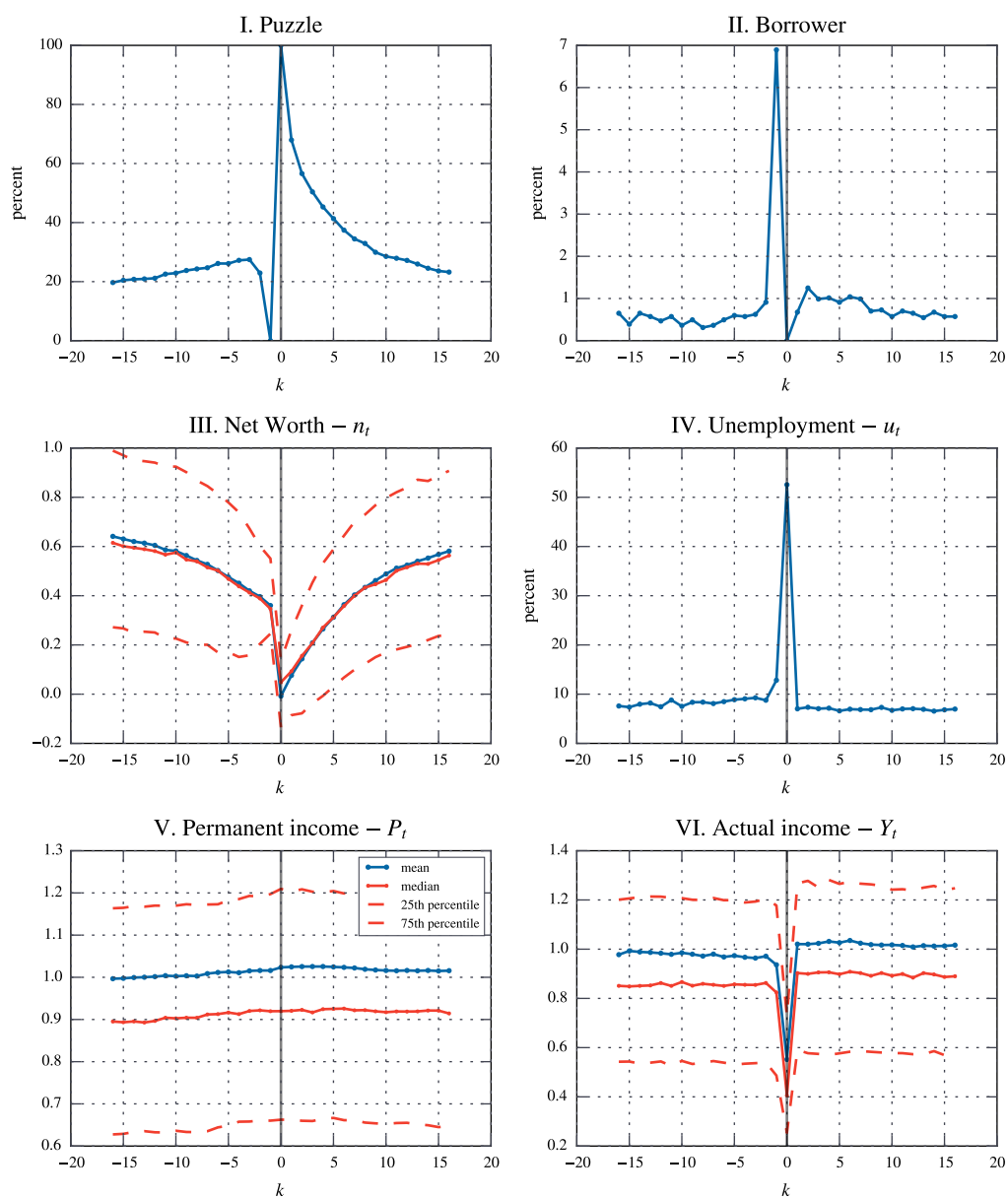
### 6.3 Before/After

Figure 6.4 provides further details on what happens before and after a household transitions into the puzzle group (i.e. the household is in the puzzle group at time  $k = 0$  but not at  $k = -1$ ). We see that persistence is limited as just above 40 percent of the households are still in the puzzle group one year on (graph I), and that most of the puzzle households (over 90 percent) were savers in the period before their transition (graph II). In more general terms, the third graph shows that the households are deaccumulating net worth at an accelerating speed in the quarters before joining the puzzle group.

Looking at the income dimension of the simulation, we see that the yearly income of the puzzle households is a bit below the total mean and median; in the data this was only true for the mean. Note however, that the permanent income of the puzzle households is actually slightly *above* both the total mean and median. The average unemployment rate of the puzzle households over the last four quarters is 17 percent, but looking at figure 6.4 (graph IV) we see that about 50 percent of the puzzle households are unemployed at transition. This shows that continuing large falls in transitory income is necessary to make households choose to be borrower-

savers (see also graph VI). On the other hand graph V in figure 6.4 shows that falls in permanent income are *not* necessary; the reason is that such shocks also lowers the optimal consumption level of the household and thus does not induce precautionary borrowing.

Figure 6.4: Before and After Transition to Puzzle Group



*Sample:* Households who are in the puzzle group at  $k = 0$ , but were not so at  $k = -1$ .



## 7 The Welfare Gain of Precautionary Borrowing

The welfare of the households can be measured as the ex ante discounted expected utility seen from an initial period. The simulation analog of this measure can be calculated taking the average over a sample of households experiencing different draws of shocks,

$$\mathcal{U}_0(P_0) = P_0^{1-\rho} \cdot \frac{1}{N} \cdot \sum_{i=0}^N \sum_{t=0}^T \cdot \beta^t \cdot \frac{\left(c^*(s_{it}) \cdot \Gamma^t \cdot \prod_{j=1}^t \psi_{ij}\right)^{1-\rho}}{1-\rho} \quad (7.1)$$

$$\mathcal{T}(s_{it}, d^*(s_{it}), c^*(s_{it})) \Rightarrow s_{i,t+1}$$

where  $s_{i,t}$  is the vector of normalized state variables of household  $i$  and  $\mathcal{T}(\bullet)$  is the stochastic transition function.<sup>18</sup>

We are now interested in the level of welfare across different values of  $\lambda$ , remembering that as  $\lambda \rightarrow 1$  we return to the canonical buffer-stock model which does not allow precautionary borrowing. Facilitating these comparisons, we can analytically derive the compensation in terms of a percentage increase ( $\tau$ ) in initial permanent income, and thus the average future path of permanent income, a household needs to receive in order to be indifferent to a change in  $\lambda$  relative to the baseline:

$$\mathcal{U}_0(P_0, \lambda_0) = \mathcal{U}_0\left(P_0 \cdot \left(1 + \frac{\tau_j}{100}\right), \lambda_j\right) \Leftrightarrow \frac{\tau_j}{100} = \left(\frac{\mathcal{U}_0(P_0, \lambda_0)}{\mathcal{U}_0(P_0, \lambda_j)}\right)^{\frac{1}{1-\rho}} - 1 \quad (7.2)$$

The results are plotted in figure 7.1; as  $\lambda$  increases the required compensation (the blue line) naturally increases as the choice set of the households only shrinks and the scope for precautionary borrowing becomes more limited. In total, the households needs a compensating increase in the path of permanent income of 1.10 percent to be indifferent between  $\lambda = 0.03$  (the baseline) and  $\lambda = 0.99$ .

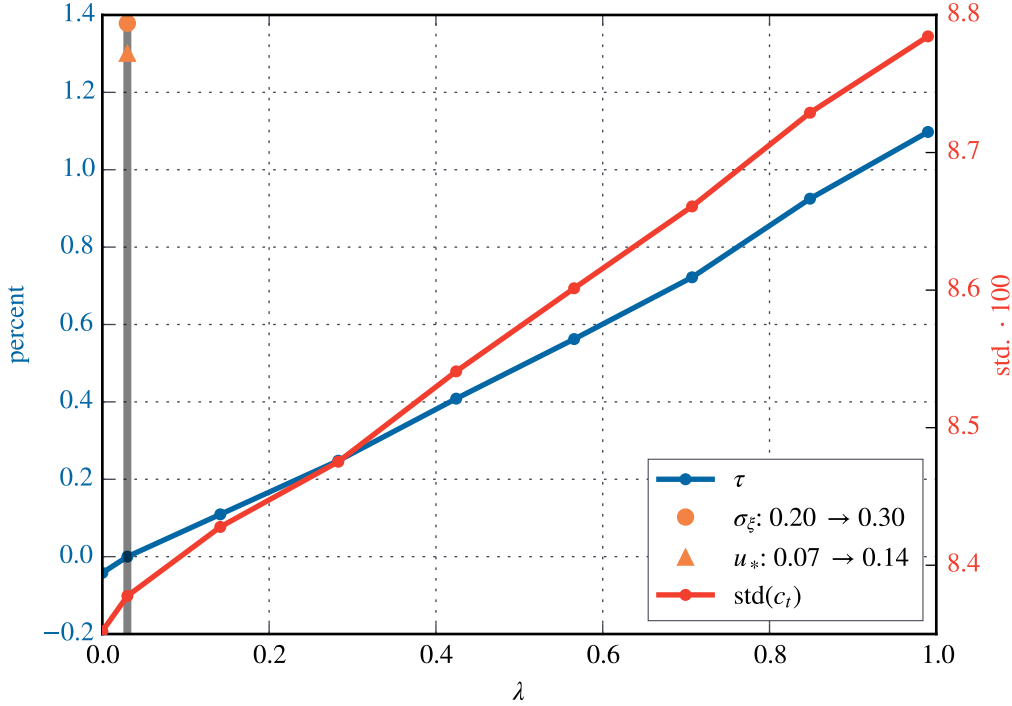
To ease comparison, figure 7.1 also depicts the compensating equivalents for changes in respectively the variance of the transitory income shock and steady state unemployment: Increasing  $\sigma_\varepsilon$  from 0.20 to 0.30 implies  $\tau = 1.37$ , while increasing  $u_*$  from 7 to 14 percent implies  $\tau = 1.30$ . The households welfare loss of losing access to precautionary borrowing is thus only a bit smaller than a doubling of the unemployment rate.

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<sup>18</sup>The average is calculated conditional on  $P_0$ , but not on the other initial states

The red line in figure 7.1 shows that a central underlying reason for the loss of welfare when the household's access to precautionary borrowing is limited is an *increase* in the standard deviation of normalized consumption, which the households dislike because of the concavity of the utility function.

Figure 7.1: Welfare



Note: Vertical gray line represents baseline  $\lambda$  value.

## 8 Robustness

### 8.1 Growth Impatience

Figure 8.1 shows how the size of the puzzle group (*blue line*) and the average net worth of both all households (*full red line*) and the puzzle group (*dashed red line*) are affected by changes in  $r_a$  and  $\Gamma$ .

In understanding the figure it is useful to consider *the growth impatience factor* as defined in Carroll (2012)

$$\bar{\beta} \equiv (\beta \cdot (1 + r_a))^{\frac{1}{\rho}} \cdot \Gamma^{-1} \quad (8.1)$$

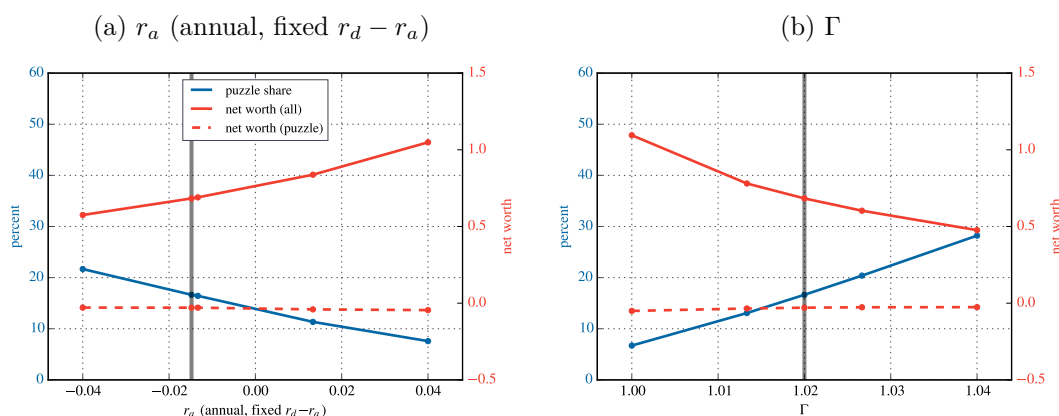
In the perfect foresight case a growth impatience factor *less than one* implies that

for an unconstrained consumer the ratio of consumption to permanent income will *fall* over time. In general, a larger growth impatience factor induces saving; these savings also satisfy the household's precautionary motive making costly *precautionary borrowing* less needed. Consequently, the puzzle group is *increasing* in  $\Gamma$ , and *decreasing* in  $r_a$ .

In figure 6.2 we likewise saw that the puzzle group was *decreasing* in patience  $\beta$  and eventually in risk aversion  $\rho$  (we always have  $\beta \cdot (1 + r_a) < 1$ ). Initially, however, an increase in the curvature of the utility function ( $\rho$ ) expands the puzzle group because it implies a stronger incentive to smooth consumption, which makes it relatively more worthwhile for the households to pay the costs of precautionary borrowing.

Summing up, the model can explain a large puzzle group if households are impatient enough, in a growth corrected sense, and are neither too risk neutral nor too risk averse.

Figure 8.1: Growth Impatience



Note: Vertical gray line represents baseline parameter value.

## 8.2 Income Uncertainty

The underlying motive for precautionary borrowing is insurance against transitory income losses. We therefore see in figure 8.2 that the size of the puzzle group is first increasing in the variance of the *transitory* income shock and risk of unemployment (higher  $\sigma_\xi$  and  $u_*$ ). At some point, however, larger transitory shocks does not increase the puzzle group because they induce too much precautionary saving. Lowering the unemployment benefits (lower  $\mu$ ) thus also only increase the puzzle group if the initial level is rather high.

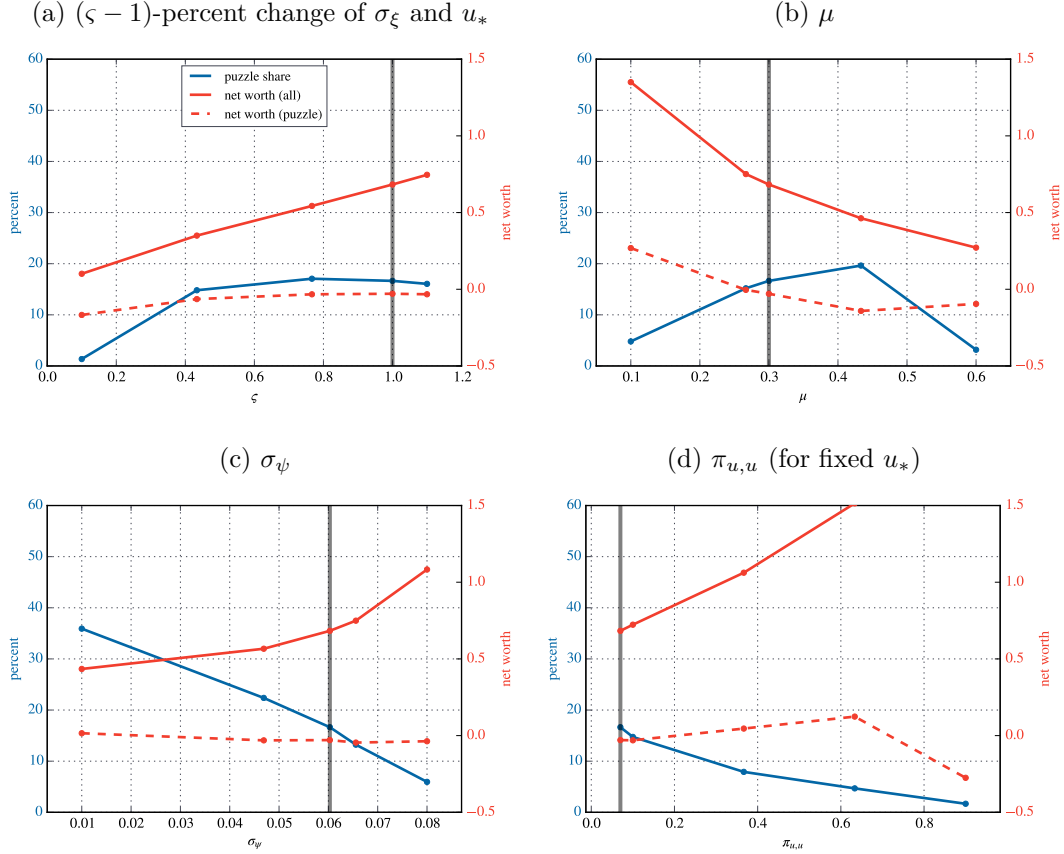
A larger variance of the *permanent* shock (higher  $\sigma_\psi$ ) on the contrary shrinks the

puzzle group because the incentive to accumulate precautionary funds imply that the average net worth increases so much that the households do not need to rely on precautionary borrowing. This can also be understood as the consequence of an increase in the *uncertainty adjusted* growth impatience factor,

$$\tilde{\beta} \equiv (\beta \cdot (1 + r_a))^{\frac{1}{\rho}} \cdot \Gamma^{-1} \cdot \mathbb{E} [\psi_{t+1}^{-1}] = \bar{\beta} \cdot \mathbb{E} [\psi_{t+1}^{-1}] \quad (8.2)$$

where the last term is increasing in the variance of the permanent shock due to Jensen's Inequality. The same mechanism moreover also implies that the puzzle group is *decreasing* in adding unemployment persistence, where  $\pi_{u,u}$  is the unemployment risk for the unemployed.

Figure 8.2: Income Uncertainty



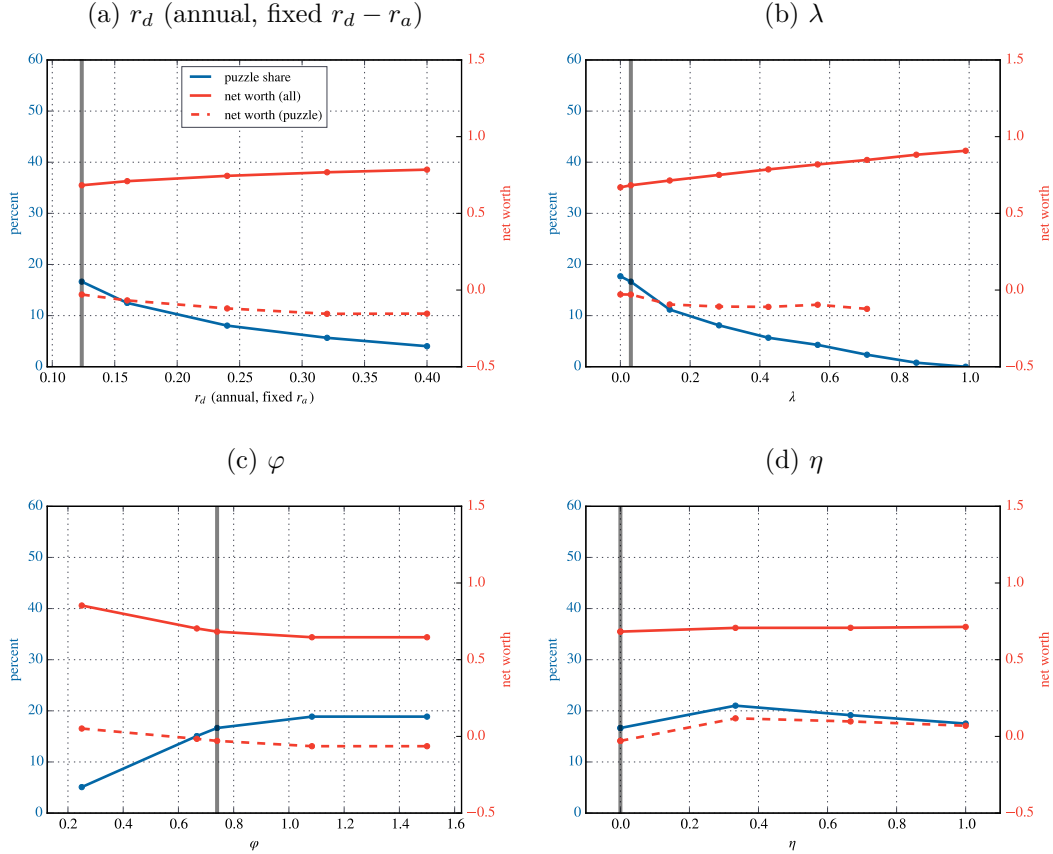
Note: Vertical gray line represents baseline parameter value.

### 8.3 Terms of Borrowing

Naturally the size of the puzzle group is decreasing if either the cost of borrowing increases (higher  $r_d - r_a$ , fixed  $r_a$ ) or the repayment rate increases (higher  $\lambda$ ).

This is shown in the two first graphs in figure 8.3. Furthermore the puzzle group is relatively small if  $\varphi$  is too small, as the extensive potential for precautionary borrowing is then limited. When  $\varphi$  reaches one this positive effect on the size of the puzzle group more or less disappears. Allowing for gearing in the form of a  $\eta > 0$  does almost not affect the results, and our formulation is thus robustness in this regard.

Figure 8.3: Terms of Borrowing



*Note:* Vertical gray line represents baseline parameter value. The net worth of the puzzle group is not shown if the puzzle group is too small.

## 8.4 Credit Risk

Figure 8.4 presents the effects of changing the *unconditional* probabilities for losing ( $\pi_{x,*}^{lose}$ ) and gaining ( $\pi_{x,*}^{gain}$ ) access to new debt. In the first graph we see that the puzzle group is naturally increasing in the risk of losing access, but that the effect is highly non-linear as a higher risk also induces more saving. Specifically we see that size of the puzzle group stops increasing at  $\pi_{x,*}^{lose} = 0.01$  indicating that our results quantitatively are very robust to a lower estimate of  $\pi_{x,*}^{lose}$ .

Table 8.1: Results ( $\pi_{x,*}^{gain} = 0.5$ ,  $\beta = 0.905$ ,  $\rho = 3.3$ )

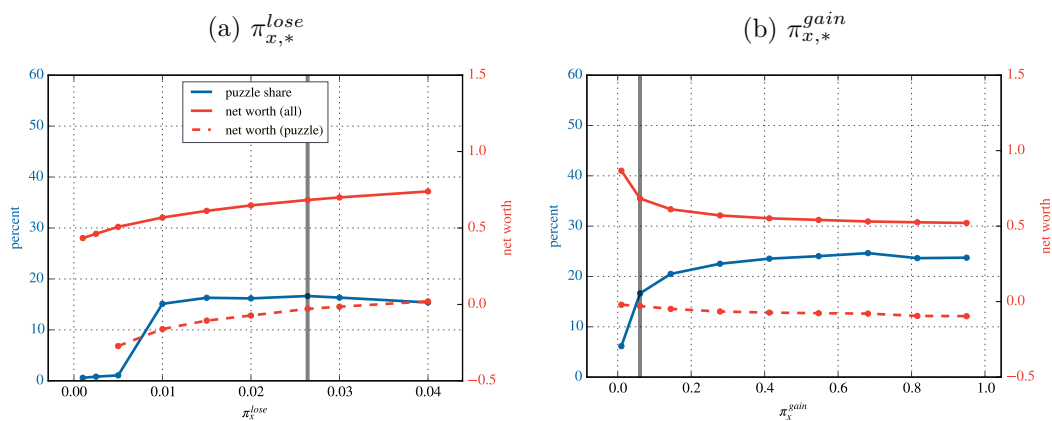
	Puzzle	Borrower	Saver	All
	<i>percent</i>			
Share	19.2	0.3	80.3	100.0
$u_t$ (4 qrt.)	18.0	42.9	4.1	7.0
$x_t$	3.5	68.7	5.0	5.0
	<i>mean / median</i>			
<i>Relative to income</i>				
$d_t$	0.30	0.33	0.00	0.06
	0.24	0.23	0.00	0.00
$a_t$	0.24	0.00	0.86	0.73
	0.23	0.00	0.74	0.58
$n_t$	-0.06	-0.32	0.85	0.67
	-0.01	-0.23	0.74	0.57
$Y_t$ (4 qrt.)	0.86	0.69	1.03	1.00
	0.79	0.62	0.92	0.89
$P_t$ (4 qrt.)	1.03	1.03	0.99	1.00
	0.94	0.93	0.90	0.91

“4. qrt.”: Average of the last four quarters.

Puzzle group definition:  $d_t, a_t > 0.04$ .

The second graphs shows that the puzzle group is (perhaps surprisingly) also increasing in the probability of re-gaining access to credit when it is lost; the intuition is that long expected exclusion spells induce more prior saving diminishing the need for precautionary borrowing. Table 8.1 show that we reach the same conclusion when we choose an expected duration of the exclusion spell of about two quarters (setting  $\pi_{x,*} = 0.5$ ) and re-calibrate  $\beta$  and  $\rho$  to match the same targets as in the baseline parametrization. It is thus clear that our results does not hinge on the assumption of a very low probability of re-gaining access.

Figure 8.4: Credit Risk



*Note:* Vertical gray line represents baseline parameter value. The net worth of the puzzle group is not shown if the puzzle group is too small.

## 9 Conclusion

We have shown that precautionary borrowing can explain a large part of the puzzle group of households who simultaneously has expensive credit card debt and hold low-return liquid assets. We have moreover shown that no knife-edge assumptions on preferences or income uncertainty are needed for this result. However, the power of the precautionary borrowing channel is strongest if households are relatively impatient in a growth and uncertainty adjusted sense, are neither too risk neutral nor too risk averse, and are subject to sizable transitory income shocks.

The strongest assumption we need in order to amplify our results, is that bad income shocks are perceived to be positively correlated with a higher risk of a fall in the availability of credit. This is not an implausible assumption, and we provide some indicative empirical evidence adding to that in [Fulford \(2015\)](#). More work on disentangling demand and supply effects in these estimates are, however, needed. Nevertheless, we show that only a very small risk of losing access to new borrowing is needed for our results to be quantitatively robust, and that the results are actually stronger if the chance of re-gaining access once lost is larger than the current estimate.

A natural extension of our model would be to include an illiquid asset subject to transaction costs as in [Kaplan and Violante \(2014\)](#). We conjecture that in such a model precautionary borrowing will still be an important tool for both poor and wealthy hand-to-mouth households. Together with a detailed life-cycle setup such an extension is probably necessary to empirically estimate the importance of precautionary borrowing with precision. This we leave for future work. Extending the model in this direction would also make it possible to study the implications of precautionary borrowing for the average marginal propensity to consume out of both income and credit shocks. Finally, the concept of precautionary borrowing is also relevant for understanding households utilization of other forms of consumer loans, including car loans and mortgages.<sup>19</sup>

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<sup>19</sup>See e.g. [Druehdahl \(2015\)](#).



## A The Euler-Equation

The purpose of the present appendix is to show that *conditional on the debt choice* the standard *Euler-equation* is *necessary* at all *interior optimal consumption choices*. This is shown for a slightly simplified version of the model from the main text using lemmas from [Clausen and Strub \(2013\)](#); the results can easily be extended to the full model. Using a method along the lines of [Fella \(2014\)](#) (building on [Edlin and Shannon \(1998\)](#)), we furthermore show that the *debt-contingent savings correspondence* is *monotonically increasing* in a specific sense, which is a necessary condition for the *endogenous grid point method* (EGM), developed in [Carroll \(2006\)](#), to work.

### A.1 Lemmas from Clausen and Strub (2013)

Using

**Definition A.1.**  $F : X \rightarrow \mathbb{R}$  is *differentiable sandwiched* between the lower and upper support functions  $L, U : X \rightarrow \mathbb{R}$  at  $\hat{x} \in \text{int}(X)$  if

$$\forall x \in X : \quad L \text{ and } U \text{ are differentiable} \quad (\text{A.1})$$

$$L(x) \leq F(x) \quad (\text{A.2})$$

$$U(x) \geq F(x) \quad (\text{A.3})$$

$$x = \hat{x} : \quad L(x) = F(x) = U(x) \quad (\text{A.4})$$

[Clausen and Strub \(2013\)](#) prove that

**Lemma A.1.** (*Differentiable Sandwich Lemma*). *If  $F$  is differentiable sandwiched between  $L$  and  $U$  at  $\hat{x}$  for an  $\mathcal{X} \subseteq X$  with  $\hat{x} \in \text{int}(\mathcal{X})$  then  $F$  is differentiable at  $\hat{x}$  with  $F'(\hat{x}) = L'(\hat{x}) = U'(\hat{x})$ .*

and

**Lemma A.2.** (*Reverse Calculus*). *Suppose  $F : X \rightarrow \mathbb{R}$  and  $G : X \rightarrow \mathbb{R}$  have differentiable lower support functions at  $\hat{x}$  then*

1. *If  $H(x) = F(x) + G(x)$  is differentiable at  $\hat{x}$ , then  $F$  is differentiable at  $\hat{x}$ .*
2. *If  $H(x) = F(x)G(x)$  is differentiable at  $\hat{x}$  and  $F(\hat{x}) > 0$  and  $G(\hat{x}) > 0$ , then  $F$  is differentiable at  $\hat{x}$ .*
3. *If  $H(x) = \max\{F(x), G(x)\}$  is differentiable at  $\hat{x}$  and  $F(\hat{x}) = H(\hat{x})$  then  $F$  is differentiable at  $\hat{x}$ .*

## A.2 Simplified Model

The simplified model is written in recursive form as

$$\begin{aligned}
 v(\bar{d}_t, \bar{n}_t) &= \max_{d_t, n_t} u(c_t) + \beta \cdot \sum_{\Xi \times \Psi} \cdot \psi^{1-\rho} \cdot v(\bar{d}_+(\bullet), \bar{n}_+(\bullet)) \quad (\text{A.5}) \\
 &\text{s.t.} \\
 u(c_t) &= \frac{c_t^{1-\rho}}{1-\rho} \Rightarrow u'_c(c_t) = c_t^{-\rho} \\
 c_t &= \bar{n}_t - n_t \\
 a_t &= \bar{n}_t + d_t - c_t = n_t + d_t \\
 d_t &\leq \max\{\bar{d}_t, \eta \cdot n_t + \varphi\} \\
 \bar{d}_+(d_t; \psi) &= \psi^{-1} \cdot (1 - \lambda) \cdot d_t \\
 \bar{n}_+(d_t, n_t; \psi, \xi) &= \psi^{-1} \cdot [(1 + r_a) \cdot n_t - (r_d - r_a) \cdot d_t] + \xi \\
 d_t, c_t, a_t &\geq 0 \\
 \Psi \times \Xi &\equiv \{\psi_b, \psi_g\} \times \{\xi_b, \xi_g\} \\
 \sum_{\Psi \times \Xi} &\equiv \sum_{(\psi, \xi) \in \Psi \times \Xi} p(\psi, \xi) = 1
 \end{aligned}$$

We denote the *optimal choice functions* by  $d^*(\bar{d}_t, \bar{n}_t)$  and  $n^*(\bar{d}_t, \bar{n}_t)$ . Furthermore we can define the *consumption function*

$$c^*(\bar{d}_t, \bar{n}_t) \equiv \bar{n}_t - n^*(\bar{d}_t, \bar{n}_t) \quad (\text{A.6})$$

Conditional on  $d_t$ , we have that the choice of  $n_t$  is constrained by

$$\begin{aligned}
 n_t &\in [\underline{n}(\bar{d}_t, d_t), \bar{n}_t] \quad (\text{A.7}) \\
 \underline{n}(\bar{d}_t, d_t) &\equiv \begin{cases} -d_t & \text{if } d_t \leq \bar{d}_t \\ -\min\{d_t, \frac{1}{\eta}(\varphi - d_t)\} & \text{if } d_t > \bar{d}_t \end{cases}
 \end{aligned}$$

Noting that  $n_t = \bar{n}_t$  implies  $c_t = 0$ , we can conclude that  $n^*(\bar{d}_t, \bar{n}_t) < \bar{n}_t$ .

## A.3 “Lazy” Household

Consider a “lazy” household who only “knows” the optimal choice functions  $d^*(\bar{d}_t, \bar{n}_t)$  and  $n^*(\bar{d}_t, \bar{n}_t)$  in the particular point  $(\widehat{d}, \widehat{n})$ . Due to its laziness it also chooses  $d_t = d^*(\widehat{d}, \widehat{n})$  and  $n_t = n^*(\widehat{d}, \widehat{n})$  for all  $(\bar{d}_t, \bar{n}_t) \neq (\widehat{d}, \widehat{n})$  whenever that it feasible.

If  $n^* \left( \widehat{d}, \widehat{n} \right) > \underline{n} \left( \widehat{d}, d^* \left( \widehat{d}, \widehat{n} \right) \right)$  then because  $n^* \left( \widehat{d}, \widehat{n} \right) < \widehat{n}$  this lazy behavior is at least feasible in a *small open interval around*  $\widehat{n}$ ,  $\mathcal{O} \left( \widehat{n} \right)$ . Hereby we can define the “lazy” household value function

$$\forall \bar{n}_t \in \mathcal{O} \left( \widehat{n} \right) : L \left( \bar{n}_t; \widehat{d}, \widehat{n} \right) = u \left( \bar{n}_t - n_L^* \left( \widehat{d}, \widehat{n} \right) \right) + \beta \cdot \sum_{\Psi \times \Xi} \psi^{1-\rho} \cdot v \left( \bar{d}_+ \left( d_L^*; \psi \right), \bar{n}_+ \left( d_L^*, n_L^*; \psi, \xi \right) \right) \quad (\text{A.8})$$

where

$$d_L^* \equiv d^* \left( \widehat{d}, \widehat{n} \right)$$

$$n_L^* \equiv n^* \left( \widehat{d}, \widehat{n} \right)$$

where the *continuation value*  $v(\bullet)$  is a *constant* depending on  $\left( \widehat{d}, \widehat{n} \right)$ .

Note that  $L \left( \bar{n}_t; \widehat{d}, \widehat{n} \right)$  is a *differentiable lower support function* for  $v \left( \widehat{d}, \bar{n}_t \right)$  at  $\widehat{n}$  as

$$\forall \bar{n}_t \in \mathcal{O} \left( \widehat{n} \right) : L \text{ is differentiable.} \quad (\text{A.9})$$

$$\forall \bar{n}_t \in \mathcal{O} \left( \widehat{n} \right) : L \left( \bar{n}_t; \widehat{d}, \widehat{n} \right) \leq v \left( \widehat{d}, \bar{n}_t \right) \quad (\text{A.10})$$

$$\bar{n}_t = \widehat{n} : L \left( \bar{n}_t; \widehat{d}, \widehat{n} \right) = v \left( \widehat{d}, \bar{n}_t \right) \quad (\text{A.11})$$

For later use we note

$$L' \left( \bar{n}_t; \widehat{d}, \widehat{n} \right) = u'_c \left( \bar{n}_t - n^* \left( \widehat{d}, \widehat{n} \right) \right) = \left( \bar{n}_t - n^* \left( \widehat{d}, \widehat{n} \right) \right)^{-\rho} \quad (\text{A.12})$$

## A.4 Euler-Equation

**Proposition A.1.** *Conditional on  $d_t = \widehat{d}$  an interior optimal consumption choice  $c_t^{*,\widehat{d}} \equiv c^* \left( \widehat{d}_t, \bar{n}_t; \widehat{d} \right)$  must satisfy the Euler-equation*

$$u'_c \left( c_t^{*,\widehat{d}} \right) = (1 + r_a) \cdot \beta \cdot \sum_{\Psi \times \Xi} \cdot \psi^{-\rho} \cdot u'_c \left( c_{t+1}^* \right) \Leftrightarrow \quad (\text{A.13})$$

$$c_t^{*,\widehat{d}} = \left[ (1 + r_a) \cdot \beta \cdot \sum_{\Psi \times \Xi} \cdot \left( \psi \cdot c_{t+1}^* \right)^{-\rho} \right]^{-\frac{1}{\rho}}$$

where  $c_{t+1}^* \equiv c^* \left( \bar{d}_+ \left( \hat{d}; \psi \right), \bar{n}_+ \left( \hat{d}, n_t^{*,\hat{d}}; \psi, \xi \right) \right)$  with  $n_t^{*,\hat{d}} \equiv n^{*,\hat{d}} \left( \bar{d}_t, \bar{n}_t; \hat{d} \right)$  as the corresponding optimal (net) savings choice.

*Proof.* Define the *value-of-choice function* conditional on the debt choice as

$$\begin{aligned} \phi \left( n_t; \bar{d}_t, \bar{n}_t, \hat{d} \right) &\equiv u \left( \bar{n}_t - n_t \right) \\ &+ \beta \cdot \sum_{\Psi \times \Xi} \cdot v \left( \bar{d}_+ \left( \hat{d}; \psi \right), \bar{n}_+ \left( \hat{d}, n_t; \psi, \xi \right) \right) \end{aligned} \quad (\text{A.14})$$

Then consider the two functions:

$$\begin{aligned} \bar{\phi} \left( n_t; \bar{d}_t, \bar{n}_t, \hat{d} \right) &\equiv u \left( \bar{n}_t - n_t^{*,\hat{d}} \right) \\ &+ \beta \cdot \sum_{\Psi \times \Xi} \cdot \psi^{1-\rho} \cdot v \left( \bar{d}_+ \left( \hat{d}; \psi \right), \bar{n}_+ \left( \hat{d}, n_t^{*,\hat{d}}; \psi, \xi \right) \right) \end{aligned} \quad (\text{A.15})$$

such that

$$\begin{aligned} \bar{\phi} \left( n_t; \bar{d}_t, \bar{n}_t, \hat{d} \right) &= \phi \left( n_t^{*,\hat{d}}; \bar{d}_t, \bar{n}_t, \hat{d} \right) \\ \bar{\phi}' \left( n_t; \bar{d}_t, \bar{n}_t, \hat{d} \right) &= 0 \end{aligned} \quad (\text{A.16})$$

$$\begin{aligned} \underline{\phi} \left( n_t; \bar{d}_t, \bar{n}_t, \hat{d} \right) &= u \left( \bar{n}_t - n_t \right) \\ &+ \beta \cdot \sum_{\Psi \times \Xi} \cdot \psi^{1-\rho} \cdot L \left( \bar{n}_+ \left( \hat{d}, n_t; \psi, \xi \right); \hat{d}, \hat{n} \right) \end{aligned} \quad (\text{A.17})$$

where

$$\begin{aligned} \hat{d} &\equiv d_+ \left( \hat{d}; \psi \right) \\ \hat{n} &\equiv \bar{n}_+ \left( \hat{d}, n_t^{*,\hat{d}}; \psi, \xi \right) \end{aligned}$$

where (A.15) is clearly a differentiable *upper* support function for (A.14) at  $n_t = n_t^{*,\hat{d}}$ , and (A.17) is a differentiable *lower* support function for (A.14) at  $n_t = n_t^{*,\hat{d}}$  because the first terms are the same in both equations, and because we showed in section A.3 that

$$L \left( \bar{n}_+ \left( \hat{d}, n_t; \psi, \xi \right); \bar{d}_+ \left( \hat{d}; \psi \right), \bar{n}_+ \left( \hat{d}, n_t^{*,\hat{d}}; \psi, \xi \right) \right)$$

is a differentiable *lower* support function for

$$v \left( \bar{d}_+ \left( \hat{d}; \psi \right), \bar{n}_+ \left( \hat{d}, n_t; \psi, \xi \right) \right) \text{ at } n_t = n_t^{*,\hat{d}}$$

Using the *differentiable sandwich lemma* A.1 we can now conclude that  $\phi \left( n_t; \bar{d}_t, \bar{n}_t, \hat{d} \right)$  is differentiable at  $n_t = n_t^{*,\hat{d}}$ , and by using the *reverse calculus lemma* A.2 repeat-

edly we can then conclude that  $v(\bar{d}_{t+1}, \bar{n}_{t+1})$  is differentiable in  $\bar{n}_{t+1}$  at  $n_t = n_t^{*, \hat{d}}$ . Finally the *differentiable sandwich lemma* A.1 also implies that the derivatives of the endogenous functions at  $n_t = n_t^{*, \hat{d}}$  is equal to the derivatives of both their upper and lower support functions. This implies

$$\begin{aligned} \phi'_n \left( n_t^{*, \hat{d}}; \bar{d}_t, \bar{n}_t, \hat{d} \right) &= 0 \Leftrightarrow \\ u'_c \left( \bar{n}_t - n_t^{*, \hat{d}} \right) &= \beta \cdot \sum_{\Xi \times \Psi} \psi^{1-\rho} \cdot v'_n \left( \bar{d}_+ (\bullet), \bar{n}_+ (\bullet) \right) \cdot \frac{\partial \bar{n}_+ \left( \hat{d}, n_t; \psi, \xi \right)}{\partial n_t} \Leftrightarrow \\ u'_c \left( c_t^{*, \hat{d}} \right) &= \beta \cdot \sum_{\Xi \times \Psi} \psi^{1-\rho} \cdot L' \left( \bar{n}_+ (\bullet); \bar{d}_+ (\bullet), \bar{n}_+ (\bullet) \right) \cdot \frac{1+r_a}{\psi} \\ &= (1+r_a) \cdot \beta \cdot \sum_{\Xi \times \Psi} \psi^{-\rho} \cdot u'_c \left( c^* \left( \bar{d}_+ \left( \hat{d}; \psi \right), \bar{n}_+ \left( \hat{d}, n_t^{*, \hat{d}}; \psi, \xi \right) \right) \right) \end{aligned}$$

where first (A.16) and secondly (A.12) were used. Simple insertions now imply equation (A.13).  $\square$

## A.5 Monotonicity of the Savings Correspondence

Fella (2014) presents the following lemma from Edlin and Shannon (1998):

**Lemma A.3.** *If  $g(x, z)$  is a function where  $\frac{\partial g}{\partial x}$  is strictly increasing in  $z$  at  $x^*(z) \in \arg \max_x g(x, z)$ , then  $x^*(z)$  is strictly increasing in  $z$ .*

To use this result we first define an *inner value function* conditional on the  $d_t$ -choice:

$$\begin{aligned} w \left( \bar{d}_t, \bar{n}_t, d_t \right) &\equiv \max_{n_t} u \left( \bar{n}_t - n_t \right) \\ &+ \beta \cdot \sum_{\Xi \times \Psi} \psi^{1-\rho} \cdot v \left( \bar{d}_+ (d_t; \psi), \bar{n}_+ (d_t, n_t; \psi, \xi) \right) \end{aligned} \quad (\text{A.18})$$

Hereby we have

$$n_t^{*, d_t} \left( \bar{d}_t, \bar{n}_t; d_t \right) = \arg \max_{n_t} w \left( \bar{d}_t, \bar{n}_t, d_t \right) \quad (\text{A.19})$$

and using proposition A.1 we get

$$\begin{aligned} \frac{\partial w}{\partial n} \Big|_{n_t = n_t^{*, d_t}} &= -u'_c \left( \bar{n}_t - n_t^{*, d_t} \right) \\ &+ (1+r_a) \cdot \beta \cdot \sum_{\Xi \times \Psi} \psi^{-\rho} \cdot v'_n \left( \bar{d}_+ (d_t; \psi), \bar{n}_+ \left( d_t, n_t^{*, d_t}; \psi, \xi \right) \right) \end{aligned} \quad (\text{A.20})$$

which is clearly *increasing* in  $\bar{n}_t$  due to the *concavity of the utility function*. Consequently lemma A.3 applies, and we get the following proposition

**Proposition A.2.** *If  $\bar{n}_H > \bar{n}_L$  then for any  $n_L \in n^{*,\hat{d}}(\bar{d}_t, \bar{n}_L; d_t)$  and any  $n_H \in n^{*,\hat{d}}(\bar{d}_t, \bar{n}_H; \hat{d})$  we have  $n_H \geq n_L$ .*

This further implies that the inverse of  $n^{*,d_t}(\bar{d}_t, \bar{n}_t; d_t)$  with respect to  $\bar{n}_t$  is a function, which is a necessity for the EGM-algorithm to work as explained in more detail by Fella (2014). Fundamentally we now know that as  $\bar{n}_t$  increases, there *cannot* be any *upward* jumps in  $c_t^{*,d_t}(\bar{d}_t, \bar{n}_t; d_t)$ . As we discuss in more detail in appendix B, we can therefore establish a *numerical criterion* for “*practical sufficiency*” of the Euler-equation, which we for “high enough” degrees of uncertainty always find to be satisfied.

## B Solution Algorithm

The purpose of the present appendix is to describe the *solution algorithm* in detail.

### B.1 Discretization

To facilitate solving the model, we consider a *discretized* version with *finite-horizon*:

$$\begin{aligned}
 v_t(u_t, x_t, \bar{d}_t, \bar{n}_t) &= \max_{d_t, c_t} u(c_t) + \beta \cdot \sum \Omega_{t+1}(\bullet) \\
 &\text{s.t.} \\
 n_t &= \bar{n}_t - c_t \\
 \Omega_{t+1}(d_t, n_t; u_+, x_+, \psi, \xi) &= (\Gamma\psi)^{1-\rho} \cdot v_{t+1}(u_+, x_+, \bar{d}_+(\bullet), \bar{n}_+(\bullet)) \\
 \bar{d}_+(d_t; \psi) &= \arg \min_{z \in \bar{\mathcal{D}}} \left| z - \frac{1}{\Gamma\psi} \cdot (1 - \lambda) \cdot d_t \right| \\
 &\quad \bar{\mathcal{D}} = \{0, \dots, \Upsilon\}, \quad |\bar{\mathcal{D}}| = N_{\bar{d}} \in \mathbb{N}, \quad \Upsilon > 0 \\
 \bar{n}_+(d_t, n_t; u_+, \psi, \xi) &= \frac{1}{\Gamma\psi} \cdot [(1 + r_a) \cdot n_t - (r_d - r_a) \cdot d_t] + \tilde{\xi}(u_+, \xi) \\
 d_t &\in \mathcal{D}(u_t, x_t, \bar{d}_t, \bar{n}_t) \\
 c_t &\in \mathcal{C}(u_t, x_t, \bar{d}_t, \bar{n}_t, d_t) \\
 v_T(\bar{n}_t) &= u(\max\{\bar{n}_t, 0\}) \\
 \sum &\equiv \sum_{U \times X \times \Psi \times \Xi} p(u_+, x_+, \psi, \xi | u_t, x_t) = 1
 \end{aligned}$$

where  $\mathcal{D}(u_t, x_t, \bar{d}_t, \bar{n}_t)$  is the choice set for  $d_t$  and  $\mathcal{C}(u_t, x_t, \bar{d}_t, \bar{n}_t, d_t)$  is the choice set for  $c_t$ :

$$d_t \in \left[ \max\{-\bar{n}_t, 0\}, \max\{\bar{d}_t, \eta \cdot \bar{n}_t + \mathbf{1}_{x_t=0} \cdot \varphi\} \right] \quad (\text{B.1})$$

$$c_t \in \left[ 0, \bar{c}(\bar{d}_t, \bar{n}_t, d_t) \right] \quad (\text{B.2})$$

$$\bar{c}(\bar{d}_t, \bar{n}_t, d_t) \equiv \begin{cases} \bar{n}_t + d_t & \text{if } d_t \leq \bar{d}_t \\ \bar{n}_t + \min\left\{d_t, \frac{1}{\eta}(\mathbf{1}_{x_t=0} \cdot \varphi - d_t)\right\} & \text{if } d_t > \bar{d}_t \end{cases}$$

The critical step is discretizing the  $\bar{d}_+(\bullet)$ -function, but we can easily verify that both a higher  $\Upsilon$  and/or a higher  $N_{\bar{d}}$  do *not* change the *optimal choice functions*  $d_t^*(u_t, x_t, \bar{d}_t, \bar{n}_t)$  and  $c_t^*(u_t, x_t, \bar{d}_t, \bar{n}_t)$ .

The shocks are discretized using *Gauss-Hermite quadrature* with node sets  $\Psi =$

$\Psi(N_\psi)$  and  $\xi = \xi(N_\xi)$ , where  $N_\psi$  and  $N_\xi$  are the number of nodes for each shock. The lower and upper supports are  $\underline{\psi} \equiv \min(\Psi)$ ,  $\bar{\psi} \equiv \max(\Psi)$ ,  $\bar{\xi} \equiv \max(\Xi)$ , and  $\underline{\xi} \equiv \min(\Xi)$ . The shock probabilities naturally sum to one, and are conditional on the  $u_t$  and  $x_t$  states.

## B.2 State Space

The discretization allows us to construct the *state space* starting from the the *terminal period*

$$\begin{aligned} \mathcal{S}_T(u_T, x_T) &= \left\{ (\bar{d}_T, \bar{n}_T) : \bar{d}_T \in \bar{\mathcal{D}}, \bar{n}_T \geq \kappa_T(u_T, x_T, \bar{d}_T) \right\} \\ &\quad \kappa_T(u_T, x_T, \bar{d}_T) = 0 \end{aligned} \quad (\text{B.3})$$

and using the *recursion*

$$\begin{aligned} \mathcal{S}_t(u_t, x_t) &= \left\{ (\bar{d}_t, \bar{n}_t) : \bar{d}_t \in \bar{\mathcal{D}}, \bar{n}_t \geq \kappa_t(u_t, x_t, \bar{d}_t) \right\} \\ &\quad \kappa_t(u_t, x_t, \bar{d}_t) = \min(\mathcal{Z}) \\ \mathcal{Z} &= \left\{ z : \exists d_t \begin{array}{l} d_t \in \mathcal{D}(u_t, x_t, \bar{d}_t, z) \text{ and} \\ \forall (\psi, \xi, u_+, x_+) : \\ \bar{n}_+(d_t, z; u_+, \psi, \xi) \geq \kappa_{t+1}(u_+, x_+, \bar{d}_+(d_t, \psi)) \end{array} \right\} \end{aligned} \quad (\text{B.4})$$

This procedure ensures that there for all *interior* points in the state space *exists* a set of choices such that the *value function is finite*. On the contrary such a set of choices does not exist on the *border* of the state space, and the value function therefore approaches  $-\infty$  as  $\bar{n}_t \rightarrow \kappa_t(u_t, x_t, \bar{d}_t) \geq -\max\left\{\bar{d}_t, \frac{\mathbf{1}_{x_t=0} \cdot \varphi}{1+\eta}\right\}$ .

A corollary is that the households will always choose  $d_t$  and  $c_t$  such that

$$n_t > \underline{n}_t(d_t) = \max_{x_+ u_+, \psi, \xi} \frac{\Gamma \psi \cdot \left[ \kappa_{t+1}(x_+, u_+, \bar{d}_+(d_t, \psi)) - \tilde{\xi}(u_+, \xi) \right] + (r_d - r_a) \cdot d_t}{1 + r_a} \quad (\text{B.5})$$



Figure B.1: State Space Border,  $\kappa_t(0, 0, \bar{d}_t)$

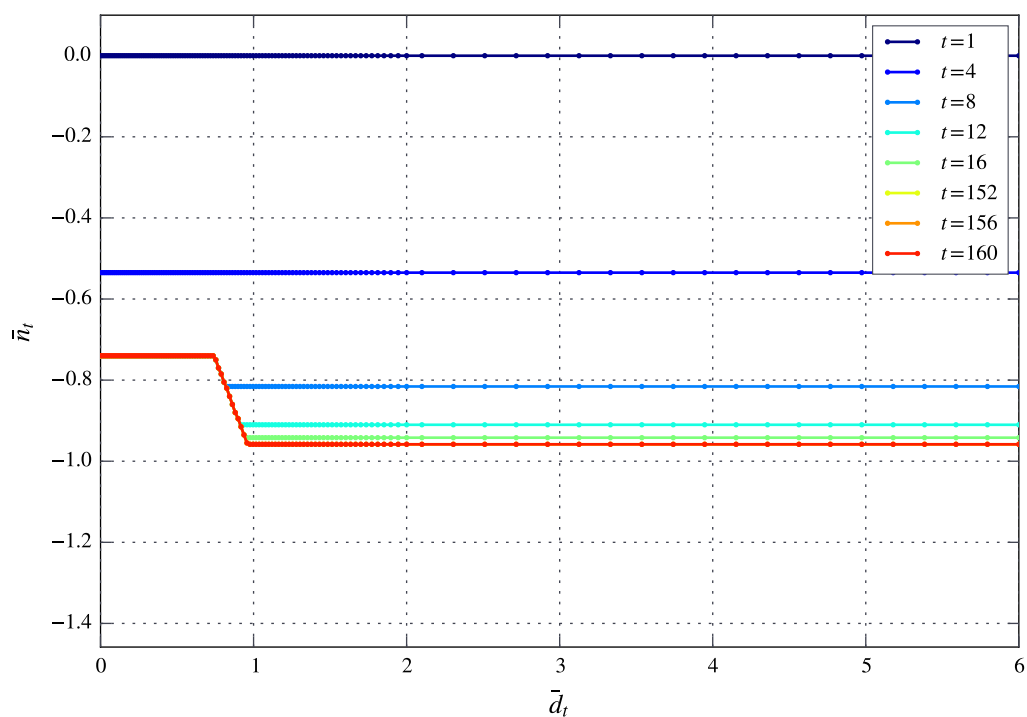
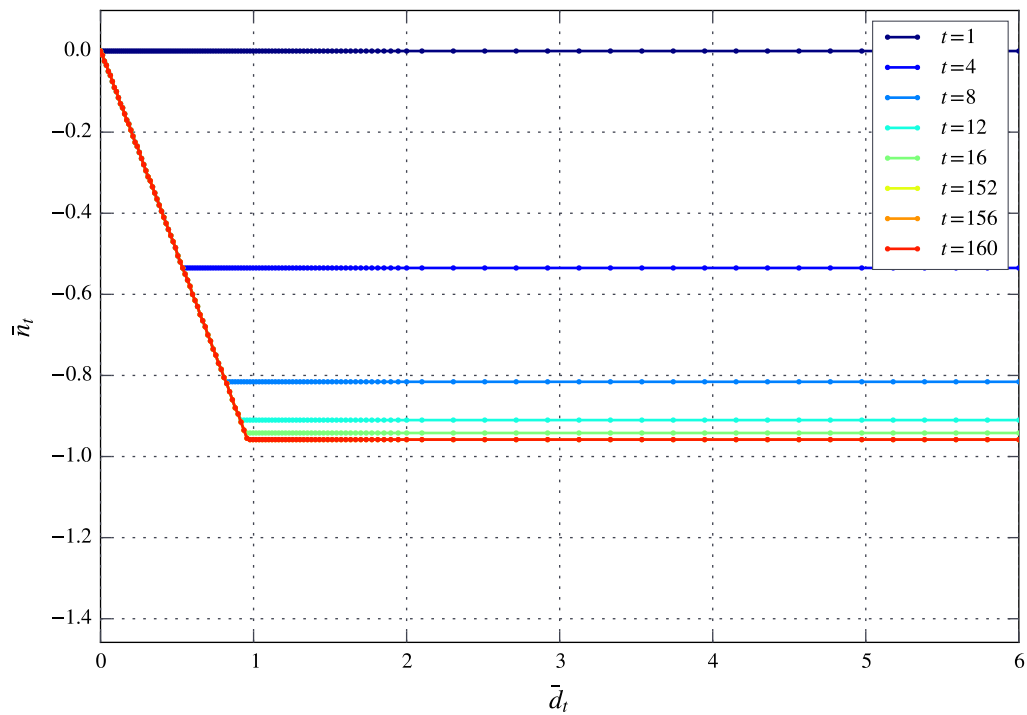


Figure B.2: State Space Border,  $\kappa_t(1, 1, \bar{d}_t)$

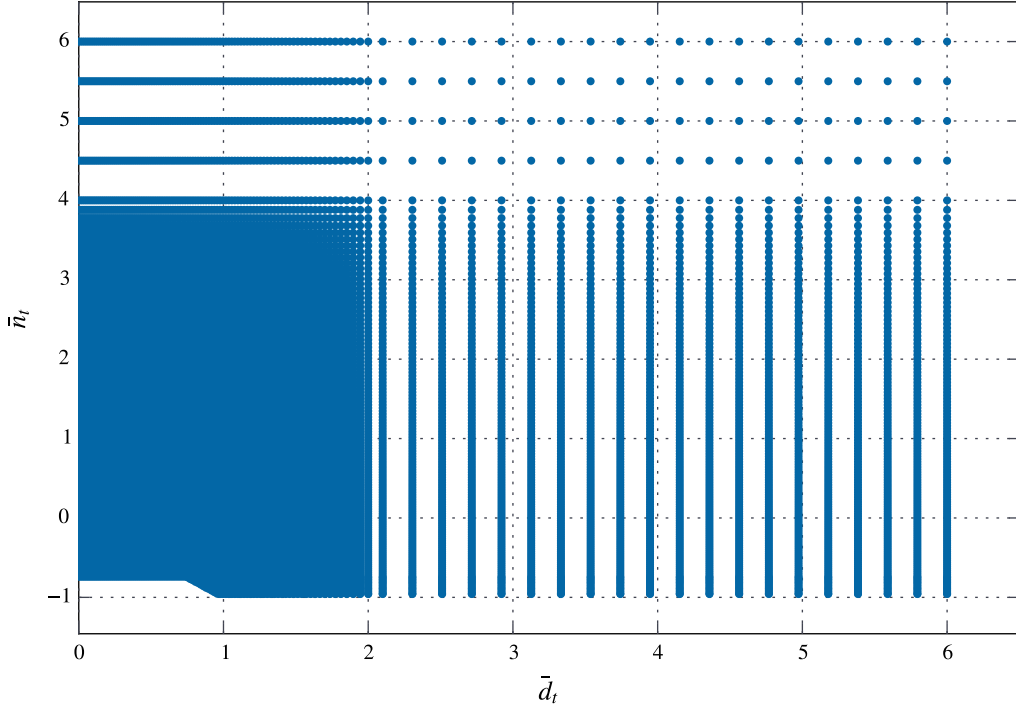


Note that the state space does not seem to have an *analytical* form, but in the limit must satisfy

$$\begin{aligned}
 \mathcal{S}_{-\infty}(u_t, x_t) &\subseteq \mathcal{S}_L \cap \mathcal{S}_S & (B.6) \\
 \mathcal{S}_L &= \left\{ (\bar{d}, \bar{n}) : \bar{n} > -\max \left\{ \bar{d}, \frac{\mathbf{1}_{x_t=0} \cdot \varphi}{1 + \eta} \right\} \right\} \\
 \mathcal{S}_S &= \left\{ (\bar{d}, \bar{n}) : \bar{n} > -(\phi + \phi^2 \dots) \min \{ \mu, \underline{\xi} \} \right\} \\
 \phi &\equiv \frac{\Gamma \psi}{1 + r_d} < 1
 \end{aligned}$$

Outside  $\mathcal{S}_L$  the household lacks *liquidity* in the current period, and outside  $\mathcal{S}_S$  it is *insolvent* under *worst case expectations*. This is also clear from figure B.1 and B.2.

The state space grid is constructed beginning with an universal  $\bar{d}_t$ -vector with  $N_{\bar{d}}$  nodes chosen such that there are relative more nodes closer to zero. For each combination of  $u_t$  and  $x_t$ , we hereafter construct a  $t$ -specific  $\bar{n}_t$ -vector as the union of a) all unique  $\kappa_t(u_t, x_t, \bar{d}_t)$ -values, and b) a  $\bar{n}_t$ -vector with  $N_{\bar{n}}$  nodes beginning in the largest  $\kappa_t(u_t, x_t, \bar{d}_t)$ -value and chosen such that there are relative more nodes closer to this minimum. The grid values of  $\bar{n}_t$  conditional on  $\bar{d}_t$  is then the  $t$ -specific  $\bar{n}_t$ -vector excluding all  $\bar{n}_t < \kappa_t(u_t, x_t, \bar{d}_t)$ , implying a total maximum of  $N_{\bar{d}} + N_{\bar{n}}$  nodes in the  $\bar{n}_t$ -dimension. The grid is illustrated in figure B.3.

Figure B.3: State Space Grid ( $u_t = 0, x_t = 0, t = 0$ )


### B.3 Value Function Iteration

The *value function iteration* is now given by  $\forall (u_t, x_t), \forall (\bar{d}_t, \bar{n}_t) \in \mathcal{S}_t(u_t, x_t)$

$$v_t(u_t, x_t, \bar{d}_t, \bar{n}_t) = \max_{d_t, c_t} \frac{c_t^{1-\rho}}{1-\rho} + \beta \cdot \sum \Omega_{t+1}(d_t, n_t; u_+, x_+, \psi, \xi) \quad (\text{B.7})$$

where when  $t$  is so low that  $\mathcal{S}_t \approx \mathcal{S}_{-\infty}$ , we could implement the following *stopping criterion*

$$\nexists (u, x, \bar{d}, \bar{n}) \in \mathcal{S}_{-\infty} : |v_t(u, x, \bar{d}, \bar{n}) - v_{t+1}(u, x, \bar{d}, \bar{n})| \geq \zeta \quad (\text{B.8})$$

where  $\zeta$  is a *tolerance parameter*. To simplify matters we instead always iterate  $T$ -periods and check that our results are unchanged when increasing  $T$ .

### B.4 Unconstrained Consumption Function

Assuming that the debt choice,  $d_t = d$ , the employment status,  $u_t = u$ , and the credit market access status,  $x_t = x$ , are given, the *Euler-equation* (see appendix

A) for the consumption choice,  $c_t$ , is

$$c_t = \left[ (1 + r_a) \cdot \beta \cdot \sum (\Gamma \psi \cdot c_{t+1}^*)^{-\rho} \right]^{-\frac{1}{\rho}} \quad (\text{B.9})$$

where  $c_{t+1}^* = c_{t+1}^*(u_+, x_+, \bar{d}_+(d, \psi), \bar{n}_+(d, n_t, u_+, \psi, \xi))$ .

Assuming that the  $c_{t+1}^*$ -function is known from earlier iterations, the *endogenous grid point method* can now be used to construct an *unconstrained consumption function*. The steps are:

1. Construct a **grid vector** of  $n_t$ -values denoted  $\vec{n}$  with the minimum value  $\underline{n}_t(d) + \epsilon$  (see equation (B.5)) where  $\epsilon$  is a small number (e.g.  $10^{-8}$ ) and of length  $N_n$  with more values closer to the minimum.

2. Construct an **associated consumption vector**

$$\vec{c} = \left( (1 + r_a) \cdot \beta \cdot \sum (\Gamma \psi \cdot c_{t+1}^*(u_+, x_+, \bar{d}_+(d, \psi), \bar{n}_+(d, \vec{n}, \psi, \xi)))^{-\rho} \right)^{-\frac{1}{\rho}}$$

3. Construct an **endogenous grid vector** of  $\bar{n}_t$ -values by

$$\vec{\bar{n}} = \vec{n} + \vec{c}$$

4. The **unconstrained consumption function**,  $c_{u,x,d}^\circ(\bar{n}_t)$  can now be constructed from the association between  $\{\underline{n}_t, \vec{\bar{n}}\}$  and  $\{0, \vec{c}\}$  together with *linear interpolation*.

Note that this can be done independently across  $d_t$ 's and does *not* depend on the states, except for  $u_t$  and  $x_t$  which affects the expectations. This step speeds up the algorithm tremendously because it avoids root finding completely.

Note that because we lack a proof of sufficiency of the Euler-equation, we cannot be certain that  $\vec{\bar{n}}$  will be *increasing* and thus only have *unique values*. If the same value is repeated multiple times in  $\vec{\bar{n}}$  the EGM-algorithm breaks down, but in practice we find that this is never the case as long as the degree of uncertainty is "large enough".

## B.5 Choice Functions

The consumption choice can now be integrated out, and the household problem written purely in terms of the debt choice, i.e.

$$\begin{aligned}
 v(u_t, x_t, \bar{d}_t, \bar{n}_t) &= \max_{d_t \in \mathcal{D}(u_t, x_t, \bar{d}_t, \bar{n}_t)} \frac{(c^\bullet(\bullet))^{1-\rho}}{1-\rho} & (B.10) \\
 &+ \beta \cdot \sum \Omega_{t+1}(d_t, n_t; u_+, x_+, \psi, \xi) \\
 &\text{s.t.} \\
 n_t &= \bar{n}_t - c^\bullet(\bullet) \\
 c^\bullet(u_t, x_t, \bar{d}_t, \bar{n}_t, d_t) &= \min \left\{ c_{u_t, x_t, d_t}^\circ(\bar{n}_t), \bar{c}(u_t, x_t, \bar{d}_t, \bar{n}_t, d_t) \right\} \\
 \bar{c}(u_t, x_t, \bar{d}_t, \bar{n}_t, d_t) &= \begin{cases} \bar{n}_t + d_t & \text{if } d_t \leq \bar{d}_t \\ \bar{n}_t + \min \left\{ d_t, \frac{1}{\eta} \cdot (\mathbf{1}_{x_t=0} \cdot \varphi - d_t) \right\} & \text{if } d_t > \bar{d}_t \end{cases}
 \end{aligned}$$

This problem can be solved using a *grid search algorithm* over a *fixed*  $d_t$ -grid with step-size  $d_{step}$ , such that  $c_{u_t, x_t, d_t}^\circ(\bar{n}_t)$  is a simple look-up table. This has to be done for all possible states, but it is possible to speed this up by utilizing some bounds on the optimal debt choice function. Specifically we use that given

$$d^*(u_t, x_t, \Upsilon, \bar{n}_t) = d_\Upsilon \quad (B.11)$$

$$d^*(u_t, x_t, 0, \bar{n}_t) = d_0 \quad (B.12)$$

$$d^*(u_t, x_t, \bar{d}_{d=d_0}, \bar{n}_t) = d_0 \quad (B.13)$$

we must have

$$\forall \bar{d}_t \in [d_\Upsilon : \Upsilon] : d^*(u_t, x_t, \bar{d}_t, \bar{n}_t) = d_\Upsilon \quad (B.14)$$

$$\forall \bar{d}_t \in [d_0 : d_\Upsilon], \epsilon \geq 0 : d^*(u_t, x_t, \bar{d}_t + \epsilon, \bar{n}_t) \geq d^*(u_t, x_t, \bar{d}_t, \bar{n}_t) \quad (B.15)$$

$$\forall \bar{d}_t \in (0, d_0) : d^*(u_t, x_t, \bar{d}_t, \bar{n}_t) \leq d_0 \quad (B.16)$$

$$\forall \bar{d}_t \in [0 : \bar{d}_{d=d_0}] : d^*(u_t, x_t, \bar{d}_t, \bar{n}_t) = d_0 \quad (B.17)$$

Over  $u_t$ ,  $x_t$  and  $\bar{n}_t$  the problem is jointly parallelizable. The value function is evaluated in the  $\bar{n}_{t+1}$ -dimension<sup>20</sup> by “negative inverse negative inverse” linear interpolation, where the negative inverse value function is interpolated linearly and the negative inverse of the result is then used; this is beneficial because the

<sup>20</sup>The other dimensions are fully discretized.

value function is then equal to zero on the border of the state space.

Note that the grid search needs to be *global* because we otherwise might find multiple *local* extrema and because there might be *discontinues* due to the non-convex choice set. This directly give us  $d^*(u_t, x_t, \bar{d}_t, \bar{n}_t)$  and therefore also

$$c^*(u_t, x_t, \bar{d}_t, \bar{n}_t) = c^\bullet(u_t, x_t, \bar{d}_t, \bar{n}_t, d^*(u_t, x_t, \bar{d}_t, \bar{n}_t)) \quad (\text{B.18})$$

## B.6 Implementation

The algorithm is implemented in *Python 2.7*, but the core part is written in *C* parallelized using *OpenMP* and called from Python using *CFFI*. Only free open source languages and programs are needed to run the code. The code-files are available from the authors upon request.

Table B.1 shows the parametric settings we use. Our results are robust to using even finer grids.

Table B.1: Algorithm Settings

Parameter	Value
Nodes for transitory income shock, $N_\xi$	8
Nodes for permanent income shock, $N_\psi$	8
Nodes for beginning-of-period debt, $N_{\bar{d}}$	80
Nodes for beginning-of-period net wealth, $N_{\bar{n}}$	100
Nodes for net wealth grid vector ( $\vec{n}$ ), $N_n$	40
Value used to calculate minimum of net wealth grid vector, $\epsilon$	$10^{-8}$
Step-size of fixed debt grid, $d_{step}$	$5 \cdot 10^{-3}$
Number of iterations, $T$	160

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# Chapter 2

The Demand for Housing over the Life-Cycle  
under Long-Term *Gross* Debt Contracts

# The Demand for Housing over the Life-Cycle under Long-Term *Gross* Debt Contracts\*

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## Abstract

This paper generalizes the model of individual demand for housing over the life-cycle in [Attanasio, Bottazzi, Low, Nesheim and Wakefield \(2012\)](#) by formulating the long-term debt contracts in *gross* terms instead of in *net* terms. This more realistic market structure have important implications for the model dynamics because it enables the households to save in financial assets instead of requiring them to save by increasing their mortgage repayments. Moreover the potential for such *precautionary balance sheet expansions* make the households able to self-insure more optimally and thus increase their ex ante expected welfare. Quantitatively the welfare gain is largest when there is no mortgage spread and no forced mortgage repayments. Qualitatively the results are robust to a substantial mortgage spread and forced repayments if just the households are impatient enough. Introducing a combination of proportional and fixed (re)financing costs does likewise not affect the central results.

**Keywords:** Housing, Uncertainty, Credit Constraints.

**JEL-Codes:** D91, R52, G11.

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## 1 Introduction

Understanding the demand for housing over the life-cycle is of uttermost importance. Firstly housing services are a central component in the utility of households, and secondly housing as collateral is a key factor in determining how well households can smooth consumption in the face of e.g. income shocks.

The purpose of this paper is to generalize the model of individual demand for housing over the life-cycle in [Attanasio, Bottazzi, Low, Nesheim and Wakefield \(2012\)](#) (henceforth ABLNW) with a borrowing constraint in *gross* debt instead of in *net* debt. The ABLNW model framework is a good starting point because it incorporates many of the most important features of the housing choice: there is an outside option of renting, houses come in different discrete sizes, buying and selling them is subject to substantial transaction costs, and housing and consumption choices are made under uninsurable income and house price uncertainty.

Most importantly ABLNW specify mortgages as long-term debt contracts. Specifically they assume that households are subject to both a *loan-to-value (LTV)* constraint and a *loan-to-income (LTI)* constraint, but that they only need to satisfy these constraints when they originate a new or refinance an existing mortgage. ABLNW argue persuasively that this is “a novel and realistic assumption” (p. 2) because it e.g. does not force highly indebted household to deleverage sharply when house prices fall and the LTV-ratio mechanically increases.

One central limitation in the original ABLNW model, however, is that it is formulated in financial *net* worth alone. This implies that households will never hold any (gross) financial assets while they have an outstanding balance on their mortgage. This assumption is especially problematic in terms of internal consistency because ABLNW also assume that the interest rate on financial assets and mortgages are equal. Hereby it is actually cost less for a household to build up its balance sheet, and it will therefore always do so in full in order to reap the option value of a large mortgage. The reason is that households with a large gross debt are certain that they in future periods will be able to choose continuing to have a large gross debt even if house prices and their income fall; the large mortgage thus provides extra liquidity.

The central contribution in this paper is to show that modeling the long-term borrowing constraint in *gross* terms rather than *net* terms has important implications for the model dynamics because it allows for *precautionary balance sheet expansions* where financial assets are accumulated instead of mortgage debt being repaid. This amplifies the overall demand for housing, and induces a marked

substitution from flats towards bigger houses. These effects are especially strong among the young, and in total the welfare gain of precautionary balance sheet expansions can be substantial for the households if the spread between the interest rate on mortgages and the return rate on financial assets is close to zero and the forced mortgage repayment rate is low. Furthermore the welfare gain remains significant even under a substantial mortgage spread and sizable forced mortgage repayments if just the households are impatient enough; the reason is that in the net stock formulation of the model impatient households have higher LTV-ratios early in life lowering the base cost of a precautionary balance sheet expansion. This increased impatience is moreover a substantial improvement in terms of matching the empirical facts on outright ownership rates and LTV-ratios in the early part of the life-cycle.

The above discussed results are robust to deviations from the baseline parameters taken from the detailed calibration in ABLNW. Finally the results are only marginally affected by introducing proportional and fixed (re)financing costs.

The paper is primarily related to a growing set of papers that aim to deepen our understanding of housing decisions in life-cycle consumption models. Apart from [Attanasio, Bottazzi, Low, Nesheim and Wakefield \(2012\)](#), some of the most recent contributions are [Li, Liu and Yao \(2014\)](#), [Halket and Vasudev \(2014\)](#), [Chambers, Garriga and Schlagenhaut \(2013\)](#), [Bajari, Chan, Krueger and Miller \(2013\)](#) and [Chen, Michaux and Roussanov \(2013\)](#).<sup>1</sup> None of these papers discuss the importance of modeling long-term debt in terms of gross debt and thus of allowing for *precautionary balance sheet expansions*. Balance sheet expansions are in principle allowed for in the models presented by [Chambers, Garriga and Schlagenhaut \(2009c\)](#), [Chen, Michaux and Roussanov \(2013\)](#) and [Sommer, Sullivan and Verbrugge \(2013\)](#), but the precautionary motive for doing so is never discussed or highlighted. Finally both [Attanasio, Leicester and Wakefield \(2011\)](#) and [Halket and Vasudev \(2014\)](#) use a mortgage setup very similar to ABLNW, and in particular also confine themselves to formulate their models in net worth alone without motivating this restrictive choice.

An additional contribution in the present paper is the novel restructuring of the

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<sup>1</sup> Older examples are [Gervais \(2002\)](#), [Yao and Zhang \(2005\)](#), [Silos \(2007a,b\)](#), [Campbell and Cocco \(2007\)](#), [Li and Yao \(2007\)](#), [Ríos-Rull and Sánchez-Marcos \(2008\)](#), [Díaz and Luengo-Prado \(2008\)](#), [Leth-Petersen and Ejarque \(2008\)](#), [Yang \(2009\)](#), [Chambers, Garriga and Schlagenhaut \(2009a,b,c\)](#), [Attanasio, Leicester and Wakefield \(2011\)](#) and [Fernandez-Villaverde and Krueger \(2011\)](#).

state space used to considerable speed up the solution algorithm, which can generally be used in consumption-saving models with long-term debt. This speed up is absolutely necessary when solving the model in gross stocks because this basically adds both a continuous state and a continuous choice to a model which it is already very time consuming to solve.

The rest of the paper is organized as follows: Section 2 introduces the life-cycle model and shows how it generalizes the ABLNW-model. Section 3 briefly discusses the solution algorithm, and the baseline parametrization taken from ABLNW. Section 4 presents the central results on the importance of formulating the model in terms of gross stocks, which are tested for robustness in section 5. Section 6 concludes. The solution algorithm is explained in more detail in appendix A, and some further details are included in appendix B.

## 2 The Model

**States and Choices** We consider unitary households living for  $L$  periods, and retiring at the end of period  $T$ . The households are all characterized by the following four *idiosyncratic* state variables:  $A_{t-1}$ , end-of-period *financial assets*,  $D_{t-1}$ , end-of-period *mortgage debt*,  $H_{t-1}$ , end-of-period *housing status*, and finally  $Y_t$ , *non-financial income*. We let  $H_{t-1} = 0$  indicate that the household was a renter in the previous period, while  $H_{t-1} \in \{1, 2\}$  indicates that it respectively owned a *flat* or a *house*. Additionally the house price  $P_t$  is an *aggregate* state variable. The price of flats is given by  $\kappa \cdot P_t$  with  $\kappa \in [0, 1]$ . The *rental price* (of flats) is given by

$$Q_t = \min \{ \alpha_Y \cdot Y_t, \alpha_P \cdot \kappa \cdot P_t \}, \quad \alpha_Y, \alpha_P > 0 \quad (2.1)$$

where the first term in the minimum operator is a rent ceiling proportional to income (e.g. due to a government subsidy). In each period the households first choose their housing status,  $H_t \in \{0, 1, 2\}$ . If they own and do not move they can choose to keep their current mortgage,  $K_t = 1$ , or refinance,  $K_t = 0$ . Secondly they choose the size of their mortgage,  $D_t \in \mathbb{R}_+$ , and how much to consume,  $C_t \in \mathbb{R}_+$ .

**Preferences** The per-period utility function is

$$u(C_t, H_t) = \frac{C_t^{1-\rho}}{1-\rho} \cdot e^{\theta \cdot \phi(H_t)} + (\phi(H_t) - 1) \cdot \mu, \quad \rho > 1, \theta, \mu \geq 0 \quad (2.2)$$

where

$$\phi(H_t) = \begin{cases} 0 & \text{if } H_t = 0 \\ \phi \in [0, 1] & \text{if } H_t = 1 \\ 1 & \text{if } H_t = 2 \end{cases}$$

Here  $\rho$  is a measure of risk aversion,  $\theta$  is a measure of the complementarity between consumption and home ownership,  $\mu$  is an absolute home ownership premium, and  $\phi$  is a scaling factor for the utility value of owning a flat relative to owning a house. Future utility is discounted exponentially with a factor  $\beta > 0$ .<sup>2</sup>

**Exogenous Processes** *Income* evolves stochastically around a deterministic life-cycle profile given by

$$\mathcal{L}_t = \log(\ell_0) + \ell_1 \cdot t + \ell_2 \cdot t^2 \quad (2.3)$$

*Before* retirement the income ( $Y_t$ ) of the households is subject to *permanent* income shocks

$$\forall t \leq T : \log Y_t = \mathcal{L}_t + \Psi_t \quad (2.4)$$

$$\Psi_t = \Psi_{t-1} + \psi_t, \quad \psi_t \sim \mathcal{N}(-\sigma_\psi^2/2, \sigma_\psi^2) \quad (2.5)$$

*After* retirement there is no shocks and income is determined by a fixed retirement replacement rate

$$\forall t > T : \log Y_t = \vartheta \cdot \log Y_T \quad (2.6)$$

The *house price* is modeled as an *AR*(1) around a trend

$$\log P_t = \log(\tau_0) + \tau_1 \cdot t + \rho_P \cdot \log P_{t-1} + \xi_t, \quad \xi_t \sim \mathcal{N}(-\sigma_\xi^2/2, \sigma_\xi^2) \quad (2.7)$$

Both exogenous processes are approximated by a discrete first order Markov process with 15 states using the method in [Tauchen \(1986\)](#).

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<sup>2</sup> As the model lacks both stochastic mortality and a bequest motive, it is *not* designed to fit the housing and consumption choices of the elderly.



**Mortgage Constraints** The mortgage constraint depends on whether the household *keeps* its current mortgage ( $K_t = 1$ ) or *not* ( $K_t = 0$ ). Specifically we have

$$D_t \leq \begin{cases} (1 - \gamma) \cdot D_{t-1} & \text{if } K_t = 1 \\ \Lambda(H_t, P_t, Y_t) & \text{if } K_t = 0 \end{cases}, \quad \gamma \in [0, 1] \quad (2.8)$$

where  $\gamma$  is the forced repayment rate and  $\Lambda(H_t, P_t, Y_t)$  is the maximum mortgage a household can take out when originating or refinancing:

$$\Lambda(H_t, P_t, Y_t) = \min \{ \Lambda_t^{LTV}, \Lambda_t^{LTI} \} \quad (2.9)$$

$$\Lambda_t^{LTV} \equiv \begin{cases} \lambda_H \cdot P_t \cdot \kappa & \text{if } H_t = 1 \\ \lambda_H \cdot P_t & \text{if } H_t = 2 \end{cases}, \quad \lambda_H > 0 \quad (2.10)$$

$$\Lambda_t^{LTI} \equiv \lambda_Y \cdot Y_t, \quad \lambda_Y > 0 \quad (2.11)$$

In the terminal period, there is a “die without debt” constraint,  $D_L \leq 0$ .

**Transaction Costs** The households pay proportional *transactions costs* ( $F_{buy}, F_{sell}$ ) when buying and selling flats and houses so that the total net cost of these transactions are

$$\begin{aligned} \Omega_t^H &= (1 + F_{buy}) \cdot [\mathbf{1}_{H_t=2, H_{t-1} \neq 2} + \mathbf{1}_{H_t=1, H_{t-1} \neq 1} \cdot \kappa] \cdot P_t \\ &\quad - (1 - F_{sell}) \cdot [\mathbf{1}_{H_{t-1}=2, H_t \neq 2} + \mathbf{1}_{H_{t-1}=1, H_t \neq 1} \cdot \kappa] \cdot P_t, \quad F_{buy}, F_{sell} \geq 0 \end{aligned} \quad (2.12)$$

This accounts for both moving costs, real estate agent fees and stamp duty.

Furthermore it is also costly for the households to originate and extend mortgages due to both fees and time costs. Specifically we assume that there are *quasi-proportional (re)financing costs*, i.e.

$$\begin{aligned} \Omega_t^D &= \mathbf{1}_{H_t=H_{t-1}, K_t=0} \cdot [S_D \cdot (\max \{ D_t - (1 - \gamma) \cdot D_{t-1}, 0 \}) + S_F] \\ &\quad + \mathbf{1}_{H_t \neq H_{t-1}, D_t > 0} \cdot [S_D \cdot D_t + S_F], \quad S_D, S_F \geq 0 \end{aligned} \quad (2.13)$$

In total, the cost-function for *non-consumption expenses* consequently is

$$\Omega_t = \Omega_t^H + \Omega_t^D + \mathbf{1}_{H_t=0} \cdot Q_t \quad (2.14)$$

**End-of-Period Assets** *End-of-period assets* are therefore given by

$$A_t = (1 + r_a) \cdot A_{t-1} - (1 + r_d) \cdot D_{t-1} + Y_t + D_t - (C_t + \Omega_t) \quad (2.15)$$

where  $r_a$  is the risk free return rate of assets and  $r_d$  is the mortgage rate; we are only interested in the case  $r_d \geq r_a$ .

The households do not have access to an overdraft facility or credit card so  $A_t \geq 0$ , and we can consequently define the maximum consumption function,  $\bar{C}(\bullet)$ , implicitly as the  $C_t$  implying  $A_t = 0$ . For future reference we also define *end-of-period financial net worth* as  $N_t \equiv A_t - D_t$  and *end-of-period total net worth* as  $W_t \equiv N_t + \mathbf{1}_{H_t=1} \cdot \kappa \cdot P_t + \mathbf{1}_{H_t=2} \cdot P_t$ .

**Recursive Form** The Bellman equation of the household problem is given by

$$V_t(H_{t-1}, D_{t-1}, A_{t-1}, Y_t, P_t) = \max_{H_t, K_t, D_t, C_t} u(C_t) + \beta \cdot \mathbb{E}_t[V_{t+1}(\bullet)] \quad (2.16)$$

s.t.

$$H_t \in \{0, 1, 2\} \quad (2.17)$$

$$K_t \in \begin{cases} \{0, 1\} & \text{if } H_t = H_{t-1} \\ \{0\} & \text{else} \end{cases} \quad (2.18)$$

$$D_t \in \begin{cases} [0, (1 - \gamma) \cdot D_{t-1}] & \text{if } K_t = 1 \\ [0, \Lambda(\bullet)] & \text{if } K_t = 0 \end{cases} \quad (2.19)$$

$$C_t \in [0, \bar{C}(\bullet)] \quad (2.20)$$

$$A_t = \bar{C}(\bullet) - C_t \quad (2.21)$$

The model can also be reformulated as the *upper envelope* of a series of discrete choice specific value functions as discussed in appendix A.

**Comparison to Attanasio, Bottazzi, Low, Nesheim and Wakefield (2012)**

The ABLNW-model differs from the model presented here in the central aspect that the households are subject to the following unmotivated restriction on their choice set

$$\text{if } D_t > 0 \text{ then } C_t = \bar{C}(\bullet) \quad (2.22)$$

Conditional on a strictly positive debt choice, the households are thus forced to consume everything, implying that  $A_t = 0$  if  $D_t > 0$ . Consequently *end-of-period financial net worth* is  $N_t = -D_t$  if  $D_t > 0$  and  $N_t = A_t$  if  $D_t = 0$ . Therefore the

whole model can be reformulated in terms of  $N_{t-1}$  alone (instead of both  $D_{t-1}$  and  $A_{t-1}$ ), where specifically the borrowing constraint in (2.8) becomes

$$N_t \geq \begin{cases} \min \{(1 - \gamma) \cdot N_{t-1}, 0\} & \text{if } K_t = 1 \\ -\Lambda_t & \text{if } K_t = 0 \end{cases} \quad (2.23)$$

Finally the ABLNW-model is formulated under the parametric restrictions that  $\gamma = 0$ ,  $S_D = S_F = 0$  and  $r_d = r_a$ .

### 3 Solution and Calibration

**Solution Algorithm** The *non-convexities* introduced by both the discrete housing choice and the various transaction costs, imply that time iteration methods are not applicable. Instead we have to rely on value function iterations to solve the model, but as it has two continuous states ( $A_{t-1}$ ,  $D_{t-1}$ ), three discretized states ( $H_{t-1}$ ,  $P_t$ ,  $Y_t$ ) and both discrete ( $H_t$ ,  $K_t$ ) and two continuous choices ( $C_t$ ,  $D_t$ ), this is generally very time consuming. One important novel speed-up trick is to introduce *beginning-of-period financial net worth* defined as<sup>3</sup>

$$M_t \equiv (1 + r_a) \cdot (A_{t-1} - D_{t-1}) - (r_d - r_a) \cdot D_{t-1} \quad (3.1)$$

The model can then be written with  $M_t$  as a state variable instead of  $A_{t-1}$ . As shown in more detail in appendix A, this implies that the optimal choices when *buying* and *renting* are all *independent* of  $D_{t-1}$ . Moreover this restructuring of the state space is helpful in proving some properties of the optimal debt choice function because  $M_t$  is now a fixed dimension. For example, it implies that  $D_{t-1}$  only have an impact on the choice set of  $D_t$  but not on the value of any choice (because its effect on net worth has been netted out in  $M_t$ ); a decrease in  $D_{t-1}$  will therefore in some cases only remove non-optimal choices which cannot change the optimal choice. See appendix A for further details on the solution algorithm.

**Calibration** We begin from the exact same parametrization as in ABLNW and with a period length of one year. Table 3.1 provides the parameters for the exogenous income and house price processes (see figure 3.1 for the implied trends and grid nodes). For the income process we use their estimates for high education

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<sup>3</sup> A similar trick is used in [Druehl and Jørgensen \(2015\)](#).

Table 3.1: Parameters I

Parameter	Value
<i>Housing</i>	
$\rho_P$	0.94
$\sigma_\psi^2$	0.008
$\tau_0$	4.67
$\tau_1$	0.0232
$\kappa$	0.60
$\alpha_Y$	0.50
$\alpha_P$	0.01
<i>Income</i>	
$\rho_Y$	1.00
$\nu$	0.70
$\sigma_{\xi,H}^2$	0.035
$\ell_{0,H}$	1.00
$\ell_{1,H}$	0.042
$\ell_{2,H}$	-0.00082
$\sigma_{\xi,L}^2$	0.044
$\ell_{0,L}$	0.80
$\ell_{1,L}$	0.022
$\ell_{2,L}$	-0.00037

H: High education.  
L: Low education.

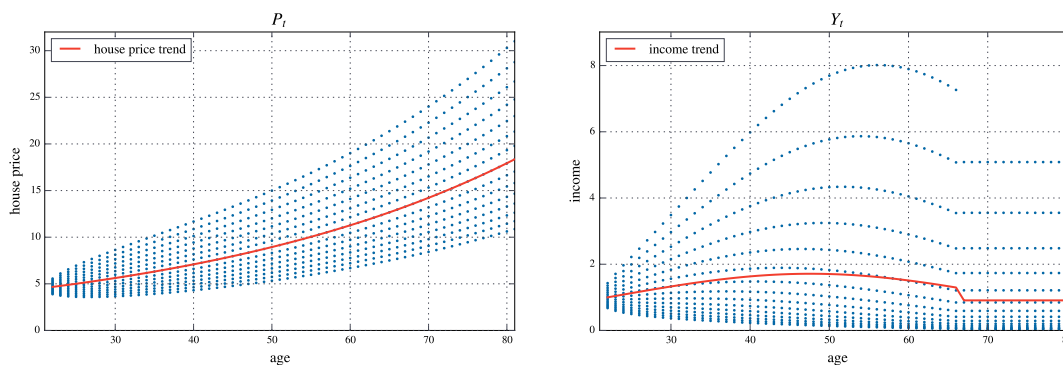
Table 3.2: Parameters II

Parameter	Value
<i>Demographics</i>	
$L$	60
$T$	45
<i>Preferences</i>	
$\beta$	$1.02^{-1}$
$\gamma$	1.430
$\theta$	0.115
$\phi$	0.90
$\mu$	0.26
<i>Borrowing and saving</i>	
$r_a$	0.018
$r_d$	0.018
$\lambda_H$	0.9
$\lambda_Y$	3.0
<i>Transaction costs</i>	
$F_{buy}$	0.05
$F_{sell}$	0.05
$S_D$	0.00
$S_F$	0.00

households (A-levels or higher). The house price process is estimated using data from the *Office of the Deputy Prime Minister* (ODPM) on national and regional house prices in the UK in years 1969-2000. The income parameters are estimated using data from the *British Household Panel Survey* (BHPS) for the years 1991-2002. The 2000 wave of the BHPS is also used to set the initial distribution of financial assets (see appendix table B.1), while households are assumed to have zero housing endowments at age 22.

Table 3.2 provides the remaining parameters for the demographics, preferences,

Figure 3.1: Exogenous Trends and Grids:  $P_t$  and  $Y_t$



Notes: See table 3.1 for the underlying parameters. The discretization is based on the method in Tauchen (1986) using 15 nodes.

borrowing and saving, and the various transaction costs. The preference parameters related to owning ( $\mu$ ,  $\phi$  and  $\theta$ ) and the symmetric fixed transaction cost ( $F = F_{buy} = F_{sell}$ ) were calibrated by ABLNW to fit data on home ownership rates in *The Family Expenditure Survey* (FES) years 1991-2000. The interest rate used is the average 90 day UK Treasury Bill discount rate in years 1968-1997.

One apparent questionable aspect of the baseline parametrization is that there is *no mortgage spread* ( $r_a = r_d$ ), and *no forced mortgage repayments* ( $\gamma = 0$ ). We will there also consider parametrizations with a mortgage spread of 0.5 percentage points<sup>4</sup> (i.e.  $r_d = 0.023$ ), and a strictly positive repayment rate ( $\gamma = 0.035$ ) implying a mortgage half-life of about 20 years, which is similar to a standard 30-year mortgage. Note, however, that if the model included a full portfolio choice with bonds and stocks, the households would be able to get a substantial higher mean return on their gross assets if they were willing to take on some risk.<sup>5</sup> Consequently a zero mortgage spread may actually be the best approximation to reality when we for computational reasons cannot include a full portfolio choice.

Another important aspect of the baseline parametrization is the interaction between the relatively high retirement replacement rate (high  $\eta$ ) and a low discount rate (high  $\beta$ ). ABLNW see their retirement replace rate of 70 percent as only covering *state* pensions. Loosely matching the model to data on non-housing net wealth closely before retirement they consequently include *private* pensions in their empirical moments. [Banks, O’Dea and Oldfield \(2010\)](#) (table 5), however, arrive at a median replacement rate of 70 percent for the full population including both state and private pensions. [Crawford and O’Dea \(2014\)](#) further show that on quintiles of life-time income the split between state and private pensions is fifty-fifty for the third quintile, while the share of state pensions falls to 40 percent for the fourth quintile and 25 percent for the highest quintile. In conclusion a case can be made for considering parametrizations which will imply less accumulation of financial assets for the median household than seen in the baseline; we will therefore also consider values of  $\beta$  smaller than  $1.02^{-1}$ , which is also often seen in the precautionary savings literature.

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<sup>4</sup> See [Miles \(2005\)](#) for a discussion of mortgage rate spreads in the UK.

<sup>5</sup> See e.g. the model with long-term debt in [Alan, Crossley and Low \(2012\)](#).

## 4 Results

### 4.1 Baseline

The central simulation results across the different parametrizations and the two formulations of the model in respectively net stocks (as in ABLNW) and gross stocks are presented in table 4.1.<sup>6</sup> To facilitate comparison with ABLNW, their simulation results are repeated in the left most column of table 4.1. Overall the results in the “Base” parametrization in net stocks match those from ABLNW fairly well.<sup>7</sup>

Comparing the second and third column of table 4.1 we see that the effect of shifting from a formulation in net stocks to one in gross stocks, i.e. of allowing for precautionary balance sheet expansions, is large under the “Base” parametrization. The overall home ownership rate increases from 62 percent to 68 percent, and as seen in figure 4.1a it is the whole ownership rate life-cycle profile which is shifting upwards; the largest increase is seen for the young, where the home ownership rate increases from 65 percent to 73 percent. Simultaneously there is also substantial substitution towards houses away from flats; in line with this, the total number of “buys per life” falls from 1.5 to 1.3 indicating that fewer households first buy a flat and then later adjust upwards to a house.

An extreme implication of the zero mortgage spread is that the rate of outright ownership drops to zero<sup>8</sup> as seen in figure 4.1b, and the median LTV-ratio hits the upper bound as seen in figure 4.1d.

All in all the implications of introducing precautionary balance sheet expansions is that the households can move more resources forward in life without putting themselves in a too risky position; in figure 4.2a we therefore see that mean consumption

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<sup>6</sup> We simulate the model for 100.000 households with independent draws of both the income and house price process.

<sup>7</sup> The remaining discrepancy is probably a purely technical issue. Apart from different implementations of grids and interpolation procedures, two central technical issues are: 1) The ABLNW figures are an average over only 40 realizations of the house price process. 2) ABLNW impose the constraint that a household can never be technically insolvent in the sense that its debt is larger than the sum of its discounted future *minimum* income plus the *minimum* discounted sale value of its home *tomorrow* (see e.g. `equatdefn.f90` line 99 and 170 in their code-files). I do *not* impose this constraint because the trend in the house price process implies that a household can be technically insolvent in a given period without being so at the end of life *even under worst case outcomes*.

<sup>8</sup> Here we are disregarding that some rich households with no liquidity problems may actually be *indifferent* between owning outright, and having any feasible balance sheet expansion.

Table 4.1: Gross vs. Net Stocks

	ABLNW	Base		Spread		Repay		Beta	
	Net	Net	Gross	Net	Gross	Net	Gross	Net	Gross
	<i>percent</i>								
Ownership rate	62	62.3	67.7	61.7	62.3	61.7	61.7	54.9	56.3
- flats	30	28.0	24.4	31.5	29.7	31.9	30.9	42.0	43.0
- houses	32	34.3	43.3	30.1	32.6	29.7	30.8	12.8	13.2
Ownership rate (age 26-35)	58	64.9	73.2	61.8	62.2	61.7	61.6	60.4	62.9
- flats	19	22.5	22.8	28.9	25.5	29.9	27.5	55.7	57.8
- houses	39	42.3	50.3	32.9	36.6	31.7	34.0	4.7	5.1
outright share		37.6	0.0	48.8	44.1	49.8	47.4	21.1	19.6
median LTV		11.5	89.0	0.7	4.7	0.3	1.4	27.8	32.2
Ownership rate (age 36-50)	80	82.6	86.6	82.9	83.4	83.0	83.0	75.6	77.5
- flats	38	36.0	30.6	42.1	39.4	42.8	41.1	58.7	60.0
- houses	42	46.5	56.0	40.8	43.9	40.2	41.8	16.8	17.4
outright share		87.9	0.0	91.7	85.0	92.1	90.3	71.7	66.3
median LTV		0.0	83.6	0.0	0.0	0.0	0.0	0.0	0.0
Ownership rate (age 51-60)	86	80.3	84.3	80.9	81.4	80.9	81.0	71.3	72.4
- flats	46	37.7	32.1	41.2	39.6	41.6	40.7	49.7	50.4
- houses	40	42.6	52.1	39.6	41.8	39.3	40.3	21.5	22.0
outright share		91.9	0.0	93.1	77.3	93.2	89.2	82.1	75.4
median LTV		0.0	72.1	0.0	0.0	0.0	0.0	0.0	0.0
	<i>mean (relative to median <math>Y_t</math>) (age 51-60)</i>								
Total Net Worth ( $W_t$ )		15.7	15.3	15.7	15.7	15.7	15.7	10.0	9.9
Financial Net Worth ( $N_t$ )	9.0	9.0	7.9	9.1	8.9	9.1	9.0	4.7	4.6
Gross Debt ( $D_t$ )		0.0	5.3	0.0	0.3	0.0	0.0	0.0	0.2
	<i>median (relative to median <math>Y_t</math>) (age 51-60)</i>								
Total Net Worth ( $W_t$ )		12.5	12.0	12.5	12.4	12.5	12.5	7.9	7.8
Financial Net Worth ( $N_t$ )	6.3	6.5	5.5	6.5	6.4	6.5	6.5	3.2	3.1
Gross Debt ( $D_t$ )		0.0	5.4	0.0	0.0	0.0	0.0	0.0	0.0
	<i>transactions</i>								
Buys per life		1.5	1.3	1.5	1.4	1.5	1.5	1.3	1.4
- flats		0.9	0.6	1.0	0.9	1.0	0.9	1.1	1.1
- houses		0.6	0.6	0.5	0.5	0.5	0.5	0.2	0.2

Base parameters: see table 2.1 and 2.2.

Spread parameters: see table 2.1 and 2.2, except  $r_d = 0.023$ .

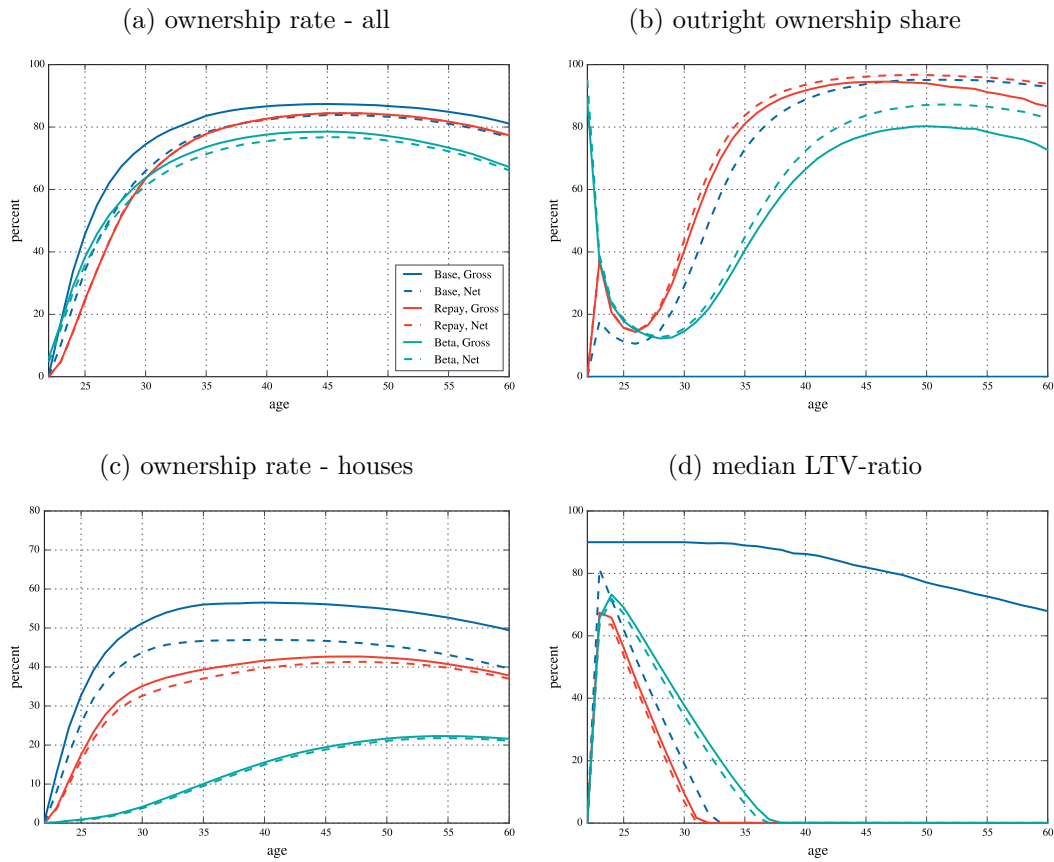
Repay parameters: see table 2.1 and 2.2. except  $r_d = 0.023$ ,  $\gamma = 0.035$ .

Beta parameters: see table 2.1 and 2.2 except  $r_d = 0.023$ ,  $\gamma = 0.035$ ,  $\beta = 0.96$ .

increases with about 1.5-2.5 percent for households under age 35 compared to the net stock formulation.<sup>9</sup> Figure 4.2b further shows that the cross-sectional standard deviation of utility consequently increases a bit initially, but then fall massively, indicating a stronger ability to self-insure.

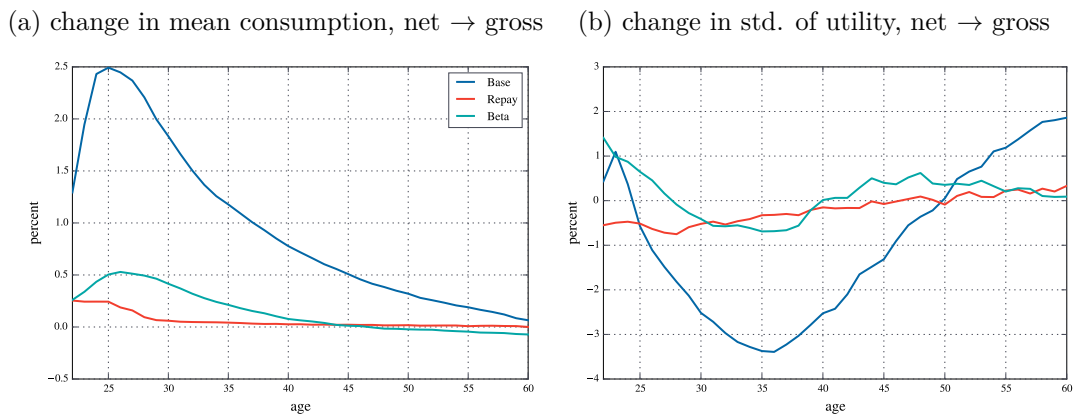
<sup>9</sup> The change in mean consumption never turns negative even at older ages because the steep trend in house prices implies that households save life time resources by buying earlier.

Figure 4.1: Simulated Life-Cycle Profiles I



*Samples:* Simulations of 100,000 households with independent draws of both the income and house price process.

Figure 4.2: Simulated Life Cycle Profiles II



*Samples:* Simulations of 100,000 households with independent draws of both the income and house price process.



## 4.2 Spread ( $r_d = 0.023$ ) and Repay (also $\gamma = 0.035$ )

The introduction of a small mortgage spread in the “Spread” parametrization ( $r_d = 0.023$ ) makes it more expensive to have a large mortgage debt, and looking across the net stock formulations we therefore also see both a fall in the overall demand for homes, a substitution towards flats and an increase in the outright ownership rate. More importantly it also implies that the differences between the net stock and gross stock formulations become much smaller. The overall homeownership rate thus now only increases by 0.6 percentage points when allowing for precautionary balance sheet expansions; among the young households the increase is only 0.4 percentage points. Further also introducing forced mortgage repayments in the “Repay” parametrization ( $\gamma = 0.035$ ) the differences between the net and gross formulations disappears almost completely with only a minor substitution towards houses left.

At first, it may seem surprising that a small 0.5 percentage points mortgage rate spread have such big effects. However, it implies that the cost of balance sheet expansions becomes linearly increasing in the extra gross debt accumulated and saved in financial assets. On the other hand the benefit of a precautionary balance sheet expansion only becomes substantial once the gross debt stock is so large that it relaxes the borrowing constraint of the households in future periods under not too unlikely decreases in income and house prices. If the households optimal LTV-ratio is low under the net stock formulation then the *base cost* of a precautionary balance sheet expansion is thus very high, and the benefits are initially small or even zero. In figure 4.1b we precisely see that for the “Repay” parametrization in net stocks, the median LTV-ratio hits zero already before age 35. The median LTV-profile shifts somewhat upwards under gross stocks, but in conclusion the linear costs of precautionary balance sheet expansions heavily outweighs the benefits when a strictly positive mortgage spread and forced mortgage repayments are added to the baseline parametrization.

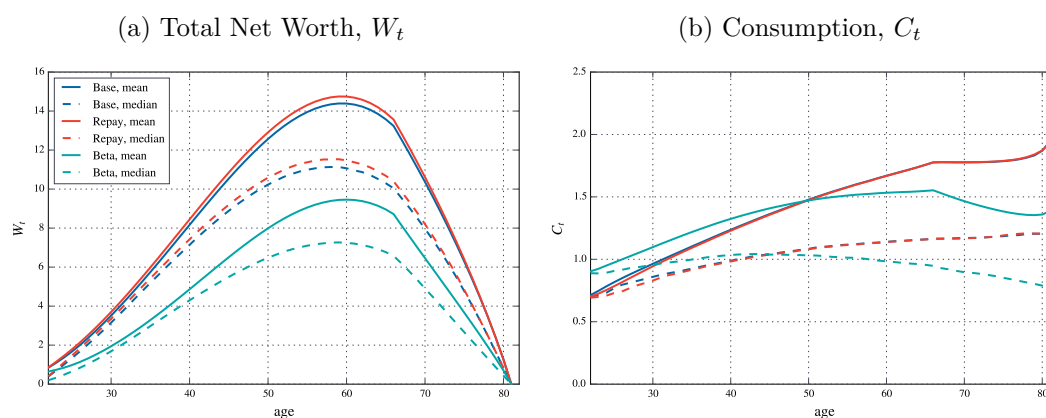
## 4.3 Outright Ownership

In subsection 3 we discussed the pros and cons of including a strictly positive mortgage spread when our model does not include a full portfolio choice. One apparent common problem with both parametrizations, however, is that they imply very large outright ownership rates and very low median LTV-ratios relatively early in life. Figure 4.1b shows that about 90 percent of age 40 home owners

own their flat or house outright. This is unrealistically high. Crossley and O’Dea (2010) (table 3.5) e.g. show that for the full population age 40-45 only 18 percent of home owners own their house outright<sup>10</sup>, and this only increases to 75 percent at age 60-64.

Outright ownership is mostly a function of the households non-retirement net wealth. The life-cycle profile of total net worth is shown in figure 4.3a for the different parametrizations (in gross stocks), while figure 4.3b shows the corresponding life-cycle profiles of consumption. Hereby we can see that the households in the baseline parametrization are assumed to be so patient that they achieve a continuing increase in consumption during retirement.

Figure 4.3: Simulated Life Cycle Profiles III (Only Gross)



*Samples:* Simulations of 100,000 households with independent draws of both the income and house price process.

#### 4.4 Beta ( $\beta = 0.96$ )

Making the households more impatient with  $\beta = 0.96$  in the “Beta” parametrization (keeping the same mortgage spread and forced mortgage repayment rate as before) naturally implies a substantial fall in the overall ownership rate, massive substitution towards flats and a much steeper increase in the home ownership rate for the young (again see table 4.1). Centrally, however, it also implies that allowing for precautionary balance sheet expansions again have important effects; e.g. it implies a 2.5 percentage points increase in the home ownership rate among the young, and a bit of relative substitution towards houses.

<sup>10</sup>The total home ownership rate in their sample at age 40-45 is 78 percent, only marginally lower than in our simulations.

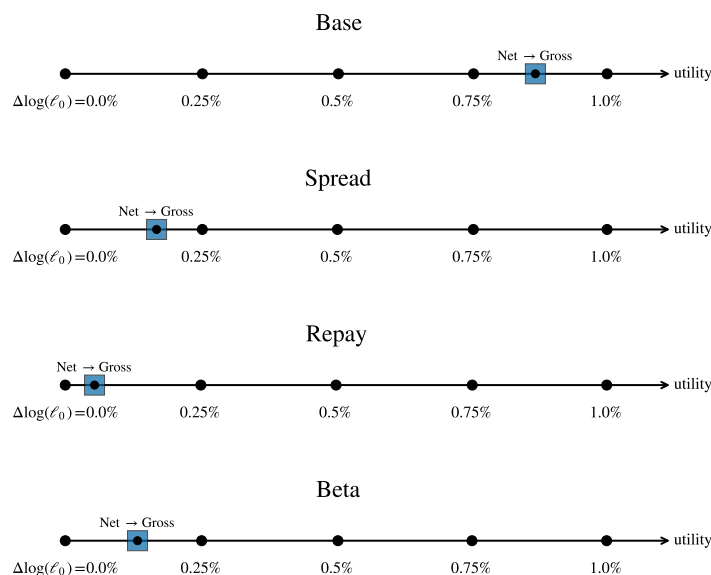
In the gross stock formulation the outright ownership rate among home owners now never increases substantially above 80 percent (see figure 4.1b), which is in line with the empirical estimates. However, it is only down to about 70 percent at age 40, which is still rather high compared to the empirical estimates. This indicates that the differences between the net stock and gross stock formulations found here is probably a *lower* bound.

In a full re-calibration of the model focused on also fitting the empirical outright ownership rates, the LTV-ratios of the households would have to increase, which would lower the base cost of precautionary balance sheet expansions making them more attractive, and thus more important to account for.

## 4.5 Welfare

Another approach to measuring the importance of allowing for precautionary balance sheet expansions is to consider their welfare implications. As a welfare criterion we look at the *ex ante expected discounted utility* seen from just before the beginning of life (i.e. before the draws of the initial wealth distribution). This welfare measure can be calculated as an ex post average over all the households in our simulation. To further clarify the quantification of the welfare gain of shifting from the net stock to the gross stock formulation, we compare it to the welfare gains in the net stock formulation implied by upward shifts in the life-cycle profile of income (i.e. increases in  $\ell_0$ , see equation (2.3)).

Figure 4.4: Welfare Gain of Net  $\rightarrow$  Gross



Notes: A value of e.g. 1.0 percent shows that the welfare gain is equivalent to the welfare gain from a 1.0 percent increase in the trend of life-cycle income profile (calculated under the net formulation of the model).

Figure 4.4 reports the results of our investigation. As expected the welfare gain is largest in the “Base” parametrization, where a household would rather have access to precautionary balance sheet expansions than get a 0.75 percent increase in their life-cycle profile of income. The welfare gain is much smaller in the “Spread” parametrization and almost disappears completely in the “Repay” parametrization. In the “Beta” parametrization, with a higher degree of impatience, the welfare gain increases a bit again, but is still relatively small and the equivalent income increase is below 0.25 percent.

## 5 Robustness

### 5.1 Different Parametrizations ( $\mu$ , $\phi$ , $\theta$ and $F$ )

Table 5.1 shows the simulation results when changing each single calibration parameter ( $\mu$ ,  $\phi$ ,  $\theta$  and  $F$ ) such that it induces more home ownership building on top of the “Beta” parametrization, where the home ownership rate was markedly too low. In all cases the differences between the net stock and gross stocks formulations remain approximately the same or become even larger indicating that the results are parametrically robust. Note that this also holds true in the cases

where respectively  $\theta$  is lowered to 0.100 and  $\mu$  is increased to 0.32 implying higher home ownership rates than in the original “Base” parametrization.

Table 5.1: Parametric Robustness

	$\mu = 0.32$		$\theta = 0.100$		$\phi = 0.80$		$F = 0.025$	
	Net	Gross	Net	Gross	Net	Gross	Net	Gross
	<i>percent</i>							
Ownership rate	65.8	67.4	63.5	65.1	56.2	57.6	57.2	58.5
- flats	49.7	51.0	49.0	50.4	40.5	41.7	36.1	36.6
- houses	16.1	16.4	14.5	14.7	15.6	15.8	21.1	21.8
Ownership rate (age 26-35)	83.2	85.6	78.9	81.7	63.6	66.4	66.3	68.6
- flats	76.7	78.9	73.6	76.1	57.8	60.4	52.6	53.7
- houses	6.4	6.7	5.3	5.5	5.7	6.0	13.7	14.8
outright share	15.8	14.6	17.2	15.9	19.7	18.2	14.8	13.7
median LTV	36.1	41.8	34.3	40.0	29.0	33.7	34.5	39.2
Ownership rate (age 36-50)	84.6	86.5	83.2	85.1	77.3	79.2	76.8	78.3
- flats	62.8	64.4	63.8	65.4	56.6	58.2	45.3	45.6
- houses	21.7	22.1	19.3	19.6	20.6	20.9	31.4	32.7
outright share	66.3	60.8	68.0	62.4	70.6	65.3	59.9	55.1
median LTV	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Ownership rate (age 51-60)	77.2	78.3	76.2	77.3	72.2	73.4	71.9	72.8
- flats	50.3	51.2	51.9	52.7	46.3	47.2	38.1	38.3
- houses	26.8	27.1	24.3	24.6	25.8	26.1	33.8	34.5
outright share	79.5	71.8	80.6	72.8	81.4	74.4	74.3	67.7
median LTV	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	<i>mean (relative to median <math>Y_t</math>) (age 51-60)</i>							
Total Net Worth ( $W_t$ )	10.4	10.2	10.3	10.2	10.1	10.0	10.1	10.0
Financial Net Worth ( $N_t$ )	4.4	4.2	4.5	4.3	4.5	4.4	4.3	4.2
Gross Debt ( $D_t$ )	0.0	0.3	0.0	0.2	0.0	0.2	0.1	0.3
	<i>median (relative to median <math>Y_t</math>) (age 51-60)</i>							
Total Net Worth ( $W_t$ )	8.2	8.0	8.1	8.0	7.9	7.8	8.0	7.9
Financial Net Worth ( $N_t$ )	2.9	2.7	3.0	2.9	3.1	3.0	2.8	2.7
Gross Debt ( $D_t$ )	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	<i>transactions</i>							
Buys per life	1.6	1.6	1.5	1.5	1.4	1.4	1.9	1.9
- flats	1.3	1.3	1.2	1.2	1.1	1.1	1.4	1.4
- houses	0.3	0.3	0.2	0.2	0.3	0.3	0.4	0.4

Parameters: see table 2.1 and 2.2, except for  $r_d = 0.023$ ,  $\beta = 0.96$ ,  $\gamma = 0.035$ .

The optimal robustness test would naturally be to re-calibrate the model to fit the original home ownership moments and e.g. add the life-cycle profile of outright ownership as a set of new moments to fit. This task is, however, beyond the scope of this paper. Furthermore it is not obvious that it will at all be possible to fit these moments because the increased impatience implies that the home ownership profile becomes too steep both in the sense of a too fast increase for the young households and a too strong decrease for households approaching retirement. Consequently it would be necessary to extend the model to make housing in general and owning in particular less valuable for younger households. An exogenous taste shifter justified by e.g. changes in family size and composition would be the simplest

solution. A more endogenous mechanism could be a strong incentive for young households to remain geographically mobile for both family and career reasons; this reduces home ownership among the young households because of the large transaction costs.<sup>11</sup>

## 5.2 (Re)financing Costs

Table 5.2 shows the simulation results when (re)financing costs are introduced. The fixed cost is set to one week of income for the young, i.e.  $S_F = \frac{1}{52}$ , and the proportional cost is assumed to be 1 percent, i.e.  $S_D = 0.01$ . In themselves these financing costs naturally reduce the home ownership rate, but their effect on the difference between the net and gross stocks formulations is more or less negligible. The total home ownership rate now increases by 5.1 percentage points in the “Base” parametrization and 1.6 percentage points in the “Beta” parametrization, while the original effects were 5.4 and 1.4 percentage points (see table 4.1). Underlying this, however, we see that allowing for precautionary balance sheet expansions now, especially in the “Base” parametrization, has a relatively smaller effect on the home ownership rate for the young households; naturally this is counter-weighted by a relatively larger positive effect for the older households. In total the results in this paper does *not* depend on assuming zero (re)financing costs.

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<sup>11</sup>See for e.g. [Halket and Vasudev \(2014\)](#) for a thorough investigation of these issues.

Table 5.2: Refinancing Costs

	Base		Beta	
	Net	Gross	Net	Gross
	<i>percent</i>			
Ownership rate	61.3	66.4	53.0	54.6
- flats	30.5	23.7	41.9	42.8
- houses	30.8	42.6	11.0	11.8
Ownership rate (age 26-35)	62.6	69.9	56.4	58.9
- flats	26.3	20.8	53.3	55.1
- houses	36.3	49.1	3.0	3.8
outright share	45.1	1.5	24.8	22.2
median LTV	3.7	79.5	24.3	29.3
Ownership rate (age 36-50)	81.8	85.7	73.1	75.7
- flats	40.7	30.2	59.1	60.5
- houses	41.1	55.5	13.9	15.1
outright share	93.5	2.1	81.5	70.7
median LTV	0.6	73.1	0.3	0.0
Ownership rate (age 51-60)	79.9	83.7	69.8	71.6
- flats	40.9	32.1	50.8	51.5
- houses	39.0	51.6	19.0	20.0
outright share	96.8	0.4	92.2	81.0
median LTV	0.0	67.5	0.4	0.9

Parameters: see table 2.1 and 2.2, except for  $S_F = 1/52$ , and  $S_D = 0.01$ .

## 6 Conclusions

The central contribution in this paper has been to show that modeling the long-term borrowing constraint in *gross* terms rather than *net* terms has important implications for the model dynamics. We have shown that letting households have access to precautionary balance sheet expansions boosts the overall demand for housing and induces a substitution from flats towards bigger houses. The further analysis showed that these results were robust to introducing both a substantial mortgage spread and marked forced mortgage repayments if just the households were impatient enough. The required level of impatience was moreover shown to make the model better match the outright ownership rates found empirically. Finally we concluded that changing individual parameters or adding refinancing costs did if anything rather amplify than dampen the importance of allowing households to do precautionary balance expansions.

A full re-calibration of the model in gross stocks was left for future work, but our results indicate that matching outright ownership rates will be central for such an exercise. Combining such a re-calibration with a discussion of the model's ability to explain the response of non-durable consumption to both house price shocks and income shocks is an interesting topic; both when looking at it in the

aggregate and across different stages of the life-cycle. Adding a stochastic interest rate process would further open up for matching the model to data on the actual refinancing behavior of households and thereby a discussion of the full welfare costs of the refinancing frictions.

Finally the whole setup is naturally only a first step in building a full general equilibrium model with endogenous house prices where allowing the households to survive being underwater and doing precautionary balance sheet expansions could be central for explaining why turnover rates fall so steeply in recessions freezing the housing market.



## A Solution Algorithm

### A.1 Restructured State Space

The Bellman equation with  $M_t$  as a state variable is given by

$$V_t(H_{t-1}, D_{t-1}, M_t, Y_t, P_t) = \max_{H_t, K_t, D_t, C_t} u(C_t) + \beta \cdot \mathbb{E}_t[V_{t+1}(\bullet)] \quad (\text{A.1})$$

s.t.

$$H_t \in \{0, 1, 2\} \quad (\text{A.2})$$

$$K_t \in \begin{cases} \{0, 1\} & \text{if } H_t = H_{t-1} \\ \{0\} & \text{else} \end{cases} \quad (\text{A.3})$$

$$D_t \in \begin{cases} [0, (1 - \gamma) \cdot D_{t-1}] & \text{if } K_t = 1 \\ [0, \Lambda(\bullet)] & \text{if } K_t = 0 \end{cases} \quad (\text{A.4})$$

$$C_t \in [0, \bar{C}(\bullet)] \quad (\text{A.5})$$

$$A_t = \bar{C}(\bullet) - C_t \quad (\text{A.6})$$

$$M_{t+1} = (1 + r_a) \cdot A_t - (1 + r_d) \cdot D_t \quad (\text{A.7})$$

where

$$\bar{C}(H_{t-1}, M_t, Y_t, P_t, H_t, K_t, D_t) = M_t + Y_t + D_t - \Omega(\bullet) \quad (\text{A.8})$$

We denote the optimal choice functions by  $H_t^*(\bullet)$ ,  $K_t^*(\bullet)$ ,  $D_t^*(\bullet)$  and  $C_t^*(\bullet)$ .

Alternatively the model can be reformulated as the *upper envelope* of a series of *choice specific value functions*, i.e

$$V_t(H_{t-1}, D_{t-1}, M_t, Y_t, P_t) = \max \{V_t^{own}, V_t^{rent}\} \quad (\text{A.9})$$

where

$$V_t^{own}(\bullet) \equiv \begin{cases} \max \{V_t^{buy, flat}(\bullet), V_t^{buy, house}(\bullet)\} & \text{if } H_t = 0 \\ \max \{V_t^{keep}(\bullet), V_t^{refi}(\bullet), V_t^{buy, house}(\bullet)\} & \text{if } H_t = 1 \\ \max \{V_t^{keep}(\bullet), V_t^{refi}, V_t^{buy, flat}(\bullet)\} & \text{if } H_t = 2 \end{cases}$$

and the choices are restricted as follows in the various cases

1. **Keep:**  $H_t = H_{t-1}$ ,  $K_t = 1$  and  $D_t \in [0, (1 - \gamma) \cdot D_{t-1}]$ .

2. **Refi(nance)**:  $H_t = H_{t-1}$ ,  $K_t = 0$  and  $D_t \in [0, \Lambda(H_t, P_t, Y_t)]$ .
3. **Buy, flat**:  $H_t = 1$  and  $D_t \in [0, \Lambda(1, P_t, Y_t)]$ .<sup>12</sup>
4. **Buy, house**:  $H_t = 2$  and  $D_t \in [0, \Lambda(2, P_t, Y_t)]$ .
5. **Rent**:  $H_t = 0$  and  $D_t = 0$ .

Interpolating the choice specific value functions *separately* (and finding the maximum) when evaluating the continuation value has a gain in terms of precision when solving the model. The gain seems to be large relative to the increased computation time.

## A.2 Choice Bounds

**Lemma A.1.** *For all  $x \in \mathbb{R}_+$  we have the following logical implication*

$$\begin{aligned} D_t^{*,keep}(\bullet, x, \bullet) = z &\rightarrow \\ \forall D_{t-1} \in \left[ \frac{z}{1-\gamma} : x \right] : D_t^{*,keep}(\bullet, D_{t-1}, \bullet) = z \end{aligned}$$

*Proof.* Decreasing  $D_{t-1}$  from  $x$  to  $\frac{z}{1-\gamma} \leq x$  only (weakly) shrinks the choice set and removes non-optimal choices. This cannot change the optimal choice.  $\square$

**Lemma A.2.** *If  $S_D = 0$  and  $S_F = 0$  then if  $D_{t-1} \in \left[ 0, \frac{1}{1-\gamma} \cdot \Lambda(H_t, P_t, Y_t) \right]$  we have*

$$V_t^{refi}(\bullet, D_{t-1}, \bullet) \geq V_t^{keep}(\bullet, 0, \bullet)$$

*Proof.* Given  $(1-\gamma) \cdot D_{t-1} \leq \Lambda(H_t, P_t, Y_t)$  the choice set is (weakly) smaller under *keeping*, so the optimal choice cannot be any better than under *refinancing* (when refinancing is cost less,  $S_D = S_F = 0$ ).  $\square$

**Lemma A.3.** *If  $S_D = 0$  then for all  $D_{t-1} \in \mathbb{R}_+$  we have*

$$\begin{aligned} V_t^{refi}(\bullet, D_{t-1}, \bullet) &= V_t^{refi}(\bullet, 0, \bullet) \\ D_t^{*,refi}(\bullet, D_{t-1}, \bullet) &= D_t^{*,refi}(\bullet, 0, \bullet) \\ C_t^{*,refi}(\bullet, D_{t-1}, \bullet) &= C_t^{*,refi}(\bullet, 0, \bullet) \end{aligned}$$

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<sup>12</sup>Note that when  $S_F > 0$  there is also a first order kink at  $D_t > 0$  vs.  $D_t = 0$ , which we for simplicity have chosen not to include as a discrete choice.

*Proof.* When  $S_D = 0$  the term with  $D_{t-1}$  disappears from (2.13).  $\square$

**Lemma A.4.** *If  $D_{t-1} \geq \frac{1}{1-\gamma} \cdot \Lambda(H_t, P_t, Y_t)$  we have*

$$\begin{aligned} V_t^{refi}(\bullet, D_{t-1}, \bullet) &= V_t^{refi}\left(\bullet, \frac{1}{1-\gamma} \cdot \Lambda(H_t, P_t, Y_t), \bullet\right) \\ D_t^{*,refi}(\bullet, D_{t-1}, \bullet) &= D_t^{*,refi}\left(\bullet, \frac{1}{1-\gamma} \cdot \Lambda(H_t, P_t, Y_t), \bullet\right) \\ C_t^{*,refi}(\bullet, D_{t-1}, \bullet) &= C_t^*\left(\bullet, \frac{1}{1-\gamma} \cdot \Lambda(H_t, P_t, Y_t), \bullet\right) \end{aligned}$$

*Proof.* Due to the maximum operator in (2.13) increasing  $D_{t-1}$  above  $\frac{1}{1-\gamma} \cdot \Lambda(H_t, P_t, Y_t)$  does not affect any choices under *refinancing*.  $\square$

### A.3 Implementation

The code is written in *C* with *OpenMP 4.0* for parallelization and called from *Python 2.7.6* using the *CFFI* interface. The C-code is compiled using the *TDM-GCC 4.9.2* compiler. Only free open source languages and programs are required to run the code. The code-files are available from the author upon request.

## B Further Details

### B.1 Discretization

The state variables are discretized as follows:

- $Y_t$ : Tauchen ( $N_Y = 15$ ). All transitions with a probability less than  $10^{-6}$  are disregarded.
- $P_t$ : Tauchen ( $N_P = 15$ ). All transitions with a probability less than  $10^{-6}$  are disregarded.
- $D_{t-1}$ : Age and  $H_{t-1}$  dependent grid (more nodes closer to zero,  $N_D = 50$ ), where the upper bound is given recursively by

$$\begin{aligned} D_{t-1} &\in [0; \overline{D}(t-1, H)] \\ \forall t \geq 1, \overline{D}(t, H) &= (1 - \gamma) \cdot \max \left\{ \overline{D}(t-1, H), \Lambda(H, \overline{P}_t, \overline{Y}_t) \right\} \\ \overline{D}(-1, H) &= 0 \end{aligned}$$

where an over-line denotes the *upper support* of a variable.

- $M_t$ : Age and  $H_{t-1}$  dependent grid (more nodes closer to lowest point,  $N_M = 100$ ), the lower bound is given by

$$\underline{M}_t = -(1 + r_d) \cdot \overline{D}(t-1, H) \quad (\text{B.1})$$

### B.2 Simulation: Initial Wealth

Table B.1: Initial Wealth

<i>Low Education</i>		<i>High Education</i>	
Cum. Prob.	$A_{-1} = x \cdot Y_{0,L}$	Cum. Prob.	$A_{-1} = x \cdot Y_{0,H}$
33.0	0.0000	22.0	0.0000
39.7	0.0000	29.8	0.0016
46.4	0.0008	37.6	0.0093
53.1	0.0070	45.4	0.0304
59.8	0.0251	53.2	0.0708
66.5	0.0435	61.0	0.1332
73.2	0.0785	68.8	0.2186
79.9	0.1566	76.6	0.4564
86.6	0.2490	84.4	0.7217
93.3	0.6598	92.2	1.2434
100.0	1.4617	100.0	4.4281

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# Chapter 3

## Heterogeneous Preferences and Wealth Inequality

# Heterogeneous Preferences and Wealth Inequality\*

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## Abstract

We perform a maximum likelihood estimation of a standard life cycle model allowing for non-parametric heterogeneity in patience and risk aversion using high quality Danish register data. We find substantial preference heterogeneity within educational strata and positive correlation between patience and risk aversion. Across the educational strata, higher educated households are found to be more patient and more risk averse. Although the model fits the average life cycle profiles of consumption and wealth quite well, it cannot explain the observed degree of wealth dispersion over the life cycle. This result suggests that heterogeneity in patience and risk aversion only explains a rather limited part of the observed wealth inequality.

**Keywords:** Preference Heterogeneity, Wealth Inequality, Consumption, Life-Cycle Models.

**JEL-Codes:** C14, D91, E21.

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# 1 Introduction

Allowing for heterogeneity in patience and risk aversion is typically found to be important in explaining wealth inequality in excess of income inequality<sup>1</sup>, understanding asset price puzzles<sup>2</sup>, and for evaluating the welfare effects of economic policies<sup>3</sup>. Experimental and survey based studies furthermore often find evidence indicating substantial heterogeneity even conditional on background variables such as cohort, gender and education.<sup>4</sup> In this paper, we instead estimate the degree of preference heterogeneity from *observational* data in the form of high quality Danish administrative register data. The Danish data provide longitudinal information on household-level income and wealth and we are thus able to estimate preference heterogeneity by systematic variation in the consumption decisions across households.

We estimate the joint distribution of subjective discount factors and relative risk aversion coefficients by a novel non-parametric maximum likelihood estimator (NPMLE). Using the standard Deaton-Carroll buffer-stock model first estimated in [Gourinchas and Parker \(2002\)](#) and [Cagetti \(2003\)](#), we let the preference parameters follow some arbitrary distribution. Our NPMLE is closely related to what [Kamakura \(1991\)](#) termed the “histogram model” for non-parametric estimation of heterogeneous preferences over discrete alternatives. The same type of estimator is studied in [Bajari, Fox and Ryan \(2007\)](#), [Fox, Kim, Ryan and Bajari \(2011\)](#), and [Fox, Kim and Yang \(2015\)](#) for *discrete* (or discretized) choice models. By allowing for measurement error in consumption, we extend the histogram estimator to *continuous* choice dynamic programming models. We provide a detailed discussion of the numerical implementation and show Monte Carlo evidence supporting the applicability of the proposed NPMLE.<sup>5</sup> The NPMLE builds on the

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<sup>1</sup> See e.g. [Krusell and Smith \(1997, 1998\)](#), [Hendricks \(2007\)](#), [Cagetti and De Nardi \(2008\)](#), [Cozzi \(2014\)](#), [Carroll, Slacalek, Tokuoka and White \(2014\)](#) and [De Nardi \(2015\)](#).

<sup>2</sup> See e.g. [Guvenen \(2006, 2009\)](#) and [Gârleanu and Panageas \(2015\)](#).

<sup>3</sup> See e.g. [Kocherlakota \(2010\)](#) and [Farhi and Werning \(2012\)](#).

<sup>4</sup> See e.g. [Barsky, Juster, Kimball and Shapiro \(1997\)](#), [Beetsma and Schotman \(2001\)](#), [Holt and Laury \(2005\)](#), [Andersen, Harrison, Lau and Rutström \(2008, 2010\)](#), [Guiso and Paiella \(2008\)](#), [Kimball, Sahn and Shapiro \(2008, 2009\)](#), [Dohmen, Falk, Huffman, Sunde, Schupp and Wagner \(2011\)](#), [Andreoni and Sprenger \(2012\)](#) and [Finke and Huston \(2013\)](#).

<sup>5</sup> [Fox, Kim, Ryan and Bajari \(2011\)](#), and [Fox, Kim and Yang \(2015\)](#) argue that discretizing the continuous choice outcome variable could be an alternative route to go. [Nevo, Turner and Williams \(2013\)](#) implement a simulated minimum distance estimator using both discrete and continuous choices over residential broadband use.

key insight that finding the optimal weights (shares in the population) over a fixed grid of preference nodes can be nested in such a way that we do not need to successively re-solve the model for trial values of the weights. This makes it feasible to allow for fine grids over preference nodes compared to other estimators where both the value and weight of each node is estimated (such as the one proposed in [Heckman and Singer, 1984](#)).

We find clear evidence of preference heterogeneity within educational strata, especially in risk aversion, although we disregard self-employed and households in the top and bottom percent of the wealth distribution. For low skilled households we estimate discount factors that vary over the range  $[0.960, 0.980]$  and relative risk aversion coefficients in the range  $[0.82, 4.14]$ . For the high skilled, the means of both marginal distributions are shifted upwards with discount factors varying over the range  $[0.976, 1.001]$  and relative risk aversion coefficients varying over  $[1.53, 4.38]$ . We also find evidence of a positive correlation between patience and risk aversion within each educational strata.

The estimated model fits the data very well and, although not targeted by our estimator, the average simulated wealth age profile is very close to that observed in the registers. The estimated preference heterogeneity can, however, *not* explain the observed inequality of wealth accumulating over the life cycle. This is in contrast to most of the existing literature which documents substantial dispersion in preferences. [Carroll, Slacalek, Tokuoka and White \(2014\)](#), for example, match selected wealth percentiles in the *Survey of Consumer Finances* (SCF) and show that this requires a *uniform* distribution of (annual) discount factors ranging from around 0.93 to 0.99.<sup>6</sup> Our results instead suggest that heterogeneity in impatience and risk aversion only explains a rather limited part of the observed wealth inequality. The broader literature on wealth inequality is surveyed in [De Nardi \(2015\)](#).<sup>7</sup> She notes that other important drivers of wealth inequality (in excess of income inequality) are *i*) inter-generational transmission of bequests and human capital, *ii*) entrepreneurship or high returns to capital coupled with borrowing

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<sup>6</sup> Closely related exercises (with comparable results) are conducted in [Samwick \(1998\)](#) and [Hendricks \(2007\)](#). [Cozzi \(2014\)](#) apply the method proposed in [Kimball, Sahm and Shapiro \(2009\)](#) to elicit the risk aversion distribution from hypothetical labor income gambles in the PSID. He finds that substantial shares of the population have risk aversion coefficients respectively below 0.5 and above 6, and show that the model matches the observed degree of wealth inequality in the U.S. In all the above cases the inequality observed at the top (e.g. the top 5 percent or top 1 percent) can, however, not be reproduced.

<sup>7</sup> For previous surveys reaching similar conclusion see [Cagetti and De Nardi \(2008\)](#) and [Heathcote, Storesletten and Violante \(2009\)](#).

constraints, and *iii*) high earnings risk for the top earners.

Few other studies document heterogeneity in discount factors and risk aversion coefficients from observed consumer behavior. To our knowledge, [Alan and Browning \(2010\)](#) is the only study documenting heterogeneity in intertemporal allocation parameters using observed consumer data.<sup>8</sup> They use a semi-structural *synthetic residual estimation* (SRE) methodology together with data from the *Panel Study of Income Dynamics* (PSID) and find evidence of much more dispersion in discount rates and relative risk aversion coefficients than we do. They do not investigate the implications of their estimated preferences on wealth inequality.

After describing the life cycle buffer-stock model in section 2, we present our proposed non-parametric estimator in section 3 and discuss the numerical implementation. The Danish register data are introduced in section 4 and in section 5 we calibrate some model parameters. The estimation results are presented in section 6 where we illustrate that the predicted level of consumption from the estimated model fits the imputed level of consumption observed in the data very well, while we cannot generate the observed dispersion in wealth. Section 7 concludes.

## 2 The Model

We rely on the canonical buffer-stock life-cycle model of [Deaton \(1991, 1992\)](#) and [Carroll \(1992, 1997, 2012\)](#). We consider unitary households indexed by  $i$  with heterogeneous preferences who work for  $T$  periods, then retire and eventually die at the end of period  $\bar{T}$ . The recursive form of the household  $i$ 's problem prior to retirement is given by

$$V_t(P_t, M_t) = \max_{C_t \geq 0} \frac{C_t^{1-\rho_i}}{1-\rho_i} + \beta_i \cdot \mathbb{E}_t [V_{t+1}(P_{t+1}, M_{t+1})] \quad (2.1)$$

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<sup>8</sup> In a working paper, [Alan, Browning and Ejrnaes \(2014\)](#) extend their approach to also consider income heterogeneity. [Bozio, Laroque and O'Dea \(2013\)](#) find significant heterogeneity in patience of older households in the English Longitudinal Survey of Ageing (ELSA). They find that households with less education or numerical ability (financial literacy) are more patient. [Lawrance \(1991\)](#), [Gourinchas and Parker \(2002\)](#) and [Cagetti \(2003\)](#) only allow for heterogeneity across educational strata and find the more educated to be more patient. [Gourinchas and Parker, 2002](#) also split the sample by four occupation groups. [Cohen and Einav \(2007\)](#) estimate a model for the choice of the level of deductibility in car insurance and find a large degree of heterogeneity in risk preferences.

subject to the intertemporal budget constraint

$$M_{t+1} = R \cdot A_t + Y_t \quad (2.2)$$

$$A_t = M_t - C_t \quad (2.3)$$

where  $A_t$  is end-of-period assets,  $M_t$  is beginning-of-period market resources,  $Y_t$  is income, and  $R$  is the gross rate of return. Consumers are allowed to be net-borrowers up to a fraction of their permanent income  $P_t$ . End-of-period wealth thus has to satisfy

$$A_t \geq -\lambda_t \cdot P_t, \quad \lambda_t = \begin{cases} 0 & t = T \\ \lambda & \text{else.} \end{cases} \quad (2.4)$$

**Income Process.** In the beginning of each period, households receive a stochastic income

$$Y_t = \xi_t \cdot P_t \quad (2.5)$$

$$P_t = G_t \cdot \psi_t \cdot P_{t-1} \quad (2.6)$$

$$\psi_t \sim \log \mathcal{N}(-0.5 \cdot \sigma_\psi^2, \sigma_\psi^2) \quad (2.7)$$

where  $P_t$  is permanent income,  $G_t$  is the age-dependent gross growth rate of permanent income,  $\psi_t$  is a mean-one *permanent* shock to income, and  $\xi_t$  is a mean-one *transitory* shock to income given by

$$\xi_t = \begin{cases} \mu & \text{with probability } \wp \\ (\epsilon_t - \mu\wp)/(1 - \wp) & \text{with probability } 1 - \wp \end{cases} \quad (2.8)$$

$$\epsilon_t \sim \log \mathcal{N}(-0.5 \cdot \sigma_\xi^2, \sigma_\xi^2). \quad (2.9)$$

**Retirement.** We implement a parsimonious account of post-retirement motives following the approach in [Gourinchas and Parker \(2002\)](#). We first assume no uncertainty in retirement and use that the optimal level of consumption in the period  $T + 1$  is then given by

$$\bar{C}_{T+1} = \varrho_0 \cdot (R \cdot A_T + \kappa \cdot \varrho_1 \cdot P_T)$$

where  $\kappa$  is the fixed retirement replacement and

$$\begin{aligned}\varrho_0 &\equiv \left( \sum_{k=0}^{\bar{T}-T} \left( R^{-1} (R\beta_i)^{\frac{1}{\rho_i}} \right)^k \right)^{-1} \\ \varrho_1 &\equiv G_R \cdot \sum_{k=0}^{\bar{T}-T} \left( R^{-1} G_R \right)^k.\end{aligned}$$

with  $G_R$  as the constant growth rate of income.

We then secondly assume that consumption in period  $T$  satisfies the following *adjusted* Euler-equation

$$C_T^{-\rho} = R\beta_i \cdot \nu \cdot \left( \bar{C}_{T+1} (P_T, A_T) \right)^{-\rho_i}, \quad (2.10)$$

in which the parameter  $\nu$  is a parsimonious way to account for the effects of post retirement factors such as a bequest motive, stochastic mortality and income risk, without modeling them directly.

**Parameters.** The full set of model parameters are given by

$$\Theta = \left\{ \underbrace{T, \bar{T}, R, \{G_t\}_{t+1}^T, \sigma_\xi, \sigma_\psi, \wp, \kappa, G_R, \lambda}_{\text{calibrated}}, \underbrace{\nu, \mu, f(\beta, \rho)}_{\text{estimated}} \right\}$$

where  $f(\beta, \rho)$  is the distribution of patience and risk aversion parameters in the population. We calibrate the parameters we have either relatively good priors on or are exogenous and can be estimated from the Danish register data (see section 5). We estimate the remaining parameters, as discussed in the following section.

### 3 Non-Parametric Maximum Likelihood Estimation

We wish to estimate  $\theta = (\nu, \mu)$  and the joint distribution of patience and risk aversion parameters in the population  $f(\beta, \rho)$ .<sup>9</sup> To do this we set up a non-parametric maximum likelihood estimator. Our data consist of household resources  $M_{it}$ , consumption  $C_{it}$  and permanent income  $P_{it}$  of an unbalanced panel of  $N$  households

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<sup>9</sup> Given a set of calibrated parameters, discussed in section 5.

indexed by  $i$  and observed in  $T_i$  periods. We construct normalized variables as  $c_{it} = C_{it}/P_{it}$  and  $m_{it} = M_{it}/P_{it}$  and denote the *model-implied* normalized level of consumption of household  $i$  in period  $t$  by

$$c_{it}^*(\theta, \beta_i, \rho_i) = c_t^*(m_{it} | \theta, \beta_i, \rho_i) \quad (3.1)$$

We then assume that the observed normalized level of consumption in the Danish register data is contaminated with *iid* additive Gaussian measurement error with zero mean and variance  $\sigma_\varepsilon^2$ , i.e.

$$c_{it} = c_{it}^*(\theta, \beta_i, \rho_i) + \varepsilon_{it}, \quad \varepsilon_{it} \sim \text{i.i.d. } \mathcal{N}(0, \sigma_\varepsilon^2) \quad (3.2)$$

The distribution  $f(\beta, \rho)$  is unobserved and we thus formulate an *expected* likelihood function which only depend on the remaining parameters:

$$\begin{aligned} \tilde{\mathcal{L}}(\theta, \sigma_\varepsilon, f(\beta, \rho)) &\equiv P(c | m, \theta, \sigma_\varepsilon, f(\beta, \rho)) \\ &= \prod_{i=1}^N \int_{\beta} \int_{\rho} \ell_i(\theta, \sigma_\varepsilon, \beta, \rho) f(\beta, \rho) d\rho d\beta \end{aligned} \quad (3.3)$$

where  $c$  is the stacked observations of  $c_{it}$ ,  $m$  is the stacked observations of  $m_{it}$ , and

$$\ell_i(\theta, \sigma_\varepsilon, \beta, \rho) = \prod_{t=1}^{T_i} \frac{1}{\sqrt{2\pi\sigma_\varepsilon}} \exp\left(-\frac{\varepsilon_{it}(\theta, \beta, \rho)^2}{2\sigma_\varepsilon^2}\right) \quad (3.4)$$

is the likelihood contribution of household  $i$  for a given set of parameters.

We approximate the integral in equation (3.3) with a discrete sum. Denoting  $\omega_j \in [0, 1]$  as the weight on node  $\{\beta_j, \rho_j\}$  and  $\omega = (\omega_1, \dots, \omega_J)$  as the stacked vector of all weights, the average log *expected* likelihood function is then given by

$$\mathcal{L}(\theta, \sigma_\varepsilon, \omega) = \frac{1}{N} \sum_{i=1}^N \log \left\{ \sum_{j=1}^J \omega_j \ell_i(\theta, \sigma_\varepsilon, \beta_j, \rho_j) \right\} \quad (3.5)$$

Importantly, the solution of the economic model is independent of the weights and the measurement error variance. We use this insight when implementing the estimator as described below.

The estimator is related to that proposed by Heckman and Singer (1984), where both the placement of the nodes  $\{\beta_j, \rho_j\}_1^J$  and the weights placed on each node  $\omega_j$  is estimated simultaneously. This approach is more computationally time consuming

than the one proposed here because the economic model needs to be re-solved for each trial value of the nodes. Consequently, empirical implementations of the Heckman-Singer approach allow for relatively few nodes and they are thus often referred to as *types*. For example, [French and Jones \(2011\)](#) implement a method of simulated moments (MSM) estimator based on the Heckman-Singer approach allowing for four types of retirees when studying the effects of health insurance and self-insurance on retirement behavior.

We allow for many nodes (or types) but fix the nodes and only estimate the weights,  $\omega$ . [Kamakura \(1991\)](#) termed this strategy the “histogram model” because it resembles how density histograms are constructed. [Bajari, Fox and Ryan \(2007\)](#), [Fox, Kim, Ryan and Bajari \(2011\)](#), and [Fox, Kim and Yang \(2015\)](#) study this type of estimator for heterogeneous agents performing *discrete* choices while we focus on the continuous consumption choice. Although the strategy applied here is similar to that of these authors, in the discrete-choice case the problem of estimating  $\omega$  can be reformulated into a least square problem with bounds and a linear equality constraint. Those types of least square problems are globally convex while ours might not be. Furthermore, we nest the estimation of weights in an outer numerical optimization routine, maximizing over the homogeneous parameters in  $\theta$ .

### 3.1 Implementation

For each candidate value of  $\theta$ , the model is solved for all  $J$  nodes of  $\{\beta_j, \rho_j\}_{j=1}^J$  using the endogenous grid method (EGM) proposed by [Carroll \(2006\)](#). We use 300 discrete points to approximate the consumption function and  $8^2$  Gauss-Hermite quadrature points to approximate expectations with respect to future transitory and permanent income shocks. The EGM solves our type of model extremely fast and accurate making it ideal for this type of nested fixed point estimator ([Jørgensen, 2013](#)).

Because the model only needs to be successively solved for trial values of  $\theta$ , we formulate a sequential problem where the log *expected* likelihood estimates of the model-independent parameters – for a given value of  $\theta$  – are

$$\begin{aligned}
 (\hat{\sigma}_\varepsilon(\theta), \hat{\omega}(\theta)) &= \arg \max_{\sigma_\varepsilon, \omega} \mathcal{L}(\theta, \sigma_\varepsilon, \omega) & (3.6) \\
 \text{s.t. } &\omega_j \in [0, 1] \forall j \\
 &\sum_{j=1}^J \omega_j = 1 \\
 &\sigma_\varepsilon > 0
 \end{aligned}$$

This is a constrained problem because all weights must lie in the unit interval and should sum to one. We use the built-in MATLAB routine *fmincon* to perform the constrained maximization. We initialize the inner maximization at five random starting values of  $\omega$  and  $\sigma_\varepsilon$  to increase the likelihood of ending in the *global* maximum.

To jointly estimate  $\theta$ ,  $\sigma_\varepsilon$  and  $f(\beta, \rho)$ , where the latter is governed by the weights  $\omega$ , we maximize the log expected likelihood function in (3.6) with respect to  $\theta$ :

$$\begin{aligned}\hat{\theta} &= \arg \max_{\theta} \mathcal{L}(\theta, \hat{\sigma}_\varepsilon(\theta), \hat{\omega}(\theta)) \\ (\hat{\omega}, \hat{\sigma}_\varepsilon) &= (\hat{\sigma}_\varepsilon(\hat{\theta}), \hat{\omega}(\hat{\theta}))\end{aligned}$$

**Gradients.** To improve accuracy, convergence of the numerical optimizer and computational speed, we supply the analytic gradients when estimating the measurement error variance and the weights in the inner most optimization step. The gradients with respect to the weights are

$$\frac{\partial \mathcal{L}(\theta, \sigma_\varepsilon, \omega)}{\partial \omega_k} = \frac{1}{N} \sum_{i=1}^N \frac{\ell_i(\theta, \sigma_\varepsilon, \beta_k, \rho_k)}{\bar{\ell}_i(\theta, \sigma_\varepsilon, \omega)}, \quad \forall k \in \{1, 2, \dots, J\}$$

where

$$\bar{\ell}_i(\theta, \sigma_\varepsilon, \omega) \equiv \sum_{j=1}^J \omega_j \ell_i(\theta, \sigma_\varepsilon, \beta_j, \rho_j)$$

is the *expected* likelihood contribution of household  $i$ . The gradient with respect to the measurement error variance is

$$\frac{\partial \mathcal{L}(\theta, \sigma_\varepsilon, \omega)}{\partial \sigma_\varepsilon} = \frac{1}{N} \sum_{i=1}^N \frac{1}{\bar{\ell}_i(\theta, \sigma_\varepsilon, \omega)} \sum_{j=1}^J \omega_j \frac{\partial \ell_i(\theta, \sigma_\varepsilon, \beta_j, \rho_j)}{\partial \sigma_\varepsilon}$$

where

$$\frac{\partial \ell_i(\theta, \sigma_\varepsilon, \beta_j, \rho_j)}{\partial \sigma_\varepsilon} = (\Xi_i \sigma_\varepsilon^{-3} - T_i \sigma_\varepsilon^{-1}) \sigma_\varepsilon^{-T_i} \exp(-0.5 \Xi_i \sigma_\varepsilon^{-2})$$

and  $\Xi_i = \sum_{t=1}^{T_i} \varepsilon_{i,t}(\theta, \beta_j, \rho_j)^2$  is the sum of squared errors.

Because  $\ell_i(\theta, \sigma_\varepsilon, \beta_j, \rho_j)$  is a product of  $T_i$  numbers between zero and one, it is very likely that for some observations  $\bar{\ell}_i(\theta, \sigma_\varepsilon, \omega) \approx 0$  leading to *numerical underflow* and, thus, division with zero. To avoid this, we have simply added a small number (say  $10^{-6}$ ) to  $\ell_i(\theta, \sigma_\varepsilon, \beta_j, \rho_j)$  before calculating the gradients.

**Domain Specification and the Number of Nodes.** Because we exogenously fix the domain of  $\beta$  and  $\rho$  and nests the estimation of the weights inside an



outer-most maximization step, estimating parameters in  $\theta$ , we have chosen the domain to be rather large. Particularly, we let  $\beta \in [.91, 1.03]$  and  $\rho \in [0.1, 7]$ . To determine how many discrete approximation nodes to include, we applied a successive approach, similar to that suggested by [Fernández-Villaverde, Rubio-Ramírez and Santos \(2006\)](#) to determine the degree of accuracy of a numerical solution method required for the approximate likelihood function to be a good approximation of the exact likelihood. Particularly, we increased the number of equally spaced nodes in each direction until the estimated likelihood function did not change “significantly”. Because we could be adding more mass at zero-regions – resulting in the estimated likelihood not changing when adding additional points – we stop when the likelihood function has not changed significantly in two consecutive estimations. Using 30 discrete nodes in each direction (yielding 900 weights to be estimated) seems sufficient by this informal metric.

### 3.2 *Conditional Joint Distribution, $g(\beta, \rho)$*

The joint distribution of patience and risk aversion parameters in the population  $f(\beta, \rho)$  is *unconditional* in sense that without having observed the behavior of a specific household it is our best guess of its preferences. We can, however, make a more tight prediction on the preferences of each household by conditioning on observed choices.<sup>10</sup> This is in particular useful when simulating the model.

First, we note that if we knew  $\beta_i$  and  $\rho_i$ , the probability of household  $i$ 's observed behavior would be

$$P(c_i | m_i, \theta, \sigma_\varepsilon, \beta_i, \rho_i) = \ell_i(\theta, \sigma_\varepsilon, \beta_i, \rho_i) \quad (3.7)$$

where  $m_i$  is the stacked vector of normalized observed resources of household  $i$ , and  $c_i$  is the corresponding stacked vector for normalized consumption.

Secondly, since we do not know  $\beta_i$  and  $\rho_i$ , the probability of the observed behavior is the integral over the joint distribution of  $\beta$  and  $\rho$ ,

$$P(c_i | m_i, \theta, \sigma_\varepsilon, f(\rho, \beta)) = \int_{\beta} \int_{\rho} \ell_i(\theta, \sigma_\varepsilon, \beta, \rho) f(\beta, \rho) d\rho d\beta \quad (3.8)$$

The *conditional joint distribution*  $g(\beta, \rho)$  can now be derived using Bayes' rule,

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<sup>10</sup>See e.g. [Train \(2009, chapter 11\)](#) for a similar approach in the case of discrete choice models.

and we get

$$g_i(\beta, \rho) \equiv g(\beta, \rho | c_i, m_i, \theta, \sigma_\varepsilon, f(\rho, \beta)) = \frac{P(c_i | m_i, \theta, \sigma_\varepsilon, \beta, \rho) \cdot f(\rho, \beta)}{P(c_i | m_i, \theta, \sigma_\varepsilon, f(\rho, \beta))} \quad (3.9)$$

With estimated parameters and weights, the *conditional weight* on node  $k$  for household  $i$  is

$$\hat{\omega}_{ki} = \frac{\ell_i(\hat{\theta}, \hat{\sigma}_\varepsilon, \beta_k, \rho_k) \cdot \hat{\omega}_k}{\sum_{j=1}^J \hat{\omega}_j \cdot \ell_i(\hat{\theta}, \hat{\sigma}_\varepsilon, \beta_j, \rho_j)} \quad (3.10)$$

These conditional weights are e.g. useful when drawing initial values for simulation of data from the model. Given that we are simulating a household of type  $j$  we will thus draw the assets and income observed for household  $i$  with probability

$$\pi_{ji} = \frac{\hat{\omega}_{ji}}{\sum_{i=1}^N \hat{\omega}_{ji}} \quad (3.11)$$

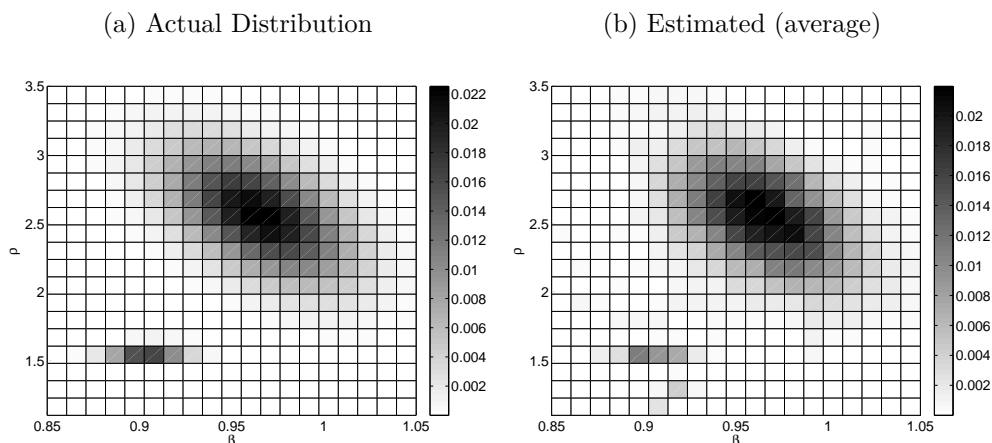
where we for simplicity have assumed that all household are observed at  $t = 1$ .

### 3.3 Small Sample Properties: A Monte carlo Study

To study how the proposed NPMLE performs on finite samples, we conduct a Monte Carlo (MC) study. Data is simulated from the model outlined in section 2 with parameter values  $\sigma_\varepsilon = \sigma_\psi = 0.1$ ,  $R = 1.04$ ,  $\wp = 0$ ,  $\mu = 0$ ,  $G_R = 1$ ,  $\kappa = 0.9$ ,  $\nu = 1.3$ ,  $\lambda = 0$ , and

$$G_t = \begin{cases} 1.10 & \text{if } age_t \leq 30 \\ 1.08 & \text{if } 31 \leq age_t \leq 35 \\ 1.03 & \text{if } 36 \leq age_t \leq 45 \\ 1.01 & \text{if } 45 < age_t. \end{cases}$$

Figure 3.1 illustrates how the estimator performs in a sample size similar to that used in the present paper. Particularly, we simulate  $N = 100,000$  households from they are 26 to 59 years old and pick  $T = 10$  random adjacent time periods to use for estimation. We then add normal measurement error with mean zero and standard deviation  $\sigma_\varepsilon = 0.03$  to simulated consumption (normalized with simulated permanent income). We do this 50 times and report the average estimated weights across these 50 estimated set of parameters in the right panel of figure 3.1.

Figure 3.1: Monte Carlo Estimation of Weights,  $\omega$ .


*Notes:* The average is over 50 repetitions.

The estimator performs very well and uncovers both the disjoint parts of the joint distribution  $f(\rho, \beta)$ . All 50 runs converged and the MC estimation results indicate that the implemented estimator can uncover the true distribution of preferences in the Danish data described below.

## 4 The Data

We use high quality Danish administrative registers covering the entire population in the period 1987-1996. We begin in 1987 to be able to consistently match individuals into couples, and we end with 1996 because the Danish wealth tax was abolished in this year. Information on, e.g., cars and boats were not collected in subsequent years leading to a significant break in the wealth measure from 1996 to 1997. All information are based on third party reports with little additional self-reporting. All self-reporting are subject to possible auditing giving reliable longitudinal information on household characteristics, assets, liabilities and income.

Income includes all monetary income net of all taxes, except any income related to ownership of financial assets. Transfers, such as child benefits and unemployment benefits, are also included to ensure that disposable income accurately measures the flow of resources available for consumption. Net wealth consists of stocks, bonds, bank deposits, cars, boats, house value for home owners and mortgage debts net of total liabilities. The house value is assessed by the tax authorities for tax purposes. Pension wealth is not included in the wealth measure.

Household consumption is not observed in the registers and is, therefore, imputed using a simple budget approach,  $C_t = \tilde{Y}_t - \Delta A_t$ , where  $\tilde{Y}_t = Y_t + r \cdot A_t$  is disposable income,  $A_t$  is end-of-period net wealth,  $r$  is the real rate of return, and  $\Delta A_t$  thus proxies savings. A very similar imputation method is evaluated on Danish data in [Browning and Leth-Petersen \(2003\)](#) and found to produce a reasonable approximation. The resulting consumption measure will, however, e.g. include some durables such as home appliances.

We restrict attention to stable married or cohabiting couples in which the husband is between age 25 and 59. This is to mitigate issues regarding educational and retirement choices. To increase homogeneity of households, we restrict the spousal age difference to be no more than five years, and require that no one in the household ever becomes self-employed or retire before age 59. To limit the effect of errors in the imputation procedure on our estimates of preference heterogeneity, we require that the households are in our data set for at least 5 years. Finally, we trim our sample from extreme observations leaving an unbalanced panel of 336.017 households observed in at most 9 time periods with a total of 2.953.594 household-time observations. Households are classified as high skilled if either member holds at least a bachelor degree (about one in four is high-skilled). All variables are deflated with the official consumer price index and expressed relative to the income of a 25 year old low-skilled household. Further details on the data are provided in appendix C.

To remove year effects from income, we first regress log-income for each education group separately on a full set of age and year dummies, i.e.

$$\log(Y_{ikt}^{raw}) = \text{cons} + \sum_{t=25}^{59} \alpha_t^{age} \mathbf{1}_{age_{ikt}=t} + \sum_{k=1987}^{1996} \alpha_k^{year} \mathbf{1}_{year_{ikt}=k \neq k_{base}} + \epsilon_{ikt} \quad (4.1)$$

where  $i$  is for couple,  $t$  is for age,  $k$  is for year, and the base-year is  $k_{base} = 1992$ . Income *adjusted for year effects* is consequently defined as

$$Y_{i,t} = \exp(\text{cons} + \alpha_t^{age} \mathbf{1}_{age=t} + \epsilon_{i,k,t}) \cdot \mathcal{T}_t \quad (4.2)$$

where the re-trending term  $\mathcal{T}_t \equiv \mathcal{G}^{(t-25)}$  is included to account for aggregate growth.<sup>11</sup> We assume a constant real annual growth rate of 1.5 percent ( $\mathcal{G} =$

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<sup>11</sup>If we have a stable life-cycle profile of income, continuing aggregate growth of  $z$  percent, and we in a cross-section observe that income at age 26 is  $x$  percent higher than at age 25, then those aged 25 today can expect income growth of approximately  $x + z$  over the following year

1.015).

Because net-wealth can be negative, we cannot remove year effects from wealth in the same way. To still achieve a proportional adjustment for year effects on wealth we first calculate the average level of wealth in each *age-year* bin

$$\tilde{A}_{k,t} \equiv \frac{1}{N_{k,t}} \sum_{i=0}^{N_{k,t}} A_{k,t}^{raw} \quad (4.3)$$

where  $N_{k,t}$  is number of households in the sample at age  $t$  in year  $k$ . The average level of wealth in a given *year* can then be expressed as

$$\bar{A}_k = \frac{1}{N_k} \sum_{t=25}^{59} N_{k,t} \cdot \tilde{A}_{k,t} \quad (4.4)$$

where  $N_t$  is the number of households in the sample in year  $t$ . Secondly, we derive the counter-factual average level of wealth in year  $k$  if *the age-bin wealth averages had been as they were in the base-year* (but the age distribution was unchanged), i.e.,

$$\bar{A}_{k,base} = \frac{1}{N_k} \sum_{t=25}^{59} N_{k,t} \cdot \tilde{A}_{k,base,t} \quad (4.5)$$

Wealth *adjusted for year effects* is finally defined as

$$A_{i,t} = \frac{\hat{A}_{k,base}}{\bar{A}_k} \cdot A_{i,k,t}^{raw} \cdot \mathcal{T}_t \quad (4.6)$$

Figure 4.1-4.3 show the resulting empirical life-cycle profiles of income  $Y_t$ , wealth  $A_t$ , and consumption  $C_t$ . Income and consumption share the same age profile, and had we not re-trended income through  $\mathcal{T}_t$  (not reported), the age profiles would be hump-shaped, respectively peaking around age 40 and 45 for low and high skilled. The average wealth age profile is monotonically increasing and there is significant dispersion in wealth across households. The poorest 10 percent are net-borrowers throughout the most of their working life.

Figure 4.1: Life Cycle Profiles -  $Y_t$

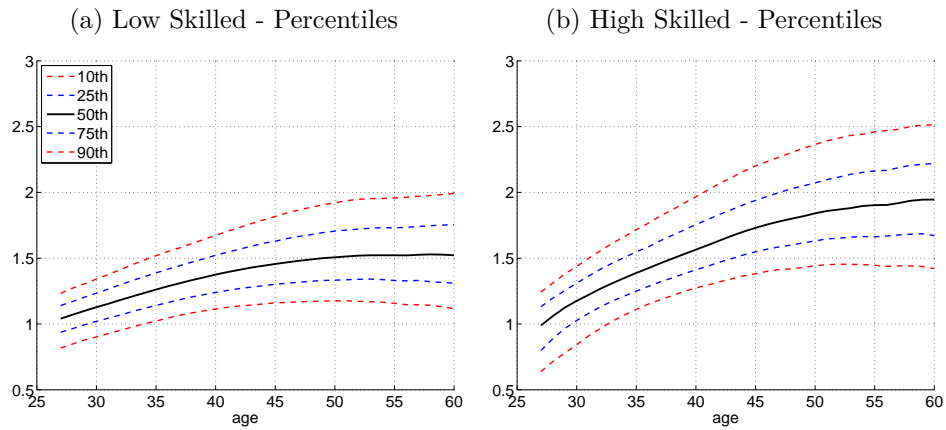


Figure 4.2: Life Cycle Profiles -  $A_t$

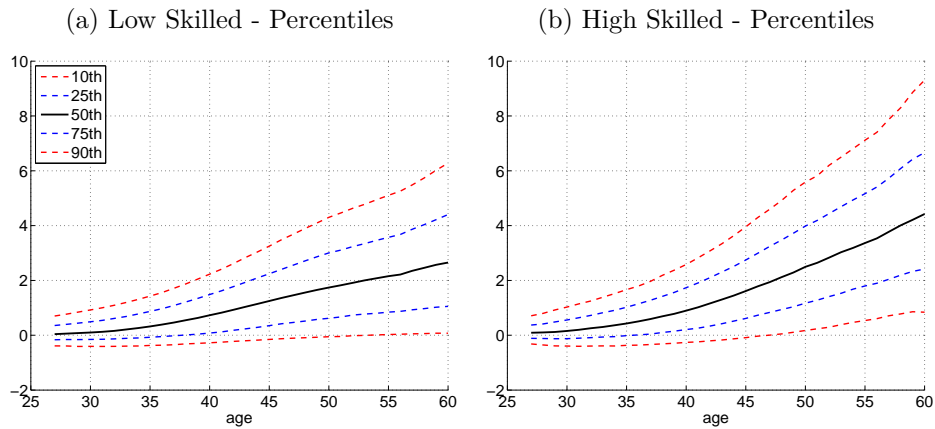
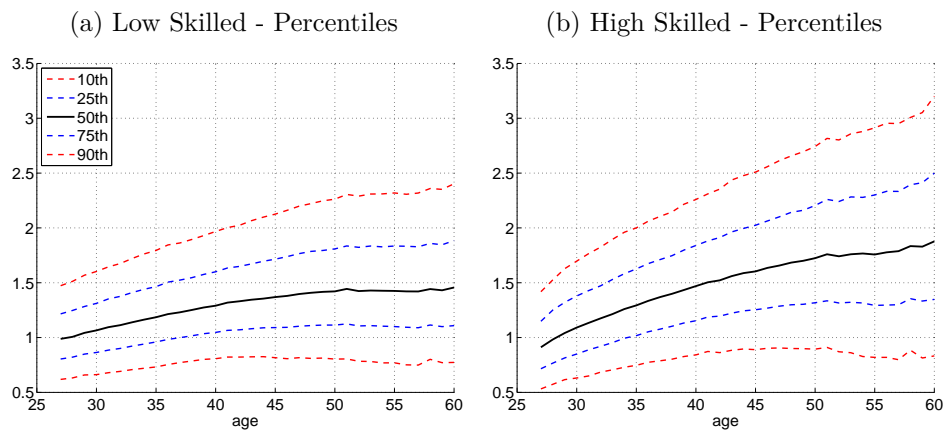


Figure 4.3: Life Cycle Profiles -  $C_t$



## 5 Calibration

**Income Process Estimations.** Following the approach in [Meghir and Pistaferri \(2004\)](#), we estimate the transitory and permanent income variances as

$$\sigma_{\psi}^2 = \text{cov} \left( \Delta \tilde{y}_{it}, \sum_{k=0}^2 \Delta \tilde{y}_{i,t-1+k} \right) \quad (5.1)$$

$$\sigma_{\xi}^2 = -\text{cov} (\Delta \tilde{y}_{it}, \Delta \tilde{y}_{i,t+1}) \quad (5.2)$$

where  $\tilde{y}_t$  is residuals from a regression of log household income (already cleaned for year effects) on a full set of age dummies. We do this separately for each educational group. The results are reported in table 5.1 The income variances of Danish households are an order of magnitude smaller than those typically estimated for the US. As argued in [Jørgensen \(2015\)](#), this is most likely due to *i*) a generous social welfare system, *ii*) progressive taxation, *iii*) a relatively high “minimum wage”, and *iv*) register data is typically less noise compared to surveys typically used. We find that the low and high skilled are subject to similar transitory shocks, but that the permanent shocks are larger for the high skilled.

Table 5.1: Income Variances

	Low skilled		High skilled	
	Est	(s.e.)	Est	(s.e.)
$\sigma_{\psi}^2 \cdot 10^3$	2.93	(0.011)	3.65	(0.023)
$\sigma_{\xi}^2 \cdot 10^3$	1.51	(0.008)	1.52	(0.017)

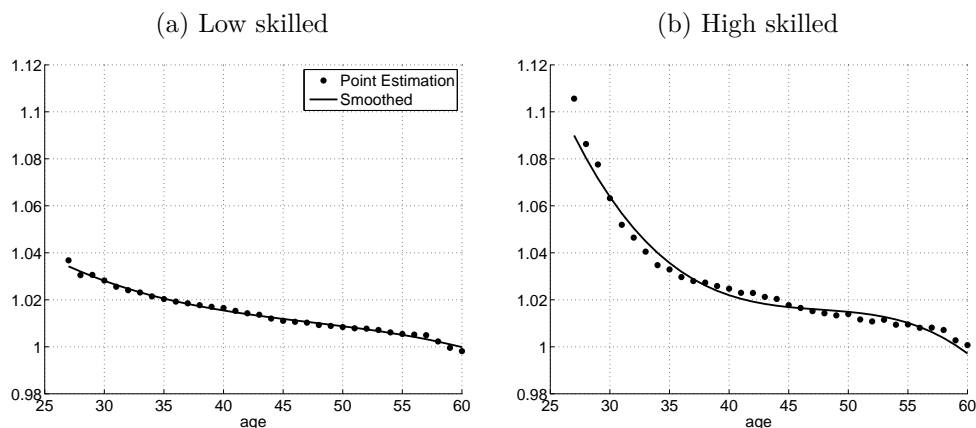
*Notes:* The income shock variances are estimated based on the approach proposed in [Meghir and Pistaferri \(2004\)](#).

The growth in income is estimated by re-arranging the income process such that

$$G_t = \exp \left( \frac{1}{N} \sum_{i=1}^N \Delta \log Y_{it} + \frac{1}{2} \sigma_{\psi}^2 \right) \quad (5.3)$$

A smoothed growth rate  $\tilde{G}_t$  is obtained using a third degree polynomial in age. The results are reported in figure 5.1. Permanent income,  $P_{it}$  is found by applying the Kalman filter on the time series of log income for each household (the resulting life cycle profile is shown in appendix A).

Figure 5.1: Gross Income Growth Rates



We finally assume that households face a one percent ( $\varphi = 0.01$ ) risk of receiving the low income shock of value  $\mu$ .

**Saving and Borrowing.** We choose an interest of  $R = 1.04$  similar to the long run real return on 10 year Danish government bonds which over the period 1987-2007 was 3.8 percent. Informally looking into the observed consumption behavior of households in debt we furthermore set the borrowing constraint to be binding at 30 percent of permanent income ( $\lambda = 0.30$ ).

**Retirement.** Based on the median Danish household from *The Danish Ministry of Finance's* report *Ældres Sociale Vilkår* (in Danish), we set the replacement rate in retirement to 90 percent ( $\kappa = 0.9$ ). We further set the income in retirement to be constant ( $G_R = 1$ ).

## 6 Results

We estimate the model both under the restriction of homogeneous preferences and allowing for heterogeneity in both patience ( $\beta$ ) and risk aversion ( $\rho$ ). The estimation results are presented in table 6.1. Figure 6.1 shows the estimated bivariate and marginal distributions of  $\beta$  and  $\rho$ .

The point estimates from the homogeneous version of the model and the means of the estimated distribution are in ranges typically found in the related literature. Particularly, the point estimate and the mean of the distribution of  $\beta$  is 0.97 and 0.98 for low and high skilled households, respectively. The estimated distributions



are relative narrow with virtually all households in the range [0.960, 0.980] for low skilled and [0.976, 1.001] for high skilled (we estimate a small weight on the lowest node at 0.91).

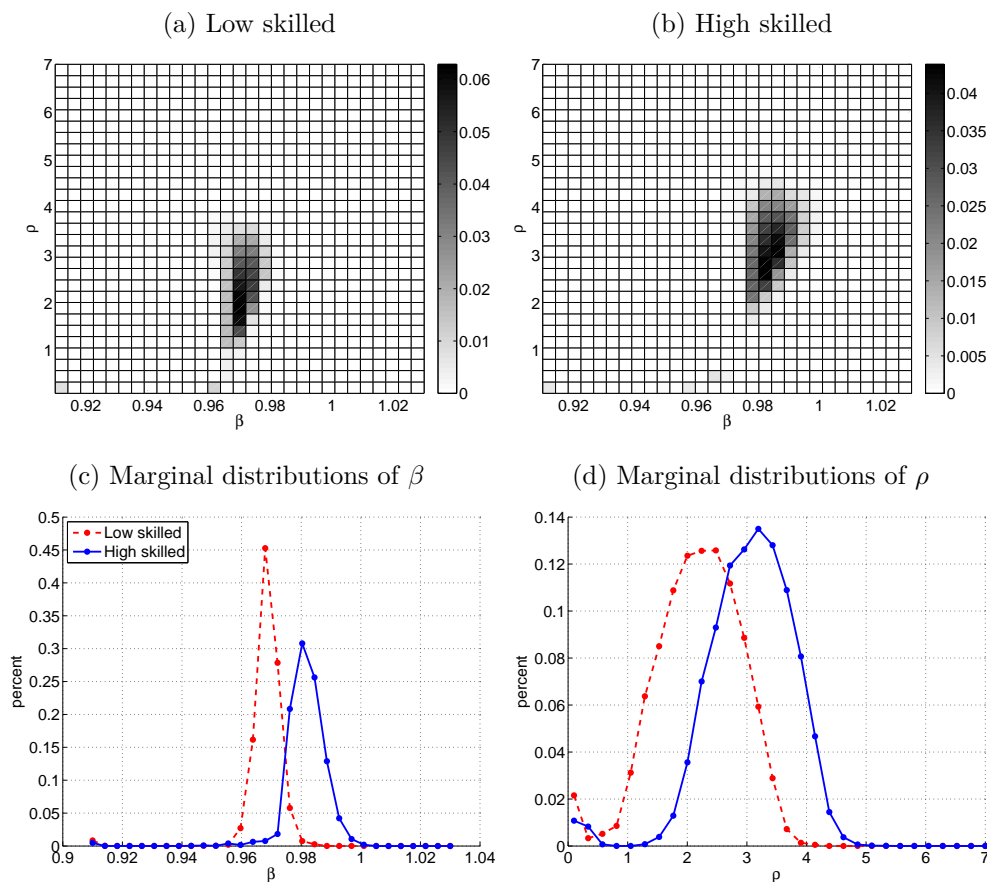
Table 6.1: Estimation Results

	Low skilled		High skilled	
	Homogenous	Heterogenous	Homogenous	Heterogenous
$\beta$	0.969 (0.0002)	0.968 <sup>a</sup>	0.983 (0.0005)	0.981 <sup>a</sup>
$\rho$	1.998 (0.0598)	2.179 <sup>a</sup>	2.841 (0.0703)	3.054 <sup>a</sup>
$\nu$	1.123 (0.0073)	1.115 (0.0019)	1.163 (0.0177)	1.197 (0.0061)
$\mu$	0.188 (0.0069)	0.188 <sup>b</sup>	0.132 (0.0026)	0.132 <sup>b</sup>
$\sigma_\varepsilon$	0.366	0.364	0.386	0.382
$\mathcal{L}$	-3.203	-3.169	-3.631	-3.610
N	253,472	253,472	82,545	82,545

Standard errors in brackets, based on the inverse of the numerical Hessian.

<sup>a</sup> Estimated mean of distribution.

<sup>b</sup> Fixed at the estimated value from the homogeneous case.

Figure 6.1: Estimated Preference Distributions,  $\widehat{f}(\rho, \beta)$ 


We estimate  $\rho$  to be 2.00 for the low skilled and 2.84 for the high skilled in the homogeneous case. The means of the estimated distributions are a bit higher at, respectively, 2.18 and 3.05. In this dimension we find much more heterogeneity with a substantial share of households with  $\rho$  in the range  $[0.82, 4.14]$  for the low skilled and  $[1.53, 4.38]$  for the high-skilled. From figure 6.1 we also see that there, especially for the high skilled, is a *positive* correlation between patience and risk aversion (i.e. a higher  $\rho$  is associated with a higher  $\beta$ ).

Although we find evidence of substantial preference heterogeneity, our estimated distributions are much more narrow than those found in [Alan and Browning \(2010\)](#), which is the only comparable study using observational data and estimating the full joint-distribution of  $\beta$  and  $\rho$ .<sup>12</sup> There are important differences

<sup>12</sup>In the case of  $\beta$ , they find a spread between the 90th and 10th percentile of 0.143 for the low-skilled and 0.134 for the high-skilled. For  $\rho$ , the 90th-10th spreads reported in [Alan and Browning \(2010\)](#) is 8.2 for the low-skilled, and 7.7 for high-skilled. Similar spreads are found in [Alan, Browning and Ejrnaes \(2014\)](#) (more narrow  $\beta$  distribution, but more wide  $\rho$  distribution)

across both the data used and the chosen estimation methodologies. In regard to the data they rely on self-reported measures of food consumption in the PSID and disregard households close to the borrowing constraint, while we use imputed consumption from high quality Danish register data on wealth and income. In regard to the chosen estimation methodology, their SRE is fully parametric and in order to construct synthetic consumption paths they assume that the distribution of Euler-residuals can be approximated by a mixture of two log-normals, and that all households, irrespective of their individual preferences, draw residuals from the same distribution.<sup>13</sup> Unlike in SRE, our fully structural approach requires an explicit specification of the income process both before and after retirement.

The remaining estimated parameters are also found to be in their expected ranges.  $\nu$  is slightly above one, indicating that the retirement saving motive is stronger than our model without bequests and uncertainty in retirement would otherwise be able to explain. [Jørgensen \(2015\)](#) estimates a similar effect (albeit slightly larger) of post-retirement motives from a comparable parametrization.  $\mu$  is estimated to be 0.19 for the low skilled and 0.13 for the high skilled which does not seem to be a too extreme value of a low income shock occurring with a one percent probability. Finally, the standard deviation of the measurement error in consumption is estimated to be between 0.36 and 0.39 across the different specifications; this is similar to but a bit lower than found earlier in [Jørgensen \(2015\)](#). In appendix [D](#), we show that our estimator is capable of catching a high degree of preference heterogeneity despite large measurement error when the variation in resources is as in the data.<sup>14</sup>

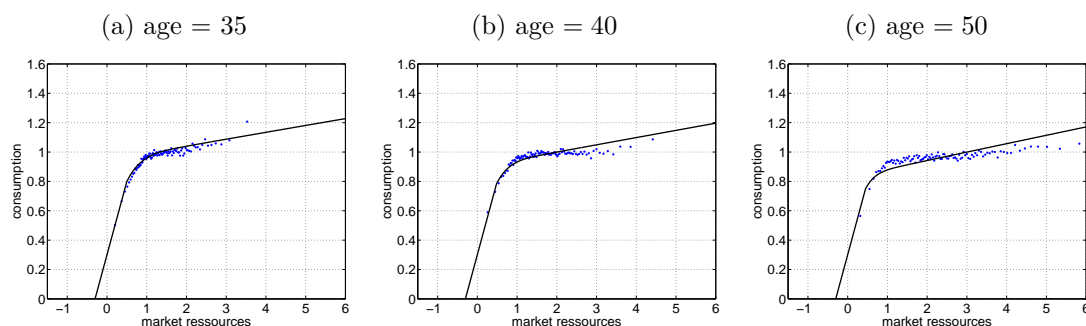
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who do not condition on education, but allow for the income heterogeneity. Compared to most of the experimental and survey based literature, our estimated degree of heterogeneity is also rather small, especially in  $\beta$ .

<sup>13</sup>Note, however, that while the mean Euler-residual (in the absence of borrowing constraints) is independent of preferences, higher order moments are generally not. This is the case even though the distribution of *pooled* Euler-residuals across heterogeneous households are well approximated by a mixture of two log-normals (as found in [Alan and Browning, 2010](#)).

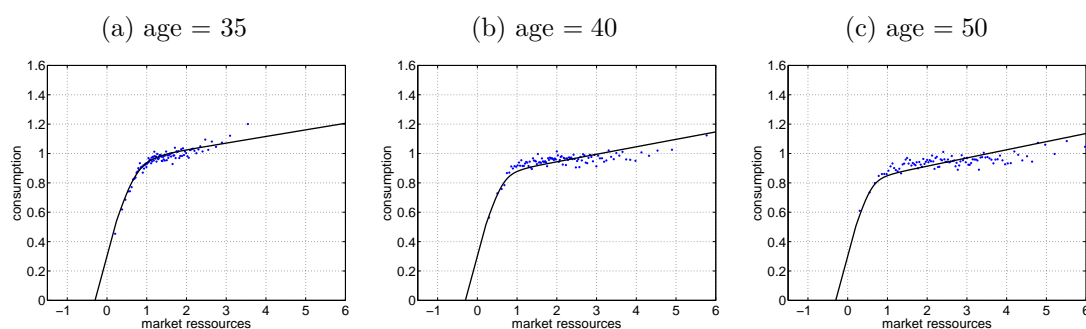
<sup>14</sup>In light of our high estimate of the measurement error in consumption, informative robustness checks might be to either estimate the model on a longer panel, or write the likelihood in terms of  $k$ -step predictions instead of exclusively in one-step predictions as used currently.

Figure 6.2: Consumption Functions - Low Skilled (No Heterogeneity)



Notes: The black lines are consumption functions from the estimated model, while the blue dots represent the average level of imputed consumption binned in percentiles of observed resources.

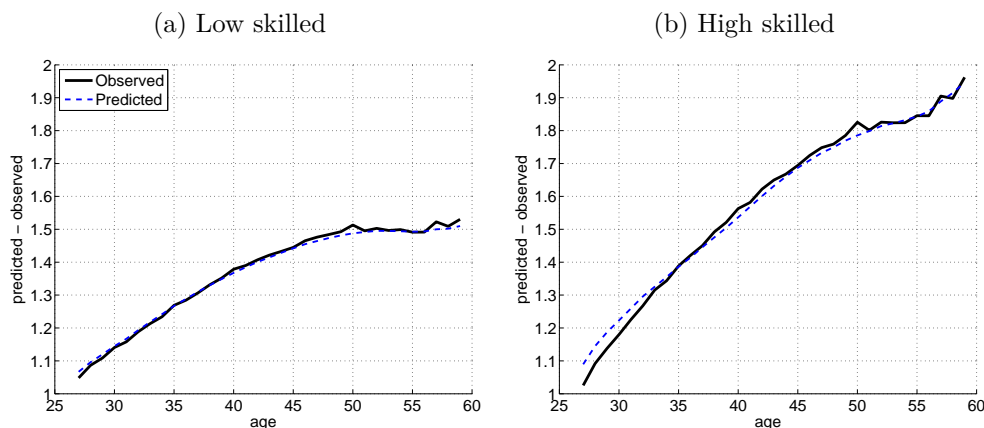
Figure 6.3: Consumption Functions - High Skilled (No Heterogeneity)



Notes: The black lines are consumption functions from the estimated model, while the blue dots represent the average level of imputed consumption binned in percentiles of observed resources.

The model fit for the case of homogeneous preferences is evaluated in figure 6.2 and 6.3, where the implied consumption functions are plotted against the average level of imputed consumption binned in percentiles of observed resources. We see that the model fits the overall pattern found in the data quite well. In figure 6.4, the *observed* and *predicted* level of consumption is plotted over the life cycle. This again indicates a quite good fit with a slight over-prediction for the young.

Figure 6.4: Prediction vs. Observed Consumption (No Heterogeneity)



## 6.1 Simulation: Matching Wealth Inequality

Although it is widely believed that preference heterogeneity is an important candidate in explaining the observed wealth inequality, relatively little research investigate how important preference heterogeneity is empirically. Most existing studies of wealth inequality and preference heterogeneity minimize some distance between the observed wealth distribution and the simulated wealth distribution from a model with heterogeneous preferences.<sup>15</sup> Those studies typically find that heterogeneity can – perhaps not surprisingly – explain almost the entire distribution of wealth (except the extreme top). Our estimator has deliberately *not* been formulated to match the empirical wealth distribution such that we can asses whether the uncovered preference heterogeneity can generate a similar dispersion in wealth as observed in the actual data.

We simulate data for 500.000 households using the estimated model. The initial distribution of resources  $M_t$  and permanent income  $P_t$  is drawn with replacement from the 26-28 aged households in our estimation sample using the weights defined in equation (3.11).<sup>16</sup> Using the *conditional* distribution of preferences defined by the weights in (3.11) allows for dependence between preferences and the initial level of resources observed in the data.

<sup>15</sup>Examples include [Hendricks \(2007\)](#) and [Carroll, Slacalek, Tokuoka and White \(2014\)](#).

<sup>16</sup>Using the *conditional* preference distribution is an improvement on just using the *unconditional* preference distribution, but we might still miss some correlation between initial resources and preferences. Given that the initial resources are small relative to those accumulated over the life-cycle we believe this is not a big problem.

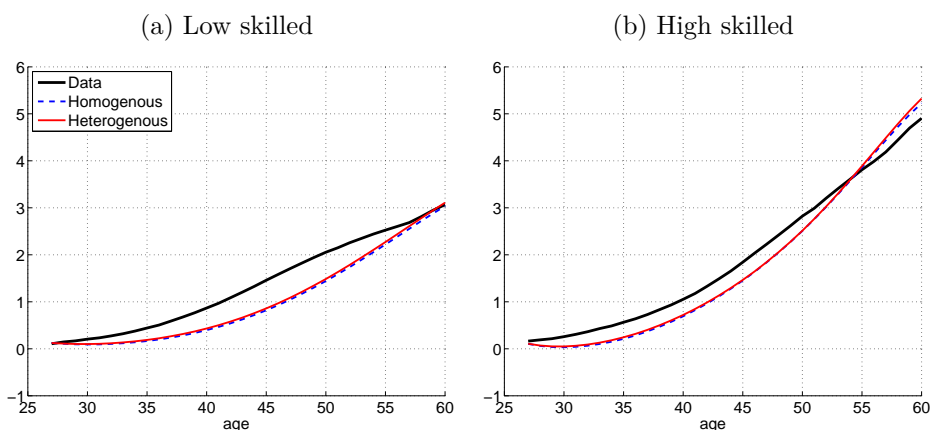
Figure 6.5: Average Simulated Wealth Age Profiles -  $A_t$ 


Figure 6.4 shows the average wealth age profiles (normalized with the average income of low-skilled 25 year-old households) in the Danish data and in simulated data from respectively the estimated homogeneous and heterogeneous models. The age profile of wealth is remarkably similar across the two models and mimics the actual age profile closely, despite it has not been targeted directly in our estimation. This is important because if we cannot simulate data from the estimated models that generate reasonably looking *average* age profiles, it seems unlikely that the simulated distribution of wealth will be anything close to the observed distribution.

Figure 6.6, shows that we wildly (and surprisingly) under-predict the level of wealth inequality. Although allowing for heterogeneous preferences increases the wealth dispersion, the interquartile range of wealth (75th - 25th percentile of  $A_t$ ) in the simulated data is significantly lower than in the data. In turn, this indicates that heterogeneity in patience and risk aversion only explains a rather limited part of the observed inequality of wealth. Particularly, while wealth inequality monotonically increases in the observed data, wealth inequality is almost constant until age 40 in the simulated data suggesting that the wealth distribution of older households are better matched by the model than that of younger households.

Figure 6.6: Interquartile Inequality - 75th-25th percentile of  $A_t$

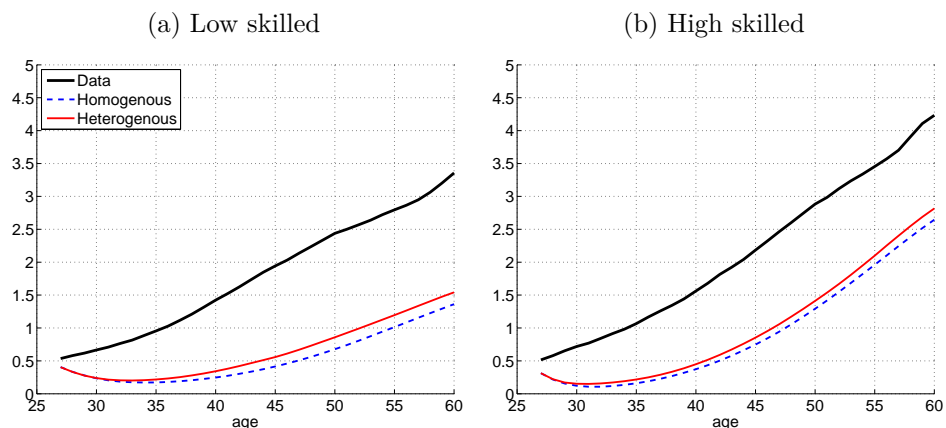
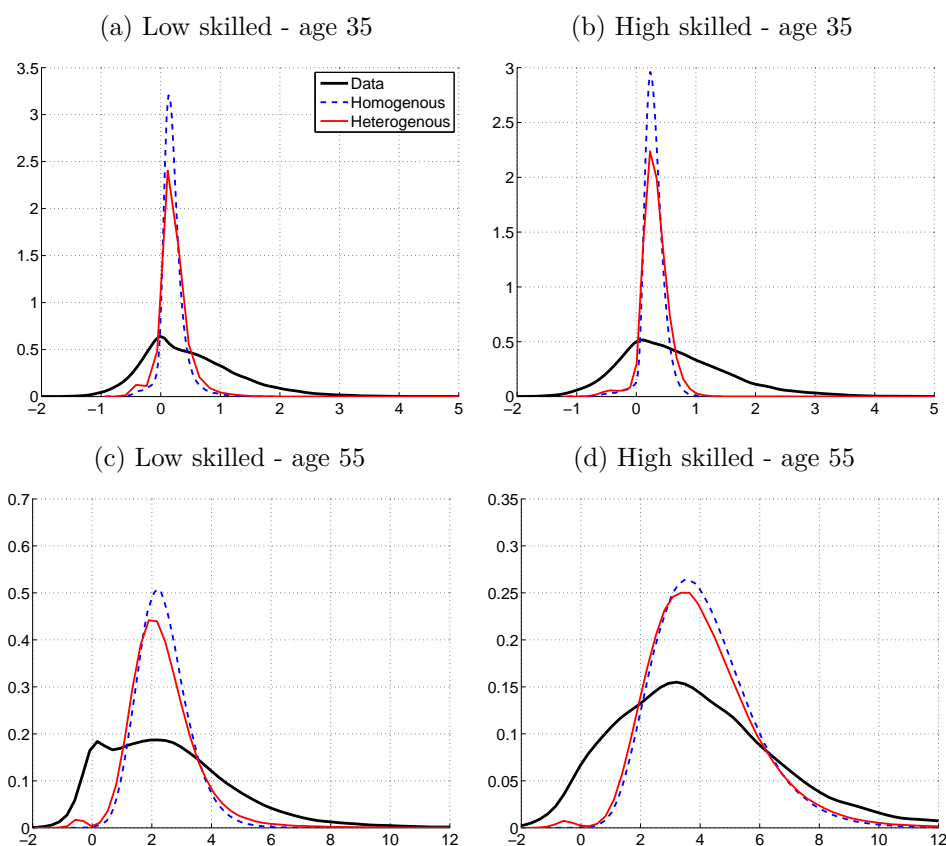


Figure 6.7 compares the distributions of wealth at age 35 and 55 in the data and the simulations. For both educational groups it is clear that the distribution of wealth at old age is much wider, but also better captured by the estimated model than for younger households.

Figure 6.7: Distributions of  $A_t$



Notes: The plots contain kernel smoothed densities.

## 7 Conclusion

Although it is widely believed that preference heterogeneity is an important candidate in explaining the observed wealth inequality, relatively little research investigate how important preference heterogeneity is empirically. A recent strand of literature estimate preference heterogeneity by matching moments (e.g., percentiles) of the wealth distribution and document significant heterogeneity in preferences.<sup>17</sup> Such an approach, however, only shows that heterogeneity *can* explain the observed wealth inequality.

We have proposed a novel non-parametric maximum likelihood estimator (NPMLE) to uncover the joint distribution of heterogeneous preferences. Our estimator does not match moments of the wealth distribution directly, and it can thus be used to uncover the importance of preference heterogeneity for wealth inequality. The non-parametric distribution of heterogeneity is instead estimated using systematic variation in the consumption decisions across households conditioning on the observed level of resources using a full structural model. We have implemented the NPMLE on high quality Danish register data and estimate a standard life cycle model allowing for heterogeneity in patience and risk aversion.

We find substantial preference heterogeneity within educational strata, especially in risk aversion. For the low skilled, we find discount factors that vary over the range  $[0.960, 0.980]$  and relative risk aversion coefficients in the range  $[0.82, 4.14]$ . For the high skilled, the means of the estimated marginal distributions are shifted upwards with a substantial share of the population in the range  $[0.976, 1.001]$  for the discount factor and  $[1.53, 4.38]$  for the relative risk aversion coefficient. Within both education strata, we also find evidence of a positive correlation between patience and risk aversion.

The estimated model fits the data quite well and the simulated average life cycle profiles of wealth are close to those observed in the registers – although our estimator does not explicitly try to match the simulated wealth age profile to the observed one. The estimated heterogeneity can, however, not explain the observed inequality of wealth over the life cycle. This result suggests that heterogeneity in patience and risk aversion only explains a limited part of the observed wealth inequality.

Admittedly, the life cycle model we estimate is rather simplistic and has been

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<sup>17</sup>See e.g. [Carroll, Slacalek, Tokuoka and White \(2014\)](#).



implemented because it is one of the workhorses in intertemporal allocation of resources. We imagine several interesting extensions for future research. Including housing, labor supply or a more detailed treatment of retirement are obvious candidates, which could improve our model's ability to explain the accumulation of wealth inequality over the life cycle. Adding additional heterogeneity – for example in the income process or in a bequest motive – could also help in this regard. It would also be interesting to thoroughly investigate whether households in the extreme ends of the wealth distribution, which we currently do not match with the model, share some common characteristics. Finally it would also be interesting to uncover the resulting correlation in preferences across generations and within families.

## A Additional Figures and Tables

Figure A.1: Life Cycle Profiles -  $P_t$

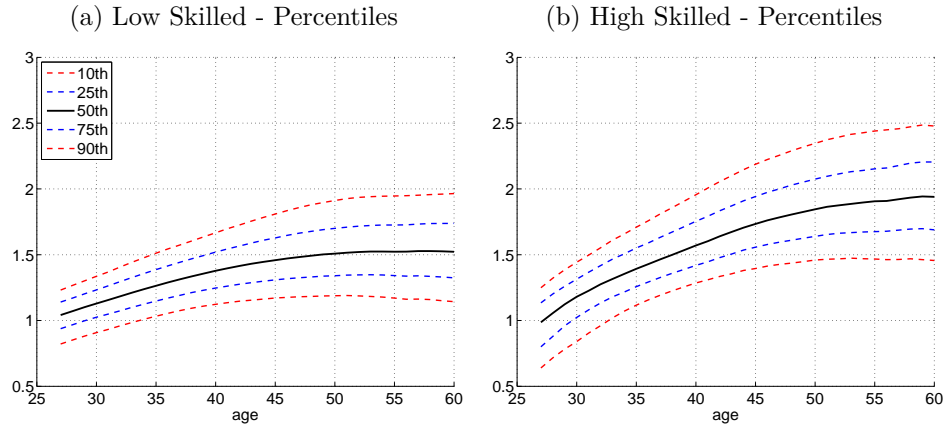
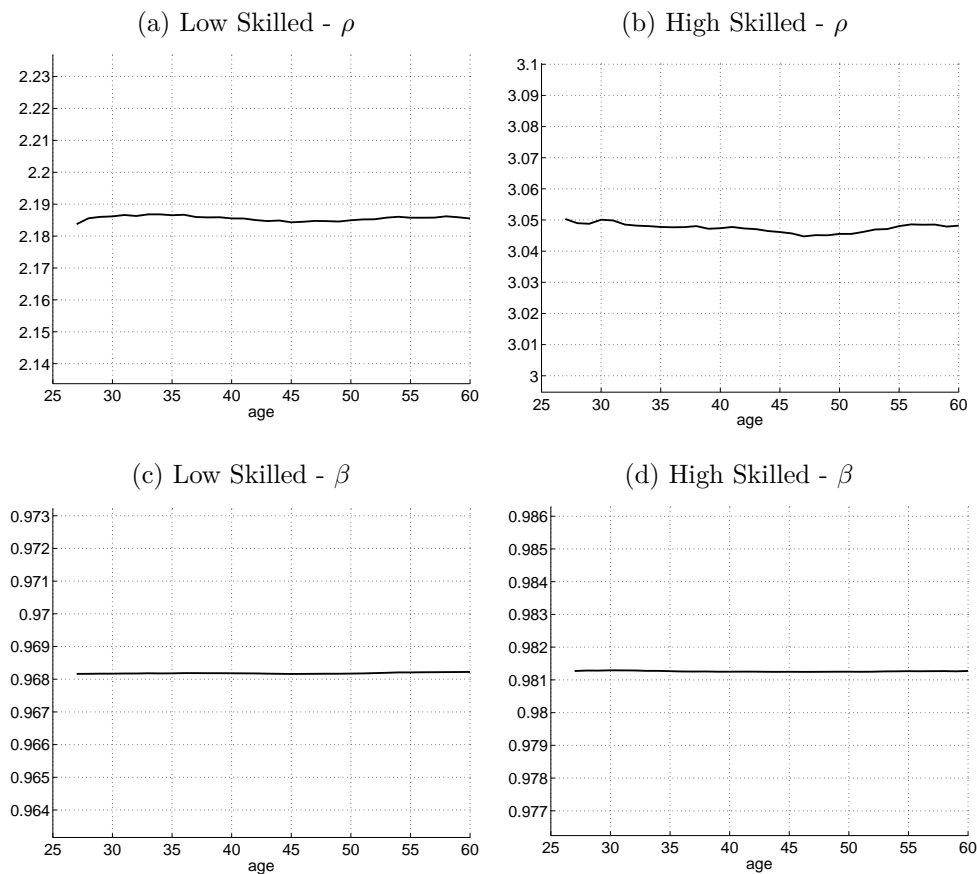


Figure A.2: Mean  $\rho$  and  $\beta$  drawing from  $g_i(\beta, \rho)$



Notes: At each  $t$  we draw  $\beta$  and  $\rho$  using a two step approach: Using equal weights we first draw (with replacement) a household who is in the sample at age  $t$ , and then we secondly draw  $\beta$  and  $\rho$  from their conditional preference distribution  $g_i(\beta, \rho)$ .

## B Model Details

**Proposition B.1.** *The optimal end-of-period asset choice satisfies*

$$A_t \geq \underline{A}_t = \begin{cases} 0 & \text{if } t = T \\ -\min\{\Omega_t, \lambda_t\} \cdot \Gamma_t & \text{if } t < T \end{cases}$$

where

$$\begin{aligned} A_t &\equiv \begin{cases} R^{-1} \cdot \Gamma_t \cdot \underline{\xi} & \text{if } t = T - 1 \\ R^{-1} \cdot [\min\{\Lambda_{T-1}, \lambda\} + \underline{\xi}] \cdot \Gamma_t & \text{if } t < T - 1 \end{cases} \\ \Gamma_t &\equiv G_t \cdot \underline{\psi} \end{aligned}$$

*Proof.* Let  $\mathbb{E}_t[\bullet]$  denote the *worst-case* expectation operator given information  $t$ . Note that any  $M_T \leq 0$  implies that the household cannot choose a  $C_t > 0$  such that  $A_t \leq 0$ . Consequently

$$\lim_{M_t \searrow 0} V_T(\bullet, M_T) = \lim_{C_t \searrow 0} \frac{C_t^{1-\rho}}{1-\rho} = -\infty$$

which the household want to avoid at any cost. Therefore we have

$$\begin{aligned} \mathbb{E}_{T-1}[M_T - \underline{A}_T] &> 0 \Leftrightarrow \\ \mathbb{E}_{T-1}[R \cdot A_{T-1} + Y_T] &> 0 \Leftrightarrow \\ R \cdot A_{T-1} + \Gamma_T \cdot \underline{\xi} \cdot P_{T-1} &> 0 \Leftrightarrow \\ A_{T-1} &> -R^{-1} \cdot \Gamma_T \cdot \underline{\xi} \cdot P_{T-1} \end{aligned}$$

Combining this with the exogenous borrowing constraint we get

$$A_{T-1} > -\min\{\Lambda_{T-1}, \lambda\} \cdot P_{T-1}$$

Similar arguments further implies

$$\begin{aligned} \mathbb{E}_{T-2}[M_{T-1} - \min\{\Lambda_{T-1}, \lambda\} \cdot P_{T-1}] &> 0 \Leftrightarrow \\ \mathbb{E}_{T-1}[R \cdot A_{T-2} + Y_{T-1}] &> -\min\{\Lambda_{T-1}, \lambda\} \cdot \mathbb{E}_{T-1}[P_{T-1}] \Leftrightarrow \\ R \cdot A_{T-2} + G_{T-1} \cdot \underline{\psi} \cdot \underline{\xi} \cdot P_{T-2} &> -\min\{\Lambda_{T-1}, \lambda\} \cdot G_{T-1} \cdot \underline{\psi} \cdot P_{T-2} \Leftrightarrow \\ A_{T-2} &> \underbrace{-R^{-1} \cdot [\min\{\Lambda_{T-1}, \lambda\} + \underline{\xi}] \cdot \Gamma_{T-1} \cdot P_{T-2}}_{=\Lambda_{T-2}} \end{aligned}$$

□

## C Data

### C.1 Income Definitions

In the Danish income registers, we have the following income variables:

$$\begin{aligned}
 \underbrace{\text{DISPON\_NY}}_{\text{disposable income}} &= \text{SAMLINK\_NY} - \underbrace{\text{SKATMVIALT\_NY}}_{\text{taxes}} \\
 &\quad - \underbrace{\text{QRENTUD2}}_{\text{interest payments}} - \underbrace{\text{UNDERHOL} + \text{TBKONTHJ}}_{\text{alimony+returned benefits}} \\
 \underbrace{\text{SAMLINK\_NY}}_{\text{total income}} &= \text{PERINDKIALT} \\
 &\quad + \underbrace{\text{OVSKEJD02\_NY} + \text{OVERSKEJD07}}_{\text{imputed rental value}} \\
 \underbrace{\text{PERINDKIALT}}_{\text{total monetary income}} &= \underbrace{\text{RENTEINDK}}_{\text{interest income}} + \underbrace{\text{PEROEVRIFORMUE}}_{\text{other property income}} + \\
 &\quad \underbrace{\text{ERHVERVSINDK}(\_GL)}_{\text{wages and profits}} + \underbrace{\text{OVERFORSINDK}}_{\text{public transfers}} \\
 &\quad + \underbrace{\text{RESUINK}(\_GL)}_{\text{other income}}
 \end{aligned}$$

We define

$$\begin{aligned}
 Y_{ikt}^{gross} &\equiv \text{PERINDKIALT} \\
 Y_{ikt}^{assets} &\equiv \text{RENTEINDK} + \text{PEROEVRIFORMUE} \\
 Y_{ikt}^{nonassets} &\equiv \text{PERINDKIALT} - Y_{ikt}^{assets} \\
 Y_{ikt}^{transfers} &\equiv \text{OVERFORSINDK} \\
 S_{ikt} &\equiv \text{SKATMVIALT\_NY} \\
 Y_{ikt}^{nom} &\equiv \begin{cases} Y_{ikt}^{gross} - S_{ikt} & \text{if } \left| \frac{Y_{ikt}^{assets}}{Y_{ikt}^{gross}} \right| < 0.1 \\ (1 - \tau_{ikt}) \cdot Y_{ikt}^{nonassets} - & \text{else} \end{cases}
 \end{aligned}$$

where  $i$  is for a couple,  $t$  is for age,  $k$  is for observation year, and  $Y_{ikt}^{nom}$  is *after-tax monetary income from all sources, except financial assets*. To approximate the after tax earnings of households with substantial income from financial assets, we use the tax rate  $\tau_{ikt} \equiv \frac{S_{ikt}}{Y_{ikt}^{gross}}$  of households *without* substantial income from financial assets, but with a similar level of non-asset income (specifically we use twenty bins of  $Y_{ikt}^{nonassets}$ ).

## C.2 Data Construction

We construct our variables as follows:

1. **Couples** are constructed using *EFALLE* (from BEF) (before 1987 we only have C\_FAELLE\_ID from FAIN).
2. **Birthyear** and **gender** is based on FOED\_DAG and KOEN (from BEF) or if not available ALDER and KOEN (from FAIN). Couple age is the age of the male.
3. **Wealth**  $A_{ikt}^{nom}$  is the total net wealth excluding pensions (FORM and FORM-REST\_NY05 (after 1996) from INDH) adjusted upwards with 10 percent of the value of any owned properties  $H_{ikt}^{nom}$  (KOEJD or if missing EJEN-DOMSVURDERING from INDH).
4. **Self-Employment** is coded as  $PSTILL \leq 20$  (from IDAP).
5. **Retirement** is coded as PSTILL in  $\{50, 55, 92, 93, 94\}$  (from IDAP).
6. A couple is coded as **high-skilled** if at least one of them has  $\geq 180$  months of education (using HFPRIA from UDDA); otherwise it is coded as **low-skilled**.

## C.3 Sample Selection

We use the following iterative selection criteria:

1. Our baseline sample is all unique couples, where the male is older than 18 and is in the income registers sometimes between 1987 and 1996 (both included).
2. Both partners are between age 25 and 59 (both included).
3. The age difference is not larger than 5 years.
4. Neither of them are ever self-employed (see definition in sub-section C.2).
5. Neither of them retire before age 59 (see definition in sub-section C.2).
6. Information on wealth and income is non-missing.
7. Income is strictly positive.
8. Education information is not missing for both partners.
9. Extreme observations are disregarded:

Table C.1: Sample Selection

	Unique Couples	Observations
1. Baseline	1.929.855	12.830.368
2. Age between 25 and 59	1.198.380	8.521.870
3. Age difference $\leq 5$ years	1051.457	6.859.912
4. Never self-employed	845.425	5.306.158
5. Not retired before age 59	809.922	5.067.903
6. Not missing income or wealth	809.922	5.067.903
7. Income is positive	808.714	5.064.849
8. Education information not missing	801.063	5.037.439
9. Extreme experiences	546.795	3.402.515
10. $\geq 5$ observations	336.017	2.953.594
<i>hereof with lagged wealth</i>	336.017	2.607.510
<i>hereof high-skilled</i>	82.545	729.106

(a) We define

$$\begin{aligned}
 M_{ikt}^{raw} &\equiv R \cdot A_{ik,t-1}^{raw} + Y_{ikt}^{raw} \\
 C_{ikt}^{raw} &\equiv M_{ikt}^{raw} - A_{ikt}^{raw} \\
 m_{ikt}^{raw} &\equiv \frac{M_{ikt}^{raw}}{Y_{ikt}^{raw}} \\
 c_{ikt}^{raw} &\equiv \frac{C_{ikt}^{raw}}{Y_{ikt}^{raw}} \\
 a_{ikt}^{raw} &\equiv \frac{A_{ikt}^{raw}}{Y_{ikt}^{raw}}
 \end{aligned}$$

and we remove all households that in *any* age-year bin has a  $Y_{ikt}^{raw}$ ,  $A_{ikt}^{raw}$ ,  $M_{ikt}^{raw}$ ,  $C_{ikt}^{raw}$ ,  $a_{ikt}^{raw}$ ,  $m_{ikt}^{raw}$ , or  $c_{ikt}^{raw}$  below the 1st percentile or above the 99th percentile.

(b) We remove all households who have

$$A_{ikt}^{raw} < \max \{-2 \cdot Y_{ikt}^{raw}, -(0.5 \cdot Y_{ikt}^{raw} + 0.2 \cdot H_{ikt}^{raw})\}$$

10. It is observed for 5 years or more.

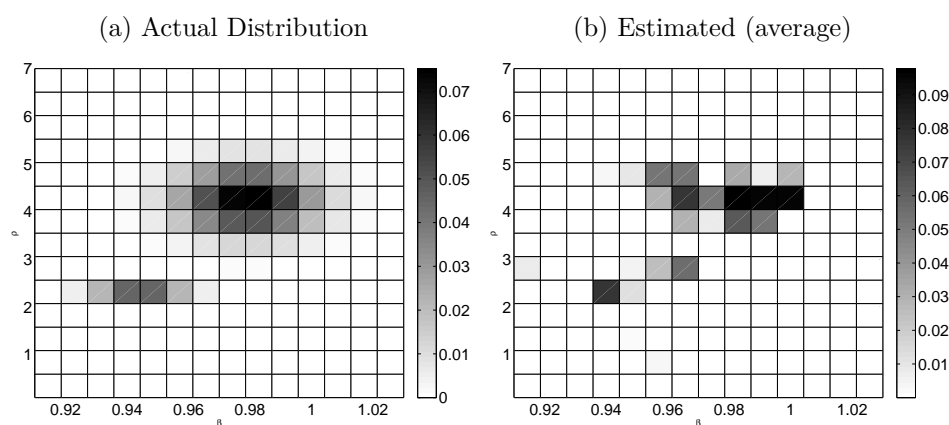
Table C.1 shows how the sample size is affected by these choices.

## D Alternative Monte Carlo Study

In our alternative Monte Carlo study, we first assume a joint distribution  $f(\rho, \beta)$  and calculate the model-implied normalized level of consumption  $c_{it}^*$  for each  $m_{it}$  actually observed in our data (also using the homogeneous parameters estimated in section 6). We then add measurement error with  $\sigma_\varepsilon = 0.30$  to the model-implied consumption, and create a new Monte Carlo data set with the exact same dimensions and  $m_{it}$ 's as the actual data set by randomly drawing which preferences each household has using  $f(\rho, \beta)$ . Finally, we run our estimation algorithm 50 times on the new data set drawing new measurement error for each repetition.

Panel (a) in figure D.1 shows the assumed joint distribution, and panel (b) shows the average estimated weights across the 50 repetitions. The estimator catches a high level of preference heterogeneity though the results are somewhat less precise than those in the Monte Carlo study in section 3.

Figure D.1: Alternative Monte Carlo Estimation of Weights,  $\omega$ .



*Notes:* We use the data and parameters for the low-skilled households. The average is over 50 repetitions.

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# Chapter 4

Business Cycle Fluctuations

in the Demand for Consumer Durables

# Business Cycle Fluctuations in the Demand for Consumer Durables\*

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## Abstract

This paper studies the effect of cyclical variation in both idiosyncratic income risk and in the tightness of the collateral constraint on the demand for consumer durables in a generalized buffer-stock consumption model. The proposed model includes both transaction costs, an outside option of renting, non-separable utility, and taste shocks. It is shown that empirically plausible fluctuations in income risk and in the down payment ratio, roughly double the model-implied cyclical drop in the share of households who adjust their durable stock during recessions compared to in expansions. This is moreover achieved without any cyclical decrease in the intensive margin in terms of expenditures per purchase, which is contrarily the case when the drop is induced by e.g. lower income growth or higher unemployment.

**Keywords:** Durable Consumption, Income Uncertainty, Collateral Constraints, Business Cycles.

**JEL-Codes:** E21, E32, D91.

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\*This paper is part of a larger project on explaining the business cycle fluctuations in car demand observed in the Danish register data, which I am conducting together with Thomas Høgholm Jørgensen and Anders Munk-Nielsen. I thank them for many fruitful discussions on the subject of the present paper. I am also thankful to Matthias Meier for some very useful discussions. Naturally the normal disclaimer applies.

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# 1 Introduction

Spending on consumer durables is highly pro-cyclical and 4-5 times as volatile as GDP over the business cycle.<sup>1</sup> Romer (1990) has even argued that a central component in explaining the onset of the *Great Depression* was the fall in purchases of consumer durables induced by the uncertainty created by the stock market crash of October 1929. Furthermore, spending on consumer durables is often found to be highly responsive to both income and wealth shocks.<sup>2</sup> Understanding the household decision process underlying this volatility is thus of central importance for understanding business cycle fluctuations and designing economic stabilization policy.

A central stylized fact is that most of the fluctuations in the demand for consumer durables is on the *extensive* margin in the form of households postponing adjustment altogether. In the case of cars, a large and important consumer durable, Hassler (2001) thus concludes that “while *average expenditures per purchase* are well predicted by the permanent income hypothesis, the *number* of purchases is much more volatile and may have a short-run elasticity with respect to permanent income several times higher than unity.”<sup>3</sup> Theoretically, it is well-known that  $(S, s)$ -type models a la Arrow, Harris and Marschak (1951) and Scarf (1959) can fit such behavior, at least *qualitatively*.<sup>4</sup>

In a recent paper, Berger and Vavra (2015) estimate a state-of-the-art version of a buffer-stock  $(S, s)$ -model with a broad durable (including both houses and cars) on household-level data from the *Panel Study of Income Dynamics* (PSID). They show that their model is able to explain a large fraction of the cross-sectional variation in the probability to adjust, but in the time dimension the model-implied movements in the hazard rate are much weaker than found empirically (e.g. comparing their figure 11 and 12).

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<sup>1</sup> See e.g. Stock and Watson (1999).

<sup>2</sup> In relation to income shocks see e.g. Browning and Crossley (2009), Aaronson, Agarwal and French (2012), Parker, Souleles, Johnson and McClelland (2013), Kreiner, Lassen and Petersen (2014), and Misra and Surico (2014). In relation to wealth shocks see in particular Mian, Rao and Sufi (2013). The evidence is especially strong for larger shocks and for households with relatively few liquid assets.

<sup>3</sup> See also Lam (1991), Bar-Ilan and Blinder (1992), Eberly (1994), Attanasio (2000) and Foote, Hurst and Leahy (2000).

<sup>4</sup> See in particular McDonald and Siegel (1986), Lam (1989), Grossman and Laroque (1990), Pindyck (1991), Caballero (1990, 1993), Hassler (1996), Adda and Cooper (2000, 2007) and Bertola, Guiso and Pistaferri (2005).

The present paper's main contribution is to formally show that adding cyclical variation in the level of idiosyncratic income risk<sup>5</sup> and in the tightness of the collateral constraint, both strongly amplify the model's ability to explain business cycle fluctuations in the share of adjusters. Neither of these mechanisms have previously been added to a full buffer-stock model a la [Deaton \(1991, 1992\)](#) and [Carroll \(1992, 1997\)](#).<sup>6</sup> Including both transaction costs and an outside option of renting make it a complicated model to solve. Using recent advances in numerical methods on solving non-convex dynamic programming models<sup>7</sup>, however, make it possible to discuss the optimal policy functions in great detail, and conduct a large number of robustness checks.

If recessions are characterized solely by a 3 percentage point drop in income growth and a 6 percentage point increase in unemployment, we find that the model imply a 2.2 percentage points drop in the yearly share of household who adjust their durable stock in recessions compared to in expansions. The drop is increased to 4.5 percentage points when adding cyclical variation in income risk and collateral tightness in the form of, respectively, an increase in the standard deviation of the permanent income shock from 0.1 to 0.2, and an increase in the down payment ratio from 10 to 20 percent. It is furthermore shown that this large effect on the *extensive* margin is achieved while affecting the *intensive* margin (the average expenditures per purchase) in the *opposite* direction; falling income growth and increasing unemployment contrarily have a large negative effect on the intensive margin compared to their effect on the extensive margin.

The effect on the cyclical drop in the adjuster share is also large when each of the two proposed mechanisms are added to the model on their own. The effect of adding them simultaneously give a combined effect which is substantially smaller than the sum of the separate effects, thus showing that they negate each other somewhat. The intuition is that a stronger precautionary saving motive, due to the increased idiosyncratic income risk, makes the households aim at a so high level of savings in recessions that changes in the collateral constraint is not important for most households.

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<sup>5</sup> As documented in [Storesletten, Telmer and Yaron \(2004\)](#) and [Guvenen, Ozkan and Song \(2014\)](#).

<sup>6</sup> See, however, [Carroll and Dunn \(1997\)](#) for an early attempt where, given purchase, the size of the durable (in the form of a house) was fixed at a pre-specified scalar relative to permanent income.

<sup>7</sup> In particular [Clausen and Strub \(2013\)](#), [Fella \(2014\)](#) and [Iskhakov, Jørgensen, Rust and Schjerning \(2014\)](#).



Finally, it is shown that the effect from increased idiosyncratic income risk is almost purely due to expectations, and that pessimism in this dimension could thus amplify the model-implied cyclical drop in the adjuster share even further.

Beginning with [Bloom \(2009\)](#), and as surveyed in [Bloom \(2014\)](#), fluctuations in uncertainty has recently also been used, very successfully, to explain business cycle fluctuations in firm investment. [McKay \(2015\)](#) has likewise shown that fluctuations in idiosyncratic income risk is important for non-durable consumption. In an empirical context [Johnson, Pence and Vine \(2014\)](#) and [Benmelech, Meisenzahl and Ramcharan \(2015\)](#) have recently argued that a contraction in the supply of credit was important for the collapse of car sales in the US during the Great Recession.

Additionally, the present paper is related to [Luengo-Prado \(2006\)](#) who analyze the properties of a similar model without fluctuations in neither idiosyncratic income risk nor in the tightness of the collateral constraint. Other related papers are [Fernandez-Villaverde and Krueger \(2011\)](#) and [Cerletti and Pijoan-Mas \(2014\)](#) who use buffer-stock models extended with a durable good not subject to any transaction costs, to study respectively life cycle dynamics and the transmission of income shocks into non-durable consumption. Finally, [Iacoviello and Pavan \(2013\)](#) study the effect of *secular* changes in idiosyncratic income risk and mortgage down payment requirements on housing investment volatility in a general equilibrium model.

The rest of the paper is structured as follows: The model is presented in section 2 and the chosen parametrization explained in section 3. Section 4 presents and discusses the optimal choice functions, while the central simulation results are presented in section 5. Section 6 contains a number of robustness checks, and section 7 concludes. Some further details on the model is included in appendix A, and the solution algorithm is described in detail in appendix B.

## 2 Model

**States** We consider unitary households who potentially live forever, but who are subject to an exogenous and constant death probability of  $\zeta \geq 0$ . The households are characterized by the following four *idiosyncratic* state variables:

1.  $A_{t-1}$ : End-of-period (net) financial assets.
2.  $D_{t-1}$ : End-of-period durable stock (depreciation rate,  $\delta \in [0, 1]$ ).

3.  $b_{t-1} \in \{E, R\}$ : A collateral constraint tightness indicator.

4.  $P_t$ : *Permanent* (non-financial) income.

Additionally,  $z_t \in \{E, R\}$  is an *aggregate* state variable. If  $z_t = E$  the economy is in an *expansion*, and if  $z_t = R$  it is in a *recession*. The probability of staying in an *expansion* is given by  $\pi_{EE}$ , and the probability of staying in a *recession* is given by  $\pi_{RR}$ .

**Choices** In each period the households have to make a *discrete choice* between:

1. Continuing to *own* the same durable ( $k_t = \textit{keep}$ ).
2. *Buying* a new durable ( $k_t = \textit{adj.}$ ).
3. Only *rent* durable services ( $k_t = \textit{rent}$ ).

The full discrete choice set is thus given by  $\mathcal{K} \equiv \{\textit{keep}, \textit{adj.}, \textit{rent}\}$ .

Durable owners furthermore choose *consumption*  $C_t$  and if adjusting also how large a *durable*  $D_t$  to purchase, where the implied level of durable services is given by  $S_t = D_t$ . Durable renters choose *consumption*  $C_t$  and how much durable service  $S_t$  to rent at the price  $r_d$ .

**Preferences** The per-period utility function is

$$u(C_t, S_t) = \frac{g(C_t, S_t)^{1-\rho}}{1-\rho}, \quad \rho > 1 \quad (2.1)$$

$$g(C_t, S_t) = C_t^\omega S_t^{1-\omega}, \quad \omega \in [0, 1] \quad (2.2)$$

where  $\rho$  is the relative risk aversion coefficient and  $\omega$  is the utility weight on non-durable consumption. Future utility is discounted by  $\beta < 1$  (including the death probability).

For a renter the choice of  $C_t$  and  $S_t$  is purely *intra-temporal*, and given *total consumption expenditures*  $E_t = C_t + r_d \cdot S_t$  standard results imply

$$C_t = c(E_t) = \omega \cdot E_t \quad (2.3)$$

$$S_t = s(E_t) = \frac{1-\omega}{r_d} \cdot E_t \quad (2.4)$$

The households are furthermore also subject to *taste shocks* over the discrete choices, which are known in the beginning of the period, and all follow extreme

value distributions, i.e.

$$\begin{aligned} \forall k_t \in \mathcal{K} : \Phi_t^k &= \sigma_\zeta \cdot \varsigma_{tk} \cdot P_t^{1-\rho} \\ \varsigma_{tk} &\sim \text{i.i.d. } EV \end{aligned} \quad (2.5)$$

Adding taste shocks in this way implies that the hazard rates of adjustment is not exclusively zero or one, but can take on any value between zero and one as in the generalized  $(S, s)$ -model proposed by Caballero and Engel (1999).

**Income Process** In the beginning of each period, households receive a stochastic income given by

$$Y_t = \tilde{\xi}_t \cdot P_t \quad (2.6)$$

$$P_t = \Gamma_{z_t} \cdot \psi_t \cdot P_{t-1} \quad (2.7)$$

where  $P_t$  is permanent income,  $\Gamma_{z_t}$  is the gross growth rate of permanent income,  $\psi_t$  is a mean-one *permanent* shock to income, and  $\tilde{\xi}_t = \tilde{\xi}(u_t, \xi_t)$  is a compound *transitory* shock to income given by

$$\tilde{\xi}(z_t, \xi_t) = \begin{cases} \mu & \text{with probability } \pi_{uz_t} \\ (\xi_t - \mu u_*) / (1 - u_*) & \text{with probability } 1 - \pi_{uz_t} \end{cases} \quad (2.8)$$

where  $\pi_{uz_t}$  is the risk of unemployment,  $\mu$  is unemployment benefits (relative to permanent income),  $u_*$  is the long-run average rate of unemployment, and  $\xi_t$  is a mean-one log-normal shock

$$\xi_t \sim \text{i.i.d. } \log \mathcal{N}(-0.5 \cdot \sigma_\xi^2, \sigma_\xi^2) \quad (2.9)$$

To be able to account for changes in both the *variance* and in the *skewness* of the permanent shocks over the business cycle, we assume that  $\psi_t$  is distributed as a mixture of two log-normals, i.e.

$$\psi_t \sim \begin{cases} \text{i.i.d. } \log \mathcal{N}(\bar{\mu}_{\psi z_t}, \sigma_{\psi z_t}^2) & \text{with probability } \pi_\psi \\ \text{i.i.d. } \log \mathcal{N}(\bar{\mu}_{\psi z_t} + \mu_{\psi z_t}, \sigma_{\psi z_t}^2) & \text{with probability } 1 - \pi_\psi \end{cases} \quad (2.10)$$

where  $\bar{\mu}_{\psi z_t}$  is a normalization constant ensuring a mean of one.<sup>8</sup>

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<sup>8</sup>  $\bar{\mu}_{\psi z_t} = -\log \left( \pi_\psi \cdot \exp \left( -0.5 \cdot \sigma_\psi^2 \right) + (1 - \pi_\psi) \cdot \exp \left( -0.5 \cdot \sigma_\psi^2 + \mu_{\psi z_t} \right) \right)$ .

To ease the parametrization we also define

$$\Delta\pi_u \equiv \pi_{uR} - \pi_{uE} \quad (2.11)$$

$$\Delta\Gamma \equiv \Gamma_E - \Gamma_R \quad (2.12)$$

and denote the long-run average aggregate growth rate by  $\Gamma_*$ .

**Transaction Costs** We assume a *monetary transaction cost function* given by

$$\begin{aligned} \Lambda(D_t, D_{t-1}) &= \mathbf{1}_{D_t \neq (1-\delta) \cdot D_{t-1}} \cdot [\tau_b \cdot D_t + \tau_s \cdot (1-\delta) \cdot D_{t-1}] \quad (2.13) \\ \tau_b &\geq 0, \tau_s \in [0, 1] \end{aligned}$$

where  $\tau_b$  is an adjustment cost on *buying* (fees, search costs etc.) and  $\tau_s$  is a *resale loss*. We also allow for a breakdown risk of  $\bar{\tau}$ . For simplicity we assume that an universal insurance company pays the households the full sale value *after* transaction costs, i.e.  $(1-\tau_s) \cdot (1-\delta) \cdot D_{t-1}$ , in the case of a breakdown.

**Assets and Borrowing** Focusing on *keepers* and *adjusters* end-of-period *financial net worth*  $A_t$  is given by

$$\begin{aligned} A_t &= (1+r(A_{t-1})) \cdot A_{t-1} \quad (2.14) \\ &\quad + Y_t - [C_t + (D_t - (1-\delta) \cdot D_{t-1}) + \Lambda(D_t, D_{t-1})] \end{aligned}$$

where the interest rate function is

$$r(A_{t-1}) = \begin{cases} r & \text{if } A_{t-1} \geq 0 \\ r_b & \text{if } A_{t-1} < 0 \end{cases} \quad (2.15)$$

The households do not have access to credit cards or overdrafts, but can borrow with their durable as collateral, implying that end-of-period assets must satisfy

$$A_t \geq -(1-\theta_{b_t}) \cdot D_t, \quad \theta_{b_t} \in [0, 1] \quad (2.16)$$

where  $b_t \in \{E, R\}$  is an indicator for the tightness of the collateral constraint, and  $\theta_{b_t}$  is the resulting down payment ratio. In practice we will have  $\theta_R \geq \theta_E$  such that the households need to pay a larger down payment in recessions than in expansions. To let the collateral constraint depend solely on the aggregate state would be problematic because most durable loans are installment loans,

where the lending company cannot require that the households pay a huge extra installment once a recession hits. Instead, we therefore let households who bought their durable in an expansion (or have refinanced during an expansion) stay with their loose collateral constraint, i.e. we assume

$$b_t = \begin{cases} z_t & \text{if } k_t \in \{adj., rent\} \\ E & \text{if } k_t = keep \text{ and } z_t = E \\ b_{t-1} & \text{if } k_t = keep \text{ and } z_t = R \end{cases} \quad (2.17)$$

This is somewhat of a short cut, but we avoid to specify a detailed loan contract and to add the debt principal as a new (continuous) state and choice variable. We have chosen to let the households refinance freely to the loose collateral constraint in expansions in order to avoid an increase in the adjustment probability in expansions for this reason.<sup>9</sup>

**Helping Variables** It is useful to introduce respectively beginning-of-period *market resources*  $M_t$  defined by

$$M_t \equiv (1 + r(A_{t-1})) \cdot A_{t-1} + Y_t \quad (2.18)$$

beginning-of-period *total net worth after liquidation*  $X_t$  defined by

$$X_t \equiv M_t + (1 - \tau_s) \cdot (1 - \delta) \cdot D_{t-1} \quad (2.19)$$

and beginning-of-period *durable stock*  $\bar{D}_t$  defined by

$$\bar{D}_t \equiv (1 - \delta) \cdot D_{t-1} \quad (2.20)$$

For later use we also define end-of-period net wealth as

$$N_t = A_t + D_t \quad (2.21)$$

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<sup>9</sup> The first best would naturally be to introduce some kind of refinancing costs, but this would also imply that the debt principal should be added as a new state and choice variable.

**Bellman Equation** In total the household optimization problem can be written as the upper envelope of the discrete choice specific value functions

$$V\left(z_t, b_{t-1}, P_t, X_t, \bar{D}_t \mid \left\{\Phi_t^k\right\}_{k \in \mathcal{K}}\right) = \max_{k \in \mathcal{K}} \left\{V^k + \Phi_t^k\right\} \quad (2.22)$$

The choice specific value functions all share the following common constraints

$$A_t = \begin{cases} X_t - [(1 - \tau_s) \cdot \bar{D}_t + C_t] & \text{if } k_t = \textit{keep} \\ X_t - [(1 + \tau_b) \cdot D_t + C_t] & \text{if } k_t = \textit{adj.} \\ X_t - E_t & \text{if } k_t = \textit{rent} \end{cases} \quad (2.23)$$

$$\bar{D}_{t+1} = \begin{cases} (1 - \delta) \cdot D_t & \text{if } \iota_{t+1} > \bar{\iota} \\ 0 & \text{else} \end{cases} \quad (2.24)$$

$$X_{t+1} = (1 + r(A_t)) \cdot A_t + Y_{t+1} + (1 - \tau_s) \cdot (1 - \delta) \cdot D_t \quad (2.25)$$

$$A_t \geq -(1 - \theta_{b_t}) \cdot D_t \quad (2.26)$$

and are otherwise given by

$$V^{\textit{keep}}(z_t, b_{t-1}, P_t, \bar{D}_t, X_t) = \max_{C_t \in \mathbb{R}_+} u(C_t, D_t) + \beta \cdot \mathbb{E}_t[W(\bullet_{t+1})] \quad (2.27)$$

s.t

$$D_t = \bar{D}_t$$

$$V^{\textit{adj.}}(z_t, b_{t-1}, P_t, X_t) = \max_{(C_t, D_t) \in \mathbb{R}_+^2} u(C_t, D_t) + \beta \cdot \mathbb{E}_t[W(\bullet_{t+1})] \quad (2.28)$$

$$V^{\textit{rent}}(z_t, b_{t-1}, P_t, X_t) = \max_{E_t \in \mathbb{R}_+} u(C_t, S_t) + \beta \cdot \mathbb{E}_t[W(\bullet_{t+1})] \quad (2.29)$$

s.t

$$D_t = 0$$

$$C_t = c(E_t)$$

$$S_t = s(E_t)$$

where the expectation  $\mathbb{E}_t[\bullet]$  is taken over the discrete transition  $z_t \rightarrow z_{t+1}$  and the realizations of  $u_{t+1}$ ,  $\xi_{t+1}$ ,  $\psi_{t+1}$  and  $\iota_{t+1}$ , and  $W(\bullet)$  is the after-taste-shock value function. Due to the extreme value distribution assumption for the taste shocks,  $W(\bullet)$  is given by

$$W\left(z_t, b_{t-1}, P_t, X_t, \bar{D}_t\right) = \sigma_\varsigma \cdot \log \left\{ \sum_{k \in \mathcal{K}} \exp \left( \frac{V^k}{\sigma_\varsigma \cdot P_t^{1-\rho}} \right) \right\} \cdot P_t^{1-\rho} \quad (2.30)$$

**Optimal Behavior** It is straightforward to show that the Bellman equation is homogeneous in  $P_t$  of degree  $1 - \rho$ , and that we can thus normalize with  $P_t$  and spare a state. We denote all variables normalized with  $P_t$  by lower-cases, e.g.  $c_t = C_t/P_t$ , and  $v = V/P_t^{1-\rho}$ .<sup>10</sup>

In general the household problem is non-convex due to the transaction costs. Using standard timing iteration methods for the consumption choice is, however, still possible by relying on a generalized envelope theorem such as that proven in [Clausen and Strub \(2013\)](#). Specifically, we can discretize the durable purchase choice, and solve the model with a value function iteration algorithm in this dimension nesting an endogenous grid point method for finding the optimal consumption choice given the durable choice. This give us both a very fast and very accurate solution algorithm. The details are provided in appendix [B](#).

We denote the optimal continuous choice functions by  $c^{*,keep}$ ,  $c^{*,adj}$ ,  $d^{*,adj}$ , and  $e^{*,rent}$ . The discrete choice probabilities are given by

$$\Pr \left[ k^* \left( z_t, b_{t-1}, x_t, \bar{d}_t \right) = k \right] = \exp \left( \frac{1}{\sigma_\varsigma} \left( v^k - \sigma_\varsigma \cdot \bar{v} \right) \right) \quad (2.31)$$

where

$$\bar{v} \equiv \log \left\{ \sum_{j \in \mathcal{K}} \exp \left( \frac{v^j}{\sigma_\varsigma} \right) \right\}$$

### 3 Calibration

The model is calibrated with a large durable, such as a car, in mind and otherwise to clearly exhibit the model's ability to explain a large cyclical drop in the share of households who adjust their durable stock. The chosen parameters are presented in table [3.1](#). In section [6](#) a number of central robustness checks will be presented.

**Preference** The discount factor  $\beta$  and risk aversion coefficient  $\rho$  are set to respectively 0.94 and 2.0 and are thus within the ranges typically used in the household consumption literature. The utility weight on non-durable consumption  $\omega$  is set to 0.92 implying an aggregate durable stock to non-durable consumption ratio ( $D_t/C_t$ ) of about 0.50. Empirically the total stock of cars relative to non-durable consumption is in a similar range (see [Attanasio \(2000\)](#) and [Bertola, Guiso and](#)

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<sup>10</sup>See appendix [A](#) for the full set of normalized equations.

[Pistaferri \(2005\)](#)). A small taste shock with scaling parameter  $\sigma_\zeta = 0.01$  is also included, and the death probability is set to achieve an average life-span of 30 years,  $\zeta = 0.033$ .

**Durable** The depreciation rate is set to  $\delta = 12.5\%$  close to the average found for cars in [Attanasio \(2008\)](#), [Aaronson, Agarwal and French \(2012\)](#) and [Munk-Nielsen \(2015\)](#). The break down risk is set to a modest value of  $\bar{\tau} = 1\%$ . The transaction costs are chosen to get an unconditional adjustment probability equal to 20 percent per year as found in e.g. [Bertola, Guiso and Pistaferri \(2005\)](#); specifically we ignore transaction costs in buying, and set  $\tau_b = 0.0$  and  $\tau_s = 0.10$ . To hit an average renter share of about 10 percent we set  $r_d = 0.175$ .

**Saving/Borrowing** The real interest rate on (fully liquid) savings is set to 2 percent ( $r = 0.02$ ), while the interest rate on debt is set to 5 percent ( $r_b = 0.05$ ), implying a spread of 3 percentage points. The households are allowed to borrow up to 90 percent of the value of their durable in expansions ( $\theta_E = 0.10$ ), and 80 percent in recessions ( $\theta_E = 0.20$ ). In recent times some households have been able to borrow almost up to 100 percent (see e.g. [Attanasio \(2008\)](#)), but to include this in the model we would also need to consider an income-to-loan constraint and differentiable interest rates.

**Income Process** The long-run average growth rate is chosen to be 2 % ( $\Gamma = 1.02$ ), while the long-run average level of unemployment is set to 7 % ( $u_* = 0.07$ ). The cyclical changes in growth and unemployment is chosen to be rather large at respectively  $\Delta\Gamma = 0.03$  and  $\Delta u = 0.06$ . The transitions probabilities are chosen to give long expansions only ending with a 10 percent probability each year ( $\pi_{EE} = 0.90$ ), and somewhat shorter recessions ending with a 25 percent probability each year ( $\pi_{EE} = 0.75$ ).

In setting the variances of the permanent shock in expansions and recessions we roughly follow the results in [Storesletten, Telmer and Yaron \(2004\)](#) and set  $\sigma_{\psi E} = 0.1$  and  $\sigma_{\psi R} = 0.2$ . In the baseline parametrization we do *not* allow for changes in skewness over the cycle ( $\pi_\psi = \mu_{\psi E} = \mu_{\psi R} = 0$ ). Recent results in [Guvenen, Ozkan and Song \(2014\)](#) have, however, indicated that it is the *skewness* rather than the *variance* of the permanent income shocks that increases during recessions. Translating the results in [Guvenen, Ozkan and Song \(2014\)](#) to an income process such as ours with a fully permanent and a fully transitory shock is, however, not straightforward (see e.g. [McKay \(2015\)](#)). We will therefore instead do a



Table 3.1: Calibrated Parameters

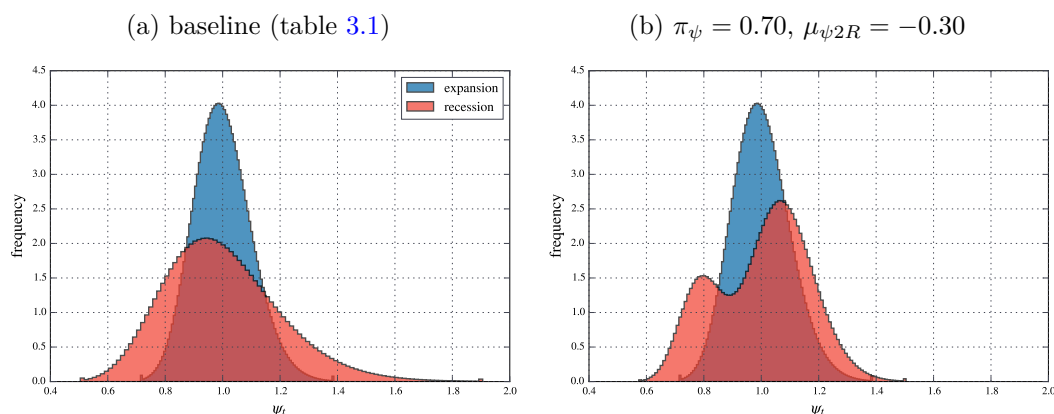
Description	Symbol	Value
<i>Preferences</i>		
Discount factor	$\beta$	0.94
Risk aversion coefficient	$\rho$	2.00
Utility weight on non-durables	$\omega$	0.92
Taste shock scale parameter	$\sigma_\zeta$	0.01
Death probability	$\zeta$	0.033
<i>Durable Good</i>		
Depreciation rate	$\delta$	0.125
Breakdown risk	$\bar{\tau}$	0.01
Transaction cost, selling	$\tau_s$	0.10
Transaction cost, buy	$\tau_b$	0.0
Rental price	$r_d$	0.175
<i>Saving/Borrowing</i>		
Interest rate, savings	$r$	0.02
Interest rate, debt	$r_b$	0.05
Down payment ratio, expansion	$\theta_E$	0.10
Down payment ratio, recession	$\theta_R$	0.20
<i>Income Process</i>		
Income growth, long-run avg.	$\Gamma_*$	1.02
Income growth, cyclical change	$\Delta\Gamma$	0.03
Unemployment, long-run avg.	$u_*$	0.07
Unemployment, cyclical change	$\Delta u$	0.06
Expansion $\rightarrow$ Expansion	$\pi_{EE}$	0.90
Recession $\rightarrow$ Recession	$\pi_{RR}$	0.75
Permanent shock (std.), expansion	$\sigma_{\psi E}$	0.10
Permanent shock (std.), recession	$\sigma_{\psi R}$	0.20
Permanent shock (weight)	$\pi_\psi$	0.0
Permanent shock (skew.), expansion	$\mu_{\psi E}$	0.0
Permanent shock (skew.), recessions	$\mu_{\psi R}$	0.0
Transitory shock (std.)	$\sigma_\xi$	0.10
Unemployment benefits	$\mu$	0.50

robustness check, where we adjust  $\pi_\psi$  and  $\mu_{\psi R}$  to roughly match only the left tail of the baseline increase in the variance. Setting  $\sigma_{\psi R} = \sigma_{\psi E}$ ,  $\pi_\psi = 0.70$  and  $\mu_{\psi R} = -0.30$  approximately achieve this as seen in figure 3.1.

For the transitory shock we set  $\sigma_\xi = 0.10$  as in e.g. [Carroll, Slacalek and Tokuoka](#)

(2015); this is lower than the  $\sigma_\xi = 0.169$  found in Storesletten, Telmer and Yaron (2004), but is chosen because the model include unemployment explicitly. The unemployment benefits is set to 50 percent of permanent income ( $\mu = 0.50$ ); Carroll, Slacalek and Tokuoka (2015) instead use a value of 15 percent but their income process is at a quarterly frequency, at a yearly frequency a value of 15 percent is very extreme.

Figure 3.1: Cyclical Variation in  $\psi_{t+1}$

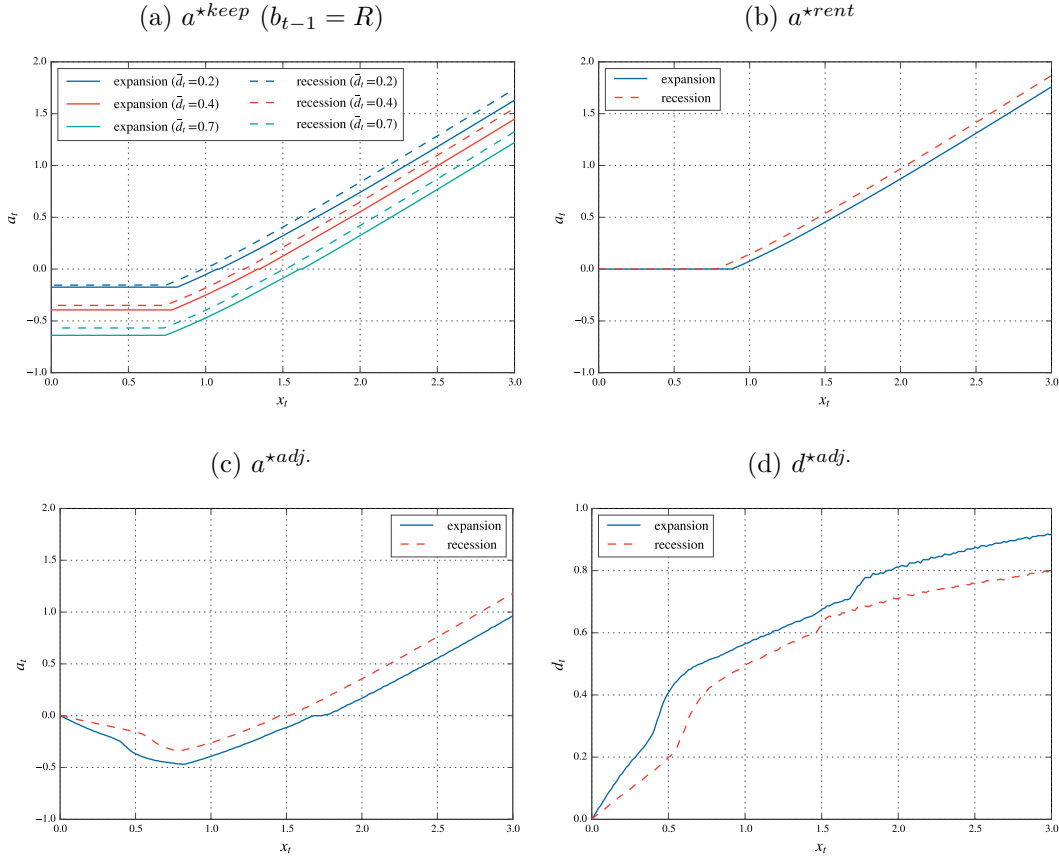


Notes: From simulations of 500,000 households for 500 periods. The shocks are bounded by the minimum and maximum of the Gauss-Hermite nodes used in the solution algorithm.

## 4 Optimal Choice Functions

Figure 4.1 shows the optimal normalized saving functions (in terms of net financial assets) for each of the three discrete choices and the normalized durable purchase choice given adjustment. We see that the optimal saving functions (panel a-c) move unilaterally *upward* in recessions, while the durable purchase choice (panel d) move *downward*.

Figure 4.1: Saving Functions and the Durable Purchase Choice



Notes: Given the parameters in table 3.1.

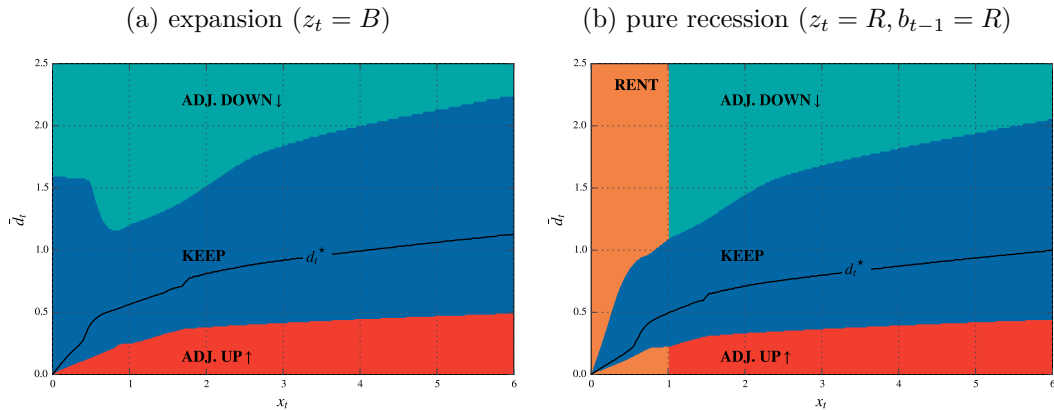
Looking at the saving functions given *keeping*, we see that they all have four regions: (1) If beginning-of-period net worth is low enough the households choose to borrow up to their borrowing constraint (determined by the size of their durable stock). (2) At some point the richer households decide to borrow less than they are able to, and eventually (4) start saving in net financial assets. In between the second and fourth regions, however, there also is a small range of intermediate levels of net worth, where (3) the households neither borrow nor save; this is because of the drop in the interest rate on savings relative to the borrowing rate (i.e.  $r < r_b$ ).

Turning to the *adjustment* case, the optimal behavior is even more complicated. For low levels of net worth the households are mostly concerned about surviving the current period, and it is therefore of central importance whether the down payment constraint is loose or tight. In *recessions*, where the collateral constraint is relatively *tight* (a large down payment is required), the poor households purchase a small durable, which yields an intra-temporally too low share of durables in util-

ity; in *expansions* we have the opposite case because the constraint is so *loose* that it is optimal for the liquidity constrained households to buy an intra-temporally too large durable. As the households get less liquidity constrained, they in general choose a larger durable, but also obtain a better intra-temporal balance with non-durable consumption. Note, additionally, that as long as the households are net borrowers the user cost is larger than for rich households (as  $r_b > r$ ), and consequently the optimal intra-temporal utility share of durables is lower for poor households. Finally, there, like under *keeping*, also exists a range of intermediate values of net worth where the households optimally choose to be neither savers nor borrowers; up to the numerical precision of the solution, the durable purchase choice is furthermore linear in normalized net worth in this region.

The disjoint sets of states, where each of the three discrete choices are optimal, are shown in figure 4.2 together with the durable purchase choice given adjustment. The figure is drawn for zero realizations of all the taste shocks; in general it is the case that the distance to a decision region is a strong determinant of the choice probability. Note also that it is the *normalized* level of net worth and the *normalized* durable stock which are on the axes. Permanent shocks *above* one consequently, all else equal, induce a *south-west* movement, while permanent shocks *below* one induce a *north-east* movement. Considering a household saving nothing and getting permanent shocks of exactly one, depreciation induces a movement *south* because the durable stock shrinks, and a bit weaker movement *west* because net wealth thus also falls (but only with a factor of  $1 - \tau_s < 1$ ).

Figure 4.2: Discrete Choice



Notes: Given the parameters in table 3.1.  $\forall k \in \mathcal{K}: s_{tk} = 0$ .

In both expansions and recessions we see that there for a given level of net worth exists an intermediate range of durable stocks such that *keeping* is optimal, while

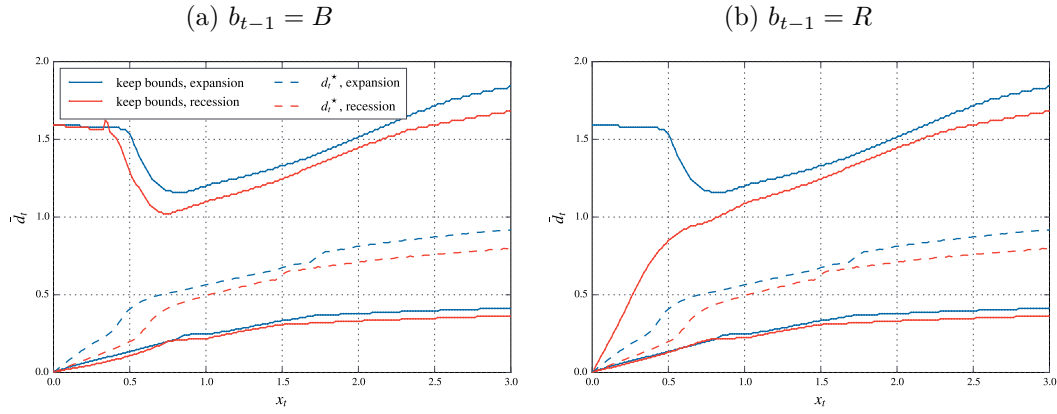
a *larger/smaller* durable stock makes it worthwhile to pay the transaction costs involved in adjusting *downwards/upwards*. For high levels of net worth the optimal discrete choice is *qualitatively* rather similar across expansions and recessions, while there are remarkable differences for low net worth. In particular, we see that the loose collateral constraint in expansions imply that it is never optimal for the poor households to rent, while it is optimal for some of them to do so in pure recessions (with  $b_{t-1} = R$ ) because the collateral constraint is tighter. However, it should also be noted that the value of renting in expansions is very close to that of adjusting, and that relative likely taste shocks thus can make it optimal to rent rather than adjust even in expansions.

Across the cycle there are important changes in the *inaction bounds* for *keeping* as illustrated more clearly in figure 4.3. Focusing on the lower bound, we see that it moves downwards in recessions, and that the households thus are more willing to keep being struck with a small durable. This is similar to the result of expanding inaction bounds due to an increase in uncertainty generally found in the real-options literature and lumpy investment models. In total, however, the inaction region becomes *more narrow* in recessions in the present model because the upper bound also moves downwards; we conjecture that this is because of both lower expected future income growth, and an increase in the precautionary saving motive.<sup>11</sup> In figure 4.4 we also see that if we start from a parametrization with *no cyclical fluctuations* ( $\Delta\Gamma = \Delta u = 0$ ,  $\sigma_{\psi R} = \sigma_{\psi E}$ , and  $\theta_R = \theta_E$ ), it is the cyclical variation in idiosyncratic income risk that by far creates most cyclical variation in the lower inaction bound.

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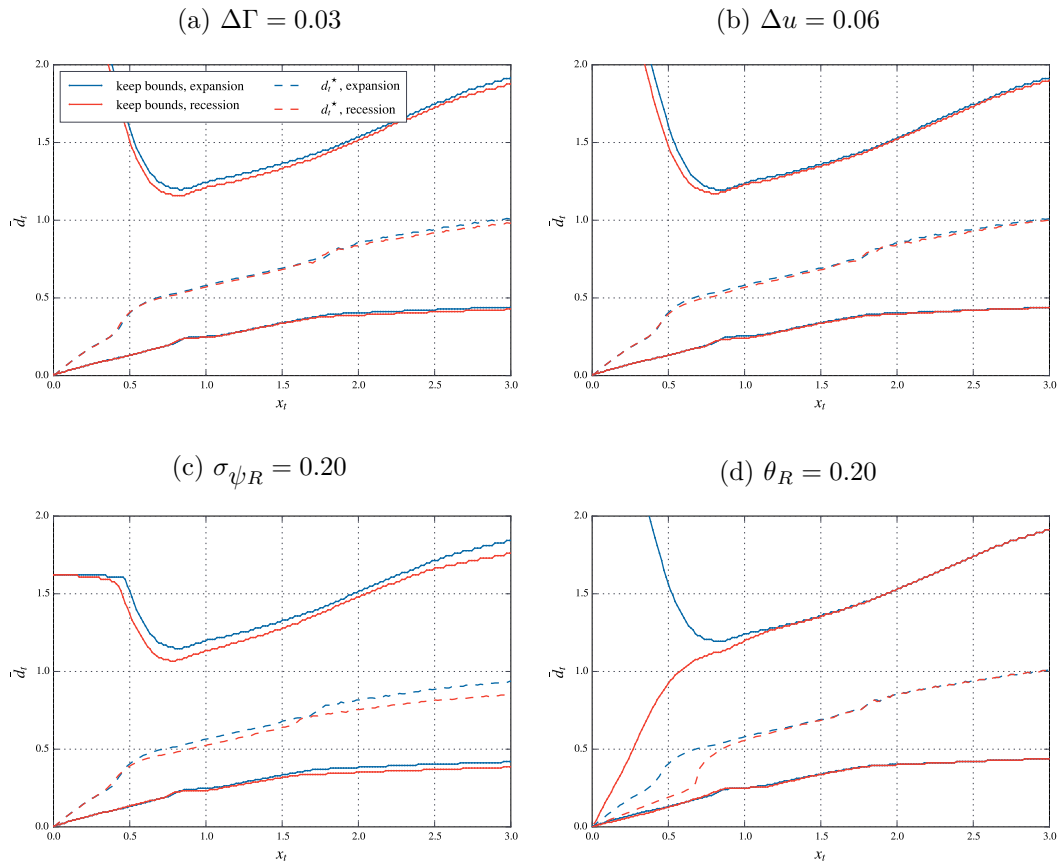
<sup>11</sup>Our assumption of non-separability between durable and non-durable consumption could also be central for this result.

Figure 4.3: Inaction Region



Notes: Given the parameters in table 3.1.  $\forall k \in \mathcal{K}: \varsigma_{tk} = 0$ .

Figure 4.4: Inaction Region ( $\Delta\Gamma = \Delta u = 0$ ,  $\sigma_{\psi R} = \sigma_{\psi E}$ ,  $\theta_R = \theta_E$ ,  $b_{t-1} = R$ )



Notes: Otherwise given the parameters in table 3.1.  $\forall k \in \mathcal{K}: \varsigma_{tk} = 0$ .

## 5 Simulation

Given the converged policy functions, we simulate a panel of 500,000 households for 800 periods, and throw away the first 300 periods. All households are born not owning a durable, with lagged permanent income of one, and with lagged financial assets drawn from a normal distribution with mean 0.25 and standard deviation 0.10 truncated at zero.<sup>12</sup> Households are replaced with new ones upon death, and their assets are “taxed away”. All monetary variables are detrended using  $\Gamma_*^t$ .

The first row in table 5.1 shows the resulting long-run averages in the simulation. We see that about one in five households buy a new durable each period, while about 9 percent choose to be renters. The average value of the durable stock is approximately 0.70 at purchase, and 0.50 just conditional on ownership. The precautionary saving motive implied by the model is so strong that the households on average hold a small reserve fund in net financial assets ( $A_t > 0$ ).

Table 5.1: Simulation: Long-Run Averages

	Adj.	Rent	$A_t$	$N_t$	$D_t^{own}$	$D_t^{adj.}$
Baseline	19.9	9.0	0.03	0.49	0.51	0.69
$\sigma_{\psi R} = \sigma_{\psi E}, \theta_R = \theta_E$	19.6	11.2	-0.09	0.35	0.50	0.67
$\sigma_{\psi R} = \sigma_{\psi E}$	19.5	11.3	-0.09	0.36	0.50	0.67
$\theta_R = \theta_E$	20.0	9.0	0.03	0.49	0.51	0.69
No Cycle <sup>a</sup>	19.6	11.1	-0.09	0.35	0.50	0.67
$\Delta\Gamma = 0.03$	19.7	11.1	-0.09	0.35	0.50	0.67
$\Delta u = 0.06$	19.6	11.1	-0.09	0.35	0.50	0.67
$\sigma_{\psi R} = 0.20$	19.9	9.1	0.00	0.46	0.51	0.69
$\theta_R = 0.20$	19.5	11.3	-0.09	0.36	0.50	0.67
$\mu_{\psi R} = -0.30$	19.9	9.9	-0.04	0.41	0.50	0.68

*Samples:* Simulations of 500,000 households for 500 periods.

<sup>a</sup> *No Cycle* is  $\Delta\Gamma = 0, \Delta u = 0, \sigma_{\psi R} = \sigma_{\psi E}$  and  $\theta_R = \theta_E$ .

In the remaining rows of table 5.1 the resulting simulation averages are shown for various changes to the baseline parametrization. We see that the precautionary motive is strong especially due to the increase in the variance of the permanent

<sup>12</sup>Experiments have shown that the results are not very sensitive to these choices.

shock during recessions; without it the households would choose to be net borrowers in financial assets (third row). The increase in the variance of the permanent shocks also, perhaps surprisingly, *decrease* the share of renters, but this could be an artifact of our assumptions regarding the possibility to freely take out collateral loans on already owned durables. The size of the durable stock given purchase or ownership is, on the other hand, almost not affected by adding cyclical variation in income risk or collateral tightness.

Turning to the cyclical fluctuations, table 5.2 shows the simulated absolute and relative changes in selected variables over the business cycle. In total, the model can explain a drop of 4.5 percentage points in the adjuster share during recessions compared to in expansions. In figure 5.1 we see that the first year impact is very large, and that the adjuster share hereafter typically increases a bit during the recession. Allowing for more aggregate states and a slower transition to full blown recession would naturally affect this pattern substantially.

Removing the cyclical variation in both income risk and collateral tightness reduce the cyclical drop in the adjuster share to 2.2 percentage points (second row), and they are thus very important for creating a large drop. Removing either only the cyclical variation in income risk, or only the cyclical variation in collateral tightness (row three and four), reduces the cyclical drop to respectively 3.9 and 4.0 percentage points. The two effects thus seem to negate each other quite a lot when included simultaneously. The reason probably is that the increase in the permanent shock variance induce so much precautionary saving that most households never choose to be close to the collateral constraint, and that changes in it is thus not very important.



Table 5.2: Simulation: Cyclical Differences

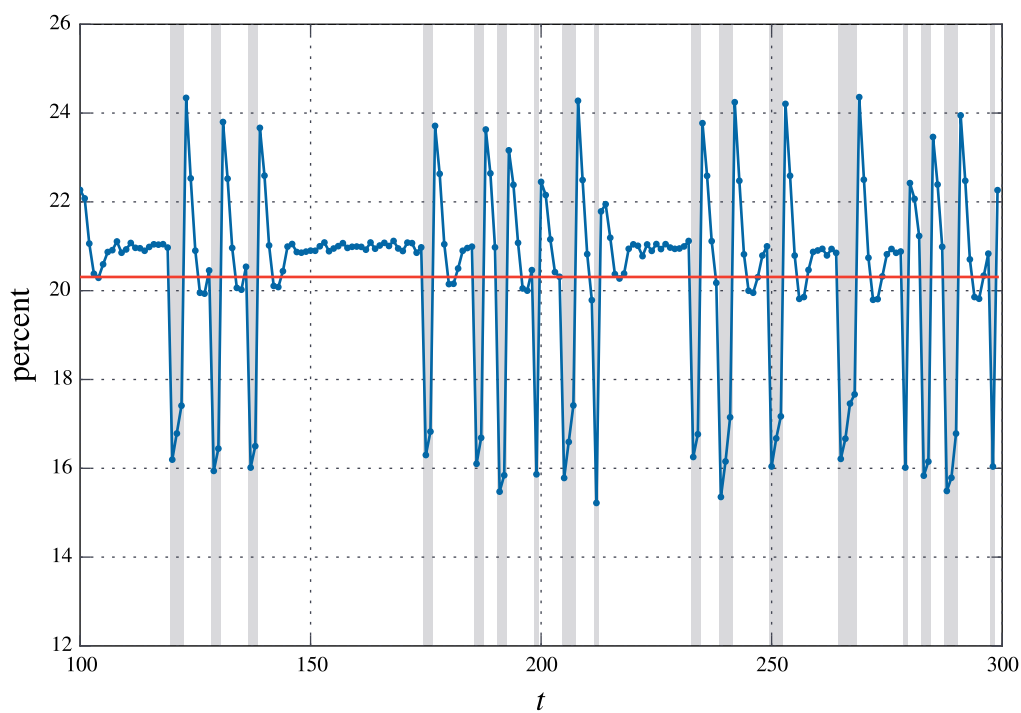
	$\Delta\text{Adj.}$	$\tilde{\Delta}\text{Adj.}^b$	$\Delta \log(Y_t)$	$\Delta \log(C_t)$	$\Delta \log(D_t^{\text{adj.}})$
Baseline	-4.5	-2.5	-9.9	-14.0	-6.3
$\sigma_{\psi_R} = \sigma_{\psi_E}, \theta_R = \theta_E$	-2.2	0.2	-9.9	-10.9	-8.3
$\sigma_{\psi_R} = \sigma_{\psi_E}$	-3.9	-1.7	-9.9	-10.9	-8.8
$\theta_R = \theta_E$	-4.0	-2.1	-9.9	-14.0	-6.1
No Cycle <sup>a</sup>	0.0	0.0	0.0	0.0	0.0
$\Delta\Gamma = 0.03$	-1.2	0.2	-7.0	-8.3	-6.3
$\Delta u = 0.06$	-1.0	-0.3	-3.1	-2.9	-2.1
$\sigma_{\psi_R} = 0.20$	-1.2	-1.3	0.0	-3.1	2.0
$\theta_R = 0.20$	-1.7	-1.8	0.0	0.0	-0.6
$\mu_{\psi_R} = -0.30$	-0.7	-0.6	0.0	-1.7	0.5

*Samples:* Simulations of 500,000 households for 500 periods.

<sup>a</sup> *No Cycle* is  $\Delta\Gamma = 0, \Delta u = 0, \sigma_{\psi_R} = \sigma_{\psi_E}$  and  $\theta_R = \theta_E$ .

<sup>b</sup>  $\tilde{\Delta}$  denotes the cyclical difference when income draws are always from the *expansion* state.

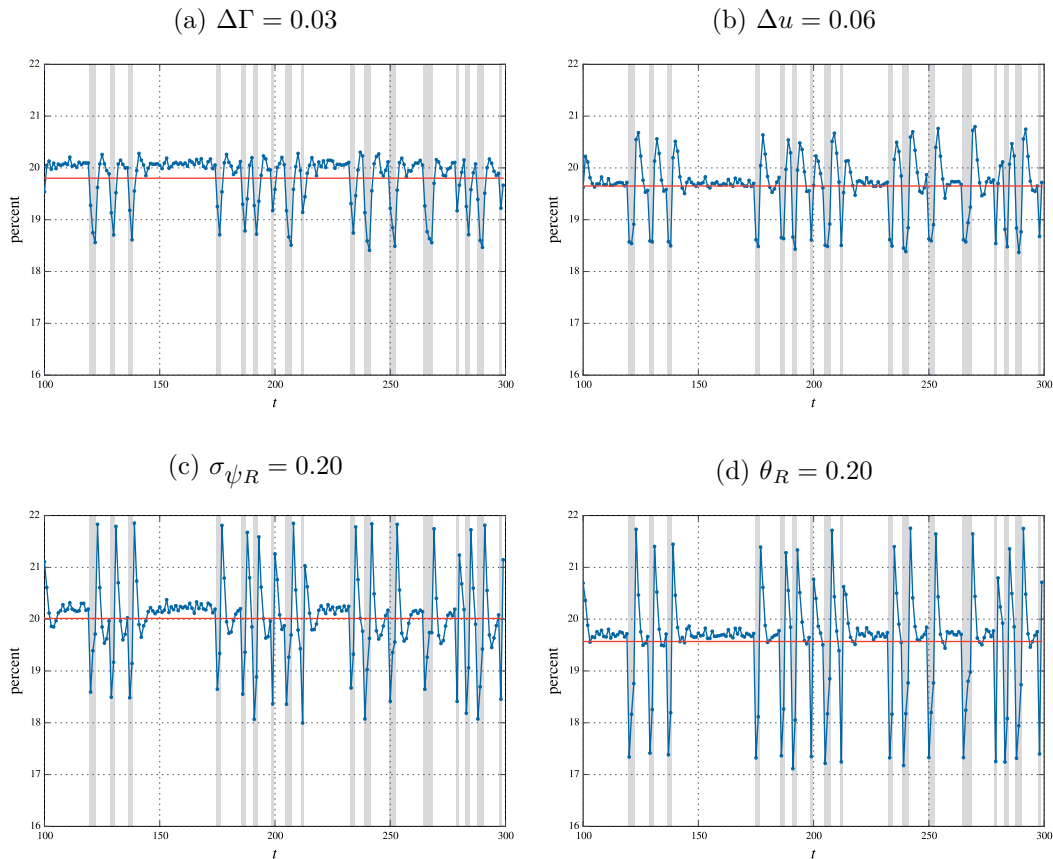
Figure 5.1: Adjuster Share



*Sample:* Simulation of 500,000 households. Shaded recessions.

Beginning from an assumption of *no cyclical fluctuations* (row five and down in table 5.2) adding cyclical variation in income risk induce a cyclical drop in the adjuster share of 1.2 percentage points, while adding cyclical variation in the tightness of the collateral constraint induces a cyclical drop of 1.7 percentage points. The remaining part of the cyclical drop in the adjuster share seems to be about equally due to variation in the average income growth rate and in the unemployment rate. Figure 5.2, however, shows that the patterns in the business cycle fluctuations in the adjuster share are somewhat different for the different mechanisms. When we only allow for changes in the growth rate of income or in the unemployment rate, the adjuster share is typically falling in the beginning of the recession, while it is typically lowest in the first year when only allowing for either cyclical variation in income risk or in the tightness of the collateral constraint.

Figure 5.2: Adjuster Share ( $\Delta\Gamma = \Delta u = 0$ ,  $\sigma_{\psi_R} = \sigma_{\psi_E}$ ,  $\theta_R = \theta_E$ )



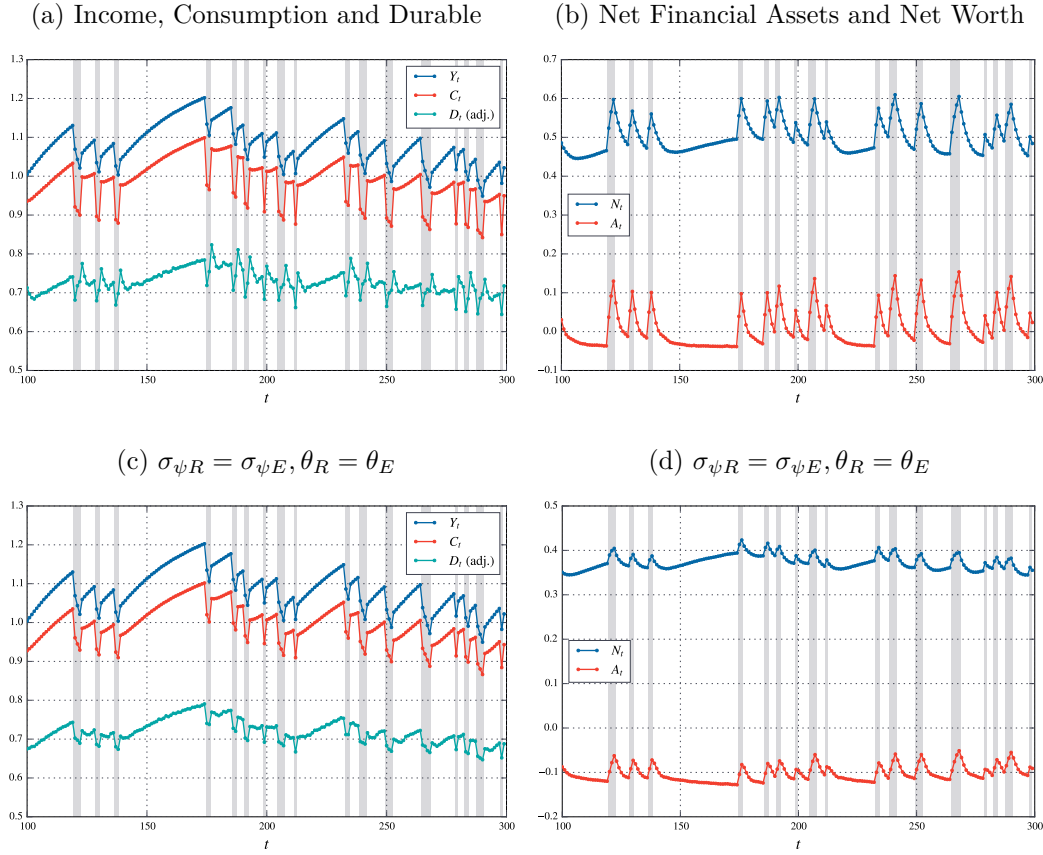
*Samples:* Simulations of 500,000 households. Shaded recessions.

To give an idea about how much of the cyclical fluctuations in the adjuster share that is due to respectively (a) actual changes in income, and (b) changes in behav-

ior because of changed collateral tightness and changed expectations about future growth rates, unemployment and income uncertainty, the second column in table 5.2 shows the cyclical change in the adjuster share from a simulation where income is always drawn as if the economy was in an *expansion*. The resulting drop in the adjuster share over the business cycle is 2.5 percentage points, and more than half of the drop in the share of adjusters is thus not due to the income changes in themselves. Cyclical variation in income risk and collateral tightness is, however, very important for this result; without it the effects from expectations are actually a bit positive. Starting from a parametrization without cyclical fluctuations, we also see in table 5.2 (row eight) that all of the effect from introducing cyclical income risk can be explained as an expectation effect. This also remains true if we instead consider our robustness check where more of the cyclical variation in income is due to increased skewness ( $\mu_{\psi R} = -0.30$ ,  $\pi_{\psi} = 0.7$ ,  $\sigma_{\psi R} = \sigma_{\psi E}$ , last row); the *increased skewness* specification, however, only add 0.7 percentage points to the cyclical drop in the adjuster share compared to 1.2. percentage points for the *increased variance* specification.

The simulation also implies that the fluctuations in consumption are somewhat larger than those in income, while the cyclical changes in the size of the durables purchased are somewhat smaller (column three to five of table 5.2). This is also shown in panel (a) of figure 5.3. The larger changes in consumption are especially due to the cyclical variation in income growth, but both the cyclical variation in income risk and in unemployment are also important. The fluctuations in the size of durable expenditures given purchase are, however, mostly explained by the changes in income growth and unemployment. This is also seen by comparing panel (a) and (c) in figure 5.3, where cyclical variation in income risk and collateral tightness is disregarded in the latter. Comparing panel (b) and (d) we also see that ignoring fluctuations in income risk and collateral tightness make the fluctuations in the precautionary motive very limited, such that the average levels of net financial assets and net worth are almost not cyclical.

Figure 5.3: Cross-Sectional Averages

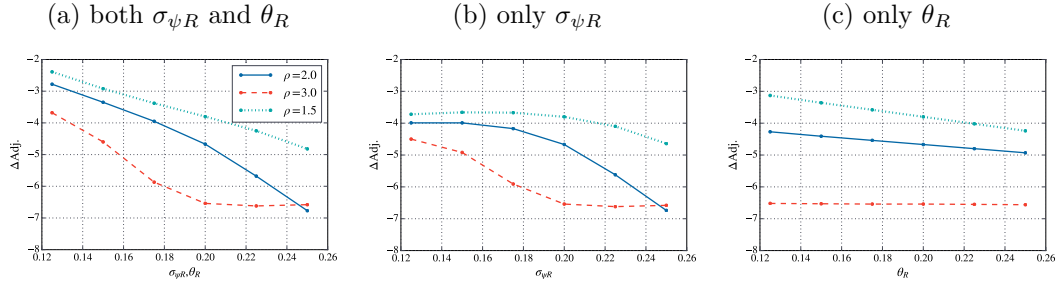


*Samples:* Simulations of 500,000 households. Detrended with  $\Gamma_*^t$ . Shaded recessions.

## 6 Robustness

Table 6.1 shows the effects on the simulation results of changing selected groups of parameters. Focusing on the cyclical drop in the share of households who adjust their durable stock, we see that it becomes larger when we remove taste shocks ( $\sigma_\zeta = 0$ ) or increase the cost of renting (higher  $r_d$ ), while shorter recessions (lower  $\pi_{RR}$ ) or higher transactions costs (higher  $\tau_s$ ) make the cyclical drop in the adjuster share smaller.

Strengthening the growth and uncertainty adjusted precautionary saving motive (e.g. higher  $\beta$ , lower  $\Gamma_*$ , higher  $r$ , higher  $\rho$ , or lower  $\mu$ ) also makes the drop larger. This is perhaps a bit surprising; on the one hand a strong precautionary saving motive naturally implies more accumulation of savings (larger  $A_t$ ), which in itself should insulate the households better against shocks, on the other hand the households with a stronger precautionary motive are also more afraid of a long

Figure 6.1: Cyclical Adjuster Share Drop,  $\Delta\text{Adj}$ .


*Samples:* Simulations of 500,000 households for 500 periods.

continuing recession implying a relatively stronger *increase* in the precautionary saving motive when a recession hits. In figure 6.1, however, we see that if the risk aversion coefficient  $\rho$  is high then further amplification of the cyclical drop in the adjuster share cannot be achieved with  $\theta_R > \theta_E$  or  $\sigma_{\psi R}$  above the baseline value of 0.20; for lower values of  $\rho$  increases in idiosyncratic income risk or collateral tightness in recessions always imply an amplification of the cyclical drop in the adjuster share for the values considered here.

More importantly, for the results presented in this paper, disregarding variation in the idiosyncratic income risk and in the tightness of the collateral constraint, always reduce the drop by more than 30 percent, and for some parameter choices by more than 60 percent (sixth column of table 6.1). The relative importance of the two proposed mechanisms for creating more cyclical fluctuations are lowest when there are no taste shocks and renting is expensive. Even without both taste shocks ( $\sigma_\zeta = 0$ ) and renting ( $r_d = \infty$ ), however, they add a total of 1.8 percentage points to the cyclical drop in the adjuster share.

Figure 6.2 also documents that a large cyclical change in idiosyncratic income risk (high  $\sigma_{\psi R}$ ) is important to get the result that a large share of the cyclical drop in the adjuster share is due to behavioral effects not related to actual changes in income (i.e. a high  $\tilde{\Delta}\text{Adj}$ . relative to  $\Delta\text{Adj}$ ., where  $\tilde{\Delta}$  denotes the cyclical change in a simulation where income is drawn as if the economy was always in the *expansion* state). Higher levels of risk aversion in general also lowers the relative importance of the changes in income per se.

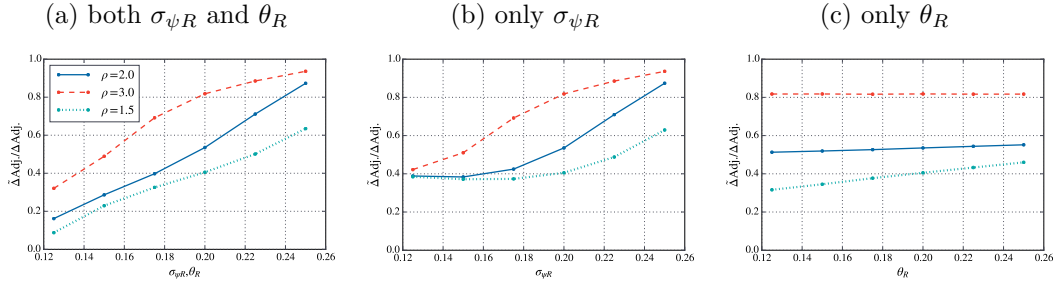
The effects on the cyclical fluctuations in the size of the durables purchased given adjustment are more varied. When the precautionary motive is strong enough including cyclical variation in income risk and collateral tightness thus imply a further fall in the size of the durables purchases in recessions than already implied by the drop in income growth and the rise in unemployment (see the last column

Table 6.1: Robustness

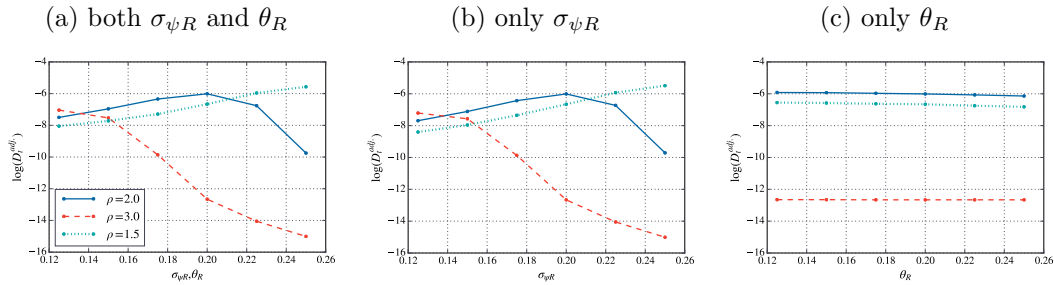
	No Other Changes				$\sigma_{\psi R} = \sigma_{\psi E}, \theta_R = \theta_E$		
	Adj.	Rent	$A_t$	$\Delta$ Adj.	$\Delta$ Adj.	$\Delta$ Adj. %	$\Delta \log(D_t^{adj.})$
	<i>diff. to "No Other Changes"</i>						
Baseline	19.9	9.0	0.03	-4.5	-2.3	51.6	2.1
$\beta = 0.92$	19.9	10.8	-0.07	-4.3	-2.3	52.7	1.6
$\beta = 0.96$	19.8	7.2	0.31	-5.7	-3.0	53.8	-1.8
$\rho = 1.5$	19.6	12.0	-0.07	-3.8	-2.0	51.8	1.5
$\rho = 3.0$	20.2	5.0	0.86	-6.5	-3.5	53.5	-5.5
$\omega = 0.93$	20.3	9.9	0.06	-4.7	-2.5	52.7	1.1
$\omega = 0.94$	20.6	11.0	0.09	-4.7	-2.5	53.3	0.1
$\sigma_\zeta = 0$	19.1	0.6	0.01	-5.6	-2.4	42.8	4.5
$\delta = 0.10, r_d = 0.145$	17.7	7.0	-0.02	-4.6	-2.2	48.7	2.5
$\delta = 0.15, r_d = 0.195$	22.3	11.7	0.07	-4.5	-2.5	54.4	0.8
$\bar{t} = 0.0001$	19.6	8.6	0.02	-4.8	-2.5	51.0	2.1
$\tau_s = 0.05$	27.0	10.4	0.03	-5.4	-2.8	51.8	0.3
$\tau_s = 0.15$	17.4	9.6	0.04	-4.0	-2.2	54.8	1.9
$\tau_s = 0.05, \tau_b = 0.02$	21.5	12.9	0.04	-4.5	-2.6	58.2	1.2
$r_d = 0.17$	19.5	12.6	0.04	-4.2	-2.5	59.8	1.3
$r_d = 0.18$	20.4	6.9	0.02	-5.0	-2.3	47.3	2.1
$r_d = \infty$	21.0	0.0	0.00	-5.8	-2.2	37.8	3.1
$r_d = \infty, \sigma_\zeta = 0$	19.2	0.0	0.01	-5.6	-1.8	32.4	4.4
$r = 0.01$	20.5	7.0	-0.02	-4.4	-2.1	47.7	3.2
$r = 0.03$	19.4	12.6	0.11	-4.8	-2.7	55.3	-0.4
$r_b = r = 0.02$	19.4	5.4	-0.11	-6.2	-2.7	43.3	-1.2
$\theta_E = 0.05, \theta_R = 0.15$	20.0	9.2	0.02	-4.5	-2.4	53.6	1.7
$\Gamma_* = 1.01$	19.3	6.9	0.26	-5.6	-2.9	52.0	-1.1
$\Delta\Gamma = 0.015$	20.0	9.3	0.01	-3.9	-2.2	57.5	1.9
$u_* = 1.01$	19.9	10.6	-0.03	-4.1	-2.5	61.3	2.4
$\Delta u = 0.015$	20.0	9.1	0.03	-4.1	-2.4	57.7	1.8
$\pi_{BB} = 1 - 1/5$	19.8	7.8	0.14	-4.7	-2.5	52.8	2.7
$\pi_{RR} = 1 - 1/2$	19.8	10.0	-0.03	-4.2	-1.9	46.3	4.6
$\mu = 0.30$	20.1	7.5	0.19	-6.0	-2.4	39.3	0.0
$\mu = 0.70$	19.6	12.1	-0.07	-3.1	-1.8	58.5	1.6

*Samples:* Simulations of 500.000 households for 500 periods.

of table 6.1). Figure 6.3 shows that it is the idiosyncratic income risk which is most important for this result, and that it also emerges for  $\rho = 2$  if  $\sigma_{\psi R}$  is higher

Figure 6.2: Cyclical Adjuster Share Drop (Behavioral Share),  $\tilde{\Delta}Adj./\Delta Adj.$ 


*Samples:* Simulations of 500,000 households for 500 periods.

 Figure 6.3: Cyclical Durable Purchase Decision Change,  $\Delta \log(D_t^{adj.})$ 


*Samples:* Simulations of 500,000 households for 500 periods.

than the baseline value of 0.20. For wide ranges of parameters the main result that the proposed mechanisms *dampen* the cyclical fluctuations in the size of the durables purchased, however, remains true.

## 7 Conclusions

This paper has shown that cyclical fluctuations in idiosyncratic income risk and in the tightness of the collateral constraint, both strongly amplify the model-implied drop in the share of households who adjust their durable stock during recessions relative to in expansions. If the precautionary saving motive is not overly strong, this is furthermore achieved with a small increase in the intensive margin in the sense of larger expenditures per purchase. Additionally it has also been shown that the effect from increased idiosyncratic income risk is purely due to changed expectations, and that the two mechanisms negate each other somewhat; the increased precautionary savings induced by the increase in idiosyncratic income risk simply make the households aim at so a high a level of savings in recessions that changes in the collateral constraint is not important for most households.

Even though the proposed model includes both transaction costs, an outside option of renting, and taste shocks, it is still simplified along many dimension. One interesting extension could be a more general specification of utility allowing for e.g. complementarity between durable and non-durable consumption or a decoupling of the risk aversion coefficient and inter-temporal substitution elasticity like in Epstein-Zin preferences. Additionally a more detailed specification of the loan contract differentiating between newly bought and old durables, and allowing for re-financing costs, would be an interesting robustness check. Introducing a portfolio choice between saving in liquid and illiquid assets (as in [Kaplan and Violante \(2014\)](#)) would likewise be interesting for studying the importance of the precautionary savings motive in more detail. Together with an explicit distinction between new and old durables, this would also make it more transparent how to calibrate or estimate the model on household-level data. Finally general equilibrium effects could be important.<sup>13</sup>

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<sup>13</sup>See [Bachmann, Caballero and Engel \(2013\)](#) for a similar discussion in the firm investment literature.



## A Model Details

### A.1 Normalization

Denoting all *normalized variables* by *lower cases* the value function is

$$v\left(z_t, b_{t-1}, x_t, \bar{d}_t \mid \{\phi_t^k\}_{k \in \mathcal{K}}\right) = \max_{k \in \mathcal{K}} \{v^k + \phi_t^k\} \quad (\text{A.1})$$

The common constraints are

$$b_t = \begin{cases} z_t & \text{if } k_t \in \{adj., rent\} \\ E & \text{if } k_t = keep \text{ and } z_t = E \\ b_{t-1} & \text{if } k_t = keep \text{ and } z_t = R \end{cases} \quad (\text{A.2})$$

$$a_t = \begin{cases} x_t - [(1 - \tau_s) \cdot \bar{d}_t + c_t] & \text{if } k_t = keep \\ x_t - [(1 + \tau_b) \cdot d_t + c_t] & \text{if } k_t = adj. \\ x_t - e_t & \text{if } k_t = rent \end{cases} \quad (\text{A.3})$$

$$\tilde{\Gamma}_{t+1} = \Gamma_{z_{t+1}} \cdot \psi_{t+1} \quad (\text{A.4})$$

$$\bar{d}_{t+1} = \begin{cases} 1/\tilde{\Gamma}_{t+1} \cdot (1 - \delta) \cdot d_t & \text{if } \iota_{t+1} > \bar{\iota} \\ 0 & \text{else} \end{cases} \quad (\text{A.5})$$

$$x_{t+1} = 1/\tilde{\Gamma}_{t+1} \cdot [(1 + r(a_t)) \cdot a_t + (1 - \tau_s) \cdot (1 - \delta) \cdot d_t] + \tilde{\xi}_{t+1} \quad (\text{A.6})$$

$$a_t \geq -(1 - \theta_{b_t}) \cdot d_t \quad (\text{A.7})$$

The discrete choice specific value functions are

$$v^{keep}(z_t, b_{t-1}, x_t, \bar{d}_t) = \max_{c_t \in \mathbb{R}_+} u(c_t, d_t) + \beta \cdot \mathbb{E}_t \left[ \tilde{\Gamma}_{t+1}^{1-\rho} \cdot w_{t+1}(\bullet_{t+1}) \right] \quad (\text{A.8})$$

s.t

$$d_t = \bar{d}_t$$

$$v_t^{adj.}(z_t, b_{t-1}, x_t) = \max_{(c_t, d_t) \in \mathbb{R}_+^2} u(c_t, d_t) + \beta \cdot \mathbb{E}_t \left[ \tilde{\Gamma}_{t+1}^{1-\rho} \cdot w_{t+1}(\bullet_{t+1}) \right] \quad (\text{A.9})$$

$$v^{rent}(z_t, b_{t-1}, x_t) = \max_{c_t \in \mathbb{R}_+} u(c_t, s_t) + \beta \cdot \mathbb{E}_t \left[ \tilde{\Gamma}_{t+1}^{1-\rho} \cdot w_{t+1}(\bullet_{t+1}) \right] \quad (\text{A.10})$$

s.t

$$d_t = 0$$

$$c_t = c(e_t)$$

$$s_t = s(e_t)$$

where

$$w(z_t, b_{t-1}, x_t, \bar{d}_t) = \sigma_\varsigma \cdot \log \left\{ \sum_{k_t \in \mathcal{K}} \exp \left( \frac{v^k}{\sigma_\varsigma} \right) \right\} \quad (\text{A.11})$$

We denote the optimal continuous choice functions by  $c^{\star, keep}$ ,  $c^{\star, adj.}$ ,  $d^{\star, adj.}$ , and  $e^{\star, rent}$ . The discrete choice probabilities are given by

$$\Pr [k^\star(z_t, b_{t-1}, x_t, \bar{d}_t) = k] = \exp \left( \frac{1}{\sigma_\varsigma} (v^k - \sigma_\varsigma \cdot \bar{v}) \right) \quad (\text{A.12})$$

where

$$\bar{v} \equiv \log \left\{ \sum_{j \in \mathcal{K}} \exp \left( \frac{v^j}{\sigma_\varsigma} \right) \right\}$$

## A.2 State Space

Define  $\underline{a}(d_t, b_t)$  as the *lowest*  $a_t$  (given a  $d_t$  and  $b_t$ ) satisfying

$$\begin{aligned} a_t &\geq -(1 - \theta_{b_t}) \cdot d_t \\ \underline{\mathbb{E}}_t [x_{t+1}] &\geq 0 \end{aligned}$$

and  $\underline{x}(\bar{d}_t, b_t)$  as the lowest  $x_t$  such that both non-negative consumption and  $a_t \geq \underline{a}(d_t, b_t)$  are feasible under *keeping*

$$\underline{x}(\bar{d}_t, b_t) \equiv \max \left\{ (\theta_{b_t} - \tau_s) \cdot \bar{d}_t, x_t - [\underline{a}(\bar{d}_t, b_t) + (1 - \tau_s) \cdot \bar{d}_t] \right\}$$

Under *adjusting* and *renting* a  $x_t \geq 0$  is both necessary and sufficient for satisfying these conditions.

The state spaces we are interested in are consequently given by

$$\begin{aligned} \mathcal{S} &\equiv \left\{ (z_t, b_{t-1}, \bar{d}_t, x_t) \mid x_t \geq 0, \bar{d}_t \geq 0, z_t, b_{t-1} \in \{E, R\} \right\} \\ \mathcal{S}^{keep} &\equiv \left\{ (z_t, b_{t-1}, \bar{d}_t, x_t) \mid x_t \geq \max \left\{ \underline{x}(\bar{d}_t, b_t), 0 \right\}, \bar{d}_t \geq 0, z_t, b_{t-1} \in \{E, R\} \right\} \\ \mathcal{S}^{adj} = \mathcal{S}^{rent} &\equiv \left\{ (z_t, b_{t-1}, x_t) \mid x_t \geq 0, z_t, b_{t-1} \in \{E, R\} \right\} \end{aligned}$$

such that defining the *transformed value function* as

$$\tilde{v}_t = -\frac{1}{v_t} \quad (\text{A.13})$$

we have

$$\begin{array}{l}
 \tilde{v}_t^{keep} (z_t, b_{t-1}, \bar{d}_t, x_t) \\
 \tilde{v}_t^{adj.} (z_t, b_{t-1}, x_t) \\
 \tilde{v}_t^{rent} (z_t, b_{t-1}, x_t) \\
 \tilde{v}_t (z_t, b_{t-1}, \bar{d}_t, x_t)
 \end{array}
 \left\{
 \begin{array}{l}
 > 0 \quad \forall \bar{d}_t > 0, \forall x_t \in \text{interior} (\mathcal{S}^{keep}) \\
 \geq 0 \quad \forall \bar{d}_t > 0, \forall x_t \in \text{boundary} (\mathcal{S}^{keep}) \\
 = 0 \quad \text{if } \theta_{b_T} \geq \tau_s, \forall \bar{d}_t > 0, \forall x_t \in \text{boundary} (\mathcal{S}^{keep}) \\
 > 0 \quad \forall x_t \in \text{interior} (\mathcal{S}^{adj.}) \\
 = 0 \quad x_t \in \text{boundary} (\mathcal{S}^{adj.}) \\
 > 0 \quad \forall x_t \in \text{interior} (\mathcal{S}^{rent}) \\
 = 0 \quad x_t \in \text{boundary} (\mathcal{S}^{rent}) \\
 > 0 \quad \forall x_t \in \text{interior} (\mathcal{S}) \\
 \geq 0 \quad \forall x_t \in \text{boundary} (\mathcal{S}) \\
 = 0 \quad \text{if } \theta_{b_t} \geq \tau_s, \forall x_t \in \text{boundary} (\mathcal{S})
 \end{array}
 \right.$$

Note that if  $\bar{v} > 0$  we do *not* need to be concerned with any  $x_t < 0$  even if *keeping* is feasible in this case because the households will never make a choice that can result in a  $x_t < 0$  even under worst case outcomes.

### A.3 Marginal Utility

Given durable ownership we have

$$\begin{aligned}
 u(C_t, S_t) &= \frac{(C_t^\omega S_t^{1-\omega})^{1-\rho}}{1-\rho} \rightarrow \\
 u'_C(C_t, S_t) &= \kappa \leftrightarrow \\
 \omega \cdot C_t^{\omega \cdot (1-\rho)-1} \cdot S_t^{(1-\omega) \cdot (1-\rho)} &= \kappa \leftrightarrow \\
 C_t &= \left( \frac{\kappa}{\omega \cdot S_t^{(1-\omega) \cdot (1-\rho)}} \right)^{\frac{1}{\omega \cdot (1-\rho)-1}} = u_C^{-1}(\kappa, S_t)
 \end{aligned}$$

Given renting we have

$$\begin{aligned}
 u(C_t, S_t) = u(E_t) &= \frac{\left( (\omega \cdot E_t)^\omega \cdot \left( \frac{1-\omega}{r_d} \cdot E_t \right)^{1-\omega} \right)^{1-\rho}}{1-\rho} = \varrho \cdot \frac{E_t^{1-\rho}}{1-\rho} \rightarrow \\
 \varrho &\equiv \left( \omega^\omega \cdot \left( \frac{1-\omega}{r_d} \right)^{1-\omega} \right)^{1-\rho} \\
 u'_E(E_t) &= \kappa \leftrightarrow \\
 \varrho \cdot E_t^{-\rho} &= \kappa \leftrightarrow \\
 E_t &= \left( \frac{\kappa}{\varrho} \right)^{-\frac{1}{\rho}} = u_E^{-1}(\kappa)
 \end{aligned}$$

## B Solution Algorithm

### B.1 Overview

The model is solved by backward induction using the DC-EGM algorithm. The DC-EGM algorithm developed in [Fella \(2014\)](#) and [Iskhakov, Jørgensen, Rust and Schjerning \(2014\)](#) builds on the EGM-algorithm but extend it to allow for discrete choices. We describe it in more detail in the following sections. More specifically:

- **Renting:** We use the DC-EGM-algorithm to find the optimal  $e_t$  over an endogenous  $x_t$ -grid.  $\bar{d}_t$  is *not* a state variable.
- **Keeping:** We use the DC-EGM-algorithm to find the optimal  $c_t$  over an endogenous  $x_t$ -grid.  $\bar{d}_t$  is a state variable with an exogenous grid.
- **Adjusting:** We use the DC-EGM-algorithm to find the optimal  $c_t$  over endogenous  $x_t$ -grids for each element in an exogenous grid of  $d_t$ -choices. We then merge these grids by interpolation and choose the optimal (discretized)  $d_t$  for each  $x_t$  in a value function iteration step.  $\bar{d}_t$  is *not* a state variable.

In the last period everything is consumed and we require  $a_t \geq 0$ . We construct the overarching value function by interpolation in order to check for convergence.

### B.2 Notation

To simplify the notation we:

1. Disregard the aggregate state and taste shocks.

2. Denote

$$R(a) = \begin{cases} 1 + r_b & \text{if } a < 0 \\ 1 + r & \text{else} \end{cases}$$

3. Use the following ordering:

(a)  $k = 0$  for *keep*.

(b)  $k = 1$  for *adj.*.

(c)  $k = 2$  for *rent*.

4. Define the following variables and functions:

$$\begin{aligned}\Omega_{t+1} &\equiv \mathbb{E}_t \left[ (\Gamma \cdot \psi_{t+1})^{1-\rho} \cdot v_{t+1}(\bar{d}_+, x_+) \right] \\ \bar{d}_+ = \bar{d}_+(d_t, \psi_{t+1}) &\equiv 1/(\Gamma \cdot \psi_{t+1}) \cdot (1 - \delta) \cdot d_t \\ x_+ = x_+(a_t, d_t, \psi_{t+1}, \xi_{t+1}) &\equiv 1/(\Gamma \cdot \psi_{t+1}) \cdot (1 + r(a_t, d_t)) \cdot a_t \\ &\quad + \xi_{t+1} + (1 - \tau_s) \cdot \bar{d}_+\end{aligned}$$

5. Introduce a *future choice weight function*,  $\kappa(k_{t+1}, \bar{d}_+, x_+)$ , where:

$$\begin{aligned}\kappa_{t+1}(0, \bar{d}_+, x_+) &\equiv \begin{cases} 1 - \bar{v} & \text{if } v_{t+1}^0(\bar{d}_+, x_+) \geq \max \{ v_{t+1}^1(\bar{d}_+, x_+), v_{t+1}^2(\bar{d}_+, x_+) \} \\ 0 & \text{else} \end{cases} \\ \kappa_{t+1}(1, \bar{d}_+, x_+) &\equiv \begin{cases} 0 & \text{if } v_{t+1}^1(\bar{d}_+, x_+) < v_{t+1}^2(\bar{d}_+, x_+) \\ \bar{v} & \text{else if } v_{t+1}^0(\bar{d}_+, x_+) \geq v_{t+1}^1(\bar{d}_+, x_+) \\ 1 & \text{else} \end{cases} \\ \kappa_{t+1}(2, \bar{d}_+, x_+) &\equiv \begin{cases} 0 & \text{if } v_{t+1}^2(\bar{d}_+, x_+) < v_{t+1}^1(\bar{d}_+, x_+) \\ \bar{v} & \text{else if } v_{t+1}^0(\bar{d}_+, x_+) \geq v_{t+1}^2(\bar{d}_+, x_+) \\ 1 & \text{else} \end{cases}\end{aligned}$$

### B.3 Euler Equations

Conditional on the current  $k_t$ -choice, the current  $d_t$ -choice, the future consumption functions,  $c_{t+1}^{*,k}(\bullet)$ , the durable service functions,  $s_{t+1}^{*,k}(\bullet)$ , and the future value functions,  $v_{t+1}^k(\bullet)$ , the optimal choice of  $c_t$  must necessarily satisfy one of the following Euler-conditions depending on the implied  $a_t$ :

$$a_t = \underline{a}(d_t) : u'_c(c_t, s_t) \geq \beta R(a_t) \cdot \Upsilon_{t+1} \quad (\text{B.1})$$

$$a_t \in (\underline{a}(d_t), 0) : u'_c(c_t, s_t) = \beta R(a_t) \cdot \Upsilon_{t+1} \quad (\text{B.2})$$

$$a_t = 0 : u'_c(c_t, s_t) \leq \beta(1 + r_b) \cdot \Upsilon_{t+1} \quad (\text{B.3})$$

$$a_t = 0 : u'_c(c_t, s_t) \geq \beta(1 + r) \cdot \Upsilon_{t+1} \quad (\text{B.4})$$

$$a_t \in (0, \infty) : u'_c(c_t, s_t) = \beta R(a_t) \cdot \Upsilon_{t+1} \quad (\text{B.5})$$

where

$$\Upsilon_{t+1} \equiv \mathbb{E}_t \left[ (\Gamma \cdot \psi_{t+1})^{-\rho} \cdot u'_c(c_{t+1}^*, s_{t+1}^*) \right]$$

Note that these Euler-conditions are necessary but *not* sufficient. All optimal

consumption choices must thus be a solution to the relevant case, but some of their solutions are not optimal choices. Turning the problem around, we can, however, use the Euler-conditions to derive a *full set of possible solutions*, and then remove the irrelevant ones using an upper envelope algorithm.

## B.4 Discretization and Interpolation

We discretize  $\psi_{t+1}$  and  $\xi_{t+1}$  using standard Gauss-Hermite quadrature into  $\psi^i$  and  $\xi^j$  with  $\#\psi$  and  $\#\xi$  nodes and  $\pi_\psi^i$  and  $\pi_\xi^j$  as associated weights.<sup>14</sup> The inner parts of the right-hand sides of equation (B.1)-(B.5) are then given by

$$\Upsilon_{t+1} \approx \sum_{i=1}^{\#\psi} \pi_\psi^i \cdot (\Gamma \cdot \psi^i)^{-\rho} \sum_{j=1}^{\#\xi} \pi_\xi^j \sum_{k_{t+1}=0}^2 \kappa_{t+1}(k_{t+1}, \bar{d}_+, x_+) \cdot u'_c(c_{t+1}^*, s_{t+1}^*) \quad (\text{B.6})$$

where  $\bar{d}_+ = \bar{d}_+(d_t, \psi^i)$  and  $x_+ = x_+(a_t, d_t, \psi^i, \xi^j)$ , and  $v_{t+1}^k(\bar{d}_+, x_+)$  (inside  $\kappa_{t+1}(\bullet)$ ),  $c_{t+1}^{k,*}(\bar{d}_+, x_+)$  and  $s_{t+1}^*(\bar{d}_+, x_+)$  are evaluated with bi-linear interpolation between grid points.

Given *adjusting* or *renting* in the future period we do not need to interpolate in the  $\bar{d}_t$ -dimension. Given *renting* in the future period intra-temporal optimization implies

$$u'_c(c_{t+1}^{*,2}(\bullet), s_{t+1}^{*,2}(\bullet)) = u'_e(e_{t+1}^{*,2}(\bullet))$$

In a similar manner, the continuation value of a joint  $a_t$  and  $d_t$  can be evaluated by

$$\Omega_{t+1} \approx \sum_{i=1}^{\#\psi} \pi_\psi^i \cdot (\Gamma \cdot \psi^i)^{1-\rho} \sum_{j=1}^{\#\xi} \pi_\xi^j \sum_{k_{t+1}=0}^2 \kappa_{t+1}(k_{t+1}, \bar{d}_+, x_+) \cdot v_{t+1}^k(\bar{d}_+, x_+) \quad (\text{B.7})$$

where  $v_{t+1}^k(\bar{d}_+, x_+)$  is again evaluated with bi-linear interpolation between grid points.

## B.5 Full Set of Possible Solution

The full set of solutions to equations (B.1) to (B.5) can be found by first evaluating the right-hand sides for given  $a_t$  (and  $d_t$ ) and then using the inverse marginal utility

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<sup>14</sup>In the case were  $\psi_{t+1}$  is a mixture of two log-normals, we use two sets of Gauss-Hermite quadrature nodes.

function to derive the implied  $c_t$ .

If general we do the following steps:

1. Construct the stacked *asset* vector  $\vec{a} = \{\vec{a}_a, \vec{a}_-, \vec{a}_0, \vec{a}_+\}$ , and the associated stacked *consumption* and *continuation value* vectors  $\vec{c} = \{\vec{c}_a, \vec{c}_-, \vec{c}_0, \vec{c}_+\}$  and  $\vec{\Omega} = \{\vec{\Omega}_a, \vec{\Omega}_-, \vec{\Omega}_0, \vec{\Omega}_+\}$ ; all have length  $\#_a = \#_a + \#_{a-} + \#_0 + \#_{a+}$ .
2. Consider the *negative* region:
  - (a)  $\vec{a}_-$  has entries from  $\underline{a}(d_t)$  to 0 (both included).
  - (b)  $\vec{c}_-$  is found using equation (B.2).
  - (c)  $\vec{\Omega}_-$  is found using equation (B.7).
3. Consider the *positive* region:
  - (a)  $\vec{a}_+$  has entries from 0 to some maximum  $\bar{a}$  (both included).
  - (b)  $\vec{c}_+$  is found using equation (B.5).
  - (c)  $\vec{\Omega}_+$  is found using equation (B.7).
4. Consider the *constrained* region (see (B.1)):
  - (a)  $\vec{a}_a$  has only  $\underline{a}(d_t)$ -entries.
  - (b)  $\vec{c}_a$  has increasing entries between 0 (included) and the first element in  $\vec{c}_-$  (not included).
  - (c)  $\vec{\Omega}_a$  has all entries equal to the first element in  $\vec{\Omega}_-$ . The first element in  $\vec{\Omega}_-$  corresponds to the point  $\underline{a}(d_t)$ .
5. Consider the *zero* region (see (B.3) and (B.4)):
  - (a)  $\vec{a}_0$  has only zero entries.
  - (b)  $\vec{c}_0$  has increasing entries between the *last* element in  $\vec{a}_-$  and the *first* element in  $\vec{a}_+$  (both not included).
  - (c)  $\vec{\Omega}_0$  has all entries equal to the first element in  $\vec{\Omega}_+$  (or equivalently the last in  $\vec{\Omega}_-$ ).

6. Construct the *value-of-choice* vector  $\vec{v}$  by

$$\vec{v} = u(\vec{d}, \vec{s}) + \beta \cdot \vec{\Omega}$$

where

$$\vec{s} = \begin{cases} \bar{d}_t & \text{if } k_t = 0 \\ d_t & \text{if } k_t = 1 \\ s(c^{-1}(\vec{c})) & \text{if } k_t = 2 \end{cases}$$

7. Construct the endogenous market resources after liquidation vector  $\vec{x}$  using

$$\vec{x} = \begin{cases} \vec{a} + [(1 - \tau_s) \cdot \bar{d}_t + \vec{c}] & \text{if } k_t = 0 \\ \vec{a} + [(1 + \tau_b) \cdot d_t + \vec{c}] & \text{if } k_t = 1 \\ \vec{a} + [\vec{c} + s(c^{-1}(\vec{c}))] & \text{if } k_t = 2 \end{cases}$$

Given *renting* in the current period we can instead use Euler-equations on the form

$$u'_e(e_t) = \beta R_b \cdot \Upsilon_{t+1}$$

and construct an expenditure vector  $\vec{e}$  instead of a consumption vector  $\vec{c}$  and use that  $\vec{x} = \vec{a} + \vec{e}$ .

We also have the following special cases:

1. **No negative region**,  $a(d_t) \geq 0$ : drop step 2 and 5.
2. **No exogenous borrowing constraint**,  $\underline{a}(d_t) > -(1 - \theta) \cdot d_t$ : drop step 4.
3. **No interest rate differential**,  $r = r_b$ : drop step 5 (to avoid duplicates the asset vector for the negative region must now *not* include  $a_t = 0$ ).

## B.6 Implementation

In the discretization we use:

- 1,000 nodes for  $d_t$ .
- 400 nodes for  $x_t$ .
- 400 nodes for  $a_t$  (100 on the constraint, 60 in the zero region)



- 250 nodes for  $\bar{d}_t$ .
- 6 nodes for both  $\psi_t$  and  $\xi_t$ .
- $T = 50$

The discretizations are all done in such a way that there are more nodes in the regions with more curvature in the choice and value functions.

The algorithm is implemented in *Python 2.7*, but the core part is written in *C* parallelized using *OpenMP* and called from Python using *CFFI*. Only free open source languages and programs are needed to run the code. The code-files are available from the authors upon request.

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