PhD Dissertation

Essays on the Interactions between Financial Markets, the Macroeconomy, and Economic Policy

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Contents

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Søren Hove Ravn
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Summary

This dissertation consists of four chapters, each of which can be read separately. All four chapters are concerned with the same overall topic though, namely the linkages between financial markets, the real economy, and economic policy. The importance of such linkages has been made painfully clear during the recent financial and economic crises. In retrospect, many macroeconomic models failed to pay sufficient attention to financial markets before the crisis. As a result, the last five years have seen the emergence of a large literature seeking to incorporate financial factors into models of the macroeconomy. In the first three chapters of this dissertation, I contribute to this literature. A particular topic that has received a certain attention in the wake of the crisis is the potential existence of non-linearities in the way financial markets affect the real economy. The first two chapters of this dissertation contain studies of the effects of various types of asymmetries or non-linearities that arise at the intersection between financial markets and the macroeconomy. In the third chapter, I introduce profit-maximizing banks and a version of relationship banking into a general equilibrium model of the macroeconomy. The absence of a banking sector was one important shortcoming of macroeconomic models before the crisis. Finally, in the fourth chapter of this dissertation, which is joint work with Morten Spange, we study the effects of fiscal policy in Denmark. In the wake of the crisis, fiscal policy has made a forceful return to the academic research agenda. Our study contributes to the recent literature about the circumstances under which the fiscal multiplier is likely to be large or small.

A topic that seems to have returned to the policy debate in the wake of the crisis is how monetary policy should deal with asset price movements, as discussed by Kuttner (2011). I study this topic in the first two chapters. In chapter 1, I study whether monetary policy in the US has featured an asymmetric reaction to stock prices in the years leading up to the crisis. To avoid endogeneity problems, I employ the method of identification through heteroskedasticity developed by Rigobon and Sack (2003). I augment their method so as to study whether the reaction to stock price increases and decreases is different. Using a daily dataset spanning the period 1998-2008 I show that during this period, interest rates were cut in response to declining stock prices, whereas monetary policy remained inactive when stock prices rose. This result is confirmed in an estimated, augmented Taylor rule using lower-frequency (monthly) data. I use both OLS

and IV methods, as recent studies have suggested that these methods are not necessarily subject to endogeneity problems when estimating contemporaneous Taylor rules. Chapter 1 was published in the *B.E. Journal of Macroeconomics* in 2012 (see Ravn, 2012).

The results in chapter 1 provide an empirical input to the recent debate about monetary policy and asset prices. Two recent studies support the finding of an asymmetric reaction to stock markets in the US (Hall, 2011; Hoffmann, 2013). In chapter 2, I use a Dynamic Stochastic General Equilibrium (DSGE) model to study the impact of such a policy on macroeconomic outcomes. Furthermore, I investigate how an asymmetric policy interacts with already existing asymmetries in the way stock markets affect the macroeconomy. For example, empirical studies have found that the financial accelerator of Bernanke *et al.* (1999) may exert a larger impact on the macroeconomy during recessions, when firms' net worth is already low and many firms are in need of external financing, than in booms. Other studies have shown that the wealth effect of stock prices on consumption is likely to be stronger during periods of falling asset prices, in line with the implications of loss aversion with respect to consumption or financial wealth. I study the circumstances under which such a policy may succeed in 'correcting for' such inherent asymmetries. The results can be summarized as follows: First, booms in inflation and output are amplified in the presence of such an asymmetric policy, while recessions are dampened. In other words, an asymmetric business cycle results. Second, such a policy gives rise to an additional rise in stock prices in the wake of positive shocks to the economy, as private agents realize that once stock prices start falling, the central bank will loosen monetary policy. Finally, an asymmetric policy is able to 'correct for' existing asymmetries in the wake of supply shocks, but not after demand shocks.

In the third chapter, I introduce monopolistically competitive banks and a reduced-form version of relationship banking into a DSGE model. The objective of this paper is to study endogenous movements in collateral requirements over the business cycle. Endogenous changes in credit standards have been documented by a number of empirical studies, and while a range of competing theories have been proposed in the finance literature to explain this, few attempts have been made to incorporate this feature into macroeconomic models. To bridge this gap, I build a DSGE model in which profit maximization by banks gives rise to endogenous, countercyclical movements in collateral requirements. The key

assumption behind this result is that firms prefer to borrow from the same bank as in previous periods. In other words, I assume that firms display 'deep habits' in their demand for bank loans, as in Aliaga-Diaz and Olivero (2010), building in turn on the work of Ravn *et al.* (2006). In the presence of deep habits, a bank that manages to increase its market share today will benefit from this in future periods, as more borrowers will have developed a 'habit' for this bank's loan products. When a persistent, positive shock hits the economy, future market shares become more valuable, so banks start to compete more intensively for potential borrowers. As non-price competition is widespread in the banking sector, I assume - in contrast to Aliaga-Diaz and Olivero (2010) - that banks compete by lowering collateral requirements rather than lending rates. As a result, collateral requirements are lowered, and loan-to-value ratios raised during an economic boom. In turn, higher loan-to-value ratios act as an amplifier of business cycle fluctuations. However, I find that this amplification is quite modest at the macroeconomic level, suggesting that endogenous movements in credit standards may be less important than previously thought.

Chapter 4 is joint work with Morten Spange. In this paper, we undertake an empirical investigation of the effects of fiscal policy in Denmark by constructing an open economy version of the structural vector-autoregressive (SVAR) model proposed by Blanchard and Perotti (2002). At the onset of the recent crisis, as interest rates quickly reached their zero lower bound, a number of countries engaged in expansionary fiscal policy measures. Later on, many of the same countries have turned to fiscal austerity. In a time of activist fiscal policy, a solid knowledge of the impact of such measures is of key importance. Yet empirical estimates of fiscal multipliers are 'all over the map', according to Leeper (2010). We seek to shed light on the size of the fiscal multiplier in Denmark, which is interesting also from a theoretical point of view. On one hand, Denmark's fixed exchange rate implies that the nominal interest rate remains fixed after a fiscal expansion. This facilitates a substantial impact of fiscal stimulus on the real economy and thus points towards a relatively large multiplier. On the other hand, the large degree of openness of the Danish economy means that a sizeable share of the fiscal stimulus will be directed towards imported goods and services. Our results suggest that the 'monetary accomodation channel' dominates the 'leakage effect' in the very short run. In our baseline specification of the SVAR model, we find that the multiplier has been around 1.3 during the period 1983-2011,

i.e. since the adoption of a fixed exchange rate. However, we also find that the expansionary effects of government spending are very shortlived. The multiplier is above 1 only in the first quarter after the expansion, and is significantly greater than zero only during the first year. Our results indicate that the effects of fiscal stimulus die out as the stimulus itself is removed, suggesting that the dynamic effects of government spending in Denmark are small. Moreover, a key finding from the recent literature, as well as from our study, is that the fiscal multiplier is far from constant over time and across economic states. We find that the multiplier was below 1 in the 1970's and 1980's, while it has been above 1 since around 1990. This is consistent with the standard view that a credibly fixed exchange rate, as well as sound public finances, may give rise to a relatively large government spending multiplier.

Resume

Denne afhandling består af fire kapitler, som kan læses uafhængigt af hinanden. Alle fire kapitler omhandler dog det samme overordnede emne, nemlig samspillet mellem de finansielle markeder, den reale økonomi og den økonomiske politik. Betydningen af dette samspil er blevet gjort ubehageligt klart i løbet af den aktuelle finansielle og økonomiske krise. I bagklogskabens lys var mange makroøkonomiske modeller tidligere ikke tilstrækkeligt opmærksomme på de finansielle markeder. Derfor har der i løbet af de seneste fem år i den økonomiske litteratur været stor fokus på at indarbejde finansielle faktorer i makroøkonomiske modeller. I de første tre kapitler af denne afhandling bidrager jeg til denne litteratur. Et emne, som har været genstand for en del opmærksomhed efter krisen, er eksistensen af potentielle ikke-lineariteter i den måde, hvorpå finansielle markeder påvirker den reale økonomi. I de første to kapitler i denne afhandling studerer jeg effekterne af forskellige asymmetrier eller ikke-lineariteter, som opstår, når de finansielle markeder og makroøkonomien støder sammen. I det tredje kapitel introducer jeg profitmaksimerende banker og en version af vedvarende bankforbindelser i en makroøkonomisk generel ligevægtsmodel. Fraværet af en banksektor udgjorde en betydelig svaghed ved makroøkonomiske modeller før krisen. I det fjerde og sidste kapitel i afhandlingen, som er udarbejdet i samarbejde med Morten Spange, studerer vi virkningen af finanspolitik i Danmark. Finanspolitikken er vendt stærkt tilbage til den akademiske dagsorden i kølvandet på den økonomiske krise. Vores studie bidrager til den nylige litteratur omkring, under hvilke omstændigheder den finanspolitiske multiplikator er henholdsvis stor eller lille.

Et emne, som tilsyneladende er vendt tilbage til den politiske debat i kølvandet på krisen, er, hvordan pengepolitikken skal håndtere udsving i aktivpriser, jvf. Kuttner (2011). Jeg studerer dette emne i de første to kapitler. I kapitel 1 undersøger jeg, hvorvidt pengepolitikken i USA indebar en asymmetrisk reaktion på bevægelser i aktiepriserne i årene op til krisen. For at undgå endogenitetsproblemer anvender jeg en metode, som kaldes identifikation via heteroskedasticitet og er udviklet af Rigobon og Sack (2003). Jeg udvider deres metode, så den kan anvendes til at studere, hvorvidt reaktionen på stigninger og fald i aktiepriserne er forskellig. På baggrund af et dagligt datasæt for perioden 1998-2008 finder jeg, at et fald i aktiepriserne i denne periode blev mødt med en sænkning af renten, mens pengepolitikken forblev uændret efter en aktieprisstigning. Jeg op-

når samme resultat ved at estimere en udvidet Taylor-regel på et lav-frekvent, månedligt datasæt. Her anvender jeg både OLS- og IV-estimation, idet nyere studier har antydet, at disse metoder kan undgå endogenitetsproblemer, når man estimerer en Taylor-regel, hvor pengepolitikken antages at reagere på samtidige bevægelser i inflation og BNP. Kapitel 1 er blevet publiceret i *B.E. Journal of Macroeconomics* i 2012 (se Ravn, 2012).

Resultaterne i kapitel 1 udgør et empirisk bidrag til den nylige debat om samspillet mellem pengepolitik og aktivpriser. To nye studier bekræfter tilstedeværelsen af en asymmetrisk politik over for aktiemarkedet i USA (Hall, 2011; Hoffmann, 2013). I kapitel 2 anvender jeg en Dynamisk Stokastisk Generel Ligevægtsmodel (DSGE-model) til at undersøge de makroøkonomiske implikationer af en sådan politik. Derudover undersøger jeg, i hvilket omfang en asymmetrisk politik interagerer med allerede eksisterende asymmetrier i den måde, hvorpå aktiemarkedet påvirker makroøkonomien. Eksempelvis har empiriske studier fundet, at den såkaldte finansielle accelerator (Bernanke m.fl., 1999) har størst effekt på makroøkonomien i en recession, hvor virksomheders aktiver har en lav værdi, og mange virksomheder har behov for ekstern finansiering. Andre studier har vist, at formueeffekten på det private forbrug som følge af ændringer i aktiepriserne tenderer mod at være stærkere, når aktiepriserne er faldende, end når de stiger. Dette er i overensstemmelse med teorien om tabsaversion med hensyn til det private forbrug eller den finansielle formue. Jeg undersøger, om en asymmetrisk politik kan tænkes at korrigere for sådanne asymmetrier. Resultaterne i dette kapitel kan sammenfattes som følger: For det første bliver stigninger i inflationsraten og BNP forstærket, mens fald i disse variable bliver dæmpet. Der opstår altså asymmetriske konjunkturcykler. For det andet fører den asymmetriske politik til kraftigere stigninger i aktiepriserne, når økonomien rammes af positive stød. Det sker, fordi de fremadskuende agenter forudser, at centralbanken vil sætte renten ned, når aktiepriserne igen begynder at falde. Endelig finder jeg, at den asymmetriske politik er i stand til at korrigere for allerede eksisterende asymmetrier, når økonomien rammes af udbudsstød, men ikke ved efterspørgselsstød.

I det tredje kapitel udvider jeg en DSGE-model med en banksektor karakteriseret ved monopolistisk konkurrence samt en reduceret form-udgave af vedvarende bankforbindelser. Målet er at studere endogene bevægelser i bankernes krav til sikkerhedsstillelse over konjunkturcyklen. Endogene udsving i bankernes

kreditstandarder er dokumenteret i en række empiriske studier. Mens der i den finansielle litteratur findes et bredt udvalg af teorier, som kan forklare dette, er der blevet gjort ganske få forsøg på at indarbejde endogene kreditstandarder i makroøkonomiske modeller. Jeg forsøger at udfylde dette hul i litteraturen ved at opstille en DSGE-model, hvor bankernes profitmaksimering medfører endogene, kontracykliske udsving i kravene til værdien af den pant, virksomhederne stiller som sikkerhed for et lån. Den centrale antagelse bag dette resultat er, at virksomheder foretrækker at låne fra den samme bank, som de tidligere har lånt hos. Med andre ord antager jeg, at virksomhedernes efterspørgsel efter banklån er karakteriseret ved 'dyb vanedannelse', jvf. Aliaga-Diaz og Olivero (2010), som igen bygger på Ravn m.fl. (2006). Under dyb vanedannelse vil en bank, som forøger sin markedsandel i indeværende periode, også profitere af denne forøgelse i kommende perioder, idet flere låntagere vil have dannet vaner over for den pågældende banks produkter. Når økonomien rammes af et positivt og persistent stød, bliver fremtidige markedsandele mere værdifulde for bankerne. Derfor begynder bankerne at konkurrere mere intenst om potentielle nutidige og fremtidige kunder. Efter som konkurrence i banksektoren ofte sker på andre parametre end prisen på et lån (udlånsrenten), antager jeg - i modsætning til Aliaga-Diaz og Olivero (2010) - at bankerne konkurrerer ved at sænke kravene til værdien af låntagerens pant i forhold til lånets størrelse, dvs. ved at tillade en stigning i belåningsgraden. Dette medfører, at bankerne hæver den tilladte belåningsgrad under en højkonjunktur, og sænker den i en recession. Dette fører til en yderligere amplificering af de økonomiske konjunkturudsving. Jeg finder dog, at denne amplificering er af moderat betydning på det makroøkonomiske niveau. Dette antyder, at endogene udsving i bankernes kreditstandarder kan være mindre betydningsfulde end tidligere antaget.

Kapitel 4 er udarbejdet i samarbejde med Morten Spange. I dette kapitel foretager vi et empirisk studie af virkningen af finanspolitik i Danmark ved at konstruere en åben økonomi-version af den strukturelle vektor-autoregressive (SVAR) model fra Blanchard og Perotti (2002). Idet renterne ramte deres nedre nulgrænse i begyndelsen af den aktuelle krise greb en række lande til ekspansiv finanspolitik. Senere har mange af de samme lande i stedet forsøgt sig med finanspolitiske besparelser. Når der føres en sådan aktiv finanspolitik, er en solid viden om virkningen heraf af afgørende betydning. Imidlertid er de empiriske estimater af finanspolitiske multiplikatorer spredt for alle vinde ifølge Leeper

(2010). Vi forsøger at belyse størrelsen af den finanspolitiske multiplikator i Danmark, som også er interessant fra et teoretisk synspunkt. På den ene siden medfører Danmarks fastkurspolitik, at den nominelle rente forbliver uændret efter en finanspolitisk lempelse. Dette giver mulighed for, at lempelsen kan have betydelige realøkonomiske effekter, og peger derfor mod en relativt stor multiplikator. På den anden side er den danske økonomi meget åben, hvilket betyder, at en stor del af en given finanspolitisk lempelse vil gå til importerede varer og tjenester. Vores resultater tyder på, at betydningen af den monetære akkomodering er større end lækageeffekten på helt kort sigt. I vores foretrukne specifikation af SVAR-modellen finder vi en multiplikator på omkring 1,3 i perioden 1983-2011, dvs. siden fastkurspolitikken blev indført. Vi finder imidlertid også, at virkningen af en finanspolitisk lempelse er meget kortvarig. Multiplikatoren er således kun større end 1 i det første kvartal efter lempelsen, og bliver insignifikant efter et år. Vores resultater indikerer, at effekten af en finanspolitisk lempelse dør ud i takt med, at lempelsen rulles tilbage, hvilket tyder på, at de dynamiske effekter af stigninger i det offentlige forbrug er små i Danmark. Endvidere er et centralt resultat fra den nyere litteratur, såvel som fra vores studie, at den finanspolitiske multiplikator langt fra er konstant over tid og på tværs af lande. Vi finder således, at multiplikatoren var mindre end 1 i 1970'erne og 1980'erne, mens den har været over 1 siden omkring 1990. Dette er konsistent med den gængse opfattelse af, at en troværdig, fast valutakurs samt sunde offentlige finanser giver mulighed for, at den finanspolitiske multiplikator kan være relativt stor.

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The paper titled "Has the Fed Reacted Asymmetrically to Stock Prices?", which constitutes chapter 1 of this dissertation, has been published in 2012 in the *B.E. Journal of Macroeconomics (Topics)*, vol. 12(1), article 14. Therefore, I am not authorized to include the paper in the published version of this dissertation. Instead, the reader is referred to the webpage of the relevant journal in order to obtain the published manuscript. This webpage can be found at:

http://www.degruyter.com/view/j/bejm

Asymmetric Monetary Policy Towards the Stock Market: A DSGE Approach*

Søren Hove Ravn

Abstract

In the aftermath of the financial crisis, it has been argued that a guideline for future policy should be to take the 'a' out of 'asymmetry' in the way monetary policy deals with asset price movements. Recent empirical evidence has suggested that the Federal Reserve may have followed an asymmetric policy towards the stock market in the pre-crisis period. The present paper studies the effects of such a policy in a DSGE model. The asymmetric policy rule introduces an important non-linearity into the model: Booms in output and inflation tend to be amplified, while recessions are dampened. We further investigate to what extent an asymmetric stock price reaction could be motivated by the desire of policymakers to correct for inherent asymmetries in the way stock price movements affect the macroeconomy.

Keywords: Asymmetries, Monetary Policy, Asset Prices, DSGE Modelling.

JEL classification: E13, E32, E44, E52, E58.

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1 Introduction

While the recent financial and economic crisis does not invalidate everything we have learned about macroeconomics since 1936, as Barro (2009) eloquently puts it, it has led economists to reconsider some ideas that once were common sense. As one example, the crisis has led to a revival of the debate about the role of asset prices in monetary policy; see Kuttner (2011) for an overview.[1] Despite some enduring disagreement, a certain degree of consensus had been reached before the crisis, according to which central banks should not lean against asset price movements; if not for other reasons then because of the practical problems in doing so. Instead, they should stand ready to cut the interest rate in response to plummeting asset prices.[2] The aftermath of the crisis has witnessed an emerging appreciation and critique of an inherent asymmetry in this approach to monetary policy (see White (2009), Mishkin (2010), and Issing (2011), among others). In the words of Stark (2011), the consensus implied that '*monetary policy should react to asset price busts; not to asset price booms*'. Issing (2011) points to the risk that such a policy might lead to moral hazard problems by covering part of the downside risk faced by investors in the stock market.

Some recent studies lend empirical support to the existence of an asymmetric monetary policy towards the stock market in the US before the crisis. Ravn (2012) finds that during the period 1998-2008, a 5 % drop in the S&P 500 index increased the probability of a subsequent 25 basis point interest rate cut by between 1/3 and 1/2. On the other hand, he finds no significant policy reaction to stock price increases. Similarly, Hoffmann (2013) reports that for the longer period 1987-2008, the Federal Reserve lowered interest rates in response to stock market drops, but did not raise rates when stock prices boomed. For the same sample period, Hall (2011) finds that stock price deflation led to a highly significant cut in the interest rate, and that the inclusion of stock price deflation improves the fit of an estimated Taylor rule.

[1] This debate goes back at least to Bernanke and Gertler (1999, 2001), who argue that monetary policy should not react to asset prices *per se*. This has been supported by, among others, Gilchrist and Leahy (2002), Tetlow (2005), and Faia and Monacelli (2007), as well as in speeches by leading Federal Reserve officials (Kohn, 2006; Mishkin, 2008). In contrast, Cecchetti *et al.* (2000) find that the optimal monetary policy rule does include a reaction to the stock market. This position has received support from Bordo and Jeanne (2002), Borio and White (2003), and recently Pavasuthipaisit (2010) and Leduc and Natal (2011).

[2] This view has been coined the 'pre-crisis consensus' by Bini Smaghi (2009), and the 'Jackson Hole consensus' by Issing (2009).

In this paper, we contribute to the recent debate by examining the effects of an asymmetric monetary policy in general equilibrium. We build a Dynamic Stochastic General Equilibrium (DSGE) model with an explicit role for asset prices through the financial accelerator of Bernanke *et al.* (1999). We then allow the central bank to follow a monetary policy rule with an asymmetric reaction to stock prices. This introduces an important discontinuity into the model that cannot be 'log-linearized away'. As a result, it is not possible to solve the model using standard techniques. Instead, we apply a numerical solution method which exploits the piecewise linearity of the model. Essentially, the model consists of two linearized systems around the same steady state; one system for when stock prices are increasing (or constant), and another for when they are decreasing. We construct a shooting algorithm to detect the switching points between these systems in order to solve the model. In this sense, we make a methodological contribution to the sparse literature on endogenous regime switching in monetary policy initiated by Davig and Leeper (2006). The solution method is similar to the one used by Bodenstein *et al.* (2009) to deal with the zero lower bound on interest rates, which in turn builds on work by Eggertson and Woodford (2003) and Christiano (2004).

The analysis uncovers some interesting implications of the asymmetric policy. By reacting only to stock price drops, the central bank induces an outcome where booms in output and inflation are amplified, while recessions are dampened. In other words, the asymmetric policy translates into an asymmetric business cycle. We briefly relate this finding to the existing literature on asymmetric business cycles. In addition, the asymmetric policy gives rise to what we call an *anticipation boom* in asset prices. In the wake of an expansionary shock, the asset price jumps up. It turns out that this jump is larger than in a model with no reaction to stock price changes, despite the fact that in both cases, the actual policy reaction to stock prices is zero during the asset price boom. The anticipation boom, which measures the *additional* rise in asset prices when the asymmetric policy is introduced, can be attributed to forward-looking agents anticipating that whenever stock prices start falling, the central bank will cut the interest rate. This implicit, partial insurance against asset price drops amplifies the rise in asset prices immediately after the shock. If the asymmetric policy reaction to stock prices is of the magnitude found in the recent empirical studies, these effects are quantitatively quite small. In the literature, an important divergence

exists between the magnitude of the reaction to asset prices found in empirical studies, which is often quite small, and the values used in theoretical investigations, which are usually a lot larger. To bridge this gap, we therefore also employ a value of the reaction parameter which is more in accordance with the values in other theoretical contributions. When this is done, the above effects are sizeable. In general, we conclude that while an asymmetric policy has the theoretical potential to generate severely skewed business cycles and important additional asset price volatility, the asymmetric reactions found in recent empirical studies are too small to have had quantitatively important macroeconomic effects.

We also discuss potential motivations for an asymmetric monetary policy. One such motivation could be an asymmetric loss function of the central bank, as previously studied in the literature. Another potential explanation is that such a policy could be an attempt by the central bank to 'correct for' other asymmetries in the economy, in particular in the way stock prices influence the macroeconomy. We therefore evaluate how an asymmetric monetary policy interacts with other potential asymmetries, such as the financial accelerator of Bernanke *et al.* (1999) and the stock wealth effect on consumption. We demonstrate that if the financial accelerator is assumed to be stronger when net worth of firms is low, as has been suggested by several authors, the asymmetric policy is able to 'cancel out' this asymmetry in the case of supply shocks, but not after demand shocks. A similar conclusion is reached under the assumption of asymmetric wealth effects.

The remainder of the paper is structured as follows. Section 2 describes the DSGE workhorse model. Section 3 illustrates the dynamics of the model and the implications of introducing an asymmetric reaction to stock prices. In section 4, we discuss possible explanations for the asymmetric policy within the model framework. Section 5 concludes. The appendix contains details about the model and the solution method.

2 The Model

The general equilibrium model is a version of the standard New-Keynesian sticky-price model with capital, augmented with the financial accelerator of Bernanke *et al.* (1999) in order to introduce a role for asset prices. An additional feature is that contracts are written in terms of the nominal interest rate as in Christensen & Dib (2008), introducing the debt-deflation channel of Fisher (1933). Christiano

et al. (2010) find that this channel is empirically relevant. The model is in large part similar to that of Christensen and Dib (2008) or Gilchrist and Saito (2008). This has the advantage that the dynamics of this class of models is well described in the literature, allowing us to isolate the effects of the asymmetric monetary policy rule. Moreover, this allows us to calibrate the model using the parameter values estimated by Christensen and Dib for the US economy for most of the parameters. Finally, this class of models is typically used in the literature on the role of asset prices in monetary policy cited above. The stochastic part of the model is quite parsimonious, as only two shocks are included: a technology shock and a monetary policy shock. These two shocks, which can loosely be interpreted as a supply and a demand shock, are sufficient to highlight the effects of the asymmetric policy.

2.1 Entrepreneurs

Entrepreneurs produce the intermediate goods that the final goods producers take as input. Each entrepreneur employs labor H_t and capital K_t, and produces output Y_t according to the following production technology:

$$Y_t \leq (A_t H_t)^{1-\alpha} {K_t}^{\alpha}. \tag{1}$$

The technology level A_t evolves according to

$$\ln(A_t) \equiv (1-\rho_a) A + \rho_a \ln(A_{t-1}) + \varepsilon_t^a, \tag{2}$$

where ε_t^a is a normally distributed shock to technology with mean zero. In each period, entrepreneurs face a constant probability $(1-\nu)$ of leaving the economy. As described by Bernanke *et al.* (1999), this assumption is made in order to ensure that entrepreneurs do not eventually accumulate enough capital to be able to finance their own activities entirely. We follow Christensen and Dib (2008) in allowing newly entering firms to inherit a portion of the net worth of those firms who exit the economy. This assumption is made in order to ensure that new entrepreneurs start out with non-zero net worth. In contrast, Bernanke *et al.* (1999) ensure this by assuming that entrepreneurs also work. This difference is of little importance for the results.

Entrepreneurs choose the inputs of capital and labor to maximize their profits,

subject to the production technology. As there is perfect competition in the entrepreneurial sector, the price which they receive for their products will be equal to the marginal cost of producing the intermediate good. This gives rise to the following first-order conditions:

$$mp_t = \alpha \frac{Y_t}{K_t} mc_t, \tag{3}$$

$$w_t = (1-\alpha) \frac{Y_t}{H_t} mc_t, \tag{4}$$

where mp_t denotes the real marginal productivity of capital, and mc_t is the real marginal production cost of entrepreneurs.

Each entrepreneur can obtain the capital needed for production in two ways: He can issue equity shares (internal financing), or he can borrow the money from a financial intermediary (external financing). Because internal financing is cheaper, as discussed below, entrepreneurs use all of their net worth, and borrow the remainder of their funding needs from the financial intermediary. The total funding needed by an entrepreneur is $q_t K_{t+1}$, where q_t is the real price of capital as measured in units of consumption. In order to ensure that any financial constraint faced by the entrepreneur applies to the capital stock as such, and not just to the investment in any given period, we assume that the entrepreneur must refinance his entire capital stock each period. If n_t denotes the net worth of the entrepreneur, the amount he needs to borrow is then $q_t K_{t+1} - n_{t+1}$. Letting f_t denote the external financing cost of one extra unit of capital, the demand for external finance must satisfy the following condition in optimum:

$$E_t \left[f_{t+1} \right] = E_t \left[\frac{mp_{t+1} + (1-\delta) q_{t+1}}{q_t} \right]. \tag{5}$$

The numerator on the right-hand side is the marginal productivity of a unit of capital plus the value of this unit of capital (net of depreciation) in the next period. If this condition was not satisfied, the capital demand of entrepreneurs would be either zero or infinite. Note that we interpret the price of capital q_t as the stock price in the model economy.[3] Equity shares are ultimately claims to the assets of firms, which in this model amounts to their capital stock. Therefore, in a model of this type, q_t is the relevant variable to enter the central bank's

[3] In the rest of the paper, we will use the terms *price of capital*, *asset price* and *stock price* interchangeably.

reaction function in order to model a reaction to stock prices.[4]

As in Bernanke *et al.* (1999), the existence of an agency problem between borrower and lender renders external finance more costly than internal finance. While entrepreneurs observe the outcome of their investments costlessly, the financial intermediary must pay an auditing cost to observe this outcome. Entrepreneurs must decide - after observing the outcome - whether to report a success or a failure of the project, i.e. whether to repay or default on the loan. If they default, the financial intermediary pays the auditing cost, and then claims the returns to the investment. Bernanke *et al.* (1999) demonstrate that the optimal financial contract involves an external finance premium (the difference between the cost of external and internal finance) which depends on the entrepreneur's net worth, and show that the marginal external financing cost is equal to the external finance premium times the opportunity cost of the investment; given by the risk-free real interest rate (the reader is referred to Bernanke *et al.* (1999) for details):

$$E_t\left[f_{t+1}\right] = E_t\left[\Psi\left(\frac{n_{t+1}}{q_t K_{t+1}}\right)\frac{R_t}{\pi_{t+1}}\right], \tag{6}$$

where the function $\Psi(\cdot)$ describes how the external finance premium depends on the financial position of the firm. $\frac{n_{t+1}}{q_t K_{t+1}}$ denotes the ratio of the firm's internal financing to its total financing, and is thus a measure of the leverage ratio. Equation (6) is the key to the financial accelerator mechanism. Bernanke *et al.* (1999) demonstrate that $\Psi'(\cdot) < 0$, implying that if firms' net worth goes up (or, equivalently, their leverage ratio goes down), the external finance premium falls, and firms get cheaper access to credit. The reason is that as the entrepreneur puts more of his own money behind the project, thus lowering the leverage ratio, the agency problem between borrower and lender is alleviated. The entrepreneur's incentive to undertake projects with a high probability of success increases, and as a result, the lender demands a lower return on the loans he makes. The drop in the external finance premium leads to an increase in the firm's demand for external finance, which in turn causes an increase in the firm's stock of capital in the next period, and thus its production level. In this way, to the extent that movements in net worth are procyclical, the financial accelerator works to amplify business cycle movements.

[4]This is standard in the literature; see for instance Tetlow (2005) or Gilchrist and Saito (2008). In Bernanke and Gertler (1999, 2001), the central bank reacts to the price of capital plus a bubble term.

The net worth of entrepreneurs consists of the financial wealth they have accumulated (i.e., profits earned in previous periods) plus the bequest Υ_t they receive from entrepreneurs leaving the economy:

$$n_{t+1} = \nu \left[f_t q_{t-1} K_t - E_{t-1} f_t \left(q_{t-1} K_t - n_t \right) \right] + (1 - \nu) \Upsilon_t. \tag{7}$$

2.2 Households

A continuum (of unit length) of households derive utility from an index of the final consumption goods produced by the retailers (C_t) and leisure ($1 - H_t$), and decide how much labor to supply to entrepreneurs producing intermediate goods. As all households are identical, they each solve the following utility maximization problem:

$$\max_{C_t, H_t, D_t} U = E_0 \sum_{t=0}^{\infty} \beta^t u \left(C_t, H_t \right), \tag{8}$$

with instantaneous utility function:

$$u \left(C_t, H_t \right) = \frac{\gamma}{\gamma - 1} \ln \left(C_t^{\frac{\gamma - 1}{\gamma}} \right) + \eta \ln \left(1 - H_t \right), \tag{9}$$

subject to the relevant budget constraint:

$$C_t + \frac{D_t - R_{t-1} D_{t-1}}{P_t} \leq \frac{W_t}{P_t} H_t + \Omega_t. \tag{10}$$

D_t are deposits which are stored at a financial intermediary at the risk-free rate of interest R_t. Ω_t denotes dividend payments deriving from households' ownership of retail firms. The first-order conditions of the household are presented in the appendix.

2.3 Capital Producers

The role of capital producers is to construct new capital K_{t+1} from invested final goods I_t and existing capital. As in Bernanke *et al.* (1999), it is implicitly assumed that capital producers rent existing capital from entrepreneurs within each period at a rental rate of zero. They face capital adjustment costs, implying

a non-constant price of capital q_t. We use the same quadratic functional form for the capital adjustment costs as Christensen and Dib (2008): $\frac{\chi}{2}\left(\frac{I_t}{K_t}-\delta\right)^2 K_t$. Profits of capital producers are then:

$$\Pi_t^c = q_t I_t - I_t - \frac{\chi}{2}\left(\frac{I_t}{K_t}-\delta\right)^2 K_t. \tag{11}$$

Choosing the level of investment that maximizes this expression results in the following equilibrium condition:

$$q_t - \chi\left(\frac{I_t}{K_t}-\delta\right) = 1. \tag{12}$$

Note that in the absence of adjustment costs, the parameter χ equals zero, so the optimality condition collapses to $q_t = 1$.[5] This illustrates that capital adjustment costs are necessary to create a time-varying price of capital. Moreover, the condition is essentially a Tobin's q-relation, ensuring that the investment level is chosen so that the 'effective' price of capital (i.e., net of capital adjustment costs) is equal to 1.

2.4 Retailers

Firms in the retail sector take intermediate goods as inputs, repackage these costlessly, and sell them. The retail sector is included in the model with the single purpose of creating price stickiness. Following Calvo (1983), price rigidity is introduced by assuming that in each period, only a fraction $(1-\xi)$ of firms in the retail sector are allowed to change their price. The price of firms who are not allowed to change their price is indexed with the steady state inflation rate π. This problem gives rise to the following first-order condition for the optimal price $P_t^n(i)$ set by firm i:

$$P_t^n(i) = \frac{\epsilon^p}{\epsilon^p - 1}\frac{E_t\left\{\sum_{s=0}^{\infty}(\beta\xi)^s \lambda_{t+s} Y_{t+s}(i) P_{t+s} mc_{t+s}\right\}}{E_t\left\{\sum_{s=0}^{\infty}(\beta\xi)^s \lambda_{t+s} Y_{t+s}(i) \pi^s\right\}}. \tag{13}$$

Here, λ_t is the Lagrange multiplier associated with the budget constraint in households' optimization, and the parameter ϵ^p measures the elasticity of sub-

[5] Recall that q_t is a real price measured in units of consumption. Hence, $q_t = 1$ will hold in the absence of adjustment costs, irrespective of the fact that the price level on consumption goods fluctuates due to the price stickiness faced by retailers.

stitution between different intermediate goods. The evolution of the aggregate price level is a weighted average of the price of those firms who are allowed to change their price in a given period, and all set the same new price, and of those who are not; whose prices are therefore indexed:

$$P_t = \left[(1-\xi) \left(P_t^n \right)^{1-\epsilon^p} + \xi \left(P_{t-1} \pi \right)^{1-\epsilon^p} \right]^{1/(1-\epsilon^p)}. \tag{14}$$

In the appendix, we demonstrate how the log-linearized versions of (13) and (14) can be combined to yield a standard version of the New-Keynesian Phillips Curve.

2.5 Monetary Policy

To introduce an asymmetric policy reaction to stock prices, we assume that the central bank follows a Taylor rule augmented with a term that captures a reaction to stock price drops. This is in line with the specifications used by Hoffmann (2013) and Hall (2011).[6] We further add interest rate smoothing, as this tends to improve the empirical performance of Taylor rules (Clarida *et al.*, 1999; Christiano *et al.*, 2010). This gives rise to the following monetary policy rule:

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R} \right)^{\rho_r} \left[\left(\frac{\pi_t}{\pi} \right)^{\phi_\pi} \left(\frac{Y_t}{Y} \right)^{\phi_y} \left\{ \left(\frac{\Delta q_t}{q} \right)^{\phi_q} \right\}^{1[\Delta q_t < 0]} \right]^{(1-\rho_r)} e^{\varepsilon_t^r}, \tag{15}$$

where $1[X]$ is the indicator function; equal to 1 if X is true and zero otherwise. This captures that the central bank is reacting to the change in stock prices only when this change is negative. ε_t^r is a normally distributed monetary policy shock with mean zero. The stated monetary policy rule allows for interest rate smoothing, as measured by the parameter ρ_r. The parameters ϕ_π and ϕ_y measure the monetary policy reaction to deviations of inflation from its target level, and of output from its steady state level, respectively. Note that the steady state or natural level of output (Y) is below the efficient level of output (Y^*) due to the

[6] Ravn (2012) attempts to control for the movements in the interest rate that are driven by macroeconomic variables such as output and inflation. Therefore, also his result is interpretable as a reaction to stock prices *on top of* the reaction to those variables, in line with the implicit assumption behind an augmented Taylor rule.

presence of monopolistic competition.

While this paper is the first to study theoretically a Taylor rule with a reaction to stock price drops, Taylor rules augmented with a symmetric reaction to stock price changes have been studied by Tetlow (2005) and Gilchrist and Saito (2008) in models largely similar to the one outlined above. A similar type of 'speed limit'-rule is also studied by Leduc and Natal (2011). The rule above is essentially a speed-limit rule with no upper speed limit.

2.6 Model Solution

The model consists of 15 equilibrium conditions in 15 variables, as described in the appendix. The equilibrium of the model consists of a vector of allocations $(C_t, H_t, Y_t, K_t, n_t, I_t)$ and prices $\left(\pi_t, R_t, w_t, mc_t, mp_t, q_t, f_t, \lambda_t, \left(\frac{P_t^n}{P_t}\right)\right)$ such that those 15 equations are satisfied. In the appendix, we present the steady state. The model is log-linearized around this steady state. However, the non-linear monetary policy rule implies that even after log-linearization, an important non-linearity remains in the model. As a result, the model cannot be solved with standard techniques. Instead, we solve the model using a numerical solution method which exploits the piecewise linearity of the model. This method follows the approach taken by Bodenstein *et al.* (2009) in order to deal with problems where the zero lower bound on interest rates is binding in a number of periods. While Bodenstein *et al.* study a one-off switch, we generalize the solution method to handle multiple switches between policy regimes. As the only non-linearity in the present model is the monetary policy reaction to asset prices, the model in effect consists of two linear systems; one for when asset prices are decreasing, and one for when they are non-decreasing. Following Bodenstein *et al.* (2009), we first build a shooting algorithm in order to identify the 'turning points' in the evolution of the asset price following a shock; i.e. when the sign of Δq_t, and thus the monetary policy regime, switches. For any initial guess of the turning points, the model is then solved using backward induction. If the initial guess turns out not to be consistent with the sign of Δq_t switching at that time, the guess is adjusted accordingly, and the process is repeated until the switching criteria are satisfied. Details of the solution method are outlined in the appendix.

It should be noted that this approach to endogenous regime switching is somewhat different from that of Davig and Leeper (2006). They solve their model, in

which the monetary policy reaction to inflation depends on the lagged level of inflation, numerically over a discrete partition of the state space. However, applying this method to our model, which is considerably larger than that of Davig and Leeper, involves substantial computational problems, as their approach suffers heavily from the curse of dimensionality. The ability to handle endogenous switching even in a medium-scale DSGE model is thus an advantage of the shooting method employed in the present paper.[7] On the other hand, the shooting method in effect combines two approximations, as the model is linearized under each regime. This is a potential drawback, albeit a small one, as the two systems are almost identical. Moreover, observe that the steady state of the model is unaffected by the non-linear policy, as the reaction to asset price changes will always be zero in steady state. As a result, we are linearizing each of the systems around the same steady state. A more substantial disadvantage of using a numerical, non-linear solution method is that it renders welfare calculations unfeasible, and thus prevents an investigation of whether the asymmetric policy is optimal in terms of welfare.

2.7 Equilibrium Determinacy

Following Blanchard and Kahn (1980), equilibrium determinacy of rational expectations models is ensured if the number of endogenous state variables in the model is equal to the number of stable eigenvalues (i.e., eigenvalues inside the unit circle) of the matrix governing the law of motion. However, due to our use of a numerical solution method, we are unable to write the solution analytically as a law of motion on the standard form. As a result, we can only make a formal check of the Blanchard-Kahn conditions for each of the two piecewise linear systems comprising our model, but not for the model as such. To make sure that equilibrium determinacy is in fact preserved in our model, we first verify that the Blanchard-Kahn conditions are satisfied for each of the two systems. This turns out to be the case, which is not surprising. First, as described below, the interest rate reaction to inflation is larger than 1 in both regimes, as we keep $\phi_\pi = 1.5$

[7] The shooting method is in fact more similar in spirit to the 'guess-and-verify' method used by Máckowiak (2007) in order to study the outbreak of currency crises. However, common to all these studies is the use of numerical methods. The task of developing analytical tools to deal with endogenous regime switching is an obvious next step, but beyond the scope of this study.

constant. In other words, the Taylor principle is satisfied in both systems. Second, Pfajfar and Santoro (2011) show that adding a reaction to asset price *growth* does not compromise equilibrium determinacy, but rather promotes it.[8] Having established that each of the two systems satisfy the Blanchard-Kahn conditions, it follows that the model as such also satisfies the criteria for equilibrium determinacy. If the model economy switches between two regimes that both satisfy equilibrium determinacy, the model itself will never be exposed to problems of indeterminacy. It is straightforward to show that in a model where the economy switches between two regimes, and the monetary policy reaction to inflation is strictly larger than 1 in both regimes, the 'long run Taylor principle' of Davig and Leeper (2007), which is a necessary and sufficient condition for equilibrium determinacy, is always satisfied, as shown in the appendix. In particular, the regime-switching probabilities, which in our case are determined by the sign of the stock price change, have no impact on the conditions for equilibrium determinacy in this case. We conclude that the asymmetric reaction to stock prices does not in itself lead to equilibrium indeterminacy.

2.8 Calibration

As already mentioned, we obtain most of the parameter values from Christensen and Dib (2008), who estimate a model largely similar to the one outlined above using US data for the sample period 1979-2004. The parameters that were not estimated by Christensen and Dib are instead calibrated. With a few minor exceptions described in the appendix to this paper, we follow the calibration in Christensen and Dib (2008). The reader is therefore referred to Christensen and Dib for a more detailed discussion of the parameter values. All parameter values are presented in the appendix.

The parameter measuring the elasticity of the external finance premium with respect to changes in firms' leverage position deserves special mention. We use the value $\psi = 0.042$ as estimated by Christensen and Dib. This value is somewhat smaller than the value used by Bernanke *et al.* (1999) and Gilchrist and Saito (2008) of $\psi = 0.05$. This implies that the financial accelerator mechanism is less strong in the present paper.

[8] As these authors demonstrate, the reason is that a reaction to asset price growth is isomorphic to a higher degree of interest rate smoothing, which has been shown to alleviate potential indeterminacy problems.

As for monetary policy, the policy rule in our model differs substantially from that of Christensen and Dib (2008). Therefore, we do not use their parameter estimates. Instead, we set $\phi_\pi = 1.5$ as suggested by Taylor (1993). Furthermore we set $\phi_y = 0.2$, whereas the interest rate smoothing parameter is set to 0.67, indicating a degree of interest rate smoothing around 2/3 as suggested by, among others, Clarida *et al.* (1999). Finally, a value must be assigned to the parameter ϕ_q, the reaction to stock price drops. Hoffmann (2013) and Hall (2011) directly estimate this parameter from comparable Taylor rules with interest rate smoothing.[9] For the case of falling stock prices ($\Delta q_t < 0$), Hoffmann (2013) reports an estimate of 0.331 for the period 1987-2008, while Hall (2011) arrives at an estimate of 0.139 in her baseline specification. Ravn (2012) reports estimates from a high-frequency study using daily data. To cast his results in terms of the Taylor rule, the point estimate needs to be transformed.[10] Following the same line of reasoning as Ravn (2012), his estimated result implies a value of $\phi_q = 0.0246$ whenever $\Delta q_t < 0$.[11]

These estimates are quite low. The DSGE literature offers little guidance on the magnitude of this parameter. However, some information can be obtained from the contributions of Tetlow (2005) and Gilchrist and Saito (2008), who augment the Taylor rule with a symmetric reaction to the change in stock prices. Tetlow evaluates a rule with a stock price reaction that is quite large; always bigger than 1. Gilchrist and Saito allow the parameter to take on values between 0.1 and 2.0. In other words, there seems to be a substantial divergence between estimated and calibrated values of this parameter.[12] To bridge this gap, we therefore perform most of the simulations below for two different values of ϕ_q; the estimate of 0.0246 obtained from Ravn (2012), which represents a lower

[9] To be exact, Hoffmann (2012) finds a reaction to stock price changes relative to the change in the underlying HP-filtered trend, while Hall (2011) finds a reaction to the *lagged* stock price change relative to the change in the fundamental value as measured by dividend yields.

[10] The interpretation offered by Ravn (2012) relies on the fact that the Federal Open Market Committee meets once every six weeks. The model of the present paper is formulated (and calibrated) in quarterly terms. This involves an implicit assumption that monetary policy can only be changed every 12 weeks; once per quarter.

[11] In fact, Ravn (2012) also estimates an augmented Taylor rule, and obtains a reaction that is somewhat larger.

[12] Indeed, if one were to use the result of Rigobon and Sack (2003) in the present setting, this would imply a (symmetric) value of $\phi_q = 0.0428$. One potential explanation of this divergence is that in theoretical investigations, the researcher is often interested only in policy reactions to stock price changes larger than some threshold value, while the empirical contributions considered here measure the (presumably smaller) reaction to stock price changes of any size.

bound for the various empirical estimates, and a value of 0.5, more in line with the theoretical literature.

3 Dynamics of the Model

In this section, we investigate the dynamics of the model when the asymmetric monetary policy rule is in place. In linear models, the impulse response to a positive shock is by construction the mirror image of the response to a negative shock of the same type and size. In this model, instead, positive and negative shocks have different dynamic effects. As the central bank reacts only to falling asset prices, a shock that drives asset prices down will induce a stronger monetary policy reaction than a shock which leads to higher asset prices. Further, the adjustment back to the steady state will also differ, depending on whether asset prices are approaching their steady state value from above or below.

Before looking into the effects of the asymmetric policy, we report the effects of each shock in the model without an asymmetric policy. Figure 1 and 2 display the impulse responses of some key endogenous variables to an orthogonalized unit shock to technology and monetary policy when the policy reaction to stock prices is always zero; $\phi_q = 0$. Following a positive technology shock, Figure 1 illustrates that output rises, as does consumption and investment (not shown). The hump-shaped pattern of output is generated by the real and nominal rigidities in the model. Inflation and the nominal interest rate both fall in response to this positive supply shock. The drop in the inflation rate is the source of the drop in net worth. Lower inflation implies a higher real cost of repaying outstanding debt, depressing the net worth of firms. This is the debt-deflation channel. The consumer price index is the relevant price index for 'deflating' net worth, since firms are eventually owned by households. As net worth goes down, the external finance premium increases due to more severe agency problems between borrower and lender, as described above. In turn, this dampens economic activity. Thus, the term financial accelerator is in fact misleading in the case of a technology shock when the debt-deflation channel is included, as in this case the fluctuations in output are actually attenuated. This was already noted by Iacoviello (2005). The presence of the debt-deflation channel is crucial for this result, as also demonstrated by Christiano *et al.* (2010). In a similar model, they

find that the debt-deflation channel and the financial accelerator mechanism reinforce each other in the wake of shocks that drive output and inflation in the same direction, whereas they counteract each other after shocks that, like the technology shock, drive output and inflation in different directions.

The technology shock leads to a boom-bust cycle in the asset price. The initial rise and fall in the price of capital is due to the investment boom following the technology shock. However, the price of capital 'undershoots' its steady state level for a number of periods. This undershooting is again due to the debt-deflation channel, as the persistent drop in net worth leads to a persistent rise in the price of external funding, lowering the demand for capital (and thus, the asset price) even many periods after the shock. It may seem counterintuitive that net worth and the price of capital move in different directions. The explanation is that the initial (and numerically quite small) increase in the price of capital is the result of two opposing effects: While the positive technology shock increases investment and the price of capital; the resulting rise in the external finance premium has the exact opposite effect.

Figure 2 illustrates the dynamics after a one-time positive innovation to monetary policy. As expected, the nominal interest rate jumps up, and then falls back gradually due to interest rate smoothing. In this case, the financial accelerator does work to amplify business cycle fluctuations. As output and inflation move in the same direction, this is in line with the predictions of Christiano *et al.* (2010). The higher interest rate depresses economic activity and in particular investment, reducing the price of capital. This leads to a drop in the net worth of firms, which is further enhanced by the drop in inflation through the debt-deflation channel. Lower net worth increases the external finance premium, which further depresses investment and output. These dynamics explain why this mechanism is referred to as the financial accelerator.

In the appendix, we report the effects of a moderate but symmetric monetary policy reaction to asset price changes, where the central bank reacts to both positive and negative stock price changes with a reaction parameter of 0.5. A symmetric reaction of this size does not lead to major changes in the dynamics of the model relative to the description above. We also report the effects of having the central bank react to stock price *deviations* from their steady state level, instead of stock price *changes*. Such a policy rule has been studied by Faia and Monacelli (2007), Gilchrist and Saito (2008), and Pavasuthipaisit (2010),

among others. Overall, this modification of the nature of the policy rule does not significantly alter the transmission of shocks within the model. In the following, we therefore stick to the original assumption of a reaction to stock price changes. A practical advantage of reacting to changes in the stock price rather than its deviations from a steady state level is that the latter can be quite difficult to determine in real time.

3.1 Dynamics under Asymmetric Policy

Having discussed the effects of each shock in the absence of asymmetric policy, we now turn to study how these effects are altered when an asymmetric monetary policy rule is introduced. When computing impulse responses, we use the value of $\phi_q = 0.5$ in order to clearly illustrate the effects of the asymmetric policy. For each shock, we compare the effects of positive and negative shocks on the dynamics of key endogenous variables. Consider first the effects of a technology shock. Figure 3 illustrates what happens after positive and negative technology shocks. The 'mirror image' of a negative shock is just the impulse responses of the negative shock multiplied by -1; facilitating comparison. As illustrated, the asymmetric policy has a dampening effect on contractions in output relative to expansions. A positive technology shock causes output to increase by more than it decreases following a similar-sized negative shock. The explanation is that in the wake of a negative technology shock, the asset price is pushed down for a number of periods (except for the effect on impact, when the asset price actually rises). Under the asymmetric policy, this drop in asset prices is met with an interest rate cut (although this cut is dominated by the increase in the interest rate as a reaction to the jump in inflation), spurring economic activity and thus dampening the initial economic slowdown. On the other hand, as asset prices rise following a positive technology shock, this induces no increase in the interest rate *per se*. In other words, output contractions following technology shocks are mitigated by an interest rate reaction to asset prices, while output expansions are not. Also for inflation, increases will be larger than drops, as the interest rate reaction to asset prices exerts an upward pressure on inflation following a negative shock, but no corresponding downward pressure after a positive shock. While the asset price still displays a boom-bust cycle, the asymmetric policy implies that the decline following a negative shock is less severe than the boom following

a positive shock. It thus seems that the policy reaction to asset price drops succeeds in mitigating these drops. The quantitative impact of the asymmetric policy on the macroeconomy is quite small, though, as indicated by the small absolute distance between the impulse responses for the positive and (mirrored) negative shocks.

It is interesting to compare the effects on the asset price to the effects of a similar-sized shock with no stock price reaction (Figure 1). As the negative shock induces a monetary policy reaction to the drop in stock prices, it is not surprising that the effects of a negative shock (Figure 3) are numerically smaller than the effects of a positive shock under no stock price reaction at all. However, we also observe that the increase in the asset price following a positive shock is larger under the asymmetric policy than in the absence of an asset price reaction. As the asset price increases immediately after a positive technology shock, both models imply no reaction of monetary policy to this increase. Under the asymmetric policy, however, agents realize that whenever asset prices start to fall, this drop will be alleviated by a monetary policy reaction. This expectation drives up the asset price more than in the case where the reaction to asset prices is always zero, giving rise to an 'anticipation boom'. This anticipation boom measures the additional increase of the asset price under asymmetric policy, relative to its increase in the case of no stock price reaction following a positive shock. Quantitatively, the anticipation boom is quite substantial under the calibration with $\phi_q = 0.5$; amounting to 23.9 % when evaluated two periods after the shock; the last period before the asset price starts to fall and monetary policy actually starts reacting to asset price changes. On the other hand, using the smallest of the estimated values ($\phi_q = 0.0246$), the number is reduced to only 1.1 %. Recall that the other empirical estimates were all in the range between these two values.

Consider finally the asymmetric effects on the two financial variables, net worth and the external finance premium. Recall that because of the debt-deflation channel, net worth is depressed after a positive technology shock, as the drop in inflation increases the real burden of firms' debt repayments. However, it is apparent that the effect on net worth is much larger following a negative shock. After a positive shock, the drop in net worth is counteracted by the rise in the asset price. In the case of a negative shock, this effect is much weaker, as the drop in asset prices is much smaller. Indeed, after a negative shock, the asset price rises in the first period, which is exactly where most of the difference arises

in the effects on net worth. As net worth is highly persistent, so is this difference. In turn, also the external finance premium is affected more by a negative shock, which is unsurprising given the movements in net worth.

Figure 4 illustrates the asymmetric effects of contractionary and expansionary monetary policy shocks. Once again, output and inflation both drop following a contractionary monetary policy shock. An expansionary shock, however, induces an even larger increase in output and inflation. As was the case for technology shocks, then, the asymmetric policy implies that when the economy is hit by monetary policy shocks, booms become larger than recessions, once again creating an asymmetric business cycle. The explanation is again linked to the movements in the asset price. Following a contractionary shock, the asset price goes down, inducing the central bank to cut the interest rate. This mitigates the initial economic downturn caused by the shock, and also pushes inflation up. On the other hand, the rise in asset prices following an expansionary shock is not met with any monetary policy reaction, so the counteracting effect is not present in that case. Furthermore, adding to the asymmetric effects on output and inflation stemming from the monetary policy reaction to asset prices, the increase in the external finance premium during expansions is much larger than the drop during contractions. In turn, this implies cheaper access to credit for firms, increasing the demand for capital, the investment level, and eventually output. Note that while the nominal interest rate does not display a large, numerical difference, the real interest rate, which matters for consumption and investment decisions, is affected differently during expansionary and contractionary phases, as implied by the impulse responses for inflation. As a result, the macroeconomic effects of the asymmetric policy are much larger than in the case of technology shocks.

As in the case of technology shocks, an expansionary shock to monetary policy leads to an anticipation boom in asset prices. This is evident when comparing the effects of an expansionary shock under asymmetric policy (Figure 4) to the effects in the case of no stock price reaction (Figure 2). In the case of monetary policy shocks, the anticipation boom is evaluated one period after the shock; the last period before the asset price starts declining. The extra rise in asset prices is substantial, 28.1 %, when ϕ_q is set to 0.5. Using instead the much smaller estimated value from Ravn (2012), the number drops to 1.1 %.

The emergence of the anticipation boom can be related to what Davig and Leeper (2006) call the *preemption dividend*. In their model, the central bank

is assumed to react stronger to inflation if the lagged inflation level is above a certain threshold (the inflation target). Rational agents will embed this non-linearity in their inflation expectations. As a consequence, monetary policy will be more effective in bringing down inflation in the wake of an inflationary shock, compared to a situation with a linear reaction to inflation. As the central bank is able to successfully manage expectations, the actual increase in the interest rate does not have to be very large. In our setup, agents embed the monetary policy reaction to stock price drops in their expectations, leading to a larger increase in asset prices immediately after a positive shock. This happens despite the fact that when asset prices are increasing, as in the first period(s) after the shock, the actual monetary policy reaction to asset prices is zero under the asymmetric policy as well as with no reaction to asset prices at all. As the preemptive dividend of Davig and Leeper (2006), the anticipation boom arises solely due to the central bank's ability to manage the expectations of private agents. In this way, the asymmetric monetary policy amplifies the boom-bust cycle in asset prices following a shock to the economy, thereby creating additional volatility in asset prices.[13]

The results above can be related to some of the results from the empirical literature on asymmetric business cycles. The finding that the asymmetric policy amplifies booms relative to recessions seems to contradict a number of empirical studies which tend to find that recessions are bigger than booms (Neftci, 1984; Acemoglu and Scott, 1997). This suggests that an asymmetric policy of the type investigated above has not historically been driving the business cycle. For several reasons, this is not particularly surprising. First, the recent empirical findings are obtained only for relatively short samples, especially in the case of Ravn (2012). Second, these results are of too little quantitative importance to be a dominant driver of the business cycle. On the other hand, Beaudry and Koop (1993) find that negative shocks to the economy are much less persistent than positive ones, implying that recessions should be shorter than booms. This is more in line with the effects of an asymmetric policy shown above, even if the quantitative differences between booms and recessions are too small to match the findings of Beaudry and Koop. Finally, the implications of an asymmetric

[13] Finally, and similar to Davig and Leeper (2006), we find substantial differences between the impulse responses shown above, which take into account that agents anticipate the possibility of future regime switches, and the impulse responses (not shown, but available upon request) obtained when agents naively expect the present regime to be in place forever.

policy are also consistent with the results of Cukierman and Muscatelli (2008) and Wolters (2012), who find that the Federal Reserve has displayed a recession avoidance preference in the recent past.[14] According to these studies, estimated reaction functions for the Federal Reserve indicate that US monetary policymakers tend to react more strongly to the output gap during recessions than during expansions. This creates outcomes that are in line with the impulse responses displayed above, suggesting that an asymmetric reaction to stock prices can be rationalized by recession avoidance preferences. This is further discussed in the next section.

4 Potential Reasons for an Asymmetric Policy

As demonstrated by the impulse responses in the previous section, reacting asymmetrically to asset prices can lead to a situation in which recessions are attenuated relative to expansions. This raises the question of whether one could think of the central bank as aiming to obtain exactly such an asymmetric outcome. This would then have to show up in the central bank's underlying loss function. Usually, it is assumed that the central bank (implicitly or explicitly) minimizes a loss function where deviations of output and inflation from their target values are punished in a fully symmetric way (see, e.g., Woodford, 2003). Given a mapping from the parameter governing the central bank's preference for output stability relative to inflation stability to the parameters of the Taylor rule, one can think of the Taylor rule as a tool used by the central bank to minimize a loss function of this type. It is, however, not given that the objective of the central bank should be perfectly symmetric. Among others, Blinder (1997), Ruge-Murcia (2004), and Surico (2003, 2007) suggest that the central bank could be seeking to minimize an asymmetric loss function. For example, Ruge-Murcia (2004) assumes that the loss arising from inflation fluctuations is symmetric, but that social loss is higher when unemployment (which he allows to enter the loss function in lieu of output) is above its natural level, compared to when it is below.[15]

[14] On the contrary, Surico (2007) finds no evidence of a recession avoidance preference in the US.

[15] A recent strand of the literature has suggested that policymakers may instead have a preference for robustness towards model uncertainty. Ellison and Sargent (2012) provide an example of how such a preference may rationalize alternative policy outcomes. A preference for robustness could potentially rationalize an asymmetric policy towards stock prices, although

If the loss function of the central bank is of such an asymmetric type, this could serve as the motivation for an asymmetric stock price reaction. Indeed, the central bank could adjust the parameters in its asymmetric Taylor rule (15) to obtain the outcome that minimizes the asymmetric loss function. In section 3, we saw how the asymmetric policy implied that booms not only in output, but also in inflation, tended to be stronger and longer than recessions. This would be consistent with a central bank that has a preference for booms and high inflation over recessions and low inflation.

Woodford (2003) shows that a symmetric loss function approximates the negative of the utility of the representative household in the basic New-Keynesian model, so that minimizing such a loss function is equivalent to maximizing the utility of the representative household. Accordingly, if the central bank minimizes an asymmetric loss function, this would also require a micro-foundation in order to be optimal. Ruge-Murcia (2004) suggests that the motivation for the asymmetric loss function could be concerns about the costs of high unemployment. Another way to micro-found an asymmetric loss function is to assume that agents are loss averse with respect to changes in financial wealth. This possibility is discussed at the end of subsection 4.2. Surico (2007) discusses other possible sources of asymmetric welfare losses. The model outlined above, however, does not include any features that could serve as a welfare-based motivation for an asymmetric loss function, and therefore is unable to explain why the central bank would adopt such a loss function. Another potential motivation for the central bank to obtain outcomes such as the ones illustrated in section 3 could be that natural or steady state output is lower than the efficient level of output. This gives the central bank an incentive to try to push output above its natural level, as in the well-known model of Barro and Gordon (1983).

Moreover, even if the loss function of the central bank is of the usual, symmetric form, this does not necessarily imply that the tools of the central bank should also be symmetric. Indeed, if the central bank believes that certain asymmetries exist in the economy, for example that stock price drops and increases have asymmetric macroeconomic effects, an asymmetric policy might be seen as an attempt to correct for this inherent asymmetry, and in turn obtain a symmetric outcome. Ravn (2012) acknowledges this possibility, and points out two potential sources of asymmetric effects of stock prices. In the following, we study

this remains to be explored.

each of them in more detail.

4.1 Asymmetric Financial Accelerator

One channel which may give rise to asymmetric effects of stock price movements is the financial accelerator. The possibility of a non-linear financial accelerator has received some attention in the literature (Bernanke and Gertler, 1989; Gertler and Gilchrist, 1994; and Bernanke *et al.*, 1996). During a recession, when asset prices tend to be falling, more firms are likely to be liquidity constrained and in need of external financing. Moreover, small changes in the net worth of firms are likely to be more costly when the collateral value of firms is already low, and the agency costs of borrowing are already large. A final reason why the financial accelerator might be stronger when net worth is low is that ultimately, as firms' net worth becomes 'low enough', a credit crunch might result. Peersman and Smets (2005) assess the empirical transmission effects of monetary policy in the euro area, and find that the financial accelerator effect does indeed seem to be stronger in recessions. Gertler and Gilchrist (1994) provide empirical evidence that the performance of small firms are more sensitive to interest rate changes during economic downturns than in booms, suggesting that financial factors are more important in bad times. As discussed by Peersman and Smets (2005), such an asymmetry could potentially explain why monetary policy exerts a stronger effect on output during recessions than in booms.

In the model of this paper, the strength of the financial accelerator is measured by the parameter ψ in equation $(A13)$ in the appendix, which is in turn the log-linearized version of equation (6) above. ψ measures the elasticity of the external finance premium with respect to the net worth of firms, so the larger is ψ, the stronger is the effect on the business cycle of a given change in net worth. In other words, an asymmetric financial accelerator can be modelled by assuming different values of ψ. In particular, in light of the above discussion, we allow ψ to take on one value (ψ_L) for the case when net worth is above its steady state value, i.e., $\widehat{n}_t > 0$, and a higher value (ψ_H) when $\widehat{n}_t < 0$. This reflects that when net worth of firms is already low, the external finance premium is more sensitive to small changes in net worth. In this way, the financial accelerator becomes a source of asymmetric business cycle fluctuations by amplifying bad economic shocks more than good ones. As described in the previous section,

the asymmetric policy reaction to stock prices had the exact opposite effects, suggesting that these two asymmetries might 'cancel each other out'.

To investigate this possibility in detail, consider first the effects of technology shocks. After a positive shock, net worth drops below its steady state value, implying that the elasticity of the external finance premium becomes high. This exerts a downward pressure on output through the accelerator effect, dampening the initial boom, while the drop in inflation is amplified. In the case of a negative technology shock, net worth instead rises, so the dampening of the initial downturn in output is small. Hence, the drop in output is large, which in turn mitigates the increase in inflation. In consequence, the effects of positive and negative shocks are asymmetric.[16] By following an asymmetric policy and cutting the interest rate in response to the drop in stock prices after a negative shock, the central bank can drive up output and inflation, and thereby mimic (the mirror image of) a positive shock. In fact, for a given 'degree' of asymmetry of the financial accelerator, there exists a magnitude of the asymmetric policy reaction (ϕ_q) that exactly eliminates the initial asymmetry after supply-side shocks.

It turns out that the same is not true after shocks originating from the demand side. An expansionary shock to monetary policy pushes up net worth, so that the financial accelerator is relatively weak. The resulting amplification of the initial boom in output is limited, while the increase in inflation is relatively large. On the other hand, the financial accelerator is much stronger following a contractionary monetary policy shock due to the drop in net worth, resulting in a large drop in output and a strong dampening of the initial drop in inflation. An asymmetric policy induces an interest rate cut in response to the stock price drop after the contractionary shock. While this dampens the drop in output, again mimicking the mirror image of a positive shock, it also mitigates further the drop in inflation, which was already 'too small' compared to the relatively large increase in inflation after a positive shock. In other words, a trade-off arises between bringing output or inflation to their 'symmetric' values. While the asymmetric policy might alleviate the effects of a non-linear financial accelerator, it never obtains the fully symmetric outcome.[17] This will in general be the case

[16] Note that the debt-deflation channel is not critical for this conclusion. Without the debt-deflation channel, net worth would be procyclical after technology shocks (Gilchrist and Saito, 2008). An asymmetric financial accelerator would then amplify recessions more than booms.

[17] To visualize these scenarios, observe that an asymmetric financial accelerator induces a 'kink' in both the aggregate demand (AD) and aggregate supply (AS) curves, as firms are on the demand side of the market for financing, but on the supply side of the goods market.

for shocks that drive output and inflation in the same direction, as demand shocks tend to do.

To shed light on the empirical relevance of these issues, it seems natural to ask: How severe should the asymmetry of the financial accelerator be in order to 'rationalize' the recent empirical findings about asymmetric monetary policy, in the sense that this policy 'cancels out' the asymmetric financial accelerator under supply shocks, or obtains the most favorable trade-off under demand shocks? In order to quantify the necessary degree of asymmetry, we fix the elasticity of the external finance premium at the baseline value of $\psi_L = 0.042$ when net worth is above its steady state value. We then use impulse response matching of output and inflation responses for positive and negative shocks to calibrate the 'optimal' value of ψ_H.[18] This value can then be compared to ψ_L. Table 1 shows the degree of asymmetry needed to optimally match the impulse responses of output and inflation to technology shocks for different values of ϕ_q, the reaction coefficient of monetary policy to stock price changes. As the table illustrates, the degree of asymmetry in the financial accelerator needed to match impulse responses is quite sensitive to the choice of ϕ_q. For the value found by Ravn (2012), the balance-sheet channel needs to be only slightly asymmetric (2 % stronger when net worth is low, compared to when it is high) in order for the two asymmetries to 'cancel each other out' under supply shocks. If instead ϕ_q is set at 0.50, this number rises to 40 %.

Through an asymmetric policy reaction of the 'right' size, the central bank can eliminate the kink in the AD curve, but not in the AS curve. As a consequence, shocks to aggregate supply shifts the AS curve along the resulting, potentially linear AD curve, giving rise to symmetric outcomes of positive and negative shocks. On the other hand, the AD curve will intersect steep or flat areas of the non-linear AS curve in response to positive or negative shocks, respectively.

[18] More specifically; for each of the two types of shocks, we focus on the impulse responses of output and inflation. We then compute the sum of squared errors (SSE) between the impulse response to a positive shock and the mirror image of the impulse response to a negative shock. For this, we use the values in the first 16 periods after the shock. Finally, we solve for the value of ψ_H that minimizes the sum of the SSE's.

Table 1: Asymmetric financial accelerator, technology shocks

Value of ϕ_q	**Value of ψ_L**	**Calibrated value of ψ_H**	**Ratio $\frac{\psi_H}{\psi_L}$**
0.0246	0.042	0.043	1.02
0.50	0.042	0.059	1.40

For the same values of ϕ_q, table 2 shows the degree of asymmetry of the financial accelerator needed to obtain the most favorable trade-off between symmetry in output and in inflation after monetary policy shocks. If the policy reaction to stock price changes is set at $\phi_q = 0.50$, the balance-sheet effect has to be much stronger (77%) during periods of low net worth in order to minimize the distance to the symmetric outcome. For a policy reaction of the size estimated by Ravn (2012), the number reduces to 8 %.

Table 2: Asymmetric financial accelerator, monetary policy shocks

Value of ϕ_q	**Value of ψ_L**	**Calibrated value of ψ_H**	**Ratio $\frac{\psi_H}{\psi_L}$**
0.0246	0.042	0.0455	1.08
0.50	0.042	0.0745	1.77

To put these numbers in perspective, we look to the empirical study of an asymmetric financial accelerator by Peersman and Smets (2005). They show that a positive innovation of 1 %-point to the interest rate causes a drop in the growth rate of output of 0.22 %-points during a boom, but a much larger drop of 0.66 %-points during a recession. They then estimate how various measures of firms' financial position contribute in explaining this asymmetry. They find that if firms' leverage ratio increases by 5 % of its average value, the *difference* between the effect on output growth of a monetary policy shock in booms and in recessions increases by 0.14 %-points; i.e. from the original 0.44 %-points to 0.58 %-points. In other words, the financial position of firms is able to account for substantial asymmetries over the business cycle, indicating that the financial accelerator effect is considerably stronger in recessions than in booms. In this light, the degrees of asymmetry computed above to 'rationalize' the results of Ravn (2012) do not seem unrealistic. The other empirical results require a somewhat higher, but not necessarily unrealistic degree of asymmetry.

4.2 Asymmetric Wealth Effects and Loss Aversion

Another possible source of asymmetric macroeconomic effects of stock price movements is the wealth effect on consumption. Shirvani and Wilbratte (2000) and Apergis and Miller (2006) provide empirical evidence that the wealth effect of stock prices is stronger when stock prices are declining than when they are increasing. One possible, theoretical explanation for this finding is provided by prospect theory (Kahneman and Tversky, 1979). Prospect theory introduces an inherent asymmetry in agents' preferences, as the utility loss from bad outcomes is assumed to be larger than the utility gain from good outcomes. If agents display such loss aversion in consumption, as suggested by, among others, Koszegi and Rabin (2009), this might give rise to non-linear effects on consumption from asset price movements. If asset prices decline, so does financial wealth and permanent income, and agents will have to cut their consumption level, painful as it is. On the other hand, following a rise in asset prices, loss averse agents are likely not to increase their consumption level by as much, but instead engage in precautionary savings to cushion themselves against the risk of a future drop in asset prices. As a result, increases in asset prices have smaller effects on consumption, and hence on the macroeconomy, than asset price declines. Gaffeo *et al.* (2012) introduce loss aversion in consumption into a Markov-switching DSGE model. They show that in this case, the optimal monetary policy reaction (to output and inflation) should be asymmetric so as to exploit that after a drop in asset prices and consumption, a more favorable trade-off between output and inflation arises, as households increase their labor supply to compensate for this loss.

Consider first what happens under demand shocks in our model. An expansionary shock to monetary policy drives up inflation, output and the asset price. However, the weak wealth effect attenuates the boom in output. At the same time, more labor is needed to satisfy the extra demand. However, because consumption is rising, the labor supply of households is relatively low, resulting in a large increase in inflation. Instead, after a monetary contraction the wealth effect is strong, so output falls by a lot. As in the model of Gaffeo *et al.* (2012), a more favorable trade-off between output and inflation arises, so the drop in inflation is relatively small. A reaction to the drop in stock prices is able to offset the direct wealth effect, but not the effect on the labor-leisure decision.

As for supply shocks, the initial rise in asset prices following a positive inno-

vation to technology leads to only a small wealth effect, moderating the boom in output and amplifying the drop in inflation. A negative technology shock instead causes a large drop in output through a strong, negative wealth effect. At the same time, the spike in inflation is modest. An asymmetric reaction to the stock price drop will tend to push up inflation and output, bringing both variables closer to the mirror image of a positive shock. In sum, if the stock wealth effect is assumed to be asymmetric over the business cycle, an asymmetric policy is able to 'correct for' this asymmetry and obtain symmetric outcomes only in the case of shocks that move output and inflation in different directions, such as shocks to the supply side, while a trade-off arises after demand-side shocks. This is similar to the previous subsection where the financial accelerator was the source of the underlying asymmetry.[19]

According to the above explanation, asymmetric wealth effects arise through the effect of stock wealth on consumption. A related line of argument, also deriving from prospect theory, is that gains and losses in financial wealth might have direct, asymmetric effects on utility. Barberis *et al.* (2001) assume that agents display loss aversion with respect to fluctuations in their financial wealth. Thus, the loss in utility following from a drop in asset prices and financial wealth is larger than the utility gain from a similar-sized increase. As illustrated in section 3, the introduction of an asymmetric policy rule implies a dampening of the drops in asset prices and an amplification of the increases. If the central bank believes that agents have preferences of the type suggested by Barberis *et al.* (2001), the asymmetric policy could therefore be an attempt to cushion agents from the utility losses when asset prices decline. As agents are assumed to derive utility from changes in asset prices (as opposed to the level), this story would be consistent with the result that the central bank is reacting to *changes* in stock prices. Note the distinction that in this case, changes in asset prices would be entering the reaction function of the central bank not because of their effects on other variables of interest, such as output and inflation, but as a separate target variable entering the underlying loss function of the central bank. Loss aversion with respect to changes in financial wealth could therefore serve as a potential welfare-based motivation for an asymmetric loss function.

[19] Similar to the explanation in footnote 18 for the financial accelerator, asymmetric wealth effects induce a kink in both the AD curve (through the direct wealth effect) and the AS curve (through the intratemporal labor-leisure choice). An asymmetric policy can eliminate the kink in the AD curve but not in the AS curve.

5 Concluding Remarks

The present paper provides some theoretical inputs to the recent debate concerning a potentially asymmetric reaction of monetary policy to stock prices. We demonstrate that an asymmetric policy towards the stock market will translate into an asymmetric business cycle. Booms in output following expansionary shocks will tend to be amplified, while recessions will be dampened. A similar pattern emerges for inflation. This could be motivated by assuming that the desire of the policymaker is to minimize an asymmetric loss function, or by the existence of other asymmetries in the economy. We show that if the financial accelerator or the stock wealth effect is assumed to be non-linear over the business cycle, an asymmetric monetary policy can obtain symmetric outcomes in response to supply shocks, but only partly alleviate such asymmetries after demand shocks.

The magnitude of the asymmetric policy reaction found in recent empirical studies is rather small, and our analysis shows that its quantitative impact on the macroeconomy is limited. This is especially the case when economic fluctuations are driven by technology shocks. The asymmetric policy exerts a small additional effect on the movements in output and inflation. It also does not lead to problems of equilibrium indeterminacy.

Although an asymmetric policy reaction to stock prices might be useful in order to eliminate or mitigate other asymmetries, it also implies a risk of creating moral hazard problems by effectively insulating stock market investors from part of their downside risk. As a matter of fact, this has been at the heart of the critique of the pre-crisis consensus and the plea to take the 'a' out of 'asymmetry' in the policy approach to asset prices (Mishkin, 2010; Issing, 2011). The potential moral hazard problems of an asymmetric monetary policy towards the stock market was already analyzed by Miller *et al.* (2001). In the present paper, this issue is linked to the anticipation boom in asset prices that arises following expansionary shocks as a result of the asymmetric policy. However, we have not attempted to analyze the potential moral hazard problems in detail. A comprehensive study of how an asymmetric monetary policy can cause moral hazard problems by distorting the incentives of the individual investor would require an even richer microfoundation than that of the present paper, explicitly modelling the investment decision. While this is surely an interesting idea for

future research, it is beyond the scope of this paper.

The present paper follows most of the modern macroeconomic literature by log-linearizing the equilibrium conditions around a steady state. Thus, by construction, the economy eventually returns to the same steady state following a shock. This inherent limitation implies that it is not possible to study whether the asymmetric policy might push the economy to a new steady state. For example, if economic booms are consistently stronger and longer than recessions, as suggested by the impulse responses, one would eventually expect a 'level' effect on output. Another question is whether the asymmetric policy will sooner or later drive the interest rate to its zero lower bound. Due to the limitations of the log-linear approach, the model above does not have much to say about such issues.

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Impulse Responses

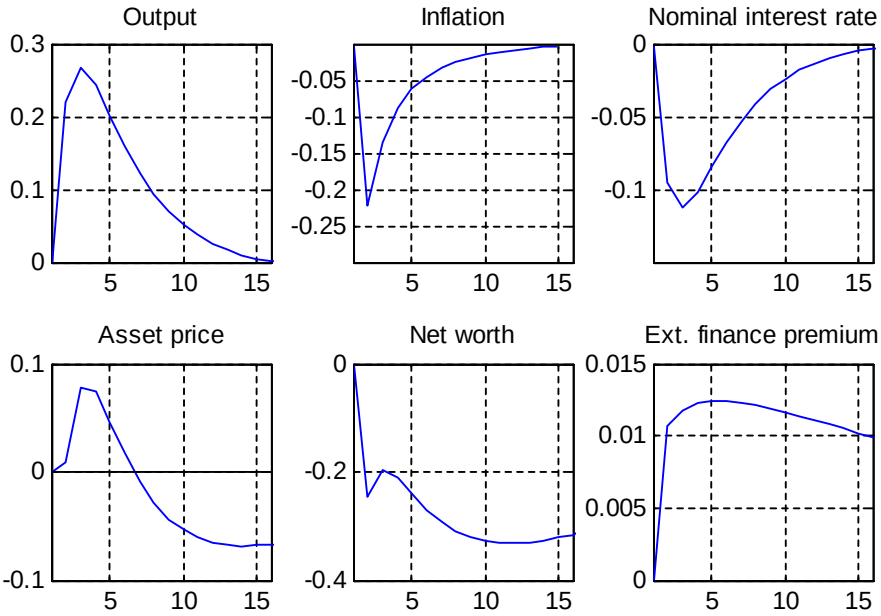

Figure 1: Effects of a positive technology shock, no policy reaction to asset prices

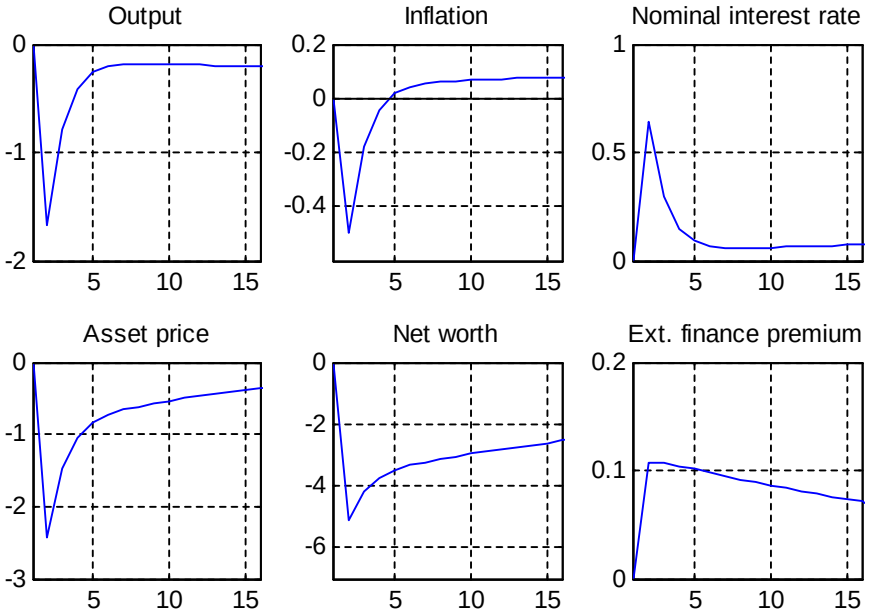

Figure 2: Effects of a contractionary monetary policy shock, no reaction to asset prices

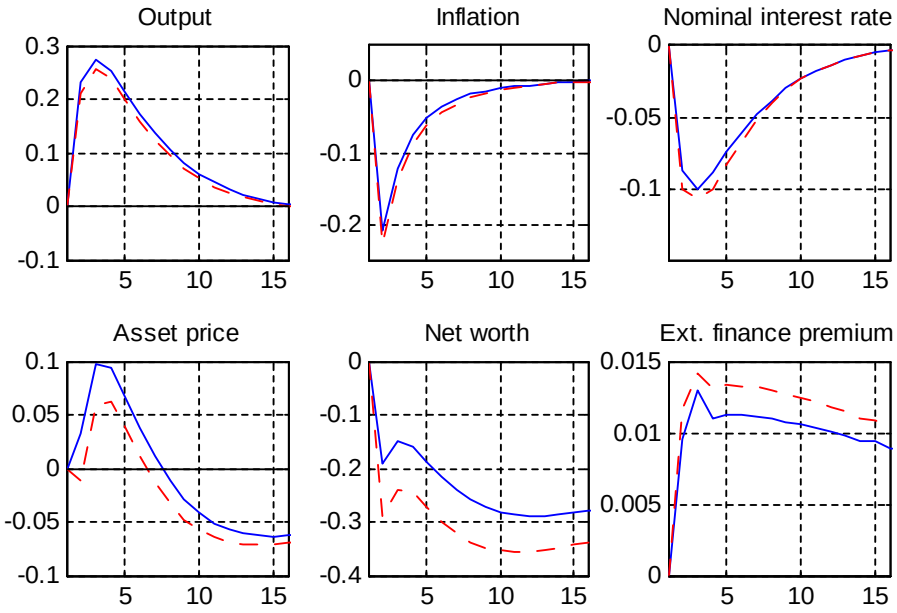

Figure 3: Effects of a technology shock, asymmetric model. *Solid blue line:* Positive shock. *Dashed red line:* Mirror image of negative shock.

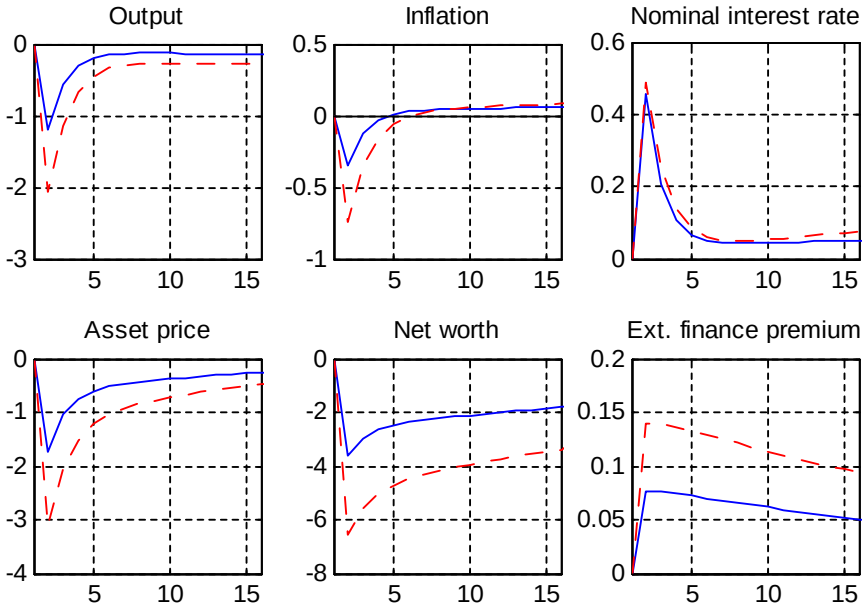

Figure 4: Effects of a monetary policy shock, asymmetric model. *Solid blue line:* Contractionary shock. *Dashed red line:* Mirror image of expansionary shock.

Deep Habits, Endogenous Credit Standards, and Aggregate Fluctuations*

Søren Hove Ravn

Abstract

The lending boom in many developed economies in the years leading up to the recent crisis and the subsequent claims of a credit crunch after the crisis provide anecdotal evidence that credit standards tend to vary over the business cycle. This is backed up by a number of formal, empirical studies. In this paper, we build a DSGE model of the macroeconomy featuring countercyclical lending standards. This is done by introducing deep habits in the demand for bank loans. We assume that banks compete on lending standards (as measured by collateral requirements) rather than on lending rates. This gives rise to countercyclical collateral requirements, in line with the data. We investigate the importance of this mechanism for macroeconomic fluctuations, and find that countercyclical lending standards in general act as an amplifier of business cycles. However, the quantitative importance of this mechanism is modest.

Keywords: Credit Standards, Collateral Assets, Deep Habits, Business Cycle Movements, DSGE Modelling.

JEL classification: E22, E32, E44.

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1 Introduction

Banks are continuously accused of easing their credit standards in good times and tightening them when the economy tanks. Empirical evidence suggests that credit conditions change over the business cycle by more than what is accounted for by automatic changes in the quality of the pool of potential borrowers, i.e. that credit *standards* vary systematically over the business cycle.[1] A number of studies have found that various measures of credit standards vary with the business cycle, such that, for example, borrowers are required to pledge more collateral or pay higher interest rates during a recession. At the theoretical level, a number of competing (though not mutually exclusive) explanations for this finding have been suggested.[2] In the macroeconomic literature, however, few attempts have been made to incorporate such theories into general equilibrium models of the business cycle in order to quantify the macroeconomic effects of endogenous changes in credit standards.

In this paper, we present a general equilibrium model in which countercyclical lending standards arise endogenously. While a number of authors have studied the macroeconomic effects of exogenous shifts in loan-to-value ratios, this model is, to the best of our knowledge, the first general equilibrium model in which such changes arise endogenously.[3] The mechanism behind this endogeneity is our assumption that entrepreneurs, who run the firms in our model economy, prefer to borrow from the same bank as in previous periods. In other words, we assume that entrepreneurs have 'deep habits' in their demand for loans from individual banks. The deep habits model was developed by Ravn *et al.* (2006) to model habit formation in consumers' demand for different varieties of consumption goods. We believe the deep habits model to be particularly well-suited to characterize the market for bank loans for a number of reasons. Firstly, many households and firms do indeed borrow from the same banks repeatedly. Sec-

[1] By changes in credit standards, we imply that borrowers of a given quality may sometimes find themselves able to obtain a loan, and at other times unable to obtain a loan of the same size, or only able to borrow at less attractive conditions, for example by pledging more collateral.

[2] See, among others, Rajan (1994), Berger and Udell (2004), Ruckes (2004), and Dell'Ariccia and Marquez (2006).

[3] To be clear, a number of authors have studied endogenous movements in credit limits arising from changes in the borrower's collateral value (the literature initiated by Kiyotaki and Moore, 1997) or his income (e.g., Ludvigson, 1999). However, the loan-to-value (or loan-to-income) ratio is typically assumed to be either constant or subject to exogenous shocks.

ondly, switching between banks is costly, not least due to informational asymmetries between lenders and unknown borrowers (Sharpe, 1990; Kim *et al.*, 2003). Ravn *et al.* (2006) suggest that the deep habits model is a natural vehicle for incorporating switching costs in general equilibrium models. Moreover, in part because of these switching costs, the market for bank loans fits well with the assumption of imperfect (monopolistic) competition inherent in the deep habits model. It is also very common, at least for firms, to have more than one banking relationship (Ongena and Smith, 2000). Deep habits in banking may be thought of as a way of modeling relationship banking, albeit in a reduced-form manner.

While banks can offer their customers a variety of borrowing conditions, we choose in this paper to focus specifically on the collateral requirements set by the banks. As discussed in the next section, collateral requirements have been shown to fluctuate over the course of the business cycle.[4] Non-price competition is widespread in the banking sector, in part because of the well-known agency problems related to price competition in banking (Stiglitz and Weiss, 1981). Furthermore, competition on collateral requirements is likely to be particularly relevant in relationship banking, where banks can offer borrowers more favourable valuations of their assets as the duration of the relationship increases and banks learn about the 'true' value of these assets.[5] We therefore assume that banks compete monopolistically by requring borrowers to pledge different amounts of collateral. Borrowers, in turn, prefer to pledge as little collateral as needed to satisfy the collateral constraint they face.

In each period, an individual bank trades off the potential gains and losses from marginally lowering its collateral requirements. The gains arise from the increase in the bank's market share. In the presence of deep habits, a larger market share today further translates into a larger market share tomorrow, as more borrowers will be 'held up' by each bank, having developed a habit for loans from this bank. On the other hand, we assume that if a bank lowers its collateral requirements, the risk that loans made by that bank will not be repaid in the next period increases, so that lowering credit standards also involves a cost. As we describe, this assumption is backed up by empirical evidence. To see how

[4]Moreover, the collateral requirement is an important part of a very large share of loan contracts. In the US, for example, around 80% of small business loans are collateralized (Avery *et al.*, 1998).

[5]For example, Berger and Udell (1995) find that borrowers with long banking relationships are less likely to pledge collateral.

deep habits in bank loans may give rise to countercyclical collateral requirements, consider how the trade-off faced by each individual bank is altered in the wake of an expansionary shock to the economy, after which output, and thus loan demand, is likely to be above average for a number of periods. This implies that a larger market share in current and future periods is more valuable for banks. All else equal, each bank will therefore be inclined to compete more intensively for current and future market shares by lowering its collateral requirements, despite the higher risk of credit default this entails. As a result, shocks that push up output will also lead to lower collateral requirements, in line with the empirical evidence.

In turn, the drop in collateral requirements during economic booms may itself act as a 'financial accelerator' by driving further up the volume of loans and thus economic activity. A central issue in our study is to what extent this mechanism may amplify macroeconomic fluctuations. To answer this question, we evaluate the impact of deep habits in the market for bank loans in the framework of a Dynamic Stochastic General Equilibrium (DSGE) model of the macroeconomy. We show that endogenous credit standards do indeed act as an amplifier of shocks to the economy for all but one of the shocks in our model. However, the quantitative importance of this amplification is rather small at the macroeconomic level.

We are not the first to study the presence of deep habits in the market for bank loans, but to our knowledge, we are the first to apply this mechanism to collateral requirements. Aliaga-Diaz and Olivero (2010) show how deep habits may generate countercyclical spreads between lending and deposit rates. The main difference between our model and their work is that we assume banks do not compete on interest rates, but instead on collateral requirements, while Aliaga-Diaz and Olivero (2010) abstract from collateral and have banks competing on lending rates. Nevertheless, our finding that deep habits in banking lead to a quantitatively modest financial accelerator is in line with their conclusion. In two recent contributions, Airaudo and Olivero (2012) and Aksoy *et al.* (2013) make the same assumptions about bank competition as Aliaga-Diaz and Olivero (2010) and study the impact of deep habits in banking in sticky-price models. They also find the mechanism to be of limited quantitative importance for macroeconomic fluctuations.

The rest of the paper is structured as follows. In the next section, we summa-

rize the empirical evidence of countercyclical lending standards, and also discuss how our model relates to other theoretical explanations of this phenomenon. In section 3, we present the model and describe how banks' profit maximization gives rise to endogenously changing credit standards. The results are presented in section 4. Finally, section 5 concludes.

2 Endogenous Credit Standards: Theory and Evidence

As described above, countercyclical movements in credit standards arise endogenously under deep habits in the market for bank loans. An important part of the explanation for this is that in the presence of deep habits, banks compete more intensively for customers during good times, thereby inducing each bank to lower its collateral requirements. In this sense, our paper is related to a number of theoretical studies that explore the link between the business cycle, bank competition, and credit standards. Dell'Ariccia and Marquez (2006) show that if the share of entirely unknown loan applicants is large enough, banks will extend credit to all applicants with no collateral requirement, whereas if there are only few genuinely new and unknown applicants, it is optimal for banks to screen out some of them by requiring a positive amount of collateral. The intuition is that banks prefer to be approached by new, untested loan applicants rather than those who have already been turned down by rival banks. When the share of unknown applicants is large, each bank is therefore willing to lower its collateral requirements in order to beat its rivals and expand its market share. As discussed by Dell'Ariccia and Marquez (2006), the share of unknown loan applicants, and thus the degree of competition among banks, is likely to be positively correlated with the business cycle, as many new entrepreneurs will try their luck (and existing firms will initiate more new investment projects) when the macroeconomic outlook is bright. In a more recent study by Hainz *et al.* (2012), it is assumed that banks are more likely to screen potential borrowers rather than have them pledge collateral when bank competition is strong, for instance because banks open up more local branches, so that the distance between borrower and lender becomes smaller. As a result, loans made when bank competition is strong are less likely to be collateralized than loans made in times of less intense competition. Hainz

et al. (2012) further provide empirical evidence based on a cross-country sample from 70 countries that strongly supports this conclusion. Finally, Berlin and Butler (2002) obtain a similar, theoretical finding, although their story is somewhat different. They assume that banks monitor their borrowers to gather information about them, and that loan contracts (and hence collateral requirements) can potentially be renegotiated. Stronger bank competition reduces the bargaining rents of banks in renegotiations, so that monitoring is less profitable, and hence less prevalent. As a result, renegotiation is less attractive for banks, who instead demand less collateral in the original contract, so as to discourage borrowers from asking for renegotiations.[6]

While these studies attribute changes in collateral requirements to variations in bank competition, the focus of the study by Ruckes (2004) is on the link between competition in the banking sector and business cycle movements. Ruckes argues that during recessions, the average quality of loan applicants is low, so each potential borrower is likely to receive only few loan offers. As the economic outlook improves, so does the quality of loan applicants, who will then each have more offers to choose among. As a result, banks have to compete more intensively to lure in good customers, for example by offering them more attractive borrowing conditions, such as lower lending rates, as considered by Ruckes (2004), or lower collateral requirements as in our model and in the studies cited above.[7]

2.1 Empirical Evidence

There is a considerable amount of empirical evidence supporting the hypothesis that credit standards, including collateral requirements, move in a countercycli-

[6] On the other hand, Manove *et al.* (2001) arrive at the opposite result. They show theoretically that collateralization is more likely under perfect competition than in the case of monopolistic competition. The key mechanism in their model is that collateral weakens the incentives of banks to screen borrowers under perfect competition, but not if banks are monopolistic competitors. In addition, Jimenez *et al.* (2006) use Spanish data to show empirically that the likelihood of collateralization is lower when the credit market is more concentrated, as measured by the Herfindahl index, i.e. when competition is less intense. See, however, Maudos and de Guevara (2007) and the references therein for a critique of the use of concentration measures as proxies for competition in the banking sector.

[7] A related story is suggested by Rajan (1994), who argues that banks have short-term concerns about their reputation relative to other banks. As a result, each bank is inclined to lower its credit standards (i.e., to enhance competition for borrowers) during booms, so as to make profits at a time when all its rivals are doing so.

cal fashion. In two related papers, Jimenez and Saurina (2006) and Jimenez *et al.* (2006) use a large dataset covering all loans over 6,000 euros made in Spain in the period 1984-2002 (around 2 million loans in total), and find that loans made during macroeconomic downturns are significantly more likely to be collateralized than loans made during booms. To get a sense of the size of this effect, they report that an increase in real GDP growth of 1 %-point lowers the probability of collateralization by 3.7 %-points. In other words, collateralization is found to be countercyclical. A similar result is obtained by Asea and Blomberg (1998), who find that higher aggregate unemployment leads to a significant rise in the probability of collateralization. They use a dataset of around 2 million commercial and industrial loans made in the US in the years 1977-1993. Dell'Ariccia *et al.* (2012) use data for the US subprime mortgage market covering the years 2000-2006, and find that lending standards are lowered when credit demand is high, as is typically the case during economic booms. For example, they find that average loan-to-income ratios of subprime borrowers are higher in areas where unemployment is low.

A number of recent studies provide further empirical evidence that lending standards are countercyclical, but with no specific focus on collateral requirements. These include Maddaloni and Peydro (2011), who use data from the Euro Area Bank Lending Survey and the Senior Loan Officer Survey for the US. In these surveys, loan officers are asked about recent changes in lending standards. Combining data from the Euro area and the US for the period 2002-2008, they find that lending standards as reported by loan officers are loosened when GDP growth is high, and vice versa.[8] Jimenez *et al.* (2011, 2012) study loan applications and granted loans, and also find that higher GDP growth tends to reduce credit standards, as measured by the probability that a given loan application is approved. Lown and Morgan (2006) also find that tighter lending standards are positively correlated with low GDP growth using the Loan Officer Opinion Survey, while an event study by Schreft and Owens (1991) confirms that such a link has been present also in the 1960's, 1970's and 1980's in the US. Mendoza and Terrones (2008) provide cross-country evidence that credit booms are systematically associated with economic expansions, and also with an increase in the share of non-performing loans, indicating that credit standards are often

[8] On the other hand, extending the sample back to 1991 and excluding the Euro area, they obtain insignificant results.

lowered during such episodes.

3 The Model

We add a banking sector and deep habits in the demand for loans to an underlying Real Business Cycle model with capital and land in the production function, investment adjustment costs, and standard habit formation in consumption. The RBC model thus closely resembles the model employed by Liu *et al.* (2012). The model economy consists of households, entrepreneurs, and banks. There is a continuum of unit length of each type. Households work, save, and consume non-durable goods as well as housing services. Entrepreneurs own the capital stock, and use it along with inputs of labor and housing services (land) to produce goods for consumption or investment. They are subject to a borrowing constraint. Finally, as in Gerali *et al.* (2010) there is a monopolistically competitive banking sector, the profits of which pertain to households.

3.1 Households

The utility function of household i takes the following form:

$$E_0 \sum_{t=0}^{\infty} \left(\beta^P\right)^t \left[\log\left(C_t^P(i) - \gamma^P C_{t-1}^P(i)\right) - \iota \frac{\left[N_t(i)\right]^{1+\varphi}}{1+\varphi} + \varsigma_t \log H_t^P(i)\right], \quad (1)$$

where C_t^P, N_t, and H_t^P denote non-durable consumption, labor and housing, respectively, $\beta^P \in (0,1)$ is a discount factor, and $\gamma^P \in (0,1)$ measures the degree of habit formation in consumption. We use superscript P because households are more patient than entrepreneurs. $\varphi > 0$ is the Frisch elasticity of labor supply, $\iota > 0$ is a measure of the households' trade-off between consumption and labor, and ς_t is a housing demand shock as in Liu *et al.* (2012), and follows the process:

$$\log \varsigma_t = (1-\rho_H)\log \varsigma + \rho_H \log \varsigma_{t-1} + \sigma_H \varepsilon_{Ht}, \quad (2)$$

where the innovation ε_{Ht} follows an i.i.d. normal process with standard deviation σ_H, and where $\varsigma > 0$, and $\rho_H \in (0,1)$. The household is subject to the following

budget constraint:

$$C_t^P(i)+Q_t^H\left[H_t^P(i)-H_{t-1}^P(i)\right]+\int_0^1 D_{ik,t}dk \leq W_t N_t(i)+\int_0^1 \Pi_t^k(i)\,dk+R_{t-1}^D\int_0^1 D_{ik,t-1}dk, \tag{3}$$

where Q_t^H is the price of one unit of housing, as measured in units of consumption goods, W_t is the real wage (in consumption units), and R_{t-1}^D is the (gross) risk-free interest rate on the stock of deposits $D_{ik,t-1}$ of household i in bank k at the end of period $t-1$. $\Pi_t^k(i)$ denotes profits obtained by household i from bank k. It is assumed that the housing stock does not depreciate.

3.2 Entrepreneurs

Entrepreneur j ultimately maximizes the utility he obtains from consuming the non-durable consumption good:

$$E_0\sum_{t=0}^{\infty}\left(\beta^E\right)^t\log\left(C_t^E(j)-\gamma^E C_{t-1}^E(j)\right). \tag{4}$$

where β^E and γ^E are defined as above. It is assumed that entrepreneurs have a lower discount factor than households; $\beta^E<\beta^P$. As we show in the appendix, this assumption implies that the collateral constraint faced by the entrepreneur is binding in the steady state. More specifically, the collateral constraint limits the entrepreneur's borrowing to a fraction of the value of his assets:

$$\int_0^1 l_{jk,t}dk \leq \frac{1}{R_t^L}\int_0^1 \theta_{kt}a_t(j)\,dk, \tag{5}$$

where $l_{jk,t}$ denotes lending of entrepreneur j from bank k, $a_t(j)$ is the expected value of the entrepreneur's assets, and R_t^L is the lending rate, which is common for all banks, as discussed below. θ_{kt} is the loan-to-value (LTV) ratio allowed by bank k, which we assume to be the same for all entrepreneurs borrowing from that bank. In turn, $a_t(j)$ is given by:

$$a_t(j)=E_t\left[Q_{t+1}^H H_t^E(j)+Q_{t+1}^K K_t(j)\right], \tag{6}$$

where Q_t^K denotes the value of installed capital in units of consumption goods.

As already discussed, we assume that entrepreneurs have a preference for

obtaining loans from the same bank as in previous periods, i.e. that they have deep habits in their banking relationships. This gives rise to a wedge between *actual* and *effective* borrowing of each entrepreneur. The difference between these two may be interpreted as switching costs paid by the entrepreneur. We let $l_{jk,t}$ denote the size of the *actual* loan obtained by an entrepreneur, and thus also the amount he must pay back with interest in the next period. It is also the relevant measure to enter the collateral constraint, as banks are interested in the actual repayment. *Effective* borrowing instead measures loans effectively available for the entrepreneur net of switching costs, i.e. funds he can actually use to pay for labour services, investments etc. This measure therefore enters the income side of the entrepreneur's budget constraint. We let $x_t(j)$ denote entrepreneur j's effective (or 'habit-adjusted') borrowing.[9,10] As there is a continuum of banks in the economy, who compete under monopolistic competition, we can write this as:

$$x_t(j) = \left[\int_0^1 \left(l_{jk,t} - \gamma^L s_{jk,t-1}\right)^{\frac{\xi-1}{\xi}} dk\right]^{\frac{\xi}{\xi-1}}, \tag{7}$$

where $s_{jk,t-1}$ is the stock of habits, which develops according to

$$s_{jk,t-1} = \rho_s s_{jk,t-2} + (1-\rho_s) l_{jk,t-1}. \tag{8}$$

$\gamma^L \in (0,1)$ denotes the degree of habit formation in the demand for loans, while $\rho_s \in (0,1)$ measures the persistence of these habits.[11] The parameter ξ denotes the elasticity of substitution between loans from different banks, and is thus a measure of the market power of each individual bank.

Bester (1985) assumes that there are linearly increasing costs to the borrower of pledging collateral. Chan and Kanatas (1985) point out that these may include transaction costs such as legal documentation, while Berger *et al.* (2011) suggest opportunity costs from tying up assets, as well as the cost of fluctuations in credit availability due to fluctuations in asset prices. Bester (1987) points out

[9] To be exact, $x_t(j)$ denotes effective borrowing net of a lump-sum transfer made to the entrepreneur, as described below.

[10] In that sense, 'habit-adjusted' borrowing is the loan measure which the entrepreneur actually benefits from, much in the same way that 'habit-adjusted' consumption enters the utility function of consumers in the model of Ravn *et al.* (2006)

[11] We follow Ravn *et al.* (2006) and assume that deep habits are external. As these authors discuss, the alternative assumption of internal deep habits would give rise to a time-inconsistent problem for the banks, as these would have an incentive to renege on past collateral promises made to borrowers.

that collateralization is inefficient for society as such, as it involves inefficient risk sharing. In the present setup, we do not model a cost of collateral, but simply assume that, all else equal, entrepreneurs prefer to pledge as little collateral as possible, for example because they dislike fluctuations in their credit availability. Introducing a cost to the entrepreneur of pledging collateral would not change our results in any fundamental way. Moreover, observe that as we assume (and later verify) that the collateral constraint is always binding, minimizing collateral is simply equivalent to each entrepreneur maximizing the credit available to him given the value of his assets.

Thus, given his total need for financing, $x_t(j)$, each entrepreneur chooses $l_{jk,t}$ so as to minimize his cost of financing, $\int_0^1 R_{kt}^L l_{jk,t} dk$ as well as the total amount of collateral he must pledge. But since all banks charge the same lending rate ($R_{kt}^L = R_t^L, \forall k$), cost minimization is irrelevant. As a result, each entrepreneur's problem reduces to one of choosing the composition of his loan portfolio so as to minimize the amount of collateral pledged. We show in the appendix that this problem gives rise to the following expression for entrepreneur j's optimal demand for loans from bank k:

$$l_{jk,t} = \left(\frac{\theta_{kt}}{\theta_t}\right)^{\xi} x_t(j) + \gamma^L s_{jk,t-1}, \tag{9}$$

where $\theta_t = \left[\int_0^1 \theta_{kt}^{1-\xi} dk\right]^{\frac{1}{1-\xi}}$ is the aggregate LTV ratio in the economy. This expression implies that the demand for loans from bank k is an increasing function of that bank's LTV ratio relative to the aggregate LTV ratio in the banking sector. In other words, if a bank relaxes its collateral requirements, thus allowing a greater LTV ratio θ_{kt}, the demand for loans from that bank increases, all else equal. Furthermore, note that due to the presence of deep habits, each entrepreneur's demand for loans from a given bank depends positively on his own past demand for loans from the same bank through the effect of past loans on the stock of habits $s_{jk,t-1}$. In the absence of deep habits ($\gamma^L = 0$), the second term would disappear, and a standard demand function under monopolistic competition would result. Moreover, as pointed out by Ravn *et al.* (2006), an additional, intratemporal effect is at play, reinforcing the intertemporal effect just described. In (9), the term arising from the stock of habits for loans from each bank is given from the previous period, and hence inelastic to changes in the LTV ratio in the current period. During a boom, when the aggregate demand for loans is high,

the importance of the first term in (9), which has a positive LTV-elasticity, increases, thereby driving up the 'average' LTV-elasticity of the demand for each bank's loans. As a result, a given decrease in a bank's LTV ratio is associated with a larger increase in that banks market share.[12]

Entrepreneurs use their accumulated capital and land as well as the labor services they hire from households to produce a final good for consumption or investment. The production function of each entrepreneur thus takes the following form:

$$Y_t(j) = A_t \left[N_t(j)\right]^{1-\alpha} \left\{ \left[H^E_{t-1}(j)\right]^{\phi} \left[K_{t-1}(j)\right]^{1-\phi} \right\}^{\alpha}, \tag{10}$$

where H^E_t is the land holdings of each entrepreneur, K_t is the accumulated capital stock, and N_t is labor services, while $\alpha \in (0,1)$ and $\phi \in (0,1)$ are factor shares. A_t is aggregate total factor productivity, which follows the process:

$$\log A_t = (1-\rho_A)\log A + \rho_A \log A_{t-1} + \sigma_A \varepsilon_{At}, \tag{11}$$

with the technology shock ε_{At} following an i.i.d. normal process with standard deviation σ_A, where $A > 0$ and $\rho_A \in (0,1)$. The accumulation of capital is subject to an investment adjustment cost, giving rise to the following law of motion for capital:

$$K_t(j) = (1-\delta) K_{t-1}(j) + \left[1 - \frac{\Omega}{2}\left(\frac{I_t(j)}{I_{t-1}(j)} - 1\right)^2\right] I_t(j), \tag{12}$$

where I_t is the investment level, $\delta \in (0,1)$ is the rate of depreciation of the capital stock, and $\Omega > 0$ is an adjustment cost parameter. This type of investment adjustment cost is in line with the form suggested by Christiano *et al.* (2005).

The problem of the entrepreneur is to maximize (4) by choosing inputs of labor, housing, and capital, as well as his borrowing, investment, and personal consumption, subject to (12), the collateral constraint, and the budget constraint below. Observe that the entrepreneur's actual borrowing appears on the expenditure side in the budget, whereas only his effective borrowing shows up on the

[12] Note also that, despite the different nature of the problem, the demand function above is of a nature similar to that derived by Ravn *et al.* (2006). In our model, the demand for loans from each bank depends *positively* on that bank's relative loan-to-value ratio, while in the setup of Ravn *et al.*, the demand for goods produced by each individual firm depends *negatively* on that firm's relative price.

revenue side.

$$C_t^E(j) + R_{t-1}^L \int_0^1 l_{jk,t-1} dk \leq Y_t(j) - W_t N_t(j) - \vartheta_t^Q I_t(j) - Q_t^H \left[H_t^E(j) - H_{t-1}^E(j)\right] + \\ + x_t(j) + \Phi_t(j) + \Psi_t(j), \tag{13}$$

where ϑ_t^Q denotes a transitory investment-specific technology shock, which evolves as:

$$\log \vartheta_t^Q = \left(1 - \rho_Q\right) \log \vartheta^Q + \rho_Q \log \vartheta_{t-1}^Q + \sigma_Q \varepsilon_{Qt}, \tag{14}$$

with the shock ε_{Qt} following and i.i.d. normal process with standard deviation σ_Q, and where $\vartheta^Q = 1$ and $\rho_Q \in (0,1)$.[13] Since there is only one type of goods in the economy, which can be used either for consumption or investment, the price of investment would always equal 1 in the absence of the investment-specific technology shock. ϑ_t^Q thus measures the price of investment in units of consumption. Moreover, in this model, the presence of investment adjustment costs introduces a wedge between the price of new capital (i.e., investment) and the price of already installed capital, Q_t^K. This implies that the model-specific expression for Tobin's q is ϑ_t^Q / Q_t^K, which is the price ratio of new relative to existing capital.

In the entrepreneur's budget constraint, the two terms $\Phi_t(j) = \gamma^L \int_0^1 \frac{\theta_{kt}}{\theta_t} s_{jk,t-1} dk$ and $\Psi_t(j) = \int_0^1 (1 - p_{kt-1}) \left(R_{t-1}^L L_{kt-1} - \tau a_{t-1}\right) dk$ are lump-sum transfers made to the entrepreneurs in order to ensure that all markets clear, as demonstrated in the appendix. The term $\Phi_t(j)$ is standard in the deep habits literature, and arises due to the wedge between actual and effective borrowing of entrepreneur j.[14] The term $\Psi_t(j)$ is specific to our model, and represents a wedge between what can be interpreted as actual and effective repayment of loans to each bank, as we discuss in the next subsection. In that sense, it resembles the 'repayment shock' considered by Iacoviello (2013). The two lump-sum transfers are exogenous to the individual entrepreneur, and thus do not affect his behaviour.

[13] Observe that we include the investment-specific technology shock in the same way as in Liu *et al.* (2012) and Christiano *et al.* (2010), which differs slightly from the way the shock is modelled by Justiniano *et al.* (2010).

[14] Following Aliaga-Diaz and Olivero (2010), we may define define entrepreneur k's *actual* borrowing as $\int_0^1 l_{jk,t} dk$, and his gross *effective* borrowing as $x_t(j) + \Phi_t(j)$, i.e. including the lump-sum transfer. We can then write the wedge between the two, which can be interpreted as a measure of the switching costs paid by entrepreneur j, as $\gamma^L \int_0^1 \frac{\theta_{kt}}{\theta_t} s_{jk,t-1} dk$. In other words, the switching costs incurred by each entrepreneur is an increasing function of his stock of habits, as well as the difference between the LTV ratios set by different banks.

3.3 The Banking Sector

The role of the banking sector is to receive deposits from households, and use these funds to make loans to entrepreneurs. The gross interest rate on deposits is given by R_t^D, and is taken as given by each bank. Moreover, as already discussed, we assume in this paper that banks compete for customers not by changing the interest rate on loans, but instead by charging different amounts of collateral from borrowers. As a result, all banks set the same interest rate on loans, and each bank therefore takes also the lending rate R_t^L as given. We assume that the lending rate is set as a mark-up over the deposit rate:

$$R_t^L = \psi R_t^D, \tag{15}$$

where $\psi > 1$ measures the mark-up, which we assume to be constant. A positive markup can be rationalized, for instance, by the presence of fixed costs in the production of loans leading to imperfect competition in the banking sector. The presence of a constant mark-up is a key difference between our model and the one of Aliaga-Diaz and Olivero (2010), and reflects our assumption that banks compete on collateral requirements rather than lending rates.

From the viewpoint of each bank, lowering the LTV ratio it allows is associated with higher profits through an increase in current and future market shares, as seen from the loan demand function (9). To make matters interesting, we need to ensure that lowering its credit standards also involves a cost for each bank. We therefore assume that there is a positive relationship between each bank's collateral requirement and the probability that loans made by that bank will be repaid in the next period. In other words, lower credit standards are associated with higher credit risk. We formalize this assumption as:

$$p_{kt} = \Xi + \varpi \left(\theta_{kt} - \theta \right), \tag{16}$$

where p_{kt} is the (bank-specific) probability that a given loan is repaid, and $\varpi < 0$ measures the elasticity of this probability with respect to changes in the bank's LTV ratio relative to the steady state LTV ratio, which is the same for all banks. $\Xi > 0$ is a constant.

We do not attempt to model default of entrepreneurs endogenously. Instead, we assume that there is a wedge between the (actual) loan repayment made

by entrepreneurs and the (effective) repayment received by banks. This wedge increases when credit standards are low. To the extent that lowering its credit standards may be seen as an attempt by the bank to obtain higher profits, which is indeed the case through the increase in its market share, the assumption of a negative relationship between credit standards and default risk is similar in spirit to the one made by Allen and Gale (2000, ch. 8) or Christiano and Ikeda (2011), among others. These authors assume a negative relationship between the potential payoff from a bank's lending activities and the probability of success of these. Our assumption is also in line with empirical evidence. Jimenez *et al.* (2006) find that in young borrower-lender relationships, where informational asymmetries are likely to be large, ex-post high quality borrowers are more likely to pledge collateral than borrowers who turn out to be of low credit quality. This suggests that loans with less collateral are likely to turn out as more risky ex-post. Similar results are obtained by Edelberg (2004), who finds that ex-post high risk borrowers pledge less collateral than low risk borrowers.[15]

We can then write the profits of bank k as:

$$\begin{aligned}\Pi_t^k &= \underbrace{[\Xi + \varpi(\theta_{kt-1} - \theta)]}_{=p_{kt-1}} R_{t-1}^L L_{kt-1} + \underbrace{[1 - \Xi - \varpi(\theta_{kt-1} - \theta)]}_{=1-p_{kt-1}} \frac{L_{kt-1}}{\int_0^1 L_{kt-1} dk} \tau a_{t-1} + \\ &+ \int_0^1 D_{ik,t} di - L_{k,t} - R_{t-1}^D \int_0^1 D_{ik,t-1} di, \end{aligned} \tag{17}$$

where L_{kt} denotes total loans made by bank k to all entrepreneurs; i.e. $L_{kt} \equiv \int_0^1 l_{jk,t} dj$, and where the presence of $a_{t-1} \equiv a_{t-1}(j)$, $\forall\ j$, reflects that all entrepreneurs own the same amount of assets. With probability p_{kt-1}, the bank receives the loans it made in the previous period with interest. With probability $(1 - p_{kt-1})$, the loan is not repaid, in which case we assume that bank k receives a share of the liquidation value of the collateral assets, with this share given by bank k's total lending relative to total lending of all firms.[16] Since we

[15] We emphasize that these results are related to the ex-post risk of default, i.e. to characteristics that were *unobservable* to the lender when the loan was made. On the contrary, there is evidence that borrowers with high *observable* risk must pledge more collateral to obtain a loan; see e.g. Jimenez *et al.* (2006).

[16] Since all entrepreneurs are identical, so is their loan composition from different banks. Moreover, recall our assumption that all banks lend to all entrepreneurs. This implies that we can replace the share of bank k's lending to a given entrepreneur relative to total loans to that entrepreneur $\left(\frac{l_{jk,t-1}}{\int_0^1 l_{jk,t-1} dk}\right)$ by the share of bank k's total lending relative to total lending of all banks $\left(\frac{L_{kt-1}}{\int_0^1 L_{kt-1} dk}\right)$.

are not modelling default of entrepreneurs, these are compensated for handing over a share of their assets through the lump-sum transfer Ψ_t. The parameter $\tau \in (0,1)$ reflects that the value of the collateral assets is lower in liquidation.[17] Finally, with probability 1, the bank must pay back the deposits it received from households in the previous period multiplied by the deposit rate.

We abstract from modelling an interbank lending market or central bank lending facilities, as well as any reserve requirements. The balance sheet of bank k is then simply:

$$L_{kt} = \int_0^1 D_{ik,t} di. \tag{18}$$

Moreover, each bank takes the demand for its loans as given:

$$L_{kt} = \int_0^1 l_{jk,t} dj = \int_0^1 \left[\left(\frac{\theta_{kt}}{\theta_t} \right)^{\xi} x_t + \gamma^L s_{kt-1} \right] dj. \tag{19}$$

Each banks chooses L_{kt} and θ_{kt} so as to maximize its profits subject to (18) and (19). Eventually, we consider symmetric equilibria, in which all banks optimally set the same LTV ratio. As we show in the appendix, the first-order conditions for this problem can then be written as:

$$\mu_t^B = \mathrm{E}_t q_{t,t+1} \left[p_t R_t^L + (1 - p_t) \frac{\tau}{\theta_t} - R_t^D + \gamma^L (1 - \rho_s) \mu_{t+1}^B \right], \tag{20}$$

$$\xi \mu_t^B \frac{x_t}{\theta_t} = -\varpi \mathrm{E}_t q_{t,t+1} \left(R_t^L L_t - \tau a_t \right), \tag{21}$$

where μ_t^B is the Lagrange multiplier on (19) in the bank's optimization problem, and can thus be interpreted as the shadow value to the bank of lending an extra dollar. $q_{t,t+1} = \beta^P \frac{\lambda_{t+1}^P}{\lambda_{t+1}^P}$ is the stochastic discount factor of banks, and is equal to that of households, as these are the owners of the banks.

The first optimality condition states that the shadow value of lending an extra dollar is given by the probability-weighted repayment to the bank in case the loan is (respectively, is not) repaid, minus the cost of borrowing that extra dollar from the household sector. Finally, the last term reflects that if bank

[17] The same argument is usually used to argue that the LTV ratio should be smaller than 1. In our setting, the presence of τ is needed to ensure that the profit of each bank is higher if a given loan is repaid than if it is not. We assume that for some reason, banks cannot write contracts that take into account that $\tau < 1$.

k lends an extra dollar in this period, the borrower of that dollar will develop a habit for loans from this bank, and will therefore borrow more from bank k also in the next period. The size of this effect depends on the strength and the persistence of deep habits in the market for bank loans. In the absence of deep habits, the latter term is zero.

The second equation pins down the optimal LTV ratio. By marginally increasing θ_t, each bank increases its market share in the current period, as firms prefer to borrow from banks who allow a high LTV ratio. The resulting increase in profits is tied to the elasticity of substitution between loans from different banks; ξ. The presence of μ_t^B on the left-hand side ensures that the bank factors in the increase in demand for its loans in subsequent periods, as seen from (20). The cost of a marginal increase in θ_t is given by the increase in credit risk brought about by easier credit standards. This can be illustrated by rewriting (21) as:

$$\xi \mu_t^B \frac{x_t}{\theta_t} = -\frac{\partial \Pi_t}{\partial p_{t-1}} \frac{\partial p_{t-1}}{\partial \theta_{t-1}}.$$

In optimum, the marginal gains and profits of a marginal increase in θ_t must be equal.

3.4 Aggregation and Market Clearing

The aggregate resource constraint for this economy is given by:

$$C_t^P + C_t^E + \vartheta_t^Q I_t = Y_t, \tag{22}$$

while the housing market clearing condition is:

$$H_t^P + H_t^E = H, \tag{23}$$

where H is the supply of housing, which is held fixed.

3.5 Equilibrium and Model Solution

In the appendix, we present a complete list of the equilibrium conditions of the model, as well as its steady state. We consider a log-linear approximation of the

model around this steady state. The list of log-linearized equations is also presented in the appendix. We then use DYNARE to solve the log-linearized model. We have checked and verified that the model has a unique, stable equilibrium.

As already mentioned, the collateral constraint is binding in steady state by assumption. However, if a shock takes the economy sufficiently far away from the steady state, the constraint may become non-binding, so that treating the constraint as an equality will be incorrect. While occasionally non-binding constraints can be handled, we want to restrict our attention to cases in which the constraint always binds. We are interested in the effects of changes in collateral requirements, but by definition, such changes will have little effect if the collateral constraint is non-binding. As a result, we only consider shocks that are small enough not to make the constraint non-binding. To check that the constraint is in fact always binding, we follow Holden and Paetz (2012) and augment our model with a set of shadow price shocks that take on non-zero values if and only if the shadow price of borrowing (μ_t^E) turns negative. We then verify that these shocks are in fact zero in virtually all periods. For the impulse responses shown in the next section, the constraint is always binding, while it becomes non-binding in very few periods in our dynamic simulation, as described below. See the appendix for details on this method.

3.6 Parametrization

The full set of parameter values is shown in table A1 in the appendix to this chapter. While most of the parameter values are quite standard, we discuss the values of some of the more important parameters in the following. We allow for a large difference between the discount factors of households and entrepreneurs in order to ensure that the steady state value of μ_t^E is not too close to zero. The degree of habit formation in consumption is set so as to match the degree of deep habit formation in banking, as described below. We set the investment adjustment cost parameter to 4, in line with empirical estimates of this parameter, although these vary from close to 0 (Liu *et al.*, 2012) to above 26 (Christiano *et al.*, 2010). We set the steady state interest rate spread between deposit and lending rates to 0.0168, following the estimate of Aliaga-Diaz and Olivero (2010). The recovery rate of assets in liquidation has been calibrated to yield a steady state aggregate LTV ratio of 0.75 in steady state, which represents

a middle ground with respect to the values used in the literature for this ratio (see for example Iacoviello (2005) or Liu *et al.* (2012)). As for the parameters related to deep habits in banking, we again rely on values estimated by Aliaga-Diaz and Olivero (2010), who report a habit formation parameter of 0.72, and an elasticity of substitution between loans from different banks of 190. While the former estimate is not very different from the value of 0.86 estimated by Ravn *et al.* (2006), the latter parameter is much higher than the elasticities of substitution usually employed in macroeconomic models with monopolistic competition, including the elasticity of 5.3 used by Ravn *et al.* (2006). However, as argued by Aliaga-Diaz and Olivero (2010), loans from different banks are likely to be much better substitutes than products of different firms in the goods market, indicating that the elasticity of substitution should indeed be much higher. As we shall see in the next section, the value of this elasticity is of key importance for the macroeconomic implications of deep habits in banking.

We use the estimate of 0.85 from Ravn *et al.* (2006) for the persistence of the stock of habits. Finally, we need to set a value for the parameter ϖ, which measures the elasticity of credit risk with respect to changes in the LTV ratio. In this respect, the literature offers little guidance. We need ϖ to be negative in order to induce a cost of lowering collateral requirements. Intuitively, the (numerically) larger ϖ is, the bigger is this cost, and the less attractive it is for banks to lower their credit standards. We therefore initially pick a relatively large numerical value of -50, which we think of as a conservative choice, in the sense that if the model is able to generate an amplification of macroeconomic fluctuations even for such a large value of ϖ, this would suggest that our mechanism may indeed give rise to a non-negligible amplification in the general case.

For the steady state values of the shocks, we set $\varsigma = 0.04$, in line with the estimate from Liu *et al.* (2012). The steady state values of the two other shocks are normalizations. Following the Real Business Cycle literature, we set the persistence of the technology shock to $\rho_A = 0.97$ as in e.g. Mandelman *et al.* (2011), while we set a somewhat smaller standard deviation of $\sigma_A = 0.001$. As will become evident below, this implies that the borrowing constraint of entrepreneurs will in fact remain binding in all periods. We set similar values for the persistence and volatility of the other shocks.

4 Results

In this section, we present the results from a number of simulation exercises using the laboratory model outlined in the previous section. Our main focus is to investigate how the amplification and propagation of technology shocks are altered in our setup. A number of papers have found that technology shocks are not amplified by the presence of financial frictions such as collateral constraints (Kocherlakota, 2000; Liu *et al.*, 2012). Hence, if the mechanism described in this paper is succesful in amplifying technology shocks, this suggests that it may be even more important in the presence of other types of shocks. Therefore, the first two subsections are concerned with technology shocks. In the final subsection, we consider two different shocks that recent studies have proposed as being important drivers of the business cycle, as well as a shock originating in the banking sector.

4.1 The Impact of Technology Shocks

We first present impulse responses of some key variables to an exogenous technological innovation. Figure 1 displays impulse responses from our baseline model, as well as the responses of a similar model with the deep habits mechanism shut off, i.e. for $\gamma^L = 0$, so as to facilitate comparison.

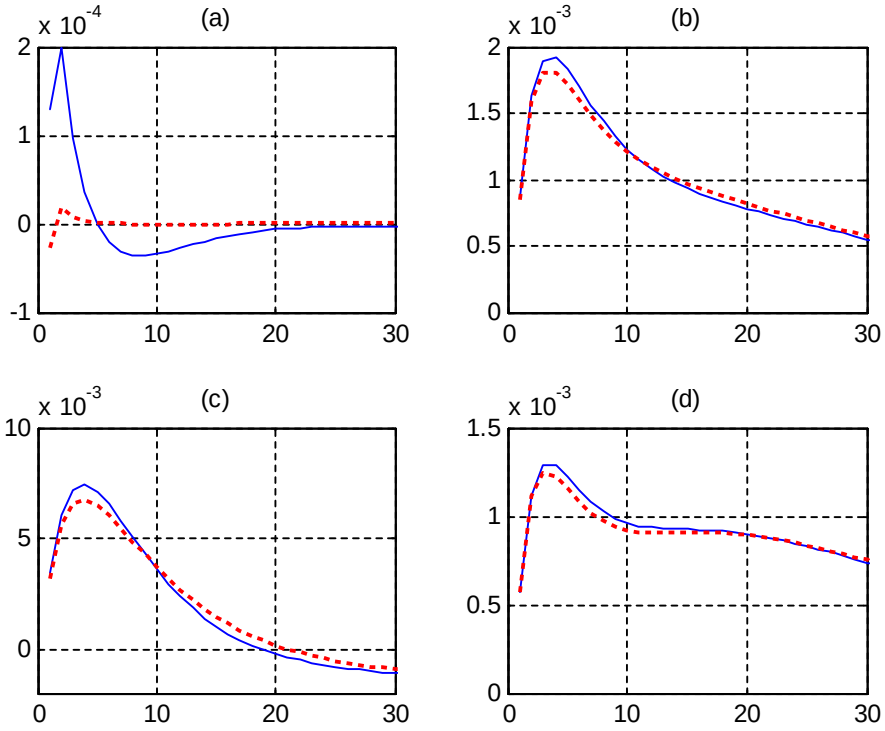

Figure 1: Effects of a technology shock. Solid blue line: $\gamma^L = 0.72$, dashed red line: $\gamma^L = 0$ (no deep habits). Shock to ε_{At} with $\sigma_A = 0.001$, $\rho_A = 0.97$. Panel (a): LTV ratio, panel (b): Output, panel (c): Investment, panel (d): Total Consumption.

As illustrated in panel (a) in figure 1, the loan-to-value ratio goes up in the wake of a positive technology shock in the presence of deep habits. In other words, banks lower their credit standards after a positive shock. This reflects that due to the persistence of the shock, aggregate output, and thus also demand for loans, will be higher than usual in the periods to come. As a result, the trade-off between a larger market share in present and future periods vis-a-vis the corresponding, higher credit risk associated with each individual loan has changed, so that it is now profitable for the bank to lower collateral requirements. The probability of repayment therefore goes down (not shown). Without the deep habits mechanism, instead, the LTV ratio stays roughly constant, as the future profits motive from current gains in market shares is not present in that case.

Panels (b)-(d) illustrate the effects on macroeconomic variables. With deep habits in banking, the boom in output and investment in the wake of a technology shock are amplified, although not by much. At its peak, the response of output is some 6 % higher than in the model without deep habits. For investment, this

number is 9 %. For aggregate consumption, the small, additional increase in the presence of deep habits is driven by the consumption of households, while entrepreneurs instead buy more land so as to increase production.

We also perform a dynamic simulation of our model with only technology shocks turned on. Table 1 summarizes the most important results from a dynamic simulation of the baseline model compared to a simulation of the same model with $\gamma^L = 0$ in order to highlight the contribution of deep habits in banking. As can be seen from the variance ratios in the table, the variance of output, investment and aggregate consumption increases when we turn on the deep habits mechanism, although again by moderate amounts. In contrast, the variance of the LTV ratio increases a great deal, which is not surprising, as the LTV ratio is practically constant in the model without deep habits. The conclusion from this exercise is therefore that deep habits in banking tend to amplify aggregate fluctuations driven by technology shocks, but that this amplification is of limited quantitative importance. This conclusion is fully in line with that of Aliaga-Diaz and Olivero (2010), who also find that deep habits generate little additional volatility of macroeconomic variables.

Table 1: Summary of simulation results

Variance ratios	
Output	1.02
Investment	1.10
Consumption (Agg.)	1.02
LTV Ratio	69.38
Correlation with output	
Bank Profits $\left(\gamma^L = 0.72\right)$	0.77
Bank Profits $\left(\gamma^L = 0\right)$	0.61

Note: The statistics are based on a dynamic simulation of 51,000 quarters, with a burn-in period of 1000 quarters which are discarded. All parameter values are as described in table A1. The variance ratios are computed by dividing the variance of the variable in question in the model with deep habits by the variance of the same variable in the model without deep habits. The entrepreneur's borrowing constraint is binding 99.99 % of the time in the simulation with deep habits, and 99.97 % of the time in the simulation without deep habits, indicating that the approximation error we make by assuming that the constraint is always binding is very small.

Finally, observe from table 1 that there is a large, positive correlation between output and bank profits, and that this correlation increases substantially when we turn on deep habits. This is an important result, as bank profits tend to be strongly procyclical in the data (Albertazzi and Gambacorta, 2009), a result that other DSGE models with banks have had a hard time reproducing; see for instance Gerali *et al.* (2010). The reason for this shortcoming is the presence of a countercyclical spread between loan and deposit rates in the models of Gerali *et al.* (2010) and Aliaga-Diaz and Olivero (2010), whereas we have made the simplifying assumption of a constant spread. Empirical studies tend to find evidence of countercyclical spreads (see e.g. Dueker and Thornton, 1997; and Olivero, 2010), suggesting that our model generates the 'right' response of bank profits, but for the 'wrong' reason.[18]

4.2 Sensitivity Analysis

In this subsection, we evaluate the sensitivity of the results in the previous subsection with respect to different values for some key parameters of our model. In each step, we alter the calibration along one dimension at a time, keeping all other parameters at their baseline values described above. For the sake of brevity, we focus on the effects on output. Figure 2 displays the results from this analysis.

[18] In addition, Albertazzi and Gambacorta (2009) report that the procyclicality of bank profits comes about for two reasons: larger loan volumes, and higher credit portfolio quality. Our model captures the first of these effects, but not the second effect.

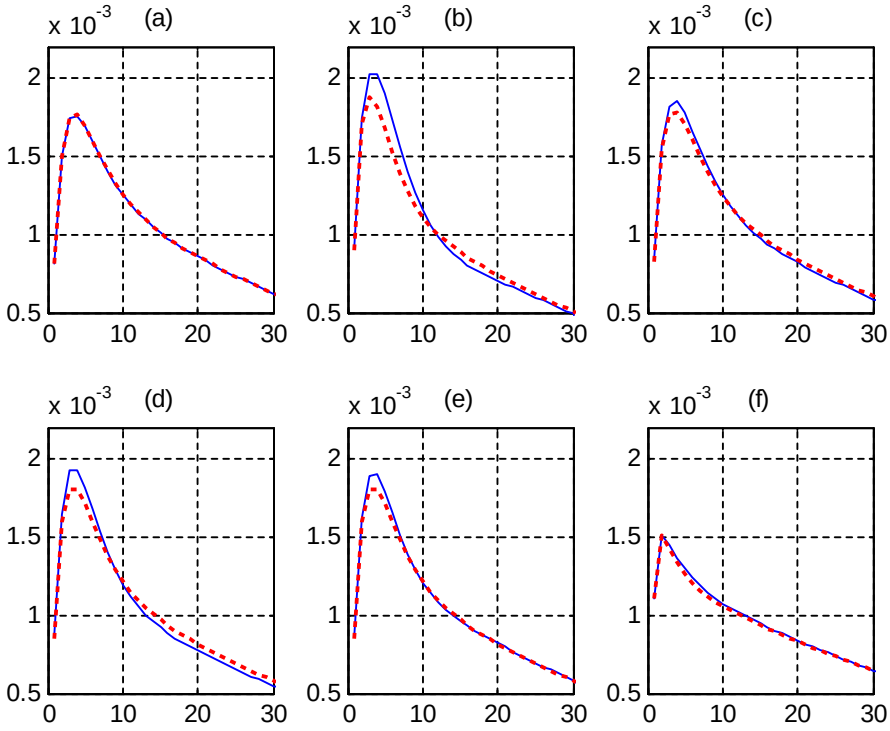

Figure 2: Response of output to a technology shock under various parameter changes. In each panel, the solid blue line represents $\gamma^L = 0.72$, and the dashed red line $\gamma^L = 0$ (no deep habits). Shock to ε_{At} with $\sigma_A = 0.001$, $\rho_A = 0.97$. Panel (a): $\xi = 25$, (b): $\varpi = -20$, (c): $\varpi = -100$, (d): $\gamma^L = 0.86$ (blue line), (e): $\rho_s = 0.5$, (f): $\gamma^P = \gamma^E = 0.2$, $\Omega = 0.5$.

Panel (a) in the figure shows the impact of deep habits in banking in the presence of a lower elasticity of substitution between banks (ξ) of 25, i.e. still a relatively high value. In that case, the two curves are almost identical. This demonstrates that a very high value of ξ is crucial for the mechanism to generate any amplification at all. Nevertheless, as already argued, we believe that loans from different banks are indeed much more substitutable than goods from different firms, which warrants a relatively high value of ξ. This is also supported by the empirical evidence provided by Aliaga-Diaz and Olivero (2010).

The next two panels are concerned with the value of ϖ, which measures the elasticity of the repayment probability to changes in the LTV ratio. Panel (b) shows that setting this value to -20 leads to a somewhat larger amplification of output fluctuations. In particular, the peak response of output is now 7.5 % larger in the presence of deep habits. Intuitively, when ϖ is (numerically)

smaller, it is less costly for banks to lower their credit standards, as the drop in the loan repayment probability is smaller. This induces banks to allow a larger increase in the LTV ratio, which in turn leads to a bigger increase in lending and output. Accordingly, panel (c) illustrates that setting $\varpi = -100$ dampens the amplification relative to our baseline model. Observe, however, that the output response in the presence of deep habits remains above the response with no deep habits. This is confirmed even when setting $\varpi = -1000$ (not reported).

In panels (d) and (e), we experiment with the deep habits parameters. We first raise γ^L to 0.86, which is the value originally used by Ravn *et al.* (2006). This has a very limited effect on the response of output. Next, we lower the habit persistence (ρ_s) to 0.5. Panel (e) illustrates that this also leads to very small changes. In other words, the results do not seem to be very sensitive to our assumptions about these parameter values.

Finally, we dampen the internal persistence of the model. Under our baseline calibration, habit formation in consumption and adjustment costs of investment play an important role in propagating external shocks. Panel (f) shows the impact of lowering habit formation in consumption to 0.2, which is substantially lower than in most studies, and at the same time setting the investment adjustment cost parameter to 0.5, which is definitely at the lower end of the range of available estimates. In this case, there is very little difference between the response of output with and without deep habits, although the response with $\gamma^L = 0.72$ remains on top in all periods except one. Interestingly, this suggests that the deep habits mechanism interacts with other 'frictions' of the model, so that its quantitative importance is magnified in the presence of these frictions.

4.3 Effects of Other Shocks

So far, we have considered only technology shocks. The recent macroeconomic literature has suggested that two other types of shocks may be important drivers of the business cycle. The first is an investment-specific technology shock, the importance of which has been emphasized by, among others, Fisher (2006) and Justiniano *et al.* (2010).[19] The second shock is a housing demand shock. Liu

[19] In a recent paper, Justiniano *et al.* (2011) study two different types of investment shocks. They find that when the relative price of investment is included in the list of observables used to estimate DSGE models, a shock to the marginal efficiency of investment takes over most of the explanatory power of the investment-specific technology shock. However, for our purposes,

et al. (2012) find that this shock, which may potentially be interpreted as a financial innovation in the mortgage market, has played an important role in the US economy in recent decades. In this section, we therefore investigate how the propagation of these two shocks are affected by the presence of deep habits in banking. Consider first the impact of an investment-specific technology shock, which is displayed in figure 3 at the end of the paper. The effect of an investment-specific technology shock is to lower the relative price of investment (in units of consumption). As a result, investment goes up, and consumption goes down on impact. After a while, output starts to go up, which allows for consumption to increase as well. In the presence of deep habits in banking, the LTV ratio initially goes down, but then starts increasing for a number of periods, mimicking the pattern of output. The initial drop in output leads banks to tighten their credit standards, as demand for loans in the near future is depressed, making it less profitable for the bank to increase its market share in these periods. After a while, output and thus loan demand pick up, so the trade-off faced by each bank again changes, making it profitable for banks to lower collateral requirements in spite of the resulting increase in credit risk. As illustrated, easier lending standards further amplifies the boom in output, although the additional increase in output in the presence of deep habits is again modest, reaching a maximum of slightly above 10%.

Figure 4 at the end of the paper presents the response of our model economy to a shock to housing demand.[20] As explained by Liu *et al.* (2012), the shock to the housing demand of households leads to an increase in the land price. This pushes up the entrepreneur's collateral value, allowing him to increase his borrowing and to invest in more capital and land. In turn, this pushes up the land price further. In fact, even though the shock hits only the household, with our parametrization of the model it is the entrepreneur who ends up with more land in equilibrium (not reported). In other words, the propagation of the housing demand shock relies heavily on the collateral constraint. Exactly for this reason, the additional increase in the entrepreneur's access to credit brought about by the observed increase in the LTV ratio in the presence of deep habits results in a non-negligible amplification of the response of output. While this is not very obvious from panel (b) in figure 4, we can verify this by 'zooming in'

we do not need to make this distinction.

[20] Liu *et al.* (2012) find that this shock is extremely persistent; practically a unit root. We set a persistence similar to that of the technology shock.

on the output response in the first periods after the shock. Between periods 3 and 5, the deep habits mechanism generates an additional amplification of the output response of between 10 and 30 %, compared to the case of $\gamma^L = 0$. In other words, the quantitative importance of this mechanism is somewhat larger in the case of housing demand shocks. Moreover, we observe from figure 4 that the amplification effect works in a symmetric way, in the sense that not only the initial boom in output and investment, but also the subsequent bust is magnified by the deep habits mechanism. Between 3 and 5 years after the shock, the drop in output is around 10-12 % larger when the deep habits mechanism is at play. As hinted above, housing demand shocks lead to an increase in house prices which, through the presence of a collateral constraint, improve the entrepreneur's ability to borrow. When our deep habits mechanism is added, housing demand shocks have an additional effect on the entrepreneur's borrowing ability by driving up the LTV ratio, thereby enhancing the macroeconomic impact of housing demand shocks. In other words, the interaction between our deep habits mechanism and housing demand shocks give rise to a non-negligible amplification of this type of shock. Moreover, we also observe a strongly procyclical movement of bank profits in response to investment-specific technology shocks as well as housing demand shocks.

Finally, we consider the effects of a shock arising within the banking sector. In particular, we allow for a shock to the elasticity of substitution between banks, which is then given by:

$$\log \xi_t = \left(1 - \rho_\xi\right) \log \xi + \rho_\xi \log \xi_{t-1} + \sigma_\xi \varepsilon_{\xi t}, \tag{24}$$

with the shock $\varepsilon_{\xi t}$ following and i.i.d. normal process with standard deviation $\sigma_\xi = 0.001$, $\rho_\xi = 0.9$, and where $\xi = 190$ now denotes the steady state value of the elasticity of substitution. Figure 5 at the end of the paper displays the response of key variables to a one standard deviation shock to $\varepsilon_{\xi t}$. An increase in the elasticity of substitution implies that each bank will be more inclined to enhance competition for potential borrowers, as a given increase in the LTV ratio allowed by the bank now gives rise to a larger increase in the demand for loans from that bank, according to (9). It is therefore optimal for banks to lower their lending standards. In the presence of deep habits, lowering credit standards in the current period is even more attractive for each bank due to the intertemporal

effect on the demand for its loans. As a result, the LTV ratio rises in both cases, but the increase is almost twice as large under deep habits. However, contrary to the other shocks, the presence of deep habits *dampen* the macroeconomic impact of a shock to $\varepsilon_{\xi t}$. The reason for this is the intratemporal effect of deep habits, as previously described. As seen from (9), the total demand for loans from a given bank is much less sensitive to changes in the elasticity of substitution under deep habits ($\gamma^L > 0$), as the second term on the right hand side is inelastic. In other words, a shock to $\varepsilon_{\xi t}$ has a much smaller effect on the demand for loans faced by each bank in the presence of deep habits as compared to the case of $\gamma^L = 0$. Of course, this is counteracted by the drop in credit standards under deep habits. In equilibrium, however, the former effect dominates, as evidenced by the responses of the macroeconomic variables, which are substantially higher in the absence of deep habits. In other words, in this case the amplification arising from countercyclical credit standards is dominated by the drop in the elasticity of the demand for loans from each bank brought about by our version of relationship banking.

5 Conclusion

In this paper, we have proposed a way to incorporate endogenous movements in credit standards into aggregate models of the macroeconomy. When the deep habits mechanism is introduced in firms' demand for bank loans, countercyclical collateral requirements arise endogenously as a result of profit maximization by each individual bank. We believe our model is well-suited to describe the market for bank loans, as this market is characterized by repeated interaction between borrowers and lenders and widespread non-price competition.

In general, countercyclical credit standards tend to amplify business cycle fluctuations. Nevertheless, our results indicate that this amplification is relatively small at the macroeconomic level. While our model may lack some features that could potentially overturn this conclusion, our results are in line with other recent studies, suggesting that countercyclical lending standards may be less important for macroeconomic fluctuations than previously thought. For one of the shocks we consider, the presence of deep habits actually dampens macroeconomic fluctuations.

Our results have implications for macroprudential policy. In order to counteract the additional fluctuations arising from deep habits in banking, such policies should be aimed at lowering the cost of switching between banks. This may be achieved through an increased transparency of borrowing conditions, allowing borrowers to compare their current conditions to the terms offered by rival banks. More fundamentally, switching costs arise because of problems of asymmetric information, which macroprudential policymakers should therefore also seek to address.

In future work, it would be interesting to incorporate some of the features that have been left out of the model. A natural first step would be to fully endogenize borrower default, so as to incorporate the effects of firm bankruptcy. This could potentially enhance the macroeconomic impact of time-varying credit standards.

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Impulse Responses

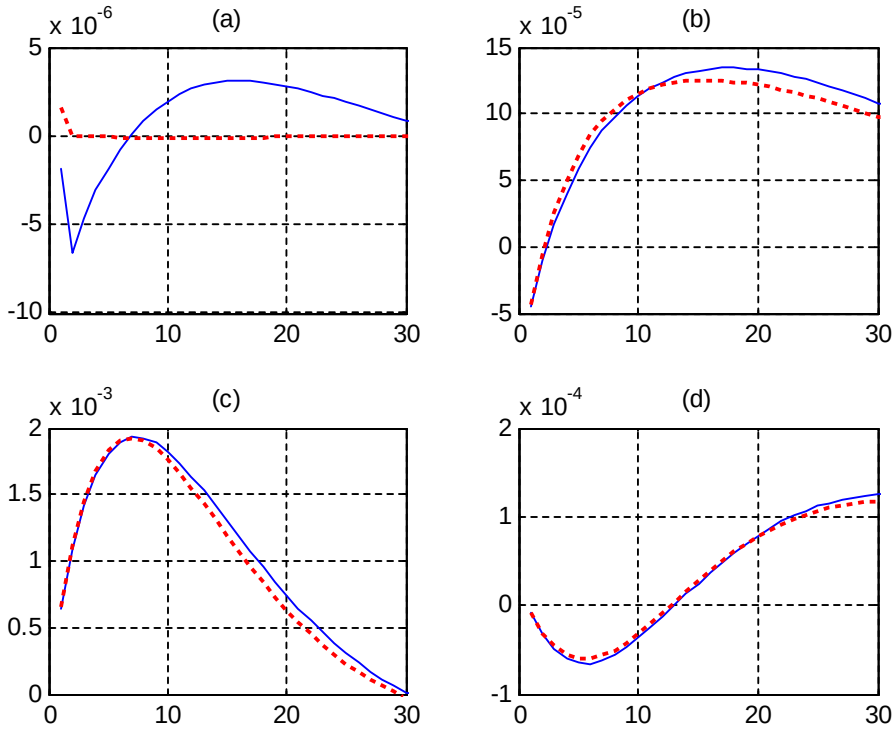

Figure 3: Effects of an investment-specific technology shock. Solid blue line: $\gamma^L = 0.72$, dashed red line: $\gamma^L = 0$ (no deep habits). Shock to ε_{Qt} with $\sigma_Q = 0.001$, $\rho_Q = 0.9$. Panel (a): LTV ratio, panel (b): Output, panel (c): Investment, panel (d): total consumption.

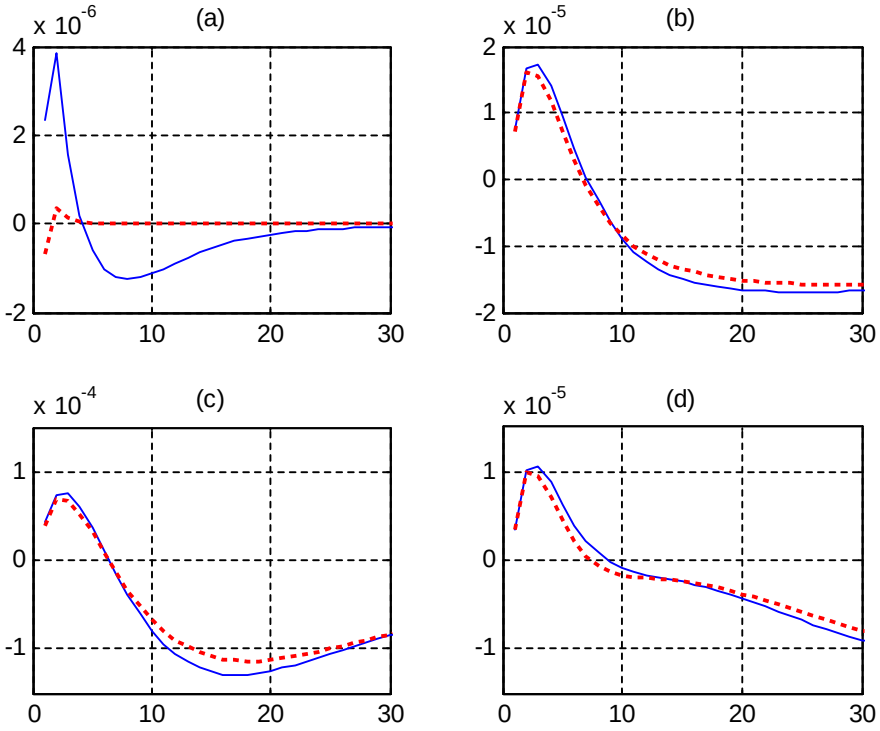

Figure 4: Effects of a shock to housing demand. Solid blue line: $\gamma^L = 0.72$, dashed red line: $\gamma^L = 0$ (no deep habits). Shock to ε_{Ht} with $\sigma_H = 0.001$, $\rho_H = 0.97$. Panel (a): LTV ratio, panel (b): Output, panel (c): Investment, panel (d): total consumption.

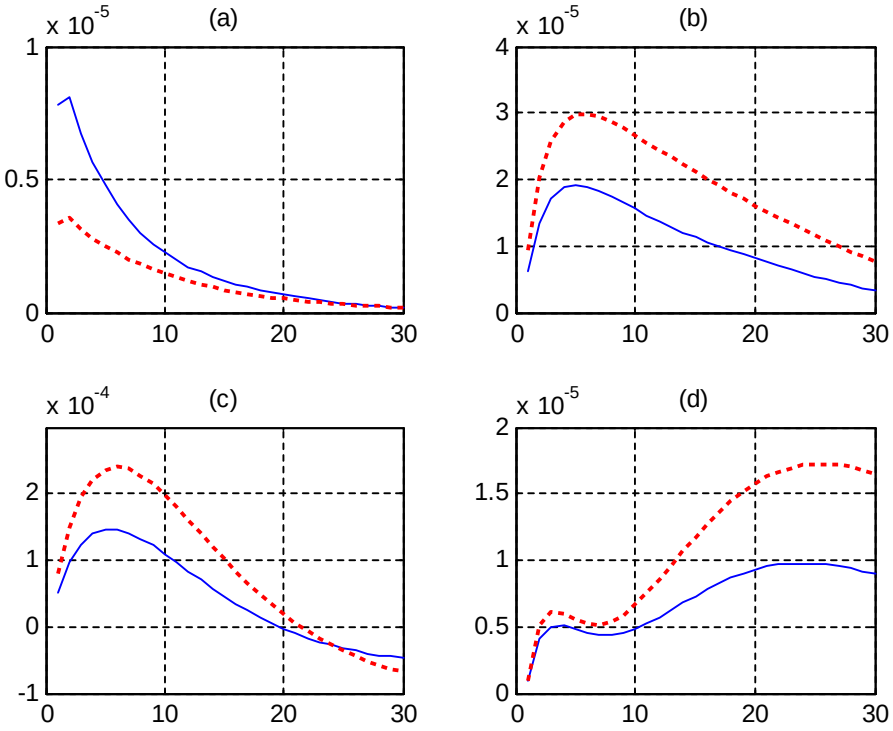

Figure 5: Effects of a shock to the elasticity of substitution between banks. Solid blue line: $\gamma^L = 0.72$, dashed red line: $\gamma^L = 0$ (no deep habits). Shock to $\varepsilon_{\xi t}$ with $\sigma_\xi = 0.001$, $\rho_Q = 0.9$. Panel (a): LTV ratio, panel (b): Output, panel (c): Investment, panel (d): total consumption.

The Effects of Fiscal Policy in a Small Open Economy with a Fixed Exchange Rate: The Case of Denmark*

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Abstract

We study the empirical effects of fiscal policy in Denmark since the adoption of a fixed exchange rate policy in 1982. Denmark's fixed exchange rate implies that the nominal interest rate remains fixed after a fiscal expansion, facilitating a substantial impact of the fiscal stimulus on the real economy. On the other hand, the large degree of openness of the Danish economy means that a sizeable share of the fiscal stimulus will be directed towards imported goods. Our results suggest that the 'monetary accomodation channel' dominates the 'leakage effect' in the short run. We demonstrate that fiscal stimulus has a rather large impact on economic activity in the very short run, with a government spending multiplier of 1.3 on impact in our preferred specification. We also find that the effects of fiscal stimulus are very short-lived in Denmark, with the effect on output becoming insignificant after around a year. We further demonstrate that while the fiscal multiplier was below 1 in the 1970's and 1980's, it has been above 1 in the 1990's and the 2000's, when Denmark has had a credibly fixed exchange rate and sound public finances.

JEL classification: E32, E62, F41.

Keywords: Fiscal Policy, Fixed Exchange Rates, Structural VARs.

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1 Introduction

The macroeconomic effects of discretionary fiscal policy have been the subject of a longstanding, academic debate.[1] The present paper adds to this debate by presenting an empirical analysis of the effects of fiscal policy in Denmark. Since 1982, Denmark has been conducting a fixed exchange rate policy, with its currency, the Krone, pegged first to the German D-mark, and since 1999 to the euro. The Danish economy is characterized by a large degree of openness, with a ratio of exports to GDP around 50 % in recent years. Hence, the effects of fiscal policy in Denmark are interesting also from a theoretical point of view. A fixed exchange rate is traditionally believed to allow for relatively large effects of fiscal policy, as this implies that the nominal interest rate is held fixed; while it is likely to be raised under a floating exchange rate (see e.g. Corsetti *et al.* (2011), or the textbook Mundell-Fleming model). In particular, to the extent that prices and wages are sticky in the short run, no nominal adjustment can take place, so an increase in government spending is likely to have a large effect on output. At the same time, however, the large degree of openness implies that a relatively large share of fiscal stimulus is likely to be spent on foreign goods or services. This 'leakage' effect is likely to dampen the size of the fiscal multiplier (Beetsma and Giuliodori, 2011).

While the interest rate effect has traditionally received more attention in the literature, the relative importance of these two opposite effects is ultimately an empirical question, which we seek to address in this paper. To this end, we employ a structural vector autoregressive (SVAR) model, and follow the identification strategy first described by Blanchard and Perotti (2002). As we are considering a country for which economic fluctuations abroad are very important, we augment the SVAR approach of Blanchard and Perotti to take into account business cycle movements in Denmark's most important trading partners; Germany and Sweden. We also control for global business cycle fluctuations, such as a global technology shock, by including US GDP as an exogenous variable in the model.

Our empirical results indicate that the fiscal multiplier in Denmark is relatively large in the very short run. For the period 1983-2011, i.e. since the introduction of the currency peg, we find an estimated government spending

[1] We refer the reader to Coenen *et al.* (2010) or Hebous (2011) for extensive surveys on this literature.

multiplier of 1.3 on impact. However, we also find that the expansionary effects of government spending die out quickly. The multiplier is above 1 only in the first quarter, and is significantly greater than zero only during the first year in our baseline specification. The cumulative multiplier, which measures the accumulated increase in output relative to the accumulated increase in government spending during the first 20 quarters, is also 1.3; indicating that the effects of fiscal stimulus die out as the stimulus itself is removed. This suggests that the dynamic effects of government spending in Denmark are small.

The relatively large impact multiplier tends to suggest that in the short run, the interest rate effect is indeed more important than the leakage effect. After a while, the opposite seems to be the case. There are, however, other potential explanations for the extremely short-lived effects of fiscal stimulus in Denmark. As prices and wages start to adjust, the relative price of Danish goods and services will rise, inducing Danish as well as foreign consumers to substitute away from these. Given the large export share in the Danish economy, the resulting drop in exports is likely to outweigh the rise in domestic government spending. Moreover, Denmark has very important automatic fiscal stabilizers. These tend to dampen the persistence of economic shocks, including shocks to government spending.

Our results are consistent with other recent, empirical findings. Ilzetzki *et al.* (2012) study a sample of 44 countries, and find a cumulative multiplier of around 1.4 in economies operating under fixed exchange rates, while the multiplier is much lower (and significantly so) in countries with floating exchange rate regimes. These authors furthermore find empirical support for the importance of the interest rate channel, as they report an increase in the nominal interest rate under flexible exchange rates. Corsetti *et al.* (2012) study the effects of fiscal policy in 17 OECD countries, and also find a significantly higher fiscal multiplier under fixed exchange rates. They find an estimated multiplier of 0.6 under fixed exchange rates, and around zero under floating rates. On the other hand, they find no direct evidence in favor of the interest rate effect. Beetsma and Giuliodori (2011) find a fiscal multiplier around 1.2 for a sample of 14 member countries of the European Union, although for the most open economies among these, including Denmark, the multiplier is found to be slightly below 1. This highlights the importance of the leakage effect, which has also been emphasized by Zhang and Zhang (2010). Nakamura and Steinsson (2011) estimate a fiscal multiplier of around 1.5 based on US data at the state and regional level. The idea is that

each state represents a small, open economy with a fixed exchange rate relative to its neighbour states. What they report is the so-called *open economy relative multiplier*, which measures the change in output in one state relative to other states when government spending in that state is increased. As such, their result is not directly comparable to ours. Finally, Bergman and Hutchison (2010) study the effects of fiscal policy in Denmark in a setup related to ours, but with a sample from 1971-2000, and with specific focus on the effects of the Danish fiscal contraction in the mid-1980's. Our results are in general consistent with their findings, although some differences arise due to the use of different model specifications and different sample periods. It should be noted, however, that our results differ from those in the literature in one important aspect. We find that private consumption drops on impact in response to an increase in government spending.[2] This is different from most studies following the approach of Blanchard and Perotti (2002), which tend to find an increase in private consumption. On the other hand, studies in the tradition of Ramey and Shapiro (1998) and Ramey (2011a) tend to find a drop in private consumption, more in line with our results.[3] However, we find the response of consumption to be significantly different from zero only on impact, after which the response is very close to zero.

A related, recent strand of the literature focuses on the effects of government spending when the zero lower bound on nominal interest rates is binding (see e.g. Christiano *et al.*, 2011). In that case, just as in a small open economy with a fixed exchange rate, the nominal interest rate does not move in response to a government spending shock. As a result, the short run real interest rate goes down due to the rise in inflation resulting from the fiscal stimulus. As argued by Nakamura and Steinsson (2011), however, the fiscal multiplier in a small open economy with a fixed exchange rate is substantially smaller than in an economy where the interest rate is at its zero lower bound. The reason is that under a fixed exchange rate, the initial rise in domestic inflation must eventually be followed by a drop in domestic (relative to foreign) inflation, so as to keep relative foreign and domestic prices constant in the long run. As a result, the long-term real interest rate is unaffected. At the zero lower bound, instead, this mechanism is not present, so the long-term real interest rate also drops. This

[2] The reason why the fiscal multiplier can be larger than 1 even though private consumption drops is that we find a rise in private investment on impact, after which it becomes insignificant

[3] This literature focuses on anticipation effects by assuming that agents react to fiscal policy shocks when they are announced, rather than when they are implemented.

stimulates current demand further, facilitating a very large government spending multiplier, as reported by Christiano *et al.* (2011), among others.

One of the key insights of the recent literature on fiscal policy is that the government spending multiplier is not constant, but differs substantially across different states of the economy, as well as over time (Favero *et al.*, 2011; Auerbach and Gorodnichenko, 2012). We corroborate this finding by studying how the fiscal multiplier in Denmark has evolved over time since 1971. We find that in the 1970's and 1980's, the impact multiplier was smaller than 1, while in the 1990's and the 2000's, the multiplier has been above 1. In the 1970's and 1980's, Denmark suffered from unsound public finances, and while the fixed exchange rate policy was adopted in 1982, a credible currency peg is not gained overnight. As a result, fiscal expansions were likely to be met by expectations of higher inflation and higher interest rates, resulting in a low spending multiplier. On the other hand, the latter two decades correspond to times of low and stable inflation, sound public finances, and a credibly fixed exchange rate, laying the ground for more effective fiscal policy. Interestingly, Billbie *et al.* (2008) reach the opposite conclusion for the US, as they document a drop in the fiscal multiplier over time. As for Denmark, however, they find that this result can (at least partly) be attributed to regime shifts in US monetary policy.

To shed additional light on the importance of fiscal policy in Denmark, we present historical decompositions of output fluctuations. The main and unsurprising result from this exercise is that the Danish business cycle is to a large extent driven by economic fluctuations abroad. In particular, and especially since the mid-1990's, the Danish business cycle has been under heavy influence from global fluctuations (as measured by US GDP). On the other hand, shocks to government spending account for a small fraction of output fluctuations. Our decomposition suggests that Danish policymakers have not always been successful in conducting a countercyclical fiscal policy that might alleviate the fluctuations coming from abroad. For example, fiscal policymakers failed to cut back on public spending in the years leading up to the recent crisis; a time when global factors, including low interest rates, exerted a large, positive impact on the Danish business cycle. Tigthening the stance of fiscal policy during economic booms is of paramount importance if fiscal policymakers wish to stimulate the economy in bad times.

The rest of this paper is structured as follows: In section 2, we introduce

our SVAR model, and discuss the data, our choice of variables etc. We present our results as well as various extensions and robustness checks in section 3. In section 4, we use the estimated SVAR-model to undertake historical variance decompositions. Finally, section 5 concludes.

2 Empirical Model

Our baseline empirical model is a VAR model with 4 endogenous variables: Foreign trade-weighted GDP (F_t), domestic government spending (G_t), domestic private consumption (C_t), and domestic output (Y_t). For F_t, we use a weighted average of GDP in Germany and Sweden, Denmark's two most important trading partners, weighted by each country's share (in 1995) in the computation of the real effective rate of the Danish Krone by Danmarks Nationalbank (Pedersen and Plagborg-Møller, 2010). The structure of the VAR is the following:

$$X_t = \Psi + \Phi D_t + \Gamma Tr_t + \sum_{i=1}^{p} A_i X_{t-i} + \sum_{j=0}^{q} B_j Z_{t-j} + u_t, \tag{1}$$

where $X_t = [F_t \;\; G_t \;\; C_t \;\; Y_t]'$ is the vector of endogenous variables. In an alternative specification, we replace government spending with a measure of tax revenues net of transfers. Our baseline specification includes a constant, a linear trend Tr_t, and a crises dummy D_t which equals 1 during the recent financial crises and zero otherwise. $u_t = [f_t \;\; g_t \;\; c_t \;\; y_t]'$ is the vector of reduced-form residuals with variance-covariance matrix $Eu_t u_t' = V$. We include current and lagged values of GDP in the US as an exogenous variable, denoted Z_t. The exogenous variable is included as a proxy for the state of the global economy, including global technology shocks. By including this variable, we control for the fact that domestic output Y_t and foreign, trade-weighted output F_t are likely to be affected by common factors (such as a global recession). Without the inclusion of Z_t, the estimated effect on X_t of a shock to F_t would likely be biased. Moreover, we also assume that foreign trade-weighted GDP, F_t, is exogenous with respect to the domestic variable. We verify that our exogeneity assumptions are in fact satisfied through block-exogeneity tests, which confirm our assumptions.[4] The specifica-

[4]More specifically, we perform an F-test of the null hypothesis that the three (lagged) domestic variables can be excluded from the regression equation for F_t against the alternative

tion with a strictly exogenous variable Z_t as well as block exogeneity of one of the variables in X_t that is exogenous to the other variables in X_t but is affected by Z_t is due to Mojon and Peersman (2003), who employ a similar specification to model the effects of monetary policy in Austria, Belgium, and the Netherlands. They argue that these three countries do not affect, but are strongly affected by economic conditions in Germany, as well as 'global' economic factors, which are in turn assumed to be exogenous also with respect to the German economy.[5] The same description applies to the Danish economy, and we therefore find it natural to follow the specification suggested by Mojon and Peersman (2003). Cushman and Zha (1997) use block exogeneity to study the effects of monetary policy in Canada, where economic conditions are heavily influenced by, but have a very small effect on the US economy.

The inclusion of a deterministic trend in the VAR allows us to use data in log-levels.[6] However, following Blanchard and Perotti (2002), as a robustness check we also perform the analysis with the data in log-differences, allowing instead for a stochastic trend. We also need to choose which number of lags p of the endogenous variables to include. Table 1 in the appendix displays a number of tests and information criteria, to which we look for guidance on this choice. The three information criteria all point towards a low number of lags; 1 or 2. The vector Portmanteau test suggests using 2 (or 3) lags, while the vector test for normality of the residuals prefers a specification with 2 (or 4) lags. Finally, the likelihood ratio tests fail to reject that p lags are sufficient when p is between 2 and 6, except for $p = 4$. This test seems to favour 2 lags as well. Thus, while the data does not speak with a single voice on this issue, a choice of $p = 2$ lags seems a reasonable compromise for our baseline specification. We later change

that the exclusion restrictions are not satisfied. We then test the null that the four (lagged) variables in X_t can be excluded from the regression equation for Z_t. The p-values for these tests are 0.404 and 0.565, respectively, implying that we fail to reject the null hypothesis of block exogeneity in both cases.

[5] Mojon and Peersman (2003) include the short-term nominal interest rate in the US and a world commodity price index along with US GDP in the strictly exogenous block.

[6] We have tested the trend-stationarity of all the variables used. Using the augmented Dickey-Fuller (1979) test, we fail to reject the null hypothesis of a unit root for three of the six variables (Y_t, G_t, Z_t), while we can reject the null even at the 1% level for C_t, F_t, and T_t. Due to the well-known problems of low power of unit root test, we also apply the stationarity test of Kwiatkowski *et al.* (1992). In this case, we fail to reject the null of trend-stationarity at the 5 % level for all variables except Z_t (for which we fail to reject trend-stationarity at the 1% level). In sum, the assumption of trend-stationarity is not rejected by the data. These results are available upon request.

the number of lags as a robustness check.

2.1 The Data

Our dataset includes quarterly national accounts data spanning the sample from 1971:Q1 to 2011:Q2. We believe, however, that a regime shift occured in 1982 when Denmark shifted from a floating to a fixed exchange rate.[7] We therefore start our baseline estimation in 1983:Q1, although we include the years 1971-1982 as a robustness check later on. Moreover, we include a dummy for the recent crises, which equals 1 from 2008:Q4 onwards, and zero otherwise. While the recent crisis may not represent a regime shift, we consider it a time of unusual circumstances, which justifies the use of a dummy variable. For the domestic variables, we use national accounts data from Danmarks Nationalbank's MONA database. We obtain GDP data for the US, Sweden and Germany from the OECD.

2.2 Identification Strategy

As already mentioned, our identification strategy follows the approach of Blanchard and Perotti (2002). They argue that it takes more than a quarter for fiscal policymakers to realize that a shock has hit the economy, decide on the appropriate response of fiscal policy, pass the relevant legislation, and implement the fiscal measures in practice. Thus, using quarterly data, there can be no within-period discretionary response of government spending to economic shocks, so any simultaneous reaction of government spending to output or other variables must be due to automatic effects. These automatic effects can then be estimated outside the system. More specifically, we set up the following system of equations, which is essentially an open-economy version of that in Blanchard and Perotti (2002); except that we exclude taxes and instead include private consumption:

$$f_t = a_1 g_t + a_2 c_t + a_3 y_t + e_t^f, \tag{2}$$

$$g_t = b_1 f_t + b_2 c_t + b_3 y_t + e_t^g, \tag{3}$$

[7] Moreover, beginning in early 1983, an automatic indexation of wages and transfers to the rate of inflation was suspended. This is likely to have played an important role in bringing down the inflation rate.

$$c_t = c_1 f_t + c_2 g_t + c_3 y_t + e_t^c, \tag{4}$$

$$y_t = d_1 f_t + d_2 g_t + d_3 c_t + e_t^y. \tag{5}$$

As mentioned above, $u_t = [f_t \; g_t \; c_t \; y_t]'$ contains the reduced-form residuals from the VAR regression, while $\varepsilon_t = \left[e_t^f \; e_t^g \; e_t^c \; e_t^y\right]'$ is the vector of orthogonalized, structural innovations to F_t, G_t, C_t, and Y_t, respectively. These two vectors are related in the following way:

$$u_t = C\varepsilon_t, \quad CC' = V, \tag{6}$$

where V is the variance-covariance matrix of the residuals, as noted above. We need to impose identifying restrictions that allow us to pin down uniquely the matrix C, as this enables us to back out the structural innovations and compute meaningful impulse responses.

In the system of equations above, (5) states that unexpected movements in domestic GDP (y_t) within a quarter can arise due to unexpected movements in foreign GDP (f_t), unexpected movements in private (c_t) or public consumption (g_t), or structural shocks to output (e_t^y). The interpretation of the other equations is similar. Given our assumption that foreign GDP is exogenous with respect to the domestic variables, we impose that $a_1 = a_2 = a_3 = 0$. Moreover, we assume that if there is any automatic effect on public spending of changes in foreign output, this effect occurs via the effect of foreign output on domestic output, so that $b_1 = 0$. Similarly, we assume that changes in private consumption does not cause automatic changes in government spending *on top of* a potential effect through output, i.e. that $b_2 = 0$.

The parameter b_3 measures the automatic effects that changes in output might have on public spending within a quarter. As discussed by Caldara and Kamps (2012), as well as in section 3.3 of the present paper, setting a value for this parameter is not innocuous, as this has substantial effects on the estimated impact multiplier of an increase in public spending. In the literature, this parameter is typically set to zero; see e.g. Blanchard and Perotti (2002), Monacelli and Perotti (2008, 2010), or Ravn *et al.* (2012). An exception is Bergman and Hutchison (2010), who set the parameter to -0.2 for Denmark, based on a study by Giorno *et al.* (1995). That elasticity is found by computing the elasticity of unemployment-related expenditures to output, and multiplying by the share of unemployment-related expenditures to total government expenditure. However,

unemployment-related expenditures are not included in the measure of public consumption used in the present study. In general, our measure of government spending should not include any components that vary automatically with output within a quarter. Caldara and Kamps (2012) argue that b_3 is likely to be positive, citing evidence by, among others, Lane (2003) that government consumption tends to be pro-cyclical in most OECD countries, including Denmark. This result is obtained at the annual level, however, and does not necessarily carry over to quarterly data. In our baseline scenario, we therefore follow the literature and set $b_3 = 0$, while we use different values for this parameter as a robustness check.

To pin down the parameters in the final two equations, we need to take a stand on whether private consumption or output is determined first. We assume that private consumption affects output within a quarter, but not the other way around; i.e. $c_3 = 0$ but $d_3 \neq 0$. This choice turns out be unimportant for our results. We then construct the cyclically adjusted government spending residuals; $g'_t = g_t - b_3 y_t$ ($= g_t$ when we set $b_3 = 0$). These residuals are uncorrelated with the structural innovations to output and consumption, e^y_t and e^c_t, allowing us to use them as instruments for g_t in regressions of c_t and y_t on the right-hand side variables in (4) and (5). Likewise, we need the structural innovations to foreign output, e^f_t, to be uncorrelated with e^y_t and e^c_t. This is ensured by the inclusion of US GDP as an exogenous variable in the original VAR, as this variable controls for global shocks such as productivity shocks that are likely to affect both the foreign and the domestic economy. Hence, we first estimate c_1 and c_2 by regressing c_t on e^f_t and g_t, with g'_t as instrument for g_t.[8] We then estimate d_1, d_2, and d_3 by regressing y_t on e^f_t, c_t and g_t, again using g'_t as an instrument.[9]

Having pinned down all parameters, it is straightforward to solve the system above for the structural shocks as functions of the reduced-form residuals obtained from the VAR and the estimated coefficients. Moreover, we can compute the coefficients in the matrix C from these parameters, allowing us to obtain the impact effects on the endogenous variables of an orthogonalized, structural innovation to one of these variables, which is needed for impulse responses. The

[8] Recall that $e^f_t = f_t$.

[9] Of course, when we set $b_3 = 0$ so that $g'_t = g_t$, the use of g'_t as an instrument in practice becomes redundant. On the other hand, when we set $b_3 \neq 0$, or in the specification with taxes instead of government spending, this step becomes relevant.

impact effects of a shock to government spending are given by:

$$f_t = 0, \tag{7}$$

$$g_t = \left[1 + b_3 \frac{d_2 + c_2 d_3}{1 - b_3 d_2 - b_3 c_2 d_3}\right] e_t^g, \tag{8}$$

$$c_t = \left[c_2 + \frac{(b_3 c_2)(d_2 + c_2 d_3)}{1 - b_3 d_2 - b_3 c_2 d_3}\right] e_t^g, \tag{9}$$

$$y_t = \frac{d_2 + c_2 d_3}{1 - b_3 d_2 - b_3 c_2 d_3} e_t^g. \tag{10}$$

It should be noted that when we set the elasticity of government spending to output $b_3 = 0$, the identification strategy collapses to a standard Choleski decomposition with government spending ordered before consumption and output. However, we use the structural identification scheme outlined above for at least two reasons: First, it allows us to replace government spending with taxes, for which the output elasticity is surely not zero. Second, we are able to relax the assumption of a zero output elasticity of government spending as a robustness check. As described in subsection 3.3, it turns out that our results are very sensitive to this parameter.

3 The Effects of Fiscal Policy

In this section, we present and discuss our results, including a number of robustness checks. We begin by computing impulse responses to an increase in government spending.

3.1 Impulse Responses and Fiscal Multipliers

We first look at orthogonalized impulse responses to a shock to government spending. Given the exogeneity of the foreign variables, these are not affected by this shock, so we report impulse responses only for the domestic variables. Consider first our baseline scenario with variables in log-levels, as depicted in Figure 1.

As the figure makes clear, the increase in government spending is quite persistent, remaining significantly above zero for some 3 years after the shock. Nev-

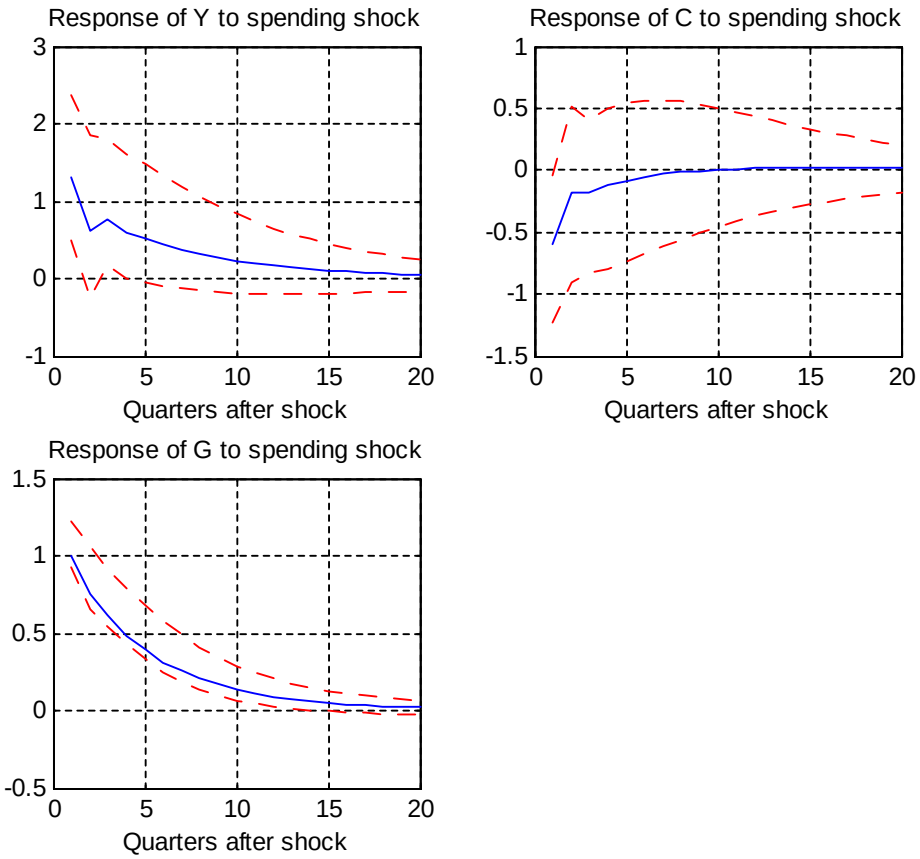

Figure 1: The dynamic response of each variable to a unit shock to government spending. Dotted lines indicate bootstrapped 95 % error bands. The error bands are computed using Hall's (1992) bootstrap method with 10,000 replications.

ertheless, the effect on output dies out much faster. After a large initial increase, output quickly reverts back to its original level. The reaction of output is significant only during the first year (except for the second quarter after the shock). Somewhat surprisingly, we observe a borderline significant drop in consumption on impact. From the second quarter onwards, the reaction of consumption is small and insignificant.

We have converted the impulse responses in Figure 1 so that the fiscal multiplier is directly observable. The impact multiplier of government spending on output is 1.31, implying that a 1 DKR rise in government spending causes an immediate increase in GDP of 1.31 DKR. This multiplier is rather high, although well within the interval 0.8-1.5 highlighted by Ramey (2011b). However, we observe that the multiplier quickly decreases. A year after the shock, the multiplier

is 0.6, after which it becomes insignificant. The government spending multiplier is above 1 only on impact, i.e. in the same quarter in which government spending is increased. These findings are in line with the theoretical arguments in the introduction. In the very short run, fiscal stimulus is quite effective in Denmark, as prices are sticky, and neither the nominal interest rate nor the nominal exchange rate can adjust. However, as soon as prices start to adjust, the effects of fiscal stimulus quickly die out, as the Danish economy loses competitiveness against its trading partners. The cumulative multiplier, computed as the accumulated increase in output divided by the accumulated increase in government spending, is found to be 1.34.[10] This number is comparable to the estimate of Ilzetzki *et al.* (2012), who study fiscal policy in 44 countries, and find a cumulative multiplier of 1.4 in countries operating under a fixed exchange rate regime. These authors estimate a much smaller impact effect, however. Our finding that the impact multiplier and the cumulative multiplier are almost identical reflects that the effect on output declines at around the same rate as the response of spending itself, as illustrated in the figure.

Most studies based on the approach of Blanchard and Perotti (2002) tend to report an increase in consumption after a shock to government spending (e.g. Gali *et al.*, 2007). Our finding of a drop in consumption is instead more in line with studies using the approach of Ramey and Shapiro (1998). In particular, it may seem puzzling that the government spending multiplier is above 1 despite the drop in consumption. In results not reported, we find that this is explained by an increase in private investment on impact.[11]

3.2 Subsample Stability

While much of the debate about the effects of fiscal policy has centered around the size of *the* government spending multiplier, it is important to note that this multiplier is far from constant. Instead, it is likely to vary substantially over time and across different economic situations. For the US, for example, Perotti (2005) and Billbie *et al.* (2008) have demonstrated that the fiscal multiplier has been declining over time. Billbie *et al.* (2008) argue that this can be explained by

[10] We compute the cumulative multiplier at a horizon of 20 quarters, after which the response of both output and spending itself is practically zero.

[11] The increase in investment is large on impact, after which it quickly reverts back around zero. In effect, the impulse response of investment mirrors that of private consumption.

three factors: Increased asset-market participation by households, a more active monetary policy since the beginning of the 1980's, and more persistent shocks to government spending. Increased asset-market participation allows households to smooth consumption over time, lowering their sensitivity to shocks affecting current income, such as fiscal policy shocks. This mitigates the effect described by Gali *et al.* (2007). A more active stance of monetary policy implies a stronger interest rate reaction to the inflationary effects of an expansionary fiscal policy, inducing an increase in the real interest rate which dampens the effect on economic activity.

To evaluate how the fiscal multiplier in Denmark has evolved over time, we extend our analysis back to 1971, and then split the entire sample into four different subsamples; one for each decade in our dataset. Table 1 shows the impact multiplier for various subsamples, i.e. the increase in output (in DKR) in the same quarter as government spending is increased by 1 DKR. As the table illustrates, the government spending multiplier varies substantially over time. First, when the years 1971-1982 are included in the baseline regression, the impact multiplier drops to 1.17. This indicates that the multiplier was lower in the 1970's, but also that our baseline result is not too sensitive to our choice of sample period. Second, we observe that fiscal stimulus seems to have become more effective in the latter two decades of our sample than in the 1970's and 1980's. In particular, the multiplier is below one in the first two decades, but above one after 1990. The confidence intervals are very wide, however, in large part because of the short subsamples with only 40 quarterly observations each. Nevertheless, we have some confidence in the finding that discretionary fiscal policy has been more effective in the last two decades, despite the interesting fact that our findings are in opposition to results obtained in studies using US data, as mentioned above. In the 1970's and well into the 1980's, Denmark's public finances were very unsound, and inflation and nominal interest rates were often in double digits. In such an economic environment, fiscal stimulus is likely to have led to expectations of higher inflation and interest rates, and in turn to expectations of a devaluation of the Danish Krone, which was not uncommon in the 1970's. While a fixed exchange rate was adopted in 1982, credibility around a currency peg is not gained overnight. On the other hand, during the 1990's and 2000's the Danish economy has generally been characterized by a credibly fixed exchange rate and sound public finances, facilitating a more

effective conduct of discretionary fiscal policy. As discussed above, larger effects of fiscal policy under fixed than under flexible exchange rates are in line with a range of theoretical models as well as empirical evidence. Furthermore, a number of reforms have increased the flexibility of the Danish labor market considerably over our sample, which is likely to have contributed to the enhanced effectiveness of fiscal policy. On the other hand, the increasing openness to trade of the Danish economy over our sample is likely to have lowered the fiscal multiplier over time, as a larger share of government spending is 'leaked' from the home economy (Beetsma and Giuliodori, 2011). In sum, while our results contrast with those of Perotti (2005) and Billbie *et al.* (2008) for the US, there is a number of reasons for this discrepancy. Furthermore, there is no reason to believe that the fiscal multiplier will be even larger in the future, as Denmark's fixed exchange rate is now surrounded by a very high credibility, as evidenced by the low interest rate spread against Germany, and its public finances are relatively solid. The channels for obtaining larger effects of fiscal policy thus seem to have been exhausted.

Table 1: Impact multipliers for different subsamples.

Subsample	Multiplier	95 % Confidence Interval
1983-2011	**1.31**	[0.50;2.36]
1971-2011	**1.17**	[0.54;1.90]
1971-1980	**0.78**	[-0.05;1.97]
1981-1990	**0.33**	[-1.31;1.97]
1991-2000	**1.03**	[-0.22;3.10]
2001-2010	**1.54**	[0.45;3.45]

Note: The crisis dummy is included in the regressions for 1983-2011 and 1971-2011, but not in the regression for 2001-2010. The confidence intervals are computed using the bootstrap method of Hall (1992) with 1000 replications. Note that because confidence intervals are bootstrapped, they are not necessarily symmetric. All specifications include a deterministic trend.

3.3 Robustness

While the previous subsection offered a first glance at the robustness of the estimated fiscal multiplier, we now investigate this issue in more detail. We display impulse responses only when these differ substantially from those in Figure 1, although all results are available upon request.

First, we allow for quarterly dummies, as suggested by Blanchard and Perotti (2002). This does not change our results in any relevant aspect. Next, we observe that our results are also practically identical if we exclude the crisis dummy from our baseline specification. The impulse responses look very much like those in Figure 1.

A more interesting robustness check is to investigate the sensitivity of our results to the number of lags in the VAR, which we set to 2 in our baseline estimation (we always choose the same number of lags for the exogenous variable as for the endogenous variables). With 1 lag, the impact multiplier is practically identical to the baseline, while with 3 lags, it rises slightly to 1.44. The impulse responses do not change much. With 4 lags, however, the results change considerably, as witnessed by Figure A.1 in the appendix. In particular, the initial drop in consumption is now small and insignificant. Instead, the response of consumption becomes positive for a number of periods; significantly so from 3 to 6 quarters after the shock. As a result, the response of output no longer reaches its peak on impact, but instead in the third quarter after the shock. The response of output is significantly positive until two years after the shock. The findings of a positive response of consumption and a delayed peak effect on output are in fact consistent with the results from a number of studies for other countries, including Blanchard and Perotti (2002) for the US. Indeed, Blanchard and Perotti use 4 lags in their study, although they do not report tests or information criteria to support this choice. Thus, while our data strongly favours the use of a model with a low number of lags, as already discussed, the fact that our results come closer to mimicking those of Blanchard and Perotti when we imitate their choice of lags is an interesting finding.

The results above were obtained with data in log-levels. To address concerns about potentially non-stationary variables, we perform a similar analysis allowing for a stochastic trend in the data instead of a deterministic trend. With data in log-differences, the VAR regression and the structural identification strategy are the same, with the exception that the linear trend Tr_t is removed from the VAR. We display the impulse responses from this analysis in Figure A.2a in the appendix. As these impulse responses fluctuate a lot, we display in figure A.2b the same impulse responses after smoothing them using a Hodrick-Prescott filter. As these figures illustrate, it is hard to draw any firm conclusions from this specification, as the impulse responses of output and consumption are insignificant

most of the time. The impact effect on output is significantly positive, though, with an impact multiplier of 1.09, i.e. somewhat lower than in the model with a deterministic trend. As for consumption, we still observe a drop on impact, but now the response turns positive in the next few quarters. The consumption response is never significantly different from zero under this specification.

Finally, we want to evaluate the consequences of different assumptions in our identification scheme. In particular, we consider the robustness of the estimated impact multiplier of government spending with respect to different values of the elasticity of government spending to output within the quarter (b_3), which was set to zero in our baseline specification. As demonstrated by Caldara and Kamps (2012), this parameter has a heavy influence on results based on US data. This turns out also to be the case for our study of Denmark. Figure A.3 in the appendix shows how our estimate for the impact multiplier changes when we vary the value of b_3. As the figure illustrates, the impact multiplier is highly sensible to the value of this parameter. For example, if b_3 is allowed to take on a modest value of 0.1, the impact multiplier drops to 0.56, compared to our baseline estimate of 1.31. Similarly, if we set $b_3 = -0.1$, the multiplier is as high as 1.92.[12] The intuitive explanation for this finding is the following: If for example the automatic elasticity of government spending to output is negative ($b_3 < 0$), the increase in output brought about by a positive shock to government spending will in itself induce a fall in government spending, all else equal. As a result, the eventual increase in government spending will be small, while the increase in output is the same (abstracting from a small second-round effect). Hence, the estimated multiplier will be larger.

We are therefore able to confirm the results of Caldara and Kamps (2012); in fact, the sensitivity of the multiplier seems even bigger in our case. The large sensitivity of the results is an obvious shortcoming of the identification strategy of Blanchard and Perotti (2002) that has until recently largely been ignored in the literature. As discussed in subsection 2.2, the parameter b_3 is likely to be close to zero, but as we have demonstrated, even small deviations from zero lead to substantially different results. This indicates that our estimated multiplier should be interpreted with care.

[12] In results not reported, we observe that consumption rises on impact when b_3 is sufficiently low. In fact, we find that this explains the divergence between the negative consumption response in the present study and the positive response obtained by Bergman and Hutchison (2010), who set $b_3 = -0.2$.

3.4 Effects of Taxes

As discussed by Blanchard and Perotti (2002), the structural VAR model presented above favours a view of fiscal policy as working primarily through the demand side of the economy. While this seems a reasonable assumption for government spending, we believe it provides only a partial account of how changes in taxes affect the economy. Changes in income taxes, for example, are likely to affect the economy's supply side through changes in labor supply as well as the demand side via a change in disposable income. Therefore, in contrast to Blanchard and Perotti (2002), we choose not to include taxes along with government spending in our baseline specification. Nevertheless, in this subsection we attempt to gain at least some insight on the effects of taxes by including them in our SVAR, although these results should therefore not be interpreted as a complete account of the effects of tax changes.

We use a measure of tax revenues net of transfers. We add direct taxes (including corporate taxes and capital gains taxes), indirect taxes, and social contributions, and subtract transfers to households. We then insert taxes (T_t) instead of government spending (G_t) in our baseline VAR as presented in (1) with two lags, a constant, a deterministic trend, and with the crises dummy included in the block of exogenous variables.

The structural system is essentially the same as the one presented in subsection 2.2, with taxes replacing government spending. We also keep the same identifying assumptions. The only difference is related to the elasticity of the tax revenue with respect to changes in output, which we denote b_3^T. In contrast to the specification with government spending, this elasticity is now unlikely to be zero, as a rise in GDP will lead to an automatic increase in the tax base and in turn, the tax revenue. In order to pin down b_3^T, we decompose the total tax revenue into different types of taxes (income taxes, corporate taxes, etc.). We then compute the elasticity of each type of tax with respect to changes in output, and weigh these together to obtain a measure of the elasticity of total tax revenues. The method is described in detail in the appendix. We arrive at a value of $b_3^T = 2.09$.

We compute impulse responses to an increase in the tax revenue. These are shown in figure A.4 in the appendix. As illustrated, an increase in taxes leads to a drop in output and consumption, although the latter is not significant. The estimated tax multiplier is 0.78 on impact, which is smaller than the government

spending multiplier. Given the focus of the SVAR approach on the demand side effects of taxes, as described above, this finding is unsurprising. However, a number of recent studies that pay more attention also to supply-side effects have challenged this result, and tend to find that the tax multiplier is at least as large as the spending multiplier (see Alesina and Ardagna, 2010; Romer and Romer, 2010; or Mertens and Ravn, 2012).

We further observe that output quickly returns to its initial level, as the response is numerically quite small from the second quarter after the shock onwards. Due to the drop in output and the resulting drop in the tax base, tax revenues quickly return to zero.[13] Finally, we explore the sensitivity of the tax multiplier to the value of the parameter b_3^T. It turns out that the tax multiplier is much more robust than the government spending multiplier, in line with the findings of Caldara and Kamps (2012). In particular, if we increase b_3^T to 2.5, the estimated impact multiplier increases only to 0.79. If instead we set $b_3^T = 1.5$, the multiplier drops to 0.65.

4 What Drives the Danish Business Cycle?

In this section, we use our estimated, structural VAR model to decompose recent business cycle fluctuations in Denmark. We first undertake a historical decomposition to shed light on the contribution of various shocks to fluctuations in output at specific points in time. Later on, we perform a variance decomposition of the endogenous variation in our VAR-model.

4.1 Historical Decompositions

Following the approach of Lindé (2003), we first obtain the trend growth in the exogenous variable Z_t (US GDP) by estimating and then simulating a VAR with Z_t as the dependent variable and two lags of Z_t as regressors, along with a constant and a deterministic trend. In the simulation, we do not add the residuals, so that we obtain a simulated variable $\overline{Z}_t$ describing the trend in US GDP. The next step is to simulate the trend of the four endogenous variables

[13] Note that the shock to tax revenues has been normalized to 1, so as to facilitate comparison with the shock to government spending. The response of tax revenues, however, is smaller than 1 already on impact, as the rise in taxes implies a drop in output, and hence in the tax base.

in X_t, which we will denote by $\overline{X}_t = \left(\overline{F}_t\ ,\ \overline{G}_t\ ,\ \overline{C}_t\ ,\ \overline{Y}_t\right)'$. This is done by simulation of the following regression:

$$\overline{X}_t = \Psi + \Phi D_t + \Gamma T r_t + \sum_{i=1}^{p} A_i \overline{X}_{t-i} + \sum_{j=0}^{q} B_j \overline{Z}_{t-j}. \tag{11}$$

We use the first two quarters in our sample to start up the simulation. We then feed our estimated VAR with lagged values of the trend in the endogenous as well as the exogenous variables. Once again, note that we do not add any residuals to the simulation. Once the trend is obtained, we can easily compute the deviations from trend in each variable by subtracting the trend from the actual, observed variables.

We can decompose these deviations from trend into contributions from each of our 4 endogenous variables, as well as from Z_t. Having already backed out the structural shocks in the previous sections, we can isolate, for example, the contribution of structural innovations to government spending to deviations of output from its trend. This is done by 'turning off' all other structural shocks than those to e_t^g; i.e., simply setting them to zero. We then perform a new simulation of (11), in which we feed the VAR with the structural shocks to government spending in each step. The same can be done for each of the four endogenous variables. Since all four shocks have a structural interpretation, including only one shock at a time in the simulation is a meaningful exercise. As for Z_t, we simply simulate (11) using the actual values of Z_t instead of the simulated trend $\overline{Z}_t$.

Figure 2 shows the deviations of output from its simulated trend over the course of our sample, as well as the share explained by structural shocks to government spending. The share of other shocks is illustrated in figure A.5-A.7 in the appendix. As the figure illustrates, shocks to government spending do not account for a very large share of output fluctuations. The reason is that by construction, the simulations above assign large explanatory power to variables that display large deviations from their trend in any given period. As government spending follows its trend growth quite closely during most of our sample, its deviations from trend are simply too small to account for a very large share of output fluctuations. Moreover, it is noteworthy that there is little evidence of systematic, countercyclical fiscal policy; at least as measured by government spending. In particular, the stance of fiscal policy appears to have been 'too

tight' during the recession in the late 1980's and early 1990's. Likewise, during the economic booms in the second half of the 1990's and the years 2004-2007, the growth rate of government spending was not reduced relative to its historical trend, despite the fact that during both episodes, as evidenced by figure A.5 in the appendix, global factors exerted a strong, positive effect on the Danish business cycle.[14]

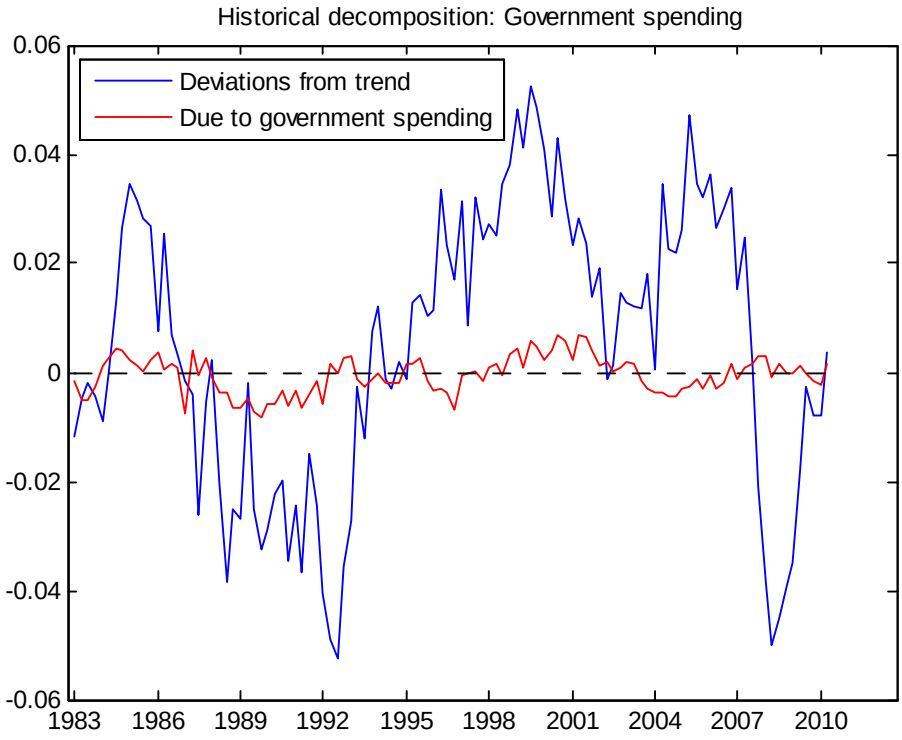

Figure 2: Historical decomposition. The blue line shows deviations in output growth relative to its trend growth. The red line shows the share explained by deviations in the growth rate of government spending relative to its trend.

Furthermore, figure A.5 in the appendix shows that the Danish business cycle has been mainly driven by global factors during the period in question. Given the size and openness of the Danish economy, this is an unsurprising finding. The figure suggests that episodes such as the US recession in 1990-91, the boom

[14]Moreover, a recent study by Ravn (2012) suggests that, as a consequence of Denmark's fixed exchange rate towards the euro, the Danish interest rate was substantially lower than what would have been prescribed by a Taylor rule for Denmark in the years 2005-2007. This would in turn have called for an even tighter stance of fiscal policy during these years.

in the 1990's, and the financial and economic crisis beginning in 2008 have large and direct spill-over effects on the Danish economy. Figure A.6 shows that the contribution from Denmark's two most important trading partners, Germany and Sweden, is much less important. The post-reunification boom in Germany in 1990-91 can be clearly identified, but its effect on the Danish economy seems to be dominated by the concurrent recession in the US. Finally, figure A.7 shows the contribution that can be attributed to other domestic shocks, i.e. fundamental shocks to Y_t or C_t. Throughout the 1980's, these shocks account for a remarkably large share of the movements in GDP, suggesting that the economic boom in Denmark in the mid-1980's and the subsequent recession to a large extent were 'homegrown'. This is consistent with historic events in the 1980's in Denmark. In 1982 the new, conservative government announced a number of economic reforms, including, as already mentioned, a currency peg towards the German D-Mark as well as the suspension of an automatic indexation of wages and transfers. This confidence boost set off an economic expansion. In 1986, as the Danish economy showed signs of overheating, a new set of reforms were enacted, including regulations on real estate mortgage lending and a tax reform, which effectively limited credit-financed consumer spending, and put a sudden end to the boom. For the remainder of the sample, domestic shocks have been less important for the business cycle; although a substantial, positive contribution appears in 1993-94 following the appointment of a new, social democratic government and a new set of reforms, including a reform of the labor market.

4.2 Variance decompositions

We can shed further light on the driving forces behind the Danish business cycle by examining the importance of each shock at various points of the frequency domain. Unfortunately, this method can be applied only to the 4 endogenous variables in X_t, as the method makes use of the variance-covariance matrix V of the structural VAR-regression in (1), in which no shock related to the exogenous variable (Z_t) appears. As a result, the variance decompositions below ignore the contributions from global factors. Nevertheless, it still offers interesting insights on the relative importance of the shocks to the remaining four variables.

We follow the approach to variance decompositions taken by Altig *et al.* (2005). The details of the method are outlined in the appendix. This method

allows us to decompose the variance of each of the four endogenous variables for any frequency ω in the frequency domain. In this way, we can investigate the relative importance of the four shocks at various frequency intervals, uncovering the importance of these shocks for short-run and long-run movements in the four variables. As an example, one could suspect that fiscal policy shocks are more important in explaining output over the span of the business cycle than in explaining the long-run trend. The present approach will allow us to answer such questions.

Table 2: Variance decomposition for output

	f_t-shocks	g_t-shocks	c_t-shocks	y_t-shocks
Low frequencies	0.2702	0.0934	0.3787	0.2577
Business cycle freq.	0.1887	0.0871	0.1763	0.5480
High frequencies	0.0187	0.0864	0.0831	0.8118
All frequencies	0.2054	0.0901	0.2652	0.4393

Table 2 shows the variance decomposition for output. Each row shows how much of the variation in output at, say, low frequencies, can be attributed to structural shocks to each of the four variables. In other words, the numbers in each row sum up to 1. We follow Altig *et al.* (2005) and define high frequencies as up to 5 quarters, business cycle frequencies as 6 to 32 quarters, and low frequencies as more than 32 quarters. The table reveals that shocks to government spending explain less than 10 % of the variation in output at all frequencies. The importance of government spending shocks is almost constant across the frequency domain. This confirms the finding from our historical decomposition that government spending has not played a very important role in driving the Danish business cycle. Furthermore, the table shows that shocks to output in Germany and Sweden are a substantial contributor to output fluctuations at low and medium frequencies, but not at high frequencies. Finally, fundamental shocks to output or private consumption are the two main drivers of output variations, especially in the short run, suggesting that domestic factors account for a somewhat surprisingly large share of output fluctuations. A similar conclusion is reached by Dam and Linaa (2005).[15] Recall, however, that the numbers in

[15]Dam and Linaa (2005) estimate a DSGE model for Denmark, and report that the main driver of output variations, especially in the long run, is stochastic movements in the labor supply. Our SVAR-model is much more rudimentary, and in particular does not feature shocks to the labor supply. In our setup, however, such shocks are likely to show up as fundamental

table 2 concern only the part of output variations that remain after controlling for global economic factors by regressing the endogenous variables on US GDP, which was shown in the previous subsection to have very large effects on the Danish economy.

5 Conclusion

We have presented an array of empirical findings about the effects of fiscal policy in Denmark that can broadly be summarized as follows: First, an increase in government spending has a rather large impact on output in the very short run, with a fiscal multiplier around 1.3. However, the expansionary effects are very short-lived, as the multiplier is above 1 only on impact, and the response of output becomes insignificant after about a year. As argued in the introduction, these results suggest that in the very short run, the monetary accomodation effect under a fixed exchange rate outweighs the leakage effect following from a large degree of openness. Second, as for the effect on consumption, our results are somewhat inconclusive, but tend to suggest that private consumption goes down after an increase in government spending. Third, the fiscal multiplier is not constant. In particular, fiscal stimulus seems to have become more effective in the last two decades compared to the 1970's and 1980's. Fourth, an increase in taxes depresses economic activity, although the tax multiplier is smaller than the spending multiplier. Finally, the estimated government spending multiplier is highly sensitive to the automatic elasticity of government spending to output.

A number of authors have used empirical results about fiscal policy to evaluate competing macroeconomic theories and models (Blanchard and Perotti; 2002, Gali *et al.*; 2007). As discussed by Blanchard and Perotti, for example, an increase in private consumption in response to a government spending shock is consistent with traditional, Keynesian models, in which a household's consumption is a function of its current income. This will tend to increase, depending on how the fiscal stimulus is financed. In contrast, a drop in consumption suggests that households behave in a Ricardian fashion, as assumed in standard neoclassical models such as the Real Business Cycle model, as well as in New-Keynesian models. In these models, consumption is instead determined by lifetime income,

shocks to y_t; or perhaps to c_t (through the consumption/labor decision of households).

which goes down due to the increase in the present value of future tax payments. The results in the present paper seem to lend more support to the latter class of models, in which households display at least some degree of Ricardian behaviour, although the data does not allow us to draw any firm conclusions in this respect.

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Appendix to "Has the Fed Reacted Asymmetrically to Stock Prices"

Abstract

This appendix contains supplemental material for Chapter 1; "Has the Fed Reacted Asymmetrically to Stock Prices".

1 Mathematical Appendix

As in the main text, the calculations in this appendix are shown for β_1. Solving for β_2 proceeds in the exact same way.

In section 2, we showed what the covariance matrix for v_t^i and v_t^s looked like for a given regime. The covariance matrix for regime i is repeated here for convenience:

$$\Omega_i = \frac{1}{(1-\alpha\beta_1)^2} \times \tag{A1}$$

$$\begin{bmatrix} (\beta_1+\gamma)^2 \sigma_{i,z}^2 + \beta_1^2 \sigma_{i,\eta}^2 + \sigma_\varepsilon^2 & (1+\alpha\gamma)(\beta_1+\gamma)\sigma_{i,z}^2 + \beta_1 \sigma_{i,\eta}^2 + \alpha\sigma_\varepsilon^2 \\ (1+\alpha\gamma)(\beta_1+\gamma)\sigma_{i,z}^2 + \beta_1 \sigma_{i,\eta}^2 + \alpha\sigma_\varepsilon^2 & (1+\alpha\gamma)^2 \sigma_{i,z}^2 + \sigma_{i,\eta}^2 + \alpha^2\sigma_\varepsilon^2 \end{bmatrix}$$

As already described, the identification involves subtracting the covariance matrices of different regimes from each other. Subtracting covariance matrices i and j from each other yields:

$$\Delta\Omega_{ij} = \frac{1}{(1-\alpha\beta_1)^2} \times \tag{A2}$$

$$\begin{bmatrix} (\beta_1+\gamma)^2 \Delta\sigma_{ij,z}^2 + \beta_1^2 \Delta\sigma_{ij,\eta}^2 & (1+\alpha\gamma)(\beta_1+\gamma)\Delta\sigma_{ij,z}^2 + \beta_1 \Delta\sigma_{ij,\eta}^2 \\ (1+\alpha\gamma)(\beta_1+\gamma)\Delta\sigma_{ij,z}^2 + \beta_1 \Delta\sigma_{ij,\eta}^2 & (1+\alpha\gamma)^2 \Delta\sigma_{ij,z}^2 + \Delta\sigma_{ij,\eta}^2 \end{bmatrix}.$$

Note in this step how, due to the assumption of homoskedasticity of the monetary policy shock ε_t across regimes, the terms involving σ_ε^2 cancel out.

As noted in the main text, all four covariance regimes are needed for the system to be fully identified. However, for our purposes, identifying β_1 is enough. For this, only three different regimes are needed, as shown below. Therefore, fix $j = 1$ and let $i = \{2, 3\}$. Moreover, we follow Rigobon and Sack (2003) in rewriting the covariance matrix in the following way:

Define:

$\theta = \frac{(1+\alpha\gamma)}{(\beta_1+\gamma)}$ and $\varpi_{z,i} = (\beta_1+\gamma)^2 \Delta\sigma_{i1,z}^2$.

Using this notation, $(A2)$ can be rewritten as:

$$\Delta\Omega_{i1} = \frac{1}{(1-\alpha\beta_1)^2}\begin{bmatrix} \varpi_{z,i} + \beta_1^2\Delta\sigma_{i1,\eta}^2 & \theta\varpi_{z,i} + \beta_1\Delta\sigma_{i1,\eta}^2 \\ \theta\varpi_{z,i} + \beta_1\Delta\sigma_{i1,\eta}^2 & \theta^2\varpi_{z,i} + \Delta\sigma_{i1,\eta}^2 \end{bmatrix}. \tag{A3}$$

Writing out the equations contained in $(A3)$ for $i = 2$ explicitly yields:

$$\Delta\Omega_{21,11} = \frac{1}{(1-\alpha\beta_1)^2}\left[\varpi_{z,2} + \beta_1^2\Delta\sigma_{21,\eta}^2\right], \tag{A4}$$

$$\Delta\Omega_{21,12} = \frac{1}{(1-\alpha\beta_1)^2}\left[\theta\varpi_{z,2} + \beta_1\Delta\sigma_{21,\eta}^2\right], \tag{A5}$$

$$\Delta\Omega_{21,22} = \frac{1}{(1-\alpha\beta_1)^2}\left[\theta^2\varpi_{z,2} + \Delta\sigma_{21,\eta}^2\right]. \tag{A6}$$

A similar system of three equations can be written for $i = 3$. Together, these are six equations in the following seven variables: $\alpha, \beta_1, \gamma, \varpi_{z,2}, \Delta\sigma_{21,\eta}^2, \varpi_{z,3}$ and $\Delta\sigma_{31,\eta}^2$. Rewriting the system $(A4) - (A6)$ in the following way, we are able to exploit the obvious symmetry in these three equations. First, insert $(A4)$ into $(A5)$:

$$\theta\left(1-\alpha\beta_1\right)^2\Delta\Omega_{21,11} - \theta\beta_1^2\Delta\sigma_{21,\eta}^2 + \beta_1\Delta\sigma_{21,\eta}^2 = \left(1-\alpha\beta_1\right)^2\Delta\Omega_{21,12} \Longleftrightarrow$$

$$\Delta\Omega_{21,12} - \theta\Delta\Omega_{21,11} = \frac{\beta_1\left(1-\theta\beta_1\right)}{\left(1-\alpha\beta_1\right)^2}\Delta\sigma_{21,\eta}^2. \tag{A7}$$

Similarly, insert $(A5)$ into $(A6)$:

$$\theta\left(1-\alpha\beta_1\right)^2\Delta\Omega_{21,12} - \theta\beta_1\Delta\sigma_{21,\eta}^2 + \Delta\sigma_{21,\eta}^2 = \left(1-\alpha\beta_1\right)^2\Delta\Omega_{21,22} \Longleftrightarrow$$

$$\Delta\Omega_{21,22} - \theta\Delta\Omega_{21,12} = \frac{\left(1-\theta\beta_1\right)}{\left(1-\alpha\beta_1\right)^2}\Delta\sigma_{21,\eta}^2. \tag{A8}$$

Next, divide $\frac{(A7)}{(A8)}$: $\quad \frac{\Delta\Omega_{21,12}-\theta\Delta\Omega_{21,11}}{\Delta\Omega_{21,22}-\theta\Delta\Omega_{21,12}} = \beta_1 \Longleftrightarrow$

$$\theta = \frac{\Delta\Omega_{21,12} - \beta_1\Delta\Omega_{21,22}}{\Delta\Omega_{21,11} - \beta_1\Delta\Omega_{21,12}}. \tag{A9}$$

Remember that a system similar to $(A4) - (A6)$ can be written for $i = 3$. Solving that system for θ then yields:

$$\theta = \frac{\Delta\Omega_{31,12} - \beta_1\Delta\Omega_{31,22}}{\Delta\Omega_{31,11} - \beta_1\Delta\Omega_{31,12}}. \tag{A10}$$

As it turns out, $(A9)$ and $(A10)$ are two equations in just two variables, β_1 and θ. This illustrates how the underidentified system of six equations collapses to a smaller system where β_1 is now identified. To solve the system for β_1, equalize the right hand sides of $(A9)$ and $(A10)$ and cross-multiply:

$$\Delta\Omega_{21,12}\Delta\Omega_{31,11} - \beta_1\Delta\Omega_{21,12}\Delta\Omega_{31,12} - \beta_1\Delta\Omega_{21,22}\Delta\Omega_{31,11} + \beta_1^2\Delta\Omega_{21,22}\Delta\Omega_{31,12} =$$

$$\Delta\Omega_{31,12}\Delta\Omega_{21,11} - \beta_1\Delta\Omega_{31,12}\Delta\Omega_{21,12} - \beta_1\Delta\Omega_{31,22}\Delta\Omega_{21,11} + \beta_1^2\Delta\Omega_{31,22}\Delta\Omega_{21,12}$$

$$\begin{aligned}\Longleftrightarrow 0 = {} & \beta_1^2\left[\Delta\Omega_{31,22}\Delta\Omega_{21,12} - \Delta\Omega_{21,22}\Delta\Omega_{31,12}\right] \\ & -\beta_1\left[\Delta\Omega_{31,22}\Delta\Omega_{21,11} - \Delta\Omega_{21,22}\Delta\Omega_{31,11}\right] + \left[\Delta\Omega_{31,12}\Delta\Omega_{21,11} - \Delta\Omega_{21,12}\Delta\Omega_{31,11}\right]\end{aligned}$$

$$\Longleftrightarrow 0 = a\beta_1^2 - b\beta_1 + c, \tag{A11}$$

- where:

$$a = \left[\Delta\Omega_{31,22}\Delta\Omega_{21,12} - \Delta\Omega_{21,22}\Delta\Omega_{31,12}\right],$$

$$b = \left[\Delta\Omega_{31,22}\Delta\Omega_{21,11} - \Delta\Omega_{21,22}\Delta\Omega_{31,11}\right],$$

$$c = \left[\Delta\Omega_{31,12}\Delta\Omega_{21,11} - \Delta\Omega_{21,12}\Delta\Omega_{31,11}\right].$$

This solves the system for the parameter of interest; β_1. As noted above, the exact same method is used to solve for β_2.

It should be noted that the quadratic equation $(A11)$ has two roots. Rigobon and Sack (2003) describe how the system of two equations in two variables $(A9)$ and $(A10)$ is solvable for β and θ whenever one of these roots is real. This condition is ensured by the positive definiteness of the covariance matrices. Rigobon and Sack then show that one set of solutions to the system gives the correct values of β and θ, while the other set gives the inverse of these values.

2 Correlations of Simulated Shocks

Figure A1 and A2 show histograms for the computed correlations of the simulated shocks. For each of 500 replications, the series are split based on the sign of the stock price change, and then by covariance regimes. By keeping track of the shocks that generated each of the observations, we can then compute the correlations conditional on the sign of the stock price change and the variance regime. In the main text, it is assumed that this correlation is zero.

The histograms show the correlation between the stock price shock and the common shock (interpretable as η_t and z_t, respectively). As argued in the main text, the concern of non-zero correlations is most relevant for these two shocks. The correlations between each of these two shocks and the monetary policy shock (ε_t) display similar patterns.

Figure A1 shows the correlations between the two shocks in each of the four variance regimes and for decreasing stock prices, while figure A2 shows the correlations in each regime for increasing stock prices. In each case, there is no clear tendency for the correlations to be systematically above or below zero. Instead, the correlations are distributed around zero. For all of the eight regimes displayed, at least around 20 % of the computed correlations are located on either side of zero. In other words, there is no sign of non-zero correlations. Note that the lower-right panel in each figure shows a few very large correlation coefficients. This is because the lower-right panels display the 'high-high'-variance regime, which in some replications have very few observations (as low as 3), giving rise to potentially very high correlations.

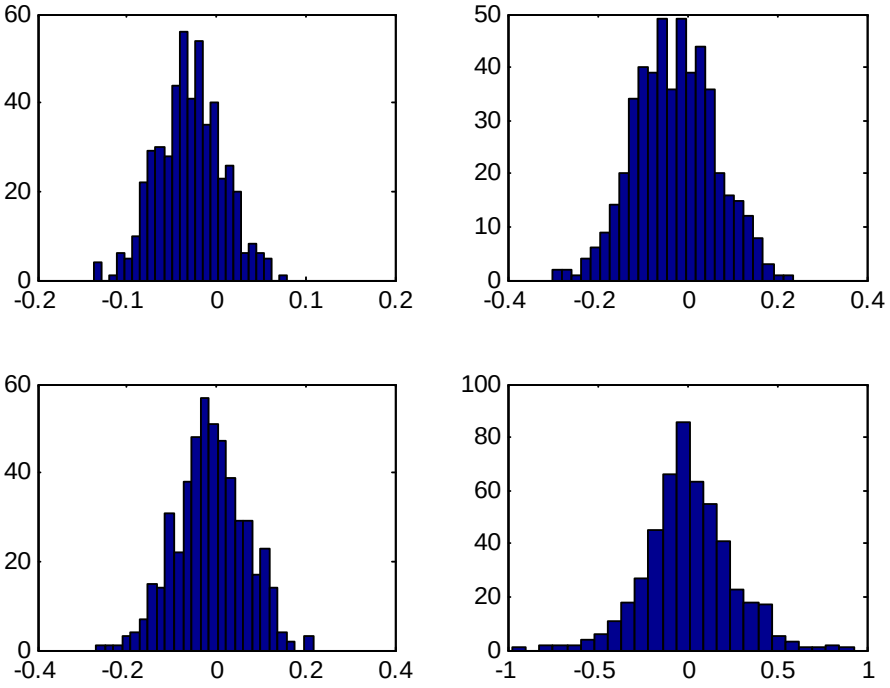

Figure A1: Histogram of simulated correlations, decreasing stock prices.

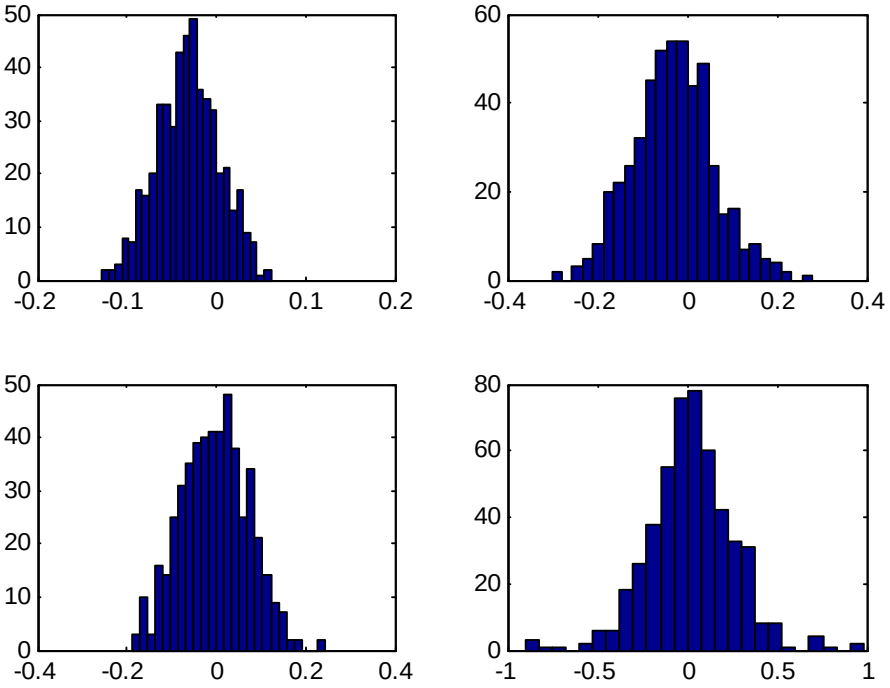

Figure A2: Histogram of simulated correlations, increasing stock prices.

3 Unit Root Tests

The table below shows the results of our unit root tests of the daily data series for the interest rate (3-month T-Bill rate).

Table A1: Unit root test statistics

	Augmented Dickey-Fuller test	KPSS test
Test statistic	-0.25	0.43
Critical Value (5 % level)	-2.86	0.46

In the augmented Dickey-Fuller test, the null hypothesis is that the series displays a unit root, while in the KPSS test, the null hypothesis is that the variable is stationary. In both cases, we fail to reject the null hypothesis at the 5 % level.

4 Scatterplots

Figure A3 illustrates the scatterplots of the residuals v_t^i and v_t^s for each of the four regimes. Viewed in isolation, each of the upper and the lower panel constitutes an empirical equivalent of the theoretical scatterplots in figure 1a and 1b in the main text. The upper left panel displays the regime where both residuals have low variance, while the upper right panel illustrates the regime with low variance of the interest rate residual but high variance of the stock price residual. In other words, the upper right panel illustrates an increase in the volatility of stock price residuals, holding fixed the volatility of interest rate residuals, relative to the upper left panel; exactly as in figure 1. The same is true for the lower panels. Indeed, there seems to be a vague tendency for the residuals in the upper right panel to be distributed along an upward-sloping line, while no clear picture seems to emerge from the upper left panel. This is supported by the slope of the trend line, which is much larger for the upper right panel. For the lower panels, the slopes of the tendency lines tell the same story, whereas the pattern is not really clear graphically; partly because of the lower number of observations. In sum, the residuals do tend to display the pattern described in section 2.1, even if the picture is a lot less pronounced in the empirical scatterplots above than in the 'slanted' illustrations in figure 1.[1]

[1] On the other hand, one should expect to see a move towards a lower slope of the tendency lines when comparing the upper and lower panels. Fixing the volatility of the stock price residuals, an increase in the volatility of the interest rate residuals should cause the residuals to better trace out a downward-sloping curve. This pattern does not emerge in the scatterplots. As the interest rate residuals are in general a lot less volatile than the stock price residuals, the shift in volatility of the former simply seems to be of too little importance to alter the picture.

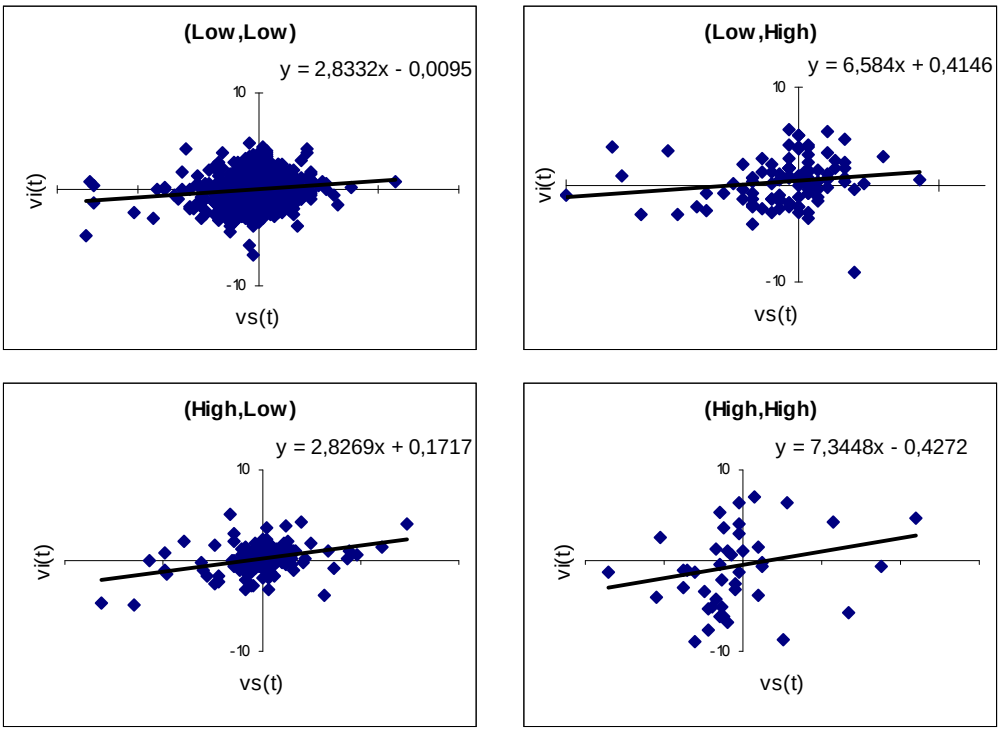

Figure A3: Scatterplots for each of the four covariance regimes of the residuals.

5 Stock Price Volatility and Covariance with Interest Rate

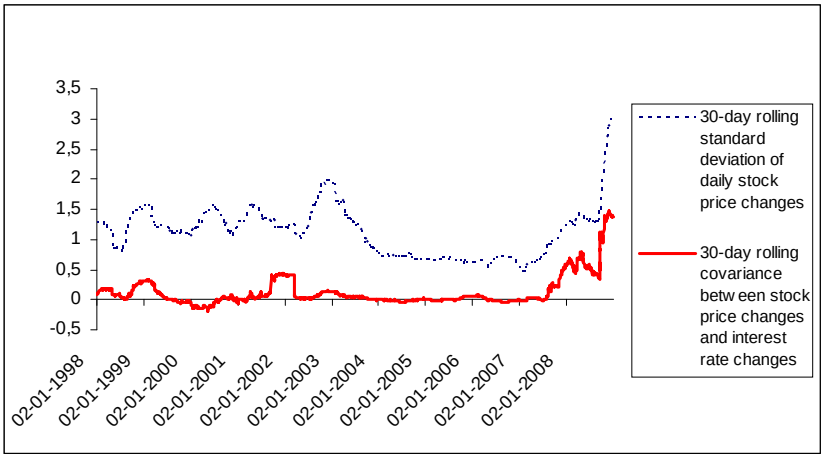

Figure A4: Link between volatility of stock prices and covariance between stock prices and interest rates. Correlation = 0.60.

6 The Bootstrap

For the purpose of this paper, we do not have to bootstrap the actual observations that enter the original VAR. (Remember that this VAR has 2 dependent variables and 16 regressors). Instead, we can bootstrap the residuals from the VAR (see Efron and Tibshirani (1994,) or Johnston and DiNardo (1997) for a treatment of bootstrapping residuals). Usually, in order to bootstrap the residuals, these first need to be standardized, as emphasized by Johnston and DiNardo (1997). However, this is only necessary when the residuals are used for computing fitted values of the dependent variable in the original regression. The fitted values can then be regressed on the regressors to obtain a large number of estimates of the regression coefficients.

However, estimating the regression coefficients of the VAR is not the primary purpose of this paper. Instead, we are interested in the residuals from the VAR themselves, as we want to impose theoretical restrictions on these. Therefore, standardizing the residuals before implementing the bootstrap is not appropriate in the current context.

We therefore use the raw residuals from the VAR to do the bootstrap. This yields 10,000 realizations of the covariance matrix for each regime. With these in hand, it is easy to obtain 10,000 estimates of β_1 and β_2, the parameters of interest.

7 Alternative Interest Rate Variable

Table A2: Estimates for β_1; the parameter measuring the reaction to stock price increases; using the 6-month T-Bill rate.

β_1	Regime 1,2,3	Regime 1,2,4	Regime 1,3,4
Mean	-0.0077	0.0553	-0.0080
Median	-0.0083	0.0013	-0.0163
Probability mass above 0	29.97 %	50.41 %	32.14 %

Table A3: Estimates for β_2; the parameter measuring the reaction to stock price decreases; using the 6-month T-Bill rate.

β_2	Regime 1,2,3	Regime 1,2,4	Regime 1,3,4
Mean	0.0131	0.0768	0.0109
Median	0.0122	0.0276	0.0105
Probability mass above 0	91.90 %	73.46 %	90.10 %

8 Alternative Taylor Rules

Table A4: Taylor rule with monetary policy shocks from Barakchian and Crowe (2010)

	OLS (1)	OLS (2)
c	0.268138*** (0.093375)	0.263273*** (0.092468)
ρ	0.967218*** (0.012759)	0.968770*** (0.012658)
ϕ_π	−1.382208* (0.822433)	−1.479063* (0.873819)
ϕ_y	1.259721*** (0.385322)	1.232669*** (0.394926)
ϕ_q^+	−46.99346 (35.44797)	−55.17527 (39.02790)
ϕ_q^-	104.2027** (48.05645)	102.9023** (49.49175)
ϕ_ε		0.043088* (0.023219)
R^2	0.989409	0.989710

Sample: 1998:01 to 2008:06. Standard errors in brackets. *, **, and *** denote significance at the 10 %, 5 % and 1 % levels.

Table A5: Forward-looking Taylor rule

	OLS (1)	IV (2)	OLS (3)	IV (4)	OLS (5)	IV (6)
c	0.176554** (0.080381)	0.226198** (0.090633)	0.184534** (0.076633)	0.199891* (0.102881)	0.228915*** (0.062773)	0.061113 (0.101987)
ρ	0.962887*** (0.012303)	0.962539*** (0.012877)	0.959214*** (0.011680)	0.961993*** (0.012761)	0.958154*** (0.009052)	0.967896*** (0.012604)
ϕ_π	−0.361866 (0.544110)	−0.808510 (0.634251)	−0.202801 (0.476225)	−0.440041 (0.712163)	−0.487636 (0.415093)	0.915941 (1.048278)
ϕ_y	1.150809*** (0.301955)	1.192588*** (0.311489)	1.164091*** (0.270336)	1.173930*** (0.301382)	1.403567*** (0.275310)	1.100610*** (0.379727)
ϕ_q^+	−21.10207 (27.44148)	−25.71486 (28.05524)	−24.97482 (24.48184)	−29.70909 (27.17239)	−34.34332 (20.56438)	−35.50547 (30.07590)
ϕ_q^-	60.10560** (28.86084)	61.53519** (30.30246)	62.06217** (28.21679)	69.42890** (34.52826)	56.85601*** (21.38624)	86.24871* (47.28013)
R^2	0.989274	0.989213	0.989707	0.989671	0.992448	0.991634
Hausman p		0.4546		0.1527		0.0002

Sample: 1998:01 to 2008:12. Standard errors in brackets. *, **, and *** denote significance at the 10 %, 5 % and 1 % levels.

The forward-looking equations in columns 1-2, 3-4, and 5-6 are estimated using CPI inflation and industrial production 1, 2, and 6 months ahead, respectively.

The p-value for the stock price drop variable (ϕ_q^-) in column 6 is p = 0.07.

References:

Efron, Bradley, and Robert J. Tibshirani. 1994. *An Introduction to the Bootstrap*. First edition, Chapman and Hall/CRC.

Johnston, Jack, and John DiNardo. 1997. *Econometric Methods*. Fourth Edition, McGraw-Hill.

Rigobon, Roberto, and Brian Sack, 2003. Measuring the Reaction of Monetary Policy to the Stock Market. *Quarterly Journal of Economics*, 118(2), p. 639-669.

Appendix to "Asymmetric Monetary Policy Towards the Stock Market: A DSGE Approach"

Abstract

This appendix contains supplemental material for Chapter 2; "Asymmetric Monetary Policy Towards the Stock Market: A DSGE Approach".

1 Equilibrium Conditions

The first step is to present the conditions which must hold in equilibrium, and the details underlying a few of them.

1.1 Household First-order Conditions

As described in the main paper, the problem of the representative household is the following:

$$\max_{C_t, H_t, D_t} U = E_0 \sum_{t=0}^{\infty} \beta^t u\left(C_t, H_t\right),$$

where the instantaneous utility function is given by

$$u\left(C_t, H_t\right) = \frac{\gamma}{\gamma - 1} \ln\left(C_t^{\frac{\gamma-1}{\gamma}}\right) + \eta \ln\left(1 - H_t\right),$$

and subject to the following budget constraint:

$$C_t + \frac{D_t - R_{t-1} D_{t-1}}{P_t} \leq \frac{W_t}{P_t} H_t + \Omega_t.$$

This problem gives rise to the following first-order conditions:

$$\lambda_t = C_t^{-1}, \tag{I}$$

$$\frac{\eta}{1 - H_t} = \lambda_t w_t, \tag{II}$$

$$\frac{\lambda_t}{R_t} = \beta E_t \frac{\lambda_{t+1}}{\pi_{t+1}}, \tag{III}$$

Conditions (I), (II), and (III) are the first-order equations describing optimal behaviour by the representative household.

1.2 Optimal Pricing Behaviour of Retail Firms

The problem of retail firm i is to set the optimal price $P_t^n(i)$ so as to maximize its profits:

$$\max_{P_t^n(i)} \; E_0 \left\{ \sum_{s=0}^{\infty} (\beta\xi)^s \frac{\lambda_{t+s}}{\lambda_t} Y_{t+s}(i) \left[P_t^n(i) \pi^s - P_{t+s} mc_{t+s} \right] \right\},$$

subject to the demand function

$$Y_{t+s}(i) = \left[\frac{P^n_{t+s}(i)}{P_{t+s}}\right]^{-\epsilon^p} Y_{t+s},$$

with aggregate demand for the final good given by:

$$Y_t = \left[\int_0^1 Y_t(i)^{(\epsilon^p-1)/\epsilon^p}\, di\right]^{\epsilon^p/(\epsilon^p-1)}.$$

Note that as in the main text, mc_t denotes the real marginal cost of the entrepreneurs. Since these operate under perfect competition, they set their output price equal to their marginal cost, which therefore becomes the input price faced by retailers. As all entrepreneurs are identical, their marginal cost is the same. The first-order condition with respect to the choice of $P^n_t(i)$ becomes:

$$E_t \sum_{s=0}^{\infty} (\beta\xi)^s \frac{\lambda_{t+s}}{\lambda_t} \Theta^p_{t+s} = 0,$$

where $\Theta^p_{t+s} =$

$$(-\epsilon^p) Y_{t+s} \left[\frac{P^n_{t+s}(i)}{P_{t+s}}\right]^{-\epsilon^p-1} \frac{1}{P_{t+s}} \left[P^n_t(i)\pi^s - P_{t+s} mc_{t+s}\right] + Y_{t+s}\pi^s \left[\frac{P^n_{t+s}(i)}{P_{t+s}}\right]^{-\epsilon^p}.$$

Using the definition of $Y_{t+s}(i)$ from the demand function above, this expression can be rewritten as:

$$0 = E_t \sum_{s=0}^{\infty} (\beta\xi)^s \frac{\lambda_{t+s}}{\lambda_t} \left[(-\epsilon^p) Y_{t+s}(i) \frac{1}{P^n_t(i)} \left[P^n_t(i)\pi^s - P_{t+s} mc_{t+s}\right] + Y_{t+s}(i)\pi^s\right] \tag{$\Leftrightarrow$}$$

$$0 = E_t \sum_{s=0}^{\infty} (\beta\xi)^s \frac{\lambda_{t+s}}{\lambda_t} \left[(1-\epsilon^p) Y_{t+s}(i)\pi^s + \epsilon^p Y_{t+s}(i) \frac{P_{t+s} mc_{t+s}}{P^n_t(i)}\right] \tag{$\Leftrightarrow$}$$

$$\frac{1}{\lambda_t} E_t \sum_{s=0}^{\infty} (\beta\xi)^s \lambda_{t+s} (\epsilon^p - 1) Y_{t+s}(i)\pi^s = \frac{1}{\lambda_t} E_t \sum_{s=0}^{\infty} (\beta\xi)^s \lambda_{t+s} \epsilon^p Y_{t+s}(i) \frac{P_{t+s} mc_{t+s}}{P^n_t(i)} \tag{$\Leftrightarrow$}$$

$$P^n_t(i) = \frac{\epsilon^p}{\epsilon^p - 1} \frac{E_t\left\{\sum_{s=0}^{\infty} (\beta\xi)^s \lambda_{t+s} Y_{t+s}(i) P_{t+s} mc_{t+s}\right\}}{E_t\left\{\sum_{s=0}^{\infty} (\beta\xi)^s \lambda_{t+s} Y_{t+s}(i)\pi^s\right\}},$$

which is the expression for the first-order condition presented in the main paper. Finally, since each firm that is allowed to change its price in a given period will

set the same price, we can drop the index i to obtain an expression for the new price being set in any period:

$$P_t^n = \frac{\epsilon^p}{\epsilon^p - 1} \frac{E_t \left\{ \sum_{s=0}^{\infty} (\beta\xi)^s \lambda_{t+s} Y_{t+s} P_{t+s} mc_{t+s} \right\}}{E_t \left\{ \sum_{s=0}^{\infty} (\beta\xi)^s \lambda_{t+s} Y_{t+s} \pi^s \right\}},$$

which is the expression used to derive the equilibrium of the model.

1.3 Equilibrium Conditions

The 15 equilibrium conditions of the model are summarized below. In equilibrium, the production technology constraint (eq. (4) below) will hold with equality. Moreover, with respect to the main paper, the law of motion for capital (eq. (10) below) and the aggregate resource constraint (15) are needed to fully describe the equilibrium. The remaining conditions have all been described in the main paper or above.

$$\lambda_t = C_t^{-1}, \tag{1}$$

$$\frac{\eta}{1 - H_t} = \lambda_t w_t, \tag{2}$$

$$\frac{\lambda_t}{R_t} = \beta E_t \frac{\lambda_{t+1}}{\pi_{t+1}}, \tag{3}$$

$$Y_t = (A_t H_t)^{1-\alpha} K_t^{\alpha}, \tag{4}$$

$$mp_t = \alpha \frac{Y_t}{K_t} mc_t, \tag{5}$$

$$w_t = (1 - \alpha) \frac{Y_t}{H_t} mc_t, \tag{6}$$

$$E_t [f_{t+1}] = E_t \left[\frac{mp_{t+1} + (1 - \delta) q_{t+1}}{q_t} \right], \tag{7}$$

$$E_t [f_{t+1}] = E_t \left[\Psi \left(\frac{n_{t+1}}{q_t K_{t+1}} \right) \frac{R_t}{\pi_{t+1}} \right], \tag{8}$$

$$n_{t+1} = \nu \left[f_t q_{t-1} K_t - E_{t-1} f_t \left(q_{t-1} K_t - n_t \right) \right] + (1 - \nu) \Upsilon_t, \tag{9}$$

$$K_{t+1} = I_t + (1 - \delta) K_t, \tag{10}$$

$$q_t - \chi \left(\frac{I_t}{K_t} - \delta \right) = 1, \tag{11}$$

$$P_t^n = \frac{\epsilon^p}{\epsilon^p - 1} \frac{E_t \left\{ \sum_{s=0}^{\infty} (\beta\xi)^s \lambda_{t+s} Y_{t+s} mc_{t+s} P_{t+s} \right\}}{E_t \left\{ \sum_{s=0}^{\infty} (\beta\xi)^s \lambda_{t+s} Y_{t+s} \pi^s \right\}}, \tag{12}$$

$$P_t = \left[(1-\xi) \left(P_t^n\right)^{1-\epsilon^p} + \xi \left(P_{t-1}\pi\right)^{1-\epsilon^p} \right]^{1/(1-\epsilon^p)}, \tag{13}$$

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R}\right)^{\rho_r} \left[\left(\frac{\pi_t}{\pi}\right)^{\phi_\pi} \left(\frac{Y_t}{Y}\right)^{\phi_y} \left\{ \left(\frac{\Delta q_t}{q}\right)^{\phi_q} \right\}^{1[\Delta q_t < 0]} \left\{ \left(\frac{\Delta q_t}{q}\right)^{0} \right\}^{1[\Delta q_t \geq 0]} \right]^{(1-\rho_r)} e^{\varepsilon_t^r}, \tag{14}$$

$$Y_t = C_t + I_t. \tag{15}$$

We further need to assume a functional form for how the external finance premium depends on firms' net worth, i.e. the function $\Psi(\cdot)$. We specify the following functional form:

$$\Psi\left(\frac{n_{t+1}}{q_t K_{t+1}}\right) = \left(\frac{n_{t+1}}{q_t K_{t+1}}\right)^{-\psi}, \tag{$\Diamond$}$$

where $\psi > 0$ measures the elasticity of the external finance premium with respect to the capital position of the firms. This specification satisfies $\Psi'(\cdot) < 0$ and follows Christensen and Dib (2008) and Gilchrist and Saito (2008).

1.4 The Steady State

The steady state of the model requires that all the endogenous variables are constant, giving rise to the following conditions:

$$\lambda = C^{-1}, \tag{16}$$

$$\frac{\eta}{1-H} = \lambda w, \tag{17}$$

$$R = \frac{\pi}{\beta}, \tag{18}$$

$$Y = (AH)^{1-\alpha} K^{\alpha}, \tag{19}$$

$$\frac{mp}{mc} = \alpha \frac{Y}{K}, \tag{20}$$

$$\frac{w}{mc} = (1-\alpha) \frac{Y}{H}, \tag{21}$$

$$f = mp + 1 - \delta, \tag{22}$$

$$f = \left(\frac{n}{qK}\right)^{-\psi} \frac{R}{\pi}, \tag{23}$$

$$1 = \nu f, \tag{24}$$

$$I = \delta K, \tag{25}$$

$$q = 1, \tag{26}$$

$$mc = \frac{\epsilon^p - 1}{\epsilon^p}, \tag{27}$$

$$Y = C + I. \tag{28}$$

In addition, recall that we calibrated the steady state values of the variable π and the ratio $\frac{K}{n}$. Equation (23) imposes the functional form for $\Psi(\cdot)$ specified in equation ($\Diamond$) above. In the steady state version of the law of motion of net worth (24), we have assumed that bequests from entrepreneurs leaving the economy (Υ) are small and drop out of the model. This follows the related literature, see Christensen and Dib (2008) or Gilchrist and Saito (2008).

1.5 Calibration

The model is calibrated using the estimated values from Christensen and Dib (2008). For those parameters that were not estimated in that study, we use the calibrated values from that study to the extent possible. As described in the main text, exceptions include the parameters of the monetary policy rule, as the rule estimated by Christensen and Dib differs substantially from that of the present model.

Moreover, Christensen and Dib (2008) do not impose steady state conditions (23) and (24) presented above. In fact, with their choice of the relevant parameters (which are calibrated, not estimated, in their study), these conditions do not hold. On the contrary, Gilchrist and Saito (2008) do impose these conditions. We therefore follow Gilchrist and Saito and set $\frac{K}{n} = 1.8$ and $\beta = 0.984$. Keeping the estimated value of the key parameter $\psi = 0.042$ found by Christensen and Dib (2008), this yields a steady state value of the external finance premium of $\Psi\left(\frac{n}{qK}\right) = \left(\frac{n}{qK}\right)^{-\psi} = 1.0250$. Equation (23) then implies a steady state external financing *cost* of $f = 1.0417$, and (24) then in turn implies that the survival rate

of entrepreneurs must be set to $\nu = 0.960$, i.e. slightly lower than the value of 0.9728 chosen by Christensen and Dib (2008).

Parameter	**Interpretation**	**Value**
α	Capital share in production	0.3384
β	Discount factor	0.984
γ	Preference for consumption	0.0598
δ	Depreciation rate	0.025
ϵ^p	Elasticity of substitution between final goods	6
η	Preference for leisure	1.315
ν	Entrepreneurs' survival rate	0.9600
ξ	Probability of not adjusting price	0.7418
χ	Importance of capital adjustment cost	0.5882
ψ	Elasticity of ext. fin. premium wrt. leverage	0.042
$\overline{\Psi}$	Steady state external finance premium	1.0250
π	Steady state inflation rate	1
$\frac{K}{n}$	Rate of capital to net worth in steady state	1.8
ρ_r	Degree of interest rate smoothing	0.67
ϕ_π	Monetary policy reaction to inflation	1.5
ϕ_y	Monetary policy reaction to output	0.2
ϕ_q	Monetary policy reaction to stock price drops (estimated)	0.0246
ϕ_q	Monetary policy reaction to stock price drops (calibrated)	0.5
ρ_a	Persistence of technology shock	0.7625
σ_a	Standard deviation of technology shock	0.0096
σ_r	Standard deviation of monetary policy shock	0.0058

1.6 Log-linearizing the Equilibrium Conditions

The next step is to log-linearize the conditions describing the equilibrium; (1)-(15), around the steady state described above. For details about log-linearization, see for example Uhlig (1999) or Woodford (2003). In the following, $\widehat{x}_t$ will denote the log-deviation of variable x_t from its value in the nonstochastic steady state; denoted x.

Below, we derive the log-linearized equations, presenting the calculations as

we find necessary. First, log-linearize (1):

$$\lambda\left(1+\widehat{\lambda}_t\right)=C\left(1-\widehat{C}_t\right)\Leftrightarrow$$

$$\widehat{\lambda}_t=-\widehat{C}_t \tag{29}$$

To log-linearize (2), first rewrite it as $\eta=\lambda_t w_t-\lambda_t w_t H_t$. Then log-linearize to get:

$$\eta=\lambda w\left(1+\widehat{\lambda}_t+\widehat{w}_t\right)-\lambda wH\left(1+\widehat{\lambda}_t+\widehat{w}_t+\widehat{H}_t\right).$$

Now use (17) to substitute in for η, cancel out terms, and rearrange to get:

$$H\widehat{H}_t=(1-H)\left(\widehat{\lambda}_t+\widehat{w}_t\right). \tag{30}$$

From (3), we get:

$$\frac{\lambda}{R}\left(1+\widehat{\lambda}_t-\widehat{R}_t\right)=\beta\frac{\lambda}{\pi}E_t\left(1+\widehat{\lambda}_{t+1}-\widehat{\pi}_{t+1}\right).$$

Using the steady state condition that $\pi=\beta R$, we get:

$$\widehat{\lambda}_t=E_t\widehat{\lambda}_{t+1}-E_t\widehat{\pi}_{t+1}+\widehat{R}_t. \tag{31}$$

The log-linearization of (4) results in:

$$Y\left(1+\widehat{Y}_t\right)=(AH)^{1-\alpha}K^{\alpha}\left(1+(1-\alpha)\widehat{A}_t+(1-\alpha)\widehat{H}_t+\alpha\widehat{K}_t\right).$$

Recall from (19) that in steady state, we have: $Y=(AH)^{1-\alpha}K^{\alpha}$. Use this to get:

$$\widehat{Y}_t=(1-\alpha)\widehat{A}_t+(1-\alpha)\widehat{H}_t+\alpha\widehat{K}_t. \tag{32}$$

From (5), and using (20), it is straightforward to get:

$$\widehat{mp}_t=\widehat{Y}_t+\widehat{mc}_t-\widehat{K}_t. \tag{33}$$

Similarly, log-linearize (6) and use (21) to get:

$$\widehat{w}=\widehat{Y}_t+\widehat{mc}_t-\widehat{H}_t. \tag{34}$$

From (7), we get:

$$fq\left(1+\widehat{f}_{t+1}+\widehat{q}_t\right)=mp\left(1+\widehat{mp}_{t+1}\right)+(1-\delta)\,q\,(1+\widehat{q}_{t+1})\,.$$

Using (22) and the fact that in steady state; $q=1$:

$$\widehat{f}_{t+1}=\frac{mp}{f}\widehat{mp}_{t+1}+\frac{1-\delta}{f}\widehat{q}_{t+1}-\widehat{q}_t. \tag{35}$$

Equation (8) log-linearized becomes:

$$f\left(1+E_t\widehat{f}_{t+1}\right)=\left(\frac{n}{qK}\right)^{-\psi}\frac{R}{\pi}\left[1+\widehat{R}_t-E_t\widehat{\pi}_{t+1}-\psi\left(\widehat{n}_{t+1}-\widehat{q}_t-\widehat{K}_{t+1}\right)\right],$$

which can be rewritten as:

$$E_t\widehat{f}_{t+1}-\left(\widehat{R}_t-E_t\widehat{\pi}_{t+1}\right)=-\psi\left(\widehat{n}_{t+1}-\widehat{q}_t-\widehat{K}_{t+1}\right). \tag{36}$$

Note that in deriving (36), we use the functional form for $\Psi(\cdot)$ imposed in $(\Diamond)$. Recall that the parameter ψ (which measures the elasticity of that function) is larger than zero.

The log-linearization of (9) is not entirely straightforward and deserves some attention. First, substitute in for $E_{t-1}f_t$ by lagging (8) one period:

$$n_{t+1}=\nu\left[f_tq_{t-1}K_t-E_{t-1}\left[\Psi\left(\frac{n_t}{q_{t-1}K_t}\right)\frac{R_{t-1}}{\pi_t}\right](q_{t-1}K_t-n_t)\right]+(1-\nu)\,\Upsilon_t.$$

As mentioned, we follow the literature and assume that bequests (Υ_t) are small and drop out of the model. Log-linearizing then yields:

$$\begin{aligned}
n\,(1+\widehat{n}_{t+1}) \;=\;& \nu fqK\left[1+\widehat{f}_t+\widehat{q}_{t-1}+\widehat{K}_t\right]\\
&-\nu\left(\frac{n}{qK}\right)^{-\psi}\frac{R}{\pi}qK\left[1+\widehat{R}_{t-1}-\widehat{\pi}_t-\psi\left(\widehat{n}_t-\widehat{q}_{t-1}-\widehat{K}_t\right)+\widehat{q}_{t-1}+\widehat{K}_t\right]\\
&+\nu\left(\frac{n}{qK}\right)^{-\psi}\frac{R}{\pi}n\left[1+\widehat{R}_{t-1}-\widehat{\pi}_t-\psi\left(\widehat{n}_t-\widehat{q}_{t-1}-\widehat{K}_t\right)+\widehat{n}_t\right].
\end{aligned}$$

Now use the steady state conditions (23) and (26) to get:

$$\begin{aligned}\frac{n}{\nu}\left(1+\widehat{n}_{t+1}\right) &= fK\left[1+\widehat{f}_t+\widehat{q}_{t-1}+\widehat{K}_t\right] \\ &\quad -fK\left[1+\widehat{R}_{t-1}-\widehat{\pi}_t-\psi\left(\widehat{n}_t-\widehat{q}_{t-1}-\widehat{K}_t\right)+\widehat{q}_{t-1}+\widehat{K}_t\right] \\ &\quad +fn\left[1+\widehat{R}_{t-1}-\widehat{\pi}_t-\psi\left(\widehat{n}_t-\widehat{q}_{t-1}-\widehat{K}_t\right)+\widehat{n}_t\right].\end{aligned}$$

Next, cancel out terms and simplify to obtain:

$$\begin{aligned}\frac{1}{\nu f}\left(1+\widehat{n}_{t+1}\right) &= \frac{K}{n}\widehat{f}_t+\left(1-\frac{K}{n}\right)\left(\widehat{R}_{t-1}-\widehat{\pi}_t\right)+\left(1-\frac{K}{n}\right)\left(\widehat{q}_{t-1}+\widehat{K}_t\right) \\ &\quad +\left[1+\psi\left(\frac{K}{n}-1\right)\right]\widehat{n}_t+1.\end{aligned}$$

Finally, from (24) we have that in steady state, $\nu f = 1$ must hold. Imposing this condition yields the log-linearized equation:

$$\widehat{n}_{t+1} = \frac{K}{n}\widehat{f}_t+\left(1-\frac{K}{n}\right)\left(\widehat{R}_{t-1}-\widehat{\pi}_t\right)+\left(1-\frac{K}{n}\right)\left(\widehat{q}_{t-1}+\widehat{K}_t\right)+\left[1+\psi\left(\frac{K}{n}-1\right)\right]\widehat{n}_t. \tag{37}$$

From (10), and using steady state relation (25), we get:

$$\widehat{K}_{t+1} = \frac{I}{K}\widehat{I}_t+(1-\delta)\,\widehat{K}_t. \tag{38}$$

The log-linear version of (11) is:

$$\widehat{q}_t = \chi\left(\widehat{I}_t-\widehat{K}_t\right). \tag{39}$$

The log-linearized version of the monetary policy rule (14) is:

$$\widehat{R}_t = \rho_r\widehat{R}_{t-1}+(1-\rho_r)\left[\phi_\pi\widehat{\pi}_t+\phi_y\widehat{Y}_t+\phi_q\left[\Delta\widehat{q}_t<0\right]\Delta\widehat{q}_t\right]+\varepsilon_t^r, \tag{40}$$

where $\Delta\widehat{q}_t = \frac{\Delta q_t}{q}$, and where $[\Delta\widehat{q}_t < 0]$ is the indicator function; equal to 1 if the change in stock prices is negative, and zero otherwise.

In log-linear terms, (15) becomes:

$$Y\widehat{Y}_t = C\widehat{C}_t+I\widehat{I}_t. \tag{41}$$

Finally, we show below how (12) and (13) can be combined to yield a log-linear version of the so-called New-Keynesian Phillips Curve (Woodford, 2003). Start by log-linearizing (13). Recall that we calibrated the steady state value of the gross inflation rate to $\pi = 1$, which we impose in the following calculations.

$$P^{1-\epsilon^p}\left(1+\left(1-\epsilon^p\right)\widehat{P}_t\right) = \left(1-\xi\right)\left(P^n\right)^{1-\epsilon^p}\left(1+\left(1-\epsilon^p\right)\widehat{P_t^n}\right)+\xi P^{1-\epsilon^p}\left(1+\left(1-\epsilon^p\right)\widehat{P}_{t-1}\right).$$

Recognizing that in steady state, it must hold that $P^n = P$, we can cancel out terms and rewrite as:

$$\left(1-\epsilon^p\right)\widehat{P}_t = \left(1-\xi\right)\left(1-\epsilon^p\right)\widehat{P_t^n} + \xi\left(1-\epsilon^p\right)\widehat{P}_{t-1},$$

which then further collapses to:

$$\widehat{P}_t = \left(1-\xi\right)\widehat{P_t^n} + \xi\widehat{P}_{t-1}. \qquad (\#)$$

Next, define $\theta \equiv \frac{\epsilon^p}{\epsilon^p-1}$, and rewrite (12) as:

$$P_t^n E_t\left\{\sum_{s=0}^{\infty}\left(\beta\xi\right)^s\lambda_{t+s}Y_{t+s}\pi^s\right\} = \theta E_t\left\{\sum_{s=0}^{\infty}\left(\beta\xi\right)^s\lambda_{t+s}Y_{t+s}mc_{t+s}P_{t+s}\right\}. \qquad (\#\#)$$

For the sake of tractability, we first consider only the left hand side of (##). Writing out the sum, we get:

$$LHS = P_t^n E_t\left[\lambda_t Y_t + \beta\xi\lambda_{t+1}Y_{t+1}\pi + ...\right].$$

Log-linearize this expression to get:

$$LHS = P^n\lambda Y\left(1+\widehat{P_t^n}+\widehat{\lambda}_t+\widehat{Y}_t\right)+\beta\xi P^n\lambda Y\pi\left(1+\widehat{P_t^n}+\widehat{\lambda}_{t+1}+\widehat{Y}_{t+1}\right)+...$$

Recollect the sums:

$$LHS = P^n\lambda Y\sum_{s=0}^{\infty}\left(\beta\xi\right)^s\widehat{P_t^n} + P^n\lambda Y E_t\sum_{s=0}^{\infty}\left(\beta\pi\xi\right)^s\widehat{\lambda}_{t+s}+\widehat{Y}_{t+s}.$$

Using the formula for an infinite sum, and the condition $\pi = 1$, this gives:

$$LHS = P^n\lambda Y\frac{1}{1-\beta\xi}\widehat{P_t^n} + P^n\lambda Y E_t\sum_{s=0}^{\infty}\left(\beta\xi\right)^s\widehat{\lambda}_{t+s}+\widehat{Y}_{t+s}. \qquad (\Delta)$$

Now, consider the right hand side of (##). Importantly, this features the *real* marginal cost. Proceeding as above, we can write this as:

$$RHS = \theta E_t \left[\lambda_t Y_t mc_t P_t + \beta\xi \lambda_{t+1} Y_{t+1} mc_{t+1} P_{t+1} + ... \right].$$

In log-linear terms, this becomes (imposing $\pi = 1$):

$$RHS = \theta\lambda Y mcP \left(1 + \widehat{\lambda}_t + \widehat{Y}_t + \widehat{mc}_t + \widehat{P}_t\right) + \theta\beta\xi\lambda Y mcP \left(1 + \widehat{\lambda}_{t+1} + \widehat{Y}_{t+1} + \widehat{mc}_{t+1} + \widehat{P}_{t+1}\right) + ...$$

Now use the steady state condition that $mc = \frac{\epsilon^p - 1}{\epsilon^p} = \frac{1}{\theta}$, and recollect the sum:

$$RHS = \lambda Y P E_t \left[\sum_{s=0}^{\infty} (\beta\xi)^s \left(\widehat{\lambda}_{t+s} + \widehat{Y}_{t+s} + \widehat{mc}_{t+s} + \widehat{P}_{t+s} \right) \right]. \qquad (\Delta\Delta)$$

Now we are ready to combine the LHS and the RHS of the original equation (##). First, use that $P = P^n$ to cancel out terms:

$$\frac{1}{1-\beta\xi} \widehat{P_t^n} + E_t \sum_{s=0}^{\infty} (\beta\xi)^s \widehat{\lambda}_{t+s} + \widehat{Y}_{t+s} = E_t \left[\sum_{s=0}^{\infty} (\beta\xi)^s \left(\widehat{\lambda}_{t+s} + \widehat{Y}_{t+s} + \widehat{mc}_{t+s} + \widehat{P}_{t+s} \right) \right].$$

This immediately collapses to:

$$\widehat{P_t^n} = (1 - \beta\xi) E_t \left[\sum_{s=0}^{\infty} (\beta\xi)^s \left(\widehat{mc}_{t+s} + \widehat{P}_{t+s} \right) \right].$$

The next step is to rewrite this condition as a first-order difference equation. This gives:

$$\begin{aligned} \widehat{P_t^n} &= (1 - \beta\xi) \left(\widehat{mc}_t + \widehat{P}_t \right) + (1 - \beta\xi) \beta\xi E_t \left(\widehat{mc}_{t+1} + \widehat{P}_{t+1} \right) \\ &= (1 - \beta\xi) \left(\widehat{mc}_t + \widehat{P}_t \right) + \beta\xi E_t \widehat{P_{t+1}^n}. \end{aligned}$$

Leading eq. (#) by one period, and isolating for $\widehat{P_{t+1}^n}$, we can substitute in the resulting expression:

$$\widehat{P_t^n} = (1 - \beta\xi) \left(\widehat{mc}_t + \widehat{P}_t \right) + \beta\xi E_t \left(\frac{\widehat{P}_{t+1} - \xi \widehat{P}_t}{(1 - \xi)} \right).$$

The final step is then to insert this expression for $\widehat{P_t^n}$ into the log-linearized price

level equation (#), which yields:

$$\widehat{P}_t = (1-\xi)(1-\beta\xi)\left(\widehat{mc}_t + \widehat{P}_t\right) + (1-\xi)\beta\xi E_t\left(\frac{\widehat{P}_{t+1} - \xi\widehat{P}_t}{(1-\xi)}\right) + \xi\widehat{P}_{t-1}.$$

Rewrite this:

$$\widehat{P}_t - \xi\widehat{P}_{t-1} = (1-\xi)(1-\beta\xi)\left(\widehat{mc}_t + \widehat{P}_t\right) + \beta\xi E_t\widehat{P}_{t+1} - \beta\xi^2\widehat{P}_t \qquad (\Leftrightarrow)$$

$$\xi\left(\widehat{P}_t - \widehat{P}_{t-1}\right) = -(1-\xi)\widehat{P}_t + (1-\xi)(1-\beta\xi)\widehat{mc}_t + \beta\xi E_t\widehat{P}_{t+1} + (1-\beta\xi-\xi)\widehat{P}_t \qquad (\Leftrightarrow)$$

$$\xi\left(\widehat{P}_t - \widehat{P}_{t-1}\right) = (1-\xi)(1-\beta\xi)\widehat{mc}_t + \beta\xi E_t\left(\widehat{P}_{t+1} - \widehat{P}_t\right).$$

Using the fact that $\left(\widehat{P}_t - \widehat{P}_{t-1}\right) = \widehat{\pi}_t$, this can then be rewritten:

$$\widehat{\pi}_t = \beta E_t\widehat{\pi}_{t+1} + \frac{(1-\xi)(1-\beta\xi)}{\xi}\widehat{mc}_t. \qquad (42)$$

This is the log-linearized New-Keynesian Phillips Curve that enters the set of log-linearized equations used to solve the model.

1.7 Summarizing the Linearized Equilibrium Conditions

The log-linearized version of the model consists of equations (29) – (42).[1] Note that the monetary policy condition (40) is not linear, as the value of the parameter ϕ_q depends on the sign of $\Delta\widehat{q}_t$. However, the model is piecewise linear, in the sense that given one of the two possible values of ϕ_q, all equations are linear. This is the key insight underlying the solution method. We can represent each of the two linear systems in the following way, stacking the 14 equations and 14 variables:

$$0 = AE_t s_{t+1} + Bs_t + Cs_{t-1} + D\varepsilon_t. \qquad (43)$$

Here, the vector s contains all the relevant variables, as measured in log-deviations from their steady state values: $s_t = \left[\widehat{K}_t, \widehat{n}_t, \widehat{q}_t, \widehat{R}_t, \widehat{C}_t, \widehat{H}_t, \widehat{\lambda}_t, \widehat{f}_t, \widehat{Y}_t, \widehat{I}_t, \widehat{w}_t, \widehat{\pi}_t, \widehat{mp}_t, \widehat{mc}_t\right]'$. The matrices A, B, and C are $N \times N$ coefficient matrices, where $N = 14$ is the

[1] While the model originally consisted of 15 equations in 15 variables, the log-linearized model has only 14 equations in 14 variables, as equations (12) and (13) were combined to yield one log-linearized equation; (42), making the variable $\left(\frac{P_t^n}{P_t}\right)$ redundant.

number of variables. Finally, $\varepsilon_t = [\varepsilon_t^a, \varepsilon_t^r]'$ is the vector of shocks, and D is a $N \times M$ coefficient matrix, with $M = 2$ representing the number of shocks. The elements of the coefficient matrices derive from the log-linear system of equations derived above.

Each of the two systems summarized as in (43) can then be solved using standard methods for solving linear rational expectations models. These methods include the Toolkit of Uhlig (1999) and the Gensys method of Sims (2002), but many other methods exist. We use Uhlig's method to solve each of the systems. This gives a solution that can be written on the form:

$$s_t = Ps_{t-1} + Q\varepsilon_t \tag{44}$$

This illustrates that at any point in time, the set of values of the endogenous variables is fully described by the set of lagged values (in particular, the lagged values of the state variables) and the realization of the shocks in that period. This explains the appeal of the state space representation. See the section concerning the solution method for details about how to solve the overall model, given the solution to each of the two linear systems that it consists of.

Finally, following Uhlig (1999), we can establish a link between the exposition of each of the two linear systems above and that in Blanchard and Kahn (1980). To do this, we need to reformulate the second-order difference equation (43) as a first-order difference equation. The method employed by Blanchard and Kahn implicitly constructs the stacked vector $x_t' = \left[s_t', s_{t-1}'\right]$, and then proceeds by analyzing the first-order difference equation:

$$E_t x_{t+1} = \Omega x_t + \Xi \varepsilon_t, \tag{45}$$

where the matrices Ω and Ξ are mappings of the matrices A, B, C, and D:

$$\underset{2N\times 2N}{\Omega} = \begin{bmatrix} \underset{N\times N}{-A^{-1}B} & \underset{N\times N}{-A^{-1}C} \\ \underset{N\times N}{0} & \underset{N\times N}{0} \end{bmatrix}, \quad \underset{2N\times N}{\Xi} = \begin{bmatrix} \underset{N\times N}{-A^{-1}D} \\ \underset{N\times N}{0} \end{bmatrix}$$

Uhlig (1999) demonstrates the equivalence between solving model (43) and (45), and further discusses pro's and con's of each of the two formulations.

1.8 Equilibrium Determinacy

According to Proposition 1 in Blanchard and Kahn (1980), there exists a unique solution to the problem, and hence a determinate equilibrium, if and only if the number of non-predetermined variables in the model exactly corresponds to the number of eigenvalues of the matrix Ω that lie outside the unit circle. Hence, examining the determinacy properties of each of the two linear systems that our model comprises is straightforward. However, as the model itself is non-linear, it cannot be represented on the form (45). As mentioned in the main text, we have verified that each of the two systems satisfy the conditions for equilibrium determinacy. It then follows that the model as such also does not suffer from indeterminacy problems. For example, it is easy to show that in a model where the economy switches between two regimes, and the monetary policy reaction to inflation is strictly larger than 1 in both regimes, the 'long run Taylor principle' of Davig and Leeper (2007), which is a necessary and sufficient condition for equilibrium determinacy, is always satisfied. Davig and Leeper (2007) show that the 'long-run Taylor principle' can be written as:

$$(1-\alpha_2)\,p_{11} + (1-\alpha_1)\,p_{22} > 1 - \alpha_1\alpha_2,$$

where α_1 and α_2 are the monetary policy reaction parameters on inflation in each regime, and $p_{11} \in (0,1)$ and $p_{22} \in (0,1)$ are the probabilities of remaining in regime 1 or 2 each period, which in our setup are governed by the sign of Δq_t. As mentioned, we assume $\alpha_1, \alpha_2 > 1$. In that case, the left-hand side of the inequality reaches its lowest possible value for $p_{11} = p_{22} = 1$. Hence, if the inequality holds in this case, it always holds. With $p_{11} = p_{22} = 1$, we get:

$$\begin{aligned} (1-\alpha_2) + (1-\alpha_1) &> 1 - \alpha_1\alpha_2 \Leftrightarrow \\ 1 - \alpha_1 - \alpha_2 &> -\alpha_1\alpha_2. \end{aligned}$$

We have assumed $\alpha_1, \alpha_2 > 1$, so the lowest possible value of α_1, α_2 is $1+\varepsilon$, where ε is positive but very small. In that case, we obtain:

$$\begin{aligned} 1-\alpha_1-\alpha_2 &> -\alpha_1\alpha_2 \Leftrightarrow \\ 1-(1+\varepsilon)-(1+\varepsilon) &> -(1+\varepsilon)^2 \Leftrightarrow \\ -1-2\varepsilon &> -1-2\varepsilon-\varepsilon^2 \Leftrightarrow \\ -\varepsilon^2 &< 0, \end{aligned}$$

which is indeed true for any $\varepsilon > 0$.

2 The Solution Method

Below, we present the details of the solution method used to solve the model outlined above. The method exploits the fact that while the model is not linear, it is piecewise linear; consisting of two linear systems. A number of authors have used solution methods that rely on piecewise linearity, see for instance Eggertson and Woodford (2003) or Christiano (2004). As the solution method we use follows the work of Bodenstein *et al.* (2009), this section builds on their Appendix A.

Assuming that the model starts out in steady state, the initial regime for monetary policy involves a zero reaction to stock price changes. As discussed above, the log-linearized conditions describing the equilibrium can be written on matrix form.

$$0 = \overline{A}E_t s_{t+1} + \overline{B}s_t + \overline{C}s_{t-1} + \overline{D}\varepsilon_t. \tag{46}$$

In this system, s is the vector containing all the endogenous variables as described above, $\overline{A}$, $\overline{B}$, $\overline{C}$ and $\overline{D}$ are coefficient matrices describing the dynamics of the system, and ε is the vector of shocks. Similarly, whenever the asset price is decreasing, the dynamics of the system is described by the following set of equations, including a non-zero reaction to asset price changes:

$$0 = A^* E_t s_{t+1} + B^* s_t + C^* s_{t-1} + D^* \varepsilon_t. \tag{47}$$

Note, however, that the only difference between the two systems is the reaction of monetary policy to asset price changes; i.e., whether $\phi_q = 0$ in equation (40) or not. This affects only the matrices multiplying s_t and s_{t-1}. In other words,

$\overline{A} = A^*$, and $\overline{D} = D^*$. Further, the matrices $\overline{B}$ and B^* differ in only one entry, and the same is true for $\overline{C}$ and C^*: If the monetary policy reaction function is listed as the n'th equation in the system, and the price of capital appears as the m'th variable in the vector s, then these matrices differ only in the (n,m)'th entry.

As each of these two systems are linear, they can be solved separately using well-known methods such as the Toolkit method of Uhlig (1999) or the Gensys method of Sims (2002). The solutions can then also be written on matrix form, as the evolution of the endogenous variables are fully described by the lagged values of the state variables and the realizations of the shocks. Hence, the solutions to the above systems are, respectively:

$$s_t = \overline{P} s_{t-1} + \overline{Q} \varepsilon_t, \tag{48}$$

$$s_t = P^* s_{t-1} + Q^* \varepsilon_t. \tag{49}$$

Assume that a shock hits the economy in period 0. As the economy starts out in the regime with no reaction to stock price changes, the first regime change will occur the first time the change in the asset price ($\Delta q_t = q_t - q_{t-1}$) becomes negative. Depending on the shock, this may happen on impact or after a number of periods.[2] Once the regime has shifted, it may shift back, or it may remain in the new regime.[3] In principle, an arbitrary number of regime shifts might take place, depending on the evolution of the asset price.

In order to illustrate the idea behind the solution method, consider the evolution of the asset price following a positive technology shock. This impulse response is repeated here for convenience:[4]

[2] Unless the asset price remains forever constant, however, it will happen sooner or later, as the asset price must return to its initial value.

[3] Of course, the economy will eventually return to its steady state, where the regime is always that of a zero reaction to stock price changes.

[4] The figure shows the impulse response of the asset price in the model without asymmetric policy. We first assume that the turning points under this policy are unchanged when the asymmetric policy is introduced. We then later verify that this is in fact the case.

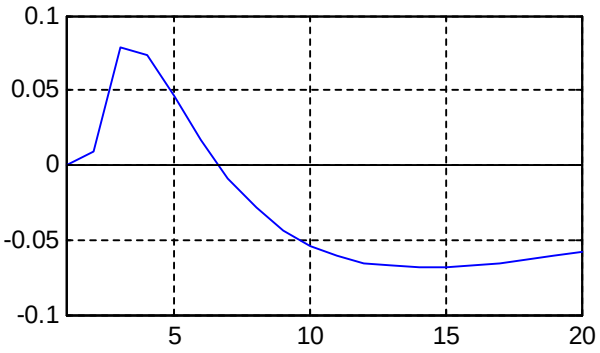

Figure A1: Impulse response of asset price to positive technology shock

Evidently, this impulse response involves two turning points; called T_1 and T_2, i.e. points where the sign of the change in the asset price switches. After the second turning point, the stock price is increasing, so the dynamics of the economy are described by the solution to the model with no reaction to asset prices (and no further shocks):

$$s_t = \overline{P} s_{t-1}, \ t > T_2. \tag{50}$$

Consider now the dynamics for $T_1 < t \leq T_2$, for which the monetary policy reaction to asset prices is non-zero. We use backward induction to trace out the evolution of the endogenous variables in these periods. As no shocks are assumed to hit the economy outside period 0, it follows from (50) that $s_{T_2+1} = \overline{P} s_{T_2}$. This is useful in the last period before the shift ($t = T_2$), where the following is true:

$$0 = \overline{A} E_t s_{T_2+1} + B^* s_{T_2} + C^* s_{T_2-1} \quad \Leftrightarrow$$

$$0 = \left(\overline{AP} + B^*\right) s_{T_2} + C^* s_{T_2-1} \quad \Leftrightarrow$$

$$s_{T_2} = -\left(\overline{AP} + B^*\right)^{-1} C^* s_{T_2-1} \quad \Leftrightarrow$$

$$s_{T_2} = \Gamma_1 s_{T_2-1} \ , \ \Gamma_1 \equiv -\left(\overline{AP} + B^*\right)^{-1} C^*. \tag{51}$$

In similar fashion, we can derive an expression for the second-last period before the shift ($t = T_2 - 1$). Let $A = -\left(B^*\right)^{-1} \overline{A}$, and $C = -\left(B^*\right)^{-1} C^*$. Then;

$$0 = \overline{A} E_t s_{T_2} + B^* s_{T_2-1} + C^* s_{T_2-2} \quad \Leftrightarrow$$

$$s_{T_2-1} = A\Gamma_1 s_{T_2-1} + C s_{T_2-2} \quad \Leftrightarrow$$

$$s_{T_2-1} = (I - A\Gamma_1)^{-1} C s_{T_2-2}. \tag{52}$$

Thus, by recursive substitutions, we can express the endogenous variables at any point in this interval as a function of their 1-period lagged values. In the general case, we get:

$$s_t = (I - A\Gamma_{T_2-t})^{-1} C s_{t-1} \qquad \Leftrightarrow$$

$$s_t = \Gamma_{T_2-t+1} s_{t-1} \ , \ T_1 < t \leq T_2, \tag{53}$$

where, for each t;

$$\Gamma_{T_2-t+1} = (I - A\Gamma_{T_2-t})^{-1} C,$$

recalling the definition of $\Gamma_1 \equiv \left(\overline{AP} + B^*\right)^{-1} C^*$. In fact, the recursivity of the problem allows us to write s_t for each period in this interval as a function of s_{T_1+1}; the first period in this interval:

$$s_t = \left(\prod_{i=1}^{t-1} \Gamma_{T_2-i} \right) s_{T_1+1}. \tag{54}$$

In period $T_1 + 1$, the values of the endogenous variables are 'inherited' from the dynamics in the previous interval. For $t \leq T_1$, when the policy reaction to asset prices is again zero, we can similarly compute the value of s_t in each period recursively as a function of s_1. From (53), we get the following expression, which is needed to describe the last period before this first shift:

$$s_{T_1+1} = \Gamma_{T_2-T_1} s_{T_1}. \tag{55}$$

Performing recursive operations in a similar fashion to above provides us with the following expression for s_t:

$$s_t = \left(I - \widehat{A}\Theta_{T_1-t}\right)^{-1} \widehat{C} s_{t-1} \qquad \Leftrightarrow$$

$$s_t = \Theta_{T_1-t+1} s_{t-1} \ , \ 2 \leq t \leq T_1, \tag{56}$$

where, for each t;

$$\Theta_{T_1-t+1} = \left(I - \widehat{A}\Theta_{T_1-t}\right)^{-1} \widehat{C},$$

and where $\widehat{A} = -\left(\overline{B}\right)^{-1} \overline{A}$; $\widehat{C} = -\left(\overline{B}\right)^{-1} \overline{C}$; and

$$\Theta_1 \equiv -\left(\overline{A}\Gamma_{T_2-T_1} + \overline{B}\right)^{-1} \overline{C}.$$

Finally, the special case where $t = 1$ is the only time at which the shocks take on non-zero values. We use (46) and (56) as well as the assumption that the economy starts out in steady state in period 0, implying that $s_0 = 0$. We then obtain an expression for s_1 as a function of the time 1-innovations:

$$0 = \overline{A}s_2 + \overline{B}s_1 + \overline{C}s_0 + \overline{D}\varepsilon_1 \qquad \Leftrightarrow$$

$$s_1 = \left(I - \widehat{A}\Theta_{T_1-1}\right)^{-1} \widehat{D}\varepsilon_1, \tag{57}$$

where $\widehat{D} = -\left(\overline{B}\right)^{-1}\overline{D}$. Finally, we then obtain:

$$s_t = \left(\prod_{i=1}^{t-1} \Theta_{T_1-i}\right) s_1 \qquad \Leftrightarrow$$

$$s_t = \left(\prod_{i=1}^{t-1} \Theta_{T_1-i}\right) \left(I - \widehat{A}\Theta_{T_1-1}\right)^{-1} \widehat{D}\varepsilon_1 \ , \ 2 \leq t \leq T_1. \tag{58}$$

As mentioned in the main text, the model is solved in practice by making use of a shooting algorithm to find the turning points. An initial guess for each of the turning points is needed. Given the initial guess, we solve for s_t, $\forall t$. It is then easy to verify whether this initial guess was correct or not by simply checking whether the sign of Δq_t actually does shift for $t = T_{initial\ guess}$. If this is the case, we keep the solution. If not, we adjust our initial guess, and we 'shoot' again, until the condition is satisfied.

3 Additional Figures

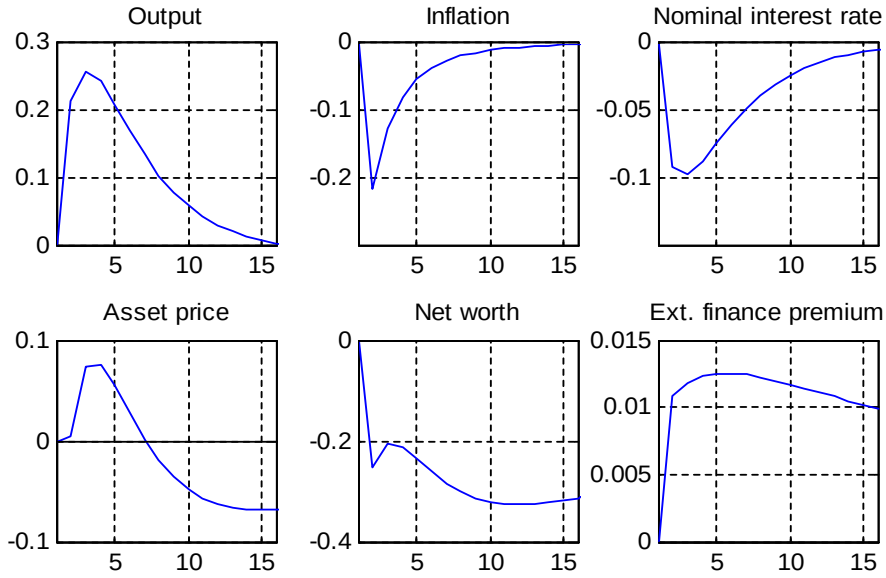

Figure A2: Positive technology shock, symmetric policy reaction to asset price changes $(\phi_q = 0.5)$

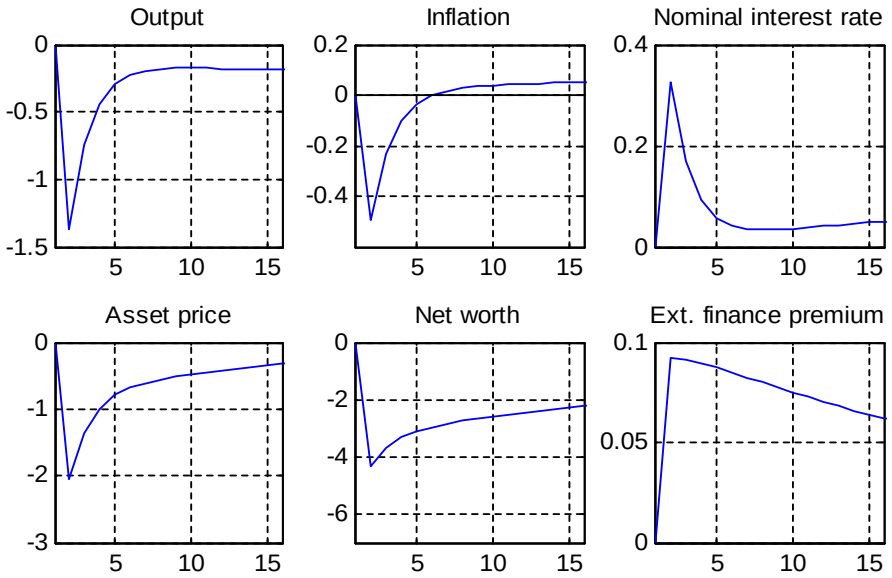

Figure A3: Contractionary monetary policy shock, symmetric reaction to asset price changes $(\phi_q = 0.5)$

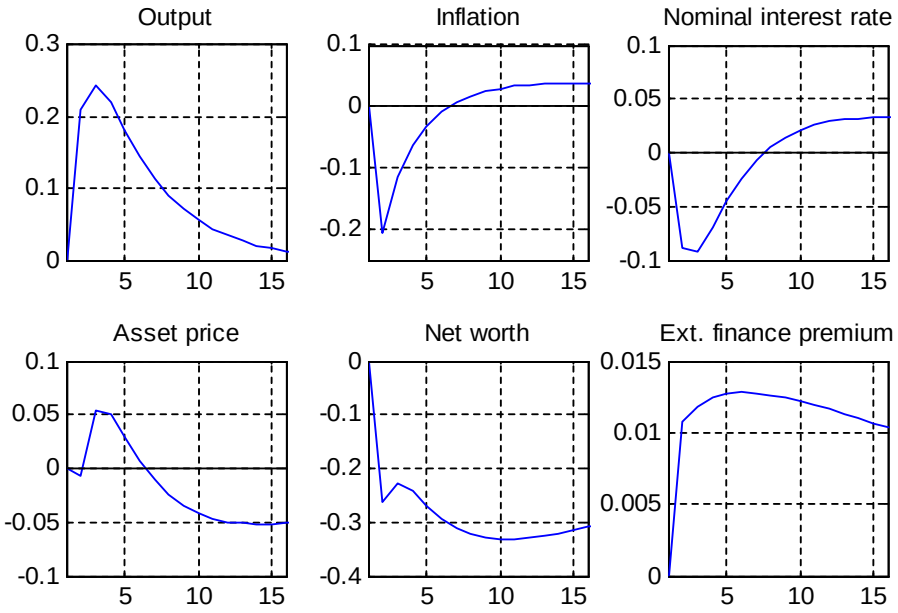

Figure A4: Positive technology shock, symmetric reaction to asset price deviation $\left(\phi_q = 0.5\right)$

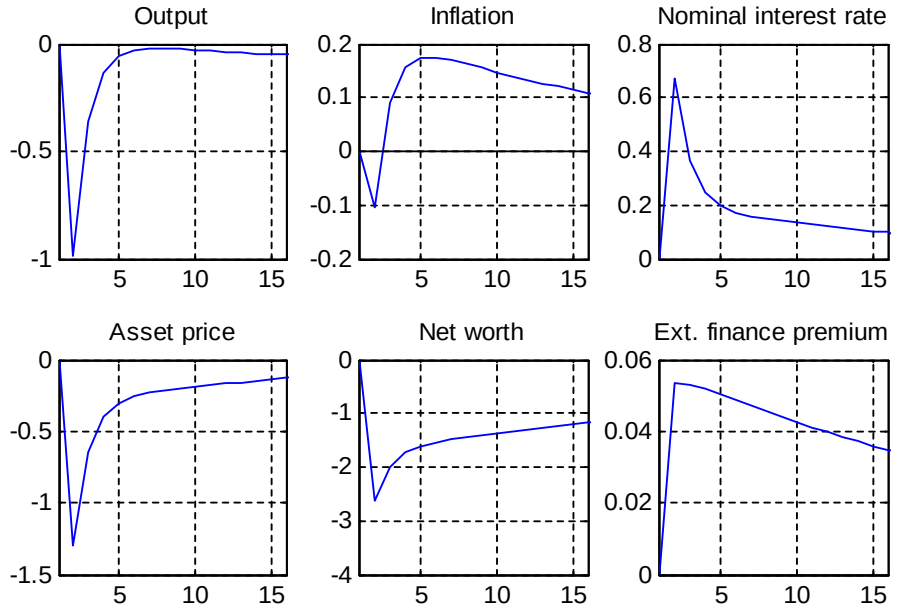

Figure A5: Contractionary monetary policy shock, symmetric reaction to asset price deviation $\left(\phi_q = 0.5\right)$

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Appendix to "Deep Habits, Endogenous Credit Standards, and Aggregate Fluctuations"

Abstract

This appendix contains supplemental material for Chapter 3; "Deep Habits, Endogenous Credit Standards, and Aggregate Fluctuations".

1 Parametrization

Table A1: Parameter values

Parameter	Interpretation	Value
Preference parameters		
β^P	Discount factor, households	0.995
β^E	Discount factor, entrepreneur	0.95
γ^P	Habits in consumption, households	0.72
γ^E	Habits in consumption, entrepreneurs	0.72
φ	Frisch elasticity of labor supply	1.5
ι	Labor preference parameter	10
Technology parameters		
α	Non-labor share of production	0.3
ϕ	Land share of non-labor input	0.15
Ω	Investment adjustment cost parameter	4
δ	Capital depreciation rate	0.02
ψ	Interest rate spread, gross	1.0168
τ	Recovery rate of assets in liquidation	0.7408
Parameters related to deep habits		
γ^L	Deep habit formation	0.72
ρ_s	Persistence of stock of deep habits	0.85
ξ	Elasticity of substitution between banks	190
ϖ	Elasticity of credit risk	-50
Steady states of shocks		
ς	Housing demand shock	0.04
A	Technology shock	10
ϑ^Q	Investment-specific technology shock	1

Table A1, continued

Persistence and volatility of shocks		
ρ_A	Persistence of technology shock	0.97
σ_A	Std. deviation of technology shock	0.001
ρ_Q	Persistence of inv.-spec. tech. shock	0.9
σ_Q	Std. deviation of inv.-spec. tech. shock	0.001
ρ_H	Persistence of housing demand shock	0.97
σ_H	Std. deviation of housing demand shock	0.001

2 Verifying that the Collateral Constraint Always Binds

As discussed in the main text, we need to check that the collateral constraint of the entrepreneur is binding not only in the steady state, but also outside it, where in principle, the constraint may be 'occasionally non-binding'. To this end, we adopt the method of Holden and Paetz (2012). This section therefore builds on their paper.

The collateral constraint implies an upper bound on the borrowing of entrepreneurs. Observe that we can reformulate the collateral constraint in terms of a restriction on the entrepreneur's shadow value of borrowing; μ_t^E. We know that $\mu_t^E > 0$ if and only if the optimal debt level of the entrepreneur is at or above his credit limit, i.e. if and only if the collateral constraint is binding. In other words, we simply need to verify that $\mu_t^E \geq 0, \forall t$. If this restriction is satisfied with inequality, the collateral constraint is in fact binding, so that we may treat it as an equality. If it holds with equality, we may of course treat it as such, even though the constraint is non-binding. If instead the constraint is violated, the entrepreneur's optimal level of debt is lower than the credit limit, so that treating his collateral constraint as an equality implies that we are forcing him to borrow 'too much'. We want to make sure we that we are avoiding the latter scenario.

To check this, we add a set of 'shadow price shocks' to the model. In the case where $\mu_t^E < 0$, the role of these is to 'push' μ_t^E back up until it exactly equals its lower limit of zero. As will become clear below, if the condition $\mu_t^E \geq 0$ is

satisfied, the shadow price shocks have no role to play.

We first describe how to compute impulse responses to, say, a technology shock. The first step is to add a set of shadow price shocks to the log-linearized collateral constraint of the entrepreneur. In this step, we need to determine the number of periods T in which we suspect that the collateral constraint may be non-binding. This number may be smaller than or equal to the number of periods for which we compute impulse responses; $T \leq T^{IRF}$. For each period $t \leq T$, we then add shadow price shocks which hit the economy in period t but become known at period 0, that is, at the same time the economy is hit by a given shock. In other words, the log-linearized collateral constraint becomes:

$$\widehat{l}_t = \widehat{\theta}_t - \widehat{R}_t^L + \frac{Q^H H^E}{Q^H H^E + Q^K K} \mathrm{E}_t \left(\widehat{Q}_{t+1}^H + \widehat{H}_t^E \right) + \frac{Q^K K}{Q^H H^E + Q^K K} \mathrm{E}_t \left(\widehat{Q}_{t+1}^K + \widehat{K}_t \right) - \sum_{s=0}^{T-1} \varepsilon_{s,t-s}^{SP},$$

where $\varepsilon_{s,t-s}^{SP}$ is the shadow price shock that hits in period $t = s$, but is anticipated (by all agents) in period $t = t - s = 0$.

We let all shadow price shocks be of unit magnitude. We then need to compute a set of weights α_{μ^E} to control the impact of each shock on μ_t^E. The 'optimal' set of weights ensures that μ_t^E is bounded below at exactly zero. The optimal set of weights is computed by solving the following quadratic programming problem:

$$\begin{aligned} \alpha^* &\equiv \left[\alpha_{\mu^E}^{*\prime} \right]' \\ &= \arg\min \left[\alpha_{\mu^E}' \right] \left[\left[\mu^E + \widetilde{\mu}^{E,A} \right] + \left[\widetilde{\mu}^{E,\varepsilon^{SP}} \right] \left[\alpha_{\mu^E} \right] \right], \end{aligned}$$

subject to

$$\alpha_{\mu^E}' \geq 0, \tag{a}$$

$$\mu^E + \widetilde{\mu}^{E,A} + \widetilde{\mu}^{E,\varepsilon^{SP}} \alpha_{\mu^E} \geq 0, \tag{b}$$

Here, μ^E and $\widetilde{\mu}^{E,A}$ denote, respectively, the steady state value and the unrestricted relative impulse response of μ^E to a technology shock, that is, the impulse-response of μ^E when the collateral constraint is assumed to always bind. In this respect, the vector $\left[\mu^E + \widetilde{\mu}^{E,A} \right]$ contains the absolute, unrestricted im-

pulse response of the shadow value. Further, the matrix $\widetilde{\mu}^{E,\varepsilon^{SP}}$ contains the relative impulse response of μ^E to the shadow price shocks, in the sense that column s in $\widetilde{\mu}^{E,\varepsilon^{SP}}$ represents the response of μ^E to a shock $\varepsilon^{SP}_{s,t-s}$, i.e. to a shadow price shock that hits in period s but is anticipated at time 0, as described above.[1]

We can explain the nature of the optimization problem as follows. First, note that $\mu^E + \widetilde{\mu}^{E,A} + \widetilde{\mu}^{E,\varepsilon^{SP}} \alpha_{\mu^E}$ denotes the combined response of μ^E_t to a given shock (here, a technology shock) *and* a simultaneous announcement of a set of future shadow price shocks for a given set of weights. Given the constraints of the problem, the objective is to find a set of optimal weights so that the impact of the (non-negative) shadow price shocks is exactly large enough to make sure that the response of μ^E_t is never negative. The minimization ensures that the impact of the shadow price shocks will never be larger than necessary to obtain this. Finally, we only allow for solutions for which the value of the objective function is zero. This ensures that at any given horizon, a positive shadow price shock occurs if and only if μ^E_t is at its lower bound of zero in that period. As pointed out by Holden and Paetz (2012), this can be thought of as a complementary slackness condition on (a) and (b). Once we have solved the minimization problem, it is straightforward to compute the bounded impulse responses of all endogenous variables by simply adding the optimally weighted shadow price shocks to the unconstrained impulse responses of the model in each period.

We rely on the same method to compute dynamic simulations. For each period t, we first generate the shock hitting the economy. We then compute the unrestricted path of the endogenous variables given that shock and given the simulated values in $t-1$. The path of μ^E_t then takes the place of the impulse response in the optimization problem. If the unrestricted path of μ^E_t never hits the bound in future periods, our simulation for period t is fine. If the bound is hit, we follow the method above and add anticipated shadow price shocks for a sufficient number of future periods. We then compute restricted values for all endogenous variables, and use these as our simulation for period t. Note that, unlike the case for impulse responses, in our dynamic simulations not all anticipated future shadow price shocks will eventually hit the economy, as other shocks may occur before the realization of the expected shadow price shocks and

[1] The matrix $\widetilde{\mu}^{E,\varepsilon^{SP}}$ needs to be a square matrix, so if the number of periods in which we suspect the constraint may be non-binding is smaller than the number of periods for which we compute impulse responses, $T < T^{IRF}$, we use only the first T rows of the matrix, i.e. the upper square matrix.

push μ_t^E away from its lower bound.

3 Market Clearing

As described in the main text, we need to add two types of lump-sum transfers to the model to make sure all markets clear. Below, we demonstrate that these transfers are exactly sufficient to clear all markets, and we show how to derive the expression for $\Psi_t(j)$ in the main text. We start by adding together the budget constraints of households and entrepreneurs, where we have summed over all households and entrepreneurs, respectively:

$$\begin{aligned}
&\int_0^1 \left(C_t^P(i) + Q_t^H \left[H_t^P(i) - H_{t-1}^P(i) \right] + \int_0^1 D_{ik,t} dk \right) di \\
&+ \int_0^1 \left(C_t^E(j) + R_{t-1}^L \int_0^1 l_{jk,t-1} dk \right) dj \\
= &\int_0^1 \left(W_t N_t(i) + \int_0^1 \Pi_t^k(i)\, dk + R_{t-1}^D \int_0^1 D_{ik,t-1} dk \right) di \\
&+ \int_0^1 \left(Y_t(j) - W_t N_t(j) - \vartheta_t^Q I_t(j) - Q_t^H \left[H_t^E(j) - H_{t-1}^E(j) \right] + x_t(j) + \Phi_t(j) + \Psi_t(j) \right) dj
\end{aligned}$$

$$\begin{aligned}
\Leftrightarrow\ & C_t^P + Q_t^H \left[H_t^P - H_{t-1}^P \right] + \int_0^1 \int_0^1 D_{ik,t} di dk + C_t^E + R_{t-1}^L \int_0^1 \int_0^1 l_{jk,t-1} dk dj \\
= &\ W_t N_t + \int_0^1 \int_0^1 \Pi_t^k(i)\, dk di + R_{t-1}^D \int_0^1 \int_0^1 D_{ik,t-1} dk di \\
&+ Y_t - W_t N_t - \vartheta_t^Q I_t - Q_t^H \left[H_t^E - H_{t-1}^E \right] + \int_0^1 x_t(j)\, dj + \int_0^1 \Phi_t(j)\, dj + \int_0^1 \Psi_t dj
\end{aligned}$$

$$\begin{aligned}
\Leftrightarrow\ & C_t^P + C_t^E + \vartheta_t^Q I_t - Y_t + Q_t^H \left[\left(H - H_t^E \right) - \left(H - H_{t-1}^E \right) \right] + Q_t^H \left[H_t^E - H_{t-1}^E \right] \\
&+ \int_0^1 \int_0^1 D_{ik,t} di dk + R_{t-1}^L \int_0^1 \int_0^1 l_{jk,t-1} dk dj \\
= &\int_0^1 \int_0^1 \Pi_t^k(i)\, dk di + R_{t-1}^D \int_0^1 \int_0^1 D_{ik,t-1} dk di + \int_0^1 x_t(j)\, dj + \int_0^1 \Phi_t(j)\, dj + \int_0^1 \Psi_t(j)\, dj.
\end{aligned}$$

Here, we use the resource constraints to cancel out terms. Moreover, we insert

for $x_t(j)$, $\Phi_t(j)$, and Π_t^k from the main text:

$$\begin{aligned}\int_0^1\int_0^1 D_{ik,t}didk &= R_{t-1}^D\int_0^1\int_0^1 D_{ik,t-1}dkdi - R_{t-1}^L\int_0^1\int_0^1 l_{jk,t-1}dkdj \\ &+\int_0^1\left[\int_0^1\left(l_{jk,t}-\gamma^L s_{jk,t-1}\right)^{\frac{\xi-1}{\xi}}dk\right]^{\frac{\xi}{\xi-1}}dj+\gamma^L\int_0^1\int_0^1\frac{\theta_{kt}}{\theta_t}s_{jk,t-1}dkdj \\ &+\int_0^1\int_0^1\left(p_{kt-1}R_{t-1}^L L_{kt-1}+(1-p_{kt-1})\frac{L_{kt-1}}{\int_0^1 L_{kt-1}dk}\tau a_{t-1}-L_{k,t}\right)dkdi \\ &+\int_0^1\int_0^1\left(\int_0^1 D_{ik,t}di-R_{t-1}^D\int_0^1 D_{ik,t-1}di\right)dkdi+\int_0^1\Psi_t(j)\,dj\end{aligned}$$

Now, let $\xi\longrightarrow\infty$, and we obtain:

$$\begin{aligned}\int_0^1\int_0^1 D_{ik,t}didk &= R_{t-1}^D\int_0^1\int_0^1 D_{ik,t-1}dkdi - R_{t-1}^L\int_0^1\int_0^1 l_{jk,t-1}dkdj \\ &+\int_0^1\int_0^1\left(l_{jk,t}-\gamma^L s_{jk,t-1}\right)dkdj+\gamma^L\int_0^1\int_0^1\frac{\theta_{kt}}{\theta_t}s_{jk,t-1}dkdj \\ &-R_{t-1}^D\int_0^1\int_0^1 D_{ik,t-1}didk+\int_0^1\Psi_t(j)\,dj \\ &+\int_0^1\int_0^1\left(p_{kt-1}R_{t-1}^L L_{kt-1}+(1-p_{kt-1})\frac{L_{kt-1}}{\int_0^1 L_{kt-1}dk}\tau a_{t-1}\right)dkdi\end{aligned}$$

$$\begin{aligned}\Leftrightarrow\quad &\int_0^1\int_0^1 D_{ik,t}didk = -R_{t-1}^L\int_0^1\int_0^1 l_{jk,t-1}dkdj \\ &+\int_0^1\int_0^1\left(l_{jk,t}-\gamma^L s_{jk,t-1}+\gamma^L\frac{\theta_{kt}}{\theta_t}s_{jk,t-1}\right)dkdj+\int_0^1(1-p_{kt-1})\frac{L_{kt-1}}{\int_0^1 L_{kt-1}dk}\tau a_{t-1}dk \\ &+\int_0^1\Psi_t(j)\,dj+\int_0^1 p_{kt-1}R_{t-1}^L L_{kt-1}dk\end{aligned}$$

$$\begin{aligned}\Leftrightarrow\quad &\int_0^1\int_0^1 D_{ik,t}didk = -R_{t-1}^L\int_0^1 L_{kt-1}dk+\int_0^1\int_0^1 l_{jk,t}dkdj+\int_0^1\Psi_t(j)\,dj+ \\ &\int_0^1(1-p_{kt-1})\tau a_{t-1}dk+R_{t-1}^L\int_0^1 p_{kt-1}L_{kt-1}dk\end{aligned}$$

$$\Leftrightarrow \int_0^1 \left(\int_0^1 D_{ik,t} di - L_{k,t} \right) dk = \int_0^1 \Psi_t(j)\, dj + \int_0^1 (1 - p_{kt-1})\, \tau a_{t-1} dk$$
$$- \int_0^1 (1 - p_{kt-1})\, R_{t-1}^L L_{k,t-1} dk$$

$$\Leftrightarrow \int_0^1 (1 - p_{kt-1})\, R_{t-1}^L L_{k,t-1} dk - \int_0^1 (1 - p_{kt-1})\, \tau a_{t-1} dk = \int_0^1 \Psi_t(j)\, dj$$

$$\int_0^1 \Psi_t(j)\, dj = \Psi_t(j) = \int_0^1 (1 - p_{tk-1}) \left(R_{t-1}^L L_{kt-1} - \tau a_{t-1} \right) dk,$$

which is the expression for Ψ_t presented in the main text. We have made use of Fubini's theorem to allow us to switch the order of integrals where necessary.

4 Derivations of first-order conditions

This section demonstrates how the first-order conditions of households, entrepreneurs, and banks are derived.

4.1 Households

The problem of each household is:

$$\underset{C_t^P(i), D_t(i), H_t^P(i), N_t^P(i)}{Max} E_0 \sum_{t=0}^{\infty} \left(\beta^P\right)^t \left[\log\left(C_t^P(i) - \gamma^P C_{t-1}^P(i) \right) - \iota \frac{\left[N_t(i)\right]^{1+\varphi}}{1+\varphi} + \varsigma_t \log H_t^P(i) \right]$$
$$-\lambda_t^P(i) \cdot$$
$$\left[C_t^P(i) + Q_t^H \left[H_t^P(i) - H_{t-1}^P(i) \right] + \int_0^1 D_{ik,t} dk - W_t N_t(i) - \int_0^1 \Pi_t^k(i)\, dk - R_{t-1}^D \int_0^1 D_{ik,t-1} dk \right]$$

The first-order conditions for $C_t^P(i), \int_0^1 D_{ik,t} dk, H_t^P(i), N_t^P(i)$ are, respectively:

$$\frac{1}{C_t^P(i) - \gamma^P C_{t-1}^P(i)} - \beta^P \mathrm{E}_t \frac{\gamma^P}{C_{t+1}^P(i) - \gamma^P C_t^P(i)} = \lambda_t^P(i), \qquad \text{(A.1)}$$

$$\beta^P E_t\left[\lambda_{t+1}^P(i)\right] = \frac{\lambda_t^P(i)}{R_t^D}, \tag{A.2}$$

$$\frac{\varsigma_t}{H_t^P(i)} + \beta^P E_t\left[\lambda_{t+1}^P(i) Q_{t+1}^H\right] = \lambda_t^P(i) Q_t^H, \tag{A.3}$$

$$\iota N_t^{\varphi}(i) = \lambda_t^P(i) W_t, \tag{A.4}$$

4.2 Entrepreneurs

Entrepreneur j's problem can be solved in two steps. First, given his total financing needs, x_t, he must choose the optimal composition of his loan portfolio from. This problem gives rise to the entrepreneur's demand function for loans from each individual bank. Second, the entrepreneur solves the dynamic problem of maximizing profits and, in turn, utility.

The first part of the problem consists of choosing the amount of borrowing from each individual bank, $l_{jk,t}$, so as to minimize the total amount of collateral the entrepreneur has to pledge, given his overall financing needs, and subject to the aggregate loan composition as well as the collateral constraint. Note that because all banks set the same interest rate on loans, the entrepreneur does not face a cost minimization problem. We can write the problem as:

$$\underset{l_{jk,t}}{Min} \frac{1}{R_t^L}\int_0^1 \theta_{kt} a_t dk - \chi_t\left[x_t(j) - \left(\int_0^1 \left(l_{jk,t} - \gamma^L s_{jk,t-1}\right)^{\frac{\xi-1}{\xi}} dk\right)^{\frac{\xi}{\xi-1}}\right] - \varrho_t\left[\int_0^1 \frac{l_{jk,t}}{\theta_{kt}} - \frac{a_t}{R_t^L}\right],$$

where we have rewritten the collateral constraint, and where χ_t and ϱ_t are Lagrange multipliers. The first-order condition of this problem is:

$$\chi_t \frac{\xi}{\xi-1}\left(\int_0^1 \left(l_{jk,t} - \gamma^L s_{jk,t-1}\right)^{\frac{\xi-1}{\xi}} dk\right)^{\frac{1}{\xi-1}} \frac{\xi-1}{\xi}\left(l_{jk,t} - \gamma^L s_{jk,t-1}\right)^{\frac{-1}{\xi}} = \varrho_t \frac{1}{\theta_{kt}}, \tag{$\triangle$}$$

which can be rewritten in the following ways:

$$\left(\int_0^1 \left(l_{jk,t}-\gamma^L s_{jk,t-1}\right)^{\frac{\xi-1}{\xi}} dk\right)^{\frac{1}{\xi-1}} \left(l_{jk,t}-\gamma^L s_{jk,t-1}\right)^{\frac{-1}{\xi}} = \frac{\varrho_t}{\chi_t \theta_{kt}} \Leftrightarrow$$

$$\left(\int_0^1 \left(l_{jk,t}-\gamma^L s_{jk,t-1}\right)^{\frac{\xi-1}{\xi}} dk\right)^{\frac{1}{\xi-1}} \left(l_{jk,t}-\gamma^L s_{jk,t-1}\right)^{\frac{\xi-1}{\xi}} = \frac{\varrho_t}{\chi_t \theta_{kt}} \left(l_{jk,t}-\gamma^L s_{jk,t-1}\right) \Leftrightarrow$$

$$\left(\int_0^1 \left(l_{jk,t}-\gamma^L s_{jk,t-1}\right)^{\frac{\xi-1}{\xi}} dk\right)^{\frac{1}{\xi-1}} \int_0^1 \left(l_{jk,t}-\gamma^L s_{jk,t-1}\right)^{\frac{\xi-1}{\xi}} dk = \int_0^1 \frac{\varrho_t}{\chi_t \theta_{kt}} \left(l_{jk,t}-\gamma^L s_{jk,t-1}\right) dk \Leftrightarrow$$

$$\underbrace{\left(\int_0^1 \left(l_{jk,t}-\gamma^L s_{jk,t-1}\right)^{\frac{\xi-1}{\xi}} dk\right)^{\frac{\xi}{\xi-1}}}_{=x_t(j)} = \frac{\varrho_t}{\chi_t} \int_0^1 \frac{1}{\theta_{kt}} \left(l_{jk,t}-\gamma^L s_{jk,t-1}\right) dk \Leftrightarrow$$

$$x_t(j) = \frac{\varrho_t}{\chi_t} \int_0^1 \frac{1}{\theta_{kt}} \left(l_{jk,t}-\gamma^L s_{jk,t-1}\right) dk. \qquad (\ddagger)$$

Now, define the aggregate LTV ratio in the economy as $\theta_t \equiv \left[\int_0^1 \theta_{kt}^{1-\xi}\right]^{\frac{1}{1-\xi}}$. Moreover, observe that at the optimum, the following condition must hold:

$$\frac{1}{\theta_t} x_t = \int_0^1 \frac{1}{\theta_{kt}} \left(l_{jk,t}-\gamma^L s_{jk,t-1}\right) dk. \qquad (*)$$

We can then rewrite ($\ddagger$) as:

$$x_t = \frac{\varrho_t}{\chi_t} \frac{1}{\theta_t} x_t \Leftrightarrow$$

$$\theta_t = \frac{\varrho_t}{\chi_t}.$$

Now, insert this in the original expression for the first-order condition ($\triangle$):

$$\chi_t \frac{\xi}{\xi-1} \left(\int_0^1 \left(l_{jk,t}-\gamma^L s_{jk,t-1}\right)^{\frac{\xi-1}{\xi}} dk\right)^{\frac{1}{\xi-1}} \frac{\xi-1}{\xi} \left(l_{jk,t}-\gamma^L s_{jk,t-1}\right)^{\frac{-1}{\xi}} = \varrho_t \frac{1}{\theta_{kt}} \Leftrightarrow$$

$$\underbrace{\left(\int_0^1 \left(l_{jk,t}-\gamma^L s_{jk,t-1}\right)^{\frac{\xi-1}{\xi}} dk\right)^{\frac{1}{\xi-1}}}_{=x_t^{\frac{1}{\xi}}} \left(l_{jk,t}-\gamma^L s_{jk,t-1}\right)^{\frac{-1}{\xi}} = \theta_t \frac{1}{\theta_{kt}} \Leftrightarrow$$

$$x_t^{\frac{1}{\xi}} \left(l_{jk,t}-\gamma^L s_{jk,t-1}\right)^{\frac{-1}{\xi}} = \frac{\theta_t}{\theta_{kt}} \Leftrightarrow$$

$$x_t^{\frac{1}{\xi}} \frac{\theta_{kt}}{\theta_t} = \left(l_{jk,t}-\gamma^L s_{jk,t-1}\right)^{\frac{1}{\xi}} \Leftrightarrow$$

$$l_{jk,t} = \left(\frac{\theta_{kt}}{\theta_t}\right)^{\xi} x_t + \gamma^L s_{jk,t-1},$$

which is exactly the equation appearing in the main text, describing entrepreneur j's optimal demand for loans from bank k.

As for the second, dynamic part of the problem, entrepreneur j chooses the optimal levels of his production factors, as well as his investment, consumption, and his total demand for financing, $x_t(j)$. Finally, we also need the entrepreneur to optimize with respect to $\int_0^1 l_{jk,t}dk$ to ensure that he takes all aspects of his optimization problem into account, as illustrated below. We can write the problem as:

$$\underset{C_t^E(j),x_t(j),\int_0^1 l_{jk,t}dk,N_t(j),H_t^E(j),K_t(j),I_t(j)}{Max} E_0 \sum_{t=0}^{\infty} \left(\beta^E\right)^t \log\left(C_t^E(j)-\gamma^E C_{t-1}^E(j)\right)$$

$$-\lambda_t^E(j)\cdot$$
$$\left[C_t^E(j)+R_{t-1}^L\int_0^1 l_{jk,t-1}dk - Y_t(j)+W_tN_t(j)+\vartheta_t^Q I_t(j)-x_t(j)-\Phi_t(j)-\Psi_t(j)\right]$$
$$-\lambda_t^E(j)\,Q_t^H\left[H_t^E(j)+H_{t-1}^E(j)\right]$$
$$-\mu_t^E(j)\left[R_t^L\int_0^1 l_{jk,t}dk-\int_0^1 \theta_{k,t}dkE_t\left[Q_{t+1}^H H_t^E(j)+Q_{t+1}^K K_t(j)\right]\right]$$
$$-\eta_t^E(j)\left[K_t(j)-(1-\delta)K_{t-1}(j)-\left[1-\frac{\Omega}{2}\left(\frac{I_t(j)}{I_{t-1}(j)}-1\right)^2\right]I_t(j)\right]$$
$$-\epsilon_t^E(j)\left[x_t(j)-\left[\int_0^1\left(l_{jk,t}-\gamma^L s_{jk,t-1}\right)^{\frac{\xi-1}{\xi}}dk\right]^{\frac{\xi}{\xi-1}}\right],$$

where $Y_t(j) = A_t [N_t(j)]^{1-\alpha} \left\{ \left[H^E_{t-1}(j)\right]^{\phi} [K_{t-1}(j)]^{1-\phi} \right\}^{\alpha}$ may be inserted for $Y_t(j)$ in the budget constraint, and $s_{jk,t-1} = \rho_s s_{jk,t-2} + (1-\rho_s) l_{jk,t-1}$ for $s_{jk,t-1}$ in the last line. The first-order condition for consumption is:

$$\frac{1}{C^E_t - \gamma^E C^E_{t-1}} - \beta^E \mathrm{E}_t \frac{\gamma^E}{C^E_{t+1} - \gamma^E C^E_t} = \lambda^E_t, \tag{A.5}$$

while for $x_t(j)$, we obtain:

$$\lambda^E_t = \epsilon^E_t,$$

and for $\int_0^1 l_{jk,t} dk$:

$$-\beta^E \mathrm{E}_t \lambda^E_{t+1} R^L_t - \mu^E_t R^L_t + \epsilon^E_t \left(\int_0^1 \left(l_{jk,t} - \gamma^L s_{jk,t-1} \right)^{\frac{\xi-1}{\xi}} dk \right)^{\frac{1}{\xi-1}} \left(l_{jk,t} - \gamma^L s_{jk,t-1} \right)^{\frac{-1}{\xi}} = 0 \Leftrightarrow$$

$$-\beta^E \mathrm{E}_t \lambda^E_{t+1} R^L_t - \mu^E_t R^L_t + \epsilon^E_t \frac{x_t(j)}{\int_0^1 \left(l_{jk,t} - \gamma^L s_{jk,t-1} \right) dk} = 0 \Leftrightarrow$$

$$-\beta^E \mathrm{E}_t \lambda^E_{t+1} R^L_t - \mu^E_t R^L_t + \epsilon^E_t = 0 \Leftrightarrow$$

$$\epsilon^E_t = \beta^E \mathrm{E}_t \lambda^E_{t+1} R^L_t + \mu^E_t R^L_t,$$

where the second-last line again uses that (*) must hold at the optimum. Combining this with the first-order condition for $x_t(j)$, we obtain:

$$\lambda^E_t = \beta^E \mathrm{E}_t \lambda^E_{t+1} R^L_t + \mu^E_t R^L_t, \tag{A.6}$$

which is a standard consumption Euler equation, as we would expect given the nature of the problem.

We now proceed with the remaining first-order conditions. In order to maximize the firm's profits, the wage rate equals the marginal product of labor:

$$W_t = (1-\alpha) \frac{Y_t(j)}{N_t(j)}, \tag{A.7}$$

The optimal choice of housing leads to:

$$\lambda_t^E(j) Q_t^H = \beta^E E_t \left[\lambda_{t+1}^E(j) \left(Q_{t+1}^H + \alpha\phi \frac{Y_{t+1}(j)}{H_t^E(j)} \right) \right] + \mu_t^E(j) \theta_t E_t \left(Q_{t+1}^H \right), \quad \text{(A.8)}$$

while the optimal choice of capital yields:

$$\eta_t^E(j) = \alpha(1-\phi)\beta^E E_t \left(\frac{\lambda_{t+1}^E(j) Y_{t+1}(j)}{K_t(j)} \right) + \beta^E(1-\delta) E_t \eta_{t+1}^E(j) + \mu_t^E(j) \theta_t E_t \left(Q_{t+1}^K \right). \quad \text{(A.9)}$$

The first-order condition for investment is:

$$\begin{aligned} \lambda_t^E(j) \vartheta_t^Q &= \eta_t^E(j) \left[1 - \frac{\Omega}{2} \left(\frac{I_t(j)}{I_{t-1}(j)} - 1 \right)^2 - \Omega \frac{I_t(j)}{I_{t-1}(j)} \left(\frac{I_t(j)}{I_{t-1}(j)} - 1 \right) \right] \\ &\quad + \beta^E \Omega E_t \left[\eta_{t+1}^E(j) \left(\frac{I_{t+1}(j)}{I_t(j)} \right)^2 \left(\frac{I_{t+1}(j)}{I_t(j)} - 1 \right) \right]. \end{aligned} \quad \text{(A.10)}$$

Finally, note also that the price of installed capital in this model must equal the shadow price of capital as measured in utility terms, i.e. it must hold at all times that $Q_t^K = \eta_t^E / \lambda_t^E$, since this is exactly the utility value of unit of installed capital.

4.3 Banks

As described in the main text, the problem of each bank is to choose its lending as well as its LTV ratio. In doing so, it takes into account that the market for bank loans is characterized by the presence of deep habits in loan demand as well as adverse selection. The latter is captured by the following expression:

$$p_{kt} = \Xi + \varpi(\theta_{kt} - \theta),$$

where θ denotes the steady state value of θ_{kt}, which is the same for all k. We assume $\Xi > 0$ and $\varpi < 0$, so that $\frac{\partial p_{kt}}{\partial \theta_{kt-1}} < 0$, while $\frac{\partial^2 p_{kt}}{(\partial \theta_{kt-1})^2} = 0$.

The bank solves the following problem:

$$\underset{L_{kt},\theta_{kt}}{Max}\ \Pi_t = \underbrace{[\Xi + \varpi(\theta_{kt-1} - \theta)]}_{=p_{kt-1}} R_{t-1}^L L_{kt-1} + \underbrace{[1 - \Xi - \varpi(\theta_{kt-1} - \theta)]}_{=(1-p_{kt-1})} \frac{L_{kt-1}}{\int_0^1 L_{kt-1}dk} \tau a_{t-1}$$
$$-R_{t-1}^D L_{kt-1} + \mu_t^B \left(\int_0^1 \left[\left(\frac{\theta_{kt}}{\theta_t} \right)^{\xi} x_t + \gamma^L s_{kt-1} \right] dj - L_{kt} \right),$$

where we have imposed the balance sheet condition of each individual bank, which states that $L_{kt} = \int_0^1 D_{ik,t} di$. If we let $\mathrm{E}_t q_{t,t+1} \equiv \beta^P \frac{\lambda_{t+1}^P}{\lambda_t^P}$ denote the stochastic discount factor of banks, which is given by the stochastic discount factor of households, who own the banks, the problem gives rise to the following first-order condition for L_{kt}:

$$\mathrm{E}_t q_{t,t+1} p_{kt} R_t^L + \mathrm{E}_t q_{t,t+1} (1 - p_{kt}) \frac{\tau a_t}{\int_0^1 L_{kt} dk} - \mathrm{E}_t q_{t,t+1} R_t^D + \gamma^L (1 - \rho_s) \mathrm{E}_t q_{t,t+1} \mu_{t+1}^B - \mu_t^B = 0 \Leftrightarrow$$

$$\mu_t^B = \mathrm{E}_t q_{t,t+1} \left[p_{kt} R_t^L + (1 - p_{kt}) \frac{\tau a_t}{\int_0^1 L_{kt} dk} - R_t^D + \gamma^L (1 - \rho_s) \mu_{t+1}^B \right],$$

and for θ_{kt}:

$$\varpi \mathrm{E}_t q_{t,t+1} R_t^L L_{kt} - \varpi \mathrm{E}_t q_{t,t+1} \frac{L_{kt}}{\int_0^1 L_{kt} dk} \tau a_t + \xi \mu_t^B \left(\frac{\theta_{kt}}{\theta_t} \right)^{\xi - 1} \frac{x_t}{\theta_t} = 0 \Leftrightarrow$$

$$\xi \mu_t^B \left(\frac{\theta_{kt}}{\theta_t} \right)^{\xi - 1} \frac{x_t}{\theta_t} = -\varpi \mathrm{E}_t q_{t,t+1} \left[R_t^L L_{kt} - \frac{L_{kt}}{\int_0^1 L_{kt} dk} \tau a_t \right].$$

As described below, we eventually consider only symmetric equilibria, in which all banks set the same LTV ratio ($\theta_{kt} = \theta_t, \forall k$), and therefore also lend the same amount ($L_{kt} = L_t, \forall k$). In that case, we can rewrite the bank's first-order conditions as:

$$\mu_t^B = \mathrm{E}_t q_{t,t+1} \left[p_{kt} R_t^L + (1 - p_{kt}) \frac{\tau a_t}{L_t} - R_t^D + \gamma^L (1 - \rho_s) \mu_{t+1}^B \right] \Leftrightarrow$$

$$\mu_t^B = \mathrm{E}_t q_{t,t+1}\left[p_{kt}R_t^L + (1-p_{kt})\frac{\tau}{\theta_t} - R_t^D + \gamma^L(1-\rho_s)\mu_{t+1}^B\right], \tag{A.11}$$

$$\xi\mu_t^B\frac{x_t}{\theta_t} = -\varpi\mathrm{E}_t q_{t,t+1}\left[R_t^L L_{kt} - \tau a_t\right], \tag{A.12}$$

where in $(A.11)$ we have imposed that $L_t = l_t$ in a symmetric equilibrium, and that the collateral constraint always holds with equality. These are the equations presented in the main text.

4.4 List of equations

The model consists of the set of equations below. We consider only symmetric equilibria, in which all banks set the same LTV ratio, and therefore lend the same amount. This allows us to drop all subscript k's. Moreover, as all household (respectively, entrepreneurs) are also identical, we can further drop the i's and j's.

First, note that under these assumptions, the expression for the aggregate loan composite can be rewritten as:

$$\begin{aligned}
x_t(j) &= \left[\int_0^1 \left(l_{jk,t} - \gamma^L s_{jk,t-1}\right)^{\frac{\xi-1}{\xi}} dk\right]^{\frac{\xi}{\xi-1}} \Leftrightarrow \\
x_t &= \left[\left(l_{0,t} - \gamma^L s_{0,t-1}\right)^{\frac{\xi-1}{\xi}} + ... + \left(l_{1,t} - \gamma^L s_{1,t-1}\right)^{\frac{\xi-1}{\xi}}\right]^{\frac{\xi}{\xi-1}} \Leftrightarrow \\
x_t &= \left[\left(l_t - \gamma^L s_{t-1}\right)^{\frac{\xi-1}{\xi}}\right]^{\frac{\xi}{\xi-1}} \Leftrightarrow \\
x_t &= \left(l_t - \gamma^L s_{t-1}\right),
\end{aligned}$$

which is equation $(A.14)$ below. The assumption of symmetry furthermore leads to equation $(A.15)$, as $L_{kt} \equiv \int_0^1 l_{jk,t}dj = l_{kt}$, so that $L_t = l_t$. Finally, note that in a symmetric equilibrium, we obtain $\Phi_t = \gamma^L s_{t-1}$ and $\Psi_t = (1-p_{t-1})\left(R_{t-1}^L L_{t-1} - \tau a_{t-1}\right)$, which enter equation $(A.19)$ below. We are then ready to summarize the equations of the model:

$$\frac{1}{C_t^P - \gamma^P C_{t-1}^P} - \beta^P \mathrm{E}_t \frac{\gamma^P}{C_{t+1}^P - \gamma^P C_t^P} = \lambda_t^P, \tag{A.1}$$

$$\beta^P E_t \left[\lambda_{t+1}^P\right] = \frac{\lambda_t^P}{R_t^D}, \tag{A.2}$$

$$\frac{\varsigma_t}{H_t^P} + \beta^P E_t \left[\lambda_{t+1}^P Q_{t+1}^H\right] = \lambda_t^P Q_t^H, \tag{A.3}$$

$$\iota N_t^{\varphi} = \lambda_t^P W_t, \tag{A.4}$$

$$\frac{1}{C_t^E - \gamma^E C_{t-1}^E} - \beta^E \mathrm{E}_t \frac{\gamma^E}{C_{t+1}^E - \gamma^E C_t^E} = \lambda_t^E, \tag{A.5}$$

$$\beta^E E_t \left[\lambda_{t+1}^E R_t^L\right] + \mu_t^E R_t^L = \lambda_t^E, \tag{A.6}$$

$$W_t = (1-\alpha)\frac{Y_t}{N_t}, \tag{A.7}$$

$$\lambda_t^E Q_t^H = \beta^E E_t \left[\lambda_{t+1}^E \left(Q_{t+1}^H + \alpha\phi \frac{Y_{t+1}}{H_t^E}\right)\right] + \mu_t^E \theta_t E_t \left(Q_{t+1}^H\right), \tag{A.8}$$

$$\eta_t^E = \alpha(1-\phi)\beta^E E_t \left(\frac{\lambda_{t+1}^E Y_{t+1}}{K_t}\right) + \beta^E (1-\delta) E_t \eta_{t+1}^E + \mu_t^E \theta_t E_t \left(Q_{t+1}^K\right), \tag{A.9}$$

$$\lambda_t^E \vartheta_t^Q = \eta_t^E \left[1 - \frac{\Omega}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2 - \Omega \frac{I_t}{I_{t-1}}\left(\frac{I_t}{I_{t-1}} - 1\right)\right] + \beta^E \Omega E_t \left[\eta_{t+1}^E \left(\frac{I_{t+1}}{I_t}\right)^2 \left(\frac{I_{t+1}}{I_t} - 1\right)\right], \tag{A.10}$$

$$\mu_t^B = \mathrm{E}_t q_{t,t+1} \left[p_{kt} R_t^L + (1-p_{kt})\frac{\tau}{\theta_t} - R_t^D + \gamma^L (1-\rho_s)\mu_{t+1}^B\right], \tag{A.11}$$

$$\xi \mu_t^B \frac{x_t}{\theta_t} = -\varpi \mathrm{E}_t q_{t,t+1} \left[R_t^L L_{kt} - \tau a_t\right], \tag{A.12}$$

$$s_t = \rho_s s_{t-1} + (1 - \rho_s)\, l_t, \tag{A.13}$$

$$x_t = \left(l_t - \gamma^L s_{t-1}\right), \tag{A.14}$$

$$L_t = l_t, \tag{A.15}$$

$$C_t^P + C_t^E + \vartheta_t^Q I_t = Y_t, \tag{A.16}$$

$$H_t^P + H_t^E = H, \tag{A.17}$$

$$L_t = D_t, \tag{A.18}$$

$$C_t^E + R_{t-1}^L l_{t-1} = Y_t - W_t N_t - \vartheta_t^Q I_t - Q_t^H \left[H_t^E - H_{t-1}^E\right] + x_t + \Phi_t + \Psi_t, \tag{A.19}$$

$$l_t = \frac{\theta_t a_t}{R_t^L}, \tag{A.20}$$

$$a_t = E_t \left[Q_{t+1}^H H_t^E + Q_{t+1}^K K_t\right], \tag{A.21}$$

$$Y_t = A_t \left[N_t\right]^{1-\alpha} \left\{\left[H_{t-1}^E\right]^{\phi} \left[K_{t-1}\right]^{1-\phi}\right\}^{\alpha}, \tag{A.22}$$

$$K_t = (1 - \delta)\, K_{t-1} + \left[1 - \frac{\Omega}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2\right] I_t, \tag{A.23}$$

$$R_t^L = \psi R_t^D, \tag{A.24}$$

$$p_t = \Xi + \varpi\left(\theta_t - \theta\right), \tag{A.25}$$

$$\eta_t^E = \lambda_t^E Q_t^K, \tag{A.26}$$

$$q_{t,t+1} = \beta^P \frac{\lambda_{t+1}^P}{\lambda_t^P}. \tag{A.27}$$

5 Derivations of steady state conditions

In the following, we show how to derive the steady state of the model. The references to numbered equations below all refer to the number assigned in the present appendix. Once again, we ignore the (i)'s, (j)'s, and (k)'s denoting each particular household, entrepreneur or bank. We get the following steady state relations: From $(A.1)$ and $(A.5)$:

$$\frac{1-\beta^P\gamma^P}{(1-\gamma^P)\,C^P} = \lambda^P, \tag{1}$$

$$\frac{1-\beta^E\gamma^E}{(1-\gamma^E)\,C^E} = \lambda^E.$$

From $(A.2)$:

$$\beta^P\lambda^P = \frac{\lambda^P}{R^D} \Leftrightarrow$$

$$R^D = \frac{1}{\beta^P}. \tag{2}$$

From $(A.24)$:

$$R^L = \psi R^D. \tag{3}$$

$(A.3)$ gives:

$$\begin{aligned} \frac{\varsigma}{H^P} + \beta^P\lambda^P Q^H &= \lambda^P Q^H \Leftrightarrow \\ \frac{\varsigma}{H^P} &= \lambda^P Q^H\left(1-\beta^P\right) \Leftrightarrow \end{aligned}$$

$$Q^H H^P = \frac{\varsigma}{\lambda^P \left(1 - \beta^P\right)}. \tag{4}$$

From $(A.4)$, we get:

$$\iota N^{\varphi} = \lambda^P W. \tag{5}$$

Next, use $(A.6)$ to get:

$$\beta^E \lambda^E R^L + \mu^E R^L = \lambda^E \Leftrightarrow$$

$$\mu^E = \frac{\lambda^E \left(1 - \beta^E R^L\right)}{R^L}. \tag{6}$$

This equation shows that the collateral constraint is binding (that is, μ^E is strictly positive) if and only if the discount factor of the entrepreneurs is smaller than $1/R^L$, where $1/R^L$ equals the discount factor of patient households.

From equation $(A.7)$ we get:

$$W = (1 - \alpha) \frac{Y}{N}, \tag{7}$$

Next, rewrite equation $(A.8)$ as follows:

$$\lambda^E Q^H = \beta^E \lambda^E \left(Q^H + \alpha\phi \frac{Y}{H^E}\right) + \mu^E \theta Q^H \Leftrightarrow$$

$$\lambda^E = \beta^E \lambda^E \left(1 + \alpha\phi \frac{Y}{Q^H H^E}\right) + \mu^E \theta \Leftrightarrow$$

$$\lambda^E = \beta^E \lambda^E + \beta^E \lambda^E \alpha\phi \frac{Y}{Q^H H^E} + \frac{\lambda^E \left(1 - \beta^E R^L\right)}{R^L} \theta \Leftrightarrow$$

$$1 = \beta^E + \beta^E \alpha\phi \frac{Y}{Q^H H^E} + \frac{1 - \beta^E R^L}{R^L} \theta \Leftrightarrow$$

$$\left(1 - \beta^E\right) = \beta^E \alpha\phi \frac{Y}{Q^H H^E} + \frac{1 - \beta^E R^L}{R^L} \theta \Leftrightarrow$$

$$R^L \left(1 - \beta^E\right) - \theta \left(1 - \beta^E R^L\right) = R^L \beta^E \alpha\phi \frac{Y}{Q^H H^E} \Leftrightarrow$$

$$\frac{Q^H H^E}{Y} = \frac{\beta^E \alpha \phi R^L}{\left(1-\beta^E\right) R^L - \theta\left(1-\beta^E R^L\right)}, \tag{8}$$

where R^L is given from (3).

We can divide the two expressions for housing demand with each other (after dividing by Y in (4)).

$$\frac{\frac{Q^H H^P}{Y}}{\frac{Q^H H^E}{Y}} = \frac{\frac{\varsigma}{Y\lambda^P\left(1-\beta^P\right)}}{\frac{\beta^E \alpha \phi R^L}{\left(1-\beta^E\right)R^L - \theta\left(1-\beta^E R^L\right)}} \Leftrightarrow$$

$$\frac{H^P}{H^E} = \frac{\varsigma}{Y\frac{1-\beta^P\gamma^P}{(1-\gamma^P)C^P}\left(1-\beta^P\right)} \frac{\left(1-\beta^E\right) R^L - \theta\left(1-\beta^E R^L\right)}{\beta^E \alpha \phi R^L} \Leftrightarrow$$

$$\frac{H^P}{H^E} = \frac{\varsigma}{\frac{\left(1-\beta^P\gamma^P\right)\left(1-\beta^P\right)Y}{(1-\gamma^P)C^P}} \frac{\left(1-\beta^E\right) R^L - \theta\left(1-\beta^E R^L\right)}{\beta^E \alpha \phi R^L} \Leftrightarrow$$

$$\frac{H^P}{H - H^P} = \frac{\varsigma\left(1-\gamma^P\right)}{\left(1-\beta^P\right)\left(1-\beta^P\gamma^P\right)} \frac{\left(1-\beta^E\right) R^L - \theta\left(1-\beta^E R^L\right)}{\beta^E \alpha \phi R^L} \frac{C^P}{Y}, \tag{9}$$

where the last step uses the steady state version of the housing market clearing condition ($A.17$), and where we can insert for R^L from (3). We then only need an expression for $\frac{C^P}{Y}$ in order to determine H^P and, in turn, H^E.

Equation ($A.9$) can be rewritten to yield:

$$\eta_t^E = \alpha\left(1-\phi\right)\beta^E E_t\left(\frac{\lambda_{t+1}^E Y_{t+1}}{K_t}\right) + \beta^E\left(1-\delta\right) E_t \eta_{t+1}^E + \mu_t^E \theta_t E_t\left(Q_{t+1}^K\right) \Leftrightarrow$$

$$\eta^E = \alpha\left(1-\phi\right)\beta^E \frac{\lambda^E Y}{K} + \beta^E\left(1-\delta\right)\eta^E + \mu^E \theta Q^K \Leftrightarrow$$

$$\eta^E\left(1-\left(1-\delta\right)\beta^E\right) = \alpha\left(1-\phi\right)\beta^E \frac{\lambda^E Y}{K} + \frac{\lambda^E\left(1-\beta^E R^L\right)}{R^L}\theta Q^K \Leftrightarrow$$

$$\frac{\eta^E}{\lambda^E}\left(1-\left(1-\delta\right)\beta^E\right) = \alpha\left(1-\phi\right)\beta^E \frac{Y}{K} + \frac{\left(1-\beta^E R^L\right)}{R^L}\theta Q^K. \tag{10}$$

Next, we obtain from $(A.10)$:

$$\begin{aligned}\lambda_t^E(j)\,\vartheta_t^Q &= \eta_t^E(j)\left[1-\frac{\Omega}{2}\left(\frac{I_t(j)}{I_{t-1}(j)}-1\right)^2-\Omega\frac{I_t(j)}{I_{t-1}(j)}\left(\frac{I_t(j)}{I_{t-1}(j)}-1\right)\right]\\ &\quad+\beta^E\Omega E_t\left[\eta_{t+1}^E(j)\left(\frac{I_{t+1}(j)}{I_t(j)}\right)^2\left(\frac{I_{t+1}(j)}{I_t(j)}-1\right)\right]\end{aligned}$$

$$\Leftrightarrow \lambda^E=\eta^E\left[1-\frac{\Omega}{2}\left(\frac{I}{I}-1\right)^2-\Omega\frac{I}{I}\left(\frac{I}{I}-1\right)\right]+\beta^E\Omega\left[\eta^E\left(\frac{I}{I}\right)^2\left(\frac{I}{I}-1\right)\right]\Leftrightarrow$$

$$\lambda^E=\eta^E$$

This can be combined with the steady version of $(A.26)$, which reads:

$$\eta^E=\lambda^E Q^K,$$

so that we have $Q^K=1$, in steady state, where $1/Q^K$ is the expression for Tobin's q in the present model. Note that the price of capital equals one, and thus the price of new investment goods, in steady state. This reflects the absence of investment adjustment costs in steady state when the investment level is constant.

Using this, we can now rewrite (10):

$$\frac{\eta^E}{\lambda^E}\left(1-(1-\delta)\beta^E\right)=\alpha(1-\phi)\beta^E\frac{Y}{K}+\frac{\left(1-\beta^E R^L\right)}{R^L}\theta Q^K\Leftrightarrow$$

$$\left(1-(1-\delta)\beta^E\right)=\alpha(1-\phi)\beta^E\frac{Y}{K}+\frac{\left(1-\beta^E R^L\right)}{R^L}\theta\Leftrightarrow$$

$$R^L\left(1-(1-\delta)\beta^E\right)-\theta\left(1-\beta^E R^L\right)=\alpha(1-\phi)R^L\beta^E\frac{Y}{K}\Leftrightarrow$$

$$\frac{K}{Y}=\frac{\alpha(1-\phi)R^L\beta^E}{R^L\left(1-(1-\delta)\beta^E\right)-\theta\left(1-\beta^E R^L\right)}. \tag{11}$$

The steady state version of $(A.27)$ reads:

$$q = \beta^P.$$

Using this, we get from $(A.11)$:

$$\mu^B = q\left[pR^L + (1-p)\frac{\tau}{\theta} - R^D + \gamma^L(1-\rho_s)\mu^B\right] \Leftrightarrow$$

$$\mu^B\left(1 - \beta^P\gamma^L(1-\rho_s)\right) = \beta^P\left[pR^L + (1-p)\frac{\tau}{\theta} - R^D\right] \Leftrightarrow$$

$$\mu^B = \frac{\beta^P\left[pR^L + (1-p)\frac{\tau}{\theta} - R^D\right]}{1 - \beta^P\gamma^L(1-\rho_s)} \tag{12}$$

and from $(A.12)$:

$$\xi\mu^B\frac{x}{\theta} = -\varpi q\left[R^L L - \tau a\right]. \tag{13}$$

From $(A.13)$, we obtain:

$$s_t = \rho_s s_{t-1} + (1-\rho_s)l_t \Leftrightarrow$$

$$(1-\rho_s)s = (1-\rho_s)l \Leftrightarrow$$

$$s = l.$$

From $(A.14)$:

$$x_t = \left(l_t - \gamma^L s_{t-1}\right) \Leftrightarrow$$

$$x = \left(l - \gamma^L s\right) \Leftrightarrow$$

$$x = \left(1 - \gamma^L\right)l.$$

$(A.15)$ and $(A.18)$ simply become:

$$L = l,$$

$$D = L.$$

We can use the expressions for L and x to rewrite (13):

$$\xi\mu^B \frac{\left(1-\gamma^L\right) l}{\theta} = -\varpi\beta^P \left[R^L l - \tau a\right].$$

In steady state, the collateral constraint binds:

$$R^L l = \theta a,$$

which we can insert to get:

$$\xi\mu^B \frac{\left(1-\gamma^L\right) l}{\theta} = -\varpi\beta^P \left[R^L l - \tau \frac{R^L l}{\theta}\right] \Leftrightarrow$$

$$\xi\mu^B \frac{\left(1-\gamma^L\right)}{\theta} = -\varpi\beta^P \left[R^L - \frac{R^L \tau}{\theta}\right] \Leftrightarrow$$

$$\xi\mu^B \left(1-\gamma^L\right) = -\varpi\theta\beta^P R^L + \varpi\beta^P R^L \tau \Leftrightarrow$$

$$\theta = \frac{\beta^P R^L \varpi\tau - \xi\mu^B \left(1-\gamma^L\right)}{\varpi\beta^P R^L}. \tag{14}$$

Together with (12), this equation constitutes two equations in two unknowns; μ^B and θ. Inserting the expression for θ in (12), we obtain a second-order equation for μ^B:

$$\mu^B \left(1-\beta^P\gamma^L\left(1-\rho_s\right)\right) = \beta^P \left(pR^L - R^D\right) + \beta^P \left(1-p\right)\tau \frac{\varpi\beta^P R^L}{\beta^P R^L \varpi\tau - \xi\mu^B \left(1-\gamma^L\right)} \Leftrightarrow$$

$$\mu^B \left[\beta^P R^L \varpi\tau - \xi\mu^B \left(1-\gamma^L\right)\right] - \frac{\beta^P \left(pR^L - R^D\right) \left[\beta^P R^L \varpi\tau - \xi\mu^B \left(1-\gamma^L\right)\right]}{\left(1-\beta^P\gamma^L\left(1-\rho_s\right)\right)}$$

$$= \frac{\left(\beta^P\right)^2 \left(1-p\right)\tau\varpi R^L}{\left(1-\beta^P\gamma^L\left(1-\rho_s\right)\right)} \Leftrightarrow$$

$$0 = -\xi\left(1-\gamma^L\right)\left(\mu^B\right)^2 + \left[\beta^P R^L \varpi\tau + \frac{\beta^P\left(pR^L - R^D\right)\xi\left(1-\gamma^L\right)}{1-\beta^P\gamma^L\left(1-\rho_s\right)}\right]\mu^B$$
$$-\left(\beta^P\right)^2 \frac{\left(pR^L - R^D\right)R^L\varpi\tau + (1-p)\tau\varpi R^L}{1-\beta^P\gamma^L\left(1-\rho_s\right)}.$$

As is well known, we can write the solution to this equation as:

$$\mu^B = \frac{-b \pm \sqrt{d}}{2a},$$

where in this case we have that $a = -\xi\left(1-\gamma^L\right)$, $b = \beta^P R^L\varpi\tau + \frac{\beta^P\left(pR^L-R^D\right)\xi\left(1-\gamma^L\right)}{1-\beta^P\gamma^L(1-\rho_s)}$, and $d = b^2 - 4ac$, with a and b as already defined, and $c = -\left(\beta^P\right)^2\frac{\left(pR^L-R^D\right)R^L\varpi\tau+(1-p)\tau\varpi R^L}{1-\beta^P\gamma^L(1-\rho_s)}$. Given the parameter values of the model, we solve this equation, and we keep the solution that returns positive values for μ^B and θ.[2]

Next, we can combine $(A.20)$ and $(A.21)$ to obtain:

$$l = \frac{\theta}{R^L}\left[Q^H H^E + Q^K K\right].$$

We can use the expressions derived above to express this in ratios to output:

$$\frac{l}{Y} = \frac{\theta}{R^L}\left[\frac{Q^H H^E}{Y} + \frac{Q^K K}{Y}\right] \Leftrightarrow$$

$$\frac{l}{Y} = \frac{\theta}{R^L}\left[\frac{\beta^E R^L\alpha\phi}{R^L\left(1-\beta^E\right)-\theta\left(1-\beta^E R^L\right)} + \frac{\alpha\left(1-\phi\right)R^L\beta^E}{R^L\left(1-\left(1-\delta\right)\beta^E\right)-\theta\left(1-\beta^E R^L\right)}\right] \Leftrightarrow$$

$$\frac{l}{Y} = \alpha\theta\beta^E\left[\frac{\phi}{R^L\left(1-\beta^E\right)-\theta\left(1-\beta^E R^L\right)} + \frac{(1-\phi)}{R^L\left(1-\left(1-\delta\right)\beta^E\right)-\theta\left(1-\beta^E R^L\right)}\right].$$

From the entrepreneur's budget constraint $(A.19)$, we get:

[2]The second-order equation provides two possible solutions for μ^B, and in turn for θ. It turns out that one pair of solutions is not well-behaved, as it returns negative value of μ^B and θ. The other pair instead yields positive values for μ^B and θ. We therefore use the second pair of values.

$$C_t^E + R_{t-1}^L l_{t-1} = Y_t - W_t N_t - \vartheta_t^Q I_t - Q_t^H \left[H_t^E - H_{t-1}^E\right] + x_t + \Phi_t + \Psi_t$$

$$C^E + R^L l = Y - WN - I + x + \Phi + \Psi \Leftrightarrow$$

Rewrite this in ratios to output:

$$\frac{C^E}{Y} + \frac{R^L l}{Y} = 1 - \frac{WN}{Y} - \frac{I}{Y} + \frac{x}{Y} + \frac{\Phi}{Y} + \frac{\Psi}{Y} \Leftrightarrow$$

$$\frac{C^E}{Y} = 1 - (1-\alpha) - \frac{I}{Y} + \frac{\left(1 - \gamma^L - R^L\right) l}{Y} + \frac{\Phi}{Y} + \frac{\Psi}{Y} \Leftrightarrow$$

$$\frac{C^E}{Y} = \alpha - \delta\frac{K}{Y} + \left(1 - \gamma^L - R^L\right)\frac{l}{Y} + \frac{\Phi}{Y} + \frac{\Psi}{Y}.$$

Now, insert the steady state expression for Φ and Ψ:

$$\frac{C^E}{Y} = \alpha - \delta\frac{K}{Y} + \left(1 - \gamma^L - R^L\right)\frac{l}{Y} + \frac{\gamma^L s}{Y} + \frac{(1-p)\left(R^L L - \tau a\right)}{Y} \Leftrightarrow$$

$$\frac{C^E}{Y} = \alpha - \delta\frac{K}{Y} + \left(1 - \gamma^L - R^L\right)\frac{l}{Y} + \frac{\gamma^L l}{Y} + \frac{(1-p)\,R^L L}{Y} - \frac{(1-p)\,\tau R^L l}{\theta Y} \Leftrightarrow$$

$$\frac{C^E}{Y} = \alpha - \delta\frac{K}{Y} + \left(1 - pR^L - \frac{(1-p)\,\tau R^L}{\theta}\right)\frac{l}{Y}, \tag{15}$$

where we can insert the expressions for $\frac{K}{Y}$ and $\frac{l}{Y}$ from above.

Now compute the steady state version of the aggregate resource constraint $(A.16)$:

$$C^P + C^E + I = Y \Leftrightarrow$$

$$\frac{C^P}{Y} = 1 - \frac{C^E}{Y} - \frac{I}{Y} \Leftrightarrow$$

$$\frac{C^P}{Y} = 1 - \frac{C^E}{Y} - \delta\frac{K}{Y}, \tag{16}$$

where we can insert for $\frac{C^E}{Y}$ and $\frac{K}{Y}$ from above. Once we have the steady state value of $\frac{C^P}{Y}$, we can insert it in (9) to find H^P. We can then use the housing market clearing condition to solve for H^E.

From the aggregate law of motion for capital $(A.23)$, we get:

$$K = (1-\delta)K + \left[1 - \frac{\Omega}{2}\left(\frac{I}{I} - 1\right)^2\right] I \Leftrightarrow$$

$$I = \delta K. \tag{17}$$

We obtain the steady state value of $(A.25)$ as:

$$p = \Xi,$$

where Ξ is therefore interpretable as the probability of default in steady state, which enters a number of the expressions above.

Now, combine (1), (5), and (7) to obtain:

$$\iota N^{\varphi} = \lambda^P W \Leftrightarrow$$

$$\iota N^{\varphi} = \frac{1 - \beta^P\gamma^P}{(1-\gamma^P)C^P}(1-\alpha)\frac{Y}{N} \Leftrightarrow$$

$$N^{1+\varphi} = \frac{\left(1-\beta^P\gamma^P\right)(1-\alpha)Y}{\iota(1-\gamma^P)C^P} \Leftrightarrow$$

$$N = \left(\frac{\left(1-\beta^P\gamma^P\right)(1-\alpha)}{\iota(1-\gamma^P)}\right)^{\frac{1}{1+\varphi}} \left(\frac{C^P}{Y}\right)^{-\frac{1}{1+\varphi}}, \tag{18}$$

which determines labor, since $\frac{C^P}{Y}$ has already been determined above. We can then pin down steady state output from the production function:

$$Y = AN^{1-\alpha}\left(H^E\right)^{\alpha\phi} K^{\alpha(1-\phi)} \Leftrightarrow$$

$$Y = AN^{1-\alpha}\left(H^E\right)^{\alpha\phi}\left(\frac{K}{Y}\right)^{\alpha(1-\phi)}Y^{\alpha(1-\phi)} \Leftrightarrow$$

$$Y^{1-\alpha(1-\phi)} = AN^{1-\alpha}\left(H^E\right)^{\alpha\phi}\left(\frac{K}{Y}\right)^{\alpha(1-\phi)} \Leftrightarrow$$

$$Y = \left[AN^{1-\alpha}\left(H^E\right)^{\alpha\phi}\left(\frac{K}{Y}\right)^{\alpha(1-\phi)}\right]^{\frac{1}{1-\alpha(1-\phi)}},$$

where we can insert for $\frac{K}{Y}$, H^E, and N, and where A is exogenously given. With Y determined, we can pin down the remaining variables that were initially expressed in ratios to output. We get the steady state wage rate from (7). Finally, we determine the house price by:

$$Q^H = \frac{\varsigma}{H^P\lambda^P\left(1-\beta^P\right)},$$

where λ^P is determined once we pin down the level of C^P. This completes the characterization of the steady state.

6 Log-linearization

The next step is to log-linearize the equilibrium conditions around the steady state. We begin with the first-order conditions of the patient household:

$$\frac{1}{C_t^P - \gamma^P C_{t-1}^P} - \beta^P \mathrm{E}_t \frac{\gamma^P}{C_{t+1}^P - \gamma^P C_t^P} = \lambda_t^P \Leftrightarrow$$

$$\left(\mathrm{E}_t C_{t+1}^P - \gamma^P C_t^P\right) - \beta^P\gamma^P\left(C_t^P - \gamma^P C_{t-1}^P\right) = \lambda_t^P\left(C_{t+1}^P - \gamma^P C_t^P\right)\left(C_t^P - \gamma^P C_{t-1}^P\right) \Leftrightarrow$$

$$\begin{aligned}
\Leftrightarrow\ & C^P\mathrm{E}_t\widehat{C}_{t+1}^P - \gamma^P C^P\widehat{C}_t^P - \beta^P\gamma^P C^P\widehat{C}_t^P + \beta^P\left(\gamma^P\right)^2 C^P\widehat{C}_{t-1}^P = \lambda^P\left(C^P\right)^2\left(\widehat{\lambda}_t^P + \widehat{C}_{t+1}^P + \widehat{C}_t^P\right) \\
& -\gamma^P\lambda^P\left(C^P\right)^2\left(\widehat{\lambda}_t^P + 2\widehat{C}_t^P\right) - \gamma^P\lambda^P\left(C^P\right)^2\left(\widehat{\lambda}_t^P + \widehat{C}_{t+1}^P + \widehat{C}_{t-1}^P\right) \\
& + \left(\gamma^P\right)^2\lambda^P\left(C^P\right)^2\left(\widehat{\lambda}_t^P + \widehat{C}_t^P + \widehat{C}_{t-1}^P\right)
\end{aligned}$$

$$
\begin{aligned}
\Leftrightarrow \quad & C^P \mathrm{E}_t \widehat{C}^P_{t+1} \left(1 - \lambda^P C^P + \gamma^P \lambda^P C^P\right) \\
& + C^P \widehat{C}^P_t \left(-\gamma^P - \beta^P \gamma^P - \lambda^P C^P + 2\gamma^P \lambda^P C^P - \left(\gamma^P\right)^2 \lambda^P C^P\right) \\
& + C^P \widehat{C}^P_{t-1} \left(\beta^P \left(\gamma^P\right)^2 + \gamma^P \lambda^P C^P - \left(\gamma^P\right)^2 \lambda^P C^P\right) \\
= \quad & \lambda^P \left(C^P\right)^2 \widehat{\lambda}^P_t \left(1 - \gamma^P - \gamma^P + \left(\gamma^P\right)^2\right).
\end{aligned}
$$

Here, we can use the steady state expression for λ^P in (1) to get the following expression for $\lambda^P C^P$:

$$
\lambda^P C^P = \frac{1 - \beta^P \gamma^P}{\left(1 - \gamma^P\right)},
$$

which we can then use to obtain:

$$
\begin{aligned}
& C^P \mathrm{E}_t \widehat{C}^P_{t+1} \left(1 - \frac{1 - \beta^P \gamma^P}{\left(1 - \gamma^P\right)} + \gamma^P \frac{1 - \beta^P \gamma^P}{\left(1 - \gamma^P\right)}\right) \\
& + C^P \widehat{C}^P_t \left(-\gamma^P - \beta^P \gamma^P - \frac{1 - \beta^P \gamma^P}{\left(1 - \gamma^P\right)} + 2\gamma^P \frac{1 - \beta^P \gamma^P}{\left(1 - \gamma^P\right)} - \left(\gamma^P\right)^2 \frac{1 - \beta^P \gamma^P}{\left(1 - \gamma^P\right)}\right) \\
& + C^P \widehat{C}^P_{t-1} \left(\beta^P \left(\gamma^P\right)^2 + \gamma^P \frac{1 - \beta^P \gamma^P}{\left(1 - \gamma^P\right)} - \left(\gamma^P\right)^2 \frac{1 - \beta^P \gamma^P}{\left(1 - \gamma^P\right)}\right) \\
= \quad & \frac{1 - \beta^P \gamma^P}{\left(1 - \gamma^P\right)} C^P \widehat{\lambda}^P_t \left(1 - \gamma^P - \gamma^P + \left(\gamma^P\right)^2\right) \Leftrightarrow
\end{aligned}
$$

$$
\begin{aligned}
& \frac{\mathrm{E}_t \widehat{C}^P_{t+1}}{1 - \gamma^P} \left[1 - \gamma^P - \left(1 - \beta^P \gamma^P\right) + \gamma^P \left(1 - \beta^P \gamma^P\right)\right] \\
& + \frac{\widehat{C}^P_t}{1 - \gamma^P} \left[-\gamma^P \left(1 + \beta^P\right) \left(1 - \gamma^P\right) - \left(1 - \beta^P \gamma^P\right) \left(1 - 2\gamma^P + \left(\gamma^P\right)^2\right)\right] \\
& + \frac{\widehat{C}^P_{t-1}}{1 - \gamma^P} \left[\gamma^P \left(\beta^P \gamma^P \left(1 - \gamma^P\right) + \left(1 - \gamma^P\right) \left(1 - \beta^P \gamma^P\right)\right)\right] \\
= \quad & \frac{\widehat{\lambda}^P_t}{1 - \gamma^P} \left(1 - \beta^P \gamma^P\right) \left(1 - \gamma^P\right)^2 \Leftrightarrow
\end{aligned}
$$

$$
\begin{aligned}
& \mathrm{E}_t \widehat{C}_{t+1}^P \left[\beta^P \gamma^P - \beta^P \left(\gamma^P \right)^2 \right] + \\
& \widehat{C}_t^P \left[-\gamma^P + \left(\gamma^P \right)^2 - \beta^P \gamma^P + \beta^P \left(\gamma^P \right)^2 - 1 + \beta^P \gamma^P + 2\gamma^P - 2\beta^P \left(\gamma^P \right)^2 - \left(\gamma^P \right)^2 + \beta^P \left(\gamma^P \right)^3 \right] \\
& + \widehat{C}_{t-1}^P \left[\beta^P \left(\gamma^P \right)^2 - \beta^P \left(\gamma^P \right)^3 + \gamma^P - \beta^P \left(\gamma^P \right)^2 - \left(\gamma^P \right)^2 + \beta^P \left(\gamma^P \right)^3 \right] \\
= \; & \widehat{\lambda}_t^P \left(1 - \beta^P \gamma^P \right) \left(1 - \gamma^P \right)^2 \Leftrightarrow
\end{aligned}
$$

$$
\begin{aligned}
& \mathrm{E}_t \widehat{C}_{t+1}^P \left(1 - \gamma^P \right) \beta^P \gamma^P + \widehat{C}_t^P \left[-1 + \gamma^P - \beta^P \left(\gamma^P \right)^2 + \beta^P \left(\gamma^P \right)^3 \right] + \widehat{C}_{t-1}^P \left[\gamma^P - \left(\gamma^P \right)^2 \right] \\
= \; & \widehat{\lambda}_t^P \left(1 - \beta^P \gamma^P \right) \left(1 - \gamma^P \right)^2 \Leftrightarrow
\end{aligned}
$$

$$
\beta^P \gamma^P \mathrm{E}_t \widehat{C}_{t+1}^P - \left(1 + \left(\gamma^P \right)^2 \beta^P \right) \widehat{C}_t^P + \gamma^P \widehat{C}_{t-1}^P = \left(1 - \beta^P \gamma^P \right) \left(1 - \gamma^P \right) \widehat{\lambda}_t^P , \quad (19)
$$

We further get:

$$
\beta^P \mathrm{E}_t \left[\lambda_{t+1}^P \right] = \frac{\lambda_t^P}{R_t^D} \Leftrightarrow
$$

$$
\mathrm{E}_t \widehat{\lambda}_{t+1}^P = \widehat{\lambda}_t^P - \widehat{R}_t^D, \quad (20)
$$

$$
\frac{\varsigma_t}{H_t^P} + \beta^P \mathrm{E}_t \left[\lambda_{t+1}^P Q_{t+1}^H \right] = \lambda_t^P Q_t^H \Leftrightarrow
$$

$$
\frac{\varsigma}{H^P} \left(\widehat{\varsigma}_t - \widehat{H}_t^P \right) + \beta^P \lambda^P Q^H \mathrm{E}_t \left[\widehat{\lambda}_{t+1}^P + \widehat{Q}_{t+1}^H \right] = \lambda^P Q^H \left(\widehat{\lambda}_t^P + \widehat{Q}_t^H \right) \Leftrightarrow
$$

$$
\varsigma \left(\widehat{\varsigma}_t - \widehat{H}_t^P \right) + \beta^P Q^H H^P \lambda^P \mathrm{E}_t \left[\widehat{\lambda}_{t+1}^P + \widehat{Q}_{t+1}^H \right] = Q^H H^P \lambda^P \left(\widehat{\lambda}_t^P + \widehat{Q}_t^H \right) \Leftrightarrow
$$

$$\varsigma\left(\widehat{\varsigma}_t-\widehat{H}_t^P\right)+\beta^P\frac{\varsigma}{\lambda^P\left(1-\beta^P\right)}\lambda^P\mathrm{E}_t\left[\widehat{\lambda}_{t+1}^P+\widehat{Q}_{t+1}^H\right]=\frac{\varsigma}{\lambda^P\left(1-\beta^P\right)}\lambda^P\left(\widehat{\lambda}_t^P+\widehat{Q}_t^H\right)\Leftrightarrow$$

$$\widehat{\varsigma}_t-\widehat{H}_t^P+\frac{\beta^P}{1-\beta^P}\mathrm{E}_t\left[\widehat{\lambda}_{t+1}^P+\widehat{Q}_{t+1}^H\right]=\frac{1}{1-\beta^P}\left(\widehat{\lambda}_t^P+\widehat{Q}_t^H\right)\Leftrightarrow$$

$$\left(1-\beta^P\right)\left(\widehat{\varsigma}_t-\widehat{H}_t^P\right)+\beta^P\mathrm{E}_t\left[\widehat{\lambda}_{t+1}^P+\widehat{Q}_{t+1}^H\right]=\widehat{\lambda}_t^P+\widehat{Q}_t^H, \tag{21}$$

$$\iota N_t^{\varphi}=\lambda_t^P W_t\Leftrightarrow$$

$$\varphi\iota\left(N\right)^{\varphi}\left(\widehat{N}_t\right)=\lambda^P W\left(\widehat{\lambda}_t^P+\widehat{W}_t\right)\Leftrightarrow$$

$$\varphi\widehat{N}_t=\widehat{\lambda}_t^P+\widehat{W}_t. \tag{22}$$

For the entrepreneur, we get:

$$\beta^E\gamma^E\mathrm{E}_t\widehat{C}_{t+1}^E-\left(1+\left(\gamma^E\right)^2\beta^E\right)\widehat{C}_t^E+\gamma^E\widehat{C}_{t-1}^E=\left(1-\beta^E\gamma^E\right)\left(1-\gamma^E\right)\widehat{\lambda}_t^E, \tag{23}$$

$$\beta^E\mathrm{E}_t\left[\lambda_{t+1}^E R_t^L\right]+\mu_t^E R_t^L=\lambda_t^E\Leftrightarrow$$

$$\beta^E\lambda^E R^L\mathrm{E}_t\left[\widehat{\lambda}_{t+1}^E+\widehat{R}_t^L\right]+\mu^E R^L\left(\widehat{\mu}_t^E+\widehat{R}_t^L\right)=\lambda^E\left(\widehat{\lambda}_t^E\right)\Leftrightarrow$$

$$\beta^E R^L\mathrm{E}_t\left[\widehat{\lambda}_{t+1}^E+\widehat{R}_t^L\right]+\left(1-\beta^E R^L\right)\left(\widehat{\mu}_t^E+\widehat{R}_t^L\right)=\widehat{\lambda}_t^E\Leftrightarrow$$

$$\widehat{R}_t^L+\beta^E R^L\mathrm{E}_t\widehat{\lambda}_{t+1}^E+\left(1-\beta^E R^L\right)\widehat{\mu}_t^E=\widehat{\lambda}_t^E, \tag{24}$$

$$W_t=\left(1-\alpha\right)\frac{Y_t}{N_t}\Leftrightarrow$$

$$W\widehat{W}_t=\left(1-\alpha\right)\frac{Y}{N}\left(\widehat{Y}_t-\widehat{N}_t\right)\Leftrightarrow$$

$$\widehat{W}_t = \widehat{Y}_t - \widehat{N}_t, \tag{25}$$

$$\lambda_t^E Q_t^H = \beta^E \mathrm{E}_t \left[\lambda_{t+1}^E \left(Q_{t+1}^H + \alpha\phi \frac{Y_{t+1}}{H_t^E} \right) \right] + \mu_t^E \theta_t E_t \left(Q_{t+1}^H \right)$$

$$\begin{aligned} \lambda^E Q^H \left(\widehat{\lambda}_t^E + \widehat{Q}_t^H \right) &= \beta^E \lambda^E Q^H \mathrm{E}_t \left(\widehat{\lambda}_{t+1}^E + \widehat{Q}_{t+1}^H \right) + \\ &\quad + \alpha\phi\beta^E \lambda^E \frac{Y}{H^E} \mathrm{E}_t \left[\widehat{\lambda}_{t+1}^E + \widehat{Y}_{t+1} - \widehat{H}_t^E \right] + \mu^E \theta Q^H \mathrm{E}_t \left(\widehat{\mu}_t^E + \widehat{\theta}_t + \widehat{Q}_{t+1}^H \right) \end{aligned}$$

$$\begin{aligned} Q^H H^E \left(\widehat{\lambda}_t^E + \widehat{Q}_t^H \right) &= \beta^E Q^H H^E \mathrm{E}_t \left(\widehat{\lambda}_{t+1}^E + \widehat{Q}_{t+1}^H \right) + \\ &\quad + \alpha\phi\beta^E Y \mathrm{E}_t \left[\widehat{\lambda}_{t+1}^E + \widehat{Y}_{t+1} - \widehat{H}_t^E \right] + \frac{\mu^E}{\lambda^E} \theta Q^H H^E \mathrm{E}_t \left(\widehat{\mu}_t^E + \widehat{\theta}_t + \widehat{Q}_{t+1}^H \right) \end{aligned}$$

$$\begin{aligned} \Leftrightarrow \quad & \frac{\beta^E \alpha\phi R^L}{\left(1-\beta^E\right) R^L - \theta \left(1 - \beta^E R^L\right)} \left(\widehat{\lambda}_t^E + \widehat{Q}_t^H \right) = \\ & \beta^E \frac{\beta^E \alpha\phi R^L}{\left(1-\beta^E\right) R^L - \theta \left(1 - \beta^E R^L\right)} \mathrm{E}_t \left(\widehat{\lambda}_{t+1}^E + \widehat{Q}_{t+1}^H \right) + \alpha\phi\beta^E \mathrm{E}_t \left[\widehat{\lambda}_{t+1}^E + \widehat{Y}_{t+1} - \widehat{H}_t^E \right] \\ & + \frac{\mu^E}{\lambda^E} \theta \frac{\beta^E \alpha\phi R^L}{\left(1-\beta^E\right) R^L - \theta \left(1 - \beta^E R^L\right)} \mathrm{E}_t \left(\widehat{\mu}_t^E + \widehat{\theta}_t + \widehat{Q}_{t+1}^H \right) \end{aligned}$$

$$\begin{aligned} \left(\widehat{\lambda}_t^E + \widehat{Q}_t^H \right) &= \beta^E \mathrm{E}_t \left(\widehat{\lambda}_{t+1}^E + \widehat{Q}_{t+1}^H \right) + \left(\frac{1}{R^L} - \beta^E \right) \theta \mathrm{E}_t \left(\widehat{\mu}_t^E + \widehat{\theta}_t + \widehat{Q}_{t+1}^H \right) \\ &\quad + \left[\left(1-\beta^E\right) - \theta \left(\frac{1}{R^L} - \beta^E \right) \right] \mathrm{E}_t \left[\widehat{\lambda}_{t+1}^E + \widehat{Y}_{t+1} - \widehat{H}_t^E \right]. \end{aligned} \tag{26}$$

As for the capital Euler equation, I begin by dividing through by λ_t^E, and use that $Q_t^K = \eta_t^E / \lambda_t^E$:

$$Q_t^K = \alpha \left(1-\phi\right) \beta^E \mathrm{E}_t \left(\frac{\lambda_{t+1}^E}{\lambda_t^E} \frac{Y_{t+1}}{K_t} \right) + \beta^E \left(1-\delta\right) \mathrm{E}_t Q_{t+1}^K \frac{\lambda_{t+1}^E}{\lambda_t^E} + \frac{\mu_t^E}{\lambda_t^E} \theta_t \mathrm{E}_t \left(Q_{t+1}^K \right) \Leftrightarrow$$

$$
\begin{aligned}
Q^K \widehat{Q}_t^K &= \alpha\left(1-\phi\right)\beta^E \frac{\lambda^E Y}{\lambda^E K}\mathrm{E}_t\left[\widehat{\lambda}_{t+1}^E - \widehat{\lambda}_t^E + \widehat{Y}_{t+1} - \widehat{K}_t\right] \\
&\quad +\beta^E\left(1-\delta\right)Q^K\frac{\lambda^E}{\lambda^E}\mathrm{E}_t\left(\widehat{Q}_{t+1}^K + \widehat{\lambda}_{t+1}^E - \widehat{\lambda}_t^E\right) + \frac{\mu^E}{\lambda^E}\theta Q^K \mathrm{E}_t\left[\widehat{\mu}_t^E - \widehat{\lambda}_t^E + \widehat{\theta}_t + \widehat{Q}_{t+1}^K\right]
\end{aligned}
$$

$$
\begin{aligned}
\Leftrightarrow\quad \widehat{Q}_t^K &= \alpha\left(1-\phi\right)\beta^E\frac{Y}{K}\mathrm{E}_t\left[\widehat{\lambda}_{t+1}^E - \widehat{\lambda}_t^E + \widehat{Y}_{t+1} - \widehat{K}_t\right] + \beta^E\left(1-\delta\right)\mathrm{E}_t\left(\widehat{Q}_{t+1}^K + \widehat{\lambda}_{t+1}^E - \widehat{\lambda}_t^E\right) + \\
&\quad +\frac{\left(1-\beta^E R^L\right)}{R^L}\theta E_t\left[\widehat{\mu}_t^E - \widehat{\lambda}_t^E + \widehat{\theta}_t + \widehat{Q}_{t+1}^K\right]
\end{aligned}
$$

$$
\begin{aligned}
\Leftrightarrow\quad \widehat{Q}_t^K &= \left(\alpha\left(1-\phi\right)\beta^E\right)\frac{R^L\left(1-\left(1-\delta\right)\beta^E\right) - \theta\left(1-\beta^E R^L\right)}{\alpha\left(1-\phi\right)R^L\beta^E}\mathrm{E}_t\left[\widehat{\lambda}_{t+1}^E - \widehat{\lambda}_t^E + \widehat{Y}_{t+1} - \widehat{K}_t\right] \\
&\quad +\beta^E\left(1-\delta\right)\mathrm{E}_t\left(\widehat{Q}_{t+1}^K + \widehat{\lambda}_{t+1}^E - \widehat{\lambda}_t^E\right) \\
&\quad +\left(1-\beta^E R^L\right)\frac{1}{R^L}\theta E_t\left[\widehat{\mu}_t^E - \widehat{\lambda}_t^E + \widehat{\theta}_t + \widehat{Q}_{t+1}^K\right]
\end{aligned}
$$

$$
\begin{aligned}
\Leftrightarrow\quad \widehat{Q}_t^K &= \left[1-\beta^E\left(1-\delta\right) - \theta\left(\frac{1}{R^L} - \beta^E\right)\right]\mathrm{E}_t\left[\widehat{\lambda}_{t+1}^E - \widehat{\lambda}_t^E + \widehat{Y}_{t+1} - \widehat{K}_t\right] \\
&\quad +\beta^E\left(1-\delta\right)\mathrm{E}_t\left(\widehat{Q}_{t+1}^K + \widehat{\lambda}_{t+1}^E - \widehat{\lambda}_t^E\right) \\
&\quad +\left(1-\beta^E R^L\right)\frac{1}{R^L}\theta E_t\left[\widehat{\mu}_t^E - \widehat{\lambda}_t^E + \widehat{\theta}_t + \widehat{Q}_{t+1}^K\right], \qquad (27)
\end{aligned}
$$

Finally, we need to log-linearize the investment equation. For this purpose, it is convenient to rewrite the first-order condition $(A.10)$ in a more general way, without imposing the functional form of the investment adjustment cost function. Instead, define $F\left(\frac{I_t}{I_{t-1}}\right) \equiv \frac{\Omega}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2$, so that the law of motion for capital becomes:

$$
K_t = \left(1-\delta\right)K_{t-1} + \left[1 - F\left(\frac{I_t}{I_{t-1}}\right)\right]I_t.
$$

Similarly, we can rewrite the first-order condition for investment $(A.10)$. At the same time, we also divide through by λ_t^E in order to get rid of the multiplier η_t^E. The first-order condition then becomes:

$$\lambda_t^E \vartheta_t^Q = \eta_t^E \left[1 - \frac{\Omega}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2 - \Omega \frac{I_t}{I_{t-1}}\left(\frac{I_t}{I_{t-1}} - 1\right)\right] + \beta^E \Omega E_t \left[\eta_{t+1}^E \left(\frac{I_{t+1}}{I_t}\right)^2 \left(\frac{I_{t+1}}{I_t} - 1\right)\right] \Leftrightarrow$$

$$\vartheta_t^Q = Q_t^K \left[1 - F\left(\frac{I_t}{I_{t-1}}\right) - F'\left(\frac{I_t}{I_{t-1}}\right)\frac{I_t}{I_{t-1}}\right] + \beta^E E_t \left[Q_{t+1}^K \frac{\lambda_{t+1}^E}{\lambda_t^E} F'\left(\frac{I_{t+1}}{I_t}\right)\left(\frac{I_{t+1}}{I_t}\right)^2\right]$$

This more general formulation is in line with the literature, see as an example Christiano *et al.* (2011, RED). Moreover, it facilitates imposing the characteristics of the investment adjustment cost function, which satisfies $F(1) = 0$ and $F'(1) = 0$, just as in the literature. This is useful in the log-linearization, which now becomes:

$$\begin{aligned}
\widehat{\vartheta}_t^Q &= Q^K \left[\widehat{Q}_t^K + \left\{1 - F\left(\frac{I_t}{I_{t-1}}\right)\widehat{-F'}\left(\frac{I_t}{I_{t-1}}\right)\frac{I_t}{I_{t-1}}\right\}\right] \\
&\quad + 2\beta^E Q^K F'(1)(1)^2 E_t\left[\widehat{Q}_{t+1}^K + \widehat{\lambda}_{t+1}^E - \widehat{\lambda}_t^E + F'\left(\widehat{I}_t - \widehat{I}_{t-1}\right) + \widehat{I}_{t+1} - \widehat{I}_t\right] \\
&\quad + \beta^E F''(1)(1)\left(E_t\widehat{I}_{t+1} - \widehat{I}_t\right),
\end{aligned}$$

where the third term on the right hand side appears because I need to take into account the second-order terms when log-linearizing the terms containing $F'(\cdot)$. Imposing $F'(1) = 0$, I get:

$$\widehat{\vartheta}_t^Q = \left[\widehat{Q}_t^K + \left\{1 - F\left(\frac{I_t}{I_{t-1}}\right)\widehat{-F'}\left(\frac{I_t}{I_{t-1}}\right)\frac{I_t}{I_{t-1}}\right\}\right] + \beta^E F''(1)\left(E_t\widehat{I}_{t+1} - \widehat{I}_t\right). \qquad (\bigstar)$$

I now consider the term $\left\{1 - F\left(\frac{I_t}{I_{t-1}}\right)\widehat{-F'}\left(\frac{I_t}{I_{t-1}}\right)\frac{I_t}{I_{t-1}}\right\}$, which gives:

$$-F(1)\left[F'\left(\widehat{I}_t - \widehat{I}_{t-1}\right)\right] - F'(1)\left[F'\left(\widehat{I}_t - \widehat{I}_{t-1}\right) + \widehat{I}_t - \widehat{I}_{t-1}\right] - F''(1)\left[\widehat{I}_t - \widehat{I}_{t-1}\right],$$

which, imposing $F(1) = 0$ and $F'(1) = 0$, collapses to

$$-F''(1)\left[\widehat{I}_t-\widehat{I}_{t-1}\right].$$

Inserting this in (★), I obtain:

$$\widehat{\vartheta}_t^Q=\widehat{Q}_t^K-F''(1)\left[\widehat{I}_t-\widehat{I}_{t-1}\right]+\beta^E F''(1)\left(E_t\widehat{I}_{t+1}-\widehat{I}_t\right). \qquad (★★)$$

Finally, I again impose the functional form for $F(\cdot)$ in order to compute $F''(\cdot)$. With $F\left(\frac{I_t}{I_{t-1}}\right)=\frac{\Omega}{2}\left(\frac{I_t}{I_{t-1}}-1\right)^2$, and recalling that I now need to differentiate with respect to the argument $\frac{I_t}{I_{t-1}}$ (as opposed to in the computations above, where I differentiated wrt. I_t or I_{t-1}), I get that $F'\left(\frac{I_t}{I_{t-1}}\right)=\Omega\left(\frac{I_t}{I_{t-1}}-1\right)$, and in turn that $F''\left(\frac{I_t}{I_{t-1}}\right)=\Omega$. Inserting this in (★★), I arrive at:

$$\widehat{\vartheta}_t^Q=\widehat{Q}_t^K-\Omega\left[\widehat{I}_t-\widehat{I}_{t-1}\right]+\beta^E\Omega\left(E_t\widehat{I}_{t+1}-\widehat{I}_t\right)\Leftrightarrow$$

$$\widehat{Q}_t^K=\widehat{\vartheta}_t^Q+\left(1+\beta^E\right)\Omega\widehat{I}_t-\beta^E\Omega E_t\widehat{I}_{t+1}-\Omega\widehat{I}_{t-1}, \qquad (28)$$

which enters the set of log-linear equations. See also the appendix of Christiano et al. (2011).

We now log-linearize the first-order conditions for the bank. First, $(A.11)$:

$$\mu_t^B=\mathrm{E}_t q_{t,t+1}\left[p_{t-1}R_t^L+(1-p_{t-1})\frac{\tau}{\theta_{t-1}}-R_t^D+\gamma^L(1-\rho_s)\mu_{t+1}^B\right]\Leftrightarrow$$

$$\begin{aligned}\mu^B\widehat{\mu}_t^B &= \beta^P pR^L\left(\mathrm{E}_t\widehat{q}_{t,t+1}+\widehat{p}_{t-1}+\widehat{R}_t^L\right)-\beta^P R^D\left(\mathrm{E}_t\widehat{q}_{t,t+1}+\widehat{R}_t^D\right)+\beta^P\frac{\tau}{\theta}\left(\mathrm{E}_t\widehat{q}_{t,t+1}-\widehat{\theta}_{t-1}\right)\\ &\quad -\beta^P\frac{\tau p}{\theta}\left(\mathrm{E}_t\widehat{q}_{t,t+1}+\widehat{p}_{t-1}-\widehat{\theta}_{t-1}\right)+\beta^P\mu^B\gamma^L(1-\rho_s)\mathrm{E}_t\left(\widehat{q}_{t,t+1}+\mu_{t+1}^B\right)\end{aligned}$$

$$\begin{aligned}\Leftrightarrow\quad &\frac{\mu^B}{\beta^P}\widehat{\mu}_t^B-\mu^B\gamma^L(1-\rho_s)\mathrm{E}_t\mu_{t+1}^B=\left[pR^L-R^D+(1-p)\frac{\tau}{\theta}+\mu^B\gamma^L(1-\rho_s)\right]\mathrm{E}_t\widehat{q}_{t,t+1}\\ &+pR^L\left(\widehat{p}_{t-1}+\widehat{R}_t^L\right)-R^D\widehat{R}_t^D-(1-p)\frac{\tau}{\theta}\widehat{\theta}_{t-1}-\frac{\tau p}{\theta}\widehat{p}_{t-1}, \qquad (29)\end{aligned}$$

and then $(A.12)$:

$$\xi\mu_t^B\frac{x_t}{\theta_t}=-\varpi\mathrm{E}_t q_{t,t+1}\left[R_t^L L_{t-1}-\tau a_{t-1}\right]\Leftrightarrow$$

$$\xi\mu^B\frac{x}{\theta}\left(\widehat{\mu}_t^B+\widehat{x}_t-\widehat{\theta}_t\right)=\beta^P\varpi\tau a\left(\mathrm{E}_t\widehat{q}_{t,t+1}+\widehat{a}_{t-1}\right)-\beta^P\varpi R^L L\left(\mathrm{E}_t\widehat{q}_{t,t+1}+\widehat{R}_t^L+\widehat{L}_{t-1}\right) \tag{30}$$

The log-linear version of $(A.13)$ is:

$$s_t=\rho_s s_{t-1}+\left(1-\rho_s\right)l_t\Leftrightarrow$$

$$s\widehat{s}_t=\rho_s s\widehat{s}_{t-1}+\left(1-\rho_s\right)l\widehat{l}_t\Leftrightarrow$$

$$\widehat{s}_t=\rho_s\widehat{s}_{t-1}+\left(1-\rho_s\right)\widehat{l}_t, \tag{31}$$

and from $(A.14)$:

$$x_t=\left(l_t-\gamma^L s_{t-1}\right)\Leftrightarrow$$

$$x\widehat{x}_t=l\widehat{l}_t-\gamma^L s\widehat{s}_{t-1}\Leftrightarrow$$

$$x\widehat{x}_t=\left(\widehat{l}_t-\gamma^L\widehat{s}_{t-1}\right)l\Leftrightarrow$$

$$\widehat{x}_t=\frac{\widehat{l}_t}{1-\gamma^L}-\frac{\gamma^L\widehat{s}_{t-1}}{1-\gamma^L}. \tag{32}$$

The next set of equations yield:

$$L\widehat{L}_t=l\widehat{l}_t\Leftrightarrow$$

$$\widehat{L}_t=\widehat{l}_t, \tag{33}$$

$$C_t^P+C_t^E+\vartheta_t^Q I_t=Y_t,$$

$$C^P\widehat{C}_t^P + C^E\widehat{C}_t^E + I\left(\widehat{I}_t + \widehat{\vartheta}_t^Q\right) = Y\widehat{Y}_t \Leftrightarrow$$

$$\widehat{Y}_t = \frac{C^P}{Y}\widehat{C}_t^P + \frac{C^E}{Y}\widehat{C}_t^E + \frac{I}{Y}\widehat{I}_t + \frac{I}{Y}\widehat{\vartheta}_t^Q, \tag{34}$$

$$H^P\widehat{H}_t^P + H^E\widehat{H}_t^E = 0, \tag{35}$$

$$\widehat{L}_t = \widehat{D}_t, \tag{36}$$

$$C_t^E + R_{t-1}^L l_{t-1} = Y_t - W_t N_t - \vartheta_t^Q I_t - Q_t^H\left[H_t^E - H_{t-1}^E\right] + x_t + \Phi_t + \Psi_t \Leftrightarrow$$

$$C_t^E + R_{t-1}^L l_{t-1} = Y_t - W_t N_t - \vartheta_t^Q I_t - Q_t^H\left[H_t^E - H_{t-1}^E\right] + x_t + \gamma^L s_{t-1} + (1-p_{t-1})\left(R_{t-1}^L L_{t-1} - \tau a_{t-1}\right) \Leftrightarrow$$

$$\begin{aligned} C^E\widehat{C}_t^E + R^L l\left(\widehat{R}_{t-1}^L + \widehat{l}_{t-1}\right) &= Y\widehat{Y}_t - WN\left(\widehat{W}_t + \widehat{N}_t\right) - I\left(\widehat{\vartheta}_t^Q + \widehat{I}_t\right) - Q^H H^E\left(\widehat{Q}_t^H + \widehat{H}_t^E\right) + \\ &+ Q^H H^E\left(\widehat{Q}_t^H + \widehat{H}_{t-1}^E\right) + x\widehat{x}_t + \gamma^L s\widehat{s}_{t-1} + R^L L\left(\widehat{R}_{t-1}^L + \widehat{L}_{t-1}\right) \\ &- \tau a\widehat{a}_{t-1} - pR^L L\left(\widehat{p}_{t-1} + \widehat{R}_{t-1}^L + \widehat{L}_{t-1}\right) + \tau pa\left(\widehat{p}_{t-1} + \widehat{a}_{t-1}\right) \end{aligned}$$

$$\begin{aligned} C^E\widehat{C}_t^E + R^L l\left(\widehat{R}_{t-1}^L + \widehat{l}_{t-1}\right) &= Y\widehat{Y}_t - WN\left(\widehat{W}_t + \widehat{N}_t\right) - I\left(\widehat{\vartheta}_t^Q + \widehat{I}_t\right) - Q^H H^E\left(\widehat{H}_t^E - \widehat{H}_{t-1}^E\right) \\ &+ x\widehat{x}_t + \gamma^L s\widehat{s}_{t-1} + R^L L\left(\widehat{R}_{t-1}^L + \widehat{L}_{t-1}\right) - \tau a\widehat{a}_{t-1} \\ &- pR^L L\left(\widehat{p}_{t-1} + \widehat{R}_{t-1}^L + \widehat{L}_{t-1}\right) + \tau pa\left(\widehat{p}_{t-1} + \widehat{a}_{t-1}\right), \end{aligned} \tag{37}$$

$$\widehat{ll}_t = \frac{\theta a}{R^L}\left(\widehat{\theta}_t + \widehat{a}_t - \widehat{R}_t^L\right) \Leftrightarrow$$

$$\widehat{l}_t = \widehat{\theta}_t + \widehat{a}_t - \widehat{R}_t^L, \tag{38}$$

$$a_t = \mathrm{E}_t \left[Q_{t+1}^H H_t^E + Q_{t+1}^K K_t \right] \Leftrightarrow$$

$$a_t = E_t \left[Q_{t+1}^H H_t^E + Q_{t+1}^K K_t \right]$$

$$a\widehat{a}_t = Q^H H^E \mathrm{E}_t \left(\widehat{Q}_{t+1}^H + \widehat{H}_t^E \right) + Q^K K \mathrm{E}_t \left(\widehat{Q}_{t+1}^K + \widehat{K}_t \right) \Leftrightarrow$$

$$\widehat{a}_t = \frac{Q^H H^E}{Q^H H^E + Q^K K} \mathrm{E}_t \left(\widehat{Q}_{t+1}^H + \widehat{H}_t^E \right) + \frac{Q^K K}{Q^H H^E + Q^K K} \mathrm{E}_t \left(\widehat{Q}_{t+1}^K + \widehat{K}_t \right) \Leftrightarrow$$

$$\widehat{a}_t = \frac{Q^H H^E}{Q^H H^E + Q^K K} \mathrm{E}_t \left(\widehat{Q}_{t+1}^H + \widehat{H}_t^E \right) + \frac{Q^K K}{Q^H H^E + Q^K K} \mathrm{E}_t \left(\widehat{Q}_{t+1}^K + \widehat{K}_t \right), \quad (39)$$

$$Y_t = A_t \left[N_t \right]^{1-\alpha} \left\{ \left[H_{t-1}^E \right]^{\phi} \left[K_{t-1} \right]^{1-\phi} \right\}^{\alpha} \Leftrightarrow$$

$$Y\widehat{Y}_t = AN^{1-\alpha} \left(H^E \right)^{\alpha\phi} K^{\alpha(1-\phi)} \left[\widehat{A}_t + (1-\alpha) \widehat{N}_t + \alpha\phi \widehat{H}_{t-1}^E + \alpha (1-\phi) \widehat{K}_{t-1} \right] \Leftrightarrow$$

$$\widehat{Y}_t = \widehat{A}_t + (1-\alpha) \widehat{N}_t + \alpha\phi \widehat{H}_{t-1}^E + \alpha (1-\phi) \widehat{K}_{t-1}. \quad (40)$$

The law of motion for capital is

$$K_t = (1-\delta) K_{t-1} + \left[1 - \frac{\Omega}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 \right] I_t,$$

which can be written

$$K_t = (1-\delta) K_{t-1} + \left[1 - F \left(\frac{I_t}{I_{t-1}} \right) \right] I_t.$$

Imposing that $F(1) = 0$, I now get the following log-linear law of motion:

$$K\widehat{K}_t = (1-\delta) K\widehat{K}_{t-1} + I\widehat{I}_t - F \left(\frac{I}{I} \right) I \left[F' \left(\widehat{I}_t - \widehat{I}_{t-1} \right) + \widehat{I}_t \right] \Leftrightarrow$$

$$\widehat{K}_t = (1-\delta)\,\widehat{K}_{t-1} + \frac{I}{K}\widehat{I}_t - 0 \Leftrightarrow$$

$$\widehat{K}_t = (1-\delta)\,\widehat{K}_{t-1} + \delta\widehat{I}_t. \tag{41}$$

The final equations are:

$$R_t^L = \psi R_t^D \Leftrightarrow$$

$$R^L \widehat{R}_t^L = \psi R^D \widehat{R}_t^D \Leftrightarrow$$

$$\widehat{R}_t^L = \widehat{R}_t^D, \tag{42}$$

$$p_t = \Xi + \varpi\,(\theta_t - \theta) \Leftrightarrow$$

$$p\widehat{p}_t = \varpi\theta\widehat{\theta}_t, \tag{43}$$

$$\eta_t^E = \lambda_t^E Q_t^K \Leftrightarrow$$

$$\widehat{\eta}_t^E = \widehat{\lambda}_t^E + \widehat{Q}_t^K, \tag{44}$$

$$q_{t,t+1} = \beta^P \frac{\lambda_{t+1}^P}{\lambda_t^P} \Leftrightarrow$$

$$\widehat{q}_{t,t+1} = \widehat{\lambda}_{t+1}^P - \widehat{\lambda}_t^P. \tag{45}$$

The log-linearized system of equations consists of 27 equations (19)-(45) in 27 variables.

Appendix to "The Effects of Fiscal Policy in a Small Open Economy with Fixed Exchange Rates: The Case of Denmark"

Abstract

This appendix contains supplemental material for Chapter 4; "The Effects of Fiscal Policy in a Small Open Economy with Fixed Exchange Rates: The Case of Denmark".

1 Additional Tables and Figures

Table A1: Specification tests for the VAR

# lags	Akaike	Schwartz	HQ	Portmanteau	Normality	Likelihood Ratio
1	**-47.80**	**-46.79**	**-47.39**	339.29 (0.0049)	24.18 (0.0071)	N/A*
2	**-47.80**	-46.16	-47.17	**299.62** (0.0172)	**18.43** (0.0482)	**29.01** (0.2635)
3	-47.66	-45.39	-46.74	275.41 (0.0122)	25.61 (0.0043)	32.75 (0.1374)
4	-47.58	-44.69	-46.41	267.28 (0.0010)	20.61 (0.0240)	50.53 (0.0018)
5	-47.56	-44.24	-46.33	231.63 (0.0027)	24.78 (0.0058)	29.34 (0.2500)
6	-47.69	-43.54	-46.00	211.79 (0.0007)	32.39 (0.0003)	29.84 (0.2303)

Note: The first 3 columns simply report the information criteria (HQ denotes Hannan-Quinn). The last 3 columns report test statistics, with p-values in brackets. In calculating these tests, we have included also the exogenous variable (US GDP) in the block of endogenous variables, as suggested by Lindé (2003). For the LR test of 1 lags versus 2, we encounter the problem that the determinant of the variance-covariance matrix of the VAR with 1 lag is too close to zero, so that the test statistic takes on a value of -1126.7, which is not very meaningful. We therefore discard this test.

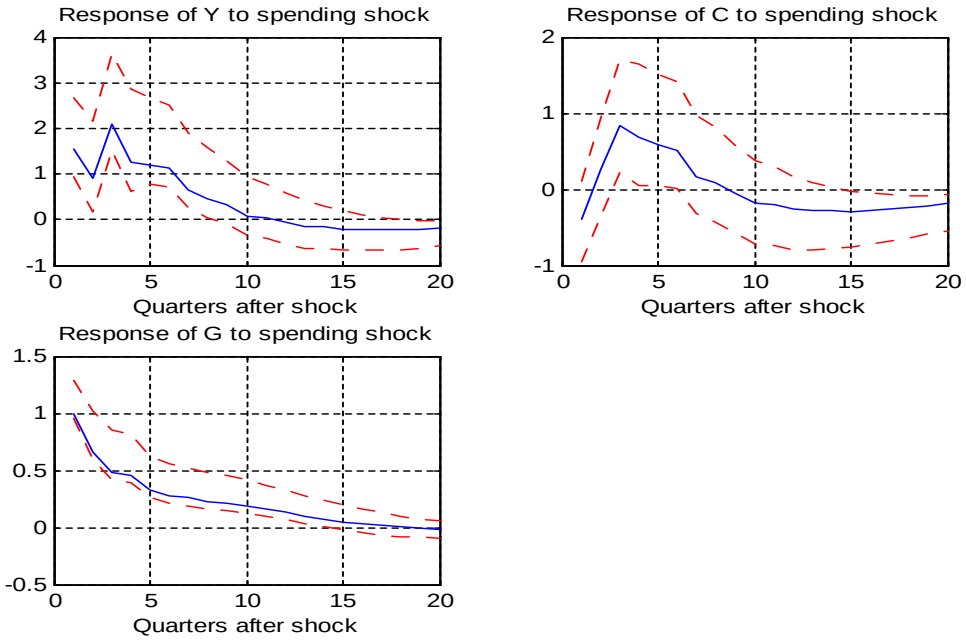

Figure A.1: The effects of a shock to government spending, specification with 4 lags instead of 2, deterministic trend.

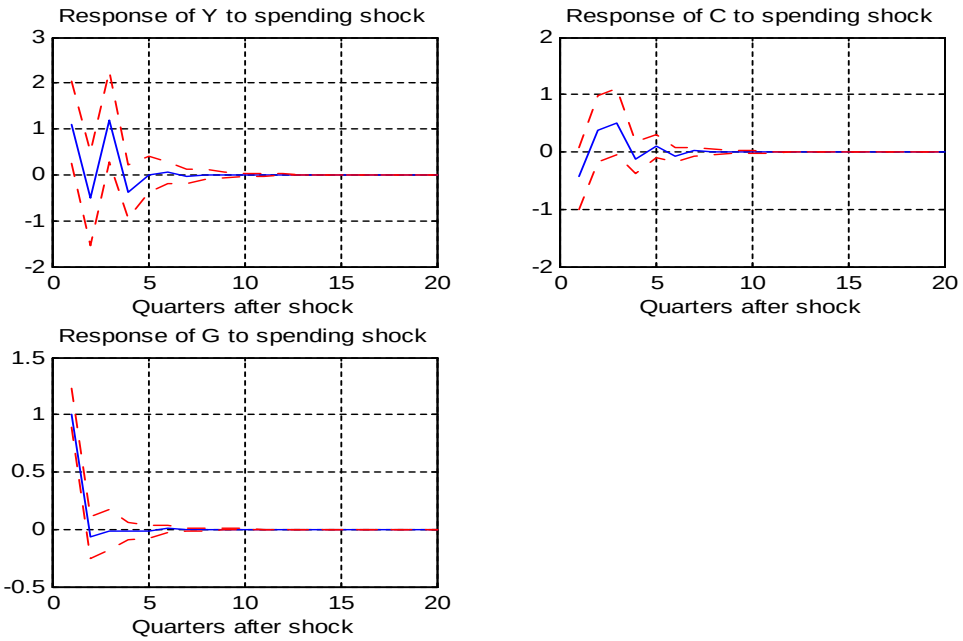

Figure A.2a: The effects of a shock to government spending, specification with stochastic trend.

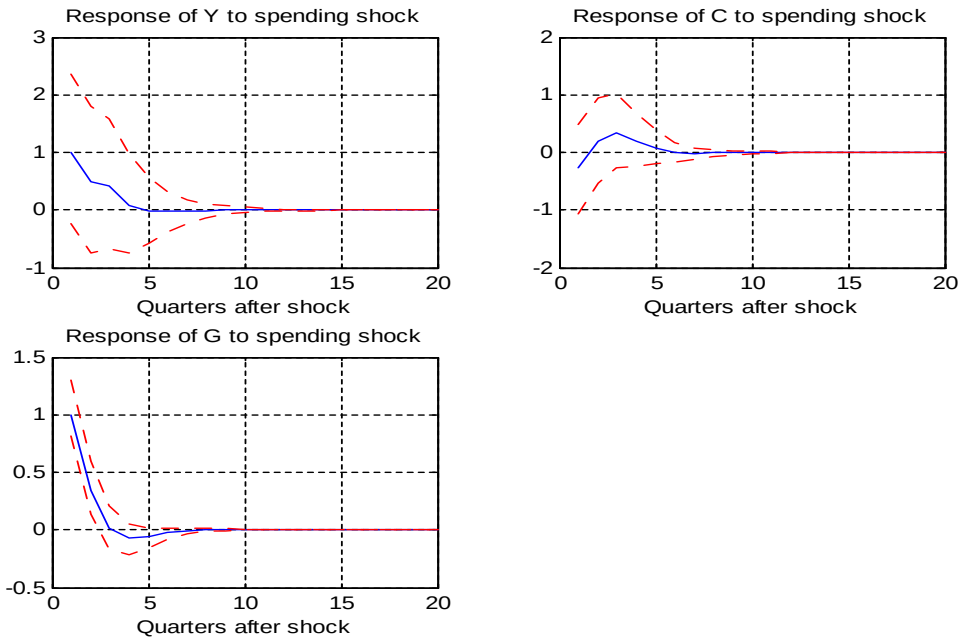

Figure A.2b: The effects of a shock to government spending, specification with stochastic trend, impulses HP-filtered with $\lambda = 1$.

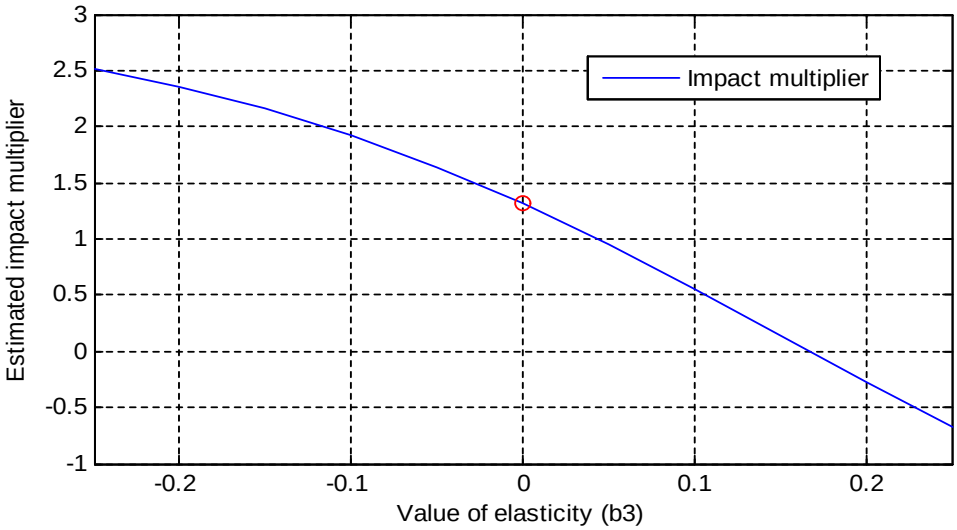

Figure A.3: The sensitivity of the estimated impact multiplier of government spending to the elasticity of government spending to output. The red dot indicates our baseline estimate of $b_3 = 0$.

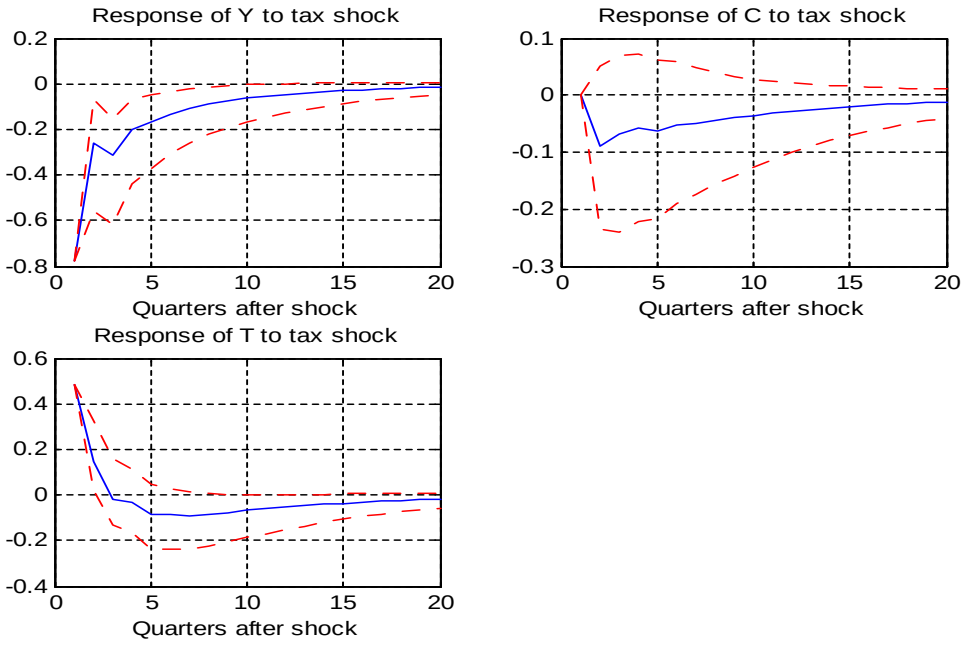

Figure A.4: The effects of an increase in net tax revenues.

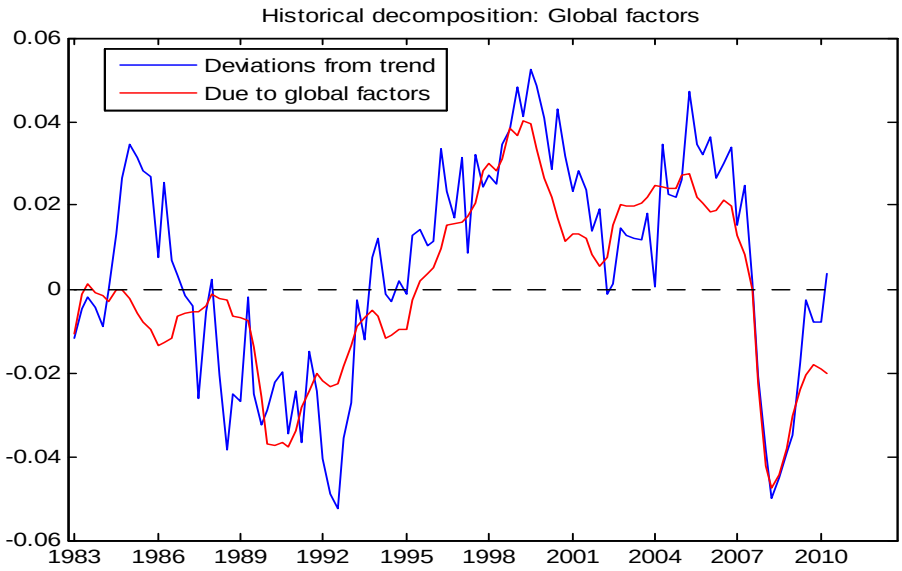

Figure A.5: Historical decomposition. The blue line shows deviations in output growth relative to its trend growth. The red line shows the share explained by deviations in the growth rate of US GDP relative to its trend.

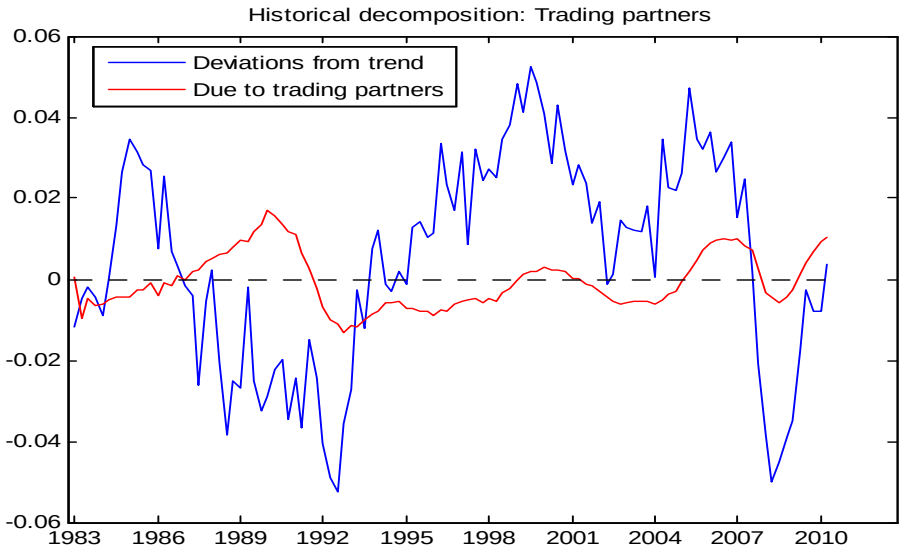

Figure A.6: Historical decomposition. The blue line shows deviations in output growth relative to its trend growth. The red line shows the share explained by deviations in the growth rate of F_t relative to its trend.

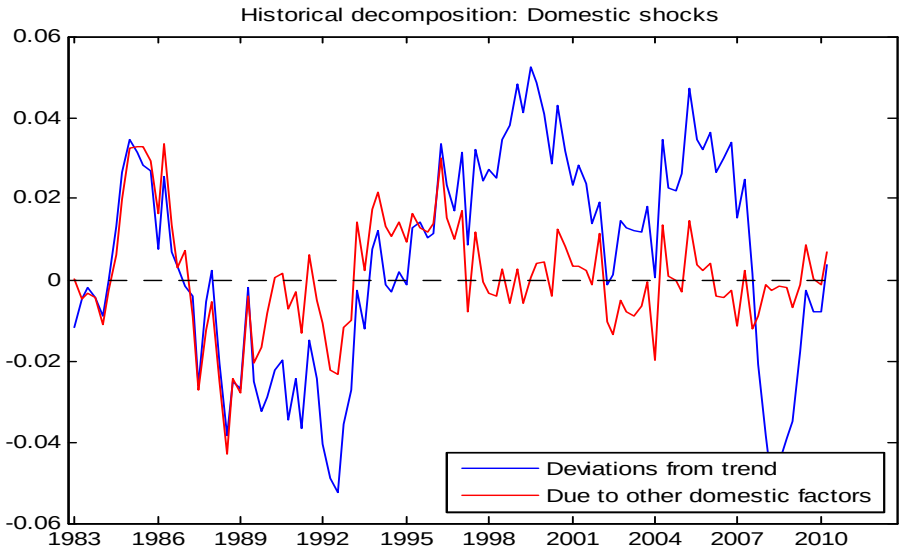

Figure A.7: Historical decomposition. The blue line shows deviations in output growth relative to its trend growth. The red line shows the share explained by deviations from trend in other domestic variables than government spending.

2 Computing the Output Elasticity of Taxes

This appendix provides a detailed account of how we obtain an estimate of the elasticity of taxes to changes in output, as employed in subsection 3.4.

We decompose the total tax revenue into four categories: Income taxes, corporate taxes, indirect taxes, and social contributions. We then obtain the elasticity of each of these types of taxes from a study by the OECD (Girouard and André, 2005). Moreover, recall that we use a measure of taxes net of transfers. We therefore also need an estimate of the elasticity of transfers to changes in output. Finally, we weigh the elasticities together according to their average share of total net revenues during our sample period.

The tax elasticities estimated by Girouard and André (2005) for Denmark are the following: Income taxes; 1.0. Indirect taxes; 1.0. Corporate taxes; 1.6. Social contributions; 0.7. We refer the reader to that study for further details.

As for transfers, we follow Girouard and André and assume that unemployment benefits is the only type of transfers that contains a significant cyclical component. We therefore compute the sample average share of unemployment-related transfers to total transfers, and multiply this share by the elasticity of unemployment with respect to the output gap, which Girouard and André estimate to -7.9 for Denmark.

As noted in the main text, we arrive at an output elasticity of net tax revenues of 2.09.

3 Computing Variance Decompositions

To perform the variance decompositions, we rely on results from spectral analysis. Recall that any covariance-stationary time series can be represented equally well in the frequency domain as in the time domain (Hamilton, 1994). In the frequency domain, the spectral density of the process is a measure of the share of the overall variance of the process accounted for at various frequencies. If the spectral density is high at low frequencies, much of the variation of the process can be interpreted as long-term movements in the data, perhaps reflecting an underlying trend.

For our VAR-model outlined in section 2, the spectral density of X_t at any frequency ω is given by:[1]

$$S_X(\omega) = \left[I - A\left(e^{-i\omega}\right)\right]^{-1} CC' \left[I - A\left(e^{-i\omega}\right)'\right]^{-1} \tag{C.1}$$

Here, A is the coefficient matrix from the VAR regression, and I is the identity matrix. C is the matrix linking the reduced-form residuals of the VAR-regression to the structural shocks of the model, with the property $CC' = V$, as described in subsection 2.2. i denotes complex i, so that $i^2 = -1$. Thus, the function assigns to any frequency ω a square matrix of complex numbers. However, as pointed out in Hamilton (1994), the complex part of the diagonal elements in this matrix will in fact be zero. The spectral density at frequency ω for each of the variables in Y_t is given exactly by these (real and non-negative) diagonal elements of the matrix.

We want to compute the variance of each of the variables in X_t that is accounted for by each of the shocks in ε_t. Recall that in the expression for the spectral density, $CC' = V$ denotes the variance-covariance matrix when all the shocks are 'turned on'. Following Altig *et al.* (2005), in order to compute the spectral density of X_t when only the j'th shock ($j = 1, .., 4$) is turned on, we can replace CC' by CI_jC', where I_j is a square matrix of zeros, except for a unit entry in the j'th diagonal element. In other words,

$$S_X^j(\omega) = \left[I - A\left(e^{-i\omega}\right)\right]^{-1} CI_jC' \left[I - A\left(e^{-i\omega}\right)'\right]^{-1} \tag{C.2}$$

denotes the spectral density of X_t when only shock j is active.

[1] See Hamilton (1994) or Altig *et al.* (2005).

As the spectral density for variable k is given by the k'th diagonal element of $S_X(\omega)$, we can then compute the fraction of the variance of the k'th variable accounted for by the j'th shock at frequency ω as:

$$var_{k,j}(\omega) = \frac{\left[S_X^j(\omega)\right]_{kk}}{\left[S_X(\omega)\right]_{kk}} \tag{C.3}$$

- where $[M]_{kk}$ denotes element (k,k) of matrix M. Observe that by construction:

$$\sum_{j=1}^{4} var_{k,j}(\omega) = 1 \tag{C.4}$$

Having decomposed the variance of any variable at any frequency, we can then sum the variance ratios over various frequency bands, for example the business cycle frequencies, and see how important each shock is for each variable within these frequency bands.

References

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