Essays on Macro-Finance and Monetary Policy

by

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Preface and Editorial Remark

This thesis consists of five individual papers and a summary. Each paper is self contained.

This has the advantage that each paper can be read independently, but also the disadvantage that notation can shift between paper to paper and that repetitions of arguments are likely to arise.

I also apologise for the use of both American-English and English-English.

The first chapter refers to "additional tables and results are available upon request from the authors". I have included this information as a seperate chapter.

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Summary

The two overall themes in this thesis are monetary policy, chapter 1, 2, and 3 and the interrelationships between macroeconomics on business-cycle frequencies and the bond market, chapter 3 and 4.

The first chapter, **"Real-Time Effects of Central Bank Interventions in the Euro Market"**, is joint work with Rasmus Fatum. The chapter investigates the real-time effects of foreign exchange intervention on the Danish kroner - Euro exchange rate using official intraday intervention data provided by the Danish central bank. Our analysis employs the two-step weighted least squares estimation procedure of (Andersen, Bollerslev, Diebold, and Vega 2003) and an array of robustness tests. We find that intervention exerts a statistically and economically significant influence on exchange rate returns when the direction of intervention is consistent with monetary policy stance, thereby illustrating that sterilized intervention is not an independent policy instrument. We also show that the exchange rate does not adjust instantaneously to the unannounced and discretionary interventions under study.

Endogenity plague intervention studies: Does the exchange rate move in response to the intervention or does the intervention occurs due to movements in the exchange rate? Our real-time intervention study is well-suited for adressing this issue as we know the exact timing of all interventions.

We control for endogeneity by estimating a reaction function for the central bank. We use the residuals from this reaction function as a proxy for unexpected intervention which we then use for obtaining an estimate of the affects of intervention on the exchange rate that is free of simultaneity bias. We find that some endogeneity is present even in our intraday analysis though we also find the resulting bias is too small to affect the results.

We lastly include macroeconomic news announcement into our regression and show that the relative influcence of intervention is similar to the relative influence of most macro surprises.

While the study of intervention effects on exchange rate *levels* perhaps is the first and foremost interest for the central banker, intervention effects on the bid-ask exchange rate spread are of great interest for market microstructure theorist and this is the theme for the second chapter, **"The Intraday Effects of Central Bank Intervention on Exchange Rate Spreads"**. This chapter investigates the intraday effects of sterilized foreign exchange intervention on exchange rate spreads using the same data set as used in chapter 1. The main result is that intervention

purchases and intervention sales both exert a significant influence on exchange rate spreads but in opposite directions: Intervention sales of EUR, on average, reduce the spread while intervention purchases of EUR, on average, increase the spread. Apparently, the market's uncertainty over the true exchange rate is higher when the DN has stepped in to defend the DKK, while the opposite occurs when the DN signals that the DKK is strong. We also show that the significant and asymmetric effects of intervention purchases and sales stem from intervention carried out on "normal" days in terms of exchange rate volatility while intervention appears to be overlooked by the market when the market is volatile.

Clearly, the findings illustrate the necessity of distinguishing between intervention purchases and intervention sales when assessing the influence of intervention on exchange rate spreads, and likewise to distinguish between periods of high/low volatility. These new results pave the way for an existing road ahead of the microstructure theorist.

Interventions in the foreign exchange market is only one policy instrument available for central banks. A more widely used instrument is the short rate of interest and this instrument plays a key role in the subsequent two chapters.

The first of these chapters, "**Monetary Policy, Housing, and Macroeconomic Effects of Changes in Long Interest Rates**", addresses the wide belief among practitioners and academics alike that aggregate demand depends upon more interest rates than (expected future) short rates. Standard models in the academic literature do not have room for a monetary policy transmission mechanism through long interest rates, and this is what this chapter tries to bring through.

One possible link between changes in long rates and the aggregate economy is the mortgage market and this is what the model exploits. The representative consumer derives utility non-seperable from both a consumer goods index and an index of housing goods. A financial constraint makes the representative consumer to issue bonds of long maturities equal to the value of new housing stock. This financial constraint contributes with sufficient requirements to move the economy away from a single interest rate model of aggregate demand determination in which monetary policy can be specified solely in terms of a path of future policy rates. The financial constraint makes the user cost of housing to be dependent upon the long interest rate, and introduces an endogenously determined wedge between bonds of different maturities. As a result, the central bank can not determine long rates perfectly through changes in short rates, and as the consumer can not bypass higher cost of financing housing goods by moving down the yield curve, the model clears the road in a consistent way for a multiple interest rate determination of aggregate demand.

I find that a one period higher than expected monetary policy rate in this framework raises the long interest rate, which in turn depresses total output through increasing costs of financing.

The emphasis on the role of the yield curve in monetary policy especially arose during the period from 2004 to 2006, which saw a decline in the long end of the yield curve while the Fed raised the short end, the so called bond yield conundrum. This for some unusual behaviour of long interest rates has partly been contributed to unusual low term premia, which in turn has sparked interest in modeling risk premia in bonds, and this is the topic of the third chapter in my thesis titled, "**Is the Bond Premium Puzzle Really a Puzzle?**".

This chapter tries to resurrect the troubles DSGE models have in modeling empirical plausible bond risk premia. Standard models within the DSGE literature either generates implausible low risk premia, (Smets and Wouters 2003), (Lawrence, Eichenbaum, and Evans 2005),(Rudebusch, Sack, and Swanson 2007), or if the model is able to fit empirical moments for bond premia, it tend to do so at the expense of the macroeconomic side of their model the implication being too variable of the macroeconomic variables, (Hordahl, Tristani, and Vestin 2006), (Ravenna and Seppala 2006), and the discussion in (Rudebusch and Swanson 2007).

The chapter addresses two interrelated aspects of previous DSGE models which are likely to complicate the modeling of risk premia. Firstly, the model in this chapter provides closed form solution for bond prices thus bypassing higher order approximations to get non-zero and non-constant bond premia. Higher order terms are small by definition, so perhaps the modest success of the DSGE literature in fitting bond risk premia is due to the approximations and not bad models per se. Secondly, I do not ask what the implication are for bond risk premia in a model set up for the study of the dynamics of macroeconomic variables, as the question implicitly is posed in the majority of the existing DSGE models. I instead analyse whether I can provide a micro foundation for models which are known to generate plausible risk premia namely the affine term structure models originated in finance.

I conclude the bond premium puzzle is not a puzzle ones closed form solutions for bond prices are formed. I find the model is able to give an explanation to the decline in bond yields and term premia from the Volker period until the present and to provide a coherent explanation for the unusual low premia during 2004-2006.

Resumé

Denne afhandling omhandler monetær politik, kapitel 1, 2 og 3, og sammenhængen imellem makroøkonomi på mellemlang sigt og obligationsmarkeder, kapitel 3, og 4.

Det første kapitel, **"Real-Time Effects of Central Bank Interventions in the Euro Market"**, er udarbejdet sammen med Rasmus Fatum. Kapitlet analyserer real tids effekter af steriliseret intervention på kroner-euro valutamarkedet ved hjælp af høj-frekvent data for interventioner udført af Nationalbanken. Vores analyse bruger *two-step weighted least squares* estimationsmetoden fra (Andersen, Bollerslev, Diebold, and Vega 2003) og en række robusthedstest. Vi finder, at interventioner påvirker valutakursafkast på en både økonomisk og statistisk signifikant måde, når retningen af interventionen er konsistent med den førte pengepolitik, således at interventioner ikke er et uafhængigt pengepolitisk instrument. Vi viser også, at valutakursen først påvirkes 20-25 minutter efter den uannonceret og diskretionære intervention.

Interventionsstudier er plaget af endogenietet: Bevæger valutakursen sig på grund af interventionen eller bliver interventionen udført på grund af valutakursbevægelser? Vi er med vores data sæt i stand til at adressere dette spørgsmål, idet vi kender det eksakte tidspunkt for udførelsen af alle interventionerne.

Vi kontrollerer for endogenietet ved at estimere en reaktionsfunktion for centralbanken. Vi bruger residualerne fra denne model som en approksimation til den uforventede intervention fri for simultanitetsbias. Vi bruger efterfølgende denne approksimation i vores økonometriske model omtalt tidligere. Vi finder, at endogenietet er til stede i vores interdagsstudie, men at vores estimater af regressionskoefficienterne er ikke kvalitativt påvirket. Endelig inkluderer vi nyheder om makroøkonomiske variable i vores regression og viser, at effekter af interventioner på valutakursen er sammenlignelige med effekter af makroøkonomiske nyheder.

Imens analyser af interventionseffekter på valutakursniveauer måske er det mest interessante aspekt ved interventionsinstrumentet for en central bank, så er effekten af en intervention på bid-ask spread af stor interesse for *market microstructure* teorien, og dette er emnet for det andet kapitel, **"The Intraday Effects of Central Bank Intervention on Exchange Rate Spreads"**. Dette kapitel analyserer intradagseffekterne af steriliserede interventioner i DKK/EUR valutakursmarkedet på forskellen imellem bid - og ask valutakurspriser ved hjælp af det samme datasæt som brugt i kapitel 1. Hovedresultatet er, at interventionskøb og interventionssalg begge påvirker *bid-ask spread* signifikant, men i forskellige retninger: Interventionssalg af EUR reducerer, i gennemsnit, spreadet imens interventionskøb af EUR

i gennemsnit udvider spreadet. Markedsusikkerheden omkring den "sande" valutakurs er åbenbart højere, når DN forsvarer valutakursen, imens det modsatte er tilfældet, når DN signallerer at DKK/EUR valutakursen er stærk. Vi viser også, at den signifikante og asymmetriske effekt af interventionskøb og interventionssalg stammer fra interventioner udført på "normale" dage, definteret som dage med lav valutakursvolatilitet, imens interventioner der er udført på dage, hvor valutakursmarkedet er volatilt, bliver overset målt med bid-ask spreadet.

Disse resultater illustrerer nødvendigheden af at skelne imellem interventionskøb og interventionssalg, når man analyserer påvirkningen på valutakursspread samtidig med, at analysen bør skelne imellem høj - og lav volatilitet. Disse nye resultater står tilbage som udfordringer for teoretikere, der arbejder med mikrostruktursteori på valutakursmarkedet.

Interventioner i valutamarkedet er kun et politik instrument, som central banker kan benytte til at opnå deres politik mål. Et mere udbredt instrument er den korte rente, og dette instrument er fundamentet bag de efterfølgende to kapitler.

Det første af disse kapitler, **"Monetary Policy, Housing, and Macroeconomic Effects of Changes in Long Interest Rates"**, adresserer opfattelsen i blandt økonomer, at den samlede efterspørgsel ikke kun afhænger af korte eller forventede fremtidige renter, men også af lange renter. Standard modeller i den akademiske litteratur har ikke en monetær transmissions mekanisme igennem en eller flere lange renter, og det er hvad dette kapitel omhandler.

En mulig sammenhæng imellem ændringer i lange renter og den samlede økonomi er realkreditmarkedet, og det er hvad modellen i dette kapitel udnytter. Den repræsentative forbruger får nytte ikke-separabelt fra både et forbrugsindeks og et boligindeks. Modellen begrænser forbrugerens valg af finansieringsmuligheder til udstedelse af obligationer med en lang løbetid samtidigt med, at al ny bolig skal finansieres fuldt ud. Disse egenskaber ved modellen opnås igennem en finansiel begrænsning, som introducerer tilstrækkelige betingelser til at bevæge økonomien væk fra en samlet efterspørgselsfunktion, som kun afhænger af (fremtidige forventede) korte renter. Den finansielle begrænsning gør forbrugsomkostningen af bolig afhængig af den lange rente og introducerer en endogent bestemt kile imellem obligationer med forskellig løbetider. Centralbanken kan som et resultat heraf ikke sætte den lange rente perfekt igennem den korte rente, og forbrugeren må nødvendigvis bruge den lange rente til finansiering af bolig, selvom den korte rente muligvis er lavere. Hermed bestemmes den samlede efterspørgsel i modellen på en konsistent måde blandt andet af den lange rente.

Jeg finder, at en højere end forventet kort pengepolitisk rente i en periode påvirker den lange rente positivt. Denne højere lange rente presser den samlede efterspørgsel nedad igennem højere omkostninger i forbindelse med finansiering af huskøb.

Den større fokus på sammenhængen imellem rentekurven og monetær politik opstod især i perioden 2004 til 2006, som oplevede en lavere lang rente selvom den amerikanske central bank strammede pengepolitikken igennem højere pengepolitiske renter; det såkaldte *bond yield conundrum.* Denne for nogen usædvanlige udvikling i rente kurven er blevet forklaret med en udsædvanlig lav risiko præmie på obligationer, hvilket har medfødt en interesse for at modellere risiko præmier på obligationsmarkedet, og dette er temaet for det tredje kapitel med titlen **"Is the Bond Premium Puzzle Really a Puzzle?"**

Dette kapitel prøver at råde bod på de problemer, som repræsentativ agent DSGE modeller har med at generere empiriske første og anden momenter af risikopræmier på obligationsmarkeder. Standard modeller fra DSGE litteraturen genererer enten meget lave risiko præmier i forhold til data, se eksempelvis (Smets and Wouters 2003), (Lawrence, Eichenbaum, and Evans 2005), (Rudebusch, Sack, and Swanson 2007), eller hvis modellen er i stand til at generere empirisk plausible momenter, så er omkostningen ved dette en makroøkonomisk side af modellen, som genererer for variable momenter af makroøkonomiske variable, såsom inflation, forbrug og korte renter, se eksempelvis (Hordahl, Tristani, and Vestin 2006), (Ravenna and Seppala 2006), og diskussionen i (Rudebusch and Swanson 2007).

Kapitlet adresserer to relaterede aspekter af eksisterende DSGE modeller, som komplicerer modelleringen af risiko præmier. For det første giver modellen lukketformsløsninger for obligationspriser hvormed at højere-ordens approksimationer undgås. Disse højereordensled er per definition små, så måske er de mindre gode resultater for modellering af risikopræmier for den eksisterer litteratur i højere grad bundet i højere-ordens approksimationer og i mindre grad i modellen. For det andet så analyserer jeg ikke model implikationerne for risikopræmier i en model bygget til at analysere dynamikken for makroøkonomiske variable, som spørgsmålet implicit er stillet i størstedelen af den eksisterende litteratur. Jeg analyserer i stedet for, om jeg kan give et mikrofundament til modeller, der er i stand til at genere plausible risiko præmier - de såkaldte affine rentekurvsmodeller fra den finansielle litteratur.

Jeg konkluderer, at *the bond premium puzzle* ikke er et puzzle, når modellen er i stand til at give lukketformsløsninger for obligationspriser. Modellen giver samtidig også en forklaring på de empirisk observerede faldende nominelle renter og risikopræmier fra Volker perioden i starten af 80erne til nu, og en sammenhængende forklaring på den usædvanligt lave risikopræmie i perioden 2004-2006.

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Real-Time Effects of Central Bank Interventions in the Euro Market

Rasmus Fatum* and Jesper Pedersen†

Abstract

This paper investigates the real-time effects of sterilized foreign exchange intervention using official intraday intervention data provided by the Danish central bank. Our analysis employs a two-step weighted least squares estimation procedure. We control for macro surprises, address the issue of endogeneity, and carry out an array of robustness tests. Our main result is that intervention exerts a significant influence on exchange rate returns only when the direction of intervention is consistent with the monetary policy stance, thereby illustrating that sterilized intervention is not an independent policy instrument.

Key words: Foreign Exchange Intervention; Intraday Data; ERM II

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1 Introduction

This paper investigates the real-time (intraday) effects of Danish central bank intervention in the Euro (EUR) market.[1] The interventions under study are carried out under the provisions of the Exchange Rate Mechanism (ERM II). Proprietary data on official, intraday intervention transactions, provided by the Danish central bank (Danmarks Nationalbank, henceforth DN), facilitates our investigation. Our investigation employs the time-series econometrics of Andersen and Bollerslev (1998) and Andersen, Bollerslev, Diebold, and Vega (2003).

It is very rare that a central bank makes official intraday intervention data available for research.[2] In fact, until now only the Bank of Canada and the Swiss National Bank have made such data available. It has, however, been a decade or more since the Bank of Canada and the Swiss National Bank last intervened.[3] By contrast, Denmark is currently pursuing an active intervention policy, i.e. DN carries out sterilized interventions on a discretionary basis when deemed necessary. The DN intraday intervention data, therefore, constitutes a unique opportunity to learn about the real-time effects of foreign exchange intervention carried out by a central bank that is currently intervening.

We investigate whether sterilized intervention is an independent policy instrument or whether it only works when backed by consistent monetary policy (e.g. sterilized intervention sales of domestic currency during a period of domestic monetary policy easing). As discussed in Ghosh (2008), most industrialized countries that intervene use monetary policy (e.g. changing interest rates) towards achieving domestic goals such as controlling inflation or stimulating growth, while they use sterilized intervention (e.g. sell domestic currency and use open market operations to off-set any impact on money supply or policy rates) towards achieving exchange rate goals. Sterilized rather than unsterilized intervention is used since monetary and exchange rate goals are not always compatible. To illustrate, suppose a country experiences an undesirable appreciation of its currency as well as inflationary pressure. The former would call for

[1] See Humpage (2003), Neely (2005), and Sarno and Taylor (2001) for recent surveys of the intervention literature.

[2] Studies of intraday effects of intervention by G3 and other countries use newswire reports of interventions as a proxy for actual interventions (see, for example, Dominguez (2003)) or impose assumptions regarding the unknown intraday timing of interventions. Fischer (2006) illustrates the lack of accuracy of newswire reports in the context of Swiss interventions.

[3] See Beattie and Fillion (1999) and Fischer and Zurlinden (1999), respectively, for early studies of the Bank of Canada and the Swiss National Bank intraday intervention data sets.

lower interest rates (monetary policy easing) while the latter would call for higher interest rates (monetary policy tightening). Monetary policy cannot possibly meet both goals. Instead, a country can pursue sterilized intervention towards the exchange rate goal and at the same time use monetary policy towards the domestic goal. As a result, it is not uncommon to observe sterilized interventions that are monetary policy inconsistent.[4] However, using sterilized intervention to pursue exchange rate goals independently of monetary goals and the associated monetary policy stance implicitly assumes that sterilized intervention constitutes an independent policy instrument.

The period under study encompasses a structural break and two distinctly different sub-periods in terms of monetary policy. The official Danish monetary policy rate as well as the key ECB interest rates were all held constant during the first sub-period, whereas during the second sub-period these rates were lowered in tandem by a total of 125 basis points. Furthermore, the Danish deposit and lending rates were lowered by a total of 140 basis points (see Nationalbank (2002), Nationalbank (2003b) and Nationalbank (2004)) during the second sub-period. Accordingly, the first sub-period is associated with a neutral Danish monetary policy stance while the second sub-period constitutes a period of Danish monetary policy easing, both in absolute and relative terms. It follows that the interventions of the first sub-period are neither consistent nor inconsistent with monetary policy, while the interventions that occur during the second sub-period are either monetary policy consistent or inconsistent. We investigate whether monetary policy consistency matters for the effectiveness of intervention by estimating the effect of intervention separately across the two sub-periods and separately across consistent and inconsistent interventions. Our main result is that intervention only works when the direction of intervention is consistent with the monetary policy stance. This illustrates that even though sterilized intervention is by construction detached from monetary policy and, therefore, it is possible to use sterilized intervention to pursue an exchange rate goal while using monetary policy towards a domestic goal that may conflict with the exchange rate goal, doing so is a futile exercise. In other words, our results show that, in reality, sterilized intervention is not an independent policy instrument; sterilized intervention only works in conjunction with monetary policy.

We also assess whether consistency with either official or what we

[4]Ghosh (2008) points to intervention in the DEM/USD exchange rate market in support of the USD around the February 1987 Louvre Agreement when US interest rates were held steady and German interest rates were increased as an example of a monetary policy inconsistent intervention episode.

refer to as "de facto" exchange rate policy matter for the effectiveness of intervention. Contrary to our findings regarding monetary policy, we find no evidence that exchange rate policy consistency is a condition for effective intervention.

In addition, we address the issue of endogeneity. Doing so reveals that some endogeneity is present even in our intraday analysis of intervention. However, we also show that the resulting simultaneity bias is too small to affect our results. We also extend our analysis to incorporate Danish, German, and Euro-area macro surprises. This allows us to get a sense of the relative influence of intervention. We show that the magnitudes of the coefficient estimates associated with scaled and thus comparable macro surprises and interventions are similar, thereby illustrating the importance of taking into account interventions when estimating exchange rate models.

Since the Danish ERM II intervention experience pertains to maintaining the exchange rate within a narrow deviation band, our findings are applicable to Denmark and other countries that intervene to keep their respective exchange rates in narrow bands. Additional research is warranted in order to shed light on whether our results also pertain to countries intervening to keep their respective exchange rates in wide deviation bands and to countries with flexible exchange rates.

The rest of the paper is organized as follows. The next section provides an overview of the institutional aspects regarding ERM II and DN intervention. Sections 3 and 4 present the data and the econometric methodology, respectively. Section 5 discusses the results. Section 6 presents several robustness checks. Section 7 concludes.

2 Institutional Aspects

With the launch of the EUR on 1 January 1999, the ERM II was introduced and replaced the exchange rate mechanism (ERM I) of the European Monetary System (EMS). According to the EU Accession Treaty, successful participation in ERM II is a requirement for joining the EMU and for adoption of the EUR. Currently Denmark as well as 4 new EU members participate in ERM II.[5]

In ERM II, a bilateral central rate and a deviation band is set for the currency of the participating country vis-à-vis the EUR, but not against the currency of the other member states. The official DKK/EUR central

[5] Denmark has participated since 1 January 1999, Estonia and Lithuania since 28 June 2004, Latvia since 2 May 2005, and Slovakia since 28 November 2005. Bulgaria, The Czech Republic, Hungary, Poland and Romania are expected to follow.

rate is 7.46038 DKK/EUR and the official deviation band is set to +/- 2.25 percent.[6] The DKK has traded within an even narrower range of +/- 0.50 percent around the Danish ERM II central rate. The official deviation band for all other ERM II member states is set to +/- 15 percent. Slovakia, however, is the only existing ERM II member using (almost) the full +/- 15 percent deviation band. Two of the other existing ERM II member states, Estonia and Lithuania, have EUR currency boards in place, and one existing ERM II member state, Latvia, similar to Denmark keeps its currency in a much narrower band than its official deviation band (the LVL has traded within a narrow range of +/- 1.00 percent of the Latvian ERM II central rate).

In order to keep its currency inside the deviation band, the ERM II member state adjusts its short term interest rates and/or intervenes in the foreign exchange market. If the currency reaches either the upper or the lower limit of the deviation band, both the European Central Bank (ECB) and the central bank of the member state in question intervene to maintain the exchange rate inside the interior of the band. The ECB is only obligated to intervene when a currency reaches one of the band limits.

In reality, interventions when the currency is at the limit of the deviation band are avoided. Instead, interventions are carried out when the currency is in the interior of the band. In the case of Denmark in ERM II, at no point has the Danish currency been near either the upper or the lower limit of the band (see Figure 1). Accordingly, our analysis pertains only to interventions carried out unilaterally by the DN. All DN interventions are discretionary, unannounced and automatically sterilized.

For additional details regarding the ERM II and the Danish foreign exchange rate policy see Nationalbank (2003a), ECB (2004), and EU (2006).

3 Data

The intervention data covers all DN interventions in the DKK/EUR market over the 1 January 2002 to 31 December 2004 period. Our sample period is determined by availability of the data. The data includes

[6] Prior to the launch of the EUR, Denmark participated in the EMS. Between 1982 and until the launch of the EUR, Denmark successfully maintained a stable exchange rate vis-à-vis a basket of other EMS currencies within its ERM I deviation band. As a result, Denmark was in a position to negotiate a tighter official ERM II band than any other ERM II member. Effectively, the official Danish ERM II deviation band is a continuation of the official Danish ERM I deviation band.

the exact amount and time-stamp to the nearest minute obtained directly from the trade-sheet of each intervention transaction. Intervention amounts are quoted in EUR and a positive amount denotes a purchase of EUR against a sale of DKK. In accordance with the ERM II provisions, the DN trader conducting an intervention operation is obligated to write the amount and the exact time of the operation on the trade-sheet immediately after the completion of each individual intervention transaction. This information is forwarded to the ECB by the end of the trading day, at the latest. Our intraday intervention data consists of this extremely reliable information.

Table 2 displays descriptive statistics of the intervention data.[7] Our sample consists of a total of 89 intervention days, encompassing a total of 220 intervention transactions. On intervention days, the average intervention amount is EUR 164 million. A total of 68 intervention days consist of EUR purchases. Figure 1 shows that all interventions occur when the DKK/EUR rate is in the interior of the deviation band.

The high-frequency DKK/EUR exchange rate data is provided by Olsen and Associates, collected from commercial banks by Tenfore and Oanda, and covers the full sample period. The data consists of the bid and the offer spot exchange rate at the end of every 5-minute interval over every 24-hour period. The quotes are indicative quotes, i.e. not necessarily traded quotes. We follow Dacorogna, Muller, Nagler, Olsen, and Pictet (1993) and filter the data for anomalies and bad quotes.

Following Andersen and Bollerslev (1998) and Andersen, Bollerslev, Diebold, and Vega (2003), our midpoint (log) exchange rate price at each 5-minute point is constructed by linearly interpolating the average of the preceding and immediately following (log) bid and offer quotes. The continuously compounded 5-minute returns (R_t) are calculated as the change in the 5-minute midpoint prices.[8]

It is standard in the intraday literature on widely traded currency pairs (e.g. the USD/EUR, the USD/DEM, the USD/JPY, and the USD/GBP) to define a trading day to start at 21.05 GMT the night before and end at 21.00 GMT on the evening of the trading day in question (see Bollerslev and Domowitz (1993)) and, furthermore, to define a weekend to start at 21.05 GMT Friday and finish 21.00 GMT Sunday,

[7]Given the confidential nature of the intervention data, we are not at liberty to display or describe this data in greater detail.

[8]Andersen, Bollerslev, Diebold, and Vega (2003) on p. 40 note that "Goodhart, Ito, and Payne (1996) and Danielsson and Payne (1999) find that the basic characteristics of 5-minute returns constructed from quotes closely match those calculated from transaction prices." Transaction prices are not available for the DKK/EUR exchange rate market.

see e.g., Andersen, Bollerslev, Diebold, and Vega (2003). The Danish exchange rate market, however, is different from the major exchange rate markets in that there is very little or virtually no trading of the Danish currency outside of standard Danish business hours (see Nationalbank (2003a) and ECB (2004)). Therefore, we define a trading day in the Danish currency market to start at 8.00 GMT+1 and finish at 17.00 GMT+1.[9] Consequently, our analysis considers a total of 752 trading days consisting of a total of 80476 5-minute DKK/EUR exchange rate returns.[10] Importantly, our trading day definition encompasses all intervention transactions in the period under study.

Table 3 summarizes key statistical properties of our 5-minute returns. As expected, the returns have fatter tails than a normal distribution, and the Bera-Jarque test for normality is strongly rejected. The rejection is mainly attributed to excess kurtosis. The 5-minute return series is far from being a white-noise process due to long memory, as evidenced by the Ljung-Box Q-test statistic which rejects that the first 540 autocorrelations (corresponding to one business-week) are jointly zero. This long memory can be attributed to daily periodicity.

Danish and Euro-Area interest rates are obtained from the websites of DN (www.nationalbanken.dk) and the ECB (www.ecb.int), respectively. Time-stamped Danish, German, and Euro-area macro announcements and preceding survey expectations are obtained from Bloomberg. Summary statistics regarding interest rates and macro news are available from the authors upon request.

4 Econometric Methodology

Clearly, the long memory and the intraday periodicity of our exchange rate series would affect the residuals of a standard OLS regression of interventions on exchange rate returns, thereby invalidating standard errors and rendering the associated test statistics unreliable. In order to obtain consistent and asymptotically efficient estimates of the response of the foreign exchange series to an intervention we instead employ the two-step WLS procedure developed by Andersen and Bollerslev (1998).

We model the response of the DKK/EUR exchange rate return, R_t,

[9] This definition of a trading day carries over naturally to a definition of a weekend, i.e. we define a weekend to start at 17.05 GMT+1 Friday and finish at 8.00 GMT+1 Monday.

[10] We also deleted the following fixed holidays from the analysis: 1 January, Easter (3 holidays), Christmas (24/25/26 December), 31 December as well as 4 Denmark-specific holidays (Store Bededag, Kristi Himmelfartsdag, Anden Pinsedag, Grundlovsdag).

as a linear function of J lagged values of the return itself and K lags of the intervention variable, I_t:

$$R_t = \beta_0 + \sum_{j=1}^{J} \beta_j R_{t-j} + \sum_{k=0}^{K} \gamma_k I_{t-k} + \varepsilon_t \ , \ t = 1...T \tag{1}$$

As noted earlier, T=80476. We choose J=5 based on the Schwartz and Akaike information criteria and we set K=6, i.e. we include 6 lags of the intervention variable (corresponding to 30 minutes) and vary this number in our robustness checks. We first estimate equation (1) by OLS in order to obtain the estimated residuals, $\hat{\varepsilon}_t$.[11]

The next step is to model the volatility pattern using the estimated residuals of equation (1). We follow Andersen, Bollerslev, Diebold, and Vega (2003) and use the following parameterization:

$$\begin{aligned} |\hat{\varepsilon}_t| = {} & c_1 + c_2 + \alpha \frac{\hat{\sigma}_t}{\sqrt{n}} + \sum_{k=0}^{K} \beta_k I_{t-k} \\ & + \left(\sum_{q=1}^{Q} \delta_q \cos\left(\frac{q2\pi n}{108}\right) + \varphi_q \sin\left(\frac{q2\pi n}{108}\right) \right) \\ & + \sum_{h=0}^{H} \eta_h \left(r_{t-h}^{EUR} - r_{t-h}^{DKK} \right) + u_t \end{aligned} \tag{2}$$

where $\hat{\varepsilon}_t$ is the residual of equation (1), and its absolute value proxies for the volatility in the 5-minute interval t, c_1 and c_2 are two normalizing constants, n is the number of intervals in a day (in our case 108), $\hat{\sigma}_t$ the one-day ahead volatility forecast for day t (i.e. the day that contains interval t), q is a specific intraday calendar effect, Q is the total number of calendar effects accounted for (Q=8, based on the Schwartz and the Akaike information criteria), $r_t^{EUR} - r_t^{DKK}$ is the EUR-DKK interest rate differential, and u_t denotes the residuals (assumed to be standard normal).[12]

[11] We also include in the conditional mean model as additional explanatory variables the distance from parity, i.e. a measure of the distance between the DKK/EUR exchange rate and the central rate, as well as the EUR-DKK interest rate differential. Both variables proved insignificant in all estimations and were thus excluded from the conditional mean model for the remainder of the analysis.

[12] We include contemporaneous and lagged values of the EUR-DKK interest rate differential (captured by the difference between the DN Folio rate and the ECB Refirate) in the volatility model. We set the number of lags to two, based on the Schwartz and the Akaike criteria. This inclusion improves the overall fit of the model. However, whether or not we include the interest rate differential in our volatility model does not affect the significance of the coefficient estimates associated with the intervention variable in either the conditional mean or the volatility model.

We model the lower frequency intraday pattern (the first term after the vector of constants) using the concept of realized volatility (RV). RV is defined as the daily sum of squared returns and constitutes an unbiased, efficient and asymptotically consistent estimate of the true daily quadratic variation. A key advantage of using RV is that this semi-parametric approach does not require additional model estimation. For our baseline analysis we calculate the RV using 30 minute returns. Different RV measures are employed as robustness checks.

Since the realized volatility forecast cannot capture the observed cyclical intraday patterns (the slow decay in the autocorrelations), we model the higher frequency periodicity by inclusion of a Fourier flexible form (see Gallant (1981)). A Fourier flexible form consists of a number of sine- and cosine terms with varying degrees of periodicity (the terms in the parenthesis of equation (2)) It allows for a model specification as flexible as possible, thereby enabling us to fit the intraday pattern of the residuals from equation (1).

Consistent with Andersen and Bollerslev (1998) and Andersen, Bollerslev, Diebold, and Vega (2003), who include their macro news variables in the volatility equation, we include the intervention variable (i.e. our main "news" variable) in the volatility model.

4.1 Structural break

We suspect that a structural break related to the change in the monetary policy stance may have occurred during our sample period. We consequently test for a structural break and parameter instability to ensure that our statistical inference is valid. We use the test procedure described in Andrews (1993).[13]

We test for parameter stability across all the estimated coefficients and find the value of the test statistic to equal 165 at $\pi = 0.2$ (where π is the percentage of the full sample). The test statistic follows the square of a standardized tied-down Bessel process of order p, where p is the number of restrictions in the hypothesis. The critical value of a squared standardized tied-down Bessel process of order 13 is 31.10, thus the null-hypothesis of parameter stability is strongly rejected at the 20 percent change point mark.[14] This translates into a structural break

[13]An advantage of the Andrews (1993) test is that, unlike a simple Wald test (or similar tests), this testing procedure does not require advance knowledge regarding the exact timing of a potential change point. See Andrews (1993) for details regarding this test.

[14]We also utilize the ability of the Andrews (1993) testing procedure to focus on only a sub-sample of the parameter vector to test for parameter stability across the

on 16 August 2002. Since the Danish deposit and lending rates were lowered (twice) in August 2002, independently of the ECB who kept its key interest rates unchanged until the end of 2002, the coinciding change in the monetary policy stance provides the economic rationale for the statistical break point.

5 Results

5.1 Benchmark Estimations

Before we investigate whether policy consistency matters for the effectiveness of intervention, we first carry out the two-step WLS estimation without taking into account the issue of policy consistency. This provides a useful benchmark against which subsequent results and robustness checks are compared. We carry out our estimations on the full sample as well as separately across the 1 January 2002 to 16 August 2002 period (sub-sample 1) and the 17 August 2002 to 31 December 2004 period (sub-sample 2). Since a positive amount of intervention constitutes a purchase of EUR against a sale of DKK, and the exchange rate is measured in terms of DKK per EUR, a positive cumulative effect of intervention implies that intervention is effective and influences the exchange rate in the intended direction.

Tables 1 displays the result of the benchmark estimation of equation (1) and tables 4-5 display the result of equation (2). Since the key goal of intervention in the context of ERM II is to ensure that the value of a currency is kept within its deviation band, the level effects of intervention displayed in table 1 are of primary interest. The first column of table 1 shows that for the full sample the coefficient estimates associated with contemporaneous and the first lag of intervention are highly significant and negative, i.e. the opposite sign of what is expected, and lags 4 through 6 are significant and positive, i.e. in the intended direction. The Wald test of the hypothesis that the (positive) sum of the estimated intervention coefficients is equal to zero cannot be rejected, i.e. intervention is, on average, not effective over the full sample. The sub-sample 1 results shown in column 2 are similar and, again, we cannot reject that the there is no cumulative effect of intervention. However,

sum of the intervention coefficients and separately for each individual intervention coefficient. This alternative procedure also rejects the null-hypothesis of parameter stability at the 20 percent change point mark. Additionally, we perform the same change point test for a specification with a full one-hour lag of interventions included (i.e. 12 lags of interventions). Again, we find significant evidence of a change point at the 20 percent mark.

the sub-sample 2 results shown in column 3 are different. While the coefficient estimates associated with contemporaneous and lagged intervention are once again of mixed signs and varying degrees of significance, the Wald test strongly rejects that the (positive) sum of the estimated intervention coefficients is equal to zero. In other words, intervention is, on average, effective and influences the exchange rate in the intended direction across sub-sample 2.

Tables 4 and 5 display the results of the estimation of the volatility model described in equation (2). Intervention has no significant volatility effects across either the full sample (column 1, table 4) or sub-sample 2 (column 3, table 4). Only when estimating the volatility model across sub-sample 1 (column 2, table 4) do we find significant effects of intervention (consistent with reduced volatility). Overall, these results do not suggest that intervention exerts a strong influence on exchange rate volatility.[15]

The validity of the WLS estimation procedure is contingent on the fit of the volatility model. As a first measure of fit we follow Andersen, Bollerslev, Diebold, and Vega (2003) and, for both sub-samples, plot the absolute average residuals (for each 5 minute interval across all the included 752 trading days) estimated from the initial OLS regression of equation (1) against the fitted absolute average residuals from the estimation of equation (2). Our plots suggest that the volatility model provides a good fit for both sub-samples. As a second measure of fit we plot the raw foreign exchange rate returns against the fitted foreign exchange rate returns from the initial OLS estimation of equation (1) and against the fitted returns from the WLS estimation of equation (1). While the OLS procedure fails to capture the pattern of the raw returns, the fitted returns from the WLS estimation match the raw returns well. This comparison confirms the necessity of the WLS procedure as well as its success.[16]

5.2 Intervention and Policy Consistency

As noted earlier, the official Danish monetary policy rate as well as the key ECB interest rates were all held constant during sub-sample 1, whereas during sub-sample 2 these rates were lowered in tandem. Furthermore, the Danish deposit and lending rates were lowered by a total

[15]While we do not detail the results of the volatility estimations associated with the subsequent model estimations, these results are available from the authors upon request.

[16]These plots as well as the similar plots associated with subsequent estimations are not shown for brevity but available from the authors upon request.

of 140 basis points (see Nationalbank (2002), Nationalbank (2003b) and Nationalbank (2004)) during sub-sample 2. Accordingly, sub-sample 1 is associated with a neutral Danish monetary policy stance while sub-sample 2 constitutes a period of Danish monetary policy easing, both in absolute and relative terms. It follows that the interventions during sub-sample 1 (all purchases of EUR) are neither consistent nor inconsistent with monetary policy, the intervention purchases of EUR during sub-sample 2 are monetary policy consistent, and the intervention sales of EUR during sub-sample 2 are policy inconsistent. This straightforward classification of all the interventions under study allows us to investigate whether monetary policy consistency matters for the effectiveness of intervention, by estimating the effect of intervention purchases of EUR and intervention sales of EUR separately across the two sub-samples.

Table 6 displays the results. As mentioned, sub-sample 1 consists of all EUR purchases and, therefore, the results displayed in the first column of table 6 are identical to the sub-sample 1 benchmark estimation results displayed in the second column of table 1. In terms of monetary policy consistency, the sub-sample 1 results suggest that interventions carried out when monetary policy is neutral are not effective. The second column of table 6 shows that the cumulative effect of the sub-sample 2 monetary policy inconsistent intervention sales of EUR is insignificant (the Wald test cannot reject that the there is no cumulative effect of intervention). By contrast, the cumulative effect of the sub-sample 2 monetary policy consistent intervention purchases of EUR are of the correct sign and significant at the 95% level (the Wald test rejects that the cumulative effect of a consistent intervention is equal to zero). Clearly, these results suggest that intervention exerts a significant influence on exchange rate returns only when the direction of intervention is consistent with the monetary policy stance.

The official Danish exchange rate policy in terms of the +/- 2.25% deviation band around the central rate of 7.4604 DKK/EUR is announced and known to the markets. In terms of intervention and consistency with official exchange rate policy, an intervention aimed at bringing the exchange rate closer to the central parity, e.g. an intervention purchase of EUR when the DKK is appreciated relative to the central rate, is official exchange rate policy consistent, whereas an intervention aimed at pushing the exchange rate further away from the central rate, e.g. an intervention purchase of EUR when the DKK is depreciated relative to the central rate, is inconsistent with the official exchange rate policy. Since the DKK/EUR exchange rate traded between 7.4150 and 7.4592 DKK/EUR throughout the 3 years under study (see Figure 1), it follows that all intervention purchases of EUR are consistent and all intervention sales are inconsistent with the official exchange rate policy. Consequently, all sub-sample 1 interventions are consistent, all sub-sample 2 intervention purchases of EUR are consistent, and all sub-sample 2 intervention sales of EUR are inconsistent with official exchange rate policy. We have already estimated separately the effects of intervention purchases and sales and shown in table 6 that only the sub-sample 2 intervention purchases are effective. Put differently, the official exchange rate policy consistent sub-sample 1 interventions are not effective while the official exchange rate policy consistent sub-sample 2 interventions are effective. This disconnect indicates that whether intervention is consistent with official exchange rate policy does not determine whether intervention systematically influences exchange rate returns.

Since actual ("de-facto") Danish exchange rate policy clearly differs from the official Danish exchange rate policy (or there would have been no intervention sales of EUR in our sample), we also employ as a measure of "de-facto" exchange rate policy consistency the position of the exchange rate at the time of the intervention relative to the unconditional full-sample mean of 7.4337 DKK/EUR. In other words, if an intervention purchase (sale) of EUR occurs when the DKK is appreciated (depreciated) relative to the unconditional full-sample mean, the intervention is deemed consistent with what we label the "de-facto" exchange rate policy. According to this measure of "de-facto" exchange rate policy consistency, all sub-sample 1 interventions (all purchases of EUR), all sub-sample 2 intervention sales of EUR, and 67 of the sub-sample 2 intervention purchases of EUR are consistent with the "de-facto" exchange rate policy. The remaining 32 sub-sample 2 intervention purchases of EUR are "de-facto" exchange rate policy inconsistent. As before, we investigate whether this consistency matters for the effectiveness of intervention by estimating the effect of consistent and inconsistent intervention separately across the two sub-samples. In order to take into account our insights regarding monetary policy consistency we also distinguish between intervention purchases and intervention sales.

The results are displayed in table 7. The new and interesting finding here is in regards to the sub-sample 2 intervention purchases of EUR. As the second column shows, whether intervention purchases of EUR are consistent or inconsistent with the "de-facto" exchange rate policy measure, the cumulative effect on the exchange rate is significant, of the right sign, and practically of the same magnitude. This finding suggests that consistency with "de-facto" exchange rate policy also does not determine whether intervention is effective.[17]

[17] All the sub-sample 1 intervention purchases of EUR and all the sub-sample 2

In sum, our investigation of intervention and policy consistency produces evidence that on the one hand effectiveness of intervention is contingent on its consistency with monetary policy, i.e. intervention only works when backed by changes in the monetary policy stance in the same direction as intervention.[18] On the other hand, we find no evidence that exchange rate policy consistency is a condition for effective intervention.

Taken together, this implies that in the context of the Danish ERM II experience there is no monetary policy/exchange rate policy trade-off. Instead, in order for intervention to be effective, monetary policy consistency is a necessity, while exchange rate policy consistency is irrelevant. Importantly, this illustrates that even though sterilized intervention is by construction detached from monetary policy and, therefore, as discussed earlier, it is possible to use sterilized intervention to pursue an exchange rate goal while using monetary policy towards a domestic goal that may conflict with the exchange rate goal, doing so is a futile exercise. In other words, our results show that, in reality, sterilized intervention is not an independent policy instrument; sterilized intervention only works in conjunction with monetary policy.

5.3 Endogeneity

While it seems reasonable to assume that intervention is not triggered by the contemporaneous exchange rate movement (i.e. the change in exchange rate return that occurs over the 5-minute interval within which intervention is carried out), intervention is likely correlated with recent (lagged) exchange rate movements and with recent (lagged) intervention. Accordingly, even our study probably does not perfectly abstract from endogeneity and, therefore, simultaneity bias could make effective intervention appear ineffective. See Neely (2005) for a general discussion of endogeneity, and Neely (2008) for a survey-based assessment of how fast intervening central banks react to exchange rate market developments.

In order to control for endogeneity, we follow the daily data studies by Humpage (1999) and Naranjo and Nimalendran (2000) and estimate a central bank reaction function in order to capture the expected component of the intervention variable. In turn, we use the residuals of the

intervention sales of EUR are consistent with both the official and the "de-facto" exchange rate policy, thus the result regarding no significant cumulative effect of intervention pertaining to these two categories of intervention has already been discussed.

[18] While it is beyond the scope of this paper to analyze through which transmission channel effective intervention works, this finding clearly is in line with the signaling channel hypothesis, i.e. by carrying out intervention the central bank informs the market about its future policy intentions and/or fundamentals.

reaction function estimation (i.e. we subtract the expected component of intervention from the actual intervention variable in intervals where the latter is non-zero) as a proxy for unexpected intervention that we then use for obtaining an estimate of intervention that is free of simultaneity bias.

The results of the reaction function estimation are displayed in table 8. The results show that the first four lags of exchange rate returns and the first lag of intervention are significant, confirming the suspicion that some endogeneity is present in any intervention study, even at the intraday frequency. The results of the re-estimation of the benchmark model with the proxy for unexpected intervention in place of the actual intervention variable are displayed in table 9. The results are qualitatively identical to the comparable estimation results from estimations that do not address endogeneity (table 1).[19]

Overall, addressing the issue of endogeneity in the context of an intraday study of intervention tells us two things. One, it shows that some endogeneity is present even in an intraday analysis of the effectiveness of intervention. Two, at least in the Danish case, the resulting simultaneity bias is too small to affect the results.

5.4 Macro Surprises

In order to compare the influence of intervention relative to the influence of macro surprises as well as to ensure that our estimated effects of intervention are not tainted by the coincidental arrival of macro news, we extend our analysis to include time-stamped Danish, German, and Euro-area macro surprises. Specifically, we include macro surprises regarding Danish Unemployment (DKUNEMP), Trade Balance (DKTB), Current Account (DKCA), CPI (DKCPI), GDP (DKGDP) and Consumer Confidence (DKCC); German IFO Index (DEIFO), GDP (DEGDP), and Industrial Production (DEIP); Euro-Area CPI (EACPI), Industrial Production (EAIP), and Business Climate Index (EABC). We measure macro surprises as the difference between macro announcement and preceding survey expectation obtained from Bloomberg. To facilitate the comparison of the coefficient estimates of the macro surprises to the coefficient estimates of intervention, we follow Andersen, Bollerslev, Diebold, and Vega (2003) and Andersen, Bollerslev, Diebold, and Vega (2007) and others and standardize the macro news as well as the intervention vari-

[19]We also control for endogeneity using alternative reaction function specifications with different lag structures and with lags of the distance from parity measure included as an additional explanatory variable. Our results are not sensitive to these alternative reaction function specifications.

able (i.e. for each variable we divide the surprise by its sample standard deviation).[20]

The results are displayed in table 10-12.[21] The results show, not surprisingly, that some but not all of the macro surprises influence the DKK/EUR exchange rate. The significant macro surprises are of the expected sign, e.g. a higher than expected Danish CPI announcement is associated with a depreciation of the DKK. The absolute magnitude of the significant macro surprise point estimates as well as the estimate of the magnitude of the cumulative effect of intervention fall in the 0.00002 to 0.00012 range. This shows that the relative influence of intervention is comparable to the relative influence of most macro surprises, thereby illustrating the importance of taking into account the effect of intervention when estimating exchange rate models. For example, the relative influence of intervention is similar to the relative influence of Danish or Euro-Area CPI surprises.[22]

6 Robustness

In order to test the robustness of our results, we carry out the analysis using a different econometric procedure, a different intervention lag-structure, fewer trigonometric terms, and different RV measures. Additionally, we address whether interventions that occur on high-volatility days impact the exchange rate differently. All results pertaining to this section are available from the authors upon request.

First, the gain in efficiency from the WLS procedure is potentially costly in terms of inconsistent estimates if the residuals from the initial

[20] Almeida, Goodhart, and Payne (1998) and Andersen, Bollerslev, Diebold, and Vega (2003) show that the conditional mean of the exchange rate generally adjusts immediately (i.e. jumps) in response to macro news. This is not surprising considering that the announcement time of macro news is known in advance. Accordingly, we include only the contemporaneous and the first lag of the macro surprises in our estimations. Since the DN interventions are unannounced and carried out on a discretionary basis, the foreign exchange market participants cannot know in advance with certainty whether an intervention will occur and if so at what time. We therefore allow for an adjustment period of 30 minutes in order to capture the full effect of the interventions.

[21] Sub-sample 1 is too short to encompass a sufficient number of macro surprises for a meaningful estimation of the influence of macro news.

[22] The magnitude of our estimates regarding macro surprises and intervention are broadly similar to the comparable estimates regarding US macro surprises found in Almeida, Goodhart, and Payne (1998), p. 391. They show (unscaled) estimates in the 0.00001 to 0.00290 range. Our estimates also appear similar to the (scaled) coefficient estimates displayed graphically in Andersen, Bollerslev, Diebold, and Vega (2007), p. 263.

estimation of equation (1) are improperly fitted in the volatility model described by equation (2). In order to address this potential concern we also estimate the effects of intervention on exchange rate returns using heteroskedasticity- and serial-correlation consistent (HAC) standard errors (i.e. we re-estimate equation (1) using HAC errors). The HAC results are qualitatively identical to the conditional mean results based on the more sophisticated two-step WLS procedure.

Second, in order to test for delayed effects of intervention beyond the 6th lag, we re-estimate our models with 12 lags of intervention included. We do so using both the WLS and the HAC procedure. The results based on the WLS procedure show that while the 7th and the 10th lags are marginally significant in sub-sample 1 and the 7th lag is significant in sub-sample 2, our previously discussed results regarding cumulative effects remain. When using the HAC procedure, the two marginally significant delayed effects in sub-sample 1 become insignificant. Overall, this robustness check confirms that it takes no longer than about 30 minutes before the exchange rate has fully adjusted to an intervention operation.

Third, our volatility model includes 8 sine and 8 cosine terms, while the volatility model of Andersen, Bollerslev, Diebold, and Vega (2003) includes only 4 sine and 4 cosine terms. In order to ensure that our volatility model isn't over-fitted, we also carry out the WLS estimation using only 4 sine and 4 cosine terms. Our results are robust to this reduction in the number of trigonometric terms included.

Fourth, while our baseline model uses a RV measure based on 30 minute frequency returns (pertaining to equation (2)), we also estimate the model using RV measures based on 10 and 60 minute frequency returns. Our results are robust across each of these RV measures. Our findings are also robust to replacing the RV series with a daily volatility series derived from a standard GARCH(1,1) model (as originally proposed by Andersen and Bollerslev (1998)).

Fifth, in order to assess whether interventions that are carried out on high-volatility days exert more or less of an influence on the exchange rate, we first define a high-volatility day as a day with either a significant intraday volatility jump (i.e. a "jump-day", as defined in Andersen, Bollerslev, and Diebold (2007)) or with a daily realized volatility that is at least the average realized volatility of the sample plus two times the standard deviation of the realized volatility. We then add to the mean equation a stand-alone 0-1 dummy that equals 1 when the 5-minute interval during which an intervention occurs falls on a high-volatility day, and re-estimate the baseline model with only the contemporaneous dummy added and with the contemporaneous dummy and six lags of this dummy added. The results show that all the dummies are insignificant across all the samples, implying that whether or not intervention is carried out when volatility is high does not affect the influence of intervention. For completeness, we also assess whether interventions that occur on low-volatility days (defined as a day when the realized volatility is less than the average realized volatility of the sample minus two times the standard deviation of the realized volatility) impact the exchange rate differently. Again, all the low-volatility day dummies are insignificant across all the samples.

7 Conclusion

This paper investigates the real-time (intraday) effects of Danish intervention in the EUR market over the 1 January 2002 to 31 December 2004 period, using proprietary intraday intervention data provided by the Danish central bank and the WLS time-series econometrics of Andersen and Bollerslev (1998) and Andersen, Bollerslev, Diebold, and Vega (2003). The Danish ERM II intervention experience pertains to maintaining the exchange rate within a narrow deviation band. Therefore, our findings are applicable to Denmark and other countries that intervene to keep their respective exchange rates in narrow bands (such as Latvia in ERM II), but not necessarily to countries intervening to keep their respective exchange rates in wide deviation bands (such as Slovakia in ERM II) or to countries with flexible exchange rates (such as Japan or the US).

We test for and find a structural break in the data in August 2002, coinciding with the Danish deposit and lending rates being lowered. Based on the structural break we separate our data into two sub-samples that are distinctly different in terms of the stance of the Danish monetary policy. Sub-sample 1 is associated with a neutral Danish monetary policy stance while sub-sample 2 constitutes a period of Danish monetary policy easing, both in absolute and relative terms. This facilitates an investigation of whether monetary policy consistency of sterilized intervention matters for whether sterilized intervention is effective. Our results show that intervention only works when the direction of intervention is consistent with the monetary policy stance.

We also assess whether consistency with either official or a measure of “de facto” exchange rate policy matter for the effects of intervention. We find no evidence that exchange rate policy consistency is a condition for effective intervention.

Furthermore, we address the issue of endogeneity by estimating a central bank reaction function in order to capture the expected component of the intervention variable. In turn, we use the residuals of the reaction function estimation as a proxy for unexpected intervention that we then use for obtaining an estimate of intervention that is free of simultaneity bias. We find that some endogeneity is present even in our intraday analysis of the effectiveness of intervention, yet we also find that the resulting simultaneity bias is too small to affect the results.

In order to compare the influence of intervention relative to the influence of macro surprises as well as to ensure that our estimated effects of intervention are not tainted by the coincidental arrival of macro news, we also include time-stamped Danish, German, and Euro-area macro surprises in our estimations. Our results show, not surprisingly, that some but not all of the macro surprises influence the DKK/EUR exchange rate. More importantly, the magnitude of the coefficient estimates associated with scaled and thus comparable macro surprises and interventions show that the relative influence of intervention is comparable to the relative influence of most macro surprises.

Our main result is that in order for intervention to be effective, monetary policy consistency is a necessity. This illustrates that even though it is technically possible to use sterilized intervention to pursue an exchange rate goal that may conflict with the domestic goal towards which monetary policy is aimed, doing so is futile. In other words, we show that sterilized intervention is not an independent policy instrument.

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8 Tables

WLS Estimation of eq. 1: Mean Equation			
	Full Sample	**Sub-sample 1**	**Sub-sample 2**
C	0.0040	-0.1500	0.0600
	(0.0333)	*(0.7142)*	*(0.2857)*
Γ_t	-0.15***	-0.32***	-0.08*
	(0.0560)	*(0.4638)*	*(0.0696)*
Γ_{t-1}	-0.16***	-0.26	-0.17***
	(0.0578)	*(0.1844)*	*(0.0369)*
Γ_{t-2}	0.06	0.06	0.04
	(0.0632)	*(0.1579)*	*(0.0702)*
Γ_{t-3}	-0.05	-0.03	-0.06
	(0.0714)	*(0.1250)*	*(0.0896)*
Γ_{t-4}	0.13**	0.12	0.1
	(0.0578)	*(0.1091)*	*(0.0676)*
Γ_{t-5}	0.12**	-0.13	0.36***
	(0.0533)	*(0.1429)*	*(0.0679)*
Γ_{t-6}	0.15**	0.16*	0.11
	(0.0641)	*(0.1185)*	*(0.0859)*
β_{t-1}	-0.64***	-0.76***	-0.61***
	(0.0067)	*(0.0097)*	*(0.0078)*
β_{t-2}	-0.45***	-0.58***	-0.43***
	(0.0075)	*(0.0116)*	*(0.0086)*
β_{t-3}	-0.31***	-0.42***	-0.29***
	(0.0070)	*(0.0119)*	*(0.0079)*
β_{t-4}	-0.2***	-0.26***	-0.19***
	(0.0063)	*(0.0107)*	*(0.0072)*
β_{t-5}	-0.1***	-0.13***	-0.1***
	(0.0050)	*(0.0082)*	*(0.0061)*
Sum	0.1	-0.39	0.31
Wald Test Statistic	1.91	0.63	7.28
Number of observations	220	58	162

Table 1: *The dependent variable is the first difference of the log of the daily DKK/EUR mid-point exchange rate.* Γ_{t-j} *denotes current and lagged interventions.* β_{t-j} *denotes lagged 5 min. fx returns. The coefficients, standard deviations and the sum for the interventions are multiplied by 1.000.000. *, **, and *** denotes significance at respectively the 10, 5, and 1 per cent level. Standard in ( ) below the point estimates; test statistic values in [ ]; Subscript denotes lags.*

	Daily Intervention		Intraday Interventions	
	Number of Interventions	*Average Amount*	*Number of Interventions*	*Average Amount*
All	89	164	220	67
Purchases	68	158	157	69
Sales	21	-182	63	61

Table 2: *This table shows summary Statistics for the Invervention Variable. The average amount is denoted in millions of Euros. Data source: Danmarks Nationalbank Sample period: 1 January 2002 31 December 2004*

Mean	**Std. dev.**	**Skewness**	**Kurtosis**
0	0.00013	0.0146	18**
(∼0)	(-)	(0.0086)	(0.0173)
Minimum	**Maximum**	**BJ-test for Normality**	**LB Q-test (5-day lag)**
-0.1724	0.2215	754720	16068
		[5.99]	[595]

Table 3: *Data runs from January 1, 2002 to December 31, 2004. The data consists of 80.476 observations on DKK/EUR exchange rate. The returns are calculated from bid- and ask prices from Olsen Financial Technologies. *, **, and *** denotes significance at respectively the 10, 5, and 1 per cent level. Standard errors in () below the point estimates. Critical values in [].*

Estimation of eq. 2: Volatility			
	Full Sample	**Sub-sample 1**	**Sub-sample 2**
C	0.0025***	-0.0004	0.0030***
	(0.0004)	*(0.0007)*	*(0.0005)*
Normalising Constant I	-0.0019***	0.0003	-0.0030***
	(0.0003)	*(0.0003)*	*(0.0005)*
Normalising Constant II	0.0002***	-0.0004	0.0003***
	(0.0000)	*(0.0006)*	*(0.0001)*
Realised Volatility	0.2700***	0.1300***	0.2700***
	(0.0050)	*(0.0160)*	*(0.0050)*
Interest differentials			
η_t	-0.1400***	-0.0070*	-0.0020
	(0.0140)	*(0.0036)*	*(0.0030)*
η_{t-1}	0.5700**	0.5500	-0.0003
	(0.2591)	*(3.4380)*	*(0.0050)*
η_{t-2}	-0.3600	-0.4800***	0.0030
	(0.2118)	*(0.0918)*	*(0.0029)*
Interventions			
Γ_t	-0.0700	-0.3300***	0.0300
	(0.0680)	*(0.0636)*	*(0.1035)*
Γ_{t-1}	-0.0700	0.0100	0.1100
	(0.0700)	*(0.1250)*	*(0.0909)*
Γ_{t-2}	0.0899	0.0800	0.0900
	(0.0899)	*(0.1231)*	*(0.1139)*
Γ_{t-3}	0.0755	-0.0900	-0.0300
	(0.0755)	*(0.0776)*	*(0.0938)*
Γ_{t-4}	0.0769	-0.1300	-0.0300
	(0.0769)	*(0.0823)*	*(0.0968)*
Γ_{t-5}	0.0833	-0.0040	0.1000
	(0.0833)	*(0.0098)*	*(0.1020)*
Γ_{t-6}	-0.0500	-0.1800***	-0.0040
	(0.0794)	*(0.0690)*	*(0.1000)*

Table 4: *The dependent variable is the absolute residual from the auxiliary regression, equation (1). The independent variables are normalizing constants, a realized volatility measure, trigonometric terms, δ_j and φ_j, the EUR-DKK interest rate differential, η_j, and interventions, Γ_{t-j}. The coefficients and standard deviations for the sine, cosine, and interest differentials are multiplied by 1.000. The coefficients and standard deviations for the interventions and their sum are multiplied by 1.000.000. *, **, and *** denotes significance at respectively the 10, 5, and 1 per cent level. Standard Errors in ( ) below the point estimates. Subscript denotes lags.*

Estimation of eq. 2: Volatility			
	Full Sample	**Sub-sample 1**	**Sub-sample 2**
	Sine terms		
δ_1	-0.1030***	0.0210	-0.1400***
	(0.0170)	*(0.0280)*	*(0.0200)*
δ_2	-0.0100***	0.0010	-0.0120***
	(0.0017)	*(0.0030)*	*(0.0020)*
δ_3	0.0060***	-0.0030	0.0080***
	(0.0014)	*(0.0020)*	*(0.0015)*
δ_4	0.0064***	-0.002	0.009***
	(0.0014)	*(0.0020)*	*(0.0017)*
δ_5	0.0040***	-0.0020*	0.0060***
	(0.0009)	*(0.0010)*	*(0.0011)*
δ_6	0.0031***	0.0010	0.0040***
	(0.0006)	*(0.0020)*	*(0.0007)*
δ_7	-0.0021***	-0.0004	-0.0030***
	(0.0005)	*(0.0007)*	*(0.0007)*
δ_8	-0.0013***	-0.0003***	-0.0020***
	(0.0005)	*(0.0001)*	*(0.0007)*
	Cosine terms		
φ_1	0.0800***	-0.0150	0.1100***
	(0.0148)	*(0.0428)*	*(0.0182)*
φ_2	0.0300***	-0.0060	0.0420***
	(0.0054)	*(0.0085)*	*(0.0067)*
φ_3	0.0130***	-0.0040	0.0170***
	(0.0024) ***	*(0.0055)*	*(0.0027)* ***
φ_4	0.0040***	-0.0010	0.0050***
	(0.0008) ***	*(0.0010)*	*(0.0009)*
φ_5	0.0004	-0.0001	0.0002
	(0.0057)	*(0.0003)*	*(0.0009)*
φ_6	-0.0030***	-0.0010	-0.0030***
	(0.0008) ***	*(0.0077)*	*(0.0008)* ***
φ_7	-0.0010*	0.0010	-0.0020**
	(0.0005) *	*(0.0018)*	*(0.0008)* **
φ_8	0.0002	0.0020	-0.0001
	(0.0004)	*(0.0035)*	*(0.0005)*
R^2	0.11	0.01	0.12
F-statistic	329	7	304

Table 5: *The dependent variable is the absolute residual from the auxiliary regression, equation (1). The independent variables are normalizing constants, a realized volatility measure, trigonometric terms, δ_j and φ_j, the EUR-DKK interest rate differential, η_j, and interventions, Γ_{t-j}. The coefficients and standard deviations for the sine, cosine, and interest differentials are multiplied by 1.000. The coefficients and standard deviations for the interventions and their sum are multiplied by 1.000.000. *, **, and *** denotes significance at respectively the 10, 5, and 1 per cent level. Standard Errors in ( ) below the point estimates. Subscript denotes lags.*

WLS Estimation of eq. 1: Sales and purchases						
	Full Sample		**Sub-sample I**		**Sub-sample II**	
	Sales	*Purchases*	*Sales*	*Purchases*	*Sales*	*Purchases*
C	0.4359	0.4359		6.9002	0.7757	0.7757
	(1.0457)	*(1.0457)*		*(12.7020)*	*(2.4969)*	*(2.4969)*
Γ_t	-0.0805	-0.1482*		-0.3200***	-0.0749	-0.1050
	(0.0854)	*(0.0776)*		*(0.0647)*	*(0.0845)*	*(0.1094)*
Γ_{t-1}	-0.2002***	-0.1622		-0.2601	-0.1966***	-0.1234
	(0.0674)	*(0.1087)*		*(0.1785)*	*(0.0670)*	*(0.1225)*
Γ_{t-2}	-0.0803	0.2090		0.0605	-0.0746	0.3046*
	(0.0528)	*(0.1163)*		*(0.1645)*	*(0.0531)*	*(0.1630)*
Γ_{t-3}	-0.0337	-0.1011		-0.0259	-0.0314	-0.1642
	(0.1231)	*(0.0858)*		*(0.1146)*	*(0.1229)*	*(0.1182)*
Γ_{t-4}	0.0381	0.1976***		0.1243	0.0380	0.2223**
	(0.0786)	*(0.0770)*		*(0.1162)*	*(0.0799)*	*(0.0999)*
Γ_{t-5}	0.2086**	0.0868		-0.1276	0.2066**	0.2166
	(0.0906)	*(0.0951)*		*(0.1426)*	*(0.0917)*	*(0.1264)*
Γ_{t-6}	0.2521	0.1006		0.1578*	0.246	0.0817
	(0.1846)	*(0.0798)*		*(0.0838)*	*(0.1855)*	*(0.1146)*
β_{t-1}	-0.6330***	-0.6330***		-0.7593***	-0.6061***	-0.6061***
	(0.0072)	*(0.0072)*		*(0.0097)*	*(0.0084)*	*(0.0084)*
β_{t-2}	-0.4466***	-0.4466***		-0.5828***	-0.4196***	-0.4196***
	(0.0079)	*(0.0079)*		*(0.0116)*	*(0.0091)*	*(0.0091)*
β_{t-3}	-0.3082***	-0.3082***		-0.4179***	-0.2875***	-0.2875***
	(0.0075)	*(0.0075)*		*(0.0119)*	*(0.0086)*	*(0.0086)*
β_{t-4}	-0.1991***	-0.1991***		-0.2616***	-0.1876***	-0.1876***
	(0.0068)	*(0.0068)*		*(0.0108)*	*(0.0078)*	*(0.0078)*
β_{t-5}	-0.1023***	-0.1023***		-0.1326***	-0.0966***	-0.0966***
	(0.0055)	*(0.0055)*		*(0.0083)*	*(0.0064)*	*(0.0064)*
Sum	0.1040	0.1824		-0.3437	0.1130	0.4327
Wald Test	0.3010	1.3108		1.7860	0.3583	4.8594**

Table 6: *This table shows the estimates from the 2WLS regression of equation 1 in which interventions are divided into purchases of EUR, a positive intervention, and sales of EUR, a negative intervention. The dependent variable is the first difference of the log of the daily DKK/EUR mid-point exchange rate. Γ_{t-j} denotes the intervention variable. β_{t-j} denotes 5 min. fx returns. The coefficients and the standard deviation for the constant and the interventions are multiplied by 1.000.000 *, **, and *** denotes significance at respectively the 10, 5, and 1 per cent level. Standard Errors in ( ) below the point estimates; test statistic values in [ ]. Subscript denotes lags.*

WLS Estimation of eq. 1: Purchases, Consistent and inconsistent sales

	Full Sample			Sub-sample 2		
	Sales	$Purc_C$	$Purc_I$	*Sales*	$Purc_C$	$Purc_I$
Γ_t	-0.0805	-0.1594	-0.0833	-0.0749	-0.1146	-0.0759
	(0.0854)	*(0.0936)*	*(0.0684)*	*(0.0845)*	*(0.1540)*	*(0.0692)*
Γ_{t-1}	-0.2003***	-0.2132	0.0422	-0.1966***	-0.2080	0.0466
	(0.0674)	*(0.1284)*	*(0.1297)*	*(0.0670)*	*(0.1677)*	*(0.1290)*
Γ_{t-2}	-0.0803	0.2369	0.1146	-0.0746	0.4093	0.1139
	(0.0528)	*(0.1428)*	*(0.1151)*	*(0.0531)*	*(0.2322)*	*(0.1138)*
Γ_{t-3}	-0.0338	-0.1425	0.0776	-0.0314	-0.2735	0.0734
	(0.1232)	*(0.0997)*	*(0.1368)*	*(0.1229)*	*(0.1539)*	*(0.1374)*
Γ_{t-4}	0.0380	0.2230**	0.1009	0.0380	0.2790**	0.0964
	(0.0786)	*(0.0912)*	*(0.1187)*	*(0.0799)*	*(0.1322)*	*(0.1199)*
Γ_{t-5}	0.2085**	0.0654	0.1612**	0.2066**	0.2420	0.1572**
	(0.0906)	*(0.1178)*	*(0.0753)*	*(0.0917)*	*(0.1919)*	*(0.0737)*
Γ_{t-6}	0.2520	0.1385	-0.0538	0.2460	0.1490	-0.0589
	(0.1846)	*(0.0961)*	*(0.0793)*	*(0.1855)*	*(0.1658)*	*(0.0797)*
Sum	0.1037	0.1487	0.3593	0.1131	0.4833	0.3527
Wald Test Statistic	0.2993	0.6270	2.8347*	0.3585	3.1716*	2.9215*
Interventions	46	113	28	46	61	28

Table 7: *This table shows the estimates from the 2WLS regression in which interventions are divided into three variables. i) Sales of EUR. ii) Consistent purchases of EUR (I>0 when fx is below unconditional mean) denoted by subscript C. iii) Inconsistent purchases of EUR (I>0 when fx is above unconditional mean) denoted by subscript I. The dependent variable is the first difference of the log of the daily DKK/EUR mid-point exchange rate. The coefficients and the standard deviation for the constant and the interventions are multiplied by 1.000.000* Γ_{t-j} *denotes the intervention variable. The coefficients for the constant and the exchange rate returns are not shown for brevity. The unconditional full sample mean is 7.4337 DKK/EUR. *, **, and *** denotes significance at respectively the 10, 5, and 1 per cent level. Standard Errors in ( ) below the point estimates; test statistic values in [ ]. Subscript denotes lags.*

Reaction function for DN Interventions	
	Full Sample
C	0***
	(0.0615)
Γ_{t-1}	0.0622***
	(0.0185)
Γ_{t-2}	0.0093
	(0.0087)
Γ_{t-3}	0.0439
	(0.0255)
Γ_{t-4}	0.006
	(0.0061)
Γ_{t-5}	0.0018
	(0.0033)
Γ_{t-6}	0.0302
	(0.0180)
β_{t-1}	-303.3497**
	(133.7626)
β_{t-2}	-338.9983**
	(145.7333)
β_{t-3}	-394.9642***
	(135.7151)
β_{t-4}	-278.0769**
	(126.6711)
β_{t-5}	50.0422
	(124.0949)
F-test	42.7194
R^2	0.0079

Table 8: *This table shows the estimates from a reaction function for DN interventions. The independent variables are lags of the exchange rate returns, current and lagged macro news, current and lagged Denmark/Euro-Area interest rate differential, and lags of the dependent variable. The coefficient estimates associated with the constant, current and lagged macro news, and current and lagged interest rate differential are not shown for ease of exposition.* Γ_{t-j} *denotes the intervention variable.* β_{t-j} *denotes 5 min. fx returns. The coefficients and standard deviations for the constant and the interventions are multiplied by 1.000.000. *, **, and *** denotes significance at respectively the 10, 5, and 1 per cent level. Standard Errors in ( ) below the point estimates; test statistic values in [ ]. Subscript denotes lags.*

WLS Estimation of eq. 1: Interventions from reaction function			
	Full Sample	**Sub-sample 1**	**Sub-sample 2**
C	0.4421	5.7819	0.6102
	(0.9418)	*(13.0090)*	*(2.2810)*
$\Gamma_{U,t}$	-1.5470***	-0.2780***	-0.0833
	(0.5736)	*(0.0695)*	*(0.0690)*
$\Gamma_{U,t-1}$	-1.6030***	-0.2610	-0.1530***
	(0.6237)	*(0.1846)*	*(0.0528)*
$\Gamma_{U,t-2}$	0.7580	0.0699	0.0390
	(0.6920)	*(0.1769)*	*(0.0685)*
$\Gamma_{U,t-3}$	-0.4750	-0.0234	-0.0541
	(0.6469)	*(0.1158)*	*(0.0790)*
$\Gamma_{U,t-4}$	1.3500**	0.1498	0.1110
	(0.5914)	*(0.1251)*	*(0.0700)*
$\Gamma_{U,t-5}$	1.7050**	-0.1381	0.2903***
	(0.7443)	*(0.1492)*	*(0.0788)*
$\Gamma_{U,t-6}$	1.2380*	0.1027	0.1106
	(0.6414)	*(0.1030)*	*(0.0842)*
β_{t-1}	-0.6386***	-0.7576***	-0.6136***
	(0.0067)	*(0.0102)*	*(0.0078)*
β_{t-2}	-0.4518***	-0.5815***	-0.4262***
	(0.0074)	*(0.0122)*	*(0.0085)*
β_{t-3}	-0.312***	-0.4169***	-0.2922***
	(0.0071)	*(0.0124)*	*(0.0080)*
β_{t-4}	-0.2013***	-0.2611***	-0.1901***
	(0.0064)	*(0.0111)*	*(0.0072)*
β_{t-5}	-0.102***	-0.1332***	-0.0963***
	(0.0052)	*(0.0086)*	*(0.0059)*
Sum	0.14	-0.38	0.26
Wald Test Statistic	1.6639	1.8852	4.6832**

Table 9: *The dependent variable is the first difference of the log of the daily DKK/EUR mid-point exchange rate. The independent variables are current and lagged unexpected intervention (denoted by $\Gamma_{U,t-j}$), and lags of the dependent variable denoted by β_{t-j}. Unexpected intervention is defined as the residual of the intervention reaction function estimation (described in Section 5 and displayed in Table 6) in an interval where actual intervention occurs, and zero otherwise. The coefficients and the standard deviations for the constant, the interventions and their sum are multiplied by 1.000.000. *, **, and *** denotes significance at respectively the 10, 5, and 1 per cent level. Standard Errors in ( ) below the point estimates; test statistic values in [ ]. Subscript denotes lags*

WLS Estimation of eq. 1 with news variables			
	Full Sample	**Sub-sample 1**	**Sub-sample 2**
C	0.4403	-	0.6013
	(0.9422)	-	*(2.2802)*
Γ_t	-0.0132***	-	-0.0069
	(0.0048)	-	*(0.0058)*
Γ_{t-1}	-0.01295***	-	-0.01268***
	(0.0050)	-	*(0.0050)*
Γ_{t-2}	0.00554	-	0.0051
	(0.0057)	-	*(0.0063)*
Γ_{t-3}	-0.00381	-	-0.00468
	(0.0055)	-	*(0.0067)*
Γ_{t-4}	0.01133**	-	0.00936
	(0.0050)	-	*(0.0059)*
Γ_{t-5}	0.01597**	-	0.02232***
	(0.0068)	-	*(0.0066)*
Γ_{t-6}	0.01061*	-	0.00992
	(0.0055)	-	*(0.0073)*
β_{t-1}	-0.6385***	-	-0.6135***
	(0.0067)	-	*(0.0078)*
β_{t-2}	-0.4519***	-	-0.4264***
	(0.0074)	-	*(0.0085)*
β_{t-3}	-0.312***	-	-0.2922***
	(0.0071)	-	*(0.0080)*
β_{t-4}	-0.2012***	-	-0.1899***
	(0.0064)	-	*(0.0072)*
β_{t-5}	-0.1019***	-	-0.0963***
	(0.0052)	-	*(0.0059)*
Sum	0.0135	-	0.0225
Wald Test Statistic	2.2369	-	4.5985**

Table 10: *This table shows the estimates from the 2WLS regression using news variables for Denmark, the Euro Area, and Germany. The dependent variable is the first difference of the log of the daily DKK/EUR mid-point exchange rate. The interventions are standardised to facilitate comparison with the news variable coefficients. All variables are standardized by dividing each variable by its sample standard deviation.* Γ_{t-j} *denotes the intervention variable.* $\beta_{t-j,n}$ *denotes 5 min. fx returns. The coefficients and the standard deviation for the constant is multiplied by 1.000.000 The coefficients and standard deviations for the news variables and interventions are multiplied by 1.000. There are not a sufficient amount of news to estimate the full model for the first model. *, **, and *** denotes significance at respectively the 10, 5, and 1 per cent level. Standard Errors in ( ) below the point estimates; test statistic values in [ ]. Subscript denotes lags.*

WLS Estimation of eq. 1 with news variables			
	Full Sample	**Sub-sample 1**	**Sub-sample 2**
DKUNEMP(0)	-0.03	-	-0.0334
	(0.0280)	-	*(0.0322)*
DKUNEMP(-1)	-0.0138	-	-0.0114
	(0.0109)	-	*(0.0106)*
DKTB(0)	-0.019	-	-0.0074
	(0.0109)	-	*(0.0111)*
DKTB(-1)	-0.0053	-	0.0069
	(0.0098)	-	*(0.0101)*
DKCA(0)	0.0087	-	0.0079
	(0.0148)	-	*(0.0136)*
DKCA(-1)	0.0125	-	-0.0161
	(0.0132)	-	*(0.0093)*
DKCPI(0)	0.0456	-	0.0482
	(0.0491)	-	*(0.0380)*
DKCPI(-1)	0.0484**	-	0.0192
	(0.0219)	-	*(0.0198)*
DKGDP(0)	-0.1132***	-	-0.1177**
	(0.0335)	-	*(0.0501)*
DKGDP(-1)	0.0183	-	-0.0021
	(0.0372)	-	*(0.0398)*
DKCC(0)	0.0152	-	0.0155
	(0.0137)	-	*(0.0144)*
DKCC(-1)	0.0209**	-	0.0213**
	(0.0100)	-	*(0.0104)*

Table 11: *This table shows the estimates from the 2WLS regression using news variables for Denmark, the Euro Area, and Germany. The dependent variable is the first difference of the log of the daily DKK/EUR mid-point exchange rate. Macro news variables capture news surprises as the difference between actual announcement and survey expectations extracted from Bloomberg. The estimations take into account news regarding Danish Unemployment (DKUNEMP), Trade Balance (DKTB), Current Account (DKCA), CPI (DKCPI), GDP (DKGDP) and Consumer Confidence (DKCC); German IFO Index (DEIFO), GDP (DEGDP), and Industrial Production (DEIP); Euro-Area CPI (EACPI), Industrial Production (EAIP), and Business Climate Index (EABC). All variables are standardized by dividing each variable by its sample standard deviation. The coefficients and standard deviations for the news variables and interventions are multiplied by 1.000. There are not a sufficient amount of news to estimate the full model for the first model. *, **, and *** denotes significance at respectively the 10, 5, and 1 per cent level. Standard Errors in ( ) below the point estimates; test statistic values in [ ]. Subscript denotes lags.*

WLS Estimation of eq. 1 with news variables			
	Full Sample	**Sub-sample 1**	**Sub-sample 2**
DEIFO(0)	0.0335**	-	0.0338
	(0.0162)	-	*(0.0222)*
DEIFO(-1)	0.0148	-	-0.0389
	(0.0139)	-	*(0.0478)*
DEGDP(0)	-0.0192	-	-0.0179
	(0.0336)	-	*(0.0329)*
DEGDP(-1)	-0.0043	-	-0.0008
	(0.0217)	-	*(0.0229)*
DEIP(0)	-0.0095	-	-0.0048
	(0.0089)	-	*(0.0067)*
DEIP(-1)	-0.026	-	-0.0245
	(0.0159)	-	*(0.0177)*
EACPI(0)	-0.08*	-	0.06
	(0.0423)	-	*(0.0378)*
EAIP(-1)	-0.02*	-	-0.03
	(0.0128)	-	*(0.0367)*
EAIP(0)	-0.0012	-	0.0074
	(0.0167)	-	*(0.0171)*
EAIP(-1)	-0.0006	-	0.0005
	(0.0098)	-	*(0.0103)*
EABC(0)	0.0606***	-	0.0697***
	(0.0198)	-	*(0.0218)*
EABC(-1)	-0.0167	-	0.0272
	(0.0254)	-	*(0.0221)*

Table 12: *This table shows the estimates from the 2WLS regression using news variables for Denmark, the Euro Area, and Germany. The dependent variable is the first difference of the log of the daily DKK/EUR mid-point exchange rate. Macro news variables capture news surprises as the difference between actual announcement and survey expectations extracted from Bloomberg. The estimations take into account news regarding Danish Unemployment (DKUNEMP), Trade Balance (DKTB), Current Account (DKCA), CPI (DKCPI), GDP (DKGDP) and Consumer Confidence (DKCC); German IFO Index (DEIFO), GDP (DEGDP), and Industrial Production (DEIP); Euro-Area CPI (EACPI), Industrial Production (EAIP), and Business Climate Index (EABC). All variables are standardized by dividing each variable by its sample standard deviation. The coefficients and standard deviations for the news variables and interventions are multiplied by 1.000. There are not a sufficient amount of news to estimate the full model for the first model. *, **, and *** denotes significance at respectively the 10, 5, and 1 per cent level. Standard Errors in ( ) below the point estimates; test statistic values in [ ]. Subscript denotes lags.*

9 Figures

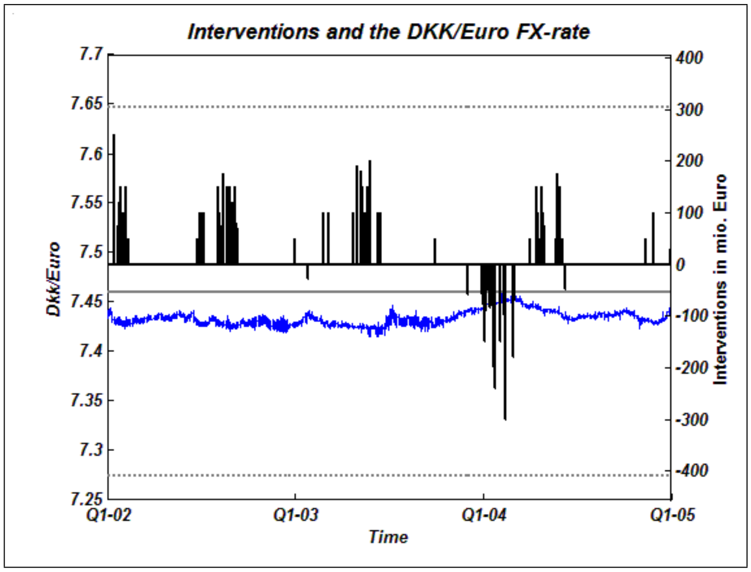

Figure 1: 5 min. DKK/EUR spot exchange rate and interventions in mill. EUR over the 1 January 2002 to 31 December 2004 period. A positive intervention corresponds to a purchase of EUR against a sale of DKK. The upper and lower lines are the ERM II deviation bands. Interventions are plotted against the central parity exchange rate of 7.46038.

The Intraday Effects of Central Bank Intervention on Exchange Rate Spreads

Rasmus Fatum* and Jesper Pedersen†and Peter Norman Sørensen‡

Abstract

This paper investigates the intraday effects of sterilized foreign exchange intervention on exchange rate spreads using official intraday intervention data provided by the Danish central bank. Our main result is that intervention purchases and intervention sales both exert a significant influence on exchange rate spreads but in opposite directions: Intervention sales of EUR, on average, reduce the spread while intervention purchases of EUR, on average, increase the spread. We also show that the significant and asymmetric effects of intervention purchases and sales stem from intervention carried out on "normal" days in terms of exchange rate volatility while intervention appears to be overlooked by the market when the market is volatilite.

Key words: Foreign Exchange Intervention; Intraday Data; Exchange Rate Spreads

JEL Classifications: D53; E58; F31; G15

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1 Introduction

This paper investigates the real-time (intraday) effects of Danish central bank intervention in the Euro (EUR) market on exchange rate spreads. The interventions under study are carried out by the Danish central bank (Danmarks Nationalbank, henceforth DN) over the 1 January 2002 to 31 December 2004 time-period under the provisions of the Exchange Rate Mechanism (ERM II).[1] Proprietary data on official intraday intervention transactions provided by the DN, along with indicative 5-minute spot bid and ask DKK/EUR exchange rate prices, facilitate our investigation. Our main result is that intervention purchases and intervention sales both exert a significant influence on exchange rate spreads but in opposite directions. The significant and asymmetric effects of intervention purchases and sales stem from intervention carried out on "normal" days in terms of exchange rate volatility while intervention appears to be overlooked by the market dealers when the market is volatile.

While the literature investigating the exchange rate level and volatility effects of intervention is vast, the literature on intervention and exchange rate spreads is extremely scarce.[2] Existing contributions to the latter literature are Naranjo and Nimalendran (2000), who analyze lower frequency (daily) intervention data, and Chari (2007), who analyzes less than accurate newswire reports of intervention as a proxy for official intraday interventions.[3] Both studies conclude that, on average, intervention increases the exchange rate spread.

Unlike their contributions, our study investigates the intraday effects of intervention on exchange rate spreads using accurate official intervention transactions data. We are the first to do so.[4]

[1] Denmark has participated in ERM II since 1 January 1999. In ERM II, a bilateral central rate and a deviation band is set for the currency of the participating country via-á-vis the EUR, but not against the currency of the other member states. The official DKK/EUR central rate is 7.46038 DKK/EUR an the official deviation band is set to +/- 2.25 percent. The DKK has traded within an even narrower range of +/- 0.50 percent around the Danish ERM II central rate. The official deviation band for all other ERM II member states is set to +/- 15 percent. For additional details on the institutional aspects of ERM II and DN intervention see Fatum and Pedersen (2007).

[2] See Humpage (2003) and Neely (2005) for recent surveys of the intervention literature. See Hasbrouck (2007) for a overview of the market microstructure theory.

[3] Fischer (2006) illustrates the lack of accuracy of newswire reports in the context of Swiss interventions.

[4] It is very rare that a central bank makes official intraday intervention data available for research. The Bank of Canada and the Swiss National Bank, and now DN are the only central banks that have made such data available. The Bank of Canada and the Swiss National Bank have not intervened since 1998 and 1995, respectively.

Our empirical research question is whether intervention exerts an intraday influence on exchange rate spreads. In order to answer this question we estimate time-series models of the exchange rate spread with intervention as the focal explanatory variable. We carry out our baseline estimations using OLS with heteroskedasticity and autocorrelation consistent (HAC) standard errors and covariances. As a methodological robustness test we also estimate the baseline model using the two-step weighted least squares (WLS) procedure developed by Andersen and Bollerslev (1998).

Using the same data and sample period as ours while assessing the intraday effects of intervention on exchange rate returns rather than on exchange rate spreads, Fatum and Pedersen (2007) show that the period under study encompasses a structural break, coinciding with a change in the Danish monetary policy stance. We carry out a standard Wald test and confirm that the break point identified in Fatum and Pedersen (2007) also applies to the context of our study. Consequently, we estimate our models over the full sample as well as separately across two sub-samples.

Fatum and Pedersen (2007) also show that while intervention purchases of EUR are effective in influencing exchange rate returns over the majority of the sample period, intervention sales of EUR are not.[5] This leads us to suspect that intervention purchases and sales may also affect the exchange rate spread differently. To test whether this is indeed the case, we also estimate all our models with intervention purchases of EUR and intervention sales of EUR entering as separate explanatory variables.

Furthermore, we take into account the possibility that the impact of intervention on exchange rate spreads depends on the state of the market around the time intervention is carried out. Particularly, we test whether interventions carried out on days when the exchange rate market is characterized by high intraday volatility are more or less influential than interventions that occur on other days. We do so by first distinguishing between intervention on "high-volatility" days (defined as a day with either a significant intraday volatility jump, i.e. a "jump-day"

By contrast, the DN intraday intervention data provides an opportunity to learn about the real-time effects of foreign exchange intervention carried out by a central bank that is currently intervening.

[5]Fatum and Pedersen (2007) note that the one sub-period is associated with a neutral Danish monetary policy stance while the other constitutes a period of Danish monetary policy easing both in absolute and relative terms (i.e. the interventions that occur during the latter sub-period are either montary policy consistent or inconsistent). Their main result is that intervention only affects exchange rate returns when the direction of intervention is consistent with the monetary policy stance.

as defined in Andersen, Bollerslev, and Diebold (2007), or with a daily realized volatility that is at least the average realized volatility of the sample plus two times the standard deviation of the realized volatility of the sample) and "non-high-volatility" days, i.e. "normal" days and, subsequently, by entering as separate variables interventions that occur on "high-volatility" days and interventions that occur on "normal" days.

In addition, we follow Naranjo and Nimalendran (2000) and others and take into account the possibility that our intervention variable contains an expected component, we control for macro news surprises, and we include lags of the intervention variables to allow for the possibility of delayed effects.

Our baseline estimations with only one intervention variable included (containing both intervention purchases and sales in one and the same variable) suggest that intervention has no significant intraday influence on the exchange rate spread. However, once we allow for the possibility that purchases and sales can have different effects, our results show that intervention purchases as well as sales do in fact significantly influence the exchange rate spread, but in opposite directions: Intervention sales of EUR, on average, reduce the spread while intervention purchases of EUR, on average, increase the spread. Apparently, the market's uncertainty over the true exchange rate is higher when the DN has stepped in to defend the DKK, while the opposite occurs when the DN signals that the DKK is strong. Clearly, our finding illustrates the necessity of distinguishing between intervention purchases and intervention sales when assessing the influence of intervention on exchange rate spreads.

Disentangling interventions on "high-volatility" days from interventions on "normal" days while also distinguishing between intervention purchases and intervention sales reveals that the significant and asymmetric effects of intervention purchases and sales are not uniform across intervention days but stem solely from the effects of interventions that are carried out on "normal" days. In other words, interventions that occur on high-volatility days have no impact on the exchange rate spread, i.e. interventions appear to be less relevant for the market dealers when the market is volatile, while interventions that occur on "normal" days significantly impact the exchange rate spread in the asymmetric manner previously discussed.

Fatum and Pedersen (2007) point out that the Danish ERM II intervention experience pertains to maintaining the exchange rate within a narrow deviation band and, therefore, their findings are applicable to Denmark and other countries that intervene to keep their respective exchange rates in narrow bands, but not necessarily to countries intervening to keep their respective exchange rates in wide deviation bands or to countries with flexible exchange rates. A similar disclaimer applies to our study, i.e. data permitting, additional research is warranted in order to shed light on whether our results are broadly applicable.

The rest of the paper is organized as follows. Sections 2 and 3 present the data and the econometric methodology, respectively. Section 4 discusses the results. Section 5 presents several robustness checks. Section 6 concludes.

2 Data

The intervention data covers all DN interventions in the DKK/EUR market over the 1 January 2002 to 31 December 2004 period. Our sample period is determined by availability of the data. The data includes the exact amount and time-stamp to the nearest minute obtained directly from the trade-sheet of each intervention transaction. Intervention amounts are quoted in EUR and a positive amount denotes a purchase of EUR against a sale of DKK. In accordance with the ERM II provisions, the DN trader conducting an intervention operation is obligated to write the amount and the exact time of the operation on the trade-sheet immediately after the completion of each individual intervention transaction. This information is forwarded to the ECB by the end of the trading day, at the latest. Our intraday intervention data consists of this extremely reliable information.

Table 1 displays descriptive statistics of the intervention data.[6] Our sample consists of a total of 89 intervention days, encompassing a total of 220 intervention transactions. On intervention days, the average intervention amount is EUR 164 million. A total of 68 intervention days consist of EUR purchases. Figure 1 shows that all interventions occur when the DKK/EUR rate is in the interior of the deviation band.

The high-frequency DKK/EUR exchange rate data is provided by Olsen and Associates, collected from commercial banks by Tenfore and Oanda, and covers the full sample period. The data consists of the bid and the offer spot exchange rate at the end of every 5-minute interval over every 24-hour period. The quotes are indicative quotes, i.e. not necessarily traded quotes. We follow Dacorogna, Muller, Nagler, Olsen, and Pictet (1993) and filter the data for anomalies and bad quotes.[7]

[6] Given the confidential nature of the intervention data, we are not at liberty to display or describe this data in greater detail.

[7] Transactions bid and ask prices are not available for the DKK/EUR exchange rate market.

It is standard in the intraday literature on widely traded currency pairs (e.g. the USD/EUR, the USD/DEM, the USD/JPY, and the USD/GBP) to define a trading day to start at 21.05 GMT the night before and end at 21.00 GMT on the evening of the trading day in question (see Bollerslev and Domowitz (1993)) and, furthermore, to define a weekend to start at 21.05 GMT Friday and finish 21.00 GMT Sunday (see Andersen, Bollerslev, Diebold, and Vega (2003)). The Danish exchange rate market, however, is different from the major exchange rate markets in that there is very little or virtually no trading of the Danish currency outside of standard Danish business hours (see Nationalbank (2003a) and ECB (2004)). Therefore, we define a trading day in the Danish currency market to start at 8.00 GMT+1 and finish at 17.00 GMT+1.[8] Consequently, our analysis considers a total of 752 trading days consisting of a total of 80476 5-minute DKK/EUR exchange rate returns.[9] Importantly, our trading day definition encompasses all intervention transactions in the period under study.

Table 2 summarizes key statistical properties of our 5-minute exchange rate spreads (defined as ask minus bid). As expected, the series has fatter tails than a normal distribution, and the Bera-Jarque test for normality is strongly rejected. The rejection is mainly attributed to excess kurtosis. The spreads series is far from being a white-noise process due to long memory, as evidenced by the Ljung-Box Q-test statistic which rejects that the first 540 autocorrelations (corresponding to one business-week) are jointly zero. This long memory can be attributed to daily periodicity.

Danish and Euro-Area interest rates are obtained from the websites of DN (www.nationalbanken.dk) and the ECB (www.ecb.int), respectively. Time-stamped Danish, German, and Euro-area macro announcements and preceding survey expectations are obtained from Bloomberg. Summary statistics regarding interest rates and macro news are available from the authors upon request.

[8] This definition of a trading day carries over naturally to a definition of a weekend, i.e. we define a weekend to start at 17.05 GMT+1 Friday and finish at 8.00 GMT+1 Monday.

[9] We also deleted the following fixed holidays from the analysis: 1 January, Easter (3 holidays), Christmas (24/25/26 December), 31 December as well as 4 Denmark-specific holidays (Store Bededag, Kristi Himmelfartsdag, Anden Pinsedag, Grundlovsdag).

3 The Empirical Model

We model the response of the DKK/EUR exchange rate spread, S_t, as a linear function of K lagged values of the spread itself and J lags of the intervention variable (in absolute terms), I_t:

$$S_t = C + \sum_{k=0}^{K} \beta_j S_{t-k} + \sum_{j=0}^{J} \gamma_j I_{t-j} + \varepsilon_t \text{ , } t = 1...T \tag{1}$$

As noted earlier, T=80476. We choose K=6 based on the Schwartz and Akaike information criteria and we set J=0 in our baseline estimations (we control for delayed effects by setting K=12 in our robustness estimations).[10]

To ensure that the long memory and the intraday periodicity of the exchange rate spreads series does not invalidate standard errors, thereby rendering the associated test statistics unreliable, we carry out the baseline estimations using OLS with heteroskedasticity and autocorrelation consistent (HAC) standard errors and covariances.

As a methodological robustness test we also estimate the baseline model using the two-step weighted least squares (WLS) procedure developed by Andersen and Bollerslev (1998). In order to do so, we first estimate equation (1) by OLS in order to obtain the estimated residuals, $\hat{\varepsilon}_t$. We then model the volatility pattern using these estimated residuals and the following parameterization:

$$\begin{aligned} |\hat{\varepsilon}_t| &= c_1 + c_2 + \alpha \frac{\hat{\sigma}_t}{\sqrt{n}} + \sum_{k=0}^{K} \beta_k I_{t-k} \\ &+ \left(\sum_{q=1}^{Q} \delta_q \cos\left(\frac{q2\pi n}{108}\right) + \varphi_q \sin\left(\frac{q2\pi n}{108}\right) \right) + u_t \end{aligned} \tag{2}$$

where $\hat{\varepsilon}_t$ is the residual of equation (1), and its absolute value proxies for the volatility in the 5-minute interval t, c_1 and c_2 are normalizing constants, n is the number of intervals in a day (in our case 108), $\hat{\sigma}_t$ the one-day ahead volatility forecast for day t (i.e. the day that contains

[10] We also include in the conditional mean model as additional explanatory variables the distance from parity, i.e. a measure of the distance between the DKK/EUR exchange rate and the central rate, as well as the EUR-DKK interest rate differential. Both variable proved insignificant in all estimations and were thus excluded form the conditional mean model for the remainder of the analysis.

interval t), q is a specific intraday calendar effect, Q is the total number of calendar effects accounted for (Q=6, based on the Schwartz and the Akaike information criteria), and u_t denotes the residuals (assumed to be standard normal).

We model the lower frequency intraday pattern (the first term after the vector of constants) using the concept of realized volatility (RV), calculated on 30 minute returns.[11] Since the realized volatility forecast cannot capture the observed cyclical intraday patterns (the slow decay in the autocorrelations), we model the higher frequency periodicity by inclusion of a Fourier flexible form, see Gallant (1981).[12] Consistent with Andersen and Bollerslev (1998) and Andersen, Bollerslev, Diebold, and Vega (2003), who include their macro news variables in the volatility equation, we include the intervention variable (i.e. our main "news" variable) in the volatility model.

3.1 Structural break

In their study of the intraday effects of DN interventions on DKK/EUR exchange rate returns, Fatum and Pedersen (2007) show that a structural break occurred on 16 August 2002.[13] They argue that since the Danish deposit and lending rates were lowered (twice) in August 2002, independently of the ECB who kept its key interest rates unchanged until the end of 2002, the coinciding change in the monetary policy stance provides the economic rationale for the statistical break point.

Since our data and sample period are the same as that of Fatum and Pedersen (2007), we also take into account the possibility that the same 16 August 2002 structural break affects our study of the intraday effects of DN interventions on DKK/EUR exchange rate bid-ask spreads. Using a standard Wald test we confirm that a structural break occurred on 16 August 2002.[14] Consequently, to allow for parameter instability

[11] RV is defined as the daily sum of squared returns and constitutes an unbiased, efficient and asymptotically consistent estimate of the true daily quadratic variation. A key advantage of using RV is that this semi-parametric approach does not require additional model estimation.

[12] A Fourier flexible form consists of a number of sine- and cosine terms with varying degrees of periodicity (the terms in the parenthesis of equation (2)) It allows for a model specification as flexible as possible, thereby enabling us to fit the intraday pattern of the residuals from equation (1).

[13] Fatum and Pedersen (2007) employ the change point test procedure of Andrews (1993). This test does not require advance knowledge regarding the exact timing of a potential change point.

[14] We use a standard Wald test rather than the Andrews (1993) procedure since, given the findings of Fatum and Pedersen (2007), we do have advance knowledge regarding the suspected exact timing of the potential change point.

and to ensure that our statistical inference is valid, we also carry out our analysis separately on two sub-samples, the 1 January 2002 to 16 August 2002 period (sub-sample 1), and the 17 August 2002 to 31 December 2004 period (sub-sample 2).

4 Results

Table 3 displays the results of the baseline estimation of equation (1). The first column of Table 3 shows that for the full sample the coefficient estimate associated with contemporaneous intervention is insignificant. Columns 2 and 3 show that this also the case for sub-samples 1 and 2. These initial estimations would suggest that intervention has no intraday influence on the exchange rate spread.

As noted earlier, in their analysis of the intraday effects of intervention on exchange rate returns, Fatum and Pedersen (2007) show that over the majority of the sample period only purchases of EUR are effective in influencing exchange rate returns while intervention sales of EUR are not. Consequently, their findings illustrate the importance of distinguishing between intervention purchases and sales when investigating the effects of intervention on exchange rate returns. To allow for the possibility of similar non-uniform effects across intervention purchases and sales in our context of analyzing exchange rate spreads, we re-estimate the baseline model with intervention purchases and sales entering as separate variables.

The results of the baseline estimation with separate intervention purchases and sales variables (both in absolute terms) are displayed in Table 4. The full sample results reported in the first column reveal that contemporaneous intervention purchases as well as contemporaneous sales do indeed significantly influence the exchange rate spread but in opposite directions. While intervention sales of EUR decrease the exchange rate spread, intervention purchases of EUR increase the spread. The second column displays the results pertaining to sub-sample 1. Since all intervention carried out during sub-sample 1 are purchases of EUR, the sub-sample 1 results are identical to the sub-sample 1 results displayed in Table 3 and thus do not reveal any new insights. Sub-sample 2, however, contain both intervention purchases and sales and, as the third column of Table 3 shows, the sub-sample 2 results are qualitatively identical to the results pertaining to the full sample (thereby implying that the full sample results stem from the effects of intervention during sub-sample 2).

Certainly, these results make clear the necessity of distinguishing between intervention purchases and intervention sales when assessing the influence of intervention on exchange rate spreads.

4.1 Intervention on High-Volatility Days

In order to test whether interventions that occur on "high-volatility" days impact the exchange rate differently, i.e. do interventions that are carried out on "high-volatility" days exert more (or less) of an influence on the exchange rate, we distinguish between intervention on "high-volatility" days (defined as a day with either a significant intraday volatility jump, i.e. a "jump-day" as defined in Andersen, Bollerslev, and Diebold (2007), or with a daily realized volatility that is at least the average realized volatility of the sample plus two times the standard deviation of the realized volatility of the sample) and "non-high-volatility" days, i.e. "normal" days. Subsequently, we enter as separate variables interventions that occur on "high-volatility" days and interventions that occur on "normal" days in our estimations.

Table 5 displays the results for all interventions divided into separate high- and non-high volatility day interventions, and Table 6 displays the results for intervention sales and purchases divided into separate high- and non-high volatility day interventions, respectively. Table 5 does not provide any new insights as the variable containing all interventions is insignificant with respect to interventions carried out on high-volatility days as well as with respect to interventions carried out on non-high-volatility days. This simply confirms the necessity of taking into account that intervention purchases and sales impact the exchange rate spread in opposite directions.

Table 6, however, reveals that the significant and asymmetric effects of intervention purchases and sales are not uniform across intervention days but stem solely from the effects of interventions that are carried out on non-high-volatility days. Particularly, our results show that interventions that occur on high-volatility days have no impact on the exchange rate spread, i.e. interventions appear to be overlooked by the market when the market is volatile, while interventions that occur on "normal" days significantly impact the exchange rate spread in the asymmetric manner previously discussed.

5 Robustness

In order to test the robustness of our results, we take into account the possibility that the intervention variable contains an expected component, control for macro news surprises, carry out the analysis using a different econometric procedure, and include lags of the intervention variables to allow for the possibility of delayed effects.

First, while there is no reason to believe that intervention is triggered by the contemporaneous exchange rate spread (i.e. the change in exchange rate spread that occurs over the 5-minute interval within which intervention is carried out). Fatum and Pedersen (2007) show that intervention is correlated with recent (lagged) exchange rate movements and with recent (lagged) intervention even at the intraday frequency.[15] Accordingly, our intervention variable is comprised of an unexpected as well as an expected component. To ensure that failure to disentangle the latter component from the intervention variable does not lead to an underestimation of the true impact of intervention on exchange rate spreads, we therefore disentangle the expected component from the intervention variable and, in turn, re-assess the effect of intervention on exchange rate spreads employing only the unexpected component of intervention.[16] Specifically, we follow Fatum and Pedersen (2007) and estimate a central bank reaction function to capture the expected component of the intraday intervention variable. In turn, we subtract the expected component of intervention from the actual intervention variable in intervals where the latter is non-zero. The resulting series constitutes a proxy for unexpected intervention variable.[17] The results of estimating the effects of unexpected intervention on exchange rate spreads are displayed in Tables 8 (all interventions in one variable) and 9 (separate intervention sales and purchases variables). As the tables show, the results are qualitatively identical to the comparable estimation results from estimations that do not distinguish between actual intervention and unexpected intervention (Tables 3 and 4).[18]

Second, to ensure that our estimated effects of intervention are not tainted by the coincidental arrival of macro news, we extend our analysis to include time-stamped Danish, German, and Euro-area macro surprises. Specifically, we include macro surprises regarding Danish Unemployment (DKUNEMP), Trade Balance (DKTB), Current Account (DKCA), CPI (DKCPI), GDP (DKGDP) and Consumer Confidence

[15] See Neely (2008) for a survey-based assessment of what prompts central banks to intervene in the foreign exchange market.

[16] In the context of exchange rates and monetary policy news, Fatum and Scholnick (2008) show that failure to disentangle the surprise component from the actual monetary policy change leads to an underestimation of the impact of monetary policy.

[17] The results of the reaction function are displayed in table 7.

[18] We also estimate the effects of unexpected intervention using proxies derived from alternative reaction function specifications with different lag structures and with lags of the distance from parity measure included as an additional explanatory variable. Our results are not sensitive to these alternative reaction function specifications.

(DKCC); German IFO Index (DEIFO), GDP (DEGDP), and Industrial Production (DEIP); Euro-Area CPI (EACPI), Industrial Production (EAIP), and Business Climate Index (EABC). We measure macro surprises as the difference between macro announcement and preceding survey expectation obtained from Bloomberg. To facilitate the comparison of the coefficient estimates of the macro surprises to the coefficient estimates of intervention, we follow Fatum and Pedersen (2007) and standardize the macro news as well as the intervention variable (i.e. for each variable we divide the surprise by its sample standard deviation).[19]

The results pertaining to the model where intervention purchases and sales enter as separate variables are displayed in Table 10 and 11.[20] Our results show that a few of the macro surprises influence the DKK/EUR exchange rate spread and, more importantly, that the baseline results regarding the asymmetric and significant effects of intervention purchases and sales remain unchanged.[21]

Third, we re-estimate the baseline model using the WLS procedure of Andersen and Bollerslev (1998). The WLS results pertaining to the conditional mean equation are displayed in Table 12 and are qualitatively identical to the conditional mean results based on the less sophisticated HAC baseline estimation procedure.[22]

Fourth, in order to test for delayed effects of intervention, we re-estimate our baseline models with 12 lags of intervention included (i.e. we set J=12 in Equation (1)). The results are displayed in Tables 16 (all interventions in one variable) and 17 (separate intervention sales and purchases variables). Table 16 shows that, with the exception of a mar-

[19] Almeida, Goodhart, and Payne (1998) and Andersen, Bollerslev, Diebold, and Vega (2003) show that the conditional mean of the exchange rate generally adjusts immediately (i.e. jumps) in response to macro news. Accordingly, we include only the contemporanuous and the first lag of the macro surprises in our estimations.

[20] Sub-sample 1 is too short to encompass a sufficient number of macro surprises for a meaningful estimation of the influence of macro news.

[21] The (absolute) magnitude of the coefficient estimates associated with the standardized macro surprise are similar to the (absolute) magnitude of the coefficient estimates associated with the standardized interventions, thereby showing that the relative influence of intervention on exchange rate spreads is comparable to the relative influence of most macro surprises. A similar result is found in Fatum and Pedersen (2007) who show that the relative influence of intervention on exchange rate returns is comparable to the relative influence of macro surprises.

[22] Tables 13 through 15 show the results of the WLS (OLS) estimation of the volatility equation (equation (2)) associated with the model where intervention purchases and sales enter as separate variables. Interestingly, while intervention purchases and intervention sales impact the conditional mean of the exchange rate spread in opposite directions, these tables show that intervention purchases and sales both reduce the volatility of the exchange rate spread.

ginally significant 11th lag associated with the full sample estimation, we find no significant effect of intervention when we do not distinguish between intervention sales and intervention purchases. Table 17 shows that, once again we find asymmetric and significant contemporaneous effects of intervention sales and purchases, consistent with what we have previously discussed. When we distinguish between purchases and sales, the 4th lag of intervention sales and the 11th lag of intervention purchases are also significant across both the full sample and sub-sample 2. Their respective signs are consistent with the respective signs associated with the previously discussed contemporaneous effects.

In sum, all our robustness checks confirm that intervention purchases and intervention sales both exert a significant influence on exchange rate spreads but in opposite directions.

6 Conclusion

This paper investigates the real-time (intraday) effects of intervention on bid-ask exchange rate spreads using proprietary intraday intervention data provided by the Danish central bank and indicative 5-minute spot bid and ask DKK/EUR exchange rate quotes. All the interventions under study are carried out in the DKK/EUR market over the 1 January 2002 to 31 December 2004 period.

It is very rare that a central bank makes official intraday intervention data available for research. It is, therefore, not surprising that existing studies of the effects of intervention on exchange rate spreads are forced to either use less than accurate newswire reports of intervention in order to analyze the intraday effects of intervention or use official daily intervention data to analyze the daily effects of intervention. Our study is the first to analyze the intraday effects of intervention on exchange rate spreads using official, time-stamped intraday intervention data.

We test whether intervention exerts an intraday influence on exchange rate spreads by estimating time-series models of the exchange rate spread with intervention as the focal explanatory variable. In particular, we estimate OLS with HAC standard errors and covariances and, as a methodological robustness check, we also employ the two-step WLS procedure developed by Andersen and Bollerslev (1998).

We take into account the possibility that intervention purchases and sales may affect the exchange rate spread differently and, consequently, estimate models with intervention purchases of EUR and intervention sales of EUR entering as separate explanatory variables. We also take into account the possibility that the impact of intervention on exchange rate spreads depends on the state of the market around the time intervention is carried out and distinguish between intervention carried out on "high-volatility" days and intervention carried out on "normal" days.

Our main result is that intervention purchases and intervention sales both exert a significant influence on exchange rate spreads, but in opposite directions: Intervention sales of EUR aimed at appreciating the domestic currency, on average, reduce the spread, while intervention purchases of EUR aimed at depreciating the domestic currency, on average, increase the spread. Clearly, this result illustrates the necessity of distinguishing between intervention purchases and intervention sales when assessing the influence of intervention on exchange rate spreads. We also show that these significant and asymmetric effects of intervention purchases and sales stem from intervention carried out on "normal" days in terms of exchange rate volatility, while intervention appears to be overlooked by the market when carried out when the market is volatile.

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7 Tables

	Daily Intervention	
	Number of Interventions	*Average Amount*
All	89	164
Purchases	68	158
Sales	21	-182
	Intraday Interventions	
	Number of Interventions	*Average Amount*
All	220	67
Purchases	157	69
Sales	63	61

Table 1: *This table shows summary Statistics for the Invervention Variable. The average amount is denoted in millions of Euros. Data source: Danmarks Nationalbank Sample period: 1 January 2002 31 December 2004*

Summary Statistics for bid-ask Spreads			
Mean	**Std. dev.**	**Skewness**	**Kurtosis**
0.019	0.0315	3.0272	12.5202**
(∼0.001)	(-)	(0.0080)	(0.0170)
Minimum	**Maximum**	**BJ-test for Normality**	**LB Q-test (5-day lag)**
0	0.3083	426800***	38942***
		[5.9915]	[3.8415]

Table 2: *Data runs from January 1, 2002 to December 31, 2004. The data consists of 80.476 observations on DKK/EUR exchange rate. The spreads are calculated from bid- and ask prices from Olsen Financial Technologies. The data consists of 80476 observations of DKK/EUR exchange rate bid - and ask prices. The exchange rate spreads are calculated as ask minus bid prices. *, **, and *** denotes significance at respectively the 10, 5, and 1 per cent level. Standard errors in () below the point estimates. Critical values in [].*

Baseline Regression and subsample			
$S_t = \alpha + \Sigma_{k=1}^{K}\beta_{t-k}S_{t-k} + \Sigma_{j=0}^{J}\Gamma_j I_{t-k} + \varepsilon_t$			
	Full Sample	**Sub-sample 1**	**Sub-sample 2**
Γ_t	1,7278	0,2822	1,9319
	(1,4013)	*(1,9821)*	*(1,7399)*
C	0,0021***	0,0042***	0,0023***
	(0,0001)	*(0,0004)*	*(0,0001)*
β_{t-1}	0,3234***	0,0987***	0,3381***
	(0,0095)	*(0,0132)*	*(0,0102)*
β_{t-2}	0,1981***	0,1243***	0,1974***
	(0,0077)	*(0,0119)*	*(0,0084)*
β_{t-3}	0,1161***	0,0714***	0,1154***
	(0,0074)	*(0,0119)*	*(0,0082)*
β_{t-4}	0,0974***	0,0910***	0,0938***
	(0,0074)	*(0,0125)*	*(0,0080)*
β_{t-5}	0,0691***	0,0783***	0,0650***
	(0,0079)	*(0,0133)*	*(0,0086)*
β_{t-6}	0,0862***	0,0896***	0,0828***
	(0,0067)	*(0,0110)*	*(0,0073)*
R^2	0,58	0,10	0,60
Number of Observations	220	58	162

Table 3: *This table shows the estimates of a regression of absolute interventions on bid-ask spreads. I denotes absolute intervention. S denotes the bid-ask spread measured in pips.* Γ_{t-j} *denotes the coefficient on the absolute intervention variable.* β_{t-j} *denotes coefficients on contemporanous and lagged 5 min. fx spreads. The coefficients for the constant, the interventions and their standard deviations are multiplied by 100.000 *, **, and *** denotes significance at respectively the 10, 5, and 1 per cent level. Standard errors in () below point estimates.*

Baseline Regression and subsample				
$S_t = \alpha + \Sigma_{k=1}^{K}\beta_{t-k}S_{t-k} + \Sigma_{j=0}^{J}\Gamma_j I_{t-j}^{Purc} + \Sigma_{j=1}^{J}\Gamma_j I_{t-j}^{Sales} + \varepsilon_t$				
	Full Sample		**Sub-sample II**	
	Sales	*Purchases*	*Sales*	*Purchases*
Γ_t	-2,1529**	3,5530*	-0,2304**	0,4965*
	(1,0658)	*(1,8655)*	*(0,1074)*	*(0,2634)*
C	0,0021***	0,0021***	0,0023***	0,0023***
	(0,0001)	*(0,0001)*	*(0,0001)*	*(0,0001)*
β_{t-1}	0,3233***	0,3233***	0,3380***	0,3380***
	(0,0095)	*(0,0095)*	*(0,0102)*	*(0,0102)*
β_{t-2}	0,1981***	0,1981***	0,1974***	0,1974***
	(0,0077)	*(0,0077)*	*(0,0084)*	*(0,0084)*
β_{t-3}	0,1161***	0,1161***	0,1155***	0,1155***
	(0,0074)	*(0,0074)*	*(0,0082)*	*(0,0082)*
β_{t-4}	0,0974***	0,0974***	0,0938***	0,0938***
	(0,0074)	*(0,0074)*	*(0,0080)*	*(0,0080)*
β_{t-5}	0,0691***	0,0691***	0,0650***	0,0650***
	(0,0079)	*(0,0079)*	*(0,0086)*	*(0,0086)*
β_{t-6}	0,0862***	0,0862***	0,0829***	0,0829***
	(0,0068)	*(0,0068)*	*(0,0072)*	*(0,0072)*
R^2		0,60		0,60
Number of Observations	157	63	99	63

Table 4: *This table shows the estimates of a regression of absolute interventions on bid-ask spreads. I denotes absolute intervention purchases or sales. S denotes the bid-ask spread measured in pips.* Γ_{t-j} *denotes the coefficient on the absolute intervention variable.* β_{t-j} *denotes coefficients on contemporanous and lagged 5 min. fx spreads. The coefficients for the constant, the interventions and their standard deviations are multiplied by 100.000 *, **, and *** denotes significance at respectively the 10, 5, and 1 per cent level. Standard errors in () below point estimates.*

Interventions and Volatility						
$S_t = \alpha + \Sigma_{k=1}^{K}\beta_{t-k}S_{t-k} + \Sigma_{j=0}^{J}\Gamma_j I_{t-j}^{L.vol} + \Sigma_{j=0}^{J}\Gamma_j I_{t-j}^{H.vol} + \varepsilon_t$						
	Full Sample		**Sub-sample 1**		**Sub-sample 2**	
	High Vol Day	*Low Vol Day*	*High Vol Day*	*Low Vol Day*	*High Vol Day*	*Low Vol Day*
Γ_t	0,7097	2,3764	1,3750	-1,7651	-0,2841	2,8280
	(2,4752)	*(1,7074)*	*(2,8177)*	*(2,0985)*	*(4,2352)*	*(1,9257)*
R^2		0,58		0.10		0,60
Number of Observations	89	131	27	31	62	100

Table 5: *This table shows the estimates of a regression of absolute interventions on bid-ask spreads in which interventions are partitioned into interventions that fall on high/low volatility days. A high volatility day is defined as a day with a significant intraday volatility "jump" as defined in Andersen, Bollerslev, and Diebold, 2007 and days with a RV higher than two times the standard deviation of the RV though the sample. A low volatility day is defined as all other intervention days. I denotes absolute intervention purchases or sales. S denotes the bid-ask spread measured in pips.* Γ_{t-j} *denotes the coefficient on the absolute intervention variable. Estimates of the coefficients for the contemporanous and lagged 5 min. fx spreads are not shown for brevity. The coefficients for the constant, the interventions and their standard deviations are multiplied by 100.000 *, **, and *** denotes significance at respectively the 10, 5, and 1 per cent level. Standard errors in () below point estimates.*

Interventions and Volatility				
$S_t = \alpha + \Sigma_{k=1}^{K}\beta_{t-k}S_{t-k} + \Sigma_{j=0}^{J}\Gamma_j I_{t-j}^{Purc,H.vol} + \Sigma_{j=1}^{J}\Gamma_j I_{t-j}^{Sales,H.vol} + \Sigma_{j=0}^{J}\Gamma_j I_{t-j}^{Purc,L.vol} + \Sigma_{j=1}^{J}\Gamma_j I_{t-j}^{Sales,L.vol} + \varepsilon_t$				
	Full Sample		**Sub-sample 2**	
	High Vol day	*Low Vol day*	*High Vol day*	*Low Vol day*
	Sales			
Γ_t	0,6682	-3,0099***	0,5164	-3,1693***
	(0,8225)	*(1,0592)*	*(0,8217)*	*(1,0660)*
Number of Observations	70	87	30	69
	Purchases			
Γ_t	0,6505	5,8077***	-1,2167	7,7709***
	(2,8653)	*(2,3141)*	*(5,4948)*	*(2,6954)*
Number of Observations	19	44	19	44
R^2		0,58		0,60

Table 6: *This table shows the estimates of a regression of absolute interventions on bid-ask spreads in which interventions are partitioned into interventions that fall on high/low volatility days. A high volatility day is defined as a day with a significant intraday volatility "jump" as defined in Andersen, Bollerslev, and Diebold, 2007 and days with a RV higher than two times the standard deviation of the RV though the sample. A low volatility day is defined as all other intervention days. I denotes absolute intervention purchases or sales. S denotes the bid-ask spread measured in pips.* Γ_{t-j} *denotes the coefficient on the absolute intervention variable. Estimates of the coefficients for the contemporanous and lagged 5 min. fx spreads are not shown for brevity. The coefficients for the constant, the interventions and their standard deviations are multiplied by 100.000 *, **, and *** denotes significance at respectively the 10, 5, and 1 per cent level. Standard errors in () below point estimates.*

Reaction Function	
	$I_t = \alpha + \Sigma_{k=0}^{K}\beta_{t-k}S_{t-k} + \Sigma_{j=1}^{J}\Gamma_j I_{t-j} + \varepsilon_t$
C	0*** *(0,0167)*
Γ_{t-1}	0,0627*** *(0,0185)*
Γ_{t-2}	0,0097 *(0,0087)*
Γ_{t-3}	0,0443 *(0,0255)*
Γ_{t-4}	0,0065 *(0,0061)*
Γ_{t-5}	0,0023 *(0,0034)*
Γ_{t-6}	0,0307 *(0,0181)*
β_{t-1}	-304,48** *(133,82)*
β_{t-2}	-340,50** *(145,83)*
β_{t-3}	-396,42*** *(135,78)*
β_{t-4}	-279,28** *(126,78)*
β_{t-5}	49,367 *(124,10)*
F-test	51***
R^2	0,0075

Table 7: Γ_{t-j} *denotes the intervention variable.* $\beta_{t-j,n}$ *denotes 5 min. fx returns. The coefficients for current and lagged macro news, and current and lagged Denmark/Euro-Area interest rate differential are not shown for ease of exposition. *, **, and *** denotes significance at respectively the 10, 5, and 1 per cent level. Standard errors in () below point estimates.*

Intervention effects from reaction function on spreads			
$S_t = \alpha + \Sigma_{k=1}^{K}\beta_{t-k}S_{t-k} + \Sigma_{j=0}^{J}\Gamma_j^{unexp}I_{t-j} + \varepsilon_t$			
	Full Sample	**Sub-sample 1**	**Sub-sample 2**
Γ_t^{unexp}	1,7278	0,2668	1,9362
	(1,4153)	*(1,9865)*	*(1,7594)*
C	0,0021***	0,0042***	0,0023***
	(0,0001)	*(0,0004)*	*(0,0001)*
β_{t-1}	0,3234***	0,0987***	0,3381***
	(0,0095)	*(0,0132)*	*(0,0102)*
β_{t-2}	0,1981***	0,1243***	0,1974***
	(0,0077)	*(0,0119)*	*(0,0084)*
β_{t-3}	0,1161***	0,0714***	0,1154***
	(0,0074)	*(0,0119)*	*(0,0082)*
β_{t-4}	0,00974	0,091***	0,0938***
	(0,0074)	*(0,0125)*	*(0,0080)*
β_{t-5}	0,0691***	0,0783***	0,065***
	(0,0079)	*(0,0133)*	*(0,0086)*
β_{t-6}	0,0862***	0,0897***	0,0828***
	(0,0067)	*(0,0110)*	*(0,0073)*
R^2	0,58	0,10	0,60
Number of Observations	220	58	162

Table 8: *This table shows the estimates of a regression of absolute unexpected interventions estimated from the reaction function on bid-ask spreads. The unexpected intervention is defined as the residual of the intervention reaction function estimation (displayed in table 7) in an interval in which actual intervention occurs, and zero otherwise. S denotes the bid-ask spread measured in pips. I denotes absolute intervention.* Γ_{t-j}^{unexp} *denotes the coefficient on the absolute unexpected intervention variable.* β_{t-j} *denotes coefficients on contemporanous and lagged 5 min. fx spreads. The coefficients for the constant, the interventions and their standard deviations are multiplied by 100.000 *, **, and *** denotes significance at respectively the 10, 5, and 1 per cent level. Standard errors in () below point estimates.*

Intervention effects from reaction function on spreads				
$S_t = \alpha + \Sigma_{k=1}^{K}\beta_{t-k}S_{t-k} + \Sigma_{j=0}^{J}\Gamma_j^{unexp}I_{t-j}^{Sale/Purc} + \varepsilon_t$				
	Full Sample		**Sub-sample II**	
	Sales	*Purchases*	*Sales*	*Purchases*
Γ_t^{unexp}	-2,2089** *(1,0720)*	3,5643* *(1.8814)*	-2,3623** *(1,0806)*	4,9926* *(2,6572)*
C	0,0021*** *(0,0001)*	0,0023*** *(0,0001)*	0,0023*** *(0,0001)*	0,0023*** *(0,0001)*
β_{t-1}	0,3233*** *(0,0095)*	0,3380*** *(0,0102)*	0,3380*** *(0,0102)*	0,3380*** *(0,0102)*
β_{t-2}	0,1981*** *(0,0077)*	0,1974*** *(0,0084)*	0,1974*** *(0,0084)*	0,1974*** *(0,0084)*
β_{t-3}	0,1161*** *(0,0074)*	0,1155*** *(0,0082)*	0,1155*** *(0,0082)*	0,1155*** *(0,0082)*
β_{t-4}	0,0974*** *(0,0074)*	0,0938*** *(0,0080)*	0,0938*** *(0,0080)*	0,0938*** *(0,0080)*
β_{t-5}	0,0691*** *(0,0079)*	0,0650*** *(0,0086)*	0,0650*** *(0,0086)*	0,0650*** *(0,0086)*
β_{t-6}	0,0862*** *(0,0067)*	0,0829*** *(0,0073)*	0,0829*** *(0,0000)*	0,0829*** *(0,0073)*
R^2		0,58		0,60
Number of Observations	157	63	99	44

Table 9: *This table shows the estimates of a regression of absolute unexpected interventions estimated from the reaction function on bid-ask spreads. The unexpected intervention is defined as the residual of the intervention reaction function estimation (displayed in table 7) in an interval in which actual intervention occurs, and zero otherwise. I denotes absolute intervention. S denotes the bid-ask spread measured in pips.* Γ_{t-j}^{unexp} *denotes the coefficient on the absolute unexpected intervention variable.* β_{t-j} *denotes coefficients on contemporanous and lagged 5 min. fx spreads. The coefficients for the constant, the interventions and their standard deviations are multiplied by 100.000 *, **, and *** denotes significance at respectively the 10, 5, and 1 per cent level. Standard errors in () below point estimates.*

Interventions and News		
$S_t = \alpha + \Sigma_{k=1}^{K}\beta_{t-k}S_{t-k} + \Sigma_{j=0}^{J}\Gamma_j^{Purc/Sales}I_{t-j} + \varepsilon_t$		
	Full Sample	**Sub-sample 2**
Interventions		
Γ_t^{Sales}	-0,0014*	-0.0015**
	(0.0007)	*(0.0007)*
$\Gamma_t^{Purc.}$	0,0017*	0.0023*
	(0.0009)	*(0.0012)*
Danish News		
$DKUNEMP_t$	0,0104	0,0108
	(0,0060)	*(0,0060)*
$DKUNEMP_{t-1}$	0,0081	0,0102**
	(0,0048)	*(0,0044)*
$DKTB_t$	0,0001	0,0045
	(0,0040)	*(0,0041)*
$DKTB_{t-1}$	-0,0027**	-0,0035
	(0,0017)	*(0,0021)*
$DKCA_t$	-0,0019	-0,0112***
	(0,0046)	*(0,0035)*
$DKCA_{t-1}$	-0,0028	-0,0044
	(0,0007)	*(0,0043)*
$DKCPI_t$	-0,0117**	0,0138
	(0,0051)	*(0,0150)*
$DKCPI_{t-1}$	0,0056	-0,0048
	(0,0056)	*(0,0078)*
$DKGDP_t$	-0,0024	-0,0085
	(0,0111)	*(0,0099)*
$DKGDP_{t-1}$	-0,0196**	-0,0274***
	(0,0088)	*(0,0054)*
$DKCC_t$	0,0122***	0,0120**
	(0,0048)	*(0,0049)*
$DKCC_{t-1}$	-0,0061**	-0,0065**
	(0,0028)	*(0,0029)*

Table 10: *This table shows the estimates of a regression of absolute interventions on bid-ask spreads and macroeconomic news. I denotes absolute interventions standardized by its sample standard deviation to facilitate comparison. S denotes the bid-ask spread measured in pips.* Γ_{t-j} *denotes the coefficient on the standardized absolute intervention variable.* $News_{h,t}$ *denotes a vector of macroeconomic news variables. Macro news variables are defined as the difference between actual announcement and survey expectations extracted from Bloomberg. The estimation includes news regarding Danish Unemployment (DKUNEMP), Trade Balance (DKTB), Current Account (DKCA); German IFO Index (DEIFO), GDP (DEGDP), and Industrial Production (DEIP); Euro-Area CPI (EACPI), Industrial Production (EAIP), and Business Climate Index (EABC). All variables are standardized by dividing each variable by its sample standard deviation. *, **, and *** denotes significance at respectively the 10, 5, and 1 per cent level. Standard errors in () below point estimates.*

Interventions and News		
$S_t = \alpha + \Sigma_{k=1}^{K} \beta_{t-k} S_{t-k} + \Sigma_{j=0}^{J} \Gamma_j^{Purc/Sales} I_{t-j} + \varepsilon_t$		
	Full Sample	**Sub-sample 2**
German News		
$DEIFO_t$	-0,0095** *(0,0039)*	-0,0101*** *(0,0038)*
$DEIFO_{t-1}$	0,0037 *(0,0037)*	0,0018 *(0,0045)*
$DEGDP_t$	-0,0150*** *(0,0026)*	-0,0155*** *(0,0026)*
$DEGDP_{t-1}$	-0,0057*** *(0,0020)*	-0,0057*** *(0,0020)*
$DEIP_t$	0,0075* *(0,0040)*	0,0035 *(0,0033)*
$DEIP_{t-1}$	0,0031 *(0,0062)*	0,0070 *(0,0075)*
Euro Area News		
$EACPI_t$	-0,0051 *(0,0033)*	-0,0053 *(0,0058)*
$EACPI_{t-1}$	0,0031 *(0,0026)*	0,0044 *(0,0033)*
$EAIP_t$	0,0036 *(0,0028)*	0,0029 *(0,0031)*
$EAIP_{t-1}$	-0,0011 *(0,0026)*	-0,0023 *(0,0023)*
$EABC_t$	-0,0212 *(0,0187)*	-0,0355 *(0,0243)*
$EABC_{t-1}$	0,0035 *(0,0093)*	0,0134 *(0,0098)*
R^2	0,58	0,60

Table 11: *This table shows the estimates of a regression of absolute interventions on bid-ask spreads and macroeconomic news announcements. I denotes absolute interventions standardized by its sample standard deviation to facilitate comparison. S denotes the bid-ask spread measured in pips. Γ_{t-j} denotes the coefficient on the standardized absolute intervention variable. $News_{h,t}$ denotes a vector of macroeconomic news variables. Macro news variables capture news surprises as the difference between actual announcement and survey expectations extracted from Bloomberg. The estimation includes news regarding Danish Unemployment (DKUNEMP), Trade Balance (DKTB), Current Account (DKCA); German IFO Index (DEIFO), GDP (DEGDP), and Industrial Production (DEIP); Euro-Area CPI (EACPI), Industrial Production (EAIP), and Business Climate Index (EABC). All variables are standardized by dividing each variable by its sample standard deviation. *, **, and *** denotes significance at respectively the 10, 5, and 1 per cent level. Standard errors in () below point estimates.*

2WLS of Baseline Regressions			
	Interventions	**Sales**	**Purchases**
Γ_t	1,0222	-2,1853*	3,0219**
	(1,0385)	*(1,1259)*	*(1,4767)*
Number of Observations	220	63	157

Table 12: *This table shows the estimates of a regression of absolute interventions on bid-ask spreads estimated using the two-stage weighted least squares methodology from Andersen et al. 2003. Only the coefficients on the absolute interventions, denoted by Γ_{t-j}, are shown for brevity. The coefficients for the constant, the interventions and their standard deviations are multiplied by 100.000.* R^2 *is not applicable to the two-stage WLS estimation procedure. *, **, and *** denotes significance at respectively the 10, 5, and 1 per cent level. Standard errors in () below point estimates.*

2WLS, Volatility			
$\hat{\varepsilon}_t = c + \alpha\frac{\hat{\sigma}_t}{\sqrt{n}} + \sum_{k=0}^{K}\beta_k I_{t-k} + \sum_{q=1}^{Q}\left(\delta_q \cos\left(\frac{q2\pi n}{108}\right) + \varphi_q \sin\left(\frac{q2\pi n}{108}\right)\right) + u_t$			
C	0.1414** *(0.0562)*	**Normalising Constant I**	-0.1000** *(0.0423)*
Normalising Constant II	0.0118** *(0.0051)*	**Realised Volatility**	55.182*** *(0.8613)*
	Sine terms		*Cosine terms*
δ_1	0.0019 *(0.0018)*	φ_1	-0.0060*** *(0.0024)*
δ_2	0.0013 *(0.0007*	φ_2	-0.0016*** *(0.0003)*
δ_3	0.0009*** *(0.0003)*	φ_3	-0.0004*** *(0.0002)*
δ_4	0.0006*** *(0.0001)*	φ_4	-0.0001*** *(0.0000)*
δ_5	0.0005*** *(0.0001)*	φ_5	0.0004 *(0.0001)*
δ_6	0.0003*** *(0.0001)*	φ_6	0.0019*** *(0.0001)*
Dummy terms			
Δ_1	0.0023** *(0.0012)*	Δ_2	0.0046** *(0.0008)*
Δ_3	0.0026** *(0.0008)*		
Interventions			
Γ_t	-0.3721*** *(0.11471)*	Γ_{t-1}	-0.3889*** *(0.1121)*
Γ_{t-2}	-0.2644* *(0.1338)*	Γ_{t-3}	-0.2102 *(0.1471)*
Γ_{t-4}	-0.3710*** *(0.1144)*	Γ_{t-5}	-0.4104*** *(0.1345)*
Γ_{t-6}	-0.2830 *(0.1571)*		
R^2			0.12

Table 13: *The dependent variable is the absolute residual from the auxiliary regression. The independent variables are normalizing constants, a realized volatility measure, trigonometric terms, δ_j and φ_j, and interventions, Γ_{t-j}. Δ_j denote dummies which captures intra-day spikes in volatility due to market openings/closings, and moneymarket clearing. *, **, and *** denotes significance at respectively the 10, 5, and 1 per cent level. The coeffcient and standard deviation for the interventions are multiplied by 10.000. Standard errors in () below point estimates. Subscript, besides for the dummies, denotes lags.*

2WLS, Volatility Equation			
$\hat{\varepsilon}_t = c + \alpha \frac{\hat{\sigma}_t}{\sqrt{n}} + \sum_{k=0}^{K} \beta_k I_{t-k}^{Purc/Sales} + \sum_{q=1}^{Q} \left(\delta_q \cos\left(\frac{q2\pi n}{108}\right) + \varphi_q \sin\left(\frac{q2\pi n}{108}\right) \right) + u_t$			
C	0.1410** *(0.0562)*	**Normalising Constant I**	-0.0997** *(0.0423)*
Normalising Constant II	0.0118** *(0.0052)*	**Realised Volatility**	55.337*** *(0.8607)*
	Sine terms		*Cosine terms*
δ_1	0.0018 *(0.0018)*	φ_1	-0.0060*** *(0.0024)*
δ_2	0.0013 *(0.0007*	φ_2	-0.0016*** *(0.0003)*
δ_3	0.0009*** *(0.0003)*	φ_3	-0.0004*** *(0.0002)*
δ_4	0.0006*** *(0.0001)*	φ_4	-0.0001*** *(0.0002)*
δ_5	0.0005*** *(0.0001)*	φ_5	0.0000 *(0.0001)*
δ_6	0.0003*** *(0.0001)*	φ_6	0.0004*** *(0.0001)*

Table 14: *The dependent variable is the absolute residual from the auxiliary regression. The independent variables are normalizing constants, a realized volatility measure, trigonometric terms, δ_j and φ_j, and interventions, Γ_{t-j}. Δ_j denote dummies which captures intra-day spikes in volatility due to market openings/closings, and moneymarket clearing. The coeffcient and standard deviation for the interventions are multiplied by 10.000. *, **, and *** denotes significance at respectively the 10, 5, and 1 per cent level. Standard errors in () below point estimates. Subscript, besides for the dummies, denotes lags.*

2WLS, Volatility Equation, continued			
	Dummy terms		
Δ_1	0.0023**	Δ_2	0.0045**
	(0.0012)		*(0.0008)*
Δ_3	0.0026**		
	(0.0008)		
	Purchases		*Int. Sales*
Γ_t	-0.2224*	Γ_t	-0.5007**
	(0.1172)		*(0.2354)*
Γ_{t-1}	-0.3704**	Γ_{t-1}	-0.4093**
	(0.1349)		*(0.2087)*
Γ_{t-2}	-0.1923	Γ_{t-2}	-0.4508
	(0.1486)		*(0.2655)*
Γ_{t-3}	-0.0282	Γ_{t-3}	-0.6038**
	(0.1829)		*(0.2206)*
Γ_{t-4}	-0.3753***	Γ_{t-4}	-0.3365
	(0.1136)		*(0.2579)*
Γ_{t-5}	-0.3951***	Γ_{t-5}	-0.4126
	(0.1316)		*(0.3174)*
Γ_{t-6}	-0.2055	Γ_{t-6}	-0.3000
	(0.1679)		*(0.2782)*
R^2			0.12

Table 15: *The dependent variable is the absolute residual from the auxiliary regression. The independent variables are normalizing constants, a realized volatility measure, trigonometric terms, δ_j and φ_j, and interventions, Γ_{t-j}. Δ_j denote dummies which captures intra-day spikes in volatility due to market openings/closings, and moneymarket clearing. The coeffcient and standard deviation for the interventions are multiplied by 10.000. *, **, and *** denotes significance at respectively the 10, 5, and 1 per cent level. Standard errors in () below point estimates. Subscript, besides for the dummies, denotes lags.*

Baseline Regression and subsamples with 12 lags			
$S_t = \alpha + \Sigma_{k=1}^{K} \beta_{t-k} S_{t-k} + \Sigma_{j=0}^{J} \Gamma_j I_{t-j} + \varepsilon_t$			
	Full Sample	**Subsample I**	**Subsample II**
Γ_t	1,6810	0,2775	1,9135
	(1,4037)	*(1,9976)*	*(1,7446)*
Γ_{t-1}	-0,5042	-2,4392**	-0,0225
	(1,1577)	*(1,1214)*	*(1,4798)*
Γ_{t-2}	0,2932	-0,8658	0,4518
	(1,2884)	*(1,8811)*	*(1,6052)*
Γ_{t-3}	-1,1781	0,7895	-1,9946
	(1,6851)	*(1,5533)*	*(2,1954)*
Γ_{t-4}	0,2541	-3,1637***	1,3806
	(1,1928)	*(0,7460)*	*(1,5351)*
Γ_{t-5}	-1,2533	-1,8947	-1,3343
	(1,4401)	*(1,3262)*	*(1,8452)*
Γ_{t-6}	-0,7760	1,2344	-1,5952
	(1,6262)	*(1,6162)*	*(2,1093)*
Γ_{t-7}	-1,3256	-2,2094	-1,0437
	(1,5610)	*(1,6510)*	*(1,9900)*
Γ_{t-8}	0,2967	-2,0304	0,7409
	(1,2793)	*(1,6057)*	*(1,6050)*
Γ_{t-9}	1,2481	-2,7234***	2,2612
	(1,6773)	*(0,9440)*	*(2,1845)*
Γ_{t-10}	0,6611	-0,7754	0,7604
	(2,0785)	*(1,4467)*	*(2,7280)*
Γ_{t-11}	3,1952*	1,0125	3,5684
	(1,7536)	*(1,8282)*	*(2,2417)*
Γ_{t-12}	0,7215	-1,2472	1,1388
	(1,8260)	*(1,5018)*	*(2,3663)*
R^2	0,58	0,10	0,60

Table 16: *This table shows the estimates of a regression of absolute interventions on bid-ask spreads. I denotes absolute intervention purchases or sales. S denotes the bid-ask spread measured in pips.* Γ_{t-j} *denotes the coefficient on the absolute intervention variable. Estimates of the coefficients for the contemporanous and lagged 5 min. fx spreads are not shown for brevity. The coefficients for the constant, the interventions and their standard deviations are multiplied by 100.000 *, **, and *** denotes significance at respectively the 10, 5, and 1 per cent level. Standard errors in () below point estimates.*

Baseline Regression and subsamples with 12 lags				
$S_t = \alpha + \Sigma_{k=1}^{K}\beta_{t-k}S_{t-k} + \Sigma_{j=0}^{J}\Gamma_j I_{t-j}^{Purc} + \Sigma_{j=1}^{J}\Gamma_j I_{t-j}^{Sales} + \varepsilon_t$				
	Full Sample		**Sub-sample II**	
	Sales	*Purchases*	*Sales*	*Purchases*
Γ_t	-2,1819*	3,4503*	-2,3237**	4,9226*
	(1,1248)	*(1,8686)*	*(1,1332)*	*(2,6384)*
Γ_{t-1}	0,7898	-1,0935	0,6903	-0,5735
	(1,4005)	*(1,5603)*	*(1,3866)*	*(2,3259)*
Γ_{t-2}	-0,3360	0,5531	-0,4756	1,0895
	(1,4027)	*(1,7141)*	*(1,4055)*	*(2,4661)*
Γ_{t-3}	-2,7695***	-0,4982	-2,8887***	-1,4359
	(0,7827)	*(2,3915)*	*(0,7967)*	*(3,6408)*
Γ_{t-4}	0,9923	-0,1608	0,9034	1,6067
	(1,9388)	*(1,4910)*	*(1,9374)*	*(2,2367)*
Γ_{t-5}	-0,5029	-1,6062	-0,6291	-1,6696
	(2,4431)	*(1,6822)*	*(2,4688)*	*(2,4661)*
Γ_{t-6}	2,2341	-2,0190	2,1208	-3,8539
	(1,8347)	*(2,1039)*	*(1,8365)*	*(3,1113)*
Γ_{t-7}	0,6056	-2,1833	0,4538	-1,9539
	(2,3389)	*(1,9113)*	*(2,3423)*	*(2,8315)*
Γ_{t-8}	0,5591	0,2652	0,4162	1,1528
	(1,1137)	*(1,7543)*	*(1,1105)*	*(2,5403)*
Γ_{t-9}	-0,6302	1,9709	-0,7766	4,2187
	(1,2147)	*(2,3117)*	*(1,2158)*	*(3,4562)*
Γ_{t-10}	1,7767	0,0934	1,6450	-0,0721
	(2,7288)	*(2,7150)*	*(2,7190)*	*(4,1744)*
Γ_{t-11}	-0,6181	4,8293*	-0,7826	6,5795*
	(1,0114)	*(2,4403)*	*(1,0110)*	*(3,5576)*
Γ_{t-12}	0,1382	1,1181	-0,0007	2,1484
	(1,8617)	*(2,5257)*	*(1,8555)*	*(3,8590)*
R^2		0,58		0,60

Table 17: *This table shows the estimates of a regression of absolute interventions on bid-ask spreads. I denotes absolute intervention purchases or sales. S denotes the bid-ask spread measured in pips.* Γ_{t-j} *denotes the coefficient on the absolute intervention variable. Estimates of the coefficients for the contemporanous and lagged 5 min. fx spreads are not shown for brevity. The coefficients for the constant, the interventions and their standard deviations are multiplied by 100.000 *, **, and *** denotes significance at respectively the 10, 5, and 1 per cent level. Standard errors in () below point estimates.*

8 Figures

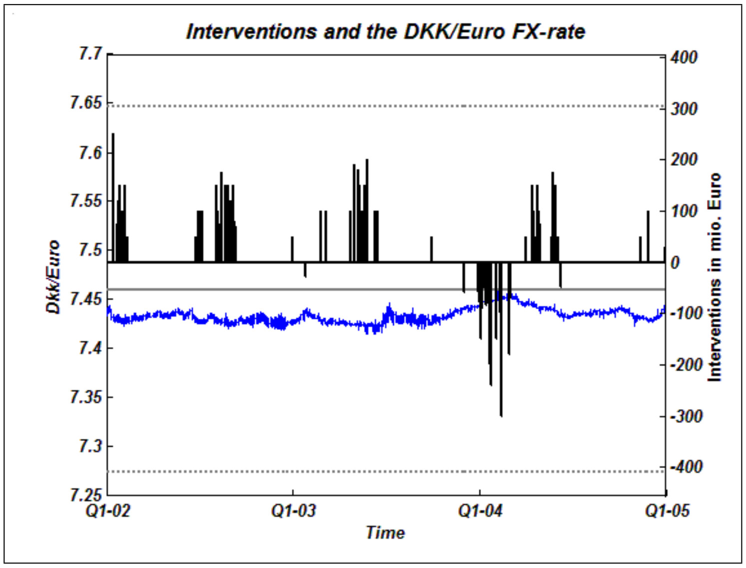

Figure 1: 5. min DKK/EUR spot exchange rate and interventions in mio. EUR over the 1 January 2002 to 31 December 2004 period. A positive intervention corresponds to a purchase of EUR against a sale of DKK. The upper and lower lines are the ERM II deviation bands. The interventions are plotted against the central parity exchange rate of 7.46038 DKK/EUR.

Monetary Policy, Housing, and Macroeconomic Effects of Changes in Long Interest Rates

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Abstract

It is widely believed, both in academia and among practitioners, that monetary policy affects the aggregate economy through changes in the long or medium end of the yield curve as well as through changes in the short rate. Standard models in the academic literature do not have such a mechanism. This paper addresses this short coming. I build and calibrate a DSGE-model in which long bond yields influence the macro economy through the housing sector. Bonds of long maturities plays an independent role for monetary policy through housing in this model as the consumer must issue long bonds to finance all purchases of housing. The friction consistently introduces a long interest rate into the economy and clears the way for an analysis of the monetary transmission mechanism through the yield curve.

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1 Introduction

> "..., if spending depends on long-term interest rates, special factors that lower the spread between short-term and long-term rates will stipulate aggregate demand. Thus, - -, "a higher short-term rate is required to obtain the long-term rate and the overall mix of financial conditions consistent with maximum sustainable employment and stable prices." [Ben Bernanke, 2006][1]
>
> "My main purpose in calling attention to the term structure puzzle here is not to resolve it, but rather to urge central bank research departments to give it high priority. It may be the piece of the monetary transmission mechanism about which we are most in the dark", [Alan S. Blinder, 2006][2]

This paper analyses the relationship between monetary policy and the macro economy through changes in long interest rates. The aggregate economy is surely not independent of the evolution of long interest rates and this interrelationship between the aggregate economy and long interest rates might also play a key element in the monetary transmission mechanism. But it is difficult to say much about this from state-of-the-art macroeconomic DSGE-models, as it is quite hard to come by a structural model in which long interest rates play a role for the economy *independently* of the short rate, as in these DSGE-models long interest rates are just expectations of future short rates and have no feed-back effects to the macro economy.

The financial constraint in this paper is sufficient to move the standard DSGE framework away from a single-interest-rate model of aggregate demand determination. The expectations hypothesis does not hold so long rates are thus not just a stand in for expected averages of future short-rates. The financial constraint restricts the consumer from bypassing the long bond market completely and move down the yield curve in the search for cheaper ways to finance housing, and lastly, the financial constraint introduces an endogenous wedge between bond yields for different maturities. This independent long interest rate and bidirectional feed-back effects from short-rates to long-rates and back onto the economy thus provide a framework for the study of monetary policy through the whole yield curve.

The main contributions of this paper are as follows. A one period

[1] Remarks by Chairman Ben S. Bernanke before the Economic Club of New York, New York, March 20, 2006

[2] Monetary Policy Today, 2006.

higher-than-expected monetary policy short rate in this model depresses output by tilting intertemporal consumption and through a higher long interest rate increases the costs of financing housing propagating the effects of the monetary policy shock. Further, the long interest rate significantly mitigates the co movement problem for given relative price rigidities in the economy through higher long interest rates which increase the cost of financing.

I utilise housing and mortgage markets to accomplish these results in the model framework. Consumers derive utility non separable from a consumption goods index and a housing goods index. I introduce a financial friction into the model such that the consumer need to finance all new housing purchases by bond issuances of a specific time-to-maturity, m, in which m is strictly above one period.

Durable goods are often considered to be interest-rate sensitive. This is especially so for durable goods which demand a price at the time of the purchase which is too big to be covered by the income of the consumer within the same period and thus demand a need for financing. One obvious example of such a durable good is housing. For a brief survey of the mortgage markets for different countries, see Calza, Monacelli, and Stracca (2006). Four things are apparent from their analysis. Firstly, the typical duration of a mortgage contract is around 15 years but varies from 10 to 30 years. These differences are in economic terms hard to explain and mostly reflect national traditions, cultural factors, as well as the institutional framework which the financial sector operates in. Second, the construction sector does play a significant role for the business-cycle. In 2002 for the U.S., new construction accounted for 412 billion U.S. dollars or around 4% of GDP, while i.e. this number is as high as around 18% for Spain. Further, fluctuations in residential investment account for more than 18% of the fluctuations in GDP. Thirdly, in absolute numbers the importance of housing is also clear: There are roughly 72 million owner occupied houses in the U.S., while there are around 1.3 million housing starts of 1-4 unit dwellings. 2/3 of households own their own home in the U.S., and the value of residential real estate makes up over 75% of total assets for the median household, Survey of Consumer Finances (2001). Lastly, Consumer Expenditure Survey conducted in 1999 for the U.S., found that on average, households spend around 19% of their total expenditures on shelter.[3]

[3]Long interest rates can also affect the aggregate economy through demands from firms, institutional investors, the government, as well as from consumers. Firms might wish to match the maturity of their assets and this need to be done by long-term bonds, as investments in capital are often long time investments. Though bond yields are highly correlated, there can be portfolio demand for longer bonds, and these long

The production side consists of two sectors, a consumption goods production sector and a construction sector. Final goods producers assemble intermediate goods into a final good, while the intermediate goods producers face monopolistic competition and Calvo-pricing. This set-up delivers two interdependent New-Keynesian Phillips curves.

I otherwise keep the model as simple as possible to identify and isolate the effects of long interest rates upon the macro economy, but this goal necessitates a variation of the degree of price stickiness in construction goods due to lack of empirical evidence for price rigidity in durables and due to the *co movement problem* found in two-sector models with durables and asymmetrical nominal rigidities.[4] The problem in these two sector models is that if one sector expands the other contracts, and this stands in sharp contrast to empirical evidence.

The model in this paper is not the only model with a long interest rate, but it is to my knowledge the first model that includes a long interest rate with an *independent* role for the macro economy in a *structural, micro founded* framework. Examples from macroeconomics includes, Fuhrer (1996) which incorporates a long rate into a macroeconomic model, but this inclusion is not build upon a structural framework and the difference between interest rates arises exogenously. Another example is McCallum (1994), who analyses the expectations hypothesis, but his model is not a general equilibrium model and has no interrelations between yields and the rest of the economy. One recent example from finance literature is Vayanos and Vila (2007), which introduces a specific form for utility such that each investor demands only a specific maturity. Their partial equilibrium model does not allow for feedback effects from yields to macroeconomic variables. Examples from macro-finance includes Ang and Piazzesi (2003), Gallmeyer, Hollifield, and Zin (2005), Ravenna and Seppala (2006), Pedersen (2008) among many. This research program models the short-rate by a Taylor-rule and prices bonds by no-arbitrage introducing an endogenously determined wedge between bond yields in terms of a (possible) time varying risk premia. While these models can tell a lot about the determinants of risk premia, the cross-section of bond yields, as well as their dynamics through time, they can not tell much about the relationship between the financial market

bonds might even be safer investments than investments in shorter maturity bonds in an environment with stable inflation as we reside in today, see e.g. Cochrane and Piazzesi (2006). Life-insurance companies as well as pension funds have preferences for bonds with maturities of much more than a year. Central banks often finance public debt by issuance of long bonds with maturities of around 10 year. This papers' focus is limited to the mortgage market.

[4] See e.g., Barsky, House, and Kimball (2007) and Monacelli (2008)

and the macro economy. The reason is the models do not allow for any feed-back between yields and the macro economy and the consumers can simply bypass the long bond market and use short rates instead.

The paper that comes close to a multiple interest rate model of aggregate demand determination is Andres, Salido, and Nelson (2004) whose model includes long interest rates with feedback effects to the macro economy. However, Andres, Salido, and Nelson (2004) need to impose quite strong assumptions to get a long interest rate with an independent role in the economy. They specifically assume that only some agents are restricted from trading in all bonds, and their wedge between a long rate and a short rate is exogenous.

This paper has the following structure. Section 2 sets up the macroeconomic model. Section 2.1 sets up the demand side in general, and analyses the financial friction, the key relationships in the economy and a new channel for the impact of the monetary policy changes in particular. Section 2.2 sets up the supply side, while section 2.3 deals with the public sector. I solve and calibrate the model in section 3. I analyse the equilibrium dynamic response to a monetary policy change in the model with sticky non-durable prices and flexible durable prices in section 3.2, and in the case of symmetric price rigidity in section 3.3. Section 4 concludes.

2 The macroeconomic model

I set up a standard, closed-economy DSGE model, see e.g., Smets and Wouters (2003), Woodford (2003), Clarida, Gali, and Gertler (1999), with two main differences. Firstly, the model consists of two sectors, a consumption sector and a housing sector. The representative consumers' utility function in these two goods is assumed to be non separable. Secondly, I impose that the representative consumer needs to issue bonds with a maturity longer than one period to finance any new purchases of housing.

2.1 The representative consumer

Let C_t denote non-durable consumption, H_t the stock of housing, $0 < \delta < 1$ the rate of depreciation, and $X_t = H_t - (1-\delta) H_{t-1}$ the flow of housing, in which the variables are composite consumption goods consisting of differentiated products produced by monopolistically competitive final goods producers. The consumer derives utility from consumption goods and housing goods, while working, N_t, gives disutility, and $\gamma > 1$, $\varphi, \upsilon, \omega > 0$:

$$U(C_t, H_t, N_t) = \frac{(C_t H_t^{\omega})^{1-\gamma} - 1}{1-\gamma} - \frac{\upsilon}{1+\varphi} N_t^{1+\varphi} \tag{1}$$

γ governs both the intertemporal rate of substitution and given the restriction $\gamma > 1$, housing and consumption are Edgeworth substitutes, $U_{CH} < 0$. The implication of (1) is that the utility from non-housing consumption depends on the size of the housing stock and that the IS-relation consequently will depend upon the net growth rate in the housing stock, $H_t - H_{t-1}$.

The consumer faces the following budget constraint:

$$P_{Ct}C_t + P_{Ht}(H_t - (1-\delta) H_{t-1}) = W_t N_t + \Pi_t - T_t - \left(\bar{P}_t \bar{B}_t - B_{t-1}\right) \tag{2}$$

P_{Ct}, P_{Ht} denotes the price of a housing unit and a consumption good respectively. $\bar{P}_t$ denotes a row vector where each element corresponds to price of a zero-coupon bond each with different maturity. $\bar{B}_t$ represents the quantity of such claims purchased by the consumer at the end of period t such that negative entries correspond to borrowing. The constraint says that the households purchases of consumption and new housing cannot exceed the income from labour, $W_t N_t$, profit shares from the production sector, Π_t, and taxes paid, T_t. Further, the consumer can purchase bonds maturing in period n, $\bar{P}_t \bar{B}_t$, and the consumer receives income from bonds purchased at previous periods maturing in this period, B_{t-1}.

2.1.1 A financial constraint for the mortgage market

The consumer also faces the following financial constraint:

$$P_{Ht}(H_t - (1-\delta) H_{t-1}) \leq P_t^m B_t^m \tag{3}$$

m denotes the time-to-maturity of the bond which I assume is longer than one period effectively ruling out the short-rate as an instrument for financing housing. The constraint forces the consumer to finance all changes in the stock of housing, $P_{Ht}(H_t - (1-\delta) H_{t-1})$, with issuances of bonds, B_t^m, to the price P_t^m such that the value of new housing purchases is balanced by the nominal value of a bond portfolio.

The financial constraint imposes an implicit adjustment cost of changing the housing stock as the consumer not only need to pay the price P_{Ht} for an extra unit of housing, but the consumer also needs to finance the value of the extra housing unit by a potentially expensive instrument, the long rate of interest. The consumers in this economy does accumulate savings through purchases and sales of bonds to smooth consumption through time, but they are restricted by (3) to use these savings to purchase housing.

I assume the government holds the mortgage bonds issued by the consumers. The financial constraint thus resemblance a cash-in-advance constraint in terms of an implicit monopoly for the public sector of providing mortgage bonds and money respectively. Specifically, mortgage bond trading in between consumers is ruled out as the government has a monopoly in buying mortgage bonds at issuance, and the consumers must purchase any housing by exactly the bond with maturity m.

The constraint captures two widely observed aspects of the real economy: Consumers usually do not have savings large enough to cover their housing purchases up front but instead finance the excess amount on the bond market, and they do so by bonds with maturities above one period.[5,6] *Some long interest rate should matter for aggregate economic variables.* I do not attempt to analyse these institutional factors nor do I try to endogenise the choices of mortgage contract - the choice between fixed versus variable contracts and between the maturity of the bond used in the contracts.[7] The financial constraint aims to capture all these choices and properties of mortgage markets in one simple and tractable equation.

2.1.2 The consumers' problem

The problem for the consumer is to maximise (1) with respect to labour, consumption and housing subject to (2) and (3) with λ_t and $\lambda_t \Phi_t$ being their respective Lagrange multipliers:

$$
\begin{aligned}
&\max_{C,H,\bar{B}} E_t \sum_{t=T}^{\infty} \beta^t E_t \left[U\left(C_t, H_t, N_t\right)\right] \\
&-\lambda_t \left[P_{Ct} C_t - W_t N_t - \Pi_t + T_t + \left(\bar{P}_t \bar{B}_t - B_{t-1}\right) + P_{Ht}\left(H_t - (1-\delta) H_{t-1}\right)\right] \\
&-\Phi_t \lambda_t \left[P_{Ht}\left(H_t - (1-\delta) H_{t-1}\right) - P_t^m B_t^m \right]
\end{aligned}
$$

[5]See the discussion in the introduction.

[6]The constraint also rules out the possibility of adding past wealth to the port-folio. I could have assumed that, say, only a certain percentage of the port-folio needs to be raised on the financial markets, but I have refrained from doing so as it would not have changed the analysis significantly.

[7]Examples from finance includes Campbell and Viceira (2001) and Campbell and Cocco (2003). Campbell and Viceira (2001) derives a partial equilibrium model for consumption and portfolio choice and show that long bonds are appropriate assets for investors who have hedging demands due to a whish for stable income. Campbell and Cocco (2003) analyses households choice of instrument for financing mortgages, and find similar conclusions. These models are however not easily incorporated into a general equilibrium framework.

The optimality conditions for optimal consumption, labour supply, bond holdings, housing, and mortgage bonds are the following:

$$\lambda_t = \frac{U_{Ct}}{P_{Ct}} \tag{4}$$

$$U_{Nt} = U_{Ct}\frac{W_t}{P_{Ct}} \tag{5}$$

$$0 = -\lambda_t \bar{P}_t + E_t\left[\beta\lambda_{t+1}\bar{P}_{t+1}\right], \forall_{n\neq m} \tag{6}$$

$$\begin{aligned} 0 = & \, U_{Ht} - \lambda_t P_{Ht} + E_t\left[\beta\lambda_{t+1}(1-\delta)P_{Ht+1}\right] \\ & -\Phi_t\lambda_t P_{Ht} + E_t\left[\beta\Phi_{t+1}\lambda_{t+1}(1-\delta)P_{Ht+1}\right] \end{aligned} \tag{7}$$

$$0 = -\lambda_t P_t^m + E_t\left[\beta\lambda_{t+1}P_{t+1}^m\right] - \Phi_t\lambda_t P_t^m \tag{8}$$

Relation (4) equates the marginal utility of consumption to the shadow value of the flow budget constraint, while (5) is the standard optimality condition for the choice between consumption and labour. (6) is the Euler-equation for optimal consumption through time. I combine (6) with (4) and define the stochastic discount factor as:

$$M_{t+1} \equiv \beta\frac{\lambda_{t+1}}{\lambda_t} = \beta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\left(\frac{H_{t+1}}{H_t}\right)^{\omega(1-\gamma)}\frac{P_{Ct}}{P_{Ct+1}} \tag{9}$$

Conditions (7) and (8) arise from the financial constraint and are new compared to the work-horse DSGE model. They together provide sufficient conditions to move this model into a multiple-interest-rate framework for determination of aggregate demand. Sections 2.1.3 through 2.1.5 will scrutinize their implications for the user cost of housing, the relation between short and long interest rates, and the IS relation, while 2.1.6 will put the pieces together and analyse monetary policy in this framework for given prices.

2.1.3 The user cost of housing and the financial constraint

The housing market and the financial constraint together bring feedback effects between bond yields and the macro economy. This section explains how.

Relation (7) could be written into the following standard expression, if (3) were not imposed such that $\Phi_t = 0$

$$\frac{U_{Ht}}{U_{Ct}} = usc_{\Phi=0} \equiv Q_t - \beta(1-\delta)E_t\left[\frac{U_{Ct+1}}{U_{Ct}}Q_{t+1}\right] \tag{10}$$

in which I have defined the relative price of housing in units of the consumption good as $Q_t \equiv \frac{P_{Ht}}{P_{Ct}}$. Relation (10) equates the marginal rate of substitution between consumption of housing and non-durables, the left hand side, to the user cost of housing, the right hand side. This user cost depends positively on the relative price in terms of consumption today and negatively on the relative price in the period to come. The latter term constitutes the expected discounted marginal utility of the gains from expanding future consumption through the resale value of the extra unit of housing relatively to using that extra income on consumption today.

Substitute (7) into (8) to see how (3) breaks this standard condition in (10):

$$\frac{U_{Ht}}{U_{Ct}} = usc_{\Phi=0} + Q_t \left\{ \Phi_t - E_t \left[\Phi_{t+1} M_{t+1} (1-\delta) \frac{P_{Ht+1}}{P_{Ht}} \right] \right\} \tag{11}$$

Hence, *the user cost of housing is positively dependent upon the cost of financing housing by the long bond.* However, expression (11) shadows the interdependence between the long rate and the user cost of housing, but an approximation and an introduction of (8) brings this relationship out in light:[8]

$$\frac{U_{Ht}}{U_{Ct}} \approx \tilde{Q}_t - \beta (1-\delta) E_t \left[\frac{U_{Ct+1}}{U_{Ct}} \tilde{Q}_{t+1} \right] \tag{12}$$

in which I have defined the *effective relative price*

$$\tilde{Q}_t \equiv Q_t \left(\frac{E_t \left[RX^m_{t+1} \right]}{R^f_t} \right)$$

I have also introduced the gross risk free rate of interest, R^f_t, given by the inverse of the conditional expectation of the stochastic discount factor, and I have rewritten (8) into

$$\Phi_t = E_t \left[M_{t+1} RX^m_{t+1} \right] - 1 \tag{13}$$

[8] (12) is an approximation as it discard covariance terms such that I can write $E_t \left[M_{t+1} RX^m_{t+1} \right] \approx \frac{E_t\left[RX^m_{t+1}\right]}{R^f_t}$ and $E_t \left[\left(E_{t+1} \left[M_{t+2} RX^m_{t+2} \right] - 1 \right) \frac{U_{Ct+1}}{U_{Ct}} Q_{t+1} \right] \approx$ $E_t \left[\frac{U_{Ct+1}}{U_{Ct}} Q_{t+1} \frac{RX^m_{t+2}}{R^f_{t+1}} - \frac{U_{Ct+1}}{U_{Ct}} Q_{t+1} \right]$. The covariance terms left out will likewise not be present in the full model solution due to the use of log-linearisations.

$E_t\left[RX_{t+1}^m\right] \equiv E_t\left[\frac{P_{t+1}^m}{P_t^m}\right]$ denotes the expected holding period return from holding mortgage bonds. Using definition of a bond yield with maturity n, $y_t^n = -\frac{1}{n}\log\left(P_t^n\right)$, I can write the holding period return as

$$E_t\left[\frac{P_{t+1}^{m-1}}{P_t^m}\right] = E_t\left[\exp\left(y_t^m - (m-1)\left(y_{t+1}^{m-1} - y_t^m\right)\right)\right] \tag{14}$$

The holding period return thus depends positively on this periods' yield and negatively on the change in the yield over the holding period.

To get the intuition behind the presence of (14) in (12), note from (10) the holding period return from holding a maturing bond implicitly is present in models with durable goods without (3), as in these models only the one period (real) rate determines the durable/non-durable margin. Hence, the second term in (14) is zero in models without (3).

With (3), the intuition behind expression (10) holds *except* that relative prices, Q_t, are altered by the presence of the long and short interest rate. The mortgage bond yield is a cost for the consumer from purchasing more housing this period as this is the implicit rate the consumer needs to finance new housing with. The consumer is however free to alter the housing stock in the period to come and close a similar value of his bond portfolio explaining the presence of the second term in (14). The reverse holds for the second holding period return in (12) when the consumer takes into account the resale value of housing in the determination of the user cost of housing this period.

Take in the model reduces to a standard model without (3) *if* long bond yields a return equal to the short rate; it is the relative price that determines optimal consumption between different goods and it is the interest rate spread that alters the relative price of housing in (14). The user cost of housing depends on the real rate with respect to consumption goods, R_t^f, as the latter reflects the opportunity cost of investing in new housing instead of putting your money in a one-period bond.

2.1.4 The expectation hypothesis and the financial constraint

The following section explains how, in contrast to standard models, long rates in this model can not simply be defined as expected future short rates due to an endogenous wedge between bonds of different maturities. As a result, the central bank can not determine long rates perfectly through changes in short rates.

That the financial constraint breaks the expectations hypothesis in this economy can be seen by rewriting (8) into a relationship between the yield of a long bond, y_t^m, future expected short interest rates, and future expected multipliers:

$$y_t^m = \frac{1}{m}\sum_{i=0}^{m-1} E_t\left[i_{t+i}\right] + \frac{1}{m}\sum_{i=0}^{m-2} E_t\left[\phi_{t+i}\right] \tag{15}$$

in which lower case letters denote log deviations from steady state. Specifically, i_t denotes the short rate of interest, and ϕ_t denotes the log linearised multiplier on the financial constraint around its steady state. The difference between short - and short interest rates thus arises endogenously in terms of an average of expected future multipliers on the financial constraint.[9,10]

When (3) binds, the long interest rate in its steady state, $y^m = \delta + \left(\frac{m-1}{m}\right)\phi$, is higher than the short rate in the steady state, $i = \delta$. Hence, from (8) the wedge between short and long term bonds can be interpreted as a measure of the cost of being forced to finance housing with a relatively expensive instrument, the long interest rate, compared to the short-rate of interest. From (13) this wedge can further be given a pecuniary value in terms of the net holding period return from holding mortgage bonds.[11]

[9] In contrast, Andres, Salido, and Nelson (2004) introduces an *exogenously* wedge, while Graeve, Emiris, and Wouters (2008) introduces an exogenous risk premia shock on long bonds.

[10] A binding financial constraint, however, does not imply the existence of a dominated asset. The price of a mortgage bond as of time t is given by (8), but after one-period, that bond is no longer a bond with maturity m, but with maturity $m-1$, and given free trading in the secondary mortgage market, any difference between a "standard" bond and a mortgage bond is traded away.

Secondly, the shadow value of borrowing short is zero after one period. The lagrange multiplier can be interpreted as the marginal value for the consumer in terms of utility of relaxing the financial constraint marginally. The financial constraint only binds for the consumer in the period where he purchases the new housing stock.

Thirdly, the consumer needs to go long in the mortgage bond and short in the short-rates to exploit possible arbitrage opportunities between the mortgage bond and other bonds. However, to exploit such arbitrage opportunities, the multiplier must be positive, so the consumer needs to go short in the mortgage bond. But in the steady-state the consumers are always short in the mortgage market, as the consumers buy housing, and they thus need to *decrease* their stock of housing to exploit the arbitrage opportunity. The latter points can be seen from (11).

[11] The net return would be zero if the investor were allowed to finance housing by the short-rate: $1 = E_t\left[M_{t+1}R_t^f\right] = E_t\left[M_{t+1}\right]R_t^f$, and hence $\Phi_t = 0$, so the standard DSGE model without the financial constraint is in fact a special case of the model, namely the case in which the maturity of the mortgage bond is one.

2.1.5 The IS relation and the financial constraint

The distinction between short-rates and long-rates matters not only for the intratemporal choice between the goods in the economy in (11), but also for *intertemporal* consumption. To see this, log linearise (4) and (6) around their steady states to obtain the model implied IS-relation

$$c_t = E_t [c_{t+1}] - \frac{1}{\gamma} E_t [i_t - \pi_{Ct+1}] - \frac{\omega (1-\gamma)}{\gamma} E_t [\Delta h_{t+1}] \tag{16}$$

in which π_{Ct+1} correspond to inflation in the price level for the consumption goods sector. I can substitute out the growth in the housing stock by the mortgage bond price in equilibria in which the financial constraint binds:

$$\begin{aligned} c_t = {} & E_t [c_{t+1}] - \frac{1}{\gamma} E_t [i_t - \pi_{Ct+1}] + \frac{\omega (1-\gamma) \delta m}{\gamma} E_t \left[y_{t+1}^m\right] \\ & - \frac{\omega (1-\gamma) \delta}{\gamma} E_t \left[b_{t+1}^m - p_{Ht+1}\right] + \frac{\omega (1-\gamma) \delta}{\gamma} h_t \end{aligned} \tag{17}$$

Hence, the IS-relation in this economy depends on both the short-rate and the long-rate in the economy in a consistent, micro founded way in which the distinction between them matters.

The first two terms on the right hand side of (17) are standard: A higher monetary policy rate, a higher i_t, increases the real rate in terms of consumption goods, as consumption good prices are sticky, causing an reallocation of consumption from the present to the future. This effect is the standard real effect from changes in the monetary policy rate. Further, a *higher* long interest rate *depresses* current consumption through the third term on the right hand side of (17). This is due to the fact that the consumers' marginal utility of extra consumption depends upon the size of the housing stock, the non-separable utility function, and that the consumer *must* borrow to satisfy the demand for the size of the house, the financial constraint. Current consumption falls in response to change in the policy rate by the elasticity of intertemporal substitution, $\frac{1}{\gamma}$. The same holds for changes in the long interest rate but the effect is further determined by the elasticity of marginal utility of an extra unit of consumption with respect to changes in the level of housing, $\frac{H_t U_{CHt}}{U_{Ct}} = \omega (1-\gamma)$, and by the rate of depreciation, δ. The next term in (17) is the debt to equity ratio, $b_{t+1}^m - p_{Ht+1}$, introducing a wealth effect in the IS-relation.

Current housing stock affects the current level of consumption in the same way as changes in the long rate does, but the effect can be given a different interpretation namely as capturing habit formation as in e.g., Campbell and Cochrane (1999), in the sense that marginal utility of consumption approaches infinity if the level of consumption approaches habits. Housing is in this paper a slow-moving state variable which affects non-durable consumption due to a non-separable utility function. As shown by Flavin and Nakagawa (2008) housing in a framework with adjustment costs for reoptimisation of new housing, as in this model, can be thought of as a microfoundation for habit formation. That is, the marginal utility of consumption approaches zero if the consumer is homeless - when the housing stock approaches zero.

In total the aggregate demand side consists of the IS-relation, (16), the financial constraint, (3), and housing demand, (11).

2.1.6 Monetary policy and the financial constraint

With the demand side in place, I can in greater detail explain what I denote the second monetary transmission mechanism. A one-period higher than expected current short rate, i_t, tilts intertemporal consumption through (17), as is standard in the DSGE framework, see e.g., Woodford (2003).

A one-period higher than expected nominal short-rate pushes bond yields up through the first part of (15). However, the expectations hypothesis is in this model broken, so higher expected future short rates are not sufficient to push long interest rates up. A higher short rate gives rise to a higher multiplier on the financial constraint as well, see (8), so both terms in (15) rises and the mortgage bond yield consequently rises in response to higher short rates.

The effect of the higher bond yield is two fold. Firstly, bond returns, RX_{t+1}^m, depends positively on bond yields and negatively on changes in future yields. A positive but temporary shock to bond yields, like a monetary policy shock, therefore gives rise to an expected positive bond holding return, and *increases* the user cost of housing, as the consumer is short in mortgage bonds. That is, the effective relative price in (12) is increased changing the optimal allocation between consumption and housing at the margin.[12]

Secondly, the higher long interest rate depresses current consumption directly, see section 2.1.5 and (17), and indirectly through lower housing stock. That is, the higher user cost of financing depresses the housing

[12]Notice the consumer is restricted from bypassing these higher prices of financing new housing by moving down the yield curve to a cheaper way of financing housing through the financial constraint.

stock downwards through time as the consumer reinvest less in new housing, see section 2.1.5 and (17), as reinvestment is more expensive due to the higher long rate.

I emphasize the role of expectations. It is the expected future path of short rates that determine consumption growth in the standard model, see e.g., Gali (2008). The same holds in this model but to a greater extend. The long rate moves in response to expectations of future short rates, the first term in (15), and the more the central bank can affect these expectations, the more long interest rate moves and the more effective is monetary policy through this channel.

The higher user cost of financing, the direct effect on current consumption through the long interest rate, the fall in the housing stock and its effect upon current consumption together give rise to a *second* interest rate channel.

The analysis in this section and the previous sections in section (2.1.2) is for given prices. I next introduce the production side and move on to general equilibrium.

2.2 The production sector

The production side consists of two sectors. One sector produces consumption goods and one sector produces housing. The latter sector is denoted the construction sector.[13]

I model the two sectors in a standard way. Intermediate firms produce goods under monopolistic competition, while a perfectly competitive final goods producer uses these inputs into the production of the respective goods. The focus of this paper is not on asymmetries between sectors and I thus keep the sectors symmetric except that I allow for asymmetric price rigidities within the respective sectors for the intermediate goods producers. That is, I assume: no sector specific shocks, free labour movement equalising sector wages, and a symmetric constant returns to scale production function for the intermediate firms.

I consider three versions of the model. Firstly and secondly a model with sticky prices in consumption and flexible prices in durables with and without the financial constraint imposed, and lastly, a model with symmetric nominal rigidities. I consider a model without the financial constraint to be able to identify and isolate the effects of (3). I vary the

[13] Capital is another plausible channel for monetary policy affects of long interest rate changes. This effect could perhaps be introduced through a financial friction on the production side resembling the friction I have imposed on the demand side in this paper. I leave the introduction of capital into such a set-up for future research.

intersectoral degree of nominal rigidities for an empirical and a theoretical reason.

While there has been done a lot of empirical work on the degree of price rigidity for non-durables, see e.g., Levy, Bergen, Dutta, and Venable (1997), Bils and Klenow (2004), and Kashyap (1995), there has not been done similar work on the degree of price rigidity for durables. Theoretical arguments have been put forward. As analysed in the introduction, housing is an important asset for both the cross-section of consumers and for the individual consumer. House prices can therefore be considered as having asset pricing behaviour and thus vary widely. The perception that prices of housing are downward sticky has at the time of writing shown not to hold either. Barsky, House, and Kimball (2007) argues that house prices overshoot in response to a monetary contraction suggesting that house prices may be quite flexible.

It is theoretically likely that durable good prices in general and house prices in particular can be quite flexible being relatively expensive on a per-unit basis. If implicit or explicit menu costs have fixed components in these markets, the incentives to reset prices on a frequent basis are relatively high compared to non-durable goods.

However, I do not find all these arguments convincing for this model framework. I model housing as an index where the price of the goods are set for the individual elements of the index. The production and pricing framework for, say, concrete or windows, are not likely to be widely different from, say, meat or clothing.

2.2.1 The co movement problem

The theoretical reason behind the consideration of two pricing frameworks for durables is the *co movement problem*, see e.g., Monacelli (2008), Barsky, House, and Kimball (2007): In response to monetary policy shocks, if price stickiness is asymmetric in the two sectors, whenever spending contracts in one sector it tends to expand in the other. This is not what is found in the data. U.S. data suggests a strongly procyclical response of durable spending to a shock to monetary policy, and a positive co movement with non-durable spending, as shown in e.g., Monacelli (2008). This is an important problem as construction accounts for a relatively big part of aggregate activity.[14]

The co movement problem generally arises in symmetric two-sector models with sticky prices in one sector and flexible prices in the other, see e.g., Barsky, House, and Kimball (2007), Calza, Monacelli, and Stracca

[14] See the introduction.

(2006). I now show the co movement problem is likely to prevail in this model. Assume for now flexible construction goods prices. Denote $\Upsilon_t \equiv U_{Ct}\tilde{Q}_t$ as the shadow value of an extra unit of the housing good and rewrite (11) as follows

$$U_{Ct}\tilde{Q}_t \equiv \Upsilon_t = U_{Ht} + \beta (1-\delta) E_t [\Upsilon_{t+1}] \tag{18}$$

This is a difference equation in Υ_t, which can be rewritten into an infinite sum of expected, discounted, marginal utilities of housing:

$$\Upsilon_t = \sum_{j=0}^{\infty} (\beta (1-\delta))^j U_{Ht+j} \tag{19}$$

Barsky, House, and Kimball (2007) argue the right hand side of (19) is approximately constant for low values of the rate of depreciation, which is likely to hold for perhaps the most durable good you can think of, housing, and thus for this model. Specifically, a high stock-flow ratio, $\frac{1}{\delta}$, implies that a change in production of housing, Y_{Xt}, only give rise to small changes in the marginal utility of housing, U_{Ht}, and if the rate of depreciation is low, the multiplier on housing is affected by housing flows in the distant future. Therefore, temporary shocks to the economy, like a monetary policy shock, are unlikely to affect the multiplier significantly, and can for long-lived durables be treated as approximately constant.

This near constancy has profound implications for the properties of the model. A close to constant left hand side of (19) implies $U_{Ct}\tilde{Q}_t$ likewise is constant such that any variation in the relative prices is matched by a variation in the consumption of the non-durable of the same sign. If non-durables goods prices are sticky and durable goods prices are flexible, the relative price, Q_t, falls in response to a one period positive monetary policy shock and construction increases.[15]

Though the analysis above gives an uncontroversial explanation of the co movement problem it is controversial in this model as I have assumed the relative interest rate terms in the effective relative price, $\tilde{Q}_t$, did not move in response to the shocks, an assumption which is unlikely to hold. It is without a full solution to the model unclear how important these movements are for the co movement problem and whether the introduction of the financial constraint can solve or mitigate the problem. What is clear is that I need to break the co movement problem if I want to identify and isolate the effects of higher long rates upon construction as the effects explained in section 2.1.6 are possibly can drown by changes

[15] The details of the arguments can be found in e.g., Monacelli (2008).

in the relative price. I therefore vary the degree of relative, nominal rigidity from one extreme, sticky consumption goods price and flexible construction goods prices, to another extreme, completely symmetric nominal price rigidity between sectors.

2.2.2 Final goods producers and Intermediate goods - *in general*

The final goods in the two sectors are produced from intermediates, which are assembled through the following production function:

$$Z_t = \left[n_z^{-\frac{1}{\theta}} \int_{N_z} z_t(i)^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}$$

in which $z = c_c, x_x$ and $Z = Y_{Xt}, Y_{Ct}$ denotes intermediate goods and final goods respectively in each sector. n_z is the number of goods in each of the sectors, and $N_C = [0; n_C], N_X = [n_C; 1)$ such that $n_C + n_X = 1$. θ governs the price elasticity of demand for the good and is assumed to be strictly above one.

The final goods producers minimise their costs and solve:

$$\min_{z(i)} \int_0^1 p_{zt}(i) z_t(i) di \ st. \left[n_z^{-\frac{1}{\theta}} \int_{N_z} z_t(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}} \geq Z_t$$

in which the price of intermediate good i in sector z is denoted by $p_{zt}(i)$ The demand for the intermediate good in sector z is given by the well-known expression

$$z_{zt}(i) = \frac{1}{n_z} Z_t \left(\frac{p_{zt}(i)}{P_{zt}} \right)^{-\theta}$$

The price index follows from this expression:

$$P_{zt} = \left[\frac{1}{n_z} \int_{N_z} p_{zt}(i)^{1-\theta} \right]^{\frac{1}{1-\theta}}$$

I assume for simplicity a linear production function with labour being the only input

$$Y_{zt}(i) = N_t^z(i)$$

in which $N_t^z(i)$ denotes labour input into sector z for firm i. With labour free to flow across sectors, and with constant returns to scale production function, the wage rate will be equalised across firms and across sectors in equilibrium in this economy, and all firms across sectors and across firms will have the same nominal marginal cost of production, $MC_t = \frac{W_t}{\frac{\partial F(N_t)}{\partial N_t}} =$

W_t, and will employ the same amount of workers, $N_{tj}(i) = N_{tk}(j) = N_t$. The profit maximisation problem for the intermediate producer is:

$$\max_{p_z(i)} \pi_z = \frac{1}{n_z} Z_t \left(\frac{p_{zt}(i)}{P_{zt}} \right)^{-\theta} (p_{zt}(i) - W_t)$$

A producer which is free to reset prices each period sets its price, $p_t(i)$, as a markup, $M^z \equiv \frac{\theta}{(\theta-1)}$, over nominal marginal cost, $MC_t \equiv W_t$:

$$p_{zt}(i) = M^z MC_t \ \forall_{ji} \tag{20}$$

Deviations from this optimality condition under completely flexible prices can arise due to nominal rigidities introduced next.

2.2.3 Intermediate goods producers in particular - *the production of consumption goods*

Following Woodford (2003) and Yun (1996), intermediate good firms in the consumption sector face Calvo pricing such that $1 - \alpha_C$ of the producers in the sector are allowed to be reset in each period, while α_C of the prices remain unchanged. Each supplier which can chose a new price for its good at time t faces the same problem as anybody else who can change its price at that date. The optimal price, p_{Ct}^*, for the individual firm that can change its price will in equilibrium equal the price chosen by the rest. The pricing index for this sector is therefore given by:

$$P_{Ct} = \left[(1 - \alpha_C) p_{Ct}^{*1-\theta} + \alpha_C P_{Ct-1}^{1-\theta} \right]^{\frac{1}{1-\theta}} \tag{21}$$

The intermediate goods producer takes into account the dynamic aspect of the price setting, and maximises the present value of his profits discounting these profits by the stochastic discount factor, (9),

$$\begin{aligned} &\max_{p_t(i)} E_t \left[\sum_{T=t}^{\infty} \alpha_C^{T-t} M_{t,T} \Pi(.) \right] \\ &= E_t \left[\sum_{T=t}^{\infty} \alpha_C^{T-t} M_{t,T} \left[\frac{1}{n_C} \left(\frac{p_{Ct}(i)}{P_{Ct}} \right)^{-\theta} \frac{Y_{Ct}}{P_{Ct}} (p_{Ct}(i) - W_t) \right] \right] \end{aligned}$$

and sets his price such that the expected mark-up equals the desired mark-up:

$$0 = E_t \left[\sum_{T=t}^{\infty} \alpha_C^{T-t} M_{t,T} \left[\begin{array}{l} \frac{Y_{Ct}}{P_{Ct}} \left(\frac{p_{Ct}(i)}{P_{Ct}} \right)^{-\theta} (1-\theta) \\ +\theta \frac{Y_{Ct}}{P_{Ct}} \left(\frac{p_{Ct}(i)}{P_{Ct}} \right)^{-\theta-1} \frac{W_t}{P_{Ct}} \end{array} \right] \right] \tag{22}$$

Together with the price index, this equation determines the evolution of the aggregate price in the production sector of consumption goods given the evolution of consumption output and the disturbances in the economy, and these equations thus constitute the aggregate-supply block of the consumption goods sector.

I can log-linearize (21) and (22) and combine them to obtain a New-Keynesian Phillips curve for the production sector of consumption goods:

$$\pi_{Ct} = \kappa^C mc_t^C + \beta E_t \left[\pi_{Ct+1}\right] \tag{23}$$
$$\kappa^C \equiv \frac{(1-\alpha_C)(1-\beta\alpha_C)}{\alpha_C}$$

I can substitute out marginal cost with some measure of the output in the economy and some measure of the natural rate of output in the economy defined as the output that would prevail in the economy under flexible prices. These expressions are derived next.

2.2.4 Output for the production of consumption goods

I start by the consumption goods sector. I can impose the equilibrium condition $P_{Ct} = p_{Ct}(i)\ \forall_i$ in (20) since all firms within a sector share the same characteristics given that all firms in a sector are free to set its prices. (2.2.2). This condition, together with the production function, $Y_{Ct}(i) = N_t^C(i)$, the identity for the labour force,

$$N_t^* = N_{Ct}^* + N_{Xt}^* = n_C N_t^* + n_X N_t^* = n_C Y_{Ct}^* + n_X Y_{Xt}^*$$

and imposing the equilibrium condition $Y_{Ct}(i) = Y_{Ct}$ in (5), allows me to write output of consumption goods as:

$$Y_{Ct} = \frac{1}{n_C}\left(\frac{1}{\upsilon M^C}\right)^{\frac{1}{\varphi}} Y_{Ct}^{-\frac{\gamma}{\varphi}} H_t^{\frac{\omega(1-\gamma)}{\varphi}} - \frac{n_X}{n_C} Y_{Xt}$$

The equilibrium condition for the durable is less simple as housing is a stock variable for the consumer but a flow variable for the producer. In equilibrium, the producers produce the flow of housing which is in demand in that period:

$$Y_{Xt} = H_t - (1-\delta) H_{t-1} = X_t \Leftrightarrow$$
$$H_t = \sum_{i=0}^{\infty} (1-\delta)^i Y_{Xt-i}$$

A log-linearization of the equations yields:

$$y_{Ct} = \left(\frac{\omega(1-\gamma)\delta - \varphi n_X}{\varphi n_C + \gamma}\right) y_{Xt} \tag{24}$$
$$+\frac{\omega(1-\gamma)}{\varphi n_C + \gamma}(1-\delta) h_{t-1} - \frac{1}{\varphi n_C + \gamma} m_t^C$$

The production of consumption goods depends on the housing stock in the previous period, h_{t-1}, production of housing in this period, y_{Xt}, and the mark-up, m_t^C. The interdependence between the sectors is due to the non-separability of housing and consumption in the utility function. I leave interpretations until after I have derived a relationship for production in the construction sector.

2.2.5 Intermediate goods producers in particular - *the construction sector*

I consider two pricing frameworks for the construction sector: One in which the producers are free to set their prices at the beginning of each period, and a second framework where only some of the producers are able to reset prices. The construction sector under the latter framework is completely symmetric to the production sector of non-durables goods in the section above.

Under flexible prices the firms simply maximise profit in each period, the analysis from section (2.2.2) holds, and the optimal choice of pricing for the intermediate goods producer in the construction sector is thus given by:

$$p_{Ht}(i) = M^H MC_t = P_{Ht} \tag{25}$$

The optimal condition for the consumers' labour supply is given by (5) as not only must the consumer in optimum be indifferent between working and consuming, but also between working and consuming in *either sector.* I note that in the flexible price equilibrium, the mark-up is always equal to the desired mark-up ($M_t^H = M^H$), and a log-linearisation of (5) can be written as:

$$y_{Xt} = \frac{\omega(1-\gamma)(1-\delta)}{(\varphi n_X - \omega(1-\gamma)\delta)} h_{t-1} - \frac{\gamma + \varphi n_C}{(\varphi n_X - \omega(1-\gamma)\delta)} y_{Ct}$$

The production in the construction sector for the case of sticky non-durables - flexible durables, can already be interpreted as the natural rate of output *if* production in the consumption sector is at its natural level. Hence, the natural level of output in construction goods can be written as:

$$\begin{aligned} y_{Xt}^N = & \frac{\omega(1-\gamma)(1-\delta)}{(\varphi n_X - \omega(1-\gamma)\delta)} h_{t-1} - \frac{\gamma + \varphi n_C}{(\varphi n_X - \omega(1-\gamma)\delta)} y_{Ct}^N \\ & - \frac{1}{(\varphi n_X - \omega(1-\gamma)\delta)} m_t^H \end{aligned}$$

This is the output block of the construction sector in which lagged housing stock captures the durability of good, while the interdependence with the consumption sector captures the non-separability between housing and consumption for the consumer.

2.2.6 The aggregate production sector

I next rewrite the output relations for the two sectors in a simpler way. I can substitute out marginal cost in (23) with (24) using:

$$mc_t^C = (\varphi n_C + \gamma)\left(y_{Ct} - y_{Ct}^N\right) = -m_t^C$$

The production side of the sticky non-durables - flexible durables model thus consists of a New Keynesian Phillips curve for the consumption sector as a function of the output gap for that sector:

$$\pi_{Ct} = (\varphi n_C + \gamma)\,\kappa^C\left(y_{Ct} - y_{Ct}^N\right) + \beta E_t\left[\pi_{Ct+1}\right]$$

$$y_{Ct}^N = \left(\frac{\omega(1-\gamma)\delta - \varphi n_X}{\varphi n_C + \gamma}\right) y_{Xt}^N + \frac{\omega(1-\gamma)}{\varphi n_C + \gamma}(1-\delta)\,h_{t-1} \qquad (26)$$

I notice that if production in the consumption sector is at its natural rate of output so is construction of new housing flows, $y_{Xt}^N = y_{Xt}$. I can also introduce into the aggregate production framework above the following proportionality between the production sectors:

$$y_{Xt} - y_{Xt}^N = \Gamma_{XC}\left(y_{Ct} - y_{Ct}^N\right) \qquad (27)$$

$$\Gamma_{XC} \equiv -\frac{\gamma + \varphi n_C}{(\varphi n_X - \omega(1-\gamma)\delta)}$$

I get an expression for the relative price in terms of consumption goods, Q_t, in the case of sticky construction goods prices and flexible consumption goods prices from a log linearisation of the optimal price for the construction goods producer, (25), the first efficiency condition for the consumers' optimal choice between labour and consumption, (5),

$$P_{Ht} = MP_{Ct}\frac{U_{Nt}}{U_{Ct}}$$

I impose equilibrium conditions and log linearise to get

$$q_t = (\gamma + \varphi n_C)\,y_{Ct} + (\varphi n_X - \omega(1-\gamma)\delta)\,y_{Xt} - \omega(1-\gamma)(1-\delta)\,h_{t-1}$$

The production side in the model in which all producers in the economy are restricted from resetting their price in each period consists of two interdependent New Keynesian Phillips curves for the consumption sector and construction sector respectively:

$$\pi_{Ct} = \kappa^C mc_t^C + \beta E_t\left[\pi_{Ct+1}\right],\ \kappa^C \equiv \frac{(1-\alpha_C)(1-\beta\alpha_C)}{\alpha_C} \qquad (28)$$

$$mc_t^C = (\varphi n_C + \gamma)\left(y_{Ct} - y_{Ct}^N\right) + (\varphi n_X - \omega(1-\gamma)\delta)\left(y_{Xt} - y_{Xt}^N\right)$$

$$\pi_{Ht} = \kappa^H mc_t^H + \beta E_t \left[\pi_{Ht+1}\right], \ \kappa^H \equiv \frac{(1-\alpha_H)(1-\beta\alpha_H)}{\alpha_H} \tag{29}$$
$$mc_t^H = (\varphi n_X - \omega(1-\gamma)\delta)\left(y_{Xt} - y_{Xt}^N\right) + (\gamma + \varphi n_C)\left(y_{Ct} - y_{Ct}^N\right)$$

2.3 The public sector: *Monetary policy and the government*

Monetary policy is conducted through a Taylor rule as thoroughly discussed in the literature, see e.g., Woodford (2003):

$$i_t = \bar{\imath} + \Gamma \pi_{Ct} + \varepsilon_t^{mp} \tag{30}$$

$i_t \equiv \log\left(R_t^f\right)$ is the policy rate and $\bar{\imath}$ denotes its steady state value. ε_t^{mp} reflects a stochastic discrepancy between the target for the short-rate, which the central aims to hit, and the actual short-rate and can thus be interpreted as a monetary policy shock. I assume the central bank obeys the Taylor rule and raises its policy rate in response to shocks to inflation such that the parameter Γ, is greater than one.

The central bank is not assumed to stabilise an aggregate pricing index, but instead stabilises inflation in the consumption index. This is not controversial in the case of sticky consumption goods prices and flexible construction goods prices, as in this case inflation in the construction sector does not give rise to any welfare losses for the consumers, see e.g., Aoki (2001).[16] I keep inflation in consumption goods in the Taylor-rule in the case of sticky prices in both sectors, and I check the robustness of the model with a Taylor rule in the construction inflation index.

I assume the government holds the mortgage bonds issued by the consumer. The interpretation is that a public institution resembling, say, Fannie Mae or Freddie Mac is set up to provide the consumers with loans and to create liquidity in the mortgage bond market.

I specify an exogenously given path for public expenditures and I consider a rule for the determination of the level of tax collections net of transfers, T_t. I assume the government runs a public debt which, for simplicity, consists of risk less, one-period nominal bonds. The government thus faces the following budget constraint:

$$M_t + P_t^1 B_t^1 + P_t^m B_t^m = B_{t-1}^1 + B_{t-1}^m + M_{t-1} + P_t g - T_t$$

[16] In the case of flexible prices in construction goods sector, the output in this sector is always equal to the natural rate in that sector defined exactly as the output that would prevail if the all producers in that sector were able to reset their prices at the beginning of each period.

As in section 2.1, B_t^1 denotes bond holding as of time t maturing after this period, which trades at price P_t^1, and likewise, B_t^m denotes mortgage bond holdings as of time t. On the right hand side, B_{t-1}^1 and B_{t-1}^m indicates public debt and mortgage bonds maturing at time t, while $P_t g \equiv P_{Ct}G_{Ct} + P_{Xt}G_{Xt}$ denotes public consumption of housing and consumption goods.

I impose a balanced budget requirement for the government whereby the primary deficit must equal the net interest payments on the outstanding debt and mortgage bond holdings following Schmitt-Grohe and Uribe (2000), and Benhabib, Schmitt-Grohe, and Uribe (2001):

$$T_t - P_t g = \left(\frac{1}{P_t^1} - 1\right) P_t^1 B_t^1 + \left(\frac{1}{P_t^m} - 1\right) P_t^m B_t^m$$

Benhabib, Schmitt-Grohe, and Uribe (2001) defines a Ricardian fiscal policy as a fiscal policy that ensures a present discounted value of total government liabilities that converges to zero under all possible, equilibrium or off-equilibrium, paths of endogenous variables. Given a monetary policy rule that ensures a strictly positive interest rate, a balanced-budget rule is Ricardian since the rule ensures that total government liabilities are constant.

A Ricardian fiscal policy together with a monetary policy that is not dependent on the paths of the debt nor taxes that conforms to the Taylor principle, $\Gamma > 1$, implies a determinate equilibrium for aggregate demand, inflation and interest rates, see e.g., Benhabib, Schmitt-Grohe, and Uribe (2001), Schmitt-Grohe and Uribe (2000), and Woodford (2003). Intuitively, in the case of a Ricardian fiscal policy rule the endogenous variables are independent of fiscal policy as the public debt is bounded and the public sector therefore does not need to, say, print an infinite amount of money nor raise taxes to excessive levels at any point in time.

2.4 Equilibrium and market clearing

Equilibrium in goods markets implies the production of the final goods to be allocated to household total expenditure and government expenditure in the respective sectors:

$$Y_{Xt} = X_t$$

$$Y_{Ct} = C_t$$

Equilibrium in the labour market requires:

$$N_t = N_t^C + N_t^X$$

Equilibrium in the government bond market requires the total net supply of bonds for all maturities is zero. The real financial constraint dictates that all new housing purchased by the consumers must be financed through the mortgage market:

$$b_t^{R,m} \equiv \frac{B_t^m}{P_{Ct}} = Q_t \frac{X_t}{P_t^m}$$

Lastly, the government sector equilibrium consists of the balanced budget requirement.

2.5 Deterministic steady state

The steady state user cost of housing can be determined from (11) and the first order condition for mortgage bond holdings, (8),

$$\begin{aligned} \frac{U_H}{U_C} &= Q\left(1-\beta\left(1-\delta\right)\right)\left(1+\Phi\right) \\ &= \tilde{Q}\left\{1-\left(1-\delta\right)\beta\right\} \end{aligned} \tag{31}$$

in which $\tilde{Q} \equiv \frac{P_H}{P_C}\frac{rx^m}{R^f}$ denotes the steady state value of the effective relative price, rx^m denotes the steady-state holding period return from a mortgage bond, and $R^f \equiv \frac{1}{\beta}$ is the gross real interest rate. (31) highlights the role of the yield curve in determining the user cost of housing. As expected, if the cost of financing new housing purchases in the mortgage market equals the real short rate, $rx^m = R^f$, the user cost of housing collapses into the standard efficiency condition for durables. But a higher cost of mortgage financing relatively to financing by the real short rate, $rx^m > R^f$, tilts the optimal choice of relative spending on housing towards more consumption on non-housing goods in the steady-state.

I assume a steady-state inflation rate of zero in both sectors. In a flex price steady-state the relative price of housing is one, $Q \equiv \frac{P_H}{P_C} = 1$ and steady-state consumption is therefore given by

$$C = \frac{1}{\omega}\left\{\frac{rx^m}{R^f} - \left(1-\delta\right)\beta\frac{rx^m}{R^f}\right\}H$$

Real steady-state mortgage bond holdings can be determined from (3):

$$b^{R,m} = \frac{\delta H}{P^m} = \frac{X}{P^m}$$

The consumer is short in mortgage bonds in the steady-state, as the consumer need to refinance the housing purchases needed to keep the housing stock constant due to the rate of depreciation, δ.

I can pin down the steady-state housing stock with the help of steady-state consumption and the budget constraint:

$$H = \frac{\omega N}{\left\{\frac{rx^m}{R^f} - (1-\delta)\,\beta \frac{rx^m}{R^f}\right\} + \delta} \tag{32}$$

Notice the steady-state housing stock depends on mortgage financing such that higher long interest rates decreases the stock of housing in the economy in its steady state. I can lastly use (5), (31), and (32) to get the steady-state labour supply:

$$N = \left[\frac{1}{\upsilon M}\left(\frac{C}{H}\right)^{-\gamma}\left(\frac{C}{H}+\delta\right)^{-\omega(1-\gamma)+\gamma}\right]^{\frac{1}{\varphi-\omega(1-\gamma)+\gamma}}$$

3 Analysing the model: *The monetary transmission mechanism through a long interest rate*

I solve, calibrate and analyse the models in the following sections. Attention will only be given to the monetary transmission mechanism through changes in the short rate, the short rates' effect upon the long rate, and the key role of the long rate for the determination of the user cost of housing. I focus on the key variables in the model: inflation, consumption of both goods, relative prices, and the mortgage bond yield.

3.1 Calibration and solution method

I analyse the model properties through a calibration using Uhlighs toolkit, see Uhlig (1995). Time is in quarters. I set the discount factor, β, to 0.99, which implies a 1 percent real rate per quarter. ω, the parameter which governs the relationship between housing and consumption in (1), must be positive to ensure positive marginal utility of housing. I set the intertemporal elasticity of substitution to 2 and I set the elasticity of labour supply around 0.6, which is standard in the literature, and which is typically found in microeconometric studies. The Calvo parameter, α_C, is set to 0.75, which gives an average duration of a price contract of one year, while I set the weight on inflation in the Taylor rule, (30), to 1.5. I likewise normalise the steady-state total production to be 1, and I assume that the size of the housing sector is 25% of the total economy following Barsky, House, and Kimball (2007). I follow Calza, Monacelli, and Stracca (2006) and set the depreciation of the housing stock equal to 0.005 or a yearly depreciation rate of 2%, which is quite conservative, see e.g., Fraumeni (1997). The duration of the mortgage bond is set to 20 quarters.

3.2 The response to a monetary policy shock in the case of *sticky* construction prices and *flexible* consumption goods prices

This section analyse the model with (3) imposed in the case of completely asymmetric nominal rigidities. Construction goods prices are fully flexible, consumption goods prices are sticky and the production sector consequently consists of relations (27) and (28) with natural rate of output given by (26). Figure 1 displays the equilibrium dynamics of the model to a monetary policy shock.[17]

A current one period higher than expected short rate lowers inflation and consumption of non-durable goods, see (17). This is what I have denoted the first monetary transmission channel. Two additional things happen. Firstly, the relative price of housing falls, as consumption goods face nominal rigidities, while construction goods prices do not. Secondly, the long interest rate rises in response to the higher short rate, see (15), thereby increasing the cost of financing new housing purchases, see (12), depressing output further through (17). This is what I have denoted the second monetary transmission channel.

The relative price effect dominates the effect of higher cost of financing and construction shoots up. Hence, the financial constraint does not solve the co movement problem presented in section 2.2 in a two-sector model with symmetries across sectors, sticky non-durable and flexible durable prices.

I thus need to diminish the relative nominal rigidity between the sectors, if I want to analyse the independent role of the financial constraint. It is nevertheless useful to compare the behaviour of construction of new housing in this model to a model without the financial constraint. That is, a model reduced to (10) and (16) in which the expectations hypothesis holds and the effective relative price equals the relative price, $\tilde{Q}_t = Q_t$.

Figure 2 shows the equilibrium dynamics of the sticky-flexible price economy in the absence of (3) to a shock to the monetary policy rate. The dynamics look *qualitatively* similar: Inflation, the relative price, and consumption jump down in response to the monetary policy tightening, while construction shoots up. However, the financial constraint does mitigate the co movement problem.

Figure 3 shows the difference between the equilibrium dynamics to

[17] I plot all impulse response functions with different legends. A bond yield is the average interest rate on a bond and the movements in the long interest rate are thus by nature small.

a monetary policy shock in the model with and without the financial constraint. A positive value for, say, construction in figure 3 implies that construction shoots up by more in the model without constraint (3) imposed. The model is not able to completely solve the co movement problem - the relative price effect dominates - but construction shoots up by around 2.5 percentage point *less* in the model with the financial constraint than in the model without it. Hence, *the co movement problem is diminished for given relative nominal rigidities between the sectors in a model with (3) imposed*.

3.3 The response to a monetary policy shock in the case of equally sticky prices

I now neutralise the relative price effect to identify and isolate the implications for the aggregate economy of the financial constraint. To achieve this, I restrict producers of construction goods from resetting their prices each period and instead force them to operate in the same environment as the consumption good producers did in the model above. The production side of the model consequently consists of relations (28) and (29). I set the Calvo parameter in the construction sector, α_H, equal to 0.75 as in the consumption goods sector, such that the degree of nominal rigidity is equal across all producers in the economy. Hence, relative prices are constant, $Q_t \approx \tilde{Q}_t$, such that the effective relative price under (3) is approximately equal to the relative interest rates, $\tilde{Q}_t \approx \frac{E_t\left[RX_{t+1}^m\right]}{R_t^f}$.

Figure 4 shows the equilibrium dynamics of this model to a one period shock to the current monetary policy rate. The higher policy rate lowers demand and inflation in the non-durable sector falls, see (17). This is the same equilibrium response to a monetary policy shock as in the basic model without the financial constraint, and the effects arise through the first monetary policy channel described in section 2.1.6. Further, in response to the higher short-rate, the long interest rate *rises* through (15) and this causes a *contraction* in the construction sector through the user cost of housing, see (11), a further downward pressure upon current consumption through (17), and this is the second channel for monetary policy.

To get a grasp of the quantitative effects of the financial constraint, I, as in section 3.2, consider a model with the same calibration and same relations but without the constraint (3) imposed.[18] Figure 5 shows the difference between the equilibrium dynamics of the model with an

[18] The impulse response functions for this model are left out of the paper for brevity. However, figure 5 shows impulse response functions for the most important variables.

equal degree of nominal rigidity with and without the financial constraint imposed. A positive value for, say, construction in figure 5 implies that construction shoots up by more in the model without constraint (3) imposed.

Construction falls in both models, but construction falls by 0.15 percentage points more in the model with the financial constraint (top-right figure in figure 5) and this is thus the quantitative effect of the financial constraint in a model with completely symmetric rigidities. Both the relatively lower housing stock and the higher bond yield feeds into the IS-relation, equation (17), in the model with the financial constraint and consumption falls relatively more in the model with the financial constraint. Inflation is relatively higher in the model without the financial constraint arising as a consequence of the both relatively higher output of consumption goods and greater activity in the construction sector. Aggregate output is consequently greater without feedback effects from the long interest rate. Specifically, relative aggregate output peaks at 20 basis points 3 quarters hence, and this is thus the quantitative effect of the financial constraint on aggregate demand.

Figure 4 and 5 crystallises the main contribution of this paper. The long interest rate plays an independent role *different* from the short-rate, as the long interest rate affects housing alone, and this monetary policy channel *is* important affecting around 25% of the total economy. This together imply the transmission mechanism *is* altered from the standard one-interest rate model, as the economy wide implications of a monetary policy shock works not only through changes in short rates and the implied effect through the intertemporal rate of substitution, but also through a downward pressure upon consumption of housing goods through a higher cost of financing goods.

4 Conclusion

This paper analyses the interaction between long interest rates, housing, and monetary policy. I build a model which takes a first step towards a greater understanding of the equilibrium feed-back effects between the yield curve and the aggregate economy. The model provides an independent role for the long interest rate through a financial friction which directly affects the user cost of housing. This effect opens up for a second monetary policy channel different from the effects of changes in real short-rates upon the intertemporal consumption of non-durable goods.

The long interest rate and its effects upon the macro economy significantly changes the way the monetary transmission mechanism works in a DSGE model. Higher long-interest rate depresses consumption of new housing through higher user cost, and as housing is big relatively to the total economy, this effect is economically important. I find the introduction of the long interest rate into a two-sector model is not by it self sufficient to solve the co movement problem though it mitigates it significantly.

The policy implication of these findings is that monetary policy makers need to study two relations further. One is the effects from changes in short interest rates onto longer interest rates. Second, it is vital for the impact of monetary policy to understand how long rates affect durables. This is not a trivial task as the properties and the foundations of the economically most important durable goods sector, the housing sector, varies considerably both across countries and through time.

I do not see this conclusion as the end of the financial constraint introduced in this paper but rather as the beginning, as it opens of for a host of possibilities to study. It could be interesting to introduce the financial constraint into a bigger model with more markets, and it could especially be interesting to include capital into the model in this paper and impose a similar financial constraint as in this paper on the capital-investment decision for firms in the economy. I leave this for future research.

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5 Figures

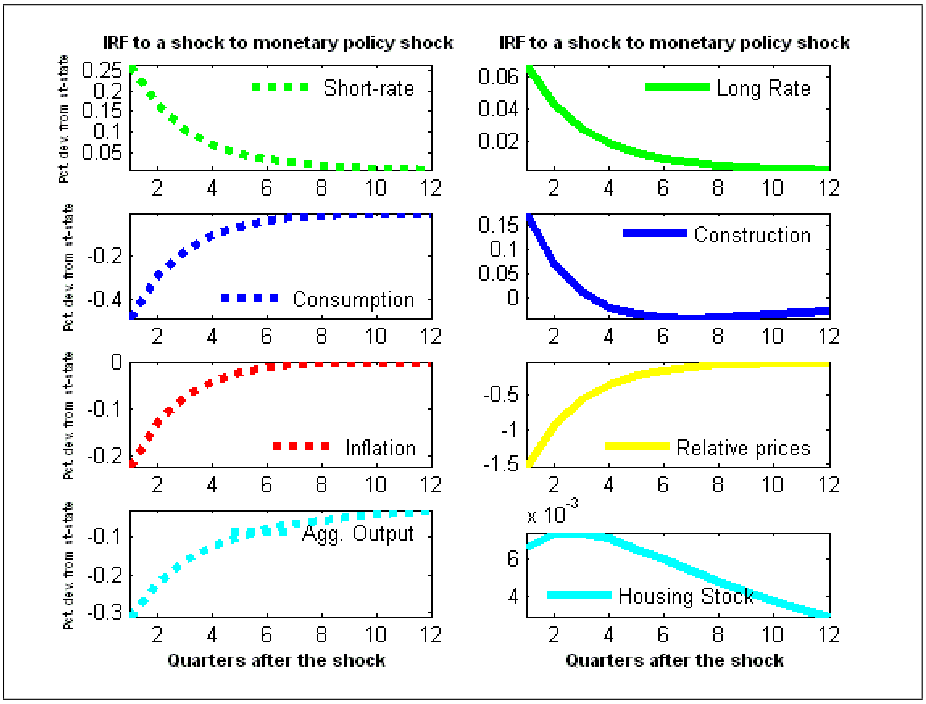

Figure 1: The effect upon key variables of an innovation to the monetary policy rate in a sticky-flexible price model in which the financial constraint is imposed

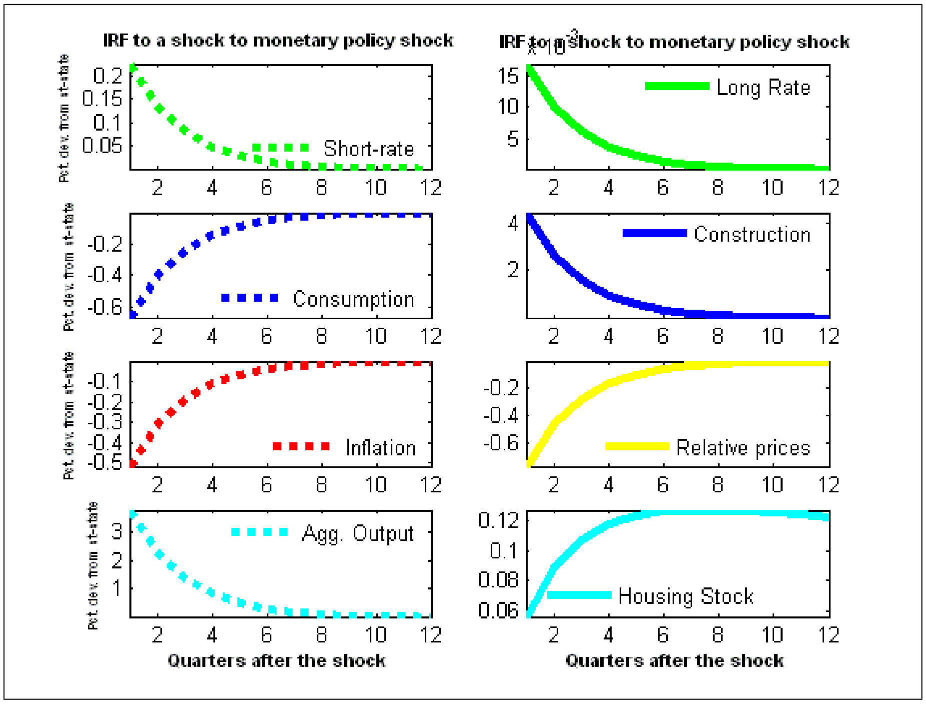

Figure 2: The effect upon key variables of an innovation to the monetary policy rate in a sticky-flexible price model without the financial constraint imposed

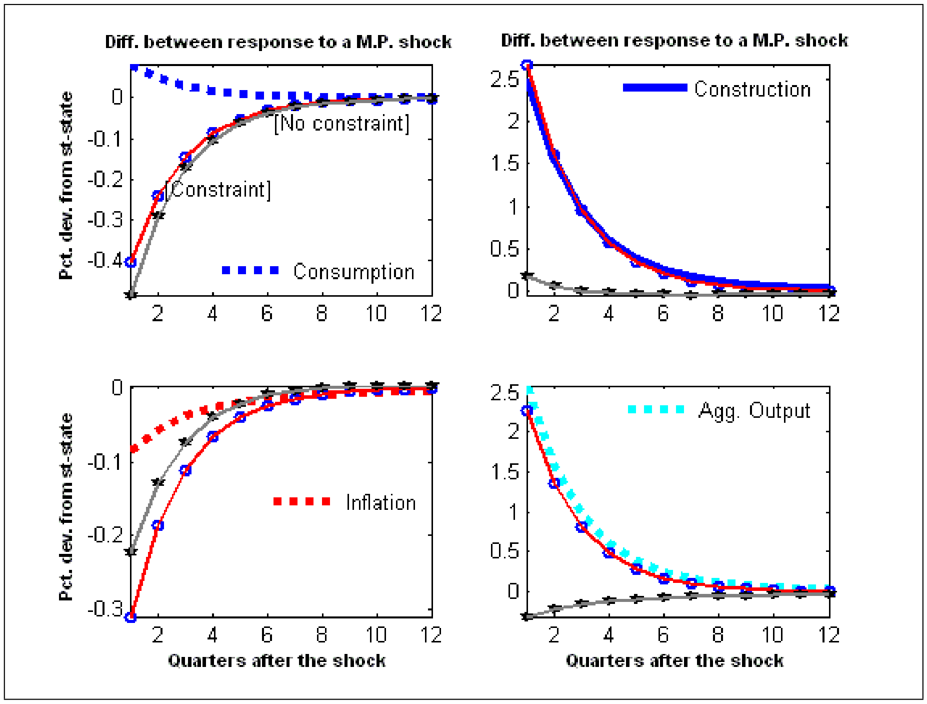

Figure 3: The difference between the effects upon key variables of an innovation to the monetary policy rate a model with sticky-flexible prices with and without the financial constraint

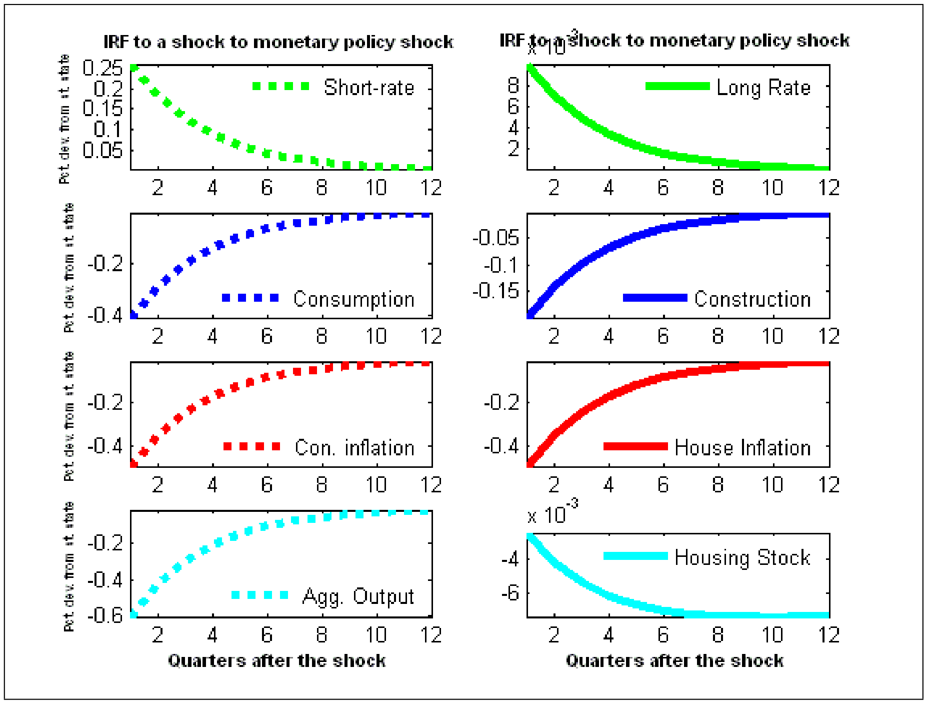

Figure 4: The effect upon key variables of an innovation to the monetary policy rate in a sticky-sticky price model in which the financial constraint is imposed

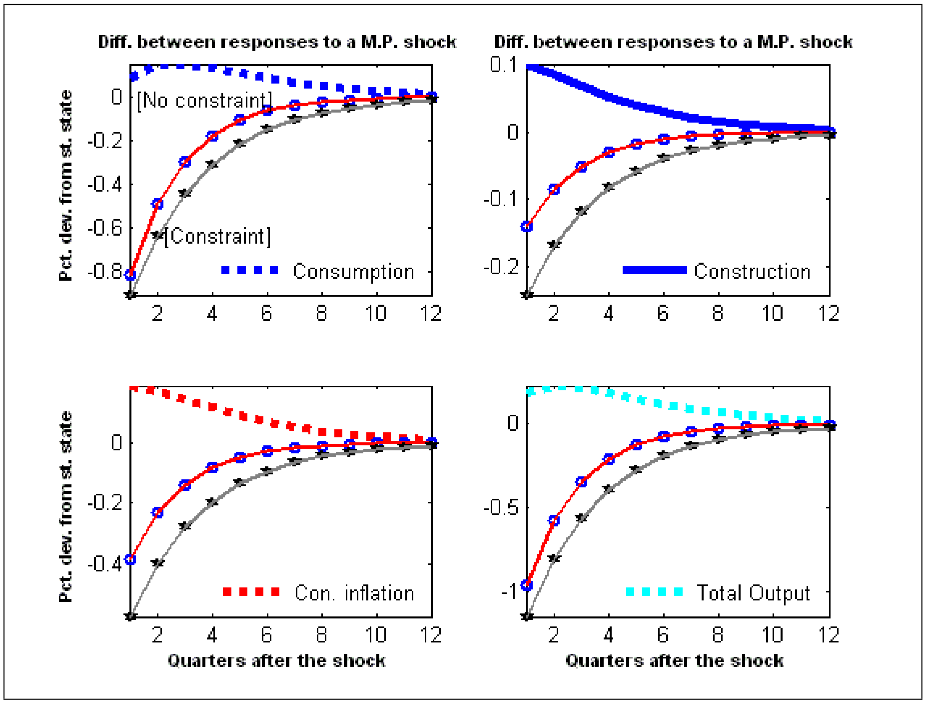

Figure 5: The difference between the effects upon key variables of an innovation to the monetary policy rate a model with symmetric nominal rigidities with and without the financial constraint

Is the Bond Premium Puzzle Really a Puzzle?

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Abstract

Empirical evidence points towards a mean term premium for a 10 year bond of around 160 basis points per year and an unconditional standard deviation of around 50. DSGE models with standard preferences have found it hard to generate these numbers. Common for these models is the use of higher-order approximations to generate non-zero, non-constant risk premia. I use closed-form solutions for bond prices, and I apply a financial model for the study of financial markets and not the other way around as in the existing DSGE literature. The parameters in the financial model are mapped to structural parameters from an underlying DSGE model. I generate plausible mean term premia, while the unconditional standard deviation resides on the margin with a model that fares no worse than a standard DSGE model. The model gives an interpretation of the decline in term premia from the Volker period in terms of better anchored inflation expectations.

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1 Introduction

DSGE models have found it hard to model bond risk premia. Empirical evidence for 10 year bonds points to a mean term premium of around 160 basis points per year with an unconditional standard deviation of around 50.[1] DSGE models find a mean term premia of around 2 basis points per year and an unconditional standard deviation of around 0.1.[2] The disability of the DSGE literature to generate moments of term premia has been denoted the *Bond Premium Puzzle*.

The DSGE literature generally looks at financial markets with an Euler equation for the study of macroeconomic quantities and less for the study of risk. The DSGE literature also needs to use higher order approximations to generate non-zero, non-constant risk premia.[3] But higher order terms are small by definition, so perhaps the modest success of the DSGE literature in fitting bond risk premia is due to the approximations and not bad models per see. I consequently introduce closed-form solutions for bond prices within the affine framework derived from an Euler equation with an eye for the fit of risk premia in a structural model. I ask in the light of these changes: *Is the bond premium puzzle really a puzzle?* My answer to this question is *no*.

It is not irrelevant for macroeconomic research that standard specifications of macroeconomic models cannot account for important features of financial markets. Sargent (2007) argues that until economists have succeeded in getting a consumption-based asset pricing model that works well, the New-Keynesian IS curve is built on sand because a representative agent's consumption Euler equation exactly *is* the IS-curve central to the policy transmission mechanism in modern models. Risk premia are also important to study in a macroeconomic framework exactly through its role as the flip-side of the financial markets.[4] If you can say a lot about macroeconomic variables in a model, which is able to get risk premia right, you can also say a lot about the underpinnings of risk and uncertainty, which are the backbone of the financial industry.

Moreover, the bond market is massive and clearly plays a key role for

[1] The term premium is defined as the difference between a forward rate of maturity n and the expected future risk free rate n-periods hence. See e.g., Rudebusch and Wu (2004), Kim and Wright (2005), Cochrane and Piazzesi (2006).

[2] See e.g., Levin, Onatski, Williams, and Williams (2005), Rudebusch, Sack, and Swanson (2007), Lawrence, Eichenbaum, and Evans (2005).

[3] The exception is Gallmeyer, Hollifield, and Zin (2005) which obtain closed form solutions for bond prices. However, their paper has a different motivation than risk and they do not analyse risk premia directly.

[4] See e.g., Cochrane (2001) and Cochrane (2007).

consumption and investment. As of 2006, the size of the international bond market is an estimated $45 trillion which roughly equals total world GDP, and bonds and interest rates act as underlying assets on a host of derivatives.[5]

The consumption-based asset pricing literature has lately been quite active in yield-curve modeling. Habit formation has been introduced into the models, see e.g., Wachter (2006), following the reverse engineering exercise in Campbell and Cochrane (1999), as well as recursive utility as in Epstein and Zin (1989) combined with learning and adaptive expectations, e.g., Piazzesi and Schneider (2007). Being partial equilibrium models they tell little about the underlying macroeconomic sources that generate the premia, and are as such less suited for the study of the interactions between the macro economy and the financial markets.

With the exception of the paper by Rudebusch, Sack, and Swanson (2007), the main focus of the DSGE literature, see e.g., Woodford (2003), Clarida, Gali, and Gertler (1999), Smets and Wouters (2003), has not been the financial markets. Papers which explicitly analyse the bond markets have used various nominal and real rigidities but end up with implausible small moments of term premia, see e.g., Rudebusch, Sack, and Swanson (2007), Rudebusch and Swanson (2007). Some successful papers in this respect, Hordahl, Tristani, and Vestin (2006) and Ravenna and Seppala (2006), tend to fit term premia moments at the expense of the macroeconomic side of their model, see also Rudebusch and Swanson (2007).

This paper successfully fits bond risk premia as a result of two different modeling strategies relative to the existing literature. Firstly, I construct a financial model for the study of financial markets starting from a class of financial models that *empirically* fits moments of term premia. Markedly, these affine term structure models (AFTM), see e.g., Dai and Singleton (2002), Duffie and Kan (1994) and Ang and Piazzesi (2003), provides closed-form solutions for bond prices and thus clears the way for avoiding higher-order approximations.

The AFTM are based upon reduced form relations while I want to study the interactions between a structural, macroeconomic model and the bond market. I therefore set up a macroeconomic model, which provides a micro-foundation for these relations and I identify structural parameters that determine the size and variability of risk premia. The question is consequently not what the implications are for bond risk premia in a model set up for the study of the dynamics of macroeconomic

[5]Outstanding U.S. Bond Market Debt Bond Market Association as of November 2006, and World Bank.

variables, as in the existing DSGE literature. The question is instead whether I can provide a micro foundation for financial models which are known to generate plausible bond risk premia.

I follow Gallmeyer, Hollifield, and Zin (2005) which uses the macroeconomic model of Clarida, Gali, and Gertler (1999), while the consumer preferences are subject to a preference shock. This preference shock takes the form of *stochastic habits* due to Dai (2003), while I extend this habit term with a *subjective inflation target*. As shown in the paper, standard preferences, like CARA preferences and habits, in a representative agent model can neither generate plausible levels nor time variation in term premia. Some extra terms in the representative consumers' preferences are therefore needed if the model aims to deliver some hope in solving the bond premium puzzle

Though stochastic habits have been applied in the existing literature, e.g., Gallmeyer, Hollifield, and Zin (2005), I stress the economic reasoning behind the inclusion of the specific form of the preference shock is dubious and its introduction into the model is a short-cut to achieve time variation in risk premia. However, I show the terms are sufficient to generate time-variation in term premia at business-cycle frequencies, the stochastic habit term, and to model long run levels of nominal bond yields and term premia, the subjective inflation target, while the mechanisms which generate risk premia *can* be given economic interpretations. Stochastic habits, the subjective inflation target, and the functional form assumptions can in this view be seen as a second best microfoundation for time variation in risk premia.

With habits, the marginal utility of consumption tomorrow depends on consumption today. With *stochastic* habits, the interdependence between consumption today and marginal utility tomorrow is stochastic and thus is marginal utility of an extra unit of consumption in the future.

Means of time-series are hard to estimate and this is particularly so for bond yields and bond risk premia, as these are both very persistent and exhibit regime shifts, and this is the motivation behind the inclusion of the subjective inflation inflation target.[6] Nominal US yields show rising yields until the beginning of the 1980s as a response of rising inflation. Thereafter, yields slowly decreased until the present. I not only want to fit means of bond yields and risk premia but also these persistent, long run swings. Shocks to the subjective inflation target does exactly this being a main determinant of the long run swings in risk

[6]See e.g. Ang, Bekaert, and Wei (2008) who estimate a regime-switching model for bond yields and identifies four regimes.

premia and bond yields through an assumption about almost unit-root behaviour, while it has almost no impact upon risk premia on business-cycle frequencies, through a specific assumption about the correlation between the subjective inflation target shock and shocks to inflation.

The subjective inflation target captures the consumers' aversion to inflation and can be thought of as a proxy for the cost of money holding. A shock to the subjective inflation target is a signal to the consumer that the economy is likely to reside in a state with relatively high inflation in the future with a monetary policy tightening on its way with higher real rates and lower consumption growth. A positive shock therefore decreases the utility of a given level of consumption and creates uncertainty about the value the consumer attach to future asset payoffs. Further, the subjective inflation shock creates *persistent uncertainty* about future consumption growth as the shock affects inflation which is combated by the central bank depressing consumption through higher real rates. Risk compensation on the bond market depends upon consumption growth because the consumers' valuation of future payoff is dependent upon past consumption through habits. The bond market therefore demands a compensation for bearing subjective inflation target risk both through habits and through the uncertainty surrounding valuation of future payoffs.

The second strategy is to reverse engineer the parameters of the macro-finance model along the lines of Campbell and Cochrane (1999). The mean term premium can not be increased through higher standard deviations on the shock process or high risk aversion parameters as such a boost also will boost the impulse responses in the underlying macro economy. Accordingly, the task is to calibrate the macro-finance model to generate plausible term premia and plausible dynamics for macroeconomic variables *simultaneously* with a *standard* calibration from the DSGE literature.

The first and most important contribution in this paper is that once a sufficiently flexible pricing model for bond prices with closed-form solutions is applied, the bond premium puzzle disappears. Secondly, all parameters in the macro-finance model can be given a clear economic interpretation and key properties in terms of means, standard deviations, determinacy, and impulse response functions are in line with both the existing literature and with empirical evidence for the macro economy and the financial market. Finally, I am able to provide an economic interpretation to the empirical observation of a decline in bond risk premia and yields from the 80s' to the present culminating with the so-called bond yield conundrum during 2004-2006, which saw not only low but also negative risk premia, higher policy rates but a decrease in long yields. I explain these movements in terms of a series of small negative shocks to the subjective inflation target through the years capturing long lasting credibility gains in the conduct of monetary policy.

This paper has the following structure. Section 2 sets up a macro-finance model of the term structure of interest rates. Section 2.1 provides an overview of the financial side of the model to which the macroeconomic is mapped into. Section 3 introduces the consumers into the economy and discusses how the functional form of their preferences transforms into the term structure model introduced in section 2.1. Section 4 introduces the supply-side. Section 5 explains how the habit specification and the subjective inflation target applied in this paper affect the macro economy and risk premia. Section 6 introduces monetary policy. Section 7, solves for the bond prices in this macro-finance model. Section 8 analyses the model properties, and compares the key properties of the model to empirical evidence, as introduced in section 8.1. This is done through a calibration, as discussed in section 8.2. Section 9 concludes.

2 A simple macro-finance model of the yield curve

2.1 An overview

The following sections set up a macro-finance model of the term structure of interest rates. The idea is to derive an *Essentially AFTM* from an underlying macro economy, as there exists strong empirical evidence in favour of this class of bond pricing models to fit key moments of bond yields including risk premia.[7] This overview explains where I am going.

The price of a zero coupon bond at time t with time-to-maturity n in an AFTM is given by:

$$P_t^n = \exp\left[A_n + \mathbf{B}_n'\mathbf{X}_t\right] \tag{1}$$

Yields are defined from prices as $y_t^n = -\frac{1}{n}\log\left(P_t^n\right) \equiv -\frac{1}{n}p_t^n$. $\mathbf{X}_t$ denotes a vector of state variables in the economy

$$\mathbf{X}_{t+1} = \boldsymbol{\mu} + \boldsymbol{\rho}\mathbf{X}_t + \boldsymbol{\Sigma}\boldsymbol{\varepsilon}_{t+1}$$

in which $\boldsymbol{\rho}$ is the autoregressive parameter matrix, $\boldsymbol{\mu}$ is its vector of constants, and $\boldsymbol{\Sigma}$ denotes the covariance matrix for the underlying shocks in the economy, $\boldsymbol{\varepsilon}_{t+1}$, specified to be homoscedastic. The coefficient A_n and the matrix $\mathbf{B}_n$ only depends upon the *maturity* of the bond, and

[7] See i.e. Piazzesi (2003) for a survey of this class of models.

respect the following recursions visualising the no-arbitrage restrictions imposed upon the financial markets by the AFTM:

$$\begin{aligned} A_{n+1} &= A_n + \mathbf{B}_n^{'}\left(\boldsymbol{\mu} - \boldsymbol{\Sigma}\boldsymbol{\lambda}_0\right) + \frac{1}{2}\mathbf{B}_n^{'}\boldsymbol{\Sigma}\boldsymbol{\Sigma}^{'}\mathbf{B}_n - \delta_0 \\ \mathbf{B}_{n+1}^{'} &= \mathbf{B}_n^{'}\left(\boldsymbol{\rho} - \boldsymbol{\Sigma}\boldsymbol{\lambda}_1\right) - \boldsymbol{\delta}_1^{'} \end{aligned} \tag{2}$$

Let R_t^f denote the *gross* risk free rate of interest and let $i_t \equiv \log\left(R_t^f\right)$ denote the log risk free rate which in the following will be denoted simply as the risk free rate. The yield of a bond which matures in the next period must equal risk free rate which in the AFTM follows:

$$i_t = \delta_0 + \boldsymbol{\delta}_1^{'}\mathbf{X}_t$$

The vector $\boldsymbol{\delta}_1^{'}$ determines the loading of the state variables in the economy to the risk free rate of interest, while δ_0 determines the level of the risk free rate of interest in the absence of any shocks.

This paper focus on risk premia and the key determinants behind risk premia are the parameters $\boldsymbol{\lambda}_0$ and $\boldsymbol{\lambda}_1$. To see this in a simple way, define the expected, log-holding period return from holding an n-period bond for one period in excess of the risk free rate of interest as

$$\begin{aligned} E_t\left[hpr_{t+1}^n\right] &\equiv E_t\left[p_{t+1}^{n-1} - p_t^n\right] - i_t \\ &= -\frac{1}{2}Var_t\left(hpr_{t+1}^n\right) - cov_t\left(m_{t+1}, hpr_{t+1}^n\right) \qquad (3) \\ &= -\frac{1}{2}\mathbf{B}_{n-1}^{'}\boldsymbol{\Sigma}\boldsymbol{\Sigma}^{'}\mathbf{B}_{n-1} + \mathbf{B}_{n-1}^{'}\boldsymbol{\Sigma}\boldsymbol{\Lambda}_t \qquad (4) \end{aligned}$$

in which I have introduced the (log) *stochastic discount factor* $m_{t+1} \equiv \log\left(M_{t+1}\right)$ which prices all assets in the economy. The first term is a Jensen inequality term while the second term is a risk premium, which arises from a non-zero covariance between the discount factor and the return on the asset. From (4), the functional form for the risk premia in an AFTM is given by $\mathbf{B}_{n-1}^{'}\boldsymbol{\Sigma}\boldsymbol{\Lambda}_t$: $\boldsymbol{\Lambda}_t \equiv \boldsymbol{\lambda}_0 + \boldsymbol{\lambda}_1^{'}\mathbf{X}_t$ denotes the *market price of risk*; the Sharpe ratio that an asset must earn if it loads on a specific shock.[8] The market price of risk is also equal to the Girsanov kernel used to change probability measure, so $\mathbf{A}_n, \mathbf{B}_n$ are functions of the stochastic processes for the state variables in the economy under the equivalent martingale measure. $\mathbf{B}_{n-1}^{'}$ is the loading on bond prices of a

[8] The Sharpe ratio is defined as the excess return above the risk free rate divided by its standard deviation. It can be shown that the standard deviation of (4) in an AFTM equals $B_{n-1}^T\Sigma$. Disregarding the Jensen-term, Λ_t therefore equals the Sharp ratio.

shock to the state-variables in the economy such that $\mathbf{B}'_{n-1}\mathbf{\Sigma}$ together is the quantity of risk or the expected fluctuation which investors can expect from bond prices.

The model is essentially affine if $\boldsymbol{\lambda}_1 \neq \mathbf{0}$, which implies state-dependent risk premia and therefore that the expectations hypothesis does not hold. A completely affine model, $\boldsymbol{\lambda}_1 = \mathbf{0}$, can clearly not generate non-zero standard deviations in risk premia. On the contrary, Dai and Singleton (2002) shows that it is the combination of time variation in market prices of risk, that is $\boldsymbol{\lambda}_1 \neq \mathbf{0}$, and correlated factors in a multifactor model that is able to generate term premia as found in data.[9]

Market prices of risk and risk aversion arise from consumer preferences so the risk premium must be a function of the underlying deep parameters in the economy. I provide an economic mapping to the reduced-form relations in the AFTM through a closed economy DSGE model along the lines of Woodford (2003), and Clarida, Gali, and Gertler (1999), following Gallmeyer, Hollifield, and Zin (2005). I introduce *external stochastic habit formation* in preferences to generate an essentially AFTM due to Dai (2003), and I introduce an extra variable into the habit term denoted the *subjective inflation target*, which captures long run swings in the term premium by introducing long run uncertainty about the growth of consumption. As stressed in the introduction, both stochastic habits and the subjective inflation target are modeled in a lax way relatively to the DSGE literature, but from (4), if the model aims to fit empirical bond premia it is of utmost importance to generate a non-zero $\boldsymbol{\lambda}_1$ from the underlying model and these terms together achieve this. CARA preferences, like used in many models within the DSGE framework, as an example only generate a complete affine model, while standard habit formation as in i.e. Campbell and Cochrane (1999) is harder to fit into the AFTM framework.

The following subsections provide the detail of the model, while propositions (1) and (2) summarise the mapping between the AFTM and the macro economy.

[9] Another way to introduce time variation in (4) is through conditional heteroskedasticity, $\Sigma = \Sigma_t$, such that the state-variables in the economy are of the CIR form. However, for these models to be admissible, the market price of risk need to be of the completely affine from, which clearly restricts the sign of risk premia in the economy. See Duffee (2002) and Dai and Singleton (2002) for more on this topic.

3 A simple macro-finance model of the yield curve: *The consumers*

The representative consumer solves the following maximisation problem with respect to labour supply, N_t, and an index of consumption goods, C_t, in which P_t denotes the price level in the economy and λ_t is the Lagrange multiplier on the budget constraint:

$$\begin{aligned}&\max_{C,N} E_t \sum_{t=T}^{\infty} \beta^t E_t \left[U\left(C_t, N_t; Q_t\right)\right] \\ &-\lambda_t \left[P_t C_t + T_t - W_t N_t - \Pi_t + \left(\bar{P}_t \bar{B}_t - B_{t-1}\right)\right]\end{aligned}$$

$\bar{P}_t$ denotes a row vector where each element corresponds to price of a zero-coupon bond each with different maturity. $\bar{B}_t$ is the quantity of these assets held at the end of period t by the consumer such that negative entries correspond to borrowing. P_t denotes the price level in the economy. The budget constraint says the consumer purchases of consumption net of taxes, T_t, cannot exceed the income from labour, $W_t N_t$, profit shares from the production sector, Π_t, and the income from bonds purchased at previous periods maturing at time t, B_{t-1}. The consumer also purchases bonds maturing in period n, $\bar{P}_t \bar{B}_t$.[10] The parameter $0 < \beta < 1$ is the subjective discount factor.

I assume the household has some market power in the wage setting process, and is free to reset wages at the beginning of each period. Consequently the only difference between a standard model with perfect competition on the labour market and this model is the inclusion of a wage mark-up, M_t^W, see e.g., Erceg, Henderson, and Levin (2000) The efficiency conditions for optimal consumption, labour supply, and bond holdings are the following:

$$\lambda_t = \frac{U_{Ct}\left(C_t; Q_t\right)}{P_t} \tag{5}$$

$$U_{Nt} M_t^W = U_{Ct}\left(C_t; Q_t\right) \frac{W_t}{P_t} \tag{6}$$

[10] The budget constraint does not say that the consumer cannot buy or sell non-maturing bonds but does include, say, a 10-year bond hold in 8 years. This can be seen from the following simplified example with only two bonds of different maturity. Consider the following budget constraint for the consumer: $C_t - P_t^1 B_t^1 - P_t^2 B_t^2 = Y_t - B_{t-1}^1 - B_{t-2}^2$. Define $B_{t-1} \equiv B_{t-1}^1 + B_{t-2}^2$ as maturing debt, and define its law of motion as $B_t = B_{t-1}^2 + \Delta B_t$, which allow me to write: $C_t - P_t^1 \Delta B_t - P_t^2 B_t^2 = Y_t - B_{t-1} \Leftrightarrow C_t - P_t^1 \left(B_t^1 + B_{t-1}^2\right) - P_t^2 B_t^2 = Y_t - B_{t-1} - B_{t-2}^2 - P_t^1 B_{t-1}^2$. The total resources are therefore not only this period bonds and maturing bonds, but also the market value of the non-maturing long bonds.

$$0 = -\lambda_t \bar{P}_t + E_t \left[\beta \lambda_{t+1} \bar{P}_{t+1} \right], \forall_n \quad (7)$$

Equation (5) equates the marginal utility of consumption to the shadow value of the flow budget constraint, while (6) is the optimality condition for the consumption/leisure choice. (7) is the Euler equation for optimal consumption through time.

I assume the following functional form for the utility function:

$$U(C_t, N_t) = \frac{C_t^{1-\gamma} Q_t - 1}{1-\gamma} - \frac{\upsilon}{1+\varphi} N_t^{1+\varphi} \quad (8)$$

in which $\gamma > 0$, determines the coefficient of relative risk aversion, $\varphi > 0$, governs the Frisch labour supply, and $\upsilon > 0$. Q_t denotes a preference shock or an *external stochastic habit* term due to Dai (2003) and introduced into the macroeconomic literature by Gallmeyer, Hollifield, and Zin (2005) to generate state dependent prices of risk, $\boldsymbol{\lambda_1} \neq 0$ in (4). The habit-term depends on aggregate consumption growth, Δc_{t+1}, defined as the log-change in aggregate consumption such that the consumer takes the habit level as given when maximising utility. I extend the preference shock with a *subjective inflation target*, $\bar{\pi}_t$. The (log) law-of-motion for the preference shock, $q_t \equiv \log(Q_t)$, is assumed to have the following form:

$$\begin{aligned} -\Delta q_{t+1} = {} & \phi_c \Delta c_t \left(\Delta c_{t+1} - E_t [\Delta c_{t+1}] \right) + \phi_{\bar{\pi}} \left(\bar{\pi}_{t+1} - E_t [\bar{\pi}_{t+1}] \right) \\ & + \frac{1}{2} \phi_{\bar{\pi}}^2 Var_t (\bar{\pi}_{t+1}) + \frac{1}{2} (\phi_c \Delta c_t)^2 Var_t (\Delta c_{t+1}) \end{aligned} \quad (9)$$

The first term in (9) makes current marginal utility dependent upon past consumption growth in the economy, Δc_t, and introduces habit formation in preferences. The parameter ϕ_c is negative such that, in the spirit of habits, high consumption yesterday coupled with a positive consumption shock today increases marginal utility today. The second term in (9) captures the consumers' aversion to inflation such that a positive shock decreases the utility of a given level of consumption in a state with high inflation relative to a state with low and stable inflation. $\phi_{\bar{\pi}}$ is thus positive.

Habits are the key to fit moments of risk premia in this model. As explained in section 2.1, without Q_t the utility-function would collapse into the standard CARA form widely applied in asset pricing literature but with modest success, see e.g., Cochrane (2001), and would give a completely AFTM unable to generate time variation in risk premia. A deeper analysis of this proposition and of the terms in the preference shock necessitates a look at the production side of the economy introduced next.

4 A simple macro-finance model of the yield curve: *The production sector*

The modeling of the production sector follows the standard DSGE paradigm closely. Appendix (10.1) provides the details of production side while this section only introduces the final equations.

A final goods producer assembles intermediate goods into a final good, while the intermediate goods producers face monopolistic competition and Calvo pricing. A producer which is free to reset prices each period sets its price, $p_t(i)$, as a markup $M^M \equiv \frac{\theta}{(\theta-1)}$ over nominal marginal cost, $MC_t \equiv W_t$

$$p_t(i) = M^M W_t$$

The parameter θ governs the price elasticity of demand for the good and is assumed to be strictly above one. Let small case letters denote the log of the variable. Real marginal cost can be substituted out with the output gap, x_t, defined as the difference between actual output and the equilibrium level of output under flexible prices and a constant *wage* mark up, M^W,

$$mc_t = (\varphi + \gamma) x_t + m_t^W = -m_t^M \tag{10}$$

in which mc_t, m_t^M denotes the log linearisations of the real marginal costs and the desired markup around their steady state respectively.

I denote u_t a *cost-push shock* as in Clarida, Gali, and Gertler (1999) representing deviations from (10) and embodies real imperfections that generate a time-varying gap between output and its efficient counterpart. These inefficient fluctuations in the wage process stems from variation in the wage mark-up from wage setting consumers and includes distortions from taxes, wage bargaining power etc.

The pricing framework gives rise to a New-Keynesian Phillips curve as a function of marginal costs, and constitutes the aggregate supply side of the model:

$$\begin{aligned} \pi_t &= \kappa mc_t + \beta E_t[\pi_{t+1}] + u_t \qquad (11) \\ \kappa &\equiv \frac{(1-\alpha)(1-\beta\alpha)}{\alpha} \end{aligned}$$

in which $\pi_t \equiv \log(P_{t+1}/P_t)$ denotes inflation in the economys' price level and α is the probability that a firm can reset its price at time t, see Calvo (1983).

5 Risk premia, habits and the subjective inflation target

The following sections explain how bond risk premia arise in this model in general and the role of habits and the subjective inflation target in particular all for given prices, while the last section provides an economic motivation behind the inclusion of the subjective inflation target into preferences. To do this, I start by defining a general asset pricing framework and then I move on to consider asset prices as implied by the macroeconomic model in this paper.

5.1 General asset pricing framework

Arbitrage free asset prices with the future uncertain pay off ζ_t, $P(\zeta_t)$, are from (7) given by the linear pricing relation:

$$P(\zeta_t) = E_t\left[M_{t+1}\zeta_{t+1}\right] \tag{12}$$

M_{t+1} is the pricing kernel which from (5) and (7) equals the representative consumers' intertemporal marginal rate of substitution

$$\bar{P}_t = E_t\left[\underbrace{\beta\frac{\lambda_{t+1}}{\lambda_t}}_{\equiv M_{t+1}}\bar{P}_{t+1}\right] \tag{13}$$

The gross risk free rate, R_t^f, equals the inverse of the conditional expectation of the stochastic discount factor:

$$R_t^f = \left(E_t\left[M_{t+1}\right]\right)^{-1} \tag{14}$$

The price of a zero coupon bond maturing in n-periods follows from (12) the recursion:

$$P_t^n = E_t\left[M_{t+1}P_{t+1}^{n-1}\right] = E_t\left[M_{t+1}...M_{t+n}\right] \tag{15}$$

Through the definition $\frac{1}{P_t^1} = R_t^f$ and the no-arbitrage restriction in (15), state variables which affect the gross risk free rate of interest also affect prices of bonds with longer maturities. From (3), if this state variable generates a non-zero correlation between the payoff of the asset and the discount factor, the state-variable further generates a non-zero risk premia in bonds. I will use these two observations to explain how habits and the subjective inflation target works in this economy.

5.2 Market price of risk

A main determinant of the correlation between the stochastic discount factor and the payoff of the assets is from (4) the market price of risk. One main task in the macro-finance literature is to identify these from the macroeconomic model. The stochastic discount factor, M_{t+1}, equals in a complete market setting contingent claims prices scaled by true probabilities, Π^P. Define the risk neutral probabilities, Π^Q, as

$$\frac{\Pi^Q}{\Pi^P} = R_t^f M_{t+1} \tag{16}$$

The left hand side of (16) thus gives the *Radon-Nikodym derivative*.[11] Given normally distributed shocks and positive probabilities that sum to one, the left-hand side equal:

$$\frac{\Pi^Q}{\Pi^P} = \exp\left\{-\tilde{\mathbf{\Lambda}}_t \mathbf{\Sigma} \boldsymbol{\varepsilon}_{t+1} - \frac{1}{2}\tilde{\mathbf{\Lambda}}_t' \mathbf{\Sigma}\mathbf{\Sigma}' \tilde{\mathbf{\Lambda}}_t\right\} \tag{17}$$

$\boldsymbol{\varepsilon}_{t+1}$ denotes a vector of structural shocks in the economy and $\tilde{\mathbf{\Lambda}}_t$ is the market price of risk which I thus can identify from (14), (13), and (16) up to functional form assumptions about the utility function

$$\frac{M_{t+1}}{E_t\left[M_{t+1}\right]} = \exp\left\{-\tilde{\mathbf{\Lambda}}_t \mathbf{\Sigma} \boldsymbol{\varepsilon}_{t+1} - \frac{1}{2}\tilde{\mathbf{\Lambda}}_t' \mathbf{\Sigma}\mathbf{\Sigma}' \tilde{\mathbf{\Lambda}}_t\right\}$$

5.3 Model specific asset pricing framework

Turning to the model specific pricing framework, the stochastic discount factor is derived from (5) and (7):

$$M_{t+1} = \beta\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\left(\frac{P_t}{P_{t+1}}\right)\left(\frac{Q_{t+1}}{Q_t}\right) \tag{18}$$

I can under log-normality write (14) and (18) as follows:

$$\begin{aligned} m_{t+1} &\equiv \log\left(M_{t+1}\right) = -\delta - \gamma\Delta c_{t+1} - \pi_{t+1} + \Delta q_{t+1} \\ i_t &\equiv \log\left(R_t^f\right) = -E_t\left[m_{t+1}\right] - \frac{1}{2}Var_t\left(m_{t+1}\right) \end{aligned} \tag{19}$$

in which $\delta \equiv -\log(\beta)$ is the steady state real interest rate, and Δc_{t+1} denotes consumption growth. (19) hides a Fisherian relationship in which the second term, $\frac{1}{2}Var_t\left(m_{t+1}\right)$, captures precautionary savings.

[11] See e.g., Duffie (1996).

Assumptions about the economy need to ensure the expectation of the right hand side of (17) is one, as it is a probability measure, and such that the market price of risk is linear, a *restriction* imposed by the AFTM. I impose conditional independence between shocks to consumption growth and shocks to the subjective inflation target due to the AFTM restriction. The variance terms in (9) make sure that $E_t[\Delta q_{t+1}] = -\frac{1}{2}Var_t(\Delta q_{t+1})$ such that the probabilities sum to one. Hence, the variance terms figures in (9) such that the model can generate term premia, while it is relatively hard to come up with a convincing economic explanation to why they should affect preferences directly though both consumption variability and inflation variability can not be thought of a being especially pleasant for consumers.

The following two sections use this framework to explain the economics behind habit and the subjective inflation target, taken their presence into preferences as given, and especially their impact upon the non-risky part in the yield curve, defined as expectations of future risk free rates, and the risky part defined through (4).

5.4 The economics behind *external stochastic habits*

To help gain intuition behind the preference shock and the terms therein, see (9), I assume temporarily that consumption growth can be written into a linear stationary process[12]

$$\Delta c_{t+1} = \rho_c \Delta c_t + \sigma_c \varepsilon^c_{t+1} \tag{20}$$

in which $\varepsilon^c_{t+1} \sim N(0,1)$, $\rho_c \in (-1,1)$. I, for ease of exposition, consider only the real part of the stochastic discount factor

$$m^R_{t+1} = -\delta - \gamma \Delta c_{t+1} - \phi_c \Delta c_t \sigma_c \varepsilon^c_{t+1} - \frac{1}{2}(\phi_c \Delta c_t \sigma_c)^2 \tag{21}$$

The marginal utility of consumption tomorrow is in this framework given by:

$$\left(\frac{\partial U(C_{t+1})}{\partial C_{t+1}}\right)_{m_{t+1}=m^R_{t+1}} = \beta C^{-\gamma}_{t+1} Q^C_t \exp\left(-\phi_c \Delta c_t \varepsilon^c_{t+1} - \frac{(\phi_c \Delta c_t \sigma_c)^2}{2}\right) \tag{22}$$

in which $Q^C_t \equiv Q^C_{t-1} \exp\left[\frac{(\phi_c \Delta c_{t-1} \sigma_c)^2}{2} + \phi_c \Delta c_t \varepsilon^c_{t+1}\right]$

Habit is the opposite of durability: Consumption today raises the marginal utility of consumption tomorrow. A shock to consumption

[12] This will up to a constant be so in the general model. See section 6.

through the stochastic habit term, $-\phi_c \Delta c_t \varepsilon^c_{t+1}$, is a direct shock to the consumers' marginal utility of consumption, like a preference shock in a non-habit model. The difference is that the effect of a preference shock in this model depends upon consumption in the previous period such that if the consumer came from a state with a booming economy, a positive consumption shock will raise the marginal utility of consumption tomorrow, and if the boom was big, so is the consumers' appetite for consumption. Risk premia arises in response to uncertainty, so it is the *stochastic* dependence of the valuation of marginal utility tomorrow and consumption today that generates risk premia, as the consumer does not know for sure how payoffs of the asset is valued in the periods to come.

5.4.1 Risk premia and habits

I note from (1) and (2) the coefficient B_{n-1} can be read off from (20). Let B^c_n denote this coefficient in (1) and write $B^c_{n+1} = B^c_n \left(\rho_c - \sigma_c \phi_c\right) - \delta^c_1, \;\; \delta^c_1 \equiv \gamma \left(\rho_c - \phi_c \sigma^2_c\right)$. The market prices of risk can be found from (17), such that the risk premium in a model under the assumptions (20) and (21) is:[13]

$$
\begin{aligned}
E^c_t \left[hpr^n_{t+1}\right] &\approx -B^c_{n-1}\sigma^2_c cov_t \left(-\gamma\varepsilon^c_{t+1} - \phi_c \Delta c_t \varepsilon^c_{t+1}, \varepsilon^c_{t+1}\right) \\
&\approx B^c_{n-1}\sigma^2_c \underbrace{\gamma}_{\equiv\lambda^c_0} + B^c_{n-1}\sigma^2_c \underbrace{\phi_c \Delta c_t}_{\equiv\lambda^c_1 X_t}
\end{aligned} \tag{23}
$$

The first term in (23) visualises the CARA part of the risk premia cannot generate any non-zero standard deviation in bond risk premia, but can only generate a maturity specific premia through B^c_{n-1}. This constant risk premia is increasing in γ. Positive risk premia are attached to risky payoff if the consumer receives positive payoffs when the consumer does not want them. That is, when the consumer already enjoys a high level of consumption thus increasing consumption variability. The more the consumer dislike this consumption variability, the higher γ is, the more the consumer want to avoid assets that pays off well when the consumer already feels wealthy, and such an asset consequently demands a higher premium for the consumer hold it. Looking at A_n and B_n in (1), λ_0 affects the long run mean of the yields under the Q-measure, and consequently the first moment of risk premia.

The stochastic habit term, Q_t, introduces state dependent risk premia through the second term in (23), and thus a potential for fitting second order moments of term premia. From (22), with habits, the valuation of payoffs, extra consumption in the period to come, depends on

[13] The approximation arises due to the ignorance of the Jensen term.

consumption in the previous period, and thence does the risk premia attached to the risky asset. From (22) and (23), the coefficient ϕ_c plays the dual role of determining the sensitivity of the representative consumers' risk aversion to the current growth rate of aggregate consumption, the market price of risk, as well as the dependence between the marginal utility of consumption and previous level of consumption growth.

5.4.2 Habits and the risk free rate of interest

I now turn to the relationship between habits and the expectations part in bond yields consisting of risk free rates of interest, which in this simplified framework is given by:

$$i_t^c = \delta + \gamma E_t \left[\Delta c_{t+1}\right] - \frac{\gamma^2}{2}\sigma_c^2 - \gamma \phi_c \sigma_c^2 \Delta c_t \tag{24}$$

The last term in (24), $\gamma \phi_c \sigma_c^2 \Delta c_t$, arises through the stochastic habit term giving rise to a non-zero correlation between consumption growth today and tomorrow. Holding consumption today and expected consumption tomorrow constant, an increase in consumption at time t increases the marginal utility of consumption today making the consumer to borrow from the future, driving up the risk free rate. Habits thus introduce persistence in the risk free rate and, through (15), further out in the yield curve.

The first three terms have standard interpretations. The short-rate is increasing in the time preference, δ, and expected consumption growth, $\gamma E_t \left[\Delta c_{t+1}\right]$, as the consumer has an incentive to try to borrow to reduce the discrepancy between consumption today and in the future, while the short-rate depends negatively on the precautionary savings term, $\frac{\gamma^2}{2}\sigma_c^2$, as the consumer build a bulk ward against swings in consumption in terms of higher savings.

5.5 The economics behind *the subjective inflation target*

The subjective inflation target is assumed to follow the stationary stochastic process

$$\bar{\pi}_{t+1} = \theta_1 + \rho_{\bar{\pi}} \bar{\pi}_t + \rho_{\pi}^{\bar{\pi}} \pi_t + \sigma_{\bar{\pi}} \varepsilon_{t+1}^{\bar{\pi}} \tag{25}$$

in which $\varepsilon_{t+1}^{\bar{\pi}} \sim N(0,1)$, $\rho_{\bar{\pi}}, \rho_{\pi}^{\bar{\pi}} \in (-1,1)$ and θ_1 is a constant term. Exogenous shocks to the subjective inflation target, $\varepsilon_{t+1}^{\bar{\pi}}$, can arise due to, say, greater political pressure upon the central bank governor to let inflation rise and keep growth high. $\bar{\pi}_t$ is not an inflation target for the monetary authority as it does not explicitly enters into any Taylor-rule nor explicitly in any central bank loss function as in e.g., Graeve, Emiris, and Wouters (2008) and Hordahl, Tristani, and Vestin (2006). However, the presence of the term $\rho_{\pi}^{\bar{\pi}}\pi_t$ into (25) implies that, say, tighter monetary policy is able to decrease the subjective inflation target.

I assume that $\rho_{\bar{\pi}}$ is close to one consequently introducing a unit-root process for the representative consumers' subjective inflation target. Credibility takes time to build and the heart of credibility is that it is long lasting.[14] Further, though persistent, movements in the subjective inflation target should a priori not affect aggregate variables in the economy to a great extend at business-cycle frequencies, and I impose this restriction upon the model as well.

I assume the following dynamics for the cost-push shock in (11)

$$u_{t+1} = \mu_u + \rho_u u_t + \rho_{\bar{\pi}}^u \bar{\pi}_t + \sigma_u \varepsilon_{t+1}^u \tag{26}$$

in which $\varepsilon_{t+1}^u \sim N(0,1)$, $\rho_u, \rho_{\bar{\pi}}^u \in (-1,1)$, and μ_u is a constant term, such that inflation depends upon the subjective inflation target. From (11) aggregate supply depends upon a cost-push shock embodying real imperfections which arise from inefficient variations in wages. I let these exogenous variations in wages be dependent upon the subjective inflation target consequently assuming that the subjective target affects labour market imperfections. I follow Gali (2008) by assuming these real imperfections arise exogenously and so does this interdependence between the labour market and long run inflation expectations. The intuition is that the wage setter can only set nominal wages but cares about the number of goods the wage is able to purchase. The subjective inflation target embeds expectations of the level of inflation thus making the decision for the wage setter dependent upon this variable.[15,16]

[14] A highly persistent process for an inflation target is likewise supported by data, see e.g., Dewachter and Lyrio (2006), Hordahl, Tristani, and Vestin (2006), and Bekaert, Cho, and Moreno (2006).

[15] The empirical literature on the interrelationship between the labour market and monetary policy regime is scarce. Abildgren (2008) provides evidence for a relationship between the monetary policy regime and the length of labour contracts for the case of Denmark such that stable regimes provides longer labour contracts. Fregert and Jonung (1998) and Fregert and Jonung (2006) provide similar evidence for the case of Sweden.

[16] The subjective inflation target thus figures explicitly in the New Keynesian Phillips curve, (11), and through this relation also in the reaction function for the central bank, see section 6. In this respect, the model does not differ much from the existing literature, see e.g., Graeve, Emiris, and Wouters (2008), Hordahl, Tristani, and Vestin (2006).

5.5.1 The macroeconomic effects of a subjective inflation target shock

The subjective inflation target causes *persistent uncertainty* about consumption growth in the economy, and this uncertainty stems from the subjection of monetary policy credibility. To see this, reintroduce the nominal part of the stochastic discount factor and, as in section 5.4, calculate the marginal utility of consumption tomorrow:

$$\left(\frac{\partial U\left(C_{t+1}\right)}{\partial C_{t+1}}\right)_{m_{t+1}=m_{t+1}} = \left(\frac{\partial U\left(C_{t+1}\right)}{\partial C_{t+1}}\right)_{m_{t+1}=m_{t+1}^{R}} * \exp\left(-\phi_{\bar{\pi}}\sigma_{\bar{\pi}}\varepsilon_{t+1}^{\bar{\pi}} - \frac{\left(\phi_{\bar{\pi}}\sigma_{\bar{\pi}}\right)^{2}}{2}\right)$$

The macroeconomic implications of a one-period shock to the inflation target are twofold. Firstly, the subjective inflation target shock decreases utility of a given level of consumption, as consumers do not like inflation. That is, the consumer enjoys consumption more if the consumption takes place in a state with stable perceptions of future inflation, as higher perceptions of future inflation is a signal of low marginal growth at some point in the future through monetary policy even though the current level of output in the economy is high with a low level of inflation.

Secondly, an exogenous shock to the subjective inflation target introduces an indirect affect on consumption growth that runs through the cost-push shock which increases through $\rho_{\bar{\pi}}^{u}$ in the period to come. This higher cost-push shock in turn pushes inflation up through (11). Monetary policy combats this shock to inflation raising the real rate tilting the path of intertemporal consumption through the IS-curve.[17] The higher level of inflation further pushes up the subjective inflation target through $\rho_{\bar{\pi}}$ in (25) repeating the effects. The subjective inflation target is close to be a unit process, $\rho_{\bar{\pi}} \approx 1$, so these effects are persistent, and hence causes uncertainty about long run consumption growth.

5.5.2 The asset pricing effects of a subjective inflation target shock

The macro economy is the flip-side of the financial market, so the macroeconomic affects of a one period shock to the subjective inflation target should also turn up in bond yields, and they do. To gain insight into the functioning of the subjective inflation target, assume at this stage that

[17] See section 6.

inflation can be written directly in terms on the cost-push shock:[18]

$$\pi_{t+1} = \rho_\pi \pi_t + \rho_{\bar{\pi}}^{\pi} \bar{\pi}_t + \sigma_\pi \varepsilon_{t+1}^{\pi}$$

in which $\varepsilon_{t+1}^{\pi} \sim N(0,1), \rho_\pi, \rho_{\bar{\pi}}^{\pi} \in (-1,1)$. Through (17) the market price of subjective inflation target risk are $\lambda_0^{\bar{\pi}} = \phi_{\bar{\pi}}$ and $\lambda_1^{\bar{\pi}} = 0$. The subjective inflation target thus affects the long run mean of the yields under the Q-measure. The more the consumer dislike inflation, the higher $\phi_{\bar{\pi}}$ is, the more the marginal utility of a given level of consumption varies in response to subjective inflation target shocks. The interpretation of the parameter $\phi_{\bar{\pi}}$ is consequently that it measures the representative consumers' level of risk aversion to the current subjective inflation target shock. As in (23), positive risk premia are attached to risky payoff if the consumer gets positive payoffs when the consumer do not want them, that is when marginal utility is low. The subjective inflation target shock decreases marginal utility, so the bond market is willing to pay a price to get rid of payoff that materialises when the subjective inflation target is high.

This is the *direct* effect on risk premia from the subjective inflation target but not the only one in a model with habits. The subjective inflation target creates persistent uncertainty in consumption growth, and consumption growth is from (23) the key determinant of the time varying part of the risk premia. Hence, the subjective inflation target through habits affects variations in the long run level of risk premia.[19]

Lastly, the short-rate can in the full model be written as:

$$i_t = i_t^c + E_t[\pi_{t+1}] - \frac{1}{2} Var_t(\pi_{t+1}) - \gamma cov_t(\Delta c_{t+1}, \pi_{t+1}) \qquad (27)$$

in which the third and forth terms combined constitutes inflation risk premia. Hence, the subjective inflation target only indirectly influences the short-rate through its effects upon consumption growth and inflation.

[18] This will up to a constant be so in the general model. See section 6.

[19] The specification of the subjective inflation target and the external stochastic habits is akin to the long run risk specification in Bansal and Yaron (2004) with one main difference: Bansal and Yaron (2004) specify a specific structure upon the consumption endowment, while I specify the growth rate of marginal utility of consumption. Bansal and Yaron (2004) model the growth rate of consumption to be dependent upon a small persistent predictable component, while fluctuations in economic uncertainty is modeled through time-varying volatility. The authors argue their model is able to explain a various financial markets puzzles like the equity premium puzzle.

5.5.3 A different interpretation of subjective inflation target and stochastic preferences

The previous sections explained the economic mechanism between risk premia, stochastic habits and the subjective inflation target. This section aims to motivate the inclusion of the subjective inflation target into preferences. I argue in this section that there exists an equivalence between the inclusion of the subjective inflation target into preferences and into the budget constraint following Feenstra (1986). Inflation surprises erode the real value of outstanding debt and wages, and as such provide an explicit explanation for inflation aversion.

I show in appendix 10.5 how the budget constraint can be rewritten into

$$c_t^r - \sum_{i=1}^{n} r_{t-i}^i b_{t-i}^{ri} = \frac{W_t N_t + \Pi_t - T_t}{P_t} - \sum_{i=1}^{n} \left(b_t^{ri} - b_{t-i}^{ri}\right) - \sum_{i=1}^{n} \left\{ \left(1 + r_{t-i}^i\right) \frac{\left[\sum_{j=0}^{i-1} \left(\pi_{t-j} - \pi_{t-j}^e\right)\right]}{\left[\prod_{j=0}^{i-1} \left(1 + \pi_{t-j}\right)\right]} b_{t-i}^{ri} \right\} \tag{28}$$

in which superscript r denotes variables deflated by the price level in the economy, P_t. I have introduced $r_{t-j} \equiv \left[\prod_{j=0}^{i-1} \left(1 + \pi_{t-j}^e\right)\right]^{-1} \left(1 + Y_{t-j}^j\right)^j - 1$ to denote the ex ante return from $t - j$ to t, in which $\left(1 + Y_{t-j}^j\right)^j$ denotes the j-period bond yield and π_{t-j}^e denotes expected inflation. The last term in (28) highlights the way unanticipated inflation can change income in that period.

Consider now condition (5) from the consumers maximisation problem repeated here for convenience

$$\frac{U_{Ct}\left(\bar{C}; Q_t\right)}{P_t} = \lambda_t \tag{29}$$

The lagrange multiplier, λ_t, can be given the interpretation as the shadow value of an extra unit of income. A positive subjective inflation target shock decreases the left hand side of (29) and therefore must the shadow value of an extra unit of income decrease as well, given the price level in the economy.

Now assume a model in which the subjective inflation target shock affects inflation in (28) directly such that a positive shock erodes the real value of bond debt. This is turn raises income pushing down the shadow value of extra income and the right hand side of (29) falls. Consumption must therefore rise *or*, as in this model, the inflation shock must decrease the marginal utility of a given level of consumption reestablishing the equality sign in (29).

Lastly, I also stress the similarity between models with money in the utility function, see e.g. Gali (2008), and a model with a subjective inflation target shock included into preferences. In money-in-the-utility models, the consumers' utility is increased by an increase in money holdings holding constant the path of real consumption for all t, and that is so even though money holdings are never used to purchase consumption. Putting money in the utility function is a useful shortcut for ensuring a demand for money. There are many models for the effects of inflation upon the well-being for the consumers like shoe-leather costs, menu costs, inflation surprises eroding purchasing power through falling real wages etc. Including an subjective inflation target shock in the preference shock is a useful shortcut for ensuring the consumer faces aversion to higher inflation, and as for money-in-the-utility function, it is just a shortcut, and as money-in-the-utility models introduces a demand for money in a simple and tractable way, the inclusion of the subjective inflation target directly as a preference shock captures inflation aversion in a simple and tractable way.

6 A simple macro-finance model of the yield curve: *Monetary policy*

The Central Bank solves the problem of minimising the expected, discounted present value of future output gaps and inflation with respect to its policy instrument, the short-rate of interest, i_t, given aggregate demand and aggregate supply

$$\min_{(i)} \frac{1}{2} E_t \left[\sum_{i=0}^{\infty} \beta^i \left(\Gamma x_{t+i}^2 + \pi_{t+i}^2 \right) \right]$$

in which Γ represents the weight of output gap fluctuations in the loss function for the central bank.

Aggregate supply is given by (11). Appendix 10.2 derives from (5) and (7) the following IS-relation, a relationship between consumption growth, and the short real interest rate, which constitute the demand side in this economy:

$$\begin{aligned}(1-\phi_c Var_t(\Delta c_{t+1}))\,x_t = & \, E_t[x_{t+1}] - \phi_c Var_t(\Delta c_{t+1})\,x_{t-1} \\ & -\frac{1}{\gamma}(i_t - E_t[\pi_{t+1}]) - \frac{\delta}{\gamma} - \frac{1}{\gamma}\Omega_{VAR} - \frac{1}{\gamma}\Omega_{COV} \\ & +\Delta z_{t+1} - \phi_c \Delta z_t Var_t(\Delta c_{t+1})\end{aligned} \tag{30}$$

I have defined

$$\begin{aligned}\Omega_{VAR} & \equiv \frac{1}{2}Var_t(\pi_{t+1}) + \gamma^2 \frac{1}{2} Var_t(\Delta c_{t+1}) \\ \Omega_{COV} & \equiv cov(-\pi_{t+1}, \gamma \Delta c_{t+1}) + cov(-\pi_{t+1}, \Delta q_{t+1})\end{aligned}$$

in which the latter two definitions denotes conditionally constant covariance and variance terms respectively. The inflation terms combined constitute the inflation risk premium discussed in section 5.5. Δz_t, denotes the change in the natural rate of output less relative government spending interpreted as a demand shock and follows the following stochastic process

$$\Delta z_{t+1} = \theta_2 + \rho_z \Delta z_t + \sigma_z \varepsilon^z_{t+1}$$

in which $\varepsilon^z_{t+1} \sim N(0,1)$, $\rho_z \in (-1,1)$, and θ_2 denotes a constant. (30) is the flip-side of the short-rate of interest presented in (5.4) and (5.5) and the same intuition applies namely that habits introduces persistence into the consumers demand for goods in the economy represented by the term $\phi_c Var_t(\Delta c_{t+1})$.

Following e.g. Clarida, Gali, and Gertler (1999) the solution to the central bank problem is given by:

$$\pi_t = \Gamma \frac{1}{(\kappa(\varphi+\gamma))^2 + \Gamma(1-\beta\rho_u)} u_t \equiv \Upsilon u_t \tag{31}$$

$$x_t = -\frac{\kappa(\varphi+\gamma)}{(\kappa(\varphi+\gamma))^2 + \Gamma(1-\beta\rho_u)} u_t \equiv -F u_t \tag{32}$$

in which Υ and F are constants. $x_t = -F u_t$ reflects a central bank that pursues a leaning against the wind policy as in Clarida, Gali, and Gertler (1999) such that a positive cost-push shock that pushes inflation above target makes the central bank to raise interest rates and thus lower the output gap. Hence, under discretion, the central bank lets the output gap and inflation to vary in proportion to the value of the cost-push shock in this period.

The macroeconomic side of the model thus consists of the New-Keynesian Phillips curve, (11), the IS-relation, (30), and central bank policy responses, (31) and (32). The next section derives bond prices consistent with this macroeconomic model.

7 A simple macro-finance model of the yield curve: *The financial market*

Appendix 10.2 shows that consumption growth can be written as a sum of the change in the output gap, Δx_{t+1}, and the demand shock, Δz_t:

$$\Delta c_{t+1} = \Delta x_{t+1} + \Delta z_{t+1} \tag{33}$$

Use (31) and (32) to rewrite inflation and growth in consumption:

$$\pi_t = \Upsilon u_t, \ \Delta c_{t+1} = -\digamma \Delta u_{t+1} + \Delta z_{t+1}$$

Shocks to u_t affect inflation and I will in view of this interpret the cost-push shock as a nominal shock though the shock originates in the real economy.

The state variables in this economy, $\mathbf{X}_{t+1}$, are thenceforward the cost-push shock, the demand shock, and the subjective inflation target shock, which together have the following dynamics consistent with the discussion in section 5.5:[20]

$$\mathbf{X}_t \equiv \begin{pmatrix} \bar{\pi}_{t+1} \\ \Delta z_{t+1} \\ u_{t+1} \\ u_t \end{pmatrix} = \underbrace{\begin{pmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_3 \end{pmatrix}}_{\equiv \boldsymbol{\mu}} + \underbrace{\begin{bmatrix} \rho_{\bar{\pi}} & 0 & -\digamma \rho_{\bar{\pi},u} & 0 \\ 0 & \rho_z & 0 & 0 \\ \theta_{\bar{\pi}} & 0 & \rho_u & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{\equiv \boldsymbol{\rho}} \begin{pmatrix} \bar{\pi}_t \\ \Delta z_t \\ u_t \\ u_{t-1} \end{pmatrix}$$

$$+ \underbrace{\begin{bmatrix} \sigma_{\bar{\pi}} & 0 & 0 & 0 \\ 0 & \sigma_z & 0 & 0 \\ 0 & 0 & \sigma_u & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{\equiv \boldsymbol{\Sigma}} \underbrace{\begin{pmatrix} \varepsilon^{\bar{\pi}}_{t+1} \\ \varepsilon^{z}_{t+1} \\ \varepsilon^{u}_{t+1} \\ 0 \end{pmatrix}}_{\equiv \boldsymbol{\varepsilon}_{t+1}}, \ \varepsilon_{t+1} \backsim N\left(0_{(4,4)}, I_{(4,4)}\right) \tag{34}$$

The following proposition summarises the mapping between the AFTM and the underlying macroeconomic model.

Proposition 1 *The structural economy admits the following expressions for the stochastic discount factor, market prices of risk, and the (short) nominal rate of interest:*

$$m_{t+1} = -i_t - \frac{1}{2}\boldsymbol{\Lambda}_t^{'}\boldsymbol{\Sigma}\boldsymbol{\Sigma}^{'}\boldsymbol{\Lambda}_t - \boldsymbol{\Lambda}_t^{'}\boldsymbol{\Sigma}\boldsymbol{\varepsilon}_{t+1}$$

[20] Note that $\rho^{\bar{\pi}}_{\bar{\pi}}$ in the simplyfied framework in (25) equals $-\digamma \rho_{\bar{\pi},u}$ in the full model. Likewise, the parameter $\rho^{u}_{\bar{\pi}}$ equals $\theta_{\bar{\pi}}$.

$$i_t = \delta_0 + \boldsymbol{\delta}_1' \mathbf{X}_t \tag{35}$$

The coefficient in the short-rate equation is given by:

$$\delta_0 \equiv \left[\begin{array}{c} \delta + \gamma (1 - \rho_z) \tilde{\theta}_2 + \Upsilon \left[(1 - \rho_u) \tilde{\theta}_3 - \tilde{\theta}_1 \theta_\pi \right] \\ -\frac{1}{2} \gamma^2 \sigma_z^2 - \frac{1}{2} (-\gamma F + \Upsilon)^2 \sigma_u^2 \end{array} \right]$$

$$\begin{aligned} \boldsymbol{\delta}_1' &\equiv \left[\delta^{\bar{\pi}}, \delta^{\Delta z}, \delta^u, 0 \right]' \\ \delta^{\bar{\pi}} &= \theta_{\bar{\pi}} (\Upsilon - \gamma F) \\ \delta^{\Delta z} &= \gamma \rho_z + \gamma \phi_c \sigma_z^2 + \phi_c F (\gamma F - \Upsilon) \sigma_u^2 \\ \delta^u &= \left[(-\gamma F + \Upsilon) \rho_u + \gamma F - \gamma \phi_c F \sigma_z^2 - \phi_c F^2 (\gamma F + \Upsilon) \sigma_u^2 \right] \end{aligned}$$

The market prices of risk are given by:

$$\boldsymbol{\Lambda}_t = \boldsymbol{\lambda}_0 + \boldsymbol{\lambda}_1 \mathbf{X}_t \tag{36}$$

$$\boldsymbol{\lambda}_0 = \left[\phi_{\bar{\pi}}, \gamma, (\gamma F + \Upsilon), 0 \right]'$$

$$\boldsymbol{\lambda}_1 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \phi_c & -F\phi_c & F\phi_c \\ 0 & -F\phi_c & F^2\phi_c & -F^2\phi_c \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \bar{\pi}_t \\ \Delta z_t \\ u_t \\ u_{t-1} \end{bmatrix}$$

The elements in the $\boldsymbol{\lambda}_1$-matrix bridge two explanations for the expectations hypothesis puzzle. McCallum (1994) shows how monetary policy can break the expectations hypothesis while Dai and Singleton (2002) contributes the empirical evidence for the puzzle to time-variation in risk premia, and both are right in this framework. The expectations hypothesis does not hold because of time-variation in risk premia, but this time variation is not something that arises out of the blue. It is an equilibrium response to monetary policy through the policy parameter F.

It is intuitive why central-bank responses to variation in the output gap influence risk premia. Risk premia arises due to the covariance between growth in marginal utility and the payoff of the asset, see (4). The representative consumers' marginal utility growth depends on variation in the output gap, so the central bank response to variations in this variable therefore must play a role for market price of risk.

As explained in section 5.4 and 5.5, the expression for the nominal short-rate is a Fisherian relationship minus inflation risk premia and precautionary savings terms. The parameter $\theta_{\bar{\pi}}$ governs the affect from the subjective inflation target onto the short-rate, so a small value for this parameter is consistent with an subjective inflation target that, through its near unit-root behaviour, affects the economy in the long-run and not on business-cycle frequencies.

The processes in this economy, (34), the short-rate, (35), and the market price of risk, (36), are affine. Hence, only the measure change from the observed probability measure to the equivalent martingale measure is needed to apply the bond price solution in (1). Using the Girsanov theorem, see e.g. Duffie (1996), p. 288, I get:

$$\begin{aligned}
\mathbf{X}_{t+1} &= \boldsymbol{\mu} + \boldsymbol{\rho}\mathbf{X}_t + \boldsymbol{\Sigma}\boldsymbol{\varepsilon}_{t+1} \Leftrightarrow \\
\mathbf{X}_{t+1} &= \boldsymbol{\mu} + \boldsymbol{\rho}\mathbf{X}_t + \boldsymbol{\Sigma}\left(\boldsymbol{\varepsilon}^{Q}_{t+1} - \boldsymbol{\Lambda}_t\right) \Leftrightarrow \\
\mathbf{X}_{t+1} &= \underbrace{(\boldsymbol{\mu} - \boldsymbol{\Sigma}\boldsymbol{\lambda}_0)}_{\equiv\boldsymbol{\mu}_Q} + \underbrace{(\boldsymbol{\rho} - \boldsymbol{\Sigma}\boldsymbol{\lambda}_1)}_{\equiv\boldsymbol{\rho}_Q}\mathbf{X}_t + \boldsymbol{\Sigma}\boldsymbol{\varepsilon}^{Q}_{t+1}
\end{aligned}$$

Subscript Q denotes the mean term, $\boldsymbol{\mu}_Q$, autoregressive matrix, $\boldsymbol{\rho}_Q$, and shocks to the economy, $\boldsymbol{\varepsilon}^{Q}_{t+1}$, under the Q-measure.

Proposition 2 summarises the closed form solution for bond prices in this economy:[21]

Proposition 2 *The state-dynamics, the pricing kernel, and the market prices of risk imply an Essentially Affine model for bond prices given by (1) and (2)*

Proof. See appendix (10.3) ■

The bond price solutions in proposition 2 are useful both because they avoid approximations but also because they allow a deeper understanding of the determination of risk premia in terms of their macroeconomic driving forces. I exploit both points in the following.

8 Analysing the model

The macro-finance model is next solved, calibrated and simulated. I analyse risk premia through a decomposition of the yield curve into

[21] It might seem odd that the time-varying part of the market prices of risk in proposition 2 does not depend on shocks to, say, inflation. One reason behind this is that in this case the proposition would no longer hold, as the state-dynamics under the Q-measure would no longer be linear due to the presence of, say, $\phi_\pi \pi_t \phi_c \Delta c_t$. Proposition 2 thus restricts the model to some extent, and the natural trade-off in macroeconomic modeling between simplicity and model properties arises.

an expectations part, ES_t^n, and a term premia part, TP_t^n, in which $f_t^n \equiv p_t^n - p_t^{n-1}$ is the forward rate:

$$y_t^n = \frac{1}{n}\sum_{i=0}^{n-1} f_t^i = \underbrace{\frac{1}{n}\sum_{i=0}^{n-1} E_t\left[i_{t+i}\right]}_{\equiv ES_t^n} + \underbrace{\frac{1}{n}\sum_{i=0}^{n-1}\left(f_t^i - E_t\left[i_{t+i}\right]\right)}_{\equiv TP_t^n} \tag{37}$$

The term $f_t^n - E_t\left[i_{t+n}\right]$ has a portfolio interpretation: Borrow for a year in the future spot market and contract today to lend in the forward market. The difference is the expected excess return or risk premia.[22]

Section 8.1 defines a good fit of the model as a model that fits empirical first and second order moments of risk premia, consumption, inflation, and the short and long rate of interest. Section 8.2 discusses the calibration of the model. Section 8.3 is devoted to a deeper analysis of the model.

8.1 A definition of a good fit for the model

The key research question in this paper is whether a macroeconomic model is able to generate term premia in the range of 160 bp. per year and an unconditional standard deviation of around 50 for a bond with 10 years to maturity. These numbers are averages of estimates for term premia from nominal yield data for the U.S. from the beginning of the 80s until the present found in Bernanke, Reinhart, and Sack (2005), Kim and Wright (2005), Rudebusch and Wu (2004), and Cochrane and Piazzesi (2006), ranging from 106 bp. to 210 bp. for the mean and 17-69 bp. for the unconditional standard deviation. The relatively large differences between these estimates reflect that estimation of risk premia is not trivial and the literature lacks a widely accepted method and model, see e.g. the discussion in Cochrane and Piazzesi (2006). I will bear this caveat in mind when I evaluate the fit of the model.

I also evaluate the model against macroeconomic data in terms of the mean and the unconditional standard deviation of the macroeconomic variables in the model. One lesson from the previous section is that market prices of risk are endogenously determined from the consumers' marginal rate of substitution between consumption today and tomorrow.

[22] TP_t^n is also equal to a sequence of holding period returns

$$\frac{1}{n}\sum_{i=1}^{n-1}\left(f_t^i - E_t\left[i_{t+i}\right]\right) = \frac{1}{n}\sum_{i=1}^{n-1} E_t\left[hpr_{t+i}^{n+1-i}\right]$$

so the analysis in (4) holds. It is however easier to implement TP_t^n using forward rates rather than using holding period returns.

Therefore, risk premia varies only when the macroeconomic variables that determines this rate of substitution varies. This point haunts the calibration of the DSGE models as almost all the parameters in the models must fit both a property on the macroeconomic side as well as on the financial side of the model.[23] It is thus not sufficient to evaluate the model against financial data. Macroeconomic data must be included as well.

Table 1 summarises the empirical moments which the model aims to fit

	Mean	**Standard Deviation**
Term Premium	162	52
Short Rate of Interest	657	296
Long Rate of Interest	761	228
Consumption	172	328
Inflation	300	300

Table 1: *This table shows moments of macroeconomic variables and bond risk premia. Moments are calculated from U.S. data from the beginning of the 80s until the present. The calculation of the moments for the term premium is explained in the text. The moments of the short interest rate, a 1-year bond, and the long interest rate, a 10-year bond, are calculated from the data in Gurkaynak, Sack, and Wright (2006). The moments for the (log) consumption growth rate are from Campbell, Lo, and MacKinlay (1997). The moments for inflation are from Hordahl, Tristani, and Vestin (2006). Everything is measured in basis points per year.*

Standard deviations, though problematic, are a good measure for the fit of the macroeconomic model in this paper, as excess variability in the aggregate economy is likely to be the result if the calibration boots term premia by boosting standard deviations and/or risk parameters. However, the macroeconomic model still needs to be determinate and provide impulse response functions that resemble closely those found in comparable models in the DSGE literature, see e.g., Woodford (2003) and Gali (2008).

[23] As an example, from (4) the key to fit the financial moments is the parameters that determines risk, say $\phi_{\bar{\pi}}, \phi_c$ or γ in this model, together with the standard deviations on the shocks in the economy, $\mathbf{\Sigma}$. A boost to these parameters is likely to induce implausible behaviour for the macro economy, as higher standard deviations and risk parameters leads to a more volatile behaviour of the macroeconomic variables, as both the New-Keynesian Phillips curve, (11), and the IS-relation, (30), depend on the same parameters as risk premia.

Further, though the utility function includes stochastic habits, γ is still both the coefficient of relative risk aversion and the inverse of the intertemporal rate of substitution, so boosting γ clearly has profound implications for the response from short-rates to consumption growth.

8.2 Calibration

The model is analysed, calibrated, and solved using Uhlighs toolkit, see Uhlig (1995). Table 2 presents the calibrated parameters. I impose near

Calibration			
Parameter	*Value*	**Parameter**	*Value*
β	0.9875	$\rho_u^{\overline{\pi}}$	0.001
α	0.79	ρ_z	0.9
γ	1.5	ρ_u	0.8
φ	1.36	$\rho_{\overline{\pi}}^u$	-0.15
Γ	0.175	$\rho_{\overline{\pi}}$	0.99
$\phi_{\overline{\pi}}$	10	$\theta_{\overline{\pi}}$	0.001
ϕ_c	-30	θ_z	0.00043
$\sigma_{\overline{\pi}}$	0.001065	θ_u	-0.0000051
σ_z	0.00115	σ_u	0.001

Table 2: *This table shows the values of the calibrated parameters. One period in the model corresponds to one quarter. The model is calibrated to US data.*

unit-root behaviour for the subjective inflation target, $\rho_{\overline{\pi}} \approx 1$. $\rho_u^{\overline{\pi}}$ is small and negative such that the central bank is able to influence the subjective inflation target but only slowly through time. The calibration of the parameters β, φ, α resemble values used in the DSGE literature. $\theta_{\overline{\pi}}$ is small in line with the discussion in section 5.5. The calibration of Γ implies a determinate macroeconomic model obeying the Taylor principle, see e.g. Woodford (2003). Both the intertemporal rate of substitution, $\frac{1}{\gamma}$, and the standard deviations are set to rather conservative values, and are no higher than what is used in the existing DSGE literature.

The calibration of the constant term in (34), $\boldsymbol{\mu}$, ensures the state-variables, $\mathbf{X}_t$, and the short-rate, i_t, converge to long-run means as found in data. The endpoints, V_t^{∞}, of a process, V_t, are the limiting conditional forecasts of the process, $V_t^{\infty} \equiv \underset{j\to\infty}{E_t} \left[V_{t+j}\right]$. The end points of $\mathbf{X}_t$ thus equal:

$$\mathbf{X}^{\infty} = \left[\mathbf{I} - \boldsymbol{\rho}\right]^{-1} \boldsymbol{\mu} = \left[-\frac{\theta_3}{\theta_\pi} - \frac{1}{F} \frac{\theta_1}{\theta_\pi \rho_{\pi,u}} \left(\rho_u - 1\right) \frac{\theta_2}{1-\rho_z} \; \frac{1}{F} \frac{\theta_1}{\rho_{\pi,u}} \, \theta_3 + \frac{1}{F} \frac{\theta_1}{\rho_{\pi,u}} \right]'$$

The end point for risk free rate equals:

$$\underset{j\to\infty}{E_t} \left[i_{t+j}\right] = \delta_0 + \boldsymbol{\delta}_1' \mathbf{X}^{\infty} \tag{38}$$

and the end-point for the individual terms in the term premia, TP_t^{∞}, equals:

$$TP_t^{\infty} \approx \boldsymbol{\delta}_1' \left\{ \left(I - \boldsymbol{\rho}_Q\right)^{-1} \boldsymbol{\mu}_Q - \left(I - \boldsymbol{\rho}\right)^{-1} \boldsymbol{\mu} \right\} \equiv \boldsymbol{\delta}_1' \left(\mathbf{X}_Q^{\infty} - \mathbf{X}^{\infty}\right)$$

Conditional forecasts clearly play a key role both for the ES_t^n-, and TP_t^n-part in (37) so the calibration of $\mathbf{X}^{\infty}$ is not innocuous.

I calibrate $\boldsymbol{\mu}$ such that the end-point of the subjective inflation target is 300 bp. per year and such that the end-point of the demand shock, Δz_t, equals 172 bp. per year, which is the average growth rate of consumption from table 1. The calibration implies an endpoint of the risk free rate of around 700 bp. per year, which is within the estimates in table 1.[24]

8.3 Results

8.3.1 The Bond Premium Puzzle is not a puzzle

The key result of this paper is stated in table 3 consisting of first and second order moments obtained from simulations of the model: *The Bond Premium Puzzle is not a puzzle.* The mean of the term premia is

	Mean	**Standard Deviation**
Term Premium	186 *(162)*	17 *(52)*
Short Rate of Interest	505 *(657)*	317 *(296)*
Long Rate of Interest	858 *(761)*	18 *(228)*
Consumption	-	247 *(328)*
Inflation	-	355 *(300)*

Table 3: *This table shows model simulated moments for key variables in the economy. The numbers in brackets are the numbers from table (1) repeated here for convenience.*

close to the goal of 162 bp. from table 1. The model gives a somewhat small standard deviation of the term premia though it is a lot bigger than what the literature so far has found, and it does reside within the interval for empirical estimates of unconditional standard deviations of

[24] Gallmeyer, Hollifield, and Zin (2005) introduces an exogenous preference shock into their habit term. I could have included such a shock into my model and through its constant term have obtained an extra free parameter. However, economic theory tells very little about the value of this parameter and such a parameter should therefore neither be a key determinant for the level of the term structure nor term premia. The inflation target in contrast explicitly provides a constant parameter with a clearer economic interpretation.

term premia. As can be seen from the standard deviations for all the macroeconomic variables, the risk free rate, consumption, and inflation, the model achieve this with a good fit of the macroeconomic side of the model.

The model does have troubles with the fit of the standard deviation of the long interest rate, which is off its empirical counterpart by a factor 12. This is a general problem in the literature, see e.g., Den Haan (1995) or Gurkaynak, Sack, and Swanson (2005) which denote this the *excess volatility puzzle.* The underlying problem is stationary and ergodic shorts rates which shows up in almost any DSGE model, implying convergence to a constant too fast to account for the empirical volatility in the long-end of the yield, see e.g. Atkeson and Kehoe (2008), and the same problem prevail in this model, see figure 3. The calibration in table 2 ensures that this constant, which in the AFTM equals (38), has an empirical plausible value, but the convergence towards it is too quick.

It is thus natural to analyse whether a different calibration is able to generate more variability in bond yields in this model. The task is to generate sufficient persistence in the short-rate and next to ensure a sufficient amount of its variability is transmitted to long rates. Boosting standard deviations is likely to raise the unconditional standard deviations of the TP_t^n-part in (37), which is low in the model, and I comment on this as well.

		Moments of Term Premia and Yields					
Parameter	*Value*	$\mathrm{E}\left[TP_t^{40}\right]$	$\mathrm{Var}\left[TP_t^{40}\right]$	$\mathrm{E}\left[y_t^4\right]$	$\mathrm{Var}\left[y_t^4\right]$	$\mathrm{E}\left[y_t^{40}\right]$	$\mathrm{Var}\left[y_t^{40}\right]$
ρ_z	*0.99*	-62	20	≈500	166	-100	49
γ	*bs*						
ρ_z	*0.99*	-57	75	≈500	222	-500	217
γ	*3*						
ρ_z	*0.9*	-35	10	≈500	275	-270	75
γ	*5*						
ρ_u	*0.85*	2000	16	≈500	176	2890	34
γ	*3*						

Table 4: *This table shows the effect on moments of term premia and yields of changing the calibration of key parameters for determination of second order moments of term premia and bond yields. bs means base-line value. Everything is measured in basis points per year.*

Table 4 only consists of key moments for yields and term premia but the picture is clear from this partial view of table 1. Take the volatility of the long end of the yield curve, $Var_t\left(y_t^{40}\right)$. I increase the autoregressive parameters in $\boldsymbol{\rho}$ to make the convergence to the end-points of both the risk free rate and the processes in the economy, i_t^{∞}, $\mathbf{X}_t^{\infty}$, slower. More persistent demand - and cost-push shocks, higher ρ_z and ρ_u, is not the solution, which can be seen from the first and last row of table 4 in which the persistence of these shocks are increased from 0.9 to 0.99 and from 0.8 to 0.85 respectively.

I next increase the coefficient of relative risk aversion, γ, from its base-line calibration of 1.5 to 3-5. The model *can* generate higher volatility of the term premia and long bond yields by a combination of doubling γ and by imposing near unit-root behaviour for the shocks, $\rho_z \approx 1$, see row 2 and 3 in table 4. The problem is that these changes in the calibration make first moments of bond yields and risk premia become more and more negative in the long end of the curves.

Hence, the model fares well in terms of matching the moments in table 1, but the model is too simple to generate more volatility in the long end of the yield curve without destroying the fit of first moments of both term premia and yields.

8.3.2 A deeper look at the macroeconomic model: *Impulse response functions*

Figure 1 shows the impulse response functions for the macroeconomic variables from a shock to each of state-variables in the economy. The same second-order moments can be obtained with different models that have different implications for the interrelationship among macroeconomic variables. The three figures together show the dynamics in this model resembles that of a basic DSGE model, see e.g., Woodford (2003).

The top figure in figure 1 is a visualisation of the theoretical discussion in section 5.5: The subjective inflation target causes long run uncertainty in consumption growth. A shock to the subjective inflation target raises inflation through its effect upon the cost-push shock in (11). This causes a monetary policy tightening and consumption falls, see (30). The effects are small at business-cycle frequencies, the subjective inflation target can only affect cost-push shocks, inflation and thus risk free rates by $\theta_{\bar{\pi}}$, see section 7, but persistent as I have imposed a near unit-root behaviour for the subjective inflation target, $\rho_{\bar{\pi}} \approx 1$, see table 3.

The impulse response function of a demand shock, figure at the bottom to the left in figure 1, shows a similar pattern as in the basic DSGE model: Higher demand now raises consumption, see (30), which put pressure on inflation through the output gap in (11), which feeds back to higher interest rates in the response function for the central bank.

The figure to the bottom right in figure1 shows the effects of a cost-push shock upon the macro economy. Two effects arise. The first effect corresponds to what is found in the basic DSGE model: Higher inflation is combated by the central bank raising the policy rate, see (31), which in turn depresses consumption growth through higher real interest rates, see (30).

The second effect of a cost-push shock works through the subjective inflation target as discussed in section 5.5. The cost-push shock pushes the subjective inflation target upwards as the cost-push shock increases inflation which signals a breach of the monetary policy target of zero, see 6. This causes the real rate to be a little higher and consumption growth a little lower, and these effects are highly persistent due to the near unit-root behaviour of the subjective inflation target. This effect visualises the cost in terms of lower consumption growth of a central bank that has lost credibility and pushed the economy into an inflationary state with higher inflation expectations.

8.3.3 A deeper look at the yield curve dynamics: *Simulated yields*

Figure 2 shows the steady-state and simulated decomposition of (37) for a 1-year and a 5-year bond, and two snap-shots of simulated decompositions. Figure 2 is a visual test of the yield curve model. I want to generate the moments for risk premia in table 1, but I do not want to do this at the expense of the yield curve dynamics.[25]

The plots in figure 2 all verify that not only is the model able to fit the numbers in table 1, but it is also able to generate yield curve dynamics as observed in post-war data, see e.g. Piazzesi (2003). The yield curve is on average upward sloping, the ES_t^n-part is on average slightly upward sloping, and the TP_t^n-part increases with maturity (top left figure). Further, the bottom plots show the model is able to generate different shapes of the yield curve, hump-shaped, flat, and steeply upward sloping, and even a downward sloping curve though on few occasions.

The top-right figure visualises the problems for this model of introducing sufficient variation in the long end of the yield curve. 5 year bond yields vary considerable less than 1-year yields, and this decline in variation as the maturity of the bonds increases is clearly due to the inability of the this model to generate sufficient variation in expectations

[25] Figure 2 does not focus on second order moments like conditional heteroskedasticity. All the factors in this economy are Gaussian and bond yield are thus homoscedastic. Hence, the model provides little hope in fitting conditional second order moments. See also Dai and Singleton (2000).

of the future state of the economy epitomised by a less variable ES_t^n for n increasing.

8.3.4 What affects the first moment of risk premia? *Slope and level effects*

This section analyses the determination of the first moment of the TP_t^n-part in (37) by looking at the underlying driving forces behind the decomposition of the yield curve complementing the partial equilibrium analysis in section 5.4 and 5.5. Figure 3 shows the response of the yield curve decomposition at time t to an annualised standard deviation shock to each of the factors.

Movements in the subjective inflation target does not introduce a lot to movements in the yield curve at *business-cycle* frequencies, as a standard deviation shock to the subjective inflation target only raises the yield curve by around 10 bp. (top-right figure), but moves yields of all maturities approximately equally and thus generates a level effect on the yield curve.

Two things in the model explain the movements. Firstly, as discussed in the section 8.3.2, the subjective inflation target shock only has a small effect upon macroeconomic variables. Secondly, the unit-root behaviour implies that subjective inflation target shocks do not die out quickly. This is reflected in a small but flat factor loading, B_n, on the yield curve from a shock to the inflation target, see top-left figure in figure 3. The subjective inflation target shock therefore affects yields of all maturities approximately equally and the effects work through both terms in (37). The ES_t^n-part is affected by the shock through the loading on the risk free rate, $\delta^{\bar{\pi}}$, due to the subjective inflation targets effect on inflation affecting the risk free rate. Intuitively, higher nominal yields are expected for all maturities if the level of inflation is higher. The TP_t^n-part is affected by the market price of risk, $\phi_{\bar{\pi}}$, generating a positive term premia curve reflecting a lower marginal utility of the same level of consumption turning into a negative correlation between consumption and bond returns and thus positive premia through the curve, see section 5.4. The market price of subjective inflation target risk is of the completely affine form, which, as explained section 5.4, affects the mean of the yield curve under the Q-measure providing a slope effect from the TP_t^n-part in the decomposition. Complementing the analysis from section 5.5, uncertainty about the subjective inflation target in 10 years time is greater than the uncertainty about the subjective inflation target tomorrow, so investors demand a larger risk premium from holding subjective inflation target risk in long term bonds than in short terms bonds.

Figure 4 shows the response of the yield curve decomposition through time to an annualised standard deviation shock to each of the factors. The near unit-root implies the subjective inflation target captures a stochastic trend in bond yields and risk premia, so although the effects of a shock to the subjective inflation target are small at business-cycle frequencies, as shown in figure 3, they continue through time, see top figure in figure 4, and can as such provide an explanation to the decline in both yields and term premia from the Volker period until the present low levels in terms of a series of shocks to the subjective inflation target that combined have pushed the level downwards.[26] Notice also from the bottom figures in figure 4 that the subjective inflation target is the sole determinant of risk premia and bond yields at longer horizons, as the effects of demand shock and cost-push shock die out relatively quickly.

However, the empirical numbers are not fully supported by the model as 25 annual standard deviation shocks to the subjective inflation target only can explain a 0.5 percent decline in risk premia and not 3 as found in data. The underlying explanation, however, is supported theoretically. As emphasized by Cochrane and Piazzesi (2006), term premia should be zero or negative in an environment with stable inflation dynamics, which in term of the model says, when the subjective inflation target is low and has not been subject to shocks in the near past. In this environment risk arises from variable real rates, and term premia should only be positive in an environment with unstable inflation, as rolling over short-term bonds runs the reinvestment risk that the short term real rate has changed. The explanation to the decline in risk premia given from this model framework *is* exactly that the US-economy today reside in an environment with more stable inflation dynamics relatively to the period in the beginning of the 80s' epitomised by the lower subjective inflation target and thus better anchoring of inflation expectations.[27]

The bottom left figure in figure 3 shows that a positive demand shock depresses term premia, while the shock increases the ES_t^n-part. Empirical evidence, e.g., Fama and French (1989) and Fama and French (1996), and theory, Cochrane (2007), says that risk premia should vary

[26]The 10-year bond yield stood at around 12 percent in the beginning of the eighties with an estimated term premia of around 3-5 percent and as I write, these figures are around 5 percent and -2-2 percent respectively. See e.g. figure 4 in Rudebusch, Sack, and Swanson (2007) for a nice plot of estimates of term premia in bond yields.

[27]Empirical estimates of inflation targets, see e.g., Hordahl, Tristani, and Vestin (2006), Dewachter, Lyrio, and Konstantijn (2006), Graeve, Emiris, and Wouters (2008), finds a more rapid decline of an inflation target from the Volker period to the present. This supports the explanation for the decline in yields and term premia given in this paper, though it also implies that to fit the empirical evidence, the subjective inflation target shock in this model should have a bigger effect upon yields.

with the business-cycle. Risk premia should be low on the top of the business-cycle when the average investor feels less risk averse and high on the bottom of a recession. A demand shock is a good proxy for *good times*, see (33), and good times in this model means low term premia, see section 2.1. Expectations of future risk free rates rise through the loading of the demand shock on the risk free rate, $\delta^{\Delta z} \approx \gamma\rho_z$, reflecting the consumer spreads the demand through time and consume less now, invest more, and consume more in the future, which in turn pushes up interest rates. The more the consumer cares about consumption variability, the higher γ is, the stronger is the effect on interest rates from a demand shock.

The last plot in figure 3 shows a cost-push shock almost only affects expectations of future short-rates and not term premia. Like the demand shock, the cost-push shock affects the slope of the yield curve. The cost-push shock affects the risk free rate by δ^u reflecting the shock affects both inflation, see (31), and the output gap, see (32), and thus risk free rates though the Fisherian relationship in (27). The risk free rate increases as the Central bank obeys the Taylor principle when conducting monetary policy thus raising the risk free rate by more than the change in inflation.

The numbers are interesting. During the previous three monetary policy tightening cycles in the U.S. prior to the last one - 1988-1989, 1994-1995, and 1999-2000 - the 10-year bond yield increased by 26 basis points for every 100 basis point increase in the Federal target. Recall that in this model the central bank reacts to the cost-push shock. Figure 3 shows that the cost-push shock give rise to a 100 basis point increase in the short-rate and a 24 basis point increase in the long-end. Hence, disregarding the last period of policy tightening, the model seems to fit the U.S. yield curve response to a monetary policy shock well.

I infer from figure 3 and figure 4 the underlying determinant of term premia in the 10-year bond yield on business-cycle frequencies is almost only demand-shocks while both demand shocks and cost-push shocks determine the expectations part in the 10 year yields. 10-year yields and 10-year term premia are in the medium to long run anchored by a sum of subjective inflation target shocks. The underlying determinants of risk premia are, as explained in section 2.1, standard, but the effects are bigger in this model, as I do not need to apply higher order approximations.

Lastly, I recall that I have introduced unconditional correlations between the inflation target and the cost-push shock under both the Q and P measure, see section 7, so the yield curve dynamics at business-cycle frequencies could be due to these correlations and not solely cost-push shocks and/or demand shocks. To analyse this, I simply exclude the inflation target from the model above and do the same analysis as shown in this section. The results are shown in appendix 10.4 and tells that the conclusions from this section hold: The yield curve behaviour at business-cycle frequencies are *not* due to shocks to the subjective inflation target while the determinant of the yield curve at long horizons *can* be contributed to the subjective inflation target.

8.3.5 What affects the second moment of risk premia? *Variance decomposition*

This section analyses the underlying forces that generates the second-order moments of risk premia. I explain below two findings about the long end of the yield curve. Firstly, fluctuations in long nominal interest rates are due mostly to fluctuations in term premia, and secondly, variation in term premia stems from nominal shocks.

The top-right figure in figure 5 shows the variance decomposition of the yield curve into the contribution of the ES_t^n-part and the TP_t^n-part of (37). 90 per cent of the movements in the long end of the yield curve are due to movements in the TP_t^n-part. On the other hand, the short end of the yield curve moves mostly because expectations about future short-rates changes.[28]

The top-left and bottom figures in figure 5 shows the variance decomposition of each term in the total decomposition, the ES_t^n-part and the TP_t^n-part, into the relative contribution of the individual factors in $\mathbf{X}_{t+1}$. Figure 5 answers how each factor contributes *both* to the total variation in the yield curve *and* to the individual terms in the decomposition.

The subjective inflation target shock does not affect the variability of yields significantly, which does not come as great surprise keeping the discussion in section 8.3.4) in mind. The ES_t^n-part is almost equally influenced by real shocks and nominal shocks through the yield curve, while the relative importance of the real and nominal shock changes with the maturity of the yields in the TP_t^n-part of (37). Real shocks do play a role for the variability of the short-end of the TP_t^n-curve (25 per cent), but the variability of the long-end of this curve is almost solely determined by nominal shocks (90 per cent).

The findings above are supported in the empirical literature. Risa (2001) finds the variability of a 10 year bond is mostly explained by inflation risk premia and term premia not due inflation. Ang, Bekaert,

[28] Figure 2 foresee this result by showing a decline of the variability in the ES_t^n-part maturity increases.

and Wei (2008) finds that around 80 percent of variation in the long end of the yield curve can be explained by a sum of expected inflation and inflation risk premia. Further, Beechey (2007) finds that for 10-15 year nominal bond yields, three quarters of the reaction to macroeconomic news announcements is due to movements in the TP_t^n-part. All these points amount in this model to a mixture of movement in the TP_t^n-part possibly due to cost-push shocks. Regarding the short end of the yield curve, Risa (2001), Beechey (2007), and Ang, Bekaert, and Wei (2008) find that movements in one year bonds can be attributed to movements in real rates and expected inflation, which in the model economy amounts to movements in the ES_t^n-part and demand shocks, which was what I also found above.

To rephrase the findings from figure 3 and 5, the central bank moves short-rates in response to inflation, see (31), and movements in the output gap, see (32). Long rates are expectations of future risk free rates and a term premium, see (37). Though the central bank can stabilise short-run inflation and output to a certain degree it can not move risk free rates indefinitely, so long rates must reveal something about long run inflation expectations. That is what I found in figure 3. This section relates fluctuations around this long run level to fluctuations in term premia, the top-right figure in figure 5, due to nominal shocks, the last three figures in figure 5, while fundamentals, that is the ES_t^n-part, are stable relatively to TP_t^n-part.

8.4 An application of the model: *The bond yield conundrum and risk premia*

The Fed began to tighten monetary policy from mid 2004 to mid 2006 but saw the yield on a 10-year bond fell. This was at that time denoted a *bond yield conundrum* (BYC) by the then chairman, Alan Greenspan. Can this simple model provide an explanation behind this behaviour of the long end of the yield curve during the BYC period? I suggest the answer is yes.

The low 10-year bond yield has partly been contributed to unusually low term premia, see Kim and Wright (2005), Rudebusch, Sack, and Swanson (2007), Wu (2008). A low term premia in this model is consistent with a series of subjective inflation target shocks driving term premia towards zero. But such an explanation is not sufficient to explain the behaviour of 10-year bond yields during this period, as the BYC period saw not only low and possibly negative term premia but also an almost zero correlation between short and long interest rates. This stands in stark contrast to the periods of monetary tightening prior to the BYC period, see section 8.3.4. Other factors must have played its part during this period unless a very big shift in long run inflation expectations occurred through the monetary policy tightening.

The macro-finance model in this paper says the difference between the BYC period and prior periods to a difference between the *types* of shocks that hit the economy. Figure 3 predicts the shocks that hit the economy prior to the BYC period where cost-push shocks raising the long-end by 24-26 basis-points. The shocks that hit the U.S. economy during the BYC period, however were demand shocks and possibly further subjective inflation target shocks, as demand shocks are the source to a large, temporary negative shock to term premia only raising the long-end of the yield curve by 12 basis points.[29]

Hence, the BYC period does not seem to be at odds with the model in this paper, and was in light of this model not really a conundrum. The behaviour of the long-end of the yield curve during the BYC period was an equilibrium reaction to a combination of demand shock(s) and credibility gains for the central bank in its conduct of monetary policy in terms of a series of negative subjective inflation target shocks.

9 Conclusion

This paper analyses the interaction between bond risk premia and macroeconomics. The key contribution is that the bond premium puzzle is not puzzling if the model provides closed form solutions for bond prices shying away from higher-order approximations. The key steps to generate plausible bond risk premia are firstly to use the empirically successful affine models originated in finance, and from a macroeconomic model, to map the parameters in the affine models' reduced form relations into a structural framework. The model provides an economic interpretation of the decline in term premia and yields observed during the previous three decades in terms of series of unit-root shocks to the consumers subjective inflation target in the economy.

This paper has addressed the macroeconomic determinants of risk premia, but as financial markets are the flip side of the macro economy, risk premia should also reveal interesting aspects of consumer behaviour. One such example is the cost of business-cycles. Lucas (2003) famous paper argues that these costs are small, but as emphasized by Cochrane (2007), asset prices reveals risk aversion, and asset prices say that there are a considerable amount of risk aversion, and hence, the

[29] Graeve, Emiris, and Wouters (2008) derive and estimate a macro-finance model for the yield curve and also explain the conundrum by demand shocks.

cost of macroeconomic fluctuations can potentially be large. This paper provides realistic term premia and thus risk aversion, and therefore has the potential to address this question possibly through a second-order approximation of the utility function as in Rotemberg and Woodford (1997). I will leave this question for future research.

Lastly, suspicion can easily arise when somebody is able to provide an answer to an economic puzzle by introducing more degrees of freedom. As stressed by Zin (2002), assumptions made about preferences in a model should be *reasonable*, but as Zin points out:

> we are not yet at the stage where there is a consensus about what types of preference assumptions are reasonable, [Zin (2002)].

Wachter (2002) provides an answer to what is reasonable: *The model must be parsimonious.* Wachter argues that a parsimonious model is a model in which the number of phenomena to be explained is much greater than the number of free parameters.

Seen in this light, I must therefore not evaluate the success of this paper on the ability of fitting term premia moments, as I introduce the parameters to fit exactly these. I must instead evaluate my model against what I do not calibrate the parameters to. As an example, I do not calibrate any parameter to fit counter-cyclical risk premia nor to fit the average response of long bond rates to a tightening of monetary policy. But that is what I get.

10 Appendix

This appendix derives the aggregate supply side, appendix (10.1), the IS-relation, appendix (10.2), and closed-form solution for bond prices, appendix (10.3).

10.1 Appendix: The aggregate supply side

The production sector in this model is quite standard and follows closely e.g., Woodford (2003) and Gali (2008), which I refer to for details.

Final goods are assembled by the following production function:

$$C_t = \left[\int_{\Omega} c_{ct}(i)^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}$$

The final goods producers minimise costs solving the following problem:

$$\min \int_0^1 p_t(i)\, c_t(i)\, di \ st. \left[\int_{\Omega} c_{ct}(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}} \geq C_t$$

The demand for the intermediate good is given by

$$c_{ct}(i) = C_t \left(\frac{p_t(i)}{P_t} \right)^{-\theta}$$

The price index follows from these equations,

$$P_t = \left[\int_{\Omega} p_t(i)^{1-\theta} \right]^{\frac{1}{1-\theta}}$$

Labour is used as input to produce intermediate goods according to the linear production function in which technology is normalised to one: $y_t(i) = N_t(i)$, where $N_t(i)$ denotes labour input into firm i. With labour free to flow across firms, with constant returns to scale production function, the wage rate will be equalised across firms in equilibrium, and all firms will have the same nominal marginal cost of production, $MC_t = \frac{W_t}{\frac{\partial F(N_t)}{\partial N_t}} = W_t$, and will employ the same amount of workers, $N_t(i) = N_t$. The profit function for the intermediate producer can therefore be written as:

$$\begin{aligned} \pi &= p_t(i)\, y_t(i) - W_t N_t(i) \\ &= Y_t \left(\frac{p_t(i)}{P_t} \right)^{-\theta} (p_t(i) - W_t) \end{aligned}$$

The demand curve follows from the optimal demand for the consumers of the individual good in the indices. The problem for the intermediate firm is to maximise profits with respect to its price, $p_t(i)$. The first order condition for the producer which is free to reset prices equals:

$$p_t(i) = \frac{\theta}{(\theta-1)} W_t = M^M MC_t$$

Deviations from this optimality condition under completely flexible prices can arise due to nominal rigidities introduced next.

Intermediate good firms face Calvo pricing such that $1-\alpha$ of the producers in the sector are allowed to be reset in each period, while α of the prices remain unchanged, following Woodford (2003) and Yun (1996). Each supplier that can chose a new price for its good at time t faces the same problem as anybody else who can change its price at that date. The optimal price, p_t^*, for the individual firm that can change its price will in equilibrium equal the price chosen by the rest. The pricing index for this sector is therefore given by $P_t = [(1-\alpha) p_t^{*1-\theta} + \alpha P_{t-1}^{1-\theta}]^{\frac{1}{1-\theta}}$. The intermediate goods producer takes into account the dynamic aspect of the price setting, and maximises the present value of his profits discounting these profits by the stochastic discount factor.

$$\begin{aligned} &\max_{p_t(i)} E_t \left[\sum_{T=t}^{\infty} \alpha^{T-t} M_{t,T} \Pi(.) \right] \\ = &E_t \left[\sum_{T=t}^{\infty} \alpha^{T-t} M_{t,T} \left[\left(\frac{p_t(i)}{P_t} \right)^{-\theta} \frac{Y_t}{P_t} (p_t(i) - W_t) \right] \right] \end{aligned}$$

The producer specifically set his price such that the expected mark-up equals the desired mark-up:

$$0 = E_t \left[\sum_{T=t}^{\infty} \alpha^{T-t} M_{t,T} \left[\begin{array}{c} \frac{Y_t}{P_t} \left(\frac{p_t(i)}{P_t} \right)^{-\theta} (1-\theta) \\ +\theta \frac{Y_t}{P_t} \left(\frac{p_t(i)}{P_t} \right)^{-\theta-1} \frac{W_t}{P_t} \end{array} \right] \right] \tag{39}$$

Together with the price index, (39) determines the evolution of the aggregate price given the evolution of output and the disturbances in the economy, and these equations thus constitute the aggregate-supply block of the production sector. I notice that the habit term drops out as its expectation equals one for all time periods.

A log linearisation of (39) yields:

$$p_t = (1-\alpha\beta) E_t \left[\sum_{i=0}^{\infty} (\alpha\beta)^i mc_{t+i}^n \right] \tag{40}$$

Log-linearising the price index yields the familiar expression:

$$p_t = \alpha p_{t-1} + (1-\alpha) p_t \tag{41}$$

(40) and (41) can be combined to obtain a New-Keynesian Phillips curve:

$$\begin{aligned}\pi_t &= \kappa mc_t + \beta E_t [\pi_{t+1}] \\ \kappa &\equiv \frac{(1-\alpha)(1-\beta\alpha)}{\alpha}\end{aligned} \tag{42}$$

Marginal cost can be substituted out with the output gap, x_t, defined as the output that would prevail in the economy under flexible prices and a constant *wage* mark up, M^W. The optimality condition for the consumers' labour supply is given by expression:

$$U_N (N_t) M_t^W = U_C (C_t) \frac{W_t}{P_t} \tag{43}$$

This condition, together with the production function, $y_t (i) = N_t (i)$, and the equilibrium conditions, $U_C (Y_t), U_N (N_t), y_t (i) = Y_t$ can be combined to write output of consumption goods as:

$$Y_t^{\varphi+\gamma} = \left(\frac{M_t^w}{\upsilon M^M}\right)^{\frac{1}{\varphi}} Q_t$$

A log linearisation of this expression yields:

$$-m_t^M = (\varphi + \gamma) y_t - \hat{q}_t + m_t^W \tag{44}$$

Marginal cost in can be substituted out with (44) in the previous expression for the Phillips-curve, (42):

$$mc_t = -m_t^M = (\varphi + \gamma) x_t + m_t^W \tag{45}$$

10.2 Appendix: The IS-relation

This section derives the IS-relation in this economy through a log-normal approximation of the stochastic discount factor. Start from the consumers' efficiency condition for the choice of savings/consumption, equations (5) and (7), and use log-normality of the shocks:

$$i_t = -E_t [m_{t+1}] - \frac{1}{2} Var_t (m_{t+1})$$

The mean term is:

$$E_t \log \left[\beta \frac{P_t}{P_{t+1}} \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} \frac{Q_{t+1}}{Q_t}\right] = E_t [\delta - \pi_{t+1} - \gamma \Delta c_{t+1} + \Delta q_{t+1}]$$

The conditional variance terms are:

$$\frac{1}{2}Var_t \log\left[\beta\frac{P_t}{P_{t+1}}\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\frac{Q_{t+1}}{Q_t}\right] = \frac{1}{2}Var_t\left(\delta - \pi_{t+1} - \gamma\Delta c_{t+1} + \Delta q_{t+1}\right)$$

$$\Omega_{VAR} \equiv \frac{1}{2}Var_t\left(\pi_{t+1}\right) + \gamma^2\frac{1}{2}Var_t\left(\Delta c_{t+1}\right)$$

Note that $E_t\left[\Delta q_{t+1}\right] = -\frac{1}{2}Var_t\left(\Delta q_{t+1}\right)$. These variance terms are all constant. The conditional covariance terms are:

$$cov\left(-\pi_{t+1}, \gamma\Delta c_{t+1}\right) + cov\left(-\pi_{t+1}, \Delta q_{t+1}\right) + cov\left(\gamma\Delta c_{t+1}, \Delta q_{t+1},\right)$$

I will later show that the terms $cov\left(-\pi_{t+1}, \gamma\Delta c_{t+1}\right)$ and $cov\left(-\pi_{t+1}, \Delta q_{t+1}\right)$ are constant, and I denote them by

$$\Omega_{COV} \equiv cov\left(-\pi_{t+1}, \gamma\Delta c_{t+1}\right) + cov\left(-\pi_{t+1}, \Delta q_{t+1}\right)$$

These covariance terms potentially involves lagged variables of the state-variable due to the state-dependence of risk. The time-varying term is:

$$\begin{aligned} cov\left(\gamma\Delta c_{t+1}, \Delta q_{t+1}\right) &= -\gamma\phi_c\Delta c_t cov\left(\Delta c_{t+1}, \Delta c_{t+1}\right) = -\gamma\phi_c\Delta c_t Var_t\left(\Delta c_{t+1}\right) \\ &= -\gamma\phi_c\Delta c_t Var_t\left(\Delta c_{t+1}\right) \end{aligned}$$

Rewrite this IS-relation into standard expression introducing the notation introduced above:

$$\begin{aligned} -i_t &= E_t\left[\delta - \pi_{t+1} - \gamma\Delta c_{t+1}\right] \\ &\quad -\gamma\phi_c\Delta c_t Var_t\left(\Delta c_{t+1}\right) + \Omega_{VAR} + \Omega_{COV} \\ &\Leftrightarrow \\ c_t &= E_t\left[c_{t+1}\right] - \frac{1}{\gamma}E_t\left[i_t - \pi_{t+1}\right] \\ &\quad +\phi_c\Delta c_t Var_t\left(\Delta c_{t+1}\right) - \frac{\delta}{\gamma} - \frac{1}{\gamma}\Omega_{VAR} - \frac{1}{\gamma}\Omega_{COV} \end{aligned} \tag{46}$$

Impose equilibrium conditions:

$$\begin{aligned} c_t &= y_t - e_t \\ e_t &\equiv -\log\left(1 - \frac{G_t}{Y_t}\right) \end{aligned}$$

$$\begin{aligned} y_t - e_t &= E_t\left[y_{t+1} - e_{t+1}\right] - \frac{1}{\gamma}\left(i_t - E_t\left[\pi_{t+1}\right]\right) \\ &\quad +\phi_c\Delta c_t Var_t\left(\Delta c_{t+1}\right) - \frac{\delta}{\gamma} - \frac{1}{\gamma}\Omega_{VAR} - \frac{1}{\gamma}\Omega_{COV} \\ y_t &= E_t\left[y_{t+1}\right] - \frac{1}{\gamma}\left(i_t - E_t\left[\pi_{t+1}\right]\right) \\ &\quad +\phi_c\Delta c_t Var_t\left(\Delta c_{t+1}\right) - \frac{\delta}{\gamma} - \frac{1}{\gamma}\Omega_{VAR} - \frac{1}{\gamma}\Omega_{COV} + e_t - E_t\left[e_{t+1}\right] \end{aligned}$$

The log form of the aggregate resource constraint can be written as $c_t = x_t + z_t$, in which x_t is the output gap and z_t is equal to the natural rate of output less government spending.

$$\begin{aligned}
x_t &= E_t\left[x_{t+1}\right] - \frac{1}{\gamma}\left(i_t - E_t\left[\pi_{t+1}\right]\right) + \phi_c \Delta c_t Var_t\left(\Delta c_{t+1}\right) \\
&\quad -\frac{\delta}{\gamma} - \frac{1}{\gamma}\Omega_{VAR} - \frac{1}{\gamma}\Omega_{COV} \\
&\quad +e_t - E_t\left[e_{t+1}\right] - z_t + E_t\left[z_{t+1}\right] \\
&\Leftrightarrow \\
\left(1 - \phi_c Var_t\left(\Delta c_{t+1}\right)\right) x_t &= E_t\left[x_{t+1}\right] - \phi_c Var_t\left(\Delta c_{t+1}\right) x_{t-1} - \frac{1}{\gamma}\left(i_t - E_t\left[\pi_{t+1}\right]\right) + \\
&\quad -\frac{\delta}{\gamma} - \frac{1}{\gamma}\Omega_{VAR} - \frac{1}{\gamma}\Omega_{COV} \qquad (47) \\
&\quad +\Delta z_{t+1} - \phi_c \Delta z_t Var_t\left(\Delta c_{t+1}\right)
\end{aligned}$$

Substitute the reaction of the central bank to inflation to rewrite this expression further. The covariance terms can be written as:

$$\begin{aligned}
\Omega_{COV} &\equiv cov\left(-\pi_{t+1}, \gamma \Delta c_{t+1}\right) + cov\left(-\pi_{t+1}, \Delta q_{t+1}\right) \\
&= F \Upsilon_\pi \sigma_u^2 - \alpha_t F^2 \sigma_u^2
\end{aligned}$$

The covariance term can be written as:

$$\begin{aligned}
\Omega_{VAR} &\equiv \frac{1}{2} Var_t\left(\pi_{t+1}\right) + \gamma^2 \frac{1}{2} Var_t\left(\Delta c_{t+1}\right) \\
&= \frac{1}{2}\Upsilon_\pi^2 \sigma_u^2 + \gamma^2 \frac{1}{2}\left(F^2 \sigma_u^2 + \sigma_{zc}^2\right)
\end{aligned}$$

Rewrite the shock term:

$$\begin{aligned}
&\left[1 - \left(\left(F + \frac{\Upsilon}{\gamma}\right) F \sigma_u^2 - \sigma_z^2\right)\phi_c\right]\left(e_t^c + z_t\right) \\
&-E_t\left[e_{t+1}^c + z_{t+1}\right] \\
&-\left(\left(F + \frac{\Upsilon}{\gamma}\right) F \sigma_u^2 - \sigma_z^2\right)\phi_c\left(e_{t-1}^c + z_{t-1}\right) \\
&= E_t\left[\Delta z_{t+1}\right] - \left(\left(F + \frac{\Upsilon}{\gamma}\right) F \sigma_u^2 + \sigma_z^2\right)\phi_c \Delta z_t \\
&= \left(\rho_z - \left(\left(F + \frac{\Upsilon}{\gamma}\right) F \sigma_u^2 - \sigma_z^2\right)\phi_c\right)\Delta z_t
\end{aligned}$$

The final expression is thus:

$$\begin{aligned}
&\left[1-\left(\left(F+\frac{\Upsilon}{\gamma}\right)F\sigma_u^2-\sigma_z^2\right)\phi_c\right]x_t \\
&= E_t\left[x_{t+1}\right]-\frac{1}{\gamma}\left(i_t-E_t\left[\pi_{t+1}\right]\right) \\
&\quad+\left(\left(F+\frac{\Upsilon}{\gamma}\right)F\sigma_u^2-\sigma_z^2\right)\phi_c x_{t-1} \\
&\quad+\left(\rho_z-\left(\left(F+\frac{\Upsilon}{\gamma}\right)F\sigma_u^2-\sigma_z^2\right)\phi_c\right)\Delta z_t
\end{aligned} \tag{48}$$

10.3 Appendix: Bond prices

This appendix derives closed form solutions for bond prices. The proof follows the proof in Ang and Piazzesi (2003).

First note that the return on a one-period bond must equal the risk free rate:

$$P_t^1 = E_t\left[M_{t+1}\right] = e^{-i_t} = e^{-\delta_0-\boldsymbol{\delta}_1^{'}\mathbf{X}_t}$$

The starting values for the recursions: $A_1 = -\delta_{0,}\mathbf{B}_1 = -\boldsymbol{\delta}_1^{'}$. Assume the price on an n-period bond is given by $P_t^n = e^{A_n+\mathbf{B}_n^{'}\mathbf{X}_t}$. I will now show that the (log) linear form also applies to the price of bonds with other maturities than n:

$$\begin{aligned}
P_t^{n+1} &= E_t\left[M_{t+1}P_{t+1}^n\right] \\
&= E_t\left[\exp\left\{-i_t-\frac{1}{2}\boldsymbol{\Lambda}_t^{'}\boldsymbol{\Sigma}\boldsymbol{\Sigma}^{'}\boldsymbol{\Lambda}_t-\boldsymbol{\Lambda}_t^{'}\boldsymbol{\Sigma}\boldsymbol{\varepsilon}_{t+1}+A_n+\mathbf{B}_n^{'}\mathbf{X}_{t+1}\right\}\right] \\
&= \exp\left\{-i_t-\frac{1}{2}\boldsymbol{\Lambda}_t^{'}\boldsymbol{\Sigma}\boldsymbol{\Sigma}^{'}\boldsymbol{\Lambda}_t+A_n\right\}E_t\left[\exp\left\{-\boldsymbol{\Lambda}_t^{'}\boldsymbol{\Sigma}\boldsymbol{\varepsilon}_{t+1}+\mathbf{B}_n^{'}\mathbf{X}_{t+1}\right\}\right] \\
&= \exp\left\{-i_t-\frac{1}{2}\boldsymbol{\Lambda}_t^{'}\boldsymbol{\Sigma}\boldsymbol{\Sigma}^{'}\boldsymbol{\Lambda}_t+A_n\right\} * \\
&\quad E_t\left[\exp\left\{-\boldsymbol{\Lambda}_t^{'}\boldsymbol{\Sigma}\boldsymbol{\varepsilon}_{t+1}+\mathbf{B}_n^{'}\left[\boldsymbol{\mu}+\boldsymbol{\rho}\mathbf{X}_t+\boldsymbol{\Sigma}\boldsymbol{\varepsilon}_{t+1}\right]\right\}\right] \\
&= \exp\left\{-i_t-\frac{1}{2}\boldsymbol{\Lambda}_t^{'}\boldsymbol{\Sigma}\boldsymbol{\Sigma}^{'}\boldsymbol{\Lambda}_t+A_n+\mathbf{B}_n^{'}\boldsymbol{\mu}+\mathbf{B}_n^{'}\boldsymbol{\rho}\mathbf{X}_t\right\} * \\
&\quad E_t\left[\exp\left\{\left(\mathbf{B}_n^{'}-\boldsymbol{\Lambda}_t^{'}\right)\boldsymbol{\Sigma}\boldsymbol{\varepsilon}_{t+1}\right\}\right]
\end{aligned} \tag{49}$$

Write the second term in the last equation as follows using normality of the shocks, ε_{t+1}:

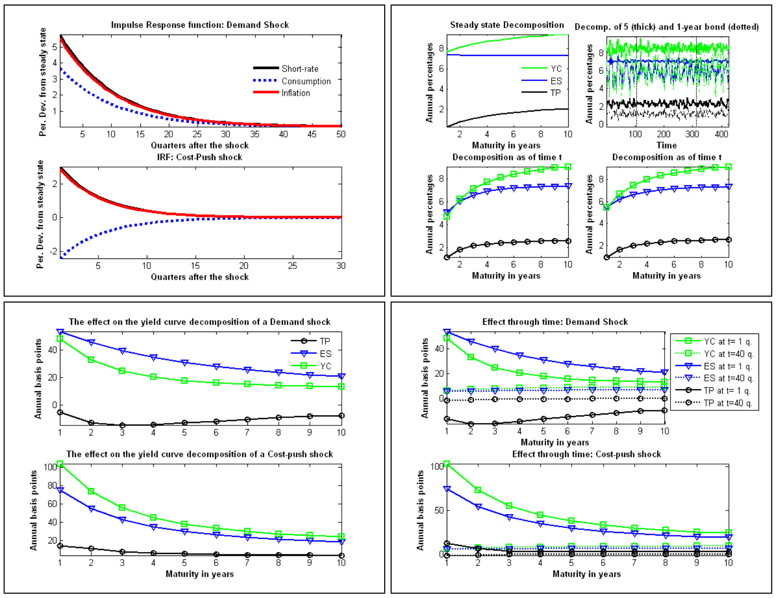
$$E_t\left[\exp\left\{\left(\mathbf{B}_n^{'}-\boldsymbol{\Lambda}_t^{'}\right)\boldsymbol{\Sigma}\boldsymbol{\varepsilon}_{t+1}\right\}\right]=\exp\left\{\frac{1}{2}\left(\mathbf{B}_n^{'}-\boldsymbol{\Lambda}_t^{'}\right)\boldsymbol{\Sigma}\boldsymbol{\Sigma}^{'}\left(\mathbf{B}_n-\boldsymbol{\Lambda}_t\right)\right\}$$
$$=\exp\left\{\left(\begin{array}{c}\frac{1}{2}\boldsymbol{\Lambda}_t^{'}\boldsymbol{\Sigma}\boldsymbol{\Sigma}^{'}\boldsymbol{\Lambda}_t\\ -\mathbf{B}_n^{'}\boldsymbol{\Sigma}\boldsymbol{\Lambda}_t+\frac{1}{2}\mathbf{B}_n^{'}\boldsymbol{\Sigma}\boldsymbol{\Sigma}^{'}\mathbf{B}_n\end{array}\right)\right\}$$

Now continue from relation (49):

$$P_t^{n+1}=\exp\left\{\begin{array}{l}-i_t-\frac{1}{2}\boldsymbol{\Lambda}_t^{'}\boldsymbol{\Sigma}\boldsymbol{\Sigma}^{'}\boldsymbol{\Lambda}_t+A_n+\mathbf{B}_n^{'}\boldsymbol{\mu}+\mathbf{B}_n^{'}\boldsymbol{\rho}\mathbf{X}_t\\ +\frac{1}{2}\left(\boldsymbol{\Lambda}_t^{'}\boldsymbol{\Sigma}\boldsymbol{\Sigma}^{'}\boldsymbol{\Lambda}_t-\mathbf{2}\mathbf{B}_n^{'}\boldsymbol{\Sigma}\boldsymbol{\Lambda}_t+\mathbf{B}_n^{'}\boldsymbol{\Sigma}\boldsymbol{\Sigma}^{'}\mathbf{B}_n\right)\end{array}\right\}$$
$$=\exp\left\{-i_t+A_n+\mathbf{B}_n^{'}\boldsymbol{\mu}+\mathbf{B}_n^{'}\boldsymbol{\rho}\mathbf{X}_t-\mathbf{B}_n^{'}\boldsymbol{\Sigma}\boldsymbol{\Lambda}_t+\frac{1}{2}\mathbf{B}_n^{'}\boldsymbol{\Sigma}\boldsymbol{\Sigma}^{'}\mathbf{B}_n\right\}$$
$$=\exp\left\{\begin{array}{c}-\delta_0+\mathbf{B}_n^{'}\left(\boldsymbol{\mu}-\boldsymbol{\Sigma}\boldsymbol{\lambda}_0\right)+\frac{1}{2}\mathbf{B}_n^{'}\boldsymbol{\Sigma}\boldsymbol{\Sigma}^{'}\mathbf{B}_n+A_n\\ +\left(-\boldsymbol{\delta}_1^{'}+\mathbf{B}_n^{'}\left(\boldsymbol{\rho}-\boldsymbol{\Sigma}\boldsymbol{\lambda}_1\right)\right)\mathbf{X}_t\end{array}\right\}$$

Matching coefficients results in the recursion in equation (1).

10.4 Appendix: Model without the subjective inflation target shock



This figure shows the impulse response function (top-left), the simulated and steady-state yield curve (top-right), responses of the yield curve decomposition to a shock to each factor at time t (bottom-left), and through time, (bottom-right), in a model without the subjective inflation target.

10.5 Appendix: A budget constraint in interest rates

Consider for now the case of two bonds, one maturing in the period to come and one in two periods time. I start from the budget constraint and I introduce interest rates instead of prices

$$P_t^1 B_t^1 + P_t^2 B_t^2 - B_{t-1}^1 = B_t^1 + B_t^2 - (1 + i_{t-1}) B_{t-1}^1 - \left(1 + Y_{t-2}^2\right)^2 B_{t-2}^2$$

in which I for simplicity with the analysis that follows consider yields defined as $P_t^n = (1 + Y_t^n)^{-n}$. Next divide by the price level, P_t, and let the left hand side of the budget constraint denote after tax income, $\Xi_t \equiv W_t N_t + \Pi_t - T_t$

$$C_t + \frac{B_t^1 + B_t^2 - (1 + i_{t-1}) B_{t-1}^1 - \left(1 + Y_{t-2}^2\right)^2 B_{t-2}^2}{P_t} = \Xi_t$$

Consider maturing bond in isolation

$$= \begin{array}{l} -\left(1+i_{t-1}\right)\dfrac{B_{t-1}^{1}}{P_t}-\left(1+Y_{t-1}^{2}\right)^{2}\dfrac{B_{t-2}^{2}}{P_t}, \\ -\left(1+i_{t-1}\right)\dfrac{B_{t-1}^{1}P_{t-1}}{P_tP_{t-1}}-\left(1+Y_{t-2}^{2}\right)^{2}\dfrac{B_{t-2}^{2}P_{t-1}P_{t-2}}{P_tP_{t-1}P_{t-2}} \end{array}$$

which can be written as

$$= \begin{array}{l} -\left(1+i_{t-1}\right)\dfrac{B_{t-1}^{1}P_{t-1}}{P_tP_{t-1}}-\left(1+Y_{t-1}^{2}\right)^{2}\dfrac{B_{t-2}^{2}P_{t-1}P_{t-2}}{P_tP_{t-1}P_{t-2}} \\ -b_{t-1}^{1}\left(\dfrac{\left(1+i_{t-1}\right)}{\left(1+\pi_t\right)}\right)-b_{t-2}^{2}\left(\dfrac{\left(1+Y_{t-2}^{2}\right)^{2}}{\left(1+\pi_t\right)\left(1+\pi_{t-1}\right)}\right) \end{array}$$

in which I have introduced inflation $(1+\pi_t)\equiv\frac{P_{t-1}}{P_t}$. I also let superscript r denote variables deflated with the price level. I can thus write

$$c_t^{r}+b_t^{r1}+b_t^{r2}-b_{t-1}^{r1}\left(\frac{\left(1+i_{t-1}\right)}{\left(1+\pi_t\right)}\right)-b_{t-2}^{r2}\left(\frac{\left(1+Y_{t-2}^{2}\right)^{2}}{\left(1+\pi_t\right)\left(1+\pi_{t-1}\right)}\right)=\Xi_t^{r}$$

I next introduce the ex post return from $t-1$ to t, $\bar{r}_{t-1}=\frac{(1-i_{t-1})}{(1+\pi_t)}-1$. Consider firstly the one-period bond

$$b_t^{r1}-\left(1+i_{t-1}\right)b_{t-1}^{r1}\left(\frac{1}{\left(1+\pi_t\right)}\right)=b_t^{r1}-b_{t-1}^{r1}\left(1+\bar{r}_{t-1}\right)=b_t^{r1}-b_{t-1}^{r1}\bar{r}_{t-1}-b_{t-1}^{r1}$$

and next the two-period bond with the ex post return from $t-2$ to t, $\bar{r}_{t-2}^{2}\equiv\frac{\left(1+Y_{t-2}^{2}\right)^{2}}{(1+\pi_t)(1+\pi_{t-1})}-1$

$$-b_{t-2}^{r2}\left(\frac{\left(1+I_{t-2}^{2}\right)^{2}}{\left(1+\pi_t\right)\left(1+\pi_{t-1}\right)}\right)=-b_{t-2}^{r2}\left(1+\bar{r}_{t-2}^{2}\right)$$

which allow me to write

$$c_t^{r}+b_t^{r1}+b_t^{r2}-b_{t-1}^{r1}\left(1+\bar{r}_{t-1}\right)-b_{t-2}^{r2}\left(1+\bar{r}_{t-2}^{2}\right)=\Xi_t^{r}$$

To highlight the respective roles of anticipated an unanticipated inflation, let r_t be the ex ante real rate of return and let π_t^{e} be the expected rate of inflation. Add and subtract the following

$$\left(r_{t-1}-\bar{r}_{t-1}\right)b_{t-1}^{r1}=\frac{\left(\pi_t-\pi_t^{e}\right)\left(1+r_{t-1}\right)}{\left(1+\pi_t\right)}b_{t-1}^{r1}$$

$$\left(r_{t-2}-\bar{r}_{t-2}\right)b_{t-2}^{r2}=\left(1+r_{t-2}^{2}\right)\left\{\frac{\left(\pi_{t-1}-\pi_{t-1}^{e}\right)+\left(\pi_t-\pi_t^{e}\right)}{\left(1+\pi_t\right)\left(1+\pi_{t-1}\right)}\right\}b_{t-2}^{r2}$$

in which I have assumed that $\pi_t \pi_{t-1} - \pi_t^e \pi_{t-1}^e \approx 0$. Take individual bond terms first

$$
\begin{aligned}
& b_t^{r1} - b_{t-1}^{r1}\left(1+\bar{r}_{t-1}\right) + \left(r_{t-1} - \bar{r}_{t-1}\right) b_{t-1}^{r1} - \left(r_{t-1} - \bar{r}_{t-1}\right) b_{t-1}^{r1} \\
= & \left(b_t^{r1} - b_{t-1}^{r1}\right) - r_{t-1} b_{t-1}^{r1} + \left(1 + r_{t-1}\right) \frac{\left(\pi_t - \pi_t^e\right)}{\left(1+\pi_t\right)} b_{t-1}^{r1}
\end{aligned}
$$

and

$$
\begin{aligned}
& b_t^{r2} - b_{t-2}^{r2}\left(1+\bar{r}_{t-2}^2\right) + \left(r_{t-2}^2 - \bar{r}_{t-2}^2\right) b_{t-2}^{r2} - \left(r_{t-2}^2 - \bar{r}_{t-2}^2\right) b_{t-2}^{r2} \\
= & \left(b_t^{r2} - b_{t-2}^{r2}\right) - r_{t-2} b_{t-2}^{r2} + \left(1 + r_{t-2}^2\right) \left\{ \frac{\left(\pi_{t-1} - \pi_{t-1}^e\right) + \left(\pi_t - \pi_t^e\right)}{\left(1+\pi_t\right)\left(1+\pi_{t-1}\right)} \right\} b_{t-2}^{r2}
\end{aligned}
$$

$$
\begin{aligned}
& c_t^r + \left(b_t^{r2} - b_{t-2}^{r2}\right) + \left(b_t^{r1} - b_{t-1}^{r1}\right) - r_{t-2}^2 b_{t-2}^{r2} - r_{t-1} b_{t-1}^{r1} + \\
& \left(1 + r_{t-2}^2\right) \left\{ \frac{\left(\pi_{t-1} - \pi_{t-1}^e\right) + \left(\pi_t - \pi_t^e\right)}{\left(1+\pi_t\right)\left(1+\pi_{t-1}\right)} \right\} b_{t-2}^{r2} + \left(1 + r_{t-1}\right) \frac{\left(\pi_t - \pi_t^e\right)}{\left(1+\pi_t\right)} b_{t-1}^{r1} = \Xi_t^r
\end{aligned}
$$

Generalising to the case of an economy with bonds of all maturities, I get

$$
\begin{aligned}
& c_t^r + \sum_{i=1}^{n} \left\{ \left(b_t^{ri} - b_{t-i}^{ri}\right) - r_{t-i}^i b_{t-i}^{ri} \right\} \\
= & \\
& \Xi_t^r - \sum_{i=1}^{n} \left(1 + r_{t-i}^i\right) \left[\prod_{j=0}^{i-1} \left(1 + \pi_{t-j}\right) \right]^{-1} \left[\sum_{j=0}^{i-1} \left(\pi_{t-j} - \pi_{t-j}^e\right) \right] b_{t-i}^{ri}
\end{aligned}
$$

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11 Figures

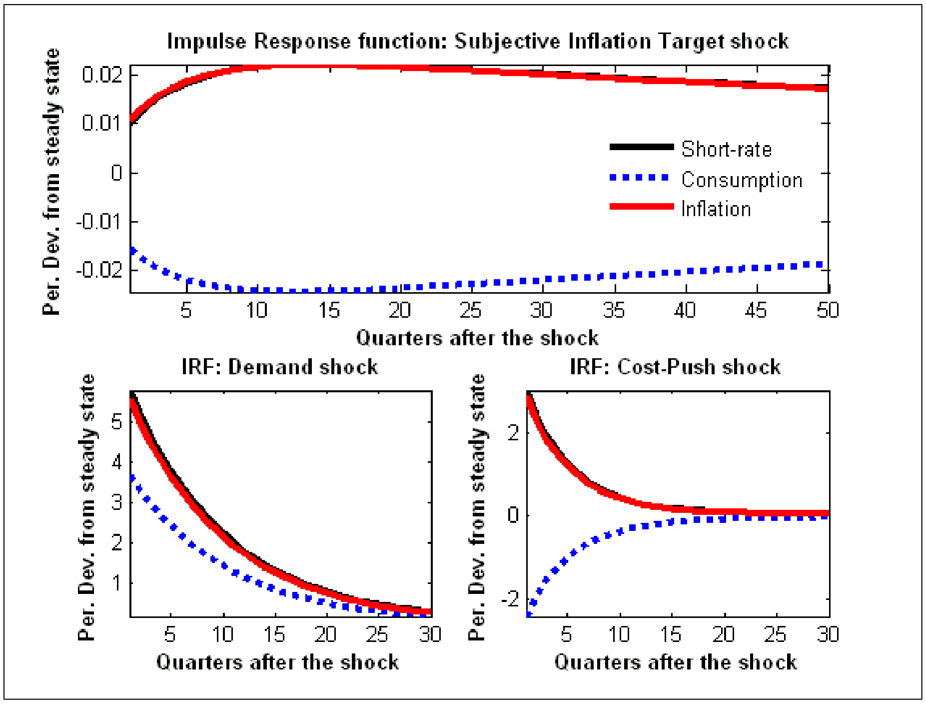

Figure 1: The effect upon the macro economy of an innovation to the subjective inflation target (top figure), demand shock (figure at the bottom left), and to the cost-push shock (figure at the bottom right).

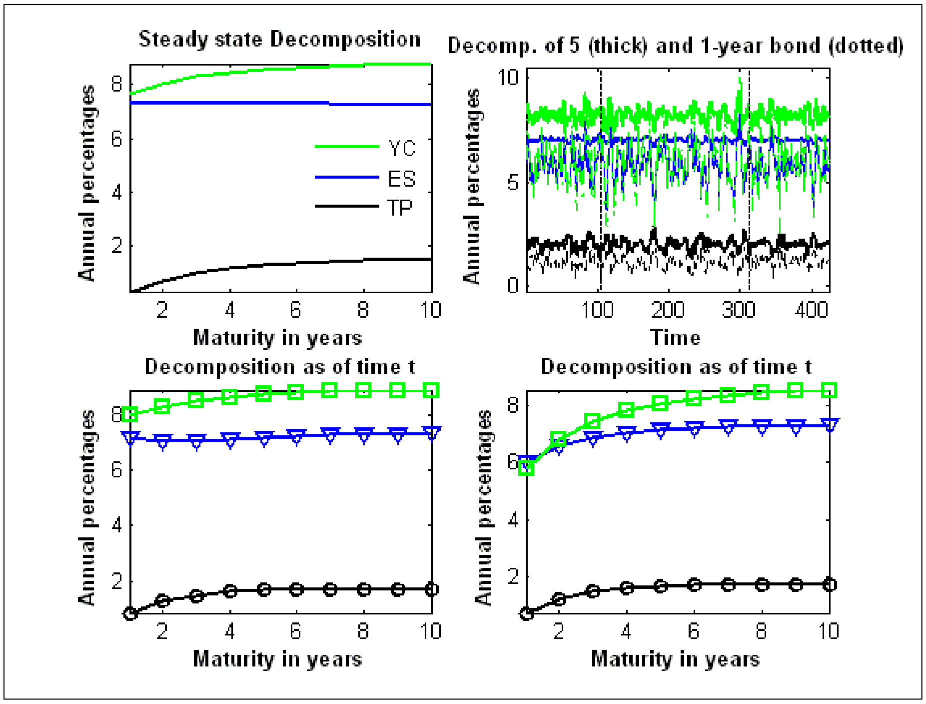

Figure 2: The top-left figure shows the decomposition of the yield curve (YC) into the ES_t^n-part and the TP_t^n-part at the steady state. The top-right figure shows simulated yields for the 5-year and 1-year bond together with their decompositions. The bottom figures show two snap-shots of the simulated yield curve decomposition.

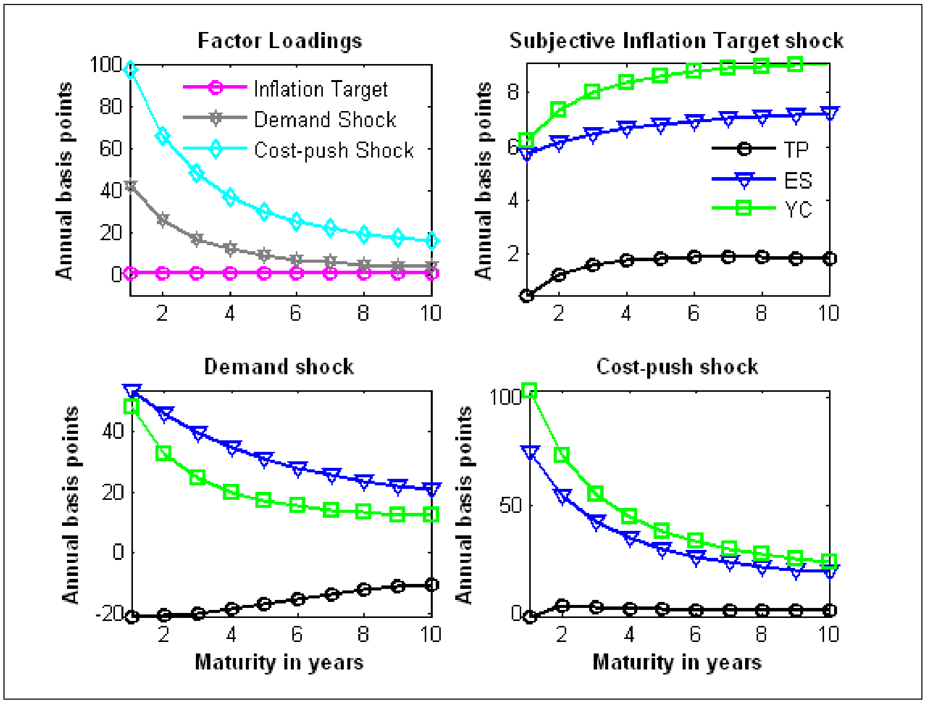

Figure 3: The top-left figure shows the factor loadings in the bond prices, $-\frac{B_n}{n}$. The factor loadings are scaled by the annualised standard deviation of the respective shocks. The top-right and bottom figures show the response of the yield curve decomposition to a one standard deviation shock to each of the factors in the economy.

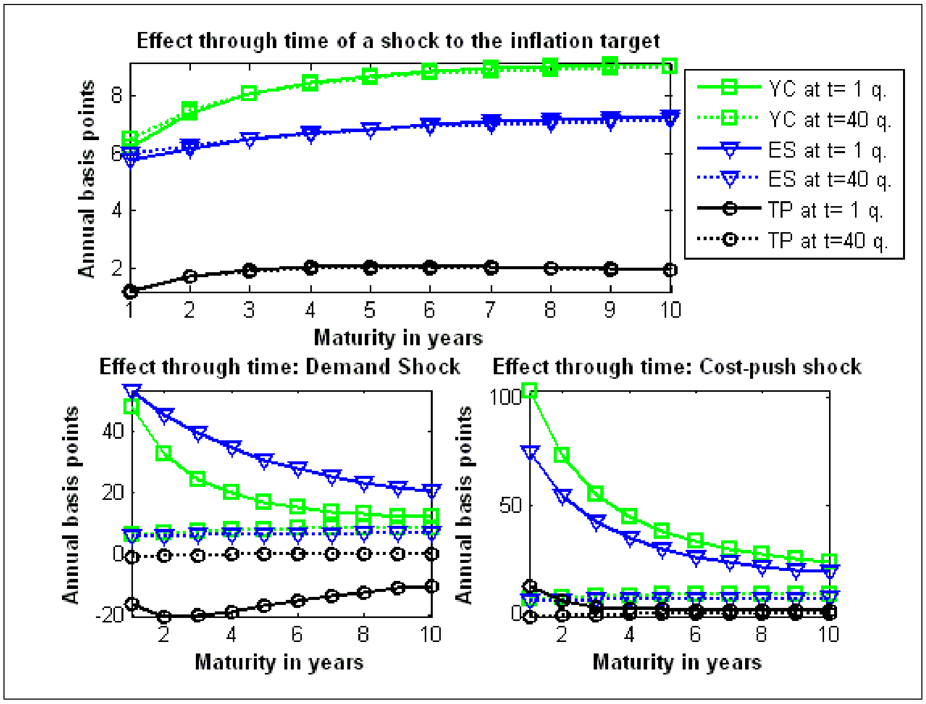

Figure 4: The figures show the response of the yield curve decomposition to a one standard deviation shock to each of the factors in the economy through time. For brevity the figure only shows the immediate response (fat lines) and the effect 40 quarters hence (dotted lines).

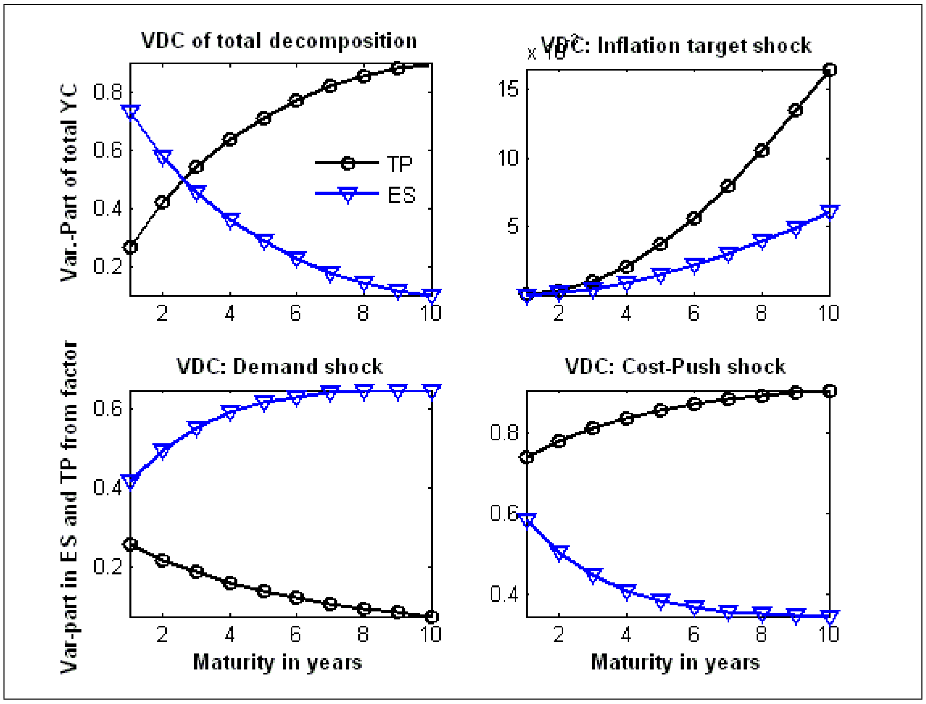

Figure 5: This top left figure shows the variance decomposition of the yield curve into the expectations part, ES_t^n, and the term premia part, TP_t^n. The top right and bottom, figures show the variance decomposition of the expectations part, ES_t^n, and the term premia part, TP_t^n, into each factor in the economy.

Additional material for: Real-Time Effects of Central Bank Interventions in the Euro Market

Rasmus Fatum* and Jesper Pedersen†

Abstract

This paper investigates the real-time effects of sterilized foreign exchange intervention using official intraday intervention data provided by the Danish central bank. Our analysis employs a two-step weighted least squares estimation procedure. We control for macro surprises, address the issue of endogeneity, and carry out an array of robustness tests. Our main result is that intervention exerts a significant influence on exchange rate returns only when the direction of intervention is consistent with the monetary policy stance, thereby illustrating that sterilized intervention is not an independent policy instrument.

Key words: Foreign Exchange Intervention; Intraday Data; ERM II

JEL Classifications: D53; E58; F31; G15

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ERM II Cross-Country Comparison

ERM II Cross-Country Comparison					
	Denmark	**Estonia**	**Lithuania**	**Latvia**	**Slovakia**
ERM II entry	0101-1999	2806-2004	2806-2004	0205-2005	1603-2007
Central Rate	7.46038	15.6466	3.45280	0.702804	35.4424
Band	2.25 pct.	15 pct.	15 pct.	15 pct.	15 pct.
De-Facto Band	+/- 0.50 pct.	+/- 0.00 pct.	+/- 0.00 pct.	+/- 1.00 pct.	+/- 14.80 pct.
Min. fx	7.4234	15.6466	3.45280	0.5533	32.866
	(2504-2002)	-	-	(0403-2002)	(1211-2007)
Min. fx	7.4630	15.6466	3.45280	0.6960	34.277
	(1812-2007)	-	-	(1603-2007)	(1106-2007)

Table 1: *Source: ECB (www.ecb.int) and own calculations. The exchange rates are quoted in amounts of the home currency needed to purchase 1 Euro. For each country, the "de-facto deviation band" is based on the larger distance between the central rate and minimum or maximum exchange rate over the respective ERM II entry date to 30 July 2008. The Danish "de-facto deviation band" of +/- 050 pct. has never been announced or officially acknowledged. Estonia and Lithuania have moved to currency boards. The Latvian "de-facto deviation band" of +/- 1.00 pct was announced by the Latvian monetary authorities on the date of the Latvian ERM II entry. The dates in brackets denote the date for the respective observation.*

WLS Estimation of Equation (1) Using Alternative Break-Point: 9 August 2002 (instead of 16 August 2002)

WLS Estimation of eq. 1: 9 August 2002 Breakpoint			
	Full Sample	**Sub-sample 1b**	**Sub-sample 2b**
C	0.4427	7.8730	0.6726
	(0.9418)	*(13.4870)*	*(2.2072)*
Γ_t	-0.1526***	-0.291***	-0.079
	(0.0570)	*(0.0631)*	*(0.0680)*
Γ_{t-1}	-0.1605***	-0.2722	-0.1604***
	(0.0582)	*(0.1808)*	*(0.0431)*
Γ_{t-2}	0.0638	0.0938	0.0335
	(0.0665)	*(0.1702)*	*(0.0660)*
Γ_{t-3}	-0.045	-0.0297	-0.0518
	(0.0640)	*(0.1163)*	*(0.0770)*
Γ_{t-4}	0.1329**	0.1295	0.1068
	(0.0589)	*(0.1148)*	*(0.0694)*
Γ_{t-5}	0.1866**	-0.1451	0.3265***
	(0.0795)	*(0.1466)*	*(0.0763)*
Γ_{t-6}	0.124*	0.1705**	0.1077
	(0.0645)	*(0.0827)*	*(0.0836)*
β_{t-1}	-0.6386***	-0.7581***	-0.6142***
	(0.0067)	*(0.0097)*	*(0.0078)*
β_{t-2}	-0.4518***	-0.5819***	-0.427***
	(0.0074)	*(0.0117)*	*(0.0085)*
β_{t-3}	-0.312***	-0.4174***	-0.2928***
	(0.0071)	*(0.0119)*	*(0.0080)*
β_{t-4}	-0.2013***	-0.2611***	-0.1905***
	(0.0064)	*(0.0108)*	*(0.0072)*
β_{t-5}	-0.102***	-0.1313***	-0.0967***
	(0.0052)	*(0.0083)*	*(0.0059)*
Sum	0.14933	-0.34407	0.2833**
Wald Test Statistic	1.96	1.74	6.09

Table 2: *This table shows the estimates from the 2WLS regression. Sub-sample 1b covers the period 01012002-08082002, while sub-sample 2b covers the period 09082002-31122004. Sub-sample 1b covers 52 interventions, while sub-sample 1 covered 49. The dependent variable is the first difference of the log of the daily DKK/EUR mid-point exchange rate.* Γ_{t-j} *denotes the intervention variable.* $\beta_{t-j,n}$ *denotes 5 min. fx returns. The coefficients for the constant, the interventions and their sum are multiplied by 1.000.000 Their standard deviations are multiplied by 1.000.000. *, **, and *** denotes significance at respectively the 10, 5, and 1 per cent level. Standard Errors in ( ) below the point estimates. Subscript denotes lags.*

WLS Estimation of Equation (1) with sub-sample 2 covering 17 August 2002 to 15 June 2004

Mean Equation for sample 15082002-15062004			
C	0.9156 *(2.6692)*	Γ_t	-0.0763 *(0.0689)*
Γ_{t-1}	-0.1783*** *(0.0277)*	Γ_{t-2}	0.058 *(0.0734)*
Γ_{t-3}	-0.0505 *(0.0819)*	Γ_{t-4}	0.1261 *(0.0714)*
Γ_{t-5}	0.3016*** *(0.0814)*	Γ_{t-6}	0.1125 *(0.0915)*
$\beta_{t-1,n}$	-0.6044*** *(0.0087)*	$\beta_{t-2,n}$	-0.4181*** *(0.0094)*
$\beta_{t-3,n}$	-0.2848*** *(0.0089)*	$\beta_{t-4,n}$	-0.1857*** *(0.0080)*
$\beta_{t-5,n}$	-0.0983*** *(0.0066)*		
Sum	0.29316	**Wald Test Statistic**	5.80**

Table 3: *This table shows the estimates from the 2WLS regression using the sample from mid-august 2002 to mid-june 2004. The dependent variable is the first difference of the log of the daily DKK/EUR mid-point exchange rate. Γ_{t-j} denotes the intervention variable. $\beta_{t-j,n}$ denotes 5 min. fx returns. The coefficients for the constant, the interventions and their sum are multiplied by 1.000.000 Their standard deviations are multiplied by 1.000.000. *, **, and *** denotes significance at respectively the 10, 5, and 1 per cent level. This table shows the estimates from the 2WLS regression in which the distance between the Euro/Dkk exchange rate and the central rate of 7.46038.. Standard Errors in ( ) below the point estimates; test statistic values in [ ]. Subscript denotes lags.*

Distance from Central Rate included in WLS Estimation of Equation (1)

WLS Estimation of eq. 1: Distance from Central Rate included			
	Full Sample	**Sub-sample 1**	**Sub-sample 2**
C	-1.7924	6.8765	-2.2084
	(1.1061)	*(12.6740)*	*(2.3208)*
Γ_t	-0.1233**	-0.2964***	-0.0364
	(0.0598)	*(0.0664)*	*(0.0662)*
Γ_{t-1}	-0.1468***	-0.2517	-0.1508**
	(0.0514)	*(0.1704)*	*(0.0637)*
Γ_{t-2}	0.0296	0.0569	0.0450
	(0.0561)	*(0.1629)*	*(0.0706)*
Γ_{t-3}	-0.0433	-0.0364	-0.0548
	(0.0673)	*(0.1159)*	*(0.0819)*
Γ_{t-4}	0.1169*	0.1139	0.0878
	(0.0596)	*(0.1172)*	*(0.0717)*
Γ_{t-5}	0.2415***	-0.1410	0.2563***
	(0.0869)	*(0.1424)*	*(0.0805)*
Γ_{t-6}	0.1186	0.1397	0.0965
	(0.0673)	*(0.0839)*	*(0.0935)*
$\beta_{t-1,n}$	-0.6435***	-0.7603***	-0.6200***
	(0.0067)	*(0.0096)*	*(0.0078)*
$\beta_{t-2,n}$	-0.4594***	-0.5819***	-0.4359***
	(0.0073)	*(0.0119)*	*(0.0083)*
$\beta_{t-3,n}$	-0.3161***	-0.4188***	-0.2969***
	(0.0071)	*(0.0118)*	*(0.0080)*
$\beta_{t-4,n}$	-0.2030***	-0.2618***	-0.1918***
	(0.0063)	*(0.0107)*	*(0.0072)*
$\beta_{t-5,n}$	-0.1024***	-0.1326***	-0.0967***
	(0.0051)	*(0.0083)*	*(0.0059)*
$\mathbf{d}\mu_{t-1,n}$	-0.0006	0.0014	-0.0009
	(0.0006)	*(0.0011)*	*(0.0007)*
$\mathbf{d}\mu_{t-2,n}$	0.0008	-0.0006	0.0011
	(0.0006)	(0.0011)	(0.0007)
Sum	0.1932	-0.4150	0.2436
Wald Test Statistic	3.0839*	2.5921	3.4515*

Table 4: *This table shows the estimates from the 2WLS regression in which the distance between the Euro/Dkk exchange rate and the central rate of 7.46038.. The dependent variable is the first difference of the log of the daily DKK/EUR mid-point exchange rate.* $d\mu_{t-j}$ *denotes deviations from the unconditional mean.* Γ_{t-j} *denotes the intervention variable.* $\beta_{t-j,n}$ *denotes 5 min. fx returns. The coefficients for the constant, the interventions and their sum are multiplied by 1.000.000 Their standard deviations are multiplied by 1.000.000. *, **, and *** denotes significance at respectively the 10, 5, and 1 per cent level. Standard Errors in ( ) below the point estimates; test statistic values in [ ]. Subscript denotes lags.*

Intervention and Policy Consistentcy

- Full Sample:

  Total 89 (220) intervention days (transactions)

  Purchases of EUR 68 (157)

  Sales of EUR 21 (63)

- Sub-Sample 1 (1 January 2002 - 16 August 2002)

  Total 23 (58) intervention days (transactions)

  Purchases of EUR 23 (58)

  Sales of EUR 0 (0)

- Policy Consistency:

  Monetary Policy Neutral (official policy rate unchanged)

  Exchange Rate Policy, parity measure All are consistent

  Exchange Rate Policy, mean measure All are consistent

- Sub-Sample 2 (17 August 2002 - 31 December 2004)

  Total 66 (162) intervention days (transactions)

  Purchases of EUR 45 (99)

  Sales of EUR 21 (63)

- Policy Consistency:

  Monetary Policy All 45 (99) purchases of EUR are consistent. All 21 (63) sales of EUR are inconsistent.

  Exchange Rate Policy, parity measure All 45 (99) purchases of EUR are consistent. All 21 (63) sales of EUR are inconsistent.

  Exchange Rate Policy, mean measure 31 (67) purchases of EUR are consistent. 14 (32) purchases of EUR are inconsistent. All 21 (63) sales of EUR are consistent.

Only Four Cosine and Four Sine Terms Included in Equation (2)

WLS Estimation of eq. 1: 4 co- and sine terms in eq. 2			
	Full Sample	**Sub-sample 1**	**Sub-sample 2**
C	0.5000	0.0683	0.8000
	(0.9430)	*(1.3000)*	*(0.2284)*
Γ_t	-0.1596***	-0.2824***	-0.0887
	(0.0565)	*(0.0654)*	*0.068*
Γ_{t-1}	-0.1622***	-0.2527	-0.1817
	(0.0572)	*(0.1759)*	*(0.0200)*
Γ_{t-2}	0.0647	0.0662	0.0391
	(0.0661)	*(0.1639)*	*(0.0670)*
Γ_{t-3}	-0.0443	-0.0211	-0.0565
	(0.0653)	*(0.1135)*	*(0.0810)*
Γ_{t-4}	0.1424**	0.1220	0.1210
	(0.0595)	*(0.1154)*	*(0.0700)*
Γ_{t-5}	0.1813**	-0.1197	0.3173
	(0.0782)	*(0.1422)*	*(0.0780)*
Γ_{t-6}	0.1234*	0.1619*	0.1158
	(0.0669)	*(0.0838)*	*(0.0880)*
$\beta_{t-1,n}$	-0.6387***	-0.7598***	-0.6139
	(0.0067)	*(0.0097)*	*(0.0078)*
$\beta_{t-2,n}$	-0.4521***	-0.5836***	-0.4268
	(0.0074)	*(0.0116)*	*(0.0085)*
$\beta_{t-3,n}$	-0.3123***	-0.4188***	-0.2927
	(0.0071)	*(0.0119)*	*(0.0080)*
$\beta_{t-4,n}$	-0.2017***	-0.2624***	-0.1906
	(0.0064)	*(0.0108)*	*(0.0072)*
$\beta_{t-5,n}$	-0.1023***	-0.1329***	-0.0968
	(0.0052)	*(0.0083)*	*(0.0059)*
Sum	0.16	-0.33	2.70
Wald Test Statistic	1.81	1.60	5.12**

Table 5: *This table shows the estimates from the 2WLS regression using only 4 lags of the Sine- and Cosine terms in the model for the absolute residuals. The dependent variable is the first difference of the log of the daily DKK/EUR mid-point exchange rate.* Γ_{t-j} *denotes the intervention variable.* $\beta_{t-j,n}$ *denotes 5 min. fx returns. The coefficients for the constant, the interventions and their sum are multiplied by 1.000.000 Their standard deviations are multiplied by 1.000.000. *, **, and *** denotes significance at respectively the 10, 5, and 1 per cent level. Standard Errors in ( ) below the point estimates; test statistic values in [ ]. Subscript denotes lags.*

Estimation of eq. 2: Volatility using 4 co- and sine terms			
	Full Sample	**Sub-sample 1**	**Sub-sample 2**
C	0.0009***	0.0002	0.0010***
	(0.0002)	*(0.0002)*	*(0.0002)*
Normalising Constant I	-0.0007***	-0.0001	-0.0008***
	(0.0001)	*(0.0002)*	*(0.0001)*
Normalising Constant II	0.0001***	0.0000	0.0001***
	(0.0000)	*(0.0001)*	*(0.0000)*
Realised Volatility	0.2737***	0.1280***	0.2691***
	(0.0053)	*(0.0162)*	*(0.0054)*
Sine terms			
δ_1	-0.0351***	-0.0044	-0.0427***
	(0.0066)	*(0.0107)*	*(0.0078)*
δ_2	-0.0045***	-0.0015	-0.0052***
	(0.0008)	*(0.0014)*	*(0.0009)*
δ_3	0.0015**	-0.0014	0.0024***
	(0.0007)	*(0.0010)*	*(0.0008)*
δ_4	0.0015**	-0.0006	0.0021***
	(0.0006)	*(0.0010)*	*(0.0007)*
Cosine terms			
φ_1	0.0250***	0.0032	0.0305***
	(0.0055)	*(0.0082)*	*(0.0065)*
φ_2	0.0107***	0.0011	0.0132***
	(0.0021)	*(0.0033)*	*(0.0025)*
φ_3	0.0041***	-0.0003	0.0052***
	(0.0009)	*(0.0015)*	*(0.0011)*
φ_4	0.0016***	0.0007	0.0017***
	(0.0005)	*(0.0009)*	*(0.0006)*

Table 6: *The dependent variable is the absolute residual from the auxiliary regression, equation (1). The independent variables are normalizing constants, a realized volatility measure, trigonometric terms, δ_j and φ_j, the EUR-DKK interest rate differential, η_j, and interventions, Γ_{t-j}. The coefficients and standard deviations for the sine, cosine, and interest differentials are multiplied by 1.000. The coefficients and standard deviations for the interventions and their sum are multiplied by 1.000.000. *, **, and *** denotes significance at respectively the 10, 5, and 1 per cent level. Standard Errors in ( ) below the point estimates; test statistic values in [ ]. Subscript denotes lags.*

Estimation of eq. 2: Volatility using 4 co- and sine terms			
	Full Sample	**Sub-sample 1**	**Sub-sample 2**
	Interest differentials		
η_1	-0.0852	-0.1404***	-0.0019***
	(0.1413)	*(0.0513)*	*(0.0001)*
η_2	-0.0935	-0.0232	-0.0004***
	(0.2058)	*(0.0713)*	*(0.0001)*
η_3	0.2477	0.2276***	0.0025***
	(0.1501)	*(0.0581)*	*(0.0001)*
	Interventions		
Γ_t	-0.0724***	-0.3316***	0.0257
	(0.0005)	*(0.0632)*	*(0.0873)*
Γ_{t-1}	-0.0724	0.0111	0.1066
	(0.0716)	*(0.1222)*	*(0.0884)*
Γ_{t-2}	0.0792	0.0786	0.0853
	(0.0740)	*(0.1191)*	*(0.1086)*
Γ_{t-3}	-0.0383	-0.0891	-0.0317
	(0.0866)	*(0.0772)*	*(0.0921)*
Γ_{t-4}	-0.0558	-0.1310	-0.0308
	(0.0731)	*(0.0828)*	*(0.0902)*
Γ_{t-5}	0.0681	-0.0396	0.0955
	(0.0712)	*(0.0932)*	*(0.0994)*
Γ_{t-6}	-0.0497	-0.1774***	0.0052
	(0.0816)	*(0.0685)*	*(0.1012)*
R^2	0.1051	0.0103	0.1192
F-statistic	450	8	415

Table 7: *The dependent variable is the absolute residual from the auxiliary regression, equation (1). The independent variables are normalizing constants, a realized volatility measure, trigonometric terms, δ_j and φ_j, the EUR-DKK interest rate differential, η_j, and interventions, Γ_{t-j}. The coefficients and standard deviations for the sine, cosine, and interest differentials are multiplied by 1.000. The coefficients and standard deviations for the interventions and their sum are multiplied by 1.000.000. *, **, and *** denotes significance at respectively the 10, 5, and 1 per cent level. Standard Errors in ( ) below the point estimates; test statistic values in [ ]. Subscript denotes lags.*

WLS Estimation of Equation (1): Volatility Dummy

We include a volatility-dummy equal to one when the 5-minute interval durring which an intervention occurs falls on a high-volatility day. A high-volatility day is defined as a day in which the daily realized volatility measure is at least twice the mean daily realized volatility of the full sample.

WLS Estimation of eq. 1: High volatility regimes			
	Full Sample	**Sub-sample 1**	**Sub-sample 2**
Γ_t	-0.0097	-0.2832***	-0.0167
	(0.0906)	*(0.0904)*	*(0.0640)*
Γ_{t-1}	-0.1583***	-0.2553	-0.1649***
	(0.0568)	*(0.1732)*	*(0.0399)*
Γ_{t-2}	0.0593	0.0628	0.0336
	(0.0657)	*(0.1640)*	*(0.0660)*
Γ_{t-3}	-0.0531	-0.0267	-0.0611
	(0.0624)	*(0.1133)*	*(0.0746)*
Γ_{t-4}	0.1311**	0.1245	0.1035
	(0.0587)	*(0.1160)*	*(0.0699)*
Γ_{t-5}	0.1967**	-0.1271	0.3683***
	(0.0802)	*(0.1386)*	*(0.0637)*
Γ_{t-6}	0.1403**	0.1559*	0.1065
	(0.0643)	*(0.0791)*	*(0.0834)*
Δ_t	-34.8700	2.2507	-27.7990
	(21.3560)	*(10.5420)*	*(23.2920)*
Sum	0.3063	-0.35	0.37
Wald Test Statistic	4.7053**	1.4468	9.8672***

Table 8: *We regress returns on lagged returns, interventions and a dummy for interventions that fall on days with "High" volatility. The dependent variable is the first difference of the log of the daily DKK/EUR mid-point exchange rate.* Γ_{t-j} *denotes the intervention variable.* Δ_{t-j} *is a dummy equal to one when the 5-minute interval durring which an intervention occurs falls on a high-volatility day. A high-volatility day is defined as a day in which the daily realized volatility measure is at least twice the mean daily realized volatility of the full sample. The coefficient estimates associated with the constant and the lags of the dependent variable are not shown for ease of exposition. The Wald test of the sum fo the intervention coefficient estimates is not applicable due to the inclusion of the interactive dummy. The coefficients and standard deviations for the constant, the interventions, the dummies and their sum are multiplied by 1.000.000 *, **, and *** denotes significance at respectively the 10, 5, and 1 per cent level. Standard Errors in ( ) below the point estimates; test statistic values in [ ]. Subscript denotes lags.*

WLS Estimation of eq. 1: High volatility regimes			
	Full Sample	**Sub-sample 1**	**Sub-sample 2**
Γ_t	-0.0984	-0.2744***	-0.0667
	(0.0637)	*(0.0891)*	*(0.0782)*
Γ_{t-1}	-0.1604**	-0.4452*	-0.1436*
	(0.0729)	*(0.2290)*	*(0.0791)*
Γ_{t-2}	0.1134	0.0432	0.1307
	(0.0766)	*(0.1799)*	*(0.0976)*
Γ_{t-3}	-0.0796	-0.1419	-0.0859
	(0.0676)	*(0.1695)*	*(0.0804)*
Γ_{t-4}	0.1379**	0.0805	0.1419*
	(0.0607)	*(0.1357)*	*(0.0726)*
Γ_{t-5}	0.0815	-0.2566	0.1863**
	(0.0676)	*(0.1993)*	*(0.0813)*
Γ_{t-6}	0.1314	0.2209*	0.1232
	(0.0841)	*(0.1117)*	*(0.1055)*
Δ_t	-8.4890	-2.8493	-4.5406
	(11.5264)	*(15.1557)*	*(16.4106)*
Δ_{t-1}	-10.3198	34.0551	-25.1566
	(14.3467)	(21.6399)	(18.4944)
Δ_{t-2}	14.5616	0.2497	24.4631
	(15.4458)	(35.6109)	(19.0203)
Δ_{t-3}	-3.3806	20.4485	-14.3984
	(11.0618)	(26.3346)	*(11.6204)*
Δ_{t-4}	7.5592	10.3994	11.7552
	(10.4696)	(16.0720)	(14.5821)
Δ_{t-5}	25.3589	22.3895	40.9772
	(15.8181)	(19.9198)	(23.5588)
Δ_{t-6}	8.4584	-23.8364	23.3115
	(15.0871)	(16.0923)	*(21.8275)*
Sum	0.1257	-0.77	0.29
Wald Test Statistic	1.1066	4.2287**	4.5124**

Table 9: *We regress returns on lagged returns, interventions and a dummy for interventions that fall on days with "High" volatility. The dependent variable is the first difference of the log of the daily DKK/EUR mid-point exchange rate.* Γ_{t-j} *denotes the intervention variable.* Δ_{t-j} *is a dummy equal to one when the 5-minute interval durring which an intervention occurs falls on a high-volatility day. A high-volatility day is defined as a day in which the daily realized volatility measure is at least twice the mean daily realized volatility of the full sample. The coefficient estimates associated with the constant and the lags of the dependent variable are not shown for ease of exposition. The Wald test of the sum fo the intervention coefficient estimates is not applicable due to the inclusion of the interactive dummy. The coefficients and standard deviations for the constant, the interventions, the dummies and their sum are multiplied by 1.000.000 *, **, and *** denotes significance at respectively the 10, 5, and 1 per cent level. Standard Errors in ( ) below the point estimates; test statistic values in [ ]. Subscript denotes lags.*

HAC Estimation of Equation (1)

HAC Estimation of eq. 1: Mean Equation			
	Full Sample	**Sub-sample 1**	**Sub-sample 2**
Γ_t	-0.1200*	-0.2800***	-0.0700
	(0.0706)	*(0.0073)*	*(0.0875)*
Γ_{t-1}	-0.18***	-0.26*	-0.18**
	(0.0776)	*(0.1857)*	*(0.0896)*
Γ_{t-2}	0.14*	0.06	0.15*
	(0.0966)	*(0.1714)*	*(0.1154)*
Γ_{t-3}	-0.08	-0.02	-0.11
	(0.0666)	*(0.1250)*	*(0.0880)*
Γ_{t-4}	0.16***	0.15	0.16*
	(0.0690)	*(0.1220)*	*(0.0842)*
Γ_{t-5}	0.14*	-0.14	0.24**
	(0.0897)	*(0.1505)*	*(0.1076)*
Γ_{t-6}	0.15*	0.1	0.15*
	(0.0938)	*(0.0990)*	*(0.1154)*
Sum	0.10	-0.39	0.35
Wald Test Statistic	1.91	2.04	6.35**

Table 10: *This table shows the estimates from a HAC regression of equation 1. The dependent variable is the first difference of the log of the daily DKK/EUR mid-point exchange rate.* Γ_{t-j} *denotes the intervention variable. The coefficient estimates associated with the constant and the lags of the dependent variable are not shown for ease of exposition. The coefficients, standard deviations and the sum are multiplied by 1.000.000. *, **, and *** denotes significance at respectively the 10, 5, and 1 per cent level. Standard Errors in ( ) below the point estimates; test statistic values in [ ]. Subscript denotes lags.*

HAC Estimation of Equation (1) using 12 lags of Intervention

HAC Estimation of eq. 1: Mean Equation		
	Sub-sample 1	**Sub-sample 2**
Γ_t	-0.3400	-0.0800
	(2.6150)	*(0.0714)*
Γ_{t-1}	-0.2500	-0.1600***
	(0.1825)	*(0.0442)*
Γ_{t-2}	0.0700	0.0500
	(0.1667)	*(0.0725)*
Γ_{t-3}	-0.0300	-0.0600
	(0.1579)	*(0.0822)*
Γ_{t-4}	0.1400	0.1100*
	(0.1111)	*(0.0714)*
Γ_{t-5}	-0.1500	0.3200***
	(0.1471)	*(0.0792)*
Γ_{t-6}	0.2000	0.1200*
	(0.2899)	*(0.0863)*
Γ_{t-7}	0.2000	0.06**
	(0.1408)	*(0.0300)*
Γ_{t-8}	-0.1400	-0.0200
	(0.1176)	*(0.2000)*
Γ_{t-9}	-0.0900	0.0500
	(0.1500)	*(0.1220)*
Γ_{t-10}	-0.1400	-0.0600
	(0.1037)	*(0.0772)*
Γ_{t-11}	0.1600	-0.0010
	(0.2857)	*(0.1111)*
Γ_{t-12}	-0.1300	0.0700
	(0.1733)	*(0.0875)*
Sum	-0.05	0.40
Wald Test Statistic	0.05	2.75*

Table 11: *This table shows the estimates from a HAC regression of equation 1 using 12 lags of the intervention variable. The dependent variable is the first difference of the log of the daily DKK/EUR mid-point exchange rate.* Γ_{t-j} *denotes the intervention variable. The coefficient estimates associated with the constant and the lags of the dependent variable are not shown for ease of exposition. The coefficients, standard deviations and the sum are multiplied by 1.000.000. *, **, and *** denotes significance at respectively the 10, 5, and 1 per cent level. Standard Errors in ( ) below the point estimates; test statistic values in [ ]. Subscript denotes lags.*

Figures

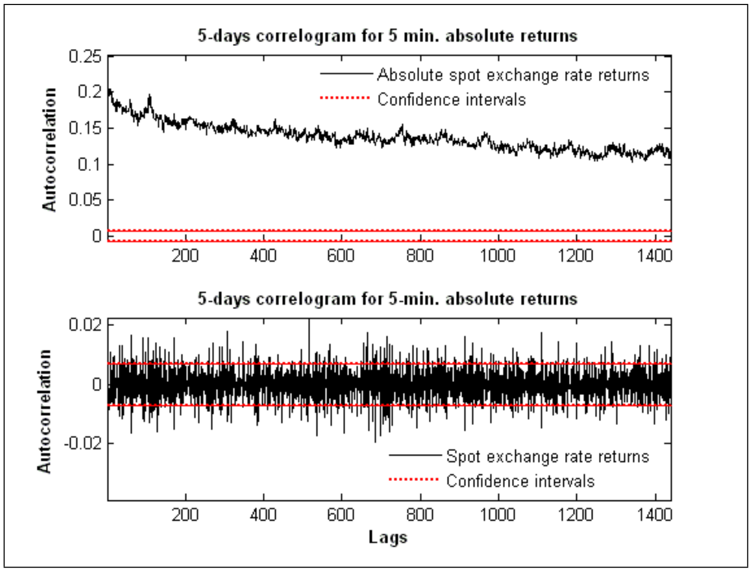

Figure 1: DKK/EUR spot exchange rate returns over the 1 January 2002 to 31 December 2004 period. Dotted lines correspond to confidence intervals for a white-noise process.

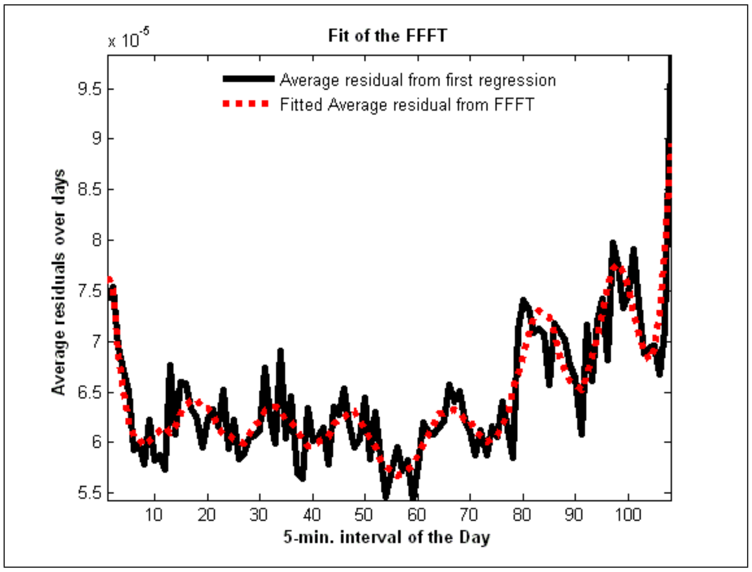

Figure 2: Average residuals for each 5 min. interval across days against the absolute average fitted residuals from Equation 2 (dotted line). Full sample.

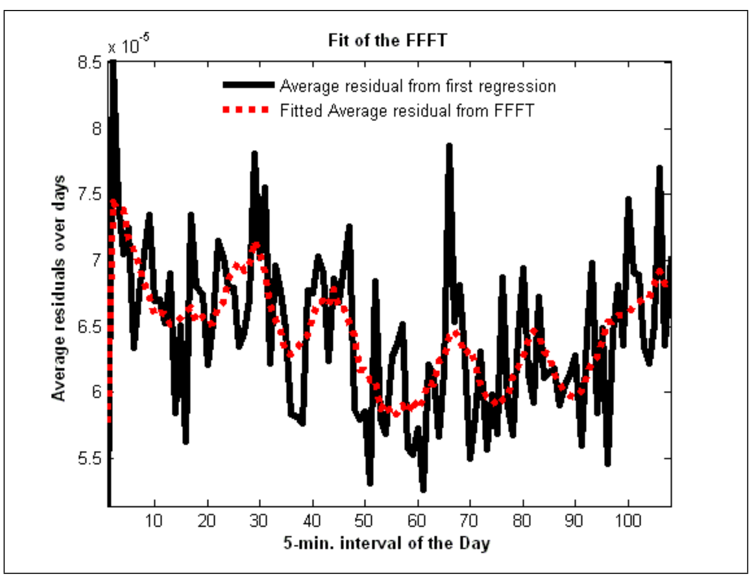

Figure 3: Average residuals for each 5 min. interval across days against the absolute average fitted residuals from Equation 2 (dotted line). Sub-sample I.

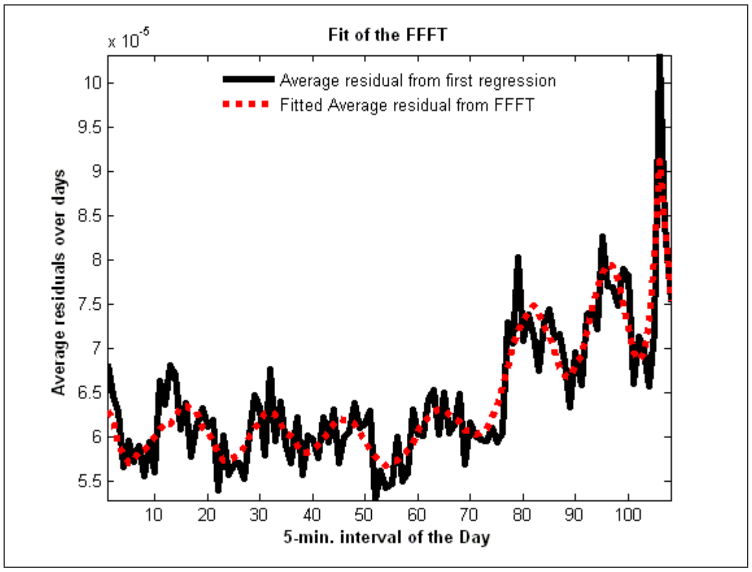

Figure 4: Average residuals for each 5 min. interval across days against the absolute average fitted residuals from Equation 2 (dotted line). Sub-sample II.

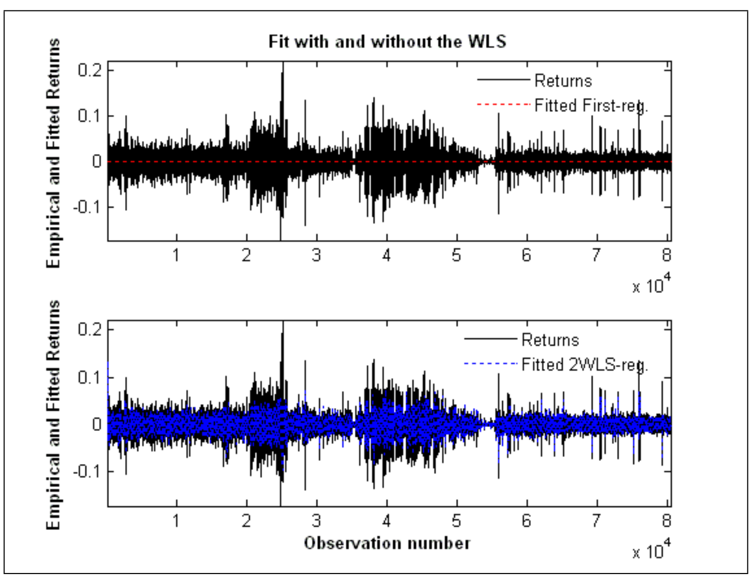

Figure 5: The dark observations are the raw exchange rate returns; the light observations are the fitted exchange rate returns. The figure at the top plots the fitted returns from the initial estimation of Equation (1). The lower graph shows the fitted returns from the WLS procedure of Equations (1) and (2).