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The latent factor VAR model: Testing for a common component in the intraday trading process

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Abstract

In this paper, we propose a framework for the modelling of multivariate dynamic processes which are driven by an unobservable common autoregressive component. Economically motivated by the mixture-of-distribution hypothesis, we model the multivariate intraday trading process of return volatility, volume and trading intensity by a VAR model that is augmented by a joint latent factor serving as a proxy for the unobserved information flow. The model is estimated by simulated maximum likelihood using efficient importance sampling techniques. Analyzing intraday data from the NYSE, we find strong empirical evidence for the existence of an underlying persistent component as an important driving force of the trading process. It is shown that the inclusion of the latent factor clearly improves the goodness-of-fit of the model as well as its dynamical and distributional properties.

Keywords: Observation vs. parameter driven dynamics, mixture-of-distribution hypothesis, VAR model, efficient importance sampling

JEL Classification: C15, C32, C52

1 Introduction

The basic idea of the mixture-of-distribution hypothesis as introduced by Clark (1973) is to explain the persistence in daily price volatility by an unknown autoregressive process of price relevant information. This idea was further developed by Tauchen and Pitts (1983) to model the relationship between daily volatility and volume. In their model, volume and volatility are jointly directed by a single autoregressive latent component. While these models and further extensions (see, for example, Andersen, 1996 and Liesenfeld, 1998, 2001) have been successfully applied to daily data, there is lacking evidence regarding their use to model intraday trading processes on financial markets.

The aim of this paper is to analyze whether on an intraday level a common autocorrelated latent component can be identified and thus evidence for the mixture-of-distribution hypothesis can be found. This objective is on the one hand motivated by the notion that the existence of an underlying autoregressive information process should be independent from the aggregation

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level. Therefore, a joint latent dynamic component should be identifiable not only based on daily data but also on an intraday level. On the other hand, the existence of an (unknown) information process which jointly affects all trading components is the fundamental underpinning of numerous theoretical market microstructure approaches.[1] A typical assumption is that market participants infer from the trading process with respect to the existence of information. This should cause systematic interdependencies between intraday price changes, volumes and trading intensities (see e.g. Easley and O'Hara, 1992). In fact, using VAR-type models, recent empirical studies confirm predictions arising from theoretical literature by providing empirical evidence for distinct relationships between these variables.[2]

The idea of this paper is to propose a framework which allows us to directly model the unknown underlying autoregressive (information) process by a latent dynamic component. Hence, we extend recent approaches by augmenting a VAR model for the intraday volatility, the trading volume and the trading intensity by a dynamic latent factor which simultaneously influences all three components. The so-called latent factor VAR (LF-VAR) model enables us to test for the existence of an underlying autoregressive component serving as a major driving force for contemporaneous relationships as well as interdependencies between volatilities, volumes and trading intensities. Therefore, the contribution of this paper to the literature is two-fold: It contributes, on the one hand, to the literature on dynamic multivariate latent factor models and, on the other hand, to recent literature on high-frequency financial data in which the modelling of intraday trading processes and a deeper understanding of market microstructure relations is an important task.

A well known result in the literature on multivariate latent factor models is that a single latent component is typically not sufficient to fully capture the dynamics of a multivariate system (see Andersen, 1996, or Liesenfeld, 1998). For this reason, Liesenfeld (2001) proposes a multi-factor model for daily volatility and volume by combining a common latent factor with additional process-specific latent components. He illustrates a significant improvement of the model's explanatory power and goodness-of-fit compared to an one-factor model. However, the major disadvantage of a latent multi-factor model is that it is computationally quite burdensome, particularly for high-dimensional processes. For this reason, we adopt the idea proposed by Bauwens and Hautsch (2003) and specify the LF-VAR model by combining a *parameter driven* dynamic for the latent factor with an *observation driven* dynamic for the individual components.[3]

[1] See e.g. Diamond and Verrecchia (1987), Admati and Pfleiderer (1988), Easley and O'Hara (1992), Blume, Easley, and O'Hara (1994) or Easley, Kiefer, O'Hara, and Paperman (1996) among others.

[2] See e.g. Manganelli (2000), Gerhard and Pohlmeier (2002), Spierdijk (2002), or Spierdijk, Nijman, and van Soest (2002).

[3] The terminology of parameter driven vs. observation driven dynamics is introduced by Cox (1981). The latter refers to the idea that a dynamic is updated based on past variables which are either directly observable or can be computed using a prediction error decomposition (like e.g. for a moving average process). In contrast, a dynamic latent factor can be alternatively interpreted as a parameter of the model which follows its own dynamics (therefore *parameter* driven dynamic). In this context, the innovations of the process are completely unobservable

This approach is motivated by the idea that there is one *common* latent factor which serves as a proxy for the unobserved information process and jointly affects volatility, volume and trading intensity. In addition, the individual trading components are influenced by process-specific (idiosyncratic) dynamics given the latent factor. In fact, it will be shown that both types of dynamics are necessary to fully capture the strong serial interdependencies in the multivariate system. The major advantage of assuming an observation driven dynamic for the idiosyncratic components is that the LF-VAR model is easily extended to higher-dimensional processes since we do not have to include additional latent factors.

The combination of a parameter driven dynamic with observation driven dynamics results in a highly flexible model. In particular, the volatility component follows a specification which encompasses a logarithmic GARCH specification as well as a stochastic volatility (SV) model (Taylor, 1982). Correspondingly, the specification of the volume and trading intensity components nest the Log-ACD model proposed by Bauwens and Giot (2000) as well as the stochastic conditional duration (SCD) model introduced by Bauwens and Veredas (2004). Moreover, the impact of the latent factor on the individual processes is allowed to be different from component to component. Therefore, the LF-VAR model enables us to analyze whether and, if yes, how strong the individual processes are influenced by the common factor.

Using this model we raise several research questions: (i) Is there evidence for a joint latent component and how strong is the persistence of this factor? (ii) How strong and in which direction influences the common component the processes of intraday volatility, trading volume and trading intensity? (iii) Is a joint unobservable factor a major source for contemporaneous correlations between the individual components of the trading process? I.e., do we find differences between conditional contemporaneous correlations given the latent component and corresponding unconditional correlations? (iv) Does the inclusion of a joint parameter driven dynamic improves the goodness-of-fit and the explanatory power of the VAR model?

The LF-VAR model is estimated by simulated maximum likelihood (SML). The computation of the likelihood requires to integrate the latent component out leading to an integral of the dimension of the sample range. For this reason, we approximate the likelihood function numerically by adopting the efficient importance sampling (EIS) algorithm proposed by Richard (1998). In our applications, this approach is shown to be efficient and computationally feasible.

The empirical analysis uses transaction data from four highly liquid stocks traded at the New York Stock Exchange (NYSE). Strong empirical evidence for the existence of an autoregressive common component is provided. Hence, from an economic point of view, we clearly confirm the mixture-of-distribution-hypothesis on an intraday level. Furthermore, it is shown that the inclusion of the latent component clearly improves the goodness-of-fit as well as the

and cannot be computed using a prediction error decomposition on the basis of past (observable) variables.

dynamical and distributional properties of the model. Moreover, we find that the unobservable factor is a major driving force of the interdependencies as well as contemporaneous relationships between the individual trading components.

The remainder of the paper is organized in the following way: Section 2 presents the LF-VAR model while Section 3 discusses its statistical properties. In Section 4, we illustrate the statistical inference. Section 5 shows the data and discusses the estimation results. Finally, Section 6 concludes.

2 The latent factor VAR model

Define $\{Y_i, V_i, \rho_i\}$, $i = 1, \ldots, N$, as the three-dimensional time series of positive random variables associated with the intraday process of returns, transaction volumes and trading intensities, respectively. Since the model can be applied either to (irregularly spaced) transaction data or to aggregated (regularly spaced) high-frequency data, the variables Y_i, V_i and ρ_i can be defined in alternative ways. In the first case, ρ_i would correspond to the time between consecutive trades, V_i would measure the trading volume per transaction and Y_i would be associated with the trade-to-trade log return standardized by $\sqrt{\rho_i}$. Then, Y_i^2 would correspond to the trade-to-trade volatility per time.[4] In the second case, which is used in this paper, Y_i corresponds to the log return measured over equi-distant time intervals (here 5 minute intervals), V_i is the cumulated or average volume traded in the i-th interval and ρ_i is the number of trades occurring during the i-th interval. Furthermore, λ_i is defined as a common unobservable component that simultaneously influences Y_i, V_i and ρ_i and follows an autoregressive process which is updated in every interval i.

Denote $\mathcal{F}_i := \sigma(\mathcal{F}_i^o \cup \mathcal{F}_i^*)$ as the information set consisting of the history of the observable processes $\mathcal{F}_i^o := (Y_i, V_i, \rho_i, Y_{i-1}, V_{i-1}, \rho_{i-1}, \ldots)$ and the history of the latent component $\mathcal{F}_i^* := (\lambda_i, \lambda_{i-1}, \ldots)$. Following Engle (2000), we propose to decompose the joint conditional density given the information set $\mathcal{F}_{i-1}$, $f(Y_i, V_i, \rho_i, \lambda_i | \mathcal{F}_{i-1})$, into the product of the corresponding conditional densities. Hence,

$$\begin{aligned} f(Y_i, V_i, \rho_i, \lambda_i | \mathcal{F}_{i-1}) &= f(Y_i | V_i, \rho_i, \lambda_i; \mathcal{F}_{i-1}) \cdot f(V_i, \rho_i, \lambda_i; \mathcal{F}_{i-1}) \qquad (1) \\ &= f(Y_i | V_i, \rho_i, \lambda_i; \mathcal{F}_{i-1}) \cdot f(V_i | \rho_i, \lambda_i; \mathcal{F}_{i-1}) \cdot f(\rho_i | \lambda_i; \mathcal{F}_{i-1}) \cdot f(\lambda_i; \mathcal{F}_{i-1}^*), \end{aligned}$$

where it is assumed that λ_i depends only on its own history $\mathcal{F}_{i-1}^*$. The chosen decomposition implies a triangular structure since Y_i is assumed to depend on both the contemporaneous volume V_i and trading intensity ρ_i, whereas V_i depends only on ρ_i. Finally, ρ_i itself is not affected by any contemporaneous variable. Of course, the order of the variables in the decomposition is arbitrary and depends on the research objective. However, researchers are typically particularly

[4] This setting was proposed by Engle (2000).

interested in the volatility process given the contemporaneous volume and the contemporaneous trading intensity (see Engle, 2000 or Manganelli, 2000). This motivates the order chosen in this approach.

The basic idea of the so-called latent factor VAR (LF-VAR) model is to combine *observation driven* dynamics with *parameter driven* dynamics. This structure is reflected in the specification of the processes Y_i, V_i and ρ_i as given by

$$Y_i = \mathrm{E}[Y_i|\mathcal{F}_{i-1}] + \xi_i, \tag{2}$$

$$\xi_i = \sqrt{h_i \lambda_i^{\delta_1}} \eta_i, \qquad \eta_i \sim \text{i.i.d.}\, N(0,1) \tag{3}$$

$$V_i = \Phi_i \lambda_i^{\delta_2} u_i, \qquad u_i \sim \text{i.i.d.}\, \mathcal{GG}(a_1, m_1) \tag{4}$$

$$\rho_i = \Psi_i \lambda_i^{\delta_3} \varepsilon_i, \qquad \varepsilon_i \sim \text{i.i.d.}\, \mathcal{GG}(a_2, m_2), \tag{5}$$

where h_i, Φ_i and Ψ_i denote (observation driven) dynamic components and η_i, u_i and ε_i are process-specific innovation terms which are assumed to be independent. We assume that the volatility innovations η_i follow a standard normal distribution whereas the volume and trading intensity innovations u_i and ε_i follow a standard generalized gamma distribution depending on the parameters a_1, m_1 and a_2, m_2, respectively[5]. The generalized gamma distribution allows for a high distributional flexibility including the cases of over-dispersion and under-dispersion as well as non-monotonic hazard shapes.

Note that the component $h_i \lambda_i^{\delta_1}$ corresponds to the conditional variance of returns given $\mathcal{F}_{i-1}$. Accordingly, up to a constant multiplicative factor[6], $\Phi_i \lambda_i^{\delta_2}$ and $\Psi_i \lambda_i^{\delta_3}$ correspond to the conditional expected volume and the conditional expected trading intensity given $\mathcal{F}_{i-1}$. Hence, the major idea of the LF-VAR model is to model these conditional moments on the basis of a multiplicative interaction of the processes $\{h_i, \Phi_i, \Psi_i\}$ and λ_i. Thus the parameters δ_1, δ_2 and δ_3 drive the process-specific impact of λ_i. Therefore, the variables h_i, Φ_i and Ψ_i correspond to the conditional variance, the conditional expected volume and the conditional expected trading intensity given $\mathcal{F}_{i-1}$ and λ_i.

Whereas specification (3) corresponds to an augmented GARCH model, (4) and (5) are associated with augmented ACD type models as proposed by Engle and Russell (1998). Note that particularly for less liquid stocks, the continuity assumption for ε_i can be questionable since the number of trades per interval is a counting variable and thus is clearly discrete. In such a case, the use of a conditional autoregressive Poisson process as proposed by Davis, Rydberg, Shephard, and Streett (2001) or Heinen (2002) could be more appropriate. However, in the

[5]The probability density function of the (standard) generalized gamma distribution is given by

$$f(x) = \frac{a}{\Gamma(m)} x^{ma-1} \exp\left[-x^a\right], \quad a > 0, m > 0.$$

[6]Note that the means of u_i and ε_i are not necessarily equal to one.

given application, the number of trades per (5 min) interval varies on average between 20 and 42[7] and thus can be considered as continuous.[8]

For the latent factor, we assume a log-linear AR(1) process, given by

$$\ln \lambda_i = a_0 \ln \lambda_{i-1} + \nu_i, \quad \nu_i \sim \text{i.i.d. } N(0,1), \tag{6}$$

where ν_i is assumed to be independent of η_i, u_i and ε_i. By reformulating the process-specific impact of the latent factor, $\ln \lambda_{i,j} := \ln \lambda_i^{\delta_j}$, as

$$\ln \lambda_{i,j} = \delta_j a_0 \ln \lambda_{i-1} + \delta_j \nu_i = a_0 \ln \lambda_{i-1,j} + \delta_j \nu_i, \tag{7}$$

it is evident that the parameters δ_j act multiplicatively on the standard deviation of the latent process.[9] Since $\frac{d \ln \lambda_{ij}}{d\nu_i} > (<)\, 0$ for $\delta_j > (<)\, 0$ with $i = 1, 2, 3$, the parameter δ_j determines the strength and the direction of a latent shock's influence on the component j. Note that because of the symmetry of the distribution of ν_i, the sign of the individual parameters δ_j are not identified. Hence, we cannot distinguish between the cases $\delta_1 > 0, \delta_2 < 0$ versus $\delta_1 < 0, \delta_2 > 0$. Nevertheless, we can identify whether the latent component influences the two components in the same or in the opposite direction. For that reason, we have to impose an identification assumption which restricts the sign of one of the parameters δ_j. Then, the signs of all other δ_j's are identified.

Whereas the latent factor follows a parameter driven dynamic, the process-specific components h_i, Φ_i and Ψ_i are assumed to follow a multivariate observation driven dynamic which is parameterized in terms of a VAR system:

$$\mu_i = \omega + A_0 z_{0,i} + \sum_{j=1}^{p} A_j z_{i-j} + \sum_{j=1}^{q} B_j \mu_{i-j}, \tag{8}$$

where

$$\begin{aligned} \mu_i &:= (\ln h_i \quad \ln \Phi_i \quad \ln \Psi_i)', \\ z_{0,i} &:= (0 \quad \ln V_i \quad \ln \rho_i)', \\ z_i &:= \left(\frac{|\xi_i|}{\sqrt{h_i}} \quad \frac{V_i}{\Phi_i} \quad \frac{\rho_i}{\Psi_i} \right)', \end{aligned}$$

ω denotes a (3×1) vector, and $A_0 = \{\alpha_{0,i}\}$ is a (3×3) triangular matrix where only the three upper right elements are nonzero. Furthermore, $A_j = \{\alpha_{j,ik}\}$ and $B_j = \{\beta_{j,ik}\}$ are (3×3) matrices of innovation and persistence parameters, respectively. The assumption of a logarithmic form ensures the positiveness of the individual processes without imposing additional parameter

[7] See the descriptive statistics in Section 6.

[8] Moreover, the individual components are modelled in a seasonally adjusted form where the original time series are divided by estimated seasonality components. This standardization step generates realizations which are clearly continuous.

[9] Hence, in order to identify the δ_j's, the latent variance $\text{Var}[\nu_i]$ is normalized to one.

restrictions. This property eases the estimation of the model particularly when $A_0 \neq 0$ and/or when additional explanatory variables are included. The triangular structure of A_0 reflects the used decomposition of the joint density in (1).

Note that the idiosyncratic innovation processes in (8) are updated based on variables which are completely observable even when there exists a latent component. This reduces the computationally effort and is particularly important to make the model applicable and estimable even for high-dimensional processes. However, on the other hand, this structure implies that the innovations z_i implicitly depend on λ_i which is evident by expressing z_i alternatively as

$$z_i = \left(|\eta_i| \lambda_i^{\delta_1/2} \quad u_i \lambda_i^{\delta_2} \quad \varepsilon_i \lambda_i^{\delta_3} \right)'. \tag{9}$$

Hence, the latent factor λ_i influences the observation driven components $\{h_i,\ \Phi_i,\ \Psi_i\}$ not only directly according to the specifications (3) to (5) but affects them also through the corresponding innovation processes. I.e., a shock in the latent factor in period i influences $\{h_i,\ \Phi_i,\ \Psi_i\}$ not only in period i, but (through z_i) also in the following periods which causes autocorrelations between the individual processes. The resulting dynamic properties of the assumed process will be discussed in Section 3.

In order to illustrate the parameterizations of the individual components in more detail, assume for simplicity $A_0 = 0$, $p = q = 1$, and diagonal parameterizations of A_1 and B_1. Then, the model is rewritten as

$$\xi_i = \sqrt{\tilde{h}_i}\eta_i, \qquad \tilde{h}_i = h_i \lambda_i^{\delta_1}, \tag{10}$$

$$V_i = \tilde{\Phi}_i u_i, \qquad \tilde{\Phi}_i = \Phi_i \lambda_i^{\delta_2}, \tag{11}$$

$$\rho_i = \tilde{\Psi}_i \varepsilon_i, \qquad \tilde{\Psi}_i = \Psi_i \lambda_i^{\delta_3}, \tag{12}$$

where

$$\ln \tilde{h}_i - \delta_1 \ln \lambda_i = \omega_1 + \alpha_{11} \frac{|\xi_{i-1}|}{\sqrt{h_{i-1}}} + \beta_{11}(\ln \tilde{h}_{i-1} - \delta_1 \ln \lambda_{i-1}), \tag{13}$$

$$\ln \tilde{\Phi}_i - \delta_2 \ln \lambda_i = \omega_2 + \alpha_{22} \frac{V_{i-1}}{\Phi_{i-1}} + \beta_{22}(\ln \tilde{\Phi}_{i-1} - \delta_2 \ln \lambda_{i-1}), \tag{14}$$

$$\ln \tilde{\Psi}_i - \delta_3 \ln \lambda_i = \omega_3 + \alpha_{33} \frac{X_{i-1}}{\Psi_{i-1}} + \beta_{33}(\ln \tilde{\Psi}_{i-1} - \delta_3 \ln \lambda_{i-1}). \tag{15}$$

Hence, it is evident that the latent component λ_i can be interpreted as an additional regressor which is statically included and is driven by its own dynamics according to (6).

We call the first component of the LF-VAR model a latent factor GARCH (LF-GARCH) model, whereas the second and third component is referred to a latent factor ACD (LF-ACD) model. These specifications nest several model classes. The LF-GARCH model encompasses a basic (Log-)GARCH specification as well as the stochastic volatility (SV) model proposed by Taylor (1986) and permits both competing models to be tested against each other.

In particular, for $\alpha_{11} = 0$, (13) can be rewritten as an SV model, while for $\delta_1 = 0$ it resembles a logarithmic GARCH specification which can be easily extended to the EGARCH model as introduced by Nelson (1991). Furthermore, for $\beta_{11} = 0$ it can be interpreted as an SV model that is mixed with a further random variable. Accordingly, the LF-ACD models as specified in (14) and (15) nest SCD models (Bauwens and Veredas, 2004) for $\alpha_{22} = 0$ and $\alpha_{33} = 0$, respectively, Log-ACD models (Bauwens and Giot, 2000) for $\delta_2 = 0$ and $\delta_3 = 0$, respectively, and, correspondingly, mixed SCD models for $\beta_{22} = 0$ and $\beta_{33} = 0$, respectively.

3 Properties of the model

3.1 Weak stationarity

The following proposition establishes the stationarity properties of the model:
Proposition 1: *Let $|a_0| < 1$ and $|\varsigma| < 1$ for all values of ς satisfying*

$$\left| I\varsigma^q - B_1\varsigma^{q-1} - B_2\varsigma^{q-2} \ldots - B_q \right| = 0,$$

where I denotes a (3×3) identity matrix. Then, the processes (2) through (5) are weakly stationary.
Proof: See Appendix A.1.

3.2 Unconditional moments

The inclusion of the latent component in the VAR model renders the analytical computation of unconditional moments and (cross-)autocorrelation functions generally quite difficult. In the following, we analyze the statistical properties of the model based on numerous simulation studies. For a wide range of different specifications of the LF-VAR model, we generate 100 sets of $50,000$ observations and analyze the distributional and dynamical properties. Tables 1 and 2[10] show the mean, standard deviation, minimum, maximum, kurtosis, different quantiles as well as the Ljung and Box (1978) statistic for (univariate) LF-GARCH and LF-ACD processes under different parameterizations.[11] Table 1 illustrates that the inclusion of a latent component has a strong influence on the standard deviation, the kurtosis as well as the serial dependence in the second moments of the simulated return process. We observe that processes generated by high parameter values of α_0 and δ_1 imply a high unconditional variance, overkurtosis, fat tails as well as a strong serial dependence in the conditional variance. It is evident that an LF-GARCH process allows for a high distributional and dynamical flexibility and captures the well known statistical properties of typical financial return series.

[10] All tables and figures are shown in the Appendix.
[11] Since the distribution of returns under a LF-GARCH process is symmetric and the conditional mean is set to zero, only the quantiles of the right tail of the distribution are shown.

Similar findings are revealed for simulated LF-ACD processes (Table 2). Again, an increase of the latent parameters α_0 and δ_3 leads to a significant rise of the unconditional variance as well as of the autocorrelations of the resulting process. As for LF-GARCH processes, it is apparent that a high serial dependence in both the observation driven component and the parameter driven component generate distributions with strong fat tail behavior. These effects are even amplified when the Weibull parameter a_3 is larger than one (specifications (15) and (16)).

Figures 1 through 16 show the autocorrelation and cross-autocorrelation functions implied by a two-dimensional LF-VAR(1,1) model for the volatility and intensity process[12]. Figures 1 through 5 show LF-VAR processes without any interdependencies between the individual observation driven components h_i and Ψ_i. It is shown that the latent component generates distinct cross-autocorrelations between both h_i and Ψ_i as well as between Y_i^2 and ρ_i. The fact that the cross-autocorrelation function (CACF) between h_i and Ψ_i is nonzero is caused by the fact that both processes are updated by innovations z_{i-1} which themselves depend on λ_{i-1} (see eq. (9)). Hence, particularly in Figures 1 through 3, the dynamics of the processes are completely dominated by λ_i. In Figures 4 and 5, both the observation driven components and the parameter driven component reveal a relatively high persistence. Here, the dynamics of λ_i amplify the dynamics of Y_i^2 and ρ_i. In contrast, Figures 6 through 8 show processes where $\lambda_i = 0$. Even though theses processes imply interactions between h_i and Ψ_i (yielding high cross-autocorrelations), the CACF of Y_i^2 and ρ_i is close to zero.[13] This picture changes clearly when we allow for a non-zero latent factor (Figures 9 and 10). Here, the persistent parameter driven dynamics imply a significant rise of the cross-autocorrelations between both processes. Figures 11 and 12 illustrate the effects when the latent factor reveals no serial dependence ($\alpha_0 = 0$), however, high standard deviations. Because of the strong impact of λ_i (implied by high values of δ_1 and δ_2) the persistence in the CACF is clearly reduced. In Figure 12 the observation driven components interact positively whereas λ_i influences both processes in opposite directions leading to a clear reduction of the CACF between Y_i^2 and ρ_i. Similar effects can be observed in Figures 13 to 15 where the observation driven and parameter driven components work in opposite directions. Finally, Figure 16 shows the impact of the latent component when the individual observation driven processes reveal a different persistence. In this case, it is evident that λ_i dominates the (joint) dynamics of the resulting processes and makes them more similar.

[12] Since the volume component is parameterized similarly, it reveals the same properties and same interactions with the other processes. For this reason, we refrain from showing the results for three-dimensional processes since it would lengthen the exposition considerably.

[13] The asymmetric cross-autocorrelations between Y_i^2 and ρ_i are caused by the fact that the return innovation ξ_i is driven by the square root of h_i, whereas ρ_i is driven by Ψ_i itself.

4 SML estimation of the LF-VAR model

Let W denote the data matrix and define $w_i := (Y_i \; V_i \; \rho_i)$ as a row of this matrix with $W_i := \{w_j\}_{j=1}^{i}$. Moreover, let θ denote the vector of parameters of the LF-VAR model.

The conditional likelihood given the realizations of the latent variable $L_i := \{\lambda_j\}_{j=1}^{i}$ is given by

$$\mathcal{L}(W;\theta|L_n) = \prod_{i=1}^{n} \frac{1}{\sqrt{2h_i\lambda_i^{\delta_1}\pi}} \exp\left[-\frac{\xi_i^2}{2h_i\lambda_i^{\delta_1}}\right] \frac{a_2 V_i^{a_2 m_2 - 1}}{\Gamma(m_2)\Phi_i^{a_2 m_2}\lambda_i^{\delta_2 a_2 m_2}} \exp\left[-\left(\frac{V_i}{\Phi_i\lambda_i^{\delta_2}}\right)^{a_2}\right] \tag{16}$$
$$\times \frac{a_3 \rho_i^{a_3 m_3 - 1}}{\Gamma(m_3)\Psi_i^{a_3 m_3}\lambda_i^{\delta_3 a_3 m_3}} \exp\left[-\left(\frac{\rho_i}{\Psi_i\lambda_i^{\delta_3}}\right)^{a_3}\right].$$

Since the latent process is not observable the conditional likelihood function must be integrated with respect to λ_i using the assumed (log-normal) distribution of the latter. Hence, the integrated log likelihood function is given by

$$\mathcal{L}(W;\theta) = \int \prod_{i=1}^{n} \frac{1}{\sqrt{2h_i\lambda_i^{\delta_1}\pi}} \exp\left[-\frac{\xi_i^2}{2h_i\lambda_i^{\delta_1}}\right] \frac{a_2 V_i^{a_2 m_2 - 1}}{\Gamma(m_2)\Phi_i^{a_2 m_2}\lambda_i^{\delta_2 a_2 m_2}} \exp\left[-\left(\frac{V_i}{\Phi_i\lambda_i^{\delta_2}}\right)^{a_2}\right] \tag{17}$$
$$\times \frac{a_3 \rho_i^{a_3 m_3 - 1}}{\Gamma(m_3)\Psi_i^{a_3 m_3}\lambda_i^{\delta_3 a_3 m_3}} \exp\left[-\left(\frac{\rho_i}{\Psi_i\lambda_i^{\delta_3}}\right)^{a_3}\right] \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\ln\lambda_i - \mu_{0,i}\right)^2\right] dL$$
$$= \int \prod_{i=1}^{n} g(w_i|\lambda_i, W_{i-1};\theta) p(\lambda_i|L_{i-1};\theta) dL = \int \prod_{i=1}^{n} f(w_i, \lambda_i|W_{i-1}, L_{i-1};\theta) dL,$$

where $\mu_{0,i} := \mathrm{E}[\ln\lambda_i|\mathcal{F}_{i-1}^*]$, $g(\cdot)$ denotes the conditional density of w_i given (λ_i, W_{i-1}) and $p(\cdot)$ denotes the conditional density of λ_i given L_{i-1}. The computation of the n-dimensional integral in (17) is performed numerically using the efficient importance sampling (EIS) method proposed by Richard (1998). This algorithm was shown to work very well in the context of the class of latent factor models (see Liesenfeld and Richard, 2002 or, Bauwens and Hautsch, 2003).

To implement the EIS algorithm, the integral (17) is rewritten as

$$\mathcal{L}(W;\theta) = \int \prod_{i=1}^{n} \frac{f(w_i, \lambda_i|W_{i-1}, L_{i-1};\theta)}{m(\lambda_i|L_{i-1}, \phi_i)} \prod_{i=1}^{n} m(\lambda_i|L_{i-1}, \phi_i) dL, \tag{18}$$

where $\{m(\lambda_i|L_{i-1}, \phi_i)\}_{i=1}^{n}$ denotes a sequence of auxiliary importance samplers indexed by auxiliary parameters ϕ_i. Then, the importance sampling estimate of the likelihood is obtained by

$$\mathcal{L}(W;\theta) \approx \hat{\mathcal{L}}_R(W;\theta) = \frac{1}{R}\sum_{r=1}^{R}\prod_{i=1}^{n} \frac{f(w_i, \lambda_i^{(r)}(\phi_i)|W_{i-1}, L_{i-1}^{(r)}(\phi_{i-1});\theta)}{m(\lambda_i^{(r)}(\phi_i)|L_{i-1}^{(r)}(\phi_{i-1}), \phi_i)}, \tag{19}$$

where $\{\lambda_i^{(r)}(\phi_i)\}_{i=1}^{n}$ denotes a trajectory of random draws from the sequence of auxiliary importance samplers m and R such trajectories are generated. The idea of the EIS approach is to choose a sequence of samplers for $m(\lambda_i|L_{i-1}, \phi_i)$ that exploits the sample information on the λ_i's revealed by the observable data. Therefore, as shown by Richard (1998), the EIS principle is to choose the auxiliary parameters $\{\phi_i\}_{i=1}^n$ in a way that provides a good match between $\Pi_{i=1}^n m(\lambda_i|L_{i-1}, \phi_i)$ and $\Pi_{i=1}^n f(w_i, \lambda_i|W_{i-1}, L_{i-1}; \theta)$ in order to minimize the Monte Carlo sampling variance of $\hat{\mathcal{L}}_R(W; \theta)$. Richard (1998) illustrates that this minimization problem can be split up into solvable low-dimensional subproblems. To define the importance sampler itself let $k(L_i, \phi_i)$ denote a density kernel for $m(\lambda_i|L_{i-1}, \phi_i)$, given by

$$k(L_i, \phi_i) = m(\lambda_i|L_{i-1}, \phi_i)\chi(L_{i-1}, \phi_i), \tag{20}$$

where

$$\chi(L_{i-1}, \phi_i) = \int k(L_i, \phi_i) d\lambda_i \tag{21}$$

denotes the integrating constant. The implementation of EIS requires to select a class of density kernels $k(\cdot)$ for the auxiliary sampler $m(\cdot)$ which provide a good approximation to the product $f(\cdot)\chi(\cdot)$. As discussed by Richard (1998), a convenient and efficient possibility is to use a parametric extension of the direct samplers, Gaussian distributions in this context. Since the function $g(\cdot)$ appearing in (17) is essentially a product of different exponential functions, we propose to approximate it by a normal density kernel

$$\zeta(\lambda_i, \phi) = \exp\left(\phi_{1,i} \ln \lambda_i + \phi_{2,i}(\ln \lambda_i)^2\right), \tag{22}$$

which is itself an exponential function in terms of $\ln \lambda_i$ based on the auxiliary parameters $\phi_i = (\phi_{1,i}, \phi_{2,i})$. Exploiting the property that the product of normal densities is itself a normal density, we parameterize $k(\cdot)$ as

$$k(L_i, \phi_i) = p(\lambda_i|L_{i-1}; \theta)\zeta(\lambda_i, \phi_i)$$

and can show that

$$\begin{aligned} k(L_i, \phi_i) &\propto \exp\left((\phi_{1,i} + \mu_{0,i}) \ln \lambda_i + \left(\phi_{2,i} - \frac{1}{2}\right)(\ln \lambda_i)^2\right) \\ &= \exp\left(-\frac{1}{2\pi_i^2}(\ln \lambda_i - \mu_i)^2\right) \exp\left(\frac{\mu_i^2}{2\pi_i^2}\right), \end{aligned} \tag{23}$$

where

$$\pi_i^2 = (1 - 2\phi_{2,i})^{-1} \tag{24}$$

$$\mu_i = (\phi_{1,i} + \mu_{0,i})\, \pi_i^2. \tag{25}$$

Hence, the auxiliary sampler $m(\cdot)$ is a normal distribution with conditional mean μ_i and conditional variance π_i^2. By omitting irrelevant multiplicative factors, we obtain the integrating constant as

$$\chi(L_{i-1}, \phi_i) = \exp\left(\frac{\mu_i^2}{2\pi_i^2} - \frac{\mu_{0,i}^2}{2}\right). \tag{26}$$

As shown by Richard (1998), the Monte Carlo variance of $\hat{\mathcal{L}}_R(W;\theta)$ can be minimized by splitting the minimization problem into n minimization problems of the form

$$\begin{aligned}\min_{\phi_{i,0},\,\phi_i} \sum_{r=1}^{R} \Big\{ \ln f\left[\left(w_i, \lambda_i^{(r)}(\theta)|W_{i-1}, L_{i-1}^{(r)}(\theta), \theta\right) \cdot \chi\left(L_i^{(r)}(\theta), \phi_{i+1}(\theta)\right)\right] \\ -\phi_{0,i} - \ln k\left(L_i^{(r)}(\theta), \phi_i(\theta)\right)\Big\}^2,\end{aligned} \tag{27}$$

where $\phi_{0,i}$ is a constant and $\{\lambda_i^{(r)}(\theta)\}_{i=1}^n$ with $\lambda_i^{(r)}(\theta) := \lambda_i^{(r)}(\phi_i(\theta))$ denotes a trajectory of random draws from the sampler m with auxiliary parameters $\phi_i(\theta)$ which themselves depend on the model parameters θ.

In practice the implementation of the ML-EIS estimator requires the following steps:

(i) Draw R trajectories of the latent factor $\{\lambda_i^{(r)}(\phi_i)\}_{i=1}^n$ using the direct sampler $p(\cdot)$.

(ii) For $i : n \to 1$ solve the least squares problem characterized by the (auxiliary) linear regression

$$D_{1,i}^{(r)} + D_{2,i}^{(r)} + D_{3,i}^{(r)} + D_{4,i}^{(r)} = \phi_{0,i} + \phi_{1,i}\lambda_i^{(r)}(\theta) + \phi_{2,i}\left[\lambda_i^{(r)}(\theta)\right]^2 + \epsilon_i^{(r)}, \quad r = 1, \ldots, R,$$

where

$$D_{1,i}^{(r)} = -\frac{1}{2}\left(\ln h_i + \delta_1 \ln \lambda_i^{(r)}(\theta)\right) - \frac{\xi_i^2}{2h_i\left(\lambda_i^{(r)}(\theta)\right)^{\delta_1}},$$

$$D_{2,i}^{(r)} = (a_2 m_2 - 1)\ln V_i - \left(a_2 m_2 \ln \Phi_i + \delta_2 a_2 m_2 \ln \lambda_i^{(r)}(\theta)\right) - \left(\frac{V_i}{\Phi_i\left(\lambda_i^{(r)}(\theta)\right)^{\delta_2}}\right)^{a_2},$$

$$D_{3,i}^{(r)} = (a_3 m_3 - 1)\ln \rho_i - \left(a_3 m_3 \ln \Psi_i + \delta_3 a_3 m_3 \ln \lambda_i^{(r)}(\theta)\right) - \left(\frac{\rho_i}{\Psi_i\left(\lambda_i^{(r)}(\theta)\right)^{\delta_3}}\right)^{a_3},$$

$$D_{4,i}^{(r)} = \ln \chi\left(L_i^{(r)}(\theta), \phi_{i+1}(\theta)\right),$$

and $\epsilon_i^{(r)}$ denotes the regression error term. These problems are solved sequentially starting at $i = n$, under the initial condition $\chi(L_n, \phi_{n+1}) = 1$ and ending at $i = 1$. Liesenfeld and Richard (2002) recommend to iterate the procedure about three to five times to improve the efficiency of the approximations.

(iii) Compute the EIS sampler $\{m(\lambda_i|L_{i-1}, \hat{\phi}(\hat{\theta})\}_{i=1}^n$ on the basis of the conditional mean and variance given in (24) and (25) in order to draw R trajectories $\{\lambda_i^{(r)}(\hat{\phi}_i(\hat{\theta}))\}_{i=1}^n$. These trajectories are used to calculate the likelihood according to (19).

The residuals of the individual processes are computed on the basis of the trajectories drawn from the sequence of auxiliary samplers which are used to compute the likelihood function in (19), leading to R residual series for each variable

$$\hat{\eta}_i^{(r)} = \frac{\hat{\xi}_i}{\sqrt{\hat{h}_i[\lambda_i^{(r)}(\hat{\phi}_i(\hat{\theta}))]^{\hat{\delta}_1}}}; \quad \hat{u}_i^{(r)} = \frac{V_i}{\hat{\Phi}_i[\lambda_i^{(r)}(\hat{\phi}_i(\hat{\theta}))]^{\hat{\delta}_2}}; \quad \hat{\varepsilon}_i^{(r)} = \frac{\rho_i}{\hat{\Psi}_i[\lambda_i^{(r)}(\hat{\phi}_i(\hat{\theta}))]^{\hat{\delta}_3}}.$$

Summary statistics of the residuals as well as residual diagnostics are computed for each of the R sequences separately.

5 Empirical results

5.1 Regression results

The empirical study uses transaction data from the AOL, Boeing, IBM and JP Morgan stock traded at the New York Stock Exchange (NYSE). The data is extracted from the 2001 CD-Roms of the Trade and Quote (TAQ) database released by the NYSE and covers a period over 5 months between 02/01/01 and 31/05/01. An aggregation level of 5 minutes is used which is regarded as a trade-off between utilizing a maximum amount of intraday information on the one hand and reducing the influence of too much market microstructure noise on the other hand. Therefore, the resulting time series consist of 8008 observations of 5 min log midquote returns, the average 5 min trading volume and the number of transactions occurring in each interval as a measure for the trading intensity. Table 3 shows the mean, standard deviation, minimum, maximum, different quantiles, kurtosis as well as the univariate and multivariate Ljung-Box statistic associated with the individual time series. The latter is computed according to Ljung and Box (1978) and is given by

$$MLB(s) := n(n+2)\sum_{j=1}^{s} \frac{1}{n-j} trace\left(\hat{C}_j' \hat{C}_0^{-1} \hat{C}_j \hat{C}_0^{-1}\right) \sim \chi^2_{ks},$$

where k denotes the number of different time series, s the number of lags taken into account and $\hat{C}_j$ is the jth residual autocovariance matrix. For $k = 1$, the multivariate Ljung-Box statistic reduces to the well known univariate one. The quite high Ljung-Box statistics in Table 3 indicate that the 5 min trading data reveal strong serial (cross-)dependencies.

In order to account for intraday seasonality effects in return volatilities, volumes and trading intensities, we standardize these variables by their corresponding seasonality component. A simultaneous estimation of seasonality effects in a LF-VAR model is theoretically possible, however, increases the computational burden considerable. For this reason, we removed intraday seasonality effects in a first step by estimating cubic spline functions based on 30 minute nodes for all individual processes separately and dividing the original variables by their corresponding seasonality component.[14] Furthermore, we reduced the complexity of the model by estimating ξ_i in a separate step as the residuals of an ARMA(1,1) process for the Y_i series.[15]

Figures 17 through 20 show the empirical autocorrelation and cross-autocorrelation functions for the plain series as well as the corresponding seasonally adjusted series. It turns out that all processes reveal significantly positive autocorrelations and a relatively high persistence. The highest serial dependence is observed for the series of volumes and trading intensities, whereas for the volatility process lower autocorrelations are found. Moreover, significantly positive cross-autocorrelations between the return volatility and the trading volume are observed whereas the interdependencies between the volatility and the trading intensity are only very weak. In contrast, significantly negative cross-autocorrelations between the trading volume and the trading intensity are found. Hence, higher volumes enter the market with a lower speed.

Tables 4 through 6 show the estimation results of univariate LF-GARCH as well as LF-ACD models for 5 min volatilities, trading volumes and trading intensities for the four stocks. To ensure model parsimony, we restrict the models to specifications with a maximal lag order of two. For all processes and all stocks, we find significant evidence for the existence of a persistent latent component. As revealed by the estimates of the parameter α_0, the strongest serial dependence in the latent component is observed for the volatility and trading intensity processes, whereas it is lower for trading volumes. It turns out that both the parameter driven dynamic as well as the observation driven dynamic interact. In particular, α_0 declines when observation driven dynamics are included. Accordingly, in the observation driven component, the innovation parameter declines and the persistence parameter is driven towards one when the latent factor is taken into account. Hence, news enter the model primarily through the latent component, which is in line with the idea that the underlying factor serves as a proxy for the unobserved information process. Furthermore, it is shown that the inclusion of the latent component increases the goodness-of-fit as well as the dynamical properties of the model. Actually, for the volatility and the volume processes, a pure parameter driven dynamic in form of a SV and SCD specification, respectively (column (3)), outperforms a pure observation driven dynamic in form of a logarithmic GARCH or ACD specification, respectively (column (2)). Nevertheless, we observe that neither the parameter driven component nor the observation driven component can be rejected. Hence, for nearly all time series, the best goodness-of-fit is obtained by specifications (4) or (5) which include both types of dynamics.

Tables 7 to 10 give the estimation results for multivariate LF-VAR models including all three trading components. In order to identify the sign of the parameters δ_j, we restrict δ_1 to be positive. As in the univariate models, we ensure model parsimony by restricting the maximal

[14]For the process of squared returns, the cubic splines are estimated based on absolute log returns.

[15]However, since for all return series the ARMA component is very close to zero, and thus ξ_i is very similar to Y_i, we refrain from showing the estimates here.

lag order to two. In addition, we restrict A_2 and B_2 to be diagonal matrices. The major findings can be summarized as following:

(i) We find significant evidence for a persistent latent *common* component with an autoregressive parameter which is on average around $\hat{a}_0 \approx 0.94$. It turns out that the impact of the latent component is quite robust over all individual specifications. This result indicates that the latent factor seems to capture an underlying common autoregressive process which is obviously not covered even by high-parameterized observation driven dynamics. The estimated parameters δ_1, δ_2 and δ_3 are significantly positive indicating that a latent shock affects the volatility, the average trading volume and the trading intensity in the same direction. However, it turns out that the underlying joint component influences primarily the volatility and volume component, whereas its impact on the trading intensity is comparably low[16].

(ii) For all four stocks, the VAR specifications without latent factor (columns (1) through (3)) are not able to completely capture the dynamics of the system as indicated by highly significant Ljung-Box statistics for the residuals. However, the introduction of the latent component clearly improves the dynamic properties of the model. Given the results discussed under (i), it is not surprising that this is particularly true for the volatility and the volume component, whereas in some cases the dynamics in the trading intensity are not completely captured by the model. Moreover, the inclusion of the latent component leads to a reduction of the multivariate Ljung-Box statistic indicating that the latent component does a good job in capturing the multivariate dynamics and interdependencies between the individual processes. Furthermore, as revealed by the Bayes information criterion (BIC), the LF-VAR model yields a clearly better goodness-of-fit compared to VAR specifications without a latent factor.

(iii) The worst performance is observed for specification (4), where any observation driven dynamics are omitted and only a parameter driven dynamic is included. Hence, a single common autoregressive component is not sufficient to completely capture the dynamics of the multivariate system which is in line with the findings by Andersen (1996) or Liesenfeld (1998). Therefore, as in the univariate models we can neither reject the parameter driven dynamic nor the observation driven dynamic. Actually, the best performance is revealed by specifications which include both types of dynamics confirming the basic idea of the proposed model.

(iv) The inclusion of the latent factor leads to a significant decline of the magnitude of the parameters α_0^{12} and α_0^{13}. This indicates that the conditional contemporaneous correlations between ξ_i^2 and V_i as well as between ξ_i^2 and ρ_i given λ_i are lower than the corresponding unconditional correlations. However, in contrast, the parameter α_0^{23} remains relatively stable indicating that the (negative) contemporaneous relation between V_i and ρ_i is widely unaffected by the latent factor. Furthermore, it is shown that the inclusion of λ_i reduces the impact of

[16]For AOL it is even insignificant.

the individual idiosyncratic innovations and increases the persistence in the observation driven dynamics. Similarly, as indicated by a decline of the magnitude of the non-diagonal elements in A_1 and B_1, the latent factor reduces the cross-autocorrelations between the individual observation driven components. Hence, in accordance with the results for the univariate models, we find evidence that news enter the model primarily through the latent factor, whereas the impact of the process-specific innovations declines.

Since our major findings are quite robust across the different stocks, we can conclude that there is clear empirical evidence for the existence of an underlying persistent factor which jointly affects all individual trading components and is a major driving force for interdependencies as well as contemporaneous relationships between the particular processes.

In order to analyze the impact of shocks on the LF-VAR process, we rely on the concept of the generalized impulse response function (GIRF) introduced by Koop, Pesaran, and Potter (1996) which is given by

$$GIRF_{X_i}(h, \delta, \mathcal{F}_{i-1}) = \mathrm{E}[X_{i+h}|\varpi_i = \delta, \mathcal{F}_{i-1}] - \mathrm{E}[X_{i+h}|\mathcal{F}_{i-1}], \tag{28}$$

where $X_i \in \{\lambda_i, Y_i^2, V_i, \rho_i\}$, $\varpi_i \in \{\nu_i, \eta_i, u_i, \varepsilon_i\}$, δ is the magnitude of the shock, and h denotes the number of periods over which the GIRF is computed. As shown in this representation, the GIRF conditions on the shock and on the history of the process whereas innovations occurring in intermediate time periods are averaged out. Then, the GIRF can be interpreted as a random variable in terms of the history $\mathcal{F}_{i-1}$. In nonlinear models, analytical expressions for the conditional expectations used in (28) are often not available and thus, Monte-Carlo simulation techniques have to be performed. Figures 21 through 24 show the generalized impulse response functions for a shock in the latent innovation ν_i with magnitude of one standard deviation. The GIRF is computed by conditioning on the unconditional means $\mathrm{E}[X_i]$ and $\mathrm{E}[\varpi_i]$ and is estimated by

$$\widehat{GIRF}_{X_i}(h, \delta, \mathcal{F}_{i-1}) = \hat{\mathrm{E}}\left[X_{i+h}|\varepsilon_i = 1, \mathcal{F}_{i-1}\right] - \hat{\mathrm{E}}\left[X_{i+h}|\mathcal{F}_{i-1}\right],$$

where the conditional expectations are estimated by sample averages based on $5,000$ simulated paths of $X_i, X_{i+1}, \ldots, X_{i+h}$ given the corresponding conditioning information and using the parameter estimates of specification (8) in Tables 7 through 10. For all processes, we observe a positive, persistent response of ξ_i^2, V_i and ρ_i due to a shock in the latent component. In most cases, the impulse response function declines monotonically and approaches zero after around 30-40 lags.

6 Conclusions

In this paper, we propose a statistical framework to model multivariate systems which are driven by an underlying common latent component. The basic idea of the so-called latent factor VAR

(LF-VAR) model is to combine a multivariate observation driven (VAR-type) dynamic with an underlying univariate parameter driven dynamic which jointly affects all individual components of the system. Whereas the observation driven dynamic is updated by process-specific innovations which are completely observable given the process history, the parameter driven component follows an autoregressive process which is updated by unobservable innovations independent from the idiosyncratic components.

The model is used to analyze whether evidence for a joint latent autoregressive component in intraday trading processes on financial markets is found. Economically this approach is motivated by the mixture-of-distribution hypothesis as postulated by Clark (1973) assuming that the trading process is driven by an unobservable autoregressive process of price relevant information. Moreover, this assumption is the underpinning of numerous theoretical market microstructure approaches such as Easley and O‘Hara (1992) or Easley, Kiefer, O‘Hara, and Paperman (1996), among others. By specifying a LF-VAR model for the return volatility, the average trading volume as well as the number of trades per 5 minutes interval, this paper investigates whether an underlying autoregressive component can be identified, how it affects the individual trading components and whether its explicit inclusion in the model leads to an improved statistical modelling.

Using intraday data from four blue chip stocks traded at the New York Stock Exchange, we find clear evidence for the existence of a joint latent dynamic component and thus a clear confirmation of the mixture-of-distribution hypothesis on an intraday level. It is shown that a latent shock simultaneously drives the return volatility, the trading volume as well as the trading intensity in the same direction. However, the strength of the latent factor's influence on the individual components differ significantly. An interesting finding is that the unobserved component primarily influences the volatility and the volume whereas its impact on the trading intensity is comparably weak.

Moreover, our results show that the latent factor is an important driving force for interdependencies between the particular components. Actually, we observe that the conditional contemporaneous correlations as well as cross-autocorrelations given the latent factor are lower than the corresponding unconditional ones. Hence, we can distinguish between effects that can be related to the unobserved information flow in the market and effects that can be attributed to trading mechanisms given the latent information. In this sense, the LF-VAR model is a valuable approach for the analysis of market microstructure relationships and provides deeper insights into the underlying data generating process.

A further important finding is that the inclusion of a latent factor significantly improves the goodness-of-fit of the model as well as its dynamic and distributional properties. Hence, taking explicitly a joint unobserved autoregressive component into account leads to an improved statistical model for financial trading processes. Nonetheless, it turns out that a single parameter driven dynamic is not sufficient to completely capture the dynamics of the multivariate system and that the inclusion of additional process-specific dynamics for the individual components is required to obtain a well-specified model. The fact that a specification which includes both the parameter driven dynamic as well as the multivariate observation driven dynamic provides the best goodness-of-fit and highest explanatory power confirms the underlying idea of the LF-VAR model.

References

Admati, A., and P. Pfleiderer (1988): "A Theory of Intraday Patterns: Volume and Price Variability," *Review of Financial Studies*, 1, 3–40.

Andersen, T. G. (1996): "Return Volatility and Trading Volume: An Information Flow Interpretation of Stochastic Volatility," *Journal of Finance*, 51, 169–204.

Bauwens, L., and P. Giot (2000): "The Logarithmic ACD Model: An Application to the Bid/Ask Quote Process of two NYSE Stocks," *Annales d'Economie et de Statistique*, 60, 117–149.

Bauwens, L., and N. Hautsch (2003): "Latent Factor Intensity Processes," Discussion paper, University of Konstanz, Université Catholique de Louvain.

Bauwens, L., and D. Veredas (2004): "The Stochastic Conditional Duration Model: A Latent Factor Model for the Analysis of Financial Durations," *Journal of Econometrics*, 119, 381–412.

Blume, L., D. Easley, and M. O'Hara (1994): "Market Statistics and Technical Analysis," *Journal of Finance*, 49 (1), 153–181.

Clark, P. (1973): "A Subordinated Stochastic Process Model with Finite Variance for Speculative Prices," *Econometrica*, 41, 135–155.

Cox, D. (1981): "Statistical Analysis of Time Series: Some Recent Developments," *Scandinavian Journal of Statistics*, 8, 93–115.

Davis, R. A., T. H. Rydberg, N. Shephard, and S. B. Streett (2001): "The CBin Model for Counts: Testing for Common Features in the Speed of Trading, Quote Changes, Limit and Market Order Arrivals," Discussion paper, Nuffield College, Oxford.

Diamond, D. W., and R. E. Verrecchia (1987): "Constraints on Short-Selling and Asset Price Adjustment to Private Information," *Journal of Financial Economics*, 18, 277–311.

Easley, D., N. M. Kiefer, M. O'Hara, and J. B. Paperman (1996): "Liquidity, Information and Infrequently Traded Stocks," *Journal of Finance*, 4.

Easley, D., and M. O'Hara (1992): "Time and Process of Security Price Adjustment," *The Journal of Finance*, 47 (2), 577–605.

Engle, R. F., and J. R. Russell (1998): "Autoregressive Conditional Duration: A New Model for Irregularly Spaced Transaction Data," *Econometrica*, 66, 1127–1162.

Engle, R. F. (2000): "The Econometrics of Ultra-High-Frequency Data," *Econometrica*, 68, 1, 1–22.

Gerhard, F., and W. Pohlmeier (2002): "On the Simultaneity of Absolute Price Changes and Transaction Durations," Discussion paper, University of Konstanz.

Heinen, A. (2002): "Modeling Time Series Count Data: An Autoregressive Conditional Poisson Model," Discussion paper, Université Catholique de Louvain.

Koop, G., M. H. Pesaran, and S. M. Potter (1996): "Impulse Response Analysis in Nonlinear Multivariate Models," *Journal of Econometrics*, 74, 119–174.

Liesenfeld, R., and J.-F. Richard (2002): "Univariate and Multivariate Stochastic Volatility Models: Estimation and Diagnostics," Discussion paper, University of Tübingen.

Liesenfeld, R. (1998): "Dynamic Bivariate Mixture Models: Modeling the Behavior of Prices and Trading Volume," *Journal of Business & Economic Statistics*, 16, 101–109.

——— (2001): "A Generalized Bivariate Mixture Model for Stock Price Volatility and Trading Volume," *Journal of Econometrics*, 104, 141–178.

Ljung, G. M., and G. E. P. Box (1978): "On a Measure of Lack of Fit in Time Series Models," *Biometrika*, 65, 297–303.

Manganelli, S. (2000): "Time, Volume and Price Impact of Trades," Discussion paper, European Central Bank, Frankfurt.

Nelson, D. (1991): "Conditional Heteroskedasticity in Asset Returns: A New Approach," *Journal of Econometrics*, 43, 227–251.

Richard, J.-F. (1998): "Efficient High-dimensional Monte Carlo Importance Sampling," Discussion paper, University of Pittsburgh.

Spierdijk, L., T. E. Nijman, and A. H. O. van Soest (2002): "The Price Impact of Trades in Illiquid Stocks in Periods of High and Low Market Activity," Discussion Paper 2002-29, CentER, Tilburg University.

Spierdijk, L. (2002): "An Empirical Analysis of the Role of the Trading Intensity in Information Dissemination on the NYSE," Discussion Paper 2002-30, CentER, Tilburg University.

Tauchen, G. E., and M. Pitts (1983): "The Price Variability-Volume Relationship on Speculative Markets," *Econometrica*, 51, 485–505.

Taylor, S. J. (1982): "Financial Returns Modelled by the Product of Two Stochastic Processes - a Study of Daily Sugar Prices," in *Time Series Analysis: Theory and Practice*, ed. by O. D. Anderson. North-Holland, Amsterdam.

——— (1986): *Modelling Financial Time Series*. Wiley, New York.

Appendix

A.1 Proof of Proposition 1

For $|a_0| < 1$, the latent process λ_i is strictly stationary. Since the innovations η_i, u_i and ε_i are i.i.d. random variables, the processes (2) through (5) are only weakly stationary if (h_i, Φ_i,

Ψ_i) are weakly stationary themselves. However, since a transformation of a weakly stationary variable is itself weakly stationary, it is sufficient to show that $\mu_i := (\ln h_i \; \ln \Phi_i \; \ln \Psi_i)'$ is weakly stationary. Without loss of generality, the process (8) can be written in terms of a VARMA(1,1) model given by

$$\mathbf{y}_i = \mathbf{C} + \mathbf{B}\mathbf{y}_{i-1} + \mathbf{A}_0\mathbf{z}_{0,i} + \mathbf{A}\mathbf{z}_{i-1}, \tag{29}$$

where

$$\mathbf{y}_i = \begin{pmatrix} \ln \mu_i \\ \ln \mu_{i-1} \\ \ln \mu_{i-2} \\ \vdots \\ \ln \mu_{i-q} \end{pmatrix}; \quad \mathbf{C} = \begin{pmatrix} \omega \\ 0 \\ \vdots \\ \vdots \\ 0 \end{pmatrix}; \quad \mathbf{B} = \begin{pmatrix} B_1 & B_2 & \cdots & B_q \\ I & 0 & \cdots & 0 \\ 0 & \ddots & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & I \end{pmatrix}; \quad \mathbf{A}_0 = \begin{pmatrix} A_0 \\ 0 \\ \vdots \\ \vdots \\ 0 \end{pmatrix};$$

$$\mathbf{z}_i = \begin{pmatrix} z_i \\ z_{i-1} \\ z_{i-2} \\ \vdots \\ z_{i-p} \end{pmatrix}; \quad \mathbf{z}_{0,i} = \begin{pmatrix} z_{0,i} \\ 0 \\ \vdots \\ \vdots \\ 0 \end{pmatrix}; \quad \mathbf{A} = \begin{pmatrix} A_1 & A_2 & \cdots & A_p \\ 0 & 0 & \cdots & 0 \\ 0 & \ddots & & \vdots \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & 0 \end{pmatrix}$$

and I denotes a (3×3) identity matrix. Under the assumption that the absolute value of the eigenvalues of $\mathbf{B}$ lie inside the unit circle, i.e. $|\varsigma| < 1$ for all values of ς satisfying

$$\left| I\varsigma^q - B_1\varsigma^{q-1} - B_2\varsigma^{q-2} \ldots - B_q \right| = 0,$$

(29) can be rewritten as

$$\mathbf{y}_i = (\mathbf{I} - \mathbf{B}L)^{-1} (\mathbf{C} + \mathbf{A}_0\mathbf{z}_{0,i} + \mathbf{A}\mathbf{z}_{i-1}),$$

corresponding to an infinite moving average representation in terms of the elements of $\mathbf{z}_{0,i}$ and $\mathbf{z}_{i-1}$, where L denotes the lag operator.

The innovation components z_{i-j} can be written as $z_{i-j} = (|\tilde{\eta}_i|, \tilde{u}_i, \tilde{\varepsilon}_i)$, where $\tilde{\eta}_i = \eta_i \lambda_i^{\delta_1/2}$, $\tilde{u}_i = u_i \lambda_i^{\delta_2}$ and $\tilde{\varepsilon}_i = \varepsilon_i \lambda_i^{\delta_3}$. Since, the product of an i.i.d. variate and a strictly stationary variable is strictly stationary, the vector $\mathbf{z}_i$ itself is strictly stationary.

It remains to show the stationarity of the component $\mathbf{A}_0\mathbf{z}_{0,i}$. This is easily verified by exploiting the triangular structure of A_0. Since the third component Ψ_i depends only on $\mathbf{z}_i$, it is itself weakly stationary. Hence, ρ_i is weakly stationary which implies weak stationarity of the second component Φ_i and thus V_i. The weak stationarity of h_i and Y_i are proven following an analogous argumentation. $\square$

A.2 Simulation results

A.2.1 Simulated moments of LF-GARCH and LF-ACD processes

Table 1: Simulated unconditional moments for differently parameterized LF-GARCH processes. The model corresponds to the specification given by (6), (10) and (13). The simulations are based on 100 sets of $50,000$ observations.
Evaluated statistics: Standard deviation, maximum, 75%-, 90%-, 95%-, 99%-quantile and kurtosis of the simulated return process as well as the Ljung-Box statistic (associated with 20 lags) for squared returns. The conditional mean return is set to zero.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
				Parameterization				
ω_1	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
α_1	0.100	0.100	0.100	0.100	0.100	0.100	0.100	0.100
β_1	0.100	0.100	0.100	0.100	0.100	0.100	0.700	0.900
α_0	0.000	0.100	0.500	0.900	0.900	0.900	0.900	0.500
δ_1	0.000	0.100	0.100	0.100	0.200	0.300	0.300	0.500
				Summary Statistics				
S.D.	1.046	1.049	1.050	1.062	1.109	1.195	1.341	1.673
Max	4.472	4.539	4.539	5.016	6.477	9.556	11.944	11.615
quant75	0.705	0.705	0.704	0.702	0.695	0.684	0.751	0.997
quant90	1.340	1.341	1.342	1.346	1.368	1.406	1.557	2.012
quant95	1.721	1.725	1.726	1.743	1.809	1.918	2.141	2.706
quant99	2.435	2.454	2.454	2.515	2.741	3.116	3.533	4.255
Kurtosis	3.010	3.046	3.055	3.194	3.798	5.209	5.812	4.488
LB(20)	109.027	112.222	122.304	384.497	2277.973	6131.453	9835.528	1706.835

	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
				Parameterization				
ω_1	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
α_1	0.100	0.100	0.100	0.100	0.100	0.100	0.200	0.500
β_1	0.900	0.950	0.950	0.950	0.950	0.950	0.700	0.500
α_0	0.100	0.100	0.500	0.700	0.900	0.900	0.900	0.900
δ_1	0.500	0.500	0.500	0.300	0.300	0.500	0.500	0.500
				Summary Statistics				
S.D.	1.617	2.457	2.554	2.416	2.978	6.860	3.206	48.175
Max	9.999	15.461	18.572	15.064	37.967	330.067	181.854	6993.980
quant75	0.999	1.507	1.509	1.499	1.497	1.536	0.836	0.949
quant90	1.980	2.996	3.053	2.961	3.226	4.026	2.078	2.518
quant95	2.634	3.995	4.126	3.928	4.581	6.544	3.255	4.202
quant99	4.036	6.158	6.558	6.014	8.137	15.854	7.237	11.448
Kurtosis	3.974	4.077	4.658	3.961	10.995	692.717	1422.349	15327.022
LB(20)	648.889	1422.047	2974.460	3837.337	29423.516	37688.351	18684.144	3758.646

Table 2: Simulated unconditional moments for differently parameterized LF-ACD processes. The model corresponds to the specification given by (6), (12) and (15). The simulations are based on 100 sets of 50,000 observations.
Evaluated statistics: Mean, standard deviation, maximum, minimum, 1%-, 5%-, 10%-, 25%-, 50%-, 75%-, 90%-, 95%-, 99%-quantile as well as the Ljung-Box statistic (associated with 20 lags).

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
				Parameterization				
ω_3	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
α_3	0.100	0.100	0.100	0.100	0.100	0.100	0.100	0.100
β_3	0.100	0.100	0.100	0.100	0.100	0.700	0.900	0.900
a_3	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
m_3	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
α_0	0.000	0.100	0.500	0.900	0.900	0.900	0.500	0.100
δ_3	0.000	0.100	0.100	0.100	0.200	0.200	0.500	0.500
				Summary Statistics				
Mean	1.124	1.131	1.133	1.167	1.310	1.795	4.585	3.771
S.D.	1.138	1.158	1.166	1.267	1.846	3.095	14.255	6.032
Max	14.371	15.559	15.732	21.927	90.964	185.843	1689.343	327.620
Min	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
quant01	0.011	0.011	0.011	0.011	0.010	0.012	0.025	0.026
quant05	0.057	0.057	0.057	0.056	0.051	0.063	0.130	0.135
quant10	0.117	0.117	0.117	0.115	0.106	0.130	0.272	0.280
quant25	0.320	0.319	0.319	0.314	0.296	0.368	0.782	0.791
quant50	0.773	0.774	0.772	0.769	0.755	0.953	2.105	2.052
quant75	1.552	1.559	1.558	1.577	1.651	2.144	5.029	4.627
quant90	2.591	2.611	2.617	2.709	3.076	4.161	10.433	8.915
quant95	3.387	3.421	3.436	3.618	4.357	6.103	16.006	12.908
quant99	5.255	5.359	5.399	5.920	8.192	12.530	36.328	25.277
LB(20)	615.291	648.092	751.600	2203.408	10739.092	26168.753	22413.452	10965.638

	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
				Parameterization				
ω_3	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
α_3	0.100	0.100	0.100	0.200	0.500	0.100	0.100	0.100
β_3	0.950	0.950	0.950	0.700	0.500	0.700	0.700	0.700
a_3	1.000	1.000	1.000	1.000	1.000	0.800	1.500	5.000
m_3	1.000	1.000	1.000	1.000	1.000	1.200	0.500	0.500
α_0	0.700	0.100	0.100	0.100	0.500	0.900	0.900	0.900
δ_3	0.300	0.500	0.300	0.300	0.300	0.200	0.200	0.200
				Summary Statistics				
Mean	11.946	12.470	9.145	2.239	6.210	1.794	1.806	1.788
S.D.	24.246	20.691	11.628	2.939	75.995	3.129	3.614	3.035
Max	1494.324	959.318	292.183	123.230	6500.553	206.091	314.030	175.815
Min	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
quant01	0.071	0.078	0.073	0.018	0.022	0.012	0.012	0.012
quant05	0.365	0.402	0.374	0.094	0.117	0.063	0.063	0.063
quant10	0.760	0.837	0.773	0.194	0.242	0.131	0.131	0.130
quant25	2.170	2.387	2.158	0.539	0.683	0.368	0.369	0.367
quant50	5.753	6.328	5.445	1.350	1.808	0.953	0.955	0.950
quant75	13.495	14.716	11.748	2.879	4.335	2.146	2.153	2.140
quant90	27.540	29.405	21.505	5.208	9.519	4.159	4.182	4.150
quant95	41.859	43.777	30.103	7.253	15.926	6.091	6.128	6.074
quant99	93.118	91.284	54.793	13.248	55.442	12.537	12.607	12.451
LB(20)	49335.797	26192.140	19098.324	8604.206	255.547	25235.396	25247.540	26940.587

A.2.2 Autocorrelation and cross-autocorrelation functions of simulated bivariate LF-VAR processes

The following figures show autocorrelation functions (ACF) and cross-autocorrelation functions (CACF) implied by bivariate LF-VAR(1,1) processes for the return volatility and the trading intensity. The model is specified as a two-dimensional version of the processes as given by (1) through (8).
From left to right: ACF of λ_i, ACF's of h_i (solid line) and Ψ_i (broken line), ACF's of Y_i^2 (solid line) and ρ_i (broken line), CACF's of h_i and Ψ_i (solid line) as well as of Y_i^2 and ρ_i (broken line). The CACF graphs show the plot of $\text{Corr}(x_i, z_{i-j})$ versus j. The conditional mean return is set to zero. The simulations are based on 100 sets of $50,000$ observations.

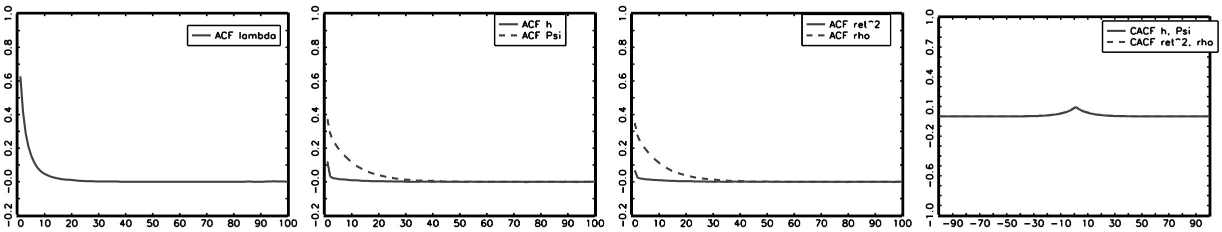

Figure 1: $\omega = (0, 0)$, $\alpha_{0,12} = 0$, $A_1 = (0.1\ 0, 0\ 0.1)$, $B_1 = (0.1\ 0, 0\ 0.1)$, $\alpha_0 = 0.9$, $\delta_1 = \delta_2 = 0.1$.

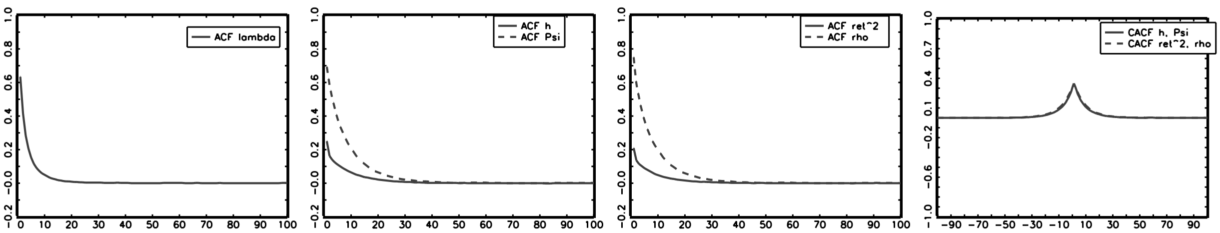

Figure 2: $\omega = (0, 0)$, $\alpha_{0,12} = 0$, $A_1 = (0.1\ 0, 0\ 0.1)$, $B_1 = (0.1\ 0, 0\ 0.1)$, $\alpha_0 = 0.9$, $\delta_1 = \delta_2 = 0.3$.

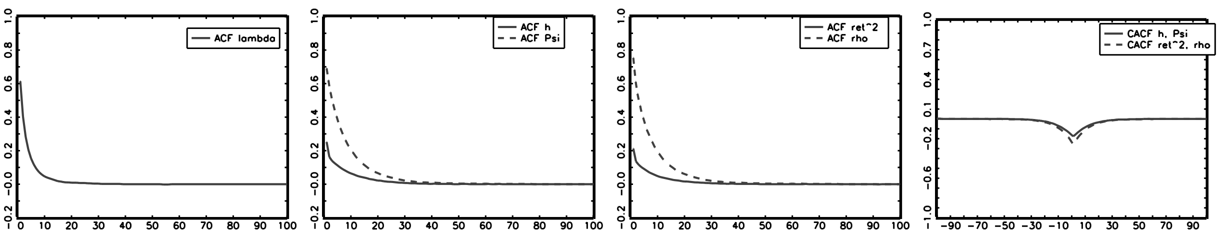

Figure 3: $\omega = (0, 0)$, $\alpha_{0,12} = 0$, $A_1 = (0.1\ 0, 0\ 0.1)$, $B_1 = (0.1\ 0, 0\ 0.1)$, $\alpha_0 = 0.9$, $\delta_1 = 0.3$, $\delta_2 = -0.3$.

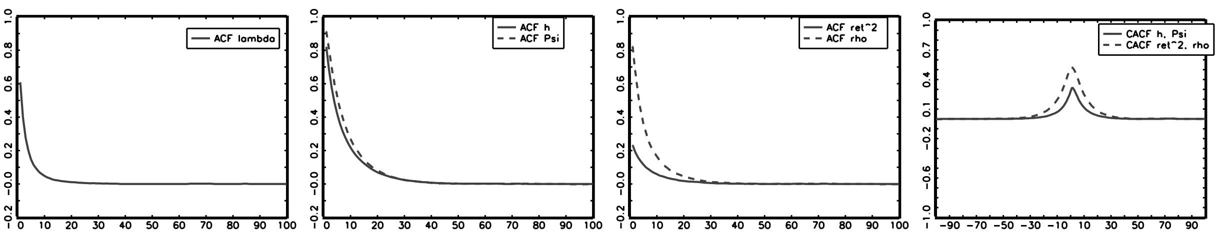

Figure 4: $\omega = (0, 0)$, $\alpha_{0,12} = 0$, $A_1 = (0.1\ 0, 0\ 0.1)$, $B_1 = (0.7\ 0, 0\ 0.7)$, $\alpha_0 = 0.9$, $\delta_1 = 0.3 = \delta_2 = 0.3$.

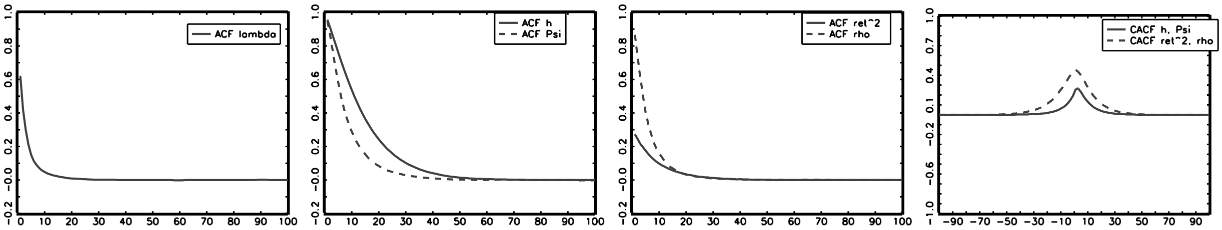

Figure 5: $\omega = (0, 0)$, $\alpha_{0,12} = 0$, $A_1 = (0.1\ 0, 0\ 0.1)$, $B_1 = (0.9\ 0, 0\ 0.9)$, $\alpha_0 = 0.9$, $\delta_1 = \delta_2 = 0.3$.

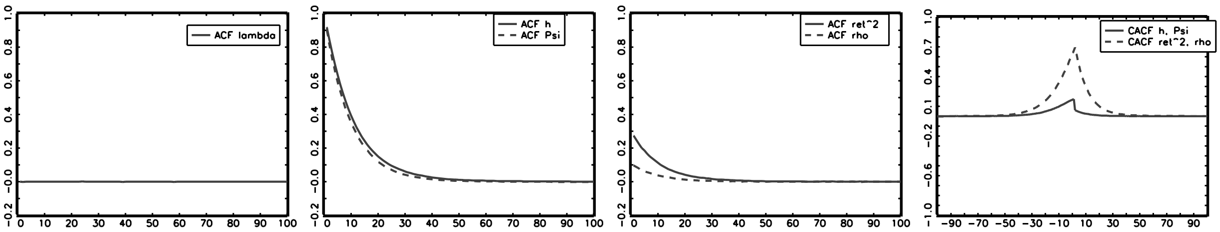

Figure 6: $\omega = (0, 0)$, $\alpha_{0,12} = 1.0$, $A_1 = (0.1\ 0, 0\ 0.1)$, $B_1 = (0.9\ 0, 0\ 0.9)$, $\alpha_0 = 0$, $\delta_1 = \delta_2 = 0$.

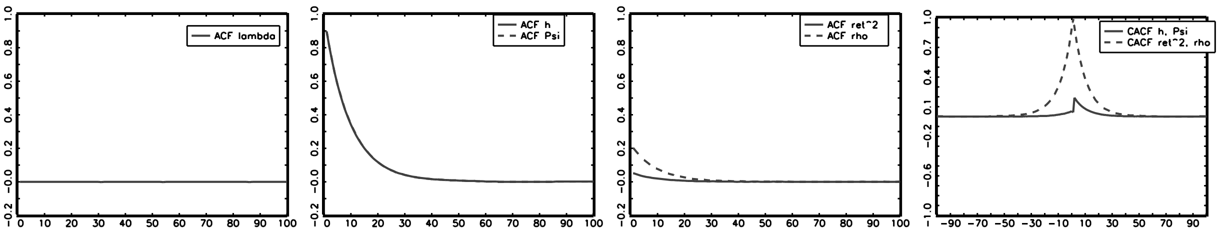

Figure 7: $\omega = (0, 0)$, $\alpha_{0,12} = 0$, $A_1 = (0.1\ 0.1, 0.1\ 0.1)$, $B_1 = (0.7\ 0.2, 0.2\ 0.7)$, $\alpha_0 = 0$, $\delta_1 = \delta_2 = 0$.

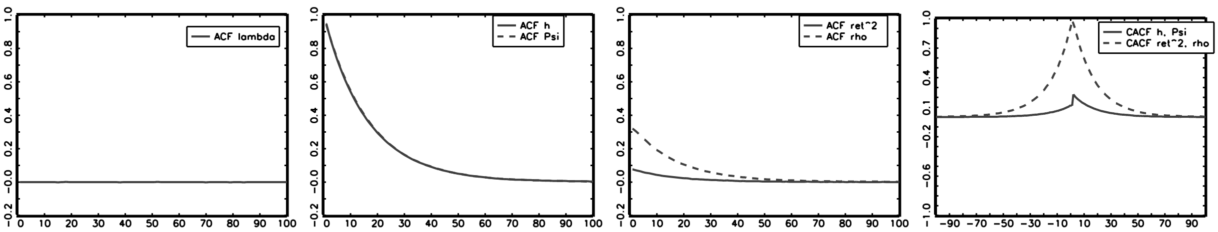

Figure 8: $\omega = (0, 0)$, $\alpha_{0,12} = 0.1$, $A_1 = (0.1\ 0.1, 0.1\ 0.1)$, $B_1 = (0.7\ 0.2, 0.2\ 0.7)$, $\alpha_0 = 0$, $\delta_1 = \delta_2 = 0$.

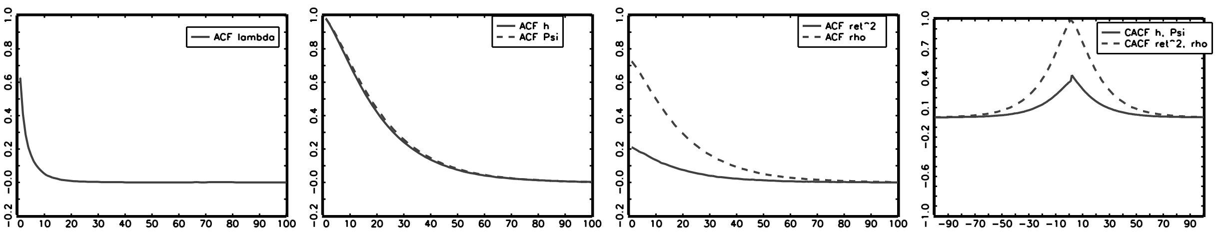

Figure 9: $\omega = (0, 0)$, $\alpha_{0,12} = 0.1$, $A_1 = (0.1\ 0.1, 0.1\ 0.1)$, $B_1 = (0.7\ 0.2, 0.2\ 0.7)$, $\alpha_0 = 0.9$, $\delta_1 = \delta_2 = 0.1$.

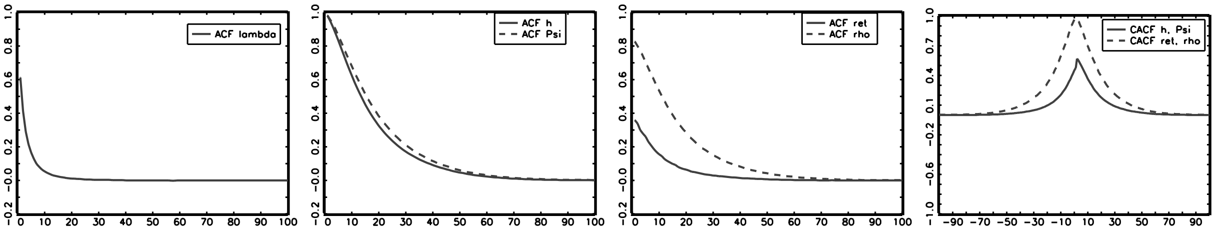

Figure 10: $\omega = (0, 0)$, $\alpha_{0,12} = 0.1$, $A_1 = (0.1\ 0.1, 0.1\ 0.1)$, $B_1 = (0.7\ 0.2, 0.2\ 0.7)$, $\alpha_0 = 0.9$, $\delta_1 = 0.3$, $\delta_2 = 0.1$.

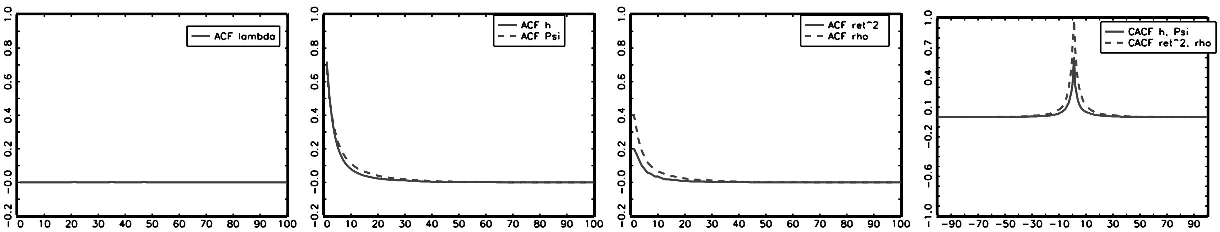

Figure 11: $\omega = (-0.2, -0.2)$, $\alpha_{0,12} = 0.1$, $A_1 = (0.1\ 0.1, 0.1\ 0.1)$, $B_1 = (0.7\ 0.2, 0.2\ 0.7)$, $\alpha_0 = 0$, $\delta_1 = \delta_2 = 1.0$.

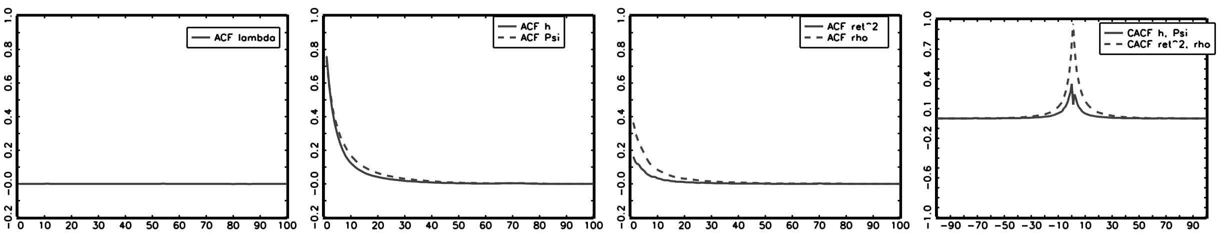

Figure 12: $\omega = (-0.2, -0.2)$, $\alpha_{0,12} = 0.1$, $A_1 = (0.1\ 0.1, 0.1\ 0.1)$, $B_1 = (0.7\ 0.2, 0.2\ 0.7)$, $\alpha_0 = 0$, $\delta_1 = 1.0$, $\delta_2 = -1.0$.

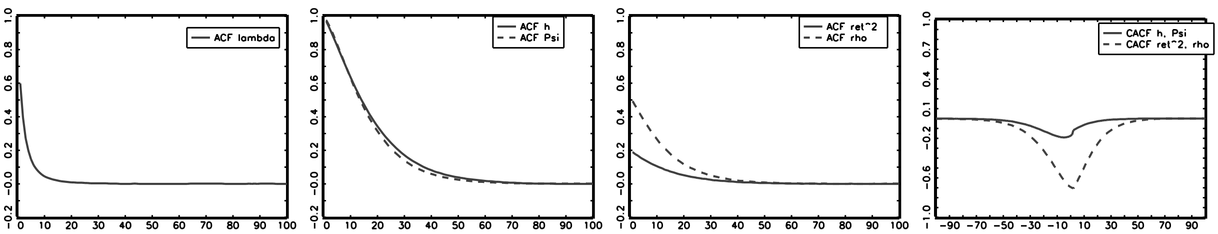

Figure 13: $\omega = (0, 0)$, $\alpha_{0,12} = -0.3$, $A_1 = (0.1\ 0, 0\ 0.1)$, $B_1 = (0.9\ 0, 0\ 0.9)$, $\alpha_0 = 0.9$, $\delta_1 = 0.1$, $\delta_2 = 0.1$.

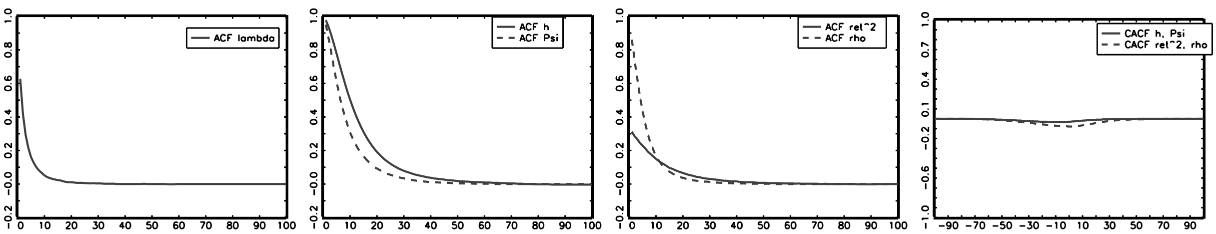

Figure 14: $\omega = (0, 0)$, $\alpha_{0,12} = -0.3$, $A_1 = (0.1\ 0, 0\ 0.1)$, $B_1 = (0.9\ 0, 0\ 0.9)$, $\alpha_0 = 0.9$, $\delta_1 = 0.3$, $\delta_2 = 0.3$.

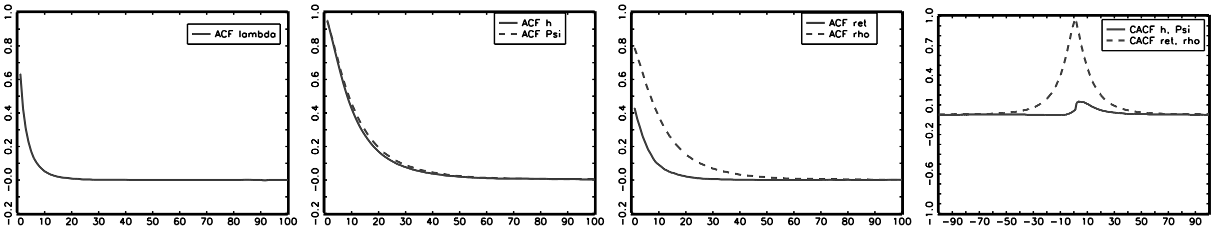

Figure 15: $\omega = (-0.2, -0.2)$, $\alpha_{0,12} = 0.1$, $A_1 = (0.1\ 0.1, 0.1\ 0.1)$, $B_1 = (0.7\ 0.2, 0.2\ 0.7)$, $\alpha_0 = 0.9$, $\delta_1 = -0.5$, $\delta_2 = 0.2$.

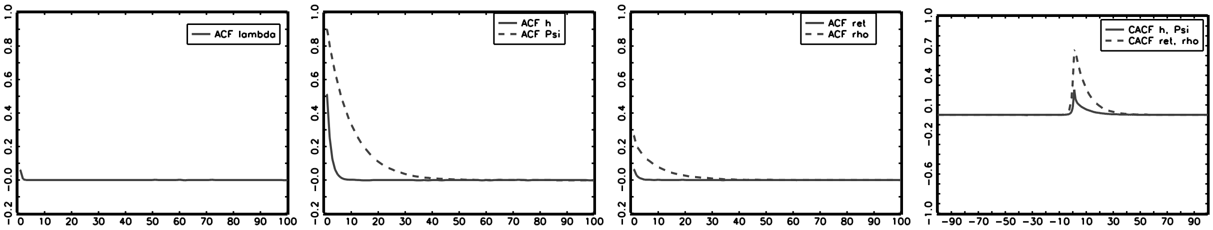

Figure 16: $\omega = (0, 0)$, $\alpha_{0,12} = 0$, $A_1 = (0.1\ 0.1, 0.1\ 0.1)$, $B_1 = (0.5\ 0, 0\ 0.9)$, $\alpha_0 = 0.1$, $\delta_1 = \delta_2 = 0.5$.

A.3 Descriptive Statistics

Table 3: Descriptive statistics of log returns (multiplied by 100), squared log returns, average volumes as well as the number of trades based on 5-min intervals for the AOL, Boeing, JP Morgan, and IBM stocks traded at the NYSE. Extracted from the 2001 TAQ data base. Sample period 02/01/01 to 31/05/01.
The following descriptive statistics are shown: Number of observations, mean, standard deviation, minimum, maximum, 5%-, 10%-, 50%-, 90%-, as well as 95%-quantile, kurtosis, univariate and multivariate Ljung-Box statistic (computed for squared log returns, volumes and number of trades) associated with 20 lags.

	AOL				Boeing			
	Returns	Sq. returns	Avg. volumes	Trades	Returns	Sq. ret.	Avg. volumes	Trades
Obs	8008	8008	8008	8008	8008	8008	8008	8008
Mean	0.005	0.143	7084.337	26.424	0.000	0.060	1829.393	19.735
S.D.	0.378	0.403	5979.153	9.537	0.245	0.153	1683.701	8.283
Min	-2.973	0.000	1.000	1.000	-1.680	0.000	1.000	1.000
Max	3.000	9.000	84250.000	75.000	1.854	3.437	24766.666	63.000
q05	-0.556	0.000	1645.000	12.000	-0.391	0.000	450.000	9.000
q10	-0.404	0.001	2131.818	15.000	-0.272	0.000	561.765	10.000
q50	0.000	0.034	5383.333	26.000	0.000	0.013	1327.273	19.000
q90	0.406	0.335	13876.471	39.000	0.269	0.150	3600.000	31.000
q95	0.593	0.581	17995.000	43.000	0.380	0.257	4900.000	35.000
Kurtosis	8.968	-	-	-	7.423	-	-	-
LB(20)	25.129	1132.700	14754.230	9868.857	42.988	1767.233	2878.382	18609.976
MLB(20)			41942.224				35931.744	

	JP Morgan				IBM			
	Returns	Sq. returns	Avg. volumes	Trades	Returns	Sq. ret.	Avg. volumes	Trades
Obs	8008	8008	8008	8008	8008	8008	8008	8008
Mean	0.002	0.099	2960.285	33.070	0.001	0.073	2375.869	41.962
S.D.	0.315	0.374	2685.456	11.204	0.270	0.186	2076.318	12.175
Min	-2.355	0.000	1.000	1.000	-1.668	0.000	1.000	1.000
Max	3.994	15.950	59153.332	78.000	2.000	4.000	45540.000	101.000
q05	-0.476	0.000	747.826	16.000	-0.430	0.000	696.774	24.000
q10	-0.334	0.000	920.000	19.000	-0.310	0.000	841.509	27.000
q50	0.000	0.021	2233.333	32.000	0.000	0.018	1794.595	41.000
q90	0.338	0.229	5729.412	48.000	0.289	0.182	4470.371	58.000
q95	0.479	0.411	7358.824	53.000	0.425	0.308	5906.667	64.000
Kurtosis	15.197	-	-	-	7.464	-	-	-
LB(20)	55.335	1401.054	9011.564	12520.965	26.568	2590.101	19751.120	18070.49
MLB(20)			43873.529				70348.102	

Empirical autocorrelation and cross-autocorrelation functions

The following figures show the autocorrelation functions (ACF) and cross-autocorrelation functions (CACF) of squared log returns, average volumes as well as the number of trades based on 5-min intervals for the AOL, Boeing, JP Morgan and IBM stocks traded at the NYSE. The upper plots are based on the plain series, whereas the lower plots are based on the seasonally adjusted series. The pictures on the left show the ACF of squared log returns (solid line), average volumes (broken line) and the number of trades (dotted line). The pictures on the right show the CACF of squared log returns and average volumes (solid line), of squared log returns and the number of trades (broken line), and of average volumes and the number of trades (dotted line). Data extracted from the 2001 TAQ data base. Sample period 02/01/01 to 31/05/01.

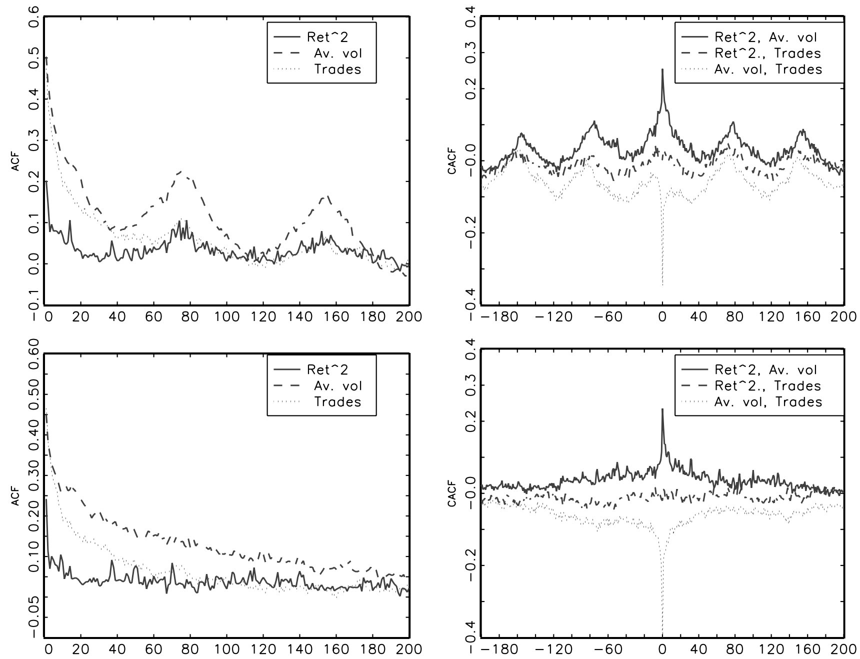

Figure 17: (Cross-)autocorrelation functions for the AOL stock.

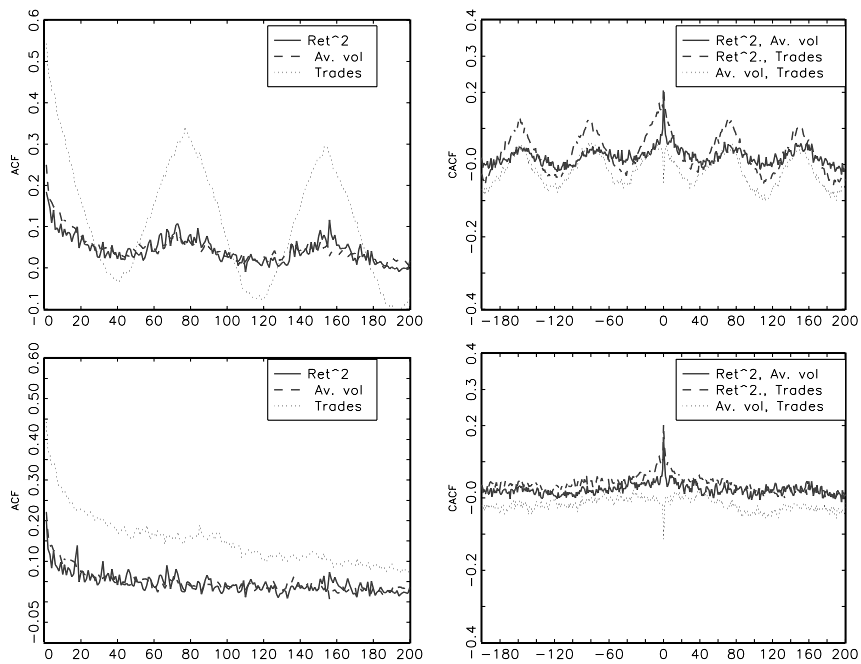

Figure 18: (Cross-)autocorrelation functions for the Boeing stock.

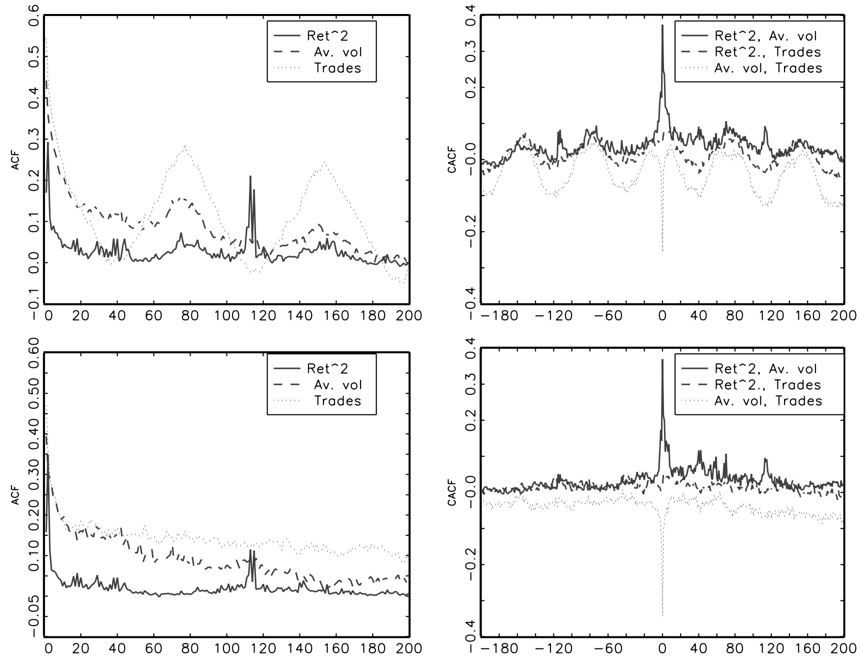

Figure 19: (Cross-)autocorrelation functions for the JP Morgan stock.

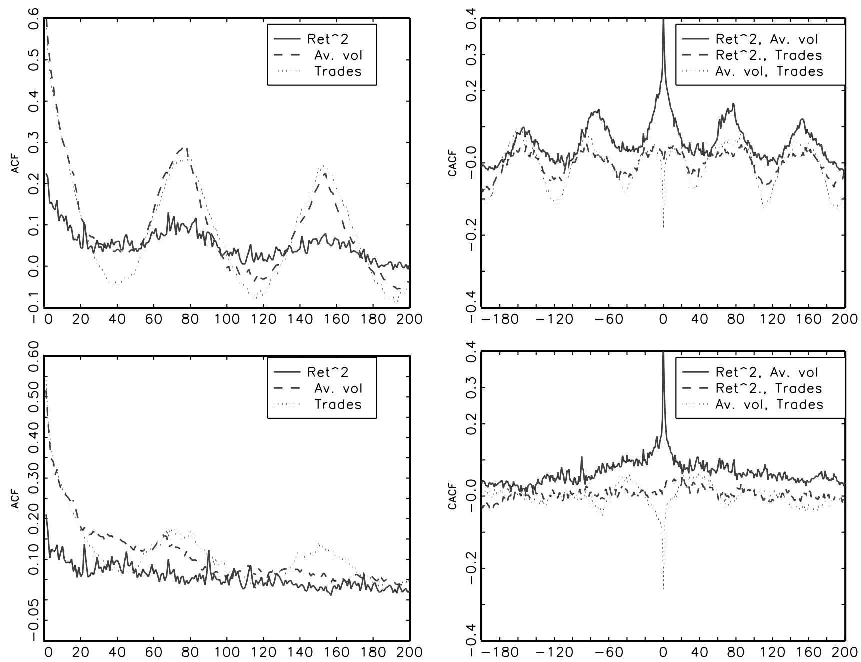

Figure 20: (Cross-)autocorrelation functions for the IBM stock.

A.4 Estimation results

A.4.1 Univariate Models

Table 4: Maximum likelihood efficient importance sampling (ML-EIS) estimates of different parameterizations of (LF-)GARCH models up to a lag order of $p = q = 2$ for 5 min log returns based on the AOL, Boeing, JP Morgan and IBM stocks traded at the NYSE. The model corresponds to the specification shown in (6), (10) and (13), where, however, higher lags are included. Data extracted from the 2001 TAQ data base. Sample period 02/01/01 to 31/05/01. Overnight returns are excluded. The models are re-initialized at every trading day. Standard errors are computed based on the inverse of the estimated Hessian. The ML-EIS estimates are computed using $R = 50$ Monte Carlo replications based on 5 EIS iterations.
Diagnostics: log likelihood function (LL), Bayes Information Criterion (BIC), average diagnostics (mean, standard deviation and Ljung-Box (LB) statistic based on 20 lags) over all trajectories of the residuals.

	AOL					Boeing				
	(1)	(2)	(3)	(4)	(5)	(1)	(2)	(3)	(4)	(5)
ω_1	-0.098***	-0.001***	0.314*	-0.028***	-0.006	-0.091***	-0.154***	0.3092***	-0.035***	-0.011**
α_1^1	0.140***	0.197***		0.034***	-0.035	0.136***	0.166***		0.042***	-0.064***
α_2^1		-0.195***			0.043*		0.060***			0.077***
β_1^1	0.987***	1.917***		0.997***	1.718***	0.982***	0.143***		0.996***	1.585***
β_2^1		-0.918***			-0.719***		0.830***			-0.586***
Latent Component										
a_0			0.961***	0.768***	0.830***			0.941***	0.675***	0.775***
δ_1			0.229***	0.428***	0.403***			0.291***	0.553***	0.513***
Diagnostics										
LL	-13343	-13278	-13115	-13060	-13057	-13456	-13449	-13217	-13151	-13145
BIC	-13357	-13300	-13129	-13082	-13088	-13469	-13471	-13230	-13173	-13177
Mean	1.000	1.002	1.007	1.009	1.011	1.000	1.000	1.009	1.012	1.014
S.D.	2.548	2.302	1.510	1.461	1.450	2.283	2.266	1.524	1.471	1.462
LB(20)	58.610***	37.567**	23.447	17.147	17.341	61.037***	56.611***	30.715*	24.426	23.546

	JP Morgan					IBM				
	(1)	(2)	(3)	(4)	(5)	(1)	(2)	(3)	(4)	(5)
ω_1	-0.149***	-0.139***	0.283**	-0.013***	-0.003***	-0.121***	-0.009***	0.323	-0.030***	-0.045**
α_1^1	0.224***	0.260***		0.016***	-0.031***	0.168***	0.228***		0.037***	0.003
α_2^1		-0.052			0.035***		-0.215***			0.051*
β_1^1	0.967***	0.972***		0.998***	1.729***	0.986***	1.826***		0.997***	0.593
β_2^1		-0.000			-0.730***		-0.827***			0.402
Latent Component										
a_0			0.951***	0.860***	0.876***			0.981***	0.860***	0.847***
δ_1			0.275***	0.379***	0.384***			0.167***	0.302***	0.320***
Diagnostics										
LL	-13351	-13349	-13071	-13028	-13023	-13040	-12998	-12883	-12849	-12848
BIC	-13365	-13371	-13085	-13050	-13055	-13054	-13021	-12896	-12871	-12879
Mean	1.000	1.000	1.007	1.008	1.010	1.000	1.004	1.004	1.006	1.007
S.D.	2.766	2.787	1.502	1.475	1.463	2.122	2.030	1.491	1.457	1.450
LB(20)	21.383	21.587	20.516	20.415	18.206	51.400***	17.026	23.200	19.907	19.083

Table 5: Maximum likelihood efficient importance sampling (ML-EIS) estimates of different parameterizations of (LF-)ACD models up to a lag order of $p = q = 2$ for 5 min average trading volumes based on the AOL, Boeing, JP Morgan and IBM stocks traded at the NYSE. The model corresponds to the specification shown in (6), (11) and (14), where, however, higher lags are included. Data extracted from the 2001 TAQ data base. Sample period 02/01/01 to 31/05/01. Overnight observations are excluded. The models are re-initialized at every trading day. Standard errors are computed based on the inverse of the estimated Hessian. The ML-EIS estimates are computed using $R = 50$ Monte Carlo replications based on 5 EIS iterations.
Diagnostics: log likelihood function (LL), Bayes Information Criterion (BIC), average diagnostics (mean, standard deviation and Ljung-Box (LB) statistic based on 20 lags) over all trajectories of the residuals.

	AOL					Boeing				
	(1)	(2)	(3)	(4)	(5)	(1)	(2)	(3)	(4)	(5)
ω_2	-0.384***	-0.180***	-3.291***	-0.080***	-0.037***	-0.403***	-0.416***	-5.841***	-0.070***	-0.057***
α_1^2	0.007***	0.013***		0.018***	-0.013***	0.001***	0.002***		0.011***	-0.006**
α_2^2		-0.007***			0.021***		-0.000			0.014***
β_1^2	0.943***	1.216***		0.986***	1.340***	0.928***	0.761***		0.981***	1.074***
β_2^2		-0.240***			-0.349***		0.165			-0.091
a_2	0.636***	0.682***	0.715***	1.315***	1.342***	0.517***	0.519***	0.450***	1.202***	1.188***
m_2	7.916***	6.974***	9.380***	4.609***	4.672***	8.032***	7.991***	12.753***	4.276***	4.321***
Latent Component										
a_0			0.934***	0.500***	0.654***			0.954***	0.260***	0.391***
δ_2			0.180***	0.373***	0.363***			0.113***	0.506***	0.494***
Diagnostics										
LL	-4896	-4859	-4665	-4594	-4568	-6211	-6200	-6019	-5954	-5943
BIC	-4919	-4890	-4688	-4626	-4608	-6234	-6232	-6041	-5985	-5983
Mean	1.003	1.002	1.001	1.003	1.004	1.011	1.010	1.007	1.004	1.005
S.D.	0.655	0.648	0.481	0.359	0.349	0.871	0.870	0.739	0.409	0.412
LB(20)	78.440***	24.061	42.693***	23.355	18.991	39.166***	23.256	46.466***	24.470	21.645

	JP Morgan					IBM				
	(1)	(2)	(3)	(4)	(5)	(1)	(2)	(3)	(4)	(5)
ω_2	-0.403***	-0.307***	-3.420***	-0.072***	-0.139***	-0.435***	-0.206***	-2.440***	-0.087***	-0.060***
α_1^2	0.004***	0.007***		0.018***	0.021***	0.008***	0.015***		0.022***	-0.002
α_2^2		-0.002***			0.020***		-0.008***			0.018***
β_1^2	0.932***	0.915***		0.984***	-0.016***	0.929***	1.242***		0.980***	1.194***
β_2^2		0.034			0.983***		-0.273***			-0.210
a_2	0.591***	0.614***	0.666***	1.388***	1.475***	0.692***	0.737***	0.888***	1.462***	1.531***
m_2	7.844***	7.314***	8.772***	3.950***	3.458***	8.696***	7.740***	7.894***	4.445***	4.371***
Latent Component										
a_0			0.919***	0.398***	0.419***			0.932***	0.512***	0.590***
δ_2			0.182***	0.429***	0.424***			0.159***	0.314***	0.317***
Diagnostics										
LL	-5454	-5429	-5260	-5164	-5158	-4229	-4201	-4038	-3973	-3956
BIC	-5477	-5460	-5282	-5196	-5199	-4251	-4232	-4060	-4004	-3996
Mean	1.005	1.005	1.001	1.004	1.006	1.001	1.001	1.001	1.003	1.003
S.D.	0.741	0.738	0.542	0.368	0.371	0.570	0.563	0.414	0.328	0.316
LB(20)	37.492**	10.981	39.256***	21.829	20.520	74.995***	24.832	35.589**	28.402	22.822

Table 6: Maximum likelihood efficient importance sampling (ML-EIS) estimates of different parameterizations of (LF-)ACD models up to a lag order of $p = q = 2$ for the number of trades in 5 min intervals based on the AOL, Boeing, JP Morgan and IBM stocks traded at the NYSE. The model corresponds to the specification shown in (6), (12) and (15), where, however, higher lags are included. Data extracted from the 2001 TAQ data base. Sample period 02/01/01 to 31/05/01. Overnight observations are excluded. The models are re-initialized at every trading day. Standard errors are computed based on the inverse of the estimated Hessian. The ML-EIS estimates are computed using $R = 50$ Monte Carlo replications based on 5 EIS iterations.
Diagnostics: log likelihood function (LL), Bayes Information Criterion (BIC), average diagnostics (mean, standard deviation and Ljung-Box (LB) statistic based on 20 lags) over all trajectories of the residuals.

	AOL					Boeing				
	(1)	(2)	(3)	(4)	(5)	(1)	(2)	(3)	(4)	(5)
ω_3	-0.307***	-0.060***	-0.226***	-0.055***	-0.047***	-0.202***	-0.025***	-0.426***	-0.035***	-0.042***
α_1^3	0.146***	0.177***		0.053***	0.104***	0.077***	0.105***		0.026***	0.046***
α_2^3		-0.143***			-0.061***		-0.092***			-0.018
β_1^3	0.892***	1.554***		0.972***	0.940***	0.953***	1.704***		0.991***	0.562***
β_2^3		-0.571***			0.035		-0.708***			0.427*
a_3	1.910***	2.016***	2.800***	3.695***	3.241***	1.632***	1.752***	2.170***	2.563***	2.366***
m_3	2.970***	2.707***	2.002***	1.326***	1.524***	3.674***	3.237***	2.639***	2.100***	2.317***
Latent Component										
a_0			0.913***	0.731***	0.793***			0.957***	0.766***	0.821***
δ_3			0.097***	0.128***	0.103***			0.067***	0.110***	0.092***
Diagnostics										
LL	-1707	-1680	-1697	-1661	-1652	-1982	-1957	-1975	-1925	-1921
BIC	-1729	-1711	-1720	-1693	-1693	-2005	-1988	-1998	-1956	-1961
Mean	1.000	0.999	1.003	1.002	1.001	0.999	0.999	1.002	1.002	1.002
S.D.	0.308	0.307	0.263	0.256	0.267	0.323	0.322	0.290	0.279	0.286
LB(20)	69.340***	37.517**	62.817***	42.030***	31.200*	55.018***	33.285**	50.428***	34.512**	25.000

	JP Morgan					IBM				
	(1)	(2)	(3)	(4)	(5)	(1)	(2)	(3)	(4)	(5)
ω_3	-0.339***	-0.013***	-0.018	-0.013***	-0.011**	-0.372***	-0.167***	-0.226***	-0.4510***	-0.022**
α_1^3	0.184***	0.212***		0.014***	0.144***	0.150***	0.183***		0.1223***	0.056***
α_2^3		-0.203***			-0.134***		-0.107***			-0.081***
β_1^3	0.847***	1.629***		0.997***	1.252***	0.908***	1.282***		0.3125***	0.811***
β_2^3		-0.631***			-0.254***		-0.316***			-0.009
a_3	2.234***	2.429***	4.174***	4.561***	3.307***	2.121***	2.220***	3.758***	2.8989***	3.436***
m_3	2.718***	2.386***	1.316***	1.214***	1.702***	4.762***	4.406***	2.492***	3.2868***	2.758***
Latent Component										
a_0			0.873***	0.739***	0.783***			0.913***	0.9506***	0.951***
δ_3			0.105***	0.126***	0.082***			0.083***	0.0464***	0.068***
Diagnostics										
LL	-966	-869	-969	-877	-854	948	987	985	1021	1022
BIC	-988	-901	-991	-909	-894	926	955	962	989	981
Mean	1.000	1.000	1.003	1.003	1.001	1.000	0.999	1.001	1.000	1.001
S.D.	0.277	0.273	0.229	0.221	0.246	0.218	0.217	0.176	0.195	0.181
LB(20)	139.049***	46.393***	103.030***	72.420***	38.232**	87.077 ***	10.400	47.357***	11.636	17.809

A.4.2 Multivariate Models

Table 7: Maximum likelihood efficient importance sampling (ML-EIS) estimates of different parameterizations of (LF)-VAR models up to a lag order of $p = q = 2$ models for the log return volatility, the average volume and the number of trades per 5 min interval for the AOL stock traded on the NYSE. Data extracted from the 2001 TAQ data base. The model is specified as given by (1) through (8). Sample period 02/01/01 to 31/05/01. Overnight observations are excluded. The models are re-initialized for every trading day. Standard errors are computed based on the inverse of the estimated Hessian. The ML-EIS estimates are computed using $R = 50$ Monte Carlo replications based on 5 EIS iterations.
Diagnostics: log likelihood function (LL), Bayes Information Criterion (BIC), average diagnostics (mean, standard deviation and univariate as well as multivariate Ljung-Box statistics (LB and MLB, respectively) based on 20 lags) over all trajectories of the residuals.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
ω_1	-0.419***	-0.041	-0.540***	0.592***	-0.062***	0.271***	0.308*
ω_2	-0.933***	-2.236***	-1.116***	-2.091***	-1.627***	-1.879***	-1.877***
ω_3	-0.265***	-0.062***	-0.023***	-0.286***	-0.307***	-0.057***	-0.076***
α^0_{12}	0.082***	0.789***	0.255***	0.365***	-0.038***	0.080***	0.064***
α^0_{13}	-0.144***	0.714***	0.201***	0.692***	-0.032***	0.112***	0.130**
α^0_{23}	-0.739***	-0.669***	-0.762***	-0.589***	-0.778***	-0.785***	-0.784***
α^1_{11}	0.136***	0.331***	0.165***		0.064***	0.110***	0.091***
α^1_{12}	0.010*	0.047***	0.030***			0.002	-0.004
α^1_{13}	0.010***	0.000	-0.004***			0.000	0.002
α^1_{21}	-0.001***	0.007***	0.001			-0.030***	-0.026***
α^1_{22}	0.018	0.014***	0.011***		0.003*	0.005***	0.009***
α^1_{23}	0.000***	0.000**	0.000***			0.000	0.000
α^1_{31}	0.156	0.028	-0.121***			-0.065*	-0.025
α^1_{32}	0.328***	-0.109***	0.020			-0.008	-0.015
α^1_{33}	0.177***	0.183***	0.228***		0.146***	0.176***	0.170***
α^2_{11}		0.293***	0.134***			0.053***	0.032*
α^2_{22}		0.010***	0.006***			0.001	-0.001
α^2_{33}		-0.151***	-0.212***			-0.144***	-0.145***
β^1_{11}	0.974***	-0.137***	0.055***		0.994***	0.508***	0.516***
β^1_{12}	0.050***		0.111***				-0.239***
β^1_{13}	-0.011***		0.000				-0.005*
β^1_{21}	-0.058***		-0.174***				0.051
β^1_{22}	0.840***	0.079*	0.298***		0.223***	0.233	0.193***
β^1_{23}	0.021***		0.001				-0.012
β^1_{31}	0.113***		-0.188***				-0.005
β^1_{32}	0.700***		0.838***				0.003
β^1_{33}	0.904***	1.574***	1.699***		0.892***	1.567***	1.551***
β^2_{11}		0.290***	0.864***			0.450***	0.473***
β^2_{22}		0.371***	0.394***			-0.025***	-0.004
β^2_{33}		-0.589***	-0.706***			-0.582***	-0.578***
a_2	0.771***	0.684***	0.732***	0.965***	0.989***	0.916***	0.898***
m_2	7.081***	8.233***	8.031***	6.248***	6.153***	6.770***	6.775***
a_3	2.020***	2.000***	2.231***	2.103***	1.910***	2.009***	2.008***
m_3	2.686***	2.746***	2.227***	2.052***	2.970***	2.728***	2.730***
Latent Component							
a_0				0.933***	0.936***	0.950***	0.943***
δ_1				0.176***	0.231***	0.263***	0.279***
δ_2				0.156***	0.143***	0.119***	0.111***
δ_3				-0.032***	0.000	0.002	0.003
Diagnostics							
LL	-18856	-19100	-18730	-19818	-18359	-18278	-18250
BIC	-18981	-19226	-18883	-19881	-18449	-18422	-18421
MLB(20)	183.004***	1390.851***	354.426***	16000.525***	258.547***	107.914***	98.147***
Diagnostics for the volatility process							
Mean	1.000	1.000	1.000	1.001	1.004	1.003	1.004
S.D.	2.384	1.850	2.450	1.754	1.600	1.571	1.531
LB(20)	40.032***	322.974***	40.399***	793.226***	20.946	16.902	18.228
Diagnostics for the volume process							
Mean	1.000	1.000	1.000	1.000	1.000	1.000	1.001
S.D.	0.536	0.554	0.531	0.424	0.418	0.433	0.445
LB(20)	116.754***	1306.748***	67.612***	92.822***	21.286	20.583	20.708
Diagnostics for the trading intensity process							
Mean	0.999	0.999	0.999	0.999	1.000	0.999	0.999
S.D.	0.308	0.307	0.308	0.341	0.308	0.307	0.307
LB(20)	67.939***	32.587***	138.980***	8166.409***	69.409***	37.849***	37.623***

Table 8: Maximum likelihood efficient importance sampling (ML-EIS) estimates of different parameterizations of (LF)-VAR models up to a lag order of $p = q = 2$ models for the log return volatility, the average volume and the number of trades per 5 min interval for the Boeing stock traded on the NYSE. Data extracted from the 2001 TAQ data base. The model is specified as given by (1) through (8). Sample period 02/01/01 to 31/05/01. Overnight observations are excluded. The models are re-initialized for every trading day. Standard errors are computed based on the inverse of the estimated Hessian. The ML-EIS estimates are computed using $R = 50$ Monte Carlo replications based on 5 EIS iterations.
Diagnostics: log likelihood function (LL), Bayes Information Criterion (BIC), average diagnostics (mean, standard deviation and univariate as well as multivariate Ljung-Box statistics (LB and MLB, respectively) based on 20 lags) over all trajectories of the residuals.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
ω_1	1.829***	0.532***	0.733***	0.625***	-0.121***	0.460***	0.712
ω_2	-1.087***	-0.809***	-0.610***	-6.611***	-2.197***	-2.531***	-2.124***
ω_3	-0.224***	-0.030***	-0.033***	-0.408***	-0.188***	-0.155***	-0.128***
α_{12}^0	0.554***	0.576***	0.565***	0.594***	-0.087***	0.020	0.004
α_{13}^0	1.037***	1.040***	1.081***	0.682***	0.034***	0.259***	0.828***
α_{23}^0	-0.235***	-0.042***	-0.214***	-0.422***	-0.381***	-0.442***	-0.257***
α_{11}^1	0.257***	0.252***	0.224***		0.134***	0.083***	0.083***
α_{12}^1	-0.005	0.032***	-0.025***			-0.032***	-0.043***
α_{13}^1	0.003	-0.001	-0.001**			-0.006**	-0.002***
α_{21}^1	-0.001***	-0.001***	-0.001***			-0.005***	-0.004***
α_{22}^1	0.000***	0.001***	0.001***		-0.001**	-0.000	-0.001**
α_{23}^1	0.000	0.000**	0.000***			-0.000***	0.000**
α_{31}^1	-0.167***	-0.173***	-0.207***			-0.236***	-0.269***
α_{32}^1	0.048***	0.016	0.078***			-0.064**	0.035
α_{33}^1	0.076***	0.109***	0.099***		0.077***	0.090***	0.085***
α_{11}^2		0.193***	0.158***			0.140***	0.116***
α_{22}^2		0.001***	-0.001***			0.000	-0.001*
α_{33}^2		-0.095***	-0.093***			-0.002	-0.084***
β_{11}^1	0.386***	0.256***	0.337***		0.948***	0.560***	0.560***
β_{12}^1	0.112***		0.066***				-0.066**
β_{13}^1	0.002		0.002***				-0.005**
β_{21}^1	0.227***		0.058				0.176
β_{22}^1	0.808***	0.221**	1.090***		0.169***	0.163	0.423**
β_{23}^1	-0.003		-0.004***				-0.035***
β_{31}^1	-0.029		-0.306***				-0.742***
β_{32}^1	0.032		0.104***				0.187**
β_{33}^1	0.950***	1.680***	1.767***		0.948***	0.606***	1.778***
β_{11}^2		0.117***	0.121***			0.301***	0.343***
β_{22}^2		0.652***	-0.197***			0.092	-0.051
β_{33}^2		-0.685***	-0.773***			0.343***	-0.783***
a_2	0.491***	0.500***	0.505***	0.396***	0.774***	0.625***	0.622***
m_2	9.167***	8.640***	8.698***	12.650***	6.750***	8.313***	8.520***
a_3	1.628***	1.786***	1.739***	2.164***	1.726***	1.879***	1.663***
m_3	3.696***	3.128***	3.293***	2.551***	3.424***	3.084***	3.626***
Latent Component							
a_0				0.950***	0.656***	0.804***	0.827***
δ_1				0.111***	0.625***	0.524***	0.453***
δ_2				0.058***	0.407***	0.305***	0.283***
δ_3				0.069***	0.046***	0.070***	0.025***
Diagnostics							
LL	-21087	-21140	-21015	-21744	-21022	-20906	-20819
BIC	-21213	-21265	-21168	-21807	-21112	-21050	-20990
MLB(20)	356.292***	159.818***	208.131***	5483.858***	181.521***	103.361***	66.069
Diagnostics for the volatility process							
Mean	0.999	1.000	0.999	1.000	1.007	1.007	1.005
S.D.	1.768	1.766	1.764	1.690	1.489	1.477	1.483
LB(20)	192.869***	154.072***	116.050***	497.411***	24.346	28.626*	19.698
Diagnostics for the volume process							
Mean	1.010	1.010	1.010	1.012	1.003	1.006	1.003
S.D.	0.851	0.862	0.849	0.869	0.514	0.596	0.591
LB(20)	44.154***	39.175***	24.091	1966.330***	33.089**	29.633*	28.638**
Diagnostics for the trading intensity process							
Mean	0.999	0.999	0.999	1.000	1.000	0.996	1.000
S.D.	0.323	0.321	0.321	0.296	0.316	0.306	0.319
LB(20)	54.691***	33.924**	33.712**	279.141***	53.238***	29.348*	34.078**

Table 9: Maximum likelihood efficient importance sampling (ML-EIS) estimates of different parameterizations of (LF)-VAR models up to a lag order of $p = q = 2$ models for the log return volatility, the average volume and the number of trades per 5 min interval for the JP Morgan stock traded on the NYSE. Data extracted from the 2001 TAQ data base. The model is specified as given by (1) through (8). Sample period 02/01/01 to 31/05/01. Overnight observations are excluded. The models are re-initialized for every trading day. Standard errors are computed based on the inverse of the estimated Hessian. The ML-EIS estimates are computed using $R = 50$ Monte Carlo replications based on 5 EIS iterations.
Diagnostics: log likelihood function (LL), Bayes Information Criterion (BIC), average diagnostics (mean, standard deviation and univariate as well as multivariate Ljung-Box statistics (LB and MLB, respectively) based on 20 lags) over all trajectories of the residuals.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
ω_1	2.394***	0.217***	1.996***	0.529***	-0.097***	0.167***	0.209
ω_2	-1.584***	-2.635***	-1.410***	-2.240***	-1.471***	-2.035***	-2.122***
ω_3	-0.292***	-0.018***	-0.016***	-0.225***	-0.337***	-0.008***	-0.005
α_{12}^0	0.817***	0.979***	0.859***	0.492***	-0.078***	0.023*	0.050***
α_{13}^0	0.841***	1.112***	0.910***	0.365***	-0.091***	0.022	-0.033
α_{23}^0	-0.885***	-0.785***	-0.882***	-1.158***	-0.953***	-0.941***	-0.958***
α_{11}^1	0.149***	0.225***	0.138***		0.097***	0.085***	0.072***
α_{12}^1	0.025***	0.068***	0.009			-0.004	-0.010
α_{13}^1	0.020***	0.000	0.001***			0.000	0.001
α_{21}^1	-0.001	0.005***	-0.003			-0.022***	-0.029***
α_{22}^1	0.003***	0.010***	0.005***		0.005***	0.005***	0.010***
α_{23}^1	0.000**	0.000	0.000**			0.000***	0.000**
α_{31}^1	0.554***	0.008	0.550***			-0.037	0.096
α_{32}^1	0.320***	-0.110***	0.388***			-0.054	-0.101**
α_{33}^1	0.208***	0.216***	0.216***		0.188***	0.212***	0.214***
α_{11}^2		0.263***	0.117***			0.053**	0.049**
α_{22}^2		0.007***	-0.002***			-0.001	-0.004**
α_{33}^2		-0.205***	-0.207***			-0.206***	-0.210***
β_{11}^1	0.007	-0.107***	0.051		0.972***	0.861***	0.789***
β_{12}^1	0.191***		0.170***				-0.246***
β_{13}^1	-0.006		0.000				-0.001
β_{21}^1	0.677***		0.646***				0.066
β_{22}^1	0.646***	0.057	0.708***		0.178***	0.115**	0.083
β_{23}^1	0.025***		0.000				-0.001
β_{31}^1	1.320***		1.166***				0.180**
β_{32}^1	0.472***		0.527***				-0.427**
β_{33}^1	0.873***	1.609***	1.623***		0.836***	1.625***	1.655***
β_{11}^2		0.089***	-0.068**			0.086	0.177
β_{22}^2		0.305***	-0.017			0.025	-0.077
β_{33}^2		-0.612***	-0.625***			-0.626***	-0.657***
a_2	0.687***	0.634***	0.720***	0.863***	1.027***	0.854***	0.890***
m_2	7.633***	8.066***	6.947***	5.800***	4.917***	6.537***	5.869***
a_3	2.308***	2.393***	2.438***	2.414***	2.287***	2.454***	2.579***
a_3	2.577***	2.451***	2.373***	1.986***	2.620***	2.364***	2.173***
				Latent Component			
a_0				0.951***	0.907***	0.941***	0.930***
δ_1				0.165***	0.339***	0.339***	0.350***
δ_2				0.122***	0.176***	0.136***	0.132***
δ_3				0.024***	0.009***	0.011***	0.015***
				Diagnostics			
LL	-18403	-18784	-18306	-19398	-18201	-18040	-18009
BIC	-18529	-18910	-18458	-19461	-18291	-18184	-18180
MLB(20)	769.914***	1830.305***	665.637***	13692.174***	266.583***	145.293***	109.606***
			Diagnostics for the volatility process				
Mean	0.999	1.000	0.999	1.000	1.002	1.002	1.003
S.D.	1.750	1.745	1.714	1.622	1.536	1.511	1.487
LB(20)	376.349***	406.581***	355.220***	382.224***	25.398	19.872	21.652
			Diagnostics for the volume process				
Mean	1.001	1.001	1.001	1.000	1.001	1.001	1.001
S.D.	0.598	0.617	0.600	0.503	0.450	0.477	0.484
LB(20)	27.975	1393.848***	22.132	161.411***	21.541	13.175	13.391
			Diagnostics for the trading intensity process				
Mean	1.000	1.000	1.000	1.000	1.000	0.999	1.000
S.D.	0.276	0.274	0.273	0.304	0.276	0.272	0.271
LB(20)	138.865***	38.987***	43.659***	6623.880***	141.237***	52.276***	55.277***

Table 10: Maximum likelihood efficient importance sampling (ML-EIS) estimates of different parameterizations of (LF)-VAR models up to a lag order of $p = q = 2$ models for the log return volatility, the average volume and the number of trades per 5 min interval for the IBM stock traded on the NYSE. Data extracted from the 2001 TAQ data base. The model is specified as given by (1) through (8). Sample period 02/01/01 to 31/05/01. Overnight observations are excluded. The models are re-initialized for every trading day. Standard errors are computed based on the inverse of the estimated Hessian. The ML-EIS estimates are computed using $R = 50$ Monte Carlo replications based on 5 EIS iterations.
Diagnostics: log likelihood function (LL), Bayes Information Criterion (BIC), average diagnostics (mean, standard deviation and univariate as well as multivariate Ljung-Box statistics (LB and MLB, respectively) based on 20 lags) over all trajectories of the residuals.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
ω_1	0.057***	0.547***	0.372***	0.504***	0.494***	0.316***	1.079***
ω_2	-0.089***	-1.267***	-0.784***	-1.420***	-1.831***	-1.307***	-1.403***
ω_3	-0.036***	-0.192***	-0.200***	-0.517***	-0.373***	-0.160***	-0.305***
α_{12}^0	0.077***	0.821***	0.792***	0.704***	0.596***	0.128***	0.140***
α_{13}^0	0.074***	0.712***	0.746***	0.182**	0.585***	0.099***	0.429***
α_{23}^0	-0.080***	-0.394***	-0.783***	-1.355***	-0.768***	-0.866***	-0.789***
α_{11}^1	0.024***	0.203***	0.183***		0.070***	0.068***	0.067***
α_{12}^1	0.003***	0.044***	0.025***			0.018**	0.015**
α_{13}^1	0.000	0.003	0.000			-0.001	0.002
α_{21}^1	-0.005***	-0.015***	-0.052***			-0.061***	-0.064***
α_{22}^1	0.002***	0.019***	0.021***		0.016***	0.012***	0.013***
α_{23}^1	0.000***	0.001***	0.001***			-0.001**	0.001**
α_{31}^1	-0.012***	-0.138***	-0.136***			-0.086**	0.015
α_{32}^1	0.031***	-0.061***	0.344***			0.110**	0.136***
α_{33}^1	0.016***	0.188***	0.191***		0.153***	0.182***	0.151***
α_{11}^2		0.230***	0.131***			0.086***	0.048**
α_{22}^2		0.014***	-0.004***			0.001	0.001
α_{33}^2		-0.110***	-0.101***			-0.105***	-0.087***
β_{11}^1	0.063***	0.340***	0.580***		-0.228***	0.471***	0.596***
β_{12}^1	0.007***		0.048***				-0.050
β_{13}^1	0.000		0.001				-0.007**
β_{21}^1	-0.006**		-0.108***				0.477***
β_{22}^1	0.081***	0.071**	0.866***		0.176***	0.400***	0.436***
β_{23}^1	0.001		0.000				-0.059***
β_{31}^1	-0.030***		-0.347***				0.114
β_{32}^1	0.068***		0.701***				0.105
β_{33}^1	0.091***	1.289***	1.211***		0.900***	1.272***	1.122***
β_{11}^2		0.133***	0.044			0.496***	0.414***
β_{22}^2		0.642***	-0.009			-0.091**	-0.115**
β_{33}^2		-0.329***	-0.252***			-0.313***	-0.229**
a_2	0.083***	0.759***	0.847***	1.168***	0.965***	1.190***	1.115***
m_2	7.271***	7.778***	6.936***	4.466***	6.492***	4.953***	5.470***
a_3	2.225***	2.222***	2.303***	2.294***	2.156***	2.278***	2.172***
a_3	4.373***	4.424***	4.131***	3.499***	4.642***	4.255***	4.620***
Latent Component							
a_0				0.942***	0.967***	0.940***	0.944***
δ_1				0.154***	0.141***	0.263***	0.256***
δ_2				0.146***	0.087***	0.133***	0.123***
δ_3				0.044***	0.004***	0.012***	0.003**
Diagnostics							
LL	-15389	-15798	-15338	-16959	-15349	-15106	-15086
BIC	-15515	-15924	-15490	-17021	-15439	-15250	-15257
MLB(20)	324.925***	686.107***	240.460***	20283.796***	1026.523***	83.109**	68.273
Diagnostics for the volatility process							
Mean	1.000	1.000	1.000	1.001	1.000	1.002	1.002
S.D.	1.720	1.724	1.705	1.609	1.553	1.475	1.471
LB(20)	175.325***	282.111***	154.973***	915.176***	537.847***	16.854	15.788
Diagnostics for the volume process							
Mean	1.000	1.000	0.999	1.000	1.000	1.001	1.000
S.D.	0.485	0.512	0.484	0.411	0.420	0.384	0.391
LB(20)	50.970***	444.839***	47.007***	407.268***	63.613***	23.199	24.031
Diagnostics for the trading intensity process							
Mean	0.999	0.999	0.999	1.000	1.000	1.000	1.000
S.D.	0.217	0.216	0.216	0.237	0.217	0.215	0.216
LB(20)	83.299***	9.630	12.195	8382.230***	84.800***	12.514	11.609

A.4.3 Estimated generalized impulse response functions

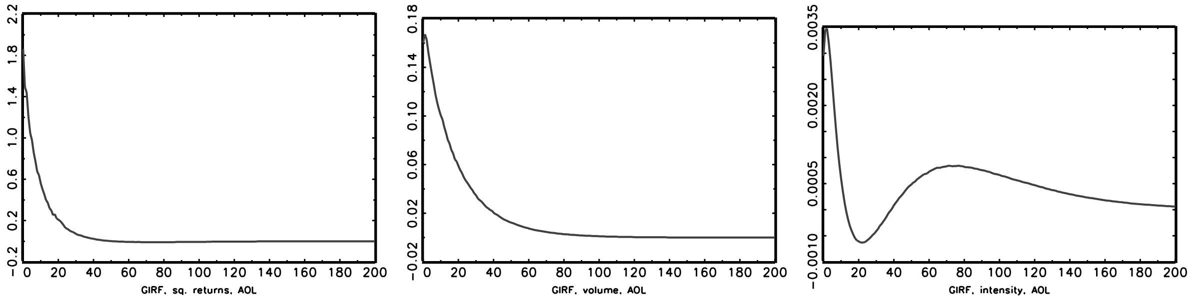

Figure 21: Generalized impulse response of a one S.D. shock of λ_i on ξ_i^2 (left), V_i (middle) and ρ_i (right) for the AOL stock. Computed based on $5,000$ Monte Carlo simulations using the estimates of specification (8) (Table 7).

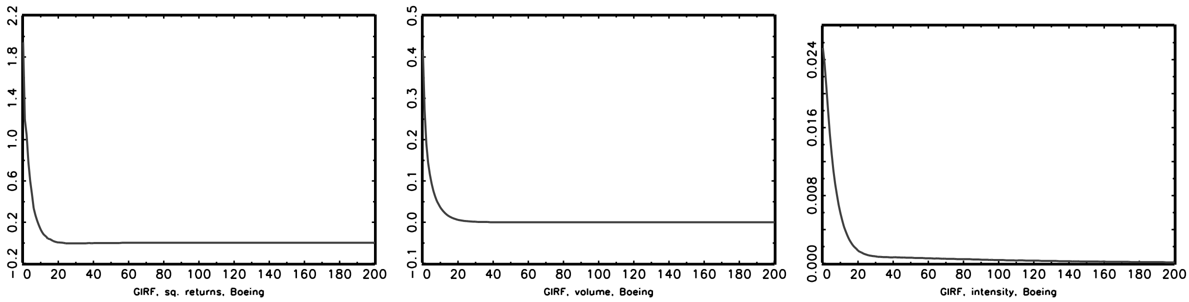

Figure 22: Generalized impulse response of a one S.D. shock of λ_i on ξ_i^2 (left), V_i (middle) and ρ_i (right) for the Boeing stock. Computed based on $5,000$ Monte Carlo simulations using the estimates of specification (8) (Table 8).

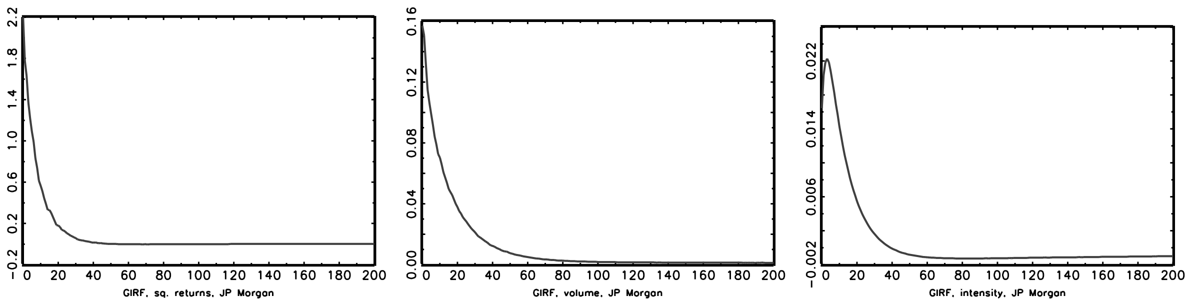

Figure 23: Generalized impulse response of a one S.D. shock of λ_i on ξ_i^2 (left), V_i (middle) and ρ_i (right) for the JP Morgan stock. Computed based on $5,000$ Monte Carlo simulations using the estimates of specification (8) (Table 9).

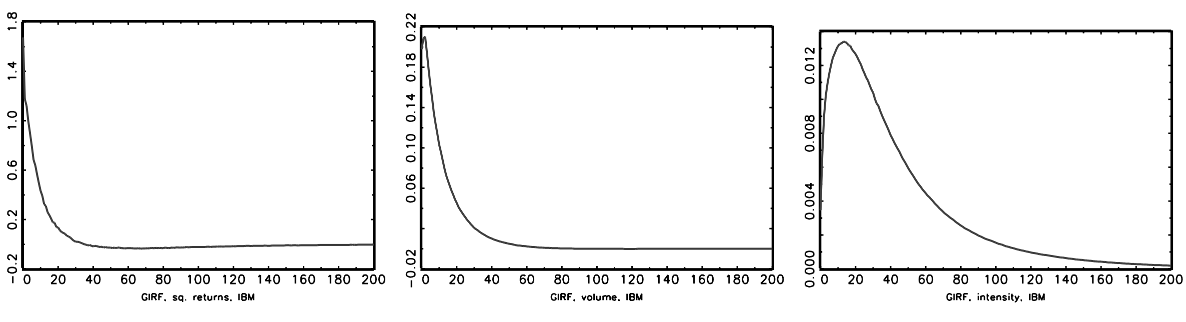

Figure 24: Generalized impulse response of a one S.D. shock of λ_i on ξ_i^2 (left), V_i (middle) and ρ_i (right) for the IBM stock. Computed based on $5,000$ Monte Carlo simulations using the estimates of specification (8) (Table 10).