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Pitfalls in Estimates of the Relationship between Share Returns and Inflation¹

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Abstract. Empirical tests of the Fisher hypothesis give conflicting results, regardless of whether income growth is accommodated in the estimates. This paper shows theoretically and empirically that standard methods of testing the Fisher hypothesis give biased results and that the bias depends on the specification of the Fisher equation, the process governing inflation, measurement of inflation expectations, and the time aggregation of the data. Alternative tests show that share markets take several years to adjust to innovations in inflation and therefore that the Fisher hypothesis cannot be maintained.

JEL Classification: G12

Key words: Fisher hypothesis, share returns, inflation persistence

1 Introduction

The puzzle of low real *ex post* share returns in the high inflation era of the 1970s and high real share returns in the low inflation period of the 1990s has sparked a substantial amount of research into the question of whether shares are hedged against expected inflation following the Fisher hypothesis (Fisher, 1911) which states that shares are hedged against expected inflation. Numerous papers have found that share returns are not hedged against expected inflation and have interpreted this as evidence against the Fisher hypothesis.² Others argue that regressions of share returns on expected inflation yield biased coefficient estimates because expected income growth has been omitted from the estimates and find that the Fisher hypothesis cannot be rejected when expected income growth is accommodated in the estimates.³ However, recent research indicates that the Fisher hypothesis is rejected even if expected income growth is accommodated in the estimates.⁴ Finally, long-run estimates and estimates for high inflation economies, have tended to show a stronger link between share returns and (expected) inflation than short-run estimates.⁵

¹ Helpful comments and suggestions from Hans Christian Kongsted, Darrel Turkington and seminar participants at the University of Western Australia and University of Konstanz are gratefully acknowledged.

² Barnes *et al* (1999), Caporale and Jung (1997), Cochran and DeFina (1993), Graham (1996), Ely and Robinson (1993), Erb *et al* (1995), Fama and Schwert (1977), Firth (1979), Gultekin (1983), Jaffe and Mandelker (1976), and Nelson (1976).

³ Benderly and Zwick (1985), Fama (1981), and Kaul (1987), Kaul and Seyhun, 1990.

⁴ Balduzzi (1995), Cochran and DeFina (1993), and Caporale and Jung (1997).

Of primary concern in this paper is the conflicting results that are obtained in the literature. The estimated coefficients of expected inflation vary substantially across countries, over time, and are overly sensitive to model specification and the inflationary environment. The sensitivity of the estimates to sample period and country has prompted some authors to explain the nexus between share returns and inflation in terms of the monetary policy rule (Geske and Roll, 1983, Graham, 1996, Kaul, 1987, 1990, and Solnik, 1983). However, what has not been questioned in the literature is the sensitivity of the results to the specification of the estimation equation, the measurement of inflation expectations, the time-series properties of inflation, and the time aggregations of the data, and the extent to which standard tests can reveal anything about the Fisher hypothesis. An implicit assumption in standard tests is that regressions of real or nominal share returns on expected inflation and other potentially important variables can be used to test the Fisher hypothesis. While Jaffe and Mandelkaer (1976) have pointed out that the theoretical coefficient of expected inflation is largely unknown due to an errors-in-variable bias, no systematic attempt has been undertaken to explain the conflicting results in the literature and why standard tests of the Fisher hypothesis can be very misleading.

This paper demonstrates theoretically and empirically that standard tests of the Fisher hypothesis can be directly misleading and often do not reveal much about the validity of the Fisher hypothesis. In the next section it is shown that the standard tests of the Fisher hypothesis tend to yield inconsistent estimates. It is further shown that the sign and size of the inconsistency depends on the time-series properties of inflation, the generation of inflation expectations, and especially whether real or nominal share returns are used as the dependent variable. Tests of the Fisher equation are, in many circumstances, a joint test of inflation persistence and the Fisher hypothesis if nominal share returns are used as the regressor. Generally, the more persistent is inflation the stronger is the evidence in favor of the Fisher hypothesis. However, if the dependent variable is real share returns, then the opposite result applies, except when actual inflation is used as a proxy for expected inflation. It follows that the author, has to a large extent, control over the outcome of the test of the Fisher hypothesis by choice of model, time aggregation of the data, and measurement of inflation expectations.

Using data on share returns for the OECD countries over the period from 1890 to 1997, the empirical estimates in Section 3 support the results that are derived in the theoretical section. The estimates reveal that tests of the Fisher hypothesis are very sensitive to the persistence of inflation, whether real or nominal share returns are used as the dependent

⁵ Boudoukh and Richardson (1993) found the Fisher hypothesis to approximately hold using 5-year frequency data. Their finding has been challenged by Barnes *et al* (1999), Boyd *et al* (1997), and Erb *et al* (1995). Choudhry (2001) found evidence of the Fisher effect for high inflation economies.

variable, choice of instruments, and time aggregation of the data. An alternative test of the Fisher hypothesis, which is not subject to the problems of standard tests, show that share returns take several years to adjust to innovations in inflation.

2 Asymptotic properties of various estimators of the Fisher equation

Consider the following equation where nominal share returns, RS_t^N is regressed against expected inflation, π_{t+1}^e , at period $t+1$ conditional on information available at period t :

$$RS_t^N = \zeta_0 + \zeta_1 \pi_{t+1}^e + \varepsilon_t$$

where ε_t is a zero-mean, finite variance, and serial uncorrelated disturbance term. The null hypothesis that the Fisher hypothesis holds is:

$$H_0: \zeta_1 = 1.$$

It is therefore an underlying assumption for the Fisher hypothesis that expectations are rational and that the real interest rate, ζ_0 , is constant. If inflation expectations could be easily measured, testing the Fisher hypothesis would be straightforward. Unfortunately, it is difficult to accurately measure inflation expectations, which, as shown below, renders tests of the Fisher hypothesis sensitive to the choice of sample period, time aggregation of the data, instruments, and country. Comparisons between tests of the Fisher hypothesis are further complicated by the use of different models that have quite different asymptotic properties. The asymptotic properties of the most common model specifications are considered in this section. The asymptotic properties of less commonly used specifications are relegated to the appendix.

Consider the following model specifications that the literature commonly uses to test the Fisher hypothesis:

$$RS_t^N = \zeta_0 + \zeta_1 \pi_{t+1}^* + \varepsilon_{1t} \tag{1}$$

$$RS_t^N = \zeta_0 + \zeta_1 \pi_t + \varepsilon_{2t} \tag{2}$$

$$RS_t^N = \zeta_0 + \zeta_1 \pi_{t+1} + \varepsilon_{3t}, \tag{3}$$

where π_{t+1}^* is either a proxy for inflation expectations at time $t+1$ or instrumented inflation expectations.⁶

Only in the very special case, where inflation expectations are measured accurately and all relevant regressors, or regressors that are orthogonal to the included regressors, are included in the estimates, do the models give consistent parameter estimates. To see this, suppose that one proxies inflation expectations using the variable, π_{t+1}^* , which consists of accurately measured inflation expectations and a stochastic measurement error term v_{t+1} :

$$\pi_{t+1}^* = \pi_{t+1}^e + v_{t+1}$$

The least squares estimates of the models yield the following probability limit of ζ_1 :

$$p \lim \hat{\zeta}_1 = \frac{\zeta_1 + \text{cov}(v_{t+1}, \varepsilon_{it})}{1 + \text{var}(v_{t+1}) / \text{var}(\pi_{t+1}^e)} \quad (4)$$

$$i = 1, 2, 3.$$

Equation (4) shows that the estimate of ζ_1 is inconsistent unless inflation expectations are accurately measured or instruments, which are uncorrelated with the error terms, ε_{it} , are used. Assuming that $\text{cov}(v_{t+1}, \varepsilon_{it}) = 0$, then Equation (4) shows that the estimate of ζ_1 is biased downwards zero if inflation expectations are measured with an error, and the more error-ridden are the data, the more downward biased is the estimate of ζ_1 . Furthermore, Equation (4) shows that the estimate of ζ_1 cannot be negative in a well-specified equation, which entails that $\text{cov}(v_{t+1}, \varepsilon_{it}) = 0$, even if the share market suffers from complete money illusion so that $\zeta_1 = 0$. It is therefore highly suspicious that several studies, which base their tests on Models 1-3, frequently get negative estimates of ζ_1 . Negative estimates of ζ_1 can only be obtained if the models are subject to specification errors and have regressors that are correlated with the error term.

Consistent estimates of the models can generally only be obtained from estimates of Model 1 *and* then only if one uses instruments for expected inflation that are uncorrelated with the measurement errors and perfectly correlated with expected inflation, unless inflation is an integrated process, as shown below. However, it is almost impossible to find instruments that are perfectly correlated with expected inflation (Griliches and Hausman, 1986). Even if good instruments are used, there is no guarantee that they are perfectly correlated with expected inflation.

⁶ Model 1 is used by Boudoukh *et al* (1994), and Boudoukh and Richardson (1993). Models (2) and (3) are used by Barnes *et al* (1999), Erb *et al* (1995), Firth (1979), and Nelson (1976).

Estimates of the models reveal more about the time-series properties of inflation than the presence of a Fisher effect under the maintained hypothesis that the Fisher hypothesis holds. Suppose that inflation follows an AR(1) process: $\pi_{t+1} = \rho\pi_t + \kappa_t$, where κ_t is a zero-mean, finite variance, and serial uncorrelated disturbance term and $0 \leq \rho \leq 1$. Under rational expectations we have that $E(\pi_{t+1}) = \rho\pi_t$. Consider Model 2 where nominal share returns are regressed on π_t . If the Fisher hypothesis holds, then $E(SR_t^N) = E(\pi_{t+1}) = \rho\pi_t$ and the probability limit of estimating Model 2 is given by:

$$p \lim \hat{\zeta}_1 = \frac{\text{cov}(RS_t^N, \pi_t)}{\text{var}(\pi_t)} = \frac{\text{cov}(\rho\pi_t, \pi_t)}{\text{var}(\pi_t)} = \rho. \quad (5)$$

Hence, under the maintained hypothesis that the Fisher hypothesis holds, the estimate of ζ_1 is simply the first order serial correlation coefficient of inflation if π_t is used as a proxy for expected inflation.⁷ It follows that Model 2 is a joint test of inflation persistence and the Fisher hypothesis. If inflation is highly serial correlated, then the Fisher hypothesis tends not to be rejected if it holds. However, if inflation is serial uncorrelated, then the Fisher hypothesis will be rejected under all circumstances. In terms of Equation (4) the higher is inflation persistence the lower is the measurement error, and the lower is the second term in the denominator. These results may explain why Barnes *et al* (1999) and Erb *et al* (1995) find that economies with high inflation have a coefficient of inflation which is very close to one, whereas low-inflation economies have coefficients of inflation that are significantly lower than one and mostly negative. Using data for four high inflation economies Choudhry (2001) is unable to reject the Fisher hypothesis. In an extensive cross-country study Anderton (1997) finds that high inflation economies have more persistent inflation than low inflation economies.

Consider Model 3, where nominal share returns are regressed on π_{t+1} . Under the assumption that the Fisher hypothesis is true, this yields the following probability limit:

$$p \lim \hat{\zeta}_1 = \frac{\text{cov}(SR_t^N, \pi_{t+1})}{\text{var}(\pi_{t+1})} = \frac{\text{var} E_t(\pi_{t+1})}{\text{var}(\pi_{t+1})} \quad (6)$$

which is equal to $p \lim R^2$ from regressing π_{t+1} on all relevant variables, which are in the investor's information set at time t . Hence, the estimated coefficient of π_{t+1} in Model 3 measures the fraction of inflation that is forecastable under the maintained hypothesis that the

⁷ The problem is essentially the same as that described McCallum's (1984) remarks on tests of Fisher effects in nominal interest rates, and Sargent's (1971) discussion of tests of the long-run slope of the Phillips curve.

Fisher hypothesis holds. Suppose ρ is close to one. Then inflation is easy to forecast and the estimate of ζ_1 is close to one under the maintained hypothesis. Conversely, if ρ is close to zero, then inflation is difficult to predict and the estimate of ζ_1 is close to zero regardless of whether the Fisher hypothesis holds. In summary it can be concluded that tests based on Models 1-3 are biased against the Fisher hypothesis and that the bias is larger the less persistent is inflation. If the Fisher hypothesis does not hold, then the coefficient of expected inflation will go towards zero independent of the persistence of inflation.

A phenomenon that has puzzled researchers is why the estimated coefficient of expected inflation in Models 1-3 tends towards one in estimates that use data of five-year frequencies or lower (see for instance Boudoukh and Richardson, 1993). The above errors-in-variables framework can be used to explain this. Inflation is more persistent in low frequency data, as shown in the empirical estimates in the next section, and is therefore easier to predict. Hence, expected inflation is close to actual inflation and the measurement error, v_{t+1} , becomes negligible. It follows from Equation (4) that ζ_1 is estimated to be closer to its true value. From this it may appear that lower frequency data are more suitable to use in testing the Fisher hypothesis than high frequency data. However, this is not necessarily true. If share prices are slow to adjust to inflation, then tests with low frequency data may give evidence for the Fisher hypothesis although it may not hold. This issue is addressed in the empirical section.

In summary, the results in this section show that tests of the Fisher hypothesis are highly sensitive to model specification, the degree of inflation persistence, time-aggregation of the data, and the quality of the instruments. The higher is the time-aggregation of the data and the better are the instruments, the more likely it is that one finds evidence for the Fisher hypothesis, using estimates of the models above. The more persistent is inflation, the more likely it is that the Fisher hypothesis is accepted, using nominal share returns as a regressand, whereas the opposite result applies when real share returns is used as the regressor. Standard tests can therefore not be used for strict testing of the Fisher hypothesis. This raises the question of whether it is possible to get a consistent test of the Fisher hypothesis. An alternative test, which overcomes the problems that are associated with standard tests of the Fisher hypothesis, is suggested in Section 3.2.

3 Empirical estimates

The first part of this section investigates the sensitivity of the test results of the Fisher hypothesis to model specification, errors-in-variables, inflation persistence, and time

aggregation of the data. An alternative test of the Fisher hypothesis is suggested in the second part of this section.

3.1 Inflation persistence and tests of the Fisher hypothesis

Models 1-3 and Models A1 and A3 in the Appendix, augmented with expected income growth following Fama (1981), are estimated:⁸

$$RS_{it}^N = a_0 + a_1\pi_{i,t+1}^e + a_2\Delta\log Y_{i,t+1}^e + u_{1it} \quad (7)$$

$$RS_{it}^N = b_0 + b_1\pi_{it} + b_2\Delta\log Y_{i,t+1}^e + u_{2it} \quad (8)$$

$$RS_{it}^N = c_0 + c_1\pi_{i,t+1}^e + c_2\Delta\log Y_{i,t+1}^e + u_{3it} \quad (9)$$

$$RS_{it}^R = d_0 + d_1\pi_{i,t+1}^e + d_2\Delta\log Y_{i,t+1}^e + u_{4it} \quad (10)$$

$$RS_{it}^R = e_0 + e_1\pi_{i,t+1}^e + e_2\Delta\log Y_{i,t+1}^e + u_{5it} \quad (11)$$

where i and t signify country i and time t , and Y is income measured as real per capita GNP. Instruments are used for expected inflation in Models 7 and 10, and actual inflation at period t or $t+1$ is used in the other models. Expected inflation is instrumented/estimated under the assumption of rational expectations using the method of McCallum (1976), which yields consistent parameter estimates. R_{t+1} is first regressed on an investor's information set at period t , where R is the nominal interest rate on long term government bonds. The predicted values from this regression are then used as regressors in the model. This method assumes constant real interest rates. However, since the Fisher hypothesis assumes constant real interest rate the test results are unaffected by this assumption. The instruments are listed in the notes to Table 1. Last month of the year share returns are used and the September figures for one-year inflation are used to allow for a 3-month publication lag. For the *pre* WWII period, consumer prices are either measured as the average during the year to September or at a certain point in time during the year. Statistical agencies first started to measure consumer prices systematically on a monthly or quarterly basis from about 1914 in the majority of the OECD countries. The equations are estimated over the following two sample periods: the period from 1890 to 1939, which is a period of relatively low inflation persistence, and the period from 1961 to 1995, which is a period of relatively high inflation persistence.⁹

⁸ Model A2 is not estimated since the parameter estimates can be inferred from estimates where nominal share returns are regressed on actual inflation as discussed in the Appendix.

⁹ Using the same estimation method, data periods, and country sample as in the estimates in Table 1 yields the following simple estimates of inflation persistence:

To gain efficiency, the equations are estimated using pooled cross-section and time series data for the 17¹⁰ OECD countries for which share returns are available over the period from 1961 to 1995, and for four OECD countries for which share returns are available for over the period from 1890 to 1939 (USA, Australia, France, and the UK). Annual data are used in all the estimates. *F*-tests are carried out to certify that the pooling does not lead to biased estimates due to cross-country coefficient variations. To gain further efficiency, the generalised instrumental variable method, where the covariance matrix is weighted by the correlation of the disturbance terms, is used. More specifically the following variance-covariance structure is assumed:

$$\begin{aligned} E\{\varepsilon_{it}^2\} &= \sigma_i^2, \quad i = 1, 2, \dots, N, \\ E\{\varepsilon_{it}, \varepsilon_{jt}\} &= \sigma_{ij}, \quad i \neq j, \end{aligned}$$

where σ_i^2 = the variance of the disturbance terms for country $i = 1, 2, \dots, N$, σ_{ij} = the covariance of the disturbance terms across countries i and j , and ε is the disturbance term. The variance σ_i^2 is assumed to be constant over time but to vary across countries and the error terms are assumed to be mutually correlated across countries, σ_{ij} , as random shocks are likely to impact on all countries at the same time. σ_i^2 and σ_{ij} are estimated using the feasible generalised least squares method described in Greene (2000, Ch. 15).

Table 1. Parameter estimates of models (7)-(11).

The results of estimating Models 7-11 over the two estimation periods are displayed in Table 1. The null hypothesis of cross-country coefficient constancy cannot be rejected at conventional significance levels for any of the models. This suggests low, if any, potential costs of pooling. Leamer's (1978, p 114) formula is used to calculate the critical *F*-values of diffuse priors, which takes into account that the likelihood of rejecting the null hypothesis grows with the sample size. The critical values are presented for each equation in Table 1.

$\hat{\pi}_t = 0.01 + 0.69\pi_{t-1}$ <small>(7.87) (23.6)</small>	$R^2(\text{mom}) = 0.76$ 1962-1995
$\hat{\pi}_t = 0.01 + 0.41\pi_{t-1}$ <small>(0.88) (6.24)</small>	$R^2(\text{mom}) = 0.17$ 1890-1939,

where the numbers in parenthesis are absolute *t*-statistics, and $R^2(\text{mom})$ is Buse's raw-moment multiple correlation coefficient. The estimates show that a large fraction of the variance of consumer price inflation can be predicted from previous inflations in the postwar period whereas only a small proportion of the variance of consumer price inflation could be predicted in the *pre* WWII period.

¹⁰ The following countries are included in the postwar data set: Canada, USA, Japan, Australia, Austria, Belgium, Denmark, Finland, France, Germany, Italy, Netherlands, Norway, Spain, Sweden, Switzerland, and the UK.

Note that the diagnostic tests are based on within country residuals to remove fixed country effects.

The residuals are serial uncorrelated of first order for all models, but the null hypotheses of structural stability with breaking points in 1910/11 and 1982/83 are rejected for most models, especially in the postwar estimates. Furthermore, heteroscedasticity is present in the postwar estimates. The diagnostic problems in the postwar estimates indicate that the equations are misspecified and suggest that the inflationary process has changed over the estimation period and that the models have therefore not fully captured the data generating process. The presence of structural instability in the estimates is predictable since the estimated coefficients of expected inflation are sensitive to the process governing inflation, especially inflation persistence that fell by a factor of 0.05 after 1983. The heteroscedasticity tests revealed that the estimated variances of the residuals are correlated with the measurement of inflation expectations (the results are not shown). These diagnostic problems reinforce the suspicion that the inflationary process interferes with the regression results. It is interesting to note that the results of tests for structural stability and heteroscedasticity are rarely presented in the literature.

Consider the postwar estimates where nominal share returns are the dependent variable. If expected inflation is proxied by π_t or π_{t+1} (Models 8 and 9), then the estimated coefficients of inflation are negative and not significantly different from zero at the 1% level. The negative estimated coefficient of π_t in Model 8 echoes the frequent finding in the literature. If π_{t+1} is instrumented, then its estimated coefficient of 0.42 is significantly higher than zero, at the 1% level (Model 7). This shows that the low estimated coefficients of π_t or π_{t+1} are partly due to an errors-in-variables problem. Conventional tests would reject the Fisher hypothesis of $\zeta_1 = 1$ in Model 7. However, as shown in the previous section, the test of $\zeta_1 = 1$ is a joint test of inflation persistence and the Fisher hypothesis. If the Fisher hypothesis holds, then ζ_1 equals the degree of inflation persistence assuming that inflation follows an AR(1) process. The hypothesis that ζ_1 equals the degree of inflation persistence cannot be rejected at any conventional significance level (Wald's $\chi^2(1) = 0.38$).¹¹ Does this mean that the Fisher hypothesis ultimately holds? Not necessarily. The test results are too sensitive to the choice of instruments and are therefore not strictly valid tests of the Fisher hypothesis. Nevertheless, the results show that estimates that take inflation persistence into account and use instruments, give results that are more favorable to the Fisher hypothesis than other standard tests.

¹¹ This test is based on joint estimates of Equation (7) and an AR(1) model of inflation using SURE.

The estimates where real share returns are used as the dependent variable, for the postwar period, are more favorable to the Fisher hypothesis, as anticipated from the exposition in the previous section. The null hypothesis is rejected if expected inflation is measured by π_{t+1} (Model 11) but not if instruments for expected inflation are used (Model 10), as indicated by the $\chi^2(1)$ tests.

Turning to the *pre* WWII estimates in the lower part of Table 1 the results clearly reflect the low inflation persistence in this period. Consider the estimates where nominal share returns are regressed on instrumented inflation in the two periods (Model 7). In the postwar estimates the coefficient of expected inflation is significantly higher than zero, but not in the *pre* WWII estimates. The difference is likely to arise because of different inflation persistence in the two periods. The joint hypothesis that the estimated coefficient of expected inflation is equal to the persistence of inflation and the Fisher hypothesis holds, cannot be rejected at any conventional significance levels (Wald's $\chi^2(1) = 0.30$). Similarly considering the estimates of Model 11, the estimated coefficient of π_{t+1} is significantly negative in the postwar period estimates, but not in the *pre* WWII estimates. This result is consistent with the results in the previous section that the more persistent is inflation the more evidence is found for (against) the Fisher hypothesis using nominal (real) share returns as the dependent variable.

Finally, to illustrate the effects of time aggregation, the log ten-year change in the accumulated share index is regressed on the log ten-year change in consumer prices. Annual overlapping data over the period from 1900 to 1997 for the US, Australia, France, and UK are used. To cater for the nine period moving average in the disturbance terms that arise because the forecast horizon exceeds the observation interval, the standard errors are based on the covariance matrix of Hansen and Hodrick (1980). The estimation results are as follows:

$$\ln ASP_t^N - \ln ASP_{t-10}^N = \underset{(11.4)}{0.70} + \underset{(7.94)}{0.54}(\ln CPI_t - \ln CPI_{t-10}), \quad (12)$$

$$T = 1900, 1901, \dots, 1997 \quad N = 1, 2, 3, 4$$

where ASP^N is the nominal accumulated share index, CPI is the consumer price index, and the numbers in parentheses are t -statistics. Income has been omitted from the estimates because its estimated coefficient was insignificant at any conventional significance level. Although the Fisher hypothesis is still rejected ($\chi^2(1) = 47.2$), the estimated coefficient of inflation is substantially higher than the estimates in Table 1. Can we, from this result, conclude that the economy is more Fisherian in long difference estimates because a large fraction of the measurement errors has been eliminated? Not necessarily, because the results in Equation (12) may arise because share markets are slow to adjust to inflation. If share markets take less

than ten years to adjust to innovations in prices, then the lack of rationality will not be identified by Equation (12).

3.2 Alternative test of the Fisher hypothesis

The estimates above show that tests of the Fisher hypothesis are sensitive to inflation persistence, model specification, errors-in-variables, and time aggregation. Common for all estimates, however, is less that proportionality between nominal share returns and expected inflation. The possibility that this result arises because of incomplete adjustment to inflationary innovations is examined in this section. The following model is estimated over the period from 1961 to 1997 for the 17 OECD countries considered above:

$$RS_t^N = f_0 + f_1 \pi_{t+1}^e + f_2 \Delta \log Y_{t+1}^e + \sum_{i=0}^8 f_{i+2} \pi_{t-i} + u_{6it}, \quad (13)$$

and the null hypothesis that the Fisher hypothesis holds is:

$$H_0: f_{i+2} = 0, \quad i = 1, 2, \dots, 8.$$

Equation (13) is estimated using the system estimator described above, and the expected inflation and income growth are instrumented using the instruments in Table 1. Up to eight lags in inflation are considered because further lags were insignificant. Estimates of Equation (13) will reveal the speed of adjustment of share prices to innovations in consumer prices. If the share market rationally incorporates all relevant information into their forecast of inflation, then the share returns will be unaffected by past inflation innovations, and the Fisher hypothesis that share markets rationally embody all relevant information in share prices, cannot be rejected. The results of estimating Equation (13) are shown in the first column in Table 2.

The estimates in the first column in Table 2 show that share prices are slow to adjust to innovations in consumer prices and that the initial response is significantly negative. An increase in inflation by one-percentage point leads initially to a 1.55-percentage reduction in share returns. Thereafter share returns gradually adjust to the innovation in inflation and the adjustment is completed after eight years to such an extent that the null hypothesis of unity of the sum of the estimated coefficients of *ex ante* and *ex post* inflations cannot be rejected at conventional significance levels. In other words the share market initially perceive innovations in inflation as impacting negatively on share returns but adjust share prices to such an extent, that proportionality between share returns and inflation holds in the long run. Since the estimated coefficients of lagged inflation are statistically highly significant, it

implies that share markets do not adequately adjust for innovations in inflation and therefore that the Fisher hypothesis is rejected.

Table 2. Parameter estimates of Model 13.

π_t	-0.99(4.13)	$R^2(\text{mom})$	0.33	ΔR_t	-3.00(9.74)	$R^2(\text{mom})$	0.39
π_{t-1}	0.87(2.87)	$F(l,m)$	1.01	π_t	-0.05(0.20)	$F(l,m)$	1.22
π_{t-2}	-0.42(1.39)	$DW(M)$	2.00	π_{t-1}	0.80(2.72)	$DW(M)$	1.99
π_{t-3}	0.89(2.94)	$\text{Chow}(o,p)$	3.11	π_{t-2}	-0.93(3.17)	$\text{Chow}(o,p)$	2.01
π_{t-4}	0.77(2.61)	$BP(k)$	19.9	π_{t-3}	0.59(2.00)	$BP(k)$	23.9
π_{t-5}	-0.59(2.04)	$\chi^2(1)$	0.3	π_{t-4}	0.66(2.25)	$\chi^2(1)$	2.7
π_{t-6}	0.95(3.34)			π_{t-5}	-0.63(2.22)		
π_{t-7}	-0.60(2.22)			π_{t-6}	1.15(4.14)		
π_{t-8}	0.79(3.85)			π_{t-7}	-0.66(2.53)		
π_{t+1}^e	-0.56(2.83)			π_{t-8}	0.76(3.78)		
Δy_{t+1}^e	1.19(4.38)			π_{t+1}^e	-0.35(1.52)		
Con	0.03(2.01)			Δy_{t+1}^e	1.18(4.34)		
				Con	0.02(1.21)		

Notes: See notes to Table 1. Estimation period: 1961-97. Number of observations = 578.

Why does the share market initially react negatively to an innovation in inflation? One reason could be that share markets expect the central bank to restrict its monetary policies in response to an unexpected increase in inflation, which will lower the discounted value of expected dividends and adversely affect expected profits due to the adverse demand effects of the real interest increase. To investigate this issue the estimates in the second column in Table 2 include the change in a long-term government bond to accommodate the effects of expected central bank reactions. If an adverse central bank reaction is anticipated, then it will be embodied in the interest rate on long bonds. The estimated coefficient of the change in the nominal interest rate is highly significant and negative. When the interest rate effect is accommodated in the estimates the estimated coefficients of expected and contemporaneous inflation are rendered insignificant, thus giving credit to the adverse central bank reaction hypothesis. It remains, however, to be explained why share markets take eight years to adjust to innovations in inflation.

4 Concluding remarks

Recently there has been an increase in the number of papers that test whether shares are hedged against inflation, and it appears that the number of conflicting results has also been increasing. This paper has shown theoretically and empirically that the conflicting results can, to a large extent, be explained by differences in model specification, time aggregation of

the data, inflation persistence in the data sample, and whether instruments have been used for expected inflation.

The interaction between model specification and inflation persistence was found to be particularly influential. The more persistent is inflation the more favorable are estimates which use nominal share returns as the dependent variable, to the Fisher hypothesis, because inflation is easier to predict. The opposite result applies using real *ex post* share returns as the dependent variable, except in the case where inflation expectations are measured by the actual rate of inflation. Furthermore, tests are more favorable to the Fisher hypothesis when low frequency data and instruments for expected inflation are used under the circumstances where nominal share returns are used as the dependent variable. Given the inflationary environment, the appropriate choice of model specification, instruments, and time aggregation, will yield any results an author desires.

An alternative test of the Fisher hypothesis, which yields consistent estimates, was suggested. The test relies only on past information and therefore does not depend on choice of instruments, time aggregation, model specification, and the degree of inflation persistence. The results reject the Fisher hypothesis, because share prices take eight years to fully adjust to the long-run equilibrium in response to an inflation innovation. This result explains the puzzle why share prices are closer indexed to consumer prices in long-run than short-run estimates. Share markets have simply adjusted to inflation innovations in long-run estimates.

Given the results in this paper, it is unlikely that further standard tests of the Fisher hypothesis will shed much more light on the validity of the Fisher hypothesis. The question for further research is not whether the Fisher hypothesis holds but a deeper understanding as to why share prices have been so slow to adjust to inflation in the postwar period. Does the slow adjustment arise because share markets are irrational or because inflation is correlated with other factors that impact negatively on share returns such as supply shocks?

APPENDIX

This section shows the asymptotic properties of the following estimates of the Fisher hypothesis:

$$RS_t^R = \varphi_0 + \varphi_1 \pi_{t+1}^* + \varepsilon_{4t}, \quad (\text{A1})$$

$$RS_t^R = \varphi_0 + \varphi_1 \pi_t + \varepsilon_{5t}, \quad (\text{A2})$$

$$RS_t^R = \varphi_0 + \varphi_1 \pi_{t+1} + \varepsilon_{6t} \quad (\text{A3})$$

RS_t^R is *ex post* real returns to shares, $RS_t^R = RS_t^N - \pi_t$, φ_0 is the real rate of return to equities, and φ_1 is defined below.¹² Unexpected inflation is frequently added as a regressor in Model A1. This augmentation is analyzed in the last part of Appendix. Finally, the null hypothesis, $H_0: \varphi_1 = 0$ defines the Fisher hypothesis that shares are hedged against inflation.

Tests of the Fisher hypothesis based on Models A1-A3 are also highly influenced by the persistence of inflation, but in a subtler manner than in Models 1-3. Consider first Model A1:

$$RS_t^R = \varphi_0 + \varphi_1 \pi_{t+1}^* + \varepsilon_{4t}. \quad (\text{A4})$$

However, from the Fisher equation it follows that the true equation is:

$$RS_t^R = \varphi_0 + \varphi_1 \pi_{t+1}^e + \varepsilon_t - \pi_t, \quad (\text{A5})$$

which yields the following least squares estimator of φ_1 :

$$p \lim \hat{\varphi}_1 = \frac{\varphi_1 - \text{cov}(\pi_{t+1}^e, \pi_t) / \text{var}(\pi_{t+1}^e)}{1 + \text{var}(v_{t+1}) / \text{var}(\pi_{t+1}^e)}. \quad (\text{A6})$$

This equation shows that the estimate of φ_1 is only consistent in the highly unlikely case where inflation expectations are uncorrelated with actual inflation *and* where inflation expectations are accurately measured. This suggests that an additional bias is introduced as compared to tests where the nominal returns is the dependent variable, namely a bias due to the correlation between actual and expected inflation. Since the two terms in the numerator of Equation (A6) go in opposite directions, this test tends to be more favorable to the Fisher hypothesis than tests that use nominal share returns as the dependent variable. If expected and actual inflation are uncorrelated, the more error-ridden is the measure of inflation

expectations the more the estimates of φ_1 are biased towards zero and therefore towards the maintained hypothesis that the Fisher hypothesis holds. By contrast, the more strongly correlated is inflation and expected inflation, the more biased are estimates of φ_1 towards -1. Hence, the more serial correlated is inflation the more are the estimates of φ_1 biased towards -1. However, this result does not apply to Model A2.

To see this consider Model A2. If the Fisher hypothesis holds, then

$$E(SR_t^N) = E(\pi_{t+1}) = \rho\pi_t$$

under the assumption that inflation follows a first-order autoregressive process. Then the probability limit of estimating Model 5 is given by:

$$p \lim \hat{\varphi}_1 = \frac{\text{cov}(RS_t^R, \pi_t)}{\text{var}(\pi_t)} = \frac{\text{cov}(\rho\pi_t, \pi_t)}{\text{var}(\pi_t)} - 1 = \rho - 1 \quad (\text{A7})$$

which is equal to $p \lim \hat{\zeta}_1 - 1$ from estimates of Model 2 where nominal share returns are regressed on π_t .¹³ Since the residual variance goes towards zero as the sample size goes towards infinity it follows from Equation (A7) that the Fisher hypothesis will always be rejected in large samples unless $\rho = 1$. Generally, the more persistent is inflation, the more likely it is that the Fisher hypothesis will *not* be rejected under the null hypothesis.

Applying the same assumptions to estimates of Model A3 yields the following probability limit:¹⁴

$$p \lim \hat{\varphi}_1 = \frac{\text{cov}(SR_t^R, \pi_{t+1})}{\text{var}(\pi_{t+1})} = \frac{\text{var } E_t(\pi_{t+1}) - \text{cov}(\pi_t, \pi_{t+1})}{\text{var}(\pi_{t+1})} = \frac{\text{var } E_t(\pi_{t+1})}{\text{var}(\pi_{t+1})} - \rho \quad (\text{A8})$$

which is equivalent to $p \lim R^2$ from regressing π_{t+1} on all relevant variables in the investor's information set at time t minus the serial correlation coefficient of inflation. A high ρ will yield a negative coefficient of inflation regardless of whether the Fisher hypothesis holds. Since the first right-hand term in Equation (A8) is positive, the expected value of φ_1 depends

¹² Equation (A1) is used by Chang and Pinegar (1987), Fama (1981), Kaul (1987, 1990), Kaul and Seyhun (1990), and Solnik (1983). Equation (A2) is used by Benderly and Zwick (1985), Caporale and Jung (1997), Cochran and DeFina (1993), Ely and Robinson (1993), and Graham (1996).

¹³ More generally it can be shown that the probability limit of the estimated coefficient of inflation in Model 5 equals the probability limit of the estimated coefficient of Equation (2) minus 1:

$$p \lim \hat{\varphi}_1 = p \lim \hat{\zeta}_1 - 1.$$

It can also be shown that the estimated variances of the two coefficients are identical.

¹⁴ Note that $\text{cov}(\pi_t, \pi_{t+1})/\text{var}(\pi_{t+1}) = \text{cov}(\pi_{t+1}, \pi_t)/\text{var}(\pi_t)$ asymptotically.

on the degree of inflation persistence, the forecastability of inflation, and whether the Fisher hypothesis holds. Generally, inflation persistence and forecastability of inflation tend to go hand-in-hand. Hence, if the Fisher hypothesis holds, then the probability limit of φ_1 tends towards zero. If not, then it will be more negative the more persistent is inflation.

Following Fama (1981) and Fama and Schwert (1977) it has become a widespread practice to regress real share returns on expected and unexpected inflation:

$$RS_t^R = \varphi_0 + \varphi_1 \pi_{t+1}^e + \varphi_2 \pi_{t+1}^u + \varepsilon_{6t}, \quad (\text{A9})$$

where π_{t+1}^u is unexpected inflation. Under the maintained hypothesis that the Fisher hypothesis holds, the literature assumes that coefficients of expected and unexpected inflation are zero: $H_0: \varphi_1 = \varphi_2 = 0$. However, this maintained hypothesis suffers from an internal inconsistency. If $\varphi_1 = \varphi_2 = 0$, then Model A9 reduces to:

$$RS_t^N = \varphi_0 + \pi_t + \varepsilon_{6t}.$$

This equation only resembles the Fisher equation if expected inflation coincides with actual inflation, which is only the case if $\rho = 1$, given that $E(\pi_{t+1}) = \rho\pi_t$ under rational expectations. Hence, the maintained hypothesis, $\varphi_1 = \varphi_2 = 0$, is only consistent with the Fisher hypothesis *if* expected inflation equals actual inflation. If $\varphi_1 \neq \varphi_2$ one cannot make any inferences about the validity of the Fisher hypothesis since this model is not derived from the standard Fisher equation framework. Coupled with the fact that the individual coefficient estimates are sensitive to inflation persistence it is not surprising that estimates of this model often give curious results.

DATA APPENDIX

Real and nominal GDP. (postwar) OECD, *National Accounts, Vol 2 (NA)* and (prewar) A. Maddison, 1995, *Monitoring the World Economy 1820-1992*, Development Centre, OECD. **Wages.** Compensation to employees (NA) divided by employment and by weekly hours worked in non-agricultural activities, ILO, *Yearbook (YB)*. **H0.** B R Mitchell, 1983, *International Historical Statistics: Americas and Australasia*, Macmillan: London and B R Mitchell, 1975, *European Historical Statistics 1750-1975*, Macmillan: London. **Consumer prices.** (postwar) IMF, *International Financial Statistics (IFS)*, and (prewar) Mitchell (1983, 1975) *op cit*. **M1. IFS. Nominal long-term interest rates on government bonds.** (postwar) *IFS*, and (prewar) S Homer and R Sylla, 1991, *A History of Interest Rates*, Rutgers University Press: London, except for Australia, where W Vamplew (ed.), 1987, *Australians: Historical Statistics*, Fairfax, is used. **Share prices.** (postwar) *IFS*, and OECD, *Main Economic Indicators (MEI)*. Prewar. **USA:** NBER Macro history data Series: 11025 from Standard and Poor's Corporation's Security Price Index Record, 1955. **Australia:** Reserve Bank of Australia database. **France:** NBER Macro history data Series. **UK.** B. R. Mitchell, 1962, *Abstract in British Historical Statistics*, Cambridge University Press: Cambridge. **Share returns.** Global Financial Data. **Population.** Maddison (1995) *op cit*.

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