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The Equity Premium Puzzle and the *Ex Post* Bias¹

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Abstract. This paper argues that the high historical excess returns to equity are a result of a severe *ex post* bias over the period from 1915 to circa 1960 because inflation surprises during this period drove a wedge between *ex ante* and *ex post* returns to bonds. Furthermore, it is shown that *ex ante* and *ex post* returns to shares are identical in steady state. Adjusting the *ex post* equity premium by the *ex post* bias reduces the equity premium to an arithmetic mean of 3.5-3.9% over the past 130 years.

JEL Classification. G0, E1, E3

Key words. Equity premium, inflation expectations, expected returns.

1 Introduction

A question that has long puzzled financial economists is why the *ex post* equity premium has been substantially higher than the equity premium predicted by standard neoclassical consumer theory (Mehra and Prescott, 1985, 2003). Several theoretical studies have assumed that the *ex ante* and *ex post* equity premium are approximately of equal size and tried to solve the puzzle by modifications of the pricing kernel or by using prospect theory (see Mehra and Prescott, 2003, and Barberis *et al.*, 2001).

Fama and French (2002) and Blanchard (1993), by contrast, argue that the *ex post* premium in the post-war period, in particular, has been driven up by a significant *ex post* bias. They find that a significant fraction of the *ex post* share returns has been unexpected and, therefore, that the dimension of the equity puzzle is either reduced or eliminated once the *ex post* bias is corrected for. Fama and French (2002) argue that an unexpected reduction in the required share returns has resulted in a positive *ex post* bias in the post-war period. Correcting for the bias they find *ex ante* equity premiums of 2.6% and 4.3% over the period from 1951 to 2000. Blanchard (1993) argues that a combination of unexpected high returns to shares and unexpected low returns to bonds resulted in an *ex post* bias.

¹ Helpful comments and suggestions from participants at seminars at University of Copenhagen, Copenhagen Business School and Aarhus Business School are gratefully acknowledged. A grant from EPRU is also gratefully acknowledged.

This paper argues that a significant *ex post* bias prevailed over the period from 1915 to circa 1960 due to unexpected low returns on bonds. It is shown that inflation was difficult to predict in the gold standard period and investors were slow to learn the inflationary consequences of the paper money regime after the abandonment of the gold standard in the beginning of the 1930s.² Quantitative, historical and inflation survey evidence suggests that inflation expectations were close to zero over the period from 1915 to circa 1960 and, therefore, that the inflation in this period was unexpected. The unanticipated inflationary shocks drove a wedge between *ex post* real share returns and *ex post* real bond returns, and the *ex post* equity premium consequently increased without affecting the *ex ante* equity premium.

The question is whether it is sufficient to adjust for an *ex post* bias in the bond market and, therefore, assume that *ex post* and *ex ante* equity returns are approximately equal. This paper argues that they are. Based on a general equilibrium model it is shown that the *ex ante* and the *ex post* real share returns are identical in steady state and, therefore, that *ex post* and *ex ante* returns only deviate outside steady states. It is shown that technology shocks, shifts in the required returns to equity and changes in corporate taxes have no long-run effects on real share prices because of endogenous adjustment of capital that certifies that the earnings per unit of capital will always tend toward the required returns to equity. An *ex post* bias, therefore, must have its genesis in the bond market when the capital stock is adjusted towards its desired level.

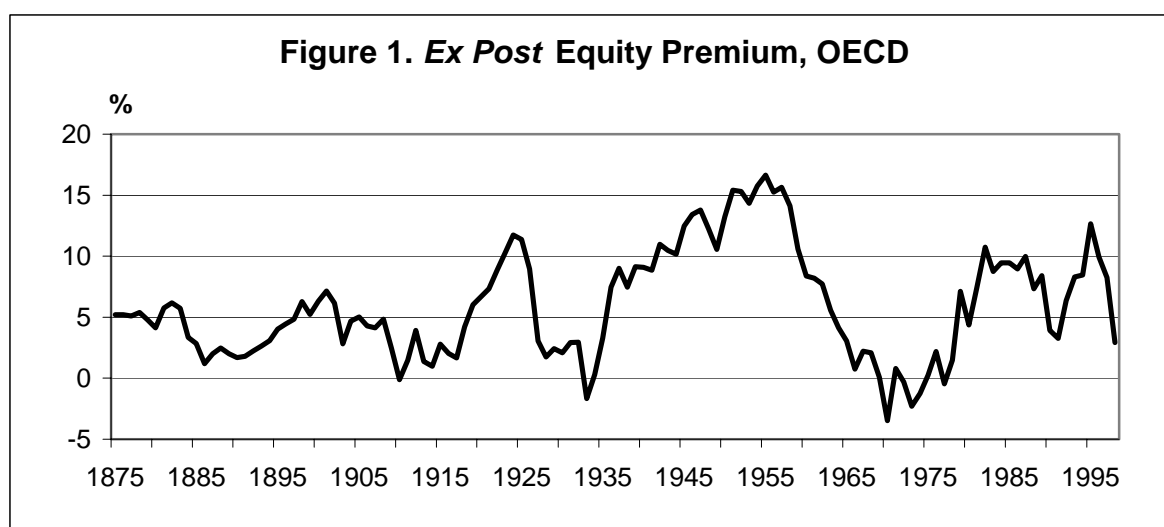
Data for a panel of 10 industrialised countries over the period from 1870 to 2002 are used to identify the *ex post* bias. The empirical estimates show that the *ex post* equity premium is significantly positively related to the realised rate of inflation on an almost one-to-one basis over the period from 1915 to 1960, after which time, inflation ceases to be an important determinant of the equity premium. Correcting the *ex post* equity premium by the inflation-induced bias reduces the historical equity premium in the OECD countries by approximately 3-percentage points to 3.5-3.9% over the past 100-130 years, which is significantly below the *ex post* equity premium.

The paper proceeds as follows. The next section presents informal evidence on the relationship between bond returns and inflation. Using historical, survey, and quantitative evidence Section 3 shows that inflation expectations were close to zero over the period from 1915 to circa 1960. The empirical Sections 4 and 5 estimate the *ex post* bias for 10 OECD countries and in Section 6 it is shown that the *ex post* and *ex ante* real returns to shares are identical in steady state.

² A similar argument has been put forward by Siegel. Siegel argues that “it is clear that the buyers of long-term bonds in the 1940s, 1950s, and early 1960s did not recognise the inflationary consequences of the change in monetary regime” (2002, p 17).

2 *Ex post* equity premium in the industrialised countries

To get an impression of the time-profile of the *ex post* equity premium consider Figure 1, which displays an weighted nine-year centred moving average of the equity premium for Canada, the US, the UK, Germany, France, Italy, Australia, Spain, the Netherlands, and Ireland, where the purchasing power parity real GDP is used as moving weights over the whole period. The hyperinflation period 1922-1924 is omitted for Germany and the civil war period 1936-1940 is omitted for Spain throughout the paper. Data definitions and data sources are relegated to the Data Appendix.



Notes: Weighted centred nine-year moving average for Canada (1901), the US (1871), UK (1871), Australia (1885), Italy (1900), France (1871), Ireland (1871), Germany (1871), the Netherlands (1901), and Spain (1900), where the numbers in parentheses indicate starting year. Purchasing power parity moving GDP weights are used. The equity premium is calculated as the percentage change in the nominal accumulated share index minus the nominal interest rate on long government bonds.

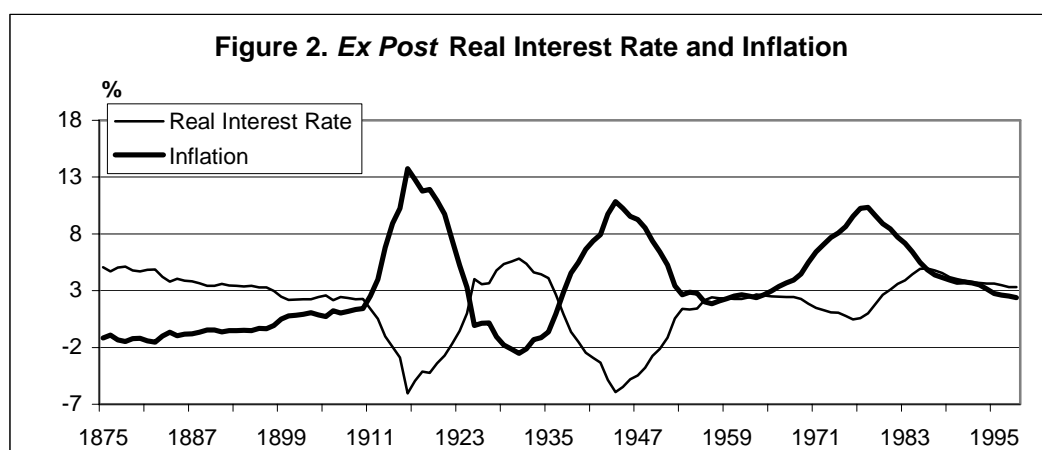
The arithmetic average of the *ex post* equity premium over the entire period is 5.56%, which is in line with most other estimates in the literature (Mehra and Prescott, 1985, 2003, Fama and French, 2002, Dimeson *et al.*, 2004).^{3,4} The equity premium was remarkably high over the period from 1915 to 1960, namely 9.30%, which is 5.73-percentage points higher than the surrounding periods where it was 3.57% (1871-1914 and 1961-2002). This suggests that the height of the *ex post* equity

³ Throughout the paper the arithmetic mean is used. See Mehra and Prescott (2003) for a discussion of the relative merits of using arithmetic and geometric means.

⁴ Jorian and Goitzman (1999) find a lower equity premium for non-US countries; however, their estimates do not include dividends, which have historically exceeded capital gains for the ten countries considered in this paper.

premium over the past 132 years has been heavily influenced by the equity premium in the 1915-1960 period.

The next question is whether the equity premium in the 1915-1960 period has been a result of high *ex post* real share returns or low real interest rates. The *ex post* real returns on long bonds/shares were, on average, 4.00/7.57 over the periods from 1871 to 1914 and from 1961 to 2002, but -1.10/10.40% over the period from 1915 to 1960, where the figures refer to weighted averages for the countries and periods covered in Figure 1. Over the whole period from 1871 to 2002 the real *ex post* return to long bonds was 1.62, whereas the number was 7.18 for shares. This suggests that the *ex post* equity premium was high in the 1915-1960 period predominantly due to lower returns to the default free asset rather than higher returns to equity; a conclusion that is also reached by Mehra and Prescott (2003). These considerations suggest that the source of the high *ex post* equity premium in the 1915-1960 period must predominantly lie with the *ex post* real interest rate.



Notes. See Notes to Figure 1. The *ex post* interest rate is the interest rate on a long government bond minus the rate of consumer price inflation.

To get an impression of the influence of inflation on real bond returns, Figure 2 displays the *ex post* real interest rate on a long government bond and the *ex post* rate of consumer price inflation. The figure shows that the real interest rate is a mirror image of the inflation rate prior to 1960. The high inflations associated with the world wars resulted in significantly negative real interest rates. After 1960, however, the nexus between the *ex post* real interest rate and inflation that prevailed prior to 1960, almost disappears because investors started to understand the inflationary bias associated with

the paper money regime that was introduced after the break-down of the gold standard in the 1930s.⁵

In the post-1960 period it appears that *ex post* real interest rates have only been influenced by changing inflation; probably because the inflation changes have been unexpected. However, this has not significantly affected the *ex post* equity premium because share returns have been equally adversely affected by changes in inflation, as discussed in the next section.

The long bond rate is used in the calculations of the equity premium here following most of the empirical literature on the equity premium. Since the short interest rate is closer to being a risk-free asset than the long interest rate, it should be the alternative asset on which the equity premium should be based. However, the empirical estimates in Section 4 show that the *ex post* bias is slightly higher if the short bond rate is used instead of a long bond. Thus, the results in this section are slightly strengthened if a short interest rate is used.

Overall the informal evidence in this section suggests that the *ex post* real interest rate prior to 1960 was driven predominantly by realised inflation because it was unexpected. The outbreak of the worldwide inflation during WWI was associated with the abandonment of the gold standard in Europe in 1914 and a sharp reduction in the gold-backup ratio in the US; thus violating the gold standard rule of constant gold back-up ratios that prevailed before 1914 (Eichengreen, 1992). Since the resulting surge in the money supply during WWI and shortly thereafter was unanticipated, bond owners suffered large real capital losses during this period. Similarly the inflationary consequences of WW2 took bond investors by surprise as argued in the next section and shown formally in the empirical section (Section 4).

3 Inflation and *ex post* returns to equity and bonds

Whereas bond yields are significantly adversely affected by unexpected inflation share returns are not. Yields on bonds at the date of issue, are, to a large extent, determined by inflation expectations over the life span of the bond. Inflation surprises consequently lower the *ex post* real returns on bonds. Shares, by contrast, are hedged against inflation surprises provided that relative prices such as real wages, taxes, and real interests on debt are neutral to inflation. Numerous studies have shown that real wages are only temporarily affected by inflation surprises following the natural rate hypothesis (see for instance Madsen, 1998).

⁵ Most countries went off gold in the first years of the 1930s whereas the Gold Block countries consisting of the Netherlands, Belgium, France, Poland, Switzerland and Italy, first abandoned gold in 1935 and 1936 (see Table 7.1 in Eichengreen, 1992).

Taxes, however, are not neutral to inflation because depreciation for tax purposes are at historical cost (Feldstein, 1980). Furthermore, Modigliani and Cohn (1979) argued that *ex post* share returns are adversely affected by changes in inflation because share investors suffer from inflation illusion. Despite adverse effects on share returns of unexpected changes in inflation being muted by tax-deductions of interest expenses on debt, and that *ex post* real interest rates tend to decrease in inflationary periods, the real *ex post* share returns are significantly negatively affected by *increasing* inflation (Campbell and Vuolteenaho, 2004). However, as shown in Section 4, the *ex post* equity premium was unaffected by the increasing inflation in the 1970s because the nominal interest rates were slow to adjust to the increasing inflation; thus counterbalancing the adverse inflationary effects on *ex post* share returns. Furthermore, the endogenous response in the capital stock to the depressed share market in the 1970s drove earnings per unit of capital up as shown formally in the general equilibrium model in Section 6, which rendered the adverse earnings-effects from inflation temporary.

The next question is the extent to which the *pre*-1960 and the *post*-1960 inflations were expected and, therefore, the prevalence of an *ex post* bias in real bond returns in these two periods. A necessary, but not a sufficient, condition for inflation to be embodied in bond prices is that inflation can be predicted. The econometric evidence of Barsky (1987) gives strong support to the hypothesis that inflation prior to 1960 could not have been predicted in the US and the UK due to low inflation persistence. Barsky (1987) shows that nominal interest rates and realised inflation rates were uncorrelated prior to 1960 but strongly correlated thereafter. The question is whether Barsky's (1987) finding, based on year-to-year evidence, carries over on a longer term basis corresponding to the life of a medium and a long-term term bond under various monetary regimes. In other words, could long-term inflation have been predicted before and after 1960?

Table 1. Five-year inflation forecasts.

Est. Period	Lagged Inflation	Growth in Oil Prices	Growth in <i>M1</i>
1893-1960	-0.04(0.68)	0.00(0.53)	-0.02(0.49)
1933-1960	-0.13(1.57)	-0.09(3.55)	0.14(1.91)
1963-2002	0.13(2.44)	0.01(1.18)	0.08(3.60)

Notes: The estimates are the sum of the lagged coefficient estimates and the numbers in parentheses are absolute *t*-statistics. All variables are measured in decimal points. Eight annual lags of all the regressors are included in the estimates. The country sample is Canada, the US, UK, Australia, Italy, France, Ireland, Germany, Spain and the Netherlands in the *post* 1960 estimates. The same countries except Ireland and Germany, are included in the *pre* 1960 estimates.

To investigate this issue five-year consumer price inflation forecasts are reported in Table 1 for the *pre* 1960 and the *post* 1960 periods and over the period from 1933 to 1960. The average rate of inflation over the subsequent five years is regressed on eight-year lags of inflation, the growth in oil prices, and the growth in *M1*. This approach is similar to the approach used by Blanchard (1993). The data are pooled using the SUR estimator, which is discussed in detail in the empirical section. The ten countries considered in Figures 1 and 2 are included in the data set except Ireland and Germany for the *pre* 1960 estimates because *M1* is not available over the whole estimation period for these two countries. The sum of the lagged coefficient estimates and their attached *t*-statistics are reported in the table. The post-1960 estimates commence in 1963 because *M1* is not available before 1949 for Germany.

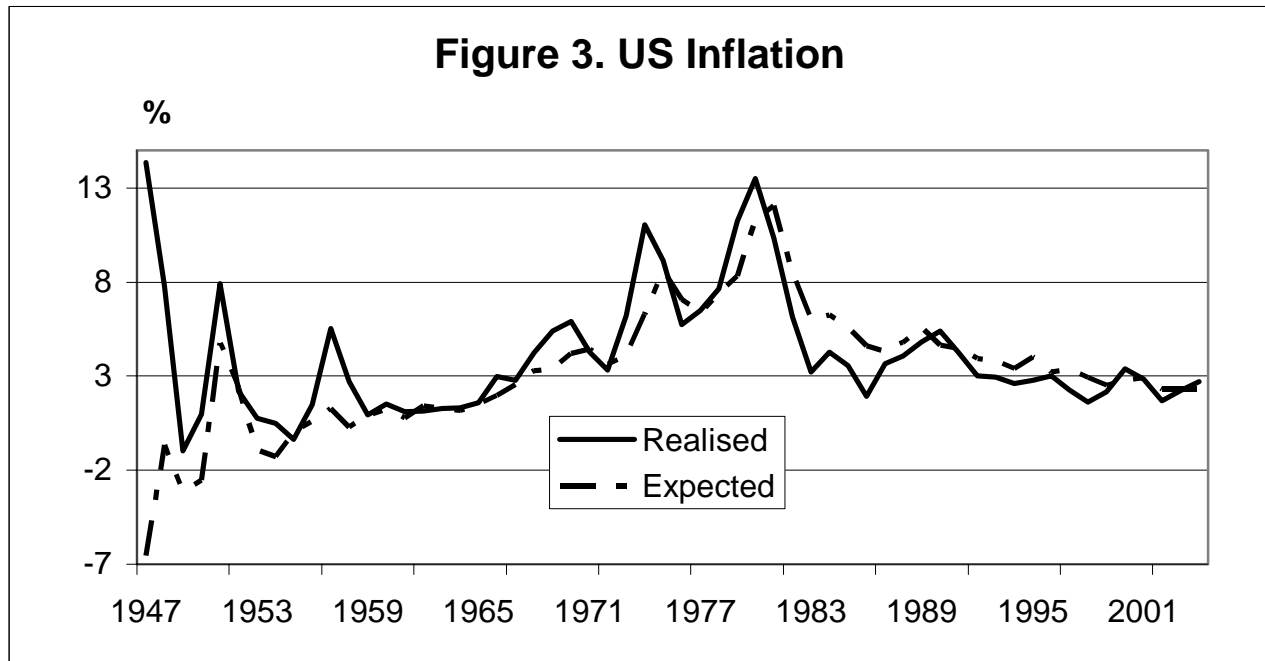
The estimation results suggest that Barsky's (1987) findings extend to five-year inflation forecasts. Over the period from 1893 to 1960 long term inflation was unrelated to lagged inflation and lagged growth in *M1* and oil prices. The results remain almost unaltered if the estimation period is narrowed down to the 1933-1960 period. In the *post* 1963 period, by contrast, the estimates indicate that inflation is, to some extent, predictable. The estimated coefficients of past inflations and past growth rates in money supply are significant at conventional significance levels. Thus, at least some of the inflation could have been predicted in the *post* 1960 period, whereas inflation could not have been predicted prior to 1960 based on information economists have traditionally highlighted as important for predicting inflation.

This evidence and the findings by Barsky (1987) are supported by survey inflation expectations. Unfortunately, survey data on inflation expectations were first made available from 1946 in the US by Livingston and much later in other countries. Inflation expectations prior to 1946 are discussed below. In the Livingston survey business people and academics are asked about expected inflation over the next 12 months.⁶ The average expected inflation is displayed together with the realised one-year growth in consumer prices in Figure 3. The survey expectations are forwarded 12 months.

The figure indicates large discrepancies between expected and realised inflation prior to circa 1960. In fact, inflation expectations were negative over the period from 1946 until 1950 despite high inflation in the same period, which suggests that the immediate post-war inflation was not expected. Clearly the respondents were, at that time, not familiar with the inflationary

⁶ The survey data are available from the Federal Reserve Bank of Philadelphia.

consequences of the post-gold-standard monetary regime and were perhaps expecting deflation in the first years after WWII based on their experience from the *post* WWI deflation in 1921 and 1922.



Notes. The realised consumer price inflation is forwarded 12 months. Realised and expected inflation are on annual basis. Expected inflation is from the Livingston Surveys.

After 1960, however, the figure indicates that actual and expected inflation coincide. These observations are consistent with results from empirical analyses in Section 4. Inflation expectations were unbiased predictors of inflation in the post-1960 period but not before then.⁷ Furthermore, the estimated coefficient of expected inflation is not significantly different from zero before 1960, which suggests that the *pre*-1960 inflation was unexpected.

Turning to the question of inflation expectations prior to 1946, the historical evidence suggests that price changes were not predicted. McGrattan and Prescott (2003), for instance, argue that the expected rate of inflation was zero in the gold standard period. In the most careful study of inflation expectations in the US over the period from 1870 to 1914, Barsky and De Long (1991) argue that the deflation from 1879 to 1896 and the inflation from 1896 to 1913 were never built into

⁷ Regressing realised inflation on 12-month lagged inflation expectations yields an estimated coefficient of expected inflation of -0.361(0.4) over the estimation period from 1947 to 1960, and an estimated coefficient of expected inflation of 0.995(9.03) over the estimation period from 1961-2003, where the numbers in parentheses are absolute *t*-statistics. Wald tests of the hypothesis of unity coefficient of inflation expectations are $\chi^2(1) = 3.36$ for the 1947-1960 period and $\chi^2(1) = 0.00$ for the 1961-2003 period. The standard errors are based on the Newey-West heteroscedastic consistent and first-order serial correlation consistent covariance matrix.

inflation expectations in the US. Their time-series evidence based on ARIMA models suggests that univariate models could not predict inflation.⁸ Although they find world gold production to be a statistically significant predictor of prices, they argue that the 1995-1914 gold boom was never build into inflation expectations based on studies of the quantity-theoretical-leaning *Economist*.

Using the model of Barsky and De Long (1991), the price changes in the period from 1914 to the Great Depression could not have been predicted. World gold production increased gradually by 2.2% annually on average over the period from 1914 to 1931 (Table 11 in Warren and Pearson, 1932), which is close to the growth in real world GDP over the same period. It is, therefore, unlikely that inflation expectations were significantly different from zero before the gold standard was abandoned in the beginning of the 1930s.

4 Empirical estimates

This section tests the implications of the hypothesis forwarded in this paper, namely, 1) that nominal interest rates were insensitive to *ex post* inflation prior to 1960; 2) that nominal share returns were positively related to inflation prior to 1960; and, therefore, 3) that the *ex post* equity premium is positively related to *ex post* inflation prior to 1960, but not thereafter.

To test these hypotheses the following three models are estimated using pooled cross section and time-series analysis:

$$ep_{it} = a_0 + a_1\pi_{it}^{1915} + a_2\pi_{it}^{1960} + a_3\pi_{it}^{2002} + \varepsilon_{1,it} , \quad (1)$$

$$i_{it} = b_0 + b_1\pi_{it}^{1915} + b_2\pi_{it}^{1960} + b_3\pi_{it}^{2002} + \varepsilon_{2,it} \quad (2)$$

$$er_{it} = c_0 + c_1\pi_{it}^{1915} + c_2\pi_{it}^{1960} + c_3\pi_{it}^{2002} + \varepsilon_{3,it} \quad (3)$$

where *ep* is the *ex post* equity premium measured in percentage points as *ex post* share returns minus the nominal interest rate on a long government bond, *i* is the nominal interest rate on long-term government bonds in percentage points, *er* is nominal equity returns in percentage points, ε is a disturbance term and π^{1915} , π^{1960} , and π^{2002} are inflation rates in percentage points prior to 1915, between 1915 and 1960, and between 1961 and 2002, respectively, and zero elsewhere. Four-year lags of all the regressors are included in the estimates to allow for slow adjustment of asset returns to innovations in inflation.

⁸ Perez and Siegler (2003) argue that changes in prices are not white noise prior to 1913 for the US and find that the serial correlation coefficient of price changes to be approximately 0.3, which suggests that some of the short-term deflation and inflation could have been predicted. However, no evidence showing that the predictive element of inflation was built into inflation expectations, is provided.

The models are estimated using annual data over the period from 1871 to 2002 for five countries (group A) and over the period from 1900 to 2002 for ten countries (group B) as dictated by data availability. Group A consists of the US, the UK, Ireland, France, and Germany. The following additional five countries are included in group B: Canada, Australia, Italy, the Netherlands, and Spain. Country-dummies were excluded from the estimates because their estimated coefficients were both individually and jointly insignificant at conventional levels, which suggests that the *ex post* equity premium is not significantly different across countries when the influence of inflation on the equity premium, share returns and bond returns is allowed for. Impulse dummies are included for Germany over the period from 1922 to 1926 to avoid the influence on the parameter estimates of the hyperinflation, and for Spain over the civil war period from 1936 to 1940.

To gain efficiency the model is estimated allowing for the correlation of the error-terms between countries. The covariance matrix is weighted by the correlation of the disturbance terms using the following variance-covariance structure:

$$\begin{aligned} E\{\varepsilon_{it}^2\} &= \sigma_i^2, \quad i = 1, 2, \dots, N, \\ E\{\varepsilon_{it}, \varepsilon_{jt}\} &= \sigma_{ij}, \quad i \neq j, \end{aligned}$$

where σ_i^2 = the variance of the disturbance terms for country $i = 1, 2, \dots, N$, σ_{ij} = the covariance of the disturbance terms across countries i and j ; and ε is the disturbance term. The variance σ_i^2 is assumed to be constant over time but to vary across countries and the error terms are assumed to be mutually correlated across countries, σ_{ij} , as random shocks are likely to impact on all countries at the same time. The terms σ_i^2 and σ_{ij} , are estimated using the feasible generalized least squares method.

The results of estimating Equations (1)-(3) are reported in Table 2. The reported coefficient estimates are the sum of all lags. R^2 is not reported since it is driven to one by the German hyperinflation dummies. The null hypothesis of cross-country coefficient homogeneity cannot be rejected at conventional significance levels for any of the estimates. Nor do the tests for first-order serial correlation and ARCH effects give evidence against the model specification. These results indicate that the models are well specified.

The estimates of the *ex post* equity premium in columns 1 and 2 are the key estimates in this paper. The estimated coefficients of inflation over the period from 1915 to 1960 are

statistically highly significant. The coefficient estimates are 0.70 for group A and 0.62 for group B, which indicate that inflation was highly influential for the *ex post* equity premium over this period. In the *post*-1960 period, however, the estimated coefficients of inflation are statistically insignificant at any conventional levels; thus, inflation ceases to affect the *ex post* equity premium past 1960. These results are consistent with the evidence in the previous section and the findings of Barsky (1987). Prior to 1915, the estimated coefficients of inflation are also insignificant, which is not surprising given inflation fluctuated very little and were, on average, close to zero in this period. Hence, little identifying variations in price changes was present in that period.

Table 2. Parameter estimates of Equations (1)-(3).

	Equity Premium		Bond Yields		Share Returns	
	A	B	A	B	A	B
π_t^{1915}	-2.57(1.33)	0.36(0.26)	0.03(0.37)	-0.06(0.54)	-2.01(1.65)	0.36(0.28)
π_t^{1960}	0.70(4.98)	0.62(6.46)	0.02(2.34)	0.00(0.78)	0.49(4.02)	0.38(4.68)
π_t^{2002}	0.61(1.41)	0.11(0.38)	0.63(10.42)	0.78(17.4)	0.97(3.35)	0.47(2.22)
Con.	3.42(2.45)	3.67(3.17)	3.99(20.9)	4.48(19.1)	6.10(4.84)	7.05(5.43)
Est. Per.	1878-2002	1907-2002	1878-2002	1907-2002	1878-2002	1907-2002
$N \cdot T$	625	960	625	960	625	960
$DW(m)$	1.98	1.97	1.98	2.00	1.98	1.93
$F(i,j)$	0.93	1.45	0.93	1.45	1.55	1.22
$ARCH$	0.00	0.03	0.00	0.03	0.04	0.27

Notes: The coefficient estimates are the sum of all lags. Absolute *t*-statistics are given in parentheses. $N \cdot T$ = number of observations, where *N* is the number of countries, *T* is the time-period. $DW(m)$ = modified Durbin-Watson test for first order serial correlation in fixed effect panel data models (see Bhargava *et al.*, 1982). $F(i,j)$ = *F*-test for cross-country coefficient constancy, and is distributed as $F(N(k-1), NT-Nk)$ under the null hypothesis of coefficient constancy, and *k* is the number of regressors (19). $ARCH$ = fixed effect model $ARCH$ test, based on within individual residuals, and is distributed as $\chi^2(1)$ under the null hypothesis of homoscedasticity. The following 5 countries are included in group A: The US, the UK, Ireland, France, and Germany. The following 10 countries are included in group B: The US, the UK, Ireland, France, Germany, Canada, Australia, Italy, the Netherlands, and Spain. Impulse dummies are included in the estimates over the period from 1922 to 1926 for Germany, and over the period from 1936 to 1940 for Spain.

Turning to nominal bond returns, columns 3 and 4 the estimated coefficients of inflation show that inflation was not embodied in nominal bond returns before 1960. This reflects either that inflation was not adequately embodied into bond prices or that inflation could not have been predicted. After 1960 the estimated coefficients of inflation become statistically highly significant. The estimated coefficients of inflation post 1960 are 0.63 (group A) and 0.78 (group B) and bonds, consequently, start to become fractionally hedged against inflation.

The estimates of the share return equation in columns 5 and 6 in Table 2 show that nominal share returns since 1915 have been significantly positively related to inflation. That the estimated coefficients of inflation are less than one is either because share prices are slow to adjust to innovations in inflation when it is unexpected or because inflation is positively correlated with omitted variables that adversely affect share returns. Madsen (2005), for instance, shows that inflationary periods are often associated with gains in labour's income shares in total income and that the null hypothesis of a unity coefficient of inflation cannot be rejected when supply shocks or labour's income share are accommodated in the estimates.

Finally, as noted in Section 2, the short-term interest rate may be a better reference rate than the long bond rate. However, estimates suggest that the *ex post* bias is slightly higher if the short interest rate is used. Basing the *ex post* equity premium on the short rate yields, in the estimates of Equation (1), an estimated coefficient on inflation of 0.74(5.24) for group A and 0.63(6.42) for group B over the period from 1915 to 1960, where the numbers in parentheses are *t*-statistics. Thus, regardless of whether a short or a long bond rate is used in the estimates, inflation remains influential for the equity premium over the period from 1915 to 1960.

5 Estimates of the *ex post* bias

The estimates of Equation (1) can be used to recover the equity premium for all countries and the countries individually. For all countries the bias-corrected equity premium is reflected in the estimated constant terms in the first two columns in Table 2. The estimated constant terms are 3.4% over the period from 1878 to 2002 (group A) and 3.7% over the period from 1907 to 2002 (group B). The estimated constant term for group A increases to 3.9% if the estimation period commences in 1907. Given that the average *ex post* return for group A over the period from 1878 to 2002 is 6.4%, the *ex post* bias is 3.0%. The *ex post* returns are also 6.4% for group B over the period from 1907 to 2002, which entail an *ex post* bias of 2.7%. Thus, the inflation-correction has reduced the *ex post* equity premium to almost half its size.

Turning to individual countries, Table 3 shows that the *ex post* equity premia varies substantially across countries from a high of 17.7% for Italy to a low of 2.3% for Spain over the period from 1915 to 1960. Although Japan has not been included in any of the estimates in the previous sections due to lack of data prior to 1914, it is included in Table 3 because the Japanese share market has always attracted much attention in the literature. The *ex post* equity premium was particularly high for four the countries that experienced the highest inflation over the period from

1915 to 1960, namely Japan (14.8%), Germany (12.5), France (12.7), and Italy (17.7), where the numbers in parentheses are *ex post* share returns. Inflation-correcting the equity premium by subtracting from the *ex post* equity premium the average rate of inflation over the period from 1915 to 1960 multiplied by the estimated coefficient of inflation for group B, reduces the cross-country variation in the equity premium substantially. The bias-correction results in a reduction in the equity premium from 14.8% to 9.8% for Japan, from 12.5% to 6.5% for Germany, from 12.7% to 4.5% for France, and from 17.7% to 7.5% for Italy. These bias-corrected values are much more plausible than the *ex post* values.

Table 3. Inflation-corrected equity premium, 1915-1960.

Country	$EP^{Ex Post}$	Inflation	EP^C	Country	$EP^{Ex Post}$	Inflation	EP^C
----- Percent -----							
USA	9.1	2.6	7.4	Canada	6.7	2.3	5.3
UK	5.1	2.9	3.3	Australia	7.4	3.4	5.3
Ireland	6.8	4.3	4.1	Italy	17.7	16.4	7.5
France	12.7	13.9	4.5	Netherlands	5.8	3.1	3.9
Germany	12.5	9.7	6.5	Spain	2.3	6.6	-1.8
Japan	14.8	8.0	9.8	Average	9.4	6.7	4.6

Notes. The coefficient of 0.62 is used in the corrections. $EP^{Ex post}$ is the *ex post* equity premium, EP^C is the inflation-corrected equity premium. The average is unweighted.

6 Are the *ex post* share returns an unbiased estimator for *ex ante* share returns?

Above it has been implicitly assumed that the *ex post* share returns are approximately equal to *ex ante* share returns. This assumption stands in contrast to the Fama and French (2002) analysis, which shows that the *ex post* equity premium over the period from 1951 to 2000 was extraordinary high because declining required returns resulted in large unexpected capital gains. The Fama-French model is based on Gordon's growth model. A problem associated with Gordon's growth model, however, is that all variables in this model are endogenous and, therefore, cannot be assumed to be exogenous over half a century. A change in the required share returns or earnings per unit of capital will trigger an endogenous response in investment, particularly, and consumption, which render their effects on share prices only short-lived, as shown below.

To show that *ex ante* and *ex post* share returns are identical in the steady a standard general equilibrium model is established. The model is used to show that earnings per share echo required returns to equity and that share prices are unaffected by shocks to earnings in steady state. The model consists of firms and consumers.

Firms. Investment and share prices are determined jointly from the following optimization problem of the representative firm, where the discount rate is taken as given:

$$\max \Pi = \int_{t=0}^{\infty} e^{-\int_{v=0}^t r_v dv} \left[(1-\tau) \{ F(z_t, L_t, K_t) - L_t W_t - \phi(I_t) \} - I_t \right] dt$$

st.

$$K_{t+1} = I_t + (1-\delta)K_t,$$

where Π is real profits, K is capital stock, I is net investment, W is the wage rate, L is labour services, r is the required returns to equity, $\phi(I_t)$ is the adjustment cost of investment, $\phi'(I_t) > 0$, $\phi''(I_t) < 0$, δ is the rate of capital depreciation, τ is the corporate tax rate, and z is a technology parameter. Furthermore, $F'_K > 0$, $F''_K < 0$, $F'_z > 0$, and $F''_z < 0$. The firm is an all equity firm and all earnings are paid out.

Solving this optimization problem yields the following first order conditions for optimum under the assumption of perfect competition:

$$(1-\tau)MP_{K,t} = (r_t + \delta)q_t - \dot{q}_t, \quad (4)$$

$$1 + \phi'(I_t)(1-\tau) = q_t, \quad (5)$$

where q is the shadow price of capital or Tobin's q , τ is the corporate tax rate, and a dot over a variable signifies first differences. The transversality condition is assumed to be satisfied and not displayed among the first order conditions to preserve space. Equation (4) is the asset market equilibrium condition and Equation (5) is the equilibrium condition in the market for fixed investment. For a given required returns to equity and employment, Equations (4) and (5) determine share prices and capital stock. Furthermore, after-tax earnings per unit of capital after depreciation are equal to the required returns in steady state.

Consumers. The representative consumer has preferences ordered by:

$$\max U = \int_{t=0}^{\infty} \frac{C_t^{1-\theta} - 1}{1-\theta} e^{-\rho t} dt$$

subject to the budget constraint

$$\dot{a}_t = W_t L_t + r_t a_t - C_t,$$

where U is utility, a is per capita assets, r is the expected returns to equity, ρ is a subjective discount factor, θ is the coefficient of relative risk aversion, and C is consumption.

A necessary and a sufficient condition for an interior solution is given by:

$$\frac{\dot{C}}{C} = \frac{1}{\theta}(r - \rho). \quad (6)$$

General equilibrium. Setting the depreciation rate equal to zero without affecting the results, the condition for equilibrium in the goods markets is given by:

$$F(K_t, L_t) = C_t + I_t + \phi(I_t). \quad (7)$$

Solving out consumption from Equation (6) using Equation (7), we arrive at the following simultaneous first-order differential equation system with the (K, q, r) -vector of endogenous variables:

$$\dot{q}_t = r_t q_t - (1 - \tau)MP_{K,t} \quad (8)$$

$$\dot{K}_t = h[(q_t - 1)/(1 - \tau)], \quad (9)$$

$$\dot{C}_t = (r_t - \rho)(F(z_t, K_t) - h[(q_t - 1)/(1 - \tau)] - \phi(h[(q_t - 1)/(1 - \tau)])) / \theta, \quad (10)$$

where labour is normalised to one following Abel and Blanchard (1983). The model can easily be extended to allow for endogenous labour supply as shown by Kiley (2005); however, this extension does not influence the steady-state properties of the model.

The equation system (8)-(10) can be used to show that *ex post* and *ex ante* share returns are identical in steady state. To show this it needs to be shown that 1) capital gains from any exogenous shock are zero in steady state; and that 2) after-tax dividends or after-tax earnings per unit of capital are equal to the required returns to equity in steady state. Note from Equation (10) that changes in the required share returns are identical to changes in the subjective discount factor.

That capital gains are independent of any exogenous shock in the steady state can be validated from the following steady-state multipliers:⁹

⁹ Summers (1981) shows that share prices in steady state are affected by changing depreciation rules because they change the effective acquisition cost of capital. However, the shares referred to in Summers' analysis are shares of established firms undertaking no investment. However, share prices of firms for which the whole capital stock has been generated under the new depreciation rules, are unaffected by changes in depreciation rules in the steady state.

$$\frac{\partial q}{\partial \rho} = \frac{\partial q}{\partial z} = \frac{\partial q}{\partial \tau} = 0. \quad (11)$$

That after-tax earnings per unit of capital change proportionally with the discount factor follows from the following steady state multiplier:

$$\frac{\partial [MP_K(1-\tau)]}{\partial \rho} = 1. \quad (12)$$

The intuition behind these results is that, due to convex adjustment costs in investment, a reduction in the subjective discount factor, for instance, increases share prices and Tobin's q above their long-run equilibriums. Tobin's q in excess of one initiate a capital deepening process, which reduces earnings per unit of capital, due to the assumption of diminishing returns to capital. The capital deepening continues until after-tax earnings per unit of capital have changed proportionally to the subjective discount factor as indicated by Equation (12). Tobin's q has returned to its initial level after the capital stock has adjusted to its new equilibrium; thus capital gains on shares are zero in the long run as established by Equation (11). Similarly, a technology shock that increases earnings per unit of capital leads to higher share prices and Tobin's q ; thus triggering a capital deepening process until earnings per unit of capital, and hence, share prices are back to their initial equilibrium.

Thus, the problem associated with the conventional analysis of the effects on *ex post* share returns of changing required returns based on Gordon's growth model is that the exogeneity assumptions are violated. A reduction in the required share returns leads simultaneously to 1) higher share prices because future earnings are discounted at a lower rate; and 2) a capital-deepening process because the marginal product of additional capital stock net of depreciation exceeds the cost of capital, which is reflected in the required returns to equity. Investment in new fixed capital terminates when the after-tax marginal product of capital net of depreciation reaches the new and lower required returns to equity. In terms of Gordon's growth model the initial decrease in the required returns results initially in increasing share prices, but a subsequent decline in share prices until their initial level is reached because the numerator always changes by the same proportion as the denominator in steady state equilibrium. Thus the capital gains are only temporary.

6 Concluding remarks

The seminal paper by Mehra and Prescott (1985) and the subsequent research on the equity premium puzzle has been motivated by a historical equity premium of approximately 6%. This paper has argued that the equity puzzle is not nearly as large as suggested by historical estimates because inflation over the period from 1915 to 1960 was unexpected and, therefore, resulted in unexpected capital losses on bonds. The estimates showed that the *ex post* equity premium was significantly positively affected by inflation over the period from 1915 to 1960 and that the inflation-induced *ex post* bias was, on average, 3-percentage points over the period from 1878 to 2002 for the major industrialised countries. Thus, half of the *ex post* equity premium has been inflation-induced, on average, for the countries considered in this paper. Furthermore, it was shown that the large cross-country discrepancies in the *ex post* equity premium over the period from 1915 to 1960 were inflation-induced; thus explaining why the *ex post* equity premia were two digit numbers in Japan, France, Germany, and Italy, but single digit numbers in other industrialised countries, over the period from 1915 to 1960.

The possibility that the *ex post* equity premium has been further biased by an unexpected reduction in the required returns to equity was ruled out. Using a general equilibrium model it was shown that *ex post* and *ex ante* share returns are identical in the steady state because earnings per unit of capital echo the required returns to shares and because real share prices will always return to a constant mean in the long provided that all earnings are paid out. The historical increase in real share prices have only been a consequence of the returns to retained earnings.

The practical implications of the findings in this paper are 1) that the bias-corrected excess returns to equity are still in excess of the premium that is predicted by standard consumer theory; and 2) that historical equity returns are bad proxies for the excess returns on shares that investors can expect in the future. The finding in this paper of a bias-corrected equity premium of slightly less than 4% is still well in excess of the predictions of standard consumer theory and, therefore, still makes research in explaining the equity premium puzzle a worthwhile exercise.

DATA APPENDIX

Accumulated share index. The data are updated from 1970 using Morgan Stanley's accumulated index. The following sources are used before 1970. Canada. 1900-1924: Dimson, Elroy, Paul Marsh and Mike Staunton, 2000, *The Millennium Book, A Century of Investment Returns*, London Business School, ABN, AMRO, 1924-99: Panjer, Harry H and K P Sharpe, 2001, "Report on Economic Statistics 1924-1998," Canadian Institute of Actuaries. USA. Jack W Wilson and Charles P Jones, 2002, "An Analysis of the S&P 500 Index and Cowles' Extensions: Price Indexes and Stock Returns, 1870-1999," *Journal of Business*, 75, 505-533. Japan. Global Financial Data. Australia. Global Financial Data. France. 1870-1900: *NBER Macroeconomic Data Base*, 1900-1970. Global Financial Data. Germany. Gregor Gielen, 1994, *Können Aktienkurse Noch Steigen?*, Wiesbaden: Gabler Verlag. The index is multiplied by 10 before 1948 to take account of the money reform that changed the old Reichsmark to a new DEM at the ratio of 1:10. Ireland. S F Whelan, 1999, *From Canals to Computers: The Friends First Guide to Long Term Investment*. Dublin: Friends First. Italy. Panetta, Fabio and Robert Violi, 1999, "Is there an Equity Premium Puzzle in Italy? A look at Asset Returns, Consumption and Financial Structure Data over the Last Century," *Termini di Discussione* 353, Bank of Italy (data received by personal correspondence with Fabio Panetta). Netherlands. Eichholtz, Piet, Kees Koedijk and Roger Otten, 2000, "De Eeuw van Het Aandeel," *Economisch Statistische Berichten*, January (data received by personal correspondence with Piet Eichholtz and Roger Otten). Spain. The data are kindly provided by Santiago Fernández Valbuena, Universidad Complutense. Madrid. United Kingdom. 1870-1913: Richard S Grossman, 2002, "New Indices of British Equity Prices, 1870-1913," *Journal of Economic History*, 62, 121-146. 1913-1970. Barclays Capital, 2001, "Equity Guild Study," Barclays Capital. **Consumer Prices**. B. R. Mitchell, 1983, *International Historical Statistics: Americas and Australasia*, Macmillan: London, B. R. Mitchell, 1975, *European Historical Statistics 1750-1975*, Macmillan: London, and B. R. Mitchell, 1982, *International Historical Statistics: Asia and Africa*, Macmillan: London. The data are updated using OECD, *Main Economic Indicators*. **Nominal interest rates on short and long term government bonds**. Canada. F. H. Leacy (ed.), 1983, *Historical Statistics of Canada*, Statistics Canada: Ottawa. USA. S. Homer and R. Sylla, 1991, *A History of Interest Rates*, London: Rudgers University Press. Corporate and municipal long-term bond yields are used over the period from 1900 to 1918. France, Germany, Netherlands, Spain and UK. Homer and Sylla, *op cit*, where the official discount rate is used for Spain before 1940. Italy. Fratianni, M, Spinelli, F, 1997, *A Monetary History of Italy*, Cambridge: Cambridge University Press. Ireland. Whelan, *op cit*. The data are updated using OECD, *Main Economic Indicators*. **Oil Prices**. The oil prices are converted into national currencies. **Exchange Rates**. 1913-46. I Svennilson, 1954, *Growth and Stagnation in the European Economy*, United Nations Economic Commission for Europe, H. Ritzmann-Blickenstorfer, 1996, *Historical Statistics of Switzerland*, Chronos: Zurich, and Geneva and League of Nations, *Monthly Bulletin of Statistics*. 1946-1950. United Nations, *Monthly Bulletin of Economic Statistics*. 1950-2002. IMF, *International Financial Statistics*. Spain. A Carreras, 1989, *Estadísticas Históricas de España*, Madrid: Fundación Banco Exterior. Italy, UK, France, Germany and Netherlands. N W Poathamus, 1946, *Inquiry into the History of Prices in Holland*, Leiden: Am. Canada. Department of Commerce, 1975, *Historical Statistics of the United States: Colonial Times to 1970*, Bureau of the Census: Washington DC. Ireland. Indexed using GBP over the period from 1870 to 1949. **M1**. Spain. Francisco Bustelo and Gabriel Tortella-Casares, 1976, "Monetary Inflation in Spain, 1800-1970," *Journal of European Economic History*, 5, 141-150. UK. Forrest Capie and Alan Webber, 1985, *A Monetary History of the United Kingdom, 1870-1982*, Boston: George Allen & Unwin. Italy. M Fratianni and F Spinelli, 1997, *A Monetary History of Italy*, Cambridge: Cambridge University Press. USA. Historical Statistics, *op cit*. Canada, France,

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