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by

Peter K. Kruse-Andersen

Øster Farimagsgade 5, Building 26, DK-1353 Copenhagen K., Denmark
Tel.: +45 35 32 30 01 – Fax: +45 35 32 30 00
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Directed Technical Change, Environmental Sustainability, and Population Growth*

By Peter K. Kruse-Andersen†

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Abstract

Population growth has two potentially counteracting effects on pollution emissions: (i) more people implies more production and thereby more emissions, and (ii) more people implies a larger research capacity which might reduce the emission intensity of production, depending on the direction of research. This paper investigates how to achieve a given climate goal in the presence of these two effects. A growth model featuring both directed technical change and population growth is developed. The model allows for simultaneous research in polluting and non-polluting technologies. Both analytical and numerical results indicate that population growth is a burden on the environment, even when all research efforts are directed toward non-polluting technologies. Thus research subsidies alone cannot ensure environmental sustainability. Instead, the analysis shows that environmental sustainability requires pollution taxes and/or population control policies.

Keywords: Directed technical change, endogenous growth, environmental policy, environmental sustainability, climate change, population growth

JEL Classification: J11, O30, O41, Q54, Q55, Q58

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†Department of Economics, University of Copenhagen. Email: peter.k.kruse-andersen@econ.ku.dk.

1 Introduction

Empirical studies find that a tighter environmental policy stimulates research in environmentally friendly technologies (e.g., Popp 2006; Haščič et al. 2012; Aghion et al. 2016). Yet many economic studies on climate change assume that technological change is governed by exogenous processes (e.g., Nordhaus and Sztorc 2013). These studies, therefore, neglect a core mechanism in climate change mitigation. In contrast, a recent strand of literature develops micro-founded growth models where the direction of technical change is affected by environmental policies. However, this literature has neglected another core factor: population growth.[1] This seems problematic as the global population size is expected to increase by 48 pct. from 2017 to 2100 (United Nations 2017, medium variant). Moreover, empirical studies find that population growth affects CO_2 emissions substantially more than income per capita growth (e.g., Liddle 2015; Casey and Galor 2017). On top of this, studies find that reducing the number of children in a household reduces CO_2 emissions much more than any other lifestyle choice (Murtaugh and Schlax 2009; Wynes and Nicholas 2017).

The present study fills this gap in the literature by developing a micro-founded growth model featuring both directed technical change and population growth. In this framework, population growth has two potentially counteracting effects on pollution emissions, and a core issue thus becomes the relative strength of these effects. The first effect is a *neo-Malthusian effect*: given the technological level, a larger population leads to a larger production and thus more pollution emissions. The second effect works through knowledge creation. Given the non-rival nature of knowledge, a larger population permits faster knowledge creation, as more resources can be allocated to research. If knowledge creation is directed toward environmentally friendly technologies, population growth has a negative effect on pollution emissions. In the words of Julian Simon:

> It is your mind that matters economically, as much as or more than your mouth or hands. The most important economic effect of population size and growth is the contribution of additional people to our stock of useful knowledge. And this contribution is great enough in the long run to overcome all the costs of population growth (Simon 1998, p. 367).

Given Simon's strong statement, it seems appropriate to name the second effect: the *Simon*

[1] These studies include Saint-Paul (2002), Hart (2004), Ricci (2007), Grimaud and Rougé (2008), Acemoglu et al. (2012), André and Smulders (2014), Hemous (2016), Kruse-Andersen (2016), Daubanes et al. (2016), Hassler et al. (2016), Van den Bijgaart (2017), Fried (2018), Greaker et al. (2018), and Hart (2019).

effect.[2] Note that both effects are scale effects, and that the Simon effect might increase or decrease pollution emissions depending on the direction of research.

The present study examines how to achieve a given climate goal in a growth model featuring both the neo-Malthusian effect and the Simon effect. Consider the Paris Agreement. The agreement states that the global temperature increase should not exceed 2 degrees Celsius. Such a policy goal can be translated into a CO_2 concentration limit. The present study investigates which environmental policies that can ensure that such a limit remains unviolated.

A central paper in the literature on directed technical change and the environment is the model analysis by Acemoglu, Aghion, Bursztyn, and Hemous (2012), hereafter AABH. In their model, production requires polluting and non-polluting inputs. Scientists can improve either polluting or non-polluting technologies, and each scientist chooses the most profitable direction of research. Environmental policy can, therefore, affect the direction of research. A tax on pollution emission will, for instance, reduce the demand for polluting inputs, thereby reducing the profitability of research aimed at polluting technologies.

The modeling strategy of the present study is closely related to that of AABH, but it differs in two important ways. First, AABH assume a constant population size, whereas the model developed in the present study allows for population growth. The introduction of population growth matters for both the qualitative and quantitative policy implications.

Second, in the model developed by AABH, research permanently targets the most advanced technology, polluting or non-polluting, under laissez-faire, implying a strong path dependency of research efforts. This strong path dependency is not only implausible given evidence of simultaneous research in environmentally and non-environmentally friendly technologies. It also leads to a wrong prediction concerning the global CO_2 intensity trend. This point is discussed further in Section 2. Additionally, AABH assume that knowledge spillovers in research are as strong as they can possibly be without implying accelerating economic growth. If population growth is introduced directly into their framework, the long-run economic growth rate increases with the population size: an implausible feature often referred to as the strong scale effect (see Jones 2005). To avoid both the strong path dependency and the strong scale effect, the present study relaxes the knowledge spillover assumptions in research. This modeling strategy is motivated by the empirical evidence presented by

[2]The positive effects of population size on innovation have long been recognized. William Petty was probably the first to realize this in 1682 (Petty 1899, p. 474). But, the relationship has also been recognized in modern economic research (e.g., Kuznets 1960; Simon 1977; Simon 1981; Kremer 1993; Jones 1995).

Kruse-Andersen (2017) and Bloom et al. (2019), and it ensures that the model can match the global CO_2 intensity trend.

The present study finds that population growth is a major burden on the environment. This is confirmed both analytically and numerically. Analytical results based on exponential population growth show that the neo-Malthusian effect always dominates the Simon effect, in the long run, even when all research efforts are directed toward environmentally friendly technologies. This finding is intimately linked to the qualitative calibration procedure, and, in particular, to the departure from the strong path dependency. Weaker spillover effects imply lower research productivity and thereby a weaker Simon effect. The neo-Malthusian effect always dominates the Simon effect in the long run, when spillovers are weak enough to break the strong path dependency.

The analytical policy implications follow directly from this result. First, subsidies can direct research efforts toward environmentally friendly technologies which strengthens the Simon effect. But since the neo-Malthusian effect always dominates the Simon effect in the long run, research subsidies cannot ensure environmental sustainability. In contrast, AABH find that even temporary research subsidies might ensure environmental sustainability. The results differ partly because the present study incorporates population growth and thereby the neo-Malthusian effect. In fact, the present study finds that permanent research subsidies can ensure environmental sustainability in the absence of population growth.

Second, a tax on pollution emission can both direct research efforts toward environmentally friendly technologies and increase the incentive to use a more environmentally friendly input mix in the production process. Together, this production input mix effect and the Simon effect can dominate the neo-Malthusian effect if the pollution penalization increases sufficiently fast. Hence a pollution tax can ensure environmental sustainability.

Third, due to the positive net contribution of population growth to pollution emissions, environmental sustainability requires a less stringent environmental tax policy for a lower population growth rate. Hence population control policies may be useful in conjunction with a pollution tax policy.

Simulations based on different population projections from the United Nations show that population growth is a major burden on the environment within this century. The simulations also show that a research subsidy alone cannot ensure that the two-degree temperature limit from the Paris Agreement remains unviolated under the baseline population growth scenario. Hence staying below a two-degree temperature increase involves either population control policies, a pollution tax, or both.

Besides the aforementioned literature, this paper is related to a substantial literature investigating how population growth relates to natural resource and pollution emission issues.[3] It is well known from economic climate models, that the expected evolution in the population size significantly affects projected emissions (Gaffin and O'Neill 1997; O'Neill et al. 2012) and optimal policy schemes (Scovronick et al. 2017). However, most economic climate models do not feature endogenous technical change.

Some studies (e.g., Nordhaus 2002; Popp 2004; Gerlagh 2008) implement directed technical change features into integrated assessment models. These studies feature both a directed technical change mechanism and population growth, but the directed technical change mechanism is not micro-founded. Thus standard problems arise when assessing the policy implications of these highly aggregated models.

The model developed by Bretschger (2013) has a flavor similar to the model developed in the present study. Population growth stimulates knowledge creation, but it also increases the scale of the economy, leaving less non-renewable resources per worker. Population growth thereby has two counteracting effects on productivity growth. In contrast to the model developed below, Bretschger's model only features one type of technology. Policymakers can therefore only affect the speed and not the direction of technical change.

In another related study, Casey (2017) develops an endogenous growth model where research efforts can improve productivity and energy efficiency of capital goods. Energy taxes incentivize energy efficiency research at the expense of productivity-increasing research. Even though the population size is allowed to grow and the direction of research efforts is endogenous, Casey's study differs notably from the present study, as the resources available for research are completely independent of the population size. Consequently, the scale of the economy has no impact on technological development which eliminates the Simon effect.

There are also other recent studies in the directed technical change literature building on the framework developed by AABH that eliminate the lock-in equilibrium. As in the present study, Daubanes et al. (2016) relax the spillover assumptions in research to ensure simultaneous development of both polluting and non-polluting technologies. Greaker et al. (2018) eliminate the lock-in equilibrium by introducing strong stepping-on-toes effects in research. Yet both studies assume a constant population size which eliminates the scale

[3]E.g., Harford (1997), Harford (1998), O'Neill and Wexler (2000), Schou (2002), Shi (2003), Asheim et al. (2007), Bréchet and Lambrecht (2009), Schäfer (2014), Bohn and Stuart (2015), Peretto and Valente (2015), Dietz et al. (2016), Scovronick et al. (2017), and Gerlagh et al. (2018).

effects investigated in the present study.

The present study features exogenous population growth and endogenous technical change. Gerlagh et al. (2018) take the opposite approach: their model features endogenous fertility choices and exogenous technical change. They find that in the social optimum, the population size is notably below the business-as-usual case. Yet, as their model does not feature endogenous technical change, population growth cannot benefit the environment through the Simon effect emphasized in the present study.

The remainder of the present study is structured as follows. Section 2 motivates the departure from the strong path dependency highlighted by AABH. Section 3 presents the main model, and Section 4 investigates the long-run policy implications. In Section 5 the robustness of the analytical results are investigated by adding relevant features to the main model. Section 6 provides simulations for the period 2015-2100 based on population projections from the United Nations. Finally, Section 7 offers reflections on the analysis.

2 Avoiding the Lock-in Equilibrium

Essentially AABH highlight the consequences of a *lock-in equilibrium* in research. In their set-up, research only focuses on the most advanced technology under laissez-faire. This technology then continues to be the most advanced technology, and only one type of research occurs. Initially the polluting technology is more advanced, implying that research has previously been locked to the polluting technology, and that research is locked to this technology in the absence of policy intervention.

However, this lock-in equilibrium is not empirically plausible. Empirical evidence suggests that both environmentally and non-environmentally friendly technologies are developed today (e.g., Dechezleprêtre et al. 2014; Noailly and Smeets 2015). And historical evidence shows that both types of technology have been developed continuously in the past. Hydropower has, for instance, been used since ancient times, and after the introduction of hydroelectric production via turbines in the 19th century, hydropower played an increasingly important role in modern industrialization (Narbel et al. 2014, p. 172-177).[4]

In addition, because of the lock-in equilibrium, the model developed by AABH cannot

[4]Another example is wind power. Prior to the Industrial Revolution, wind power was a major source of energy in Europe. Although wind power was to a large degree replaced by fossil-based technologies after the Industrial Revolution, the development of windmills continued. A significant development was the introduction of scientific testing and evaluation in the 18th century (Manwell et al. 2009, ch. 1).

match the evolution in the global CO_2 intensity, defined as (anthropogenic) CO_2 emissions divided by GDP. Consider the identity:

$$CO_2 \text{ emissions} \equiv \underbrace{CO_2 \text{ emissions/GDP}}_{CO_2 \text{ intensity}} \times \underbrace{\text{GDP/Population}}_{\text{GDP per capita}} \times \text{Population.}$$

The identity decomposes global CO_2 emissions into three factors: (i) CO_2 intensity, (ii) GDP per capita, and (iii) population size. The left panel of Figure 1 shows that global CO_2 emissions increased by a factor larger than seven from 1890 to 2015. The decomposition shown in the right panel of Figure 1 indicates that the increase was caused by economic and population growth, while a reduction in the CO_2 intensity had a dampening effect.

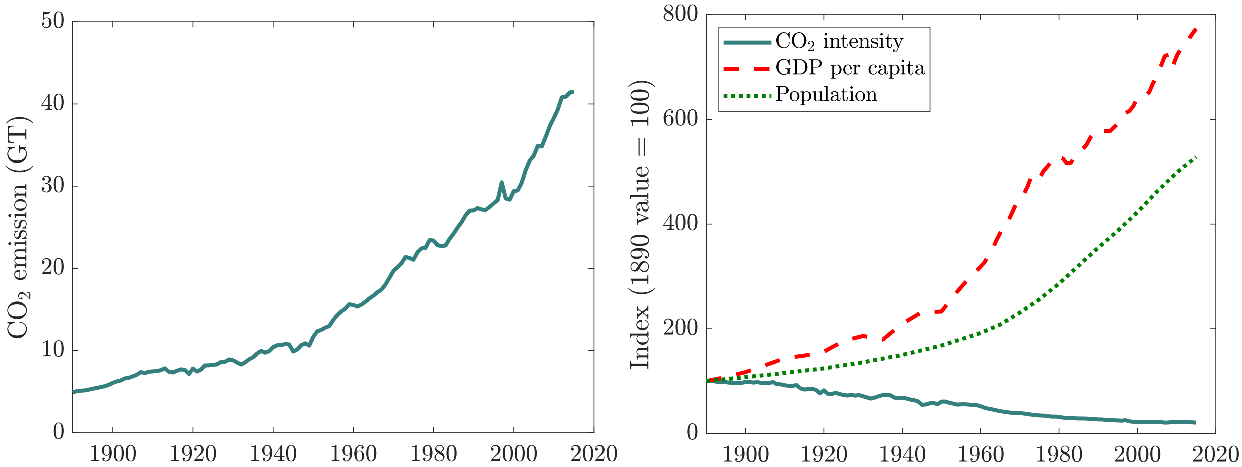

FIGURE 1: Global CO_2 emissions, CO_2 intensity, GDP per capita, and population size, 1890-2015.
Data sources: See Appendix E.

Theoretical models designed to assess climate change issues should be able to match the empirical tendencies presented in Figure 1. The model developed by AABH features GDP per capita growth, but it does not allow for population growth. Additionally, due to the lock-in equilibrium, their model predicts that the pollution intensity increases and converges to a constant which is inconsistent with the empirical evidence in Figure 1. This is shown formally in Appendix A. In contrast, the model developed in the present study can replicate these empirical tendencies, but it requires a departure from the lock-in equilibrium.

To avoid the lock-in equilibrium, the present study relaxes the spillover assumptions in research in accordance with recent empirical evidence (Bloom et al. 2019; Kruse-Andersen 2017). If the spillover effects in research are sufficiently weak, it becomes increasingly less attractive to research in a specific technology, the more advanced this technology becomes. Thus if one type of technology becomes sufficiently advanced, researchers switch their focus to another technology. In the end, all technologies are developed continuously under laissez-faire, and the lock-in equilibrium is avoided.

3 The Model

The model features a growing labor force, and labor has two potential uses: manufacturing and research. In the manufacturing sector, consumption goods are produced from polluting and non-polluting intermediate goods. These inputs are in turn produced by intermediate good specific machines. The machines are produced by labor, and thus manufacturing labor is indirectly devoted to either polluting or non-polluting intermediate good production. In the research sector, scientists develop new machine varieties for either polluting or non-polluting intermediate good production. The increase in machine varieties causes labor productivity in manufacturing to grow. The direction of technical change is determined by the relative profitability of research in the two machine types. Finally, the production of consumption goods is negatively affected by the concentration of pollution in the atmosphere, and the use of polluting intermediate goods increases this concentration.

3.1 Structure

Time is discrete and denoted $t \geq 0$. Consumers spend their entire income each period on consumption goods, and utility is strictly increasing in consumption. Each consumer supplies one unit of labor inelastically, and aggregate labor supply, $\bar{L}_t$, evolves according to:

$$\bar{L}_t = (1+n)^t \bar{L}_0, \quad n \geq 0, \quad \bar{L}_0 > 0,$$

where n is the constant population growth rate.[5]

Consumption goods are produced using the production technology:

$$C_t = (1 - D_t) \left[\left(Y_t^c \right)^{\frac{\epsilon-1}{\epsilon}} + \left(Y_t^d \right)^{\frac{\epsilon-1}{\epsilon}} \right]^{\frac{\epsilon}{\epsilon-1}}, \quad \epsilon > 0, \quad \epsilon \neq 1, \tag{1}$$

where C_t is aggregate consumption, $D_t \in [0, 1)$ is the share of produced consumption goods destroyed by climate change, Y_t^c measures "clean" intermediate good input, and Y_t^d measures "dirty" intermediate good input. Use of dirty intermediate goods causes pollution emissions,

[5] Assuming a constant population growth rate simplifies the theoretical analysis considerably. When the model is simulated in Section 6, population projection data from the United Nations are used directly to measure $\bar{L}_t$.

while the use of clean intermediate goods does not. Aggregate pollution emission equals Y_t^d. The parameter ϵ is the (constant) elasticity of substitution between clean and dirty intermediate goods. When $\epsilon > 1$ the two intermediate good types are gross substitutes, and when $\epsilon < 1$ they are gross complements.[6]

The interpretation of clean and dirty intermediate goods is rather broad. Clean (dirty) intermediate goods are inputs in the production process which reduce (increase) the pollution intensity of production defined as:

$$\frac{Y_t^d}{C_t} = \left(\frac{1}{1-D_t}\right)\left[1+\left(\frac{Y_t^c}{Y_t^d}\right)^{\frac{\epsilon-1}{\epsilon}}\right]^{-\frac{\epsilon}{\epsilon-1}}.$$

It follows directly that the pollution intensity increases in Y_t^d and decreases in Y_t^c.

Policymakers set a climate goal, $\bar{E} > 0$, in terms of the pollution stock E_t. *Environmental sustainability* is obtained if $E_t < \bar{E}$ for all $t \geq 0$. The pollution stock is given by:

$$E_t = \mu \sum_{s=t-\bar{v}}^{t} Y_s^d \xi_{t-s}, \quad 0 \leq \xi_j \leq 1 \text{ for } j > 0, \quad 0 < \xi_0 \leq 1, \quad \bar{v} \in \mathbb{N},$$

where one unit of pollution emission directly increases the pollution stock by μ units and ξ_j reflects pollution stock decay. This formulation of the pollution stock is both general and transparent: the pollution stock cannot exceed cumulated emissions, while the pollution stock may decay in various ways.[7,8] Environmental sustainability is ensured if $\bar{E}$ is sufficiently large and Y_t^d decreases in the long run, as shown in Lemma 4 in Appendix B.1. Correspondingly, environmental sustainability is not obtained if Y_t^d grows at a positive rate in the long run.

Damages from climate change are a function of the pollution stock: $D_t = D(E_t)$, where $D(E_t) \in (0,1)$ and $D' > 0$ for $E_t > \underline{E}$ and $D(E_t) = 0$ for $E_t \leq \underline{E}$. The climate damages are only present if the pollution stock exceeds some level $\underline{E}$ (e.g. pre-industrial CO_2 concentration), and damages are strictly increasing in the pollution stock after this point.[9]

[6] When $\epsilon > 1$ the demand for clean (dirty) intermediate goods increases if the price of dirty (clean) intermediate goods increases which implies that the two intermediate goods are gross substitutes. The opposite is true when $\epsilon < 1$ which implies that the two intermediate goods are gross complements.

[7] The main results also carry through using a more general formula for the pollution stock a la Van den Bijgaart (2017) as shown by Kruse-Andersen (2018, ch. 3). The functional form used here is chosen for its intuitive appeal.

[8] A fraction of the CO_2 emitted today stays in the atmosphere for centuries (Archer et al. 2009), implying that $\bar{v}$ is a very large number.

[9] It turns out that the climate damages are not important for sustainability in this model, but it does affect the costs of different policies which are investigated in Section 6.

Clean and dirty intermediate goods are produced by machines. Machines used to construct clean and dirty intermediate goods are indexed i and h, respectively. The outputs of clean and dirty intermediate goods are given by

$$Y_t^j = A\left(\left(N_t^j\right)^{\alpha-1+\alpha\psi}\int_0^{N_t^j}(x_{kt}^j)^{\alpha}\,\mathrm{d}k\right)^{\frac{1}{\alpha}}, \quad A>0, \quad 0<\alpha<1, \quad \psi>0, \tag{2}$$

where x_{kt}^j denotes the quantity of machine $k \in \{i,h\}$ used in subsector $j \in \{c,d\}$, N_t^j measures the varieties of machines in subsector j such that $i \in [0, N_t^c]$ and $h \in \left[0, N_t^d\right]$, and $1/(1-\alpha)$ is the elasticity of substitution between machine varieties.[10] The parameter ψ reflects the productivity gains associated with machine varieties, cf. (11) below. Holding the total machine input constant, more varieties results in more intermediate good output. Accordingly, the machine variety measures, N_t^j, are referred to as the technological levels. The factor N_t^j to the power of $(\alpha-1+\alpha\psi)$ ensures that an arbitrary parameter link between the elasticity of substitution and the productivity gains from machine varieties is broken (cf. Alvarez-Pelaez and Groth 2005). Breaking this parameter link eases the interpretation of the obtained results without complicating the math notably.

Machines are produced one-for-one by labor input such that

$$L_t = L_t^c + L_t^d, \quad L_t^c = \int_0^{N_t^c} x_{it}^c\,\mathrm{d}i, \quad \text{and} \quad L_t^d = \int_0^{N_t^d} x_{ht}^d\,\mathrm{d}h, \tag{3}$$

where L_t measures labor input in manufacturing, and L_t^j measures labor input used to produce machines for subsector j.

The R&D sector is bifurcated into two subsectors: one for each machine type. In both subsectors, scientists conduct research to invent new machine varieties a la Romer (1990). Scientists can switch between R&D subsectors at the beginning of period t, and the innovation process occurs through period t. There is a standing-on-shoulders effect in research such that a scientist in R&D subsector $j \in \{c,d\}$ starts $\bar{\eta}N_t^j$ projects in the beginning of period t, where $\bar{\eta}>0$. A project can either fail or succeed. For each successful project, the scientist develops a new machine variety that can be produced and used from the beginning of period $t+1$. The success probability for each project is: $\eta^j(N_t^j)^{-\phi^j}$<1, where $0<\eta^j<1$, and $0<\phi^j<1$. The success probability decreases with the technological level, as the easiest ideas are invented first: a fishing-out effect. The parameters ϕ^c and ϕ^d measure the strengths

[10]These machines are better interpreted as intermediate goods, as they depreciate fully after use. The word "machines" is used to clearly distinguish them from clean and dirty intermediate goods.

of the fishing-out effects in the two R&D subsectors. The inclusion of fishing-out effects is motivated by the empirical evidence from Kruse-Andersen (2017) and Bloom et al. (2019). The fishing-out effects allow the model to eliminate both the implausible lock-in equilibrium discussed in Section 2 and the strong scale effect discussed by Jones (2005).[11]

The structure described above leads to the following evolutions in the technological levels:

$$N_{t+1}^j = \left(1 + \eta^j \bar{\eta}\left(N_t^j\right)^{-\phi^j} s_t^j\right) N_t^j, \quad N_0^j \geq 1, \quad 0 < \eta^j < 1, \quad \bar{\eta} > 0, \quad 0 < \phi^j < 1, \tag{4}$$

where s_t^j measures scientist input in R&D subsector $j \in \{c, d\}$. The number of machine varieties in subsector j in period $t+1$ equals the number of varieties in the subsector in period t plus the varieties developed through period t. The latter is given by the total number of projects (number of scientists times the number of projects per scientist), $s_t^j \bar{\eta} N_t^j$, times the success probability per project, $\eta^j (N_t^j)^{-\phi^j}$.

The market clearing condition for scientist input requires that:

$$s_t = s_t^c + s_t^d. \tag{5}$$

Finally, for reasons outside the model, a constant fraction ω of the population is allocated to research and the remaining fraction, $(1-\omega)$, is allocated to manufacturing. Thus,

$$L_t = (1-\omega)\bar{L}_t \quad \text{and} \quad s_t = \omega \bar{L}_t, \quad 0 < \omega < 1. \tag{6}$$

This last assumption can be motivated within an overlapping generations framework as shown in Section 5.[12]

3.2 The market economy

In the market economy, consumption goods and intermediate goods are produced under perfect competition and the labor market is perfectly competitive. New machine varieties

[11] In addition to the fishing-out effects, this structure deviates from that of AABH in one important way. AABH assume that innovations occur at the beginning of period t and that these innovations can be used through period t. The present study deviates from this type of instantaneous coin-flip innovation to reflect that it takes time to innovate. As a consequence, research becomes an investment as in the overlapping generations model presented in Section 5.

[12] The fixed sectoral labor allocation can also be motivated in the following way. Given a highly skewed distribution of research skills, reallocating labor from manufacturing to research might have little effect on the effective research input (see Jaimovich and Rebelo 2017). Thus the central question continues to be how the effective research input is allocated within the R&D sector.

are produced under monopolistic competition, while old machine varieties are produced under perfect competition.

The first-order conditions of the representative firm producing consumption goods imply

$$\left(\frac{Y_t^c}{Y_t^d}\right) = \left(\frac{p_t^c}{p_t^d}\right)^{-\epsilon}, \tag{7}$$

where p_t^c and p_t^d denote the prices of clean and dirty intermediate goods, respectively. The consumption good is numéraire, implying that: $(1-D_t)^{-1}\left[(p_t^c)^{1-\epsilon}+(p_t^d)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}} = 1$.

The first-order conditions of the two representative intermediate good producers give the implicit demand functions:

$$A^{\alpha}\left(N_t^j\right)^{\alpha-1+\alpha\psi}\left(Y_t^j\right)^{1-\alpha}\left(x_{kt}^j\right)^{\alpha-1}p_t^j = p_{kt}^j, \tag{8}$$

where p_{kt}^j denotes the price of machine $k \in \{i,h\}$ used in subsector $j \in \{c,d\}$.

If a scientist develops a new machine variety in period t, he/she receives a one-period patent on that machine variety. Hence the scientist becomes a monopolist of that machine variety in period $t+1$. Thereafter, the machine variety is produced under perfect competition. To correct the market failure associated with the monopoly power on new machine varieties, the government pays a share $(1-\alpha)$ of the production costs for new machine producers. This subsidy is financed through lump-sum taxes.[13]

The monopolists maximize profits subject to (8). The supply of machines - both new and old varieties - is given by[14]

$$x_{kt}^j = \left(\frac{A^{\alpha}\left(N_t^j\right)^{\alpha-1+\alpha\psi}p_t^j}{w_t}\right)^{\frac{1}{1-\alpha}} Y_t^j \equiv \hat{x}_t^j, \tag{9}$$

where w_t is the wage rate. It then follows that the price of any machine equals the wage rate. Per-period profits for new machine producers are then given by

$$\pi_{kt}^j = (1-\alpha)\, w_t \hat{x}_t^j \equiv \hat{\pi}_t^j, \tag{10}$$

[13] These assumptions simplify the math considerably, as they ensure that all machine varieties - no matter when they are invented - are produced in the same quantity.

[14] The new machine producers are monopolists. They solve the problem: $\max p_{kt}^j(x_{kt}^j)x_{kt}^j - \alpha w_t x_{kt}^j$ wrt. x_{kt}^j, where $p_{kt}^j(x_{kt}^j)$ is given by (8). Meanwhile, the old machine producers operate under perfect competition and solve the problem: $\max p_{kt}^j x_{kt}^j - w_t x_{kt}^j$ wrt. x_{kt}^j, where p_{kt}^j is considered exogenous.

where π_{it}^c and π_{ht}^d denote profits for monopolist $i \in \left(N_{t-1}^c, N_t^c\right]$ and $h \in \left(N_{t-1}^d, N_t^d\right]$, respectively.[15] Meanwhile, old machine producers obtain zero profits, as these machines are produced under perfect competition.

It follows from (2), (3), and (9) that

$$Y_t^c = A\left(N_t^c\right)^{\psi} L_t^c \quad \text{and} \quad Y_t^d = A\left(N_t^d\right)^{\psi} L_t^d. \tag{11}$$

From (3), (7), (9), and (11) it follows that

$$\left(\frac{p_t^c}{p_t^d}\right) = \left(\frac{N_t^c}{N_t^d}\right)^{-\psi}, \quad \left(\frac{Y_t^c}{Y_t^d}\right) = \left(\frac{N_t^c}{N_t^d}\right)^{\epsilon\psi}, \quad \text{and} \quad \left(\frac{L_t^c}{L_t^d}\right) = \left(\frac{N_t^c}{N_t^d}\right)^{(\epsilon-1)\psi}. \tag{12}$$

The supply of consumption and intermediate goods are computed from (1), (3), (6), (11), and (12):

$$Y_t^c = A\left[\left(N_t^c\right)^{-(\epsilon-1)\psi} + \left(N_t^d\right)^{-(\epsilon-1)\psi}\right]^{-1} \left(N_t^c\right)^{\psi} \left(N_t^d\right)^{-(\epsilon-1)\psi} (1-\omega)\bar{L}_t, \tag{13}$$

$$Y_t^d = A\left[\left(N_t^c\right)^{-(\epsilon-1)\psi} + \left(N_t^d\right)^{-(\epsilon-1)\psi}\right]^{-1} \left(N_t^d\right)^{\psi} \left(N_t^c\right)^{-(\epsilon-1)\psi} (1-\omega)\bar{L}_t, \quad \text{and} \tag{14}$$

$$C_t = A(1-D_t)\left[\left(N_t^c\right)^{-(\epsilon-1)\psi} + \left(N_t^d\right)^{-(\epsilon-1)\psi}\right]^{\frac{1}{\epsilon-1}} \left(N_t^c\right)^{\psi} \left(N_t^d\right)^{\psi} (1-\omega)\bar{L}_t. \tag{15}$$

The struggle between the neo-Malthusian effect and the Simon effect is already apparent from the expression for Y_t^d. Assume that all scientists work in the clean R&D subsector. In this case, N_t^c increases while N_t^d remains constant. If $\epsilon > 1$, pollution emissions per capita, $Y_t^d/\bar{L}_t$, unambiguously decrease in the long run. Meanwhile, the evolution of aggregate pollution emissions depends on how fast the population grows compared to how fast pollution emissions per capita decrease.

Let $\tilde{\pi}_t^c$ and $\tilde{\pi}_t^d$ denote the expected discounted profits for scientists conducting research in the clean and dirty R&D subsector, respectively. These discounted profits depend on four factors: (i) the discount rate, (ii) the number of projects, (iii) the success probability per

[15] One possible concern is that the wage gap between manufacturing workers and scientists would increase over time, calling the exogenous labor allocation assumption into question. However, as shown in Section 5, the model can be modified such that the labor allocation becomes endogenous - resulting in a wage ratio equal to one - without changing the main policy implications of the model.

project, and (iv) the value per obtained patent. Accordingly,

$$\tilde{\pi}_t^j = \underbrace{\frac{1}{(1+r_{t+1})}}_{\text{Discount rate}} \times \underbrace{\bar{\eta}\left(N_t^j\right)}_{\text{Projects per scientist}} \times \underbrace{\eta^j\left(N_t^j\right)^{-\phi^j}}_{\substack{\text{Success probability}\\ \text{per project}}} \times \underbrace{\hat{\pi}_{t+1}^j}_{\substack{\text{Profit per}\\ \text{one-period patent}}}, \tag{16}$$

where r_{t+1} is the real interest rate.

The ratio of expected discounted profits guides the scientists' decisions, as the scientists maximize expected discounted profits. It follow from (10) and (16) that the (expected discounted) profit ratio amounts to

$$\left(\frac{\tilde{\pi}_t^c}{\tilde{\pi}_t^d}\right) = \underbrace{\left(\frac{\eta^c(N_t^c)^{-\phi^c}}{\eta^d(N_t^d)^{-\phi^d}}\right)}_{\text{Success probability effect}} \times \underbrace{\left(\frac{\hat{x}_{t+1}^c}{\hat{x}_{t+1}^d}\right)}_{\text{Market size effect}} \times \underbrace{\left(\frac{N_t^c}{N_t^d}\right)}_{\text{Standing-on-shoulders effect}}. \tag{17}$$

Three effects determine the profit ratio. The *success probability effect* has not been emphasized in the previous literature, where it is state independent. Scientists are prone to research in a subsector, the greater the success probability per project. Due to fishing-out effects, the success probability per project decreases with the technological level. The *market size effect* reflects that innovation is directed toward the relatively largest subsector.[16] Finally, the *standing-on-shoulders effect* reflects that researchers can start more projects in the technologically leading subsector. The success probability effect drags innovation toward the technologically less advanced subsector, while the market size effect and the standing-on-shoulders effect drag innovation toward the technologically leading subsector.

The success probability effect turns out to be crucial. If the success probability effect is sufficiently strong, it can dominate the market size effect and the standing-on-shoulders effect such that the implausible lock-in equilibrium is avoided. This requires relatively strong fishing-out effects represented by ϕ^c and ϕ^d.

The equilibrium profit ratio is derived from (3), (4), (5), (6), (12), and (17):

$$F(s_t^c, \bar{L}_t, N_t^c, N_t^d) = \left(\frac{\eta^c}{\eta^d}\right) \times \left(\frac{1+\eta^c\bar{\eta}(N_t^c)^{-\phi^c}s_t^c}{1+\eta^d\bar{\eta}(N_t^d)^{-\phi^d}(\omega\bar{L}_t - s_t^c)}\right)^{(\epsilon-1)\psi-1} \times \frac{(N_t^c)^{(\epsilon-1)\psi-\phi^c}}{(N_t^d)^{(\epsilon-1)\psi-\phi^d}},$$

where $F(\cdot) \equiv \tilde{\pi}_t^c/\tilde{\pi}_t^d$. The profit ratio is determined by the labor input in the clean R&D subsector given the state variables $\bar{L}_t$, N_t^c, and N_t^d. The following lemma shows how the

[16]The market size effect emphasized here essentially includes both the market size effect and the price effect emphasized by AABH. This is clear from (9). But since there is no parameter link between α and ψ, the market size effect emphasized in the present study incorporates an additional technological component, cf. (9).

profit ratio determines the allocation of scientists.

Lemma 1. *Assume that* $(\epsilon - 1)\psi < 1$*:*

1. *If* $1 \leq F(\omega \bar{L}_t, \cdot)$ *then* $(s_t^c, s_t^d) = (\omega \bar{L}_t, 0)$ *is a unique equilibrium in the R&D sector.*
2. *If* $F(0, \cdot) \leq 1$ *then* $(s_t^c, s_t^d) = (0, \omega \bar{L}_t)$ *is a unique equilibrium in the R&D sector.*
3. *If* $F(\omega \bar{L}_t, \cdot) < 1 < F(0, \cdot)$ *then* $(s_t^c, s_t^d) = (s_t^*, \omega \bar{L}_t - s_t^*)$ *is a unique equilibrium in the R&D sector, where* s_t^* *is the unique solution to* $F(s_t^*, \cdot) = 1$*.*

Proof. See Appendix B.2. □

The assumption $(\epsilon - 1)\psi < 1$ ensures a unique equilibrium, as the profit ratio is strictly decreasing in s_t^c.

3.3 Qualitative calibration

To focus the analysis on empirically plausible cases, this section restricts certain parameter values further. The global population size has been increasing for thousands of years (Kremer 1993), and the United Nations (2017) expects that the global population size continues to grow at least until 2100. Thus a positive population growth rate, $n > 0$, is assumed through most of this analysis. Based on empirical evidence presented by Papageorgiou et al. (2017), clean and dirty intermediate goods are assumed gross substitutes, $\epsilon > 1$. This assumption remains controversial, and the results presented below therefore represent the optimistic case, where it is relatively easy to substitute between polluting and non-polluting production technologies. Additionally, the parameter restriction $(\epsilon - 1)\psi < 1$ is imposed to ensure a unique equilibrium in the R&D sector.

The following parameter restrictions summarize these assumptions:

Parameter Restriction 1. $n > 0, \quad \epsilon > 1, \quad$ *and* $\quad (\epsilon - 1)\psi < 1.$

The model dynamics are strongly affected by the initial conditions and the parameter values: ϵ, ψ, ϕ^c, and ϕ^d. While the initial conditions matter quantitatively, the long-run qualitative behavior of the profit ratio is determined by the parameter values. Consider the following lemma.

Lemma 2. *Assuming that Parameter Restriction 1 holds, the profit ratio will, in the long run, equal one or fluctuate around one under laissez-faire if* $(\epsilon - 1)\psi < \phi^c$ *and* $(\epsilon - 1)\psi < \phi^d$*.*

Proof. See Appendix B.3. □

If the conditions from Lemma 2 are not fulfilled, the profit ratio might go to infinity or converge to zero depending on the initial conditions, and the model would feature the implausible lock-in equilibrium discussed above.

What is the empirically plausible case? Consider the equilibrium pollution intensity of the manufacturing sector:

$$\frac{Y_t^d}{C_t} = \left(\frac{1}{1-D_t}\right)\left(N_t^c\right)^{-\epsilon\psi}\left[\left(N_t^c\right)^{-(\epsilon-1)\psi} + \left(N_t^d\right)^{-(\epsilon-1)\psi}\right]^{-\frac{\epsilon}{\epsilon-1}}.$$

When research is locked to the dirty R&D subsector, the pollution intensity increases over time under Parameter Restriction 1.[17] This is clearly at odds with the decreasing pollution intensity shown in the right panel of Figure 1.[18] Thus research cannot be locked to the dirty R&D subsector under laissez-faire. Research cannot be locked to the clean R&D subsector either, as this would imply decreasing CO_2 emissions per capita, cf. (14). This prediction is counterfactual, as global CO_2 emissions per capita have been increasing at least since 1890. Thus research must, in the long run, be conducted in both R&D subsectors in the absence of significant policy interventions. To ensure this, the following parameter restrictions are imposed based on Lemma 2.

Parameter Restriction 2. $(\epsilon-1)\psi < \phi^c$ *and* $(\epsilon-1)\psi < \phi^d$.

These parameter restrictions are crucial for the policy implications which are discussed further in the subsequent section. The restrictions also allow the model to match the empirical patterns from Figure 1 (see Section 6).

4 Policy Implications

This section investigates which policies that are able to ensure environmental sustainability. Here, the long-run evolution in pollution emissions is key. To understand the core mechanisms governing the policy implications, this section first investigates the relative strengths of the neo-Malthusian effect and the Simon effect. First consider the following definition.

[17]This is clear since $1/(1-D_t)$ increases, when research is locked to the dirty R&D subsector, while the second part of the equation converges to one from below.

[18]If $\epsilon < 1$, the pollution intensity approaches infinity, as the dirty technological level approaches infinity.

Definition 1. *Research has* ***clean bias*** *if it is permanently and fully directed toward the clean R&D subsector:* $(s_t^c, s_t^d) = (\omega \bar{L}_t, 0)\ \forall t$. *Likewise, research has* ***dirty bias*** *if it is permanently and fully directed toward the dirty R&D subsector:* $(s_t^c, s_t^d) = (0, \omega \bar{L}_t)\ \forall t$.

The full strength of the Simon effect is obtained when research has clean bias. Thus if the neo-Malthusian effect dominates the Simon effect in this case, clean bias research is not sufficient to ensure environmental sustainability.

Now consider the following decomposition of pollution emissions:

$$Y_t^d = \frac{Y_t^{d,\text{neo}} \times Y_t^{d,\text{Simon}}}{Y_0^d}, \quad Y_t^{d,\text{neo}} = \left(\frac{Y_0^d}{\bar{L}_0}\right) \bar{L}_t, \quad \text{and} \quad Y_t^{d,\text{Simon}} = \left(\frac{Y_t^d}{\bar{L}_t}\right) \bar{L}_0,$$

where $Y_t^{d,\text{neo}}$ is pollution emissions holding technology constant, and $Y_t^{d,\text{Simon}}$ is pollution emissions with a constant population size, but technological development as under population growth. The neo-Malthusian effect and the Simon effect become apparent when computing the growth factor of pollution emissions:

$$(1 + g_{Y^d,t}) = \underbrace{(1 + g_{Y^{d,\text{neo}},t})}_{\text{neo-Malthusian effect}} \times \underbrace{(1 + g_{Y^{d,\text{Simon}},t})}_{\text{Simon effect}},$$

where the growth factor of a variable V is denoted $(1 + g_{V,t})$.

To formally investigate the long-run relative strengths of the two effects under clean bias, the following lemma states the long-run technological growth rates when research is biased.

Lemma 3. *Assume that Parameter Restriction 1 holds. If research has clean bias, the long-run growth factor of* N_t^c *denoted* $(1 + g_{N^c})$ *equals* $(1+n)^{\frac{1}{\phi^c}}$, *while the long-run growth factor of* N_t^d *denoted* $(1 + g_{N^d})$ *equals one. Likewise, if research has dirty bias* $(1 + g_{N^c})$ *equals one and* $(1 + g_{N^d})$ *equals* $(1+n)^{\frac{1}{\phi^d}}$.

Proof. See Appendix B.4. □

From Lemma 3 it follows that if research has clean bias, the long-run growth factor of pollution emissions amounts to:

$$(1 + g_{Y^d,t}) = \underbrace{(1+n)}_{\text{neo-Malth. effect}} \times \underbrace{\frac{Y_{t+1}^d}{Y_t^d}(1+n)^{-1}}_{\text{Simon effect}} \xrightarrow[t\to\infty]{} \underbrace{(1+n)}_{\text{neo-Malth. effect}} \times \underbrace{(1+n)^{-\frac{(\epsilon-1)\psi}{\phi^c}}}_{\text{Simon effect}}.$$

The expression illustrates the struggle between the neo-Malthusian effect and the Simon effect. The neo-Malthusian effect directly affects pollution emissions through an increase in the amount of workers in manufacturing (effect on extensive margin). The Simon effect comes into play as a higher population growth rate leads to a faster increase in the research capacity of the economy and thereby a faster development of clean technologies. Since there is no development of dirty technologies due to clean bias research, the relative use of dirty technologies decreases, implying less pollution emissions per worker (effect on intensive margin). This effect is weakened by the fishing-out effects in the clean R&D subsector, as stronger fishing-out effects imply slower technological development. In contrast, the effect is amplified by a higher elasticity of substitution between clean and dirty technologies, ϵ, as this implies less costly input substitution, and thereby larger changes in the input mix for changes in the relative technological level, N_t^c/N_t^d. Additionally, the Simon effect is strengthened by the productivity gains from machine varieties, represented by ψ. Larger productivity gains from machine varieties imply larger productivity gains from research, and thus, a faster change in the relative productivity of clean and dirty technologies.

The following proposition formalizes these considerations.

Proposition 1. *Assuming that Parameter Restriction 1 holds and that research has clean bias, then aggregate pollution emissions:*
(i) decreases in the long run at a constant rate if and only if $(\epsilon - 1)\psi > \phi^c$*,*
(ii) increases in the long run at a constant rate if and only if $(\epsilon - 1)\psi < \phi^c$*, and*
(iii) remains constant in the long run if and only if $(\epsilon - 1)\psi = \phi^c$*.*

Proof. See Appendix B.5. □

It follows from Proposition 1 that in the empirically plausible case where Parameter Restriction 2 holds (see Section 3.3), the neo-Malthusian effect always dominates the Simon effect in the long run even when research has clean bias.

Intuitively, to avoid the implausible lock-in equilibrium, the fishing-out effects represented by ϕ^c and ϕ^d must be relatively strong. Strong fishing-out effects imply low research productivity which implies a weak Simon effect. In fact, the requirement ensuring a departure from the lock-in equilibrium implies that the Simon effect is weaker than the neo-Malthusian effect in the long run, even when research has clean bias.

4.1 Research subsidies

The research subsidy considered is a subsidy to profits for new clean machine producers which ensures that research has clean bias. The subsidy is combined with a profit tax for new dirty machine producers to reduce the revenue cost of the subsidy. The profit tax would not provide any revenue since research has clean bias, but it reduces the cost of the subsidy, as it makes the clean R&D subsector relatively more attractive.

A temporary subsidy can ensure that research has clean bias for as long as it lasts. But when it expires, it follows from Lemma 2 that in the empirically plausible case, research is conducted in both R&D subsectors in the long run. Hence a temporary subsidy cannot ensure clean bias. In contrast, a permanent research subsidy can ensure clean bias. But in the empirically plausible case where Parameter Restriction 2 holds, Proposition 1 implies that these subsidies cannot ensure environmental sustainability.

These results are summarized in the following proposition.

Proposition 2. *Assuming that Parameter Restriction 1 and 2 hold:*
(i) a temporary subsidy to the clean R&D subsector cannot permanently direct all research toward the clean R&D subsector.
(ii) neither a temporary nor a permanent research subsidy to the clean R&D subsector can ensure environmental sustainability.

Proof. See Appendix B.6. □

Intuitively, research subsidies cannot ensure environmental sustainability, as the neo-Malthusian effect always dominates the Simon effect in the long run. Thus even though all research efforts aim at reducing pollution emissions per worker, aggregate pollution emissions still grow, in the long run, due to a growing workforce. To ensure environmental sustainability, it is necessary to use an additional policy instrument to tip the balance. In the next section, it is shown that a tax on pollution emission can do just that.

The policy implications summarized in Proposition 2 contrast to previous results. AABH find that an environmental disaster can be avoided through a *temporary* subsidy to clean research. The results obtained in the present study differ for two reasons. First, the introduction of population growth implies that aggregate pollution emissions may increase despite decreasing pollution emissions per worker caused by clean bias research. In fact, a *permanent* research subsidy can ensure environmental sustainability in the absence of population growth, cf. Section 4.3 below. Second, the parameter restrictions ensuring that the implausible lock-in equilibrium is avoided implies that the profit ratio is attracted to one under laissez-faire. Hence, in contrast to AABH, only a permanent research subsidy can ensure clean bias research (even if the population size is constant).

4.2 A pollution tax

Suppose the government imposes a tax, $\tilde{\tau}_t$, per unit of pollution emission. The tax is paid by the consumption good producers, and the tax revenue is transferred lump-sum to the consumers and/or used to finance research subsidies. The price for purchasing and using a dirty intermediate good becomes: $p_t^d+\tilde{\tau}_t$. This price is rewritten as: $p_t^d\tau_t$, where $\tau_t \equiv 1+\tilde{\tau}_t/p_t^d$. The variable τ_t is referred to as the *pollution penalty*, as it reflects the penalty (introduced by the pollution tax) associated with dirty intermediate good use.

When the pollution penalty is introduced, dirty intermediate good use amounts to:

$$Y_t^d = A\left[\left(N_t^c\right)^{-(\epsilon-1)\psi}\tau_t^{-\epsilon}+\left(N_t^d\right)^{-(\epsilon-1)\psi}\right]^{-1}\left(N_t^c\right)^{-(\epsilon-1)\psi}\tau_t^{-\epsilon}\left(N_t^d\right)^{\psi}(1-\omega)\bar{L}_t.$$

The pollution penalty reduces the incentive to use dirty intermediate goods in the production process: a *production input mix effect.* This effect is amplified by the elasticity of substitution, as a higher elasticity implies a lower cost of input substitution.

First consider a combination of a permanent research subsidy ensuring clean bias and a constant pollution penalty. In this case, the pollution emission tax would increase over time at the same rate as p_t^d which increases over time since research has clean bias. Environmental sustainability is not obtained in this case which is formally stated in the following proposition.

Proposition 3. *Assuming that Parameter Restriction 1 and 2 hold, and that research has clean bias, a constant pollution penalty, $\tau_t = \tau$, cannot ensure environmental sustainability.*

Proof. See Appendix B.7. □

The constant pollution penalty affects the level but not the long-run growth rate of dirty intermediate good use. As the long-run growth rate of pollution emissions is positive under Parameter Restriction 2, environmental sustainability is not obtained.

To ensure environmental sustainability, the pollution penalty must increase over time. The following policy rule is considered:

$$\tau_t = (1+g_\tau)^t\tau_0, \quad g_\tau > 0, \quad \tau_0 > 0, \tag{18}$$

where g_τ is the constant growth rate of the pollution penalty.

A tax on pollution emission can permanently direct research toward the clean R&D subsector if the pollution penalty is sufficiently large initially and grows sufficiently fast. In this case, the pollution tax also ensures environmental sustainability given that $\bar{E}$ is sufficiently large. This result is summarized in the following proposition.

Proposition 4. *Assuming that Parameter Restriction 1 and 2 hold, a pollution tax can ensure both clean bias and environmental sustainability if it is sufficiently large initially, the pollution penalty grows by a constant factor above* $(1+n)^{\frac{\phi^C-(\epsilon-1)\psi}{\epsilon\phi^C}}$*, and* $\bar{E}$ *is sufficiently large.*

Proof. See Appendix B.8. □

To understand Proposition 4, consider the long-run growth factor of pollution emissions under clean bias:

$$(1+g_{Y^d,t}) \xrightarrow[t\to\infty]{} \underbrace{(1+n)}_{\text{Neo-Malthusian effect}} \times \underbrace{(1+n)^{-\frac{(\epsilon-1)\psi}{\phi^C}}}_{\text{Simon effect}} \times \underbrace{(1+g_\tau)^{-\epsilon}}_{\text{Production input mix effect}} .$$

Pollution emissions decrease in the long run if the Simon effect and the production input mix effect dominate the neo-Malthusian effect. As the neo-Malthusian effect dominates the Simon effect, this requires a sufficiently fast increase in the pollution penalty. Both the neo-Malthusian effect and the Simon effect are amplified by faster population growth. Hence environmental sustainability requires a lower pollution penalty growth rate, when the population growth rate is reduced.

4.3 Population control policies

Since the neo-Malthusian effect dominates the Simon effect in the long run, the above findings suggest that the environmental sustainability problem is largely caused by population growth. Thus intuitively it should be easier to ensure environmental sustainability in the absence of population growth. This intuition is confirmed by the following proposition.

Proposition 5. *Assuming that Parameter Restriction 2 holds,* $n=0$*,* $\epsilon>1$*, and* $(\epsilon-1)\psi<1$*, then environmental sustainability can be obtained using a permanent research subsidy ensuring that research has clean bias if* $\bar{E}$ *is sufficiently large.*

Proof. See Appendix B.9. □

Even with a population growth rate of zero ($n=0$), there is still perpetual growth in the clean technological level. The growth rate decreases over time, but the clean technological level approaches infinity as time approaches infinity (see Groth et al. [2010] for a thorough examination of less-than-exponential growth). Since the dirty technological level is constant given clean bias, pollution emissions per worker decrease over time, while the number of workers remains constant. Aggregate pollution emissions decrease in the long run and approaches zero. Environmental sustainability is ensured if $\bar{E}$ is sufficiently large.

5 Robustness

This section introduces relevant extensions to the model presented in Section 3 to test the robustness of the analytical results derived in Section 4.

5.1 An overlapping generations model

This extension introduces a micro foundation for the fixed labor allocation between manufacturing and research. All the policy implications presented above are still valid within this extension.

The model builds on the overlapping generations framework, and thus saving is introduced into the model. There are at all times two generations present in the economy: the young and the old. The young work, consume and save, while the old consume their entire savings. In period t, there are $\bar{L}_t$ young and $\bar{L}_t/(1+n)$ old. The young solve the problem:

$$\begin{aligned}
&\max_{c_t^{\mathrm{y}}, c_{t+1}^{\mathrm{o}}} U_t = \ln c_t^{\mathrm{y}} + \ln G_t + \beta \left(\ln c_{t+1}^{\mathrm{o}} + \ln G_{t+1} \right) \\
&\text{st. } \; c_t^{\mathrm{y}} = (1-\tau^w) w_t + \bar{\pi}_t - b_t, \;\; c_{t+1}^{\mathrm{o}} = (1+r_{t+1}) b_t, \;\; (c_t^{\mathrm{y}}, c_{t+1}^{\mathrm{o}}, b_t) \geq 0, \\
&0 < \beta < 1, \;\; 0 < \tau^w < 1,
\end{aligned}$$

where U_t is welfare, c_t^{y} is consumption as young, c_{t+1}^{o} is consumption as old, G_t is a public good provided by the government, β is the discount factor, τ^w is a wage tax, b_t is saving, $\bar{\pi}_t$ is profits from old machine firms (equals zero in equilibrium), and r_{t+1} is the real interest rate. The generational set-up implies long time periods which eases consumption smoothing. This motivates the low intertemporal elasticity of substitution implied by the per-period utility function.

The saving of the young is used to finance research which is conducted by a fraction of the young generation. Due to labor mobility manufacturing workers and scientists are paid the same wage. The R&D investments of the young turn into valuable assets when the young turn old in the subsequent period. Specifically, the old generation owns the patents on new machine varieties which generate the return on savings.

Consumption goods can either be consumed as private goods or as a public good. Clearing on the market for consumption goods requires that: $C_t = c_t^{\mathrm{y}} \bar{L}_t + \bar{L}_{t-1} c_t^{\mathrm{o}} + G_t$. The government keeps a balanced budget such that G_t equals the residual of government spending on subsidies and the tax revenue of the government.[19]

Otherwise the model from Section 3 is unchanged. Optimizing behaviour implies: $b_t = (\beta/(1+\beta))\,(1-\tau^w) w_t$. As all saving is used to finance R&D expenditures: $b_t \bar{L}_t = w_t s_t$. From these two equations it follows that:

$$s_t = \tilde{\omega} \bar{L}_t, \quad \tilde{\omega} \equiv \left(\frac{\beta}{1+\beta} \right) (1-\tau^w).$$

Accordingly, a constant fraction, $\tilde{\omega}$, of the workforce is employed in the R&D sector. As $\tilde{\omega}$ is unaffected by the environmental policies discussed above, the policy implications remain unchanged.

5.2 Stepping-on-toes effects and intersectoral spillovers

Appendix C shows how the policy implications are robust to the introduction of stepping-on-toes effects in research. The stepping-on-toes effects introduce decreasing returns to idea production such that a doubling of the research input does not double the number of new ideas created. In the end, the stepping-on-toes effects slow down technological development and work much like the fishing-out effects in the long run.[20]

Hart (2019) introduces intersectoral spillovers in a model featuring directed technical change which weakens the path dependency of research. In Appendix D a model with knowledge spillovers between the two R&D subsectors is presented. In this model, there might exist a parameter space in which the lock-in equilibrium can be avoided, while the Simon effect dominates the neo-Malthusian effect in the long run under clean bias. Whether this parameter space is empirically relevant is beyond the scope of the present study. However,

[19] From a technical point of view, the public good is added for tractability purposes. One can think of the public good as a way that the government can transfer resources back to the households without distorting relative prices and trade-offs. In that way, the public good works somewhat like a lump-sum transfer in the neoclassical growth model framework.

[20] Greaker et al. (2018) introduce stepping-on-toes effects in a AABH-style model which eliminate the lock-in equilibrium. However, the stepping-on-toes effects considered by Greaker et al. (2018) are very strong such that the marginal research productivity approaches infinite as the research input approaches zero. The stepping-on-toes effects considered here are less potent.

the above findings carry through outside this parameter space.

6 Simulations

The theoretical analysis conducted above has two limitations. First, the results are based on the asymptotic properties of the model. Keeping the global temperature increase well below 2 degrees Celsius as stated in the Paris Agreement requires substantial action within this century (IPCC 2018). Thus, it might be insufficient to only consider these asymptotic policy implications. Second, the analytical results are derived assuming exponential population growth which is at odds with expected future population growth patterns. To confront these matters, this section provides a quantitative analysis for the period 2015-2100, based on population projections from the United Nations.[21] To ensure empirical relevance, the model is calibrated to match historical trends in global CO_2 emissions, GDP, and carbon concentration as described below.

6.1 Model adjustments

The model from Section 3 is adjusted to improve its empirical relevance. First, global workforce data are loaded directly into the model to measure $\bar{L}_t$. One could also have used population size data, but in the end, emissions are a by-product of production and thereby supply determined.[22]

Secondly, the carbon cycle follows the specification from Golosov et al. (2014):

$$E_t = \sum_{s=t-\bar{v}}^{t} \Big(\xi_P + (1-\xi_P)(1-\xi_A)(1-\xi_D)^{t-s}\Big)\mu Y_s^d,\ \mu > 0,\ 0 < \xi_h < 1,\ h = \{P, A, D\},$$

where μ equals one GT carbon per model unit of emission such that μY_t^d is period t emission in GT carbon. The mechanism is as follows. A fraction ξ_P of the carbon emitted today practically stays in the atmosphere permanently, as climate science suggests that $\bar{v}$ is a very large number (Archer et al. 2009). Of the remaining carbon, a fraction ξ_A is absorbed by the biosphere and surface oceans, while the fraction $(1-\xi_A)$ decays at the rate ξ_D.

[21]Simulations based on population projections from the Wittgenstein Centre for Demography and Global Human Capital yield similar results.

[22]As the workforce grows slower than the global population size due to a changing age distribution, this choice reduces projected emission growth leading to a somewhat more optimistic emission growth path compared to other models in the literature like the DICE model.

Third, climate damages are modelled as in Golosov et al. (2014):

$$D_t = 1 - \mathrm{e}^{-\sigma\zeta E_t}, \quad \sigma > 0,$$

where ζ equals ppm (parts per million) per GT carbon such that ζE_t is the carbon concentration in the atmosphere in ppm.

Fourth, it is necessary to compute GDP in order to calibrate the model. GDP is the sum of value added in manufacturing and research. Value added in manufacturing equals the sum of value added by machine producers, intermediate good producers, and consumption good producers which amounts to the value of aggregate consumption, C_t. Scientists in the research sector generate valuable assets: patents. Value added by the R&D sector is therefore the net present market value of all patents obtained within the period. Accordingly:

$$GDP_t = C_t + \left(N_{t+1}^c - N_t^c\right) \frac{\pi_{t+1}^c}{1 + r_{t+1}} + \left(N_{t+1}^d - N_t^d\right) \frac{\pi_{t+1}^d}{1 + r_{t+1}},$$

where $(N_{t+1}^j - N_t^j)$ is the number of new patents obtained in R&D subsector j, and the present value of a subsector j patent is given by $\pi_{t+1}^j/(1 + r_{t+1})$.

Finally, a welfare function is specified which is used to evaluate different policies. Let total welfare, U, be given by:

$$U = \sum_{t=0}^{t_{\max}} \frac{c_t^{1-\theta} - 1}{1 - \theta} \bar{L}_t^{\kappa} \beta^t, \quad c_t \equiv C_t/\bar{L}_t, \quad \theta > 0, \quad \theta \neq 1, \quad 0 \leq \kappa \leq 1, \quad 0 < \beta < 1.$$

In the following, two cases are considered: $\kappa = 0$ and $\kappa = 1$. These cases represent average and total utilitarianism, respectively. In the first case ($\kappa = 0$), a social planner maximizes average welfare, while a social planner maximizes total welfare in the second case ($\kappa = 1$).[23]

In most simulations considered, the two welfare functions yield similar policy implications. The main simulation results are shown for the total utilitarian case, as this is the typical approach taken in the literature (e.g., Nordhaus 2018). These results are then compared to the average utilitarian case at the end of this section.

[23] See IPCC (2014, ch. 3) for a discussion on different perspectives on the population size and aggregate welfare measures.

6.2 Calibration

The model is calibrated to match the evolutions in global CO_2 emissions, GDP, and CO_2 concentration for the period 1890-2015 using one-year periods. Data on these variables and the global workforce are collected from several sources as described in Appendix E.

The technological levels in period 0 (the year 1890) are set to one. The scale parameter A is also normalized to one. The parameter α reflects the market power of new machine producers. Specifically, the price set by new machine producers is the mark-up $(1/\alpha)$ multiplied by their marginal production costs. Loecker and Eeckhout (2017) find that the average mark-up in the US was around 1.25 until the 1990s. Accordingly, α is set to 0.8. The parameter ψ is (together with ϵ) limited by Parameter Restriction 2. It is set to 0.75 to allow some freedom for ϵ, but the exact value does not seem important for the general conclusions. The calibration procedure described below identifies $\eta^c\bar{\eta}$ and $\eta^d\bar{\eta}$, and thus, increases in $\bar{\eta}$ are completely offset by reductions in η^c and η^d and vice versa. To ensure that the parameter restrictions on η^c and η^d are satisfied, $\bar{\eta}$ is set to 10. Following much of the literature (e.g., Golosov et al. 2014; Nordhaus 2018), β is set to 0.985, implying a pure time discount rate of 1.5 pct. Consistent with the meta-analysis by Havránek (2015), θ is set to 1.5. The rate of return is derived from the consumer preferences: $r_{t+1} = \beta^{-1}(c_{t+1}/c_t)^{\theta} - 1$. UNESCO (2015) finds that almost 2 pct. of global GDP was allocated to R&D in 2013. Here it is assumed that 2 pct. of the workforce is allocated to R&D: $\omega = 0.02$.

Archer et al. (2009) find that 20-40 pct. of the CO_2 emitted stays in the atmosphere for centuries. The IPCC (2014) finds that the figure is 20 pct. The parameter $\bar{\upsilon}$ is therefore set to a very large number (the exact value is irrelevant here). Using the lower bound estimate from Archer et al. (2009) and IPCC (2014): $\xi_P = 0.2$. According to Archer (2005), the remaining part of the CO_2 has a mean lifetime of approximately 300 years. Following the calibration approach of Golosov et al. (2014), it is assumed that $(1 - \xi_D)^{300} = 0.5$ which implies that ξ_D equals about 0.0023. In addition, ξ_A is set to around 0.67 based on the estimation procedure described in Appendix F. Based on the conversion factors from Clark (1982, p. 467), ζ is set to 1/2.13. Finally, the climate damage function parameter is taken from Golosov et al. (2014) and adjusted to the present setting: $\sigma = 5.3 \cdot 10^{-5}/\zeta = 1.129 \cdot 10^{-4}$.

6.3 Estimation

The values of η^c, η^d, ϕ^c, ϕ^d, and ϵ are estimated through the following three-step procedure. In the first step, the parameters η^c and η^d are computed such that the model matches consumption and emission growth from 1890 to 1891.[24] Given some value of ϵ, this can be done without knowing ϕ^c and ϕ^d given the normalizations of the technological levels. In the second step, the values of η^c, η^d, and ϵ are given, and the values ϕ^c and ϕ^d are obtained by minimizing the sum of squared differences between the model generated average growth rates of CO_2 emissions and GDP from 1890 to 2015 and their actual values computed from the dataset. These estimation targets ensure that the model matches the decreasing pollution intensity trend discussed above. In the third step, the value of ϵ is determined by minimizing the difference between the actual and predicted CO_2 concentration in 2015. Essentially, the procedure consists of a minimization problem for ϵ. For each ϵ, the procedure finds parameter values consistent with that ϵ. In that sense, the minimization problem is one dimensional, although the procedure results in five parameter estimates.

The resulting parameter values are: $\eta^c \approx 0.230$, $\eta^d \approx 0.231$, $\phi^c \approx 0.489$, $\phi^d \approx 0.538$, and $\epsilon \approx 1.619$.[25,26] The model simulation for the period 1890-2015 essentially match the estimation targets: the average growth rates of simulated GDP and CO_2 emissions deviate from the actual growth rates by less than 0.03 percentage points, while the CO_2 concentration in 2015 is around 0.05 ppm above the observed level of around 400 ppm. The baseline simulation yields an average annual growth rate of GDP per worker over the present century of 1.6 pct., while GDP per capita grows at an annual rate of 1.5 pct. The last figure is well within one standard deviation of the median GDP per capita growth forecast for this century computed from an expert survey conducted by Christensen et al. (2018).

6.4 Simulation results

Laissez-faire

The left panel of Figure 2 shows projected CO_2 concentration paths for a laissez-faire economy under different population scenarios. The baseline scenario is based on the medium

[24]For simplicity, consumption and GDP growth is assumed equal in 1891.

[25]The estimates of ϕ^c and ϕ^d coincide well with recent micro-level evidence, indicating that spillovers from clean innovations are relatively larger than spillovers from dirty innovations (Dechezleprêtre et al. 2014).

[26]The estimate of ϵ is lower than the point estimate obtained by Papageorgiou et al. (2017). However, their estimate is probably upwards biased, as they assume technological neutrality between clean and dirty energy inputs, implying that all variation in their dataset must be explained by the elasticity of substitution.

variant of the United Nations population projections. In this scenario, the CO_2 concentration in 2100 is just above 630 ppm: far above the range (425-520 ppm) consistent with the Paris Agreement.[27] The figure also shows a substantial difference between the concentration levels in 2100 for the four population scenarios, highlighting the importance of population growth for long-run sustainability. Although the 2100 concentration level is substantially lower in the low and constant population scenarios, the concentration levels are still far above the range consistent with the Paris Agreement.

The concentration differences between the three UN population growth scenarios are quite small over the next 50 years. The reason is that in the short run, the evolution in the workforce is basically known, as the people entering and exiting the labor market are already born. Accordingly, the three scenarios have the same short-run workforce forecast, while the medium- and long-run forecasts are substantially different, as shown in the right panel of Figure 2.

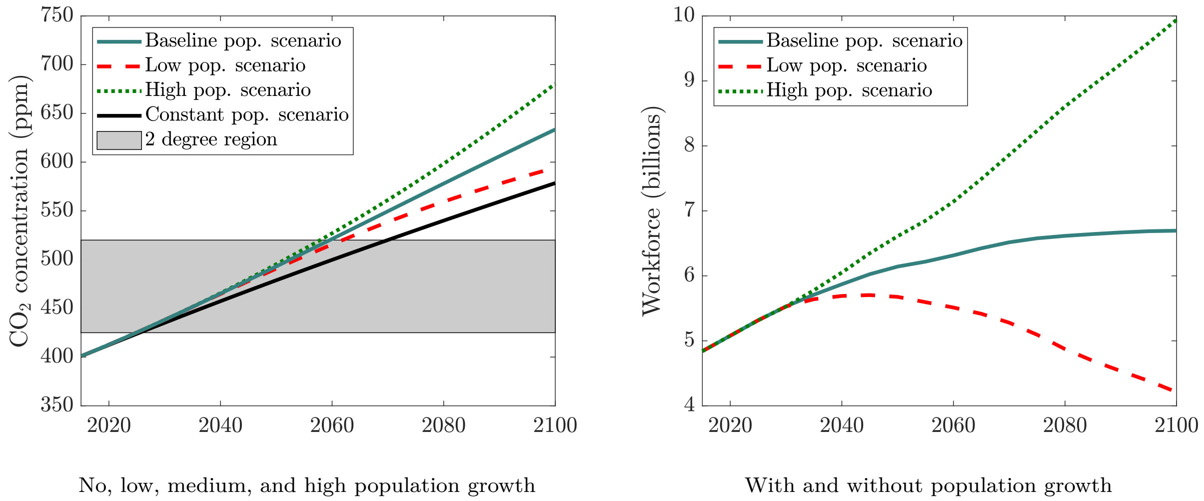

FIGURE 2: ***Left panel:*** Projected CO_2 concentration paths for a laissez-faire economy under different population scenarios, 2015-2100. ***Right panel:*** Evolution in the workforce under different population growth scenarios.
Notes: The low, medium/baseline, and high population growth scenarios are based on the low, medium, and high population projections from the United Nations (2017). The "Constant" scenario holds the population and workforce constant from 2015.

[27] The range is based on IPCC (2014, Table 6.3). The range is also affected by the evolutions in other greenhouse gases which are treated as exogenous. There is a 32 to 84 pct. probability of exceeding the two-degree temperature limit within this range. In the context of the present analysis, the range illustrates how difficult it is to ensure that the two-degree temperature limit remains unviolated with a reasonably high probability. The specific limit values should be interpreted with caution.

Environmental policy: a CO_2 tax

The baseline scenario indicates that strong environmental policies are needed to ensure that the temperature increase remains below 2 degrees Celsius within this century. The theoretical analysis highlights the effectiveness of a CO_2 tax in climate change mitigation. This subsection examines how a CO_2 tax is efficiently implemented to reach a given CO_2 concentration target in 2100.

For comparability with the analytical results, consider a tax policy following the tax rule (18). Following this rule, the (constrained) optimal environmental policy is computed by maximizing welfare with respect to the initial pollution penalty and the pollution penalty growth rate under the constraint of a specific CO_2 concentration in 2100. The implied CO_2 tax rate is derived from the pollution penalty and the price of dirty intermediate goods.

Figure 3 shows optimal CO_2 tax rate paths for the two CO_2 concentration targets 500 and 550 ppm under different population scenarios taking a total utilitarian perspective ($\kappa = 1$). A 500 ppm concentration ensures that the two-degree limit is met with a reasonable probability. In contrast, there is little or no chance of staying below a two-degree temperature increase for a 550 ppm concentration, whilst a three-degree temperature increase is unlikely.

As shown in the left panel of Figure 3, the tax rate for the 500 ppm target increases fast under the baseline population scenario, indicating that this target might be politically unfeasible at this stage. However, the tax rate increases much slower under low or no population growth. If the concentration target is set to 550 ppm, the tax rate paths are substantially lower. Again, lower population growth implies lower CO_2 tax rate paths.

Computing optimal tax policies over a range of concentration targets reveals that the optimal concentration target under baseline population growth is around 550 ppm. Hence the tax rate in 2015 for the 550 ppm target is presumably close to the social cost of carbon in 2015. Although the simulation implies a relatively low social cost of carbon - about 26 US dollars (2010 prices) per tonne of CO_2 in 2015 - it is well within the typical range of estimates from the literature as reported by Tol (2013, 2018).

The optimal tax policy suggested by the model is probably inconsistent with the Paris Agreement, although it ensures a temperature increase below 3 degrees Celsius with a high probability. Other studies in the literature also find that the temperature limit from the Paris Agreement is non-optimally low (e.g., Nordhaus 2018). Nevertheless, the optimal concentration levels implied by the present model should be interpreted with caution. If climate damages are more potent as suggested by Howard and Sterner (2017), the optimal concentration target is lower. In addition, there are large uncertainties associated with cost benefit analysis in climate change economics (Pindyck 2007). This might motivate a cautious policy approach, i.e. a tighter concentration limit (Weitzman 2012).

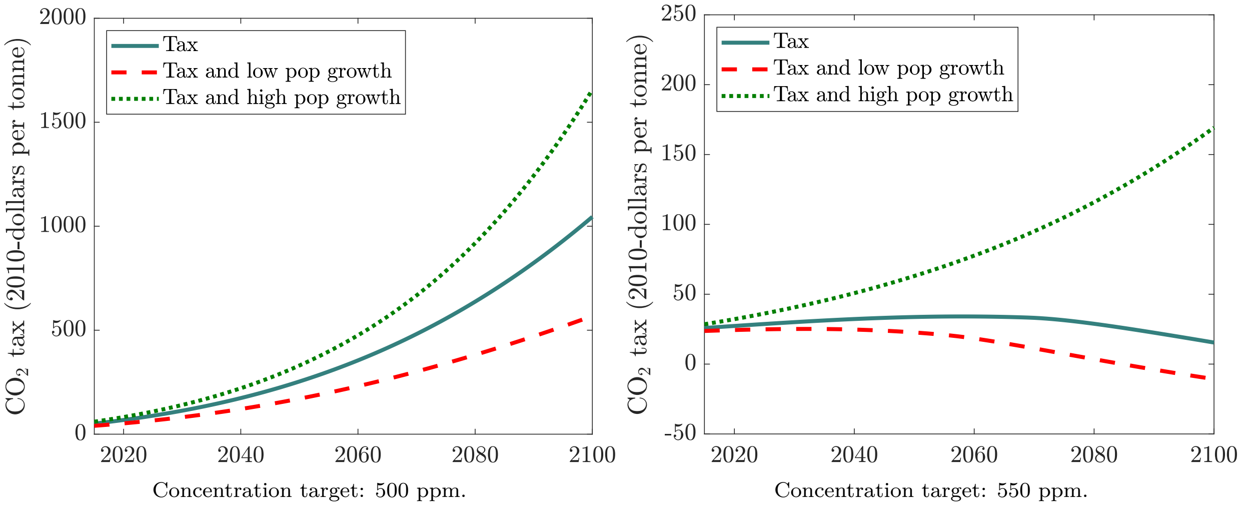

FIGURE 3: Optimal CO_2 tax rate paths under different population scenarios and CO_2 concentration targets seen from a total utilitarian perspective, 2015-2100.
Notes: The tax policies follow the simple tax rule (18). The policies are optimal in the sense that they maximize welfare, while achieving the CO_2 concentration target.

It is also worth noticing the kink on the optimal tax rate path under the baseline population growth scenario in the right panel of Figure 3. The kink occurs due to a switch in the allocation of scientists in the R&D sector from 2068 to 2069. From 2015 to 2068 all scientists are working in the clean R&D subsector due to the CO_2 tax imposed from 2015. However, the tax does not increase fast enough to ensure a permanent redirection of research efforts. As the dirty technologies become relatively less advanced, it becomes increasingly more attractive to develop them, which causes a switch from pure clean R&D to a mixed R&D effort from 2069.

More generally, there is a threshold level for the 2100 concentration target, where targets below the threshold result in a total and permanent redirection of research efforts toward the clean R&D subsector. As the R&D channel is fully utilized for targets below this threshold, further reductions can only come through a cleaner manufacturing process (the production input mix effect discussed above). As a consequence, it requires a much higher CO_2 tax for each additional ppm reduction in 2100.

Environmental policy: alternative instruments

CO_2 concentration paths for different policies are shown in Figure 4. A subsidy ensuring clean bias research results in a CO_2 concentration of 546 ppm in 2100. In the absence of population growth, the subsidy results in a CO_2 concentration of 522 ppm in 2100. Thus seen from a Paris Agreement perspective, a combined subsidy and population control policy might be a relevant alternative to a pure tax policy.

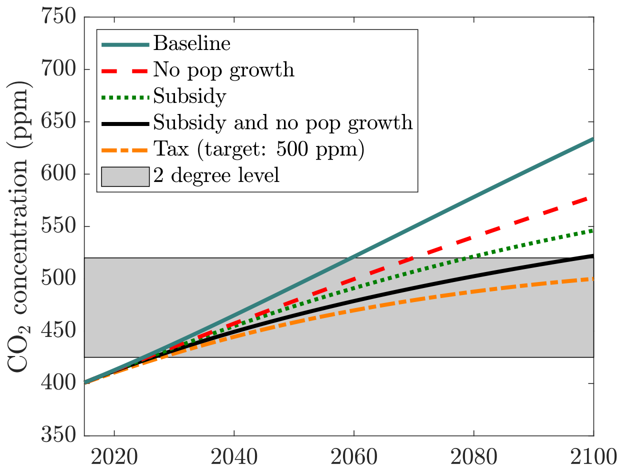

FIGURE 4: CO_2 concentration paths under different policies, 2015-2100.
Notes: All subsidy policies ensure that research has clean bias. The tax policy is optimal from a total utilitarian perspective given a 500 ppm concentration target in 2100.

Consumption losses

The left panel of Figure 5 shows per capita consumption losses associated with the optimal tax policies relative to the baseline scenario. The tax policy ensuring a CO_2 concentration of 500 ppm in 2100 reduces consumption through most of the 21st century, as this concentration target requires substantial regulation. Nonetheless, the simulations indicate that these consumption losses might be quite small: less than 0.5 pct. compared to baseline in all years. The net consumption loss is the result of reduced climate damages on the one hand and distortions from regulation on the other.

The tax policy ensuring a 550 ppm target in 2100 results in relatively small consumption losses at the beginning of the period and relatively larger consumption gains at later dates. The intuition is the same as before, but in this case, the consumption gains from a lower CO_2 concentration dominates through most of the period.

Given the significant uncertainty about the climate damage function, one might interpret the more ambitious 500 ppm target as an insurance policy again catastrophic climate risks (Weitzman 2012). The additional consumption losses associated with the 500 ppm target are then the associated insurance premium.

The right panel of Figure 5 compares consumption losses from the subsidy policy ensuring clean bias and an optimal (total utilitarian) tax policy ensuring the same 2100 concentration level. The tax policy is preferred, but both policies improve welfare. The tax policy leads to a lower consumption level in the short run and a higher consumption level in the long run. This is because the tax policy not only directs all research toward the clean R&D subsector in the short run; it also distorts the production sector through the production input mix effect. Yet, the tax policy does not direct all research toward the clean R&D subsector through the entire period. Hence the tax policy does not reduce the productivity of research as much as the subsidy policy, leading to smaller consumption losses in the long run.[28]

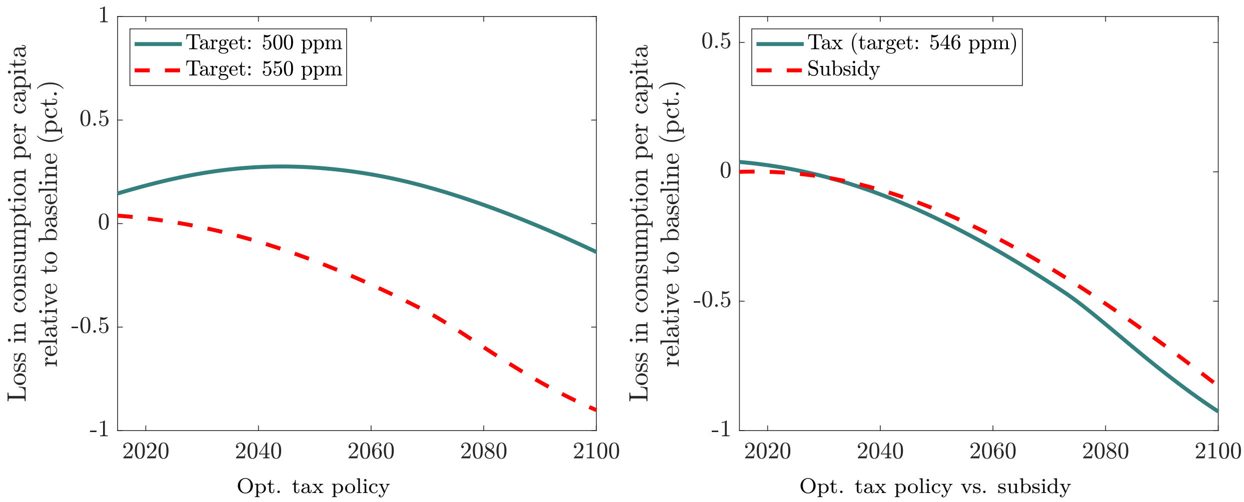

FIGURE 5: Consumption per capita losses compared to the baseline scenario, 2015-2100.

Total vs. average utilitarianism

The optimal tax rate paths shown in Figure 3 are only slightly changed if the optimal tax policy is derived from an average utilitarian welfare function. This is clear from Figure 6 in Appendix G. The average utilitarian will reduce (increase) the initial tax rate when moving from the baseline to the high (low) population growth scenario, whilst the opposite holds for the total utilitarian. This is because a higher population growth rate also results in a higher consumption per capita level in the future. The average utilitarian, therefore, increases initial average consumption by reducing the initial tightness of the environmental policy. In contrast, the total utilitarian will put more weight on future consumption, as there are also more people to benefit from future consumption under a higher population growth rate. Still, the tax rate paths are quite similar and so are the consumption loss patterns.

The simulations also show that neither an average nor a total utilitarian would prefer population reduction policies. The reason is that a higher population growth rate results

[28] As the CO_2 concentration targets in 2100 are the same, there is little difference in the emission levels over time. The climate damages are therefore virtually the same for both policies.

in a faster technological development and thus a higher consumption per capita level. The burden on the environment from a larger population is not large enough to offset this effect. Hence even an average utilitarian is willing to have more stringent environmental policies instead of reducing the population growth rate. Of course things might look very different if it is possible to reduce the population size without reducing the research input as discussed below.

7 Concluding remarks

The present study shows that population growth places a heavy burden on the environment even when accounting for positive effects of the population size on technological development. Although population reducing policies are not necessarily optimal, the present study shows that they might substantially reduce carbon accumulation in the atmosphere over this century. However, even if global population growth is reduced substantially, other instruments like a tax on CO_2 emissions are needed to ensure a CO_2 concentration level consistent with the Paris Agreement. In this regard, subsidies to environmentally friendly technologies do not seem sufficient unless they are combined with a stagnant population.

The primary limitation of the present study is the treatment of the world economy as a single entity. The main issue is that research intensities and future population growth rates are generally negatively correlated. Developed regions tend to have high R&D intensities and low population growth prospects, while the opposite holds for developing regions. Africa is the most striking example. The United Nations expects the global population to increase by 3.6 billion from 2017 to 2100. Of these 3.6 billion extra individuals, 3.2 are expected to be Africans (United Nations 2017). Meanwhile only a tiny fraction of the global R&D investments are made in Africa (UNESCO 2015). Reducing population growth in developing regions is therefore likely to have a small impact on frontier technological development. Hence such population policies might be more effective than predicted by the present analysis both in terms of consumption per capita losses and CO_2 emission reductions. On the other hand, CO_2 emissions per capita are relatively low in developing regions, reducing the climate impact of such policies. In addition, future climate change mitigation depends heavily on the ability of developing regions to adopt existing environmentally friendly production technologies. To investigate these issues further, the next natural step is to develop a two-region version of the model presented above.

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Appendix

A Pollution Intensity in Acemoglu et al. (2012)

In this appendix, it is shown that the pollution intensity increases over time under laissez-faire in the model developed by Acemoglu et al. (2012, p. 134-141). All references in this appendix are references to Acemoglu et al. (2012).

There is no savings in the model, and thus GDP equals aggregate consumption. From (8):

$$GDP_t = C_t = Y_t - \psi \left(\int_0^1 x_{it}^c \,\mathrm{d}i + \int_0^1 x_{ht}^d \,\mathrm{d}h \right),$$

where C_t is aggregate consumption, Y_t is aggregate output of final goods, x_{it}^c is clean machine i, and x_{ht}^d is dirty machine h. It takes ψ units of final goods to produce one unit of any machine.

The demand for machine $k \in \{i, h\}$ is given by (see page 160):

$$x_{kt}^j = \left(\frac{\alpha^2 p_t^j}{\psi} \right)^{\frac{1}{1-\alpha}} L_t^j A_{kt}^j \quad \Leftrightarrow \quad \left(\frac{x_{kt}^j}{A_{kt}^j} \right) = \left(\frac{\alpha^2 p_t^j}{\psi} \right)^{\frac{1}{1-\alpha}} L_t^j \equiv \hat{x}_t^j,$$

where $j \in \{c, d\}$, p_t^j is the price of intermediate j, L_t^j is labor input in sector j, and A_{kt}^j is the productivity associated with machine k.

The amount of final goods used to produce machines for a given sector is given by:

$$\psi \int_0^1 x_{kt}^j \,\mathrm{d}k = \psi \int_0^1 \left(\frac{x_{kt}^j}{A_{kt}^j} \right) A_{kt}^j \,\mathrm{d}k = \psi \hat{x}_t^j A_t^j, \quad A_t^j \equiv \int_0^1 A_{kt}^j \,\mathrm{d}k,$$

where A_t^j is the average productivity associated with the machines in sector j.

From the production function for intermediate good j, (5), it now follows that:

$$\begin{aligned} Y_t^j &= \left(L_t^j\right)^{1-\alpha} \int_0^1 \left(x_{kt}^j\right)^{\alpha} \left(A_{kt}^j\right)^{1-\alpha} \mathrm{d}k = \left(L_t^j\right)^{1-\alpha} \left(\hat{x}_t^j\right)^{\alpha} A_t^j \quad \Leftrightarrow \\ \hat{x}_t^j &= \left(Y_t^j\right)^{\frac{1}{\alpha}} \left(A_t^j\right)^{-\frac{1}{\alpha}} \left(L_t^j\right)^{\frac{\alpha-1}{\alpha}}, \end{aligned}$$

where Y_t^j is intermediate good j.

Using the above expressions for $\psi \int_0^1 x_{kt}^j \, \mathrm{d}k$ and $\hat{x}_t^j$:

$$\psi \int_0^1 x_{kt}^j \, \mathrm{d}k = \psi \left(Y_t^j\right)^{\frac{1}{\alpha}} \left(A_t^j\right)^{\frac{\alpha-1}{\alpha}} \left(L_t^j\right)^{\frac{\alpha-1}{\alpha}},$$

It follows from (7), (19), and (A.5), that in equilibrium:

$$\begin{aligned}
L_t^c &= \left(A_t^d\right)^{\varphi} \left[\left(A_t^c\right)^{\varphi} + \left(A_t^d\right)^{\varphi}\right]^{-1}, \quad L_t^d = \left(A_t^c\right)^{\varphi} \left[\left(A_t^c\right)^{\varphi} + \left(A_t^d\right)^{\varphi}\right]^{-1}, \\
Y_t^c &= \left(A_t^c\right)\left(A_t^d\right)^{\alpha+\varphi} \left[\left(A_t^c\right)^{\varphi} + \left(A_t^d\right)^{\varphi}\right]^{-\frac{\alpha+\varphi}{\varphi}}, \quad \text{and} \\
Y_t^d &= \left(A_t^c\right)^{\alpha+\varphi}\left(A_t^d\right) \left[\left(A_t^c\right)^{\varphi} + \left(A_t^d\right)^{\varphi}\right]^{-\frac{\alpha+\varphi}{\varphi}},
\end{aligned}$$

where $\varphi \equiv (1-\epsilon)(1-\alpha) < 0$.

It then follows that:

$$\begin{aligned}
\psi \int_0^1 x_{it}^c \, \mathrm{d}i &= \psi \left(A_t^c\right)\left(A_t^d\right)^{1+\varphi} \left[\left(A_t^c\right)^{\varphi} + \left(A_t^d\right)^{\varphi}\right]^{-\frac{1+\varphi}{\varphi}} \quad \text{and} \\
\psi \int_0^1 x_{ht}^d \, \mathrm{d}h &= \psi \left(A_t^c\right)^{1+\varphi}\left(A_t^d\right) \left[\left(A_t^c\right)^{\varphi} + \left(A_t^d\right)^{\varphi}\right]^{-\frac{1+\varphi}{\varphi}}.
\end{aligned}$$

Using the above and (19), it is straightforward to show that:

$$\psi \left(\int_0^1 x_{it}^c \, \mathrm{d}i + \int_0^1 x_{ht}^d \, \mathrm{d}h\right) = \psi \left(A_t^c\right)\left(A_t^d\right) \left[\left(A_t^c\right)^{\varphi} + \left(A_t^d\right)^{\varphi}\right]^{-\frac{1}{\varphi}} = \psi Y_t.$$

Thus, aggregate consumption/GDP amounts to:

$$C_t = (1-\psi) Y_t = (1-\psi)\left(A_t^c\right)\left(A_t^d\right) \left[\left(A_t^c\right)^{\varphi} + \left(A_t^d\right)^{\varphi}\right]^{-\frac{1}{\varphi}}.$$

As a result, the pollution intensity is given by:

$$\left(\frac{Y_t^d}{C_t}\right) = \left(\frac{1}{1-\psi}\right) \left[\left(A_t^c\right)^{\varphi} + \left(A_t^d\right)^{\varphi}\right]^{-\frac{\epsilon}{\epsilon-1}} \left(A_t^c\right)^{\alpha+\varphi-1}.$$

If research is conducted in the dirty sector only, then $A_t^d \to \infty$ for $t \to \infty$, while A_t^c remains constant over time. In this case:

$$\left(\frac{Y_t^d}{C_t}\right) \uparrow \left(\frac{1}{1-\psi}\right) \quad \text{for} \quad t \to \infty.$$

That is, the pollution intensity increases over time and converges to a positive constant which is inconsistent with empirical evidence. Note that research is directed toward the dirty sector from the outset due to Assumption 1. If research was directed toward the clean sector from the outset, the pollution intensity would decrease over time and approach zero which is compatible with the empirical evidence. The policy implications would, however, change substantially.

B Proofs

B.1 Lemma 4

Lemma 4. *Environmental sustainability is obtained if $\bar{E}$ is sufficiently large and Y_t^d decreases in the long run.*

Proof. The maximum value of E_t, denoted $E_t^{\max}$, given a sequence $\left\{Y_s^d\right\}_{s=t-\bar{v}}^{t}$ is:

$$E_t^{\max} = \mu \sum_{s=t-\bar{v}}^{t} Y_s^d \geq \mu \sum_{s=t-\bar{v}}^{t} Y_s^d \xi_{t-s} = E_t.$$

As $E_t^{\max}$ is given by a finite sequence, $E_t^{\max}$ is a finite number. If Y_t^d decreases over time, then $E_t^{\max}$ cannot continue to grow over time. In fact, at some point in time $E_t^{\max}$ must start to decrease as well. Accordingly, it is possible to choose a sufficiently large $\bar{E}$ such that $E_t^{\max} < \bar{E}$ for all t, implying that sustainability is obtained according to the definition given above. And since $E_t^{\max} \geq E_t$ then $E_t < \bar{E}$ for all t. □

B.2 Proof of Lemma 1

Proof. It can easily be verified that:

$$\frac{\partial F(s_t^c, \cdot)}{\partial s_t^c} \gtreqless 0 \quad \text{if} \quad (\epsilon - 1)\psi \gtreqless 1.$$

Consider the case $(\epsilon - 1)\psi < 1$. In this case $F(\cdot)$ is strictly monotonically decreasing in s_t^c. It follows that: $F(0, \cdot) > F(\omega \bar{L}_t, \cdot)$. If $F(0, \cdot) > F(\omega \bar{L}_t, \cdot) > 1$ then $s_t^c = \omega \bar{L}_t$, as the profit ratio is greater than one for any value of s_t^c. If $1 > F(0, \cdot) > F(\omega \bar{L}_t, \cdot)$ then $s_t^c = 0$, as the profit ratio is less than one for any value of s_t^c. If $F(0, \cdot) > 1 > F(\omega \bar{L}_t, \cdot)$ then $s_t^c = s_t^*$, where s_t^* is the unique solution to $F(s_t^*, \cdot) = 1$. □

B.3 Proof of Lemma 2

Proof. From Lemma 3 that if research is permanently directed towards the clean R&D subsector:

$$\frac{N_{t+1}^j}{N_t^j} \equiv (1 + g_{N^j,t}) \xrightarrow[t\to\infty]{} (1+n)^{\frac{1}{\phi^j}}.$$

Consider the growth factor of the profit ratio when research is permanently directed towards the clean R&D subsector:

$$\frac{F\left(\omega \bar{L}_{t+1}, \bar{L}_{t+1}, N_{t+1}^c, N_t^d\right)}{F\left(\omega \bar{L}_t, \bar{L}_t, N_t^c, N_t^d\right)} = \left(\frac{(1+g_{N^c,t+1})}{(1+g_{N^c,t})}\right)^{(\epsilon-1)\psi-1} \times \ (1+g_{N^c,t})^{(\epsilon-1)\psi-\phi^c}.$$

Given the long-run evolution in $(1+g_{N^c,t})$, it follows that if research is permanently directed toward the clean R&D subsector then:

$$\frac{F\left(\omega \bar{L}_{t+1}, \bar{L}_{t+1}, N_{t+1}^c, N_t^d\right)}{F\left(\omega \bar{L}_t, \bar{L}_t, N_t^c, N_t^d\right)} \xrightarrow[t\to\infty]{} (1+n)^{\frac{(\epsilon-1)\psi-\phi^c}{\phi^c}}.$$

When $(\epsilon-1)\psi < \phi^c$ the growth factor is less than one, and the profit ratio converges to zero. Thus, the profit ratio cannot stay above one permanently.

Now consider the growth factor of the profit ratio when research is permanently directed towards the dirty R&D subsector:

$$\frac{F\left(0, \bar{L}_{t+1}, N_t^c, N_{t+1}^d\right)}{F\left(0, \bar{L}_t, N_t^c, N_t^d\right)} = \left(\frac{\left(1+g_{N^d,t+1}\right)}{\left(1+g_{N^d,t}\right)}\right)^{1-(\epsilon-1)\psi} \times \ \left(1+g_{N^d,t}\right)^{\phi^d-(\epsilon-1)\psi}.$$

Given the long-run evolution in $(1+g_{N^d,t})$ (cf. Lemma 3), it follows that if research is permanently directed toward the dirty R&D subsector then:

$$\frac{F\left(0, \bar{L}_{t+1}, N_t^c, N_{t+1}^d\right)}{F\left(0, \bar{L}_t, N_t^c, N_t^d\right)} \xrightarrow[t\to\infty]{} (1+n)^{\frac{\phi^d-(\epsilon-1)\psi}{\phi^d}}.$$

When $(\epsilon-1)\psi < \phi^d$ the growth factor is above one, and the profit ratio increases over time and approaches infinity for time approaching infinity. Thus, the profit ratio cannot stay below one permanently.

As the profit ratio cannot stay permanently above or below one, the profit ratio must equal one or fluctuate around one in the long run. □

B.4 Proof of Lemma 3

Proof. The growth rate of N_t^j is given by:

$$\frac{N_{t+1}^j - N_t^j}{N_t^j} \equiv g_{N^j,t} = \eta^j \bar{\eta} s_t^j \left(N_t^j\right)^{-\phi^j}$$

If all researchers are permanently directed toward sector j, then $(s_{t+1}^j / s_t^j) = (1+n)$. Accordingly, the evolution in $g_{N^j,t}$ is described by the difference equation:

$$g_{N^j,t+1} = g_{N^j,t}(1 + g_{N^j,t})^{-\phi^j}(1+n) = f(g_{N^j,t}).$$

Consider the equilibrium:

$$g_j^* = (1+n)^{\frac{1}{\phi^j}} - 1 = f(g_j^*).$$

The difference equation is locally asymptotically stable in g_j^* if $|f'(g_j^*)| < 1$. It is easy to verify that

$$f'(g_j^*) = 1 - \phi^j \left[1 - (1+n)^{-\frac{1}{\phi^j}}\right].$$

As $0 < \phi < 1$ and $0 < (1+n)^{-\frac{1}{\phi^j}} < 1$ then $0 < f'(g_j^*) < 1$. Thus, the difference equation is locally asymptotically stable in g_j^*.

To prove global stability consider the expression:

$$f'(g_{N^j,t}) = (1+n)(1+g_{N^j,t})^{-\phi^j}\left[1 - \phi^j \frac{g_{N^j,t}}{1+g_{N^j,t}}\right].$$

Clearly, $f'(g_{N^j,t}) > 0$ for all $g_{N^j,t} \geq 0$ as:

$$1 > \phi^j \frac{g_{N^j,t}}{1+g_{N^j,t}}.$$

The inequality is true since $g_{N^j,t}/(1+g_{N^j,t}) \in [0,1)$ and $\phi \in (0,1)$.

It therefore holds that for $g_{N^j,t} \in (0, g_j^*)$:

$$\begin{aligned} g_{N^j,t+1} - g_j^* &= f(g_{N^j,t}) - f(g_j^*) \\ &= -\int_{g_{N^j,t}}^{g_j^*} f'(g)\,\mathrm{d}g < 0, \end{aligned}$$

where the last equality follows from the fundamental theorem of calculus, while the last inequality follows from the fact that $f'(g_{N^j,t}) > 0$ for all values of $g_{N^j,t} \geq 0$. Thus $g_{N^j,t+1}$ is always lower than g_j^* if $g_{N^j,t} \in (0, g_j^*)$. In the same way, it can be shown that $g_{N^j,t+1}$ is always above g_j^* if $g_{N^j,t} \in (g_j^*, \infty)$.

In addition, it can be shown that g_j^* is growing for $g_{N^j,t} \in (0, g_j^*)$, while g_j^* decreases for $g_{N^j,t} \in (g_j^*, \infty)$. Here only the first is shown. If $g_{N^j,t} \in (0, g_j^*)$, then

$$\frac{g_{N^j,t+1} - g_{N^j,t}}{g_{N^j,t}} = (1 + g_{N^j,t})^{-\phi^j}(1+n) - 1 > 0 \Rightarrow \underbrace{(1+n)^{\frac{1}{\phi^j}}}_{(1+g_j^*)} > (1 + g_{N^j,t}),$$

where the last inequality is true given that $g_{N^j,t} \in (0, g_j^*)$.

All in all, the equilibrium g_j^* is unique, and $g_{N^j,t}$ always approach g_j^* either from above (if $g_{N^j,t} \in (g_j^*, \infty)$) or from below (if $g_{N^j,t} \in (0, g_j^*)$). Thus $g_{N^j,t}$ is globally asymptotically stable, and:

$$(1 + g_{N^j,t}) \xrightarrow[t \to \infty]{} (1+n)^{\frac{1}{\phi^j}} \equiv (1 + g_j^*).$$

□

B.5 Proof of Proposition 1

Proof. Consider the ratio, Y_{t+1}^d / Y_t^d, when research has clean bias:

$$\frac{Y_{t+1}^d}{Y_t^d} = \Xi\Big(N_t^c, N_{t+1}^c, N^d\Big)(1 + g_{N^c,t})^{-(\epsilon-1)\psi}(1+n), \quad \text{where}$$
$$\Xi\Big(N_t^c, N_{t+1}^c, N^d\Big) \equiv \frac{\Big(N_t^c\Big)^{-(\epsilon-1)\psi} + \Big(N^d\Big)^{-(\epsilon-1)\psi}}{\Big(N_{t+1}^c\Big)^{-(\epsilon-1)\psi} + \Big(N^d\Big)^{-(\epsilon-1)\psi}}.$$

Clearly, $\Xi(N_t^c, N_{t+1}^c, N^d)$ approaches one when N_t^c and N_{t+1}^c become large. It follows from Lemma 3 that when research has clean bias: $(1 + g_{N^c,t}) \to (1+n)^{\frac{1}{\phi^c}}$. Thus,

$$\frac{Y_{t+1}^d}{Y_t^d} \xrightarrow[t \to \infty]{} (1+n)^{\frac{\phi^c - (\epsilon-1)\psi}{\phi^c}} \begin{cases} < 1 & \text{if} \quad (\epsilon-1)\psi > \phi^c \\ = 1 & \text{if} \quad (\epsilon-1)\psi = \phi^c \\ > 1 & \text{if} \quad (\epsilon-1)\psi < \phi^c \end{cases}.$$

□

B.6 Proof of Proposition 2

Proof. (i) It follows directly from Lemma 2 that without environmental policies, the profit ratio equals or fluctuates around one in the long run. As this property is independent of the technological level, a temporary subsidy cannot permanently direct research toward the clean R&D subsector.

(ii) If research has clean bias due to a permanent research subsidy, it follows from Proposition 1 that aggregate pollution emission approaches infinity for time approaching infinity. Hence for some $\bar{t} \in t$ it must hold that $E_{\bar{t}} > \bar{E}$. Accordingly, environmental sustainability is not obtained.

Under clean bias a variable V is denoted $\bar{V}$, while the same variable is denoted $\tilde{V}$ when research is not clean bias. The last case include a situation where research is temporarily directed toward the clean R&D subsector as well as a situation where research in clean R&D is just stimulated compared to the lassiez-faire equilibrium. If it can be proven that $\tilde{Y}_T^d > \bar{Y}_T^d$ for large values of T, then a temporary subsidy cannot ensure environmental sustainability.

If research is temporarily directed toward the clean R&D subsector by a temporary subsidy, it follows from Proposition 2 that research is conducted in both R&D subsectors in the long run. Hence if T is large, it must hold that: $\bar{N}_T^c > \tilde{N}_T^c$ and $\bar{N}_T^d < \tilde{N}_T^d$. These two inequalities are now used to show that:

$$\tilde{Y}_T^d > \bar{Y}_T^d \quad \Leftrightarrow \quad \left[1 + \left(\frac{\tilde{N}_T^c}{\tilde{N}_T^d}\right)^{(\epsilon-1)\psi}\right]^{-1} \left(\tilde{N}_T^d\right)^{\psi} > \left[1 + \left(\frac{\bar{N}_T^c}{\bar{N}_T^d}\right)^{(\epsilon-1)\psi}\right]^{-1} \left(\bar{N}_T^d\right)^{\psi}.$$

The statement is clearly true since

$$\left[1 + \left(\frac{\tilde{N}_T^c}{\tilde{N}_T^d}\right)^{(\epsilon-1)\psi}\right]^{-1} > \left[1 + \left(\frac{\bar{N}_T^c}{\bar{N}_T^d}\right)^{(\epsilon-1)\psi}\right]^{-1} \quad \Leftrightarrow \quad \frac{\bar{N}_T^c}{\tilde{N}_T^c} > \frac{\bar{N}_T^d}{\tilde{N}_T^d}.$$

Hence the pollution emission level is, in the long run, larger under a temporary research subsidy compared to a situation with clean bias. As pollution emission grows at a positive rate in the long run (cf. Proposition B.5), there exists a $\bar{t} \in t$ such that $E_{\bar{t}} > \bar{E}$. Accordingly, sustainability is not obtained. □

B.7 Proof of Proposition 3

Proof. When research has clean bias:

$$\frac{Y^d_{t+1}}{Y^d_t} = \Xi(\cdot)\left(1+g_{N^c,t}\right)^{-(\epsilon-1)\psi}(1+n), \quad \text{where}$$
$$\Xi(\cdot) \equiv \frac{\left(N^c_t\right)^{-(\epsilon-1)\psi}\tau^{-\epsilon}+\left(N^d\right)^{-(\epsilon-1)\psi}}{\left(N^c_{t+1}\right)^{-(\epsilon-1)\psi}\tau^{-\epsilon}+\left(N^d\right)^{-(\epsilon-1)\psi}}.$$

From the dynamics of N^c_t it follows that: $\Xi(\cdot) \to 1$ for $t \to \infty$. From Lemma 3, it then follows that

$$\frac{Y^d_{t+1}}{Y^d_t} \xrightarrow[t\to\infty]{} (1+n)^{\frac{\phi^c-(\epsilon-1)\psi}{\phi^c}} > 1.$$

As pollution emission grows at a positive rate in the long run, environmental sustainability is not obtained, as there exists a $\bar{t} \in t$ such that $E_{\bar{t}} > \bar{E}$. □

B.8 Proof of Proposition 4

Proof. The profit ratio is given by

$$F(s^c_t,\cdot) = (1+g_\tau)^\epsilon \tau^\epsilon_t \times \left(\frac{\eta^c}{\eta^d}\right) \times \left(\frac{1+\eta^c\bar{\eta}(N^c_t)^{-\phi^c}s^c_t}{1+\eta^d\bar{\eta}(N^d_t)^{-\phi^d}(\omega\bar{L}_t - s^c_t)}\right)^{(\epsilon-1)\psi-1} \times \frac{(N^c_t)^{(\epsilon-1)\psi-\phi^c}}{(N^d_t)^{(\epsilon-1)\psi-\phi^d}}.$$

If τ_0 is sufficiently large, research is directed toward the clean R&D subsector from the outset. In the long run, the profit ratio can only stay above one, if the pollution penalty grows sufficiently fast. Consider the long-run growth rate of the profit ratio under clean bias:

$$\frac{F(\omega L_{t+1},\cdot)}{F(\omega L_t,\cdot)} \xrightarrow[t\to\infty]{} (1+n)^{\frac{(\epsilon-1)\psi-\phi^c}{\phi^c}}(1+g_\tau)^\epsilon.$$

The profit ratio grows at a positive rate, in the long run, if and only if:

$$(1+g_\tau)^\epsilon(1+n)^{\frac{(\epsilon-1)\psi-\phi^c}{\phi^c}} > 1 \quad \Leftrightarrow \quad (1+g_\tau) > (1+n)^{\frac{\phi^c-(\epsilon-1)\psi}{\epsilon\phi^c}}.$$

Thus if the above condition is fulfilled, the pollution penalty ensures clean bias if it is sufficiently large initially.

When research has clean bias:

$$\frac{Y_{t+1}^d}{Y_t^d} = \Xi(\cdot)\,(1+g_\tau)^{-\epsilon}(1+g_{N^c,t})^{-(\epsilon-1)\psi}\,(1+n), \quad \text{where}$$
$$\Xi(\cdot) \equiv \frac{\left(N_t^c\right)^{-(\epsilon-1)\psi}\tau_t^{-\epsilon} + \left(N^d\right)^{-(\epsilon-1)\psi}}{\left(N_{t+1}^c\right)^{-(\epsilon-1)\psi}\tau_{t+1}^{-\epsilon} + \left(N^d\right)^{-(\epsilon-1)\psi}}.$$

From the dynamics of N_t^c and τ_t it follows that: $\Xi(\cdot) \to 1$ for $t \to \infty$. From Lemma 3, it then follows that

$$\frac{Y_{t+1}^d}{Y_t^d} \xrightarrow[t\to\infty]{} (1+n)^{\frac{\phi^c-(\epsilon-1)\psi}{\phi^c}}(1+g_\tau)^{-\epsilon}.$$

Environmental sustainability is ensured if $\bar{E}$ is sufficiently large and pollution emission decreases at a constant rate in the long run, cf. Lemma B.1. The latter condition is fulfilled when:

$$(1+n)^{\frac{\phi^c-(\epsilon-1)\psi}{\phi^c}}(1+g_\tau)^{-\epsilon} < 1 \quad \Leftrightarrow \quad (1+g_\tau) > (1+n)^{\frac{\phi^c-(\epsilon-1)\psi}{\epsilon\phi^c}}.$$

□

B.9 Proof of Proposition 5

Proof. Clean bias research can be obtained through a permanent research subsidy. As $0 < \phi^c < 1$, then

$$N_{t+1}^c - N_t^c = \eta^c\bar{\eta}\omega\bar{L}(N_t^c)^{1-\phi^c} < \eta^c\bar{\eta}\omega\bar{L}(N_{t+1}^c)^{1-\phi^c} = N_{t+2}^c - N_{t+1}^c \tag{19}$$

Thus, $(N_{t+1}^c - N_t^c)$ grows over time, i.e. the absolute per period increase in N_t^c becomes larger over time. Accordingly, $N_t^c \to \infty$ for $t \to \infty$.

Under clean bias:

$$Y_t^d = A\left[\left(N_t^c\right)^{-(\epsilon-1)\psi} + \left(N^d\right)^{-(\epsilon-1)\psi}\right]^{-1}\left(N_t^c\right)^{-(\epsilon-1)\psi}\left(N^d\right)^{\psi}(1-\omega)\bar{L}.$$

Clearly, $Y_t^d \to 0$ for $N_t^c \to \infty$, and thus environmental sustainability is obtained if $\bar{E}$ is sufficiently large, cf. Lemma B.1. □

C Stepping-on-toes Effects

Due to the non-rival nature of knowledge, doubling the research input might not double the output. Scientists working in different research labs might come up with the same ideas, implying decreasing returns to knowledge creation. This is often referred to as stepping-on-toes effects, and empirical evidence suggests that these effects are important.[29]

To model this feature, it is assumed that a scientist can start $N_t^j(1+s_t^j)^{-\chi}$ projects in R&D subsector $j \in \{c, d\}$ where $0 \leq \chi < 1$. The expected discounted profits for scientists conducting research in R&D subsector j amounts to:

$$\tilde{\pi}_t^j = \underbrace{\frac{1}{(1+r_{t+1})}}_{\text{Discount rate}} \times \underbrace{\bar{\eta}\left(N_t^j\right)(1+s_t^j)^{-\chi}}_{\text{Projects per scientist}} \times \underbrace{\eta^j\left(N_t^j\right)^{-\phi^j}}_{\text{Success probability per project}} \times \underbrace{\hat{\pi}_{t+1}^j}_{\text{Profit per one-period patent}}.$$

The resulting profit ratio is given by:

$$F(s_t^c, \bar{L}_t, N_t^c, N_t^d) = \left(\frac{\eta^c}{\eta^d}\right) \times \left(\frac{1+s_t^c}{1+\omega\bar{L}_t - s_t^c}\right)^{-\chi} \times \frac{(N_t^c)^{(\epsilon-1)\psi-\phi^c}}{(N_t^d)^{(\epsilon-1)\psi-\phi^d}} \times \left(\frac{1+\eta^c\bar{\eta}(N_t^c)^{-\phi^c}(1+s_t^c)^{-\chi}s_t^c}{1+\eta^d\bar{\eta}(N_t^d)^{-\phi^d}(1+\omega\bar{L}_t - s_t^c)^{-\chi}(\omega\bar{L}_t - s_t^c)}\right)^{(\epsilon-1)\psi-1}.$$

The long-run growth factor of N_t^j under j bias is given by:

$$(1+g_{N^j,t}) = 1+\eta^j\bar{\eta}\left(N_t^j\right)^{-\phi^j}(1+s_t^j)^{-\chi}s_t^j \xrightarrow[t\to\infty]{} (1+n)^{\frac{1-\chi}{\phi^j}}.$$

To avoid the lock-in equilibrium, Parameter Restriction 2 is substituted by:

Parameter Restriction 3. $(1-\chi)(\epsilon-1)\psi < \phi^c$ *and* $(1-\chi)(\epsilon-1)\psi < \phi^d$.

If research has clean bias:

$$\frac{Y_{t+1}^d}{Y_t^d} \xrightarrow[t\to\infty]{} (1+n)^{\frac{\phi^c-(1-\chi)(\epsilon-1)\psi}{\phi^c}} > 0.$$

Thus the neo-Malthusian effect dominates the Simon effect even when research has clean bias. From here, it is relatively easy to see that this model has the same qualitative policy implications, as the model presented in Section 3.

[29]Using manufacturing industry data, Venturini (2012) finds substantial stepping-on-toes effects.

D Knowledge Spillovers Between R&D Subsectors

The basic model features no knowledge spillovers between the two R&D subsectors. To assess the role of such spillovers, consider the following evolutions in the technological levels:

$$N_{t+1}^{j} = \left(1 + \eta^{j}\bar{\eta}\left(N_{t}^{f}\right)^{\gamma}\left(N_{t}^{j}\right)^{-\phi^{j}-\varphi} s_{t}^{j}\right) N_{t}^{j}, \quad 0 < \varphi < 1, \quad 0 < \gamma < 1, \quad j \in \{c, d\}, \tag{20}$$

where $0 < (1 - \varphi - \phi^{j} + \gamma) < 1$, N_t^f is the technological level of the other R&D subsector, and γ reflects the usefulness of subsector f knowledge for subsector j scientists. A scientist in R&D subsector j starts $\bar{\eta}(N_t^j)^{1-\varphi}(N_t^f)^{\gamma}$ new projects in period t. Each with the success probability $\eta^j(N_t^j)^{-\phi^j}$.

The intersubsector spillovers drag the profit ratio toward one, since technological advances in one R&D subsector increases the productivity of scientists in the other. This is clear from the equilibrium profit ratio:

$$F(\cdot) = \left(\frac{\eta^{c}}{\eta^{d}}\right) \times \left(\frac{1 + \eta^{c}\bar{\eta}(N_{t}^{d})^{\gamma}(N_{t}^{c})^{-\phi^{c}-\varphi} s_{t}^{c}}{1 + \eta^{d}\bar{\eta}(N_{t}^{c})^{\gamma}(N_{t}^{d})^{-\phi^{d}-\varphi}(\omega\bar{L}_{t} - s_{t}^{c})}\right)^{(\epsilon-1)\psi-1} \times \frac{(N_{t}^{c})^{(\epsilon-1)\psi-\phi^{c}-\varphi-\gamma}}{(N_{t}^{d})^{(\epsilon-1)\psi-\phi^{d}-\varphi-\gamma}}.$$

To avoid the lock-in equilibrium, Parameter Restriction 2 is substituted by:

Parameter Restriction 4. $(\epsilon - 1)\psi < \phi^{c} + \varphi + \gamma$ *and* $(\epsilon - 1)\psi < \phi^{d} + \varphi + \gamma$.

If research has clean bias:

$$\frac{Y_{t+1}^{d}}{Y_{t}^{d}} \xrightarrow[t\to\infty]{} (1+n)^{\frac{\phi^{c}+\varphi-(\epsilon-1)\psi}{\phi^{c}+\varphi}}.$$

Thus given that $\phi^{c} + \varphi < (\epsilon - 1)\psi < \phi^{c} + \varphi + \gamma$, the Simon effect dominates the neo-Malthusian effect in the long run under clean bias. The intuition is the following. The intersubsector spillovers reduce research productivity under clean bias, but they also increase the research productivity of the stagnant R&D subsector. Both effects reduce the incentive to permanently research in only one R&D subsector. The former effect is similar to the fishing-out effect, while the latter effect is substantially different, as it does not affect the research productivity when research has clean or dirty bias. Intersubsector spillovers thereby create a parameter space in which avoiding the lock-in equilibrium under laissez-faire is consistent with a dominating Simon effect under clean bias. Within this parameter space, permanent research subsidies can ensure environmental sustainability. The policy implications outside this parameter space are unchanged.

E Data

Population scenarios and the workforce

The global workforce is defined as all individuals, who are between 15 and 65 years old. The workforce is computed directly for the period 1950-2015 from the age distributed population size data from the United Nations (2017). Missing years are obtained using linear interpolation. The age distribution is quite stable for this period, and the workforce constitutes about 59.5 pct. of the total population in all reported years. The workforce prior to 1950 is computed by multiplying the population size data from the History Database of the Global Environment (HYDE)(Goldewijk et al. 2010, 2011) by this ratio. Missing years are obtained using linear interpolation.

The projected workforce after 2015 is computed from the age distributed population projections from the United Nations (2017). Four scenarios are considered. In the first three scenarios, the workforce is computed from the high, medium, and low population projections by the United Nations (2017). The last scenario illustrates the hypothetical effect of no population growth, where the workforce is held constant from 2015.

CO_2 concentration data

The CO_2 concentration data cover the period 1744-2015, and the concentration level is measured in parts per million (ppm). It is constructed from two sources. Concentration data for the period 1744-1953 are taken from Neftel et al. (1994), and concentration data for the period 1959-2015 are taken from Mauna Loa, Hawaii Observatory (2016). Due to missing observations, the time series is interpolated using linear interpolation.

CO_2 emission data

The time series for global CO_2 emissions is based on Boden et al. (2017). Total (anthropogenic) CO_2 emission is defined as CO_2 emissions from fossil fuel use and industrial processes plus land-use change emissions. The time series provided by Boden et al. (2017) go back to 1751. The present study extrapolates it to 1750, where the 1750 value is set equal to the 1751 value. This seems like a reasonable assumption given that the emission levels are the same in all the years over the period 1751-1770. There is no data on emissions from land-use changes before 1850, and emissions from this source is set to zero before this date. This last point is only relevant for the calibration of the carbon cycle discussed in Appendix F.

GDP data

Data on global GDP is obtained from two sources. GDP figures for the period 1960-2015 are downloaded from the World Bank, World Development Indicators. These figures are reported in 2010 US-dollars. Figures for the period 1890 to 1960 are obtained from the History Database of the Global Environment (HYDE)(Goldewijk et al. 2010, 2011). These GDP figures are reported in 1995 US-dollars. To construct one consistent time series, the HYDE figures are used until 1960 and extended using GDP growth rates implied by the World Bank GDP figures.

F Carbon Cycle

This appendix describes how ξ_A is computed based on historical carbon emissions and concentration levels. The idea is to ensure that the carbon cycle implies the observed 2015 CO_2 concentration given the historical emission path.

The law of motion for E_t can be expressed as:

$$E_t = \xi_P \sum_{s=t-\bar{v}}^{t} \mu Y_s^d + (1-\xi_P)(1-\xi_A) \sum_{s=t-\bar{v}}^{t} (1-\xi_D)^{t-s} \mu Y_s^d.$$

Define the two stock variables:

$$E_{1,t} \equiv \xi_P \sum_{s=t-\bar{v}}^{t} \mu Y_s^d \quad \text{and} \quad E_{2,t} \equiv (1-\xi_P)(1-\xi_A) \sum_{s=t-\bar{v}}^{t} (1-\xi_D)^{t-s} \mu Y_s^d.$$

Assuming that $\bar{v}$ is a very large number and that emissions far back in time are zero, then over some time interval E_t is given by:

$$\begin{aligned}
E_{t+1} &= E_{1,t+1} + E_{2,t+1} \\
E_{1,t+1} &= E_{1,t} + \xi_P \mu Y_{t+1}^d \\
E_{2,t+1} &= (1-\xi_D) E_{2,t} + (1-\xi_P)(1-\xi_A) \mu Y_{t+1}^d.
\end{aligned}$$

The pre-industrial values of these three stock variables are zero by definition. Thus, it is straightforward to compute the evolutions in these three stock variables from 1750 to 2015 for some parameter values given the CO_2 emission data described in Appendix E.

As stated in the main text, $\bar{v}$ is set to some very large number such that the exact number becomes irrelevant and emissions far back in time are assumed equal to zero. Hence $E_t = E_{1,t} + E_{2,t}$ for the time interval relevant for the simulation exercise. Meanwhile the values of ξ_P and ξ_D are set to 0.2 and around 0.0023, respectively, based on insights from climate science.

To obtain a data consistent estimate of ξ_A, the squared difference between the observed and predicted CO_2 concentration in 2015 is minimized with respect to ξ_A given the above parameter values and observed carbon emission levels for the period 1750-2015. The resulting parameter value is around 0.6746 which is slightly above the equivalent value (0.607) used by Golosov et al. (2014). The difference between the observed and the predicted concentration is essentially zero in 2015.

The 1889 values of $E_{1,t}$ and $E_{2,t}$ used in the simulation follow directly from the calibration procedure described in this appendix. The resulting 1889 values of $E_{1,t}$ and $E_{2,t}$ are around 7.8 and 9.7 GT carbon, respectively.

G Other Simulation Results

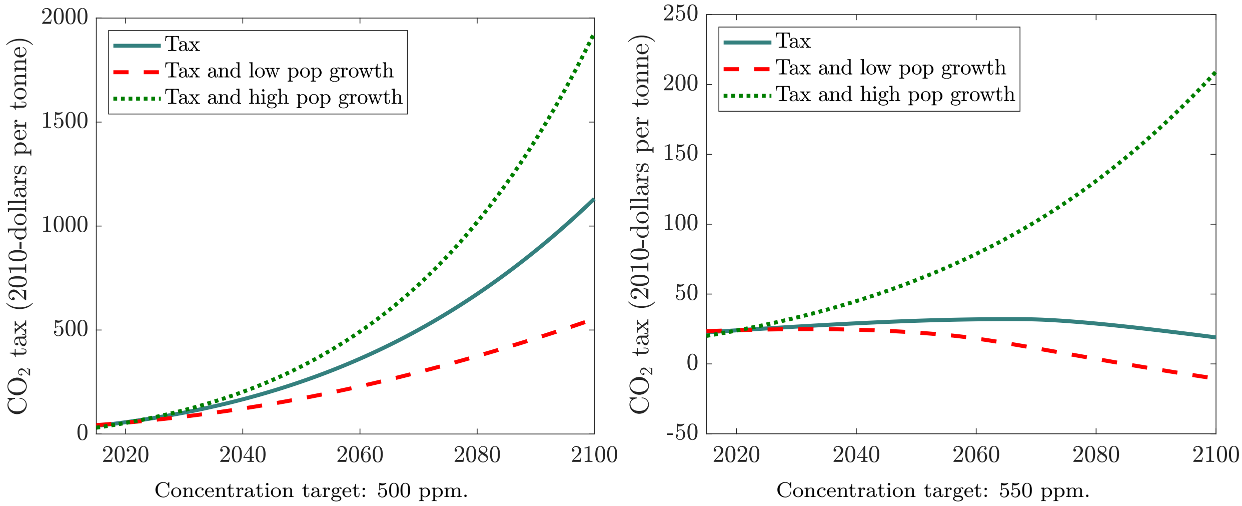

FIGURE 6: Optimal CO_2 tax rate paths under different population scenarios and CO_2 concentration targets seen from an average utilitarian perspective, 2015-2100.
Notes: The tax policies follow the simple tax rule (18). The policies are optimal in the sense that they maximize welfare, while achieving the CO_2 concentration target.