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A CVAR scenario for a standard monetary model using theory-consistent expectations*

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Abstract

A theory-consistent CVAR scenario describes a set of testable regularities capturing basic assumptions of the theoretical model. Using this concept, the paper considers a standard model for exchange rate determination and shows that all assumptions about the model's shock structure and steady-state behavior can be formulated as testable hypotheses on common stochastic trends and cointegration. While the scenario was rejected on essentially all counts, the results were informative about the cause of the empirical failure. It was the stationarity assumptions that were too restrictive to explain the long persistent swings in the real exchange rate and the interest rate differential.

Keywords: Theory-Consistent CVAR, Expectations, International Puzzles, Long Swings, Persistence, Imperfect Knowledge.

JEL Classification: F31, F41, G15, G17

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1 Introduction

Rational expectations are typically applied to ensure theoretical consistency of an economic model. This paper argues that empirical consistency is equally important. The latter entails linking basic assumptions underlying the theoretical model with the empirical regularities of a well-specified statistical model, requiring as a minimum that the model can adequately account for the long-run properties of data.

Most standard monetary models assume "rational expectations" and have often been taken to the data using calibration and Bayesian priors, restricting attention to a few specific features of the theoretical model which are then tested. But, as forcefully argued by among others Hendry and Mizon (2000) and Spanos (2009), such tests can make sense only if the assumed structure of the economic model is correct. The econometric procedures are valid *only* to the extent that the probabilistic assumptions of the underlying statistical model are satisfied vis-a-vis the data in question. But, when testing rational expectations based models in the context of a statistically fully specified model, they have often been rejected. See for example the articles in the special issue "Using Econometrics for Assessing Economic Models" (Juselius, 2009). Juselius and Franchi (2007) shows that essentially all conclusions of a real business cycle model in Ireland (2004) change when the hypotheses are tested in the context of a fully specified statistical model.

Hence, a convincing test of the hypotheses underlying a theoretical model needs to be carried out in the context of a statistical model that is an adequate description of the data generating process. A well-specified Cointegrated Vector AutoRegression (CVAR) model is a broad description of the data generating process and, therefore, is an obvious candidate for such a model (Juselius, 2006, 2015 and Hoover et al., 2008). Because the statistical model and the theoretical model represent two different entities, a bridging principle is needed. The paper argues here that a theory-consistent CVAR scenario (Juselius, 2006 and 2015, Juselius and Franchi, 2007, Møller, 2008) offers such a principle. One may say that a scenario describes a set of empirical regularities one would expect to see in the data if the theoretical model is empirically valid. A theoretical model that passes the first check of such basic properties is potentially an empirically relevant model. Hoover and Juselius (2014) argue that a theory-consistent CVAR scenario can be thought of as a designed experiment for data obtained by passive observations in the sense of Haavelmo (1944).

One major problem when associating a theoretical model with a CVAR model is that forward expectations often represent a defining feature of the economic model but expectations are generally not observed. The paper proposes a simple procedure based on which basic hypotheses about expectations formation can be translated into testable hypotheses on a CVAR model. As an illustration, the paper derives a CVAR scenario for a standard monetary model for exchange rate determination using theory-consistent expectations and tests all main hypotheses based on data on interest rates, prices, and the nominal exchange rate for Germany and USA in the post-Bretton Woods - pre-EMU period. The results show that the standard monetary model can be empirically rejected on essentially all counts. This is specifically because the model assumptions are too restrictive to explain the long and persistent swings typical of the foreign currency market.

The paper is organized as follows. Section 2 discusses principles underlying a theory-consistent CVAR scenario, Section 3 introduces a standard monetary model for exchange rate determination, Section 4 proposes a rule for associating expectations with observables and Section 5 formulates a theory-consistent CVAR scenario. Section 6 introduces the empirical CVAR model, finds that the vector process is $I(2)$, tests hypotheses on the order of integration of individual variables/relations and finds that the results are generally violating basic assumptions underlying the theoretical model. Section 7 concludes.

2 On the formulation of a theory-consistent CVAR scenario[1]

The basic idea is to derive theoretically consistent persistency properties of variables and relations and compare these with observed magnitudes measured by the order of integration, such as $I(0)$ for a highly stationary process, $I(1)$ or near $I(1)$ for a first order highly persistent process, and $I(2)$ or near $I(2)$ for a second order highly persistent process.[2] One may argue that it is implausible that economic variables move away from their equilibrium values for infinite times and, hence, that most economic relations should be classi-

[1] This section is similar to Juselius (2017).

[2] A highly persistent process is one for which a characteristic root is either close to or on the unit circle. See for example Elliot (1198) and Franchi and Johansen (2017).

fied as either stationary, *near* $I(1)$ or *near* $I(2)$. But this does not exclude the possibility that over finite samples they exhibit a persistence that is indistinguishable from a unit root or a double unit root process. In this sense the classification of variables into single or double unit roots should be seen as a useful way of classifying the data into more homogeneous groups. For a more detailed analysis, see Juselius (2012).

Unobservable expectations are often a crucial part of a theoretical model, whereas the empirical regularities to be uncovered by a CVAR analysis are based on the observed data. Therefore, we need a rule for how to associate the persistency property of expectations with the one of the observed variable. We assume here that agents form expectations which are broadly consistent with the underlying theory in the sense that they know the theory-consistent order of integration of the forecast variable, for example $x_t \sim I(1)$ or $x_t \sim I(2)$. While this is a less restrictive assumption compared to model-based rational expectations, economic agents are nonetheless assumed to be rational by not making systematic forecast errors. We illustrate the procedure for $x_t \sim I(1)$ and $x_t \sim I(2)$.

1. $x_t \sim I(1)$ can for example be the nominal exchange rate. For simplicity we assume that $x_t = x_{t-1} + \varepsilon_t$. A consistent forecasting rule is $x^e_{t+1|t} = x_t$, where $x^e_{t+1|t}$ is the expected value of the variable x at time t for $t+1$. In this case the *forecast shock* $v_t = x^e_{t+1|t} - x_t$ would be zero, but for a more general autoregressive model it would be stationary. The *forecast error* is $x_{t+1} - x^e_{t+1|t} = \varepsilon_{t+1}$ where ε_t is white noise, i.e. non-systematic. Inserting $x^e_{t+1|t} = x_t$ gives $x_{t+1} - x_t = \varepsilon_{t+1}$. Thus, non-systematic forecast errors do not change the underlying process.

2. $x_t \sim I(2)$ can for example be a Consumer Price Index (CPI) price. For simplicity we assume that $x_t = x_{t-1} + \Delta x_{t-1} + \varepsilon_t$. A consistent forecasting rule is $x^e_{t+1|t} = x_t + \Delta x_t$. Hence, the difference between the observed value and the forecast, $v_t = x^e_{t+1|t} - x_t = \Delta x_t$, is an $I(1)$ process. The forecast error is again assumed to be a non-systematic white noise process, i.e. $x_{t+1} - x^e_{t+1|t} = \varepsilon_{t+1},$. Inserting $x^e_{t+1|t} = x_t + \Delta x_{t-1} + \varepsilon_t$ in the above gives $\Delta^2 x_{t+1} = \varepsilon_{t+1}$. Thus, the process remains unchanged also in this case.

Assumption A exploits this simple idea:

Assumption A When $x_t \sim I(1)$, $(x^e_{t+1|t} - x_t) = v_t$ is $I(0)$ (or even zero in the random walk model). When $x_t \sim I(2)$ it is $I(1)$.

Note that Assumption A disregards $x_t \sim I(3)$, as it is considered empirically implausible, and $x_t \sim I(0)$, as it defines a non-persistent process for which cointegration and stochastic trends have no informational value.

Note also that $x_t \sim I(1)$ implies that $\Delta x_t \sim I(0)$, whereas $x_t \sim I(2)$ implies that $\Delta x_t \sim I(1)$ and $\Delta^2 x_t \sim I(0)$. Given Assumption A, we have that:

Corollary When $x_t \sim I(1)$, x_t, x_{t+1} and $x^e_{t+1|t}$ share the same common stochastic trend of order $I(1)$, i.e. they have the same persistency property. When $x_t \sim I(2)$, Δx_t, Δx_{t+1} and $\Delta x^e_{t+1|t}$ share the same common stochastic $I(1)$ trend, i.e. they have the same persistency property.

Consequently, when $x_t \sim I(1)$, $\beta' x_t$ has the same persistency property as $\beta' x^e_{t+1|t}$ or $\beta' x_{t+1}$. When $x_t \sim I(2)$, $\beta' x_t + d' \Delta x_t$, has the same order of integration as $\beta' x_t + d' \Delta x^e_{t+1|t}$ and $\tau' \Delta x_t$ has the same order of integration as $\tau' \Delta x^e_{t+1|t}$ and $\tau' \Delta x_{t+1}$.[3] Hence, Assumption A allows us to make valid inference about a long-run equilibrium relation in a theoretical model even though the postulated behavior is a function of expected rather than observed outcomes.

Based on the above, the steps behind a theory-consistent CVAR scenario can be formulated as follows:

1. Express the expectations variable(s) as a function of observed variables. For example, according to Uncovered Interest Rate Parity (UIP), the expected change in the nominal exchange rate is equal to the interest rate differential. Hence, the persistency property of the latter is also a measure of the persistency property of the unobservable expected change in nominal exchange rate and can, therefore, be empirically tested.

2. Translate the postulated behavioral relations of a theoretical model into a set of hypothetical conditions on their persistency properties. The next section illustrates that a standard monetary models is consistent with the purchasing power parity and the uncovered interest rate parity holding as stationary (or at most as a near $I(1)$) conditions.

[3] Section 6 provides a definition of β, d and τ.

3. For a given order of integration of the unobserved expectations variable and of the forecasting shocks derive the theory-consistent order of integration for all remaining variables.

4. Translate the stochastically formulated theoretical model into a theory-consistent CVAR scenario by formulating the basic assumptions underlying the theoretical model as a set of testable hypotheses on cointegration relations and common trends.

5. Estimate a well-specified VAR model and check the empirical adequacy of the derived theory-consistent CVAR scenario.

The following notation will be used to discriminate between different types of shocks: $\varepsilon_t \sim Niid(0, \sigma_\varepsilon^2)$ is a white noise process; $v_t = x_{t+1|t}^e - x_t$; and $u_t = f(\varepsilon_t)$ is an unobserved 'structural' shock assumed to be a linear function of the shocks to the system.

3 A standard monetary model for the real exchange rate

In the class of standard monetary models often based on rational expectations, the overshooting model by Dornbush (1976) and Dornbush and Frankel (1988) is standard in international macro. It attempts to address the long swings in the real exchange rate by assuming price rigidities that cause the nominal exchange to overshoot its equilibrium value. Another feature that characterize this type of models is the assumption that the rate of equilibrium adjustment to the PPP is identical for relative prices and nominal exchange rates (Frydman et al., 2008).[4] The rational bubble version of the monetary model (Blanchard and Watson, 1982), assumes that the nominal exchange rate is overshooting because at some point agents' forecasting behavior happens to become unrelated with fundamentals. This drives the nominal exchange rate away from fundamental values in an explosive way until the market realizes its mistake, the bubble bursts, and the nominal exchange rapidly returns to its fundamental value.

[4] Cheung, Lai and Bergman (2004) find that this feature is not supported by empirical evidence.

While these models differ in various aspects, they share the assumptions that long-run equilibrium in the goods market is characterized by Purchasing Power Parity, that Uncovered Interest Parity (UIP) is a market clearing mechanism, and that the international Fisher parity holds as a stationary condition. These basic features will be exploited when formulating a theory-consistent CVAR scenario for this class of models.

PPP states that $S_t = P_{d,t}/P_{f,t}$, implying that the nominal exchange rate, S_t, should reflect relative prices, $P_{d,t}/P_{f,t}$. The log of real exchange rate is defined as:

$$q_t = s_t - p_{d,t} + p_{f,t} \tag{1}$$

where lower cases stand for logarithmic values and a subscript d stands for a domestic and f for a foreign economy. In equilibrium, the real exchange rate, q, is defined by relative prices being equal to the nominal exchange rate, i.e. $q_{ppp} = 0$. When prices are measured by a price index, the equilibrium value, q_{ppp}, is undefined and the observed average real exchange rate can be different from zero. The real exchange rate is assumed to deviate from its long-run equilibrium value by an equilibrium error $(q_t - \bar{q})$, which in the Dornbush/Frankel type of models is assumed to be an AR(1) process:

$$\Delta q_t = -\alpha(q_{t-1} - \bar{q}) + \varepsilon_{q,t} \tag{2}$$

where $\bar{q}$ is the sample average, $0 < \alpha < 1$ measures the speed of adjustment and $\varepsilon_{q,t}$ is white noise. Even though q_t in (2) describes a stationary process some versions of the monetary model allow α to be very close to zero and, hence, the real exchange rate to be a near $I(1)$ process. For simplicity, the focus here is on the stationary case.

The Uncovered Interest Rate Parity (UIP) is defined as:

$$i_{d,t} - i_{f,t} = (s^e_{t+1|t} - s_t) \tag{3}$$

where i stands for a nominal interest rate and a superscript e denotes an expected value.

The Fisher Parity states that the nominal interest rate is equal to the expected inflation rate plus an independent real interest rate. The latter is assumed to reflect the ratio of average profit per capital in the economy, something which is difficult to measure on an aggregate level. In practise, it has often been approximated with the real GDP growth rate which is usually assumed to be stationary with a non-zero mean. Accordingly, the real interest rate is considered stationary with a constant mean.

The Fisher parity is defined as:

$$r_{j,t} = i_{j,t} - \Delta p^e_{j,t+1} +, \qquad j = d, f \tag{4}$$

where $r_{j,t}$ is an (unobserved) real interest rate and $p^e_{j,t+1}$ is a shortcut for $p^e_{j,t+1|t}$.

Finally, (2) and (3) together with Assumption A corresponds to the International Fisher Parity:

$$(i_{d,t} - i_{f,t}) = (s^e_{t+1|t} - s_t) = (\Delta p^e_{d,t+1} - \Delta p^e_{f,t+1}) \tag{5}$$

implying equality between the real interest rates in equilibrium.

4 Anchoring expectations to observables

The purpose of this section is to derive theory-consistent time-series properties for the relevant variables and relations in the monetary model using Assumption A to handle unobserved expectations.

The UIP condition states that

$$s^e_{t+1|t} - s_t = i_{d,t} - i_{f,t}, \tag{6}$$

implying that the interest rate differential is a measure of the expected change in the nominal exchange rate. Thus, in accordance with step 1 above, we can use the observed persistency properties of the nominal interest rates as a measure of the persistency of the expected change in the nominal exchange rate.

Interest rates are assumed unpredictable and can, therefore, be described by:

$$i_{j,t} = i_{j,t-1} + \varepsilon_{j,t}, \qquad j = d, f \qquad t = 1, ..., T \tag{7}$$

where $\varepsilon_{j,t}$ is a stationary error term. Integrating (7) over the sample period gives:

$$i_{j,t} = i_{j,0} + \sum_{i=1}^{t} \varepsilon_{j,i}, \qquad j = d, f \qquad t = 1, ..., T \tag{8}$$

where the cumulation of the interest rate shocks measures a stochastic trend in the interest rate.

Under Assumption A $(s^e_{t+1|t} - s_t) = v_{s,t}$ is stationary (provided $s_t \sim I(1)$). Thus, for UIP to hold as a market clearing mechanism $(i_{d,t} - i_{f,t})$ must be stationary. This implies that the stochastic trend in the interest rates must be identical, so that $\sum_{i=1}^{t}(\varepsilon^p_{d,i} - \varepsilon^p_{f,i}) = 0$. The interest rate differential can then be expressed as:

$$i_{d,t} - i_{f,t} = i_{d,0} - i_{f,0}. \tag{9}$$

The Fisher parity (4), $r_{j,t} = i_{j,t} - \Delta p^e_{j,t+1}$,can equivalently be expressed as

$$\Delta p^e_{j,t+1} = i_{j,t} - r_{j,t}, \qquad j = d, f. \tag{10}$$

As mentioned above, the real interest rate is assumed to be stationary with a constant mean, $r_{j,t} = r_j + \varepsilon_{rj,t}$. Under Assumption A, $\Delta p^e_{t+1} - \Delta p_t = \nu_{p_t}$ where $\nu_{p_t} \sim I(0)$ so the inflation rate can be expressed as:

$$\Delta p_{j,t} = i_{j,t} - r_{j,t} - \nu_{p,j,t}, \qquad j = d, f. \tag{11}$$

Inserting (8) in (11) gives an expression for the stochastic properties of the inflation rates:

$$\Delta p_{j,t} = i_{j,0} - r_{j,t} + \sum_{s=1}^{t} \varepsilon_{j,s} - \nu_{p,j,t}, \quad j = d, f \tag{12}$$

Hence, inflation rate is $I(1)$ with the same stochastic trend as the interest rate.

An expression for the price level is obtained by integrating $\Delta p_{j,t}$ over t:

$$p_{j,t} = (i_{j,0} - r_j)t + \sum_{s=1}^{t}\sum_{i=1}^{s} \varepsilon_{j,i} + \sum_{i=1}^{t} \varepsilon_{rj,i} - \sum_{s=i}^{t} \nu_{p_{j,i}} + p_{j,0}, \qquad j = d, f \tag{13}$$

Thus, the price level contains a linear time trend, a second order stochastic trend originating from twice cumulated shocks to the interest rate and first order stochastic trends originating from cumulated inflation forecast shocks. The linear trend in prices derives from the initial value of the nominal interest rate corrected for the mean value of the real interest rate, so the slope of the linear trend is approximately equal to the initial value of the inflation rate.

The international Fisher parity can be found using (12):

$$(\Delta p_{d,t} - \Delta p_{f,t}) = (i_{d,t} - i_{f,t}) - (r_d - r_f) - (\nu_{p_{d,t}} - \nu_{p_{f,t}}) \quad (14)$$
$$= (i_{d,0} - i_{f,0}) - (\nu_{p_{d,t}} - \nu_{p_{f_v}}). \quad (15)$$

Since international parity conditions imply equalization of the mean of real interest rates, $(r_d - r_f) = 0$ in (15). All components on the r.h.s. are stationary, hence the inflation differential is stationary. $(\Delta p_{d,t} - \Delta p_{f,t}) \sim I(0)$ implies $(p_{d,t} - p_{f,t}) \sim I(1)$, hence prices are cointegrated (1, -1) from $I(2)$ to $I(1)$ and, therefore, satisfy long-run price homogeneity. Integrating (14) over t gives an expression for relative prices

$$p_{d,t} - p_{f,t} = (i_{d,0} - i_{f,0})t - \sum_{i=1}^{t} (\nu_{p_{d,i}} - \nu_{p_{f,i}}) + p_{d,0} - p_{f,0}. \quad (16)$$

Thus, the relative prices contain a linear time trend due to the initial value of relative interest rates and a stochastic $I(1)$ trends originating from cumulated inflation forecast shocks.

An expression for the nominal exchange rate can be obtained from the uncovered interest rate parity in (6) using Assumption A to replace $\Delta s^e_{t+1|t}$ with $\Delta s_t + v_{s,t}$:

$$\Delta s_t = i_{d,t} - i_{f,t} - v_{s,t}. \quad (17)$$

Inserting (9) into (17):

$$\Delta s_t = (i_{d,0} - i_{f,0}) - v_{s,t}$$

Integrating (17) over t gives an expression for the level of nominal exchange rate:

$$s_t = (i_{d,0} - i_{f,0})t - \sum_{i=1}^{t} v_{s,t} + s_0 \quad (18)$$

showing that the nominal exchange rate contains a local linear trend originating from the initial values of the interest rate differential, a stochastic $I(1)$ trend originating from cumulated forecast shocks to the nominal exchange rate.

An expression for the real exchange rate can be found by subtracting (18) from (16):

$$p_{d,t} - p_{f,t} - s_t = \sum_{i=1}^{t} (\nu_{p_{d,i}} - \nu_{p_{f,i}} - \nu_{s,i}) + (p_{d,0} - p_{f,0} - s_0).$$

The forecast shocks to relative prices are likely to approximately equal the forecast shocks to the nominal exchange rate under purchasing power parity, so that $\nu_{p_{d,i}} - \nu_{p_{f,i}} \simeq v_{s,i}$. Hence, the real exchange rate is stationary consistent with the assumptions underlying the monetary model.

5 A theory-consistent scenario

According to the stochastic properties derived above, prices are $I(2)$, the nominal exchange rate and the interest rates are $I(1)$. Based on this, the behavioral equilibrium equations underlying the theoretical model can now be translated into a set of testable hypotheses on cointegration in the CVAR model.

The derivations of the theory-consistent time-series properties of the variables in the previous section were based on the assumption that the expected change of the nominal exchange rate, $s^e_{t+1|t} - s_t = i_{d,t} - i_{f,t}$. Assumption A implies that $(i_d - i_f) \sim I(0)$, so the two interest rates share one common stochastic trend. Since the stochastic properties of the other variables are directly related to the stochastic properties of the interest rates, this is the main stochastic trend in the system. It was shown that the twice cumulated interest rate shocks generate an $I(2)$ trend in prices. While forecast shocks were found to cumulate once in the system, Section 2 showed that theory-consistent expectations do not change the process when the forecast model corresponds to the true process and the latter remains unchanged over time. Thus, expectational shocks should have no autonomous effect on the long-run properties of the system implying that the system is driven by one common stochastic trend of order two and, therefore, equilibrium correcting to $p - 1 = 4$ cointegration relations. There is one common autonomous shock, $u_{1,t}$, measured by a linear combination of the estimated VAR residuals, $\hat{u}_{1,t} = \alpha'_{\perp}\hat{\varepsilon}_t$.

As discussed above in Section 4, the common shock, $u_{1,t}$, cumulates once in the interest rates and the nominal exchange rate, but twice in the price variables (see also Juselius, 2006, Chapter 2.5). Hence, the theory-consistent CVAR scenario is consistent with $\{r = 4, s_1 = 0, s_2 = 1\}$ and is formulated as follows:

$$\begin{bmatrix} p_d \\ p_f \\ s \\ i_d \\ i_f \end{bmatrix} = \begin{bmatrix} c_1 \\ c_1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} \Sigma\Sigma u_1 \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ c_1 \\ c_1 \end{bmatrix} [\Sigma u_1] + \begin{bmatrix} d_1 \\ d_2 \\ d_1 - d_2 \\ 0 \\ 0 \end{bmatrix} t + Z_t. \qquad (19)$$

where $\Sigma\Sigma u_1$ is a shorthand for $\sum_{s=1}^{t}\sum_{j=1}^{s} u_{1,j}$, Σu_1 a shorthand for $\sum_{j=1}^{t} u_{1,j}$ and Z_t is a catch-all for the short-term effects in the vector process. The common stochastic $I(2)$ trend affects both prices with identical coefficients, c_1, so $(p_d - p_f) \sim I(1)$ consistent with long-run price homogeneity. The condition for PPP to be stationary is that $b_3 = b_1 - b_2$.

The coefficients c_i, b_i and d_i are not expressed as functions of the parameters of the theory model as this requires the short-run dynamics to be specified. Thus, the CVAR scenario is informative only about the conditions under which the postulated long-run behavior of the model is empirically valid. M. Juselius (2010) shows that only such models that satisfactorily describe the long-run properties of the data need to be tested for their short-run implications.

Given the condition for long-run price homogeneity, one can apply the nominal-to-real transformation (Kongsted, 2005) without loss of information:

$$\begin{bmatrix} p_d - p_f \\ s \\ \Delta p_d \\ i_d \\ i_f \end{bmatrix} = \begin{bmatrix} b_1 - b_2 \\ b_3 \\ c_1 \\ c_1 \\ c_1 \end{bmatrix} [\Sigma u_1] + \begin{bmatrix} d_1 - d_2 \\ d_1 - d_2 \\ 0 \\ 0 \\ 0 \end{bmatrix} t + Z_t. \qquad (20)$$

Because $(p_d - p_f) \sim I(1)$, it follows that $(\Delta p_d - \Delta p_f) \sim I(0)$. Thus under long-run price-homogeneity, inflation spread is stationary consistent with (15).

The scenario contains $r = 4$ stationary cointegration relations. For example, the following relations are irreducible in the sense of Davidson (1998):

1. $(s - p_d + p_f) \sim I(0)$,
2. $(i_d - i_f) \sim I(0)$
3. $(i_d - \Delta p_d) \sim I(0)$

4. $(i_d - a_1(p_d - p_f) + a_2 t) \sim I(0)$ where $a_1 = c_1/(b_1 - b_2)$ and $a_2 = c_1(d_1 - d_2)/(b_1 - b_2)$.

Linear combinations of these relations are also stationary and, hence, would also qualify as cointegration relations.

If relative prices and the nominal exchange rate are homogeneously related in all cointegration relations, then the scenario can, without loss of information, be formulated for the real exchange rate as follows:

$$\begin{bmatrix} s - p_d + p_f \\ \Delta p_d \\ \Delta p_f \\ i_d \\ i_f \end{bmatrix} = \begin{bmatrix} 0 \\ c_1 \\ c_1 \\ c_1 \\ c_1 \end{bmatrix} [\Sigma u_1] + Z_t. \tag{21}$$

showing as before that the four irreducible cointegration relations correspond to the PPP, the UIP, and the Fisher parities:

1. $p_d - p_f - s \sim I(0)$
2. $i_d - i_f \sim I(0)$
3. $i_d - \Delta p_d \sim I(0)$,
4. $i_f - \Delta p_f \sim I(0)$,

Again, other irreducible relations can be found by linear combinations. For example, $(i_d - i_f) - (i_d - \Delta p_d) + (i_f - \Delta p_f) = (\Delta p_d - \Delta p_f) \sim I(0)$ is a linear combination of 2, 3, and 4.

Note that the $r = 4$ cointegrated relations in the transformed scenario (20) can be thought of as $r = 4$ polynomially cointegrated relations in the $I(2)$ scenario (19) where the relations 1 and 2 are formulated as directly stationary relations and the relations 3 and 4 as a polynomially cointegrated relations. See Johansen (1995) and Juselius (2006, Chapters 16-18).

6 The specification of the *CVAR* model

The empirical analysis is based on German-US data for the post Bretton Woods, pre-EMU period[5]. The sample starts in 1975:8 and ends in 1998:12.

[5] All calculations are done using the software program CATS 3 in Oxmetrics (Doornik et al., 2017).

The VAR has two lags, and a few dummy variables:

$$\Delta^2 x_t = \Gamma\Delta x_{t-1} + \Pi x_{t-1} + \mu_0 + \mu_{01} Ds_{91.1,t} + \mu_1 t + \mu_1 t_{91.1} \\ + \phi_1 D_{tax,t} + \phi_2 Dp_{86.2} + \phi_3 Dp_{91.2} + \varepsilon_t, \tag{22}$$

where $x_t = [p_{d,t}, p_{f,t}, s_t, b_{d,t}, b_{f,t}]$ and p_t stands for CPI prices, s_t for the Dmk/dollar exchange rate, b_t for long-term bond rates, a subscript d for Germany and a subscript f for USA, $t_{91:1,t}$ is a linear trend starting in 1991:1 and $Ds_{91:1,t}$ is a step dummy also starting in 1991:1. Both control for the reunification of East and West Germany. $D_{tax,t}$ is an impulse dummy accounting for three different excise taxes levied to pay for the German reunification, $Dp_{86.2}$ is controlling for a large shock to the US price and bond rate in connection with the Plaza Accord, and $Dp_{91.2}$ accounts for a large shock to the exchange rate after the reunification.

The hypothesis that x_t is $I(1)$ is formulated as a reduced rank hypothesis on $\Pi = \alpha\beta'$, where α is $p \times r$ and β is $p_1 \times r$ with $p_1 = p + 2$. The hypothesis that x_t is $I(2)$ is formulated as an additional reduced rank hypothesis $\alpha'_\perp \Gamma \beta_\perp = \xi\eta'$, where ξ, η are $(p - r) \times s_1$ and $\alpha_\perp, \beta_\perp$ are the orthogonal complements of α, β respectively. See Johansen (1992).

Since the $I(2)$ condition is formulated as a reduced rank on the transformed $\mathbf{\Gamma}$ matrix, the latter is no longer unrestricted as in the $I(1)$ model. To circumvent this problem we use the following parameterization (see Johansen, 1997, Doornik and Juselius, 2017):

$$\Delta^2 x_t = \alpha \left[\begin{pmatrix} \beta \\ \tau_{01} \\ \tau_0 \end{pmatrix}' \begin{pmatrix} x_{t-1} \\ t_{91:1,t-1} \\ t-1 \end{pmatrix} + \begin{pmatrix} d \\ d_{01} \\ d_0 \end{pmatrix}' \begin{pmatrix} \Delta x_{t-1} \\ D_{s91:1,t-1} \\ 1 \end{pmatrix} \right]$$
$$+\zeta \begin{pmatrix} \tau \\ \tau_{01} \\ \tau_0 \end{pmatrix}' \begin{pmatrix} \Delta x_{t-1} \\ D_{s91:1,t-1} \\ 1 \end{pmatrix} + \Phi_1 D_{tax,t} + \Phi_2 Dp_{86.2,t} + \phi_3 Dp_{91.2,t} + \varepsilon_t,$$
$$t = 1975.09 - 1998.12 \tag{23}$$

where $\tau = [\beta, \beta_{\perp 1}]$ and d is proportional to $\tau_\perp$. In (22), an unrestricted constant (and step dummy) will cumulate twice to a quadratic trend, and a linear (broken) trend to a cubic trend. By specifying the broken trend to be restricted to the β part and the differenced broken trend to the d part of model (23) these undesirable effects are avoided. For more details, see Doornik and Juselius (2017), Kongsted et al. (1999), Juselius (2006, Chapter 17).

6.1 Rank determination

The theory-consistent scenario in section 5 showed that we should expect to find $(r = 4, s_1 = 0, s_2 = 1)$ if a standard monetary model is consistent with the empirical regularities in the data. The standard trace test procedure proposed by Nielsen and Rahbek (2007) is used to check this condition. It starts with the most restricted model $(r = 0, s_1 = 0, s_2 = 5)$, continues to the end of the row, and proceeds similarly row-wise from left to right until the first non-rejection. Because the trace tests of $r = 0, 1$ were all rejected, the first two rows have been omitted from Table 1. The first non-rejection is at $(r = 2, s_1 = 1, s_2 = 2)$ with a p-value of 0.25. As a robustness check, Table 1 also report the characteristic roots of the model. The unrestricted VAR contains five large roots, four of which are almost on the unit circle while the fifth is large but not equally close to one. Assuming no $I(2)$ trends and $r = 2$, $p - r = 3$ would leave two very large roots (0.96) in the model. Thus, the choice of reduced rank indices should be consistent with five unit roots. The case $\{r = 2, s_1 = 1, s_2 = 2\}$ restricts five of the characteristic roots to be on the unit circle with the largest unrestricted root equal to 0.48. Thus, this choice accounts for all persistent movements in the data whereas the choice of $\{r = 2, s_1 = 2, s_2 = 1\}$ leaves a large unrestricted root in the model (0.94). Thus, in addition to the stochastic $I(2)$ trend in prices, there seems to be another near $I(2)$ trend in the system. The derived scenario is consistent with the former, whereas not with the latter. Thus, the long persistent swings in the nominal and real exchange rate seem to require a modification of the standard monetary model.

6.2 Testable hypotheses on integration and cointegration

This section tests some hypotheses on the persistency properties derived in Section 4, albeit recognizing that the trace tests were not consistent with the derived scenario. For example, according to (20) the system could be transformed into $x_t' = [p_d - p_f, s, b_d, b_f, \Delta p_d]_t$ and according to (21) it could

Table 1: Determination of the two rank indices

Rank Test Statistics								
$p-r$	r	$s_2=5$	$s_2=4$	$s_2=3$	$s_2=2$	$s_2=1$	$s_2=0$	
3	2			143.0 [0.00]	**59.7** [0.25]	45.4 [0.25]	39.5 [0.33]	
2	3				38.4 [0.35]	18.8 [0.81]	15.8 [0.78]	
Six largest characteristic roots:								
Unrestricted VAR			**0.98**	**0.98**	**0.97**	**0.97**	**0.81**	0.49
$r=2, p-r=3$			1.0	1.0	1.0	**0.96**	**0.96**	0.49
$r=2, s_1=2, s_2=1$			1.0	1.0	1.0	1.0	**0.94**	0.50
$r=2, s_1=1, s_2=2$			1.0	1.0	1.0	1.0	1.00	0.48

be further transformed into $x_t' = [p_d - p_f - s, b_d, b_f, \Delta p_d, \Delta p_f]_t$ without loss of information. Finally, the derivations of Section 4 also implied stationarity of the interest rate differential so the system could be transformed to $x_t' = [p_d, p_f, s, b_d - b_f, \Delta b_d]_t$. If all hypotheses are empirically correct, then the system could be transformed into $x_t' = [p_d - p_f - s, b_d - b_f, \Delta p_d, \Delta p_f, \Delta b_d]_t$.

The above hypotheses are tested by imposing the same restriction on all τ (Johansen, 2006 and Johansen et al., 2010) and formulated as $R'\tau = 0$ where $\tau = [\beta, \beta_{\perp 1}]$ and R is a restrictions design matrix. For example, $R' = [1, 1, 0, 0, 0, 0, 0]$ formulates the hypothesis of long-run price homogeneity. The test results are as follows:

1. Long-run price-homogeneity was rejected based on $\chi^2(3) = 9.66[0.02]$.

2. Purchasing Power Parity was borderline not rejected based on $\chi^2(6) = 12.09[0.06]$.

3. The interest rate spread was rejected based on $\chi^2(3) = 32.75[0.00]$.

The empirical support for these basic hypotheses is weak or non-existent. Thus, both the rank and the time-series properties tests suggest that a standard monetary model cannot satisfactorily account for the regularities in the data.

In a companion paper, Juselius (2017) derives a theory-consistent scenario for a similar monetary model in which the assumption of rational expectations has been replaced by imperfect-knowledge-based expectations. This model is consistent with two stochastic trends of order two, one describing the long smooth movements in relative prices and the other the long persistent swings around equilibrium values. The imperfect knowledge scenario also shows that all variables, prices, interest rates and the nominal exchange rate, should be *I(2)* or near *I(2)*. All in all, this model obtains a remarkable support for all testable hypotheses.

7 Conclusions

The paper demonstrates that structuring the data according its (near) unit root properties using the Cointegrated VAR (CVAR) model provides a powerful way of confronting a theory model with the data. This is formalized as a theory-consistent CVAR scenario which describes the empirical regularities we should expect to see if the theory model is empirically relevant. To overcome the problem of unobserved expectations, the paper demonstrates how basic hypotheses about the expectation's formation can be translated into testable hypotheses in the scenario.

As an illustration, the paper translates most of the basic hypotheses underlying a standard monetary model for nominal exchange rate determination into testable hypotheses on a well specified CVAR model. The empirical findings show that the model is not able to explain the persistent movements in the data suggesting that the informationally less demanding imperfect-knowledge-based models might be superior in this respect. For example, Johansen et al. (2011) finds for a similar data set that an imperfect knowledge based model is in line with the persistency properties of the data. In a companion paper Juselius (2017) derives a CVAR scenario for an imperfect knowledge based monetary model for exchange rate determination and finds overwhelmingly strong empirical support for this model using the same data as in this paper.

The real exchange rate and the real interest rate are among the most important determinants for the real economy, which emphasizes the importance of understanding the causes underlying their long persistent movements away from fundamental values. The failure of extant models to foresee the financial and economic crises in 2007-2008 and to propose adequate policy measures in its aftermath signifies the crucial role of financial fluctuations on our economies. The subsequent rise of populism and the political turmoil that followed is a strong warning against neglecting this important issue.

8 References

Blanchard, O and M. Watson (1982), "Bubbles, Rational Expectations, and Financial Markets", in Paul Wachter (ed.) *Crises in Economic and Financial Structures*, Lexington MA, Lexington Books, 295-315.

Cheung, Y., K.S. Lai, and M. Bergman (2004), "Dissecting the PPP Puzzle: The Unconventional Roles of Nominal Exchange Rate and Price Adjustments," *Journal of International Economics* 64, 135-150.

Davidson, J. (1998), "Structural Relations, Cointegration and Identification: Some Simple Results and Their Applications". *Journal of Econometrics*, 87(1), 87-113.

Doornik, J. and K. Juselius (2017). *Cointegration Analysis of Time Series Using CATS 3 for OxMetrics.* Timberlake Consultants Ltd.

Dornbusch, R.(1976), "Expectations and Exchange Rate Dynamics", *Journal of Political Economy*, December, 1161-1174.

Dornbusch, R. and J.A. Frankel (1988), "The Flexible Exchange Rate System: Experience and Alternatives", in S. Borner, ed., *International Finance and Trade*, London: Macmillan, reprinted in J.A. Frankel ed., *On Exchange Rates*, Cambridge MA: The MIT Press, chapter 1,1995.

Elliot, G (1998), "On the Robustness of Cointegration methods When Regressors Almost Habve Unit Roots", *Econometrica* 66, 149-158.

Franchi, M. and S. Johansen (2017), "The Behavior of Cointegration Methods in a Vector Autoregression With near Unit Roots, Under Review for *Econometrics*

Frydman, R. and M. Goldberg (2007), *Imperfect Knowledge Economics: Exchange rates and Risk.* Princeton University Press, Princeton.

Frydman, R. and M. Goldberg (2011), *Beyond Mechanical Markets: Risk and the Role of Asset Price Swings*, Princeton University Press, Princeton.

Frydman, R. and M. Goldberg (2013), "Change and Expectations in Macroeconomic Models: Recognizing the Limits to Knowability", *Journal of Economic Methodology* 20, 118-138.

Frydman, R., M. Goldberg, K. Juselius, and S. Johansen (2008), "Reexamining the Purchasing Power Parity Puzzle with Imperfect Knowledge Economics", Working Paper, University of Copenhagen.

Haavelmo, T. (1944), "The Probability Approach to Econometrics". *Econometrica*, 12 (Supplement), 1-118.

Hendry, D. F. & G. E. Mizon (2000). "Reformulating Empirical Macroeconomic Modelling". *Oxford Review of Economic Policy*, Oxford University

Press, vol. 16(4), pages 138-59.

Hoover, K. and K. Juselius (2015), "Trygve Haavelmo's Experimental Methodology and Scenario Analysis in a Cointegrated Vector Autoregression". *Econometric Theory,* 31, 2, pp. 249-274.

Hoover, K., S. Johansen, and K. Juselius (2008), "Allowing the Data to Speak Freely: The Macroeconometrics of the Cointegrated VAR". *American Economic Review*, 98, pp. 251-55.

Ireland, P. N. (2004), "A Method for Taking Models to the Data". *Journal of Economic Dynamics & Control*, 28(6), pp. 1205-26.

Johansen, S. (1992), "A Representation of Vector Autoregressive Processes Integrated of Order 2". *Econometric Theory* 8, 188–202.

Johansen, S. (1995), *Likelihood-Based Inference in Cointegrated Vector Autoregressive Models.* 2.ed. 1996 Oxford University Press, Oxford.

Johansen, S. (1997), "Likelihood Analysis of the $I(2)$ Model". *Scandinavian Journal of Statistics* 24, 433-462.

Johansen, S. (2006), "Statistical Analysis of Hypotheses on the Cointegrating Relations in the $I(2)$ Model". *Journal of Econometrics* 132, 81–115.

Johansen, S., Juselius, K., Frydman, R. and M. Goldberg, (2010), "Testing Hypotheses in an$I(2)$Model With Piecewise Linear Trends. An Analysis of the Persistent Long Swings in the Dmk/$ Rate". *Journal of Econometrics* 158: 117-129. DOI: 10.1016/j.jeconom.2010.03.018

Juselius, K. (2006), *The Cointegrated VAR Model: Methodology and Applications.* Oxford University Press, Oxford.

Juselius, K. (2012), "Imperfect Knowledge, Asset Price Swings and Structural Slumps: A Cointegrated VAR Analysis of Their Interdependence", in eds. E. Phelps and R. Frydman: *Rethinking Expectations: The Way Forward for Macroeconomics*, Princeton University Press, Princeton.

Juselius, K. (2015), "Haavelmo's Probability Approach and the Cointegrated VAR model". *Econometric Theory,* 31, 2, pp 213 - 232

Juselius (2017), "Using a Theory-Consistent CVAR Scenario to Test an Exchange Rate Model Based on Imperfect Knowledge", Submitted to the E-journal *Econometrics.*

Juselius, K. and Franchi, M. (2007), "Taking a DSGE Model to the Data Meaningfully", *Economics*, 4, 1.

Juselius, M. (2010), "Testing Steady-State Restrictions of Linear Rational Expectations Models when Data are Highly Persistent". *Oxford Bulletin of Economics and Statistics,* 73, 3, pp. 315-334

Kongsted, H. C. (2005), "Testing the Nominal-to-Real Transformation". *Journal of Econometrics* 124, 205–225.

Kongsted, H. C., Rahbek, A., and C. Jørgensen (1999), "Trend Stationarity in the $I(2)$ Cointegration Model". *Journal of Econometrics* 90, 265–289.

Møller, F. N. (2008), "Bridging Economic Theory Models and the Cointegrated Vector Autoregressive Model". *Economics: The Open-Access, Open-Assessment E-Journal*, Vol. 2, 2008-36.

Nielsen, H. B. and A. Rahbek, (2007), "The Likelihood Ratio Test for Cointegration Ranks in the $I(2)$ Model". *Econometric Theory* 23, 615–637.

Spanos, A. (2009), "The Pre-Eminence of Theory versus the European CVAR Perspective in Macroeconometric Modeling". *Economics: The Open-Access, Open-Assessment E-Journal*, Vol. 3.

Tabor, M. N. (2017), "Stochastic Cointegration Parameters as a Source of Persistence in the Cointegrated VAR Model: A Simulation Study," Submitted to the E-journal *Econometrics*.