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Testing R&D-Based Endogenous Growth Models

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Testing R&D-Based Endogenous Growth Models

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Abstract

R&D-based growth models are tested using US data for the period 1953-2014. A general growth model is developed which nests the model varieties of interest. The model implies a cointegrating relationship between multifactor productivity, research intensity, and employment. This relationship is estimated using cointegrated VAR models. The results provide evidence against the widely used fully endogenous variety and in favor of the semi-endogenous variety. Forecasts based on the empirical estimates suggest that the slowdown in US productivity growth will continue. Particularly, the annual long-run growth rate of GDP per worker converges to between zero and 1.1 pct.

Keywords: Endogenous growth, semi-endogenous growth, total factor productivity (TFP), research and development (R&D), time series econometrics, cointegration
JEL Classification: C32, E24, O31, O41, O47, O51

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1 Introduction

R&D-based growth models continue to be the foundation for much economic growth research. This includes studies on the US college wage premium (e.g., Acemoglu 1998, 2002), environmental sustainability (e.g., Acemoglu et al. 2012), and economic growth effects of tax policies (e.g., Jaimovich and Rebelo 2017). In such studies, the obtained results are often strongly affected by the underlying growth mechanism. It is, therefore, crucial that researchers base their studies on plausible growth model varieties. However, within the R&D-based framework important controversies remain unresolved. Jones (1999, 2005) points out that the widely used fully endogenous variety is nonrobust, as its long-run properties depend crucially on knife-edge assumptions. This view is, however, not generally accepted, and the fully endogenous variety remains widely used in economic growth research.[1] The purpose of this paper is not to assess this theoretical debate. Instead, this paper tests different R&D-based growth model varieties using empirical data.

This paper has two objectives. The first objective is to test a broad class of R&D-based growth models. To achieve this, a general R&D-based growth model nesting both the fully and semi-endogenous varieties is developed. The theoretical model predicts a cointegrating relationship between multifactor productivity (MFP), research intensity (R&D expenditures over GDP), and employment. This relationship is used to identify the parameters of interest within a cointegrated vector autoregressive (VAR) model. Empirical results based on US data provide support for R&D-based growth models in general. However, the fully endogenous variety is clearly rejected, while the empirical results support the semi-endogenous variety.

The second objective is to assess the growth-forecast implications of the empirical results. Assuming that the US R&D intensity remains at its 2014 level, forecasts based on the empirical estimates suggest that the long-run US growth rate of GDP per worker will converge to between zero and 1.1 pct. per year: a notable growth slowdown. Hence, the forecasts suggest that the recent slowdown in US productivity growth will continue in the coming decades unless the US research intensity is increased considerably.

The main result of this paper is the empirical rejection of the widely used fully endogenous variety. The key prediction of this variety is that the productivity growth rate tracks the fraction of GDP spent on R&D (Aghion and Howitt 2005, p. 94-95). As illustrated in Figure 1, US R&D expenditures were between 2 and 3 pct. of GDP for the period 1960-2014, while

[1] Romer (1994, p. 17-19) claims that the fully endogenous variety is a convenient approximation, while Dalgaard and Kreiner (2003) and Peretto (2017) defend the fully endogenous variety theoretically.

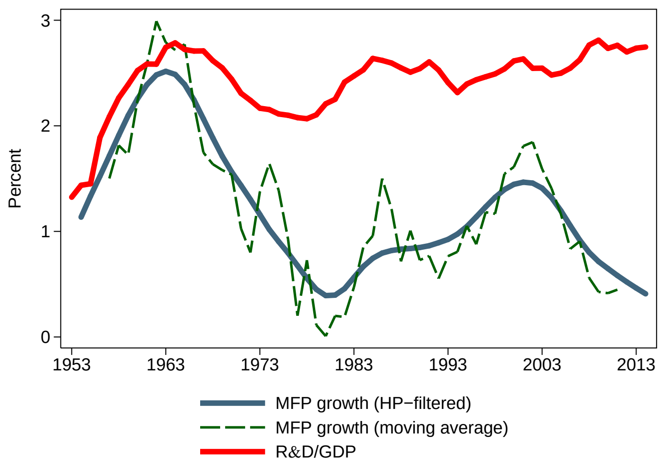


FIGURE 1: US R&D expenditures over GDP and smoothed US MFP growth rate, 1953-2014.
Data Sources: BEA, BLS, and NSF.
Notes: The smoothing factor of the HP filter is set to 100. The moving average is a 7-year centered moving average.

the smoothed US MFP growth rate decreased from over 2 pct. in 1960 to less than 0.5 pct. in 2014. Hence the productivity growth rate decreased, while the research intensity remained approximately constant. Seen from this perspective, the rejection of the fully endogenous variety might not be that surprising.

Besides the empirical results, the present study contributes to the existing literature in at least two important ways. First, even though the empirical strategy is closely related to that of Ha and Howitt (2007), the theoretical framework developed below allows a broader class of R&D-based growth models to be tested. In particular, the framework allows a more general version of the semi-endogenous variety compared to previous studies. Second, the present study puts more emphasis on certain econometric considerations largely ignored by the previous literature. This includes the examination of a potential structural transition in US R&D expenditures during the 1950s. Given the technical nature of these contributions, a more thorough discussion is postponed to Section 7.

Before starting the analysis, three issues related to the empirical approach are worth addressing further. First, it is relevant to discuss the advantages associated with the cointegrated VAR model. As emphasized by Ha and Howitt (2007), R&D-based growth models predict a cointegrating relationship between MFP, effective R&D input, and employment. Thus it seems natural to employ a cointegrated VAR model to estimate the parameters of interest. Furthermore, the cointegrated VAR approach has several advantages when testing R&D-based growth models: (1) potential mutual dependencies between the main variables are accommodated, (2) the parameters are estimated by an efficient estimator, and (3) the framework allows for a distinction between short and long-run relationships. The last point is especially important, as the focus of this paper is exclusively on the long-run relationship.

Second, it seems appropriate to address the focus on the US economy. R&D-based growth models are often the foundation for single-country analyses.[2] Accordingly, R&D-based growth models should work on a single-country basis. The US economy is especially interesting not only because of the availability of long high-quality MFP and R&D expenditure time series, but also because the US economy is typically regarded as the leading-edge economy. While economic growth in other countries over the last 50 years might have been driven mainly by catching-up effects, US economic growth must have been driven mainly by knowledge expansion given its leading position. Hence if R&D-based growth models are useful in a single-country setting, they should at least work in the US case. Nevertheless, an important underlying assumption of this analysis is that knowledge spillovers from other countries either played a negligible role in US productivity growth over the investigated period, or that these spillovers were sufficiently systematic such that they are picked up by the deterministic terms of the empirical model.

Third, it is worth emphasizing the advantages associated with long time series when testing R&D-based growth models. Growth models are designed to explain macroeconomic tendencies over long time spans. Thus their predictions should be evaluated based on data sources that also stretch over long time spans. In addition, the essential difference between the fully and semi-endogenous varieties lies in the time dimension. In particular, the long-run productivity growth rate is state-dependent in the semi-endogenous variety, while being state-independent in the fully endogenous variety. If the state-dependency is weak, it might not be detectable in cross-country or cross-industry data covering relatively short time spans.

This paper proceeds as follows. First, the theoretical model is developed, and the implied cointegrating relationship is integrated into the empirical model (Section 2). Next, the data are presented (Section 3), and the theoretical model is tested using both single-equation and VAR models (Section 4). It is subsequently shown that the empirical results are robust to justifiable changes in the empirical approach (Section 5). The empirical estimates are then used to forecast US MFP and GDP per worker figures (Section 6). The paper finishes with a discussion (Section 7) and some concluding remarks (Section 8).

[2]For instance, Jaimovich and Rebelo (2017) develop an R&D-based growth model where workers have heterogeneous entrepreneurial skills. In this setup, they find that the effect of taxation on economic growth might be highly nonlinear. They argue that these highly nonlinear effects might explain why tax rates and growth performances are uncorrelated across countries. Clearly, their argument rests on the interpretation that their growth model represents a particular economy and not the world as a single entity.

2 Theory

The identification strategy is as follows. A simple R&D-based growth model nesting different R&D-based growth model varieties is developed. The theoretical model predicts a cointegrating relationship between MFP, research intensity, and employment. This cointegrating relationship is estimated using a cointegrated VAR model. This strategy ensures that the main hypotheses distinguishing the different model varieties are tested within a coherent framework.

The theoretical model is designed to capture the long-run dynamics of the system, i.e. the long-run predictions of the different model varieties. The short-run dynamics of the empirical model are left unrestricted, allowing various adjustment processes to the cointegrating relationship.

2.1 Theoretical model

Time is continuous and indexed $t \geq 0$. Final goods can be consumed, transformed into raw capital, or be used as research input:

$$Y(t) = C(t) + J(t) + R(t), \tag{2.1}$$

where $Y(t)$ is aggregate output of final goods, $C(t)$ is aggregate consumption, $J(t)$ is aggregate capital investment, and $R(t)$ is aggregate investment in R&D.

Final goods are produced from labor and specialized capital goods:

$$Y(t) = Q(t)^{\eta+\alpha} \left(\frac{1}{Q(t)} \int_0^{Q(t)} m(i,t)^{\alpha} A(i,t) \, \mathrm{d}i \right) L(t)^{1-\alpha}, \quad \alpha \in (0,1), \quad \eta \geq 0, \tag{2.2}$$

where $Q(t)$ is a measure of the specialized capital good varieties, $m(i,t)$ is the service of specialized capital good $i \in [0, Q(t)]$, $A(i,t)$ is the productivity associated with specialized capital good i, and $L(t)$ is aggregate labor input. The parameter η captures productivity gains from horizontal innovation, cf. (2.7) below.[3]

Following Aghion and Howitt (1998, p. 94-96), it requires $A(i,t)$ units of raw capital to produce specialized capital good i, i.e. it requires more raw capital to produce more advanced specialized capital goods. The market clearing condition for raw capital, $K(t)$,

[3]The production function from Aghion and Howitt (1998, p. 407) is nested as the special case $\eta = 0$, where horizontal innovation has no productivity-enhancing effects. Additionally, the production function from Romer (1990, p. 83) is nested as the special case $\eta = (1 - \alpha)$ and $A(i,t)$ equals a constant, so that there are no quality improvements of specialized capital goods.

amounts to:

$$K(t)=\int_0^{Q(t)} A(i,t)m(i,t)\,\mathrm{d}i. \tag{2.3}$$

The stock of raw capital evolves according to

$$\frac{\mathrm{d}K(t)}{\mathrm{d}t}\equiv \dot{K}(t)=J(t)-\delta K(t),\quad \delta>0,\quad K(0)>0 \text{ given}, \tag{2.4}$$

where δ is the capital depreciation rate.

The expansion of specialized capital good varieties has been motivated in different ways in the literature (see Young 1998; Aghion and Howitt 1998, ch. 12; Howitt 1999). The common feature of these mechanisms is that the range of varieties increases with the size of the labor input. Following Jones (1999), the range of specialized capital good varieties expands according to:

$$Q(t)=L(t)^{\beta},\quad \beta\in[0,1]. \tag{2.5}$$

Let the technological level, $A(t)$, be defined as the average productivity associated with the specialized capital goods:

$$A(t)\equiv\frac{1}{Q(t)}\int_0^{Q(t)} A(i,t)\,\mathrm{d}i.$$

Investments in R&D increase the technological level such that

$$\frac{\mathrm{d}A(t)}{\mathrm{d}t}\equiv \dot{A}(t)=\lambda A(t)^{\phi}\left(\frac{R(t)}{A(t)Q(t)}\right)^{\sigma},\ \lambda>0,\ \phi\leq 1,\ \sigma\in(0,1),\ A(0)>0 \text{ given}, \tag{2.6}$$

where $A(t)^{\phi}$ captures knowledge spillovers and the effective research input is given by $R(t)/\left(A(t)Q(t)\right)$. Note that each unit of final goods spent on research becomes less effective over time, as it is spread over more specialized capital good varieties, and as the technological level becomes more advanced. Thus, the absolute amount of resources allocated to R&D must increase over time to keep the effective research input constant.

Specialized capital good varieties are produced under monopolistic competition, while final goods are produced under perfect competition. It follows that in equilibrium, all specialized capital good varieties are produced in the same quantity. Hence $m(i,t)=\bar{m}(t)$ for all i. It follows from (2.2) and (2.3) that: $Y(t)=A(t)Q(t)^{\eta+\alpha}\bar{m}(t)^{\alpha}L(t)^{1-\alpha}$ and $K(t)=A(t)Q(t)\bar{m}(t)$. These two relations imply that:

$$Y(t)=A(t)^{1-\alpha}Q(t)^{\eta}K(t)^{\alpha}L(t)^{1-\alpha}. \tag{2.7}$$

Assuming a constant savings rate, a constant research intensity, and a constant labor input growth rate,[4] the economy converges to a long-run equilibrium featuring a constant capital-output ratio (see Appendix A). To focus on the long-run equilibrium, the capital-output ratio is from now assumed constant: $K(t)/Y(t) = \kappa > 0$. This seems appropriate given that the US capital-output ratio has been approximately constant since the 1950s (Jones 2015).

From (2.5), (2.6), and (2.7) it then follows that:

$$g_A(t) \equiv \frac{\dot{A}(t)}{A(t)} = \bar{\lambda} A(t)^{\phi-1} X(t)^{\sigma} L(t)^{\sigma\left(1-\beta+\beta\frac{\eta}{1-\alpha}\right)}, \quad \bar{\lambda} \equiv \lambda \kappa^{\sigma\frac{\alpha}{1-\alpha}}, \quad X(t) \equiv \frac{R(t)}{Y(t)}, \tag{2.8}$$

where $g_A(t)$ is the technological growth rate and $X(t)$ is the *research intensity*. Hence the model describes the evolution of $A(t)$ given the paths of $X(t)$ and $L(t)$.

The parameters ϕ and β are crucial for the model's long-run predictions. If $\phi > 0$ it gets easier to innovate as the technological level increases: a standing-on-shoulders effect. In contrast, it becomes increasingly more difficult to innovate if $\phi < 0$: a fishing-out effect. According to (2.8), the technological growth rate is negatively affected by the technological level (state-dependence) unless $\phi = 1$. In this limiting case, the standing-on-shoulders effect is as strong as it can possibly be without leading to accelerating economic growth for a constant research intensity and labor input.

It appears from (2.8) that a larger labor input has two opposing effects on the technological growth rate. On the one hand, a larger labor input implies that more resources can be devoted to the creation of technical knowledge: a non-rival good. On the other hand, a larger labor input implies that more specialized capital good varieties are developed. As a result, research efforts are spread over a larger range of goods which dilutes the effect of R&D expenditures. The first effect dominates the second except for in the special case $\eta = 0$ and $\beta = 1$, where the two effects balance out. And the technological growth rate is only independent of the labor input (state-independence) in this special case.

Unfortunately the technological level, $A(t)$, cannot be observed empirically. Instead, an economist will observe the labor and capital inputs as well as the final output. From these measures and (2.7), the economist computes MFP as: $\tilde{A}(t) \equiv A(t)^{1-\alpha} Q(t)^{\eta}$. As an alternative productivity measure, the economist computes GDP (=output) per worker, $y(t)$, from the formula: $y(t) \equiv Y(t)/L(t) = \kappa^{\frac{\alpha}{1-\alpha}} \tilde{A}(t)^{\frac{1}{1-\alpha}}$. It then follows from (2.5), (2.7), and

[4] This case seems relevant as both investment in physical capital and R&D relative to GDP were approximately constant over the investigated period (Jones 2015), while the labor input grew at an approximately constant rate (see Section 3).

(2.8) that the growth rates of MFP and GDP per worker are given by

$$g_{\tilde{A}}(t) = \beta\eta g_L(t) + (1-\alpha)\underbrace{\bar{\lambda}A(t)^{\phi-1}X(t)^{\sigma}L(t)^{\sigma\left(1-\beta+\beta\frac{\eta}{1-\alpha}\right)}}_{g_A(t)} \quad \text{and} \quad g_y(t) = \frac{g_{\tilde{A}}(t)}{1-\alpha},$$

where the growth rate of a variable $z(t)$ is denoted $g_z(t)$.

There are four growth model varieties nested in the model presented above. To simplify an examination of the model's long-run predictions, assume that the labor input grows at a non-negative constant rate, $g_L(t) = n \geq 0$, and that the research intensity is constant over time, $X(t) = \bar{X} > 0$. The long-run predictions of the four growth model varieties are described below, while further documentation is provided in Appendix B.

- **First-generation fully endogenous:** $[n = 0,\ \phi = 1,\ \beta = 0,\ \eta = 0]$.
  The growth rates of MFP and GDP per worker are increasing in research intensity, and they grow at constant rates given a constant research intensity. The labor input is assumed constant, as a growing labor input implies an increasing economic growth rate. This variety essentially includes the models developed by Romer (1990), Grossman and Helpman (1991), and Aghion and Howitt (1992).

- **Second-generation fully endogenous:** $[n \geq 0,\ \phi = 1,\ \beta = 1,\ \eta = 0]$.
  As in the previous model variety, the growth rates of MFP and GDP per worker are increasing in research intensity, and they grow at constant rates given a constant research intensity. The labor input is allowed to grow, but the growth rate of the labor input has no direct effect on the growth rates on MFP and GDP per worker.[5] This variety essentially includes the models developed by Aghion and Howitt (1998, ch. 12), Dinopoulos and Thompson (1998), Peretto (1998), Young (1998), and Howitt (1999).

- **Semi-endogenous:** $[n \geq 0,\ \phi < 1,\ \beta < 1,\ \eta \geq 0]$ or $[n \geq 0,\ \phi < 1,\ \beta = 1,\ \eta > 0]$.
  The long-run growth rate is proportional to the growth rate of the labor input. Thus policies that permanently increase the research intensity only increase the growth rates of MFP and GDP per worker temporarily. These temporary effects might last for a long time depending on the parameter values. This variety essentially includes the models developed by Jones (1995), Kortum (1997), Segerstrom (1998), and Li (2000).[6]

[5] Typically, the growth rate of the labor input affects investment in R&D. The growth rates of MFP and GDP per worker are thereby indirectly affected by the growth rate of the labor input.

[6] If $n = 0$ the semi-endogenous variety exhibits quasi-arithmetic growth, and the model dynamics are similar to the less-than-exponential growth case.

- **Less-than-exponential:** $[n \geq 0,\ \phi < 1,\ \beta = 1,\ \eta = 0]$.
  The growth rates of MFP and GDP per worker decrease over time. The growth rates might, however, decrease at a slow pace depending on the value of ϕ. The model exhibits perpetual economic growth in the sense that GDP per worker approaches infinity for time approaching infinity. Thus the model belongs to a class of growth models exhibiting quasi-arithmetic growth (see Groth et al. 2010).

Based on US data, the first-generation fully endogenous growth models are clearly rejected. The US labor input more than doubled from 1960 to 2014, while the research intensity remained approximately constant. The economic growth rate declined, while first-generation fully endogenous growth models predict an increase. Hence the present study focuses on the last three model varieties.

2.2 Empirical model

The empirically relevant version of (2.8) is given by:

$$g_A(t) = \bar{\lambda} \left(\tilde{A}(t)^{\tilde{\phi}} X(t) L(t)^{\tilde{\beta}} \right)^{\sigma}, \quad \tilde{\phi} \equiv \frac{\phi - 1}{\sigma(1-\alpha)}, \quad \tilde{\beta} \equiv 1 - \beta + \beta \frac{\eta}{1-\alpha} \left(\frac{\sigma + 1 - \phi}{\sigma} \right). \quad (2.9)$$

When relating the model to data, time becomes discrete and a time dependent variable z is denoted z_t. The log discrete time version of (2.9) amounts to:

$$\Delta \ln A_{t+1} = \gamma + \psi t + \sigma \left(\tilde{\phi} \ln \tilde{A}_t + \ln X_t + \tilde{\beta} \ln L_t \right), \quad (2.10)$$

where $\bar{\lambda}$ is allowed to change over time at a constant growth rate ψ. The inclusion of the trend ensures that the model is balanced in terms of the Johansen test. In addition, if knowledge spillovers from other countries become stronger or weaker over time in a systematic way, this would be captured by the trend. The hypothesis that $\bar{\lambda}$ is constant corresponds to the hypothesis $\psi = 0$. This hypothesis cannot be rejected by any of the cointegrated VAR models considered below, further strengthening the assumption that $\bar{\lambda}$ is constant over the investigated period.

Through the rest of this section, it is assumed that $\ln \tilde{A}_t$, $\ln X_t$, and $\ln L_t$ are $I(1)$ (integrated of order one). This assumption is backed up by empirical evidence in Section 4.1. As $\ln \tilde{A}_t$ and $\ln L_t$ are $I(1)$, $\ln A_t$ is $I(1)$ as well. Hence the left hand side of (2.10) is stationary. Consequently, the right hand side must be stationary as well. For the right hand side to be stationary, $\ln \tilde{A}_t$, $\ln X_t$, and $\ln L_t$ must cointegrate. When normalizing in terms of σ, the cointegrating relationship amounts to:

$$\tilde{\gamma} + \tilde{\psi} t + \tilde{\phi} \ln \tilde{A}_t + \ln X_t + \tilde{\beta} \ln L_t \sim I(0), \quad \tilde{\gamma} \equiv \frac{\gamma}{\sigma}, \quad \tilde{\psi} \equiv \frac{\psi}{\sigma}. \tag{2.11}$$

The cointegrating relationship given by (2.11) is the identifying relationship used to estimate the parameters of interest: $\tilde{\psi}$, $\tilde{\phi}$, and $\tilde{\beta}$. The parameter assumptions distinguishing the four model varieties discussed above are implicitly tested. In particular, the hypotheses associated with the parameter assumptions of these four model varieties are given by:

- First-generation fully endogenous: $\tilde{\psi} = 0$, $\tilde{\phi} = 0$, and $\tilde{\beta} = 1$.
- Second-generation fully endogenous: $\tilde{\psi} = 0$, $\tilde{\phi} = 0$, and $\tilde{\beta} = 0$.
- Semi-endogenous: $\tilde{\psi} = 0$, $\tilde{\phi} < 0$, and $\tilde{\beta} > 0$.
- Less-than-exponential growth: $\tilde{\psi} = 0$, $\tilde{\phi} < 0$, and $\tilde{\beta} = 0$.

At first glance, it is tempting to use a single-equation cointegration procedure. But a single-equation approach is problematic, as there might be feedback effects from MFP to the other two variables in the short run. In fact, all three variables might error correct, making the single-equation procedure invalid.

Therefore, a VAR model is employed and the cointegrating relationship (2.11) is exploited within this framework which has the benefits discussed above. The VAR model is given by:

$$\Delta Z_t = a\left(b', b_1\right) \begin{pmatrix} Z_{t-1} \\ t \end{pmatrix} + \sum_{i=1}^{k-1} \Gamma_i \Delta Z_{t-i} + \Phi D_t + \chi + \nu_t, \quad \nu_t \sim IN_3\left[0, \Omega\right], \tag{2.12}$$

where $Z_t = (\ln \tilde{A}_t, \ln X_t, \ln L_t)'$; a and b are $(3 \times r)$ matrices and the rank of $\Pi \equiv ab'$ is $r \leq 3$; k is the number of lags; $\Gamma_1, ..., \Gamma_{k-1}$ are matrices of parameters; b_1 and χ are vectors of constants; D_t is a matrix of dummies; Φ is a matrix of unrestricted coefficients; ν_t is a vector of error terms; and $IN_3\left[0, \Omega\right]$ is a 3-dimensional independent normal distribution with mean zero and covariance matrix Ω (symmetric and positive definite).

According to the theory, there is a single cointegrating relationship between the three variables given by (2.11). Consequently, a single cointegrating relationship is used if the hypothesis of a single cointegrating relationship cannot be rejected by a Johansen test at the 5 pct. level of significance. This turns out to be the case for all considered VAR models.

The cointegrated VAR model used to estimate the parameters of interest is given by:

$$\begin{pmatrix} \Delta \ln \tilde{A}_t \\ \Delta \ln X_t \\ \Delta \ln L_t \end{pmatrix} = a \begin{pmatrix} \tilde{\phi} \\ 1 \\ \tilde{\beta} \\ \tilde{\psi} \end{pmatrix}' \begin{pmatrix} \ln A_{t-1} \\ \ln X_{t-1} \\ \ln L_{t-1} \\ t \end{pmatrix} + \sum_{i=1}^{k-1} \Gamma_i \begin{pmatrix} \Delta \ln A_{t-i} \\ \Delta \ln X_{t-i} \\ \Delta \ln L_{t-i} \end{pmatrix} + \Phi D_t + \chi + \nu_t, \tag{2.13}$$

where a captures short-run adjustments to the long-run relation, while the long-run relationship is captured by the vector: $\left(\tilde{\phi}, 1, \tilde{\beta}, \tilde{\psi}\right)$. Based on the data presented below, the constant is unrestricted while the trend is restricted to the cointegrating space.

3 Data

The empirical analysis is based on US data for the period 1953-2014. The length of the sample is restricted by the R&D expenditure data which are only available for the period 1953-2014 at the time of writing. It appears from (2.1) that Y_t equals GDP and R_t equals aggregate R&D expenditures. Research expenditure data are provided by the National Science Foundation (NSF) and covers total US R&D expenditures.[7] The GDP data are obtained from the Bureau of Economic Analysis (BEA).[8] The research intensity, X_t, is computed by dividing the R&D expenditures by GDP. The labor input, L_t, is measured by full-time equivalent employment.[9] There are several ways to compute the productivity measure, $\tilde{A}_t$. To ensure that the results are robust to different productivity measures, three generally accepted productivity measures are used: the multifactor productivity measure from the Bureau of Labor Statistics (BLS), the total factor productivity (TFP) measure from Fernald (2014a), and GDP per (full-time equivalent) worker.[10] From here after, MFP refers to the BLS multifactor productivity measure, while TFP refers to the total factor productivity measure from Fernald (2014a). MFP and TFP data are indexed such that the 1953 value equals 100.

The three productivity measures in logs are displayed in levels and in differences in Figure 2. All three measures grew fast during the 1950s and 1960s, while the growth rate was

[7] This study exploits the research expenditure data used in Boroush (2016). Values from 2014 are preliminary, but qualitatively similar estimation results can be obtained for the sample 1953-2013.

[8] US Bureau of Economic Analysis, Current-dollar and "real" GDP, retrieved on November 30, 2016.

[9] US Bureau of Economic Analysis, Full-time equivalent employees, retrieved on November 30, 2016 from FRED, Federal Reserve Bank of St. Louis.

[10] MFP and TFP data are retrieved on November 30, 2016. The TFP data are obtained from http://www.frbsf.org/economic-research/indicators-data/total-factor-productivity-tfp/. The MFP data are obtained from the historical series at http://www.bls.gov/mfp/mprdload.htm.

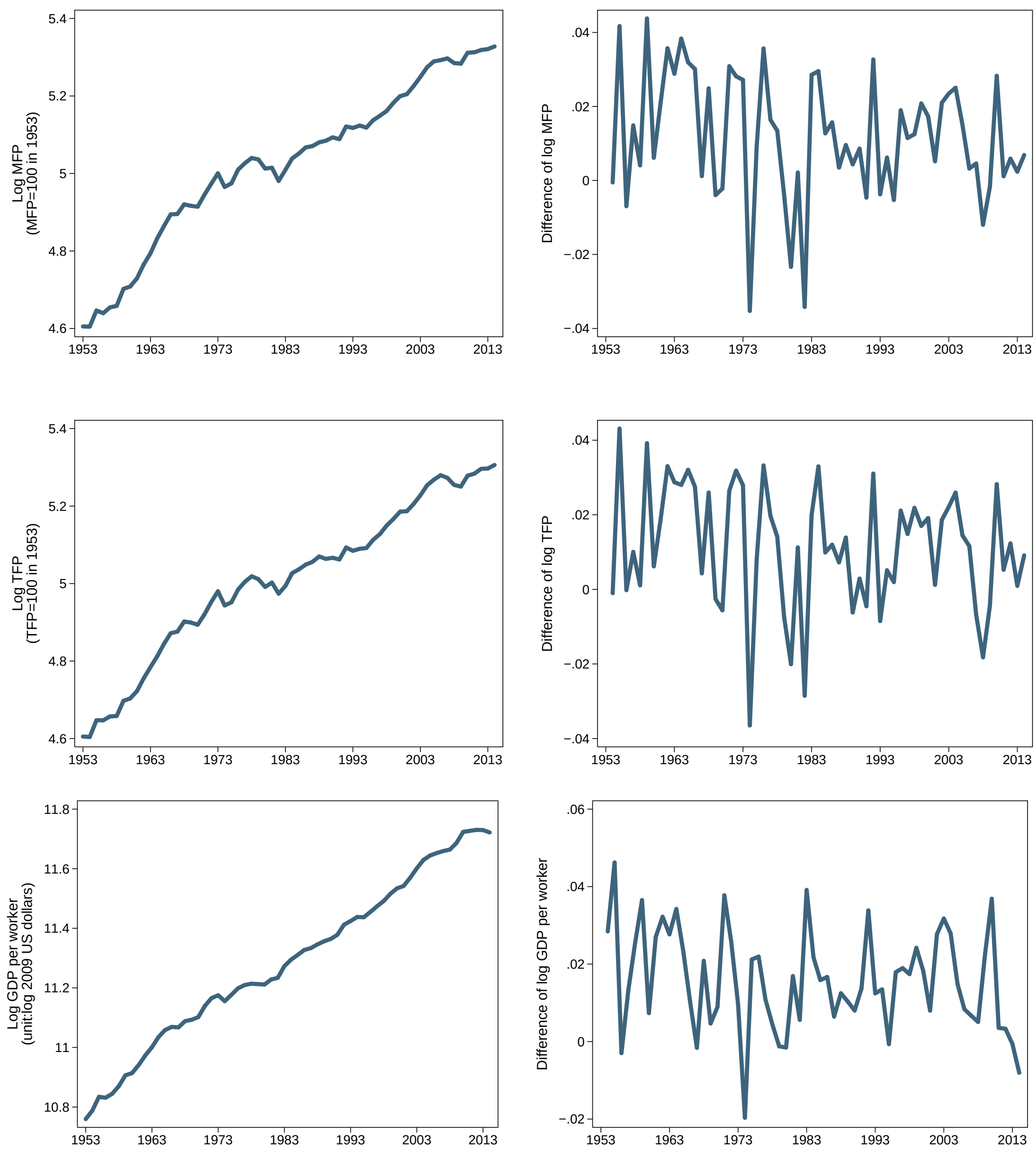


FIGURE 2: Productivity measures, $\tilde{A}_t$, (MFP, TFP, and GDP per worker) in logs: levels and differences, 1953-2014.
Data sources: BEA, BLS, Fernald (2014a), and FRED.

reduced during the 1970s and 1980s. The growth rate increased again during the mid-1990s until the mid-2000s. Finally, the growth rate slowed down after the mid-2000s. And as noted by Fernald (2014b), the slowdown began several years prior to the financial crisis of 2007–2008.

Transitory drops in all three productivity measures occurred in 1974; a shock associated with the 1973 oil embargo. There are also large transitory drops in MFP and TFP in 1982 and 1993. These drops are associated with the 1980s and 1990s recessions. The large growth rates of 1955 and 1959 are associated with the recoveries after the 1953 and 1958 recessions.

The research intensity in logs is displayed in levels and in differences in Figure 3. The research intensity increased rapidly in the 1950s and remained at a permanently higher level after 1957, possibly indicating a structural change following World War II. The research intensity increased especially fast from 1955 to 1956, creating an exceptionally large spike in the differenced data. This surge in research intensity was caused by large increases in R&D expenditures funded by the business sector and the federal government. The research intensity also grew fast from 1962 to 1963: a consequence of a historically large increase in federally funded R&D expenditures.

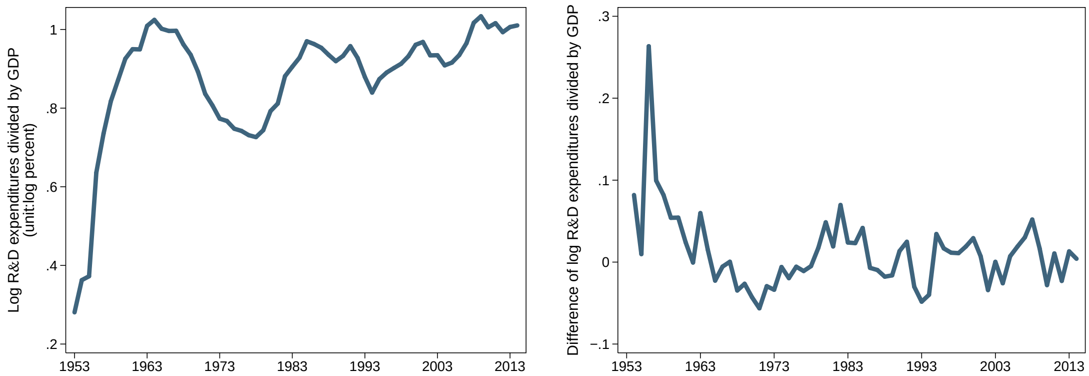


FIGURE 3: Research intensity, X_t, in logs: levels and differences, 1953-2014.
Data sources: BEA and NSF.

Employment in logs is displayed in levels and in differences in Figure 4. Employment seems to increase at an approximately constant growth rate through the period. Yet, the generally smooth increase was permanently interrupted by a large drop in 2009 caused by the financial crisis of 2007–2008. In addition, employment seems temporarily affected by the same historical events as the productivity measures.

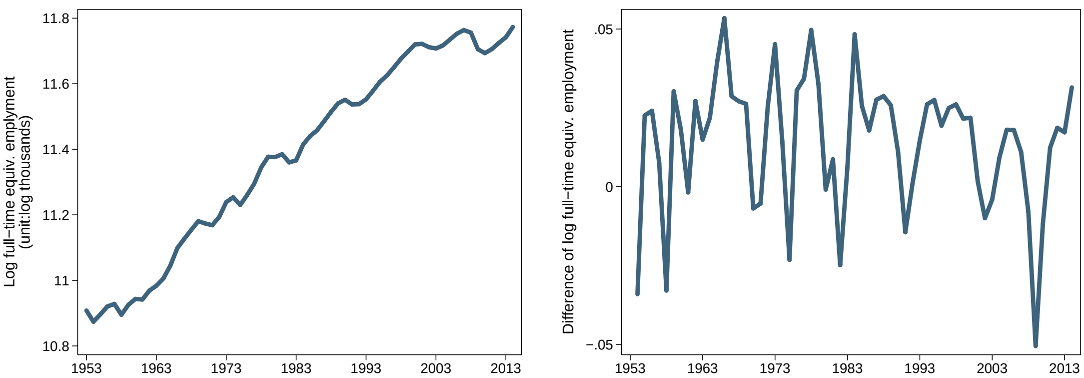


FIGURE 4: Full-time equivalent employment, L_t, in logs: levels and differences, 1953-2014.
Data source: FRED.

The inspection of the data indicates a structural change in the US research intensity after World War II. When analyzing the data, one has to carefully take this transition into account (see Section 4.1). The inspection also indicate a large permanent effect of the financial crisis of 2007–2008 on employment. In addition, the inspection indicates transitory effects on the productivity measures coinciding with distinct macroeconomic events. Both the permanent and transitory effects caused by distinct events should be accounted for in the cointegrated VAR model analysis. Finally, employment and the productivity measures are clearly trending which motivates the deterministic terms in (2.11), see Juselius (2006, ch. 6).

4 Empirical Analysis

The first part of the empirical analysis indicates that the fully endogenous variety can be rejected using single-equation time series models. These models also document the structural change in the research intensity discussed above which motivates a restricted sample for the cointegrated VAR model analysis. In the second part, the cointegrating relationship (2.10) is estimated using (2.13). The results clearly support the semi-endogenous variety.

A 5 pct. level of significance is used throughout the paper. A two-sided alternative is used to evaluate the null hypotheses: $\tilde{\phi} = 0$ and $\tilde{\beta} = 0$. A one-sided alternative might, however, be considered more appropriate given the alternative hypotheses: $\tilde{\phi} < 0$ and $\tilde{\beta} > 0$. Yet, the distinction between one and two-sided alternatives is seldomly important.

4.1 Unit root tests

From (2.10) it follows that the second-generation fully endogenous growth models predict that: $\Delta \ln \tilde{A}_{t+1} = (1-\alpha)\gamma + (1-\alpha)\sigma \ln X_t$. Thus according to this variety, $\ln X_t$ must be stationary if $\ln \tilde{A}_t$ is an $I(1)$ process. The results obtained by Ha and Howitt (2007) indicate that this is the case. Yet as illustrated below, their findings seem strongly affected by a structural change in research intensity occurring in the 1950s.

Unit root tests for the main variables are conducted using augmented Dickey-Fuller tests. The tests are based on autoregressive (AR) models, where the lag length is determined by the Schwarz criterion. The models contain both a constant and an intercept. If the process contains a unit root, the trend will feed into the process as a quadratic term, and thus it might be more appropriate to test the joint hypothesis of a unit root and the exclusion of the trend component. This joint test is evaluated using a LR (likelihood ratio) statistic, whereas the standard Dickey-Fuller test is assessed based on a t-statistic.

TABLE 1: Augmented Dickey-Fuller test results

Sample: 1953-2014					
	$\ln \tilde{A}$			$\ln X$	$\ln L$
	MFP	TFP	GDPW		
t-statistic	-1.93	-2.03	-2.78	**-3.95**	-1.10
LR statistic	3.79	4.21	7.77	**14.74**	1.31
Unit root	Yes	Yes	Yes	No	Yes
AR(p) process	1	1	2	2	3
Sample: 1960-2014					
	$\ln \tilde{A}$			$\ln X$	$\ln L$
	MFP	TFP	GDPW		
t-statistic	-3.34	-3.21	-2.95	-2.26	-1.38
LR statistic	10.68	9.96	8.64	5.27	2.05
Unit root	Yes	Yes	Yes	Yes	Yes
AR(p) process	1	1	2	2	3

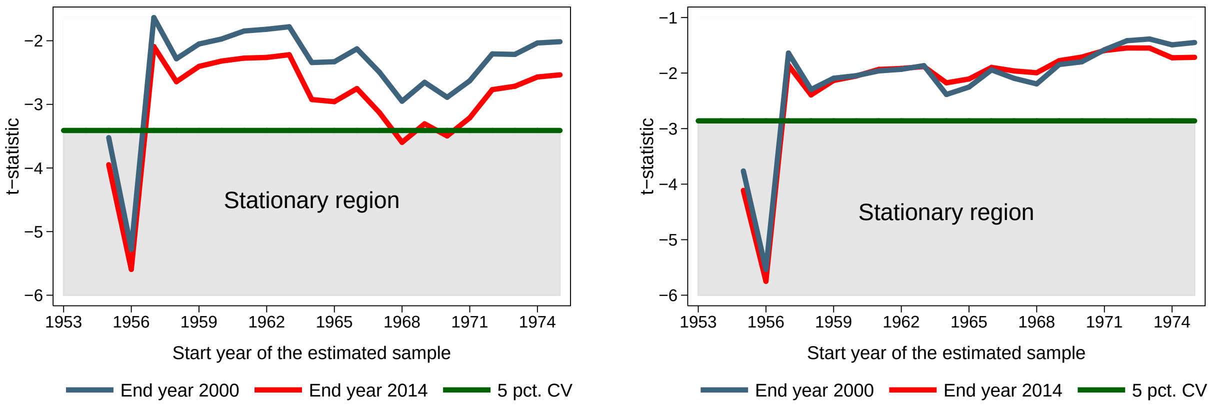
Notes : GDPW is GDP per worker. Bold indicates significance at the 5 pct. level. The models have been estimated with a trend and an intercept. The lag length is determined by the Schwarz criterion using the full sample. The 5 pct. critical values are -3.4 and 12.4 for the t-statistic and the LR statistic, respectively.

Table 1 reports unit root test results for the main variables. The results support the hypotheses that $\ln \tilde{A}_t$ and $\ln L_t$ are $I(1)$ for the periods 1953-2014 and 1960-2014. Using the whole sample (1953-2014), the test results clearly reject a unit root in $\ln X_t$, and thus the hypothesis that $\ln X_t$ is $I(1)$ is rejected. The stationarity of $\ln X_t$ coincides well with second-generation fully endogenous growth models. However, this stationarity result is not very robust. As shown in the lower part of Table 1, the hypothesis of a unit root in $\ln X_t$ cannot be rejected for the sample 1960-2014.[11]

In fact, backward recursive estimation indicates that $\ln X_t$ contains a unit root, when the estimated sample starts after 1956. Figure 5 shows backward recursively estimated t-statistics from AR(2) models with or without trends for $\ln X_t$ for samples ending in 2000 or 2014. In almost all cases, the hypothesis of a unit root in $\ln X_t$ cannot be rejected when the estimated sample starts after 1956. Similar patterns are obtained based on AR(1) and AR(3) models (see Figure 11 in Appendix C). Test statistics for samples ending in 2000 are included to illustrate the nonrobustness of the results obtained by Ha and Howitt (2007).

The rejection of a unit root in $\ln X_t$ using the full sample is a consequence of the large permanent increase in the level of $\ln X_t$ occurring in the 1950s. The test procedures interpret this as strong mean reversion which makes the time series appear more stationary than it

[11] Augmented Dickey-Fuller tests (without trends) clearly reject unit roots in $\Delta \ln \tilde{A}_t$ and $\Delta \ln L_t$ for both periods, further indicating that $\ln \tilde{A}_t$ and $\ln L_t$ are $I(1)$. Likewise, an augmented Dickey-Fuller test rejects a unit root in $\Delta \ln X_t$ for the period 1960-2014, indicating that $\ln X_t$ is $I(1)$ over this period.



(a) With trend **(b)** Without trend

FIGURE 5: Backward recursive estimation of t-statistics for AR(2) models with and without a trend for $\ln X_t$ for estimated sample ending in 2000 or 2014.
Notes: The 5 pct. critical value (5 pct. CV) is -3.41 with a trend and -2.86 without a trend.

actually is. It seems natural to assume that the rapid growth in the US research intensity occurring in the 1950s was caused by structural changes following World War II. To avoid potentially large effects from these unique structural changes, the main empirical analysis concentrates on the period 1960-2014.

The unit root test results clearly provide evidence against the fully endogenous variety. Still, the single-equation approach has some clear limitations like the inability to estimate $\tilde{\phi}$ and $\tilde{\beta}$. These limitations motivate the cointegrated VAR approach taken in the next section.

4.2 Cointegrated VAR analysis

Overview

This section provides estimation results for the VAR and cointegrated VAR models presented in Section 2.2. Estimation results are provided for all three productivity measures for the period 1960-2014. Three results are worth emphasizing before diving into the details of the estimation procedure. First, the trend can always be excluded from the cointegrating relationship (2.10). Second, when the trend is excluded, $\tilde{\phi}$ is estimated to be negative and significant, while the estimate of $\tilde{\beta}$ is positive and significant. Hence the results clearly favor the semi-endogenous variety. Third, the joint hypothesis $\tilde{\psi} = \tilde{\beta} = 0$ is accepted using MFP, borderline accepted using TFP, and rejected using GDP per worker, providing mixed evidence for the model of less-than-exponential growth.

Analysis

Estimating (2.12) using different samples, lag lengths, and dummies reveals some robust findings. First, the Schwarz criterion often suggests two lags (and sometimes three lags), while LR tests suggest three lags using a general-to-specific approach. Yet, the model with three lags is preferred as its estimates are more robust to various changes in the empirical approach. Second, Johansen tests never reject the hypothesis of a single cointegrating relationship which seems to support the general theoretical model. Johansen test results for all models are provided in Appendix C.1. Third, the models systematically detect outliers in 1963, 1974, 1982, 1993, and 2009. These outliers correspond to distinct events in the US economic history as discussed in Section 3.

As the cointegrated VAR model (2.13) is formulated in terms of first differences, a transitory dummy is given by a vector (0,...0,1,-1,0,...,0), while a permanent dummy is given by a vector (0,...0,1,0,...,0), see Juselius (2006, ch. 6). Transitory dummies are included to account for transitory effects to the system caused by distinct events like the oil price shock of 1974 caused by the 1973 oil embargo. This oil price shock had a strong negative effect on the MFP and TFP levels in 1974, where MFP and TFP were reduced by 3.6 and 3.5 pct., respectively. These drops in the MFP and TFP levels were followed by fast growth indicating that the shock had only a transitory effect.

A permanent dummy for 2009 is included in all models to account for the permanent drop in employment caused by the financial crisis of 2007–2008 (see Figure 4). Following the literature, the remaining dummies are detected from the estimated residuals. A dummy is included in the year with the largest residual and estimated again. This process continues until the largest (normalized) residual is below ± 2.5 (see Hendry and Juselius 2001). Using this procedure, transitory dummies for 1963, 1974, 1982, and 1993 are included using the MFP and TFP measures, while transitory dummies for 1963, 1974, and 1982 are included for the GDP per worker measure.

Table 2 reports estimation results from a cointegrated VAR model with three lags, transitory dummies for 1963, 1974, 1982 and 1993, and a permanent dummy for 2009 for the sample 1960-2014 using the MFP measure. The rank test and WS columns are based on the VAR model (2.12). The rank test column indicates that the hypothesis of a single cointegrating relationship cannot be rejected using the Johansen test for cointegration. The WS column indicates that the VAR model is well specified. A model is defined as well specified if the hypotheses of normal errors, no autocorrelation, and no ARCH effects cannot be rejected using multivariate tests at the 5 pct. level. Specification test descriptions and test statistics for the three models estimated in this section are provided in Appendix C.2.

TABLE 2: Estimation results from a cointegrated VAR model using MFP, sample 1960-2014

Model	N	Dummies		Lags	WS	Rank Test	Estimates			p-value
		Transitory	Permanent				$\tilde{\phi}$	$\tilde{\beta}$	Trend	
A1	55	1963, 1974, 1982, 1993	2009	3	+	+	-0.768	**1.493**	**-0.021**	
							(-0.932)	(3.549)	(-2.053)	
A1 (R1)							**-3.459**	**1.679**	0.000	0.607
							(-3.752)	(3.018)	-	
A1 (R2)							**-0.814**	0.000	0.000	0.124
							(-3.386)	-	-	
A1 (R3)							0.000	**-0.453**	0.000	0.045
							-	(-2.660)	-	

Notes: N denotes the number of observations, WS indicates whether a model is well specified ('+' if it is), and the rank test indicates whether the hypothesis of a single cointegrating relationship can be rejected by the Johansen test at the 5 pct. level ('+' if it cannot). The numbers in brackets are t-statistics. (R1) indicates that the trend coefficient is restricted to equal zero, (R2) indicates that the trend and $\tilde{\beta}$ are restricted to equal zero, and (R3) indicates that the trend and $\tilde{\phi}$ are restricted to equal zero. The p-values of the LR tests are adjusted for small sample size (Bartlett corrected p-values). All dummies are unrestricted.

The parameter estimates in Table 2 are based on the cointegrated VAR model (2.13). The unrestricted model, Model A1, estimates $\tilde{\phi}$ to be negative but insignificant. The Wald test suggest that the trend is significant, but the LR test suggest that it can be excluded from the model. As the LR test performs better than the Wald test for finite samples (Haug 2002), one should prioritize the LR test results. In a model where the trend is excluded from the cointegrating relationship, Model A1 (R1), $\tilde{\phi}$ is estimated to be negative and significantly different from zero which provides evidence against the fully endogenous variety. The estimate of $\tilde{\beta}$ is positive and significantly different from zero which provides evidence against second-generation fully endogenous growth models as well as the model of less-than-exponential growth. Yet, a LR test suggests that the employment variable can be excluded together with the trend, Model A1 (R2), which supports the case of less-than-exponential growth. In this case $\tilde{\phi}$ remains negative and significantly different from zero. Finally, the trend and the MFP variable cannot be excluded jointly, Model A1 (R3), and when they are, the employment variable becomes negative and significant which is inconsistent with the general model.

Table 3 provides results from a cointegrated VAR model using the TFP measure. It is clear from a comparison between Table 2 and 3 that the two models generate similar results both qualitatively and quantitatively. A notable exception is the LR test for restriction (R2), where the p-value is much lower using TFP compared to MFP. Thus the less-than-exponential growth case seems to demand too much of the data.[12]

TABLE 3: Estimation results from a cointegrated VAR model using TFP, sample 1960-2014

Model	N	Dummies		Lags	WS	Rank Test	Estimates			p-value
		Transitory	Permanent				$\tilde{\phi}$	$\tilde{\beta}$	Trend	
B1	55	1963, 1974, 1982, 1993	2009	3	-	+	-0.464	**1.537**	**-0.025**	
							(-0.622)	(4.010)	(-2.758)	
B1 (R1)							**-3.441**	**1.637**	0.000	0.268
							(-3.729)	(2.985)	-	
B1 (R2)							**-0.809**	0.000	0.000	0.063
							(-3.447)	-	-	
B1 (R3)							0.000	**-0.434**	0.000	0.020
							-	(-2.713)	-	

Notes: See notes to Table 2.

Estimation results using the GDP per worker measure are provided in Table 4. The general patterns from Table 2 are repeated except that the model of less-than-exponential growth, Model C1 (R2), is clearly rejected. Note that: $\ln y_t = \text{constant} + 1/(1-\alpha)\ln \tilde{A}_t$. Hence, this model estimates $(1-\alpha)\tilde{\phi}$ instead of just $\tilde{\phi}$. The point estimate of $(1-\alpha)\tilde{\phi}$ in Model C1 (R1) coincides surprisingly well with the point estimates of $\tilde{\phi}$ in Model A1 (R1) and Model B1 (R1) given that α equals about 0.34 empirically. The point estimate of $\tilde{\beta}$ in Model C1 (R1) is also close to the point estimates from Model A1 (R1) and Model B1 (R1).

TABLE 4: Estimation results from a cointegrated VAR model using GDP per worker, sample 1960-2014

Model	N	Dummies		Lags	WS	Rank Test	Estimates			p-value
		Transitory	Permanent				$(1-\alpha)\tilde{\phi}$	$\tilde{\beta}$	Trend	
C1	55	1963, 1974, 1982	2009	3	+	+	-0.233	**1.938**	-0.034	
							(-0.215)	(4.693)	(-1.609)	
C1 (R1)							**-2.199**	**1.603**	0.000	0.369
							(-6.337)	(5.287)	-	
C1 (R2)							**-0.868**	0.000	0.000	0.006
							(-3.704)	-	-	
C1 (R3)							0.000	**-0.933**	0.000	0.002
							-	(-2.734)	-	

Notes: See notes to Table 2.

A potential concern is the impact of the dummy variables. Yet, it turns out that the dummy variables are not crucial for the main conclusions. As the transitory dummies are added consecutively, three to four other models were in each case estimated before arriving

[12] Model B1 does not appear well specified as the hypothesis of normally distributed errors is borderline rejected. The normal distribution of errors increases the efficiency of the estimator, but it is not crucial for its properties (see Johansen 1991).

at the final model. Estimation results for these other models as well as models without any dummies are reported in Table 7, 8, and 9 in Appendix C. It appears that the general conclusions can be reached from these models as well. In particular, $\tilde{\phi}$ is always estimated to be negative and significant when the trend is excluded. Likewise, $\tilde{\beta}$ is always positive and significant when the trend is excluded. Furthermore, the trend can always be excluded from the cointegrating relationship. Meanwhile, excluding the trend and productivity variable jointly, or the trend and employment variable jointly, results in low p-values for the LR test.

All in all, the evidence seems to favor the semi-endogenous variety. The evidence supporting the model of less-than-exponential growth is fragile, indicating that this case demands too much of the data. Yet, the estimates clearly reject the hypothesis $\tilde{\phi} = 0$, providing further evidence against the fully endogenous variety. As shown in the next section, the main empirical results are robust to justifiable changes in the empirical approach.

5 Robustness Checks

In this section, it is shown that the results from Section 4.2 are robust to justifiable changes in the empirical approach. In general the patterns from Section 4.2 repeat: (1) the trend can be excluded, (2) when the trend is excluded, the estimate of $\tilde{\phi}$ is negative and significant while the estimate of $\tilde{\beta}$ is positive and significant, (3) when both the trend and the employment variable are excluded, the p-value of the LR test falls notably, (4) when excluding both the trend and the employment variable the estimate of $\tilde{\phi}$ becomes larger but remains negative and significant, and (5) both the trend and productivity variable are not or borderline excludable, and $\tilde{\beta}$ becomes negative when they are excluded.

Full sample

When the full sample is employed, it is necessary to add a permanent dummy for 1956 to account for the large permanent shock hitting the research intensity that year (see Figure 3). In addition, the data indicate that a transitory dummy should be added in 1959, where the economy was exiting the Eisenhower Recession. A multivariate test indicate that $\ln X_t$ is non-stationary for the sample 1953-2014 for a VAR model with three lags, transitory dummies for 1959, 1963, 1974, 1982 and 1993, and permanent dummies for 1956 and 2009. Accordingly, the main results from Section 4.2 should carry through using this VAR model and the entire sample. Estimation results using the MFP measure are provided in Table

5. In general, the qualitative patterns from the main analysis are repeated. One difference being that the hypothesis $\tilde{\beta} = 0$ is only rejected using a one-sided alternative (which seems appropriate given the alternative hypothesis $\tilde{\beta} > 0$). Similar results can be obtained using the TFP and GDP per worker measures, see Table 10 and 11 in Appendix C.

TABLE 5: Estimation results from a cointegrated VAR model using MFP, sample 1953-2014

Model	N	Dummies		Lags	WS	Rank Test	Estimates			p-value
		Transitory	Permanent				$\tilde{\phi}$	$\tilde{\beta}$	Trend	
A2	62	1959, 1963, 1974, 1982, 1993	1956, 2009	3	+	+	**-3.249** (-2.041)	1.230 (1.309)	0.005 (0.265)	
A2 (R1)							**-2.764** (-2.518)	1.284 (1.871)	0.000 -	0.944
A2 (R2)							**-0.762** (-3.285)	0.000 -	0.000 -	0.600
A2 (R3)							0.000 -	**-0.464** (-2.894)	0.000 -	0.403

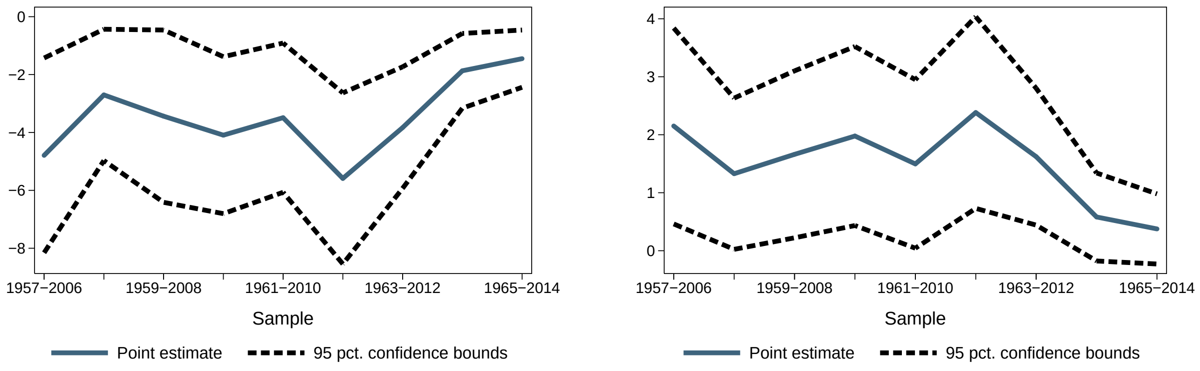
Notes: See notes to Table 2.

Moving window estimation

It is important to verify that the start and end years of the sample are not essential for the results. To verify that a cointegrated VAR model with three lags systematically rejects the hypotheses $\tilde{\phi} = 0$ and $\tilde{\beta} = 0$, Model A1 (R1) is estimated for a 50-year rolling window starting with the sample 1957-2006 and ending with the sample 1965-2014. The moving window approach ensures comparability between estimates as the power is kept constant. The starting sample 1957-2006 is chosen such that the large permanent shock to the research intensity in 1956 is avoided. The results are presented in Figure 6. The null hypothesis $\tilde{\phi} = 0$ is consistently rejected. Meanwhile, the point estimate of $\tilde{\beta}$ is systematically positive, while the hypothesis $\tilde{\beta} = 0$ can be rejected for most subsamples. In addition, all estimates for both $\tilde{\phi}$ and $\tilde{\beta}$ are consistent with the equivalent estimates for the period 1960-2014 (see Model A1 (R1) in Table 2). Finally, similar patterns can be obtained using Model B1 (R1) and C1 (R1), see Figure 12 and 13 in Appendix C.

Business sector R&D

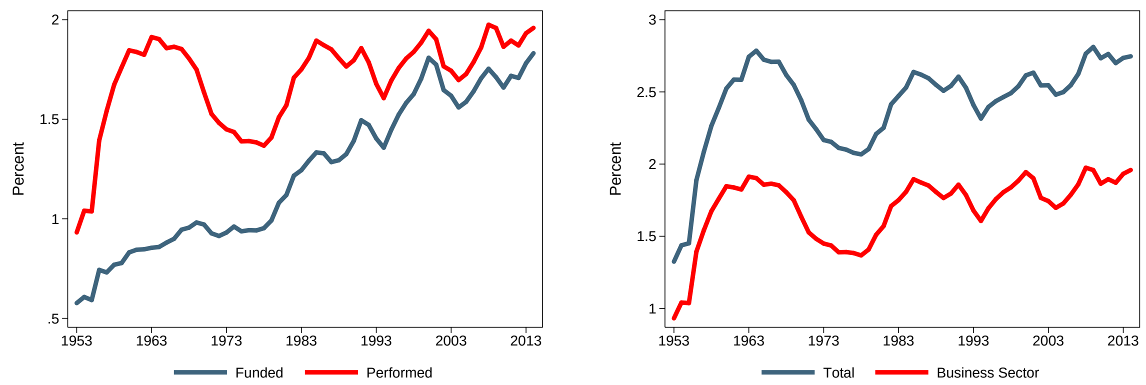
R&D-based growth models usually focus on research activities funded by private investors and conducted by private firms. The role of the government is confined to subsidizing these



(a) Estimates of $\tilde{\phi}$ **(b)** Estimates of $\tilde{\beta}$

FIGURE 6: Estimates of $\tilde{\phi}$ and $\tilde{\beta}$ based on Model A1 (R1) for 50-year samples, (1957-2006)-(1965-2014).

private research activities. Accordingly, one might argued that these models should be tested using business sector R&D expenditure data.



(a) Business sector R&D **(b)** R&D performance

FIGURE 7: US R&D expenditures over GDP, 1953-2014.
Data Sources: BEA and NSF.

The left panel of Figure 7 shows US R&D expenditures funded and performed by the business sector over GDP. US R&D expenditures funded by the business sector has increased much faster than GDP since 1953. As productivity growth slowed down over this period, fully endogenous models have no chance if R&D expenditures funded by the business sector is the correct measure to use. However, the correct measure appears to be R&D expenditures performed by the business sector.

Nevertheless, the variation in R&D expenditures performed by the business sector seems to mimic that of total R&D expenditures. This is clear from the right panel of Figure 7 which shows R&D expenditures performed in total and by the business sector over GDP. The two time series seem strongly correlated, and R&D expenditures performed by the business sector constitute roughly 70 pct. of total R&D expenditures in all years.

Given the high correlation between total R&D expenditures and R&D expenditures performed by the business sector, the obtained results using the business sector R&D expenditure data are similar to those obtained using total R&D expenditures. This is evident from the evidence provided in Table 6, and thus, the main results are robust to this alternative R&D expenditure measure. The same patterns can be obtained using the other two productivity measures, see Table 12 and 13 in Appendix C.

TABLE 6: Estimation results from a cointegrated VAR model using MFP and R&D expenditures performed by the business sector, sample 1960-2014

Model	N	Dummies		Lags	WS	Rank Test	Estimates			p-value
		Transitory	Permanent				$\tilde{\phi}$	$\tilde{\beta}$	Trend	
A3	55	1963, 1974, 1982, 1993	2009	3	+	+	-2.305	**1.766**	-0.014	
							(-1.784)	(2.582)	(-0.823)	
A3 (R1)							**-4.132**	**1.948**	0.000	0.819
							(-3.604)	(2.811)	-	
A3 (R2)							**-0.916**	0.000	0.000	0.198
							(-3.781)	-	-	
A3 (R3)							0.000	**-0.500**	0.000	0.065
							-	(-3.089)	-	

Notes: See notes to Table 2.

Further considerations

Based on the theoretical model, the present study uses R&D expenditures over GDP to measure the research intensity. Alternatively, one could formulate a model where the research intensity is given by researchers over population. This alternative measure would, however, not take potential changes in the capital-labor intensity of research into account. In addition, this measure increased substantially through the investigated period (Jones 2015). Thus the second-generation fully endogenous growth models have little chance if this alternative measure is employed.

One might argue that the LR tests should be conducted using bootstrap techniques given the small sample size and potential misspecifications (see Cavaliere et al. 2015; Boswijk et al. 2016). As p-values based on wild bootstrapping are generally higher compared to the Bartlett corrected counterparts, the model of less-than-exponential growth gains some additional support from these test results. Still, the p-values are reduced substantially when moving from restriction (R1) to restriction (R2).

Finally, it could be argued that the rank tests should be evaluated based on bootstrap methods (see Cavaliere et al. 2010a, 2010b). Yet rank tests based on wild bootstrapping confirm the previous finding; the hypothesis of a single cointegrating relationship cannot be rejected at the 5 pct. level in any of the presented models. The p-values based on bootstrapping are close to their Bartlett corrected counterparts. The only notable differences arise for the models based on the full sample. In either case, the hypothesis of a single cointegrating relationship is clearly accepted.

6 Simulation Results

This section provides US productivity forecasts based on the theoretical model and empirical estimates. The simulation exercise is based on the empirical estimates from Model A1 (R1) and Model A1 (R2). The former represents the semi-endogenous case, where long-run exponential growth is possible given exponential employment growth, while the latter represents the less-than-exponential growth case, where long-run exponential growth cannot be obtained.

The log discrete time approximation of (2.8) amounts to:

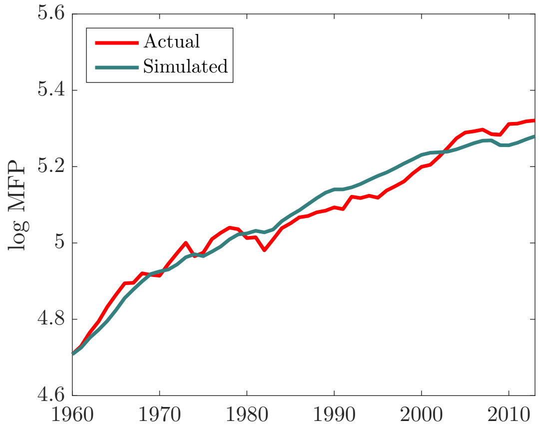
$$\ln A_{t+1} = \bar{\gamma} + \phi \ln A_t + \sigma \ln X_t + \sigma \left(1 - \beta + \beta \frac{\eta}{1-\alpha}\right) \ln L_t. \tag{6.1}$$

Given $\ln A_t$, the log of MFP, $\ln \tilde{A}_t$, can be computed using the formula:

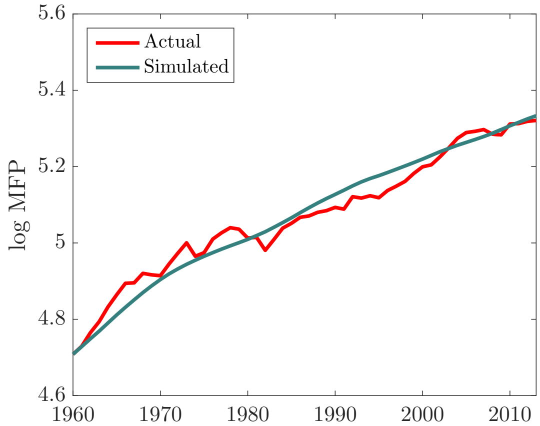
$$\ln \tilde{A}_t = (1-\alpha) \ln A_t + \beta\eta \ln L_t. \tag{6.2}$$

As the empirical results do not allow for a direct identification of α, β, η, $\bar{\gamma}$, ϕ, and σ, these values are determined in four steps. First, α and σ are set equal to 0.34 and 0.042, respectively. The former value is based on the US capital share of income for the period 1960-2014 as measured by the BLS, and the latter is taken from Venturini (2012). Second, it is assumed that $\beta = 1$, as it seems intuitive that the range of specialized capital good varieties is proportional to employment. Third, the parameters η and ϕ are computed from α, β and σ, and the empirical estimates of $\tilde{\phi}$ and $\tilde{\beta}$. Finally, $\bar{\gamma}$ is estimated by minimizing the sum of squared errors between actual and simulated MFP values for the period 1960-2014. The initial technological level, $\ln A_{1960}$, is computed from (6.2). Given $\ln A_{1960}$, $\{\ln X_t\}_{1960}^{2014}$, and $\{\ln L_t\}_{1960}^{2014}$ the predicted technological level is computed using (6.1), and the predicted MFP is computed using (6.2).

The actual and simulated MFP values for the period 1960-2014 are shown in Figure 8. The left panel shows the semi-endogenous case where $\eta \approx 0.34$ and $\phi \approx 0.90$, while the right panel shows the less-than-exponential growth case where $\eta = 0$ and $\phi \approx 0.98$. In both cases, the simulated values seem to capture the long-run trend in MFP. Note that in both cases, ϕ is close to one which might explain why previous studies have been unable to reject the hypothesis $\phi = 1$.



(a) Semi-endogenous case

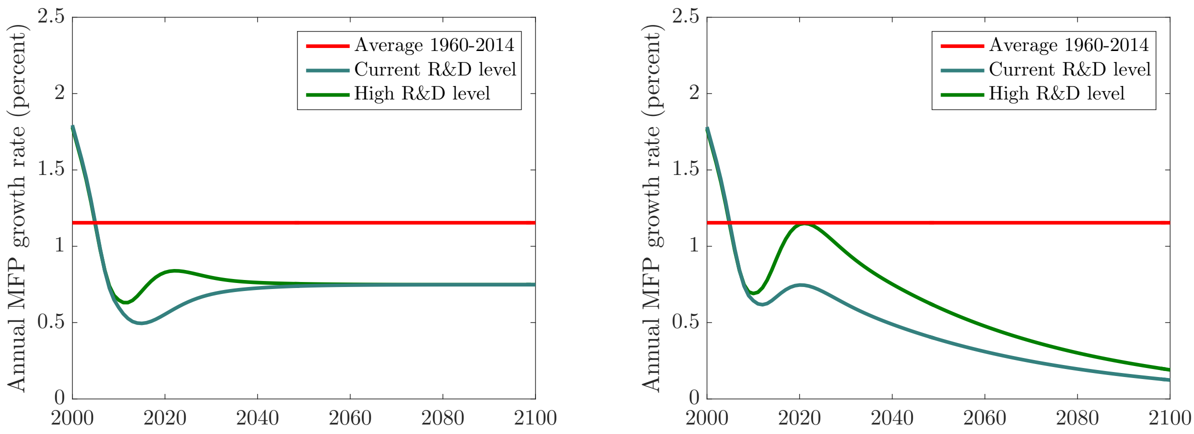


(b) Less-than-exponential growth case

FIGURE 8: Actual and simulated US MFP, 1960-2014.
Panel (a): Semi-endogenous case: $\eta \approx 0.34$ and $\phi \approx 0.90$.
Panel (b): Less-than-exponential growth case: $\eta = 0$ and $\phi \approx 0.98$.
Notes: All simulations are conducted using (6.1) and (6.2) as well as the actual 1960 log MFP value ($\ln \tilde{A}_{1960}$) and the actual time series for $\ln X_t$ and $\ln L_t$. The simulations are based on the empirical estimates from Model A1 (R1) (left panel) and Model A1 (R2) (right panel), the parameter values $\alpha = 0.34$, $\beta = 1$ and $\sigma = 0.042$, and estimated $\bar{\gamma}$ values.

To forecast MFP, assumptions on the future research intensity and employment are necessary. The research intensity is assumed to be at its 2014 level at all future dates (2.75 pct.). Employment is assumed to grow at its average (compound) growth rate from 1960 to 2014 (1.55 pct. per year). Yet due to declining labor force participation and an aging population, the BLS expects slower employment growth in the coming years (Richards 2015). Accordingly, the resulting MFP growth rate forecast should be viewed as an upper bound.

MFP growth forecasts based on the empirical estimates from Model A1 (R1) and Model A1 (R2) are shown in Figure 9. Actual MFP growth data are used for the period 2000-2014, while forecasted values are used for the period 2015-2100, and the entire series is smoothed using the HP filter. According to the semi-endogenous forecast, the MFP growth rate converges quickly to around 0.75 pct. per year. The corresponding GDP per worker growth rate is around 1.1 pct. per year, i.e. a 0.4 percentage point decline compared to the average growth rate from 1960 to 2014. However, the BLS expects employment to grow at 0.6 pct. per year over the period 2014-2024, and if the future employment growth rate is set equal to this value, the growth rates of MFP and GDP per worker converge to around 0.3



(a) Semi-endogenous case **(b)** Less-than-exponential growth case

FIGURE 9: Actual and forecasted US MFP growth rates, 2000-2100.
Panel (a): Semi-endogenous case: $\eta \approx 0.34$ and $\phi \approx 0.90$.
Panel (b): Less-than-exponential growth case: $\eta = 0$ and $\phi \approx 0.98$.
Notes: Actual values are used for the period 2000-2014 and forecasted values are used for the remaining period. The actual and forecasted MFP growth rates are smoothed using the HP filter with a smoothing factor of 100. All forecasts are conducted using (6.1) and (6.2) as well as the actual 2014 log MFP value ($\ln A_{2014}$). The research intensity, X_t, is set equal to its 2014 value (2.75 pct.) for the period 2015-2100, while the research intensity is 0.5 percentage points higher in the high R&D scenario. The employment level is assumed to grow at 1.55 pct. per year from 2014. The simulations are based on the empirical estimates from Model A1 (R1) (left panel) and Model A1 (R2) (right panel), the parameter values $\alpha = 0.34$, $\beta = 1$ and $\sigma = 0.042$, and estimated $\bar{\gamma}$ values.

and 0.45 pct. per year, respectively.

The less-than-exponential growth case is even more pessimistic. The MFP growth rate converges to zero which implies that the growth rate of GDP per worker converges to zero as well. The convergence is, however, slow such that the growth rates of MFP and GDP per worker are around 0.1 and 0.2 pct. per year, respectively, in 2100.

A natural policy response to these dismal MFP growth projections is to increase the research intensity. To check how an R&D boost affects future MFP growth, the MFP growth rate is simulated under the assumption that R&D expenditures over GDP are increased by 0.5 percentage points from the 2014 level (3.25 pct.). From Figure 9 it appears that this policy affects the MFP growth rate much more in the less-than-exponential growth case. In the semi-endogenous case, a boost to R&D expenditures increases MFP growth notably over about 25 years. In contrast, the effect is strong through most of the period in the less-than-exponential growth case.

7 Discussion

The empirical estimates obtained above reflect that spillovers in research are weaker than assumed in the fully endogenous variety. This result coincides with recent micro-level evidence. Bloom et al. (2017) show that research productivity seems to be declining in various industries, products, and firms. That is, it requires increasingly more research input to ensure constant exponential growth which coincides with the spillover assumption in the semi-endogenous variety.

Nevertheless, the results obtained in the present study contrasts with those obtained in many previous studies.[13] Explaining how these differences arise naturally leads to a discussion of the present study's main contributions. Starting with the theoretical model, previous macro-level studies have tested the semi-endogenous variety under the restrictions: $\beta = 0$ and $\eta = 0$. That is, previous studies eliminate the role of horizontal innovation in the semi-endogenous variety. These studies, therefore, only provide evidence against a particular subclass of the semi-endogenous variety. In addition, previous studies test the second-generation fully endogenous growth models under the restriction: $\eta = 0$. Thus productivity gains from horizontal innovation are assumed away from the outset, making a rejection of the fully endogenous variety less likely.

The present study also put more emphasis on two econometric considerations compared to previous macro-level studies. First, previous studies do little to check for structural changes. For instance, Ha and Howitt (2007) ignore that the US research intensity increased substantially during the 1950s (see Figure 1). As shown in Section 4.1, ignoring this transition has large implications for the obtained results. Second, previous studies provide little evidence supporting the main assumptions of the cointegrated VAR model. For example, Ha and Howitt (2007) use a Johansen test for cointegration without ensuring that the VAR model - which the Johansen test is based on - is well specified. To be valid, the Johansen test requires no (or weak) ARCH effects and no residual autocorrelation. As mentioned above, standard specification tests indicate that the VAR models used in the present study are well specified, that is the hypotheses of no ARCH effects and no residual autocorrelation are accepted, but only when including appropriate dummy variables to account for distinct events like the 1973 oil embargo and the financial crisis of 2007–2008. Yet, such dummy variables have not been employed in previous studies.

[13] These studies include Laincz and Peretto (2006), Ha and Howitt (2007), Madsen (2008) and Ang and Madsen (2011) based on macro-level data, and Ulku (2007) and Venturini (2012) based on micro-level data.

Naturally, the analysis conducted above has some important limitations. Knowledge spillovers between the US and other countries were largely ignored. However, if these spillovers were systematic, they should have been picked up by either the trend or the constant in the empirical model. Furthermore, the obtained results are based on relatively few observations. Given the data available today, the results appear robust. It is, however, not unthinkable that some of the conclusions reached in this paper might be challenged when another decade of data become available. Still, the employed dataset contains a lot of information on long-run economic growth, as it stretches over a long and relatively stable period in US economic history. Finally, the theoretical model focuses on the long-run relationship between the main variables. Transitional dynamics are thereby indirectly assumed to play a minor role which is motivated by the approximately constant capital-output ratio observed over the investigated period. Yet it is important to emphasize that the empirical model does not restrict the short-run dynamics, and thus adjustments to the long-run relationship might take various forms. Since the empirical model can distinguish between short and long-run relationships, the long-run parameters should be estimated appropriately.

8 Concluding Remarks

The results obtained in this paper support the view that long-run economic growth can be modeled using R&D-based growth models. Yet the fully endogenous variety is clearly rejected by the empirical evidence. In contrast, the empirical results support the semi-endogenous variety, while there seems to be mixed support for the less-than-exponential growth model. Thus the results suggest that future economic growth research builds on the semi-endogenous variety. Still, second-generation fully endogenous growth models are mathematically simpler, and thus, this variety can be useful in applications, where the main results are largely unaffected by the underlying growth model variety.

Forecasts based on the empirical results point toward a dismal future productivity growth perspective. This seems to support the innovation pessimism presented by Gordon (2012), who argues that innovation in the future does not have the same potential to create growth as innovation in the past. The results also seem to support the view that the slow recovery after the financial crisis of 2007–2008 was caused - at least partly - by slow growth in potential GDP (see Gordon 2014).

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A Equilibrium Stability

In this appendix, it is shown that with a constant savings rate, a constant research intensity, and a constant labor input growth rate, the long-run equilibrium is asymptotically stable and features a constant capital-output ratio. Only the semi-endogenous case ($\beta < 1$, $\phi < 1$, $\eta > 0$) is considered here, but the same properties hold for the other model varieties.

Let s_k denote the constant savings rate in physical capital, and let $\bar{X}$ denote the constant research intensity. To ensure positive consumption: $s_k + \bar{X} < 1$. The constant population growth rate is denoted $n > 0$ which implies that: $L(t) = L(0)e^{nt}$.

Consider the normalization: $\bar{z}(t) \equiv Z(t)/(A(t)L(t)^{1+\frac{\beta\eta}{1-\alpha}})$. From (2.4), (2.5), (2.7), and (2.8), it follows that the economy is described by the system:

$$\dot{\bar{k}}(t) = s_k \bar{k}(t)^{\alpha} - [\delta + \tilde{n} + g_A(t)]\,\bar{k}(t) \quad \text{and} \quad g_A(t) = \lambda A(t)^{\phi-1}\bar{X}^{\sigma}\bar{k}(t)^{\alpha\sigma}L(t)^{\sigma\left(1-\beta+\frac{\beta\eta}{1-\alpha}\right)},$$

where $\tilde{n} \equiv (1 + \beta\eta/(1-\alpha))n$ and $g_A(t) \equiv \dot{A}(t)/A(t)$. In the steady state equilibrium:

$$\bar{k}^* = \left(\frac{s_k}{\delta + \tilde{n} + g_A^*}\right)^{\frac{1}{1-\alpha}} \quad \text{and} \quad g_A^* = \frac{\sigma\left(1-\beta+\frac{\beta\eta}{1-\alpha}\right)}{1-\phi}n.$$

In this equilibrium, the capital-output ratio is constant: $K(t)/Y(t) = s_k/(\delta + \tilde{n} + g_A^*)$.

To show that the steady state equilibrium $(\bar{k}^*, g_A^*)$ is asymptotically stable, the above system is reformulated into an autonomous system. Consider the normalization: $\tilde{z}(t) \equiv Z(t)/(A^*(t)L(t)^{1+\frac{\beta\eta}{1-\alpha}})$, where $A^*(t) = A^*(0)e^{g_A^* t}$. It follows directly that: $\tilde{k}(t) = \bar{k}(t)a(t)$, where $a(t) \equiv A(t)/A^*(t)$. In the steady state equilibrium, $\bar{k}(t)$ and $a(t)$ are constant. Hence $\tilde{k}(t)$ must be constant as well. The system is rewritten in autonomous form:

$$\dot{\tilde{k}}(t) = s_k a(t)^{1-\alpha}\tilde{k}(t)^{\alpha} - \chi\tilde{k}(t) \quad \text{and} \quad \dot{a}(t) = \tilde{\lambda}a(t)^{\phi-\alpha\sigma}\tilde{k}(t)^{\alpha\sigma} - g_A^* a(t),$$

where $\chi \equiv \delta + \tilde{n} + g_A^*$ and $\tilde{\lambda} \equiv \lambda\bar{X}^{\sigma}A^*(0)^{\phi-1}L(0)^{\sigma\left(1-\beta+\frac{\beta\eta}{1-\alpha}\right)}$.

Define the functions: $\dot{\tilde{k}}(t) = g\left(\tilde{k}(t), a(t)\right)$ and $\dot{a}(t) = f\left(\tilde{k}(t), a(t)\right)$. In addition, define the matrix:

$$B \equiv \begin{pmatrix} \frac{\partial g(\tilde{k}^*, a^*)}{\partial \tilde{k}(t)} & \frac{\partial g(\tilde{k}^*, a^*)}{\partial a(t)} \\ \frac{\partial f(\tilde{k}^*, a^*)}{\partial \tilde{k}(t)} & \frac{\partial f(\tilde{k}^*, a^*)}{\partial a(t)} \end{pmatrix} = \begin{pmatrix} \alpha s_k (a^*)^{1-\alpha}(\tilde{k}^*)^{\alpha-1} - \chi & (1-\alpha)s_k(a^*)^{-\alpha}(\tilde{k}^*)^{\alpha} \\ \alpha\sigma\tilde{\lambda}(a^*)^{\phi-\alpha\sigma}(\tilde{k}^*)^{\alpha\sigma-1} & (\phi-\alpha\sigma)\tilde{\lambda}(a^*)^{\phi-\alpha\sigma-1}(\tilde{k}^*)^{\alpha\sigma} - g_A^* \end{pmatrix}.$$

Using that $(a^*)^{\phi-\alpha\sigma-1}(\tilde{k}^*)^{\alpha\sigma} = g_A^*/\tilde{\lambda}$ and $(\tilde{k}^*/a^*)^{\alpha-1} = \chi/s_k$:

$$B = \begin{pmatrix} (\alpha-1)\chi & (1-\alpha)s_k\left(\frac{\tilde{k}^*}{a^*}\right)^{\alpha} \\ \alpha\sigma g_A^*\left(\frac{\tilde{k}^*}{a^*}\right)^{-1} & (\phi-\alpha\sigma-1)g_A^* \end{pmatrix}.$$

The trace and determinant of B are given by:

$$tr(B) = -(1-\alpha)\chi - (1-\phi+\alpha\sigma)g_A^* < 0 \quad \text{and} \quad |B| = (1-\phi)(1-\alpha)\chi g_A^* > 0.$$

Thus the equilibrium point $(\tilde{k}^*, a^*)$ is locally asymptotically stable.

Figure 10 shows the phase diagram of the system. For the open set $\Re_+^2$, the two curves only intercept in the stable equilibrium $(\tilde{k}^*, a^*)$. Hence this equilibrium is unique and globally asymptotically stable.

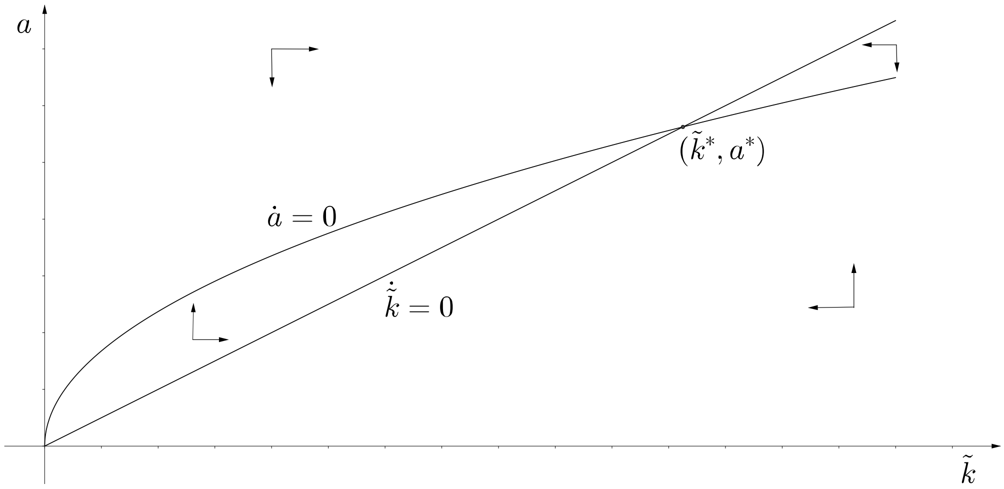


FIGURE 10: Phase diagram.

B The Four Growth Model Varieties

First-generation fully endogenous: $[n = 0,\ \phi = 1,\ \beta = 0,\ \eta = 0]$.

The growth rates of MFP and GDP per worker are given by

$$g_{\tilde{A}}(t) = (1-\alpha)\bar{\lambda}\bar{X}^{\sigma}L^{\sigma} \quad \text{and} \quad g_y(t) = \bar{\lambda}\bar{X}^{\sigma}L^{\sigma}.$$

Increasing the research intensity, $\bar{X}$, increases the growth rates of MFP and GDP per worker. The labor input is assumed constant, but if the labor input was allowed to grow at an exponential rate, the growth rates of MFP and GDP per worker would also increase at exponential rates. Economic growth would continuously accelerate which is clearly inconsistent with empirical evidence.

Second-generation fully endogenous: $[n \geq 0,\ \phi = 1,\ \beta = 1,\ \eta = 0]$.

The growth rates of MFP and GDP per worker are unaffected by the labor input, but otherwise the relations from the first-generation fully endogenous growth models carry through:

$$g_{\tilde{A}}(t) = (1-\alpha)\bar{\lambda}\bar{X}^{\sigma} \quad \text{and} \quad g_y(t) = \bar{\lambda}\bar{X}^{\sigma}.$$

Increasing the research intensity increases the growth rates of MFP and GDP per worker, but changes in the labor input have no effects on the growth rates. In fact, the growth rates are completely unaffected by the two state variables: $A(t)$ and $L(t)$.

Semi-endogenous: $[n \geq 0,\ \phi < 1,\ \beta < 1,\ \eta \geq 0]$ or $[n \geq 0,\ \phi < 1,\ \beta = 1,\ \eta > 0]$.
The technological growth rate evolves according to the differential equation:

$$G_A(t) \equiv \frac{\dot{g}_A(t)}{g_A(t)} = (\phi - 1)g_A(t) + \sigma\left(1 - \beta + \beta\frac{\eta}{1-\alpha}\right)n.$$

If $G_A(t)$ is positive, then $g_A(t)$ increases over time which reduces $G_A(t)$. Likewise, if $G_A(t)$ is negative, then $g_A(t)$ decreases over time which increases $G_A(t)$. As a consequence, $G_A(t)$ converges to zero regardless of the initial value of $g_A(t)$. Accordingly,

$$g_A(t) \to \sigma\left(1 - \beta + \beta\frac{\eta}{1-\alpha}\right)\left(\frac{n}{1-\phi}\right).$$

It follows that the long-run growth rates of MFP and GDP per worker are given by

$$g_{\tilde{A}}(t) = \left\{\beta\eta + (1-\alpha)\sigma\left(1 - \beta + \beta\frac{\eta}{1-\alpha}\right)\left(\frac{1}{1-\phi}\right)\right\}n \quad \text{and} \quad g_y(t) = \frac{g_{\tilde{A}}(t)}{1-\alpha}.$$

Thus the long-run growth rates of MFP and GDP per worker are proportional to the growth rate of the labor input. Still, both growth rates increase temporarily if the research intensity is increased. Over time both growth rates converge to their long-run values, but they might converge slowly.

Less-than-exponential: $[n \geq 0,\ \phi < 1,\ \beta = 1,\ \eta = 0]$.
The growth rates of MFP and GDP per worker approach zero as time goes to infinity. This is clear from the following expressions:

$$g_{\tilde{A}}(t) = (1-\alpha)\bar{\lambda}A(t)^{\phi-1}\bar{X}^{\sigma} \quad \text{and} \quad g_y(t) = \bar{\lambda}A(t)^{\phi-1}\bar{X}^{\sigma}.$$

To determine the path of $A(t)$, define the variable: $B(t) \equiv A(t)^{1-\phi}$. Differentiating $B(t)$ with respect to time yields: $\dot{B}(t) = (1-\phi)\bar{\lambda}\bar{X}^{\sigma}$. The solution to this differential equation is given by:

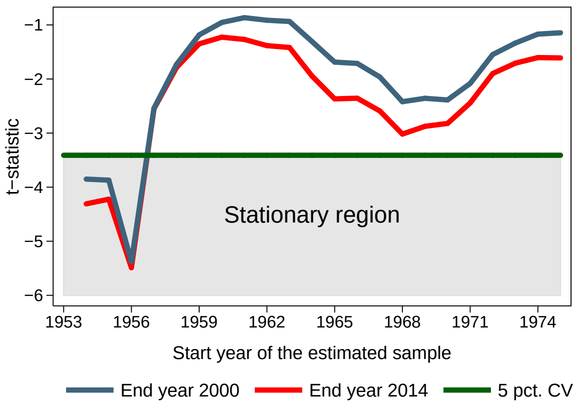
$$B(t) = B(0) + (1-\phi)\bar{\lambda}\bar{X}^{\sigma}t.$$

The technological level is, therefore, determined by:

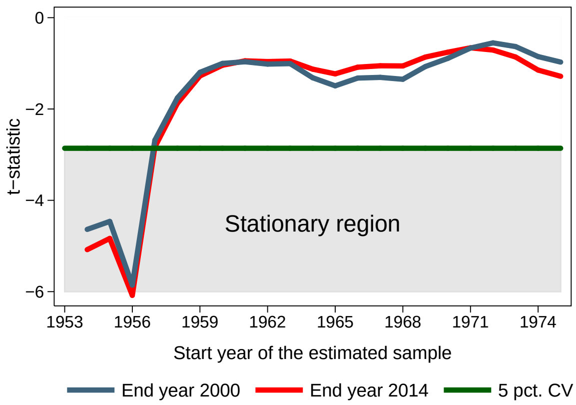
$$A(t) = A(0)\left[1 + (1-\phi)\bar{\lambda}\bar{X}^{\sigma}A(0)^{\phi-1}t\right]^{\frac{1}{1-\phi}}.$$

It follows from the expression that the model does feature perpetual growth, as MFP and GDP per worker go to infinity as time goes to infinity. Stagnation and exponential growth are given by the two limiting cases: $\phi \to -\infty$ and $\phi \to 1$. See Groth et al. (2010) for further discussion of models featuring less-than-exponential growth.

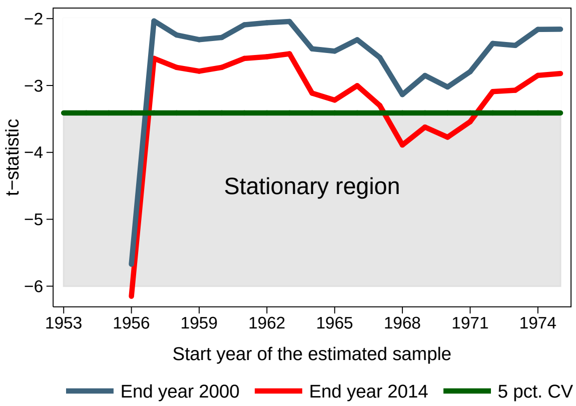
C Additional Empirical Results



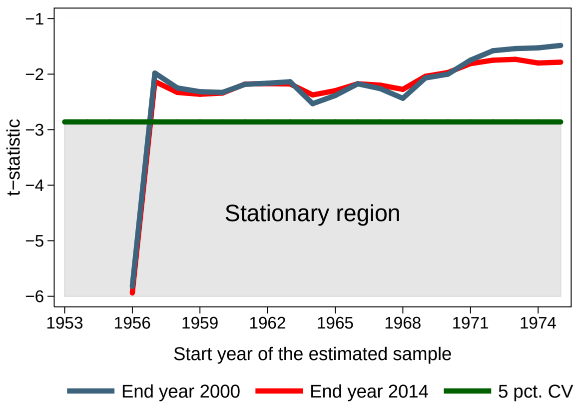
(a) AR(1) model with trend



(b) AR(1) model without trend



(c) AR(3) model with trend



(d) AR(3) model without trend

FIGURE 11: Backward recursive estimation of t-statistics for AR(1) and AR(3) models with and without a trend for $\ln X_t$ for estimated samples ending in 2000 or 2014.
Notes: The 5 pct. critical value (5 pct. CV) is -3.41 with a trend and -2.86 without a trend.

TABLE 7: Estimation results from a cointegrated VAR model using MFP with different dummy variables, sample 1960-2014

Model	N	Dummies		Lags	WS	Rank Test	Estimates			p-value
		Transitory	Permanent				$\tilde{\phi}$	$\tilde{\beta}$	Trend	
A1.0	55			3	-	+	**4.460**	**1.028**	**-0.060**	
							(5.461)	(2.337)	(-5.718)	
A1.0 (R1)							**-2.998**	**1.327**	0.000	0.093
							(-3.216)	(2.335)	-	
A1.0 (R2)							**-0.900**	0.000	0.000	0.068
							(-3.720)	-	-	
A1.0 (R3)							0.000	**-0.495**	0.000	0.024
							-	(-2.973)	-	
A1.1	55		2009	3	-	+	**8.043**	0.813	**-0.089**	
							(5.117)	(0.972)	(-4.421)	
A1.1 (R1)							**-3.490**	**1.620**	0.000	0.289
							(-3.607)	(2.758)	-	
A1.1 (R2)							**-0.905**	0.000	0.000	0.122
							(-3.720)	-	-	
A1.1 (R3)							0.000	**-0.502**	0.000	0.045
							-	(-2.974)	-	
A1.2	55	1974	2009	3	-	+	**13.693**	0.152	**-0.129**	
							(4.880)	(0.103)	(-3.607)	
A1.2 (R1)							**-3.712**	**1.776**	0.000	0.423
							(-3.853)	(3.040)	-	
A1.2 (R2)							**-0.876**	0.000	0.000	0.112
							(-3.610)	-	-	
A1.2 (R3)							0.000	**-0.480**	0.000	0.040
							-	(-2.833)	-	
A1.3	55	1974, 1982	2009	3	-	+	**56.800**	-4.484	**-0.441**	
							(4.329)	(-0.654)	(-2.645)	
A1.3 (R1)							**-3.756**	**1.825**	0.000	0.699
							(-3.879)	(3.115)	-	
A1.3 (R2)							**-0.851**	0.000	0.000	0.142
							(-3.453)	-	-	
A1.3 (R3)							0.000	**-0.464**	0.000	0.055
							-	(-2.688)	-	
A1.4	55	1974, 1982, 1993	2009	3	+	+	-1.469	**1.585**	-0.016	
							(-1.547)	(3.242)	(-1.371)	
A1.4 (R1)							**-3.657**	**1.785**	0.000	0.770
							(-3.865)	(3.122)	-	
A1.4 (R2)							**-0.828**	0.000	0.000	0.128
							(-3.407)	-	-	
A1.4 (R3)							0.000	**-0.455**	0.000	0.047
							-	(-2.663)	-	
A1	55	1963, 1972, 1982, 1993	2009	3	+	+	-0.768	**1.493**	**-0.021**	
							(-0.932)	(3.549)	(-2.053)	
A1 (R1)							**-3.459**	**1.679**	0.000	0.607
							(-3.752)	(3.018)	-	
A1 (R2)							**-0.814**	0.000	0.000	0.124
							(-3.386)	-	-	
A1 (R3)							0.000	**-0.453**	0.000	0.045
							-	(-2.660)	-	

Notes: See notes to Table 2.

TABLE 8: Estimation results from a cointegrated VAR model using TFP with different dummy variables, sample 1960-2014

Model	N	Dummies		Lags	WS	Rank Test	Estimates			p-value
		Transitory	Permanent				$\tilde{\phi}$	$\tilde{\beta}$	Trend	
B1.0	55			3	-	+	**4.505**	**1.081**	**-0.060**	
							(4.438)	(1.990)	(-4.789)	
B1.0 (R1)							**-2.632**	**1.107**	0.000	0.407
							(-2.831)	(1.984)	-	
B1.0 (R2)							**-0.859**	0.000	0.000	0.290
							(-3.657)	-	-	
B1.0 (R3)							0.000	**-0.460**	0.000	0.123
							-	(-2.993)	-	
B1.1	55		2009	3	-	+	**-5.956**	1.547	0.022	
							(-2.873)	(1.396)	(0.847)	
B1.1 (R1)							**-3.306**	**1.473**	0.000	0.860
							(-3.244)	(2.417)	-	
B1.1 (R2)							**-0.929**	0.000	0.000	0.307
							(-3.707)	-	-	
B1.1 (R3)							0.000	**-0.511**	0.000	0.127
							-	(-3.043)	-	
B1.2	55	1974	2009	3	-	+	**-2.993**	**1.643**	-0.005	
							(-2.299)	(2.376)	(-0.313)	
B1.2 (R1)							**-3.642**	**1.682**	0.000	0.937
							(-3.541)	(2.738)	-	
B1.2 (R2)							**-0.921**	0.000	0.000	0.200
							(-3.629)	-	-	
B1.2 (R3)							0.000	**-0.500**	0.000	0.073
							-	(-2.919)	-	
B1.3	55	1974, 1982	2009	3	-	+	-0.838	**1.566**	**-0.022**	
							(-1.018)	(3.614)	(-2.205)	
B1.3 (R1)							**-3.718**	**1.762**	0.000	0.497
							(-3.760)	(2.991)	-	
B1.3 (R2)							**-0.864**	0.000	0.000	0.100
							(-3.505)	-	-	
B1.3 (R3)							0.000	**-0.459**	0.000	0.033
							-	(-2.757)	-	
B1.4	55	1974, 1982, 1993	2009	3	-	+	-0.482	**1.558**	**-0.025**	
							(-0.634)	(3.964)	(-2.717)	
B1.4 (R1)							**-3.625**	**1.730**	0.000	0.302
							(-3.760)	(3.019)	-	
B1.4 (R2)							**-0.831**	0.000	0.000	0.068
							(-3.431)	-	-	
B1.4 (R3)							0.000	**-0.443**	0.000	0.023
							-	(-2.702)	-	
B1	55	1963, 1974, 1982, 1993	2009	3	-	+	-0.464	**1.537**	**-0.025**	
							(-0.622)	(4.010)	(-2.758)	
B1 (R1)							**-3.441**	**1.637**	0.000	0.268
							(-3.729)	(2.985)	-	
B1 (R2)							**-0.809**	0.000	0.000	0.063
							(-3.447)	-	-	
B1 (R3)							0.000	**-0.434**	0.000	0.020
							-	(-2.713)	-	

Notes: See notes to Table 2.

TABLE 9: Estimation results from a cointegrated VAR model using GDP per worker with different dummy variables, sample 1960-2014

Model	N	Dummies		Lags	WS	Rank Test	Estimates			p-value
		Transitory	Permanent				$(1-\alpha)\tilde{\phi}$	$\tilde{\beta}$	Trend	
C1.0	55			3	-	+	0.924	**1.916**	**-0.050**	
							(0.949)	(4.980)	(-2.621)	
C1.0 (R1)							**-1.968**	**1.381**	0.000	0.243
							(-5.501)	(4.398)	-	
C1.0 (R2)							**-0.860**	0.000	0.000	0.015
							(-3.886)	-	-	
C1.0 (R3)							0.000	**-1.035**	0.000	0.004
							-	(-2.935)	-	
C1.1	55		2009	3	-	+	-0.431	**1.869**	-0.030	
							(-0.384)	(4.323)	(-1.373)	
C1.1 (R1)							**-2.172**	**1.561**	0.000	0.478
							(-6.097)	(5.007)	-	
C1.1 (R2)							**-0.884**	0.000	0.000	0.011
							(-3.891)	-	-	
C1.1 (R3)							0.000	**-0.966**	0.000	0.003
							-	(-2.928)	-	
C1.2	55	1982	2009	3	-	+	-0.358	**1.884**	-0.031	
							(-0.333)	(4.541)	(-1.484)	
C1.2 (R1)							**-2.162**	**1.568**	0.000	0.420
							(-6.330)	(5.240)	-	
C1.2 (R2)							**-0.869**	0.000	0.000	0.007
							(-3.848)	-	-	
C1.2 (R3)							0.000	**-0.953**	0.000	0.002
							-	(-2.864)	-	
C1.3	55	1963, 1982	2009	3	+	+	-0.177	**1.884**	-0.033	
							(-0.166)	(4.627)	(-1.630)	
C1.3 (R1)							**-2.110**	**1.524**	0.000	0.367
							(-6.183)	(5.108)	-	
C1.3 (R2)							**-0.844**	0.000	0.000	0.008
							(-3.838)	-	-	
C1.3 (R3)							0.000	**-0.942**	0.000	0.002
							-	(-2.875)	-	
C1	55	1963, 1974, 1982	2009	3	+	+	-0.233	**1.938**	-0.034	
							(-0.215)	(4.693)	(-1.609)	
C1 (R1)							**-2.199**	**1.603**	0.000	0.369
							(-6.337)	(5.287)	-	
C1 (R2)							**-0.868**	0.000	0.000	0.006
							(-3.704)	-	-	
C1 (R3)							0.000	**-0.933**	0.000	0.002
							-	(-2.734)	-	
C1.4	55	1963, 1974, 1982, 1993	2009	3	+	+	-0.394	**1.927**	-0.031	
							(-0.358)	(4.632)	(-1.467)	
C1.4 (R1)							**-2.197**	**1.607**	0.000	0.388
							(-6.436)	(5.390)	-	
C1.4 (R2)							**-0.871**	0.000	0.000	0.006
							(-3.704)	-	-	
C1.4 (R3)							0.000	**-0.970**	0.000	0.002
							-	(-2.741)	-	

Notes: See notes to Table 2.

TABLE 10: Estimation results from a cointegrated VAR model using TFP, sample 1953-2014

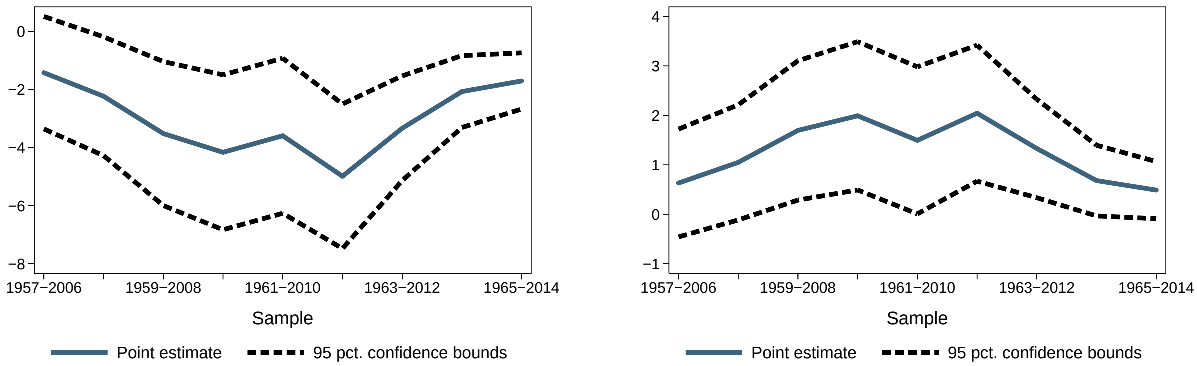
Model	N	Dummies		Lags	WS	Rank Test	Estimates			p-value
		Transitory	Permanent				$\tilde{\phi}$	$\tilde{\beta}$	Trend	
B2	62	1959, 1963, 1974, 1982, 1993	1956, 2009	3	+	+	-1.249 (-1.043)	**1.565** (2.241)	-0.019 (-1.286)	
B2 (R1)							**-2.781** (-2.349)	1.266 (1.741)	0.000 -	0.672
B2 (R2)							**-0.774** (-3.199)	0.000 -	0.000 -	0.557
B2 (R3)							0.000 -	**-0.463** (-2.849)	0.000 -	0.375

Notes: See notes to Table 2.

TABLE 11: Estimation results from a cointegrated VAR model using GDP per worker, sample 1953-2014

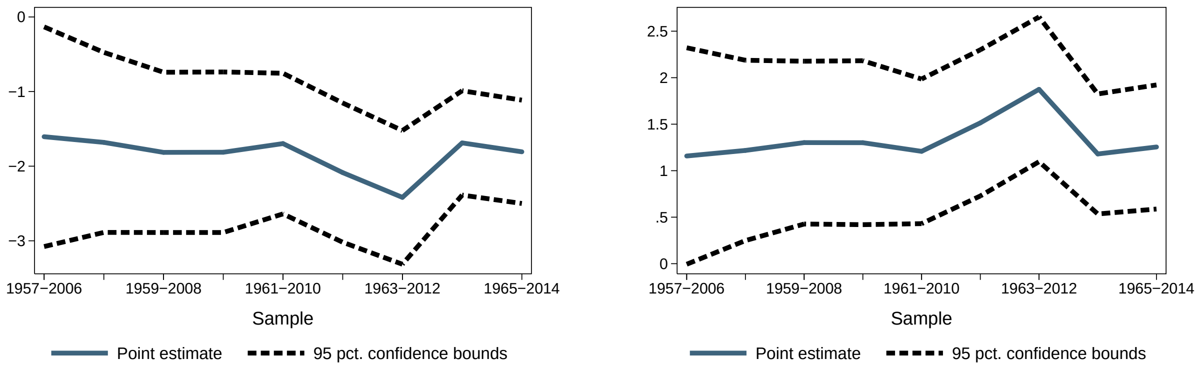
Model	N	Dummies		Lags	WS	Rank Test	Estimates			p-value
		Transitory	Permanent				$(1-\alpha)\tilde{\phi}$	$\tilde{\beta}$	Trend	
C2	62	1959, 1963, 1974, 1982, 1993	1956, 2009	3	+	+	0.337 (0.264)	**2.444** (5.024)	**-0.051** (-2.102)	
C2 (R1)							**-2.588** (-5.553)	**1.947** (4.795)	0.000 -	0.252
C2 (R2)							**-0.646** (-3.481)	0.000 -	0.000 -	0.013
C2 (R3)							0.000 -	**-0.587** (-2.822)	0.000 -	0.007

Notes: See notes to Table 2.



(a) Estimates of $\tilde{\phi}$ **(b)** Estimates of $\tilde{\beta}$

FIGURE 12: Estimates of $\tilde{\phi}$ and $\tilde{\beta}$ based on Model B1 (R1) for 50-year samples, (1957-2006)-(1965-2014).



(a) Estimates of $(1-\alpha)\tilde{\phi}$ **(b)** Estimates of $\tilde{\beta}$

FIGURE 13: Estimates of $(1-\alpha)\tilde{\phi}$ and $\tilde{\beta}$ based on Model C1 (R1) for 50-year samples, (1957-2006)-(1965-2014).

TABLE 12: Estimation results from a cointegrated VAR model using TFP and R&D expenditures performed by the business sector, sample 1960-2014

Model	N	Dummies		Lags	WS	Rank Test	Estimates			p-value
		Transitory	Permanent				$\tilde{\phi}$	$\tilde{\beta}$	Trend	
B3	55	1963, 1972, 1982, 1993	2009	3	+	+	-0.762	**1.723**	**-0.027**	
							(-0.784)	(3.345)	(-2.269)	
B3 (R1)							**-4.297**	**1.995**	0.000	0.360
							(-3.648)	(2.846)	-	
B3 (R2)							**-0.937**	0.000	0.000	0.101
							(-3.830)	-	-	
B3 (R3)							0.000	**-0.494**	0.000	0.028
							-	(-3.111)	-	

Notes: See notes to Table 2.

TABLE 13: Estimation results from a cointegrated VAR model using GDP per worker and R&D expenditures performed by the business sector, sample 1960-2014

Model	N	Dummies		Lags	WS	Rank Test	Estimates			p-value
		Transitory	Permanent				$(1-\alpha)\tilde{\phi}$	$\tilde{\beta}$	Trend	
C3	55	1963, 1974, 1982, 1993	2009	3	+	+	-0.886	**1.990**	-0.027	
							(-0.655)	(-3.882)	(-1.045)	
C3 (R1)							**-2.492**	**1.736**	0.000	0.545
							(-6.016)	(4.785)	-	
C3 (R2)							**-0.832**	0.000	0.000	0.010
							(-4.422)	-	-	
C3 (R3)							0.000	**-0.854**	0.000	0.002
							-	(-3.180)	-	

Notes: See notes to Table 2.

C.1 Johansen Test Results

The hypothesis of a single cointegrating relationship is accepted if it cannot be rejected by the Johansen test at the 5 pct. level of significance. The critical values are simulated using CATS in WinRATS 9.0 (5,000 replications and a random walk length of 400). See Juselius (2006, ch. 8) for further details. The test results are provided in Table 14. The hypothesis of a single cointegrating relationship is clearly accepted at the 5 pct. level for all nine models.

TABLE 14: Johansen test results for a single cointegrating relationship

Model	Trace	Trace*	CV_{95}	p-value	p-value*
Model A1	17.0	14.7	25.2	0.39	0.58
Model B1	12.7	11.0	25.3	0.74	0.86
Model C1	15.9	12.1	25.0	0.46	0.77
Model A2	13.4	10.0	26.4	0.72	0.92
Model B2	10.6	8.0	26.2	0.89	0.98
Model C2	15.3	9.4	25.8	0.57	0.94
Model A3	16.3	14.2	25.4	0.45	0.62
Model B3	12.7	10.9	25.1	0.74	0.86
Model C3	14.0	11.3	25.3	0.63	0.84

Notes: * indicates correction for small sample size, and CV_{95} is the 95 pct. critical level.

C.2 Specification Tests

A model is considered well specified if the hypotheses of normal errors, no autocorrelation, and no ARCH effects cannot be rejected at the 5 pct. level of significance using multivariate tests. This Appendix provides both multivariate and univariate test results. The univariate results are useful when diagnosing the source of misspecification. All test results are obtained from CATS in WinRATS 9.0. See Juselius (2006, ch. 4) for further details about the test procedures.

Tests for no autocorrelation: LM tests for first and second-order autocorrelation.

Tests for no ARCH effects: Multivariate LM tests for first and second-order ARCH effects, and univariate tests for ARCH effect.

Normality: Univariate and multivariate tests for normality.

TABLE 15: Model A1: Multivariate Statistics

Autocorrelation		Normality	ARCH	
LM(1)	LM(2)		LM(1)	LM(2)
13.4	8.1	7.4	19.0	48.5
[0.15]	[0.53]	[0.29]	[0.99]	[0.99]

Notes: p-values in brackets.

TABLE 16: Model A1: Univariate Statistics

Variable	ARCH	Normality	R^2
$\Delta \ln \tilde{A}$	0.64	1.60	0.68
	[0.89]	[0.45]	
$\Delta \ln X$	1.49	6.92	0.60
	[0.69]	[0.03]	
$\Delta \ln L$	2.57	2.18	0.69
	[0.46]	[0.34]	

Notes: p-values in brackets.

TABLE 17: Model B1: Multivariate Statistics

Autocorrelation		Normality	ARCH	
LM(1)	LM(2)		LM(1)	LM(2)
16.2	6.8	12.7	42.0	67.5
[0.06]	[0.66]	[0.05]	[0.23]	[0.63]

Notes: p-values in brackets.

TABLE 18: Model B1: Univariate Statistics

Variable	ARCH	Normality	R^2
$\Delta \ln \tilde{A}$	1.21	2.60	0.65
	[0.75]	[0.27]	
$\Delta \ln X$	1.05	8.03	0.60
	[0.79]	[0.02]	
$\Delta \ln L$	2.19	3.94	0.72
	[0.54]	[0.14]	

Notes: p-values in brackets.

TABLE 19: Model C1: Multivariate Statistics

Autocorrelation		Normality	ARCH	
LM(1)	LM(2)		LM(1)	LM(2)
10.3	7.7	4.3	23.9	61.5
[0.33]	[0.57]	[0.64]	[0.94]	[0.81]

Notes: p-values in brackets.

TABLE 20: Model C1: Univariate Statistics

Variable	ARCH	Normality	R^2
$\Delta \ln \tilde{A}$	1.99	1.20	0.79
	[0.57]	[0.55]	
$\Delta \ln X$	1.29	2.98	0.57
	[0.73]	[0.23]	
$\Delta \ln L$	4.88	2.62	0.71
	[0.18]	[0.27]	

Notes: p-values in brackets.