Discussion Papers
Department of Economics
University of Copenhagen

No. 16-06

Directed Technical Change and Economic Growth Effects of Environmental Policy

Peter K. Kruse-Andersen

Øster Farimagsgade 5, Building 26, DK-1353 Copenhagen K., Denmark
Tel.: +45 35 32 30 01 – Fax: +45 35 32 30 00
http://www.econ.ku.dk

ISSN: 1601-2461 (E)

Directed Technical Change and Economic Growth Effects of Environmental Policy

Version 2.7

By Peter K. Kruse-Andersen*

June, 2016

Abstract

A Schumpeterian growth model is developed to investigate how environmental policy affects economic growth when environmental policy also affects the direction of technical change. In contrast to previous models, production and pollution abatement technologies are embodied in separate intermediate good types. A set of stylized facts related to pollution emission, environmental policy, and pollution abatement expenditures is presented, and it is shown that the developed model is consistent with these stylized facts. It is shown analytically that a tightening of the environmental policy unambiguously directs research efforts toward pollution abatement technologies and away from production technologies. This directed technical change reduces economic growth and pollution emission growth. Simulation results indicate that even large environmental policy reforms have small economic growth effects. However, these economic growth effects have relatively large welfare effects which suggest that static models and exogenous growth models leave out an important welfare effect of environmental policy.

Keywords: Directed technical change, endogenous growth, pollution, environmental policy, Schumpeterian growth model

JEL Classification: O30, O41, O44, Q55, Q58

---

[*] Department of Economics, University of Copenhagen, Øster Farigmagsgade 5, Bygning 26, 1353 København K, Denmark; e-mail: peter.kjer.kruse-andersen@econ.ku.dk; phone: +45 35 32 38 87; fax: +45 35 32 35 32. I would like to thank Christian Groth, Sjak Smulders, Inge van den Bijgaart, Poul Schou, Samuel J. Okullo, Ingmar Schumacher, Leonardo Esteban Salazar Vergara as well as conference, workshop, and seminar participants at the Public Economic Theory Association Conference 2015 (PET15), the DGPE Workshop 2015, the University of Copenhagen, Tilburg Sustainability Center (TSC), and Sustainable Resource Use and Economic Dynamics (SURED 2016) for useful comments and suggestions. Earlier versions of this project have circulated under the titles "Growth Effects of Environmental Policy, when Production and Abatement Technologies are Embodied in Separate Intermediates" and "Growth Effects of Environmental Policy, when Production and Pollution Abatement Technologies are Embodied in Separate Intermediates".

1 Introduction

Extensive empirical work provides evidence suggesting that a tighter environmental policy stimulates environmental innovation (e.g., Brunnermeier and Cohen 2003; Johnstone and Labonne 2006; Popp 2006; Arimura et al. 2007; Ambec et al. 2011; Haščič et al. 2012; Aghion et al. 2016). The question is: does this stimulation come at the expense of other types of research? If so, environmental policy might have a negative effect on economic growth.

In this paper, I develop a Schumpeterian growth model to investigate how a tighter environmental policy affects economic growth when environmental policy also affects the direction of research efforts. The model is constructed such that it matches several stylized facts concerning pollution emission, environmental policy, and pollution abatement expenditures. I show analytically that a tighter environmental policy unambiguously reduces the economic growth rate as well as the growth rate of pollution emission. However, when the model is calibrated to the US economy, simulations indicate that even large environmental policy reforms barely affect the economic growth rate. This finding appears remarkably robust to changes in parameter values and calibration targets. Nevertheless, the simulations indicate that the economic growth effects constitute a large share of the overall welfare effects of environmental policy changes, as even small changes in growth rates have large level effects in the long run. Thus, the results indicate that static models and exogenous growth models (like the DICE model) leave out an important welfare effect of environmental policy.

Besides the policy implications, this analysis also contributes to the literature on directed technical change and the environment by developing a novel modeling strategy. Specifically, I develop a Schumpeterian growth model where production and pollution abatement technologies are embodied in separate intermediate good types. The R&D sector is bifurcated into two subsectors: one for production technologies and one for pollution abatement technologies. In contrast, Hart (2004, 2007) and Ricci (2007a) assume that production and pollution abatement technologies are embodied in the same intermediate goods. In their work, intermediate goods can be improved along two dimensions: productivity and environmental friendliness. As more environmentally friendly intermediates are less productive, R&D firms face a design trade-off when attempting to develop higher intermediate good qualities.

To illustrate the difference between the two modeling strategies, imagine a firm obtaining a patent on a new quality of a certain engine type. If production and pollution abatement technologies are embodied in the same intermediate goods (as assumed by Hart [2004, 2007] and Ricci [2007a]), the new engine quality is both more powerful and more environmentally friendly compared to the previous qualities. If the two technologies are embodied in separate intermediates (as in this analysis), one firm would obtain a patent on a more powerful engine, while another firm would obtain a patent on a catalyst, that could be implemented into the engine to reduce pollution emission. Hence, innovation arrivals of pollution abatement technologies are detached from innovation arrivals of production technologies. If a firm develops both production and pollution abatement technologies, these will be developed in separate R&D units. One unit can be successful during a certain time interval while the other fails. If the two technologies are embodied in the same intermediate, the firm has only one R&D unit. Either the firm develops a more productive and more environmentally friendly intermediate good quality, or no innovation occurs.

Separating the two technology types results in a more realistic representation of the innovation process for at least two reasons. Firstly, it seems natural to assume that the innovation arrivals of production and pollution abatement technologies are independent. Certainly, it is possible to invent a better catalyst without also inventing a more powerful engine. Secondly, to a large extent, private firms are only willing to conduct research, when the developed ideas can be protected. As patents are granted very specific components rather than entire systems, it seems appropriate to assume that production and pollution abatement technologies are developed separately.

Additionally, this framework seems to foster tractability and empirical relevance. The policy implications for economic growth are derived analytically, and they are unambiguous. In addition, the model presented below matches several stylized facts concerning pollution emission, environmental policy, and pollution abatement expenditures. Finally, in contrast to some other models in the literature, research occurs simultaneously in both R&D subsectors which seems like the more empirically relevant case.

Besides the works by Hart (2004, 2007) and Ricci (2007a), this analysis relates to several strands of literature. Firstly, it is related to a large body of literature investigating how environmental policy affects economic growth (e.g., Gradus and Smulders 1993; Bovenberg and de Mooij 1997; Nielsen et al. 1995; Hettich 1998; Schou 2002). These studies are typically novel in their identification of channels through which environmental policy might enhance growth, but they do not feature directed technical change.[1]

Secondly, the methodology used in this paper is closely related to works by Smulders and de Nooij (2003), Brock and Taylor (2010), and André and Smulders (2014). Specifically, these works develop growth models which match certain stylized facts to answer environmentally related questions. The main goal of this paper is not to explain the stylized facts presented below. Rather, the model is designed to match the stylized facts to ensure that it is empirically relevant. Other models in the literature have been developed with little or no regards to empirical tendencies. As a result, it is difficult to assess the usefulness of their policy implications.

Thirdly, this paper relates to a growing body of literature investigating how environmental pol-

---

[1] See Ricci (2007b) for a survey about channels through which environmental policy affects economic growth.

icy affects the direction of technological change in endogenous growth models.[2] In these works, it is usually assumed that output is produced using a constant elasticity of substitution production technology with two input types: clean and dirty. Pollution emission is an unavoidable by-product associated with dirty input use while no pollution emission comes from clean input use. Environmental policy can then skew incentives such that it becomes relatively more attractive to conduct research in clean input technologies. The modeling strategy developed in this paper might be a useful alternative to the clean-dirty-input approach. Specifically, the modeling strategy developed in this paper seems appropriate when considering pollutants, where emissions can be substantially reduced by end-of-pipe technologies.[3]

The model developed in this paper has the advantage that it clearly illustrates how environmental policy influences the intersectoral labor allocation between production and research as well as the intrasectoral labor allocations for these two sectors. In contrast, some studies have definitively shut down intersectural labor allocation effects. Acemoglu et al. (2012) assume a constant labor input in both production and research. Thus, environmental policy can only affect the intrasectoral labor allocations. As the trade-off between production and research is eliminated, their model does not qualify as a (complete) endogenous growth model (Pottier et al. 2014). In this paper, I show that environmental policy will affect the intersectoral labor allocation except for in a special knife-edge case. This intersectoral effect is important when investigating economic growth effects of environmental policy changes, as the labor reallocation between production and research implies intertemporal changes in the production capacity.

The paper proceeds as follows. The next section presents stylized facts related to pollution emission, environmental policy, and pollution abatement expenditures. Section 3 presents a Schumpeterian growth model consistent with the stylized facts. The policy implications of the model are derived analytically in Section 4, and natural extensions of the model are discussed in Section 5. Section 6 provides a quantitative analysis, and the results are discussed in Section 7. Section 8 concludes.

2 Stylized Facts

Some of the stylized facts presented in this section serve as motivation for model assumptions made in the subsequent section. The remaining facts serve as a first empirical test of the model.[4] In

---

[2] E.g., Saint-Paul (2002), Smulders and de Nooij (2003), Grimaud and Rouge (2008), Acemoglu et al. (2012), André and Smulders (2014), van den Bijgaart (2014), and Hemous (2015).

[3] An example would be SO_2 emissions. de Bruyn (1999, p. 172-175) argues that end-of-pipe technologies were the most important determinant for the substantial reduction in SO_2 emissions from 1980 to 1990 for West Germany and the Netherlands.

[4] Some of the stylized facts presented below are to my knowledge new to the literature. The rest have been presented in either Brock and Taylor (2005, 2010) or Botta and Kozluk (2014).

Section 3.3, I show how the model presented in Section 3 matches the stylized facts presented in this section.

2.1 Pollution emissions and intensities

Following Brock and Taylor (2005, 2010), I focus on air pollution, and in particular, the air pollutants CO, NO_x, SO_2 (or SO_x), VOC (volatile organic compounds), and CO_2. As shown in the left panel of Figure 1, US pollution intensities - defined as emissions over GDP - decreased almost monotonically for all five air pollutants over the period 1940-2014. The right panel of Figure 1 shows that US pollution emissions did not exhibit the same clear trend. Instead, emissions for the four pollutants CO, NO_x, SO_2, and VOC increased until around 1970. After 1973, CO, SO_2, and VOC emissions decreased rapidly. The exception is CO_2 emission which increased systematically over the period, while NO_x emissions did not decrease notably before the late 1990es. As income increased with an approximately constant growth rate throughout the period, it has been hypothesized that there exists an inverted U-shaped relationship between income and pollution emission. This relationship is usually referred to as the environmental Kuznets curve in the literature (see Stern 2014). These tendencies lead to the first stylized fact.

Stylized Fact 1. *Pollution emissions might increase or decrease while income increases. Meanwhile, pollution intensities decrease with income.*

A model consistent with Stylized Fact 1 predicts that pollution emissions grow slower than income.

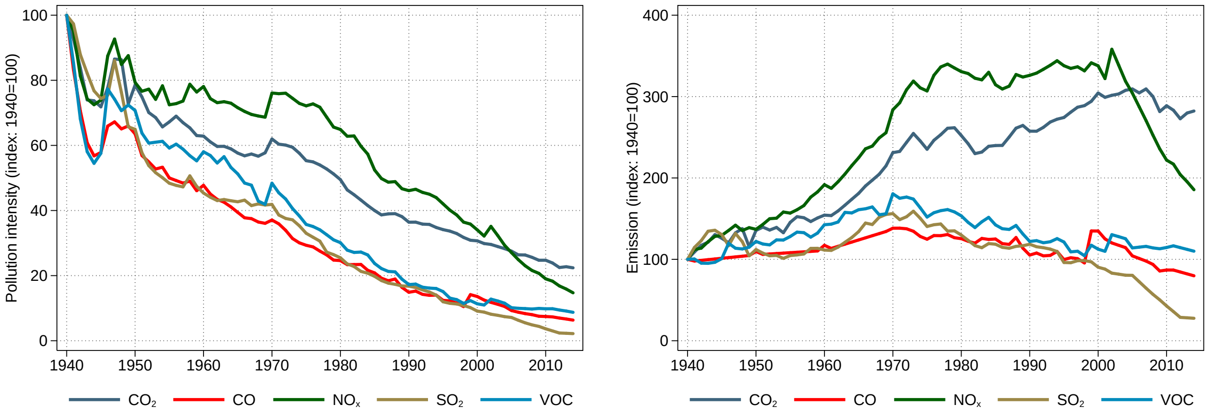

FIGURE 1: US pollution emissions and intensities, 1940-2014.
Data sources: EPA, BEA, and CDIAC.
Notes: Pollution intensity defined as emission divided by GDP. There is a data break between 1998 and 1999 for CO, NO_x, SO_2, and VOC. The data source for the period 1940-1998 is the EPA report National Air Pollutant Emission Trends, 1900-1998. Data for the remaining period can be obtained from EPA's website.

Generally, OECD countries experienced a decline in the five pollution intensities during the period 1990-2012.[5] In fact, it seems like pollution intensities are decreasing over time and with

---

[5] Disregarding Mexico due to insufficient data, the exceptions are Portugal, Span, and Turkey for CO_2; Iceland for SO_2; and Chile for VOC.

environmental policy stringency. Let environmental policy stringency be measured by the economy-wide environmental policy stringency (EPS) index described by Botta and Kozluk (2014). The EPS index is defined from zero to six, where zero is when environmental policy is nonexistent and six is a very stringent environmental policy.

TABLE 1: Log transformed pollution intensities regressed on time and/or the EPS index for OECD countries, 1990-2012.

	ln(CO_2/GDP)			ln(CO/GDP)			ln(NO_x/GDP)			ln(SO_x/GDP)			ln(VOC/GDP)		
Time	-0.02*		-0.02*	-0.07*		-0.07*	-0.05*		-0.04*	-0.10*		-0.08*	-0.06*		-0.06*
	(0.00)		(0.00)	(0.00)		(0.00)	(0.00)		(0.00)	(0.00)		(0.00)	(0.00)		(0.00)
EPS		-0.14*	-0.04*		-0.43*	0.04*		-0.31*	-0.05*		-0.64*	-0.08*		-0.39*	-0.01
		(0.00)	(0.01)		(0.01)	(0.01)		(0.01)	(0.01)		(0.02)	(0.04)		(0.01)	(0.01)
Country FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Observations	594	510	510	594	510	510	594	510	510	594	510	510	594	510	510
R^2	0.70	0.63	0.73	0.91	0.68	0.93	0.84	0.71	0.86	0.78	0.63	0.77	0.88	0.69	0.89
Adjusted R^2			0.72			0.93			0.85			0.76			0.89

Data sources: OECD.stat and Botta and Kozluk (2014).
Notes: Regressions conducted using the OLS estimator. Standard errors in brackets. (*) indicates significance at the 5 pct. level. Environmental policy stringency is measured using the EPS index presented by Botta and Kozluk (2014). Pollution intensity is emission divided by GDP (unit: tons of emission per 1,000 US 2005-dollars).

Table 1 shows OLS regression results for log transformed pollution intensities regressed on time and/or the EPS index for the OECD countries for the period 1990-2012. The regression results indicate that both time and environmental policy stringency are negatively correlated with pollution intensities. The coefficients for both time and the EPS index are negative and significant at the five pct. level in 14 out of 15 regressions. Note that in most cases the EPS coefficient is significant when controlling for time which indicates that the negative correlation between pollution intensities and the EPS index is not only caused by the fact that both measures are correlated with time. This leads to the following stylized fact.

Stylized Fact 2. *Pollution intensities fall over time and with environmental policy stringency.*

2.2 Environmental policy

Figure 2 depicts the evolution of the EPS index over time. From the left panel, it is clear that the EPS values for all individual OECD countries (for which data are available) have increased from 1995 to 2012. The right panel shows that the average EPS value in the OECD increased systematically through the period 1990-2012. This evidence leads to the following stylized fact.

Stylized Fact 3. *Environmental policy stringency increases over time.*

Figure 3 depicts tax revenues from environmentally related taxes as share of GDP for some of the largest OECD countries in the period 1994-2012. The tax revenues from environmentally related taxes as share of GDP remain remarkably constant over the period despite many economic events, e.g. business cycles, policy changes, and the Financial Crisis of 2008. This evidence is summarized in the following stylized fact.

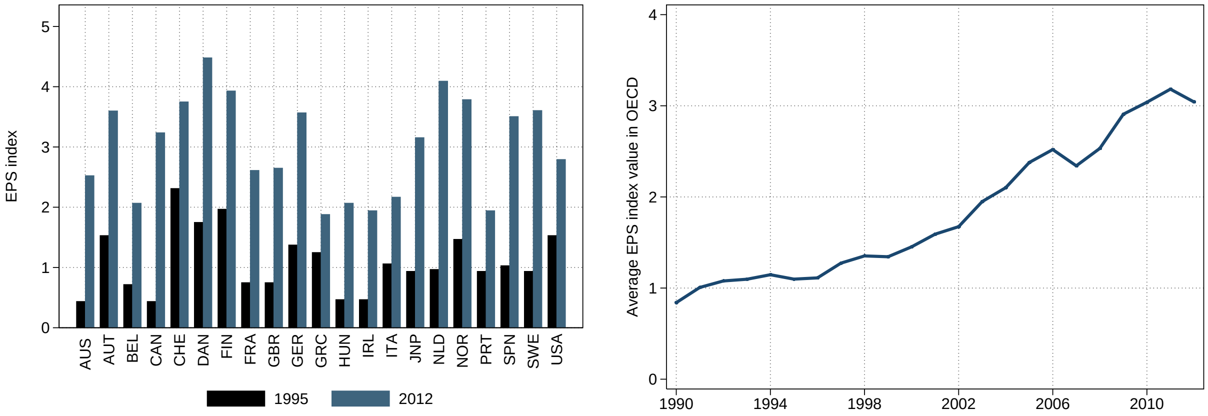

FIGURE 2: The economy-wide environmental policy stringency (EPS) index.
Data source: Botta and Kozluk (2014).
Notes: Environmental policy stringency is measured using the EPS index presented by Botta and Kozluk (2014). Due to missing data, the 1990 value is used instead of the 1995 value for IRL.

Stylized Fact 4. *The tax revenue from environmentally related taxes is approximately a constant share of GDP for long periods of time.*

The theoretical model focuses on environmental taxes. As the tax revenues from environmentally related taxes seem to be a constant share of GDP, I will assume that the government adjusts environmental tax rates to ensure this relation. Using this policy rule, environmental tax rates must increase over time as pollution intensities decrease. Higher environmental tax rates translate into a stricter environmental policy, and thus, the EPS index value increases over time.

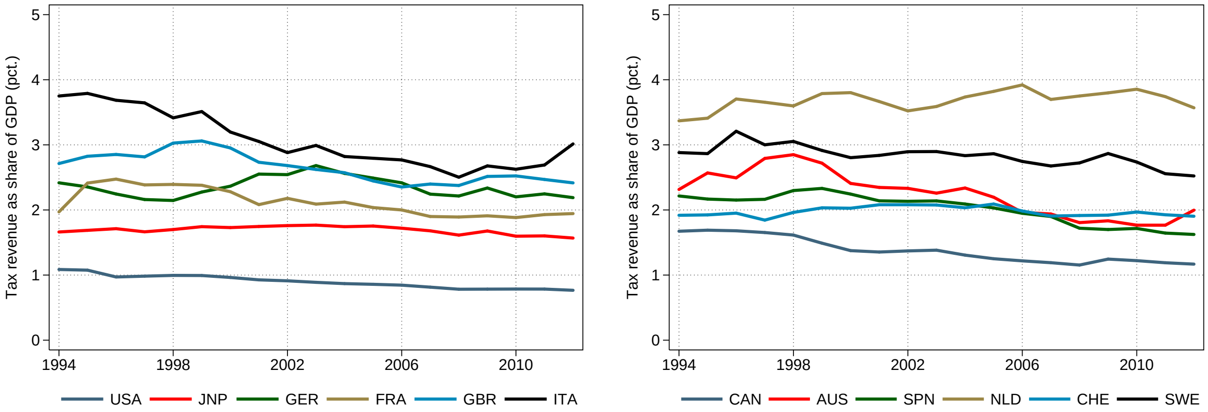

FIGURE 3: Revenues from environmentally related taxes, 1994-2012.
Data source: OECD.stat.

2.3 Pollution abatement expenditures

An important component in most growth models designed to answer environmentally related questions is the pollution abatement expenditures. Data on this subject are relatively scarce. I focus on the US case for which the longest time series are available. The left panel of Figure 4 shows that aggregate US pollution abatement expenditures were approximately a constant share of GDP over the period 1975-1994. The notable increase in pollution abatement expenditures as share of GDP from 1972 to 1975 can probably be attributed to the Clean Air Act of 1970 which changed US air pollution policy substantially.[6] The evidence presented in the left panel of Figure 4 together with the evidence from Figure 1 lead to the following stylized fact.

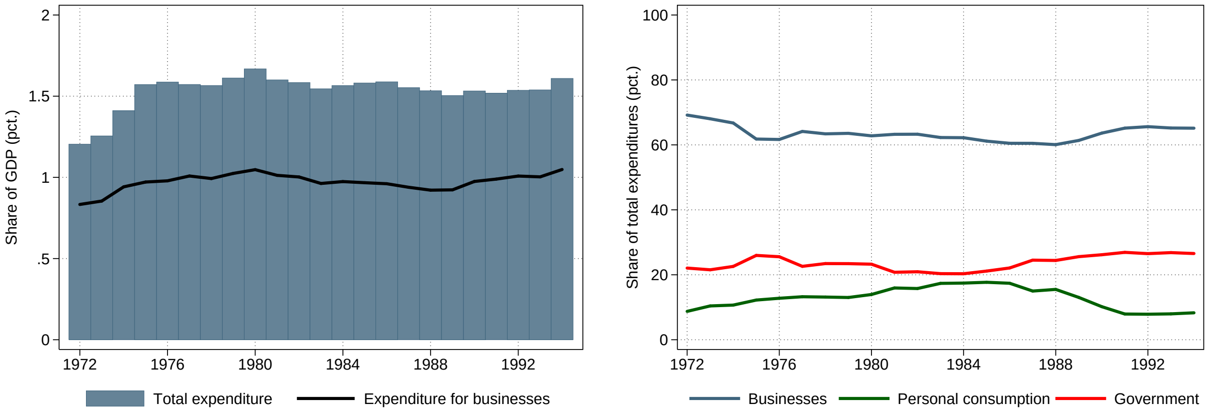

FIGURE 4: US pollution abatement expenditures, 1972-1994.
Data sources: Vogan (1996) and BEA.

Stylized Fact 5. *Pollution abatement expenditures can be a roughly constant share of GDP while pollution emissions fall.*

Stylized Fact 5 provides strong evidence against models where pollution abatement expenditures must take up an increasing share of economic output over time to reduce pollution emission. This model class includes the model developed by Stokey (1998).

The left panel of Figure 4 also shows that pollution abatement expenditures were roughly a constant share of GDP for the business sector during the period. The right panel of Figure 4 depicts pollution abatement expenditure shares by sector. The business sector had a share of over 60 pct. for the whole period, whereas each of the other two sectors never had a share above 30 pct. This evidence leads to the final stylized fact.

Stylized Fact 6. *Pollution abatement expenditures fall primarily on the business sector. In addition, the pollution abatement expenditures of the business sector can be a roughly constant share of GDP while pollution emissions fall.*

Stylized Fact 5 and 6 have two important implications. Firstly, a theoretical model should allow for decreasing pollution emissions and intensities, when aggregate and business sector pollution

---

[6] The Air Act Amendments of 1977 and 1990 added major amendments to the Clean Air Act of 1970, but it seems like they did not have a large effect on the pollution abatement expenditures. For more information about the Clean Air Act, see Davidson and Norbeck (2012)

abatement expenditures are some constant share of GDP. Secondly, as the business sector accounts for most of the aggregate pollution abatement expenditures, it seems natural to focus on this sector in theoretical work.

3 The Model

The model is based on the Schumpeterian growth model presented by Aghion and Howitt (1998, p. 85-92). I first present the general framework, then I solve for the market equilibrium, and finally I relate the model to the stylized facts presented above.

3.1 General framework

Time is continuous and denoted $t \geq 0$. The economy admits a representative household, and welfare, U, is given by

$$U = \int_0^\infty u(C_t, P_t)\, \mathrm{e}^{-\rho t}\, \mathrm{d}t \quad \rho > 0,$$

where C_t is aggregate consumption, P_t is aggregate pollution emission, and $u(\cdot)$ is increasing and strictly concave in C_t and decreasing in P_t.

Production of final goods is conducted by N_t production units (firms in the market economy). Each production unit produces final goods using the production technology:

$$Y_{jt} = \max\left(0; \int_0^1 x_{ijt}^{\alpha} A_{it}\, \mathrm{d}i - F A_t^{\mathrm{MAX}}\right), \quad i \in [0,1], \quad \alpha \in (0,1), \quad F > 0, \tag{3.1}$$

where $j \in [0, N_t]$ is an index for the production units, Y_{jt} is final output, x_{ijt} is intermediate good i input, A_{it} is a parameter capturing the productivity of intermediate good i, and $A_t^{\mathrm{MAX}} \equiv \max\left((A_{it})_{i\in[0,1]}\right)$ is the leading-edge production technology. The term FA_t^{MAX} is a quasi-fixed cost associated with production. The quasi-fixed cost reflects that a minimum input is required to produce, and that this requirement increases, as the production process becomes more advanced.[7]

All final goods are consumed such that aggregate output, Y_t, equals aggregate consumption:

$$Y_t \equiv \int_0^{N_t} Y_{jt}\, \mathrm{d}j = C_t \tag{3.2}$$

Pollution is an unavoidable by-product of production. Inspired by the pollution function used

---

[7] The quasi-fixed cost ensures that a market equilibrium with a finite number of production units exists. The proportionality between the leading-edge technology and the quasi-fixed cost ensures that the number of production units is constant over time in the market equilibrium. Substituting the leading-edge technology with the average technological level does not change the qualitative results, as the two technology measures grow at the same rate in the market equilibrium.

by Gradus and Smulders (1993), pollution emission of production unit j, P_{jt}, amounts to:

$$P_{jt} = \frac{Y_{jt}^{\beta}}{Z_{jt}^{\chi}}, \quad \beta \in (0,1), \quad \chi \in (0,1), \tag{3.3}$$

where Z_{jt} is pollution abatement activity of the production unit. Aggregate pollution emission is given by: $P_t = \int_0^{N_t} P_{jt}\,\mathrm{d}j$.

Pollution abatement activities are conducted using abatement intermediates like filters and catalysts. Pollution abatement activity of production unit j amounts to

$$Z_{jt} = \int_0^1 z_{hjt}^{\mu} B_{ht}\,\mathrm{d}h, \quad h \in [0,1], \quad \mu \in (0,1), \tag{3.4}$$

where z_{hjt} is abatement intermediate h, and B_{ht} is a parameter capturing the effectiveness of abatement intermediate h.

Note that there are two intermediate good types: one used for production indexed $i \in [0,1]$ and one for pollution abatement indexed $h \in [0,1]$. These two intermediate good types will be referred to as production and abatement intermediates, respectively. All intermediate goods are produced by labor. Given the technological design, one unit of labor can produce one unit of any intermediate. Hence, the manufacturing clearing condition requires that labor devoted to production, L_t, equals intermediate goods produced:

$$L_t = x_t + z_t, \quad x_t \equiv \int_0^{N_t} \left(\int_0^1 x_{ijt}\,\mathrm{d}i \right) \mathrm{d}j, \quad z_t \equiv \int_0^{N_t} \left(\int_0^1 z_{hjt}\,\mathrm{d}h \right) \mathrm{d}j. \tag{3.5}$$

Labor can also be used to conduct research. As there are two intermediate good types, the R&D sector is divided into two subsectors: one developing higher production intermediate good qualities, and one higher abatement intermediate good qualities. Each of these subsectors is again divided into a continuum of subsectors as there are labs developing higher qualities of each i and h intermediate.

Innovations arrive randomly following Poisson processes. Denote labor input in sub-subsector i by n_{it}^A, and labor input in sub-subsector h by n_{ht}^B. The Poisson arrival rates in the sub-subsectors i and h, are given by λn_{it}^A and ηn_{ht}^B, respectively. When a new intermediate good quality is developed, the leading-edge technology for that intermediate good type is increased due to positive spill-over effects. Specifically, the leading-edge technologies evolve according to

$$\dot{A}_t^{\mathrm{MAX}} = A_t^{\mathrm{MAX}} \lambda \ln(\gamma) n_t^A, \quad A_0^{\mathrm{MAX}} > 0 \quad \text{given}, \quad \lambda > 0, \quad \gamma > 1 \quad \text{and} \tag{3.6}$$

$$\dot{B}_t^{\mathrm{MAX}} = B_t^{\mathrm{MAX}} \eta \ln(\xi) n_t^B, \quad B_0^{\mathrm{MAX}} > 0 \quad \text{given}, \quad \eta > 0, \quad \xi > 1, \tag{3.7}$$

where $B_t^{\mathrm{MAX}} \equiv \max\left((B_{ht})_{h\in[0,1]} \right)$ is the leading-edge abatement technology, $n_t^A \equiv \int_0^1 n_{it}^A\,\mathrm{d}i$, $n_t^B \equiv \int_0^1 n_{ht}^B\,\mathrm{d}h$, the dots denote derivatives with respect to time, and $\ln(\gamma)$ and $\ln(\xi)$ capture the sizes of the spill-over effects.[8]

A new quality of an intermediate good developed at time t' will have the same technological level as the leading-edge technology of that intermediate good type. Hence, a production and an abatement intermediate developed at time t' have the technological levels $A_{t'}^{\text{MAX}}$ and $B_{t'}^{\text{MAX}}$, respectively. In the market equilibrium, research is conducted equally on each intermediate good given the type. As there exists a continuum of intermediates, the expected number equals the actual number of innovation arrivals due to the law of large numbers. Thus, the leading-edge technologies evolve according to non-stochastic processes while the technological level for each intermediate is stochastic. The distributions of the relative technology parameters $a_{it} \equiv A_{it}/A_t^{\text{MAX}}$ and $b_{ht} \equiv B_{ht}/B_t^{\text{MAX}}$ will converge to the distributions (see Aghion and Howitt 1998):

$$G_A(a) \equiv a^{\frac{1}{\ln(\gamma)}}, \quad \forall a \in [0,1] \quad \text{and} \quad G_B(b) \equiv b^{\frac{1}{\ln(\xi)}}, \quad \forall b \in [0,1]. \tag{3.8}$$

For simplicity, let a and b be distributed according to $G_A(a)$ and $G_B(b)$ from $t = 0$.

Finally, the economy is endowed with $\bar{L}$ units of labor which can be allocated to either production or research. The labor market clearing condition amounts to

$$\bar{L} = L_t + n_t^A + n_t^B. \tag{3.9}$$

3.2 The market equilibrium

A production unit now corresponds to a firm. Firms in the final goods sector operate under perfect competition, and the government imposes a pollution (emission) tax. Profits, π_{jt}^f, are given by:

$$\pi_{jt}^f = Y_{jt} - \int_0^1 x_{ijt} p_{it} \,\mathrm{d}i - \int_0^1 z_{hjt} q_{ht} \,\mathrm{d}h - P_{jt}\tau_t, \tag{3.10}$$

where p_{it} is the price of production intermediate i, q_{ht} is the price of abatement intermediate h, and $\tau_t > 0$ is the pollution tax rate. All prices are in terms of final goods. Each firm j maximizes π_{jt}^f with respect to $(x_{ijt})_{i\in[0,1]}$ and $(z_{hjt})_{h\in[0,1]}$ taking $(p_{it})_{i\in[0,1]}$, $(q_{ht})_{h\in[0,1]}$, and τ_t as given. The first-order conditions imply that

$$p_{it} = \alpha x_{ijt}^{\alpha-1} A_{it}\left(1 - \beta\frac{P_{jt}}{Y_{jt}}\tau_t\right), \quad \forall i \in [0,1] \quad \text{and} \tag{3.11}$$

$$q_{ht} = \chi\mu z_{hjt}^{\mu-1} B_{ht}\frac{P_{jt}}{Z_{jt}}\tau_t, \qquad \forall h \in [0,1]. \tag{3.12}$$

According to the following lemma, the final goods sector is always in a symmetric equilibrium.

---

[8] The knife-edge assumptions in (3.6) and (3.7) imply that the model belongs to a set of endogenous growth models often referred to as first-generation fully endogenous growth models (e.g., Romer 1990; Grossman and Helpman 1991; Aghion and Howitt 1992). As emphasized by Jones (1995, 2005) these models predict that larger economies grow faster in the long run which seems to be at odds with the data. A modified model without this (strong) scale effect - but with the same environmental policy implications - can be constructed by following the strategy of Aghion and Howitt (1998, p. 407-415) and Howitt (1999). Details are available upon request.

Lemma 1. *In a market equilibrium where firms in the final goods sector can obtain nonnegative profits, all firms in the final goods sector produce the same quantity,* $Y_{jt} = \hat{Y}_t = \frac{1}{N_t} Y_t$*, use the same amount of each production intermediate,* $x_{ijt} = \hat{x}_{it}$*, emits the same amount of pollution,* $P_{jt} = \hat{P}_t = \frac{1}{N_t} P_t$*, use the same amount of each abatement intermediate,* $z_{hjt} = \hat{z}_{ht}$*, and conduct the same amount of pollution abatement activities,* $Z_{jt} = \hat{Z}_t$*.*

Proof. See Appendix A.1. □

Firms enter the final goods sector until profits are driven to zero. Thus, the following equilibrium condition is imposed:

$$\pi^f_{jt} = 0, \quad \forall j \in [0, N_t]. \tag{3.13}$$

Motivated by Stylized Fact 4, I assume that the government adjusts the pollution tax rate, such that the tax revenue from the pollution tax remains a constant share of output:

$$P_t \tau_t = \phi Y_t, \quad \phi \in (0,1), \tag{3.14}$$

where ϕ is the tax revenue from environmentally related taxes as share of output. I refer to the parameter ϕ as the *environmental tax revenue parameter*. A higher ϕ value means a tighter environmental policy.

An expression for N_t is derived by substituting (3.11), (3.12), and (3.14) into (3.10) and employing equilibrium condition (3.13):

$$N_t = \frac{1 - \alpha(1 - \phi\beta) - \chi\mu\phi - \phi}{\alpha(1 - \phi\beta)F} \frac{Y_t}{A_t^{\text{MAX}}}. \tag{3.15}$$

To ensure that $N_t > 0$, the following parameter restriction is imposed:

Parameter Restriction 1. $1 - \alpha(1 - \phi\beta) - \chi\mu\phi - \phi > 0$.

Using (3.1), (3.2), and (3.15) a more convenient expression for output is obtained:

$$Y_t = \frac{\alpha(1 - \phi\beta)}{1 - \chi\mu\phi - \phi} N_t \int_0^1 \hat{x}_{it}^{\alpha} A_{it} \, \mathrm{d}i. \tag{3.16}$$

In the intermediate goods sector, previously successful entrepreneurs from the R&D sector produce intermediate goods. They operate under monopolistic competition, as each monopolist has the exclusive right to produce the highest existing quality of a certain intermediate good. Innovations are assumed to be drastic such that a monopolist can charge any price for his/her intermediate goods without fearing entry from previous monopolists. The sector is divided into two subsectors: one for each intermediate good type. I refer to the subsector producing production intermediates as the *production intermediate subsector*, whereas the subsector producing abatement intermediates is referred to as the *abatement intermediate subsector*.

Profits in the production intermediate subsector is given by: $\pi_{it}^{A} = (p_{it} - w_t)\hat{x}_{it}N_t$, where w_t denotes the wage rate and it is used that $\hat{x}_{it} = x_{ijt}$ according to Lemma 1. The monopolists face the inverse demand function given by (3.11) when maximizing profits. Combining the first-order condition for firm i with (3.14) yields:

$$\hat{x}_{it} = \left(\frac{\alpha^2(1-\phi\beta)}{w_t}A_{it}\right)^{\frac{1}{1-\alpha}}. \tag{3.17}$$

The price of production intermediate i is derived by substituting (3.14) and (3.17) into (3.11): $p_{it} = p_t = (1/\alpha)w_t$. The price is the marginal cost of production multiplied by a constant markup. Using the expression for the price, (3.8), (3.16), and (3.17), the equilibrium profit of monopolist i is obtained:

$$\tilde{\pi}_{it}^{A} = (1-\alpha)(1-\chi\mu\phi-\phi)\Lambda_A^{-1}a_{it}^{\frac{1}{1-\alpha}}Y_t, \quad \Lambda_A \equiv \frac{1-\alpha}{1-\alpha-\ln(\gamma)}. \tag{3.18}$$

Profits in the abatement intermediate subsector is given by: $\pi_{ht}^{B} = (q_{ht} - w_t)\hat{z}_{ht}N_t$, where it is used that $\hat{z}_{ht} = z_{hjt}$ according to Lemma 1. The monopolists face the inverse demand function given by (3.12) when maximizing profits. Combining the first-order condition for firm h with (3.14) yields:

$$\hat{z}_{ht} = \left(\frac{\chi\mu^2\phi\hat{Y}_t}{\hat{Z}_t w_t}B_{ht}\right)^{\frac{1}{1-\mu}}. \tag{3.19}$$

Pollution abatement activity per final good firm is obtained from (3.4), (3.8), and (3.19):

$$\hat{Z}_t = \left(\frac{\chi\mu^2\phi\hat{Y}_t}{w_t}\right)^{\mu}\Lambda_B^{1-\mu}B_t^{\mathrm{MAX}}, \quad \Lambda_B \equiv \frac{1-\mu}{1-\mu+\ln(\xi)}. \tag{3.20}$$

Substituting (3.20) into (3.19) and rearranging terms:

$$\hat{z}_{ht} = \frac{\chi\mu^2\phi}{w_t}\Lambda_B^{-1}b_{ht}^{\frac{1}{1-\mu}}\hat{Y}_t. \tag{3.21}$$

The price of abatement intermediate h is derived by substituting (3.14) and (3.21) into (3.12): $q_{ht} = q_t = (1/\mu)w_t$. The price is the marginal cost of production multiplied by a constant markup. Using the expression for the price and (3.21), the equilibrium profit of monopolist h is obtained:

$$\tilde{\pi}_{ht}^{B} = (1-\mu)\chi\mu\phi\Lambda_B^{-1}b_{ht}^{\frac{1}{1-\mu}}Y_t. \tag{3.22}$$

In the R&D sector, entrepreneurs conduct research to invent higher intermediate good qualities. If an entrepreneur is successful, he/she receives a patent (of infinite duration) on that intermediate good quality and becomes the new monopolist of that intermediate good. The value of a patent on the second highest quality is thereby destroyed. Hence, innovation is associated with creative destruction.

There is free entry into both R&D subsectors, and all R&D firms take the wage rate as given. As the probability of inventing a new intermediate good quality is proportional to the labor input, firms will enter the R&D sector until the following research arbitrage conditions are fulfilled:

$$w_t \geq \lambda V_{it}^{T_{it}+1} \quad \text{for} \quad n_{it}^A \geq 0 \quad \text{and} \quad w_t = \lambda V_{it}^{T_{it}+1} \quad \text{if} \quad n_{it}^A > 0; \quad \text{and} \tag{3.23}$$

$$w_t \geq \eta \bar{V}_{ht}^{\bar{T}_{ht}+1} \quad \text{for} \quad n_{ht}^B \geq 0 \quad \text{and} \quad w_t = \eta \bar{V}_{ht}^{\bar{T}_{ht}+1} \quad \text{if} \quad n_{ht}^B > 0; \tag{3.24}$$

where T_{it} and $\bar{T}_{ht}$ denote the number of qualities of intermediate i and h, respectively, developed at time t, and $V_{it}^{T_i+1}$ and $\bar{V}_{ht}^{\bar{T}_h+1}$ denote the values of inventing higher qualities of intermediate good i and h, respectively. The research arbitrage conditions ensure that in equilibrium, the cost of having a researcher working for one unit of time equals the expected value of his/her work.

The value of a patent on the highest existing intermediate good quality is the net present value of the expected future profit stream associated with the monopoly on that intermediate good quality:

$$V_{it}^{T_{it}} = \int_t^\infty \tilde{\pi}_{is}^A \, \mathrm{e}^{-\int_t^s (r_u + n_{iu}^A \lambda)\,\mathrm{d}u} \,\mathrm{d}s \quad \text{and} \quad \bar{V}_{ht}^{\bar{T}_{ht}} = \int_t^\infty \tilde{\pi}_{hs}^B \, \mathrm{e}^{-\int_t^s (r_u + n_{hu}^B \eta)\,\mathrm{d}u} \,\mathrm{d}s, \tag{3.25}$$

where r_t is the (risk-free) real interest rate. The instantaneous profit streams are discounted by modified interest rates which reflect the opportunity cost of investing in R&D firms (the real interest rate) and the risk of losing the monopoly position (the flow probabilities).

As new intermediate good qualities of a certain type invented at the same point in time are associated with the same technological level, the profit streams associated with monopolies on any of these qualities are the same. It then follows from (3.23), (3.24), and (3.25) that the same amount of research is conducted on each intermediate good, given the type. Hence, $n_t^A \equiv \int_0^1 n_{it}^A \,\mathrm{d}i = n_{it}^A$ and $n_t^B \equiv \int_0^1 n_{ht}^B \,\mathrm{d}h = n_{ht}^B$ in equilibrium.

To simplify the analysis and ensure balanced growth, I assume that: $u(C_t, P_t) = \ln(C_t) - \kappa E(P_t)$, where $E'(P_t) > 0$ and $\kappa \geq 0$. The parameter κ reflects the relative weight of pollution in the instantaneous utility function. A representative household solves the problem:

$$\max_{(C_t)_{t=0}^\infty} \int_0^\infty \left(\ln(C_t) - \kappa E(P_t)\right) \, \mathrm{e}^{-\rho t} \,\mathrm{d}t,$$

$$\text{st.} \;\; \dot{\tilde{W}}_t = w_t \bar{L} + r_t \tilde{W}_t + I_t - C_t, \quad \tilde{W}_0 > 0 \;\text{ given,} \quad \lim_{t \to \infty} \tilde{W}_t \, \mathrm{e}^{-\int_0^t r_s \,\mathrm{d}s} \geq 0, \quad C_t \geq 0,$$

where $\tilde{W}_t$ is financial wealth, and I_t is a lump-sum government transfer. Using CRRA preferences yields similar qualitative results (see Section 5). Optimal behavior requires:

$$\lim_{t \to \infty} \tilde{W}_t \, \mathrm{e}^{-\int_0^t r_s \,\mathrm{d}s} = 0 \quad \text{and} \quad g_{C,t} \equiv \frac{\dot{C}_t}{C_t} = r_t - \rho. \tag{3.26}$$

Following Ricci (2007a), the government keeps a balanced budget by transferring the entire tax revenue to the representative household at all moments in time: $I_t = \phi Y_t$.

The financial wealth of the representative household is the net present value of all existing intermediate good firms: $\tilde{W}_t = \int_0^1 V_{it}^{T_{it}}\,\mathrm{d}i + \int_0^1 \bar{V}_{ht}^{\bar{T}_{ht}}\,\mathrm{d}h$. The saving of the representative household finances R&D activities which increase the value of active intermediate good firms and thereby the representative household's financial wealth.

The equilibrium labor allocation is determined by employing the clearing conditions for the labor market and the research arbitrage conditions. The wage rate is determined by substituting (3.16), (3.17), and (3.21) into the manufacturing clearing condition (3.5):

$$w_t = \Omega_1 \frac{Y_t}{L_t}, \quad \Omega_1 \equiv (1 - \chi\mu\phi - \phi)\alpha + \chi\mu^2\phi. \tag{3.27}$$

Note that Parameter Restriction 1 ensures that $\Omega_1 > 0$. Using (3.18), (3.22), and (3.25), the values of new innovations in each of the two R&D subsectors are expressed as

$$V_{it}^{T_{it}+1} = \int_t^\infty \frac{(1-\alpha)(1-\chi\mu\phi-\phi)}{\Lambda_A} \left(\frac{A_t^{\mathrm{MAX}}}{A_s^{\mathrm{MAX}}}\right)^{\frac{1}{1-\alpha}} Y_s\, \mathrm{e}^{-\int_t^s (r_u + n_u^A \lambda)\,\mathrm{d}u}\,\mathrm{d}s \quad \text{and} \tag{3.28}$$

$$\hat{V}_{ht}^{\hat{T}_{ht}+1} = \int_t^\infty \frac{(1-\mu)\chi\mu\phi}{\Lambda_B} \left(\frac{B_t^{\mathrm{MAX}}}{B_s^{\mathrm{MAX}}}\right)^{\frac{1}{1-\mu}} Y_s\, \mathrm{e}^{-\int_t^s (r_u + n_u^B \eta)\,\mathrm{d}u}\,\mathrm{d}s. \tag{3.29}$$

It is shown in Appendix A.2 that dividing with w_t on both sides and differentiating with respect to time (using Leibniz's rule), (3.28) and (3.29) can be rewritten as

$$1 = \frac{\lambda\Omega_2 L_t}{\rho + n_t^A \lambda \Lambda_A^{-1} + g_{L,t}}, \quad \Omega_2 \equiv \frac{1-\alpha}{\Lambda_A} \frac{1-\chi\mu\phi-\phi}{(1-\chi\mu\phi-\phi)\alpha + \chi\mu^2\phi} \quad \text{and} \tag{3.30}$$

$$1 = \frac{\eta\Omega_3 L_t}{\rho + n_t^B \eta \Lambda_B^{-1} + g_{L,t}}, \quad \Omega_3 \equiv \frac{(1-\mu)\chi\mu}{\Lambda_B} \frac{\phi}{(1-\chi\mu\phi-\phi)\alpha + \chi\mu^2\phi}, \tag{3.31}$$

where $g_{L,t}$ is the growth rate of labor devoted to manufacturing, L_t.

Consider the following lemma:

Lemma 2. *If the parameters ensure that* $n_0^A > 0$*,* $n_0^B > 0$*, and* $L_0 > 0$*, then* $L_t = L$*,* $n_t^A = n^A$*, and* $n_t^B = n^B$ *for all* $t \geq 0$*.*

Proof. See Appendix A.3. □

To ensure that $n^A > 0$ and $n^B > 0$ in equilibrium, the following two parameter restrictions are imposed:

Parameter Restriction 2. $\bar{L} + \frac{\rho}{\eta}\Lambda_B > \frac{1-\phi-(1-\alpha)(1-\chi\mu\phi-\phi)}{(1-\alpha)(1-\chi\mu\phi-\phi)} \frac{\rho}{\lambda}\Lambda_A$.

Parameter Restriction 3. $\bar{L} + \frac{\rho}{\lambda}\Lambda_A > \frac{1-\phi-(1-\mu)\chi\mu\phi}{(1-\mu)\chi\mu\phi} \frac{\rho}{\eta}\Lambda_B$.

The equilibrium labor allocation is derived using, (3.9), (3.30), (3.31), and Lemma 2:

$$L = \frac{(1-\chi\mu\phi-\phi)\alpha+\chi\mu^2\phi}{1-\phi}\,\Gamma, \quad \Gamma \equiv \bar{L}+\rho\left(\frac{\Lambda_A}{\lambda}+\frac{\Lambda_B}{\eta}\right), \tag{3.32}$$

$$n^A = \frac{(1-\alpha)(1-\chi\mu\phi-\phi)}{1-\phi}\,\Gamma-\frac{\Lambda_A}{\lambda}\rho, \tag{3.33}$$

$$n^B = \frac{(1-\mu)\chi\mu\phi}{1-\phi}\,\Gamma-\frac{\Lambda_B}{\eta}\rho. \tag{3.34}$$

I define a balanced growth path as a path where the variables C_t, Y_t, Y_{jt}, A_t^{MAX}, w_t, B_t^{MAX}, P_t, and P_{jt} all grow at constant rates while N_t is constant. According to the following proposition, the economy is always on a balanced growth path:

Proposition 1. *The variables* C_t, Y_t, Y_{jt}, A_t^{MAX}, *and* w_t *all grow at the constant rate* $g_A = n^A\lambda\ln(\gamma)$, N_t *is constant over time,* B_t^{MAX} *grows at the constant rate* $g_B = n^B\eta\ln(\xi)$, *and* P_t *and* P_{jt} *grow at the constant rate* $g_P = \beta n^A\lambda\ln(\gamma)-\chi n^B\eta\ln(\xi)$.

Proof. See Appendix A.4. ☐

3.3 Stylized facts revisited

The model focuses on the business sector's pollution abatement expenditures which is motivated by Stylized Fact 6. In addition, I used Stylized Fact 4 to motivate the policy function (3.14). The model matches the remaining stylized facts endogenously. First, define pollution intensity as: $\mathcal{P}_t \equiv P_t/Y_t$. It follows directly from Proposition 1 that: $\dot{\mathcal{P}}_t/\mathcal{P}_t \equiv g_{\mathcal{P}} = (\beta-1)n^A\lambda\ln(\gamma)-\chi n^B\eta\ln(\xi) < 0$. Thus, the model predicts that the pollution intensity decreases with income/over time which matches Stylized Fact 1 and 2. Meanwhile, pollution emission might in- and decrease with income depending on the environmental tax revenue parameter, ϕ, as the growth rate of pollution emission is decreasing in ϕ. According to the evidence presented in Figure 1, pollution emission increased until 1970 and then decreased for most pollutants. For the model to match this pattern, an increase in ϕ around 1970 is needed. That is, according to the model, US environmental policy was tightened around 1970 which fits well with the implementation of the Clean Air Act of 1970.

Next, the environmental policy stringency measured by the EPS index corresponds to some monotone transformation of the pollution tax rate, τ_t. Consider the government's tax rule expressed in terms of pollution intensity: $\phi = \mathcal{P}_t\tau_t$. As the pollution intensity decreases, the pollution tax rate must increase which matches both Stylized Fact 2 and 3, i.e. the EPS index is negatively correlated with the pollution intensity and the EPS index increases over time.

Finally, aggregate and business sector pollution abatement expenditures are the total expenditures on pollution abatement for the final goods sector, as this is the only sector that pollutes. The pollution abatement expenditures are given by the cost of purchasing abatement intermediates for all final good firms: $(1/\alpha)z_tw_t = (1/\alpha)\chi\mu^2\phi Y_t$. Hence, aggregate and business sector pollution abatement expenditures are a constant share of output which fits well with the data presented in Figure 4. Also, the model matches Stylized Fact 5 and 6, as aggregate and business sector pollution abatement expenditures can be a constant share of output while pollution emission falls (if ϕ is sufficiently large).[9]

4 Policy Implications

To study how a tightening of the environmental policy affects economic growth, an unexpected increase in the environmental tax revenue parameter, ϕ, is analyzed. In reality, the government possesses many other environmental policy instruments besides environmental taxes. For the mechanisms described in this model, what matters is that pollution emission becomes more expensive when the environmental policy is tightened. The environmental tax revenue parameter can therefore be interpreted broadly as the technology-adjusted environmental policy stringency.

Consider the following proposition.

Proposition 2. *If Parameter Restriction 1, 2 and 3 hold, then:*
(1) $\frac{\mathrm{d}x}{\mathrm{d}\phi} < 0$ *and* $\frac{\mathrm{d}z}{\mathrm{d}\phi} > 0$*;* *(2)* $\frac{\mathrm{d}n^A}{\mathrm{d}\phi} < 0$ *and* $\frac{\mathrm{d}n^B}{\mathrm{d}\phi} > 0$*;* *and* *(3)* $\frac{\mathrm{d}L}{\mathrm{d}\phi} \gtreqless 0$ *for* $\alpha \lesseqgtr \mu$*.*

Proof. See Appendix A.5. ☐

The proposition highlights three labor allocation effects of a tightening of the environmental policy. The first effect is a *production direction effect*: reallocation of labor used for manufacturing. A tightening of the environmental policy increases the price of pollution emission. This causes the demand for abatement intermediates to increase while the demand for production intermediates decreases. To ensure that supply equals demand, labor must be reallocated from the production to the abatement intermediate subsector.

The second effect is a *research direction effect*: reallocation of labor used for research. As the demand for abatement intermediates increases, so does the value of a patent in the abatement intermediate subsector relative to the cost of conducting research (the wage rate). The opposite holds for the production intermediate subsector. Labor is reallocated toward the abatement R&D subsector due to research arbitrage. This reallocation of labor affects the value of patents in the two subsectors through the expected duration of a monopoly position.

The final effect is a *labor force allocation effect*: reallocation of labor between manufacturing and research. Labor might be reallocated from manufacturing to R&D and vice versa, depending on the relative sizes of α and μ. If $\alpha < \mu$ the total profits in the intermediate goods sector decrease, given the labor allocation between manufacturing and R&D. The reason is that intermediate goods

---

[9] Abatement research expenditures are not part of the aggregate pollution abatement expenditures (see Vogan 1996).

with the price $(1/\alpha)w_t$ are substituted for intermediates with the relatively lower price $(1/\mu)w_t$. As total profits in the intermediate goods sector are reduced, the incentive to conduct research is diminished. Labor then flows from R&D to manufacturing until the research arbitrage conditions are fulfilled.

The labor allocation effects described above govern the effects on output growth and pollution emission growth. The following proposition follows almost immediately from Proposition 2:

Proposition 3. *If Parameter Restriction 1, 2 and 3 hold, then:* $\frac{\mathrm{d}g_A}{\mathrm{d}\phi} < 0$ *and* $\frac{\mathrm{d}g_P}{\mathrm{d}\phi} < 0$.

According to Proposition 3, a tightening of the environmental policy decreases the growth rates of output and pollution emission. The question is then: why can the potential inflow of labor from manufacturing to R&D never be strong enough to increase the growth rate of output? When a tightening of the environmental policy causes labor to flow from manufacturing to R&D, the reason is that profits in the intermediate goods sector have increased. However, this overall increase is entirely driven by the abatement intermediate subsector, as profits in the production intermediate subsector decrease unambiguously due to the *production direction effect.* Hence, the labor flow from manufacturing is directed entirely to the abatement R&D subsector, and the labor allocated to the production R&D subsector is reduced unambiguously.

As in other models featuring directed technical change, the relative profit between the two intermediate goods subsectors is determined by a *price effect* and a *market size effect.* The relative profit is given by:

$$\frac{\pi_t^A}{\pi_t^B} = \underbrace{\left(\frac{p_t - w_t}{q_t - w_t}\right)}_{\text{Price effect}} \times \underbrace{\left(\frac{x_t}{z_t}\right)}_{\text{Market size effect}} = \underbrace{\left(\frac{1/\alpha - 1}{1/\mu - 1}\right)}_{\text{Price effect (equilibrium)}} \times \underbrace{\left(\frac{\alpha(1-\chi\mu\phi-\phi)}{\chi\mu^2\phi}\right)}_{\text{Market size effect (equilibrium)}},$$

where $\pi_t^A \equiv \int_0^1 \pi_{it}^A \,\mathrm{d}i$ and $\pi_t^B \equiv \int_0^1 \pi_{ht}^B \,\mathrm{d}h$ are the total profit streams from the production and abatement intermediate subsectors, respectively.

The size of the *price effect* is determined by the relative size of the two markups. Hence, the *price effect* is determined by technology and market power. Both are unaffected by environmental policy. Thus, the relative profit is only directly affected by environmental policy through the *market size effect* which is simply a different formulation of the *production direction effect.* The *price effect* still affects how strong the relative profit reacts to changes in the environmental policy. This insight is directly linked to the *labor force allocation effect.* When the *price effect* is large ($\alpha < \mu$) labor flows from R&D to manufacturing when the environmental policy is tightened which amplifies the effect of environmental policy on relative profits and the economic growth rate.

Meanwhile, the *research direction effect* ensures that research is conducted in both R&D subsectors. If, for instance, the profit stream associated with a patent on a new abatement intermediate becomes larger, more researchers will enter the abatement R&D subsector. As a result, the expected duration of a monopoly position in the abatement intermediate subsector is reduced which ensures that in equilibrium, it is equally attractive to conduct research in both R&D subsectors. This effect is eliminated in models with one-period patents, and it might cause innovation to permanently occur only in one R&D subsector.

5 Model Extensions

The assumption of logarithmic preferences in the instantaneous utility function might be considered problematic, as extensive empirical evidence suggests that the degree of relative risk aversion is above one (see Attanasio and Weber 1993; Okubo 2011; Havránek 2015). To relax this assumption, consider the model from Section 3 with the instantaneous utility function

$$u(C_t) = \frac{C_t^{1-\theta}}{1-\theta} - \kappa E(P_t), \quad \theta > 1, \quad \kappa \geq 0, \quad E'(P_t) > 0, \tag{5.1}$$

where θ is the degree of relative risk aversion. Most of the derivations from Section 3 are still valid. However, the real interest rate is now given by: $r = \theta g_C + \rho$. In Appendix A.6 I show that the environmental policy implications with respect to the economic growth rate as well as the growth rate of pollution emission are unchanged by this extension.

Furthermore, the absence of physical capital in the model might seem unappealing, as accumulation of physical capital usually plays a major role in economic growth models. In Appendix A.7 I sketch a model similar to the one presented in Section 3, but where physical capital is part of the production process. In this model, labor becomes an input in the final goods sector, and I assume that final good firms operate under monopolistic competition a la Dixit and Stiglitz (1977).[10] Also, intermediate goods are produced using physical capital instead of labor. The long-run policy implications of the model are very similar to those of the model presented in Section 3: a tighter environmental policy will unambiguously dampen long-run economic growth and the long-run growth rate of pollution emission.

6 Simulations

In this section, the model from Section 3 with the instantaneous utility function given by (5.1) is simulated to investigate its quantitative policy implications. The model contains several parameters that are new to the literature. To obtain somewhat plausible parameter values, the model is calibrated to the US economy for which the most comprehensive data are available.

---

[10] The monopolistic element is necessary to ensure nonnegative profits for final good firms.

6.1 Calibration

The calibration procedure is divided into four steps. First, parameters present in other growth models are assigned values from previous papers: $\alpha = 0.4$ and $\lambda = 0.5$ (from Ricci 2007a), and $\theta = 2.5$ and $\rho = 0.2$ (from Groth and Ricci 2011). Assuming that it is equally difficult to invent new production and abatement intermediate good qualities: $\eta = \lambda = 0.5$. The parameter β is estimated to be 0.7 using the procedure described in Appendix A.8. Total labor supply is normalized to one, and the parameter F (which only has a strong impact on the variable N) is set to 0.1.

Second, the US tax revenues from environmentally related taxes were 1.09 pct. of GDP in 1994. As ϕ should be interpreted as a measure of the overall environmental policy stringency and not just the tax revenues from environmentally related taxes as share of output, the ϕ value for the US, ϕ^{US}, must have been at least 1.09 pct. In fact, ϕ^{US} is probably considerably higher, as US environmental policy relies little on environmental taxes. As no empirical studies have estimated ϕ^{US}, I use $\phi^{\text{US}} = 1.09\%$ and $\phi^{\text{US}} = 10.9\%$. That is the minimum ϕ value and ten times the minimum value. The former serves as a lower bound, whereas the latter serves as an upper bound.

Third, the parameter χ is computed by matching aggregate pollution abatement expenditures with that of the US in 1994. These expenditures constituted about 1.61 pct. of GDP which correspond to $(1/\alpha)\chi\mu^2\phi = 1.61\%$ in the model.[11] Given values for ϕ and μ, χ can be computed. I set μ equal to 0.8 as lower μ values results in a χ value above one for $\phi^{\text{US}} = 1.09\%$ which is inconsistent with the parameter restrictions. In Section 6.3 I show that the main results of this section are robust to a lower μ value given that the ϕ^{US} value allows for a lower μ value.

Finally, the model is calibrated such that it matches the US growth rates of real GDP and pollution emission. Unfortunately, it is difficult to operationalize the variable P_t, as it is unclear how to construct a single pollution emission measure. As in Section 2, I focus on air pollution. I construct a pollution emission index by indexing pollution emissions for CO, NO_x, SO_2, VOC, and CO_2 to 100 in 1970 and taking the unweighted average of this index. From the pollution emission index, I compute g_P for the period 1970-2012 which is about -1.42 pct. As shown in Section 6.3, setting g_P equal to the growth rate of the fastest or slowest growing pollutant does not change the main conclusions of this exercise.[12] The values of $\ln(\gamma)$ and $\ln(\xi)$ are then calibrated such that the model matches the growth rates of US real GDP (2.85 pct.) and pollution emission (-1.42 pct.) for the period 1970-2012.

---

[11] It might be argued that $(1/\alpha)\chi\mu^2\phi$ corresponds to the business sector's pollution abatement expenditures. Using the business sector's pollution abatement expenditures does not change the main conclusions of this exercise.

[12] The index could also be computed using shadow prices as relative weights. However, the results are not sensitive to changes in the pollution growth rate used to calibrate the model. Putting all weight on either the fastest (CO_2) or the slowest (SO_2) growing pollutant does not change the main conclusions of the exercise indicating that the main conclusions can be obtained using any weighted average of the five growth rates.

6.2 Simulation results

Figure 5 depicts the change in the growth rates of output and pollution emission when the tax revenue from the pollution tax is increased for the two calibrations: $\phi^{\text{US}} = 1.09\%$ and $\phi^{\text{US}} = 10.9\%$. For both calibrations, the growth rate of output is weakly affected by environmental policy changes compared to the growth rate of pollution emission. Consider an increase of one percentage point (pp) in the tax revenue from the pollution tax as share of output (a substantial policy change). This reform would decrease the growth rates of output by 0.025 and 0.003 pp, where the growth rates of pollution emission are reduced by 4.3 and 0.37 pp for $\phi^{\text{US}} = 1.09\%$ and $\phi^{\text{US}} = 10.9\%$, respectively. The changes in both growth rates are strongly affected by the initial environmental tax revenue parameter value, but the economic growth effects of a tighter environmental policy are always small.

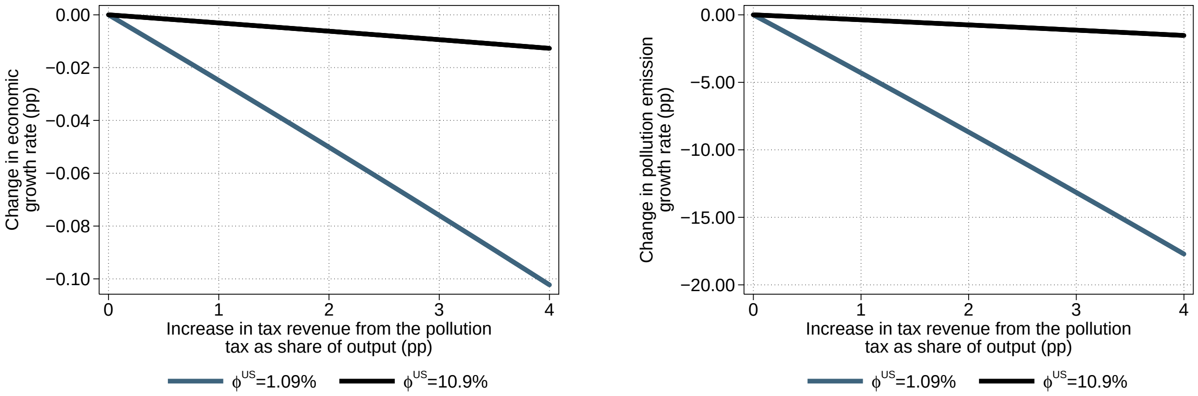

FIGURE 5: Changes in the growth rates of output and pollution emission when the tax revenue from the pollution tax as share of output is increased.

Intuitively, the results should be understood in the following way. As the model matches the US pollution abatement expenditures as share of GDP which is a small number, the market for abatement technologies is relatively small. This results in a small labor input in the abatement R&D subsector. Since US pollution intensities have decreased notably during the last plus 40 years, pollution abatement research must have had a strong impact on pollution emission despite the relatively small input. Consequently, $\ln(\xi)$ becomes large when calibrating the model which explains the relatively large impact of environmental policy on the growth rate of pollution emission.[13]

The relatively weak effect of environmental policy on economic growth is also a consequence of the relatively small market for abatement technologies. Only a small fraction of the labor force works in the two abatement subsectors. If environmental policy doubles or triples the market size for abatement technologies, the implied labor reallocation is relatively small. As a result, the input

---

[13] Since $\ln(\xi) > \ln(\gamma)$ spill-over effects are larger in the abatement R&D subsector compared to the production R&D subsector. This result aligns well with recent empirical evidence from patent citation data which suggests that knowledge spill-over effects from clean technologies are larger than knowledge spill-over effects from dirty technologies (Dechezleprêtre et al. 2014).

in the production R&D subsector is almost unaffected by significant environmental policy changes which lead to a relatively weak economic growth effect.

Another issue is how environmental policy affects the initial level of central variables given the technological level. Figure 6 shows the change in central variables as function of increases in ϕ given the technological level $(A_t^{\mathrm{MAX}}, B_t^{\mathrm{MAX}}) = (1,1)$. The simulation results indicate that a tighter environmental policy: (1) reduces profits in the production intermediate subsector and increases profits in the abatement intermediate subsector, (2) reduces output and the wage rate, and (3) reduces pollution emission and intensity.

For $\phi^{\mathrm{US}} = 1.09\%$ the increase in profits in the abatement intermediate subsector is sizable. If, for instance, ϕ is increased from 1.09 to 2.09 pct., profits in this subsector almost double. The result is intuitive. The tax revenue from the pollution tax almost doubles which means that the quantities of abatement intermediates produced increases by almost 100 pct. As the price of abatement intermediates is a constant markup over marginal costs and the wage rate is only weakly affected by the policy, the profits in the abatement intermediate subsector increase by almost 100 pct. On the other hand, the change in profits in the production intermediate subsector is much smaller (about 3 pct.), as the relative change in demand for production intermediates is only weakly affected by the policy.

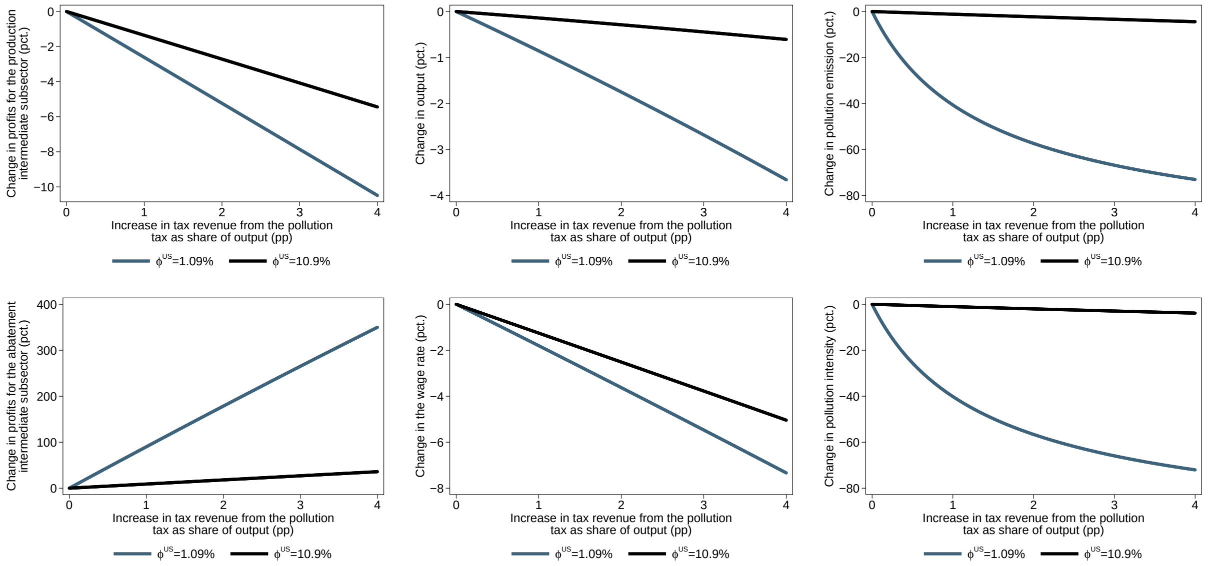

FIGURE 6: Relative changes in central variables for increases in the tax revenue from the pollution tax as share of output given the technological level $(A_t^{\mathrm{MAX}}, B_t^{\mathrm{MAX}}) = (1,1)$.

It appears from Figure 5 and 6 that the growth and level effects are amplified when ϕ^{US} is reduced. A low ϕ^{US} implies a large χ value due to the calibration procedure. A larger χ value implies more efficient pollution abatement. Thus, final good firms react with relatively more abatement, when the environmental policy is tightened for higher χ values. As the response is stronger, the growth and level effects are amplified.

It is often argued that a tighter environmental policy will decrease future growth substantially. For instance, Gordon (2012) argues that energy and environmental issues will reduce the US economic growth rate by 0.2 pp over the period 2007-2027. The model developed in this paper can be used to calculate how much tighter the environmental policy should be to impose such a growth drag on the economy. Simulating the model using $\phi^{\mathrm{US}} = 1.09\%$, it follows that the tax revenue from environmentally related taxes must be increased from 1.09 to 3.93 pct. of GDP to impose a growth drag of 0.2 pp over the period 2007-2027. Such a revenue increase is substantial but not necessarily unrealistic, as the recent climate policy initiated by the Obama administration might mark the beginning of an environmental policy change in the US. This example also illustrates the importance of the level effect. Increasing the tax revenue from environmentally related taxes from 1.09 to 3.93 pct. of GDP only reduce the long-run economic growth rate by 0.07 pp. The remaining part of the growth drag is caused by the level shift in output.

Despite the numerically small economic growth effects of environmental policy, the welfare implications might be substantial as even small changes in a growth rate have large level effects in the long run. The left panel of Figure 7 shows the output level loss 50 years after an environmental policy change for the calibration based on ϕ^{US}=1.09 %. Calibrating the model for other allowable ϕ^{US} values only reduce the economic growth effects of environmental policy, and thus, the simulation results presented in Figure 7 can be viewed as the most pessimistic case. Holding the current output level constant, a one pp increase in the tax revenue from the pollution tax as share of output reduces output 50 years later by at most 1.21 pct. When the initial level effect is included, output is reduced by at most 2.09 pct. 50 years later.

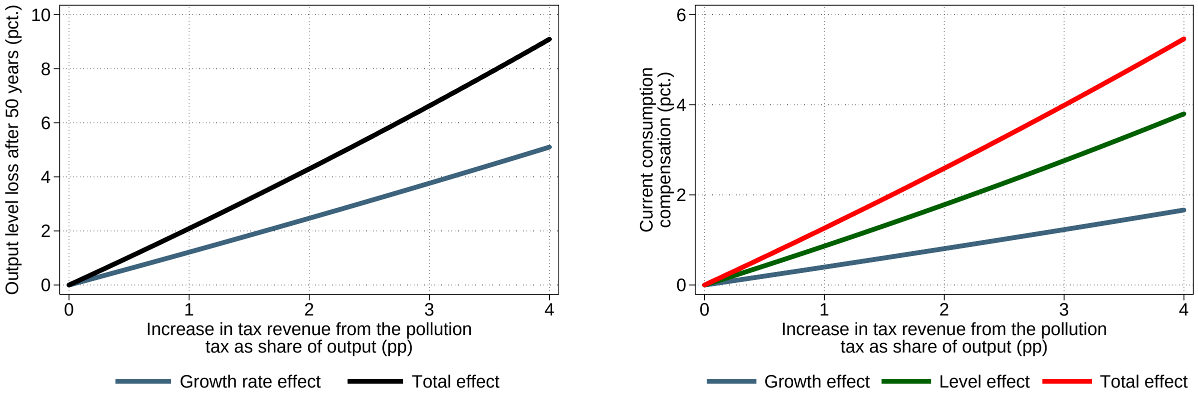

FIGURE 7: Output loss after 50 years and current consumption compensation when the tax revenue from the pollution tax as share of output is increased assuming that ϕ^{US}=2.55.
Notes: The current consumption compensation measures how much the current consumption level must increase to keep utility constant.

Another way to evaluate the welfare cost of environmental policy reforms is to consider how much the current consumption level must change to keep utility constant. I refer to this figure as the current consumption compensation (CCC). For this welfare measure, the pollution element in the instantaneous utility function matters. To avoid further assumptions on the instantaneous utility function, I assume that $\kappa = 0$. Accordingly, the computed CCC should be viewed as an upper bound. Details on how the CCC is computed are reported in Appendix A.9.

The right panel of Figure 7 shows CCC divided into a growth and a level component as well as the total CCC. The loss in growth accounts for about 30 pct. of the CCC, where the level component accounts for about 70 pct. Assuming that the policy change has no level effect on consumption, the CCC for a one pp increase in the tax revenue from the pollution tax as share of output is 0.4 pct. That is, the representative household demands a 0.4 pct. higher current consumption level with the new economic growth rate to be as well off as before the policy change.

The calculations presented in Figure 7 indicate that static models and exogenous growth models leave out a substantial part of the welfare effects of environmental policy reforms. The growth effect accounts for more than 50 pct. of the output level loss 50 years later, while it accounts for about 30 pct. of the CCC. Thus, despite the small effect on the economic growth rate, it is important to take the growth effects into account when evaluating the welfare effects of environmental policy reforms. The relatively large welfare effects from seemingly small reductions in the economic growth rate are intuitive. When the economic growth rate is reduced, consumption is reduced at all future dates. Despite the consumption discounting and the curvature of the instantaneous utility function, these consumption losses have a relatively large present value.

6.3 Sensitivity analysis

It is natural to wonder how sensitive the obtained results are to changes in the baseline parameter values and calibration targets. Table 2 shows reductions in the economic growth rates from a one percentage point increase in the tax revenue parameter, ϕ, from 2.18 to 3.18 pct. for simulations based on $\phi^{\mathrm{US}} = 2.18\%$ and the baseline parameter values and calibration targets, and where only one parameter assumption or calibration target deviates from its baseline value. Attanasio and Weber (1993) find that $1.25 \leq \theta \leq 3.33$ for the US, and I investigate the two extreme cases $\theta = 1.25$ and $\theta = 3.33$. For the pollution emission growth rate (calibration) target, I use the growth rates of the fastest (CO_2) and the slowest (SO_2) growing air pollutants. I use $\mu = 0.4$ and $\mu = 0.9$ since the model behaves very differently for μ close to zero or one, as the markup in the abatement intermediate goods subsector becomes extremely large (for $\mu \approx 0$) or almost disappears (for $\mu \approx 1$). The remaining parameters as well as the pollution abatement (calibration) target, $(1/\alpha)\chi\mu^2\phi^{\mathrm{US}}$, are multiplied and divided by two.

The model is calibrated for $\phi^{\mathrm{US}} = 2.18\%$ (twice the minimum value) as some parameter assumptions are violated for $\phi^{\mathrm{US}} = 1.09\%$ for several of the considered parameter and calibration target changes. Typically, the parameter assumptions ensuring that $n^B > 0$ are violated. Using

$\phi^{\text{US}} = 2.18\%$ allow sufficient flexibility for this sensitivity analysis.

TABLE 2: Economic growth rate reductions associated with a one percentage point increase in the tax revenue parameter, ϕ, from 2.18 to 3.18 pct. The simulations are based on ϕ^{US} equal to 2.18 pct. and the baseline parameter values and calibration targets, where one parameter assumption or calibration target is changed from its baseline value in each simulation.

Deviating value	α	β	$\lambda = \eta$	μ	θ	ρ	F	g_P	$(1/\alpha)\chi\mu\phi^{US}$
Simulations	0.20	0.35	0.50	0.40	1.25	0.01	0.05	-4.18 %	0.805 %
Growth reduction (pp)	0.01	0.01	0.01	0.03	0.01	0.01	0.01	0.01	0.01
Change from baseline (pp)	-0.01	0.00	0.00	0.01	0.00	0.00	0.00	0.00	-0.01

Deviating value	α	β	$\lambda = \eta$	μ	θ	ρ	F	g_P	$(1/\alpha)\chi\mu\phi^{US}$
Simulations	0.80	0.99	1.00	0.90	3.33	0.04	0.20	0.40 %	3.22 %
Growth reduction (pp)	0.03	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.03
Change from baseline (pp)	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01

Notes: The baseline parameter assumptions: $\alpha = 0.4$, $\beta = 0.7$, $\lambda = \eta = 0.5$, $\mu = 0.8$, $\theta = 2.5$, $\rho = 0.02$, *and* $F = 0.1$. *The baseline calibration targets:* $g_A = 2.85\%$, $g_P = -1.42\%$, *and* $(1/\alpha)\chi\mu\phi^{US} = 1.61\%$.

The results shown in Table 2 indicate that the main conclusions from Section 6.2 are robust to a variety of individual parameter and calibration target changes. Specifically, none of the simulations show large growth rate reductions, and the growth rate reductions are close to that of the baseline model. The results indicate that the simulations are most sensitive to changes in α, μ, and the pollution abatement expenditure target, $(1/\alpha)\chi\mu^2\phi^{\text{US}}$. The parameters α and μ have relatively large effects as they directly influence the markups and thereby profits in the two intermediate goods subsectors. Thus, the effects are directly linked to the *price effect* (see Section 4). The pollution abatement target determines the relative size of the abatement sector which directly influence profits in the intermediate goods sector. Hence, the effect is directly linked to the *market size effect* (see Section 4). Finally, it is worth noticing that changing the pollution emission growth rate target has little effect on the obtained results. The reason is that a change in this target primarily feeds into the calibrated value of $\ln(\xi)$.

7 Discussion

It is somewhat surprising that the economic growth effects of environmental policy changes are small in the simulation exercises, as several model assumptions amplify these effects. Starting with technology, there are three assumptions worth emphasizing. First, pollution has no negative effects on production.[14] As a result, environmental policy cannot boost productivity through a reduction in pollution emissions. Second, abatement technologies have no productivity-enhancing effects, as they can only reduce pollution emission. In reality, they might improve productivity through a decrease in resource use (see Porter and van der Linde 1995). Third, spill-over effects between

---

[14] Pollution emission can, for example, reduce production through its effect on labor supply (due to short-term effects on health, see Hanna and Olival [2015]) and labor productivity (Zivin and Neidell 2012).

production and abatement technologies are ignored which increases the economic growth impact of labor reallocation within the R&D sector.

Two assumptions related to allocation mechanisms also amplify the economic growth effects of environmental policy changes. Firstly, the tax revenue from the pollution tax is transferred to the representative household. Alternatively, this tax revenue could finance subsidies to productivity-enhancing research activities and thereby reduce the negative economic growth effect of a tighter environmental policy. Secondly, the representative household takes pollution emission as given and has additive preferences concerning consumption and pollution emission. If, instead, consumption and a clean environment are compliments, environmental policies reducing pollution emission increase the savings rate and thus the economic growth rate (see Mohtadi 1996).

Given these considerations, it appears puzzling that economic growth effects of large environmental policy reforms are almost absent in the simulation exercises. Intuitively, the results can be explained the following way. As US pollution abatement expenditures only constitute a small fraction of GDP, the market for abatement technologies is small. Hence, environmental policy reforms that increase the market size of abatement technologies substantially, only reallocates a small fraction of the economies total resources. As a consequence, even large environmental policy changes have a small impact on research input in productivity-enhancing technologies.

All in all, this analysis suggests that environmental policy has a small impact on economic growth. Additionally, even the numerically small growth effects estimated in the simulation exercises are likely to be exaggerated given that several modeling assumptions amplify the economic growth effects of environmental policy. It should, however, be emphasized that even numerically small effects on a growth rate have large level effects in the long run. The simulation exercises indicate that the economic growth effect of an environmental policy change constitutes a substantial part of the overall welfare effect. These results suggest that static models and exogenous growth models leave out important welfare effects of environmental policy.

8 Concluding Remarks

In the light of this analysis, several directions of future research appear fruitful. First, this study illustrated the importance of a technology-adjusted environmental policy stringency measure (like the environmental tax revenue parameter, ϕ, in the above analysis). If such measures were estimated empirically, simulation exercises like the one conducted above would be much more precise. Next, I assumed that the entire tax revenue from the pollution tax was transferred to the representative household. It would be interesting to examine how economic growth is affected if the tax revenues are used to finance subsidies to parts of the R&D sector. Finally, the model developed in this paper might be useful when studying how pollution emissions interact with economic growth in developing economies. Pollution levels in developing economies are often high, and it is therefore likely that a tighter environmental policy could boost economic growth in these economies by reducing the negative effects on labor supply and productivity from pollution emission.

References

D. Acemoglu, P. Aghion, L. Bursztyn, and D. Hemous. The Environment and Directed Technical Change. *American Economic Review*, 102(1):131–166, 2012.

P. Aghion and P. Howitt. A Model of Growth Through Creative Destruction. *Econometrica*, 60(4):323–351, 1992.

P. Aghion and P. Howitt. *Endogenous Growth Theory.* MIT Press, Cambridge, MA, 1998.

P. Aghion, A. Dechezleprêtre, D. Hemous, R. Martin, and J. V. Reenen. Carbon Taxes, Path Dependency, and Directed Technical Change: Evidence from the Auto Industry. *Journal of Political Economy*, 124(1), 2016.

S. Ambec, N. Johnstone, P. Lanoie, and J. Laurent-Lucchetti. Environmental Policy, Innovation and Performance: New Insights on the Porter Hypothesis. *Journal of Economics and Management Strategy*, 20 (3):803–842, 2011.

F. J. André and S. Smulders. Fueling Growth when Oil Peaks: Directed Technical Change and the Limits to Efficiency. *European Economic Review*, 69:18–39, 2014.

T. H. Arimura, A. Hibiki, and N. Johnstone. An empirical study of environmental R&D: what encourages facilities to be environmentally innovative? In N. Johnstone, editor, *Environmental Policy and Corporate Behaviour*, pages 142–173. Edward Elgar, 2007.

O. P. Attanasio and G. Weber. Consumption Growth, the Interest Rate and Aggregation. *The Review of Economic Studies*, 60(3):631–649, 1993.

E. Botta and T. Kozluk. Measuring Environmental Policy Stringency in OECD Countries: A Composite Index Approach. *OECD Economics Department Working Papers, No. 1177*, 2014.

A. L. Bovenberg and R. A. de Mooij. Environmental Tax Reform and Endogenous Growth. *Journal of Public Economics*, 63:207–237, 1997.

W. A. Brock and M. S. Taylor. Economic Growth and the Environment: A Review of Theory and Empirics. In P. Aghion and S. N. Durlauf, editors, *Handbook of Economic Growth*, volume 1B, pages 1749–1821. Elsevier, 2005.

W. A. Brock and M. S. Taylor. The Green Solow model. *Journal of Economic Growth*, 15(2):127–153, 2010.

S. B. Brunnermeier and M. A. Cohen. Determinants of Environmental Innovation in US Manufacturing Industries. *Journal of Environmental Economics and Management*, 45:278–293, 2003.

J. Davidson and J. M. Norbeck. *An Interactive History of the Clean Air Act.* Elsevier, 2012.

S. M. de Bruyn. *Economic Growth and the Environment: An Empirical Analysis.* Tinbergen Institute Research Series, 1999.

A. Dechezleprêtre, R. Martin, and M. Mohnen. Knowledge Spillovers from Clean and Dirty Technologies. *CEP Discussion Paper No 1300*, 2014.

A. K. Dixit and J. E. Stiglitz. Monopolistic Competition and Optimum Product Diversity. *American Economic Review*, 67(3):297–308, 1977.

R. J. Gordon. Is U.S. Economic Growth Over? Faltering Innovation Confronts the Six Headwinds. *NBER Working Paper*, 2012.

R. Gradus and S. Smulders. The Trade-off Between Environmental Care and Long-term Growth - Pollution in Three Prototype Growth Models. *Journal of Economics*, 58(1):25–51, 1993.

A. Grimaud and L. Rouge. Environment, Directed Technical Change and Economic Policy. *Environmental and Resource Economics*, 41:439–463, 2008.

G. M. Grossman and E. Helpman. *Innovation and Growth in the Global Economy.* MIT Press, Cambridge, MA, 1991.

C. Groth and F. Ricci. Optimal Growth when Environmental Quality is a Research Asset. *Research in Economics*, 65:340–352, 2011.

R. Hanna and P. Olival. The Effect of Pollution on Labor Supply: Evidence from a Natural Experiment in Mexico City. *Journal of Public Economics*, 122:68–79, 2015.

R. Hart. Growth, Environment and Innovation - A Model with Production Vintages and Environmentally Oriented Research. *Journal of Environmental Economics and Management*, 48:1078–1098, 2004.

R. Hart. Can Environmental Regulations Boost Growth? In L. Bretschger and S. Smulders, editors, *Sustainable Resource Use and Economic Dynamics*, pages 53–70. Springer, 2007.

T. Havránek. Measuring Intertemporal Substitution: The Importance of Method Choices and Selective Reporting. *Journal of the European Economic Association*, 12(6):1180–1204, 2015.

I. Haščič, M. Hemar, N. Johnstone, C. Michel, and J. Poirier. Environmental Policy Stringency and Technological Innovation: Evidence from Survey Data and Patent Counts. *Applied Economics*, 44:2157–2170, 2012.

D. Hemous. The Dynamic Impact of Unilateral Environmental Policies. *INSEAD Working Paper*, 2015.

F. Hettich. Growth Effects of a Revenue-neutral Environmental Tax Reform. *Journal of Economics*, 67(3): 287–316, 1998.

P. Howitt. Steady Endogenous Growth with Population and R&D Inputs Growing. *Journal of Political Economy*, 107(4):715–730, 1999.

N. Johnstone and J. Labonne. Environmental policy management and R&D. *OECD Economic Studies*, 2006/1:169–203, 2006.

C. I. Jones. R&D-based Models of Economic Growth. *Journal of Political Economy*, 103(4):759–784, 1995.

C. I. Jones. Growth and Ideas. In P. Aghion and S. N. Durlauf, editors, *Handbook of Economic Growth*, volume 1B, pages 1063–1111. Elsevier, 2005.

H. Mohtadi. Environment, Growth, and Optimal Policy Design. *Journal of Public Economics*, 63:119–140, 1996.

S. B. Nielsen, L. H. Pedersen, and P. B. Sørensen. Environmental Policy, Pollution, Unemployment, and Endogenous Growth. *International Tax and Public Finance*, 2:185–205, 1995.

M. Okubo. The Intertemporal Elasticity of Substitution: An Analysis Based on Japanese Data. *The Review of Economic Studies*, 78:367–390, 2011.

D. Popp. International Innovation and Diffusion of Air Pollution Control Technologies: The Effect of NO_x and SO_2 Regulation in the US, Japan and Germany. *Journal of Environmental Economics and Management*, 51:46–71, 2006.

M. E. Porter and C. van der Linde. Toward a New Conception of the Environment-Competitiveness Relationship. *Journal of Economic Perspectives*, 9(4):97–118, 1995.

A. Pottier, J.-C. Hourcade, and E. Espagne. Modelling the Redirection of Technical Change: The Pitfalls of Incorporeal Visions of the Economy. *Energy Economics*, 42:213–218, 2014.

F. Ricci. Environmental Policy and Growth when Inputs are Differentiated in Pollution Intensity. *Environmental and Resource Economics*, 38:285–310, 2007a.

F. Ricci. Channels of transmission of environmental policy to economic growth: A survey of the theory. *Ecological Economics*, 60:688–699, 2007b.

P. M. Romer. Endogenous Technological Change. *Journal of Political Economy*, 98(5):71–102, 1990.

G. Saint-Paul. Environmental Policy and Directed Innovation in a Schumpeterian Growth Model. *IDEI Working Papers, 153*, 2002.

P. Schou. Pollution Externalities in a Model of Endogenous Fertility and Growth. *International Tax and Public Finance*, 9:709–725, 2002.

S. Smulders and M. de Nooij. The impact of energy conservation on technology and economic growth. *Resource and Energy Economics*, 25:59–79, 2003.

D. I. Stern. The Environmental Kuznets Curve: A Primer. *CCEP Working Paper 1404*, 2014.

N. L. Stokey. Are There Limits to Growth? *International Economic Review*, 39(1):1–31, 1998.

I. van den Bijgaart. The Unilateral Implementation of a Sustainble Growth Path with Directed Technical Change. *CREE Working Paper*, 2014.

C. R. Vogan. Pollution Abatement and Control Expenditures, 1972–94. *Survey of Current Business*, 76: 48–67, 1996.

J. G. Zivin and M. Neidell. The Impact of Pollution on Worker Productivity. *American Economic Review*, 102(7):3652–3673, 2012.

A Appendices

A.1 Proof of Lemma 1

Proof. From (3.4), (3.8), and (3.12) it follows that:

$$Z_{jt} = \left(\frac{\chi \mu P_{jt} \tau_t}{q_{ht} Z_{jt}} \right)^{\mu} B_t^{\mathrm{MAX}} \Lambda_B^{1-\mu}, \quad \Lambda_B \equiv \frac{1-\mu}{1-\mu+\ln(\xi)}.$$

From the above and (3.3) it can be shown that:

$$P_{jt}(Y_{jt}) = Y_{jt}^{\frac{\beta}{1+\chi\mu}} K_{1,t}, \quad K_{1,t} \equiv \left(\frac{q_{ht}}{\chi \mu \tau_t} \right)^{\frac{\chi\mu}{1+\chi\mu}} \left(B_t^{\mathrm{MAX}} \right)^{\frac{-\chi}{1+\chi\mu}} \Lambda_B^{\frac{(\mu-1)\chi}{1+\chi\mu}}.$$

It follows directly that: $P'_{jt}(Y_{jt}) > 0$ and $P''_{jt}(Y_{jt}) < 0$. Define pollution intensity as: $\mathcal{P}_t \equiv P_t / Y_t$. Pollution intensity is then given by: $\mathcal{P}_{jt}(Y_{jt}) = Y_{jt}^{\frac{\beta-1-\chi\mu}{1+\chi\mu}} K_{1,t}$. It follows that: $\mathcal{P}'_t(Y_{jt}) < 0$ and $\mathcal{P}''_t(Y_{jt}) > 0$. Substituting (3.11) and (3.12) into (3.10) yields:

$$\pi^f_{jt}(Y_{jt}) = (1-\alpha) Y_{jt} + \alpha \beta F \tau_t A_t^{\mathrm{MAX}} \mathcal{P}_t(Y_{jt}) - \alpha F A_t^{\mathrm{MAX}} - (1 + \chi\mu - \alpha\beta) \tau_t P_{jt}(Y_{jt}).$$

There is a global maximum for $\pi^f_{jt}(Y_{jt})$. Specifically, in the global maximum for $\pi^f_{jt}(Y_{jt})$:

$$(1-\alpha) = (1 + \chi\mu - \alpha\beta) \tau_t P'_{jt}(Y_{jt}) - \alpha \beta F \tau_t A_t^{\mathrm{MAX}} \mathcal{P}'_t(Y_{jt})$$

The left hand side is constant. Meanwhile, the right hand side approaches infinity for Y_{jt} approaching zero. The right hand side is monotonically decreasing in Y_{jt} and approaches zero asymptotically. Thus, a unique $Y_{jt} = \hat{Y}_t$ solves the equation. As only one Y_{jt} value is consistent with profit maximization, all firms in the final goods sector produce the same output: $Y_{jt} = \hat{Y}_t$ for all $j \in [0, N_t]$. As P_{jt} is a monotonically increasing function of Y_{jt}: $P_{jt} = \hat{P}_t$ for all $j \in [0, N_t]$. It then follows from the two first-order conditions that: $x_{ijt} = \hat{x}_{it}$ and $z_{hjt} = \hat{z}_{ht}$ for all $j \in [0, N_t]$. From (3.4) it follows that: $Z_{jt} = \hat{Z}_t$ for all $j \in [0, N_t]$. From the expressions for aggregation output and pollution emission it follows that: $\hat{Y}_t = (1/N_t) Y_t$ and $\hat{P}_t = (1/N_t) P_t$. ☐

A.2 Deriving (3.30) and (3.31)

The method used to obtain (3.30) and (3.31) are identical. Due to space constraints, I focus on (3.30). As $C_t = Y_t$ it follows from (3.26) that: $Y_s = Y_t \ \mathrm{e}^{\int_t^s (r_u - \rho)\,\mathrm{d}u}$, where $s \geq t$. From (3.6) it follows that: $A_s^{\mathrm{MAX}} = A_t^{\mathrm{MAX}} \mathrm{e}^{\lambda \ln(\gamma) \int_t^s n_u^A \,\mathrm{d}u}$. Plotting these expressions plus (3.18) into (3.25) yields

$$V_{it}^{T_{it}+1} = \int_t^{\infty} \frac{(1-\alpha)(1-\chi\mu\phi-\phi)}{\Lambda_A} \left(\frac{A_t^{\mathrm{MAX}}}{A_t^{\mathrm{MAX}}} \right)^{\frac{1}{1-\alpha}} Y_t \ \mathrm{e}^{-\int_t^s \left(r_u + n_u^A \lambda \Lambda_A^{-1} - r_u + \rho\right) \mathrm{d}u} \,\mathrm{d}s.$$

Dividing the value function by w_t and using (3.27):

$$v_{it} = \Omega_2 L_t \int_t^{\infty} \mathrm{e}^{-\int_t^s \rho + n_u^A \lambda \Lambda_A^{-1} \,\mathrm{d}u} \,\mathrm{d}s, \quad v_{it} \equiv \frac{V_{it}^{T_{it}+1}}{w_t}.$$

Taking the derivative with respect to time yields

$$0 = g_{L,t} v_{it} + \Omega_2 L_t \frac{\mathrm{d}}{\mathrm{d}t} \left(\int_t^{\infty} e^{M(s,t)} \,\mathrm{d}s \right), \quad M(s,t) \equiv - \int_t^s \rho + n_u^A \lambda \Lambda_A^{-1} \,\mathrm{d}u, \tag{A.1}$$

where it is used that $v_{it} = 1/\lambda$ according to (3.23). The main difficulty is to differentiate the integral term. According to Leibniz's rule

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\int_t^b e^{M(s,t)}\,\mathrm{d}s\right) = \int_t^b \frac{\partial e^{M(s,t)}}{\partial M(s,t)}\frac{\partial M(s,t)}{\partial t}\,\mathrm{d}s - 1,$$

where $b >> t$. The partial derivative of $M(s,t)$ with respect to t is given by

$$\frac{\partial M(s,t)}{\partial t} = \frac{\partial}{\partial t}\left(-\int_t^s \rho + n_u^A\lambda\Lambda_A^{-1}\,\mathrm{d}u\right) = \rho + n_t^A\lambda\Lambda_A^{-1},$$

where the last equality follows from the formula: $\frac{\partial}{\partial a}\left(\int_a^b f(x)\,\mathrm{d}x\right) = -f(a)$. Thus,

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\int_t^b e^{M(s,t)}\,\mathrm{d}s\right) = \left(\rho + n_t^A\lambda\Lambda_A^{-1}\right)\int_t^b e^{M(s,t)}\,\mathrm{d}s - 1.$$

Plotting the expression into (A.1) yields: $0 = \left(\rho + n_t^A\lambda\Lambda_A^{-1} + g_{L,t}\right)v_{it} - \Omega_2 L_t$. The expression can easily be rewritten as (3.30).

A.3 Proof of Lemma 2

Proof. From (3.30) and (3.31):

$$n_t^A = \Omega_2\Lambda_A L_t - \frac{\Lambda_A}{\lambda}\rho - \frac{\Lambda_A}{\lambda}g_{L,t}, \quad \text{and} \quad n_t^B = \Omega_3\Lambda_B L_t - \frac{\Lambda_B}{\eta}\rho - \frac{\Lambda_B}{\eta}g_{L,t} \tag{A.2}$$

Substituting the expression from (A.2) into (3.9):

$$\dot{L}_t = \left(\frac{L_t}{\left(\frac{\Lambda_A}{\lambda} + \frac{\Lambda_B}{\eta}\right)}\right)\underbrace{\left((1 + \Omega_2\Lambda_A + \Omega_3\Lambda_B)\,L_t - \left(\bar{L} + \rho\left(\frac{\Lambda_A}{\lambda} + \frac{\Lambda_B}{\eta}\right)\right)\right)}_{\equiv D_t}. \tag{A.3}$$

It is clear from (A.3) that if $L_0 = 0$ then $L_t = 0$ for all $t \geq 0$. This violates (3.30) and (3.31). Hence, L_0 must be positive.

If $D_0 > 0$, then $\dot{L}_0 > 0$ and $D_t, \dot{L}_t > 0$ for all $t \geq 0$. In fact, it follows from (A.3) that $\frac{\mathrm{d}g_{L,t}}{\mathrm{d}t} > 0$. Hence, L_t will continue to grow with an increasing growth rate. For some $t > 0$, $L_t > \bar{L}$ which violates (3.9). Thus, $D_0 > 0$ leads to a contradiction.

If $D_0 < 0$, then $\dot{L}_0 < 0$ and $D_t, \dot{L}_t < 0$ for all $t \geq 0$. It follows from (A.3) that $\frac{\mathrm{d}g_{L,t}}{\mathrm{d}t} < 0$. Hence, L_t will decrease over time at an ever decreasing growth rate. For some $t > 0$, $L_t < 0$. Thus, $D_0 < 0$ leads to a contradiction.

If $D_t = 0$, then $\dot{L}_0 = 0$ and $D_t, \dot{L}_t = 0$ for all $t \geq 0$. This does not lead to a contradiction. Hence, D_t must be zero, which implies that

$$L_t = \frac{\bar{L} + \rho\left(\frac{\Lambda_A}{\lambda} + \frac{\Lambda_B}{\eta}\right)}{1 + \Omega_2\Lambda_A + \Omega_3\Lambda_B}. \tag{A.4}$$

It follows from (A.4) that $L_t = L$, i.e. labor input in manufacturing is constant over time. Hence, $g_{L,t} = g_L = 0$ in equilibrium. □

A.4 Proof of Proposition 1

Proof. From (3.15), (3.16), (3.17), and (3.27) it follows that

$$Y_t = \left(\frac{1-\alpha(1-\phi\beta)-\chi\mu\phi-\phi}{F(1-\chi\mu\phi-\phi)}\Lambda_A\right)^{\frac{1-\alpha}{\alpha}}\left(\frac{\alpha^2(1-\phi\beta)}{\Omega_1}\right)L_tA_t^{\mathrm{MAX}}.$$

As $L_t = L$ according to Lemma 2, Y_t grows at the same rate as A_t^{MAX}, which is denoted g_A and given by: $g_A = n^A\lambda\ln(\gamma)$. This implies that $N_t = N$ according (3.15). It follows immediately that $\hat{Y}_t$ grows at the rate g_A. As $C_t = Y_t$, C_t grows at the rate g_A, and from (3.27) it follows that w_t grows at the rate g_A. As $\hat{Y}_t$ and w_t grow at the same rate, it follows from (3.20) that $\hat{Z}_t$ grows at the same rate as B_t^{MAX}, which is denoted g_B and given by: $g_B = n^B\eta\ln(\xi)$. As $P_t = (1/N)\hat{P}_t$, it follows from (3.3) that P_t and $\hat{P}_t$ grow at the same rate denoted g_P and given by: $g_P = \beta g_A - \chi g_B = \beta n^A\lambda\ln(\gamma) - \chi n^B\eta\ln(\xi)$. □

A.5 Proof of Proposition 2

Proof. Differentiating (3.32), (3.33), and (3.34) with respect to ϕ:

$$\frac{\mathrm{d}n^A}{\mathrm{d}\phi} = -\frac{(1-\alpha)\chi\mu}{(1-\phi)^2}\Gamma < 0; \quad \frac{\mathrm{d}n^B}{\mathrm{d}\phi} = \frac{(1-\mu)\chi\mu}{(1-\phi)^2}\Gamma > 0; \quad \text{and}$$

$$\frac{\mathrm{d}L}{\mathrm{d}\phi} = \frac{(\mu-\alpha)\chi\mu}{(1-\phi)^2}\Gamma \gtreqless 0 \quad \text{for} \quad \alpha \lesseqgtr \mu.$$

Labor inputs in the production and abatement intermediate subsectors are given by

$$x = \frac{\alpha(1-\chi\mu\phi-\phi)}{1-\phi}\Gamma \quad \text{and} \quad z = \frac{\chi\mu^2\phi}{1-\phi}\Gamma,$$

where the first equation follows from (3.16), (3.17), and (3.27), and the second equation follows from (3.21). Differentiating x and z with respect to ϕ yields

$$\frac{\mathrm{d}x}{\mathrm{d}\phi} = \frac{-\alpha\chi\mu}{(1-\phi)^2}\Gamma < 0 \quad \text{and} \quad \frac{\mathrm{d}z}{\mathrm{d}\phi} = \frac{\chi\mu^2}{(1-\phi)^2}\Gamma > 0.$$

□

A.6 Model with CRRA utility

In this appendix, I show the main relations and results of the model from Section 3 when the instantaneous utility function is given by (5.1). The labor allocation of the economy can be expressed by the following three equations:

$$L = \frac{\bar{L} + \frac{\rho\Lambda_A}{\Sigma_1\lambda}(1-\Sigma_2) + \frac{\rho\Lambda_B}{\eta}}{1 + \frac{(1-\alpha)(1-\chi\mu\phi-\phi)}{\Sigma_1\Omega_1}(1-\Sigma_2) + \frac{(1-\mu)\chi\mu\phi}{\Omega_1}}, \tag{A.5}$$

$$n^A = \frac{(1-\alpha)(1-\chi\mu\phi-\phi)}{\Sigma_1\Omega_1}L - \frac{\rho\Lambda_A}{\Sigma_1\lambda}, \quad \text{and} \tag{A.6}$$

$$n^B = \frac{(1-\mu)\chi\mu\phi}{\Omega_1}L - \frac{\rho\Lambda_B}{\eta} - \Sigma_2 n^A, \tag{A.7}$$

where $\Sigma_1 \equiv 1 + (\theta-1)\ln(\gamma)\Lambda_A$ and $\Sigma_2 \equiv (\theta-1)(\lambda/\eta)\ln(\gamma)\Lambda_B$.

It turns out that the qualitative policy implications of the extended model are very similar to those of the model presented in Section 3. Consider the following proposition.

Proposition 4. *Assuming that parameters are such that n^A, n^B, and L are positive:*

(i) $\frac{\mathrm{d}L}{\mathrm{d}\phi} \gtreqless 0$ *for* $\frac{\alpha}{1-\alpha} \lesseqgtr \left(\frac{\mu}{1-\mu}\right)\left(\frac{1-\Sigma_2}{\Sigma_1}\right)$;

(ii) $\frac{\mathrm{d}n^A}{\mathrm{d}\phi} < 0$ *and* $\frac{\mathrm{d}n^B}{\mathrm{d}\phi} > 0$; *and*

(iii) $\frac{\mathrm{d}g_A}{\mathrm{d}\phi} < 0$ *and* $\frac{\mathrm{d}g_P}{\mathrm{d}\phi} < 0$.

Proof. Differentiating (A.5) with respect to ϕ yields

$$\frac{\mathrm{d}L}{\mathrm{d}\phi} = \left(\frac{-L}{1+\frac{(1-\alpha)(1-\chi\mu\phi-\phi)}{\Sigma_1\Omega_1}(1-\Sigma_2)+\frac{(1-\mu)\chi\mu\phi}{\Omega_1}}\right)\left(\frac{(1-\mu)\chi\mu\alpha}{\Omega_1^2} - \frac{1-\Sigma_2}{\Sigma_1}\frac{(1-\alpha)\chi\mu^2}{\Omega_1^2}\right).$$

The expression is positive if

$$\frac{(1-\mu)\chi\mu\alpha}{\Omega_1^2} < \frac{1-\Sigma_2}{\Sigma_1}\frac{(1-\alpha)\chi\mu^2}{\Omega_1^2} \quad \Leftrightarrow \quad \frac{\alpha}{1-\alpha} < \left(\frac{\mu}{1-\mu}\right)\left(\frac{1-\Sigma_2}{\Sigma_1}\right).$$

The expression is negative for the opposite inequality. Hence, (i) in Proposition 4 is proved.

The derivative of n^A with respect to ϕ amounts to

$$\frac{\mathrm{d}n^A}{\mathrm{d}\phi} = \frac{1-\alpha}{\Sigma_1}\left(\frac{(1-\chi\mu\phi-\phi)}{\Omega_1}\frac{\mathrm{d}L}{\mathrm{d}\phi} - \frac{\chi\mu^2}{\Omega_1^2}L\right).$$

If the derivative of L with respect to ϕ is negative or zero, it follows immediately that the above expression is negative. It then follows from the labor market clearing condition that n^B is increasing in ϕ.

Assume that the derivative of L with respect to ϕ is positive. Then it is not straightforward to evaluate the sign of the derivative of n^A with respect to ϕ. Instead, consider the following expression obtained from (A.6) and (A.7):

$$\left(\frac{n^B}{L}\right) = \left(\frac{(1-\mu)\chi\mu\phi}{\Omega_1} - \frac{(1-\alpha)(1-\chi\mu\phi-\phi)}{\Omega_1}\frac{\Sigma_2}{\Sigma_1}\right) - \frac{\rho\Lambda_B}{\eta\Sigma_1}\left(\frac{1}{L}\right)$$

The derivative of (n^B/L) with respect to ϕ amounts to

$$\frac{\mathrm{d}\left(\frac{n^B}{L}\right)}{\mathrm{d}\phi} = \frac{(1-\mu)\chi\mu\alpha}{\Omega_1^2} + \frac{(1-\alpha)\chi\mu^2}{\Omega_1^2}\left(\frac{\Sigma_2}{\Sigma_1}\right) + \frac{\rho\Lambda_B}{\eta\Sigma_1}\left(\frac{1}{L}\right)^2\frac{\mathrm{d}L}{\mathrm{d}\phi}.$$

As the derivative of L with respect to ϕ is positive, this derivative is also positive. It is straightforward to show that this implies that the derivative of n^B with respect to ϕ is positive, and then it follows from the labor market clearing condition that n^A is decreasing in ϕ. This finishes the proof of (ii), and the proof of (iii) follows directly from (ii), (3.6), and (3.7). ☐

A.7 Model with physical capital

In this appendix, I sketch a model closely related to the model presented in Section 3, but with a production process which incorporates physical capital. Relations similar to those presented in

Section 3 will not be repeated.

In the final goods sector firms operate under monopolistic competition a la Dixit and Stiglitz (1977). Firms produce using the production technology

$$Y_{jt} = L_{jt}^{1-\alpha} \int_0^1 x_{ijt}^{\alpha} A_{it} \, \mathrm{d}i - F A_t, \quad A_t = \int_0^1 A_{it} \, \mathrm{d}i,$$

where L_{jt} is labor input for firm j. Final goods are aggregated as follows:

$$Y_t = \left(\int_0^{N_t} Y_{jt}^{\frac{\epsilon-1}{\epsilon}} \, \mathrm{d}j \right)^{\frac{\epsilon}{\epsilon-1}}, \quad \epsilon > 1.$$

Final goods can either be consumed or transformed into (physical) capital: $Y_t = C_t + \bar{I}_t$, where $\bar{I}_t$ is final goods used for investment. Consumers face the intra-temporal problem of maximizing Y_t with respect to $(Y_{jt})_{j \in [0,N_t]}$ given prices $(m_{jt})_{j \in [0,N_t]}$ subject to a given budget constraint. The standard solution is

$$Y_{jt} = Y_t \left(\frac{m_{jt}}{M_t} \right)^{-\epsilon}, \quad M_t \equiv \left(\int_0^{N_t} m_{jt}^{1-\epsilon} \, \mathrm{d}j \right)^{\frac{1}{1-\epsilon}}, \tag{A.8}$$

where M_t is the ideal price index, and thus, the price of final goods. I set M_t equal to one, such that all prices are in terms of final goods.

Following Aghion and Howitt (1998, p. 93-99), I assume that a monopolist needs $A_{it} x_{it}$ and $B_{ht} z_{ht}$ units of capital to produce x_{it} and z_{ht} units of intermediate good i and h, respectively. The idea is that it requires more capital to produce more advanced intermediate goods.

Capital evolves according to the differential equation: $\dot{K}_t = \bar{I}_t - \delta K_t$, where K_t is aggregate capital and $\delta > 0$ is the rate of depreciation. The monopolists rent capital from the representative household at the price $R_t = r_t + \delta$.

The labor and capital market clearing conditions are given by

$$\bar{L} = \int_0^{N_t} L_{jt} \, \mathrm{d}j + n_t^A + n_t^B \quad \text{and} \quad K_t = \int_0^{N_t} \left(\int_0^1 x_{ijt} A_{it} \, \mathrm{d}i + \int_0^1 z_{hjt} B_{ht} \, \mathrm{d}h \right) \mathrm{d}j,$$

In a steady state equilibrium, the labor allocation is given by

$$n^A = \tilde{\Omega}_1 L - \frac{\rho}{\lambda \left(1 + \ln(\gamma)\right)}, \quad \tilde{\Omega}_1 \equiv \frac{\alpha \bar{a}}{1 + \ln(\gamma)}, \quad n^B = \tilde{\Omega}_2 L - \frac{\rho}{\eta \left(1 + \ln(\xi)\right)},$$

$$\tilde{\Omega}_2 \equiv \frac{(1-\mu)\chi\mu\bar{b}}{(1-\alpha)\left(1+\ln(\gamma)\right)} \frac{\phi}{1 - \phi - \chi\mu\phi}, \quad \text{and} \quad L = \frac{\bar{L} + \frac{\rho}{\lambda(1+\ln(\gamma))} + \frac{\rho}{\eta(1+\ln(\xi))}}{1 + \tilde{\Omega}_1 + \tilde{\Omega}_2}$$

where $\bar{a} = A_t^{\mathrm{MAX}}/A_t$ and $\bar{b} = B_t^{\mathrm{MAX}}/B_t$. The derivative of $\tilde{\Omega}_1$ with respect to ϕ is clearly zero, while the derivative of $\tilde{\Omega}_2$ with respect to ϕ is negative. It then follows that

$$\frac{\mathrm{d}L}{\mathrm{d}\phi} < 0, \quad \frac{\mathrm{d}n^A}{\mathrm{d}\phi} < 0 \quad \text{and} \quad \frac{\mathrm{d}n^B}{\mathrm{d}\phi} = -\left(\frac{\mathrm{d}L}{\mathrm{d}\phi} + \frac{\mathrm{d}n^A}{\mathrm{d}\phi} \right) > 0.$$

Thus, in the long run the growth rates of output and pollution emission are reduced unambiguously, when the environmental policy is tightened.

A.8 Calibrating the model

The growth rate of pollution emission is given by

$$\frac{\dot{P}_t}{P_t} = \beta\frac{\dot{Y}_t}{Y_t} - \chi\frac{\dot{\hat{Z}}_t}{\hat{Z}_t}. \tag{A.9}$$

Assuming that the growth rate of $\hat{Z}_t$ is constant (as predicted by the model), the discrete time version of (A.9) is approximated as: $\tilde{P}_t = \sigma + \beta\tilde{Y}_t + \epsilon_t$, where σ is a constant, $\tilde{X}_t \equiv \ln\left(X_{t+1}/X_t\right)$, and ϵ_t is the error term. Output and pollution emission are approximated by real GDP and the pollution emission index, respectively. Using data for the period 1970-2012 an OLS regression suggests that β is around 0.7 and statistically significant at the one pct. level.

A.9 Computing CCC

In steady state with $\kappa = 0$ welfare is given by:

$$U = \int_0^\infty \frac{C_t^{1-\theta}}{1-\theta}\mathrm{e}^{-\rho t}\,\mathrm{d}t = \frac{C_0^{1-\theta}}{(1-\theta)(\rho-(1-\theta)g_A)}.$$

The current consumption level that makes the representative household indifferent between the old and new economic growth rate is the $\tilde{C}_0$ that solves the equality:

$$\frac{C_0^{1-\theta}}{(1-\theta)(\rho-(1-\theta)g_A)} = \frac{\tilde{C}_0^{1-\theta}}{(1-\theta)(\rho-(1-\theta)g_A^R)},$$

where g_A^R is the economic growth rate after the reform. Using this expression it is straightforward to compute $\tilde{C}_0/C_0$, and thereby the CCC stemming from a reduction in the economic growth rate. The level effect on current consumption can be added directly to obtain the total CCC.