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Abstract

According to a classical argument, an employer handicaps herself if she bases hiring decisions on factors unrelated to productivity; therefore, discrimination is undermined by competition. The present paper, in contrast, argues that being discriminatory can be a commitment device that helps an employer and its rivals to partially segment the labor market, which leads to lower wages and higher profits. Discrimination can thus be an endogenous response to (changes in) competition. Indeed, the relationship between discrimination and competition can be non-monotone. Moreover, a ban on wage discrimination (which may be feasible, as such discrimination is easily detectable) may lead to discrimination in hiring (which cannot be banned because it is harder to observe).

Keywords: discrimination, competition, strategic interaction, market segmentation

JEL classification: J71 (Labor Discrimination), D43 (Oligopoly and Other Forms of Markets Imperfection)

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A business man or an entrepreneur who expresses preferences in his business activities that are not related to productive efficiency is at a disadvantage compared to other individuals who do not. Such an individual is in effect imposing higher costs on himself than are other individuals who do not have such preferences. Hence, in a free market they will tend to drive him out. (Milton Friedman, 1962, pp. 109-110)

[...] by "binding oneself" [thus also imposing a cost on oneself] a party can credibly commit to a pattern of competitive actions or reactions, and therefore affect the expectations and actions of other parties and the resulting competitive dynamics. (Metin Sengul et al., 2012, p. 378, crediting Thomas Schelling, 1956)

1 Introduction

Suppose a prejudiced employer refuses to hire members of some particular group—say, blacks. Then, according to a classical argument due to Becker (1971), this employer will handicap herself and thus earn less profits, relative to a color-blind employer who always hires the most productive available person. As a consequence, if there is sufficiently much competition in the market in which the employer sells her products, she will in the long run be driven out of that market by a non-discriminating competitor. More generally, the Becker argument suggests that more intense competition leads to less discrimination.

On the other hand, we know from game theory that, in certain environments, an economic agent can *benefit* from handicapping herself. This insight is sometimes illustrated with an anecdote about the Spanish conqueror Hernán Cortéz who, when landing in Mexico to fight the Aztecs, ordered his men to burn the ships they had arrived with. Although this action limited the Spanish side's opportunities, it also carried a strategic benefit: Since the Aztecs now knew that their enemy's only options were to win or die, the former's morale was weakened. As a consequence the Aztecs retreated to the surrounding hills and, thanks to his side's self-inflicted handicap, Cortéz had won a victory.[1]

If the act of discriminating against blacks constitutes a handicap, could it be that also this kind of disability carries an (indirect) strategic benefit? If so, could this benefit outweigh the immediate costs associated with not always hiring the most productive worker? To explore

[1]For an account of this anecdote, see e.g. Ross (2012). For more on the value of commitment, which implies that handicapping oneself can carry a strategic benefit, see Schelling (1956, 1960). There are also many very well-known applications of this idea in the microeconomics literature. Examples of such applications include: (i) the firms in Hotelling's (1929) linear city model have an incentive to strategically locate far off from each other with the purpose of softening future price competition (see also d'Aspremont et al. (1979)); (ii) the firms in Kreps and Scheinkman's (1983) oligopoly model have an incentive to strategically invest in small capacities, again with the purpose of softening future price competition; (iii) in the literature on strategic delegation (Vickers, 1985; Fershtman and Judd, 1987; Sklivas, 1987), owners of firms instruct their managers to maximize revenues instead of profits, in order to make them behave more aggressively and thereby gain market shares and profits; and (iv) in the Stackelberg (1934) model, one firm chooses its quantity before its rival and strategically exploits this opportunity in order to increase its profits.

these questions, I study an oligopsonistic labor market where employers are wage setters, which means that strategic interaction between them matters.[2] I show that in this environment an employer can indeed, in net terms, benefit from being discriminatory in her hiring decisions. Broadly speaking, the reason for this result is that the act of discriminating against black workers helps to segment the labor market, which softens the wage competition between the employer and her rival(s). However, the logic is somewhat subtle (relying, for example, on a strategic complementarity in the wage-setting game) and it will be discussed in greater detail later in this Introduction.

In the formal model that I study there are two firms that hire labor in a duopsonistic labor market.[3] The firms compete with each other over the available workers by simultaneously posting wages. After this each worker chooses which firm, if any, to be employed at. The workers differ from each other with regard to an observable, binary feature (like their skin color or gender), which is unrelated to technology and preferences; each worker thus belongs to a majority group or a minority group. In the spirit of Hotelling's (1929) linear city model, the workers are assumed to differ from each other also with respect to an unobservable "mismatch cost" that determines their non-wage related utility of working for each of the two firms; depending on the importance of this mismatch cost (relative to the worker's wage-related utility), the labor supply elasticity in this labor market may be high or low.[4] At the first stage of the game, prior to the wage competition at stage 2, the firms simultaneously choose whether and how to discriminate.[5] Each firm can (i) refuse to hire members from the minority group, (ii) refuse to hire members from the majority group, or (iii) not refuse to hire workers from either one of the groups. In addition, if having chosen (iii), the firm decides whether to post the same wage to members of the two groups or to be free to practice wage discrimination.

The analysis shows that, at stage 2, the equilibrium wages can be *lower*, which both firms benefit from, if at stage 1 one of them (say firm 1) discriminates in hiring against the minority group and firm 2 practices neither wage discrimination nor discrimination in hiring. How does this outcome arise? The fact that firm 1 hires only majority workers means that firm 2 is a monopsonist in the minority group market. This lowers the labor supply elasticity that firm 2 faces and it therefore optimally lowers its posted wage. Since in this environment the firms' reaction functions are upward-sloping,[6] firm 1's equilibrium wage adjusts in the same direction,

[2]There may still be very intense competition in the markets in which the employers sell their products.

[3]See Manning (2003) for a discussion of imperfect competition in labor markets.

[4]However, in order to guarantee existence of a pure strategy equilibrium, I require the mismatch cost not to be too low. Moreover, to rule out cases where each firm is effectively a monopsonist and thus not interacting with its rival, I exclude the possibility that the mismatch cost is very high.

[5]The firms are assumed to be able to *commit* to these choices. This commitment assumption should be thought of as an analytical shortcut that can help us understand the relative profitability of being a discriminatory and a non-discriminatory firm. In a richer, alternative setting we could have assumed (following Becker, 1971) that some firm owners have a taste for discrimination against a particular group, which is why they are committed to not hiring those workers. In addition we could have modeled an entry/exit process, where the firms' incentives would be determined in part by the strategic gains that are studied in this paper. See Section 5 for further discussion of the commitment assumption.

[6]That is, the employers' choice variables are strategic complements, as defined by Bulow et al. (1985).

i.e., it also drops. This is an indirect, strategic benefit that firm 1 derives from the fact that it discriminates in hiring: Its (per-worker) wage costs are reduced.

Of course, there is also a direct cost that firm 1 incurs, due to the fact that minority workers are not available and thus the firm's output is lower. However, the indirect benefit can dominate the direct cost, thus making it profitable for firm 1 to practice discrimination in hiring against the minority group, given that the rival firm does not discriminate at all. Reversely, it can be optimal for firm 2 not to discriminate, given that firm 1 discriminates in hiring against minority workers. Indeed, I show that such behavior on the part of the firms is an equilibrium of the overall game if the mismatch cost (and thus the labor supply elasticity) is neither too low nor too high. I use the label *discrimination equilibrium* to refer to this kind of equilibrium. (Note that there are actually two such equilibria—one where firm 1 discriminates and one where firm 2 discriminates).

The discrimination equilibrium co-exists with another equilibrium, which I will refer to as a *no discrimination equilibrium.* In such an equilibrium both firms at stage 1 choose (i) not to discriminate in hiring, in combination with (ii) the action that makes them free to do wage discrimination. At stage 2, however, the firms do not have an incentive to use the possibility in (ii), which means that no worker is refused employment and all are offered the same wage. For the part of the parameter space where the two kinds of equilibria co-exist, I argue that the discrimination equilibrium is the most plausible outcome—from the point of view of the two firms, this equilibrium payoff dominates the other one. Employing this equilibrium selection criterion, we can thus conclude that for intermediate levels of the mismatch cost the surviving equilibrium outcome involves discrimination. In the other parts of the parameter space, only the no discrimination equilibrium exists. One consequence of these results is that the model predicts a non-monotonicity in terms of the presence of discrimination: As the exogenous level of the mismatch cost decreases (and thus the labor supply elasticity increases), there is initially no discrimination, then there is discrimination, and finally there is again no discrimination.[7] Hence, in this analysis, more intense competition can to lead (locally) to *more* discrimination, which stands in sharp contrast to the Becker argument.

The model as described above assumes that a firm, if having chosen the right action at stage 1, in principle is allowed to post different wages to the minority and to the majority workers. However, in many economies there are legal constraints that prevent firms from setting discriminatory wages for the same work; moreover, such laws may be difficult to circumvent as wage discrimination often is easy to detect. Discrimination in hiring, in contrast, is a more subtle phenomenon and is therefore likely to be harder for law enforcers to find out about. Motivated by these arguments, I also study a variation of the model where the firms, by assumption, cannot practice wage discrimination. I show that in this version of the model, also without invoking any equilibrium selection criterion, there is a unique equilibrium outcome (up

[7] As explained above, the fact that one of the firms chooses to discriminate is due to a (successful) attempt to segment the market and thus lower the amount of competition. Discrimination, in this model, is thus an endogenous response to an exogenous increase in competition. This analysis highlights the potential problems with a failure to distinguish, in empirical work, between different sources of variation in competition. For further discussion of this, see Section 5.

to the labeling of the firms). If the level of competition in the labor market is relatively low, this equilibrium involves no discrimination. If competition is more intense, the equilibrium involves discrimination, with one firm refusing to hire members of a certain group of workers[8] and the other firm not discriminating at all. In particular, an equilibrium with discrimination in hiring exists for a larger set of parameter values than in the original model. The reason is that the non-availability of the option to wage discriminate eliminates an otherwise profitable deviation. The results thus suggest that the introduction of a ban on wage discrimination may instead lead to discrimination of another kind—the ban facilitates for the firms to discriminate in hiring, which effectively helps them to segment the labor market, lower their labor costs, and increase their profits.

1.1 Related Literature

As mentioned in the introductory paragraph, the argument that competition undermines discrimination is typically associated with Becker (1971). In his analysis of taste-based discrimination, employers are assumed to incur a utility cost if employing workers who belong to a minority group. Arrow (1972) pointed out, in a famous critique of this approach, that "[Beckers employer discrimination model] predicts the absence of the phenomenon it was designed to explain." As a response to this problem, there were two developments in the literature. One of these was the emergence of the literature on statistical discrimination, which was first proposed by Phelps (1972) and Arrow (1973) and recently surveyed by Fang and Moro (2011). This approach relies on a completely different logic than taste-based discrimination does and therefore avoids the criticism. The other development stayed within the framework of taste-based discrimination but argued that there are search frictions that at least partially hinder prejudiced employers from being competed out of business. Early papers in this literature are Borjas and Bronars (1989) and Black (1995). As summarized by Altonji and Blank (1999, p. 3168), these papers "point out that imperfect information about the locations and preferences of customers, employees, and employers will limit the ability of competition and segregation to eliminate the effects of prejudice on labor market outcomes."

Other papers that have identified economic environments that alleviate the incentives of discriminating employers to exit the market include Goldberg (1982) and Holden and Rosén (2014). Goldberg tweaks Becker's (1971) setting by assuming that employers have a taste for whites instead of against blacks (so nepotism rather than prejudices). Being the owner of a business that hires whites is, by assumption, the only way in which a nepotistic employer can indulge her preferences. Therefore she has no incentive to sell her business to a non-nepotistic employer. Holden and Rosén study a search model with costly layoffs and in which a firm may regret a hire. In this environment also non-prejudiced employers avoid hiring workers that are discriminated against, because these workers are less likely to leave the firm if the match turns out to be bad.

The two contributions in the existing literature that are closest to the present paper are

[8] For a part of the parameter space, it is actually the majority group that is discriminated against. However, this requires that the majority group is not too large.

Bhaskar, Manning and To (2002) and Galeotti and Moraga-Gonzalez (2008). In their *Journal of Economic Perspectives* article, Bhaskar et al. discuss racial pay gaps in a setting with taste-based discrimination and imperfect labor market competition. They demonstrate that "discrimination can persist and even enhance employer profits, rather than being competed away" (p. 166). The logic behind their argument is similar to the one in the present paper, relying on a strategic effect that arises when a firm owner has prejudiced preferences and thus offers the minority workers a lower wage. However, their discussion concerns wage discrimination only and not discrimination in hiring (therefore it can also not say anything about the interaction between these two ways of discriminating, as discussed at length above). Moreover, their analysis, which is part of broad discussion of the implications of employers' having market power, is indeed very brief: a figure that depicts the firms' reaction functions and two paragraphs of text. The analysis in the present paper significantly expands on Bhaskar et al.'s discussion and also takes it in a new direction.

Galeotti and Moraga-Gonzalez (2008) study informative and targeted advertising in a model with two firms that compete in homogenous-good product market. At stage 1 the firms choose which consumer group to target in their advertising. At stage 2 the firms compete in prices. The authors show that there is an equilibrium in which the firms can soften the price competition by targeting different consumer segments.[9] The logic is again similar to the one studied in the present paper. However, their application is obviously quite different (for example, Galeotti and Moraga-Gonzalez model an advertising cost that matters for their analysis and which has no natural equivalent in the discrimination setting). Moreover, because of the fact that they assume no product differentiation, there exist only mixed-strategy equilibria in Galeotti and Moraga-Gonzalez's model. This means that (i) their analysis is less tractable and transparent than the one in the present paper and (ii) they are unable to do any comparative statics on the level of product differentiation (i.e., on the magnitude of the mismatch cost).

The remainder of the paper is organized as follows. The next section describes the baseline model. Section 3 carries out the analysis and presents and discusses the results of this model. Section 4 considers the variation of the model where wage discrimination is ruled out by assumption. Section 5 offers conclusions and a broad discussion. Many of the proofs are collected in the Appendix, and some algebra-heavy derivations can be found in the online Supplementary Material (Lagerlöf, 2016).

2 A Model of Discriminating Firms

Consider a labor market in which two firms (indexed by $i = 1, 2$) compete for workers. The firms are exogenously located at each end of the unit line; formally, firm i's location is denoted by $\widehat{x}_i$, with $\widehat{x}_1 = 0$ and $\widehat{x}_2 = 1$. There are two groups of workers: majority workers (A) and minority workers (B). The firms are able to distinguish between A and B workers as they differ from each other with respect to some observable characteristic, like their skin color or gender. Within each group, workers differ from each other with respect to their location on the unit

[9] Other papers in the literature on targeted advertising and market segmentation include Roy (2000), although this is less close to the present paper.

line, and for both groups the distribution of locations is uniform. The locations of individual workers cannot be observed by the firms. The mass of all workers is normalized to one, and the fraction of workers belonging to group j (for $j = A, B$) is denoted by γ_j, with $\gamma_A + \gamma_B = 1$ and $\gamma_B \in \left(0, \frac{1}{2}\right)$.

Workers want to be employed by one of the two firms or by no firm. In particular, the utility of a worker who is located at point $x \in [0, 1]$ on the line is given by

$$u(x) = \begin{cases} w_i - t\,|x - \widehat{x}_i| & \text{if working at firm } i \\ 0 & \text{if working at neither firm,} \end{cases} \tag{1}$$

where $w_i \geq 0$ is the wage posted by firm i and $t > 0$ is a parameter. The location x thus determines the worker's non-wage related utility (or cost) of working at a particular firm. For example, the worker may have to incur a cost of commuting between his home and the workplace; in this interpretation, x is the location of the worker's home and $t\,|x - \widehat{x}_i|$ is the cost of commuting to employer i. An alternative interpretation of x is that it determines, more generally, the worker's "mismatch cost": the extent to which the worker likes his colleagues, his specific work tasks, the corporate culture at each of the two firms, etc. The parameter t measures the relative importance of the commuting or mismatch costs. For relatively low values of t, the worker is primarily concerned about the wage when choosing between the two firms (the labor supply elasticity is high). Hence the smaller is t, the more intense is the competition between the firms in the recruitment of workers.

The assumptions made above imply that an A and a B worker with the same location x have identical preferences. Moreover, the distribution of x values is the same (namely, uniform) across the two groups. The productivity of members of the two groups is also the same. All in all, the two groups are identical in terms of preferences and productivity.

Let $l_i(w_1, w_2)$ denote the mass of people working for firm i. It is assumed that each firm has access to a constant returns to scale production technology with labor as the only input. Moreover, it can sell its produce at an exogenous price $p > 0$. This means that firm i's profits can be written as

$$\pi_i = (p - w_i)\, l_i(w_1, w_2). \tag{2}$$

The timing of events is as follows.

1. The two firms simultaneously and independently commit to an action $y_i \in S$, where $S \stackrel{\text{def}}{=} \{A, B, C, D\}$. The action $y_i = A$ means that firm i can hire workers only from group A, while $y_i = B$ means that firm i can hire workers only from group B. That is, taking either one of these two actions amounts to a choice to *discriminate in the hiring decision*, against members of a certain group. The action $y_i = C$ means that firm i is free to hire workers from both groups and that it is committed to paying the two types of workers the same wage. That is, this action amounts to a choice not to discriminate in any way. The action $y_i = D$ means that firm i is free to hire workers from both groups and to pay the two types of workers wages that may differ. That is, this action means that the firm does not discriminate in the hiring decision but is free to practice *wage discrimination*.[10]

[10]Note that, for all four actions, wage discrimination is not feasible *within* a group. This is because the location

2. The stage 1 decisions, y_1 and y_2, are observed by the two firms and then they simultaneously post their wages. For the cases where $y_i \in \{A, B, C\}$, firm i's (single) posted wage is denoted by w_i. For the case $y_i = D$, firm i's two posted wages are denoted by w_i^A and w_i^B (with the obvious meaning).

3. The workers observe the outcomes of stages 1 and 2 and then decide which firm to work for (or not to work for any firm at all). The firms' stage 1 actions may restrict a worker's opportunity to work for a particular firm. For example, if $(y_1, y_2) = (A, C)$, then workers who belong to group B are not allowed to work for firm 1; instead their only choice is between working for firm 2 or not at all. If $(y_1, y_2) = (A, A)$, then group B workers are not allowed to work for either one of the firms. Etcetera.

I will impose some restrictions on the parameter space. To this end, define the following short-hand notation:

$$\varphi(\gamma_B) \stackrel{\text{def}}{=} \frac{18\gamma_B (1-\gamma_B)}{(3+\gamma_B)^2 + 18\gamma_B (1-\gamma_B)}.$$

The assumption below ensures that a pure-strategy equilibrium exists in all possible stage 2 subgames.

Assumption 1. *The parameters p, t, and γ_B satisfy $\varphi(\gamma_B) \leq \frac{t}{p} \leq \frac{2}{3}$.*

To conclude the model description, assume that the workers maximize their utilities (as stated in (1)) and that the two firms maximize their profits (as stated in (2)). The solution concept that will be employed is that of subgame perfect Nash equilibrium.

3 Equilibrium Analysis

We can solve for the (subgame perfect Nash) equilibria of the model by first, for each subgame $(y_1, y_2) \in S^2$, study the formation of the workers' labor supply at stage 3 of the game and the wage competition between the firms at stage 2. Then, using these equilibrium wages, we can compute the firms' equilibrium profits in each subgame. Finally, with the help of the profit expressions, we can solve the reduced-form game at the first stage. Denoting by $\pi_{j|k}$ the equilibrium profit of a firm that has chosen $y_i = j$ when the other firm has chosen $y_{-i} = k$, this reduced-form game is shown in Fig. 1.

In the next subsection I derive the profit expressions that are indicated in each cell of Fig. 1. Thereafter, in subsection 3.2, I study the solutions to the game shown in this figure.

3.1 Labor Supply Formation and Wage Competition

Depending on the firms' stage 1 decisions about whether and how to discriminate, they may at stages 2 and 3 of the game be in anyone of 16 different subgames. Although this number is quite large, many of the subgames are qualitatively very similar to each other.

of an individual worker is not observable to the firm, only the worker's group membership (A or B) is.

		Firm 2			
		$y_2 = A$	$y_2 = B$	$y_2 = C$	$y_2 = D$
Firm 1	$y_1 = A$	$\pi_{A\|A}, \pi_{A\|A}$	$\pi_{A\|B}, \pi_{B\|A}$	$\pi_{A\|C}, \pi_{C\|A}$	$\pi_{A\|D}, \pi_{D\|A}$
	$y_1 = B$	$\pi_{B\|A}, \pi_{A\|B}$	$\pi_{B\|B}, \pi_{B\|B}$	$\pi_{B\|C}, \pi_{C\|B}$	$\pi_{B\|D}, \pi_{D\|B}$
	$y_1 = C$	$\pi_{C\|A}, \pi_{A\|C}$	$\pi_{C\|B}, \pi_{B\|C}$	$\pi_{C\|C}, \pi_{C\|C}$	$\pi_{C\|D}, \pi_{D\|C}$
	$y_1 = D$	$\pi_{D\|A}, \pi_{A\|D}$	$\pi_{D\|B}, \pi_{B\|D}$	$\pi_{D\|C}, \pi_{C\|D}$	$\pi_{D\|D}, \pi_{D\|D}$

Figure 1: The reduced-form game at stage 1.

3.1.1 The Firms Addressing Different Segments of Workers

Suppose first that both firms discriminate in the hiring decision, and they do this against different groups: $(y_1, y_2) \in \{(A, B), (B, A)\}$. This means that each firm is a monopsonist employer. What is the labor supply that firm i faces, if having made the stage 1 choice $y_i = j \in \{A, B\}$? Given a posted wage w_i, a worker who is a member of group j, and who is thus allowed to work at firm i, will choose to indeed work at firm i (rather than not work at all) if, and only if, $w_i - t\,|x - \widehat{x}_i| \geq 0$. By the assumption that the consumer locations x are uniformly distributed, with a total mass of one, the labor supply function facing firm i therefore is $l_i(w_i) = \gamma_j \min\{\frac{w_i}{t}, 1\}$ and its profit function is $\pi_i = \gamma_j\,(p - w_i) \min\{\frac{w_i}{t}, 1\}$. The firm's problem is to maximize this profit function with respect to w_i.

Let $w_{j|k}$ denote the optimal wage of a firm that has chosen $y_i = j$ when the other firm has chosen $y_{-i} = k$. By solving firm i's profit-maximization problem, one obtains the result that

$$w_{A|B} = w_{B|A} = \begin{cases} t & \text{if } \frac{t}{p} < \frac{1}{2} \\ \frac{p}{2} & \text{if } \frac{t}{p} \geq \frac{1}{2}. \end{cases} \tag{3}$$

The first line of equation (3) corresponds to the case where the workers' mismatch cost t is relatively low, which means that the firm optimally hires all workers belonging to the group that it does not discriminate against. The second line corresponds to the case where the mismatch cost is relatively high, which means that some workers choose to stay out of employment. Plugging the optimal wage in (3) into the profit expression yields

$$\pi_{j|k} = \begin{cases} \gamma_j(p - t) & \text{if } \frac{t}{p} < \frac{1}{2} \\ \frac{\gamma_j p^2}{4t} & \text{if } \frac{t}{p} \geq \frac{1}{2} \end{cases} \qquad \text{for } (j, k) \in \{(A, B), (B, A)\}\,. \tag{4}$$

Note that these profit expressions are proportional to γ_j. This means that, quite intuitively, if a firm discriminates against a relatively large group (meaning that γ_j is small), then its profit must be low.

3.1.2 The Firms Addressing the Same Segments of Workers

Suppose next that the firms either have chosen the same first-period action or that one has chosen C and the other one D: $(y_1, y_2) \in \{(A, A), (B, B), (C, C), (D, D), (C, D), (D, C)\}$. Consider, for concreteness, the case $(y_1, y_2) = (A, A)$. Here we have two possibilities: The A market is covered (i.e., all A workers are employed by a firm) or this market is not covered. One can show that the latter case is ruled out by Assumption 1.[11] Thus consider the possibility that the A market *is* covered. Then, for any posted wages w_1 and w_2, there is a worker with some location, say, $\overline{x}$ such that he is indifferent between the two employers: $w_1 - t\overline{x} = w_2 - t(1-\overline{x})$, which simplifies to

$$\overline{x} = \frac{w_1 - w_2 + t}{2t}. \tag{5}$$

The threshold value $\overline{x}$ lies strictly inside the unit interval if, and only if, $w_1 \in (w_2 - t, w_2 + t)$. When this condition holds, the labor supply function facing firm 1 is $l_1(w_1, w_2) = \gamma_A \overline{x}$ and the profit function is $\pi_1 = \gamma_A (p - w_1)\overline{x}$. Firm 2's profit function is $\pi_2 = \gamma_A (p - w_2)(1 - \overline{x})$.

Solving firm i's profit-maximization problem yields the following reaction function:

$$R_i(w_{-i}) = \begin{cases} \frac{w_{-i}+p-t}{2} & \text{if } w_{-i} > p - 3t \\ w_{-i} + t & \text{if } w_{-i} \leq p - 3t. \end{cases} \tag{6}$$

The second line of (6) corresponds to the case where firm i hires all workers in the A market (and the rival firm hires no one). It is easy to verify that the Nash equilibrium of the wage setting game between the two firms (i.e., the intersection of the reaction functions) is symmetric with $w_{A|A} = p - t$. The assumption that the market is covered is, given this wage, satisfied if, and only if, $w_{A|A} - \frac{1}{2}t \geq 0$ or $\frac{t}{p} \leq \frac{2}{3}$ (which is consistent with Assumption 1). We can thus conclude that the subgame under consideration has an equilibrium in which the A market is covered and each firm chooses the wage $w_{A|A} = p - t$. Moreover, plugging this equilibrium wage into the profit expression yields

$$\pi_{A|A} = \frac{\gamma_A t}{2}. \tag{7}$$

By a reasoning that is analogous to the one above, we also have

$$\pi_{B|B} = \frac{\gamma_B t}{2} \qquad \text{and} \qquad \pi_{C|C} = \pi_{D|D} = \pi_{C|D} = \pi_{D|C} = \frac{t}{2}. \tag{8}$$

Notice, in particular, that the reasoning indeed applies also to the cases where one firm chooses C and the other D. Even though one firm is free to choose different wages for the two groups, at the equilibrium it will not have an incentive to do so.

3.1.3 One firm discriminating in hiring and one practising wage discrimination

Suppose now that one of the two firms has chosen to discriminate in hiring and the other one does not do that but is free to choose different wages for the two groups: $(y_1, y_2) \in \{(A, D), (B, D), (D, A), (D, B)\}$. Here there will effectively be two separate markets—one with

[11] The analysis of the case where the A market is not covered is very similar to the analysis in the previous subsection and must lead to the same optimal wages and thus the same optimized profits as there. For the A market indeed not to be covered, a worker who is located exactly in the middle between the firms must prefer not to be employed, given the equilibrium wage: $w_{A|A} - \frac{1}{2}t \leq 0$ or, using the second line of (3), $t \geq p$. However, this is inconsistent with Assumption 1. We can conclude that there does not exist an equilibrium where the A market is not covered.

competition and one with a monopsony employer, and the wage is not required to be the same across the markets. Each one of these two situations was studied above. By combining the results in subsections 3.1.1 and 3.1.2 above, we immediately obtain the following results:

$$\pi_{D|j} = \begin{cases} \gamma_k(p-t) + \frac{\gamma_j t}{2} & \text{if } \frac{t}{p} < \frac{1}{2} \\ \frac{\gamma_k p^2}{4t} + \frac{\gamma_j t}{2} & \text{if } \frac{t}{p} \geq \frac{1}{2} \end{cases} \qquad \text{for } (j,k) \in \{(A,B),(B,A)\}, \tag{9}$$

$$\pi_{A|D} = \frac{\gamma_A t}{2}, \qquad \pi_{B|D} = \frac{\gamma_B t}{2}. \tag{10}$$

3.1.4 One firm discriminating in hiring and one not discriminating at all

Suppose finally that one firm has chosen to discriminate in hiring and the other one does not discriminate at all: $(y_1, y_2) \in \{(A,C),(B,C),(C,A),(C,B)\}$. For concreteness, consider the subgame $(y_1, y_2) = (A, C)$. Here we have, again, two possibilities: The market in which there is no discrimination, i.e., the A market, is covered or this market is not covered. The latter case is very similar to the corresponding case discussed in subsection 3.1.2 above and one can check that, as before, such an equilibrium does not exist under Assumption 1.[12] Thus suppose the A market is covered. This means that labor supply in the A market is determined by the threshold value $\overline{x}$ specified in (5). In particular, the profit of the firm that is able to hire in the A market only, which is firm 1, is $\pi_1 = \gamma_A (p - w_1) \overline{x}$. Maximizing this profit expression yields, again, the reaction function (6) in subsection 3.1.2 above (with $i = 1$).

For firm 2, which is a monopsonist in the B market, there are two possibilities. If firm 2 posts a relatively low wage, $w_2 < t$, then the B market will not be covered and the firm's profit is given by the first line of (11) below. If firm 2 posts a relatively high wage, $w_2 \geq t$, then the B market *will* be covered and the firm's profit is given by the second line of equation (11).

$$\pi_2 = \begin{cases} (p - w_2)\left[\gamma_A (1 - \overline{x}) + \gamma_B \frac{w_2}{t}\right] & \text{if } w_2 < t \\ (p - w_2)\left[\gamma_A (1 - \overline{x}) + \gamma_B\right] & \text{if } w_2 \geq t. \end{cases} \tag{11}$$

We obtain firm 2's reaction function by maximizing the profit expression in (11) with respect to w_2 (while keeping w_1 fixed). This reaction function is weakly upward-sloping. However, the function may for a range of intermediate w_1 values be constant at $w_2 = t$. That is, for those values of w_1, firm 2's optimal wage is high enough to ensure that the B market is covered, but not higher.

An equilibrium is given by an intersection of the two firms' reaction functions. In the Appendix it is shown that such an intersection can, depending on parameter values, occur *left* of the constant range (so where $w_2 < t$), *within* this range (where $w_2 = t$) or *right* of it (where $w_2 > t$). I will refer to these different kinds of equilibria as a low-wage equilibrium, a middle-wage equilibrium, and a high-wage equilibrium, respectively. For the two subgames where one of the firms hires only A workers and the other firm does not discriminate at all, we have the following result, which is also illustrated in panel (a) of Fig. 2.

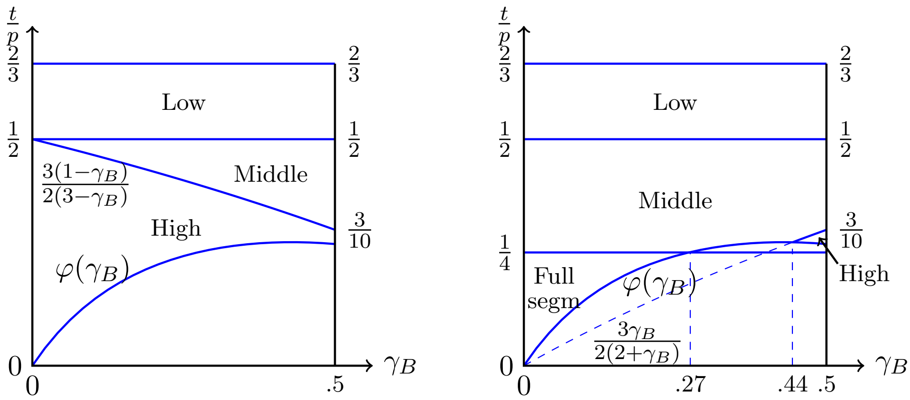
[12] The requirement we need to rule out an equilibrium where the market is not covered is that $t < p$.

Table 1: Equilibrium wages and profits in the subgame where $(y_1, y_2) \in \{(A,C),(C,A)\}$

Eq. behavior	Condition	$w_{A\|C}$	$w_{C\|A}$	$\pi_{A\|C}$	$\pi_{C\|A}$
Low-wage eq.	$\frac{t}{p} \in \left(\frac{1}{2}, \frac{2}{3}\right]$	$\frac{3(1+\gamma_B)p-(3+\gamma_B)t}{3+5\gamma_B}$	$\frac{(3+\gamma_B)p-3(1-\gamma_B)t}{3+5\gamma_B}$	$\frac{\gamma_A}{2t}\left[\frac{2\gamma_B p+(3+\gamma_B)t}{3+5\gamma_B}\right]^2$	$\frac{1+\gamma_B}{2t}\left[\frac{4\gamma_B p+3(1-\gamma_B)t}{3+5\gamma_B}\right]^2$
Middle-wage eq.	$\frac{t}{p} \in \left[\frac{3(1-\gamma_B)}{2(3-\gamma_B)}, \frac{1}{2}\right]$	$\frac{p}{2}$	t	$\frac{(1-\gamma_B)p^2}{8t}$	$\frac{(p-t)[4t-(1-\gamma_B)p]}{4t}$
High-wage eq.	$\frac{t}{p} \in \left[\varphi(\gamma_B), \frac{3(1-\gamma_B)}{2(3-\gamma_B)}\right)$	$p-\frac{(3-\gamma_B)t}{3(1-\gamma_B)}$	$p-\frac{(3+\gamma_B)t}{3(1-\gamma_B)}$	$\frac{t(3-\gamma_B)^2}{18(1-\gamma_B)}$	$\frac{t(3+\gamma_B)^2}{18(1-\gamma_B)}$

Table 2: Equilibrium wages and profits in the subgame where $(y_1, y_2) \in \{(B,C),(C,B)\}$

Eq. behavior	Condition	$w_{B\|C}$	$w_{C\|B}$	$\pi_{B\|C}$	$\pi_{C\|B}$
Low-wage eq.	$\frac{t}{p} \in \left(\frac{1}{2}, \frac{2}{3}\right]$	$\frac{3(2-\gamma_B)p-(4-\gamma_B)t}{8-5\gamma_B}$	$\frac{(4-\gamma_B)p-3\gamma_B t}{8-5\gamma_B}$	$\frac{\gamma_B}{2t}\left[\frac{2(1-\gamma_B)p+(4-\gamma_B)t}{8-5\gamma_B}\right]^2$	$\frac{(2-\gamma_B)}{2t}\left[\frac{4(1-\gamma_B)p+3\gamma_B t}{8-5\gamma_B}\right]^2$
Middle-wage eq.	$\frac{t}{p} \in \left[\max\left\{\frac{3\gamma_B}{2(2+\gamma_B)}, \frac{1}{4}, \varphi(\gamma_B)\right\}, \frac{1}{2}\right]$	$\frac{p}{2}$	t	$\frac{\gamma_B p^2}{8t}$	$\frac{(p-t)(4t-\gamma_B p)}{4t}$
Full segmentation	$\frac{t}{p} \in \left[\varphi(\gamma_B), \frac{1}{4}\right]$	$2t$	t	$\gamma_B(p-2t)$	$\gamma_A(p-t)$
High-wage eq.	$\frac{t}{p} \in \left[\varphi(\gamma_B), \frac{3\gamma_B}{2(2+\gamma_B)}\right)$	$p-\frac{(2+\gamma_B)t}{3\gamma_B}$	$p-\frac{(4-\gamma_B)t}{3\gamma_B}$	$\frac{t(2+\gamma_B)^2}{18\gamma_B}$	$\frac{t(4-\gamma_B)^2}{18\gamma_B}$



(a) Eq. when $(y_1, y_2) = (A, C)$ (b) Eq. when $(y_1, y_2) = (B, C)$

Figure 2: Different kinds of equilibria when firm 2 does not discriminate at all and firm 1 discriminates in hiring against one group. In panel (a) the discrimination is against the minority group, and in panel (b) it is against the majority group.

Lemma 1. *Suppose $(y_1, y_2) \in \{(A, C), (C, A)\}$. Then the equilibrium wages and profits of this subgame are as indicated in Table 1.*

The analysis of the two subgames where one of the firms hires only B workers and the other firm does not discriminate at all is similar to the one above. The results are reported in the following lemma and they are illustrated in panel (b) of Fig. 2.

Lemma 2. *Suppose $(y_1, y_2) \in (B, C), (C, B)\}$. Then the equilibrium wages and profit levels of this subgame are as indicated in Table 2.*

The equilibrium wages reported in Lemma 2 and Table 2 have similar features to the ones in Lemma 1 and Table 1. One important difference is that when there is discrimination against the majority group, a high-wage equilibrium exists only for a relatively small set of parameter values. Moreover, in a subgame with discrimination against the majority group and for a part of the parameter space, there exists an equilibrium with full segmentation: The non-discriminating firm hires only minority workers, whereas its rival hires only majority workers.

3.2 The first-stage discrimination game

An equilibrium of the reduced-form stage 1 game is a pair (y_1^*, y_2^*) that satisfies the following two Nash conditions:

$$\pi_{y_1^*|y_2^*} \geq \pi_{y_1|y_2^*} \quad \text{and} \quad \pi_{y_2^*|y_1^*} \geq \pi_{y_2|y_1^*} \quad \text{for all } (y_1, y_2) \in S^2.$$

With the help of Fig. 1 and the profit expressions derived in the previous subsection, it is straightforward to identify the equilibria (although doing this requires a fair amount of algebra). Proposition 1 below summarizes the results. This proposition uses the notation Ω_I, Ω_{II}, and

Table 3: Formal definitions of Ω_I, Ω_{II}, and Ω_{III}.

$\Omega_I \stackrel{\text{def}}{=} \left(\frac{\sqrt{1-\gamma_B}}{2}, \frac{2}{3}\right)$,	$\Omega_{II} \stackrel{\text{def}}{=} \left(\max\left\{\frac{9(1-\gamma_B)}{21-13\gamma_B}, \frac{1}{3}\right\}, \frac{\sqrt{1-\gamma_B}}{2}\right)$,	$\Omega_{III} \stackrel{\text{def}}{=} \left(\varphi(\gamma_B), \min\left\{\frac{9(1-\gamma_B)}{21-13\gamma_B}, \frac{1}{3}\right\}\right)$

Ω_{III}, which denote three disjoint and collectively exhaustive parts[13] of the parameter space. These three parameter regions are shown in panel (a) of Fig. 3; the formal definitions are stated in Table 3.

Proposition 1. *The equilibria of the reduced-form stage 1 game are as follows.*

(i) Suppose $\frac{t}{p} \in \Omega_I$. *Then* $(y_1^*, y_2^*) \in \{(C,C), (C,D), (D,C), (D,D)\}$.

(ii) Suppose $\frac{t}{p} \in \Omega_{II}$. *Then* $(y_1^*, y_2^*) \in \{(A,C), (C,A), (D,D)\}$.

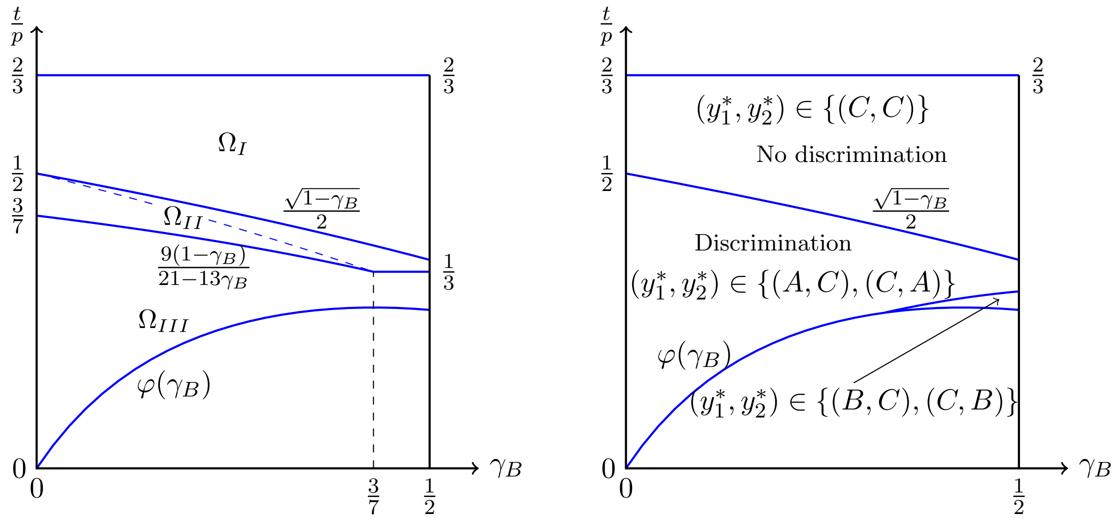
(iii) Suppose $\frac{t}{p} \in \Omega_{III}$. *Then* $(y_1^*, y_2^*) \in \{(D,D)\}$

The results reported in Proposition 1 tell us that, in all parts of the parameter space that are consistent with Assumption 1, both firms' choosing D is an equilibrium. That is, there always exists an equilibrium in which no firm discriminates in hiring. (Given the choice $y_i = D$, firm i is free to practice wage discrimination but, as argued in subsection 3.1.2, it will not have an incentive to do that.) For the parts of the parameter space where competition is relatively intense (region Ω_{III} in Fig. 3, panel (a)), $(y_1^*, y_2^*) = (D,D)$ is the only equilibrium. In the opposite part of the parameter space (region Ω_I) there are, in addition, three other equilibria: both parties choosing C, or one choosing C and the other D; however, all these four equilibria yield exactly the same wages and profit levels. Importantly, for intermediate levels of competition (region Ω_{II}) there exist, in addition to $(y_1^*, y_2^*) = (D,D)$, equilibria where one of the firms discriminates in hiring against the minority group, while the other firm does not discriminate at all. Indeed, as the figure illustrates, this kind of equilibrium can exist also when the sizes of the two groups are virtually the same (i.e., in the limit as $\gamma_B \to \frac{1}{2}$).[14]

How can $(y_1^*, y_2^*) = (A,C)$ be an equilibrium? Why does not the firm that discriminates (i.e., firm 1) want to deviate to C? Intuitively, one might think that such a deviation should be profitable, as then firm 1 could employ workers also from the minority group and thereby be able to produce more and sell a larger quantity in its product market. In order to understand these questions, it is useful to first have a closer look at the two equilibrium wages $w_{A|C}$ and $w_{C|A}$. Panel (a) of Fig. 4 plots these wages against the (normalized) competition parameter t/p; as a reference, the figure also shows the wage $w_{C|C}$, the symmetric equilibrium wage in the subgame where no firm discriminates. The figure shows that, for all $\frac{t}{p} < \frac{1}{2}$, the wages $w_{A|C}$

[13]However, to simplify the statement of the results, points exactly on the border between different parameter regions are ignored.

[14]One can check that this kind of equilibrium indeed exists also for $\gamma_A = \gamma_B = \frac{1}{2}$. However, to make it easier to state all results reasonably succinctly, I have in the model description ruled out this knife-edge case and assumed that $\gamma_B < \frac{1}{2}$.



(a) The case $S = \{A, B, C, D\}$.

(b) The case $S = \{A, B, C\}$.

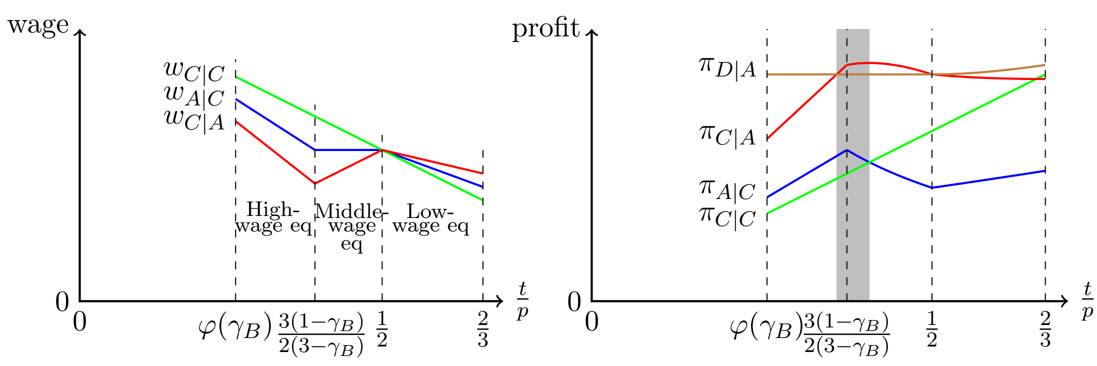
Figure 3: For the baseline model depicted in panel (a), discrimination occurs in region Ω_{II}. For the model shown in panel (b), where wage discrimination is not possible, discrimination occurs for a larger set of parameter values.

and $w_{C|A}$ are lower than $w_{C|C}$: *The fact that one firm discriminates in hiring makes it possible for both firms to post lower wages at the equilibrium.*

How can we understand this result? Broadly speaking, the act of discriminating against B workers helps to segment the labor market, which softens the wage competition between the firms. More specifically, the fact that firm 1 has chosen A, as opposed to C, makes firm 2 a monopsonist in the market for B workers. This lowers the labor supply elasticity that firm 2 faces and it therefore optimally lowers its posted wage. Since the firms' reaction functions are upward-sloping, firm 1's equilibrium wage adjusts in the same direction, i.e., it also drops. Thus both firms pay lower salaries, thanks to the fact that one of them discriminates. This clearly has a positive effect on the discriminating firm's profits.

This firm also, of course, loses revenue and profit from the fact that it cannot hire B workers. However, the effect that discrimination has on the equilibrium wage can dominate the effect on lost revenues that is due to lower production. Panel (b) of Fig. 4 plots the profit levels in some of the key subgames against the (normalized) competition parameter t/p. For sufficiently large values of t/p, the discriminating firm would indeed have an incentive to deviate to the non-discrimination strategy (for there $\pi_{C|C} > \pi_{A|C}$). For sufficiently low values of t/p, it would be the non-discriminating firm that wanted to deviate to the wage-discrimination strategy (because there $\pi_{D|A} > \pi_{C|A}$). However, for an intermediate range of t/p values—the shadowed region in the figure—neither one of the firms has an incentive to deviate, so here $(y_1^*, y_2^*) = (A, C)$ is indeed an equilibrium.

Thus, for a subset of the parameter space, the model has multiple equilibria. How can we choose among these? For the part of the parameter space that is labeled region Ω_I in panel (a) of Fig. 3, multiplicity of equilibria is not an issue, as all equilibria give rise to the same



(a) Wages (b) Profits

Figure 4: Wages and profits in some of the subgames. The shadowed area in panel (b) is the parameter region where $(y_1, y_2) = (A, C)$ is an equilibrium of the overall game.

wage and profit outcomes. Similarly, in region Ω_{III} there is a unique equilibrium. However, in region Ω_{II} there are indeed co-existing equilibria with different outcomes. One plausible criterion for selecting among these equilibria is to disregard an equilibrium if its outcome is, from the perspective of the two firms, Pareto-dominated by another equilibrium outcome. Such a criterion has bite, for we have:

Proposition 2. *Suppose $\frac{t}{p} \in \Omega_{II}$, which means that the first-stage equilibria $(y_1^*, y_2^*) = (A, C)$, $(y_1^*, y_2^*) = (C, A)$, and $(y_1^*, y_2^*) = (D, D)$ co-exist. From the perspective of the firms, each one of the first two equilibrium outcomes payoff-dominates the third one: $\pi_{A|C} > \pi_{D|D}$ and $\pi_{C|A} > \pi_{D|D}$.*

The payoff-dominance criterion and the result in Proposition 2 thus yield a unique equilibrium outcome for the whole parameter space. Of course, however, the proposition does not tell us which one of the two firms that discriminates and which one that does not do this (as the roles of the two firms are completely symmetrical in this model, we cannot expect it to be able to say anything about this issue).

If we are willing to accept the equilibrium selection criterion referred to above, then we can study the effect of a change in the level of labor market competition (t) on the equilibrium wages and profit levels.

Proposition 3. *Assume that an equilibrium that is not payoff-dominated is played.*

(i) Suppose $\frac{t}{p} \in \Omega_I \cup \Omega_{III}$. Then neither one of the firms discriminates. The (common) equilibrium wage is decreasing in t and the (common) equilibrium profit is increasing in t.

(ii) Suppose $\frac{t}{p} \in \Omega_{II}$ and $\frac{t}{p} > \frac{3(1-\gamma_B)}{2(3-\gamma_B)}$. Then one of the firms discriminates in hiring against B workers and the other firm does not discriminate. The discriminating firm's equilibrium wage is independent of t, while the non-discriminating firm's equilibrium wage is increasing in t. Moreover, the discriminating firm's equilibrium profit is decreasing in t, and the non-discriminating firm's equilibrium profit is increasing in t.

(iii) Suppose $\frac{t}{p} \in \Omega_{II}$ and $\frac{t}{p} < \frac{3(1-\gamma_B)}{2(3-\gamma_B)}$. Then one of the firms discriminates in hiring against B workers and the other firm does not discriminate. Both firms' equilibrium wages are decreasing in t, and both firms' equilibrium profits are increasing in t.

The results in parts (i) and (iii) of Proposition 3 are standard and unsurprising: As competition becomes stiffer, wages go up and profits go down. The surprising statements can be found in part (ii). For a subset of the parameter space that gives rise to discrimination, the comparative statics results are turned upside down: Stiffer competition leads to lower or unchanged wages and, for the discriminating firm, to a higher profit level. Referring to panel (a) of Fig. 3, this phenomenon occurs above the dashed curve in region Ω_{II}. The logic behind this result is related to the fact that in this part of the parameter space, the non-discriminating firm's wage is at a corner solution, in the sense that the firm sets its wage high enough to ensure that all minority workers prefer to work, but not higher (cf. eq. (11) and the surrounding discussion on page 10). This results in the wage $w_{C|A} = t$, for this is the mismatch cost that must be incurred by a worker who is located at the opposite end of the line relative to the non-discriminating firm (hence, at the distance one from this employer). It follows that as competition gets tougher (t goes down), a lower wage is required to induce the worker to take the job.

We may, at first glance, feel uncomfortable with this odd and upside down comparative statics result. However, it should actually not look so strange in light of the fact that, in the present model, discrimination is an endogenous response to an exogenous change in competition: Stiffer competition induces a firm to make an attempt, by discriminating, at eluding competition, which if successful can lead to a higher profit. The conclusion is that not only the presence of discrimination can exhibit a non-monotonicity with respect to the level of exogenous competition—this is true also, and for a similar reason, for the discriminating firm's profit level.

The last proposition in this section considers the welfare effects of a hypothetical and effective ban of all kinds of discrimination.

Proposition 4. *Suppose $\frac{t}{p} \in \Omega_{II}$ and that the firms coordinate on an equilibrium that, from their point of view, is not payoff-dominated. Thus $(y_1^*, y_2^*) \in \{(A,C),(C,A)\}$. Suppose further that there exists a policy that effectively bans the actions A, B, and D. This policy (trivially) gives rise to the outcome $(y_1, y_2) = (C, C)$. The introduction of this policy would (i) make both firms strictly worse off, (ii) make each and every A and B worker strictly better off, and (iii) make total surplus strictly larger.*

That is, discrimination as understood in this paper shifts economic resources from workers to firms. Moreover, it leads to an aggregate welfare loss in the sense that it lowers total surplus. The source of this decrease in total surplus is the change in mismatch cost that is incurred by the workers. Regardless of whether the ban is in effect or not, both the A and B markets are covered (assuming $\frac{t}{p} \in \Omega_{II}$), which means that all workers on the unit line work for one of the two firms. Hence, total employment and production in the economy is the same with and without discrimination.[15] However, under discrimination the average worker must incur a higher mismatch (or transportation) cost. First, in the minority market the workers have only one employer, which means that these workers' costs obviously must be higher. Second, in the majority market the share of workers across the two employers is not split even, which means that total mismatch costs are not minimized; this adds further to the increase in mismatch costs.

4 What if wage discrimination is not feasible?

In the model studied in Sections 2 and 3, the firms could discriminate in two ways: by refusing to hire members of a certain group, and by paying a lower wage to members of a particular group. Arguably, however, firms may often be prevented from practicing wage discrimination, as doing this would be a blatant and easily detectable violation of anti-discrimination laws. In their model of racial discrimination, Lang, Manove, and Dickens (2005) assume that making the wage contingent on race is not possible, and they justify this as follows (p. 1328):

> Race-contingent posted wage offers would be an egregious and public violation of civil rights legislation, which most employers wish to avoid. Furthermore, in white racist social environments, wage discrimination in favor of blacks would be a gross violation of social norms, and wage discrimination against blacks would inevitably lead to hiring discrimination in their favor, also socially proscribed. Thus we should not expect to see race-contingent wage offers, even in the absence of civil rights legislation.

Coate and Loury (1993, p. 1222) use a similar argument to justify their assumption that firms can discriminate in job assignments but not in wages:

> Discriminatory wages for the same work is a flagrant violation of equal-employment laws, and relatively easy to detect. Discrimination in job assignment [...] is a more subtle phenomenon.

In light of these arguments it is interesting to explore how the analysis in the previous section is altered if we assume that wage discrimination—that is, the stage 1 action $y_i = D$—is not feasible. Thus, consider a model that is identical to the one before, except that now the strategy set available to a firm at stage 1 does not include the D action. That is, here

[15] That is, discrimination does not lead to lower employment in this model. However, I conjecture that discrimination as understood here would indeed lead to lower employment among both minority and majority workers under a somewhat different (but less tractable) model specification. In particular, if, following Bhaskar and To (1999), workers were differentiated also with respect to their outside options—so that those with the highest outside option effectively chose only between working for the nearest firm and not at all—then discrimination would, via the lower equilibrium wages, lead to lower employment and production. The labor market segment made up of workers with a low outside option would still be covered (as in the present model), which ensures that the firms are interacting with each other rather than being local monopsonists.

$S \stackrel{\text{def}}{=} \{A, B, C\}$. It is obvious that this new assumption must change the set of equilibria compared to the original model, as one of the equilibria there involved both firms choosing D. However, the fact that the D action is unavailable has an impact on the set of equilibria also through a more subtle channel. In particular, the reason why, in our previous analysis and for $\frac{t}{p} \in \Omega_{III}$, the outcome where one firm discriminates in hiring could not be an equilibrium, was that the non-discriminating firm had an incentive to deviate to D (cf. panel (b) of Fig. 4). Now, when this is not feasible, discrimination in hiring is an equilibrium under a broader set of parameter configurations.

In order to state the formal results in this version of the model, the following notation is convenient:

$$\widehat{\Omega}_{III} \stackrel{\text{def}}{=} \left(\varphi(\gamma_B), \frac{3\sqrt{\gamma_B(1-\gamma_B)}}{2(3-\gamma_B)} \right).$$

Consider the following result.

Proposition 5. *Assume that wage discrimination is not feasible: $S = \{A, B, C\}$. The equilibria of the reduced-form stage 1 game are as follows.*

(i) Suppose $\frac{t}{p} \in \Omega_I$. Then $(y_1^*, y_2^*) \in \{(C, C)\}$.

(ii) Suppose $\frac{t}{p} \in \Omega_{II} \cup \Omega_{III}$ and $\frac{t}{p} \notin \widehat{\Omega}_{III}$. Then $(y_1^*, y_2^*) \in \{(A, C), (C, A)\}$.

(iii) Suppose $\frac{t}{p} \in \widehat{\Omega}_{III}$. Then $(y_1^*, y_2^*) \in \{(B, C), (C, B)\}$.

Proposition 5, which is illustrated by panel (b) of Fig. 3, tells us that when wage discrimination is banned, as opposed to being feasible, the set of parameter values for which discrimination in hiring (by one of the firms) is an equilibrium unambiguously increases. The implication of this logic is that an effective ban on wage discrimination may—as an equilibrium response from the employers—lead to discrimination in hiring. This outcome would of course not arise if also discrimination in hiring were banned. Arguably, however, effectively banning discrimination in hiring may be quite difficult whereas a ban on wage discrimination is relatively easy to enforce, as suggested in the above quotations. It is curious to note that in this version of the model, when discrimination in hiring is part of an equilibrium, then this discrimination is not necessarily directed against the minority: For a part of the parameter space, the majority group of workers is actually discriminated against. However, for this phenomenon to occur, the majority must not be too large (cf. panel (b) of Fig. 3).

5 Concluding Discussion

This paper has explored the idea that being discriminatory can be a commitment device that helps an employer gain strategic advantages in its interaction with other employers. In particular, it was investigated whether being discriminatory may enable the employer and its rival to partially segment the labor market; this, in turn, would lead to lower equilibrium wages.

An employer of course also incurs a direct cost of discriminating, in terms of forfeited production and revenues. But the lower wage expenditures could conceivably compensate for the lost revenue and, if so, the act of discriminating would lead to higher profit.

The formal analysis showed that, for a subset of the parameter space, the logic described above indeed works: One firm discriminating in hiring against minority workers and the other firm not discriminating at all constitutes an equilibrium. The analysis also showed that, if the firms by assumption are unable to practice wage discrimination, an equilibrium with discrimination in hiring exists for a larger set of parameter values. The reason is that a ban on wage discrimination makes an otherwise profitable deviation unavailable. Thus, the logic suggests that effective legislation against wage discrimination (which may be feasible, as such discrimination is easily detectable) may lead to discrimination in hiring (which cannot be banned because it is harder to observe).

In addition a number of other potentially useful observations and ideas follow from the analysis. First, *discrimination can be an endogenous response to (changes in) competition.* As the exogenous level of competition in the labor market increases, firm profits decline and this makes it more attractive for a firm to try to escape from the competitive environment by discriminating and thus partially segment the market. This observation highlights the importance, in empirical work, of finding an appropriate measure of the level of competition in the market. To see this point clearly, suppose we had access to observations of the equilibrium outcome of the model studied in the paper. If we then related the level of observed discrimination to some competition measure that is based on the equilibrium wages or equilibrium profit levels (as opposed to the normalized mismatch costs, t/p), then this would lead to very misleading conclusions.

Second, *the relationship between discrimination and competition can be non-monotone.* In the analysis, we obtained this result in the baseline model where wage discrimination was possible. For sufficiently low levels of competition, choosing to discriminate in hiring was not profitable (cf. the reasoning in the above paragraph). For sufficiently high levels of competition, on the other hand, the non-discriminating firm would have an incentive to deviate from the discrimination equilibrium by choosing to practice wage discrimination. However, for intermediate levels of competition, discrimination in hiring was indeed an equilibrium. The result that more intense competition can to lead (locally) to more discrimination stands in sharp contrast to the Becker (1971) argument and to most of the results in the existing literature.

Third, *when empirically studying the relationship between discrimination and competition, it is important to distinguish between product and labor market competition.* In the present analysis, only the level of competition in the labor market mattered, while the mode of competition in the product market did not play any role (although the price p was assumed to be exogenous, which suggests that the firms are price takers in their product markets). Moreover, it was in the labor market that discrimination occurred. We could re-interpret the model by thinking of the workers that are distributed on the unit interval as consumers in a product market. The firms, in that interpretation, would make choices about whether to refuse to serve a certain group of consumers. In that story, it would be the level of competition in the product market that mattered, and that would also be the market in which we could observe discrimination. Regardless of which version of the model we have in mind it is, for a given interpretation, crucial that we choose the right market when searching for measures of competition and for evidence of discrimination, if we want to test the implications of the discrimination logic discussed in the present paper.

Fourth, *for discrimination to occur for the reasons studied here, the firms' choice variables must be strategic complements.* Admittedly, the paper has not offered a model that nests a situation with strategic complementarity and one with strategic substitutability, and shown that discrimination cannot occur in the latter case. However, the intuition (as discussed in the Introduction and in Section 3.2) strongly suggests that strategic complementarity is a necessary ingredient in the logic. The model feature with strategic complementarity arises naturally in this setting with wage competition (as it does in many oligopoly models with price competition). Yet one could in principle imagine a setting with strategic substitutability, and if so we should not expect discrimination to arise (for the reasons identified here). It would be desirable, in empirical work, to estimate the nature of the interaction between employers (strategic complementarity/substitutability) simultaneously with studying the possible presence of discrimination, in order to help asses the relevance of the logic.

Fifth and finally, *for a subset of the parameter space where discrimination occurs, comparative statics exhibit an unusual pattern.* In particular, in that part of the parameter space, the equilibrium wages are weakly decreasing (for the non-discrimnating firm, strictly decreasing) in the level of competition in the labor market. And the equilibrium profit level for the discriminating firm is strictly increasing in the level of competition. The explanation for these counterintuitive results is related to the fact that the non-discriminating firm's wage choice is at a corner solution: The wage is high enough to ensure that the minority market is covered, but not higher. These comparative statics results do not necessarily arise when discrimination occurs in equilibrium: For other parts of the parameter space where there is discrimination in hiring, the comparative statics results follow the standard pattern. An implication of the results is that the relationship between the level of labor market competition and the discriminating firm's profit can be non-monotone.

In the model studied here it was assumed that the firms can simply make a binding commitment to discriminate or not to discriminate. As argued in footnote 5, this commitment assumption should be thought of as an analytical shortcut that can help us understand the relative profitability of being a discriminatory and a non-discriminatory firm. In a richer, alternative setting we could have assumed (following Becker, 1971) that some firm owners have a taste for discrimination against a particular group, which is why they are committed to not hiring those workers. Depending on how high profits a firm can generate given those preferences, the firm owners would then tend to be selected or deselected in an entry/exit process (alternatively, through a process of cultural transmission where parents choose what preferences to instill in their children).[16] To model this selection process has been beyond the scope of the present paper. The paper has instead focused on understanding under what circumstances, if any, being a discriminatory firm can be profitable. Thanks to the choice not to model the se-

[16] Another interpretation of the commitment assumption could be that firm owners delegate the task of choosing whether and how to discriminate to a manager, who may or may not have a taste for discrimination.

lection/deselection process, other aspects of the model could be made richer—for example, the firms were allowed to discriminate also against the majority group. Nevertheless, to explicitly model the selection process may yield further insights and it would be an interesting avenue for future work.

Some of the arguments made above naturally raise questions about how other economic agents can learn about whether a particular firm owner has a taste for discrimination or not (or about the strength of this taste). Similarly, in the present model where the firms could commit to a choice whether to discriminate, one may wonder how the rival firm is able to learn about this choice. If the choice were not observable but if the firms interacted over at least two time periods, an incentive to *signal* (à la Spence, 1973) some particular preferences would arise.[17] To model such signaling incentives may very well be an interesting and fruitful exercise. However, we should expect that at least after a large number of periods with signaling, a separating equilibrium would ensue and all the information that was initially private would be fully revealed. Therefore it may also be natural (and, depending on the question that we want to study, more appropriate) to simply assume that the choice whether to discriminate is observable.

Such an analytical shortcut was chosen for the model in the present paper, as the main goal was to understand whether being discriminatory can at all serve as a commitment device. We found that it can indeed.

Appendix

In this Appendix, Lemmas 1 and 2 as well as Propositions 2 and 4 are proven. The proofs of Propositions 1, 3, and 5 can be found in the online Supplementary Material (Lagerlöf, 2016).

Proof of Lemma 1

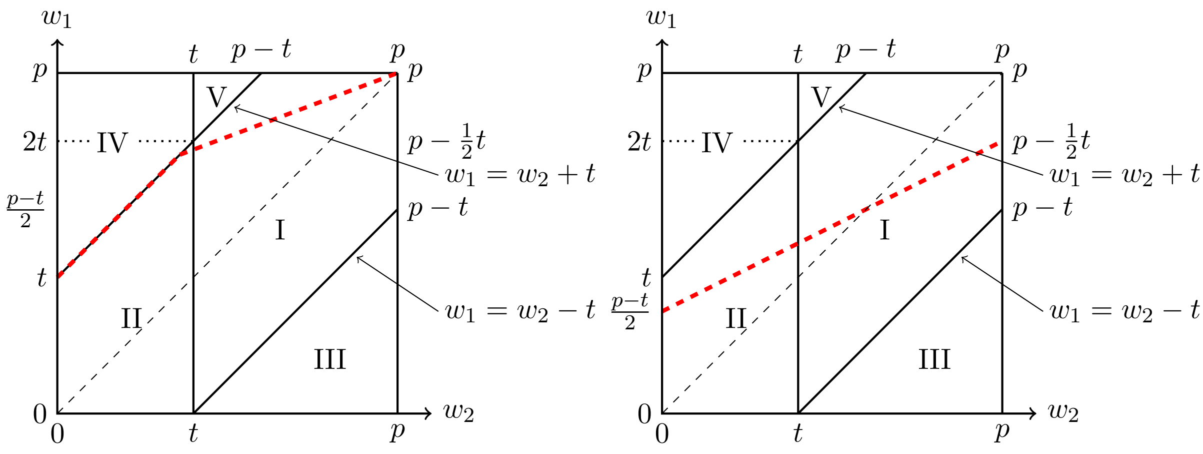
In order to prove the lemma, it suffices to show the claims about the subgame $(y_1, y_2) = (A, C)$. The results for $(y_1, y_2) = (C, A)$ then follow by symmetry of the game.

Thus consider the case $(y_1, y_2) = (A, C)$. Panels (a) and (b) of Fig. 5 illustrate the possible stage 2 outcomes in the (w_2, w_1)-space. These figures make use of some of the information stated in subsection 3.1—for example, the fact that the threshold value $\overline{x}$ lies strictly inside the unit interval if and only if $w_1 \in (w_2 - t, w_2 + t)$. In region I of the figure, the A market is covered and shared by the two firms; moreover, the B market is covered. In region II, the A market is covered and shared by the two firms, but the B market is *not* covered. And so on for the other indicated regions. Firm 1's reaction function, as stated in equation (6), is graphed in Fig. 5 as a thick dashed (red) line; panel (a) shows the case where $t/p < 1/3$, meaning that for low enough values of w_2 firm 1 employs all workers in market A, while panel (b) shows the case where $t/p \geq 1/3$.[18]

It is clear that firm 1's reaction function passes through regions I and II. It may also be located on the line $w_1 = w_2 + t$. We can therefore conclude that an equilibrium must lie: (i) in the interior of region I; (ii) in the interior of region II; (iii) on the border between regions I and II, where $w_2 = t$; or (iv) on the line where

[17] The logic identified in the present paper would play a crucial role in determining these incentives.

[18] Both panels assume that $t/p < 1/2$. If $t/p \geq 1/2$, then region V in the figures disappears, but there are no qualitative changes that affect the reasoning below.



(a) The case $\frac{t}{p} < \frac{1}{3}$.

(b) The case $\frac{t}{p} \geq \frac{1}{3}$.

Figure 5: Finding an eq. of the subgame $(y_1, y_2) = (A, C)$.

$w_1 = w_2 + t$. Below I will investigate under what circumstances, if any, there is a pure strategy equilibrium in each one of these regions.

Finding an eq. in region I (where the B market is covered)

In (the interior of) region I there cannot be equilibrium where firm 1's wage choice is "in a corner" (i.e., given by the second line of (6)). Thus firm 1's best reply is interior (i.e., given by the first line of (6)). Given that we are in region I, firm 2's profit is given by the second line of (11) and, hence, the associated first-order condition is:

$$\frac{\partial \pi_2}{\partial w_2} = -\left[1 - \frac{\gamma_A}{2t}(w_1 - w_2 + t)\right] + \frac{\gamma_A}{2t}(p - w_2) = 0,$$

which simplifies to

$$2t - \gamma_A (w_1 - w_2 + t) = \gamma_A (p - w_2). \tag{12}$$

Equation (12) and the first line of (6) define a linear equation system in w_1 and w_2. Solving this yields

$$w_1 = p - \frac{(2+\gamma_A)t}{3\gamma_A} = p - \frac{(3-\gamma_B)t}{3(1-\gamma_B)}, \qquad w_2 = p - \frac{(4-\gamma_A)t}{3\gamma_A} = p - \frac{(3+\gamma_B)t}{3(1-\gamma_B)}. \tag{13}$$

Also, using (13), we can compute firm 1's profit and firm 2's profit at the possible equilibrium:

$$\pi_1^* = \frac{\gamma_A}{2t}(p - w_1)^2 = \frac{\gamma_A}{2t}\left(\frac{2+\gamma_A}{3\gamma_A}t\right)^2 = \frac{t\,(2+\gamma_A)^2}{18\gamma_A} = \frac{t\,(3-\gamma_B)^2}{18(1-\gamma_B)}, \tag{14}$$

$$\pi_2^* = \frac{\gamma_A}{2t}(p - w_2)^2 = \frac{\gamma_A}{2t}\left(\frac{4-\gamma_A}{3\gamma_A}t\right)^2 = \frac{t\,(4-\gamma_A)^2}{18\gamma_A} = \frac{t\,(3+\gamma_B)^2}{18(1-\gamma_B)}. \tag{15}$$

We can now check the conditions that are required for being in (the interior of) region I. First,

$$w_1 > w_2 - t \Leftrightarrow p - \frac{(2+\gamma_A)t}{3\gamma_A} > p - \frac{(4-\gamma_A)t}{3\gamma_A} - t \Leftrightarrow 2 + \gamma_A > 0,$$

which always holds. Second,

$$w_1 < w_2 + t \Leftrightarrow p - \frac{(2+\gamma_A)t}{3\gamma_A} < p - \frac{(4-\gamma_A)t}{3\gamma_A} + t \Leftrightarrow \gamma_A > \frac{2}{5},$$

which also always holds. Third, the B market must indeed be covered:[19]

$$w_2 - t > 0 \Leftrightarrow p - \frac{(4-\gamma_A)t}{3\gamma_A} > t \Leftrightarrow \frac{t}{p} < \frac{3\gamma_A}{2(2+\gamma_A)}. \tag{16}$$

Finally, firm 1's best response must indeed be given by the first line of (6):

$$w_2 > p - 3t \Leftrightarrow p - \frac{(4-\gamma_A)t}{3\gamma_A} > p - 3t \Leftrightarrow \gamma_A > \frac{2}{5},$$

which again always holds.

There are two kinds of deviations that potentially could be profitable: Firm 2 could give up its ambition to hire anyone in the A market and instead choose the wage that maximizes its profits when hiring only in the B market; or firm 2 could stay in the A market but choose some wage $w_2 < t$, yielding a profit given by the first line of (11). The second kind of deviation is never profitable. If it were, the derivative of firm 2's profit function, as stated in the first line of (11) and evaluated at firm 1's wage and at $w_2 = t$, would be negative:

$$\frac{\partial \pi_2^{dev}}{\partial w_2} \mid_{(w_1,w_2)=\left(p-\frac{(2+\gamma_A)t}{3\gamma_A},t\right)} < 0 \Leftrightarrow \frac{t}{p} > \frac{3}{7-\gamma_A}.$$

But the above inequality is inconsistent with (16).

Thus consider the first kind of deviation, where firm 2 gives up on the A market. Here firm 2 could choose $w_2 = t$ or it could choose some $w_2 \in (0,t)$. If making the latter deviation, the best deviation maximizes $\pi_2 = \frac{\gamma_B}{t}(p - w_2) w_2$, i.e., it is given by $w_2 = \frac{p}{2}$. For this wage to indeed be interior, we must have

$$\frac{p}{2} < t \Leftrightarrow \frac{t}{p} > \frac{1}{2}, \tag{17}$$

which is inconsistent with (16). This means that the best possible deviation is $w_2 = t$. Making this deviation, given that w_1 is given by (13), would yield the profit

$$\pi_2^{dev} = \gamma_B (p - w_2) = \gamma_B (p - t). \tag{18}$$

Thus, there is no incentive to deviate if, and only if,

$$\pi_2^* \geq \pi_2^{dev} \Leftrightarrow \frac{t(4-\gamma_A)^2}{18\gamma_A} \geq \gamma_B(p-t) \Leftrightarrow \frac{t}{p} \geq \frac{18\gamma_A\gamma_B}{(4-\gamma_A)^2 + 18\gamma_A\gamma_B} = \varphi(\gamma_B). \tag{19}$$

We can conclude that if (16) and (19) hold, then there is an equilibrium where the prices are given by (13), and the associated profit levels are given by (14) and (15). This yields the bottom line in Table 1.

Finding an eq. in region II (where the B market is not covered)

Again, in (the interior of) region II there cannot be an equilibrium where firm 1's wage choice is "in a corner" (i.e., given by the second line of (6)). Thus firm 1's best reply is interior (i.e., given by the first line of (6)). Given that we are in region II, firm 2's profit is given by the first line of (11) and, hence, the associated first-order condition is:

$$\frac{\partial \pi_2}{\partial w_2} = -\frac{1}{2t}\left[\gamma_A (w_2 - w_1 + t) + 2\gamma_B w_2\right] + \frac{\gamma_A + 2\gamma_B}{2t}(p - w_2) = 0,$$

which simplifies to

$$\gamma_A (w_2 - w_1 + t) + 2\gamma_B w_2 = (\gamma_A + 2\gamma_B)(p - w_2). \tag{20}$$

Equation (20) and the first line of (6) define a linear equation system in w_1 and w_2. Solving this yields

$$w_1 = \frac{3(1+\gamma_B)p - (3+\gamma_B)t}{3+5\gamma_B}, \qquad w_2 = \frac{(3+\gamma_B)p - 3(1-\gamma_B)t}{3+5\gamma_B}. \tag{21}$$

[19]This implies that also the A market is covered, since the worker who has the most distant location must travel farther in a monopsony market.

We can now check the conditions that are required for being in (the interior of) region II. First,

$$w_2 < t \Leftrightarrow \frac{(3+\gamma_B)p - 3(1-\gamma_B)t}{3+5\gamma_B} < t \Leftrightarrow \frac{t}{p} > \frac{1}{2}. \tag{22}$$

Second,

$$w_1 < w_2 + t \Leftrightarrow \frac{3(1+\gamma_B)p - (3+\gamma_B)t}{3+5\gamma_B} < \frac{(3+\gamma_B)p - 3(1-\gamma_B)t}{3+5\gamma_B} + t \Leftrightarrow \frac{t}{p} > \frac{2\gamma_B}{3(1+3\gamma_B)},$$

which is implied by the condition above that $\frac{t}{p} > \frac{1}{2}$. Third, firm 1's best response must indeed be given by the first line of (6):

$$w_2 > p - 3t \Leftrightarrow \frac{(3+\gamma_B)p - 3(1-\gamma_B)t}{3+5\gamma_B} > p - 3t \Leftrightarrow \frac{t}{p} > \frac{2\gamma_B}{3(1+3\gamma_B)},$$

which is identical to the condition immediately above. Fourth, the A market must indeed be covered:

$$w_1 - t\overline{x} \geq 0 \Leftrightarrow w_1 \geq \frac{t}{2t}(w_1 - w_2 + t) \Leftrightarrow w_1 + w_2 \geq t \Leftrightarrow \frac{t}{p} \leq \frac{2(3+2\gamma_B)}{3(3+\gamma_B)},$$

which is implied by the assumption $\frac{t}{p} \leq \frac{2}{3}$.

Calculate firm 1's and firm 2's profit at the possible equilibrium:

$$\pi_1^* = \frac{\gamma_A}{2t}(p-w_1)^2 = \frac{\gamma_A}{2t}\left[p - \frac{3(1+\gamma_B)p - (3+\gamma_B)t}{3+5\gamma_B}\right]^2 = \frac{\gamma_A}{2t}\left[\frac{2\gamma_B p + (3+\gamma_B)t}{3+5\gamma_B}\right]^2, \tag{23}$$

$$\pi_2^* = \frac{(\gamma_A + 2\gamma_B)(p-w_2)^2}{2t} = \frac{(1+\gamma_B)}{2t}\left[p - \frac{(3+\gamma_B)p - 3(1-\gamma_B)t}{3+5\gamma_B}\right]^2 = \frac{(1+\gamma_B)}{2t}\left[\frac{4\gamma_B p + 3(1-\gamma_B)t}{3+5\gamma_B}\right]^2. \tag{24}$$

There is one kind of deviation that we must check: Firm 2 could give up its ambition to hire in the A market and instead choose the wage that maximizes its profit when hiring only in the B market. If making this deviation, the best deviation maximizes $\pi_2 = \frac{\gamma_B}{t}(p-w_2)w_2$, i.e., it is given by $w_2 = \frac{p}{2}$. (This wage is indeed interior, for $\frac{p}{2} < t \Leftrightarrow \frac{t}{p} > \frac{1}{2}$, which is identical to (22).) Making this deviation would yield the profit

$$\pi_2^{dev} = \gamma_B\left(p - \frac{p}{2}\right)\frac{p}{2t} = \frac{\gamma_B p^2}{4t}. \tag{25}$$

Thus, there is no incentive to deviate if, and only if,

$$\pi_2^* \geq \pi_2^{dev} \Leftrightarrow \frac{(1+\gamma_B)}{2t}\left[\frac{4\gamma_B p + 3(1-\gamma_B)t}{3+5\gamma_B}\right]^2 \geq \frac{\gamma_B p^2}{4t} \Leftrightarrow (1+\gamma_B)\left[\frac{8\gamma_B + 6(1-\gamma_B)\frac{t}{p}}{3+5\gamma_B}\right]^2 \geq 2\gamma_B. \tag{26}$$

It is easy to verify that the left-hand side of the last inequality is increasing in $\frac{t}{p}$ and, evaluated at $\frac{t}{p} = \frac{1}{2}$, equals $1+\gamma_B$; hence the inequality holds for all $\frac{t}{p} > \frac{1}{2}$. This means that there is no profitable deviation.

We can conclude that if $\frac{t}{p} \in \left(\frac{1}{2}, \frac{2}{3}\right]$, then there is an equilibrium where the wages are given by (21), and the associated profit levels are given by (23) and (24). This yields the first line in Table 1.

Finding an eq. on the border between regions I and II (B market exactly covered)

In an equilibrium on the border between regions I and II, firm 2 chooses $w_2 = t$. Firm 1's reaction function is, as before, given by the first line of (6).[20] This means that in an equilibrium of this kind, firm 1's wage is given by $w_1 = \frac{w_2+p-t}{2} = \frac{p}{2}$. For $w_2 = t$ to be optimal for firm 2, given $w_1 = \frac{p}{2}$, the following two conditions must hold:

$$\frac{\partial \pi_2}{\partial w_2}\Big|_{(w_1,w_2)=\left(\frac{p}{2},t\right)} \geq 0 \Leftrightarrow \frac{1}{2t}\left[\gamma_A\left(t - \frac{p}{2} + t\right) + 2\gamma_B t\right] \leq \frac{\gamma_A + 2\gamma_B}{2t}(p-t) \Leftrightarrow \frac{t}{p} \leq \frac{1}{2},$$

[20]The case under consideration (i.e., $w_2 = t$) is also consistent with firm 1's reaction function being given by the *second* line of (6). But, if so, we have $w_1 = w_2 + t$, which is the case dealt with below.

$$\frac{\partial \pi_2}{\partial w_2}\mid_{(w_1,w_2)=\left(\frac{p}{2},t\right)} \leq 0 \Leftrightarrow 1-\frac{\gamma_A}{2t}\left(\frac{p}{2}-t+t\right) \geq \frac{\gamma_A}{2t}(p-t) \Leftrightarrow \frac{t}{p} \geq \frac{3\gamma_A}{2(2+\gamma_A)} = \frac{3(1-\gamma_B)}{2(3-\gamma_B)}. \tag{27}$$

The profit expression that is differentiated in the first condition is given by the first line of (11), while the profit expression that is differentiated in (27) is given by the second line of (11).

We can now check the remaining conditions that are required for $(w_1, w_2) = \left(\frac{p}{2}, t\right)$ to be an equilibrium. First, firm 1's best response must indeed be given by the first line of (6):

$$w_2 > p - 3t \Leftrightarrow t > p - 3t \Leftrightarrow \frac{t}{p} > \frac{1}{4},$$

which is implied by (27) above. Second, the A market must indeed be covered:

$$w_1 - t\overline{x} \geq 0 \Leftrightarrow w_1 \geq \frac{t}{2t}(w_1 - w_2 + t) \Leftrightarrow w_1 + w_2 \geq t \Leftrightarrow \frac{p}{2} + t \geq t,$$

which always holds.

Now calculate firm 1's profit at the equilibrium:

$$\pi_1^* = (p - w_1)\gamma_A \overline{x} = (p - \frac{p}{2})\gamma_A \frac{p}{4t} = \frac{\gamma_A p^2}{8t} = \frac{(1-\gamma_B)p^2}{8t}. \tag{28}$$

And calculate firm 2's profit at the equilibrium:

$$\pi_2^* = (p - w_2)(1 - \gamma_A \overline{x}) = (p - t)\left(1 - \gamma_A \frac{p}{4t}\right) = \frac{(p-t)\left[4t - (1-\gamma_B)p\right]}{4t}. \tag{29}$$

We can conclude that if $\frac{t}{p} \in \left[\frac{3(1-\gamma_B)}{2(3-\gamma_B)}, \frac{1}{2}\right]$, then there is an equilibrium where the wages are given by $(w_1, w_2) = \left(\frac{p}{2}, t\right)$, and the associated profit levels are given by (28) and (29). This yields the middle line in Table 1.

Finding an equilibrium where $w_1 = w_2 + t$ ***holds***

Consider finally the possibility of an equilibrium where the equality $w_1 = w_2 + t$ holds (and, as before, the A market is covered). In such an equilibrium, firm 1 is the only one hiring in the A market (cf. panel (a) of Fig. 5).

A first condition that must be satisfied for this kind of equilibrium to exist is that firm 1's reaction function is given by $w_1 = w_2 + t$, i.e., by the second line of (6). This requires that $w_2 < p - 3t$. Note that for this inequality to hold for some $w_2 \geq 0$, we must have $\frac{t}{p} < \frac{1}{3}$. We also know that firm 2 is active only in the B market, and it is a monopsonist in that market. Therefore firm 2's optimally chosen wage must equal $w_2 = t$ (this follows from (3) and the fact that $\frac{t}{p} < \frac{1}{3}$ implies $\frac{t}{p} < \frac{1}{2}$). This in turn means, since $w_1 = w_2 + t$, that $w_1 = 2t$. Firm 2's profits if $(w_1, w_2) = (2t, t)$ are given by

$$\pi_2 = \gamma_B(p - t).$$

When is indeed $(w_1, w_2) = (2t, t)$ an equilibrium? A first requirement is that, evaluated at $w_2 = t$, we have $w_2 \leq p - 3t$; this is equivalent to $\frac{t}{p} \leq \frac{1}{4}$. Second, firm 2 must not have an incentive to make a global deviation by entering the A market. An entry into the A market must involve an increase of w_2 from $w_2 = t$ to some higher wage, which in particular means that firm 2 will still employ all workers in the B market. The optimal deviation thus maximizes the profit expression in the second line of (11), and the associated first-order condition is given by (12). Plugging $w_1 = 2t$ into this first-order condition and then solving for w_2, we have

$$w_2^{\text{dev}} = \frac{\gamma_A p - (2 - 3\gamma_A)t}{2\gamma_A}. \tag{30}$$

One can verify that $\frac{t}{p} < \frac{1}{3}$ and $\gamma_A > \frac{1}{2}$ guarantee that $w_2^{\text{dev}} > t$ holds. Firm 2's profit if deviating to w_2^{dev} is

$$\pi_2^{\text{dev}} = \frac{\gamma_A}{2t}(p - w_2^{\text{dev}})^2 = \frac{\gamma_A}{2t}\left[\frac{\gamma_A p + (2 - 3\gamma_A)t}{2\gamma_A}\right]^2.$$

Therefore firm 2 has no incentive to deviate if, and only if,

$$\pi_2 \geq \pi_2^{\text{dev}} \Leftrightarrow \gamma_B(p - t) \geq \frac{\gamma_A}{2t}\left[\frac{\gamma_A p + (2 - 3\gamma_A)t}{2\gamma_A}\right]^2$$
$$\Leftrightarrow 8\gamma_A\gamma_B(p - t)t \geq [\gamma_A(p - t) + 2(1 - \gamma_A)t]^2 \Leftrightarrow [\gamma_A(p - t) - 2(1 - \gamma_A)t]^2 \leq 0.$$

The last inequality is always violated (it holds with equality if $\frac{t}{p} = \frac{\gamma_A}{2-\gamma_A}$, but this is inconsistent with $\frac{t}{p} \leq \frac{1}{4}$ and $\gamma_A > \frac{1}{2}$). We can conclude that there does not exist an equilibrium with $w_1 = w_2 + t$. □

Proof of Lemma 2

In order to prove the lemma, it suffices to show the claims about the subgame $(y_1, y_2) = (B, C)$. The results for the subgame $(y_1, y_2) = (C, B)$ then follow by symmetry of the game.

Thus suppose that $(y_1, y_2) = (B, C)$; that is, firm 1 discriminates in hiring against the majority group, group A, while firm 2 does not discriminate at all. The analysis of this case is very similar to the analysis in the proof of Lemma 1. Basically, we have to replace γ_A with γ_B (and vice versa) everywhere in our previous analysis. We also must re-examine the conditions for the various kinds of equilibria to exist, since these may now look different (for we have $\gamma_A > \frac{1}{2}$, while $\gamma_B < \frac{1}{2}$).

First consider an equilibrium in (the interior of) region I. By using (13), and by replacing γ_A with γ_B, we have

$$w_1 = p - \frac{(2 + \gamma_B)t}{3\gamma_B} \qquad \text{and} \qquad w_2 = p - \frac{(4 - \gamma_B)t}{3\gamma_B}. \tag{31}$$

Similarly, using (14) and (15), we obtain the following profit expressions:

$$\pi_1^* = \frac{t\,(2 + \gamma_B)^2}{18\gamma_B} \qquad \text{and} \qquad \pi_2^* = \frac{t\,(4 - \gamma_B)^2}{18\gamma_B}. \tag{32}$$

We now check all the conditions. The requirement that $w_1 > w_2 - t$ still always holds. The requirement that $w_1 < w_2 + t$ is equivalent to $\gamma_B > \frac{2}{5}$. The condition in (16) now becomes

$$w_2 - t > 0 \Leftrightarrow \frac{t}{p} < \frac{3\gamma_B}{2(2 + \gamma_B)}. \tag{33}$$

One can check that the next few arguments in the proof of Lemma 1 do not add any new condition to the analysis here. For example, the condition in (19) becomes

$$\frac{t}{p} \geq \frac{18(1 - \gamma_B)\gamma_B}{(4 - \gamma_B)^2 + 18(1 - \gamma_B)\gamma_B}, \tag{34}$$

which is implied by the assumption $\frac{t}{p} \geq \varphi(\gamma_B)$. Moreover, one can verify that the two inequalities (33) and (34) jointly imply $\gamma_B > \frac{2}{5}$. We can thus conclude that if $\frac{t}{p} \in \left[\varphi(\gamma_B), \frac{3\gamma_B}{2(2+\gamma_B)}\right)$, then there is an equilibrium where the wages are given by (31), and the associated profit levels are given by (32). This yields the bottom line in Table 2.

Next consider an equilibrium in (the interior of) region II. By using (21) and by replacing γ_A with γ_B, we have

$$w_1 = \frac{3(1 + \gamma_A)p - (3 + \gamma_A)t}{3 + 5\gamma_A} = \frac{3(2 - \gamma_B)p - (4 - \gamma_B)t}{8 - 5\gamma_B}, \tag{35}$$

$$w_2 = \frac{(3 + \gamma_A)p - 3(1 - \gamma_A)t}{3 + 5\gamma_A} = \frac{(4 - \gamma_B)p - 3\gamma_B t}{8 - 5\gamma_B}. \tag{36}$$

Similarly, using (23) and (24), we obtain the following profit expressions:

$$\pi_1^* = \frac{\gamma_B}{2t}\left[\frac{2\gamma_A p + (3+\gamma_A)t}{3+5\gamma_A}\right]^2 = \frac{\gamma_B}{2t}\left[\frac{2(1-\gamma_B)p + (4-\gamma_B)t}{8-5\gamma_B}\right]^2, \tag{37}$$

$$\pi_2^* = \frac{(1+\gamma_A)}{2t}\left[\frac{4\gamma_A p + 3(1-\gamma_A)t}{3+5\gamma_A}\right]^2 = \frac{2-\gamma_B}{2t}\left[\frac{4(1-\gamma_B)p + 3\gamma_B t}{8-5\gamma_B}\right]^2. \tag{38}$$

We now check all the conditions. The requirement that $w_2 < t$ still holds if, and only, if $\frac{t}{p} > \frac{1}{2}$. The requirement that $w_1 < w_2 + t$ is equivalent to

$$\frac{t}{p} > \frac{2\gamma_A}{3(1+3\gamma_A)} = \frac{2(1-\gamma_B)}{3(4-3\gamma_B)},$$

which is implied by $\frac{t}{p} > \frac{1}{2}$. The condition that firm 1's best response is given by the first line of (6) is, as before, identical to the condition immediately above. The requirement that the B market (this is, for the subgame under consideration, the market in which both firms are active) is covered can be written as

$$w_1 - t\overline{x} \geq 0 \Leftrightarrow \frac{t}{p} \leq \frac{2(3+2\gamma_A)}{3(3+\gamma_A)} = \frac{2(5-2\gamma_B)}{3(4-\gamma_B)},$$

which is implied by the assumption $\frac{t}{p} \leq \frac{2}{3}$. Finally consider the condition required for firm 2 not to have an incentive to deviate globally (by giving up its ambition to hire in the B market). It is clear that the arguments in the proof of Lemma 1 apply also here: There is no profitable such deviation (to see this, note that if we replace γ_A with γ_B in (26), the resulting inequality always holds, given $\frac{t}{p} > \frac{1}{2}$ and $\gamma_A < 1$). We can thus conclude that if $\frac{t}{p} \in \left(\frac{1}{2}, \frac{2}{3}\right]$, then there is an equilibrium where the wages are given by (35) and (36), and the associated profit levels are given by (37) and (38). This yields the first line in Table 2.

Next consider an equilibrium on the border between regions I and II. Here, as in the proof of Lemma 1, the wages are given by $(w_1, w_2) = \left(\frac{p}{2}, t\right)$. The profits are obtained by swapping γ_A and γ_B in (28) and (29):

$$\pi_1 = \pi_{B|C} = \frac{\gamma_B p^2}{8t}, \qquad \pi_2 = \pi_{C|B} = (p-t)\left(1 - \gamma_B \frac{p}{4t}\right). \tag{39}$$

Among the conditions required for $(w_1, w_2) = \left(\frac{p}{2}, t\right)$ to be an equilibrium, only one is affected when we replace γ_A with γ_B. This is condition (27), which now becomes:

$$\frac{\partial \pi_2}{\partial w_2}\Big|_{(w_1,w_2)=\left(\frac{p}{2},t\right)} \leq 0 \Leftrightarrow \frac{t}{p} \geq \frac{3\gamma_B}{2(2+\gamma_B)}. \tag{40}$$

The conditions that are the same as in the proof of Lemma 1 are $\frac{t}{p} \leq \frac{1}{2}$ and $\frac{t}{p} > \frac{1}{4}$. In addition, we have assumed that $\frac{t}{p} \geq \varphi(\gamma_B)$. Of the two latter conditions and of the condition in (40), either one can (depending the value of γ_B) be the most stringent one. We can thus conclude that if

$$\frac{t}{p} \in \left[\max\left\{\frac{3\gamma_B}{2(2+\gamma_B)}, \frac{1}{4}, \varphi(\gamma_B)\right\}, \frac{1}{2}\right],$$

then there is an equilibrium where $(w_1, w_2) = \left(\frac{p}{2}, t\right)$, and the associated profit levels are given by (39). This yields the second line in Table 2.

Finally we must investigate the possibility of an equilibrium where $w_1 = w_2 + t$ holds. It follows from the arguments in the proof of Lemma 1 that in this kind of equilibrium, $(w_1, w_2) = (2t, t)$. Moreover, it follows that we must have $\frac{t}{p} \leq \frac{1}{4}$. Similarly to the Lemma 1 proof, firm 2 must not have an incentive to make a global deviation by entering the B market. An entry into the B market must involve an increase of w_2 from $w_2 = t$ to some higher wage (so firm 2 would still employ all A workers). The optimal deviation must be given by (30), but with γ_A replaced by γ_B:

$$w_2^{\text{dev}} = \frac{\gamma_B p - (2-3\gamma_B)t}{2\gamma_B}.$$

This expression does not exceed t if, and only if,

$$\frac{t}{p} \geq \frac{\gamma_B}{2-\gamma_B}. \tag{41}$$

One can show that, given $\frac{t}{p} \leq \frac{1}{4}$, (41) is implied by the assumption that $\frac{t}{p} \geq \varphi(\gamma_B)$. Hence, $\frac{t}{p} \in \left[\varphi(\gamma_B), \frac{1}{4}\right]$ guarantees that firm 2 does not have a profitable deviation and therefore that $(w_1, w_2) = (2t, t)$ is an equilibrium. We can thus conclude that if $\frac{t}{p} \in \left[\varphi(\gamma_B), \frac{1}{4}\right]$, then there is an equilibrium where $(w_1, w_2) = (2t, t)$. The associated profit levels can be computed as $\pi_1 = \pi_{B|C} = \gamma_B(p - 2t)$ and $\pi_2 = \pi_{C|B} = \gamma_A(p - t)$. This yields the third line in Table 2. □

Proof of Proposition 2

To prove the claim it suffices to show that, given $\frac{t}{p} \in \Omega_{II}$, we have $\pi_{C|A} \geq \pi_{A|C} > \pi_{D|D}$. From eq. (8) we know that $\pi_{D|D} = \frac{t}{2}$. The expressions for $\pi_{A|C}$ and $\pi_{C|A}$ depend on whether (i) $\frac{t}{p} \in \left(\frac{9(1-\gamma_B)}{21-13\gamma_B}, \frac{3(1-\gamma_B)}{2(3-\gamma_B)}\right]$ or (ii) $\frac{t}{p} \in \left(\max\left\{\frac{3(1-\gamma_B)}{2(3-\gamma_B)}, \frac{1}{3}\right\}, \frac{\sqrt{1-\gamma_B}}{2}\right)$. For case (i) we have a high-wage equilibrium and, by Table 1,

$$\pi_{A|C} > \pi_{D|D} \Leftrightarrow \frac{t\,(3-\gamma_B)^2}{18\,(1-\gamma_B)} > \frac{t}{2} \Leftrightarrow (3-\gamma_B)^2 > 9\,(1-\gamma_B) \Leftrightarrow 3\gamma_B + \gamma_B^2 > 0$$

and

$$\pi_{C|A} > \pi_{A|C} \Leftrightarrow \frac{t\,(3+\gamma_B)^2}{18\,(1-\gamma_B)} > \frac{t\,(3-\gamma_B)^2}{18\,(1-\gamma_B)}.$$

Clearly, both conditions hold for all $\gamma_B \in \left(0, \frac{1}{2}\right)$.

For case (ii) we have a middle-wage equilibrium and, by Table 1,

$$\pi_{A|C} > \pi_{D|D} \Leftrightarrow \frac{(1-\gamma_B)\,p^2}{8t} > \frac{t}{2} \Leftrightarrow \frac{t}{p} < \frac{\sqrt{1-\gamma_B}}{2}$$

and

$$\pi_{C|A} > \pi_{A|C} \Leftrightarrow \frac{(p-t)\,[4t-(1-\gamma_B)\,p]}{4t} > \frac{(1-\gamma_B)\,p^2}{8t} \Leftrightarrow 2\left(1-\frac{t}{p}\right)\left[4\frac{t}{p}-(1-\gamma_B)\right] > 1-\gamma_B.$$

The first condition clearly holds for all $\gamma_B \in \left(0, \frac{1}{2}\right)$. The left-hand side of the second condition is increasing in $\frac{t}{p}$ for all $\gamma_B \in \left(0, \frac{1}{2}\right)$; hence the condition holds if it is satisfied when evaluated at the lowest possible value of $\frac{t}{p}$, namely $\frac{t}{p} = \max\left\{\frac{3(1-\gamma_B)}{2(3-\gamma_B)}, \frac{1}{3}\right\}$. Indeed, it suffices to check that it holds for $\frac{t}{p} = \frac{3(1-\gamma_B)}{2(3-\gamma_B)}$:

$$2\left(1-\frac{3\,(1-\gamma_B)}{2\,(3-\gamma_B)}\right)\left[4\frac{3\,(1-\gamma_B)}{2\,(3-\gamma_B)}-(1-\gamma_B)\right] = \frac{(3+\gamma_B)^2\,(1-\gamma_B)}{(3-\gamma_B)^2} > 1-\gamma_B,$$

which is satisfied for all $\gamma_B \in \left(0, \frac{1}{2}\right)$. □

Proof of Proposition 4

To prove claim (i) it suffices to show that $\pi_{C|A} \geq \pi_{A|C} > \pi_{C|C}$. But, since $\pi_{C|C} = \pi_{D|D}$, this follows from the proof of Proposition 2.

To prove claim (ii), note from eq. (1) that the workers care about their wage and their mismatch cost. Also note that, given $\frac{t}{p} \in \Omega_{II}$, all workers are employed, both with and without the anti-discrimination policy described in the proposition; hence, in both scenarios, all workers earn a wage and incur a mismatch cost. From subsection 3.1.2 and Table 1 it follows that, for all $\frac{t}{p} \in \Omega_{II}$, $w_{C|C} > w_{A|C} > w_{C|A}$. That is, the wage utility that accrues to any given worker is higher with the policy. Moreover, with the policy the two firms' wages are the same, which means that the threshold value $\overline{x}$ defined in (5) is given by one-half: All workers left (right, respectively) of the midpoint of the unit interval chooses firm 1 (2, respectively). On the other hand, without the policy some workers will choose an employer that is farther away, while others choose the same employer as with the policy. That is, the mismatch cost that any given worker incurs is either the same or strictly lower with the policy. Those things imply that each one of the workers is strictly better off with the policy than without.

To prove claim (iii), note again that, given $\frac{t}{p} \in \Omega_{II}$, all workers are employed both with and without the policy. Moreover, the wage does not matter for total surplus, since it is only a transfer from a firm to a worker. Those things imply that only the aggregate mismatch costs matter for total surplus. As argued in the paragraph immediately above, however, these mismatch costs are strictly higher without the policy. □

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