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Post-Malthusian Dynamics in Pre-Industrial Scandinavia*

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Abstract

Theories of economic growth hypothesize that the transition from pre-industrial stagnation to sustained growth is associated with a post-Malthusian phase in which technological progress raises income and spurs population growth while offsetting diminishing returns to labor. Evidence suggests that England was characterized by post-Malthusian dynamics preceding the Industrial Revolution. However, given England's special position as the forerunner of the Industrial Revolution, it is unclear if a transitory post-Malthusian period is a general phenomenon. Using data from Denmark, Norway and Sweden, this research provides evidence for the existence of a post-Malthusian phase in the transition from stagnation to growth in Scandinavia.

Keywords Demography, Post-Malthusian Dynamics, Malthus, Pre-Industrial Scandinavia, Demographic Transition, Economic Growth, Unified Growth Theory, Malthusian Stagnation, Co-integration, Time Series Analysis

JEL Classification Codes C32, N3, O1

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1 Introduction

Recent long-run economic growth theory has focused on the transition of societies from millennia of economic stagnation to an era of sustained economic growth (Galor, 2011). While income per capita has stagnated for the vast part of human history, the world as a whole has experienced a ten-fold increase in income per capita in the last two centuries, following the Industrial Revolution and the Demographic Transition. However, the differential timing of this transition from stagnation to growth has led to large differences in income across the globe. In light of the fact that millions of people remain to live in severe poverty, gaining insights into the mechanisms that brought some countries out of stagnation and lead them into a state of sustained economic growth remains a central topic in the field of economic growth.[1]

Theories building upon a Malthusian interpretation of the pre-industrial era, such as unified growth theory (e.g. Galor and Weil, 2000; Galor, 2011), hypothesize that technological advances in production spurred population growth, thereby offsetting the positive effects of technology on income per capita through decreasing marginal returns to labour. According to this view, which interprets the pre-industrial, pre-demographic transition era as a *Malthusian era*, improvements in production technology therefore led to increases in population size, while income per capita remained stagnant.

Inspired by ideas laid forth by Smith (1776), Boserup and Kaldor (1965), and others, unified growth theories also suggest that, in connection with industrialization, the increasing population size accelerated the technological growth rate. This gave gave rise to a *post-Malthusian era* in which population growth did not entirely counterbalance the growth in income. Eventually, the population growth and associated acceleration of technological growth gave rise to the *modern growth regime*, where the demand for human capital – as a consequence of the acceleration in technological development – elevated investment in the human capital of children, resulting in a negative association between income per capita and population growth.

This research first of all provides time series evidence that post-Malthusian dynamics were operating in Scandinavia in the later part of the pre-industrial era. Secondly, the analysis suggests that a negative association between income and fertility eventually replaced the post-Malthusian positive association between income and fertility. These findings are consistent with the hypothesis that the evolution of societies from stagnation to growth is characterized by a transition period of post-Malthusian dynamics.

While the vast majority of existing empirical investigations of Malthusian stagnation are not directly concerned with the post-Malthusian phase and the transition from stagnation to growth, Møller and Sharp (2014) formulate and test a simple dynamic post-Malthusian model. Using annual series of real wages, marriage rates, birth rates and death rates for pre-industrial England, they find evidence consistent with post-Malthusian dynamics emerging as early as the mid-sixteenth century. Likewise, Klemp (2012) formulated and estimated a two-sector Malthusian model on pre-industrial

[1] The transition from stagnation to growth has been analyzed by (Galor and Weil, 1999, 2000; Galor and Moav, 2002; Hansen and Prescott, 2002; Lucas, 2002; Lagerlöf, 2003; Doepke, 2004; Galor, 2005; O'Rourke et al., 2014; Strulik and Weisdorf, 2008; Dalgaard and Strulik, 2010) and others. The Malthusian epoch has been analyzed separately by (Clark, 2007; Weisdorf, 2008; Vollrath, 2011; Tabata, 2013) and others.

English data and found evidence in favor of Malthusian dynamics exhibiting some post-Malthusian characteristics.[2]

However, in light of England's special position as the forerunner of the Industrial Revolution, the extent to which the existence of a post-Malthusian regime can be considered a more general phenomenon is still unknown. In particular, the occurrence of the Industrial Revolution in England may have instigated a fundamental change in the process of development in economies that took off later. Furthermore, in the period leading up to the Industrial Revolution, some characteristics of the English economy were markedly different from those of other European economies. For example, England had a relatively high level of income per capita.

For these reasons, it seems of great interest to investigate whether the findings of Møller and Sharp (2014) generalize beyond England. Adopting their methodology, the present research therefore employs, for the first time, a set of high-quality economic and demographic historical data to investigate the existence of a post-Malthusian period in three Scandinavian countries: Denmark, Norway and Sweden.[3] Since long time series from each country are available and since a primary objective of the study is to maintain comparability to the analysis of the English data in Møller and Sharp (2014), each country is analyzed separately.

A main advantage of studying the Scandiavian countries, compared to existing analyses of Malthusian dynamics in other countries, lies with the fact that official national statistics of births and deaths were often kept already from the 1730s. Indeed, the demographic birth and death rate series from Denmark, Norway and Sweden are known to have a high accuracy (see Gille, 1949). In contrast to data sources for other countries, which are often based on subsets of the national population, such as individual towns or parishes (e.g. Wrigley and Schofield, 1989), the Scandinavian data cover the entire Scandinavian populations. Moreover, one may add that, due to the fact that the Scandinavian countries took off much later than England (see below), data on income from the pre-industrial era are newer and therefore presumably of a higher quality.

Table 1 gives a broad indication of how the Scandinavian countries compared to the United Kingdom in terms of economic development. The table reports that the Norwegian and Swedish levels of GDP per capita in 1700 were the same as for the United Kingdom in the 1500s for which the analysis in Møller and Sharp (2014) begins. Likewise, the level of income per capita of Denmark in 1820 was at the level of that in the UK in 1700. As in many European countries, factory production, modeled on that in the UK, as well as railroads arrived in Scandinavia in the mid-1800s. However, the onsets of economic take-offs occurred later. In particular, Denmark took off around the 1880s, about a century after the UK, largely through highly mechanized food production and exports to the UK. Norway and Sweden, which at that time constituted a union, took of soon afterwards through a more typical expansion of the manufacturing sector. However, the economic state of

[2]Other time series analyses, most of which focus on England, include Bailey and Chambers (1993); Lee and Anderson (2002); Nicolini (2007); Crafts and Mills (2009); Rathke and Sarferaz (2010). Ashraf and Galor (2011) establish evidence in the cross-country context, for which the Malthusian theory predicts that technologically more advanced countries should be more populated but no richer.

[3]The analysis in Møller and Sharp (2014) is the first formal time series-based econometric framework designed to describe the transition period and potential post-Malthusian dynamics. However, other empirical studies that focus on the strict Malthusian model have found results that are compatible with the existence of a non-Malthusian economy before the Industrial Revolution (see e.g. Nicolini, 2007; Pfister and Fertig, 2010; Klemp, 2012).

Table 1: GDP per capita for the United Kingdom and the Scandinavian Countries, 1500–1900

Year	1500	1600	1700	1820	1850	1870	1900
Denmark	738	875	1039	1274	1767	2003	3017
Norway	610	665	722	801	956	1360	1877
Sweden	651	700	750	819	1019	1359	2209
United Kingdom	714	974	1250	1706	2330	3190	4492

This table presents estimates of GDP per capita in 1990 International Geary-Khamis dollars from Maddison (2010).

all three Scandinavian economies resembled that of the UK prior to their respective transitions into industrial production. Economically, they were predominantly agricultural, based largely on small owner occupied farms (i.e., yeoman agriculture); and demographically, they were typified by the "European Marriage Pattern" Hajnal (1965), whereby marriage predominantly took place when income was sufficient to support the family in a new household, consistent with the positive relationship between income and fertility postulated by Malthus.[4]

Thus, on the one hand, the three Scandinavian countries could be expected to be fundamentally different from the UK. They were poorer and industrialized much later – to a large extent as growth followers rather than growth leaders. On the other hand, there seems to be some similarities with the UK, when it comes to the demographic-economic interaction. Hence, *a priori* it is not obvious, but nevertheless possible, that the late pre-industrial period in these economies could have been post-Malthusian.

The results of the present research clearly suggest that the Scandinavian countries did undergo a post-Malthusian period prior to industrialization. Indeed, our findings below are strikingly similar to the findings for England presented in Møller and Sharp (2014): for all three Scandinavian countries the research finds evidence of a non-stagnating income per capita positively affecting population size and an absence of a feedback effect from population size to income per capita. Furthermore, the present analysis is markedly unambiguous in comparison to the existing evidence from England, which – in light of the superior quality of the Scandinavian data – further strengthens the reliability of the analysis and the validity of the findings. In particular, for all three countries, we find evidence of a preventive check (i.e., a positive effect of income per capita on the birth rate), and furthermore, for Denmark and Sweden the positive check (i.e., a negative effect of income per capita on the mortality rate) is supported. We are able to document parameter stability for relatively long periods. Moreover, the main conclusions do not depend on the few dummy variables, which we argue should be included in a statistically well-specified model, nor do they depend on the the exclusion of a few early observations, which apparently have been subject to extraordinary historical circumstances.

The next section describes the theory, Section 3 derives the time series implications and the relevant statistical model, Section 4 describes the data and the estimation results are presented in Section 5. Finally, Section 6 concludes the paper.

[4] A positive effect of income on marriage rates in England is supported by Møller and Sharp (2014).

2 The model

Unified growth theory hypothesizes that the transition of societies from an era of stagnation to a period of sustained economic growth is associated with a post-Malthusian phase. In this phase scale effects of population growth prompt a rising rate of technological development, which offsets the adverse Malthusian effects of population size on income per capita (via diminishing returns), while there is a positive feedback effect from income per capita on population growth.[5] Thus, in contrast to a pure Malthusian phase where technological advances induce a growing population size while income remains stagnating, in the post-Malthusian phase, technological advances induce both a growing population and a higher income per capita.

Based on Møller and Sharp (2014), this section presents a simple post-Malthusian economic model, while Section 3 shows that this has a direct correspondence with a set of parameter restrictions on a flexible class of time series econometric models, the co-integrated vector autoregressive model.[6] Hence, the post-Malthusian hypothesis can be parsimoniously assessed using a joint test of a set of parameter restrictions on the econometric model. Furthermore, the latter provides estimates of the central parameters of the post-Malthusian model.

To illustrate the essential properties of the model it is kept as simple as possible in this section. That is, we simplify with respect to the dynamics of adjustment, such as the potentially gradual responses of the demographic rates to changes in the real wage rate. However, in the empirical analysis in Section 5, the estimated model has the same long-run dynamics but is embedded in an Unrestricted Vector Auto-Regression, meaning that we allow for more realistic gradual dynamics of adjustment.

2.1 The General Malthusian Model and the post-Malthusian Hypothesis

This section will outline a simple Malthusian-based model that includes what may be referred to as the "standard" Malthusian model as well as the post-Malthusian model as special cases. In the following section, the model is reformulated as a dynamic econometric model and we derive the parameter restriction consistent with the post-Malthusian hypothesis, i.e. the absence of an effect of population size on income per capita.

The standard Malthusian model typically assumes a closed economy with endogenous population size. What one often has in mind is an agricultural-based economy for which the supply of land is exogenous and fixed over time, although the model may also describe non-agricultural economies. In every period, the economy produces a single homogeneous good using labour as an input, and it is assumed that there is decreasing returns to labour. Income per capita is represented by the real

[5] For scale effects, see (Boserup and Kaldor, 1965; Kremer, 1993).

[6] Although the exposition is largely self-contained a full account of the model and the econometric details is found in Møller and Sharp (2014). Their analysis demonstrate that, for the period from the mid-1500s to around 1800, English time series data is consistent with the post-Malthusian hypothesis. Thus, their analysis finds evidence of positive scale effects of population size offsetting the negative feedback from a larger population while growth in income per capita induce births. In essence, their results suggest that real wage growth leads to population growth (primarily through more births resulting from more marriages) that will not only negatively affect real wages due to diminishing marginal returns to labour, but also positively affect real wages by inducing increases in the technological level, with the end result that there is no net effect of population size on wages.

wage, which is determined by the equilibrium wage in the aggregate labour market. The evolution of labour supply is determined by households' fertility decisions and the death of individuals in the preceding periods.

Under the assumptions of a standard a Cobb-Douglas constant-returns-to-scale production function, inelastic labour supply and labour market clearing, the natural logarithm of the real wage, w_t, is given by

$$w_t = c_0 - c_1 \ln N_t + \ln A_t, \tag{1}$$

where (here and below) the coefficients are positive (unless otherwise stated), N_t is the total population size, and A_t is the level of technology, which determines the efficacy of the production process.[7] In general, A_t may be interpreted broadly as comprising all determinants of labour demand at given real wages, such as capital and production technology. Equation (1), therefore, describes how population affects living standards: Wages are suppressed by population growth due to diminishing returns (i.e., $c_1 > 0$), but increase with the technological sophistication, A_t.

The following two equations in turn describe the feedback effects from living standards to the demographic rates. In particular, equation (2) describes the so-called Malthusian *preventive check* whereby fertility is positively affected by real wages,

$$b_t = a_0 + a_1 w_t + \varepsilon_{bt}, \tag{2}$$

where b_t is the crude birth rate and ε_{bt} is a random error term (see below). Furthermore, equation (3) describes the so-called Malthusian *positive check*, whereby mortality is negatively affected by real wages,

$$d_t = a_2 - a_3 w_t + \varepsilon_{dt}, \tag{3}$$

where d_t is the crude death rate and ε_{dt} is a random error term (see below).[8]

Several explanations of the preventive check, ranging from rational economic behavior to purely biological circumstances, have been proposed in the literature on demographics and Malthusian dynamics (see e.g. Cinnirella et al. (2013) and the introduction in Lee (1977)). The positive check has a purely biological underpinning and is based on the adverse effects of lower income per capita on nutrition, infant mortality, and more (see e.g. Lee, 1997; Schultz, 1981).

The exogenous error terms, ε_{bt} and $\varepsilon_{dt}, t = 1, \ldots, T$, reflect unmodeled unsystematically fluctuating influences on births and deaths respectively. In the econometric analysis these variables are treated as independent and identically distributed Gaussian random variables.

[7] The equations of the model are linear and should generally be interpreted as linear approximations to more complex non-linear relations.

[8] The terms "preventive check" and "positive check" can be understood in the context of a decrease in income. When income is reduced, and simple adaptive expectations of future income are lowered, individuals will tend to delay their marriage and the onset of having children. In this way population is intentionally kept down ("checked"), *preventing* an otherwise larger population from being *positively* checked by mortality in the future.

Under an assumption of no international migration, equation (4) gives the log-approximation of the population growth rate. Hence, this is determined by the difference between the birth rate and the mortality rate,

$$\ln N_t = \ln N_{t-1} + b_{t-1} - d_{t-1}. \tag{4}$$

The relations (1)–(4) are standard in most models of Malthusian stagnation. To determine the short-run equilibrium, note that, in each period, the stocks, A_t and N_t, are predetermined and hence determine the positions of the labor demand and supply schedules, and thus, the short-run equilibrium real wage, w_t, by relation (1). The demographic rates, b_t and d_t, are then determined from the relations (2) and (3). Given these rates and N_t, the population level for the next period, N_{t+1}, can then be determined from forwarding equation (4) by one period. Depending on how technology, and hence labour demand, evolves, the real wage rate and demographic rates for the next period can subsequently be determined and so on. In the absence of any shocks, the variables will converge to their steady state values, provided that the parameters lie within the stability range (see Møller and Sharp, 2014).

In the basic model of Malthusian stagnation, stated so far, technology, A_t, is exogenous. Moreover, it is assumed that A_t evolves in a persistent manner and in Møller and Sharp (2014) this is approximated by a random walk stochastic process. The property of persistence can be interpreted as reflecting the cumulative nature of technical knowledge, as well as the slow changes in other inputs, such as capital.

In order to formulate the post-Malthusian hypothesis the basic Malthusian model above is augmented by allowing technology, A_t, to contain an endogenous part related positively to the level of the population, i.e. the scale, in addition to an exogenous part. The exogenous part is denoted X_t and is assumed to follow a random walk. Equations (5) and (6) describe this,

$$\ln A_t = c_3 \ln N_t + \ln X_t, \tag{5}$$

$$\ln X_t = \ln X_{t-1} + \varepsilon_{Xt}, \tag{6}$$

where $0 \leq c_3 \leq c_1$ and the shocks, $\varepsilon_{Xt}, t = 1, 2, ..., T$, are assumed to be independent and identically Gaussian distributed. The parameter c_3 accounts for the effect of population size on technology.

First note that the case in which there are no scale effects ($c_3 = 0$) corresponds to a standard model of Malthusian Stagnation. In this case A_t coincides with X_t, and hence, evolves as an exogenous random walk. In contrast, as in accordance with unified growth theory, inspired by Boserup and Kaldor (1965) and Smith (1776), the post-Malthusian hypothesis posits that the positive scale effect, i.e. c_3, counteracts the diminishing returns to labour, determined by c_1. In other words, the post-Malthusian hypothesis is that the general Malthusian model is operating and that furthermore $c_3 = c_1$. Although both Boserup-based or Kaldor-based interpretations of the scale effect, $c_3 > 0$, are broadly consistent with our model, we prefer a Smithian interpretation. Simply stated this suggests that a larger population implies a larger market and thus naturally leads to a higher degree of labor division (specialization), which in turn increases productivity. Such a mechanism is likely relevant not only for the agricultural sector but also for manufacturing and other sectors. Moreover, a Smithian scale effect is likely to have been relevant on the country level, and in both large economies, such as England, as well as in the smaller Scandinavian economies.

The next section shows that if we impose the post-Malthusian parameter restriction ($c_3 = c_1$) this implies *non-stationarity* in the form of a testable co-integration restriction in a VAR model.

3 Time-Series Implications of the Model

To derive the testable parameter restriction consistent with the post-Malthusian hypothesis, we first solve the above model, i.e. the equations (1)–(6), with respect to the observable variables in first-differences, $\Delta w_t, \Delta b_t$, and Δd_t.[9] Collecting the variables in a vector $x_t = (w_t, b_t, d_t)'$ and using the identity, $x_t = x_{t-1} + \Delta x_t$, the resulting equations may be written as a special case of the 3-dimensional Vector Auto-Regression (VAR) model with k lags, error-correction form (see Lütkepohl, 2005),

$$\Delta x_t = \Pi x_{t-1} + \Gamma_1 \Delta x_{t-1} + \cdots + \Gamma_{k-1} \Delta x_{t-(k-1)} + \mu + \varepsilon_t \tag{7}$$

with the following long-run restriction, i.e. restriction on the Π matrix,

$$\Pi = \begin{pmatrix} 0 & -\bar{c}_1 & \bar{c}_1 \\ a_1 & -(1 + a_1\bar{c}_1) & a_1\bar{c}_1 \\ -a_3 & a_3\bar{c}_1 & -(1 + a_3\bar{c}_1) \end{pmatrix}, \tag{8}$$

where $\bar{c}_1 \equiv c_1 - c_3$.

It is shown in Møller and Sharp (2014) that when the pure Malthusian model, for which $\bar{c}_1 = c_1$, has a stable steady state (assumed by regularity), all characteristic roots are located outside the complex unit disc and Π has full rank. This in turn implies that x_t, viewed as a stochastic process, is stationary. In contrast, under the post-Malthusian hypothesis, $\bar{c}_1 = 0$, the Π matrix has reduced rank, corresponding to roots at 1, which implies non-stationarity with co-integration.[10] To see this, we note that the determinant of Π is $\det(\Pi) = -\bar{c}_1 (a_1 + a_3)$ which is zero when $\bar{c}_1 = 0$. Inserting $\bar{c}_1 = 0$ in (8), it follows that the rank is reduced to 2. This restriction is parameterized as, $\Pi = \alpha\beta'$, where α and β are both 3×2 matrices with rank, 2.[11] In particular,

$$\alpha = \begin{pmatrix} 0 & 0 \\ -1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \beta' = \begin{pmatrix} -a_1 & 1 & 0 \\ a_3 & 0 & 1 \end{pmatrix}. \tag{9}$$

Hence, this is the testable parameter restriction corresponding to the post-Malthusian hypothesis.

[9] To do this, first take the first difference in (1), which gives $\Delta w_t = -c_1 \Delta lnN_t + \Delta lnA_t$. Then, since (4) implies that, $\Delta lnN_t = b_{t-1} - d_{t-1}$, while $\Delta lnA_t = c_3 \Delta lnN_t + \Delta lnX_t$ and $\Delta lnX_t = \varepsilon_{Xt}$ follow respectively from (5) and (6), we get, $\Delta w_t = -(c_1 - c_3)(b_{t-1} - d_{t-1}) + \varepsilon_{Xt}$. To solve for Δb_t, and Δd_t simply insert the identities, $b_t = b_{t-1} + \Delta b_t$ in (2), and $d_t = d_{t-1} + \Delta d_t$ in (3), and isolate the first differences.

[10] A general treatment of co-integrated VAR models is found in Johansen (1996).

[11] In general, these matrices are $p \times r$ where p is the dimension of the model and r is the *co-integration rank.*

The elements of the α matrix are usually referred to as *adjustment coefficients*, since they describe how the variables will react to equilibrium deviations, which in turn correspond to $\beta' x_t$. From (9) note that the zero row in α means that real wages do not respond to disequilibrium. Furthermore, we see that the coefficients in the co-integration vectors identify the preventive and positive checks. In the empirical implementation, we impose the zeros in α and β'. However, the two non-zero adjustment coefficients in α are allowed to deviate from -1, reflecting the fact that adjustment to equilibrium will realistically require more than one period (see Møller and Sharp, 2014). Thus, in the empirical analysis, the α matrix is given by

$$\alpha = \begin{pmatrix} 0 & 0 \\ -\rho_1 & 0 \\ 0 & -\rho_2 \end{pmatrix}, \tag{10}$$

with ρ_1 and ρ_2 varying freely.

To understand the intuition in why non-stationarity and co-integration arise under the post-Malthusian hypothesis, consider the following: Suppose an exogenous technological improvement, $\varepsilon_{Xt} > 0$, occurs. This will increase real wages so that population will grow, which in turn, will feedback negatively on real wages, due to diminishing marginal returns, just as in the basic Malthusian model. In isolation this feedback will thus cause stagnation or, in the context of time series processes, stationarity. However, in addition to this negative feedback there is also a positive feedback, in that the rising population increases the overall technological level (cf. the Smithian scale effect discussed in Section 2.1). If this is about the same magnitude as the negative feedback it will offset the latter, and thereby non-stagnation (non-stationarity) emerges. In fact, this non-stationarity implies that the real wage rate follows a random walk (see Footnote 9). This means that it has no steady state, and hence, the demographic rates have no steady state, i.e., a state towards which these variables would converge in the absence of any shocks. However, the restrictions also imply that, although these variables drift in this non-stationary random walk manner, they do drift together, that is, they co-integrate (with the two co-integration vectors in (9)). Altogether these restrictions therefore describe a post-Malthusian regime in which income continues to spur population growth (as in the pure Malthusian phase) but is no longer stagnant (stationary).

4 Data

The estimation is based on yearly observations of real wages, crude birth rates (i.e., the number of births per thousand inhabitants) and crude death rates (i.e., the number of births per thousand inhabitants) for Denmark, Norway and Sweden. The crude demographic rates for Denmark are taken from (Mitchell, 1998, Table A6), for Norway from the official Norwegian statistics bureau, Statistics Norway,[12] and for Sweden from Lund University Macroeconomic and Demographic Database.[13] The time series of the Danish real wages consist of wages of urban workers given by

[12]The data can be found at www.ssb.no/histstat/tabeller/3-13.html.

[13]The data can be found at www.ekh.lu.se/en/research/shna1560-2010/lu-madd/population.

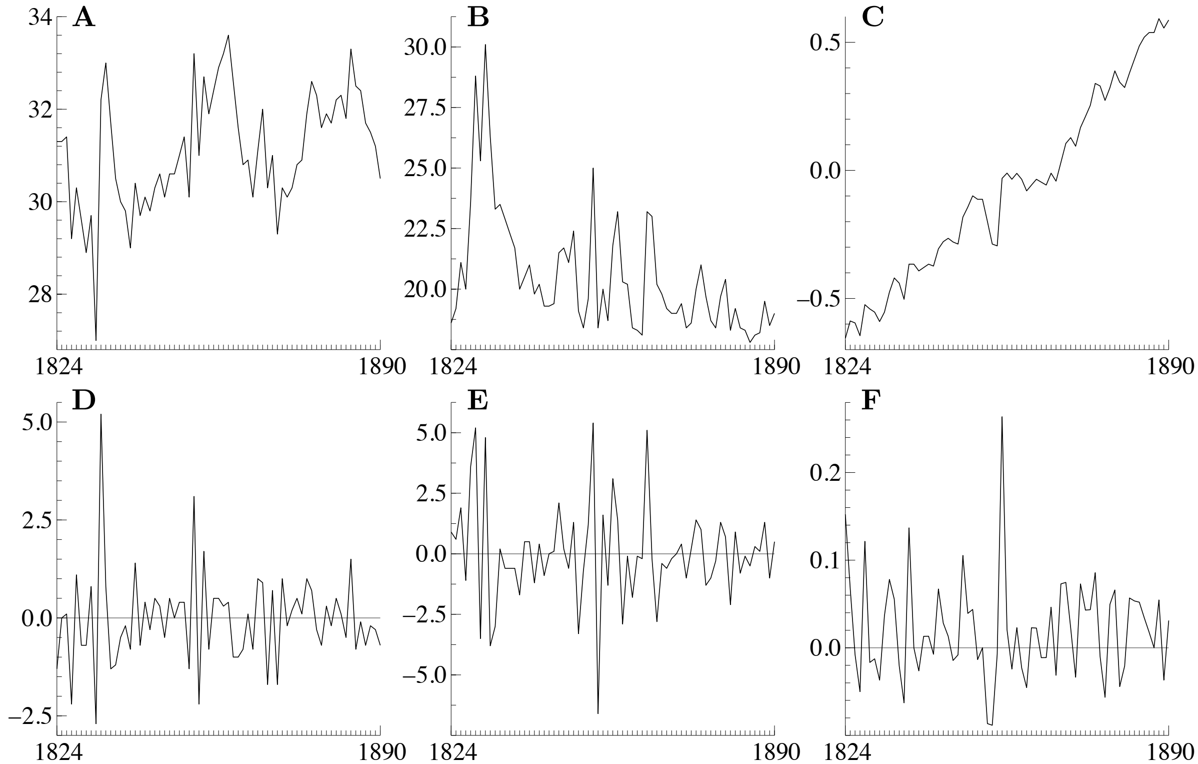

Figure 1: Plots of the Danish data in levels and first differences. A) Level of the crude birth rate. B) Level of the crude death rate. C) Level of the nautral logarithm of the real wage rate. D) First differences of the crude birth rate. E) First differences of the crude death rate. F) First differences of the nautral logarithm of the real wage rate.

(Hansen, 1984, Table 20) deflated by the consumer price index given by (Mitchell, 1998, Table H2). For Norway wages in the industrial sector, given by Grytten (2009), are used, and for Sweden the series of day rates for male agricultural workers, given by (Jörberg, 1972, pp. 710–714), are used and deflated by the historical consumer price index maintained by the Swedish central bank, The Riksbank.[14]

The time series for the analyzed samples are plotted in Figure 1–3, corresponding to Denmark, Norway and Sweden, respectively. For each country the figure indicates drifting levels with first-differences that are more stable, suggesting that these series can be econometrically modeled as realizations of an I(1) co-integrated VAR process. Secondly, there is some indication of linear deterministic trends in some of the series. In the empirical analysis we therefore allow for this, and furthermore that, since the different variables may have different trends, these may not necessarily cancel in the co-integrating relations between wages and the vital rates. Thirdly, there are some large spikes in the series which suggest that there could be some extraordinary events that the model cannot explain, and hence, a need to include a few indicator (i.e., dummy) variables. The details are explained below.

[14]The consumer price index is CPI 1. The data can be found at www.riksbank.se/templates/Page.aspx?id=26813.

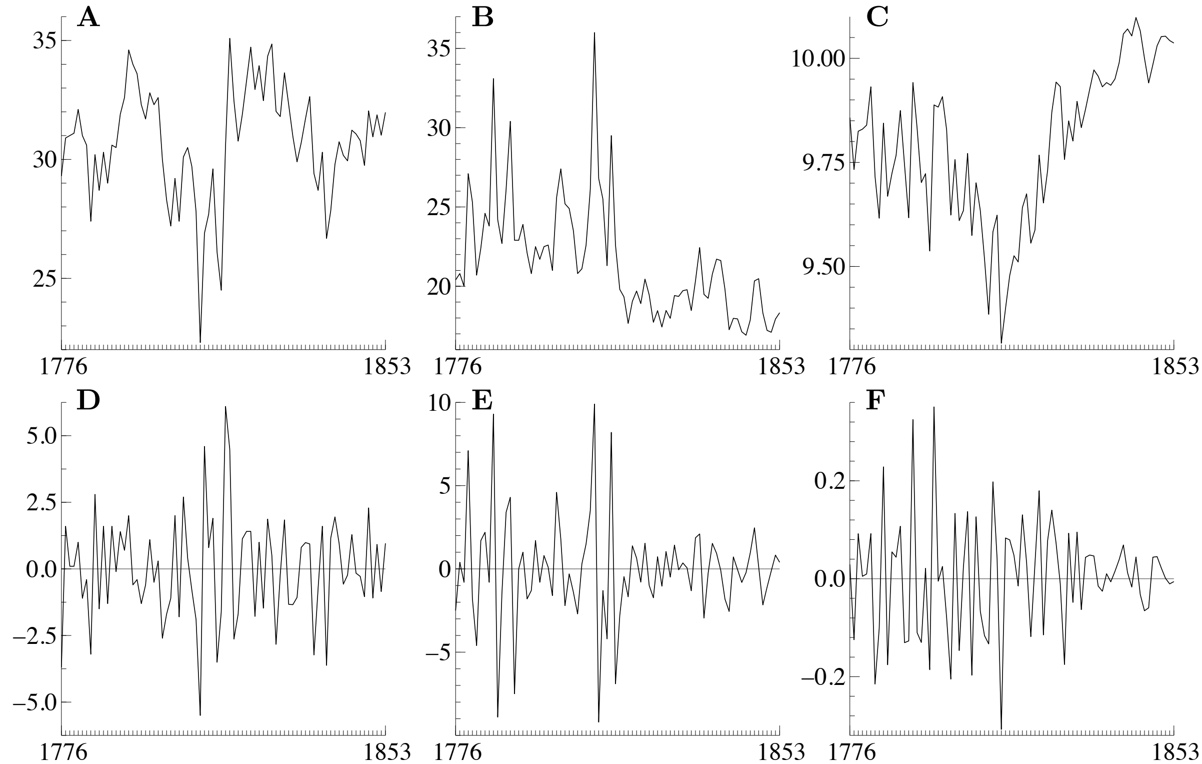

Figure 2: Plots of the Norwegian data in levels and first differences. A) Level of the crude birth rate. B) Level of the crude death rate. C) Level of the nautral logarithm of the real wage rate. D) First differences of the crude birth rate. E) First differences of the crude death rate. F) First differences of the nautral logarithm of the real wage rate.

5 Empirical Analysis

The analysis is performed in three steps. The first step consists of the formulation of statistically well-specified unrestricted co-integrated VAR models for relatively stable time periods that will function as the underlying general statistical models against which the post-Malthusian hypothesis will be tested. The second step consists of testing the joint restrictions imposed by the post-Malthusian hypothesis and at the same time estimate the parameters of interest. The third step consists of examining (*i*) parameter stability and robustness of test conclusions, (*ii*) timings of transitions out of the post-Malthusian regimes, and (*iii*) robustness of the results to specifications with no dummy variables and extended samples. Two types of recursive estimations are performed: forward-recursive estimation and rolling-windows estimation. Furthermore, to examine to what extent the obtained conclusions may depend on the use of dummy variables and the exclusion of a few extreme observations, the estimation is performed without dummy variables and where the extreme observations re-enter the estimation sample.

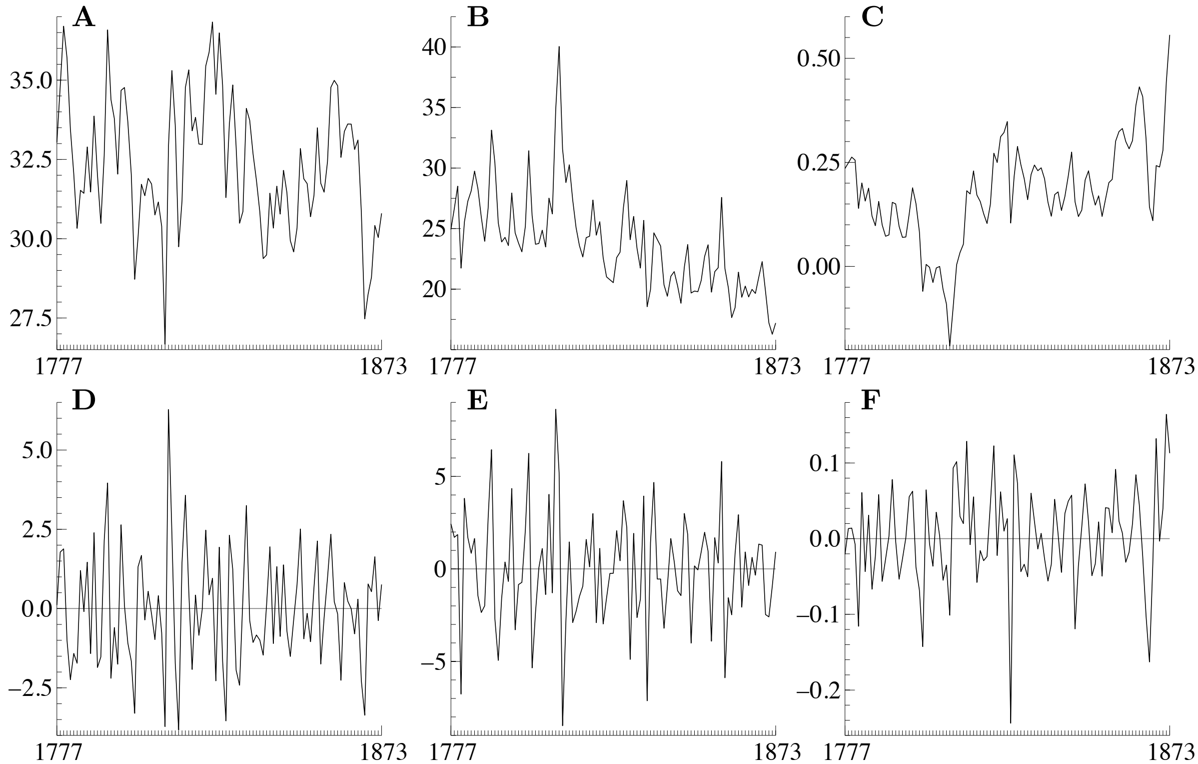

Figure 3: Plots of the Swedish data in levels and first differences. A) Level of the crude birth rate. B) Level of the crude death rate. C) Level of the nautral logarithm of the real wage rate. D) First differences of the crude birth rate. E) First differences of the crude death rate. F) First differences of the nautral logarithm of the real wage rate.

5.1 Unrestricted Co-integrated VAR Models

The first step of the analysis consists of formulating statistically well-specified unrestricted co-integrated VAR models the three countries and for for relatively stable time periods. For each country, the respective unrestricted co-integrated VAR model functions as the general statistical model in which the post-Malthusian hypothesis is tested as a set of parameter restrictions.

Two factors determine the appropriate sample periods for analyzing the post-Malthusian hypothesis. First, in accordance with the fact that the Scandinavian countries have experienced demographic transitions, unified growth theory predicts that the post-Malthusian dynamics are followed by a reversal (with respect to the sign) in the association between income and fertility. Therefore, to prevent an erroneous rejection of the post-Malthusian model based on an inappropriate inclusion of a part of the modern growth period, the end year of the analyzed samples are in the initial data inspection determined to be one decade before the fertility transition (as given in Reher, 2004).

Second, in order to maintain valid statistical inference when testing the post-Malthusian hypothesis, it is crucial to first formulate a well-specified unrestricted VAR model. That is, a statistical model for which it seems reasonable, based on the residual analysis, to assume that the errors do not exhibit auto-correlation, non-normality or auto-regressive conditional heteroscedasticity (ARCH).

However, it should be noted that in practice, non-normality due to excess kurtosis of the errors is less problematic than non-normality due to excess skewness (Juselius, 2006). Moreover, moderate ARCH effects have also been shown to have little impact on the determination of the co-integrating rank, r (Rahbek et al., 2002). Thus, the present analysis is primarily concerned with the assumption of no auto-correlation in the errors since this is clearly the most important assumption.[15] Below, residual-based misspecification tests are used to assess these important aspects of the statistical model specification.

In practice, obtaining a well-specified model requires taking account of events that the model is not intended to explain and that may obscure and bias the estimation of the post-Malthusian dynamics, for example by excluding extraordinary time periods. In the present cases, although the full sample periods start in year 1749 for Norway and Sweden, and year 1818 for Denmark, initial sub-periods are omitted due to two historical circumstances that prevent the specification of statistically satisfactory unrestricted VAR models. In particular, in the case of Denmark, the initial three-year period 1818–1820 is omitted due to the unusually high wage inflation which followed the bankruptcy of the Danish state in 1813. Furthermore, in the case of Norway and Sweden, the initial period 1749–1775 is omitted due to the severe harvest failures and the associated disease and famine in 1772 and 1773 (Drake, 1969, p. 68).[16] In addition, based on the initial data inspection, the end dates are chosen to permit the formulation of statistically well-specified models for the longest possible periods. In particular, in the case of Denmark and Norway, the end years of the estimation samples are decreased, and in the case of Sweden, the end year is increased, compared to the end years chosen in the initial data inspection.[17]

Following standard procedure in applied time series econometrics (see e.g. Juselius, 2006), the analysis accounts for extraordinary events by including indicator variables or dummy variables, in the form of impulse dummies and level shifts.[18] These dummy variables are initially identified from large residuals. This approach will work satisfactory if these residuals result from important *exogenous* events that are thus unrelated to the theory model. By confronting non-sample information, i.e. historical references, it is often straightforward to identify the exogenous event. Interestingly, the periods under investigation prompt the use of very few dummy variables in order to obtain a statistically satisfactory specification.[19] Furthermore, in almost all cases the cause corresponding can be unambiguously attributed to exogenous events. For example, in the case of Denmark, the model includes a dummy variable accounting for the effects of the Second Schleswig War in 1864, in which Denmark lost a large proportion of its land and population. As further examples, in the Norwegian case, a dummy variable proxies the effects of the blockade of Norway by England in 1813

[15] Recall that, in dynamic statistical models, auto-correlated errors lead to inconsistent estimators.

[16] Separate models for the period 1749 to 1772 in Norway and Sweden were estimated, yielding results consistent with those from the later periods. However, in light of this short sample period and the associated low statistical power, these results are omitted.

[17] The subsequent recursive analyses establish that the qualitative conclusions are robust to the choice of shorter sample periods (see Figure 4–6 and Figure A.1–A.3 in the appendix).

[18] An impulse dummy has the form (0,...,0,0,1,0,0,....,0) while a (permanent) level shift has the form (0,...,0,0,1,1,1,....,1). When the latter is present in the model we use the most general formulation which allows for the possibility that the level shifts may not cancel in the co-integrating relations (see e.g. Juselius, 2006). The latter is often the case, for example when it is only one of the variables that shifts.

[19] As established in Table A.5 in the appendix, the results are robust to the use of models with no dummies.

– during the Napoleonic wars – and the consequent famine, and another dummy variable accounts for the shift in fertility in the years 1830–1854, which likely could be due to a marked change in the age composition over this time period as described by Drake (1969), p. 45. Finally, examples from the Swedish case includes dummy variables accounting for the effects of the Russo-Swedish War of 1808–1809, and the smallpox outbreak of 1825.

The unrestricted VAR models for the three Scandinavian countries are reasonably well-specified according to the results of the residual-based tests for no auto-correlation, normality and homoscedasticity, presented in Table A.1–A.3 in the appendix.[20] Importantly, as it appears from Table A.1 in the appendix, auto-correlation in the errors is convincingly rejected. In particular, the vector test for no auto-correlation is not rejected at the 5% significance level in 11 out of the 12 performed tests. Furthermore, the tests of homoscedasticity are not rejected in the majority of cases for Norway and Sweden, while it is rejected in the Danish case (Table A.1 in the appendix). The univariate tests of normality in Table A.2 in the appendix are accepted in most cases and the few exceptions are primarily a result of excess kurtosis in the residuals. Since the vector test for normality is a function of the univariate tests, it thus appears that in general the rejection of normality is due to excess kurtosis, and hence less of a problem (Table A.3 in the appendix). Overall, in light of the long time periods, the misspecification analysis indicates that the unrestricted VAR models are sufficiently well-specified.

As established in Section 3, an implication of the post-Malthusian hypothesis is that the Π-matrix of the VAR model has a co-integration rank of $r = 2$, meaning that the system of variables is non-stationary. Table 2 presents the p-values for the Johansen trace tests of non-stationarity ($r < 3$) against stationarity ($r = 3$) both with and without dummy variables.[21] The table establishes that $r = 2$ can be accepted at the standard 5% significance level in the models without dummy variables for all the three countries. Furthermore, the table establishes that $r = 2$ can be accepted in the models including dummy variables at the 5% level for Sweden and at the 1% level for Denmark and Norway. It should be noted that the trace test is susceptible to a bias towards over-rejection due to measurement errors which are inevitable in historical data for the relevant time period.[22] Furthermore, as is sound practice, it is recommendable to consider other criteria in the process of determining the co-integration rank, such as the characteristic roots. (Johansen, 1996; Juselius, 2006). In particular, the assessment of the characteristic roots clearly suggested that $r = 2$ in the Danish case, whereas this criteria was less clear in the Norwegian case. Overall, this assessment suggests that it is a reasonable statistical approximation to impose the restriction $r = 2$.

Overall, the unrestricted VAR models for each of the three countries are well-specified and the data appear non-stationary with a co-integration rank of two. Therefore, the analysis can proceed to test the restrictions associated with the Post-Malthusian model on the VAR models.

[20]All estimation is based on PcGive, OxMetrics 6.10 and CATS in RATS (see respectively Doornik and Hendry, 2001; Doornik, 2009; Dennis et al., 2006).

[21]The test distribution has been simulated using CATS in RATS to take account of deterministic components (dummy variables and trend components).

[22]For more details, see Møller and Sharp (2014).

Table 2: Results of trace tests of co-integration for rank $r = 2$

	Denmark		Norway		Sweden	
Dummy variables	No	Yes	No	Yes	No	Yes
P-value	0.67	0.01	0.32	0.01	0.32	0.15

5.2 Estimation Results and Recursive Analyses

This section presents the estimation results of the restricted co-integrated VAR model, corresponding to the post-Malthusian hypothesis, for the three Scandinavian countries: Denmark, Norway and Sweden.

Furthermore, while the full-sample estimation exploits the maximal number of observations and therefore provides the maximal statistical precision, it does not reveal potential changes in the parameters which are assumed to be constant. Nor does it say anything about whether the test conclusion from the overall post-Malthusian restriction, or the significance of the estimates, change when based on sub-samples. The analysis therefore uses *forward-recursive* estimation and *rolling-windows* estimation to investigate these important issues. This may not only establish robustness of the (full-sample) conclusions and parameter constancy, but may also be used to detect the point in time, around which the post-Malthusian dynamics start to break down, and of particular interest, the point when the relation between births and real wages changes sign as predicted by unified growth theory.

In the forward-recursive estimation the post-Malthusian model is estimated based on recursive samples, starting from a fixed baseline sample of 20 years, and recursively adding one observation at a time. An estimate can then be plotted against the end points of the recursive samples, and the resulting graph should reflect changes in the underlying parameter. Likewise, when plotting the recursively estimated test statistic (or the associated p-value), corresponding to the post-Malthusian hypothesis, we can assess whether the test conclusion is robust across sub-samples, and when the hypothesis starts to be consistently rejected. However, since this procedure keeps observations from the earlier part of the period, it will tend to underestimate potential parameter changes towards the end of the sample. Furthermore, it also makes it difficult to assess the timing of the changes in the coefficients that are related to the transition out of the post-Malthusian Era. Therefore, as a supplement to the forward-recursive estimation, a rolling-windows analysis is conducted, where the model is estimated, still recursively, but instead based on a 40-year rolling sub-sample moving along the time axis.

Since both recursive estimations are based on shorter samples, the estimates and test values should be interpreted with some care. In particular, it should be noted that given the smaller number of observations in these estimations, the power of the significance tests are reduced, and statistical insignificance of estimates should therefore not be misinterpreted as evidence against non-zero effects in the sub-periods. However, the recursive methods, in particular the rolling-

Table 3: Estimation results for Denmark, Norway and Sweden

	Denmark	Norway	Sweden
Preventive check (a_1)	3.900*	9.097*	7.656*
	(1.133)	(2.184)	(2.982)
Positive check (a_3)	5.651*	1.831	9.567*
	(2.514)	(1.530)	(3.553)
First adjustment coefficient (ρ_1)	.527*	.665*	.528*
	(.125)	(.092)	(.097)
Second adjustment coefficient (ρ_2)	.429*	0.769*	.722*
	(.126)	(.067)	(.103)
Number of years	68	79	98
P-value of post-Malthusian test	0.86	0.30	0.06

Upper part: The estimates of the coefficients in the long-run matrix, β, from equation (9), and the adjustment matrix, α, from equation (10), corresponding to the post-Malthusian hypothesis. Lower part: Number of observations and the p-value of the overall test of the post-Malthusian hypothesis, i.e. the joint test of the imposed restrictions on α and β. Standard errors are reported in parentheses. Significance at the 5% significance level is marked by an asterisk.

windows estimation, may still be used to gauge the overall trends and tendencies in the estimated coefficients (and their significance) as well as in the test of the post-Malthusian hypothesis.

5.2.1 Denmark

Estimation Results The analysis for Denmark is performed using three lags on the effective sample 1824–1890. The analysis includes linear time trends that were found to be insignificant in the co-integrating relations, whereby they were excluded from the latter. The results of the estimation under the restrictions given by (9) and (10) are shown in Table 3, column 1.[23]

As it appears from the table the joint test of the post-Malthusian model is accepted with a p-value as high as 0.86 meaning that the restricted statistical model describes the data as well as the highly flexible unrestricted co-integrated VAR model for $r = 2$. Furthermore, the estimates of the coefficients associated with the preventive and the positive check are both significant and have the expected signs. Finally, the estimates of the adjustment coefficients, ρ_1 and ρ_2, are both positive and significant, consistent with equilibration to deviations from the long-run equilibrium. Overall, the analysis provides evidence in favor of post-Malthusian dynamics in Denmark operating in the time period, 1824–1890

Recursive Analyses Figure 4 depicts the results of the forward-recursive estimation. The graphs clearly suggest that the parameters are remarkably stable over time, recalling that some instability (of the estimates) in the beginning of the graphs is inevitable when the baseline sample is relatively short. The lower panel shows the test statistic, corresponding to the post-Malthusian restriction, together with the 95% quantile marked by the straight line and thus shows that the model is clearly

[23] Table A.4 in the appendix presents the estimates of the preventive and positive checks as elasticities and compare them to existing estimates from other studies.

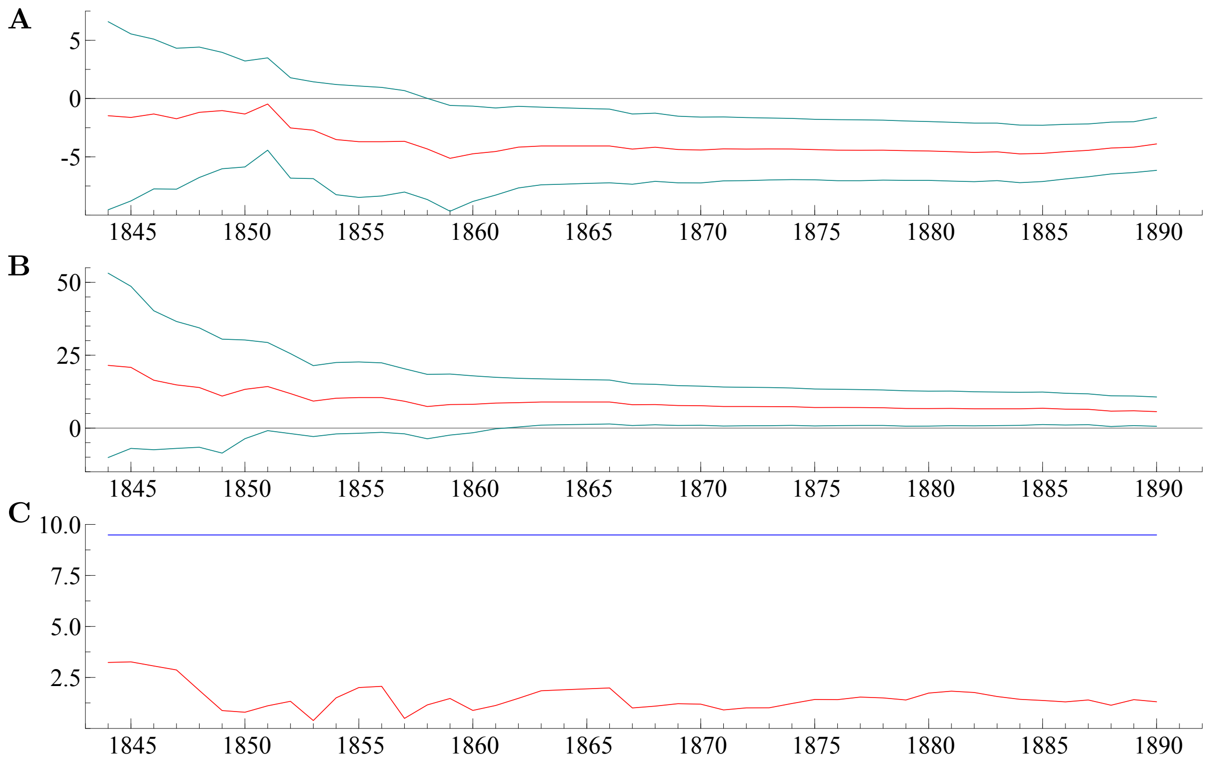

Figure 4: Results of forward-recursive estimations for the Danish data. The figure depicts the estimation results against the end point of the recursive samples. A) The point estimate of the preventive check coefficient, stated as $-a_1$, together with the 95% confidence limits. B) The point estimate of the positive check coefficient, a_3, together with the 95% confidence limits. C) The test statistic corresponding to the post-Malthusian hypothesis, where values above the blue line indicate a rejection of the hypothesis at the 5% significance level.

accepted at the 5% level for all sub-samples. In fact, the restrictions are accepted for sub-samples up until the First World War. For a large part of the consecutive sub-samples the estimate of the preventive check parameter is significant, and, going beyond the graph, it turns out that it becomes insignificant after the turn of the 19th century. Under the inclusion of data after this point in time, the estimate flips sign, consistent with the notion of a negative association between income and fertility due to the existence and increasing influence of a child quantity-quality trade-off in the process of industrialization (Galor, 2011).

Figure A.1 in the appendix depicts the results of rolling-windows estimations of the Danish data. To a satisfactory extent the figure confirms an overall degree of stability of the preventive check parameter and the estimate is significant for all but the last sub-sample point (panel A). Consistent with a transition out of a post-Malthusian era, the estimations reveal a falling tendency of the estimates based on 40-year sub-samples with starting years after 1846, eventually leading to an insignificant estimate in the estimation based on the 40-year sub-sample starting in 1851. The figure also establishes overall positive estimates of the positive check, though some insignificance is present (panel B). However, consistent with a transition out of a post-Malthusian era, the figure establishes a falling tendency of the estimates of the positive check, over the various 40-year sub-samples. Finally, the figure reassuringly confirms that the post-Malthusian hypothesis is not rejected for the vast majority of the 40-years sub-samples.

5.2.2 Norway

Estimation Results For Norway the analysis is based on a VAR with one lag and the effective sample 1776–1853. The results of the estimation under the post-Malthusian restrictions, given by equations (9) and (10), are shown in Table 3, column 2. The model includes linear trends in the variable and in the co-integrating relations. The joint test of the restrictions corresponding to the post-Malthusian model in Norway does not reject the theory ($p = 0.30$). As in the case of Denmark, the estimation provides clear evidence of the existence of the preventive check. However, although the coefficient capturing the positive check has the expected sign it is statistically insignificant. Finally, the estimates of the adjustment coefficients, ρ_1 and ρ_2, are both positive and significant. Thus, as for the Danish data the analysis provides evidence in favor of post-Malthusian dynamics in Norway over the time period, 1776–1853.

Recursive Analyses Figure 5, which depicts the forward-recursive estimation, is clearly consistent with constancy of the parameters of interest. Furthermore, the full sample conclusions of a significant preventive check estimate and an insignificant positive check (with the expected sign), are clearly confirmed. Moreover, the post-Malthusian model can be accepted throughout most of the period. In fact, going beyond the end point in the figure, the p-value is above 5% for all sub-samples stopping as late as 1920. However, for samples including the few years leading up to that point in time, the post-Malthusian model gives a poor fit to the data, and the estimate of the preventive check parameter becomes insignificant for periods stopping after 1910. Under the inclusion of data after this point in time, the estimate flips sign, consistent with the notion of an increasingly influential child quantity-quality trade-off.

Figure A.2 in the appendix depicts the results of rolling-windows estimations of the Norwegian data. Again, parameter stability is clearly supported and the conclusion of an insignificant positive check (Panel B) and the overall acceptance of the post-Malthusian hypothesis (Panel C) are clearly supported. For most of the sample the preventive check estimate is significant, but there is slight falling tendency (consistent with a transition out of a post-Malthusian era) resulting in insignificance around 1800 (panel A). Panel B also shows a falling (albeit insignifcant) positive check also consistent with a transition out of a post-Malthusian era.

5.2.3 Sweden

Estimation Results The analysis for Sweden is based on the effective sample, 1777–1873, and implies a VAR with two lags. The model includes linear trends in the variable and in the co-integrating relations. The results of the estimation under the restrictions given by equations (9) and (10) are shown in Table 3, column 2.

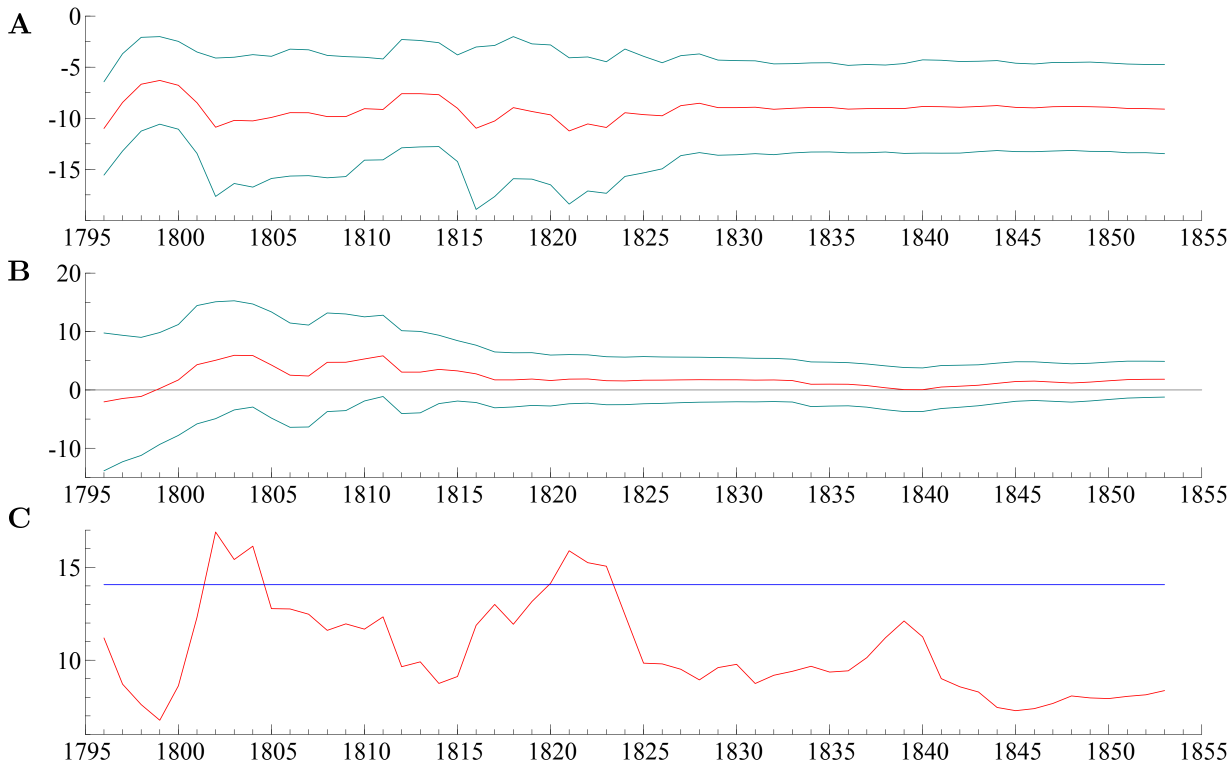

Figure 5: Results of forward-recursive estimations for the Norwegian data. The figure depicts the estimation results against the end point of the recursive samples. A) The point estimate of the preventive check coefficient, stated as $-a_1$, together with the 95% confidence limits. B) The point estimate of the positive check coefficient, a_3, together with the 95% confidence limits. C) The test statistic corresponding to the post-Malthusian hypothesis, where values above the blue line indicate a rejection of the hypothesis at the 5% significance level.

The joint test of the restrictions corresponding to the post-Malthusian model in Sweden do not reject the theory ($p = 0.06$). Note that, consistent with Figure 6, the p-value is much higher for earlier samples, but extending the data until 1873 gives the largest sample for which it is above 5 percent. Thus, the analysis provides evidence in favor of post-Malthusian dynamics in Sweden over the time period 1776–1853. Again, the estimates of the adjustment coefficients, ρ_1 and ρ_2, are both positive and significant. As in the case of Denmark, the estimates of the coefficients associated with the preventive and the positive check are both highly significant with the expected sign.

Recursive Analyses The forward-recursive estimation in Figure 6 clearly suggest that parameters are stable and that the all test conslusions are remarkably robust. In particular, the post-Malthusian model can be accepted throughout the period and both estimated check coefficients are significant for the majority of the sample. Extending the sample beyond 1890, the estimate flips sign, consistent with the notion of an increasingly influential child quantity-quality trade-off.

The rolling-windows estimation depicted in Figure A.3 in the appendix supports constant parameters for most of the sample consistent with the findings based on the forward-recursive es-

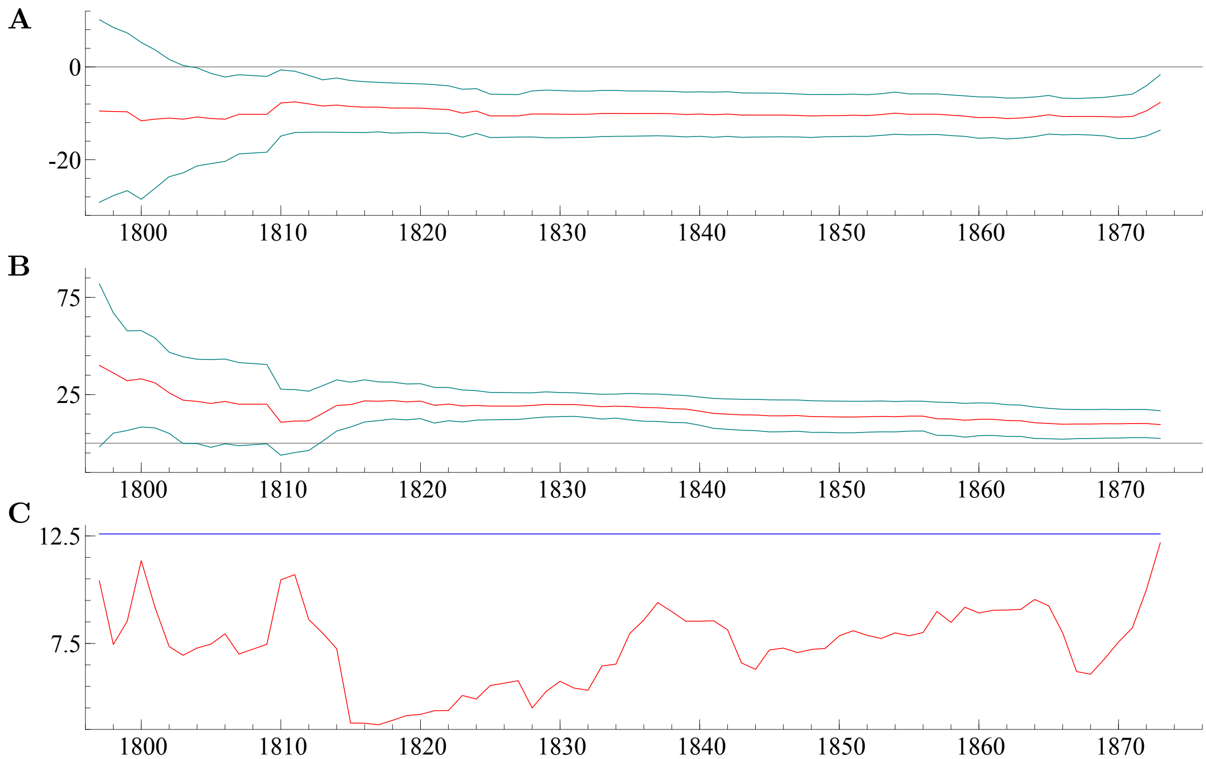

Figure 6: Results of forward-recursive estimations for the Swedish data. The figure depicts the estimation results against the end point of the recursive samples. A) The point estimate of the preventive check coefficient, stated as $-a_1$, together with the 95% confidence limits. B) The point estimate of the positive check coefficient, a_3, together with the 95% confidence limits. C) The test statistic corresponding to the post-Malthusian hypothesis, where values above the blue line indicate a rejection of the hypothesis at the 5% significance level.

timation. However, there is a falling tendency in the final part of the sample, consistent with a transition out of a post-Malthusian era (panel A). Furthermore, panel B supports a rather constant positive check for the majority of the period, but also in this case, the effect seems to diminish in size towards the end of the period. Note also that the significance of the checks, as found for the full sample holds for most of the sample, although the falling tendency eventually implies insignificance. Finally, panel C of Figure A.3 in the appendix reassuringly establishes that the test of the post-Malthusian hypothesis is not rejected for the majority of the 40-years sub-samples.

5.3 Robustness to Exclusion of Dummy Variables and Inclusion of Longer Samples

Although we argue in favor of including dummy variables (and excluding a few initial extreme observations) to account for extraordinary circumstances, in order to make sure that our conclusions are not critically dependent on this we also estimate the model with no dummy variables and where the initially excluded extraordinary periods are included. Table A.5 in the appendix establishes that the qualitative conclusions of the analysis of the Danish, Norwegian and Swedish data are robust to exclusion of dummy variables.

5.4 General Findings of the Empirical Analysis

Overall, the results for all three Scandinavian countries support the post-Malthusian model in which technological progress raises income and spurs population growth while offsetting diminishing returns to labour.

Furthermore, the recursive analyses, i.e., the forward-recursive estimations and the rolling-window estimations, establish that the estimation results are rather robust to the use of alternative sample periods. In addition, the recursive analyses establish that the preventive check tends to be more stable and more significant than the positive check and that both checks diminish in magnitude towards the end of the post-Malthusian era. Finally, as a further robustness check we have also shown that our conclusions are not sensitive to the omission of dummy variables and the inclusion of a few extreme initial observations.

6 Conclusion

Theories of economic growth hypothesize that the transition from stagnation to sustained growth is associated with a post-Malthusian phase in which income increases and spurs population growth while scale effects offset diminishing returns to labour. Evidence suggests that England was characterized by post-Malthusian dynamics as early as two centuries preceding the Industrial Revolution. However, given England's special position as the forerunner of the Industrial Revolution, it is unclear if the existence of a post-Malthusian period is a more general phenomenon.

Analyzing data from Denmark, Norway and Sweden, this research provides evidence that the Scandinavian countries were characterized by post-Malthusian dynamics in the period leading up to their industrialization. Thus, population growth was still positively affected by income per capita but the latter was not stagnant, possibly due to positive scale effects from population on the technological level that counterbalanced the Malthusian negative feedback effects arising from diminishing returns to labour. Furthermore, the breakdown of the post-Malthusian dynamics was associated with a reversal with respect to the sign of the association between income per capita and fertility, consistent with the notion of an increasingly influential child quantity-quality trade-off.

These results are strikingly similar to the previous findings of post-Malthusian dynamics in England, to the extent that the elasticities of the preventive and positive checks are of similar magnitudes, that the estimates are remarkably stable over a relatively long period, and that the post-Malthusian dynamics seem to have come into play long before the Industrial Revolution (Møller and Sharp, 2014). Furthermore, the assumption of parameter stability is generally supported, and our conclusions are robust with respect to the choice of subsample, which is assessed with both forward-recursive estimation and rolling 40-year sample-windows. Moreover the conclusions are not depending on the use of dummy variables and the exclusion of a few extreme initial observations. The recursive estimations are also used to detect the period of transition out of the post-Malthusian period and into the modern growth regime, which thus seems to occur some decades before the ends of the samples. Hence, given this high degree of similarity and the fact that the Scandinavian data are of superior quality, increasing confidence can be put in the generality of the results obtained in Møller and Sharp (2014) as a relevant description of the economic and demographic dynamics during the transition from stagnation to growth. Furthermore, the analysis establishes, that in spite of Denmark, Norway and Sweden being less developed than England in the pre-industrial era, these economies were indeed more post-Malthusian than Malthusian before their economic take-offs.

While the present model successfully describes the data at hand, future research may include additional relevant aspects of economic-demographic dynamics. In particular, while the present model abstracts from international migration, since migration over the modeled time period was relatively limited, the model could be extended to describe immigration and emigration analogously to the birth and death dynamics, potentially enabling the model to be applied to more recent time periods characterized by more prominent international population flows. Other paths are also open to future research. For example, one could augment the analysis with a two-sector model which could take account of issues such as sectoral shifts, e.g. a gradual migration from agricultural to urban industry. Such a migration could have been relevant for the period under study and may help explain the lack of an estimated effect of population size on the wage rate. In particular, one hypothesis to investigate could be whether the independence of the real wage rate on population, as found here, could partly be attributed to the mechanism that increased population pressure may have induced migration into cities where output conceivably was less subjected to diminishing returns to labor (consistent with the findings in Klemp, 2012). We hope that the present paper will inspire future research on the mechanics of the post-Malthusian era.

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A Mis-specification Tests of the Unrestricted VAR Models

The mis-specification tests presented below are based on the unrestricted VAR model with dummy variables and are estimated using CATS in RATS, version 2.

Table A.1: Lag-specific tests for auto-correlation and heteroscedasticity (ARCH)

	P-value of test of absence of	
	Auto-correlation	Heterosce-dasticity
	Denmark	
Lag 1	0.99	0.00
Lag 2	0.35	0.00
Lag 3	0.37	0.00
Lag 4	0.25	0.00
	Norway	
Lag 1	0.82	0.57
Lag 2	0.31	0.07
Lag 3	0.59	0.11
Lag 4	0.04	0.00
	Sweden	
Lag 1	0.05	0.98
Lag 2	0.46	1.00
Lag 3	0.33	0.96
Lag 4	0.24	0.86

Table A.2: Univariate statistics

	Mean	S.D.	Skewness	Kurtosis	Max.	Min.	R^2	P-value of test of absence of ARCH	P-value of test of absence of Normality
				Denmark					
Δw	0.00	0.04	0.12	2.61	0.11	-0.08	0.48	0.69	0.87
Δb	0.00	0.86	0.53	4.00	2.57	-1.93	0.46	0.01	0.08
Δd	-0.00	1.52	1.22	5.37	5.31	-2.75	0.46	0.05	0.00
				Norway					
Δw	0.00	0.09	-0.16	4.53	0.28	-0.33	0.34	0.91	0.01
Δb	-0.00	1.40	-0.61	3.61	3.50	-4.45	0.49	0.42	0.08
Δd	0.00	1.51	0.61	3.45	4.70	-2.76	0.78	0.12	0.08
				Sweden					
Δw	0.00	0.05	-0.10	3.37	0.15	-0.14	0.39	0.90	0.39
Δb	0.00	1.25	-0.20	3.29	2.76	-3.90	0.52	0.58	0.43
Δd	-0.00	2.06	0.59	4.53	6.51	-5.77	0.52	0.56	0.01

Table A.3: Overall tests for normality

	P-value of overall test of normality
Denmark	0.00
Norway	0.00
Sweden	0.01

B Comparison of Elasiticities

Table A.4: Estimated Absolute Elasticities from Various Sources

	Preventive check	Positive check
PRESENT STUDY		
Denmark 1821–1890	0.13	0.28
Norway 1775–1853	0.30	0.08
Sweden 1775–1873	0.25	0.47
OTHER STUDIES		
England[a] 1560–1760	0.21–0.32	0.10–0.22
England[b] 1701–1759	0.31	Not analyzed
England[c] 1548–1834	0.14	0.10
England[d] 1540–1870	0.12	0.08
Europe[e] 1540–1870	0.14	0.16

Sources: a) Møller and Sharp (2014), b) Klemp (2012), c) Lee (1981), d) Lee and Anderson (2002), e) Galloway (1988).

The results for all three Scandinavian countries support the post-Malthusian model in which income is non-stagnating, increases fertility and lowers mortality. The table presents the estimates for the preventive and positive checks converted into long-run elasticities, taking account of the level of the data series (see Møller and Sharp (2014) for details).

The table establishes that the estimates produced by the present analysis are of the same order of magnitude, but typically large, compared to those found in other studies. The preventive check is somewhat weaker in Denmark compared to the other nations. Furthermore, the present analysis finds evidence that the preventive check became weaker as Denmark moved into the nineteenth century.

C Estimation with no Dummy Variables and Inclusion of Longer Samples

Table A.5: Estimation Results – Without Dummy Variables

	Denmark	Norway	Sweden
Preventive check (a_1)	1.486*	7.021*	6.216*
	(.714)	(2.393)	(2.798)
Positive check (a_3)	3.248*	8.314*	15.612*
	(1.357)	(3.007)	(4.502)
First adjustment coefficient (ρ_1)	.427*	.484*	.470*
	(.113)	(0.070)	(.076)
Second adjustment coefficient (ρ_2)	.436*	.752*	.652*
	(.137)	(.090)	(.101)
Number of years	71	103	124
P-value of post-Malthusian test	0.605	0.060	0.060

This table corresponds to the estimation behind Table 3 but with the difference being that the dummy variables have been excluded and the sample periods include the otherwise excluded initial sample periods (a few extreme observations). Upper part: The estimates of the coefficients in the long-run matrix, β, from equation (9), and the adjustment matrix, α, from equation (10), corresponding to the post-Malthusian hypothesis. Lower part: Number of observations and the p-value of the overall test of the post-Malthusian hypothesis, i.e. the joint test of the imposed restrictions on α and β. Standard errors are reported in parentheses. Significance at the 5% significance level is marked by an asterisk.

D Rolling-Window Estimations

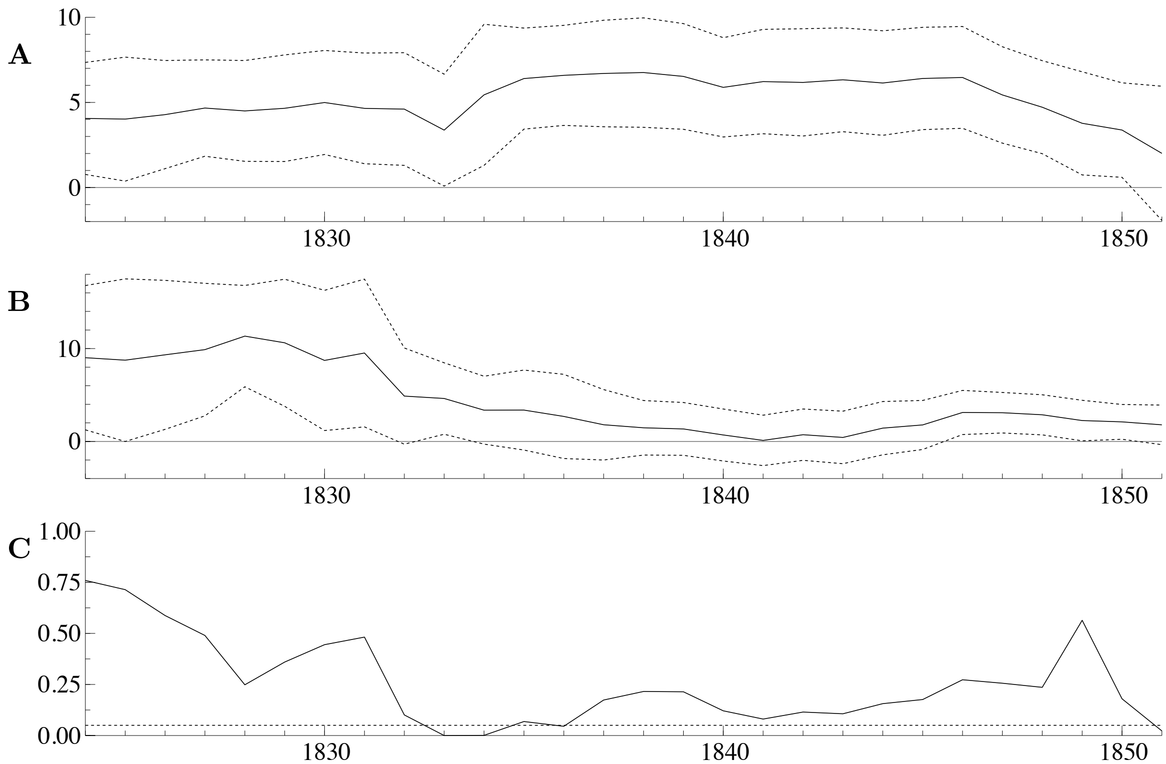

Figure A.1: Results of rolling-windows estimations of the Danish data: The x-axis marks the beginning year of the 40-year sub-samples. Panel A depicts the point estimate and limits of the 95% confidence interval of the preventive check coefficient, a_1. Panel B depicts the point estimate and limits of the 95% confidence interval of the positive check coefficient, a_3. Panel C depicts the p-value of the test of the Post-Malthusian hypothesis (solid line) with the 5 percent significance level (dotted line).

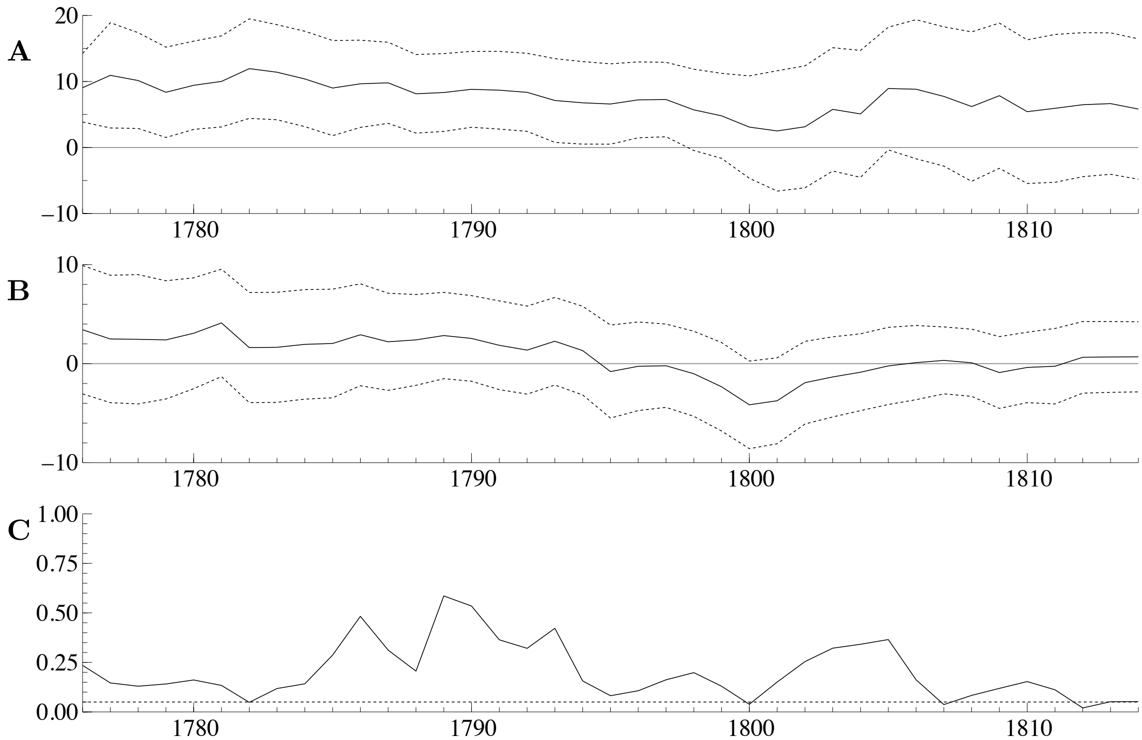

Figure A.2: Results of rolling-windows estimations of the Norwegian data: The x-axis marks the beginning year of the 40-year sub-samples. Panel A depicts the point estimate and limits of the 95% confidence interval of the preventive check coefficient, a_1. Panel B depicts the point estimate and limits of the 95% confidence interval of the positive check coefficient, a_3. Panel C depicts the p-value of the test of the Post-Malthusian hypothesis (solid line) with the 5 percent significance level (dotted line).

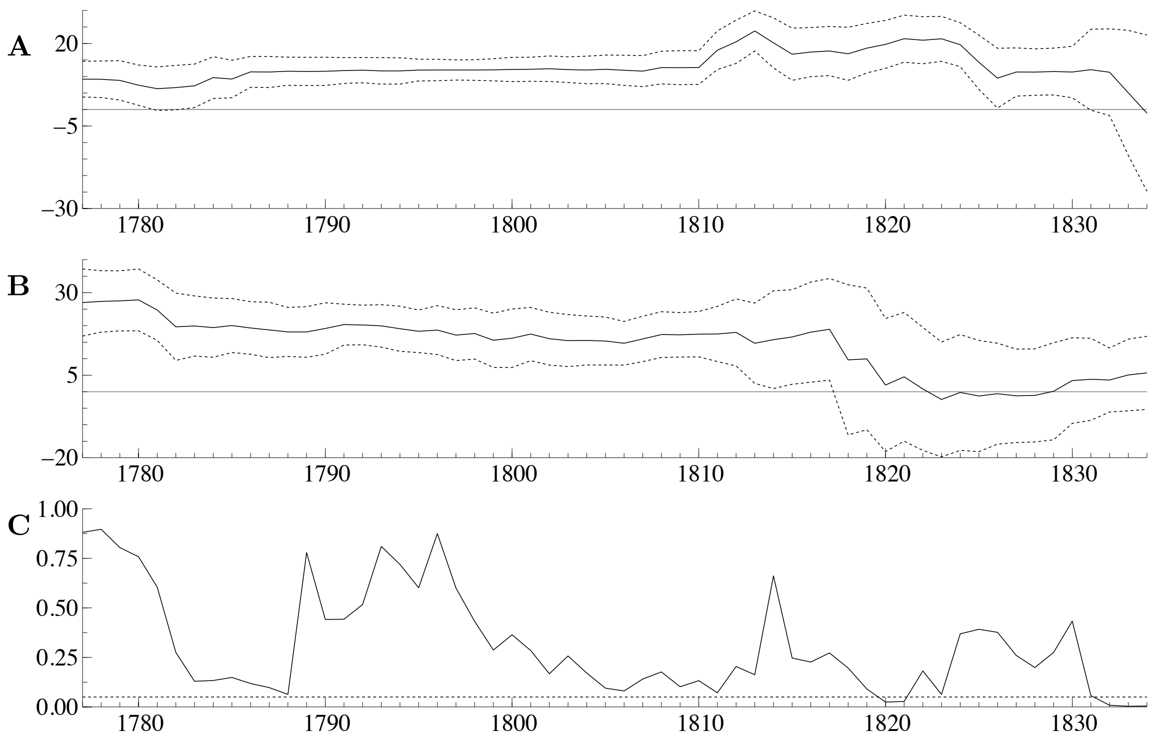

Figure A.3: Results of rolling-windows estimations of the Swedish data: The x-axis marks the beginning year of the 40-year sub-samples. Panel A depicts the point estimate and limits of the 95% confidence interval of the preventive check coefficient, a_1. Panel B depicts the point estimate and limits of the 95% confidence interval of the positive check coefficient, a_3. Panel C depicts the p-value of the test of the Post-Malthusian hypothesis (solid line) with the 5 percent significance level (dotted line).