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Real exchange rate persistence: the case of the Swiss franc-US dollar rate

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Abstract

Asset prices tend to undergo wide swings around long-run equilibrium values which can have detrimental effects on the real economy. To get a better understanding of how the financial sector and the real economy interact this paper models the long swings in the Swiss franc-US dollar foreign currency market using the $I(2)$ Cointegrated VAR model. The results show strong evidence of self-reinforcing feed-back mechanisms in the Swiss-US foreign exchange market consistent with the observed pronounced persistence in the Swiss-US parity conditions. Generally, the results provide support for models allowing expectations formation in financial markets to be based on imperfect information.

Keywords: Long swings, Imperfect Knowledge, I(2) analysis, Self-reinforcing feed-back.

JEL codes: C32, C51, F31

1 Introduction

It is a well-established fact that the ratio of domestic to foreign goods prices typically changes only slowly while the nominal exchange rate undergoes large, persistent swings. As a result, the real and nominal exchange rate exhibits similar large swings away and towards long-run benchmark values. For example, observed real exchange rate fluctuations of more than +/-30% are huge but far from unusual in countries with floating exchange rates.

Phelps (1994) forcefully argues that fluctuations in the real exchange rate (and the real interest rate abroad) affect the real economy through their effect on domestic real interest rates, output, wages and unemployment and may trigger off structural slumps in the economy.

The observed long swings in real exchange rates, i.e. the large and highly persistent deviations from long-run fundamental values, are hard to reconcile with standard monetary models based on Rational Expectations (RE) by representative agents endowed with essentially perfect knowledge. This has resulting in a burgeoning literature proposing alternative models in which this problem is addressed within a heterogenous agents model framework, which in this paper we call “imperfect knowledge” models since they depart from the basic assumption of the RE representative agent setting. For a detailed overview, see the handbook chapter by Hommes (2006).

We propose the Cointegrated VAR (CVAR) framework as a convenient tool for modeling empirical relations that arise in a context of heterogenous agents and possibly structural change. Juselius (2014a) demonstrates that the long persistent swings typical of asset prices (which are indicative of self-reinforcing feedback mechanisms) are consistent with equilibrium error increasing behavior (positive feedback) in the medium run but error correcting behavior (negative feedback) in the long run. Importantly, in a world populated by actors with rational expectations based on perfect information, the CVAR would still be an appropriate empirical framework, but would give results showing pure equilibrium error correction (Juselius, 2014a).

In all these cases, today’s asset price depends on future prices which, in varying degree, are forecast under imperfect knowledge and thus can deviate from the expected future prices under RE. Hommes (2005) and Hommes et al. (2005a, 2005b) develop models for a financial market populated by fundamentalists and chartists, where fundamentalists use long-term expectations based on economic fundamentals and chartists are trend-followers using short-term expectations. Positive feedback can arise at times when the latter dominate the market. Adam and Marcet (2011) propose a separation of standard RE rationality into an internal and external component and show that positive feedback can arise in a model where internal rationality is maintained but external rationality is relaxed due to imperfect market knowledge. Heemeijer et al. (2009) show by experiments that prices converge to the fundamental level under negative feedback but fail to do so under positive feedback. For similar results see Hommes (2013), Frydman and Goldberg (2013) and Anufriev et al. (2013)[1].

When heterogenous agents make forecasts under imperfect knowledge, causal relationships are, however, unlikely to remain constant in the aggregate and one would in general expect model parameters to be changing. Frydman and Goldberg (2007, 2011) develops a theoretical framework where agents' expectations are formed in the context of imperfect knowledge about causal relations being inherently subject to structural change. If the true model parameters change in a stochastically non-trending manner, Tabor (2013) demonstrates by simulation that constant parameter models fitted to such data exhibit a pronounced persistence.

Though structural breaks and regime shifts can be hard to predict, they are often detectable *ex post* using econometric tools. The CVAR model is tailor-made to identify and estimate persistent fluctuations in the data as well as structural breaks, features which are consistent with imperfect knowledge. For example, Johansen et al. (2010) and Juselius (2014a) estimate a near $I(2)$ CVAR model for German-US data in the post Bretton Woods/pre-EMU period and find that the persistency features of the data strongly support imperfect knowledge rather then rational expectations based theory models.

This paper aims to investigate whether similar results can be found when comparing the dominant world economy (the US) with a much smaller, but financially important country, such as Switzerland. Because Switzerland is not a member of the euro area, we do not need to deal with the structural break resulting from the introduction of the euro but can extend the sample to the present date. In addition, the information set is expanded by also including the short-term interest rates.

More specifically, the purpose is to gain a better understanding of the pulling and pushing forces that have generated the long and persistent swings in the data by exploring the dynamics of positive and negative feedback mechanisms in a CVAR model of prices, the nominal exchange rate, long- and short-term interests.

The paper is organized as follows: Section 2 discusses a theoretical framework for real exchange rate persistence. Section 3 documents the departure from basic parity conditions in the Swiss-US currency markets in the data. The $I(2)$ CVAR model framework is outlined in Section 4 and Section 5

[1] A special issue of the Journal of Economic Methodology (2013) on Reflexivity and Economics discusses various aspects of positive and negative feedback mechanisms, see Hands (2013).

presents its empirical specification for the Swiss-US data. Section 6 reports tests of various hypotheses about the long-run relations and Section 7 discusses the pulling and the pushing forces in the system. Finally, Section 8 concludes.

2 Real exchange rate persistence: theoretical considerations

There are several reasons for real exchange rates to be non-stationary, such as trends in productivity differentials between two countries (the Balassa-Samuelson effect), trends in relative shares of government consumption or population growth. While such factors can explain long-term trends in the real exchange rate, they have problems to account for the large and persistent swings away from PPP that are observable in the data (see e.g. Figure 1). We therefore argue that expectation formation in financial markets based on imperfect knowledge is consistent with these persistent movements away and towards long-run fundamental values we see in the data.

Standard models generally interpret the economy's behavior through the lens of a representative agent with rational expectations. Somewhat simplified, a model based on RE assumes that economic actors have a complete knowledge of economic structures, which are supposed not to change over time. Uncertainty about future outcomes can be reduced to quantifiable risk that rational agents can insure against. There are stochastic shocks to the model's causal variables, but these are assumed to follow a known probability distribution implying that the representative agent is aware of all possible future outcomes and their probabilities. When the model is assumed known, a rational agent will secure that prices return to equilibrium when exogenous shocks have pushed them away. This implies that the economic system is always moving towards a model-specific equilibrium, albeit possibly with some sluggishness. See Frydman and Goldberg (2007, 2011) for a more detailed discussion. Under these assumptions the deviation of the nominal exchange rate from its equilibrium PPP value, $e_{12t} - (p_{1,t} - p_{2,t})$, would be stationary (or at most near $I(1)$) and excess return on foreign exchange, i.e. the deviation from uncovered interest parity (UIP),

$$er_t = (i_{1,t} - i_{2,t}) - (e^e_{12,t+1} - e_{12,t}) \tag{1}$$

would be stationary. Empirical evidence suggests, however, that the excess return as defined in (1) behaves like a non-stationary process. To to shed light on this, a simple asset pricing model with imperfect knowledge expectations will be used as a motivating example. The idea is to first derive testable hypotheses on the long-run relationship between the nominal exchange rate and its long-run value measured by the relative price levels of the two economies. By assuming that agents base their forecasts of the future exchange rate on the expected value of its long-run value, $p_1 - p_2$, as well as on other information, v_t, we get:

$$e^e_{12,t+1} = B_{0,t} + B_t(p_1 - p_2)^e_{t+1} + v^e_t. \tag{2}$$

By inserting (2) into (1) we get an expression of excess return under imperfect knowledge:

$$er^{ik}_t = (i_{1,t} - i_{2,t}) - B_{0,t} - B_t(p_1 - p_2)^e_{t+1} + e_{12,t} - v^e_t \tag{3}$$

Adding and subtracting $B_t(p_1 - p_2)_t$ to (3) gives:

$$er^{ik} = (i_{1,t} - i_{2,t}) - B_{0,t} - B_t(\Delta p_1 - \Delta p_2)^e_{t+1} - B_t(p_1 - p_2)_t + e_{12,t} - v^e_t, \tag{4}$$

To be able to associate the above model with the data using the CVAR model we need to relate unobserved expected values with observed values as well as address the problem of time-varying parameters inherent in (4). First, we assume that the forecast error of the inflation differential $(\Delta p_1 - \Delta p_2)^e_{t+1} - (\Delta p_1 - \Delta p_2)_{t+1}$ is $I(0)$, which is plausible because individuals should adapt their forecasting rules if they observe pronounced deviations of expected from actual values. Second, we assume that the relative inflation rate between the two countries, $(\Delta p_1 - \Delta p_2)_t$, is at most $I(1)$, so that $(\Delta p_1 - \Delta p_2)_{t+1} - (\Delta p_1 - \Delta p_2)_t$ is $I(0)$. Under these assumptions the cointegration properties are robust to using actual values $(\Delta p_1 - \Delta p_2)_t$ instead of the (generally unknown) forecasted values $(\Delta p_1 - \Delta p_2)^e_{t+1}$. See Juselius (2014a) for a further discussion.

Taking structural change into account would imply that the coefficients in (4) could be time-varying. A constant-parameter CVAR model in this case would be misspecified. To study the properties of this type of model, Tabor (2014, Chapter 3) simulates a simple model, $y_t = \beta'_t x_t + \varepsilon_t$, where the coefficient β_t is stochastically non-trending ($\beta_t = \beta_0 + \rho\beta_{t-1} + \varepsilon_{\beta,t}$ with $\rho < 1$) and where the adjustment back to $\beta'_t x_t$ is immediate. He shows that the stochastically varying β coefficients produce a pronounced movement away from long-run mean values, $y_t - \beta' x_t$, where $\beta = E(\beta_t)$. Furthermore, by fitting a constant parameter CVAR model to the simulated data, thereby disregarding $(\beta_t - \beta)x_t$, he shows that the latter translates into persistence in the gap term $y_t - \beta' x_t$, and into a small, but significant, adjustment coefficient α. The closer ρ is to 1.0, the smaller is the adjustment coefficient and the more pronounced is the gap term. Slowly time-varying β coefficients due to forecasting under imperfect knowledge can, thus, generate the pronounced persistence that often characterizes constant parameter models of asset prices. Suppose, for example, that the change in the nominal exchange rate is related to relative inflation rates with a time-varying coefficient β_t

$$\Delta e_{12,t} = \beta_{0,t} + \beta_t \Delta(p_{1,t} - p_{2,t}) + u_{e_{12},t}$$

where $\beta_t = \rho\beta_{t-1} + \varepsilon_{\beta,t}$. If $E(\beta_t) = 1$, then $\Delta e_{12,t} - \Delta(p_{1,t} - p_{2,t})$ is likely to be $I(1)$ or near $I(1)$ persistent when ρ is close to 1.0, implying that $(e_{12,t} - p_{1,t} + p_{2,t})$ is one degree more persistent, i.e. $I(2)$ or near $I(2)$. This is also what we find empirically to be the case in Section 6.

Replacing β_t with $E(\beta_t) = 1$, (4) becomes:

$$er^{ik} = (i_{1,t} - i_{2,t}) - B_0 - (\Delta p_1 - \Delta p_2)^e_{t+1} - (p_1 - p_2 - e_{12})_t - v^e_t. \qquad (5)$$

Equation (5) essentially describes the standard expression for excess return (1) augmented by the PPP gap. The latter can be thought of as a proxy for an *uncertainty* premium measuring agents' *loss aversion* as suggested by Frydman and Goldberg (2007).[2] By introducing an uncertainty premium in the foreign exchange markets, they suggest that the standard UIP condition should be replaced by the Uncertainty Adjusted UIP (UAUIP) condition:

$$(i_{1,t} - i_{2,t}) = e^e_{12,t+1} - e_{12,t} + up_t, \qquad (6)$$

where up_t stands for an *uncertainty* premium. In (5) the PPP gap can be interpreted as a measure for up_t and $(\Delta p_1 - \Delta p_2)^e_{t+1} + v^e_t$ is a measure for $e^e_{12,t+1} - e_{12,t}$. Thus, UAUIP is consistent with an economy where all speculators require a minimum return – an uncertainty premium – to speculate in the foreign exchange market. A key result is that the expected change in the nominal exchange rate is not directly associated with the observed

[2] The assumption that agents are loss averse, rather than risk averse, builds on the prospect theory by Kahneman and Tversky (1979).

interest rate differential, but with the interest rate differential corrected for the uncertainty premium.

Under the above assumptions, we can formulate an empirically testable relation among the interest rate differential, the inflation rate differential and the PPP gap as follows:

$$\widehat{er}_t^{ik} = (i_{1,t} - i_{2,t}) - \omega_1(\Delta p_{1,t} - \Delta p_{2,t}) - \omega_2(p_{1,t} - p_{2,t} - e_{12,t}) \qquad (7)$$

where the PPP gap, $(p_{1,t} - p_{2,t} - e_{12,t})$, is likely to be highly persistent due to agents' forecasting behavior based on imperfect knowledge.

Thus, the assumption of imperfect knowledge leads to a number of hypotheses that can be formulated in terms of the integration properties of the data and be tested statistically. The most important hypothesis is that the real exchange rate is likely to exhibit persistent deviations from long-run benchmark values and hence, that the real interest rate differential between two countries exhibit similar persistent swings.

The next section will demonstrate graphically that such persistent swings characterize the Swiss-US real exchange rate and interest-rate differentials.

3 An graphical analysis of the basic parity conditions between Switzerland and USA

For the empirical analysis, we use quarterly data from 1976 to 2013.[3] Switzerland left the Bretton-Woods-System in January 1973 with an undervalued currency. In the following years, the Swiss franc appreciated strongly against the US dollar. In addition, the first oil-price shock in 1975 lead to considerable volatility in interest rates and inflation. For this reason, we choose 1976 as the start of our estimation period.

Figure 1 illustrates the tendency of the Swiss-US nominal exchange rate to undergo persistent swings around long-run benchmark values over the last 35 years. The upper panel shows the Swiss-US nominal exchange rate together with relative prices as a measure of its long-run benchmark (purchasing power parity) value. While the nominal exchange rate follows the same downward trending movement as relative prices over the very long run, in the medium run it fluctuates around its long-run benchmark value with long persistent swings sometimes lasting more than ten years. Consequently, the real and

[3] Details on the definition of the data and their sources can be found in the Appendix.

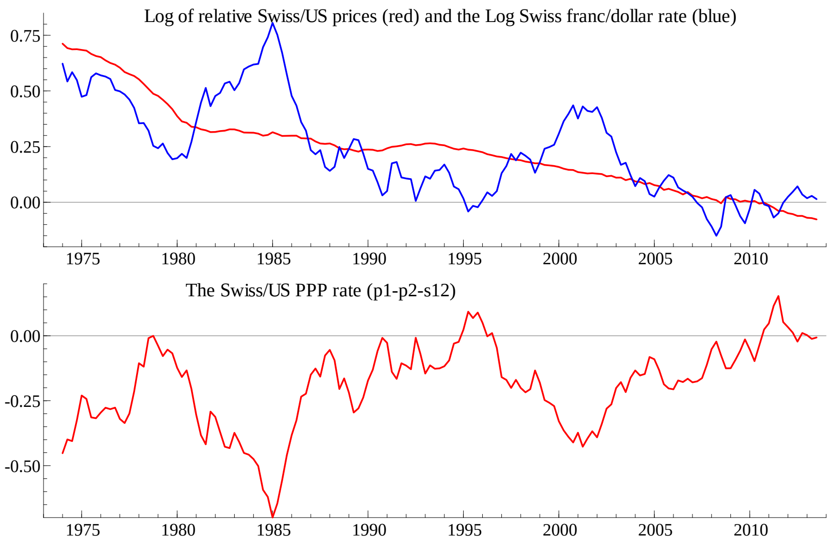

Figure 1: The log exchange rate and the log relative price between Switzerland and US (upper panel) and the deviation from PPP (lower panel)

the nominal rates are almost equally persistent, which seems untenable with standard theories (Rogoff, 1996). The upper panel of Figure 1 shows that it is the nominal exchange rate that has exhibited the long persistent swings in contrast to the relative price level that has moved much more smoothly over the sample period. Since the largest part of the foreign currency transactions in the Swiss franc-US dollar market are related to financial speculation and only a small part to the trade with goods, the large fluctuations in the real exchange rate are likely to be associated with financial behavior.

The prolonged movements away from long-run parity conditions, lasting 5-6 years or longer, are likely to trigger off compensating movements in other variables associated with the real exchange rate such as the interest rates. The upper panel of Figure 2 shows the departures from PPP, defined as $ppp_t = p_{1,t} - p_{2,t} - e_{12,t}$,[4] together with the short-term real interest rate differential ($m = 3$ months) and the lower panel with the long-term differential ($m = 10$ years). In both cases we use the actual inflation rate as a proxy

[4] $p_{1,t}$ is the log of the Swiss CPI, $p_{2,t}$ is the log of the US CPI, and $e_{12,t}$ is the log of the spot Swiss franc-US dollar exchange rate. A subscript 1 stands for the domestic country and a subscript 2 for the foreign country.

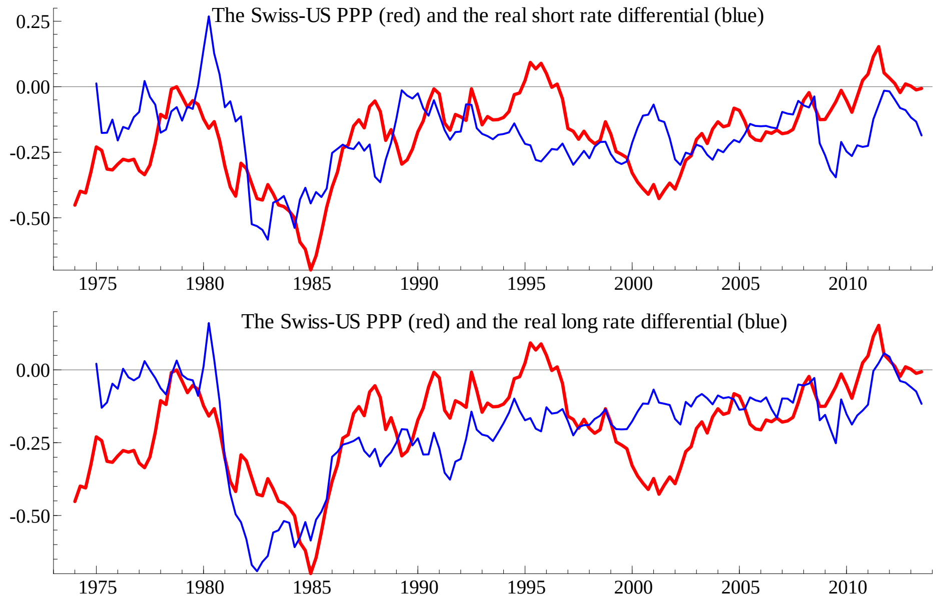

Figure 2: Deviation from PPP together with the real three-month interest rate differential (upper panel) and the real ten-year interest rate differential (lower panel). The graphs have been scaled to have the same range and mean.

for the expected one. The deviation from the PPP co-moves quite closely with both the short-term and the long-term interest rate differential though possibly less so in the latter case.

Figure 3 shows the nominal short and the long-term interest rate differential together with the inflation differential. It appears that the interest rate differentials have narrowed considerably since the nineties and that the inflation spread has become small and quite stable since 1983. The latter does not seem to be strongly co-moving with the former.

The short and the long-term interest rate differentials differ to some extent as they embody somewhat different information: The short rates react more strongly to changes in central bank policy decisions whereas the long rates react more strongly to changes in financial markets. Since both of them are likely to be important for movements in the nominal and real exchange rate the subsequent empirical analysis will include both.

Figure 4 plots the domestic long-short interest rate spread. Based on

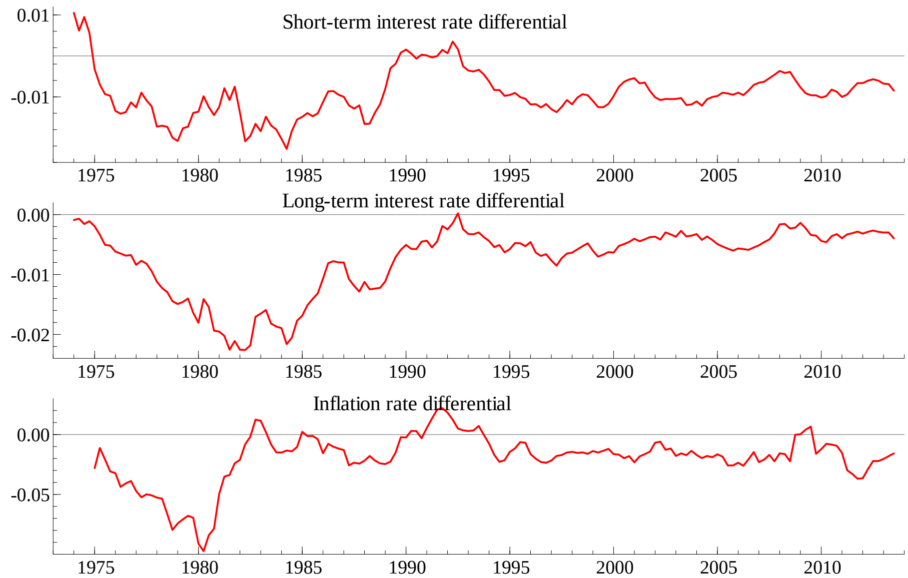

Figure 3: Three-month interest rate differential (upper panel), ten-year bond rate differential (middle panel), and the inflation rate differential (lower panel).

the expectations hypothesis of the term structure the long rate should be a weighted average of current and expected short-term interest rates. Short rates in this view 'drive' long rates and the term spread should be stationary (Campbell and Shiller, 1987), i.e:

$$b_t - s_t = e_t, \tag{8}$$

where b_t is a long interest rate of maturity l, s_t a short interest rate of maturity s and e_t denotes a error term which is stationary under the expectations hypothesis. Empirically, both the Swiss and the US spread look nonstationary, or at least very persistent, suggesting that the short and the long rate may not share one common stochastic trend over this sample period. The Fisher parity sheds light on this:

$$i_t^m = r_t^m + (\Delta_m p_{t+m}^e)/m, \tag{9}$$

where r_t^m denotes a real interest rate of maturity m. The real interest rate

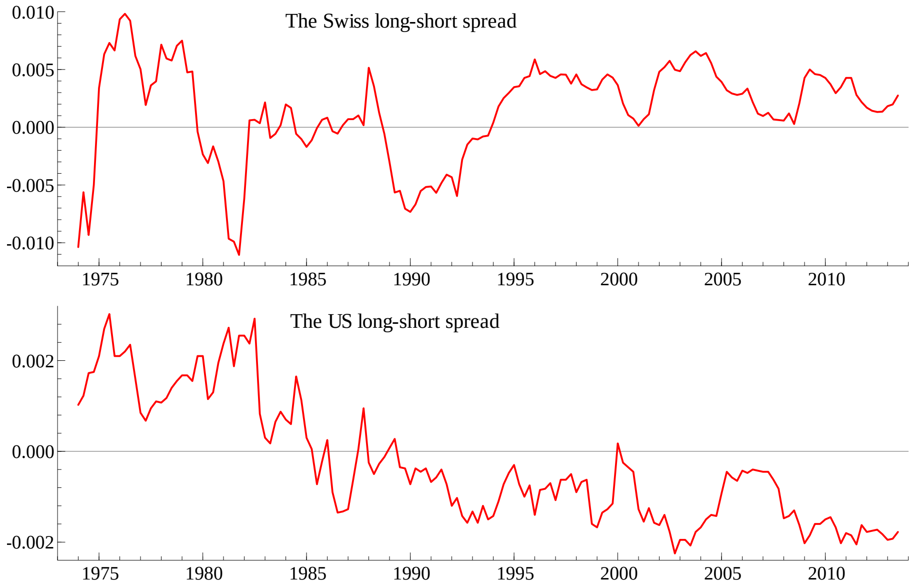

Figure 4: The spread between the ten-year bond rate and the three-month rate for Switzerland (upper panel) and the US (lower panel).

parity is defined as:

$$(i_{1,t}^m - \Delta_m p_{1,t+m}^e/m) = (i_{2,t} - \Delta_m p_{2,t+m}^e/m)$$

where i_t^m is a nominal interest rate of maturity m and $\Delta_m p_{t+m}^e/m$ is the average expected inflation rate over m periods. Using (9) the term spread can be formulated as:

$$i_t^l - i_t^s = r_t^l - r_t^s + (\Delta_{l-s} p_{t+l}^e)/(l-s)\},$$

showing that a nonstationary term spread is logically consistent with a nonstationary expected inflation rate.[5] The graph of actual inflation and interest rates in the Appendix show that they look quite nonstationary over the full sample period.

[5] Expectations are generally not observable but the cointegration results are unaffected when replacing expectations with actual values under the following two conditions: (i) the difference between $E_t(x_{t+l})$ and x_{t+l} is stationary (i.e. agents do not make systematic forecast errors forever), (ii) the differenced process $(x_{t+l} - x_t)$ is stationary.

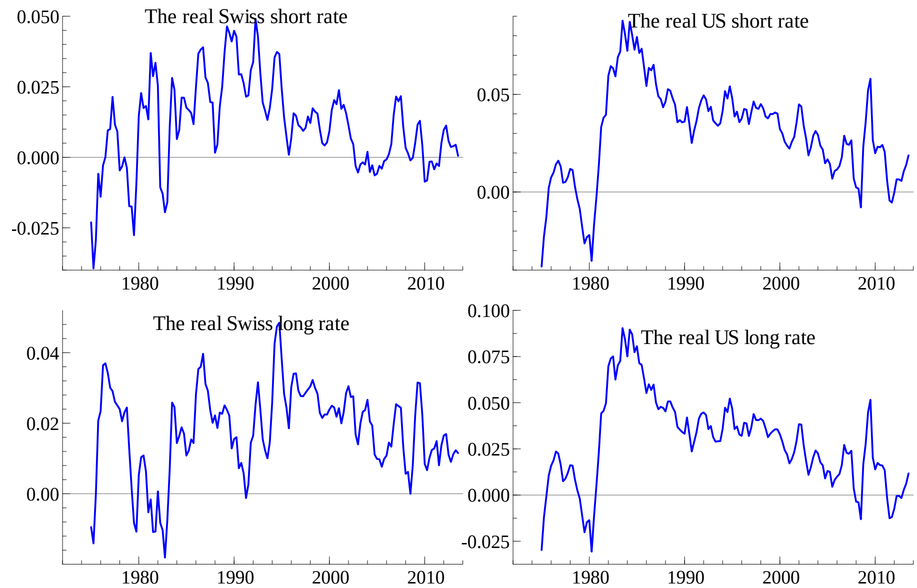

Figure 5: Real three-month interest rates (upper panels) and real ten-year bond rates (lower panels).

To study the time-series properties of the Fisher parities for Switzerland and the US at different maturities, we plot the short and the long-term real interest rate for both countries in Figure 5. The visual impression is that all four real interest rates drift off in a nonstationary manner; the US rates and the Swiss rates, respectively, behave similarly in this respect.

Of course, the empirical support for stationarity versus nonstationarity depends crucially on the sample period chosen: It is almost always possible to choose a sample period in which things have been very stable so that the variables can be considered stationary. But if we choose a short period to study a slowly adjusting variable like the real exchange rate, we would always find it to be a unit root process. This is because if it takes a long time to return to the steady-state value, say more than ten years, then one would always need a long sample to get a statistically significant adjustment effect. Whether a variable can be considered nonstationary or not for the present sample period will subsequently be determined based on statistical testing.

To summarize, Figures 1-5 illustrate a pronounced persistence in exchange rates, prices, and interest rates in both economies, suggesting that the simple parity conditions are best approximated as nonstationary or near unit root processes, which could be reconciled with the imperfect knowledge theory, but not with RE.

4 The CVAR approach

The CVAR represents a "general-to-specific" approach that does not impose a-priori restrictions without first testing them. One may say that the data are allowed to speak as freely as possible about the empirical relevance of (often competing) theoretical models. This is in contrast to the more conventional "specific-to-general" approach, where many untested restrictions from a theoretical model are imposed on the data from the outset. In the latter case it is difficult to know which results represent empirical facts and which reflect these a-priori assumptions. By its very nature, such a model analysis is less open to signals in the data suggesting that the theory is incorrect or needs to be modified (see Juselius and Franchi, 2007 for an illustration).

Here we have basically two competing hypotheses for how to explain the pronounced persistence in exchange rate data, first the RE theory claiming that the real exchange rate is a stationary process or at most a near $I(1)$ process, second, the imperfect knowledge theory claiming that the *change* of the real exchange rate is a highly persistent near $I(1)$ process, i.e. the *level* of the real exchange is a near $I(2)$ process. Because we can formulate hypotheses of $I(0)$, $I(1)$, and $I(2)$ within the unrestricted VAR without imposing any of them from the outset, it is natural to start from this model. Without loss of generality, it can be formulated in the Vector Equilibrium Correction form:

$$\Delta x_t = \Gamma_1 \Delta x_{t-1} + \Pi x_{t-2} + \mu_0 + \mu_1 t + \Phi D_t + \varepsilon_t, \tag{10}$$

where $x_t = [p_{1,t}, p_{2,t}, e_{12,t}, b_{1,t}, b_{2,t}, s_{1,t}, s_{2,t}]$ and p_t stands for CPI prices, $e_{12,t}$ for the nominal Swiss franc-US dollar rate, b_t for the ten-year bond rates, s_t for the three-month interest rates, a subscript 1 for Switzerland and a subscript 2 for the US. The model includes seasonal dummies, and D_t is vector of dummy variables defined as follows: $Dp80.02_t$ is 1 in 1980:2, 0 otherwise, $Dp82.04_t$ is 1 in 1982:4, 0 otherwise, $Dp0804_t$ is 1 in 2008:4, 0 otherwise, and three additional seasonal dummies that are activated at 2000:1 when there was a change in the seasonal pattern of the Swiss CPI. The lag length is two and all parameters are unrestricted in (10).

The hypothesis that x_t is $I(1)$ is formulated as a reduced rank hypothesis on Π:

$$\Pi = \alpha\beta' \tag{11}$$

where α, β are $p \times r$ matrices, with α representing the adjustment coefficients and β the long-run relationships among the variables.

For the $I(2)$ model it is natural to formulate the CVAR in acceleration rates, changes and levels (see Juselius 2006):

$$\Delta^2 x_t = \Gamma \Delta x_{t-1} + \alpha\beta' x_{t-2} + \mu_0 + \mu_1 t + \Phi D_t + \varepsilon_t, \tag{12}$$

where $\Gamma = (I - \Gamma_1)$. The hypothesis that x_t is $I(2)$ is formulated as an additional reduced rank hypothesis

$$\alpha'_\perp \Gamma \beta_\perp = \xi\eta', \text{ where } \xi, \eta \text{ are } (p-r) \times d_1 \tag{13}$$

and $\alpha_\perp, \beta_\perp$ are the orthogonal complements of α, β. The first reduced rank condition (11) is associated with the levels of the variables and the second (13) with the differenced variables. The intuition is that the differenced process also contains unit roots when data are $I(2)$.

Because the second rank condition is formulated as a reduced rank on the transformed Γ matrix, its coefficients in (12) are no longer unrestricted as in the $I(1)$ model. This is is the reason why the maximum likelihood estimation procedure needs a different parameterization. Paruolo and Rahbek (1999) suggested the following parameterization:

$$\Delta^2 x_t = \alpha(\beta' x_{t-1} + \delta' \Delta x_{t-1}) + \zeta\tau' \Delta x_{t-1} + \mu_0 + \varepsilon_t \tag{14}$$

where $\tau = [\beta, \beta_{\perp 1}]$, δ is a $p \times m_2$ matrix of polynomially cointegrating parameters, such that $(\beta' x_{t-1} + \delta' \Delta x_{t-1}) \sim I(0)$, and ζ is a $p \times p - m_2$ matrix of medium run adjustment coefficients.

Thus, in the $I(2)$ model, $\beta' x_{t-1} \sim I(1)$ and $\delta' \Delta x_{t-1} \sim I(1)$ become $I(0)$ by cointegration. By combining levels and differences of the variables, the fundamental equation (7) has exactly this feature. By rewriting $\beta'_j x_{t-1} + \delta'_j \Delta x_{t-1},\ j = 1, ..., r$ as:

$$\underbrace{\Delta x_{i,t}}_{I(1)} = -\delta_{ij}^{-1} \underbrace{\beta'_j x_{t-1}}_{I(1)} + \underbrace{e_{i,t}}_{I(0)}, \quad i = 1, ..., p$$

it is easy to see that a polynomial cointegration relation describes how the growth rates adjust to a persistent static equilibrium error. If $\delta_{ij}\beta_{ij} < 0$, then the variable $x_{i,t}$ will exhibit error increasing behavior in the medium run and, hence, would be prone to persistent swings around its long-run equilibrium values. This explains why the $I(2)$ model is so well-suited to test hypotheses of imperfect knowledge models.

The moving average representation of (14) subject to (11) and (13) expresses the variables x_t as a function of once and twice cumulated errors and stationary and deterministic components. It is given by:

$$\begin{aligned} x_t &= C_2 \sum_{j=1}^{t} \sum_{i=1}^{j} (\varepsilon_i + \Phi D_i + \mu_0) + C_1 \sum_{j=1}^{t} (\varepsilon_j + \Phi D_j + \mu_0) \\ &\quad + C^*(L)(\varepsilon_t + \Phi D_t + \mu_0) + A + Bt. \end{aligned} \tag{15}$$

The parameters are complicated functions of the parameters in (12), defined in Johansen (1992). For the purpose of this paper it suffices to focus on the matrix C_2 :

$$C_2 = \beta_{\perp 2}(\alpha'_{\perp 2}\Psi\beta_{\perp 2})^{-1}\alpha'_{\perp 2}, \tag{16}$$

where $\beta_{\perp 2}, \alpha_{\perp 2}$ are $(p \times m_2)$ matrices which are orthogonal to $\beta, \beta_{\perp 1}$ and $\alpha, \alpha_{\perp 1}$, respectively, and Ψ is a function of the parameters of VAR model. It is useful to denote C_2:

$$C_2 = \breve{\beta}_{\perp 2}\alpha'_{\perp 2}. \tag{17}$$

where $\breve{\beta}_{\perp 2} = \beta_{\perp 2}(\alpha'_{\perp 2}\Psi\beta_{\perp 2})^{-1}$. It is now straightforward to interpret the double summation $\alpha'_{\perp 2}\sum_{j=1}^{t}\sum_{i=1}^{j}\varepsilon_i$ as an estimate of the d_2 second order stochastic trends which load into the variables x_t with the weights $\breve{\beta}_{\perp 2}$.

From (15) it follows that an unrestricted constant will cumulate twice to a quadratic trend and similarly for the dummies. Thus, the coefficients of the deterministic components need to be appropriately restricted in the model equations to avoid undesirable effects in the process (see Rahbek et. al, 1999). The subsequent empirical model will be estimated subject to the restriction that all quadratic trends are zero.

To summarize, the CVAR parameterization allows us to distinguish between various degrees of persistence in the data such as type $I(0)$, type $I(1)$ and type $I(2)$ persistence. The difference between a type $I(1)$ and type $I(2)$ CVAR model is that deviations from the long-run equilibrium relations are stationary in the former case, but non-stationary in the latter.

Table 1: I(2) trace tests

Rank test statistics									
$p-r$	r	$d_2=5$	$d_2=4$	$d_2=3$	$d_2=2$	$d_2=1$	$d_2=0$		
4	3		213.38 [0.00]	149.00 [0.00]	101.08 [0.01]	86.90 [0.01]	84.43 [0.00]		
3	4			97.60 [0.01]	65.09 [0.11]	54.14 [0.05]	50.46 [0.01]		
2	5				**35.17** [0.53]	27.05 [0.30]	23.09 [0.11]		
1	6					15.46 [0.20]	7.49 [0.31]		
The modulus of the seven largest characteristic roots for $r=5$									
$r=5, d_2=0$			**1.00**	**1.00**	**0.96**	**0.88**	**0.88**	0.69	0.66
$r=5, d_2=2$			**1.00**	**1.00**	**1.00**	**1.00**	0.75	0.69	0.69

5 Specifying the CVAR model for the Swiss-US data

The number of stationary relations, r, and the number of $I(1)$ trends, d_1, and $I(2)$ trends, d_2, are determined based on the maximum likelihood procedure in Nielsen and Rahbek (2007). Table 1 reports the joint tests of the $I(1)$ and the $I(2)$ rank conditions as given by (11) and (13). Because the tests of $r=0,1,2$ were all strongly rejected, we have omitted the first three rows from the table. The standard test procedure starts with the most restricted model ($r=2, d_1=0, d_2=5$) in the upper left hand corner, continues to the end of the row ($r=2, d_1=5, d_2=0$), and proceeds similarly row-wise from left to right until the first acceptance at ($r=4, d_1=1, d_2=2$). However, this can only be borderline accepted based on a p-value of 0.11, whereas the next acceptable choice, ($r=5, d_1=0, d_2=2$), has a p-value of 0.53. In both cases the result is consistent with five $CI(2,1)$ relations which cointegrate from $I(2)$ to $I(1)$. In the former case there are four polynomially cointegrating relations combining $\beta' x_t$ with a linear combination of the differences, $\delta'\Delta x_t$ and one relation, $\beta'_{\perp 1}\Delta x_t$, which can only become stationary by differencing. In the latter case stationarity is achieved exclusively by polynomial cointegration. From a practical point of view, the difference between the two cases is not a major one. We continue with ($r=5, d_1=0, d_2=2$) as it was more strongly supported by the test.

The moduli of the seven largest roots for the choice of $r=5$ with $d_2=0,2$

are given in the lower part of Table 1. The last row in Table 1 shows that the choice of $d_2 = 2$ eliminates all large roots in the model, whereas if, instead, we had treated the variables in our model as $I(1)$ and chosen $d_2 = 0$ (and, hence $d_1 = 2$) our model would have contained a very large root with a modulus of 0.96 and a complex pair of roots with a modulus of 0.88. The former is likely to describe the long smooth movements in relative prices and the latter the long persistent swings in nominal exchange rates around the relative prices.[6]

While the null of a double unit root cannot be rejected based on a p-value of 0.53 it does not exclude the possibility that the alternative hypothesis of a near unit root[7] might have an equally high p-value. Since the long swings, while very persistent, should be bounded according to the IKE theory, the near unit root case would be more consistent with the theoretical framework. In spite of this, we shall approximate the near unit roots with a unit root as this allows us to exploit the different persistency profiles of the data and to study the dynamics of the error increasing/correcting behavior of the system.

Another important question is whether the Likelihood Ratio test suffers from size distortions when the process is near $I(2)$ rather than $I(2)$. Preliminary and unpublished research seems to indicate that in this case the χ^2 distribution needs a correction which depends on the closeness of the ρ parameter to the unit circle and the sample size. While this may change the p-values to some extent, it does not affect the basic idea of associating variables/relations with a similar persistency profile.

6 Integration and cointegration properties

Johansen et al. (2010) argues that the 'consensus' conclusion in the literature that real exchange rates are stationary but highly persistent, should be replaced by the conclusion that the *change* in real exchange rate is stationary but highly persistent and that an empirical understanding of the persistent currency swings is not likely to be reached without allowing for an $I(2)$ analysis of the data. The same conclusion is reached by Juselius (2014a) which uses a so called "theory consistent CVAR scenario", to demonstrate how such near $I(2)$ persistence can derive from the existence of a Frydman-Goldberg type of uncertainty premium in (7) proxied by the departure from PPP. The

[6]Similar results were also obtained in Johansen et al. (2010) based on US-German data for the pre-EMU period.

[7]A near unit root is inside but close to the unit circle.

assumption in Hommes (2005) that the proportion of chartists relative to fundamentalists decrease as the PPP gap grows is likely to capture a similar gap effect.

To derive the time-series properties of nominal interest rates assuming that the market is demanding an uncertainty premium for holding an asset in any of the two currencies, Juselius (2014a) suggested the following data generating process:

$$\Delta i_{j,t} = \omega_{j,t} + \varepsilon_{j,t}, \qquad j = 1, 2 \tag{18}$$

where $\varepsilon_{j,t} \sim Niid(0, \sigma^2_{\varepsilon,j})$ and $\omega_{j,t}$ is a change in the uncertainty premium assumed to be a near $I(1)$ process:

$$\omega_{j,t} = \bar{\rho}_j \omega_{j,t-1} + \varepsilon_{\omega j,t}, \qquad j = 1, 2 \tag{19}$$

where $\bar{\rho}_j$ is an average of time-varying coefficients with $\rho_{t,j} \approx 1.0$ in periods when the PPP gap is moderately sized (i.e. when the proportion of chartists is high), and $\rho_{t,j} \ll 1.0$ when the gap is large (i.e. when the proportion of fundamentalists is high). In the latter case the uncertainty premium for holding the currency becomes large enough to cause a swing of the exchange rate back towards its long-run benchmark value. Since periods when $\rho_{t,j} \ll 1.0$ are assumed to be short $\bar{\rho}_j$ is assumed to be fairly close to 1.0.

Integrating (18) over t gives:

$$i_{j,t} = i_{j,0} + \sum_{s=1}^{t} \varepsilon_{j,s} + \sum_{s=1}^{t} \omega_{j,s}, \qquad j = 1, 2 \tag{20}$$

Under the near $I(1)$ assumption of $\omega_{j,t}$, $\sum_{s=1}^{t} \omega_{j,s}$ is a near $I(2)$ process implying that nominal interest rates are near $I(2)$. Such a process would describe persistent swings of shorter and longer durations similar to what we saw in Figures 2-4.

Thus, under imperfect knowledge, the best predictor for the interest rate next period is not just the present interest rate level (as in the random walk) but also the rate of change has predictive content:

$$E_t(i_{t+1} \mid X_t) = i_t + \Delta i_t \tag{21}$$

where X_t stands for the information available at time t. REH-based models, in contrast, would generally assume that the best predictor is the present level of interest rate:

$$E_t(i_{t+1} \mid X_t) = i_t \tag{22}$$

i.e. the direction of change has no predictive content.

In foreign exchange markets, the short term noise component, ε_t, is likely to be large relative to the drift component, ω_t, capturing the smooth trading along the trend and the signal-to-noise ratio is likely to be small, i.e. $\sigma_\omega^2 \ll \sigma_\varepsilon^2$. In this case it is often difficult to detect the tiny drift term (19) by econometric testing. For example, Juselius (2014b) shows by simulations that the univariate Dickey-Fuller tests essentially never finds the second large root when $\bar{\rho} = 0.9$ and $\sigma_\omega/\sigma_\varepsilon = 0.15$ (a typical value for many foreign exchange markets) whereas the multivariate trace tests finds it in the majority of all cases.

Starting from (18), Juselius (2014a) derived the time-series properties of the remaining variables and showed that the deviations from basic parities such as the PPP, the Fisher parities, and the terms spreads are all likely to be near $I(2)$. Thus, given the assumption of imperfect knowledge, the parities are expected to be one degree more persistent than under the RE hypothesis. Under the RE hypothesis, they would generally be stationary, or at most near $I(1)$. Thus, assuming imperfect knowledge leads to the following testable hypotheses:

- $(p_{1,t} - p_{2,t}) \sim$ near $I(2)$,
- $e_{12,t} \sim$ near $I(2)$
- $(i_{1,t} - i_{2,t}) \sim$ near $I(2)$, $i = s, b$
- $(e_{12,t} - p_1 - p_2)_t \sim$ near $I(2)$,
- $(i_{j,t} - \Delta p_{j,t}) \sim$ near $I(2)$, $i = s, b$, $j = 1, 2$

In terms of cointegration, the following relationships would be empirically consistent with imperfect knowledge:

- $\{(i_{1,t} - i_{2,t}) - b_1(e_{12,t} - p_{1,t} - p_{2,t})\} \sim \ I(1)$
- $\{(\Delta p_{1,t} - \Delta p_{2,t}) - b_2(i_{1,t} - i_{2,t}) + b_1(e_{12,t} - p_{1,t} - p_{2,t})\} \sim I(0)$

Table 2: Testing hypotheses of $I(1)$ versus $I(2)$

	p_1	p_2	e_{12}	b_1	b_2	s_1	s_2	$\chi^2(v)$	$p-val$
Are relative prices $I(1)$?									
$\mathcal{H}_1$	1.0	-1.0	-	-	-	-	-	21.5(3)	0.00
Is the nominal/real exchange rate $I(1)$?									
$\mathcal{H}_2$	-	-	1.0	-	-	-	-	21.6(3)	0.00
$\mathcal{H}_3$	1.0	-1.0	-1.0	-	-	-	-	22.1(3)	0.00
Is the Swiss bond rate $I(1)$?									
$\mathcal{H}_4$	-	-	-	1.0	-	-	-	27.7(3)	0.00
Is the US bond rate $I(1)$?									
$\mathcal{H}_5$	-	-	-	-	1.0	-	-	22.1(3)	0.00
Is the US/CH bond rate differential $I(1)$?									
$\mathcal{H}_6$	-	-	-	1.0	-1.0	-	-	23.4(3)	0.00
Is the Swiss short rate $I(1)$?									
$\mathcal{H}_7$	-	-	-	-	-	1.0	-	9.6(3)	0.02
Is the US short rate $I(1)$?									
$\mathcal{H}_8$	-	-	-	-	-	-	1.0	22.6(3)	0.00
Is the US/CH short rate differential $I(1)$?									
$\mathcal{H}_9$	-	-	-	-	-	1.0	-1.0	23.0(3)	0.00
Is the Swiss short-long interest $I(1)$?									
$\mathcal{H}_{10}$	-	-	-	1	-	-1	-	2.5(3)	0.48
Is the US short-long interest $I(1)$?									
$\mathcal{H}_{11}$	-	-	-	-	1	-	-1	7.1(3)	0.07

All of the above hypotheses can be formulated as a known vector k_1 in β, i.e. $\beta = (k_1, k_{1\perp}\varphi)$ where $k_{1\perp}\varphi$ defines the remaining unrestricted vectors to lie in the orthogonal space of k_1. For example $k_1 = [1, -1, 0, 0, 0, 0, 0, 0, 0]$ is a test whether the relative price is a unit vector in β. If accepted, it implies that $p_1 - p_2$ can be considered $I(1)$. The tests which are Likelihood Ratio tests are described in more detail in Johansen et al. (2010).[8]

Table 2 reports the test results. Consistent with imperfect knowledge, the hypotheses that the relative price, the nominal and the real exchange

[8] Note, however, that applying a near unit root correction to these tests is likely to increase the p-values to some extent. Such a correction has not been applied here as the correct size is not yet known. Applying the correction may change the rejection of the short Swiss rate to be $I(1)$.

rate, the nominal interest rates[9], the long and the short interest differentials are $I(1)$ are all rejected, whereas the hypotheses that the Swiss and the US short-long spreads are $I(1)$ cannot be rejected with a p-value of 0.48 and 0.07 respectively. Thus, the results suggest a considerable degree of persistence in prices and interest rates and in co-movements between them. However, it needs to be emphasized that this persistence is primarily associated with the shock to the drift term, $\omega_{j,t}$, in (18). When these shocks are tiny compared to the shocks to the process itself, as in the case of the nominal exchange rate, the drift term might be hard to catch sight of because of the large short-run volatility. See Figure 1, panel 1.

7 The pulling and pushing forces

7.1 The pushing forces

Assuming imperfect knowledge, Juselius (2014a) derived a theory consistent CVAR scenario for prices, long-term interest rates, and the nominal exchange rate. The scenario below is an adapted version in which also the short-term interest rates are allowed to enter the model:

$$\begin{bmatrix} p_{1,t} \\ p_{2,t} \\ e_{12,t} \\ b_{1,t} \\ b_{2,t} \\ s_{1,t} \\ s_{2,t} \end{bmatrix} = \begin{bmatrix} c_{11} & 0 \\ c_{21} & 0 \\ c_{31} & c_{32} \\ 0 & c_{42} \\ 0 & c_{52} \\ 0 & c_{42} \\ 0 & c_{52} \end{bmatrix} \begin{bmatrix} \Sigma\Sigma u_1 \\ \Sigma\Sigma u_2 \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \\ b_{41} & b_{42} \\ b_{51} & b_{52} \\ b_{61} & b_{62} \\ b_{71} & b_{72} \end{bmatrix} \begin{bmatrix} \Sigma u_1 \\ \Sigma u_2 \end{bmatrix} + Z_t, \qquad (23)$$

where u_1 is assumed to describe the twice cumulated shock to the drift term in the relative price and u_2 to the forecast of the nominal exchange rate (v_t, in (2)), and Z_t is a catch-all for stationary and deterministic components. All variables are considered near $I(2)$, consistent with the results in Section 6. The scenario (23) mimics the MA trend decomposition (15) of the CVAR model for $I(2)$ data. The loadings to the relative price trend (c_{i1}) reflect the prior that only the two prices and the nominal exchange rate should be

[9] Applying the near unit root correction may change the conclusion regarding the short Swiss rate in the direction of rather beeing $I(1)$.

Table 3: The pushing forces

	p_1	p_2	e_{12}	b_1	b_2	s_1	s_2
$\hat{\sigma}_{\varepsilon}$	0.00381	0.00379	0.04464	0.00059	0.00102	0.00137	0.00097
The loadings to the $I(2)$ trends							
$\tilde{\beta}'_{\perp 2,1}$	**0.69**	**1.03**	**-0.34**	0.01	0.02	0.01	0.03
$\tilde{\beta}'_{\perp 2,2}$	-0.00	**1.40**	**0.38**	**-0.04**	**0.11**	**-0.02**	**0.11**
The common stochastic $I(2)$ trends $\alpha'_{\perp 2}\sum\sum\varepsilon_j$							
$\alpha'_{\perp 2,1}$	-0.07	0.02	0.04	**0.59**	**1.44**	**-0.53**	**-1.11**
$\alpha'_{\perp 2,2}$	0.01	0.03	-0.03	**0.22**	**-1.45**	**0.38**	**0.59**

affected by the relative price trend. This would be consistent with the long-term movements in relative prices and the nominal exchange rate visible in Figure 1. The loadings to the speculative currency trend (c_{i2}) reflect the prior that only the nominal exchange rate and the interest rates should be affected by the speculative currency trend. This would be consistent with the long swings in these variables visible in Figure 1-3. The loadings to the short and the long rate are assumed equal for both countries reflecting the test results of the previous section, namely that the short-long spreads can be treated as $I(1)$. The loadings to the $I(1)$ trends (b_{ij}) are left unconstrained as we have no strong prior hypotheses about them.

The scenario is useful for checking whether prior hypotheses are logically consistent with the features of the data-generating process. For example, the hypothesis that the relative price $I(2)$ trend cancels in ($p_1 - p_2 - e_{12}$) requires that $(c_{11} - c_{21}) = c_{31}$, implying that $c_{11} \neq c_{21}$ unless $c_{31} = 0$, i.e. the loadings to the two prices should not be equal in a world of imperfect knowledge. Note, however that even though $(c_{11} - c_{21}) = c_{31}$, the *ppp* term would still be near $I(2)$ as long as $c_{32} \neq 0$. The hypothesis that $(p_1 - p_2 - e_{12})$ and $(s_1 - s_2)$ are cointegrating to $I(1)$ would in addition imply that $c_{62} - c_{72} = c_{32}$.

Table 3 reports the estimated common trends formulation of the CVAR model based on a just-identified $I(2)$ representation using the decomposition (17). The loadings to the first trend, $\tilde{\beta}_{\perp 2,1}$, are derived subject to the restriction $(c_{11} - c_{21}) = c_{31}$, i.e. that the long-run relative price trend cancels in $p_1 - p_2 - e_{12}$. The remaining free coefficients show that the interest rates are not strongly affected by this trend consistent with the hypothetical scenario (23). Furthermore, the loading to the relative price trend, -0.34, is consistent with the downward sloping trend in Figure 1. The estimate of the relative price shock $u_1 = \hat{\alpha}'_{\perp 2,1}\varepsilon_{i,j}$ suggests that shocks to the US and Swiss short-long spreads have generated the first $I(2)$ trend. This points to the importance of monetary policy shocks for the long-term price development.[10]

The second stochastic trend is derived subject to the restriction that $c_{12} = 0$, i.e. that Swiss prices have not been affected by the speculative imperfect knowledge trend. It was, however, not possible to jointly restrict the loadings of both prices to zero without violating the information in the data, As such it implies a deviation from the hypothetical scenario, but the result that US prices have been strongly affected by the speculative trend was very robust based on different identification schemes. One explanation could be that the role of the dollar as an international reserve currency has allowed prices in the US to grow more than those of its competitors, as the large US trade deficits might suggest. Altogether the loadings show that the speculative trend has had a positive effect on US short and long-term interest rates, US prices and the exchange rate (an appreciation of the dollar) whereas the effect on the Swiss interest rates is negative. The estimate of the relative price shock $u_1 = \hat{\alpha}'_{\perp 2,1}\varepsilon_{i,j}$ suggests that the shocks to the US long-term bond rate relative to the other three interest rates have primarily generated this trend.

How can this be interpreted? According to Juselius (2013) a shock to the long-term interest rate (for example, as a result of a domestic increase in sovereign debt) without a corresponding increase in the inflation rate, is likely to increase the amount of speculative capital moving into the economy. The exchange rate would first appreciate, jeopardizing competitiveness in the tradable sector, the trade balance would worsen, and the pressure on the interest rate would increase. Under this scenario, the interest rate is likely to keep increasing as long as the structural imbalances are growing, thus generating persistent movements in the real interest rate differentials and the real exchange rate. Figures 1-2 illustrate such historical co-movements. But, because risk averse individuals will require increasingly larger risk premiums for holding the domestic currency as the macroeconomic imbalances grow, the persistent movements of the exchange rate in one direction will sooner or later reverse causing a long period of real appreciation to become similarly a long period of real depreciation (Frydman and Goldberg, 2007, Hommes,

[10] When interpreting the coefficients of the stochastic trend it is important to keep in mind that the magnitude of the standard errors varies a lot among the variables. For example, the standard error of the nominal exchange rate is almost 13 times as large as the ones of prices and the latter are larger than the ones of interest rates.

2013). Note also that the deviations from benchmark PPP values can be long-lasting as long as the persistent movements in the real exchange rate are compensated by persistent movements in the interest rate differential corrected for the inflation rate differential (see Figure 2).

7.2 The pulling forces

The case $\{r = 5, d_1 = 0, d_2 = 2\}$ defines five stationary polynomially cointegrating relations, $\beta_i^{'} x_t + \delta_i' \Delta x_t$, $i = 1, .., 5$. They can be interpreted as dynamic equilibrium relations in the following sense: When data are $I(2)$, $\beta^{'} x_t$ is generally $I(1)$ and can be given an interpretation as an equilibrium error that exhibits pronounced persistence. In such a case, it is relevant to ask how the growth rates, Δx_t, dynamically react to these equilibrium errors. $\delta' \Delta x_t$ is an answer to this question. Thus, when discussing the adjustment dynamics in the $I(2)$ model, it is useful to interpret the coefficients α and δ as two levels of equilibrium correction: The δ adjustment describes how the growth rates, Δx_t, adjust to the long-run equilibrium errors, $\beta^{'} x_t$; the α adjustment describes how the acceleration rates, $\Delta^2 x_t$, adjust to the dynamic equilibrium relations, $\beta^{'} x_t + \delta' \Delta x_t$. This is illustrated below for the variable $x_{i,t}$:

$$\Delta^2 x_{i,t} = \cdots \sum_{i=1}^{r} \alpha_{ij} (\delta_i' \Delta x_{t-1} + \beta_i' x_{t-2}) + \cdots, j = 1, ..., p \tag{24}$$

where $\delta_i' = [\delta_{i1}, ..., \delta_{ij}, ..., \delta_{ip}]$ and β_i' is similarly defined.

The long and persistent swings away from fundamental PPP values implies equilibrium error increasing behavior somewhere in the system. This can be empirically studied by checking the signs of β, δ, and α in the following way: If $\delta_{ij} \beta_{ij} > 0$, then the changes, $\Delta x_{i,t}$, are equilibrium correcting to the levels, $x_{i,t}$; if $\alpha_{ij} \beta_{ij} < 0$ then the acceleration rates, $\Delta^2 x_{i,t}$, are equilibrium correcting to the long-run levels relation $\beta_i' x_{t-2}$; if $\alpha_{ij} \delta_{ij} < 0$ then the acceleration rates, $\Delta^2 x_{i,t}$, are equilibrium correcting to the differences, $\delta_i' \Delta x_{t-1.}$ In all other cases the system is equilibrium error increasing. Whether a variable is equilibrium error correcting or equilibrium error increasing is crucial for the inference on self-reinforcing dynamic feedback mechanisms in the system. This is also called a positive and negative feedback mechanism (Soros, 2010). We shall focus on this particular feature when discussing the results.

Since all characteristic roots were inside the unit circle[11] the system is sta-

[11] Roots inside the unit disk imply non-explosive behavior as they are calculated as

ble, as it should be, implying that any equilibrium error increasing behavior is compensated by error correcting behavior somewhere else in the system. Thus, while variables can move away from their long-run stable equilibrium path for extended periods of time, sooner or later the equilibrating forces will set in, for example due to an increasing uncertainty premium, and pull the variable back towards equilibrium.

The finding of two stochastic near $I(2)$ trends in Section 5 suggests that neither relative prices and the nominal exchange rates nor Swiss and US prices have moved closely together over this period. We have, therefore, chosen to identify the five cointegration relations by associating each of the five variables, the *ppp* term, the nominal exchange rate, e_{12}, the price differential, $p_1 - p_2$, Swiss prices, p_1, and US prices, p_2, with suitable combinations of the four interests rates. Since all relations need a linear trend to become stationary, the latter variables should be interpreted as deviations from their long-term trends, basically capturing business cycle fluctuations in prices and exchange rates. For example, in the *ppp* relation the trend may describe a small but significant productivity differential between the two countries. Figure 6-11 in the Appendix plots the deviations from a long-run linear trend for the real and nominal exchange rate, the price differential and the two price levels.

The structure in Table 4 imposes one over-identifying restriction on β, accepted with a p-value of 0.92 but, because of the complexity of the asymptotic distribution of the δ coefficients, the latter have not been subject to over-identifying restrictions. Since the $\beta' x_t$ relations are generally defining cointegration from $I(2)$ to $I(1)$, the above relations need to be combined with the growth rates to become stationary. To facilitate interpretation, statistically significant coefficients of β and α (and large coefficients of δ) are given in bold face.[12] Error increasing coefficients are given in italics.

The first β relation is identified by incorporating the main features of the mechanism that governs the behavior of prices, interest rates and the exchange rate as expressed by (7). The second vector related the price differential to the short-long interest rate spreads. The third β vector links the Swiss short-term rate to the exchange rate while the remaining β vectors describe the relation of domestic policy rates with domestic bond rates and

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eigenvalues of the characteristic polynomials (Juselius, 2006).

[12] Note that all β coefficients have very large t ratios, actually sufficiently large to be statistically significant also after having applied a near unit root correction. However, the significance of the δ coefficients is at this stage a guesswork.

Table 4: The pulling forces

An overidentified β structure $\chi^2(1) = 0.01[0.92]$								
	p_1	p_2	e_{12}	b_1	b_2	s_1	s_2	*trend*
β_1	**0.02**	**−0.02**	**−0.02**	**0.63**	**−0.63**	**−1.00**	**1.00**	**−0.00**
	[19.09]	[−19.09]	[−19.09]	[11.09]	[−11.09]			[−15.62]
δ_1	**0.53**	**−0.55**	**−0.67**	**0.04**	**−0.09**	*0.03*	*−0.08*	0.00
α_1	*1.43*	−0.29	8.96	**−0.39**	*−0.54*	−0.23	**−0.56**	
	[2.67]	[−0.54]	[1.37]	[−4.67]	[−3.78]	[−1.16]	[−4.07]	
β_2	**0.01**	**−0.01**	−	**−0.43**	**1.00**	**0.43**	**−1.00**	**0.00**
	[23.56]	[−23.56]		[−16.67]		[16.67]		[17.08]
δ_2	*−0.19*	**−0.11**	**0.14**	−0.01	0.01	−0.01	0.00	−0.00
α_2	**−5.14**	−0.09	**−39.05**	−0.23	0.27	**−2.34**	0.18	
	[−5.49]	[−0.10]	[−3.42]	[−1.57]	[1.08]	[−6.86]	[0.75]	
β_3	−	−	**0.05**	−	−	**1.00**	−	**0.00**
			[25.23]					[34.77]
δ_3	**−0.29**	**1.56**	**0.78**	−0.06	**0.15**	*−0.04*	**0.14**	−0.07
α_3	**0.59**	**−0.38**	*3.65*	*−0.13*	**−0.21**	−0.07	**−0.21**	
	[3.19]	[−2.09]	[1.62]	[−4.41]	[−4.22]	[−1.09]	[−4.40]	
β_4	**−0.02**	−	−	**−0.54**	−	**1.00**	−	**0.00**
	[−82.73]			[−6.48]				[8.36]
δ_4	**−0.76**	0.04	**0.72**	−0.04	0.06	*−0.03*	0.05	0.08
α_4	**3.02**	0.17	16.74	−0.13	**−0.39**	*0.51*	**−0.38**	
	[6.67]	[0.39]	[3.03]	[−1.89]	[−3.22]	[3.10]	[−3.21]	
β_5	−	**−0.00**	−	−	**−0.89**	−	**1.00**	**0.00**
		[−51.96]			[−14.31]			[10.19]
δ_5	−0.01	*0.08*	0.03	−0.00	0.01	−0.00	0.01	0.01
α_5	−7.59	−1.93	**−63.14**	−0.16	0.58	−3.01	0.14	
	[−4.85]	[−1.24]	[−3.31]	[−0.63]	[1.38]	[−5.26]	[0.35]	

prices. The α and the δ coefficients describe the dynamics of the system and allow us to investigate how changes, $\Delta x_{i,t}$, and acceleration rates, $\Delta^2 x_{i,t}$, respond to imbalances in the medium and the long run.

To illustrate how to interpret the results in the table, the first relation is reproduced below.

$$\begin{aligned} s_{1,t} - s_{2,t} &= 0.63(b_{1,t} - b_{2,t}) + 0.02\widehat{ppp}_t + 0.55(\Delta p_{1,t} - \Delta p_{2,t}) - 0.67\Delta e_{12,t} + \\ &\quad +0.04\Delta b_{1,t} + 0.03\Delta s_{1,t} - 0.09\Delta b_{2,t} - 0.08\Delta s_{2,t} + e_t \end{aligned}$$

where e_t is a stationary process and $\widehat{ppp}$ stands for trend-adjusted. It describes that the short-term interest rate differential has been co-moving with the bond rate differential and with deviations from the trend-adjusted ppp. The latter two can be interpreted as measures of the uncertainty premium agents require to enter a trade consistent with the hypothetical relation (7). However, to become stationary the β_1 relation needs to be combined with the differenced process. The estimates of δ_1 suggest that the inflation rate differential is an important determinant in this respect, possibly measuring short-term exchange rate expectations as postulated by (7). The remaining components in δ_1, in particular the changes in the interest rates that enter with coefficients of similar size, are likely to be associated with the catch-all residual, v_t, in (7). Altogether, the estimated coefficients provide strong support for the relations discussed in Section 2.

In terms of adjustment behavior, all variables except the two short-term interest rates are equilibrium error correcting in the medium run. This would be consistent with monetary policy driving the relation away from equilibrium but market-determined variables adjusting back over the medium run. The nominal exchange rate, the Swiss bond rate, and the US short-term rate are equilibrium error correcting in the long run, though the coefficient on the exchange rate is not significant. That prices are equilibrium error increasing even in the long run can seem surprising, but might reflect the finding in the previous section that both price differentials were near $I(2)$. The Swiss bond rate and the Swiss short-term rate are both error increasing in the long run, though the Swiss short-term rate not very significantly so.

The second relation is a relationship between the trend-adjusted price differential and the two short-long spreads that can be interpreted as reflecting the expectations hypothesis of the term structure and relative monetary policy in the two countries. The size of the δ coefficients show that changes in Swiss and US prices and the nominal exchange rate also enter this relation in an economically significant way. Swiss prices and the Swiss and US short-term rates are error increasing in the medium run but error correcting in the long run. Most adjustment coefficients on the US variables, however, are not very significant implying that it is primarily a Swiss relation. When the Swiss short-long spread is above its equilibrium value, the Swiss franc depreciates in the medium run. This could be a situation in which Swiss short-term rates are relatively low, reflecting an expansive monetary policy. Capital flows out of Switzerland could lead to a depreciation of the Swiss franc in such a situation. In the long run, the Swiss franc appreciates.

The third relation shows that the trend-adjusted nominal exchange rate has been negatively related to the Swiss short rate, meaning that the Swiss franc tends to be stronger when Swiss short-term rate is high. The exchange rate is error correcting in the medium run but error increasing in the long run.

When the short rate is above its equilibrium value, Swiss prices have tended to go down in the medium run but up in the long run. US prices tend to move in the opposite way. The Swiss short rate is not significantly adjusting, suggesting that it is exogenous to this relation. Instead the Swiss bond rate has decreased significantly both in the medium and the long run, probably reflecting capital inflows in situations with relatively tight Swiss monetary policy. The US rates have been increasing in the medium run which would be consistent with a capital-flows interpretation. They have been decreasing in the long run.

The fourth relation shows that Swiss trend-adjusted prices have been positively co-moving with the Swiss short-term relative to the long-term interest rate. Hence, the short-term relative to the long-term rate has been higher when Swiss prices have been above their long-term trend and vice versa. This relation can be seen as a characterization of Swiss domestic monetary policy, influencing the slope of the yield curve. Swiss prices are equilibrium correcting in the medium and the long run, whereas the coefficients on the US price level are small and insignificant. The Swiss franc has appreciated when Swiss prices have been above their equilibrium value, probably reflecting relatively tight monetary policy and the resulting capital flows. Though the Swiss bond rate is equilibrium error correcting in the medium run, both Swiss interest rates are error increasing in the long run (the long rate less significantly so). The two US interest rates increase in the medium run (probably because of capital flows), but fall in the long run.

The fifth relation presents the US equivalent to the fourth relation. Trend-adjusted US prices have been positively associated with the US short-term relative to the long-term interest rate. The US short rate is not significantly adjusting and, hence, is exogenous to this relation. By contrast, the US long rate is error correcting though not very significantly so. The most obvious difference to the fourth relation is that US prices have been equilibrium error increasing both in the medium and the long run, but not very significantly so. The US dollar has appreciated when the US short-term rate has been above its equilibrium value, which would be consistent with capital flows into the US. This matches with the results of Bruno and Shin (2014) who provide evidence that monetary policy shocks in the US lead to international spillover through capital flows in the banking sector. Swiss prices and the Swiss short-term rate have decreased when the US short-term rate has been above its equilibrium value.

Altogether the results suggest that the persistent movements away from equilibrium values visible in Figure 1-5 are associated with lack of equilibrium correcting behavior in prices as well as interest rates. The US interest rates, in particular, showed a tendency for self-reinforcing behavior in the hypothetical relation (7) pointing to the importance of the US dollar as an international reserve currency. Overall, divergent monetary policies and the resulting international spillovers seem to be responsible for the long swings in interest rate differentials and the real exchange rate.

8 Concluding discussion

Real exchange rates, real interest rates and interest rate differentials tend to exhibit a pronounced persistence which seems untenable with standard models based on RE. Here we argue that expectation formation based on imperfect knowledge and fundamental uncertainty is consistent with these long, persistent swings around long-run equilibrium values we see in the data. Such swings signal the presence of self-reinforcing feedback mechanisms somewhere in the economic system. The econometric problem is to identify the channels through which they work. For this purpose, the Cointegrated VAR for $I(2)$ data is tailor-made as it can describe equilibrium error increasing adjustment behavior in the medium run combined with error correction in the long run.

We found a number of such positive and negative feedback mechanisms in the foreign exchange market. While essentially all variables showed some evidence of error increasing behavior, the strongest and most significant error increasing behavior was associated with US interest rates, suggesting that speculative asset flows in to and out of the US play a significant role for the determination of exchange rates, interest rates and prices. In spite of the strong evidence of self-reinforcing feedback mechanisms, the system itself was still found to be stable in the sense that all characteristic roots were either on or inside the unit circle. In terms of cointegration we found that persistent movements in one parity were counteracted by similar persistent movements in another. For example, persistent movements in the short-term interest rate differential were cointegrated with persistent movements in the long-term bond differential and deviations from PPP. By interpreting the latter two as a proxy for an uncertainty premium in the foreign exchange market, the results provided strong empirical support for uncovered interest parity being stationary once an adjustment for uncertainty is allowed for. Thus, much of the forward premium puzzle seem to disappear when accounting for an uncertainty premium in the foreign exchange markets.

9 References

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Table 5: Misspecification tests and characteristic roots

Multivariate tests:							
Residual autocorr.	LM_1	$\chi^2(49)$	=	52.6		p-val.	0.34
Normality:	LM	$\chi^2(14)$	=	65.8		p-val.	0.00
Univariate tests:	Δp_1	Δp_2	Δe_{12}	Δb_1	Δb_2	Δs_1	Δs_2
ARCH(2)	**13.57**	0.40	0.05	3.81	0.96	0.36	2.86
Jarq.Bera(2)	3.16	0.16	0.87	2.66	**8.23**	**26.18**	**9.36**
Skewness	0.34	-0.06	0.07	0.26	0.57	-0.36	0.58
Ex. Kurtosis	3.25	2.94	2.59	2.72	3.53	**5.40**	**4.12**
$\hat{\sigma}_\varepsilon \times 0.01$	0.37	0.37	4.53	0.06	0.10	0.14	0.10

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10 Appendix 1: Misspecification tests

Table 5 reports a number of multivariate and univariate misspecification tests. A significant test statistic is given in bold face. The multivariate LM test for first order residual autocorrelations is not significant, whereas multivariate normality is clearly violated. Normality can be rejected as a result of skewness (third moment) or excess kurtosis (forth moment) and we report the univariate Jarque-Bera test statistics together with the third and fourth moment around the mean. It turns out that the rejection of normality is essentially due to excess kurtosis (rather than skewness) in the short-term interest rates. The $ARCH(2)$ tests for second order autoregressive heteroscedastisity is (somewhat surprisingly) rejected for Swiss prices. Since cointegration results have been found to be quite roboust to moderate ARCH and excess kurtosis we regard the present model specification to be acceptable.

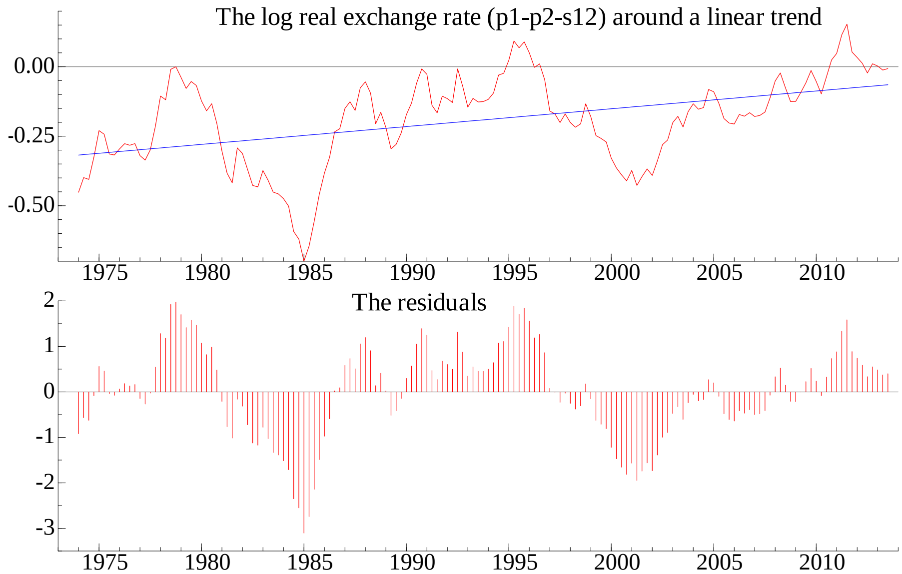

Figure 6: The real exchange rate around a linear trend.

11 Appendix 2: data

All data series are collected from the IFS database.

p_1	Swiss CPI: all country, quarterly average
p_2	US CPI, all items, city average, quarterly average
e_{12}	official rate, Swiss franc per US dollar, quarterly average
b_1	Swiss government bond yield, quarterly average
b_2	US government bond yield, 10 year, quarterly average
s_1	Swiss 3 months interbank rate, quarterly average
s_2	US 3 months interbank rate, quarterly average

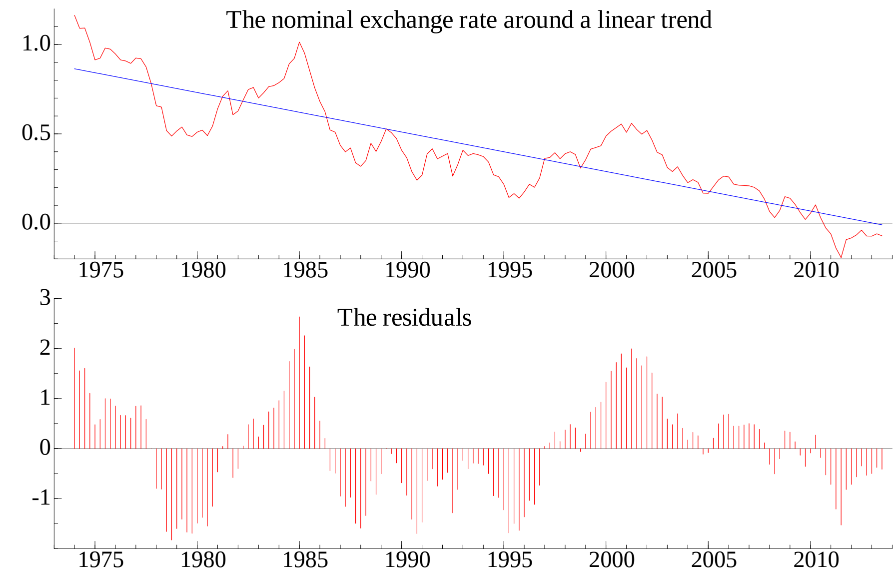

Figure 7: The nominal exchange rate around a linear trend.

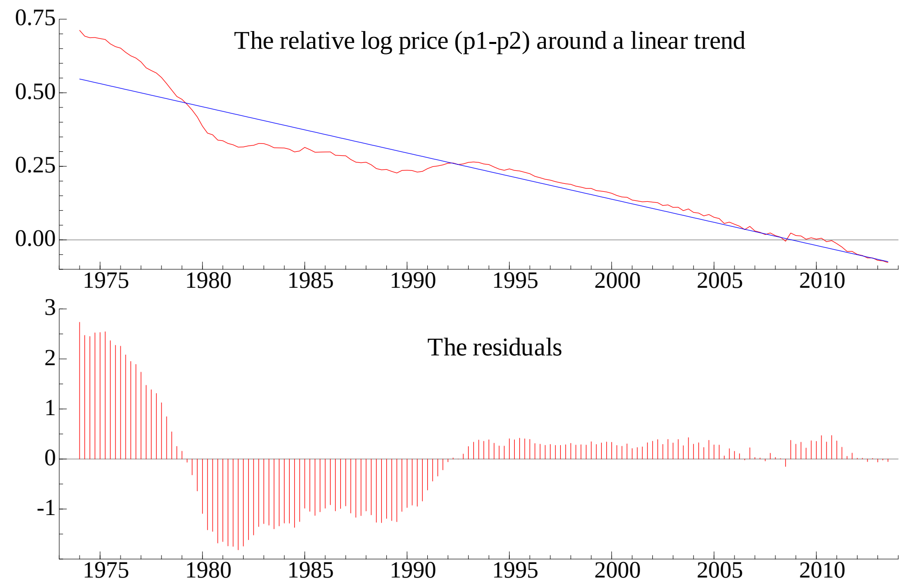

Figure 8: The Swiss-US relative price around a linear trend.

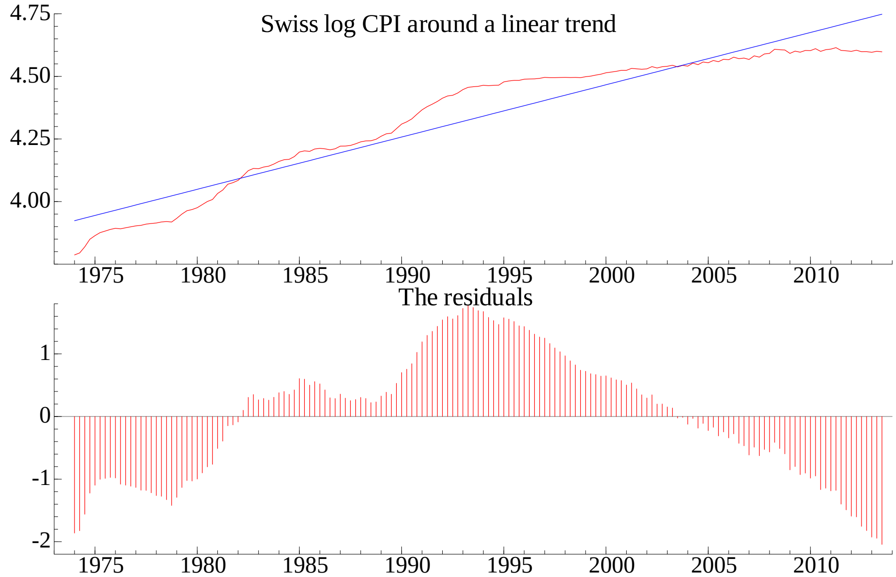

Figure 9: The Swiss CPI around a linear trend.

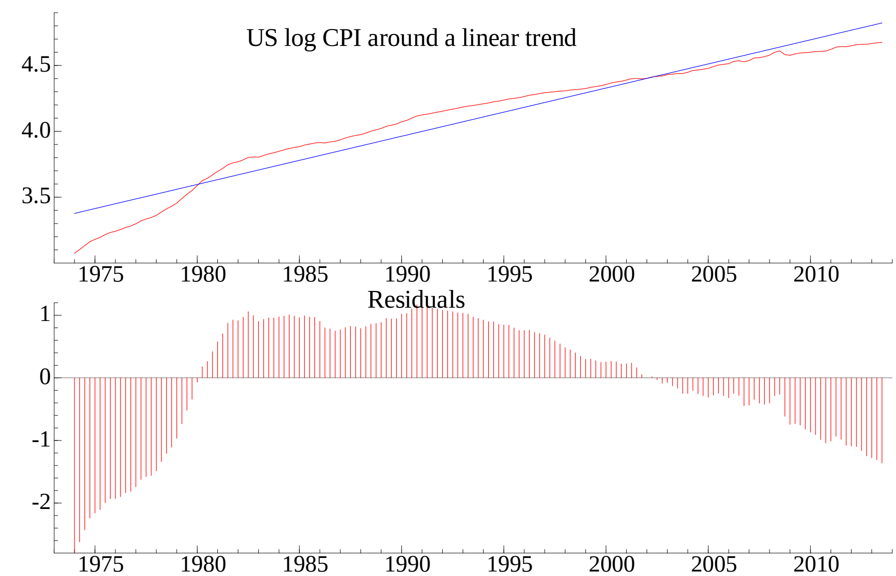

Figure 10: The US CPI price around a linear trend.

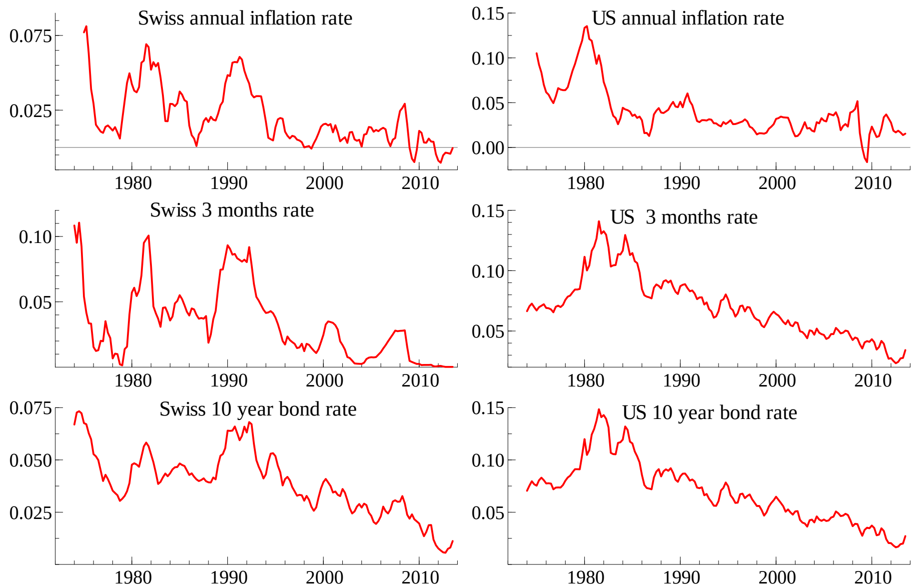

Figure 11: Inflation, three-month interest rate and ten-year bond yield for Switzerland and the US.