Discussion Papers
Department of Economics
University of Copenhagen

No. 14-25

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ISSN: 1601-2461 (E)

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September 5, 2014

Abstract

I show that conventional estimators based on the consumption Euler equation, intensively used in studies of intertemporal consumption behavior, produce biased estimates of the effect of children on consumption if potentially binding credit constraints are ignored. As a more constructive contribution, I supply a tractable approach to obtaining bounds on the effect of children and estimate these bounds using the Panel Study of Income Dynamics (PSID). Results suggest that children might not affect household consumption in the same magnitude previously assumed.

Keywords: Consumption, Euler Equation Estimation, Credit Constraints, Children, Demographics, Life Cycle, Bounds.

JEL-Codes: D12, D14, D91.

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I am grateful to Mette Ejrnæs, Bertel Schjerning and Jeppe Druedahl for fruitful discussions. This paper also benefited from discussions with Martin Browning, Christopher Carroll, John Rust, Richard Blundell, Søren Leth-Petersen, Miles Kimball, Dennis Kristensen, Daniel Borowczyk-Martins, Kjetil Storesletten, Jérôme Adda, Anders Munk-Nielsen, Mette Foged, Damoun Ashournia and conference participants at the EEA in Gothenburg 2013, the CAM December workshop in Copenhagen, 2013, the IAAE in London, 2014, and the ESEM in Toulouse, 2014. All errors are my own.

This paper extends the first part of the overly long working paper: Jørgensen, T. H. (2014): "Life-Cycle Consumption and Children," CAM Working Paper No. 2014-02, Department of Economics, University of Copenhagen.

1 Introduction

This study investigates what can be learned from Euler equation estimation of the effect of children on household consumption when households are potentially credit constrained. Although these estimators are now work horses in the analysis of intertemporal consumption behavior, little is known about their performance when households face potentially binding credit constraints, invalidating the standard Euler equation. Particularly, the effect of demographics on consumption and the extent to which children affect consumption behavior have received great attention the last two decades. Through numerous Euler equation estimations, a consensus has been reached in the literature that children are important drivers of consumption over the life cycle.[1]

The present study offers three contributions to this literature. First, I show *how* conventional Euler equation estimation methods produce biased estimates of the effect of children on consumption if consumers face possibly binding credit constraints. This has not been subject to a thorough analysis and the volume of work in the field of intertemporal consumption behavior merits one.[2]

Secondly, I supply a tractable approach to obtaining bounds on the effect of children on consumption that allows households to be affected by constraints. Specifically, if the effect of children on consumption is *large*, the credit constraint likely restrains households from increasing consumption as much as desired had (additional) borrowing been possible, producing a downwards bias. To the contrary, if the effect of children is relatively *low*, conventional methods will overestimate the effect of children. Even if children does not affect consumption, the inability to borrow against future income growth produce a positive correlation between consumption growth and changes in household demographics because children often arrive while households are young and affected by credit constraints the most.

I propose to split the sample into young households, in which children might arrive, and older households, in which children might move. Comparing older households with and without children produce a lower bound for the reason discussed above. Using the cohort average number of children as instrument produces an upper bound due to the positive correlation between the growth in the average number of children and income growth of young households.

Finally, I find that the effects of children reported in the existing literature are

[1] Thurow (1969) might be the first study investigating the consumption age profile to mention both children and constraints as potential explanations for the hump. Some important contributions to the literature on the effect of children are due to Browning, Deaton and Irish (1985); Blundell, Browning and Meghir (1994); Attanasio and Weber (1995); Attanasio and Browning (1995); Attanasio, Banks, Meghir and Weber (1999); Fernández-Villaverde and Krueger (2007) and Browning and Ejrnæs (2009).

[2] The fact that ignoring credit constraints produce biased Euler equation estimates is not new. Adda and Cooper (2003) show how Euler equation estimation of the intertemporal elasticity of substitution is overestimated if credit constraints are ignored.

above the proposed upper bound estimated from the Panel Study of Income Dynamics (PSID). In contrast to what I find, it seems broadly accepted that children play an important role in generating the observed consumption profiles. In an influential study by Attanasio, Banks, Meghir and Weber (1999), the number of children is found to be important in order to describe the consumption behavior of US consumers, using the Consumer Expenditure Survey (CEX). This is supported by the results in Attanasio and Browning (1995) using the UK Family Expenditure Survey (FES). Browning and Ejrnæs (2009) find that the number and age of children can explain completely the hump in consumption in the FES. However, all existing studies apply Euler equation estimation techniques ignoring potentially binding credit constraints. As I show, if the effect of children is relatively low the applied estimators overestimates the effect of children on consumption if households face potentially binding credit constraints.

The present study is related to a recent strand of literature investigating the validity of Euler equation estimation. For example, Carroll (2001) argues that using a log-linearized Euler equation for estimation of the intertemporal elasticity of substitution (IES) suffers from an omitted variable bias if consumers face sufficient income uncertainty. Attanasio and Low (2004) find, however, that the critique in Carroll (2001) is unwarranted. Recently, Alan, Atalay and Crossley (2012) investigate how measurement error affects Euler equation estimation results and unify the seemingly contradictory results of Carroll (2001) and Attanasio and Low (2004). They argue that the contradictory results are due to differences in the time series dimension in the implemented Monte Carlo studies and that the bias in Euler equation estimators might be small when interest rates vary sufficiently over time and the time dimension is (unrealistically) long, as in Attanasio and Low (2004). All these studies focus on the IES and, contrary to the present study, ignore potentially binding credit constraints.

A growing empirical literature finds evidence consistent with credit constrains being important for observed behavior. Interpreting the "excess sensitivity" in consumption growth to income as evidence of credit constraints, Hall and Mishkin (1982) estimate that around 20 percent of households in the PSID are credit constrained. If the excess sensitivity is due to credit constraints, households with high wealth levels should display significantly less effect of lagged income on consumption growth compared to low wealth households. This is what Zeldes (1989a) finds while Runkle (1991) does not find significant differences.

A second strand of literature exploits random variation to identify the importance of credit constraints. Using the random receipt timing of the 2001 federal income tax rebate in the US, Johnson, Parker and Souleles (2006) find that consumption in the CEX responds to the transitory income increase generated by the rebate. Including also the 2008 tax rebate, Gross, Notowidigdo and Wang (2014) show that low wealth and low income households used their tax rebates to file for bankruptcy. Both results are consistent with an important role for credit constraints. Gross and Souleles (2002) analyze how changes in credit card debt limits increase debt holdings. They find that debt increases with 13 percent of the change in the debt limit. Using Danish register data, Leth-Petersen (2010) estimates the effect of an unanticipated reform in 1992 that allowed Danish house owners to use their house as collateral to take up consumption loans. He estimates that around 12 percent of Danish households, many of which were young households, were affected by credit constraints.

A third strand of literature identifies credit constrained households from direct survey measures on credit availability. Using information on whether a request for credit had been declined Jappelli (1990) estimates that around 19 percent of households in the Survey of Consumer Finances (SCF) are credit constrained. Jappelli, Pischke and Souleles (1998) extrapolate the likelihood of being credit constrained in the SCF to the PSID. Based on observable characteristics in both the SCF and the PSID, they find that the excess sensitivity of (food) consumption to lagged income of households who are more likely to be constrained is three times that of households who are less likely to be credit constrained. Recently, Crossley and Low (forthcoming) find that around 6-14 percent of job losers in the Canadian Out of Employment Panel (COEP) survey are credit constrained, depending on how households are classified as being constrained.

The present results generalize to cases in which consumers do not face explicit credit constraints. If there instead is a probability of receiving a zero-income shock (as in Carroll, 1997 and Gourinchas and Parker, 2002), most results still hold. This is because risk averse consumers will instead face a "self-imposed" no-borrowing constraint stemming from the fear of receiving zero income in all future periods with consumption of zero as a consequence (Schechtman, 1976; Zeldes, 1989b). In turn, consumption will respond substantially to transitory income shocks and the log-linearized Euler equation will be a poor approximation.[3]

The rest of the paper proceeds as follows. The following section presents the constrained consumption Euler equation and discusses the most commonly applied estimators derived from it when ignoring credit constraints. Section 3 illustrates how these estimators fail to uncover the effect of children on consumption when households face potentially binding credit constraints and suggests how bounds can be estimated using these methods. Section 4 shows that existing estimates of the effect of children on consumption are above the proposed upper bounds estimated using the PSID. Section 5 discusses the robustness of the bounds and section 6 concludes.

[3]This is the point of Carroll (2001) where he illustrates how this poor first (and second) order approximation of the non-linear Euler equation results in poor estimates of the intertemporal elasticity of substitution. His result shows that the result in Adda and Cooper (2003) using an explicit no-borrowing constraint generalizes to cases with a self-induced constraint.

2 Euler Equation Estimation of Demographic Effects

Consider a life cycle model where consumers have time-separable utility over (a single) consumption good and are restricted in how much negative wealth they can accumulate. As most of the existing literature, I follow Attanasio, Banks, Meghir and Weber (1999) and let children affect the *marginal value* of consumption through a multiplicative taste shifter, $v(\mathbf{z}_t;\theta)$, in which $\mathbf{z}_t$ contains variables describing household demographics and θ is their loadings. As is standard in the literature, I let $v(\mathbf{z}_t;\theta) = \exp(\theta'\mathbf{z}_t)$ throughout. Alternatively, the household composition could be included as a scaling of resources and consumption (equivalence scaling), as done in, e.g., Fernández-Villaverde and Krueger (2007).[4]

The *constrained* Euler equation is

$$\begin{aligned} u'(C_t)v(\mathbf{z}_t;\theta) - \lambda_t &= R\beta\mathbb{E}_t\left[u'(C_{t+1})v(\mathbf{z}_{t+1};\theta) - \lambda_{t+1}\right] \\ &\Downarrow \\ R\beta\frac{u'(C_{t+1})}{u'(C_t)}\frac{v(\mathbf{z}_{t+1};\theta)}{v(\mathbf{z}_t;\theta)} &= \underbrace{\epsilon_{1,t+1} + \epsilon_{2,t+1}}_{\equiv\epsilon_{t+1}}, \end{aligned} \tag{1}$$

where $\mathbb{E}_t[\cdot]$ denotes expectations conditional on information available in period t, λ_t is the shadow price of resources in period t, R is the real gross interest rate, β is the discount factor, C_t denotes consumption, $u(C_t) = C_t^{1-\rho}/(1-\rho)$ is the utility function, assumed to be constant relative risk aversion (CRRA) where ρ is the inverse of the IES. The structural Euler error, ϵ_{t+1}, satisfies

$$\begin{aligned} \mathbb{E}_t[\epsilon_{1,t+1}] &= 1, \\ \mathbb{E}_t[\epsilon_{2,t+1}] &= -\frac{\lambda_t - R\beta\mathbb{E}_t[\lambda_{t+1}]}{u'(C_t)v(\mathbf{z}_t;\theta)}. \end{aligned}$$

From the Kuhn-Tucker conditions we know that $\lambda_t \geq 0$ in all time periods. Hence, the mean expectational errors in (1) equals one only if consumers are not constrained in the current period and know with perfect certainty that the borrowing constraint will *not* bind in the future. In such a case, $\mathbb{E}_t[\epsilon_{2,t+1}] = 0 \forall t$. Generally, however, consumers are *not* certain that they will be unaffected by constraints and the expectational error in (1) is a function of information today,

$$\mathbb{E}_t[\epsilon_{t+1}] = f(C_t, \mathbf{z}_t) \neq 1,$$

and serially correlated through the presence of λ_t and λ_{t+1} in $\epsilon_{2,t+1}$.

[4]See Bick and Choi (2013) for an analysis of different approaches to and implied behavior from inclusion of household demographics in life cycle models. Alternative parametrizations would require reformulating the estimable equations accordingly.

In the existing literature on intertemporal consumption allocation and the effect of children on consumption, credit constraints are often ignored or assumed away. It is clear from (1), that estimators ignoring credit constraints suffer from something similar to an "omitted variable bias". Below, I discuss the two most common estimators.

2.1 Conventional Euler Equation Estimators: Ignoring Constraints

Consider having longitudinal information on consumption and demographics for households $i = 1, \ldots, N$ in time periods $t = 1, \ldots T$. Ignoring potentially binding credit constraints (i.e., imposing $\lambda_s = 0 \, \forall s$) and inserting the standard functional form assumptions mentioned above, a non-linear GMM estimator of θ could be

$$\theta_{GMM} = \underset{\theta}{\operatorname{argmin}} \left[\frac{1}{NT} \sum_{i}^{N} \sum_{t}^{T} \left(R\beta \left(\frac{C_{i,t+1}}{C_{i,t}} \right)^{-\rho} \exp(\theta \Delta \mathbf{z}_{i,t+1}) - 1 \right) \cdot Z_{i,t+1} \right]^2, \quad (2)$$

such that θ_{GMM} is the parameter that satisfy the sample equivalent of $\mathbb{E}[(\epsilon - 1)' Z] = 0$, where Z contain instrument(s) assumed uncorrelated with the Euler residual. Ignoring measurement error, the estimator in (2) produce consistent estimates if a suitable instrument is available and, importantly, households do not face credit constraints.[5]

Using food consumption from the PSID, Alan, Attanasio and Browning (2009) estimate the effect of children to be around .18 from a similar estimator as (2) and as large as .9 using estimators allowing for measurement error in consumption.

Most existing studies work with a log-linearized version of the Euler equation since it yields estimable equations linear in parameters which can easily be estimated with synthetic cohort panels (Browning, Deaton and Irish, 1985) and handle measurement error through instrumental variables estimation. The log-linearized Euler equation is

$$\Delta \log C_{it} = \text{constant} + \rho^{-1} \theta' \Delta \mathbf{z}_{it} + \tilde{\epsilon}_{it}, \quad (3)$$

where the first term is a constant as a function of structural parameters (β, ρ) and the interest rate (assumed constant throughout), the second term is the effect of children (times the IES) and the last term is a reduced form residual, $\tilde{\epsilon}_t = -\rho^{-1} \log \epsilon_t$.

In the influential study by Attanasio, Banks, Meghir and Weber (1999), θ and ρ is estimated from the CEX by a log-linearized Euler equation using lagged changes in $\mathbf{z}_t$ as instruments along with lagged changes in income and consumption. The effect of the number of children is found to be around $\theta \approx .33$. Several studies use food consumption from the PSID to estimate versions of the log-linearized Euler equation, see, e.g., Hall and Mishkin (1982); Runkle (1991) and Lawrance (1991). The latter reports

[5] Alan, Attanasio and Browning (2009) supply modified GMM estimators to allow for measurement error while still ignoring possibly binding credit constraints.

estimates suggesting a value of θ of around 0.5. Dynan (2000), also using the PSID, estimates the effects of children to be around .7. Browning and Ejrnæs (2009) allow for a more flexible functional form of $v(\mathbf{z}_t;\theta)$ when estimating the effect of children consumption using the FES and find that the number and age of children can explain completely the hump in consumption.

Other estimators have been proposed to estimate Euler equations. For example, Alan and Browning (2010) propose a method in which they fully parameterize the Euler residuals and simulate these residuals and consumption paths simultaneously. Their Synthetic Residual Estimation (SRE) procedure does not allow for credit constraints in a coherent way. Since the GMM and log-linearized estimation methods are the conventional methods used in the literature, I focus exclusively on these.

Some empirical studies of intertemporal consumption behavior do recognize that credit constraints might affect household behavior. Potentially binding credit constraints are often handled by discarding households in which nothing is carried over from period t to $t+1$ (see, e.g., Alan, Attanasio and Browning, 2009). This strategy is clearly not a satisfactory approach because expectations about the credit constraint potentially binding in future periods still affect present consumption behavior through $\mathbb{E}_t[\lambda_{t+1}]$. Determining at which level of wealth households are completely free of the credit constraint is not trivial.

3 Bias and Bounds from Euler Equation Estimation

In this section, I illustrate how conventional Euler equation estimators, (2) and (3), produce biased estimates of the effect of children on consumption and can be used to construct bounds of this parameter. I first formulate a four-period model from which I can derive analytical expressions for the log-linearized Euler equation estimator and show how bounds can be calculated from splitting the sample into young and older households. To confirm the results from the four-period model, I formulate and numerically solve a standard life cycle model of buffer-stock savings behavior. By simulating data from this model, I estimate the proposed bounds and show that they are very similar to the bounds from the four-period model.

The present exposition is based on the absolutely best of all circumstances in which *i)* a panel of consumers is available, *ii)* consumption is observed without measurement error, *iii)* researchers know the underlying model consumers solve, and *iv)* researchers know the preferences of consumers except the effect of children on consumption.

3.1 Evidence from A Four Period Model

Here, I setup a four-period model with an analytical solution to illustrate how Euler equation estimation performs when households face potentially binding credit constraints. In the initial period, $t = 0$, all households are childless. In period $t = 1$, the "young" stage, a child arrives, $\mathbf{z}_1 = 1$, in p percent of the households and the remaining $1 - p$ percent remain childless, $\mathbf{z}_1 = 0$. In period $t = 2$, the "old" stage, the child moves (if present in period one) such that $\mathbf{z}_2 = 0$ for all households. Households die with certainty in the end of period $t = 3$ and consume all available resources in this terminal period.

Utility is CRRA and the taste shifter is assumed to be given by $v(\mathbf{z}_t; \theta) = \exp(\theta \mathbf{z}_t)$ with $\mathbf{z}_t \in \{0, 1\}$, and with baseline parameters of $\rho = 2$ and $\theta = 0.5$. To reduce unnecessary cluttering, the gross real interest rate and the discount factor both equal one, $R = \beta = 1$. Households receive a deterministic income of Y_t in beginning of every period. Income grows with G_1 between period zero and period one ($Y_1 = G_1 Y_0$) and is constant otherwise ($Y_t = Y_{t-1}$, $t = 2, 3$). The beginning-of-period resources available for consumption is the sum of household income and end-of-period wealth carried over from last period, $M_t = A_{t-1} + Y_t$.

Formally, households solve, for a given value of $\mathbf{z}_1 \in \{0, 1\}$, the problem,

$$\max_{C_0, C_1, C_2} \frac{C_0^{1-\rho}}{1-\rho} + \exp(\theta \mathbf{z}_1) \frac{C_1^{1-\rho}}{1-\rho} + \frac{C_2^{1-\rho}}{1-\rho} + \frac{(M_2 - C_2 + Y_3)^{1-\rho}}{1-\rho},$$

subject to a no-borrowing constraint, $A_t \geq 0$, $\forall t$. Appendix A in the online supplementary material solves the model analytically and reports the resulting optimal consumption functions.

Using the optimal consumption behavior from this model, Figure 1 presents consumption and wealth profiles for households initiated with no wealth in the initial period, $A_{-1} = 0$, period $t = 0$ income normalized to one ($Y_0 = 1$), and early life income growth of eight percent, $G_1 = 1.08$. Panel 1a presents consumption profiles for models with a credit constraint (solid) and without constraints (dashed) for households with children in period one (black) and without children (red). Panel 1b illustrates the associated wealth profiles. Potentially binding credit constraints affect the consumption and wealth profiles significantly.

Childless households increase consumption exactly as much as income grows and is in effect only potentially credit constrained in period $t = 0$ because they are unable to borrow against future income growth. Households in which a child arrives in period $t = 1$, on the other hand, might also be credit constrained in period $t = 1$ since they might want to increase consumption by more than their available resources.

The OLS estimator of the effect of children using consumption growth from $t - 1$

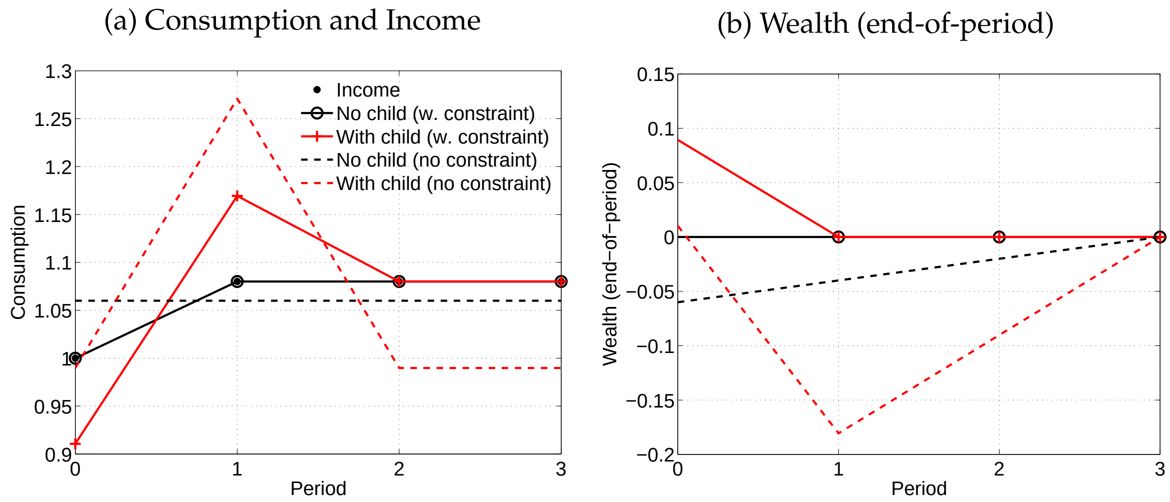


Figure 1 – Consumption and Wealth Profiles from the Four-period Model.

Notes: Figure 1 illustrates the consumption and wealth age profile from the four period model with paramters $\rho = 2$, $G_1 = 1.08$, and $\theta = 0.5$. Households are initiated with zero wealth and initial income is normalized to one, $A_{-1} = 0$ and $Y_0 = 1$, respectively. Panel a presents the income and consumption profiles for models with credit constraints (solid) and without constraints (dashed) for households with children in period one (black) and without children (red). Panel b illustrates the associated wealth profile.

to t from the log-linearized Euler equation (3) is given by

$$\hat{\theta}^t_{OLS} = (\Delta \log C_t|_{\mathbf{z}_1=1} - \Delta \log C_t|_{\mathbf{z}_1=0})\rho.$$

Using the (cohort) average number of children as instrument ($Z = p$) should be less affected by idiosyncratic uncertainty and, thus, credit constraints. The IV estimator is[6]

$$\hat{\theta}^t_{IV} = \frac{1}{p}(p\Delta \log C_t|_{\mathbf{z}_1=1} + (1-p)\Delta \log C_t|_{\mathbf{z}_1=0})\rho.$$

Appendix A in the online supplementary material derives explicit formulas for each estimator when using either young households or old households to estimate the

[6] Since there is only one cohort here (p is constant) no constant is included in the regression. Of course, in general, there will also be included a constant in such a regression.

effect of children on consumption. The resulting estimators are

$$
\begin{aligned}
\hat{\theta}^{young}_{OLS} &= \begin{cases} \theta - \log(G_1)\rho & \text{if } \theta > \log(G_1)\rho, \\ 0 & \text{if } 0 \le \theta \le \log(G_1)\rho, \end{cases} \\
&\le \theta, \\
\hat{\theta}^{young}_{IV} &= \begin{cases} \theta + (1-p)/p\log(G_1)\rho & \text{if } \theta > \log(G_1)\rho, \\ \log G_1\rho/p & \text{if } 0 \le \theta \le \log(G_1)\rho, \end{cases} \\
&\ge \theta, \\
\hat{\theta}^{old}_{OLS} &= \begin{cases} \rho\log(\frac{1+G_1}{G_1}) - \rho\log(1+\exp(-\rho^{-1}\theta)) & \text{if } \theta > \log(G_1)\rho, \\ 0 & \text{if } 0 \le \theta \le \log(G_1)\rho, \end{cases} \\
&\le \theta, \\
\hat{\theta}^{old}_{IV} &= \hat{\theta}^{old}_{OLS} \le \theta.
\end{aligned}
$$

It is immediately clear from these estimators that neither OLS nor IV estimators will in general yield consistent estimates of the true θ.[7] Interestingly, as the effect of children goes towards zero, the OLS (and IV) estimator using only older households comes close to the true value of θ. Similarly, as the effect of children on consumption gets increasingly large, the bias-part, $(1-p)/p\log(G_1)\rho$, of the young-IV estimate, $\hat{\theta}^{young}_{IV}$, becomes relatively less important. Therefore, I propose to split the sample into "young" households, in which children arrive, and "older" households, in which children leave, and use the IV estimate from the young sample as an upper bound and use the OLS estimate from the older sample as a lower bound.

The OLS estimate using young households could alternatively be used as a lower bound. However, as I show in the robustness exercise, if children arrive probabilistically, OLS from the young sample will underestimate the effect of children on consumption even if there is *no* credit constraint. I have chosen a lower bound that delivers the true effect of children on consumption when either there is no effect of children or no credit constraint, irrespectively if children arrive deterministically or probabilistically.[8]

The results are intuitive. Young households will accumulate wealth in the initial period zero, but not necessarily enough to ensure that the credit constraint is not binding in period one, in which a child arrives. Even if they do accumulate enough wealth, the fact that the childless households also increase consumption create a downwards bias in the estimate. When children subsequently leave, households with children are

[7] The bias is constant, independent of the observations and does, therefore, not vanish asymptotically. Hence, this suggests that the estimators might not even be consistent. In the more realistic life cycle model studied below, no closed form expressions can be derived and arguments are made through Monte Carlo simulation of finite samples and only the bias can be illustrated.

[8] Note, as I will show in the robustness exercise, this is only correct for the non-linear GMM estimator. The log-linear estimator will *not* be able to uncover the true effect of children if there is no credit constraint but instead a probability of a low income shock.

likely to go from being constrained in period one to unconstrained in period two (since they prefer consumption when children are present). The resulting drop in consumption will be smaller compared to the situation without a constraint, resulting in the OLS estimator being downwards biased. The fact that income and the the average number of children are positively correlated in the early part of the life cycle produce an upwards bias in the IV estimator.

Interestingly, for low levels of θ ($0 \leq \theta \leq \log(G_1)\rho$) the bounds are flat illustrating how the inability to borrow against future income growth prevents identification of the effect of children on consumption. The bounds are tightened for lower levels of income growth (G_1) and lower levels of intertemporal smoothing (ρ).

The importance of the combination of income growth and a credit constraint is clear from the analysis of the four period model. If income is constant, the OLS and IV estimators using young households deliver the correct θ. The same is true if households do not care about intertemporal smoothing of marginal utility ($\rho = 0$ and IES $= \infty$). The assumptions of income growth and finite intertemporal elasticity of substitution seem reasonable, however.

3.2 Evidence from a Multi-Period Life Cycle Model

To confirm the results from the four-period model, I setup a standard life cycle (buffer-stock) model, used intensively for analysis of intertemporal consumption behavior. The model captures the main consumption and savings incentives of households over the life cycle prior to retirement. Specifically, the model is similar to those in Attanasio, Banks, Meghir and Weber (1999); Gourinchas and Parker (2002) and Cagetti (2003).

Households work until an exogenously given retirement age, T_r, and die with certainty at age T where they consume all available resources. In all preceding periods, households solve the optimization problem

$$\max_{C_t} \mathbb{E}_t \left[\sum_{\tau=t}^{T_r-1} \beta^{\tau-t} v(\mathbf{z}_t;\theta) u(C_\tau) + \gamma \sum_{s=T_r}^{T} \beta^{s-t} v(\mathbf{z}_t;\theta) u(C_\tau) \right]. \tag{4}$$

Following Gourinchas and Parker (2002), survival and income uncertainty are omitted post retirement and the parameter γ (referred to as the retirement motive) in equation (4) is a parsimonious way of adjusting for these elements. Gourinchas and Parker (2002) ignore the post-retirement consumption decisions and adjust the perfect foresight approximation by a parameter similar to γ through a retirement value function. Although I focus on consumption behavior *prior* to retirement, the potential presence of children at retirement forces the model to be specific about post retirement behavior.

Households solve (4) subject to the intertemporal budget constraint, $M_{t+1} = R(M_t - C_t) + Y_{t+1}$, where M_t is resources available for consumption in beginning of period t

and Y_t is beginning-of-period income. End-of-period wealth, $A_t = M_t - C_t$, must be greater than a fraction $-\kappa$ of permanent income in all time periods, $A_t \geq -\kappa P_t \, \forall t$, $\kappa \geq 0$. Following Gourinchas and Parker (2002), retired households are not allowed to be net borrowers, $A_t \geq 0$, $\forall t \geq T_r$.

Prior to retirement, income follows a transitory-permanent income shock process,

$$\begin{aligned} Y_t &= P_t \varepsilon_t, \, \forall t < T_r, \\ P_t &= G_t P_{t-1} \eta_t, \, \forall t < T_r, \end{aligned}$$

where G_t is the real gross income growth, P_t denotes permanent income and $\eta_t \sim \log \mathcal{N}(-\sigma_\eta^2/2, \sigma_\eta^2)$ is a mean one permanent income shock. ε_t is a mean one transitory income shock taking the value μ with probability $\wp$ and otherwise distributed $(1-\wp)\varepsilon_t \sim \log \mathcal{N}(-\sigma_\varepsilon^2/2 - \mu\wp, \sigma_\varepsilon^2)$.[9] When retired, the income process is a deterministic fraction $\varkappa \leq 1$ of permanent income and permanent income grows with a constant rate of G_{ret} once retired, $Y_t = \varkappa P_t$, $\forall t \geq T_r$, and $P_t = G_{ret} P_{t-1}$, $\forall t \geq T_r$.

Households can have at most three children and no infants arrive after the wife turns 43 years old. For notational simplicity, the age of each child is contained in $\mathbf{z}_t$,

$$\mathbf{z}_t = (\text{age of child } 1_t, \text{ age of child } 2_t, \text{ age of child } 3_t) \in \{\text{NC}, [0, 20]\}^3,$$

where "NC" refers to "No Child" and the oldest child is denoted child one, the second oldest child as child two and the third oldest child as child three. When a child is aged 21 the child does not influence household consumption in subsequent periods regardless of the value of θ. Following Browning and Ejrnæs (2009), the arrival of an infant is deterministic in the sense that households know with perfect foresight how many children they will have and when these children arrive.[10]

Unlike the simple four-period model, the life cycle model does not have an analytical solution. Therefore, to simulate synthetic data, I solve the model using the Endogenous Grid Method (EGM) proposed by Carroll (2006) with "standard" parameters presented in Table 1. The technical details of the solution method are provided in Appendix B in the online supplemental material. The solution is then used to generate data for 50,000 households from age 22 to 59 in each of the 1,000 Monte Carlo (MC) runs. All households are initiated at age 22 with zero wealth, $A_{21} = 0$, permanent in-

[9] This formulation allows for both an explicit and self-imposed credit constraint. Depending on the value of κ and $\wp$ and μ, either the explicit or the self-imposed constraint will be the relevant one. This is discussed further in Appendix B in the supplemental material. In the baseline specification, $\kappa = 0$, $\wp = 0$ and $\mu = 0$ such that only the explicit credit constraint matters. I show in the robustness exercise, that the results regarding the log-linarized Euler equation is robust to letting $\wp = 0.003$ and $\mu = 0$ such that the self-imposed no-borrowing constraint is the relevant one rather than the explicit constraint.

[10] In the robustness analysis in Section 5, I allow children to arrive probabilistically, as in Blundell, Dias, Meghir and Shaw (2013), and find that the bounds are robust to this alternative fertility process.

come of one (normalization), $P_{22} = 1$, and no previous children, $\mathbf{z}_{21} = (\text{NC,NC,NC})$. Children are distributed across households and age according to the observed arrival of children in the PSID, as illustrated in Figure 2b, and the income profile is calibrated to be concave (Figure 2a) and constant from age 40 to mimic empirical income profiles.

Table 1 – Parameter Values Used to Simulate Data.

G_t	R	σ_ε^2	σ_η^2	κ	$\wp$	μ	β	ρ	γ	$\varkappa$	G_{ret}	θ
Fig. 2a	1.03	.005	.005	0	0	0	.95	2	1.1	.8	1.0	$\in [0,1]$

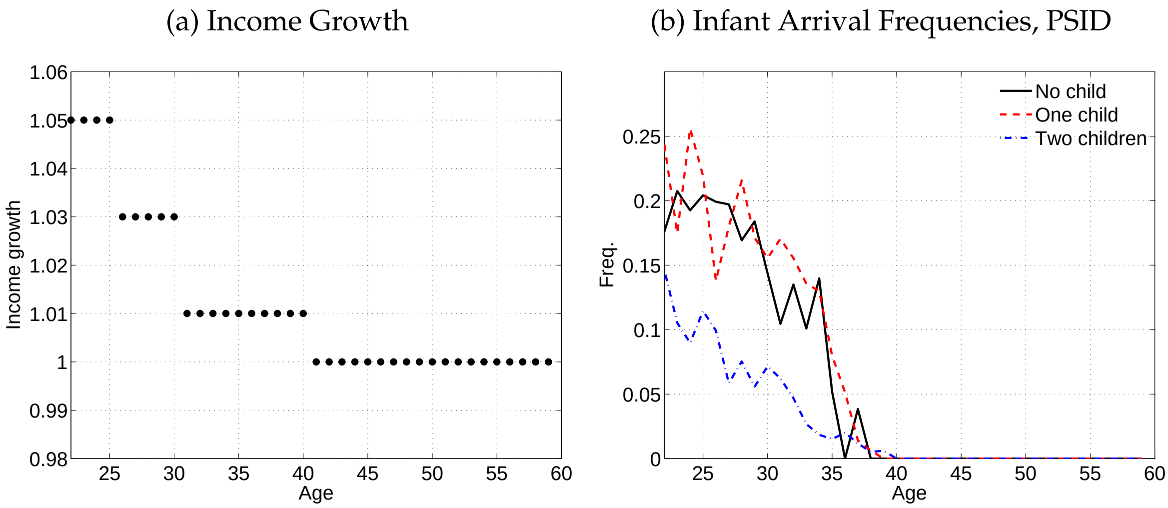


Figure 2 – Calibrated Income Growth and Arrival of Children.

Notes: Figure 2a reports how permanent income grows in the life cycle model while panel b shows how the arrival of children is calibrated using the PSID. The arrival of children is based on the PSID data described in Section 4.

Figure 3 presents simulated age profiles for income, consumption and wealth for different values of θ. All consumption profiles (even if children do not affect consumption) exhibit a hump when households are in the mid-40s, as typically observed in real data. If children affect consumption, the hump is more pronounced by a steeper consumption profile for young households and a subsequent larger decrease in consumption after the mid-40s. Income uncertainty, income growth and credit constraints produce an increasing consumption profile early in life, even if children do not affect consumption. The retirement motive produces an incentive (depending on the size of γ) to accumulate wealth for retirement later in life producing a downward sloping consumption profile after the mid-40s.

The consumption profiles are very similar for young households across θ-values. This is because credit constraints prevent households from borrowing against future income growth to increase consumption when children arrive – despite they would

(a) Simulated Income and Consumption (b) Simulated Wealth

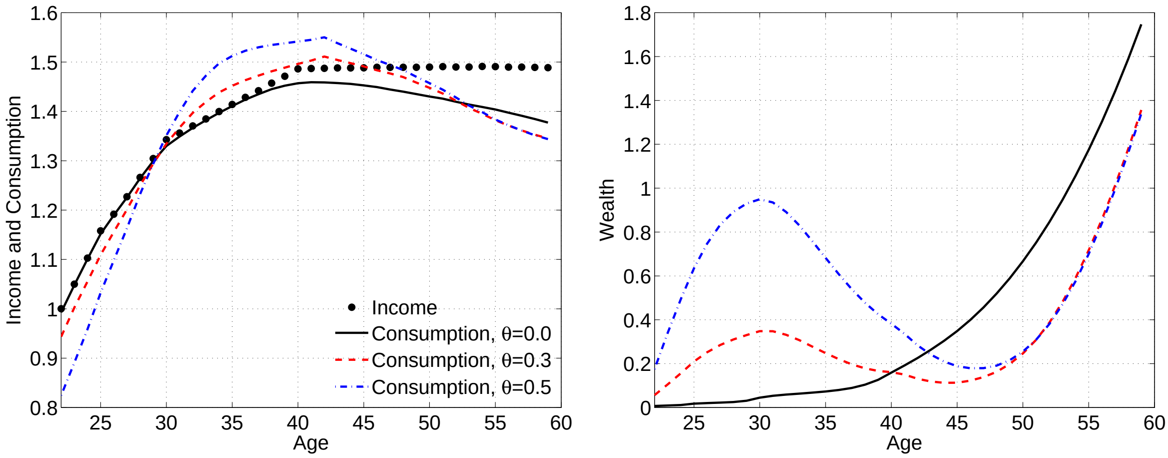


Figure 3 – Simulated Income, Consumption and Wealth Profiles.

Notes: Figure 3 illustrates the average age profile of income, consumption and wealth for 50,000 simulated households for different values of θ. Panel a shows how consumption profiles change relatively little across models with no effects of children, $\theta = 0$, through a model in which children are important, $\theta = 0.5$. Panel b shows how the wealth accumulation, on the other hand, is greatly affected by the importance of children. Particularly, a hump in the wealth profile emerges as children becomes more important.

want to, had unlimited borrowing been possible. Hence, the effect of children would in general be underestimated using young households, as shown earlier. Noticeably, young households accumulate large amounts of wealth in anticipation of children arriving in the future. When children subsequently arrive, wealth is almost depleted such that the credit constraint is binding for many households when children eventually leave. The relative drop in consumption from a *constrained* level to an (potentially) unconstrained level, when children leave, will in general be less than the relative change if households had never been constrained. Hence, the effect of children would be underestimated when only using older households as shown using the simple four-period model above.

Empirical age profiles of observed household wealth is typically not hump-shaped as illustrated in Figure 3 but rather monotonically increasing (Cagetti, 2003). This suggests that children might not be as important for consumption over the life cycle as previously found in the existing literature. I confirm this in section 4 below where I estimate the proposed bounds using the PSID.

Table 2 reports the average estimate of θ using all households, both young and old, from 1,000 MC runs and the standard deviation across these runs. For each run, data are simulated from the life cycle model for 50,000 households from age 22 through 59 and 20 random adjacent time-observations are drawn for each household from this simulation. It is clear that for low levels of θ, both the log-linearized and non-linear

GMM estimators overestimate the effect of children on consumption while they underestimate the effect if θ is large. This is true irrespectively if the actual change in number of children ($\Delta\mathbf{z}_t$) are used in the estimation or the cohort average number of children ($\Delta\bar{\mathbf{z}}_t$) is used as instrument.

Table 2 – Monte Carlo Results, Both Young and Old Included.

	$\theta = 0.0$		$\theta = 0.1$		$\theta = 0.5$		$\theta = 1.0$	
Instr.	LogLin	GMM	LogLin	GMM	LogLin	GMM	LogLin	GMM
$\Delta\mathbf{z}_t$	0.015	0.006	0.086	0.078	0.227	0.221	0.397	0.374
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.002)	(0.002)
$\Delta\bar{\mathbf{z}}_t$	0.125	0.038	0.156	0.075	0.255	0.166	0.475	0.310
	(0.002)	(0.001)	(0.002)	(0.001)	(0.002)	(0.002)	(0.003)	(0.003)

Notes: The average of all MC estimates and standard deviations (in parenthesis) across Monte Carlo runs are reported. All results are based on 1,000 independent estimations on simulated data from the life cycle model described in Section 3.2 with the parameters presented in Table 1. For each run, data are simulated for 50,000 households from age 22 through 59 and a random adjacent period of length time-observations are drawn from this simulation. All individuals are initiated at age 22 with zero wealth, $A_{21} = 0$, permanent income of one, $P_{22} = 1$, and no children. Children are assigned following the estimated arrival probabilities estimated from the PSID, reported in Figure 2b.

Figure 4 illustrates the proposed bounds based on the four period model in panel 4a and the multi-period life cycle model in panel 4b. The 45°–line represents the true value of θ while blue lines represent lower bounds and red lines represent upper bounds using the (cohort) average number of children as instrument. Young households are defined as those younger than 41.

The bounds derived from the simple four period model are very similar to the numerical bounds from the richer life cycle model. The bounds are fairly narrow for lower values of θ and the lower bound equals the true effect when $\theta = 0$ as expected. As the effect of children becomes larger, the bounds become wider and the upper bound is closest to the truth. The non-linear GMM estimator produces almost identical bounds as the log-linearized Euler equation, indicating that the nonlinear Euler equation ignoring credit constraints is an equally poor approximation to the true constrained Euler equation as the log-linearized Euler equation is.

The results show that the standard Euler equation estimators cannot in general estimate the effect of children on consumption when households face potentially binding credit constraints. Further, the multi-period life cycle model confirms that the proposed bounds are sensible in a more realistic framework. Below, I apply the bounds to the PSID and in section 5 I discuss the robustness of the proposed bounds.

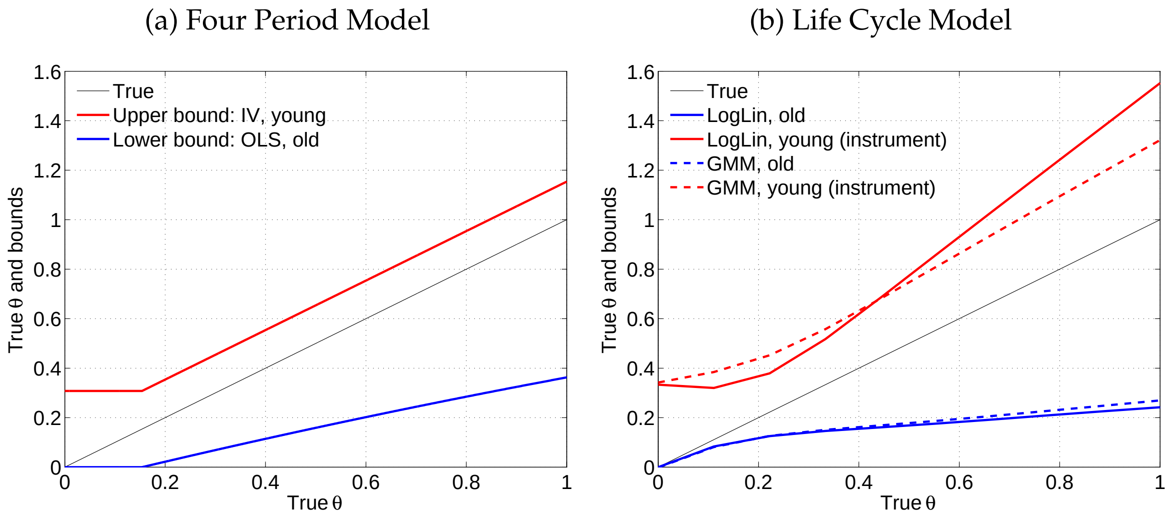


Figure 4 – Proposed Bounds, Four-Period Model and Life Cycle Model.

Notes: Figure 4 illustrates the proposed bounds based on the four period model in panel a and the life cycle model in panel b. The 45°–line represents the true value of θ while blue lines represent lower bounds and red lines represent upper bounds using the (cohort) average number of children as instrument. Solid lines are based on the log-linearized Euler equation (3) and dashed lines are based on the non-linear Euler equation estimated with the GMM estimator (2). Young households are defined as younger than 41.

4 Empirical Results from the PSID

The Panel Study of Income Dynamics (PSID) contains information on food consumption and has been used for a wide range of studies, including estimation of the effect of children on consumption. To study the evolution and link between income and consumption inequality over the 1980s, Blundell, Pistaferri and Preston (2008) impute total non-durable consumption for PSID households using food consumption measures in the CEX and the PSID. I use their final data set and refer the reader to their discussion of the PSID data.

The sample period is 1978 to 1992 and only male headed continuously married couples are used. The years 1987 and 1988 are not used because consumption measures were not collected those years. Since the present study focus on the effect of children on household consumption, I restrict the sample to cover households in which the wife is aged 20 to 59.[11] The supplementary low-income sub sample (SEO) is excluded from the analysis. All sample selection criteria leaves an unbalanced panel of 1,808 households observed for at most 13 periods in the final sample of in total 13,516 non-missing observations.[12] Households are classified as high skilled if the male head has

[11] Blundell, Pistaferri and Preston (2008) use households in which the husband is aged 30 to 65.

[12] In an earlier working paper (Jørgensen, 2014), for tractability of an alternative estimation procedure, I restricted the sample further and removed year trends prior to estimation. The results presented here will, therefore, differ slightly from those reported in the earlier working paper.

ever enrolled in college, including college drop-outs.

Table 3 – Log-Linear Euler Equation Estimates, PSID.

	Low skilled		High skilled	
	OLS, age$\geq$ 45†	IV, age$\leq$45‡	OLS, age$\geq$ 45†	IV, age$\leq$45‡
	Food consumption			
$\Delta\#kids$	0.031	0.122	0.035	0.127
	(0.037)	(0.057)	(0.028)	(0.048)
Constant	-0.097	-0.049	-0.046	-0.016
	(0.032)	(0.018)	(0.027)	(0.023)
Obs	1304	4425	1700	3351
R2	0.005	0.020	0.009	0.018
	Non-durable consumption (imputed)			
$\Delta\#kids$	0.058	0.130	-0.033	0.047
	(0.063)	(0.079)	(0.026)	(0.048)
Constant	-0.668	-0.101	-2.011	-1.993
	(0.116)	(0.073)	(0.028)	(0.023)
Obs	1304	4425	1700	3351
R2	0.062	0.002	0.708	0.675

Notes: Reported are estimates of $\widehat{\rho^{-1}\theta}$ and a constant from a log-linear Euler equation estimation of food consumption in the top panel and total non-durable consumption, imputed by Blundell, Pistaferri and Preston (2008). Robust standard errors in parenthesis. All regressions include year-dummies. Households are classified as high skilled if the male head has ever enrolled in college, including college drop-outs. Age refers to the wife's age.

† This corresponds to the suggested lower bound of $\rho^{-1}\theta$.

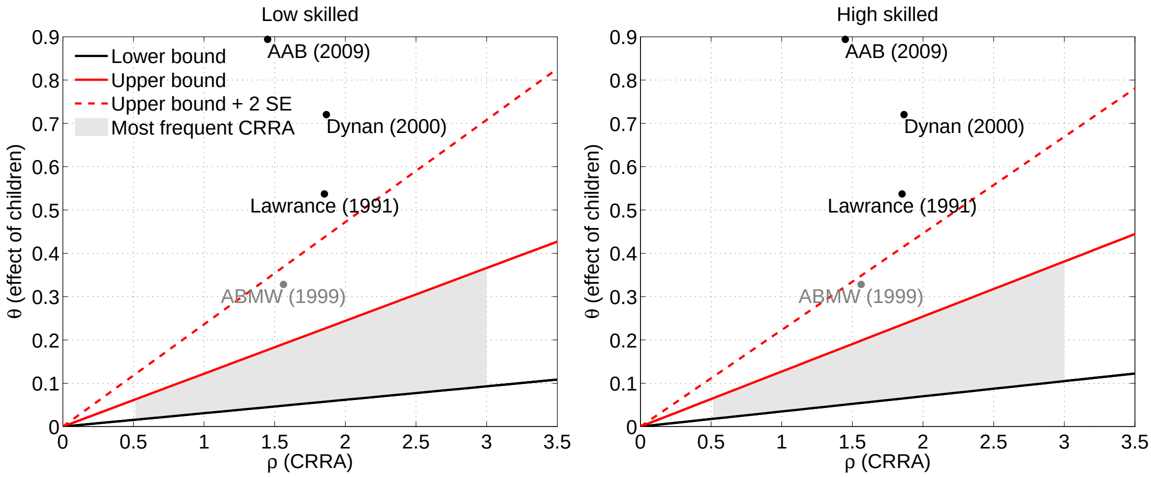
‡ This corresponds to the suggested upper bound of $\rho^{-1}\theta$. The number of children is instrumented with the cohort-average number of children.

Table 3 reports the estimated bounds of $\rho^{-1}\theta$ for the PSID data using the log-linearized Euler equation (3) with year-dummies included in all regressions. Recall that the suggested lower bound on θ can be estimated using the change in number of children ($\Delta \mathbf{z}_t$) while restricting the sample to include only older households and an upper bound can be found by using the cohort-average number of children ($\Delta\bar{\mathbf{z}}_t$) as an instrument while restricting the sample to younger households.

To illustrate how these bounds provide valuable information, imagine having estimated the IES simultaneously (as done in, e.g., Attanasio, Banks, Meghir and Weber, 1999 using interest rate variation) or having information of this parameter from elsewhere. For example, Gourinchas and Parker (2002) estimate $\rho \approx 0.87$ for high school graduates (low skilled by my definition) and $\rho \approx 2.29$ for college graduates (high skilled by my definition). Using these values, we could infer that the effect of children on total non-durable consumption is between .05 and .11 for low skilled and between

−.08 and .11 for high skilled.

(a) Food consumption



(b) Total non-durable consumption (imputed)

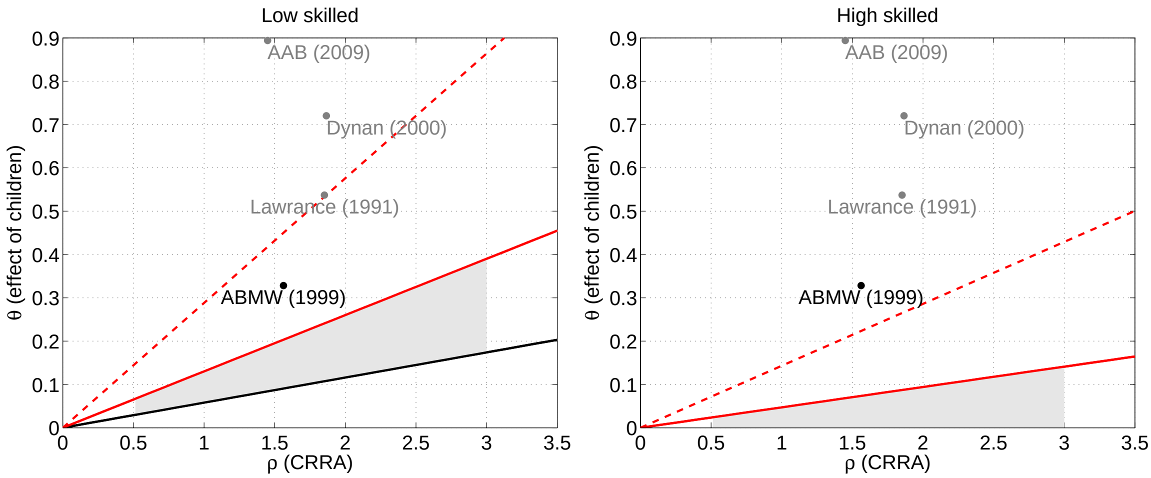


Figure 5 – Estimated Bounds of the Effect of Children on Consumption from the PSID.

Notes: Figure 5 reports the upper and lower bounds for low and high skilled households when varying ρ, the inverse of elasticity of intertemporal substitution. The top panel a reports results when using changes in log food consumption while the bottom panel b reports results when using changes in log non-durable consumption, imputed by Blundell, Pistaferri and Preston (2008). "Lawrance (1991)" refers to from Lawrance (1991), "ABMW (1999)" refers to Attanasio, Banks, Meghir and Weber (1999), "Dynan (2000)" refers to Dynan (2000) and "AAB (2009)" refers to Alan, Attanasio and Browning (2009). Gray dots illustrate that a different measure of consumption was used in the associated study.

Figure 5 reports how the upper and lower bounds for low and high skilled vary with the coefficient of relative risk aversion (the inverse of the IES). The top panel (panel 5a) reports results when using changes in log food consumption while the bottom panel (panel 5b) reports results when using changes in log non-durable consumption, imputed by Blundell, Pistaferri and Preston (2008).

The estimated effects of children on consumption reported in the existing literature are outside the proposed bounds. Specifically, the reported estimate of $\rho^{-1}\theta$ in the influential study by Attanasio, Banks, Meghir and Weber (1999) of 0.21 and their estimated ρ^{-1} of .64 produce an effect of children on *non-durable consumption* in the CEX around .33. This is outside the upper bounds reported in Figure 5 for values of ρ in a neighborhood of their estimated value of 1.56. Likewise, Figure 5 maps the implied estimated effect of children on *food consumption* in the PSID reported in Lawrance (1991); Dynan (2000) and Alan, Attanasio and Browning (2009). The latter is based on a non-linear GMM estimator allowing for log-normal measurement error in consumption while the two former studies are based on the log-linearized Euler equation. All these estimates are outside the upper bound. Adding two times the standard error of the estimated $\widehat{\rho^{-1}\theta}$ reported in Table 3 widens the bounds significantly, but only the estimate from Attanasio, Banks, Meghir and Weber (1999) is now included in the bounds.

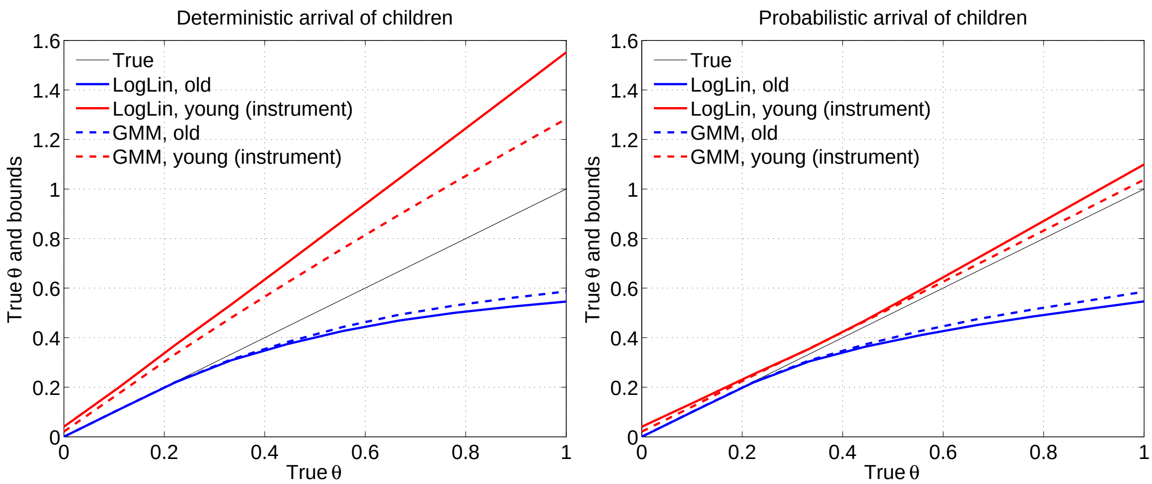
5 Robustness of Bounds

Choosing the age at which to split the sample into young and older households is not obvious. One choice could be to choose the age at which the average number of children starts to decline since the behavior of households should differ when children arrive from when they leave, c.f. the above discussion. Simply estimating different parameters related to when children arrive and move could be a route to pursue. Alternatively, the age at which average net wealth is significantly larger than average income could be chosen since around this point (on average) households are less affected by credit constraints.

A crucial assumption when calculating the bounds above is that of the researcher having knowledge on other structural parameters. Using the exact GMM estimation approach, both the discount factor, β, and the relative risk aversion, ρ, should be estimated simultaneously or qualified guesses on these parameters should be used. Log-linearized Euler equation estimation requires information only on the risk aversion parameter. This is a drawback but varying these preference parameters in "accepted" ranges produces a set of bounds with information on the size of the effect of children on consumption.

The bounds are robust to changing the calibrated parameters in Table 1. Figure 6 illustrates the bounds from models in which the discount factor, β, is .975 and .99. The bounds are robust and for larger values of the discount factor the bounds become increasingly tight. Especially for low values of θ, both the upper and lower bounds are close to the true value when households are more patient. This stems from the fact that more patient households put more emphasis on future marginal utility and, thus, accumulate more wealth prior to the arrival of children. In turn, the credit constraint has less bite.

(a) $\beta = 0.975$



(b) $\beta = 0.990$

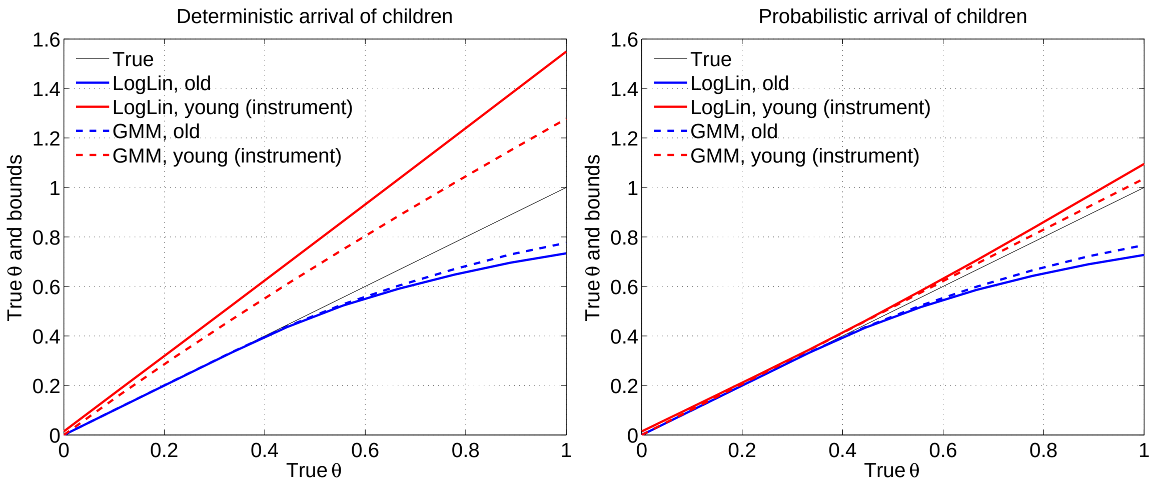


Figure 6 – Bounds, $\beta = \{0.975, 0.99\}$.

Notes: Figure 6 illustrates the proposed bounds based the life cycle model with a discount factor of .975 in panel a and .99 in panel b. The figures to the left illustrates the bounds from the baseline deterministic model, in which children are perfectly foreseen and figures to the right illustrates the bounds from a model in which children arrive probabilistically. The 45°–line represents the true value of θ while blue lines represent lower bounds and red lines represent upper bounds using the (cohort) average number of children as instrument. Solid lines are based on the log-linearized Euler equation (3) and dashed lines are based on the non-linear Euler equation estimated with the GMM estimator (2).

Below, I argue that the results are robust to alternative fertility processes, labor market costs of children and to replacing the explicit credit constraint with a zero-income shock. The robustness results are important since they stress that the true underlying effect of children on consumption is in between the lower and upper bound in realistic circumstances. An alternative route to estimating bounds could be to utilize the moment *inequality* rather than the equality in the GMM estimator (2). Assuming that an instrument is potentially positively correlated with the Euler residual, the inequality $\mathbb{E}[(\epsilon - 1)'Z] \geq 0$ could be used as a moment inequality to estimate bounds (Moon and Schorfheide, 2009). This approach is very interesting for future research but I do not pursue that strategy here.[13]

5.1 Alternative Fertility Processes

All results have been derived assuming that children are perfectly foreseen. This assumption has primarily been deployed for tractability of the four period model since that model could then be solved analytically. Versions of the model in which children arrive probabilistically as in Blundell, Dias, Meghir and Shaw (2013) produce qualitatively unchanged results.

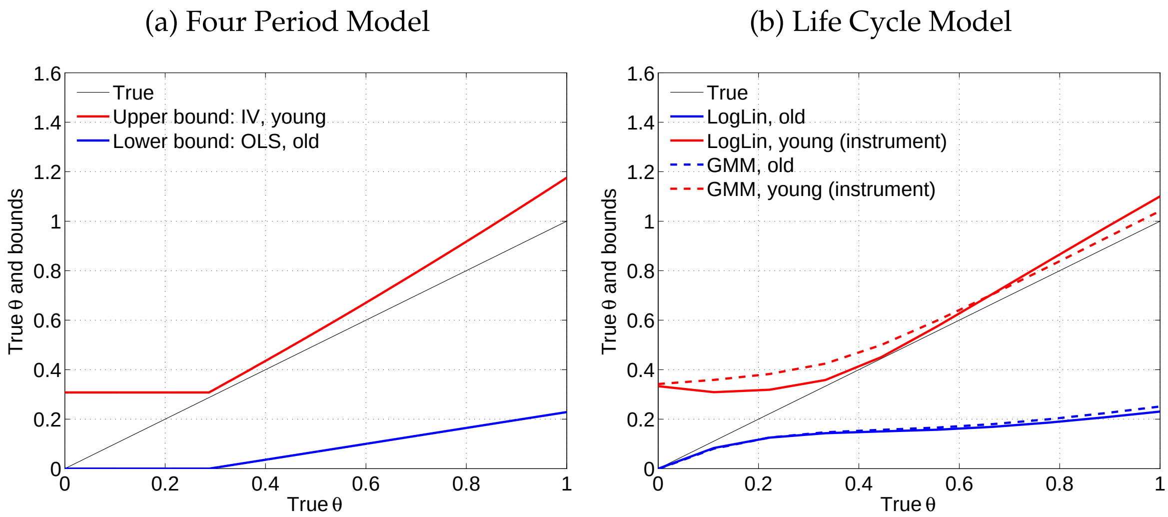


Figure 7 – Bounds, Probabilistic Arrival of Children.

Notes: Figure 7 illustrates the proposed bounds based on a model in which children arrive probabilistically rather than being perfectly foreseen as in the "deterministic" baseline model. Panel a illustrates the bounds from the four period model and the bounds from the life cycle model is illustrated in panel b. The 45°–line represents the true value of θ while blue lines represent lower bounds and red lines represent upper bounds using the (cohort) average number of children as instrument. Solid lines are based on the log-linearized Euler equation (3) and dashed lines are based on the non-linear Euler equation estimated with the GMM estimator (2). Young households are defined as younger than 41.

Figure 7 illustrates the proposed bounds based on the four period model in panel 7a and the life cycle model in panel 7b for the probabilistic version of the models. The bounds are very similar to those presented from the baseline model and the upper bound is close to the true effect of children when children arrive probabilistically.

In the probabilistic version, households are identical prior to arrival of children. In the four period model, all households save exactly the same in period zero, prior to a

[13] I am grateful to Dennis Kristensen for pointing this out to me.

child potentially arrives in period one. In period one, households who do not receive a child increase consumption due to the fact that it has been revealed for them that they will not have children. Their accumulated wealth from period zero is now distributed across remaining periods. This increase in consumption of childless households will bias the estimates downwards. This is true even if households do *not* face credit constraints and motivates the use of the OLS estimate from older households rather than the OLS estimate from young households to estimate a lower bound.

Children could, alternatively, be chosen endogenously. Endogenous fertility would significantly alter the economic environment and is typically *not* implemented in empirical work on the effect of children on consumption. It is important to stress that households in the deterministic life cycle model have strong incentives to accumulate wealth to finance increased consumption when children arrive. Thus, the baseline model is the one in which credit constraints has the *least* of an effect on the implied consumption behavior, compared to the probabilistic version. Further, the biological "constraint" on female fertility will interplay with the financial constraints and the latter is, thus, still likely to be important for household behavior in a model in which children are perfectly chosen by households (Almlund, 2013).

Importantly, the bounds will still be valid even if the financial credit constraint has less bite when households perfectly chose when to have children. Specifically, in the extreme case when households are perfectly able to break free of the constraint and is *never* affected by constraints, the lower bound will overlap with the true effect of children and the upper bound will be slightly above the true effect (see discussion in section 5.3).

5.2 Children and the Labor Market

As in the rest of the literature on the effect of children on consumption, income is assumed *independent* of household composition. If income depends on household composition, the results will change depending on in which ways children affect the labor market income of household members. The focus on the present study is on how Euler equation estimation fails to uncover the underlying effect of children if potentially binding credit constraints (either explicit or self-imposed) are ignored. Although allowing income to vary with household composition is an interesting avenue for future research, I have not pursued that here. The primary reason is that how children should affect labor market outcomes is not obvious and the results, in turn, would be hard to relate to existing estimates of the effect of children on consumption.

Children might, however, affect the number of hours worked. Calhoun and Espenshade (1988) estimate a substantial decrease in labor market hours of American females in response to childbearing. In a more recent working paper, Adda, Dustmann and Stevens (2012) analyze in a life cycle model of German households, the career cost of children and find that children can explain a substantial portion of the male-female gender wage gap. In their model of fertility, occupational choice and labor supply, consumption is assumed linear in income. In turn, all households are constrained in their model illustrating that implementing *all* features into one model is not nearly as feasible as it is interesting.

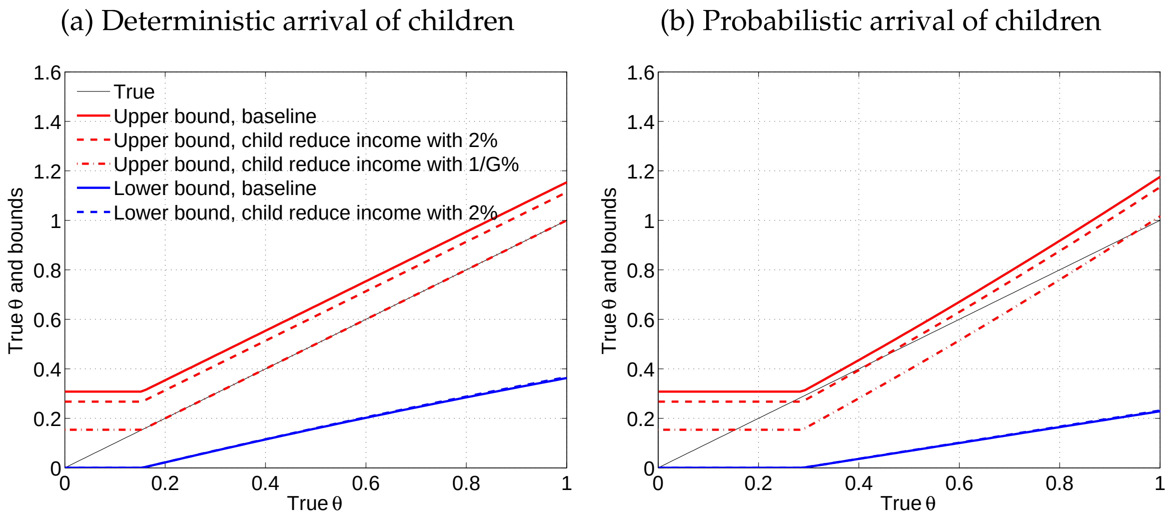


Figure 8 – Bounds if Children Reduce Labor Market Income.

Notes: Figure 8 illustrates the proposed bounds from the four period model when children reduce labor market income with 2 percent. Panel a illustrates the bounds from the model in which children arrive deterministically and the probabilistic version is illustrated in panel b. The 45°–line represents the true value of θ while blue lines represent lower bounds and red lines represent upper bounds using the (cohort) average number of children as instrument. Solid lines are based on the baseline case where children does not affect labor market income and dashed lines represent the extreme case where children reduce labor market income with 2 percent.

The bounds are still valid if children reduce permanent income, as suggested by the results above. This is true as long as children does not reduce permanent income by more than the permanent income growth as illustrated in Figure 8a. If children arrive deterministically and children reduce income to a degree that only childless households experience income growth, the upper bound equals the true effect. If children arrives probabilistically, however, the upper bound might be below the true effect, as illustrated by panel 8b.

5.3 Self-imposed No-borrowing vs. Explicit Credit Constraint

The results generalize to cases in which consumers do not face credit constraints. If risk averse consumers instead face a positive probability of receiving a zero-income shock (as in Carroll, 1997 and Gourinchas and Parker, 2002), all results concerning the log-linearized Euler equation (3) still hold. This is basically because risk averse consumers will instead face a "self-imposed" no-borrowing constraint stemming from the fear of receiving zero income in all future periods with consumption of zero as a consequence (Schechtman, 1976; Zeldes, 1989b; Carroll, 1992). In turn, consumption will respond substantially to negative income shocks if either explicit or self-imposed credit constraints affect consumers, increasing the variance in consumption growth. Because higher order moments (such as something like the variance of consumption growth, Carroll, 2001) enters the reduced form residual, $\tilde{\varepsilon}$, log-linearized Euler equation estimation will not be able to uncover the effect of children on consumption. This result supports and extends the critique in Carroll (2001) on the inability of log-linearized Euler equation estimation to uncover the IES to the inability to uncover demographic effects on consumption.

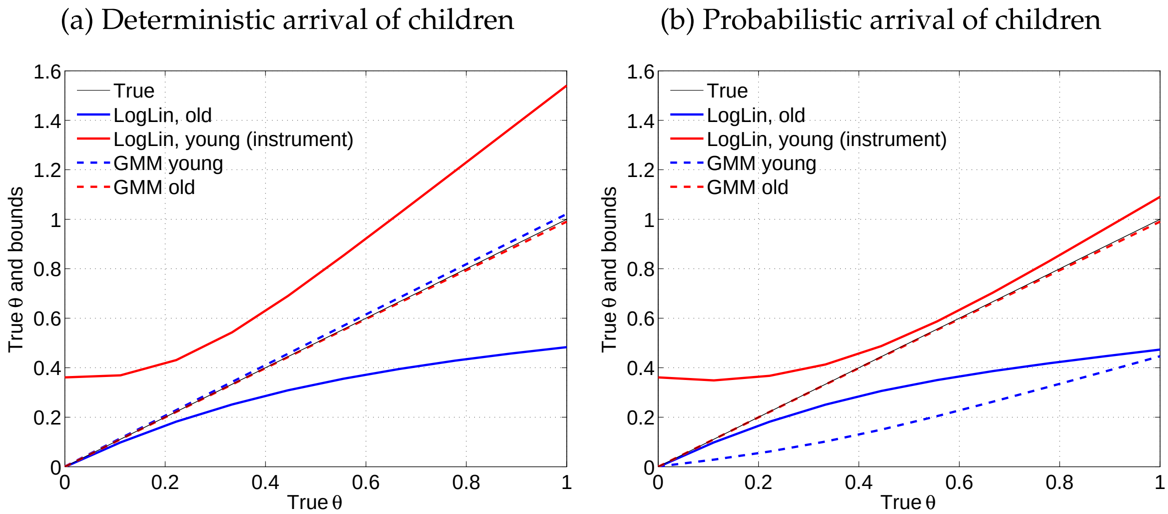


Figure 9 – Bounds, No Explicit Constraint but Self-Imposed No-borrowing.

Notes: Figure 9 illustrates the proposed bounds based on a model in which there is unlimited borrowing but a positive probability of receiving zero income. Panel a illustrates the bounds from the baseline deterministic model, in which children are perfectly foreseen and panel b illustrates the bounds from a model in which children arrive probabilistically. The 45°–line represents the true value of θ while blue lines represent lower bounds and red lines represent upper bounds using the (cohort) average number of children as instrument. Solid lines are based on the log-linearized Euler equation (3) and dashed lines are based on the non-linear Euler equation estimated with the GMM estimator (2). Young households are defined as younger than 41.

In absence of "explicit" credit constraints, the Euler equation in (1) has mean one because $\lambda_t = 0$, $\forall t$ and the exact GMM estimator is expected to produce unbiased estimates of the effect of children. Figure 9 illustrates the log-linearized Euler equation bounds along with GMM estimates from a model with a .3 percent risk of zero household income (as calibrated in Gourinchas and Parker, 2002). Panel 9a illustrates the bounds from the deterministic model while panel 9b illustrates the bounds from the probabilistic version of the model. It is clear that when children are perfectly foreseen (panel 9a), the non-linear GMM produces the correct estimate.

Interestingly, if children arrive probabilistically, using young households to uncover the effect of children on consumption is impossible even if there is no explicit constraint on borrowing, as shown by the fact that the blue dashed (GMM on young) in Figure 9b is significantly below the 45°–line. This stems from the feature of the probabilistic model that households who turn out childless have accumulated as much wealth in their youth as those households who eventually had children. Comparing consumption growth of households with and without children will, thus, underestimate the true effect of children, if children arrive probabilistically. Appendix A.4 in the online supplemental material proves this result (using the four period model): If children arrive probabilistically, using the cohort average number of children as instrument when estimating the log-linearized Euler equation (proposed upper bound) overestimates the effect of children even if there is *no* explicit credit constraint.

Table 4 – Monte Carlo Results, No Explicit Constraint.

	$\theta = 0.0$		$\theta = 0.1$		$\theta = 0.5$		$\theta = 1.0$	
Instr.	LogLin	GMM	LogLin	GMM	LogLin	GMM	LogLin	GMM
	Deterministic arrival of children							
$\Delta \mathbf{z}_t$	0.015	−0.000	0.099	0.100	0.358	0.500	0.580	0.995
	(0.001)	(0.003)	(0.001)	(0.003)	(0.001)	(0.011)	(0.002)	(0.024)
$\Delta \bar{\mathbf{z}}_t$	0.118	−0.000	0.167	0.104	0.311	0.518	0.530	1.024
	(0.002)	(0.007)	(0.002)	(0.008)	(0.002)	(0.017)	(0.003)	(0.031)
	Probabilistic arrival of children							
$\Delta \mathbf{z}_t$	0.015	−0.000	0.086	0.084	0.283	0.424	0.447	0.852
	(0.001)	(0.003)	(0.001)	(0.003)	(0.001)	(0.011)	(0.001)	(0.023)
$\Delta \bar{\mathbf{z}}_t$	0.118	−0.000	0.166	0.100	0.272	0.501	0.497	1.000
	(0.002)	(0.007)	(0.002)	(0.008)	(0.002)	(0.017)	(0.002)	(0.028)

Notes: The average of all MC estimates and standard deviations (in parenthesis) across Monte Carlo runs are reported. All results are based on 1,000 independent estimations on simulated data from the life cycle model described in Section 3.2 with the parameters presented in Table 1. For each run, data are simulated for 50,000 households from age 22 through 59 and a random adjacent period of length 20 time-observations are drawn from this simulation. All individuals are initiated at age 22 with zero wealth, $A_{21} = 0$, permanent income of one, $P_{22} = 1$, and no children. The results are based on a life cycle model in which there is no explicit constraint but instead a .3 percent risk of a zero income shock, producing a self-imposed no-borrowing constraint. In the top panel, children arrive with perfect foresight while in the bottom panel children arrive probabilistically, following the estimated arrival probabilities estimated from the PSID, reported in Figure 2b.

Table 4 reports the Monte Carlo results from pooling young and older households, using the life cycle model *without* an explicit credit constraint. Th top panel reports results from the baseline model and the bottom panel reports the results if children arrive probabilistically. It is clear than the non-linear GMM estimator can uncover the correct estimate while the log-linearized Euler equation cannot when children are perfectly foreseen. The results in the bottom panel, where children arrive probabilistically, confirm that in this case, the GMM estimator using both young and older households cannot uncover the true effect of children on consumption (unless it is zero). Using only older households, in which children leave, will, however, lead the GMM estimator to produce unbiased estimates when children arrive probabilistically (Figure 9b).

6 Concluding Discussion

Many studies use estimators derived from the consumption Euler equation. Especially the log-linearized Euler equation is popular since it yields estimable equations linear in parameters which can easily be estimated with synthetic cohort panels and handle measurement error through instrumental variables estimation. Although these estimators have now become work horses in the analysis of intertemporal consumption behavior, little is known about their performance when households face potentially binding credit constraints and the standard Euler equation, thus, no longer holds.

I have showed how both the non-linear and the log-linearized Euler equation estimators fail to uncover the true underlying effect of children on consumption when potentially binding credit constraints are ignored. Through splitting the sample into young households, in which children arrive, and older households, in which children leave, I propose a tractable approach to uncovering bounds of the effect of children on consumption using these conventional estimators.

Estimating the proposed bounds on PSID data indicates that *all*, to the best of my knowledge, existing estimates of the effect of children on consumption are above the upper bound. In turn, these results suggest that the importance of children in intertemporal consumption behavior, found in previous studies, might simply proxy for the inability of households to borrow against future income growth.

Arguably, the proposed bounds suffer from many of the same assumptions as most existing empirical literature analyzing the intertemporal consumption behavior. Particularly, it has been assumed throughout (and in the related literature) that fertility is exogenous and children do *not* affect labor market outcomes. Although the bounds are somewhat robust to these assumptions, they have been invoked for tractability and comparability with existing studies of the effect of children on consumption. Allowing for endogenous fertility and endogenous labor market supply with children affecting the dis-utility from work is extremely interesting for future research.

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Online Appendix (Not for Publication)

Euler Equation Estimation: Children and Credit Constraints

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September 5, 2014

A Solving the Four-Period Model

All variables are normalized with income such that small letter variables are normalized ones. Hence, e.g., $m_1 = (A_0 + Y_1)/Y_1 = G_1^{-1}a_0 + 1$ since $Y_1 = G_1 Y_0$. In all other periods, income is constant. This normalization facilitates solving the model analytically for *all* possible values of income. The resulting consumption function should be multiplied with current period income to give the un-normalized level of consumption, $C_t^\star = Y_t c_t^\star$. The consumption functions in periods $t = 1, 2, 3$ are independent of whether children arrive deterministically or probabilistically since it is assumed that children, if present in period $t = 1$, will move with certainty in period $t = 2$. Therefore, I first solve for optimal consumption in period $t = 1, 2, 3$ which are identical for the deterministic and probabilistic versions and then turn to the initial period consumption, prior to potential arrival of children. This analysis is split between the model in which children arrive deterministically and the model in which children arrive probabilistically.

In the terminal period, period three, all resources are consumed ($c_3^\star = m_3$) and the *unconstrained* Euler equation linking period two and period three consumption is then

$$c_2^{-\rho} = m_3^{-\rho}$$

such that inserting normalized resources, $m_3 = m_2 - c_2 + 1$ and re-arranging shows that optimal consumption in period two is the minimum of available resources, m_2, and $\frac{1}{2}(m_2 + 1)$. Since income does not fall between period one and two and because negative wealth is not allowed, $m_2 \geq 1$ and optimal consumption is then

$$c_2^\star(m_2) = \frac{1}{2}(m_2 + 1). \tag{A.1}$$

In period one, a child may be present and the *unconstrained* Euler equation is given by

$$c_1^{-\rho} \exp(\theta \mathbf{z}_1) = c_2^{-\rho},$$

such that inserting normalized resources and re-arranging yields,

$$c_1^\star(m_1|\mathbf{z}_1) = \min\left\{ m_1 \, , \frac{m_1 + 2}{1 + 2\exp(-\rho^{-1}\theta \mathbf{z}_1)} \right\}, \tag{A.2}$$

where the constraint is binding if $m_1 \leq \underline{m}_1 \equiv \exp(\rho^{-1}\theta \mathbf{z}_1)$. Note, this is certainly the case if nothing is saved from period zero.

Optimal consumption in period $t = 0$ depends on whether children arrive deterministically or probabilistically in period one. I first derive optimal consumption in the case where children arrive deterministically and then turn to the probabilistic arrival of children.

A.1 Initial Period Consumption: Deterministic Arrival of Children

In the first period, the *unconstrained* Euler equation is

$$c_0^{-\rho} = G_1^{-\rho} \exp(\theta \mathbf{z}_1) c_1^{-\rho},$$

since income grows with a factor G_1 from period zero to period one. Since consumption in period one is potentially constrained, this has to be explicitly taken into account. First, assuming that period one consumption is less than available resources, $c_1 < m_1$, inserting the optimal consumption found in (A.2) and tedious re-arranging yields optimal consumption in this case,

$$c_0^{\star}(m_0|\mathbf{z}_1)^{\text{det}}|_{c_1<m_1} = \frac{m_0 + 3G_1}{3 + \exp(\rho^{-1}\theta\mathbf{z}_1)}. \tag{A.3}$$

If, on the other hand, consumption in period one is constrained ($c_1 = m_1$), inserting this in the Euler equation and re-arranging yields,

$$c_0^{\star}(m_0|\mathbf{z}_1)^{\text{det}}|_{c_1=m_1} = \frac{m_0 + G_1}{1 + \exp(\rho^{-1}\theta\mathbf{z}_1)}. \tag{A.4}$$

To determine which of the consumption functions is relevant, note that equation (A.3) would imply a too high level of consumption in period zero if ignoring, that at some point, consumption in period one would be constrained because "too little" is saved in period zero. Hence,

$$\tilde{c}_0^{\star}(m_0|\mathbf{z}_1)^{\text{det}} = \min\left\{m_0\,, \frac{m_0 + G_1}{1 + \exp(\rho^{-1}\theta\mathbf{z}_1)}\,, \frac{m_0 + 3G_1}{3 + \exp(\rho^{-1}\theta\mathbf{z}_1)}\right\},$$

where the level of period $t = 0$ resources implying that consumption in period one is constrained is the level of resources, $\underline{m}_0^1 = \exp(\rho^{-1}\theta\mathbf{z}_1)G_1$, that makes the expression in (A.4) to be less than that in (A.3). Hence, when $m_0 \leq \underline{m}_0^1$ optimal consumption in period $t = 0$ is given by equation (A.4) and when $m_0 > \underline{m}_0^1$ optimal consumption is given by equation (A.3).

When households are initiated with zero wealth ($a_{-1} = 0$) available normalized resources in period zero is one, $m_0 = 1$, and only equation (A.4) is relevant since $m_0 = 1 \leq \underline{m}_0^1$ for all values of $\theta \geq 0$ and $G_1 \geq 1$. Therefore, assuming no initial wealth and deterministic arrival of children, optimal consumption in period zero is given by

$$c_0^{\star}(m_0|\mathbf{z}_1)^{\text{det}} = \min\left\{m_0\,, \frac{m_0 + G_1}{1 + \exp(\rho^{-1}\theta\mathbf{z}_1)}\right\}, \tag{A.5}$$

where for $m_0 \leq \underline{m}_0^2 \equiv \exp(-\rho^{-1}\theta\mathbf{z}_1)G_1$, the constraint is binding and it is optimal to consume everything. This is very intuitive: If income growth is very high, resources next period is much higher and saving today is less attractive. On the other hand, if children affect marginal utility a lot (θ large), the level of resources should be very low before it is optimal not save anything for next period, in which a child arrives.

Note, focusing on the situation in which a child arrives in period one, if $m_0 \leq \underline{m}_0^2$ next-period resources is $m_1 = G_1^{-1}(m_0 - \frac{m_0+G_1}{1+\exp(\rho^{-1}\theta)}) + 1$ and we can check whether this is less than $\underline{m}_1$ which is the case as long as $\theta \geq 0$ and $G_1 \geq 1$. Hence, if $m_0 = 1 \leq \underline{m}_0$ we know that $m_1 \leq \underline{m}_1$ and $c_1^\star(m_1|\mathbf{z}_1 = 1) = m_1$. If a child do not arrive, optimal consumption in all periods would be to consume available resources, since in period $t = 0$, borrowing against future income growth is not allowed. This is used when calculating the OLS and IV estimators below.

A.2 Initial Period Consumption: Probabilistic Arrival of Children

The analysis, if children arrive probabilistically, is slightly more complicated than the above where children arrive deterministically. The *unconstrained* Euler equation is here given by

$$c_0^{-\rho} = G_1^{-\rho} c_1^{-\rho}(p\exp(\theta) + 1 - p),$$

such that in case where period one consumption is unconstrained ($c_1 < m_1$), inserting optimal consumption from equation (A.2) and re-arranging yields

$$c_0^\star(m_0)|_{c_1<m_1} = \frac{m_0 + 3G_1}{1 + \left[p\left(\exp(\rho^{-1}\theta) + 2\right)^\rho + (1-p)3^\rho\right]^{\frac{1}{\rho}}}. \tag{A.6}$$

However, if households are potentially credit constrained if a child arrives next period, the model has in general no analytical solution because the Euler equation is

$$\begin{aligned} c_0^{-\rho} &= G_1^{-\rho}\left[c_1^{-\rho}(1-p) + p\exp(\theta)m_1^{-\rho}\right], \\ &= G_1^{-\rho}\left[[\frac{1}{3}(G_1^{-1}(m_0 - c_0) + 3)]^{-\rho}(1-p) + p\exp(\theta)\left[G_1^{-1}(m_0 - c_0) + 1\right]^{-\rho}\right], \end{aligned}$$

with no general analytical solution for c_0. To complete arguments, I solve for the optimal consumption numerically using the EGM proposed by Carroll (2006), and use that solution, denoted $c_0^\star(m_0)|_{c_1=m_1}^{\text{num}}$. In turn, optimal period zero consumption is given by

$$c_0^\star(m_0) = \min\left\{m_0\,, c_0^\star(m_0)|_{c_1=m_1}^{\text{num}}\,, \frac{m_0 + 3G_1}{1 + \left[p\left(\exp(\rho^{-1}\theta) + 2\right)^\rho + (1-p)3^\rho\right]^{\frac{1}{\rho}}}\right\}. \tag{A.7}$$

Figure A.1a reports the consumption function in the deterministic case for the baseline parameters used herein ($p = 0.5$, $\rho = 2$, and $\theta = 0.5$) and Figure A.1b reports the consumption function in the probabilistic case. The numerical solutions to both models are reported to complete the solution and confirm the analytical results.

(a) Deterministic Arrival of Children (b) Probabilistic Arrival of Children

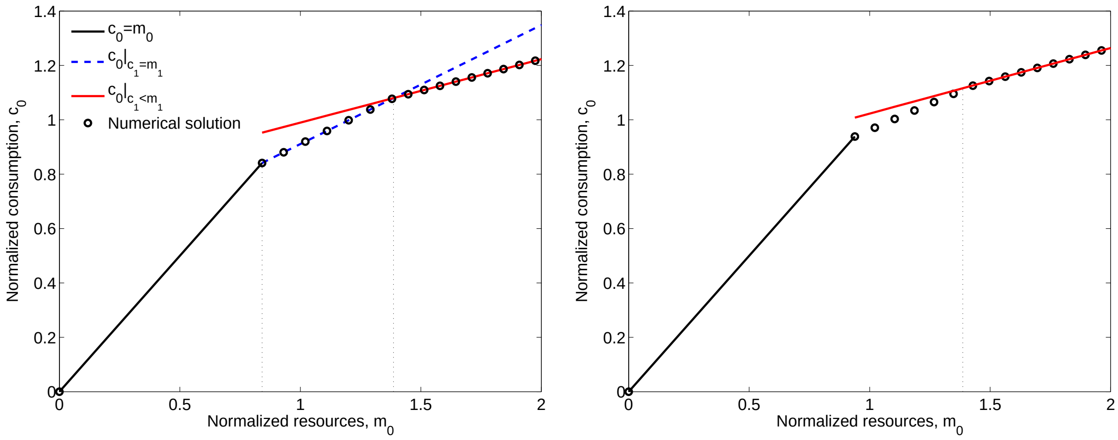

Figure A.1 – Period Zero Optimal Consumption Functions.

A.3 OLS and IV Estimates from the Four Period Model

We have that optimal consumption is given by

$$
\begin{aligned}
c_0^\star(m_0|\mathbf{z}_1) &= \begin{cases} m_0 & \text{if } m_0 \leq \exp(-\rho^{-1}\theta\mathbf{z}_1)G_1 \\ \frac{m_0+G_1}{1+\exp(\rho^{-1}\theta\mathbf{z}_1)} & \text{else} \end{cases} \\
c_1^\star(m_1|\mathbf{z}_1) &= \begin{cases} m_1 & \text{if } m_1 \leq \exp(\rho^{-1}\theta\mathbf{z}_1) \\ \frac{m_1+2}{1+2\exp(-\rho^{-1}\theta\mathbf{z}_1)} & \text{else} \end{cases} \\
c_2^\star(m_2) &= \frac{1}{2}(m_2+1) \\
c_3^\star(m_3) &= m_3.
\end{aligned}
$$

The OLS estimator is given as

$$
\theta_{OLS}^t = (\Delta \log C_t|_{\mathbf{z}_1=1} - \Delta \log C_t|_{\mathbf{z}_1=0})\rho,
$$

while the IV estimator, using the (cohort) average number of children as instrument, $Z = p$, is

$$
\begin{aligned}
\theta_{IV}^t &= \frac{\mathbb{E}[\Delta \log C_t' p]}{\mathbb{E}[p'p]}\rho, \\
&= \frac{1}{p}(p\Delta \log C_t|_{\mathbf{z}_1=1} + (1-p)\Delta \log C_t|_{\mathbf{z}_1=0})\rho.
\end{aligned}
$$

Inserting the optimal consumption for given set of parameters. Let $m_0 > \exp(-\rho^{-1}\theta\mathbf{z}_1)G_1$ (saves in period zero) and note that $m_0 = 1$, such that this implies that $\theta > \log(G_1)\rho$. The growth in log consumption is then (using the result that consumption is, then, constrained in period one)

$$\begin{aligned}\theta_{OLS}^{young} &= \begin{cases} \theta - \log(G_1)\rho & \text{if } \theta > \log(G_1)\rho \\ 0 & \text{if } 0 \leq \theta \leq \log(G_1)\rho \end{cases} \\ &\leq \theta\end{aligned}$$

Hence, OLS estimates will under estimate the true effect of children on consumption. The IV estimator is

$$\begin{aligned}\theta_{IV}^{young} &= \begin{cases} \theta + (1-p)/p\log(G_1)\rho & \text{if } \theta > \log(G_1)\rho \\ \log G_1\rho/p & \text{if } 0 \leq \theta \leq \log(G_1)\rho, \end{cases} \\ &\geq \theta\end{aligned}$$

such that IV estimation over-estimates the effect. However, as θ increases - for a fixed p and G_1 - the over estimation becomes potentially small.

Turning to older households, when children leave, the OLS estimate is

$$\begin{aligned}\theta_{OLS}^{old} &= \begin{cases} \rho\log(\frac{1+G_1}{G_1}) - \rho\log(1+\exp(-\rho^{-1}\theta\mathbf{z}_1)) & \text{if } \theta > \log(G_1)\rho \\ 0 & \text{if } 0 \leq \theta \leq \log(G_1)\rho, \end{cases} \\ &\leq \theta\end{aligned}$$

such that only if $\theta = 0$ will OLS produce a consistent estimate. Since consumption does not change between period one and two if there was no child in period one, the IV estimator is identical to OLS,

$$\theta_{IV}^{old} = \theta_{OLS}^{old}.$$

A.4 Upwards Bias of IV using Young Households Without Credit Constraints

Here, I show that in the model where children arrive probabilistically and there is *no* explicit credit constraints, there is still a (small) positive bias from IV estimation. Inserting optimal consumption in absence of credit constraints,

$$C_0 = Y_0 \frac{m_0 + 3G_1}{1 + \left[p\left(\exp(\rho^{-1}\theta) + 2\right)^{\rho} + (1-p)3^{\rho}\right]^{\frac{1}{\rho}}} \quad , \quad C_1(\mathbf{z}_1) = Y_1 \frac{m_1 + 2}{1 + 2\exp(-\rho^{-1}\theta\mathbf{z}_1)},$$

into the IV estimator yields

$$\begin{aligned}
\hat{\theta}_{IV}^{young} &= \log\left(Y_1 \frac{m_1+2}{1+2\exp(-\rho^{-1}\theta)}\right) - \log\left(Y_0 \frac{m_0+3G_1}{1+\left[p\left(\exp(\rho^{-1}\theta)+2\right)^{\rho}+(1-p)3^{\rho}\right]^{\frac{1}{\rho}}}\right) \\
&+ (1-p)/p\left(\log\left(Y_1\frac{1}{3}(m_1+2)\right) - \log\left(Y_0\frac{m_0+3G_1}{1+\left[p\left(\exp(\rho^{-1}\theta)+2\right)^{\rho}+(1-p)3^{\rho}\right]^{\frac{1}{\rho}}}\right)\right) \\
&= \rho^{-1}\theta - \left(\log\left(\exp(\rho^{-1}\theta)+2\right) + (1-p)/p\log(3)\right) \\
&\quad + p^{-1}\left[\log\left(\frac{m_0+3G_1-c_0}{m_0+3G_1}\right) + \log\left(1+\left[p\left(\exp(\rho^{-1}\theta)+2\right)^{\rho}+(1-p)3^{\rho}\right]^{\frac{1}{\rho}}\right)\right],
\end{aligned}$$

where inserting again c_0 from equation (A.6) and re-arranging finally gives the IV estimator as

$$\begin{aligned}
\hat{\theta}_{IV}^{young} &= \theta + \omega\rho, \\
&\geq \theta,
\end{aligned}$$

where

$$\begin{aligned}
\omega \equiv p^{-1}\Big[&\rho^{-1}\log\left(p\left(\exp(\rho^{-1}\theta)+2\right)^{\rho}+(1-p)3^{\rho}\right) \\
&-\left(p\log\left(\exp(\rho^{-1}\theta)+2\right)+(1-p)\log(3)\right)\Big] \geq 0
\end{aligned}$$

such that defining $\omega_1 \equiv (\exp(\rho^{-1}\theta)+2)^{\rho}$ and $\omega_2 \equiv 3^{\rho}$, the bias of the IV estimator can be seen to be the difference in the log-expected value and the expected log value;

$$\omega = p^{-1}\rho^{-1}(\log(p\omega_1+(1-p)\omega_2) - (p\log\omega_1+(1-p)\log\omega_2)),$$

which is always positive since the logarithm is a concave function.

B Solving the Life Cycle Model

To reduce the number of state variables, all relations are normalized by permanent income, P_t, and small letter variables denote normalized quantities (e.g., $c_t = C_t/P_t$). The model is solved recursively by backwards induction, starting with the terminal period, T. Within a given period, optimal consumption is found using the Endogenous Grid Method (EGM) by Carroll (2006).

The EGM constructs a grid over end-of-period wealth, a_t, rather than beginning-of-period resources, m_t. Denote this grid of Q points as $\hat{a}_t = (\underline{a}_t, a_t^1, \ldots, a_t^{Q-1})$ in which $\underline{a}_t$ is a lower bound on end-of-period wealth that I will discuss in great detail below. The endogenous level of beginning-of-period resources consistent with end-of-period assets, $\hat{a}_t$, and optimal consumption, $c_t^\star$, is given by $m_t = \hat{a}_t + c_t^\star(m_t, \mathbf{z}_t)$.

In the terminal period, independent of the presence of children, households consume all their remaining wealth, $c_T = m_T$. In preceding periods, in which households are retired, consumption across periods satisfy the Euler equation

$$u'(c_t) = \max\left\{u'(m_t)\ ,\ R\beta\frac{v(\mathbf{z}_{t+1};\theta)}{v(\mathbf{z}_t;\theta)}u'(c_{t+1})\right\},\ \forall t \in [T_r, T],$$

where consumption cannot exceed available resources. When retired, households do not produce new offspring and the age of children ($\mathbf{z}_t$) evolves deterministically.

The normalized consumption Euler equation in periods prior to retirement is given by

$$u'(c_t) = \max\left\{u'(m_t+\kappa)\ ,\ R\beta\mathbb{E}_t\left[\frac{v(\mathbf{z}_{t+1};\theta)}{v(\mathbf{z}_t;\theta)}u'(c_{t+1}G_{t+1}\eta_{t+1})\right]\right\},\ \forall t < T_r,$$

such that when $\hat{a}_t > -\kappa$ optimal consumption can be found by inverting the Euler equation

$$c_t^\star(m_t,\mathbf{z}_t) = \left(\beta R\mathbb{E}_t\left[\frac{v(\mathbf{z}_{t+1};\theta)}{v(\mathbf{z}_t;\theta)}(G_{t+1}\eta_{t+1})^{-\rho}\check{c}_{t+1}^\star(\underbrace{(G_{t+1}\eta_{t+1})^{-1}R\hat{a}_t+\varepsilon_{t+1}}_{=m_{t+1}},\mathbf{z}_{t+1})^{-\rho}\right]\right)^{-\frac{1}{\rho}},$$

where $\check{c}_{t+1}^\star(m_{t+1},\mathbf{z}_{t+1})$ is a linear interpolation function of optimal consumption next period, found in the last iteration. Since $\hat{a}_t$ is the constructed grid, it is trivial to determine in which regions the credit constraint is binding and not. I will discuss this in detail below.

The expectations are over next period arrival of children ($\mathbf{z}_{t+1}$) and transitory (ε_{t+1}) and permanent income shocks (η_{t+1}). Eight Gauss-Hermite quadrature points are used for each income shock to approximate expectations. $Q = 80$ discrete grid points are used in $\hat{a}_t$ to approximate the consumption function with more mass at lower levels of wealth to approximate accurately the curvature of the consumption function. The number of points was chosen such that the change i the optimized log likelihood did not change significantly, as proposed in Fernández-Villaverde, Rubio-Ramírez and Santos (2006).

The arrival probability of a child next period is a function of the wife's age and number of children today, $\pi_{t+1}(\mathbf{z}_t)$. No more than three children can live inside a household at a given point in time and infants cannot arrive when the household is older than 43. The next period's state is therefore calculated by increasing the age of children by one and if the age is 21, the child moves. In principle, there is $22^3 = 10,648$ combinations three children can be either not present (NC) or aged zero through 20. To reduce computation time, children are organized such that child one is the oldest at all times, the second child is the second oldest and child three is the youngest child. To illustrate, imagine a household which in period t has, say, two children aged 20 and 17, $\mathbf{z}_t = (\text{age}_{1,t} = 20,\ \text{age}_{2,t} = 17,\ \text{age}_{3,t} = \text{NC})$, then, in period $t+1$, only one child will be present; $\mathbf{z}_{t+1} = (\text{age}_{1,t+1} = 18,\ \text{age}_{2,t+1} = \text{NC},\ \text{age}_{3,t+1} = \text{NC})$, given no new offspring arrives. Had new offspring arrived, then $\text{age}_{2,t+1} = 0$.

B.1 Credit Constraint and Utility Induced Constraints

Since the EGM works with end-of-period wealth rather than beginning-of-period resources, credit constraints can easily be implemented by adjusting the lowest point in the grid, $\underline{a}_t$. The potentially binding credit constraint next period is implemented by the rule, $c^\star_{t+1} = m_{t+1}$ if m_{t+1} is lower than some threshold level, $\mathsf{m}^\star_{t+1}$. Including the credit constraint as the lowest point, $\underline{a}_{t+1} = -\kappa$, the lowest level of resources endogenously determined in the last iteration, $\underline{m}_{t+1}$, is the exact level of resources where households are on the cusp of being credit constrained, i.e., $\mathsf{m}^\star_{t+1} = \underline{m}_{t+1}$. This ensures a very accurate interpolation and requires no additional handling of shadow prices of resources in the constrained Euler equation, denoted λ_{t+1} in Section 2.

Besides the exogenous credit constraint, κ, a "natural" or utility induced self-imposed constraint can be relevant such that the procedure described above should be modified slightly. This is because households want to accumulate enough wealth to buffer against a series of extremely bad income shocks to ensure strictly positive consumption in all periods even in the worst case possible.

Proposition 1. *The lowest possible value of normalized end-of-period wealth consistent with the model, periods prior to retirement, can be calculated as*

$$\underline{a}_t = -\min\{\Omega_t, \kappa\}\ \forall t \leq T_r - 2$$

where, denoting the lowest possible values of the transitory and permanent income shock as $\underline{\varepsilon}$*, and* $\underline{\eta}$*, respectively,* Ω_t *can be found recursively as*

$$\Omega_t = \begin{cases} R^{-1} G_{T_r} \underline{\varepsilon}_{T_r} \underline{\eta}_{T_r} & \textit{if } t = T_r - 2, \\ R^{-1}(\min\{\Omega_{t+1}, \kappa\} + \underline{\varepsilon}_{t+1}) G_{t+1} \underline{\eta}_{t+1} & \textit{if } t < T_r - 2. \end{cases}$$

Proof. To see this, define $\underline{\mathbb{E}}_t[\cdot]$ as the *worst-case* expectation given information in period t and note that in the last period of working life, $T_r - 1$, households must satisfy $A_{T_r-1} \geq 0$. In the second-to-last period during working life, households must then leave a positive amount of resources in the worst case possible,

$$\begin{aligned}
\underline{\mathbb{E}}_{T_r-2}[M_{T_r-1}] &> 0, \\
\underline{\mathbb{E}}_{T_r-2}[RA_{T_r-2} + Y_{T_r-1}] &> 0, \\
RA_{T_r-2} + G_{T_r-1}P_{T_r-2}\underline{\varepsilon}_{T_r-1}\underline{\eta}_{T_r-1} &> 0, \\
&\Updownarrow \\
A_{T_r-2} &> \underbrace{-R^{-1}G_{T_r-1}\underline{\varepsilon}_{T_r-1}\underline{\eta}_{T_r-1}}_{\equiv\Omega_{T_r-2}} P_{T_r-2}.
\end{aligned}$$

Combining this with the exogenous credit constraint, κ, end-of-period wealth should satisfy

$$A_{T_r-2} > -\min\{\Omega_{T_r-2}, \kappa\}P_{T_r-2}.$$

In period $T_r - 3$, households must save enough to insure strictly positive consumption next period while satisfying the constraint above, in the worst case possible,

$$\begin{aligned}
\underline{\mathbb{E}}_{T_r-3}[M_{T_r-2}] &> -\min\{\Omega_{T_r-2}, \kappa\}\underline{\mathbb{E}}_{T_r-3}[P_{T_r-2}], \\
RA_{T_r-3} + G_{T_r-2}P_{T_r-3}\underline{\varepsilon}_{T_r-2}\underline{\eta}_{T_r-2} &> -\min\{\Omega_{T_r-2}, \kappa\}G_{T_r-2}P_{T_r-3}\underline{\eta}_{T_r-2}, \\
&\Updownarrow \\
A_{T_r-3} &> \underbrace{-R^{-1}(\min\{\Omega_{T_r-2}, \kappa\} + \underline{\varepsilon}_{T_r-2})G_{T_r-2}\underline{\eta}_{T_r-2}}_{\equiv\Omega_{T_r-3}} P_{T_r-3},
\end{aligned}$$

such that end of period wealth in period $T_r - 3$ should satisfy

$$A_{T_r-3} > -\min\{\Omega_{T_r-3}, \kappa\}P_{T_r-3}.$$

Hence, we can find Ω_t recursively by the formula in Proposition 1 and calculate the lowest value of the grid of normalized end-of-period wealth as $\underline{a}_t = -\min\{\Omega_t, \kappa\}$. $\square$