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Do we see monopoly or duopoly? The influence of perception on entry deterrence*

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Abstract

Consumers have bounded perception and treat similar goods as homogeneous. The interaction between this bias and the structure of firms is studied in a vertically differentiated duopoly with market entry. With fixed costs of quality, natural monopoly and entry deterrence occurs at lower entry costs and incumbent profit is higher. With marginal costs of quality, natural monopoly occurs at higher entry costs or not at all. Deterrence occurs at higher entry costs for mild perceptual limitations and at lower costs for severe limitations. Incumbent profit is generally lower, although for a narrow range of parameter values it may be higher. The incumbent may opt not to enter and no market is created.

JEL: D03, D42, D43

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1 Introduction

Can we always tell similar goods apart? The quality of a good is a nebulous attribute which is harder to assess at first glance, especially if relevant information is not readily available. Given this limitation to our perception, it is interesting to examine whether it may influence the goods we are in fact presented with. This question is addressed by looking at entry into a market in which consumers are bounded in their ability to distinguish between goods of similar quality. Intuitively, it is not clear whether bounded perception should help or hinder a potential entrant. On the one hand, it is harder for the entrant, since it is more difficult to distinguish its product. On the other hand, it could make it easier for the entrant, since it can produce a "knock off" good and ride on the incumbent's success. It is demonstrated that both intuitions may be correct, depending on the cost structure of a good's quality, specifically whether it is fixed or marginal. A clear demonstration is thus given of the importance of studying the interaction between biased decision makers and other economic agents, as the results are often not straightforward, and not necessarily robust to changes in the market structure.

With fixed costs of quality, the entrant firm has to produce a good that consumers perceive to be different, as otherwise it falls into the Bertrand trap and makes a loss. Perceptual limitations mean that the entrant firm finds it harder to distinguish itself, and this is exploited by the incumbent. The entry cost at which the market becomes a natural monopoly is lower, as is the cost at which the incumbent deters entry into the market. Unsurprisingly, incumbent profit is greater than in the case of perfect perception.

With marginal costs of quality, the entrant firm may exploit bounded perception to produce a good that is perceived to be homogeneous, but produced at a lower marginal cost than the incumbent's. The entry cost at which the market is a natural monopoly is higher, and natural monopoly is never observed given sufficiently severe limitations. For mild perceptual limitations, entry is only deterred at a greater entry cost, but for severe limitations, it is deterred at a lower cost. Incumbent profit is generally lower than in the standard case, with the exception of a narrow range of parameters due to it deterring entry with bounded perception, whereas with perfect perception the market is shared with the entrant. The reduction in incumbent profit means that for certain parameter values it chooses not to produce, and no market is created.

Bounded perception is formalized using Rubinstein (1988)'s concept of a *similarity relation*, which specifies which elements of a set are sufficiently similar to be regarded as identical. Similarity relations are related to earlier work by Luce (1956) on semi-orders and are consistent with much psychophysical research on stimulus detection, particularly the Weber-Fechner law (Falmagne, 2002). They have been employed to explain anomalies in lottery choice (Azipurua, Ishiishi, Nieto, & Uriarte, 1993; Leland, 1994; Buschena & Zilberman, 1999) and intertemporal choice (Leland, 2002). Webb (2014) uses an identical behavioural mechanism to this article in a vertically differentiated market, but with simultaneous rather than sequential quality choice.

The introduction of a psychological bias into how individuals regard goods is related to recent research on attention and salience. Bordalo, Gennaioli, and Shleifer (2012, 2013b, 2013c, 2013a, 2013d) and Kőszegi and Szeidl (2013) examine how the focus of individuals' attention is drawn towards attributes for which there is greater variation within the choice set and are overweighted in decision making. Eliaz and Spiegler (2011) construct a model in which firms use "pure attention grabbers", goods which attract customers' attention but give no utility. There has also been some investigation of consumers' perception in the marketing literature. (For example Chandon and Ordabayeva (2009) study how individuals' assessment of a product's volume can be influenced by packaging shape and Kwortnik, Creyer, and Ross (2006) examine the effects of labeling on consumer choice.) However, the studies in this field are mostly targeted at very specific effects with very specific product types.

The economic institution utilized is a vertically differentiated product market. Models of vertical differentiation are ideal settings in which to examine perceptual limitations, since firms' profits depend heavily on their ability to distinguish their goods in the eyes of consumers. Mussa and Rosen (1978) first introduced the vertical differentiation framework, and assumed that firms engage in Cournot competition. This paper though, follows the lead of Shaked and Sutton (1982), who adapted the framework for Bertrand competition. Hung and Schmitt (1988) were the first to use it to study market entry. There are now a profusion of theoretical models of vertical differentiation and many empirical applications. A common behavioural approach when incorporating psychological insights into economics is followed: a standard economic model is taken and an extra parameter is added such that the original model is nested within the new. To do this, a standard model of vertical differentiation and entry deterrence is required, and the particular model used as a baseline is most similar to that in Lutz (1997).

Section 2 specifies the behaviour of consumers with bounded perception. The effect of such consumers on a model of market entry is then described, with section 3 giving results for fixed costs of quality and section 4 for marginal costs of quality. Section 5 discusses the contrasting results and section 6 concludes.

2 Bounded perception and consumer behaviour

A good $c = (q, -p)$ has two attributes, quality q and price p, with $Q = [0, \infty)$ the set of qualities and $P = [0, \infty)$ the set of prices. The set of goods is then $C = Q \times -P$. $\succsim_u$ is a preference relation on C satisfying the standard asumptions. Let $\sim_s$ be a *Rubinstein similarity relation* (Rubinstein, 1988) on Q. If $q \sim_s q'$, then q and q' are sufficiently similar that an individual regards them as identical. If $q \nsim_s q'$ then an individual regards them as dissimilar. Together $\succsim_u$ and $\sim_s$ induce a decision preference relation $\succsim_d$ on X in the following way:

i If $q \nsim_s q'$ and $c \succsim_u c'$, then $c \succsim_d c'$.

ii If $q \sim_s q'$ and $p \leq p'$, then $c \succsim_d c'$.

Note that $\succsim_d$ is complete, but not generally transitive. $\succsim_u$ may be thought of as a "true" underlying preference relation and $\succsim_d$ as the relation actually used by in individual in decision making, given the biases captured by $\sim_s$.

Let $\succsim_u$ be represented by the utility function $u = \alpha q - p$, $\alpha \in \mathbb{R}_+$ and let $\sim_s$ be represented by the *perception threshold* $\delta \in [1, \infty)$. Let $q_H, q_L \in Q$ with $q_H \geq q_L > 0$, then

i If $\frac{q_H}{q_L} \geq \delta$, $q_H \nsim_s q_L$.

ii If $\frac{q_H}{q_L} < \delta$, $q_H \sim_s q_L$.

This decision making process is interpreted as consumers having a bounded ability to detect differences in quality. Hence a similarity ratio is not introduced for price, although it is plausible that consumers often act as if very similar prices are identical. Quality is generally a much harder to assess attribute than price, and so the range of prices similar enough to be treated as the same is negligable compared to the corresponding range of qualities. If $\frac{q_H}{q_L} \geq \delta$ and the ratio of qualities excedes the perception threshold, consumers perceive them as heterogeneous. If $\frac{q_H}{q_L} < \delta$ however, so that the ratio of goods' qualities falls below the threshold, then the individual is unable to perceive the difference between the goods and regards them as homogeneous. It is assumed that in this case, the individual perceives both goods to have quality $q' = \lambda q_H + (1 - \lambda) q_L$. This assumption will be revisited later in section 4, and it chould be noted that although it is a natural assumption, the only restriction on consumer behaviour in section 3 is that they never purchase the higher priced good when $\frac{q_H}{q_L} < \delta$.

The functional form bounded perception takes means that as the absolute level of quality increases, so does the absolute difference in quality required for an individual to perceive them as dissimilar. This is consistent with psychophysical research which has found that, in many cases, to a good approximation the perceived intensity of a stimulus increases logarithmically with the physical intensity (Falmagne, 2002).

Although perception is the one adhered to throughout this article, the decision making process is open to other interpretations. For example, when qualities are close enough together individuals find prices particularly salient, and due to an attentional bias use only price information in decision making.

Having specified the behaviour of consumers with bounded perception, it will now be examined what effect they have on a model of vertically integrated entry deterrence, with the cases of fixed and marginal costs of quality being contrasted.

3 Fixed costs of quality

Two identical firms, 1 and 2, may produce a good with quality $q_i \in Q$ which is sold at price $p_i \in P$, $i \in \{1, 2\}$. Consumers may purchase a single unit of the good from which they gain utility $u(q, p) = \alpha q - p$, $\alpha \in \mathbb{R}_+$. Their payoff from the outside option of not consuming is normalized to 0. There is a unit mass of consumers uniformly distributed in the interval $[0, 1]$. The timing is as follows:

Period 1: If firm 1 enters the market, it incurrs cost $E \in (0, \infty)$ and chooses quality. If it does not, the game ends and both firms receive a payoff of 0. If it enters, it chooses quality.

Period 2: Firm 2 observes firm 1's quality choice. If it enters, it incurrs cost $E \in (0, \infty)$ and chooses quality. If it does not, it receives a payoff of 0.

Period 3: If only firm 1 enters, it sets its price. If both firms enter, they compete in prices.

Firm 2 may only enter the market if firm 1 chooses to enter first. This assumption is made to reflect the fact that if firm 1 declines to enter the market and then firm 2 enters, firm 2 should be considered the first mover. The firms are identical, so it is conunterintuitive for firm 1 not to enter and then firm 2 to enter in the same circumstances. This may be formalized by allowing a potentially infinate number of periods in which the opportunity to enter the market alternates between firms. (Although only a single trading period takes place if entry occurs.)

As firm 1 chooses its quality before firm 2, it may exploit this to lower its profit. If the minmax profit of firm 2 is less than the entry cost, firm 1 may deter firm 2 from entering the market.

If firms enter the market, they incur fixed costs of quality of the form $c(q_i) = \frac{1}{2}q_i^2$, with all other production costs normalized to 0. Results for fixed costs will be contrasted with marginal costs of quality in section 4.

Let both firms enter the market and let $H \in \{1,2\}$ ($L \in \{1,2\}$) denote the firm producing the high (low) quality, $H \neq L$. Assume that $\frac{q_H}{q_L} \geq \delta$, so that consumers perceive the goods as distinct. The consumer with taste parameter $\alpha' = \frac{p_H - p_L}{q_H - q_L}$ is indifferent between the high and the low quality goods, and the consumer with $\alpha'' = \frac{p_L}{q_L}$ is indifferent between the low quality good and not consuming. $1-\alpha'$ is then demand for the high quality firm and $\alpha' - \alpha''$ is demand for the low quality firm. Profits[1] are

$$\pi_H(q_H, q_L) = p_H\left(1 - \frac{p_H - p_L}{q_H - q_L}\right) - \frac{1}{2}q_H^2, \qquad \pi_L(q_H, q_L) = p_L\left(\frac{p_H - p_L}{q_H - q_L} - \frac{p_L}{q_L}\right) - \frac{1}{2}q_L^2. \quad (1)$$

In the appendix, it is shown that equilibrium prices are

$$p_H = 2q_H\left(\frac{q_H - q_L}{4q_H - q_L}\right) \qquad p_L = q_L\left(\frac{q_H - q_L}{4q_H - q_L}\right) \quad (2)$$

implying profits for given qualities are

$$\pi_H(q_H, q_L) = \frac{4q_H^2(q_H - q_L)}{(4q_H - q_L)^2} - \frac{1}{2}q_H^2, \quad \text{(3a)} \qquad \pi_L(q_H, q_L) = \frac{q_H q_L (q_H - q_L)}{(4q_H - q_L)^2} - \frac{1}{2}q_L^2. \quad \text{(3b)}$$

When only firm 1 enters, its profit is obtained from equation (3a) by setting $q_L = 0$. From this, it is found that the monopoly profit is $\frac{1}{32}$. The entry cost for each firm is then restricted

[1] For linguistic and notational convenience, *profit* refers to profit net of entry cost and *total profit* refers to profit inclusive of entry cost.

to be in the range $E \in \left(0, \bar{E}_f\right]$, $\bar{E}_f = \frac{1}{32}$.

Consumers have an identical perception threshold $\delta > 0$. Suppose both firms enter the market and choose qualities such that $\frac{q_H}{q_L} < \delta$, so that consumers perceive them to be homogeneous. Bertrand competition in period 3 drives prices down to marginal cost (i.e. 0) and so firms will make a loss. This leads to a key result:

Lemma 1. *With fixed costs of quality, qualities such that $\frac{q_H}{q_L} < \delta$ are never observed.*

All proofs are contained in the appendix. Note that this result does not depend on the assumption that when the quality ratio lies below the perception threshold, firms perceive both goods as having quality $q' = \lambda q_H + (1 - \lambda) q_L$. The only necessary restriction on consumers' behaviour given a quality ratio below the perception threshold is that they never purchase the higher priced good.

Firm 2, as it observes firm 1's quality choice, may choose to be either the high or low quality firm. Suppose firm 2 enters the market and chooses $q_{2H} > q_1$. By lemma 1, it must be that $\frac{q_{2H}}{q_1} \geq \delta$. If $\text{argmax}_{q_{2H}} \pi_{2H}(q_1, q_{2H}) \geq \delta q_1$, this must be a best response. If, on the other hand, $\text{argmax}_{q_{2H}} \pi_{2H}(q_1, q_{2H}) < \delta q_1$, then firm 2, as $\frac{\partial^2 \pi_{2H}(q_1, q_{2H})}{\partial q_{2H}^2} < 0$ and its profit is single-peaked, will choose the lowest quality such that consumers perceive the goods as heterogeneous. A similar argument can be made if firm 2 enters with some $q_{2L} < q_1$: it chooses $\text{argmax}_{q_{2L}} \pi_{2L}(q_1, q_{2L})$ if $\text{argmax}_{q_{2L}} \pi_{2L}(q_1, q_{2L}) \geq \frac{q_1}{\delta}$ and otherwise chooses $q_{2L} = \frac{q_1}{\delta}$, the highest quality such that consumers perceive the goods as heterogeneous. Firm 2's best responses conditional on entering as the high and low quality firm are thus respectively

$$q_{f2H}^{BR}(q_1) = \max\left\{\underset{q_{2H}}{\text{argmax}}\, \pi_{2H}(q_1, q_{2H}), \delta q_1\right\} \tag{4a}$$

$$q_{f2L}^{BR}(q_1) = \min\left\{\underset{q_{2L}}{\text{argmax}}\, \pi_{2L}(q_1, q_{2L}), \frac{q_1}{\delta}\right\}. \tag{4b}$$

Equilibrium will now be derived as follows: the condition under which the market is a natural monopoly is found. It is then found when entry deterrence is feasible if the market is not a natural monopoly, followed by comparing firm 1's profit from deterring and allowing entry, making it possible to show when it will deter entry in equilibrium.

3.1 Natural monopoly

The market is a natural monopoly if the entry cost is sufficiently high that firm 1's equilibrium behaviour is unaffected by the presence of firm 2, i.e. if firm 1 chooses the monopoly quality $q_f^M = \frac{1}{4}$ and firm 2 does not enter the market. Firm 2's best response to q_f^M is termed the *monopoly best response* (MBR) quality. In the appendix, this is derived to be

$$q_{f2}^{MBR}(\delta) = \begin{cases} \dfrac{1}{4\mu_f^M} & \text{for } \delta \leq \delta'_{fM} \\ \\ \dfrac{1}{4\delta} & \text{for } \delta > \delta'_{fM} \end{cases} \tag{5}$$

where μ_f^M is the ratio $\frac{q_f^M}{q_f^{MBR}(\delta)}$ and is the unique solution to equation (A.2) which is greater than 1 and $\delta'_{fM} = \mu_f^M$ The constant μ_f^M is approximately $\mu_f^M \approx 5.200$. The natural monopoly condition is then be found by requiring firm 2's total profit from entering to be less than 0.

Proposition 1. *The market is a natural monopoly if $E \in \left[E_f^M(\delta), \bar{E}_f\right)$, where*

$$E_f^M(\delta) = \begin{cases} \dfrac{8\mu_f^M - 24\mu_f^{M2} + 8\mu_f^M - 1}{32\mu_f^{M2}\left(4\mu_f^M - 1\right)^2} & \textit{for } \delta \leq \delta'_{fM} \\ \\ \dfrac{8\delta^3 - 24\delta^2 + 8\delta - 1}{32\delta^2\left(4\delta - 1\right)^2} & \textit{for } \delta > \delta'_{fM}. \end{cases} \tag{6}$$

With fixed costs of quality, there is a range of entry costs for which the market is a natural monopoly, and the natural monopoly condition is weakly decreasing in δ.

The natural monopoly condition is illustrated in figure 1.

From lemma 1, firm 2 will never choose a quality such that the quality ratio is below the perception threshold, as the Bertrand trap would lead to a loss. In the standard case, its best response to $q_f^M = \frac{1}{4}$ is to choose a quality such that $\frac{q_f^M}{q_{f2}^{MBR}} \approx 5.200$. Thus when the threshold excedes this value, it is forced to choose a quality further away from firm 1. This in turn lowers its profit, so that the market is a natural monopoly with lower entry costs than was required without bounded perception.

3.2 Entry deterrence

Now let $E < E_f^M(\delta)$, so that the market is not a natural monopoly. Assume that firm 1 always enters the market (this assumption is shown to hold in proposition 4). Firm 1 is able to deter entry if it can choose a quality such that firm 2's profit if it enters is not sufficient to cover the cost of entering. Initially, the condition under which it is feasible for entry to be deterred is derived, and whether deterrence is observed in equilibrium is examined subsequently. In period 2, firm 2 may choose either to be the high or low quality firm. Conditional on entering as the high quality firm, firm 2's profit is decreasing in the quality choice of firm 1. Conversely, its profit conditional on entering as the low quality firm is increasing in firm 1's quality choice. The profit of firm 2 is hence minimized if firm 1 chooses its quality such that firm 2 is indifferent between entering as the high or low quality firm. If this minimized profit does not excede the entry cost, then entry deterrence is feasible.

The quality at which firm 1 minimizes firm 1's profit from entering is termed the *entrant profit minimizing* (EPM) quality. Firm 2's best response to this, conditional on entering as the high (low) quality firm is termed the *high (low) EPM best response* or HEPM (LEPM) quality. Since at the EPM quality firm 2 is indifferent between being the high or low quality firm, these qualities are obtained from $\pi_{2H}\left(q_1, q_{f2H}^{BR}(q_1, \delta)\right) = \pi_{2L}\left(q_1, q_{f2L}^{BR}(q_1, \delta)\right)$. In the appendix, the EPM, HEPM and LEPM qualities are derived to be

$$q_{f1}^{D} = \begin{cases} b_{f1} & \text{for } \delta \leq \delta_f' \\ \dfrac{\mu_f^{D3}\left(4\mu_f^D - 7\right)}{\left(4\mu_f^D - 1\right)^3} & \text{for } \delta_f' < \delta \leq \delta_f'' \\ \dfrac{2\delta^2\left(\delta - 1\right)\left(4\delta^2 - 1\right)}{\left(\delta^4 - 1\right)\left(4\delta - 1\right)^2} & \text{for } \delta > \delta_f'' \end{cases} \tag{7a}$$

$$q_{f2H}^{D}\left(\delta\right) = \begin{cases} b_{fH} & \text{for } \delta \leq \delta_f' \\ \delta q_{1f}^{D}\left(\delta\right) & \text{for } \delta > \delta_f' \end{cases} \tag{7b}$$

$$q_{f2L}^{D}\left(\delta\right) = \begin{cases} b_{fL} & \text{for } \delta \leq \delta_f' \\ \dfrac{q_{f1}^{D}\left(\delta\right)}{\mu_f^D} & \text{for } \delta_f' < \delta \leq \delta_f'' \\ \dfrac{q_{f1}^{D}\left(\delta\right)}{\delta} & \text{for } \delta > \delta_f'' \end{cases} \tag{7c}$$

where $\delta_f' \approx 1.533$, $\delta_f'' \approx 3.287$, $b_{f1} \approx 0.161$, $b_{fH} \approx 0.289$, $b_{fL} \approx 0.042$ and μ_f^D is the ratio $\left.\frac{q_{f1}^D(D)}{q_{f2L}^D(\delta)}\right|_{\delta_f' < \delta \leq \delta_f''}$ and is the unique real root of equation (A.4) greater than 1. Substituting the EPM, HEPM and LEPM qualities into firm 2's profit function, the entry deterrence condition is found.

Proposition 2. *Firm 1 is able to deter entry by firm 2 if* $E \in \left[E_f^D\left(\delta\right), E_f^M\left(\delta\right)\right)$, *where*

$$E_f^D\left(\delta\right) = \begin{cases} \dfrac{b_{fH}^2\left(8\left(b_{fH} - b_{f1}\right) - \left(4b_{fH} - b_{f1}\right)^2\right)}{2\left(4b_{fH} - b_{f1}\right)^2} & \textit{for } \delta \leq \delta_f' \\ \dfrac{4\delta^2\left(\delta - 1\right)\mu_f^{D3}\left(4\mu_f^D - 7\right)}{\left(4\delta - 1\right)^2\left(4\mu_f^D - 1\right)^3} - \dfrac{\delta^2\mu_f^{D6}\left(4\mu_f^D - 7\right)^2}{2\left(4\mu_f^D - 1\right)^6} & \textit{for } \delta_f' < \delta \leq \delta_f'' \\ \dfrac{2\delta^4\left(4\delta^2 - 1\right)\left(\delta^2 - 4\right)}{\left(\delta^3 + \delta^2 + \delta + 1\right)^2\left(4\delta - 1\right)^4} & \textit{for } \delta > \delta_f''. \end{cases} \tag{8}$$

When costs of quality are fixed, the entry deterrence condition is weakly decreasing in δ.

The entry deterrence condition is ilustrated in figure 1.

As can be seen in equation (4), the best response of firm 2 is dependent on the size of the perception threshold, as it must ensure that its good is perceived as distinct, and this leads to the piecewise nature of the EPM, HEPM and LEPM qualities. With a low perception threshold ($\delta \leq \delta'_f$), firm 2's standard best reponses lie above the perception threshold, and the entry deterrence condition is unchanged from the standard case. When $\delta'_f < \delta < \delta''_f$ however, if firm 2 enters as the high quality firm, it must select a higher quality than in the standard case. It then makes a greater profit from entering as the low than as the high quality firm. Firm 1 takes advantage of this by reducing its quality, thus lowering the profit from being the low quality firm. Firm 2's minmax profit is lower and deterrence is thus feasible at lower entry costs than in the standard case. If $\delta > \delta''_f$, then both firm 2's high and low best responses are dependent on δ. As may be seen in figure 1, entry deterrence becomes easier at a faster rate than before.

Equation (8) gives the condition under which it is feasible for firm 1 to deter entry, but it is as yet unclear when deterrence will be observed in equilibrium. The first stage in determining this is to find the profit of firm 1 from allowing entry. Comparison to its profit from deterring entry will then reveal when deterrence is optimal.

3.3 Allowing entry

Assume that firm 1 anticipates firm 2 entering the market. As costs are symmetric, it takes advantage of its leader status to produce the high quality good. It then chooses quality to satisfy $\frac{\partial \pi_{1H}\left(q_1, q_{f2L}^{BR}(q_1)\right)}{\partial q_1} = 0$. This results in

$$q_{f1}^{A}(\delta) = \begin{cases} \ell_{f1} & \text{for } \delta \leq \delta'_{fA} \\ \dfrac{4\delta(\delta-1)}{(4\delta-1)^2} & \text{for } \delta > \delta'_{fA} \end{cases}, \qquad q_{f2L}^{A}(\delta) = \begin{cases} \ell_{f2} & \text{for } \delta \leq \delta'_{fA} \\ \dfrac{q_{f1}^{D}(\delta)}{\delta} & \text{for } \delta > \delta'_{fA} \end{cases} \tag{9}$$

where $\delta'_{fA} \approx 4.941$, $l_{f1} \approx 0.245$ and $l_{f2} \approx 4.78 \times 10^{-2}$. Substituting these qualities into the profit function of firm 1 (equation (3a)) then shows its profit given that it allows entry is

$$\pi_{1A}(q_1, q_2) = \begin{cases} \dfrac{4\ell_{f1}^2(\ell_{f1} - \ell_{f2})}{(4\ell_{f1} - \ell_{f2})^2} - \dfrac{\ell_{f1}^2}{2} & \text{for } \delta \le \delta'_{fA} \\ \left(\dfrac{8\delta^2(\delta - 1)^2}{(4\delta - 1)^4}\right) & \text{for } \delta > \delta'_{fA}. \end{cases} \tag{10}$$

3.4 Equilibrium entry deterrence

The profit from allowing entry is compared to the profit from deterring entry in order to determine when entry deterrence is observed in equilibrium. Let $E \in \left[E_f^D(\delta), E_f^M(\delta)\right)$ so that deterrence is feasible. From equation (3a), the profit that firm 1 makes if it deters entry is $\pi_{1ED}(q_1) = \frac{q_1}{4}(1 - 2q_1)$. If the cost of entry is sufficiently high, firm 1 enjoys a natural monopoly and chooses its ideal quality of $q_1 = q_f^M = \frac{1}{4}$. As the cost of entry becomes lower, firm 1 must reduce its quality to deter firm 2 from entering, which also lowers its own profit. It follows that it must be feasible for firm 1 to deter entry at a sufficiently high quality for deterrence to be observed in equilibrium.

The minimum q_1 at which it is optimal for firm 1 to deter entry is termed the *minimum deterrence optimality* (MDO) quality. In the appendix, this quality and firm 2's best response to it are found to be

$$q_{f1}^{MDO}(\delta) = \begin{cases} c_{f1} & \text{for } \delta \le \delta'''_{fE} \\ \dfrac{1}{4}\left(1 - \dfrac{\sqrt{1 - 16\delta - 160\delta^2 + 256\delta^3}}{(4\delta - 1)^2}\right) & \text{for } \delta > \delta'''_{fE} \end{cases} \tag{11a}$$

$$q_{f2L}^{MDO}(\delta) = \begin{cases} c_{f2} & \text{for } \delta \le \delta''_{fE} \\ \dfrac{q_{f1}^E(\delta)}{\delta} & \text{for } \delta > \delta''_{fE} \end{cases} \tag{11b}$$

where $\delta''_{fE} \approx 3.455$, $\delta'''_{fE} \approx 4.941$, $c_{f1} = \frac{1}{4}\left(1 - \sqrt{1 - 32\,\pi_{1A}\left(q^A_{f1}(\delta), q^A_{f2}(\delta)\right)\Big|_{\delta<\delta'_{fA}}}\right) \approx 0.134$ and $c_{f2} \approx 0.0386$.

If entry deterrence is feasible at $q > q^{MDO}_{f1}(\delta)$, then it is optimal. If deterrence is only possible if firm 1 lowers its quality to some $q < q^{MDO}_{f1}(\delta)$, then allowing entry is optimal. The lowest quality at which deterrence is feasible is $q^D_{f1}(\delta)$ It follows that, if $q^{MDO}_{f1}(\delta) < q^D_{f1}(\delta)$, then deterrence is only possible at qualities such that it is optimal: the equilibrium deterrence condition is identical to the feasibility condition. Otherwise, the equilibrium deterrence condition is found by requiring firm 2's total profit to be 0 when best responding to $q^{MDO}_{f1}(\delta)$. This leads to

Proposition 3. *If $E \in \left[E^{D*}_f(\delta), E^M_f(\delta)\right)$, then if firm 1 enters the market it does so by deterring entry, where*

$$E^{D*}_f(\delta) = \begin{cases} E^D_f(\delta) & \text{for } \delta \leq \delta'_{fE} \\ \dfrac{c_{f1}c_{f2}(c_{f1}-c_{f2})}{(4c_{f1}-c_{f2})^2} - \dfrac{c^2_{f2}}{2} & \text{for } \delta''_{fE} \geq \delta > \delta'_{fE} \\ \dfrac{(\delta-1)}{(4\delta-1)^2}c_{f1} - \dfrac{c^2_{f1}}{2\delta^2} & \text{for } \delta'''_{fE} \geq \delta > \delta''_{fE} \\ \dfrac{(\delta-1)}{4(4\delta-1)^2}\left(1 - \dfrac{\sqrt{1-16\delta-160\delta^2+256\delta^3}}{(4\delta-1)^2}\right) - & \\ \quad -\dfrac{1}{32\delta^2}\left(1 - \dfrac{\sqrt{1-16\delta-160\delta^2+256\delta^3}}{(4\delta-1)^2}\right)^2 & \text{for } \delta > \delta'''_{fE} \end{cases} \tag{12}$$

where $\delta'_{fE} \approx 2.883$. The equilibrium deterrence condition is weakly decreasing in δ.

The equilibrium deterrence condition is illustrated in figure 1.

In the standard case, as in Lutz (1997), the incumbent firm will always deter entry if feasible. By proposition 4, bounded perception leads to deterrence becoming feasible at lower entry costs. In order to deter entry at these lower costs however, firm 1 must choose a low quality. Thus with a sufficiently high perception threshold ($\delta > \delta'_{fE} \approx 2.883$), there is a range of entry costs for which deterrence is possible, but the incumbent prefers to allow entry and share the market.

As may be seen in figure 1, for $\delta'_{fE} < \delta \leq \delta''_{fE} \approx 3.455$, the equilibrium deterrence condition is flat: both the best response to the MDO quality and firm 1's profit from allowing entry are not directly influenced by bounded perception: a higher perception threshold affects the feasible actions of the incumbent but does not qualitatively affect equilibrium. For $\delta > \delta'_{fE}$, however, as can be seen in equation (11b), the best response of firm 2 is affected by the necessity of keeping the quality ratio above the perception threshold. For a given q_1 then, firm 1 makes a greater profit from deterring entry, and so it will choose to do so for lower entry costs than previously. From equation (10), if the perception threshold is sufficiently high, the incumbent profit from allowing entry is increasing in the threshold. This explains why, for $\delta > \delta'''_{fE} \approx 4.941$, the equilibrium deterrence condition decreases at a slower rate: higher δ implies both higher profit from deterring entry at a given q_1 and higher profit from deterring entry.

3.4.1 Equilibrium and incumbent profit

Having derived firms' optimal actions given the assumption that firm 1 enters the market, it can now be stated that

Proposition 4. *Firm 1 chooses to enter the market in equilibrium.*

That the market is always created is not especially surprising. However, it will be contrasted in section 4 with the analagous result for marginal costs of quality (see proposition 9).

The results for fixed costs of quality may be summarized in a characterization of equilibrium:

i Natural monopoly *If* $E \in \left[E_f^M(\delta), \bar{E}_f\right)$, *firm 1 enters the market and chooses* $q_1 = q_f^M = \frac{1}{4}$. *Firm 2 does not enter.*

ii <u>*Entry deterrence*</u> *If* $E \in \left[E_f(\delta), E_f^M(\delta)\right)$, *firm 1 enters the market and chooses the* q_1 *that satisfies* $\pi_{2L}\left(q_1, q_{f2L}^{BR}(q_1)\right) = E$. *Firm 2 does not enter.*

iii <u>*Duopoly*</u> *If* $E \in \left(0, E_f^{D*}(\delta)\right)$, *both firms enter, choosing qualities* $q_1 = q_{f1}^A(\delta)$ *and* $q_2 = q_{f2}^A(\delta)$.

[Figure 1 about here]

Turning to the effect of bounded perception on profit, it is found that

Proposition 5. *For fixed costs of quality, incumbent profit is greater than or equal to the standard case when consumers have bounded perception.*

From lemma 1, firm 2 must respond to bounded perception by selecting quality that other consumers perceive to be distinct. This allows the incumbent to exploit bounded perception. The market is a natural monopoly at a lower entry cost, entry is deterred at a lower entry cost, and even when duopoly is still observed in equilibium, firm 2 is forced to choose a lower quality than in the standard case, increasing incumbent profit.

In the next section, these results will be contrasted with results for marginal costs of quality.

4 Marginal costs of quality

In a similar manner to the previous section, the interaction between customers with bounded perception and profit maximizing firms will be examined, but now with marginal rather than fixed costs of quality. The market structure is first examined, along with the nature of the entrant's best response quality choice. The condition under which the market is a natural monopoly is found, followed by the condition under which the incumbent deters entry. It is then examined when firm enters the market, after which equilibrium may be fully characterized.

Let the market be as in section 3, but now assume marginal costs of quality of the form $c(q_i) = \frac{1}{2} q_i^2 D_i$, where D_i is the demand for firm i, and assume all other production costs are 0. Assume both firms enter the market with qualities such that $\frac{q_H}{q_L} \geq \delta$. Profits in period 3 are

$$\pi_H(q_H, q_L) = \left(1 - \frac{p_H - p_L}{q_H - q_L}\right)\left(p_H - \frac{1}{2} q_H^2\right) \tag{13a}$$

$$\pi_L(q_H, q_L) = \left(\frac{p_H - p_L}{q_H - q_L} - \frac{p_L}{q_L}\right)\left(p_L - \frac{1}{2} q_L^2\right). \tag{13b}$$

Equilibrium prices are

$$p_H = q_H \frac{\left(4(q_H - q_L) + 2q_H^2 + q_L^2\right)}{2(4q_H - q_L)}, \quad (14a) \qquad p_L = q_L \frac{\left(2(q_h - q_L) + q_H(q_H + 2q_L)\right)}{2(4q_H - q_L)} \quad (14b)$$

which means profits are

$$\pi_H(q_H, q_L) = q_H^2 \frac{(4 - 2q_H - q_L)^2 (q_H - q_L)}{4(4q_H - q_L)^2} \tag{15a}$$

$$\pi_L(q_H, q_L) = q_H q_L \frac{(2 + q_H - q_L)^2 (q_H - q_L)}{4(4q_H - q_L)^2}. \tag{15b}$$

Note that as q_H becomes large, demand for the high quality good becomes 0. The restriction $q_H \leq 2 - \frac{1}{2} q_L$ must therefore be imposed to ensure non-negative demand. Unlike the profit functions for fixed costs of quality, these functions are not concave, however it is possible to show

Lemma 2. *$\pi_H(q_H, q_L)$ and $\pi_L(q_H, q_L)$ have unique local maxima in the range $0 \leq q_L \leq q_H$, $q_L \leq q_H \leq 2 - \frac{1}{2} q_L$.*

From equation (15a) it is found that the monopoly profit is $\frac{2}{27}$, so entry costs are restricted to be in the range $E \in (0, \bar{E}_m)$, $\bar{E}_m = \frac{2}{27}$.

Suppose both firms enter with qualities such that $\frac{q_H}{q_L} < \delta$. Consumers regard the goods as homogeneous, but unlike in section 3, marginal costs are not identical for each firm. It is a standard result that in period 3, Bertrand-Nash equilibrium prices are

$$p_H \in \left(\frac{1}{2}q_H^2, \infty\right) \qquad\qquad p_L = \frac{1}{2}q_H^2. \tag{16}$$

The low quality firm has an advantage in marginal cost, so if it sets its price equal to the marginal cost of the high quality firm, its rival has no incentive to undercut it. Thus the low quality firm captures the entire market and earns positive revenue, whereas the high quality firm earns 0 revenue.

Firm 1 is the first mover, and so firm 2 always has the option of entering as the low quality firm with a good that consumers perceive as identical to firm 1's and capturing the entire market. Using the strategy of choosing a lower, but indistinguishable quality is referred to as firm 2 *imitating* firm 1.

Thus far the only necessary assumption about consumer behaviour when $\frac{q_H}{q_L} < \delta$ is that they never purchase the higher priced good. Now, however, firm 2's profit when imitating depends on the quality consumers perceive when unable to distinguish between similar goods, $q' = \lambda q_H + (1 - \lambda)\, q_L$. Consumers' perception of the goods is "blurred" and so they perceive both as having a weighted average of the high and low quality good.

The main focus of this article is the contrast between the cases of fixed and marginal costs of quality. The contrast is largely driven by firm 2's ability to imitate firm 1 with marginal costs, and its profit from imitation is increasing in λ. Therefore, to emphasize the contrast between the two cases and to greatly simplify the analysis, it is assumed that $\lambda = 1$. This implies that $\frac{q_H}{q_L} < \delta$ consumers perceive both goods as being of quality $q' = q_H$. All conclusions are qualitatively unchanged under the oppositely extreme assumption of $\lambda = 0$.

If it imitates, firm 2 minimizes its cost by choosing the lowest quality such that consumers are unable to distinguish it from firm 1's good. However, this is undefined, as firm 2 can choose q_2 arbitrarily close to $\frac{q_1}{\delta}$. Assume that there is some minimum technologically feasible difference in quality ε. Firm 2 will then maximize its profit, conditional on imitating, by choosing $q_2 = \frac{q_1}{\delta} + \varepsilon$. As ε becomes very close to 0, q_2 is approximately $\frac{q_1}{\delta}$, but with consumers still unable to perceive the difference between q_1 and q_2. Thus in the following section, when it is stated that firm 2 imitates by choosing $q_2 = \frac{q_1}{\delta}$, it should be read as an approximation of choosing $q_2 = \frac{q_1}{\delta} + \varepsilon$ with ε very close to 0. To distinguish between firm 2 imitating and selecting $\frac{q_1}{\delta}$ as a *perceivably distinct* product, the latter is notated $\frac{\hat{q}_1}{\delta}$. Given that approximation, when imitating firm 1, firm 2 makes profit

$$\pi_{2I}(q_1, \delta) = \frac{q_1^2}{4}(2 - q_1)\left(\frac{\delta^2 - 1}{\delta^2}\right). \tag{17}$$

Firm 2's best responses conditional on entering as the high and low quality firm are then

$$q_{m2H}^{BR}(q_1, \delta) = \max\left\{\underset{q_{2H}}{\operatorname{argmax}}\, \pi_{2H}(q_1, q_{2H}), \delta q_1\right\} \tag{18a}$$

$$q_{m2L}^{BR}(q_1, \delta) = \begin{cases} \hat{q}_{m2L}^{BR}(q_1, \delta) & \text{if } \pi_{2L}\left(q_1, \hat{q}_{m2L}^{BR}(q_1, \delta)\right) \geq \pi_{2I}(q_1, \delta) \\ \dfrac{q_1}{\delta} & \text{if } \pi_{2L}\left(q_1, \hat{q}_{m2L}^{BR}(q_1, \delta)\right) < \pi_{2I}(q_1, \delta). \end{cases} \tag{18b}$$

where $\hat{q}_{m2L}^{BR}(q_1) = \min\left\{\operatorname{argmax}_{q_{2L}} \pi_{2L}(q_1, q_{2L}), \frac{\hat{q}_1}{\delta}\right\}$ is firm 2's best response conditional on producing a low quality that consumers perceive as different to q_1.

4.1 Natural monopoly

The nature of the entrant's best response having been examined, it can now be found when the market is a natural monopoly. The monopoly quality with marginal costs is $\frac{2}{3}$, and given this it is shown in the appendix that firm 2's MBR is

$$q_{m2L}^{MBR}(\delta) = \begin{cases} \dfrac{1}{3} & \text{for } \delta \leq \delta'_{mM} \\ \dfrac{2}{3\delta} & \text{for } \delta > \delta'_{mM} \end{cases} \tag{19}$$

where $\delta'_{mM} = 2\sqrt{\frac{2}{7}} \approx 1.069$. Given these qualities, it can be determined when the market is a natural monopoly.

Proposition 6. *With marginal costs of quality, the market is a natural monopoly for* $E \in \left[E_m^M(\delta), \bar{E}_m\right)$*, where*

$$E_m^M(\delta) = \begin{cases} \dfrac{1}{54} & \text{for } \delta \leq \delta'_{mM} \\ \dfrac{4}{27}\left(\dfrac{\delta^2 - 1}{\delta^2}\right) & \text{for } \delta > \delta'_{mM}. \end{cases} \tag{20}$$

For $\delta \geq \sqrt{2}$ *there is no* $E \in \left(0, \bar{E}_m\right)$ *such that the market is a natural monopoly.*

The natural monopoly condition is illustrated in figure 2.

In contrast to the fixed costs case, bounded perception may be exploted by the entrant, rather than the incumbent, since the entrant can imitate the incumbent's product. Thus a higher perception threshold increases firm 2's profit in response to the monopoly quality, meaning that the range of entry costs for which the market is a natural monopoly shrinks and for a sufficiently high threshold, the market is never a natural monopoly.

4.2 Entry deterrence

Let $E < \max\left\{E_m^M(\delta), \bar{E}_m\right\}$, so that firm 1 does not enjoy a natural monopoly. As with fixed costs, deterrence is feasible if firm 1 can lower firm 2's profit such that it does not excede the entry costs. It again minimizes firm 2's profit by choosing q_1 such that firm 2 is indifferent between entering as the high or low quality firm, so the EPM, HEPM and LEPM qualities are found from solving $\pi_{2L}\left(q_1, q_{m2L}^{BR}(\delta)\right) = \pi_{2H}\left(q_1, q_{m2H}^{BR}(\delta)\right)$. The solutions are

$$q_{m1}^{D}(\delta) = \begin{cases} b_{m1} & \text{for } \delta \leq \delta'_m \\ \dfrac{4\left(4\mu_m^{D2} - 3\mu_m^{D} + 2\right)}{\left(24\mu_m^{D3} - 22\mu_m^{D2} + 5\mu_m^{D} + 2\right)} & \text{for } \delta''_m \geq \delta > \delta'_m \\ -\dfrac{B(\delta) - \sqrt{B^2(\delta) - 4A(\delta)C(\delta)}}{2A(\delta)} & \text{for } \delta > \delta''_m \end{cases} \tag{21a}$$

$$q_{m2H}^{D}(\delta) = \begin{cases} b_{mH} & \text{for } \delta \leq \delta'_m \\ \mu_m^{D} q_{1D}^{D}(\delta) & \text{for } \delta''_m \geq \delta > \delta'_m \\ \delta q_{m1}^{D}(\delta) & \text{for } \delta > \delta''_m \end{cases} \tag{21b}$$

$$q_{m2L}^{D}(\delta) = \begin{cases} b_{mL} & \text{for } \delta < \delta'_m \\ \dfrac{q_{m1}^{D}(\delta)}{\delta} & \text{for } \delta > \delta'_m \end{cases} \tag{21c}$$

where $\delta'_m \approx 1.071$, $\delta''_m \approx 2.336$, $b_{m1} \approx 0.611$, $b_{mH} \approx 0.939$ and $b_{mL} \approx 0.309$. $A(\delta)$, $B(\delta)$ and $C(\delta)$ are functions of δ given by equation (A.10) and μ_m^{D} is the ratio $\left.\frac{q_{m2H}^{D}(\delta)}{q_{m1}^{D}(\delta)}\right|_{\delta'_m < \delta \leq \delta''_m}$ and is given by the unique root of equation (A.9) which takes a value greater than 1. Substituting the EPM, HEPM and LEPM qualities into firm 2's profit gives the entry deterrence condition.

Proposition 7. *If $E \in \left[E_m^{D}(\delta), \max\left\{E_m^{M}(\delta), \bar{E}_m\right\}\right)$, it is feasible for firm 1 to deter entry, where*

$$E_m^{D}(\delta) = \begin{cases} \dfrac{b_{m1}b_{mL}\left(2 + b_{m1} - b_{mL}\right)^2\left(b_{m1} - b_{mL}\right)}{4\left(4b_{m1} - b_{mL}\right)^2} & \textit{for } \delta \leq \delta'_m \\ \dfrac{1}{4}q_{m1}^{D2}(\delta)\left(2 - q_{m1}^{D}(\delta)\right)\left(\dfrac{\delta^2 - 1}{\delta^2}\right) & \textit{for } \delta > \delta'_m. \end{cases} \tag{22}$$

With marginal costs of quality, the entry deterrence condition is weakly increasing in δ for $\delta \leq \delta''_m$ and decreasing $\delta > \delta''_m$.

The entry deterrence condition is illustrated in figure 2.

Whether deterrence is feasible or not depends on the ability of firm 1 to minimize firm 2's profit, which it does by equalizing the profit the entrant makes from being the high and low quality firm. With marginal costs, firm 2 has the ability to imitate firm 1 if it enters as the low quality firm. Hence, as was seen with natural monopoly, its profit from entering as the low quality firm increases with the perception threshold. This causes the increase in the perception threshold for $\delta'_m < \delta \leq \delta''_m$: there is greater profit from imitating, meaning that deterrence is only possible at higher entry costs.

However, when entering as the high quality firm, the situation is much the same as with fixed costs. If $\frac{q_{2H}}{q_1} < \delta$, firm 2 captures none of the market, and so it must always produce a perceivably heterogeneous quality. For $\delta > \delta''_m$ therefore, its profit from entering becomes lower with an increased perception threshold. This effect dominates the increased benefits from imitating, and hence the entry deterrence condition begins to decrease for $\delta > \delta''_m$, leading to the "hump" shape in figure 2.

Proposition 7 determines the feasible actions of firm 1, but does not state when deterrence will be observed in equilibrium. The intial step in addressing this question is to find the incumbent's profit when allowing entry.

4.3 Allowing entry

Assume firm 1 anticipates firm 2 entering the market. With a sufficiently low perception threshold, firm 2 opts not to imitate and firm 1 can take advantage of being the first mover to be the high quality firm. When the threshold is high enough, firm 2 will imitate firm 1 if it enters as the low quality firm, meaning it captures the entire market. Firm 1 must hence choose a quality such that firm 2 opts not to imitate and cedes the opportunity to be the high quality firm to its rival.

Using this insight, the equilibrium qualities firms choose conditional on allowing entry are derived in the appendix as

$$q_{m1}^{A}(\delta)=\begin{cases}\ell_{m1} & \text{for } \delta\leq\delta'_{mA}\\ \\ q_{m1}^{D}(\delta) & \text{for } \delta>\delta'_{mA}\end{cases}\qquad q_{m2}^{A}(\delta)=\begin{cases}\ell_{m2} & \text{for } \delta\leq\delta'_{mA}\\ \\ q_{m2H}^{D}(\delta) & \text{for } \delta>\delta'_{mA}\end{cases}\tag{23}$$

where $\delta'_{mA}\approx 1.073$, $\ell_{m1}\approx 0.567$ and $\ell_{m2}\approx 0.289$. Substituting these qualities into equation (15a) then shows firm 1's profit from allowing entry to be

$$\pi_1^{A}(q_1,q_2)=\begin{cases}\ell_{m1}^{2}\dfrac{(4-2\ell_{m1}-\ell_{m2})^{2}(\ell_{m1}-\ell_{m2})}{4(4\ell_{m1}-\ell_{m2})^{2}} & \text{for } \delta\leq\delta'_{mA}\\ \\ q_{m1}^{D2}(\delta)\dfrac{\left(4-2q_{m1}^{D}(\delta)-q_{m2L}^{D}(\delta)\right)^{2}}{4\left(4q_{m1}^{D}(\delta)-q_{m2L}^{D}(\delta)\right)^{2}}\times & \\ \\ \times\left(q_{m1}^{D}(\delta)-q_{m2L}^{D}(\delta)\right) & \text{for } \delta>\delta'_{mA}.\end{cases}\tag{24}$$

4.4 Equilibrium, market creation and incumbent profit

Comparing firm 1's profit from deterring and allowing entry, it is revealed that

Proposition 8. *With marginal costs of quality, firm 1 always deters entry when it is feasible.*

Unlike with fixed costs, the conditions for equilibrium and feasible deterrence coincide.

Having found firm 1's optimal actions under the assumption that it enters the market, it is possible to revisit that assumption.

Proposition 9. *If $E\in\left(E_m^{MC1}(\delta),\bar{E}_m\right)\bigcup\left(E_m^{MC2}(\delta),E_m^{D}(\delta)\right)$, firm 1 does not enter the market, where*

$$E_m^{MC1}(\delta)=\frac{8\delta^2\left(\delta^2-1\right)^2}{\left(5\delta^2-4\right)^3}\tag{25}$$

and $E_m^{MC2}(\delta)=\pi_{1A}(q_1,q_{2L})$. There is a range of E satisfying $E\in\left(E_m^{MC1}(\delta),\bar{E}_m\right)$ for $\delta>\sqrt{2}$ and a range of E satisfying $E\in\left(E_m^{MC2}(\delta),E_m^{D}(\delta)\right)$ for $\delta'_{MC}<\delta<\delta''_{MC}$, where $\delta'_{MC}\approx 1.184$ and $\delta''_{MC}\approx 4.211$.

The regions in which firm 1 does not enter are illustrated in figure 2. The necessity of avoiding firm 2 imitating its quality means that firm 1's profit may be greatly reduced, and this leads to ranges of entry costs for which it cannot earn enough to make it worthwhile entering the market.

[Figure 2 about here]

The results for marginal costs of quality can be summarized in a characterization of equilibrium:-

i <u>No market created</u> *If* $E \in \left(E_m^{MC1}(\delta), \bar{E}_m\right) \bigcup \left(E_m^{MC2}(\delta), E_m^M(\delta)\right)$ *firm 1 does not enter.*

ii <u>Natural monopoly</u> *If* $E \in \left(E_m^M(\delta), \bar{E}_m\right)$*, then firm 1 enters the market and chooses* $q_1 = \frac{2}{3}$*. Firm 2 does not enter.*

iii <u>Entry deterrence</u> *If* $E \in \left(E_m^D(\delta), \max\left\{E_m^M(\delta), E_m^{MC1}(\delta)\right\}\right]$*, firm 1 enters the market and chooses the* q_1 *that satisfies* $\pi_{2L}\left(q_1, q_{m2L}^{BR}(q_1)\right) = E$*. Firm 2 does not enter.*

iv <u>Allowing entry</u> *If* $E \in \left(0, E_m^D(\delta)\right) \setminus \left[E_m^{MC2}(\delta), E_m^D(\delta)\right)$ *both firms enter, choosing qualities* $q_1 = q_{m1}^A(\delta)$ *and* $q_2 = q_{m2}^A(\delta)$*.*

Turning to the effects of bounded perception, it is found that

Proposition 10. *With marginal costs of quality, incumbent profit is weakly lower than in the standard case, with the following exception:* $\delta \geq E_m^{D-1}(E)$ *and* $E \in (E_m^{\pi}(\delta), \pi_A(q_1, q_2)|_{\delta=1})$*, where* $E_m^{\pi}(\delta)$ *is the unique solution to*

$$\frac{4\delta^2\left(\delta^2-1\right)^2 E}{\left(\delta^2\left(E - 4\,\pi_{1A}(q_1,q_2)|_{\delta=1}\right) - 4\,\pi_{1A}(q_1,q_2)|_{\delta=1}\right)^3} = 1 \qquad (26)$$

that satisfies $E \in \left(0, \bar{E}_m\right)$*.*

The region in which incumbent profit is higher than in the standard case is illustrated in figure 2. Generally, the ability to imitate firm 1 leads to lower incumbent profit. However, for some entry costs firm 1 may deter entry with sufficiently severe bounded perception, whereas it would allow entry in the standard case. Thus for some entry costs there is the possibility for incumbent profit to be higher than in the standard case.

5 Discussion

Conusmer behaviour for given qualities and prices is fully determined, and so they are not strategic players in the market: it is a game between firms only. Thus the disperate results in sections 3 and 4 are due to the differing abilities of each firm to exploit bounded perception. The key difference is whether, when consumers perceive goods as homogeneous, firms are identical when competing in prices, or whether one firm has an advantage in marginal cost. Bertrand competition with identical firms leads to a loss for both, but with differing marginal costs, the low cost firm makes a profit. Thus with fixed costs, firm 1 as the first mover finds bounded perception an advantage. It picks its quality knowing that firm 2 must position itself so that consumers perceive the goods as distinct, as otherwise it makes a loss. With marginal costs or quality, the low quality/cost firm may make a profit from choosing a good perceived as identical to its rival's. This grants an advantage to firm 2 as the second mover, since it can always choose to be the low quality firm.

[Table 1 about here]

A summary of the contrasting results for fixed and marginal costs is given in table 1. The most clearcut difference is observed for natural monopoly, as the condition for natural monopoly is determined by firm 2's best response as a low quality firm. The feasibility of entry deterrence, on the other hand, depends on firm 2's best responses both as a low and high quality firm. Its best response as a high quality firm is the same in character for both fixed and marginal costs, leading to the entry deterrence condition increasing in δ for a sufficiently high threshold.

With fixed costs, firm 1 is able to exploit entry deterrence to increase its profits. With marginal costs, the ability to deter entry as lower entry costs does nor neccessarily lead to greater profit for the incumbent. Firm 1 must ensure that firm 2 does not imitate its product. It does thus by choosing a quality so low that the product is not worth imitating. It reduces the overall value of the market, and hence its own profit.

Firm 1 unsurprisingly always enters the market with fixed costs. With marginal costs, however, there are circumstances in which the prospect of being imitated causes it not to enter. Analysis of consumer welfare when consumers do not precisely perceive perception of their goods is problematic.[2] Yet the stark result of no market being created allows at least the definitive conclusion that when this is the case that consumers are left worse off than in the standard case as they are never presented with the chance to purchase. The somewhat counterintuitive result is arrived at that consumer welfare would be greater if the incumbent had a natural monopoly.

It is easy to observe real world firms taking actions intended to influence consumer perception. This may either try to help perception, for example by adopting a clear colour scheme to make quality discernible at a glance, or to hinder it, for example important nutritional information is often hidden away on the back of food packaging. Although perception is treated exogenously in the current framework, firms have clear and conflicting incentives to influence perception. The threshold δ may be thought of as the frame in which consumers see goods. It should thus be possible in future research to analyse how firms compete over the frame in which consumers see their goods in a similar way to Piccione and Spiegler (2012) and Spiegler (2014).

[2]For example, should welfare depend on the perceived quality of a good or the objective quality? If the former, then welfare measures will be highly context dependent. If the latter, then paradoxes arise such as an individual being measured as better off when provided with good i rather than j, despite her regarding i and j as perfectly homogeneous.

In equilibrium, consumers when presented with a choice of goods can always perceive the difference between them, since forward looking firms anticipate that price competition between effectively homogeneous goods will lead to disastrous results for one or both firms. However, in the real world, it does happen that imitation goods exist which are obviously designed in the hope that consumers will not perceive the difference, as anyone who has been offered a "genuine" Rolex at a knock-down price will attest. That this model does not predict the existence of such goods is largely due to the assumption of an identical perception threshold, meaning either all consumers perceive goods as heterogeneous or none do. "Knock-off" goods are usually aimed at those with low perceptual abilities whereas more discerning consumers purchase the genuine article. A heterogeneous perception threshold may hence allow for the coexistence of genuine and imitation goods.

Aside from the specific conclusions regarding entry into a vertically differentiated market, conclusions can also be drawn about the impact of individual choice biases in a market. Consumers are identical in both sections 3 and 4, and firms differ only in whether the cost of producing a given quality is per-unit or independent of demand. The interaction between consumers' biases and firms is vastly different in both cases, however, with the same bias leading to a natural monopoly for the incumbent in one case and in the other case to a market so unprofitable the incumbent never enters. This highlights the importance of considering decision making biases not only in the context of individual choice, but also examining the consequences for interactions with other actors.

6 Conclusion

The impact on individuals' decision making when their perception is imperfect is a growing area of study in the field of economics. Here, the impact beyond that on the individual is considered. It has been demonstrated that the interactions between perceptual limitations and the cost structure of firms is a complex one, with disparate effects on market entry, equilibrium quality choices, profit and consumer welfare.

That consumers are not perfect in their perception of the world is of consequence, and should not be neglected when analysing market structure.

Appendix

A.1 Fixed costs of quality

The first order conditions of equation (3) with respect to price are

$$\frac{\partial \pi_H\left(q_H,q_L\right)}{\partial p_H}=1-\frac{2p_H-p_L}{q_H-q_L}, \qquad \text{(A.1a)} \qquad \frac{\partial \pi_L\left(q_H,q_L\right)}{\partial p_L}=\frac{p_H-2p_L}{q_H-q_L}-\frac{2p_L}{q_L} \qquad \text{(A.1b)}$$

from which it may readily be seen that the second order conditions are negative. Equating these conditions to 0 and rearranging gives the prices in equation (2).

A.1.1 Proof of lemma 1

If $\frac{q_H}{q_L}<\delta$, Bertrand competition with effectively homogeneous goods and identical marginal costs of 0 for each firm occurs. Firms earn no revenue and make a loss for any $q_H, q_L>0$. Then as each firm can make 0 profit from selecting 0 quality, $\frac{q_H}{q_L}<\delta$ cannot be an equilibrium. Any $q_H>0$ is perceivably different to $q_L=0$, so that $q_H=0$ is not a best response to $q_L=0$. For any $q_H>0$ it is possible to find some $0<q_L\leq\frac{q_H}{\delta}$ which is strictly positive and perceivably different to q_H. From the first order condition of $\pi_L\left(q_H,q_L\right)$, $\left.\frac{\partial \pi_L(q_H,q_L)}{\partial q_L}\right|_{q_L=0}>0$, so $q_L=0$ is not a best response to any $q_H=0$. □

A.1.2 Monopoly best response (MBR) quality

Assume firm 2's best response to q_f^M is to enter as the low quality firm. This is shown to hold in the derivation of the EPM quality. Let $\mu=\frac{q_f^M}{q_{2L}}$. Substituting $q_1=q_f^M$ and $q_{2L}=\frac{q_f^M}{\mu}$ into equation (A.1b). and rearranging gives

$$16\mu^4-92\mu^3+48\mu^2-12\mu+1=0 \qquad \text{(A.2)}$$

Define $\mu_f^M = 5.200$ as the unique root of this equation with $\mu > 1$. Then $q_{f2}^{MBR}(\delta) = \frac{1}{4\mu_f^M} \approx 0.048$. For $\delta > \mu_f^M$, $\frac{q_f^M}{\delta} > \frac{1}{4\mu_f^M}$ and from $q_{f2L}^{BR}(q_1)$, $q_{f2}^{MBR}(\delta) = \frac{1}{4\delta}$, which completes the derivation.

A.1.3 Proof of proposition 1

Substituting $q_f^M = \frac{1}{4}$ and equation (5) into equation (3b) gives equation (6). $E_f^M(\delta)\big|_{\delta<\delta_f^M} \approx 0.002 < \bar{E}_f$ and $\frac{\partial}{\partial\delta} E_f^M(\delta)\big|_{\delta>\delta_f^M} \geq 0$ can be reduced to $-16\delta^4 + 92\delta^3 - 48\delta^2 + 12\delta - 1 \geq 0$, which has no solutions for $\delta > \delta_f^M$. □

A.1.4 EPM, HEPM and LEPM qualities

For sufficiently low δ, $q_{f2H}^{BR}(q_1) = \text{argmax}_{q_{f2H}} \pi_{2H}(q_1, q_{2H})$ and $q_{2L}^{BR}(q_1) = \text{argmax}_{q_{2L}} \pi_{2L}(q_1, q_{2L})$. Denote the constants which solve these equations simultaneously as $b_{f1} \approx 0.161$, $b_{fH} \approx 0.289$, $b_{fL} \approx 0.042$. $\frac{b_{fH}}{b_{f1}} \approx 1.792$ and $\frac{b_{f1}}{b_{f2L}} \approx 3.862$, so for $\delta > \delta_f' = \frac{b_{fH}}{b_{f1}}$, $q_{2H} = b_{fH}$ is not a best response to b_{f1}. Let $\delta > \delta_f'$ and be sufficiently small that $q_{f2L}^{BR}(\delta) = \text{argmax}_{2L} \pi_{2L}(q_1, q_{2L})$. Let $\mu = \frac{q_1}{q_{2L}}$. Inserting $q_1 = \mu q_{2L}$ into equation (A.1a) and rearranging gives $q_{2L} = \frac{\mu^2(4\mu-7)}{(4\mu-1)^3}$. Inserting $q_{2H} = q_{f2H}^{BR}(q_1) = \delta q_1$ and $q_1 = \mu q_{2L}$ into $\pi_{2L}(q_1, q_{2L}) = \pi_{2H}(q_1, q_{2H})$ and rearranging gives

$$q_{2L} = \frac{2\mu}{(\delta^2\mu^2 - 1)} \left(\frac{4\delta^2(\delta-1)}{(4\delta-1)^2} - \frac{\mu-1}{(4\mu-1)^2} \right). \tag{A.3}$$

Equating the two expressions for q_{2L} then yields

$$A(\delta)\mu^4 + B(\delta)\mu^3 + C(\delta)\mu^2 + D(\delta)\mu + E(\delta) = 0 \tag{A.4}$$

where

$$A(\delta) = 4\delta^2, \qquad B(\delta) = -\left(\frac{512\delta^2(\delta-1)}{(4\delta-1)^2} + 7\delta^2\right) \tag{A.5a}$$

$$C(\delta) = \frac{384\delta^2(\delta-1)}{(4\delta-1)^2} + 4, \qquad D(\delta) = -\left(\frac{96\delta^2(\delta-1)}{(4\delta-1)^2} + 3\right) \tag{A.5b}$$

$$E(\delta) = \frac{8\delta^2(\delta-1)}{(4\delta-1)^2} + 2. \tag{A.5c}$$

Define μ_f^D as the unique root of this equation taking values greater than 1. The LEPM, EPM and HEPM qualities are found successively from $q_{2L} = \frac{\mu^2(4\mu-7)}{(4\mu-1)^3}$, $q_1 = \mu q_{2L}$ and $q_{2H} = \delta q_1$. Define $\delta_f'' \approx 3.287$ as the solution to $\delta = \mu_f^D$ and let $\delta > \delta_f''$. Inserting $q_{2H} = q_{f2H}^{BR}(q_1) = \delta q_1$ and $q_{2L} = q_{f2L}^{BR}(q_1) = \frac{q_1}{\delta}$ into $\pi_{2L}(q_1, q_{2L}) = \pi_{2H}(q_1, q_{2H})$ and rearranging gives the EPM quality, the HEPM and LEPM qualities follow.

Note that by construction firm 2 is indifferent between entering with high or low quality in response to $q_{f1}(\delta)$. $q_f^M > q_{f1}^D(\delta)$, so the assumption in the derivation of the MBR quality that firm 2's best response to q_f^M is to enter with a lower quality holds.

A.1.5 Proof of proposition 4

Substituting equation (7) into equation (3b) gives equation (8). For $\delta < \delta_f'$, deterrence is as in the standard case. From the best response functions of firm 2, for a given q_1, $\pi_{2H}\left(q_1, q_{f2H}^{BR}(q_1)\right)$ and $\pi_{2L}\left(q_1, q_{f2L}^{BR}(q_1)\right)$ are weakly decreasing in δ. As by definition $q_{f1}^D(\delta)$ minimizes the entrant profit, its dependence on δ implies $\frac{\partial}{\partial\delta} E_f^D(\delta)\big|_{\delta\geq\delta_f'} < 0$. □

A.1.6 Allowing entry

For sufficiently low δ, $q_{f2L}^{BR}(q_1) = \text{argmax}_{q_{2L}} \pi_{2L}(q_1, q_{2L})$. Denote the constants solving this simultaneously with $\frac{\partial \pi_{1H}\left(q_1, q_{f2L}^{BR}(\delta)\right)}{\partial q_1} = 0$ as $\ell_{f1} \approx 0.245$ and $\ell_{f2} \approx 4.78 \times 10^{-2}$. Substituting this into equation (3a), gives $\pi_{1A}(q_1, q_2)$ for low δ. Assume $q_{f2L}^{BR}(q_1) = \frac{q_1}{\delta}$. Substituting this into equation (A.1a) and rearranging gives $q_1 = \frac{4\delta(\delta-1)}{(4\delta-1)^2}$. Further substitution into equation (3a) yields $\pi_{1A}(q_1, q_2)$ for high δ. Equating the upper and lower parts of $\pi_{1A}(q_1, q_2)$ and rearranging results in

$$8\delta^4(1-32\ell_{f\pi})-16\delta^3(1-16\ell_{f\pi})+8\delta^2(1-12\ell_{f\pi})+16\delta\ell_{f\pi}-\ell_{f\pi}=0 \tag{A.6}$$

where $\ell_\pi = \pi_{1A}(q_1,q_2)|_{q_1=\ell_{f1},q_2=\ell_{f2}} \approx 0.024$. Define $\delta'_{fA} \approx 4.941$ as the unique root of this equation taking a value greater than 1. Let $q_1 = \left.\frac{4\delta(\delta-1)}{(4\delta-1)^2}\right|_{\delta=\delta'_{fA}}$. $\text{argmax}_{q_{2L}}\,\pi_{2L}(q_1,q_{2L}) \approx 0.0465$ and $\frac{q_1}{\delta} \approx 0.0448$, which verifies the assumption that $q^{BR}_{f2L}(q_1) = \frac{q_1}{\delta}$.

A.1.7 Minimum deterrence optimality (MDO) quality

From $\pi_{1ED}(q_1) = \pi_{1A}(q_1,q_2)|_{\delta<\delta'_{fA}}$, $q_1 = c_{f1} = \frac{1}{4}\left(1-\sqrt{1-32\pi_{1A}(q_1,q_2)|_{\delta<\delta'_{fA}}}\right)$. Let δ be sufficiently small that $q^{BR}_{f2L}(\delta) = \text{argmax}_{q_{2L}}\,\pi_{2L}(q_1,q_{2L})$ and denote the constant solving $\text{argmax}_{q_{2L}}\,\pi_{2L}(q_1,c_{f1})$ as $c_{f2} \approx 0.0386$. Let $\delta''_{fE} = \frac{c_{f1}}{c_{f2}} \approx 3.455$. Then for $\delta > \delta''_{fE}$ firm 2's best response to $q_1 = c_{f1}$ is $\frac{q_1}{\delta}$. From $\pi_{1ED}(q_1) = \pi_{1A}(q_1,q_2)|_{\delta>\delta'_{fA}}$, the MDO quality for $\delta > \delta'_{fA}$ is obtained.

A.1.8 Proof of proposition 12

For $\delta < \delta'_f$, $q^D_{f1}(\delta) \approx 0.161$ and $q^{MDO}_{f1}(\delta) \approx 0.134$, so $E^{D*}_f(\delta) = E^D_f(\delta)$. Let δ'_{fE} be the solution to $q^D_{f1}(\delta)|_{\delta'_f<\delta\leq\delta''_f} = q^{MDO}_{f1}(\delta)|_{\delta<\delta''_{fE}}$ with $\delta'_{fE} \approx 2.883$. For $\delta > \delta'_{fE}$, $E^{D*}_f(\delta)$ is then found from substituting equation (11) into equation (3b). By proposition 4, E^{D*}_f is decreasing in δ for $\delta \leq \delta'_{fE}$ and for $\delta'_{fE} < \delta \leq \delta''_{fE}$ it is not dependent on δ. $\frac{\partial}{\partial\delta}\,E^{D*}_f(\delta)|_{\delta''_{fE}<\delta\leq\delta'''_{fE}} > 0$ may be rearranged to become $-4c_{f1}\delta^4 + c_{f1}(7+64c_{f1})\delta^3 - 48c_{f1}\delta^2 + 12c^2_{f1}\delta + c^2_{f1} > 0$ which has no solutions for $\delta > \delta''_{fE}$. $\frac{\partial}{\partial\delta}\,E^{D*}_f(\delta)|_{\delta>\delta''_f} > 0$ becomes a lengthy polynomial in δ which is omitted for reasons of space, and has no solutions for $\delta > \delta'''_{fE}$. □

A.1.9 Proof of proposition 4

By proposition 5, firm 1's profit is weakly greater than in the standard case, so it is sufficient to show it enters when $\delta = 1$. By assumption E is less than the monopoly profit, so firm 1 always enters of $E \in \left[E^M_f(\delta), \bar{E}_f\right)$. Firm 1's profit must be at least $\pi_{1A}(q_1,q_2)|_{\delta=1} \approx 0.0245$, so it enters unless $E \gtrsim 0.0245$ and the market is a natural monopoly. The market is a natural monopoly for $E \geq E_f^M(\delta)\big|_{\delta=1} \approx 0.0015$, so firm 1 enters. $\square$

A.1.10 Proof of proposition 5

$\frac{\partial}{\partial \delta} \pi_{1A}(q_1, q_2)|_{\delta > \delta'_{fA}} > 0$ may be reduced to $2\delta^2 - \delta - 1 > 0$ which holds for $\delta > \delta'_{fA}$, so profit from allowing entry is weakly increasing in δ. $\pi_{1ED}(q_1)$ is increasing in q_1 for $q_1 < q_f^M$. When deterring entry, firm 1's quality solves $\pi_{2L}\left(q_1, q_{f2L}^{BR}(\delta)\right) = E$ so as firm 2's best response is weakly increasing in δ, q_1 is weakly increasing in δ, and so is profit from deterring entry, as $\frac{\partial \pi_{1ED}(q_1)}{\partial q_1} > 0$ for $q_1 < q_f^M$. Natural monopoly profit is constant. As $E_f^M(\delta)$ and $E_f^{D*}(\delta)$ are weakly decreasing in δ, firm 1 may transition from allowing entry to deterring entry and from deterring entry to natural monopoly, both of which increase profit. Incumbent profit is thus weakly increasing in δ and in particular is greater than in the standard case. $\square$

A.2 Marginal costs of quality

A.2.1 Proof of lemma 2

The first order condition of $\pi_H(q_H, q_L)$ is

$$\frac{\partial \pi_H(q_H, q_L)}{\partial q_H} = \frac{q_H(q_H + q_L - 4)\left(24q_H^3 + 2q_L^2(q_L - 4) + q_H q_L(5q_L + 12) - 2q_H^2(11q_L + 8)\right)}{4(4q_H - q_L)} \tag{A.7}$$

so there are at most five stationary points, two of which are $q_H = 0$ and $q_H = 2 - \frac{1}{2}q_L$. The polynomial in the rightmost bracket of the numerator has discriminant $\Delta = -71964q_L^6 + 415008q_L^5 - 627392q_L^4 + 121856q_L^3 - 94208q_L^2$. $\Delta \geq 0$ if $q_L = \frac{8}{3}$, $q_L \approx 3.005$, both of which lie outside the range, or $q_L = 0$, in which case the roots of the polynomial are $q_H = \frac{2}{3}$ and $q_H = 2$. Thus $\Delta < 0$ for $q_L < q_H < 2 - \frac{1}{2}q_L$ and there is a unique maximum of $\pi_H(q_H, q_L)$ in this range.

$\pi_L(q_H, q_L)$ is continuous in the range $0 \leq q_L \leq q_H$, is 0 at $q_L = 0$ and $q_L = q_H$ and is positive for some q_L within the range. Thus if there is a single stationary point in the interior of the range, it is a maximum. The first order condition of $\pi_L(q_H, q_L)$ is

$$\frac{\partial \pi_L(q_H, q_L)}{\partial q_L} = \frac{q_H(2q_H - q_L)\left(4q_H^3 + q_H^2(8 - 19q_L) - 2q_L^3 + q_H q_L(17q_L - 14)\right)}{4(4q_H - q_L)} \tag{A.8}$$

which shows there are at most four stationary points, one of which is $q_L = 2q_H$. The polynomial in the rightmost bracket of the numerator has discriminant $\Delta = 15633q_H^6 - 4380q_H^5 + 28900q_H^4 - 21952q_H^3$. $\Delta \geq 0$ if either $q_H = 0$, which implies no q_L such that $0 < q_L < q_H$, or if $q_H = \frac{2}{3}$, in which case the sole root of the first order condition is $q_L = \frac{1}{3}$. Otherwise $\Delta < 0$ and there is a single root of the polynomial and there is a unique maximum of $\pi_{2L}(q_H, q_L)$ for $0 < q_L < q_H$. □

A.2.2 Monopoly best response (MBR) quality

Assume firm 2's best response is to enter as the low quality firm (this will be shown to hold in the derivation of the EPM quality). Let δ be sufficiently small that $q_{m2L}^{BR}(q_1) = \operatorname{argmax}_{q_{2L}} \pi_{2L}(q_1, q_{2L})$. From equation (A.8), $q_{2L}^{MBR} = \frac{1}{3}$. Let δ'_{mM} be the solution to $\pi_{2L}(q_1, q_{2L})\big|_{q_1=\frac{2}{3}, q_{2L}=\frac{1}{3}} = \pi_{2I}(q_1, \delta)\big|_{q_1=\frac{2}{3}}$ with $\delta'_{mM} = 2\sqrt{\frac{2}{7}} \approx 1.069$. As $\delta'_{mM} < \frac{q_m^M}{q_m^{MBR}}\Big|_{\delta \leq \delta'_{mM}} = 2$ this completes the derivation.

A.2.3 Proof of proposition 6

Equation (20) is obtained by substituting equation (19) into firm 2's profit function. $E_m^M\big|_{\delta \leq \delta'_{mM}} < \bar{E}_m$, so natural monopoly is possible. $\frac{\partial}{\partial \delta} E_m^M(\delta)\big|_{\delta > \delta'_{mM}} = \frac{8}{27\delta^3} > 0$ and $E_m^M(\delta) = \bar{E}_m$ has the solution $\delta = \sqrt{2}$. □

A.2.4 EPM, HEPM and LEPM qualities

For sufficiently small δ, $q_{m2H}^{BR}(q_1) = \operatorname{argmax}_{q_{2H}} \pi_{2H}(q_1, q_{2H})$ and $q_{m2L}^{BR}(q_1) = \operatorname{argmax}_{q_{2L}} \pi_{2L}(q_1, q_{2L})$. Denote the constants solving these equations simultaneously with $\pi_{2H}(q_1, q_{2H}) = \pi_{2L}(q_1, q_{2L})$ as $b_1 \approx 0.611$, $b_{mH} \approx 0.939$ and $b_{mL} \approx 0.309$. Let δ'_m be the solution to $\pi_{2I}(q_1, \delta)\big|_{q_1=b_{m1}} = \pi_{2L}(q_1, q_{2L})\big|_{q_1=b_{m1}, q_{2L}=b_{mL}}$. $\delta'_m \approx 1.071$ and $\frac{b_{mH}}{b_{m1}} \approx 1.533$, $\frac{b_{m1}}{b_{mL}} \approx 1.980$, so for $\delta > \delta'_m$ firm

2's best response to $q_1 = b_{m1}$ is to immitate. Let $\delta > \delta'_m$ but be sufficiently small that $q^{BR}_{m2H}(q_1) = \text{argmax}_{q_{2H}} \pi_{2H}(q_H, q_L)$. Let $\mu = \frac{q_{2H}}{q_1}$ and substitute $q_{2H} = \mu q_1$ into equation (A.7), which after rearrangement gives $q1 = \frac{4\mu(4\mu^2-3\mu+2)}{24\mu^3-22\mu^2+5\mu+2}$. Substitution of this expression and $q_{2H} = \mu q_1$ into $\pi_{2I}(q_1, \delta) = \pi_{2L}(q_1, q_{2L})$ yields

$$\begin{aligned} &512\delta^2\mu^9(\delta) - 1792\delta^2\mu^8(\delta) + \left(1536 + 800\delta^2\right)\mu^7(\delta) - \\ &- \left(3840 - 2464\delta^2\right)\mu^6(\delta) + \left(4544 - 4192\delta^2\right)\mu^5(\delta) - \\ &- \left(3264 - 3232\delta^2\right)\mu^4(\delta) - 1438\left(1 - \delta^2\right)\mu^3(\delta) - \\ &- 389\left(1 - \delta^2\right)\mu^2(\delta) + 60\left(1 - \delta^2\right)\mu(\delta) - 4\left(1 - \delta^2\right) = 0. \end{aligned} \tag{A.9}$$

Although no analytical solutions exist, numerical approximations are possible to find for given values of δ. Define μ^D_m as the unique root of this equation taking a value greater than 1, from which the EPM and HEPM qualities are found, with the LEPM quality following from $q_{2L} = \frac{q_1}{\delta}$. Define δ''_m as the solution to $\mu^D_m = \delta$, with $\delta''_m \approx 2.336$. Let $\delta > \delta''_m$. Substituting $q_{2H} = \delta q_1$ and $q_{2L} = \frac{q_1}{\delta}$ into $\pi_{2H}(q_1, q_{2H}) = \pi_{2I}(q_1)$ and rearranging yields $q^D_{m1}(\delta)\big|_{\delta > \delta''_m} = \frac{-(\delta) - \sqrt{B^2(\delta) - 4A(\delta)C(\delta)}}{2A(\delta)}$, where

$$A(\delta) = \left(\delta^2 - 1\right)(4\delta - 1)^2 + \delta^4(\delta - 1)(1 + 2\delta) \tag{A.10a}$$

$$B(\delta) = -2\left(\delta^2 - 1\right)(4\delta - 1)^2 - 8\delta^4(\delta - 1)(1 + 2\delta) \tag{A.10b}$$

$$C(\delta) = 16\delta^4(\delta - 1). \tag{A.10c}$$

The HEPM and LEPM qualities then follow directly.

$\pi_{2I}(q_1, \delta)$ is decreasing in q_1 for $q_1 > \frac{4}{3} > q^M_m$. To show that firm 2's profit is not minimized at $\pi_{2I}(q_1, \delta) = \pi_{2L}\left(q_1, q^{BR}_{m2L}(q_1)\right)$, note that as $\frac{\partial \pi_{2L}(q_1, q_{2L})}{\partial q_1} > 0$, a necessary conditions for this to be the case is that $\pi_{2L}\left(q_1, q^{BR}_{m2L}(q_1)\right)\big|_{q_1 = q^M_1} < \pi_{2I}(q_1, \delta)\big|_{q_1 = q^D_{m1}(\delta)}$, which does not hold from $E^M_m(\delta) > E^D_m(\delta)$.

A.2.5 Proof of proposition 7

Substituting equation (21) into equation (14a) for $\delta \leq \delta'_m$ and $\pi_{2I}(q_1, \delta)$ for $\delta > \delta'_m$ gives equation (22). If $q^{BR}_{m2H}(q_1) = \text{argmax}_{q_{2H}} \pi_{2H}(q_1, q_{2H})$ and $q^{BR}_{m2L}(q_1) = \frac{q_1}{\delta}$, firm 2's maximized profit for a given q_1 must be increasing in δ, so that the entry deterrence condition is increasing in δ for $\delta'_m < \delta \leq \delta''_m$. $\frac{\partial E^D_m(\delta)}{\partial \delta}$ can be found to be negative at some $\delta > \delta''_m$, $\frac{\partial}{\partial \delta} E^D_m(\delta)\big|_{\delta > \delta''_m}$ is continuous and it can be shown numerically that $\frac{\partial}{\partial \delta} E^D_m(\delta)\big|_{\delta > \delta''_m} = 0$ has no solutions for $\delta > \delta''_m$. □

A.2.6 Allowing entry

Let δ be sufficiently small that $q^{BR}_{m2L}(q_1) = \text{argmax}_{q_{2L}} \pi_{2L}(q_1, q_{2L})$. Denote the constants that solve $\max_{q_1} \pi_{1H}\left(q_1, q^{BR}_{m2L}(q_1)\right)$ as $\ell_{m1} \approx 0.567$ and $\ell_{m2} \approx 0.289$. Define $\delta'_{mA} \approx 1.073$ as the solution to $\pi_{2I}(q_1, \delta)\big|_{q_1 = \ell_{m1}} = \pi_{2L}(q_1, q_{2L})\big|_{q_1 = \ell_{m1}, q_{2L} = \ell m2}$. For $\delta > \delta'_{mA} \approx 1.073$, firm 1 makes 0 unless (i) q_1 is sufficiently low that $\pi_{2I}(q_1 \delta) \leq \pi_{2H}\left(q_1, q^{BR}_{m2H}(q_1)\right)$ or (ii) q_1 is sufficiently high that $\pi_{2I}(q_1) \leq \pi_{2L}\left(q_1, q^{BR}_{m2L}(q_1)\right)$. Assume (i) is satisfied, then firm 1 chooses the q_1 such that it holds with equality, which is simply $q^D_{m1}(\delta)\big|_{\delta > \delta'_m}$ (equation (21a)) with firm 2's best response being $q^D_{m2H}(\delta)\big|_{\delta > \delta'_m}$. Thus $q^A_{m1}(\delta)$ and $q^A_{m2}(\delta)$ are arrived at, and substitution gives $\pi_{1A}(q_1, q_2)$. Suppose instead firm 1 chooses q_1 such that (ii) holds with equality. By a similar method to the derivation of equation (21), the profit of the firm is

$$\tilde{\pi}_{1A}(q_1, q_2)\big|_{\delta > \delta'_{mA}} = \frac{2\tilde{\mu}^A_m \left(4\tilde{\mu}^A_m - 7\right)\left(\tilde{\mu}^A_m - 1\right)\left(16\tilde{\mu}^{A3}_m - 48\tilde{\mu}^{A2}_m + 27\tilde{\mu}^A_m + 4\right)^2}{\left(4\tilde{\mu}^A_m - 1\right)\left(2 - 17\tilde{\mu}^A_m + 19\tilde{\mu}^{A2} - 4\tilde{\mu}^{A3}_m\right)^2} \tag{A.11}$$

where $\tilde{\mu}^A_m$ is the solution to

$$2(\mu - 1)\left(4\mu^2 - 5\mu + 1\right)^2 - \left(\frac{\delta^2 - 1}{\delta^2}\right)\mu^2 (4\mu - 1)^2 (4\mu - 7)\left(8\mu^3 - 26\mu^2 + 17\mu - 2\right) = 0. \tag{A.12}$$

$\left(\pi_{1A}(q_1, q_2) - \tilde{\pi}(q_1, q_2)\right)\big|_{\delta > \delta'_{mM}} > 0$ for some $\delta > \delta'_{mA}$ and numerically there are no roots of $\left(\pi_{1A}(q_1, q_2) - \tilde{\pi}(q_1, q_2)\right)\big|_{\delta > \delta'_{mA}} = 0$ such that $\delta > \delta'_{mA}$.

A.2.7 Proof of proposition 8

From equation (15a), if firm 1 deters entry it earns profit $\pi_{1ED}(q_1) = \frac{1}{16}q_1(2-q_1)^2$. If $\delta = 1$, $\pi_{1ED}(q_1)|_{q_1=q^D_{m1}(\delta)} = \pi_{1A}(q_1,q_2)$ and numerically the roots of $\pi_{1ED}(q_1)|_{q_1=q^D_{m1}(\delta)} - \pi_{1A}(q_1,q_2) = 0$ lie outside the feasible range of quality. □

A.2.8 Proof of proposition 9

Let $E \in \left[E^D_m(\delta), \min\left\{E^M_m(\delta), \bar{E}\right\}\right)$, so that conditional on entering firm 1 sets q_1 to satisfy $\max\left\{\pi_{2L}\left(q_1, \hat{q}^{BR}_{m2L}(q_1)\right), \pi_{2I}(q_1,\delta)\right\} = E$, so if $\pi_{1ED}(q_1) < E$ at this quality it will not enter. $\pi_{1ED}(q_1) = \pi_{2L}\left(q_1, \hat{q}^{BR}_{m''L}(q_1)\right)$ has no solutions for $q_1 < q^M_m$. Let δ be sufficiently high that $\pi_{2L}\left(q_1, \hat{q}^{BR}_{m2L}(q_1)\right) < \pi_{2I}(q_1,\delta)$. Solving $\pi_{1ED}(q_1) = \pi_{2I}(q_1,\delta)$ gives $q^{MC1}_1(\delta) = \frac{2\delta^2}{5\delta^2-4}$. Substitution into $\pi_{1ED}(q_1)$ gives $E^{MC1}_m(\delta)$. To find when $E^{MC1}_m(\delta)$ lies within the range $\left(E^D_m(\delta), \min\left\{E^M_m(\delta), \bar{E}_m\right\}\right)$, $q^{MC1}_1(\delta) > q^M_m$ for $\delta < \sqrt{2}$. $E^{MC1}_m(\delta) = \bar{E}_m$ has the solution $\delta > \sqrt{2}$ and $\frac{\partial E^{MC1}_m(\delta)}{\partial\delta} = -32\delta\frac{\left(\delta^4-\delta^2+2\right)}{(5\delta^2-4)^4}$ so that $\frac{\partial E^{MC1}_m(\delta)}{\partial\delta} < 0$ for $\delta > \sqrt{2}$. Numerically $E^D_m(\delta) - E^{MC1}_m(\delta) = 0$ has no solutions for $\delta > \sqrt{2}$ and so $E^{MC1}_m(\delta) \in \left(E^D_m(\delta), \bar{E}_m\right)$ for $\delta > \sqrt{2}$.

Let $E \in \left(0, E^D_m(\delta)\right)$ so that conditional on entering firm 1 allows entry. For $\delta \leq \delta'_{mA}$, $\pi_{1A}(q_1,q_2) \approx 0.0379$ and $E^D_m(\delta) \approx 0.0166$, so firm 1 enters. Let $E^{MC2}_m(\delta) = \pi_{1A}(q_1,q_2)$ so that firm 1 does not enter for $E \in \left(E^{MC2}_m(\delta), E^D_m(\delta)\right)$. Define the unique root of $E^{MC2}_m(\delta)\big|_{\delta'_m<\delta\leq\delta''_m} - E^D_m(\delta)\big|_{\delta'_m<\delta\leq\delta''_m} = 0$ as $\delta'_{MC} \approx 1.184$ and the unique root of $E^{MC2}_m(\delta)\big|_{\delta>\delta''_m} - E^D_m(\delta)\big|_{\delta>\delta''_m} = 0$ as $\delta''_{MC} \approx 4.211$. Then $E^{MC2}_m(\delta) \in \left(0, E^D_m(\delta)\right)$ for $\delta'_{MC} < \delta < \delta''_{MC}$. □

A.2.9 Proof of proposition 10

Let $E \in \left(0, E^D_m\big|_{\delta=1}\right)$, so that entry is allowed in the standard case. For $\delta \geq E^{D-1}_m(E)$, entry is deterred. Firm 1 deters entry by choosing q_1 such that $\pi_{2I}(q_1,\delta) = E$, and its profit is equal to that in the standard case if $\pi_{1ED}(q_1) = \pi_{1A}(q_1,\delta)|_{\delta=1}$. Combining these two equations results in $q_1 = \frac{2\delta^2 E}{E\delta^2+4(\delta^2-1)\,\pi_{1A}(q_1,q_2)|_{\delta=1}}$. Substitution into $\pi_{1ED}(q_1) = \pi_{1A}(q_1,q_2)|_{\delta=1}$ yields equation (26) from which $E^{\pi}_m(\delta)$ is found.

For $E \in (0, E_m^{\pi}(\delta))$, if it deters entry firm 1 makes less than in the standard case, and if it allows entry, from equation (23) for $\delta > \delta'_{mA}$ it shares the market by producing the same quality as when it deterred entry when $E = E_m^D(\delta)$, implying a further reduction in profit. If $E \in \left[E_m^D(\delta)\big|_{\delta=1}, E_m^M\big|_{\delta=1} \right)$, for $\delta \leq \delta'_m$ profit is as in the standard case and for $\delta > \delta'_m$ firm 1 either allows entry, implying lower profit than in the standard case, or deters entry by selecting q_1 such that $\pi_{2I}(q_1, \delta) = E$. From $\frac{\partial \pi_{2I}(q_1,\delta)}{\partial \delta} > 0$ and $\frac{\partial}{\partial q_1} \pi_{1ED}(q_1)\big|_{q_1 < q_m^M} < 0$, profit is lower than in the standard case. If $E \in \left[E_m^M(\delta), \bar{E}_m \right)$, profit is as in the standard case for $\delta \leq \delta'_{mM}$ and for $\delta > \delta'_{mM}$ and for $\delta > \delta'_{mM}$ the market is not a natural monopoly, implying lower profit than in the standard case. $\square$

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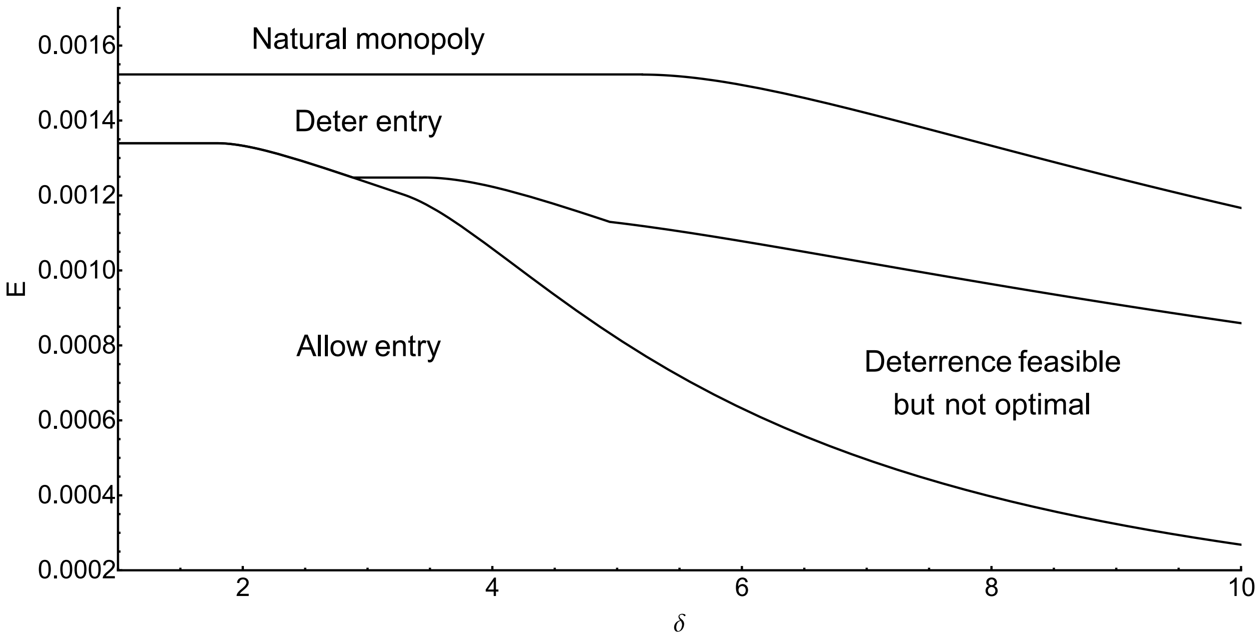

Figure 1: Entry deterrence when quality costs are fixed.

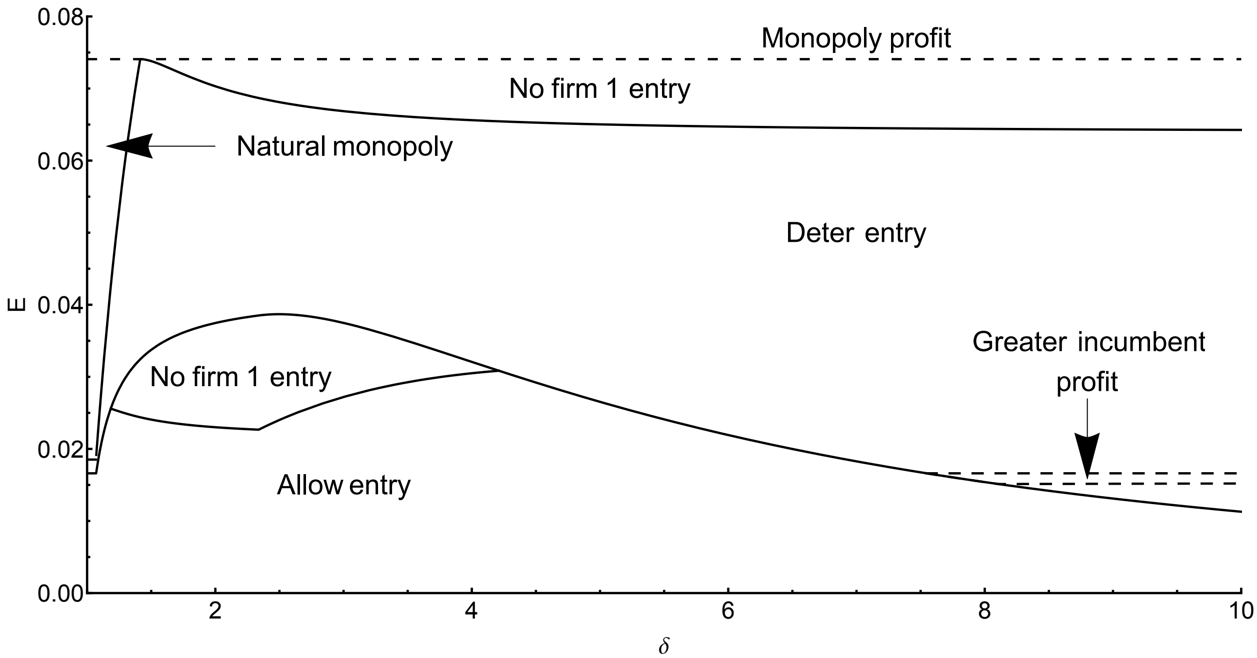

Figure 2: Entry deterrence when costs of quality are marginal.

Table 1: Summary of results contrasting fixed and marginal costs of quality

	Fixed costs	Marginal costs
Natural monopoly	Decreasing in δ	Increasing in δ
		Not observed for $\delta > \sqrt{2}$
Entry deterrence	Decreasing in δ	Increasing for low δ
		Decreasing for high δ
Incumbent profit	Weakly greater than standard case	Weakly lower than standard case apart from small set of parameters
Market creation	Market always created	Market not always created