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Correlated equilibria in homogenous good Bertrand competition

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Abstract

We show that there is a unique correlated equilibrium, identical to the unique Nash equilibrium, in the classic Bertrand oligopoly model with homogenous goods. This provides a theoretical underpinning for the so-called "Bertrand paradox" and also generalizes earlier results on mixed-strategy Nash equilibria. Our proof generalizes to asymmetric marginal costs and arbitrarily many players.

JEL: C72, D43, L13

Keywords: Bertrand paradox, correlated equilibrium, price competition

A substantial body of theory in industrial organization and other fields of economics is built on the idea that there are no equilibria with positive expected profits in a simple Bertrand competition model with homogenous goods and symmetric firms—in other words, that there are no profitable cartels and that price competition between $n > 1$ firms will drive prices down to marginal cost in one-shot price competition. The fact that price competition between two firms is equivalent to perfect competition is often referred to as the "Bertrand paradox".

Yet the theoretical foundation for this idea is not fully clear, especially where correlated equilibria are concerned. In a correlated equilibrium, players can construct a correlation device which gives each player a private recommendation before the players

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choose their actions. In correlated equilibrium, the device is such that it is an equilibrium for the players to follow the recommendation. Every (mixed strategy) Nash equilibrium is a correlated equilibrium where the recommendations are independent. Players can in many games achieve higher payoffs in correlated equilibrium than in Nash equilibrium because the device is able to correlate recommendations; see Aumann (1974).

Milgrom and Roberts (1990) show for a large class of demand functions that the unique Nash equilibrium is also the unique correlated equilibrium of Bertrand games with differentiated goods, but their reasoning only applies to supermodular games. A Bertrand game with homogenous goods is not supermodular since the profit functions (i) do not have increasing differences and (ii) are not order upper semi-continuous in the firm's price.

In this note, we show that no correlated equilibrium (and hence also no mixed Nash equilibrium) with positive expected profits can exist in a Bertrand game with homogenous products and bounded monopoly profits. While this is not entirely unexpected (and certainly a desirable property), it is also not trivial given the large set of rationalizable actions: In symmetric, homogenous good Bertrand competition all non-negative prices are rationalizable.[1] This is in stark contrast to Milgrom and Roberts' differentiated Bertrand example in which only the unique Nash equilibrium price is rationalizable.

Our proof is by contradiction: We show that if there was a correlated equilibrium in which prices higher than marginal cost were played with positive probability, then there would be an interval of recommendations in which each player prefers to deviate downwardly from his recommendation. This interval consists of the highest recommendations that a player might get in the assumed equilibrium from the correlation device, and recommendations in this interval are received with positive probability.

The contribution of this paper lies in the proof that in Bertrand games with arbitrary demand functions (in which the set of rationalizable actions is infinite), the Bertrand Nash equilibrium is the unique correlated equilibrium.

Apart from that, it is also a generalization (by different methods) of results of Baye

[1]Every $p_i \in \mathbb{R}_+$ is in our model rationalizable because p_i is – assuming zero marginal costs – a best response to $p_j = 0$ which is the Bertrand equilibrium price and therefore itself rationalizable.

and Morgan (1999) and Kaplan and Wettstein (2000) on mixed-strategy equilibria in Bertrand games. Baye and Morgan (1999) show that if monopoly profits are unbounded, any positive finite payoff vector can be achieved in a symmetric mixed-strategy Nash equilibrium, and Kaplan and Wettstein (2000) prove that unboundedness of monopoly profits is both necessary and sufficient for the existence of such mixed-strategy Nash equilibria. These insights have led Klemperer (2003, section 5.1) to conclude that "there are other equilibria with large profits, for some standard demand curves." We show that expected profits in any correlated equilibrium (and therefore in any mixed Nash equilibrium) are zero if demand is such that monopoly profits are bounded. Finally, unlike the cited results, our proof is generalizable to games with asymmetric costs and arbitrarily many players, as we show in the supplementary material.

A related result is derived in Liu (1996). Liu shows that the unique Nash equilibrium in Cournot competition with linear demand and constant marginal costs is also the unique correlated equilibrium.

Model

There are two firms with constant marginal costs which are normalized to zero. As we show in the supplementary material, neither the assumption of identical marginal costs nor the restriction to two firms is necessary to obtain our result, but they ease notation and exposition. Firms set prices simultaneously. The price of firm i is denoted by p_i. If $p_i < p_j$, consumers buy quantity $D(p_i)$ of the good from firm i (and 0 units from firm j). If both firms quote the same price p', consumers buy $D(p')/2$ from each firm. $D(p)$ denotes market demand where $D : \mathbb{R}_+ \to \mathbb{R}_+$ is a (weakly) decreasing, measurable function and $\mathbb{R}_+$ denotes the non-negative real numbers. We assume that the demand function is such that a strictly positive monopoly price $p^{mon} = arg \max_p pD(p)$ exists.[2] Firms maximize expected profits.

A correlated equilibrium in this game is a probability distribution F on $\mathbb{R}_+ \times \mathbb{R}_+$. This probability distribution is interpreted as a correlation device. The correlation

[2]If there are several prices maximizing $pD(p)$, p^{mon} denotes the supremum of all the profit maximizing prices. For notational convenience, we assume that the monopoly profit is attained at $p^{mon}D(p^{mon})$ also in this case.

device sends recommended prices (r_1, r_2) to the two firms. Each firm i observes r_i but does not observe the other firm's recommendation r_j. $F(p_1, p_2)$ is the probability that $(r_1, r_2) \leq (p_1, p_2)$. Roughly speaking, a distribution F is called a *correlated equilibrium* if both firms find it optimal to follow the recommendation.

To be more precise denote the profits of firm i given prices p_i and p_j with $i, j \in \{1, 2\}$ and $i \neq j$ as

$$\pi_i(p_i, p_j) = \begin{cases} p_i D(p_i) & \text{if } p_i < p_j \\ p_i D(p_i)/2 & \text{if } p_i = p_j \\ 0 & \text{else.} \end{cases} \tag{1}$$

Note that we define the profit function such that the *own* price is the first argument, i.e. the first argument of π_2 is p_2.

A strategy for firm i is a mapping from "recommendations" to prices. Both recommendations and prices are in $\mathbb{R}_+$. Hence, a strategy is a measurable function $\zeta_i : \mathbb{R}_+ \to \mathbb{R}_+$. The identity function represents the strategy of following the recommendation. F is a correlated equilibrium if no firm can gain by unilaterally deviating from a situation where both firms use $\zeta_i =$identity function. More formally, we follow the definition of correlated equilibrium for infinite games given in Hart and Schmeidler (1989) and also used in Liu (1996): A correlated equilibrium is a distribution F on $\mathbb{R}_+ \times \mathbb{R}_+$ such that for all measurable functions $\zeta_i : \mathbb{R}_+ \to \mathbb{R}_+$ and all $i \in \{1, 2\}$ and $i \neq j \in \{1, 2\}$ the following inequality holds:

$$\int_{\mathbb{R}_+ \times \mathbb{R}_+} \pi_i(p_i, p_j) - \pi_i(\zeta_i(p_i), p_j) \; dF(p_1, p_2) \geq 0. \tag{2}$$

In words, a distribution F is a correlated equilibrium if no player can achieve a higher expected payoff by unilaterally deviating to a strategy ζ_i instead of simply following the recommendation. Last, we define a *symmetric* correlated equilibrium as a correlated equilibrium F in which $F(p_1, p_2) = F(p_2, p_1)$ for all $(p_1, p_2) \in \mathbb{R}_+ \times \mathbb{R}_+$.

It is well known that both firms set prices equal to zero in the unique Nash equilibrium of this game (usually this is called "Bertrand equilibrium"); see, for example, Kaplan and Wettstein (2000).

Analysis and Result

We start the analysis by noting that whenever there is a correlated equilibrium F then there is a symmetric correlated equilibrium G in which the aggregated expected profits are the same as in F. This result is, of course, due to the symmetry of our setup. It will allow us later on to focus on symmetric correlated equilibria.[3]

Lemma 1. *Let F be a correlated equilibrium. Then there exists a symmetric correlated equilibrium G such that*

$$\int_{\mathbb{R}_+\times\mathbb{R}_+} \pi_1(p_1,p_2)+\pi_2(p_2,p_1)\; dF(p_1,p_2) = \int_{\mathbb{R}_+\times\mathbb{R}_+} \pi_1(p_1,p_2)+\pi_2(p_2,p_1)\; dG(p_1,p_2).$$

Proof. Let F be a correlated equilibrium. Define $\tilde{F}(p_1,p_2)=F(p_2,p_1)$. Then, $\tilde{F}$ is also a correlated equilibrium as for any measurable function $\zeta:\mathbb{R}_+\to\mathbb{R}_+$

$$\begin{aligned}
&\int_{\mathbb{R}_+\times\mathbb{R}_+} \pi_i(p_i,p_j)-\pi_i(\zeta(p_i),p_j)\; d\tilde{F}(p_1,p_2)\\
=&\int_{\mathbb{R}_+\times\mathbb{R}_+} \pi_i(p_j,p_i)-\pi_i(\zeta(p_j),p_i)\; dF(p_1,p_2)\\
=&\int_{\mathbb{R}_+\times\mathbb{R}_+} \pi_j(p_j,p_i)-\pi_j(\zeta(p_j),p_i)\; dF(p_1,p_2)\geq 0
\end{aligned}$$

where the first equality holds by the definition of $\tilde{F}$, the second holds by the symmetry of the setup, i.e. $\pi_1(x,y)=\pi_2(x,y)$, and the inequality holds as F is a correlated equilibrium.

Define $G(p_1,p_2)=\frac{1}{2}F(p_1,p_2)+\frac{1}{2}\tilde{F}(p_1,p_2)$. Then G is a correlated equilibrium as for any measurable function $\zeta:\mathbb{R}_+\to\mathbb{R}_+$

$$\begin{aligned}
&\int_{\mathbb{R}_+\times\mathbb{R}_+} \pi_i(p_i,p_j)-\pi_i(\zeta(p_i),p_j)\; dG(p_1,p_2)\\
=&\ \frac{1}{2}\int_{\mathbb{R}_+\times\mathbb{R}_+} \pi_i(p_i,p_j)-\pi_i(\zeta(p_i),p_j)\; dF(p_1,p_2)+\frac{1}{2}\int_{\mathbb{R}_+\times\mathbb{R}_+} \pi_i(p_i,p_j)-\pi_i(\zeta(p_i),p_j)\; d\tilde{F}(p_1,p_2)\\
\geq&\ \frac{1}{2}0+\frac{1}{2}0=0
\end{aligned}$$

where the equality follows from the definition of G and the inequality follows from the fact that F and $\tilde{F}$ are correlated equilibria. Clearly, G is symmetric as $G(p_1,p_2)=$

[3]Intuitively, we make use of the fact that the set of correlated equilibria in this game is convex—as could be shown by generalizing the following lemma with arbitrary weights instead of $\frac{1}{2}$ and $\frac{1}{2}$.

$\frac{1}{2}F(p_1,p_2)+\frac{1}{2}\tilde{F}(p_1,p_2)=\frac{1}{2}F(p_1,p_2)+\frac{1}{2}F(p_2,p_1)=\frac{1}{2}\tilde{F}(p_2,p_1)+\frac{1}{2}F(p_2,p_1)=G(p_2,p_1)$ by the definition of G and $\tilde{F}$. Finally, expected profits under F and G are the same as

$$\begin{aligned}
& \int_{\mathbb{R}_+\times\mathbb{R}_+} \pi_1(p_1,p_2)+\pi_2(p_2,p_1)\; dG(p_1,p_2) \\
= & \frac{1}{2}\int_{\mathbb{R}_+\times\mathbb{R}_+} \pi_1(p_1,p_2)+\pi_2(p_2,p_1)\; dF(p_1,p_2)+\frac{1}{2}\int_{\mathbb{R}_+\times\mathbb{R}_+} \pi_1(p_2,p_1)+\pi_2(p_1,p_2)\; dF(p_1,p_2) \\
= & \frac{1}{2}\int_{\mathbb{R}_+\times\mathbb{R}_+} \pi_1(p_1,p_2)+\pi_2(p_2,p_1)\; dF(p_1,p_2)+\frac{1}{2}\int_{\mathbb{R}_+\times\mathbb{R}_+} \pi_2(p_2,p_1)+\pi_1(p_1,p_2)\; dF(p_1,p_2) \\
= & \int_{\mathbb{R}_+\times\mathbb{R}_+} \pi_1(p_1,p_2)+\pi_2(p_2,p_1)\; dF(p_1,p_2)
\end{aligned}$$

where the first equality follows from the definition of G and $\tilde{F}$ and the second equality follows from the symmetry of setup, i.e. $\pi_1(x,y)=\pi_2(x,y)$. □

Let F be a symmetric correlated equilibrium. Define $\bar{p}:=\inf\{p' : \int_{(p',\infty)^2} dF(p_1,p_2)=0\}$. Intuitively, $\bar{p}$ is the price such that (i) the probability that the market price is greater than $\hat{p}$ is strictly positive for any $\hat{p}<\bar{p}$ and (ii) the probability that the market price is greater than $\hat{p}$ is zero for any $\hat{p}>\bar{p}$. That is, if we consider the distribution of prices that consumers pay in the correlated equilibrium F, $\bar{p}$ is the essential supremum of this "market price distribution". The following lemma establishes that $\bar{p}$ exists by showing that $\int_{(p^{mon},\infty)^2} dF(p_1,p_2)=0$ in any correlated equilibrium F. This implies $\bar{p}\le p^{mon}$ and consequently a finite $\bar{p}$ exists. The intuitive reason for lemma 2 is that setting a prices above p^{mon} is a weakly dominated strategy.

Lemma 2. *In a correlated equilibrium F, $\int_{(p^{mon},\infty)^2} dF(p_1,p_2)=0$.*

Proof. Consider the strategy

$$\zeta_1(r_1)=\begin{cases} r_1 & \text{if } r_1\le p^{mon} \\ p^{mon} & \text{if } r_1>p^{mon}. \end{cases}$$

Firm 1's payoff difference between following the recommendation and using the deviation strategy ζ_1 is

$$\int_{(p^{mon},\infty)^2} [\pi_1(p_1,p_2)-p^{mon}D(p^{mon})]\; dF(p_1,p_2)+\int_{(p^{mon},\infty)\times\{p^{mon}\}} -p^{mon}D(p^{mon})/2\, dF(p_1,p_2).$$

The integrand of the first integral is strictly negative by the definition of p^{mon}. The second integral is non-positive. Consequently, F can only be a correlated equilibrium, i.e. satisfy (2), if $\int_{(p^{mon},\infty)^2} dF(p_1,p_2)=0$. □

Before we proceed, it is useful to define the following sets which will serve as the domain of integration multiple times in the following proofs. For some $\hat{p} \in (0, \bar{p})$ and $\varepsilon \in (0, 1)$, define the sets

$$\begin{aligned}
A(\hat{p}) &= \{(p_1, p_2) : \ p_1 \in (\hat{p}, \bar{p}] \text{ and } p_2 \in [p_1, \bar{p}]\} \\
B(\hat{p}) &= \{(p', p') : \ \hat{p} < p' \leq \bar{p}\} \\
C(\hat{p}, \varepsilon) &= \{(p_1, p_2) : \ p_1 \in (\hat{p}, \bar{p}] \text{ and } p_2 \in [\varepsilon p_1, \bar{p}]\} \\
E(\hat{p}) &= \{(p_1, p_2) : \ p_1 \in (\hat{p}, \bar{p}] \text{ and } p_2 \in [\hat{p}, \bar{p}]\} \\
E'(\hat{p}) &= \{(p_1, p_2) : \ p_1 \in (\hat{p}, \bar{p}] \text{ and } p_2 \in (\hat{p}, \bar{p}]\}.
\end{aligned}$$

Figure 1 depicts the sets.

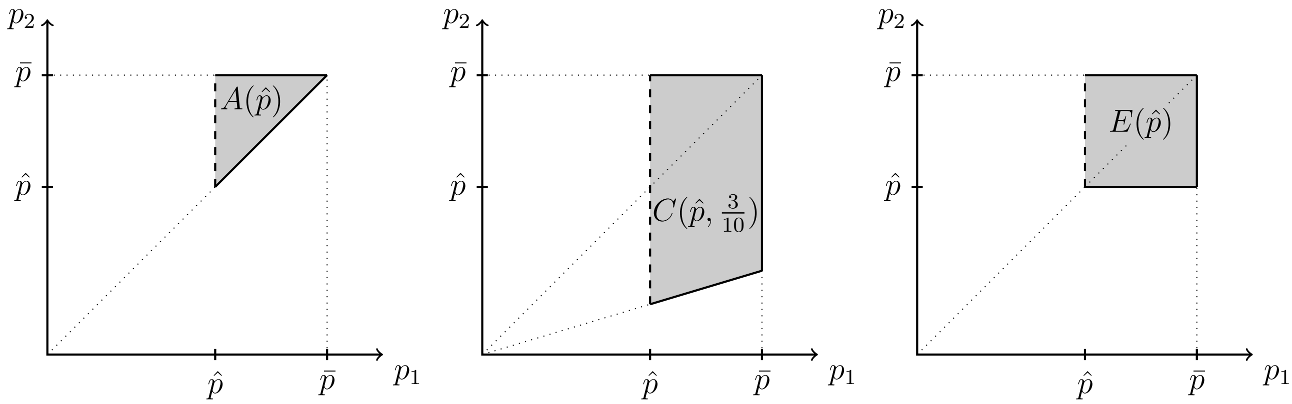

Figure 1: $A(\hat{p})$ is shown in panel 1, while $B(\hat{p})$ is simply the diagonal between $(\hat{p}, \hat{p})$ and $(\bar{p}, \bar{p})$, including the latter but not the former point. Panel 2 shows $C(\hat{p}, 0.3)$. Panel 3 shows $E(\hat{p})$; $E^{'}(\hat{p})$ is identical to $E(\hat{p})$ except that the border where $p_2 = \hat{p}$ is not part of the set.

It follows immediately from the definition of $\bar{p}$ and the symmetry of the setup that $\int_{A(\hat{p})} dF(p_1, p_2) > 0$ for any $\hat{p} \in (0, \bar{p})$. That is, a firm deviating by charging $\hat{p} < \bar{p}$ given any recommendation will sell with positive probability. This observation will be important later on.

The following lemma shows that there is no probability mass on the diagonal of the distribution F above $(\hat{p}, \hat{p})$ if F is a symmetric correlated equilibrium.

Lemma 3. *Let F be a symmetric correlated equilibrium. Then, $\int_{B(\hat{p})} dF(p_1, p_2) = 0$ for any $\hat{p} \in (0, \bar{p})$.*

Proof. The proof is by contradiction. Suppose to the contrary that $\int_{B(\hat{p})} dF(p_1, p_2) > 0$. Recall that π_1 is discontinuous at points on the diagonal of the (p_1, p_2) plane. Therefore, (2) is violated for

$$\zeta^{\varepsilon}(r_1) = \begin{cases} r_1 & \text{if } r_1 \notin (\hat{p}, \bar{p}] \\ \varepsilon r_1 & \text{if } r_1 \in (\hat{p}, \bar{p}] \end{cases}$$

for $\varepsilon \in (0, 1)$ sufficiently close to 1: Firm 1's payoff difference between following the recommendation and playing ζ^{ε} can be written as

$$\begin{aligned} \Delta &= \int_{A(\hat{p})} \pi_1(p_1, p_2)\, dF(p_1, p_2) - \int_{C(\hat{p},\varepsilon)} \pi_1(\varepsilon p_1, p_2)\, dF(p_1, p_2) \\ &= \int_{A(\hat{p})\setminus B(\hat{p})} \pi_1(p_1, p_2) - \pi_1(\varepsilon p_1, p_2)\, dF(p_1, p_2) + \int_{C(\hat{p},\varepsilon)\setminus A(\hat{p})} -\pi_1(\varepsilon p_1, p_2)\, dF(p_1, p_2) \\ &\quad + \int_{B(\hat{p})} \frac{p_1 D(p_1)}{2} - \varepsilon p_1 D(\varepsilon p_1)\, dF(p_1, p_2). \end{aligned}$$

The first term continuously approaches 0 as $\varepsilon \nearrow 1$. To see this, note that the first term equals $(1 - \varepsilon) \int_{A(\hat{p})\setminus B(\hat{p})} \pi_1(p_1, p_2) dF(p_1, p_2)$ because $p_1 < p_2$ in $A(\hat{p}) \setminus B(\hat{p})$. The second term is non-positive and the third term is strictly negative and bounded away from 0 as $\varepsilon \nearrow 1$ because $\int_{B(\hat{p})} dF(p_1, p_2) > 0$. Consequently, $\Delta < 0$ for sufficiently high $\varepsilon < 1$. This contradicts that F is a correlated equilibrium and therefore $\int_{B(\hat{p})} dF(p_1, p_2) = 0$ has to hold. $\square$

After this auxiliary result, we come to the main result: In any correlated equilibrium, both firms set prices equal to zero with probability 1 and therefore make zero profits. That is, every correlated equilibrium is essentially equivalent to the Bertrand Nash equilibrium.[4]

Theorem 1. *In every correlated equilibrium F, $\bar{p} = 0$. That is, $p_1 = p_2 = 0$ with probability 1 in every correlated equilibrium.*

Proof. By lemma 1, it is sufficient to show that in any *symmetric* correlated equilibrium F, we have $\bar{p} = 0$. Therefore, we concentrate on symmetric F in the remainder of the proof.

[4]The qualifier "essentially" stems from the definition of correlated equilibrium in infinite games: A strategy ζ_i that differs from the identity function on a set of points that has zero probability under F is also an equilibrium strategy.

The proof is by contradiction. Suppose to the contrary that $\bar{p} > 0$. Define $\hat{p} = \frac{3}{4}\bar{p}$. As F is a correlated equilibrium, player 1 must get a higher expected payoff from following the recommendation r_1 than from following the deviation strategy

$$\zeta(r_1) = \begin{cases} r_1 & \text{if } r_1 \notin (\hat{p}, \bar{p}] \\ \hat{p} & \text{if } r_1 \in (\hat{p}, \bar{p}]. \end{cases}$$

Making use of the sets $E(\hat{p})$ and $E'(\hat{p})$ as defined above, the difference between the expected payoff when following the recommendation and the expected payoff under ζ is

$$\begin{aligned} \Delta &= \int_{A(\hat{p})} \pi_1(p_1, p_2)\, dF(p_1, p_2) - \int_{E(\hat{p})} \pi_1(\hat{p}, p_2)\, dF(p_1, p_2) \\ &\leq \int_{A(\hat{p})} D(\hat{p}) p_1\, dF(p_1, p_2) - \int_{E'(\hat{p})} \pi_1(\hat{p}, p_2)\, dF(p_1, p_2) \\ &= D(\hat{p}) \int_{A(\hat{p})} (p_1 - \hat{p})\, dF(p_1, p_2) - D(\hat{p})\hat{p} \int_{E'(\hat{p}) \setminus A(\hat{p})} dF(p_1, p_2) \\ &= D(\hat{p})\hat{p} \left(\int_{A(\hat{p})} \frac{p_1 - \hat{p}}{\hat{p}}\, dF(p_1, p_2) - \int_{A(\hat{p})} dF(p_1, p_2) \right) \end{aligned}$$

where the last equality follows from the symmetry of F and lemma 3 (which states that $\int_{B(\hat{p})} dF(p_1, p_2) = 0$). By the definition of $\hat{p} = \frac{3}{4}\bar{p}$, $\frac{p_1 - \hat{p}}{\hat{p}} < 1$ for all $p_1 \in (\hat{p}, \bar{p}]$. Therefore,

$$\int_{A(\hat{p})} \frac{p_1 - \hat{p}}{\hat{p}}\, dF(p_1, p_2) < \int_{A(\hat{p})} dF(p_1, p_2) \tag{3}$$

as $\int_{A(\hat{p})} dF(p_1, p_2) \neq 0$ by the definition of $\bar{p}$ and $\hat{p} < \bar{p}$. Note that (3) implies $\Delta < 0$ which contradicts that F is a correlated equilibrium. □

The result also generalizes to Bertrand settings with n firms and non-identical marginal costs, as we show in the supplementary material. In this case, the market price paid by consumers is less or equal to the second lowest marginal costs with probability one. Hence, correlated equilibrium is essentially equivalent to the Bertrand Nash equilibrium also in this more general framework.

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Supplementary Material

In this supplementary material, we replicate the result from the main text for the case of asymmetric costs and n firms. That is, we allow the firms to have different marginal costs c_i and allow for an arbitrary finite number of firms. The main idea of the proof still carries through and we get the result that the market price, i.e. the lowest price charged by any firm (or group of firms), is lower or equal to the second-lowest marginal cost with probability 1 in every correlated equilibrium. However, we cannot utilize symmetry and symmetric equilibria anymore which inevitably complicates proofs and notation a bit.

Model

Market demand is $D(p)$ where $D : \mathbb{R}_+ \to \mathbb{R}_+$ is a weakly decreasing, measurable function and $\mathbb{R}_+$ is used to denote the non-negative real numbers. There are n firms. All firms have constant marginal costs c_i, $i \in \{1, \ldots, n\}$, where – without loss of generality – we assume $c_1 \leq c_2 \leq \ldots \leq c_n$. Firms set prices simultaneously. If $p_i < p_j$ for all $j \neq i$, consumers buy quantity $D(p_i)$ units of the good from firm i (and 0 units from the other firms).

If $k \geq 2$ firms post the same lowest price $p' = \min\{p_1, \ldots, p_n\}$, we assume that consumers do the following: The firms with the lowest marginal costs among those k firms quoting p' share the demand $D(p')$ equally. More formally, denote the k firms quoting p' as $\{m_1, \ldots, m_k\}$ and let – without loss of generality – the ordering be such that $c_{m_1} \leq c_{m_2} \leq \cdots \leq c_{m_k}$. Define $\tilde{k}$ as $max_{j \in \{1,\ldots,k\}}\{j : \ c_{m_1} = c_{m_j}\}$. Then firms m_1 to $m_{\tilde{k}}$ sell $D(p')/\tilde{k}$ units and all other firms sell zero units. We assume that the demand is such that the monopoly price $p^{mon} = \max\{p_1^{mon}, \ldots, p_n^{mon}\}$, where $p_i^{mon} = \arg\max_p (p - c_i)D(p)$, is finite.[5]

The assumption that all consumers buy from the low cost firms in case several firms charge the same price deserves some comment. We make this assumption to ensure

[5] If there are several prices maximizing $(p - c_i)D(p)$, p_i^{mon} is the supremum of all these maximizers. For notational convenience, we assume that the monopoly profit is attained at $(p_i^{mon} - c_i)D(p_i^{mon})$ also in this case.

the existence of the standard Bertrand Nash equilibrium. This well known equilibrium postulates that $p_1 = p_2 = c_2$ (and arbitrary $p_i \geq c_i$ for $i \in \{3, \ldots, n\}$). This is indeed a Nash equilibrium with our tie-breaking rule above but can fail to be an equilibrium with other tie-breaking rules. If, for example, $c_1 < c_2$ and a mass of consumers does not buy from firm 1 whenever $p_1 = p_2$, then $p_1 = p_2 = c_2$ is not an equilibrium as firm 1 could increase its profits by decreasing its price by a sufficiently small amount. Assuming a tie-breaking rule such that a Nash equilibrium exists has two advantages: First, it gives us a benchmark to which we can compare correlated equilibria. Second, as every Nash equilibrium can be interpreted as a correlated equilibrium, we know that a correlated equilibrium exists.[6] Finally, note that the behavior of the consumers that corresponds to this assumption is optimal, and that the Nash equilibrium would therefore also be a Nash equilibrium of the wider game in which a group of consumers acts as players.

Our setup gives therefore the following profits for firm i at a price vector $p = (p_1, \ldots, p_n)$:

$$\pi_i(p) = \begin{cases} (p_i - c_i)D(p_i) & \text{if } p_i < p_j \text{ for all } j \neq i \\ (p_i - c_i)D(p_i) & \text{if } p_i = p_{m_1} = \cdots = p_{m_k} < p_j \text{ for all } j \notin \{i, m_1, \ldots, m_k\} \\ & \qquad \text{and } c_i < c_l \text{ for all } l \in \{m_1, \ldots, m_k\} \\ (p_i - c_i)D(p_i)/\tilde{k} & \text{if } p_i = p_{m_1} = \cdots = p_{m_k} < p_j \text{ for all } j \notin \{i, m_1, \ldots, m_k\} \\ & \qquad \text{and } c_i = c_{m_1} = \cdots = c_{m_{\tilde{k}}} < c_{m_{\tilde{k}+1}} \leq \cdots \leq c_{m_k} \\ 0 & \text{else.} \end{cases}$$

As in the main text, a strategy for firm i is a measurable function $p_i : \mathbb{R}_+ \to \mathbb{R}_+$ and a distribution F on $\mathbb{R}^n_+$ is a correlated equilibrium if it satisfies (2) for all firms and all deviation strategies.

Analysis and Result

Given a correlated equilibrium F, we define $\bar{p} \in \mathbb{R}_+$ in the following way: $\bar{p} = \inf\{p' : \int_{(p',\infty)^n} dF(p) = 0\}$ where $p = (p_1, \ldots, p_n)$. Intuitively, $\bar{p}$ is the price such that (i) the

[6] It should be noted that the equal sharing assumption (in case $\tilde{k} > 1$) is not important for our analysis and any other rule would work as well.

probability that the market price is greater than $\hat{p}$ is strictly positive for any $\hat{p} < \bar{p}$ and (ii) the probability that the market price is greater than $\hat{p}$ is zero for any $\hat{p} > \bar{p}$. That is, if we consider the distribution of prices that consumers pay in the correlated equilibrium F, $\bar{p}$ is the essential supremum of this "market price distribution".

$\bar{p}$ is weakly below p^{mon} where $p^{mon} = \max\{p_1^{mon}, \ldots, p_n^{mon}\}$ and p_i^{mon} is the monopoly price of a firm with costs c_i: If $\bar{p} > p^{mon}$, the event that all firms charge a price above p^{mon} would have positive probability. Hence, at least one firm i would – with positive probability – sell goods at a price higher than p_i^{mon}. For this firm, it would be a profitable deviation to charge p_i^{mon} whenever receiving a recommendation r_i above p_i^{mon}. This can be shown more formally as in lemma 2 in the main text. The main point is that $\bar{p} \le p^{mon}$ exists because $\int_{(p^{mon},\infty)^n} dF(p) = 0$.

Define the following sets analogously to the main text (again p denotes a vector of prices): $A(\hat{p}) = \{p : p_1 \in (\hat{p}, \bar{p}] \text{ and } p_1 \le p_i \text{ for all } i = 1, 2, \ldots, n\}$ is the set of price vectors for which firm 1 sells with a price between $\hat{p}$ and $\bar{p}$; $K(\hat{p}) = \{p : p_2 \in (\hat{p}, \bar{p}] \text{ and } p_2 \le p_i \text{ for all } i = 2, \ldots, n \text{ and } p_2 < p_1\}$ is the set of price vectors where firm 2 sells at a price between $\hat{p}$ and $\bar{p}$ (and firm 1 does not sell). Furthermore, define $B(\hat{p}) = \{p : p_1 \in (\hat{p}, \bar{p}] \text{ and } p_1 = p_2 \le p_i \text{ for all } i = 1, \ldots, n\}$, i.e. B is the set of price vectors where firm 1 and 2 charge both the same price above $\hat{p}$ and all other firms set weakly higher prices.

Lemma S1. *Let F be a correlated equilibrium and suppose $\bar{p} = \inf\{p' : \int_{(p',\infty)^n} dF(p) = 0\} > c_2$. Then, $\int_{B(\hat{p})} dF(p) = 0$ for any $\hat{p} \in (c_2, \bar{p})$.*

Proof. Suppose to the contrary that there exists a $\hat{p} < \bar{p}$ such that $\int_{B(\hat{p})} dF(p) > 0$. We will show that it is then profitable for firm 2 to use the following deviation strategy for $\varepsilon > 0$ sufficiently small

$$\zeta_2^{\varepsilon}(r_2) = \begin{cases} r_2 & \text{if } r_2 \notin (\hat{p}, \bar{p}] \\ (1-\varepsilon) r_2 & \text{if } r_2 \in (\hat{p}, \bar{p}]. \end{cases}$$

The profit difference of firm 2 between sticking to the recommendation and using ζ_2^ε is

$$
\begin{aligned}
\Delta_2^\varepsilon &= \int_{K(\hat{p})\cup B(\hat{p})} \pi_2(p)\, dF(p) - \int_{[(1-\varepsilon)\hat{p},\bar{p}]\times(\hat{p},\bar{p}]\times[(1-\varepsilon)\hat{p},\bar{p}]^{n-2}} \pi_2(p_1,(1-\varepsilon)p_2,p_3,\ldots,p_n)\, dF(p) \\
&\le \int_{K(\hat{p})} \pi_2(p) - \pi_2(p_1,(1-\varepsilon)p_2,p_3,\ldots,p_n)\, dF(p) \\
&\quad + \int_{B(\hat{p})} \pi_2(p) - \pi_2(p_1,(1-\varepsilon)p_2,p_3,\ldots,p_n)\, dF(p) \\
&\le \int_{K(\hat{p})} D((1-\varepsilon)p_2)\varepsilon p_2\, dF(p) + \int_{B(\hat{p})} \pi_2(p) - \pi_2(p_1,(1-\varepsilon)p_2,p_3,\ldots,p_n)\, dF(p) \\
&\le \varepsilon\int_{K(\hat{p})} D((1-\varepsilon)p_2)p_2\, dF(p) + \int_{B(\hat{p})} \frac{D(p_2)(p_2-c_2)}{2} - D((1-\varepsilon)p_2)((1-\varepsilon)p_2-c_2)\, dF(p).
\end{aligned}
$$

Note that the first integral in the last line continuously converges to 0 as $\varepsilon \to 0$. The second integral in the last line is, however, negative and bounded away from 0: First, we show that the integrand is strictly negative and bounded away from zero. $\frac{D(p_2)(p_2-c_2)}{2} - D((1-\varepsilon)p_2)((1-\varepsilon)p_2-c_2) < D((1-\varepsilon)p_2)\left(\frac{-(p_2-c_2)}{2}+\varepsilon p_2\right)$ which for $\varepsilon < \frac{p_2-c_2}{4\bar{p}}$ is less than $D((1-\varepsilon)p_2)\frac{-(p_2-c_2)}{4} < D(\bar{p})\frac{-(\hat{p}-c_2)}{4}$. Hence, the integrand is bounded from above by $-D(\bar{p})\frac{\hat{p}-c_2}{4} < 0$ if $\varepsilon \in (0,\frac{\hat{p}-c_2}{4\bar{p}})$ because $\frac{\hat{p}-c_2}{4\bar{p}} < \frac{p_2-c_2}{4\bar{p}}$ for all elements of $B(\hat{p})$. By assumption, $\int_{B(\hat{p})} dF(p) > 0$ which implies that the second integral is bounded from above by $-D(\bar{p})\frac{\hat{p}-c_2}{4}\int_{B(\hat{p})} dF(p) < 0$ for $\varepsilon \in (0,\frac{\hat{p}-c_2}{4\bar{p}})$. Consequently, $\Delta_2^\varepsilon < 0$ for $\varepsilon > 0$ small enough which contradicts that F is a correlated equilibrium. □

Before coming to the main result, one further auxilliary result is needed. Roughly speaking, the result says that in a correlated equilibrium firm 1 will sell at a price in $(\hat{p},\bar{p}]$ with positive probability for any $\hat{p} < \bar{p}$. Given the definition of $\bar{p}$, this should be hardly surprising.

Lemma S2. *Let F be a correlated equilibrium such that $\bar{p} = \inf\{p' : \int_{(p',\infty)^n} dF(p) = 0\} > c_1$. Then, $\int_{A(\hat{p})} dF(p) > 0$ for any $\hat{p} \in (c_1,\bar{p})$.*

Proof. For $\hat{p} \in (c_1,\bar{p})$, consider the following deviation strategy for firm 1:

$$
\zeta_1(r_1,\hat{p}) = \begin{cases} r_1 & \text{if } r_1 \notin (\hat{p},\bar{p}] \\ \hat{p} & \text{if } r_1 \in (\hat{p},\bar{p}]. \end{cases}
$$

The payoff difference between sticking to the recommendation and using ζ_1 is[7]

$$
\Delta = \int_{A(\hat{p})} \pi_1(p) - \hat{p}D(\hat{p})\, dF(p) + \int_{(\hat{p},\bar{p}]\times[\hat{p},\bar{p}]^{n-1}\setminus A(\hat{p})} -\pi_1(\hat{p},p_{-1})\, dF(p).
$$

[7]We use $p_{-1} = p_2,\ldots,p_n$ to denote the prices of all firms but firm 1.

By the definition of $\bar{p}$, $\int_{(\hat{p},\bar{p}]\times[\hat{p},\bar{p}]^{n-1}} dF(p) > 0$. If $\int_{A(\hat{p})} dF(p) = 0$, this would imply that the second integral in Δ is strictly negative while the first integral in Δ would be zero. Hence, ζ_1 is a profitable deviation if $\int_{A(\hat{p})} dF(p) = 0$ contradicting that F is a correlated equilibrium. □

The following observation is related to lemma S2: For any $\hat{p} < \bar{p}$, a firm using the strategy

$$\zeta_i(r_i, \hat{p}) = \begin{cases} r_i & \text{if } r_i \notin (\hat{p}, \bar{p}] \\ \hat{p} & \text{if } r_i \in (\hat{p}, \bar{p}] \end{cases}$$

will sell $D(\hat{p})$ units at price $\hat{p}$ with positive probability: By the definition of $\bar{p}$, the event that all firms get a recommendation above $\hat{p}$ has positive probability. Hence, firm i sells with positive probability at price $\hat{p}$ when using the strategy ζ_i.

Using lemma S1, we can now show the main result: In any correlated equilibrium, $\bar{p} \leq c_2$. This means that the price that conumers pay will be weakly less than c_2 with probability 1. Consequently, the expected profits for firms $2, \ldots, n$ are zero and the expected profits of firm 1 are bounded from above by $c_2 - c_1$ in any correlated equilibrium.

Theorem S2. *Let F be a correlated equilibrium. Then,* $\bar{p} = \inf\{p' : \int_{(p',\infty)^n} dF(p) = 0\} \leq c_2$.

Proof. Suppose to the contrary $\bar{p} > c_2$ in a correlated equilibrium F. Let $\hat{p} = \frac{1}{4}c_2 + \frac{3}{4}\bar{p}$ and distinguish the two cases

1. $\int_{K(\hat{p})} dF(p) \geq \int_{A(\hat{p})} dF(p)$
2. $\int_{K(\hat{p})} dF(p) < \int_{A(\hat{p})} dF(p)$.

In the first case, the profit difference of firm 1 from using $\zeta_1(r_1, \hat{p})$ (see above) and from following the recommendation is

$$\begin{aligned}
\Delta_1 &= \int_{A(\hat{p})} \pi_1(p_1, p_{-1})\, dF(p) - \int_{(\hat{p},\bar{p}]\times[\hat{p},\bar{p}]^{n-1}} \pi_1(\hat{p}, p_{-1})\, dF(p) \\
&\leq \int_{A(\hat{p})} D(\hat{p})(p_1 - c_1)\, dF(p) - \int_{(\hat{p},\bar{p}]^n} D(\hat{p})(\hat{p} - c_1)\, dF(p) \\
&= D(\hat{p})(\hat{p} - c_1) \left(\int_{A(\hat{p})} \frac{p_1 - \hat{p}}{\hat{p} - c_1}\, dF(p) - \int_{(\hat{p},\bar{p}]^n \setminus A(\hat{p})} dF(p_1, p_2) \right) \\
&\leq D(\hat{p})(\hat{p} - c_1) \left(\int_{A(\hat{p})} \frac{p_1 - \hat{p}}{\hat{p} - c_1}\, dF(p) - \int_{K(\hat{p})} dF(p_1, p_2) \right).
\end{aligned}$$

By $\hat{p} = \frac{1}{4}c_2 + \frac{3}{4}\bar{p}$, $\frac{p_1 - \hat{p}}{\hat{p} - c_1} \in (0, 1)$ for all $p_1 \in (\hat{p}, \bar{p}]$. Therefore,

$$\int_{A(\hat{p})} \frac{p_1 - \hat{p}}{\hat{p} - c_1}\, dF(p_1, p_2) < \int_{K(\hat{p})} dF(p_1, p_2) \tag{S2}$$

because $0 < \int_{A(\hat{p})} dF(p_1, p_2) \leq \int_{K(\hat{p})} dF(p_1, p_2)$ by the definition of case 1 and lemma S2. Note that (S2) implies $\Delta_1 < 0$ which contradicts that F is a correlated equilibrium.

In the second case, the profit difference of firm 2 from using $\zeta_2(r_2, \hat{p})$ and from following the recommendation is

$$\begin{aligned}
\Delta_2 &= \int_{K(\hat{p}) \cup B(\hat{p})} \pi_2(p_2, p_{-2})\, dF(p) - \int_{[\hat{p},\bar{p}] \times (\hat{p},\bar{p}] \times [\hat{p},\bar{p}]^{n-2}} \pi_2(\hat{p}, p_{-2})\, dF(p) \\
&= \int_{K(\hat{p})} \pi_2(p_2, p_{-2})\, dF(p) - \int_{[\hat{p},\bar{p}] \times (\hat{p},\bar{p}] \times [\hat{p},\bar{p}]^{n-2}} \pi_2(\hat{p}, p_{-2})\, dF(p) \\
&\leq \int_{K(\hat{p})} D(\hat{p})(p_2 - c_2)\, dF(p) - \int_{A(\hat{p}) \cup K(\hat{p})} \pi_2(\hat{p}, p_{-2})\, dF(p) \\
&= \int_{K(\hat{p})} D(\hat{p})(p_2 - \hat{p})\, dF(p) - \int_{A(\hat{p})} D(\hat{p})(\hat{p} - c_2)\, dF(p) \\
&= D(\hat{p})(\hat{p} - c_2) \left(\int_{K(\hat{p})} \frac{p_2 - \hat{p}}{\hat{p} - c_2}\, dF(p_1, p_2) - \int_{A(\hat{p})} dF(p_1, p_2) \right).
\end{aligned}$$

Note that the step from the first to the second line uses lemma S1.

$\hat{p} = \frac{1}{4}c_2 + \frac{3}{4}\bar{p}$ implies that $\frac{p_2 - \hat{p}}{\hat{p} - c_2} \in (0, 1)$ for all $p_2 \in (\hat{p}, \bar{p}]$. The definition of case 2 therefore implies $0 \leq \int_{K(\hat{p})} \frac{p_2 - \hat{p}}{\hat{p} - c_2}\, dF(p_1, p_2) \leq \int_{K(\hat{p})} dF(p_1, p_2) < \int_{A(\hat{p})} dF(p_1, p_2)$. Hence, $\Delta_2 < 0$ which contradicts that F is a correlated equilibrium. $\square$