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Sellouts, Beliefs, and Bandwagon Behavior

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# Sellouts, Beliefs, and Bandwagon Behavior

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#### Abstract

This paper examines how a firm can strategically use sellouts to influence beliefs about its good's popularity. A monopolist faces a market of conformist consumers, whose willingness to pay is increasing in their beliefs about aggregate demand. Consumers are broadly rational but have limited strategic reasoning about the firm's incentives. I show that in a dynamic setting, the firm can use current sellouts to mislead consumers about future demand and increase future profits. Sellouts tend to occur when demand is low, they are accompanied by introductory pricing, and certain consumers benefit from others being misled.

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#### 1 Introduction

Consumers considering whether to buy particular products are often influenced by social or image concerns. Specifically, one important concern that drives consumer choice is the desire to conform (Lascu and Zinkhan, 1999). Many consumers prefer to buy products that they believe are popular, either in certain reference groups, or across the general population (Vigneron and Johnson, 1999; Chaudhuri and Majumdar, 2006). This desire to fit in should naturally affect firms' strategic and marketing activities. Theoretical work in both marketing and economics has examined this question, looking at consumers whose willingness to pay is increasing in aggregate demand, and showing how firms can use advertising and pricing to encourage bandwagon behavior (Becker, 1991; Karni and Levin, 1994; Amaldoss and Jain, 2005a,b; Buehler and Halbheer, 2011, 2012).<sup>1</sup>

Up until now, work on consumer conformity has neglected an issue that appears important in practice: firms looking to build a favorable buzz around their products and emphasize their popularity often draw attention to past sellouts. In the music industry, concert promoters putting new tickets on sale will prominently display information on sold out performances. Fans in professional sports also actively discuss and compare the consecutive sellout streaks of different teams.<sup>2</sup> The Boston Red Sox marked the occasion of 600 straight sellouts at Fenway Park with a widely publicized ceremony, where principal owner John W. Henry threw 600 commemorative baseballs into the crowd.<sup>3</sup>

I address this issue by showing how a firm can strategically use sellouts to increase demand from conformist consumers. A key assumption is that consumers cannot directly observe demand but they can observe sales. For example, ticket sales for concerts and sporting events are consistently reported in the press, but precise information about the

<sup>&</sup>lt;sup>1</sup>The study of bandwagon effects in consumption dates back to Leibenstein (1950). By bandwagon behavior, I refer to a situation where consumers become more willing to buy a product because they expect others to buy as well.

 $<sup>^{2}</sup>$ For example, see the numerous online discussion threads on hfboards.hockeysfuture.com.

<sup>&</sup>lt;sup>3</sup>See "The Red Sox nurture a 'Sellout' Streak" (BusinessWeek, July 29, 2010).

extent of any excess demand is not. As a result, selling out may influence consumer beliefs about aggregate demand, potentially pushing up willingness to pay. This mechanism is consistent with a view commonly held in the concert industry, as reported by Courty and Pagliero (2012), that empty seats can reveal negative information about the tour that can damage future sales. Artists and promoters may then take this effect into account when choosing venues and setting ticket prices to ensure that they systematically sell out.

Specifically, I consider a monopolist that serves a market in each of two periods, where the market size is know to the firm but not to all consumers. Willingness to pay in each period depends on consumers' expectation of aggregate demand. Aggregate demand, ceteris paribus, is increasing in market size, which is the same in each period. The firm sets capacity and an initial price, after which first-period consumers choose whether to buy and then leave the market. Second-period consumers observe period 1 sales, in particular whether a sellout occurred, and update their beliefs about the market size. The firm then sets a new price and second-period consumers choose whether to buy. In line with recent work on bounded rationality and strategic obfuscation, I allow for the possibility that the firm misleads consumers about the true market size, and solve for a cursed equilibrium (Eyster and Rabin, 2005).

The main results are as follows. First, the firm sells out more often than in the baseline, where consumers directly observe demand and cannot be mislead. It sells out specifically to manipulate beliefs, strategically setting price and capacity in period 1 so that consumers overestimate demand in period 2, which in turn increases demand through bandwagon behavior. Second, sellouts tend to occur when demand is low rather than high. Third, sellouts are accompanied by introductory pricing, with a period 1 discount compared to the baseline, followed by a period 2 premium. Fourth, the firm's decision to manipulate beliefs will always benefit some consumers, including some who are misled.

The central feature of cursed equilibrium is that consumers are broadly rational but

do not fully grasp all the subtleties of equilibrium reasoning. Consumers act optimally given their beliefs, and they update their beliefs using Bayes' rule, but without taking into account how the firm's actions may be correlated with its private information. Effectively, these consumers do not understand that the firm may signal through its price and capacity. From a theoretical perspective, such reasoning can explain otherwise curious behavior in a variety of settings, such as the winner's curse in auctions and trade in markets with adverse selection (Eyster and Rabin, 2005; Spiegler, 2011).<sup>4</sup>

The more immediate empirical motivation comes from Brown et al. (2012) and Brown et al. (2013), who find evidence of limited strategic reasoning in the movie industry. They show that consumers consistently overestimate the quality of films that studios shield from early reviews. Consumers fail to realize that the decision to shield a film from reviews might signal information about its quality, which leads studios to shield precisely the low-quality films. These results echo Li and Hitt (2008)'s analysis of online markets, showing that early product reviews tend to come from high-valuation consumers, and that later consumers fail to correct for this self-selection when interpreting the reviews.

In a similar spirit, certain papers have analyzed how firms can exploit consumers who systematically misjudge product quality.<sup>5</sup> Spiegler (2006) considers consumers who fail to realize that products are worthless and essentially update their beliefs from an incorrect prior. In Armstrong and Chen (2009), naive consumers always believe that quality is high, even though some firms set low quality in equilibrium and accompany it with low prices. Firms take advantage of consumers in both settings, but do not actively withhold information about product quality. In contrast, I show that a firm can purposely use sellouts to withhold information from conformist consumers, leaving them unable to infer the true level

 $<sup>^{4}</sup>$ For related models of coarse reasoning, see Esponda (2008) and Jehiel and Koessler (2008), along with Mullainathan et al. (2008) and Ettinger and Jehiel (2010) for applications to persuasion and deception.

<sup>&</sup>lt;sup>5</sup>The Red Sox were also accused of misleading consumers about their sellout streak by selling tickets directly to resellers. These sales served to artificially extend the streak, even if resellers failed to sell all tickets on to fans. See "Red Sox Ticket Policy Keeps Sellout Streak Alive With Resellers" (www.bloomberg.com, July 30, 2010)

of demand. Put another way, the firm uses sellouts to engage in obfuscation.

Obfuscation has received much attention of late in the literature on behavioral industrial organization. One strand of this literature, building on Gabaix and Laibson (2006), considers whether firms will directly withhold information about add-on pricing to naive consumers who are unaware of its use (see, e.g., Kosfeld and Schuwer 2011; Armstrong and Vickers 2012; Heidhues et al. 2012; Dahremoller 2013; Heidhues et al. 2014). Another strand, including Eliaz and Spiegler (2011), Piccione and Spiegler (2012), and Chioveanu and Zhou (2013), explores how firms may indirectly withhold information from consumers by presenting their products in different frames, limiting comparability with their rivals. Others still take a search-theoretic approach to obfuscation where firms choose the search costs associated with their products (Wilson 2010; Carlin and Manso 2011; Ellison and Wolitzky 2012). I contribute to this literature by showing that a firm can indirectly use sellouts as a tool for obfuscation in order to profitably mislead conformist consumers.

In relation to the literature on conformist consumption, this paper introduces the idea that sellouts can influence consumer beliefs about demand. Grilo et al. (2001) shows that consumer conformity can intensify price competition in duopoly and lead to multiple equilibria. Amaldoss and Jain (2005a) and Amaldoss and Jain (2005b) look at firm pricing decisions and relative market shares, in situations where conformists may interact with other consumers who are snobs. Buehler and Halbheer (2011) and Buehler and Halbheer (2012) take this same approach but investigate the role of persuasive advertising. In related work, Becker (1991) shows that the optimal price for a capacity-constrained firm may lead to rationing, if bandwagon effects lead to upward-sloping demand. The analysis here differs from the literature by considering endogenous capacity, demand uncertainty, and a dynamic setting.

This paper is also related to work showing that firms may strategically create shortages to increase consumer willingness to pay. Stock and Balachander (2005) analyze how a firm can signal high product quality by reducing output enough for consumers to expect shortages. Papanastasiou et al. (2013) show that a firm may use past shortages to manipulate consumer beliefs based on early product reviews, when consumers are boundedly rational in the sense of Li and Hitt (2008) described above. Both results depend on the firm being unable to adjust its price over time. In contrast, I allow for dynamic pricing, and describe how a firm that strategically sells outs will optimally adjust its price. I also make the link between sellouts and consumer conformity, and consider uncertainty not about quality but about demand.<sup>6</sup>

Finally, this paper relates to the extensive literature on network goods.<sup>7</sup> An important difference lies with the interpretation of consumption externalities. For network goods, these externalities typically arise from technological complementarities, whereas with conformity, externalities arise through consumer social concerns. This distinction is relevant in a context where rationing is possible. The relevant network for conformist consumers is the number of people who demand the good, which may differ from the number who buy. This explains why conformist consumers may accept to pay a high price for a good they believe is popular even if rationing limits sales.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 carries out the analysis, where I derive how aggregate demand depends on consumer beliefs and solve for the firm's optimal strategy. Section 4 then concludes. All proofs can be found in the appendix.

<sup>&</sup>lt;sup>6</sup>The broader literature on rationing has mainly looked at how shortages can dissuade consumers from strategically delaying their purchases. See, e.g., DeGraba 1995; Denicolo and Garella 1999; Liu and Ryzin 2008; Courty and Nasiry 2013. I follow Stock and Balachander (2005) and Papanastasiou et al. (2013) in considering a setting where strategic delay is not a concern, and instead focus on the link between sellouts, consumer beliefs, and willingness to pay.

<sup>&</sup>lt;sup>7</sup>For reviews of this literature, see Katz and Shapiro (1994) and Farrell and Klemperer (2007).

#### 2 The Model

A monopolist produces a good of fixed quality at constant marginal cost, normalized to zero. In each period  $t \in \{1, 2\}$ , the firm faces a market with a measure  $m \in [0, 1]$  of consumers, where the market size m is drawn from an atomless distribution F with full support on [0, 1]. All consumers have unit demand. The first group of m consumers face a one-off decision to buy in period 1, and the second group of m consumers face a one-off decision to buy in period 2. The firm has a discount factor  $\delta > 0$ .

A consumer's net payoff from buying at price  $p_t$  is  $U(\theta) - p_t$ , with

$$U(\theta) = \underbrace{\theta}_{\text{intrinsic payoff}} + \underbrace{\lambda d_t}_{\text{extrinsic payoff}}.$$
 (1)

The first term in (1) is the intrinsic payoff from buying, represented by a consumer's type  $\theta$ , uniformly distributed on  $\Theta = [-(1-A), A]$ , with  $A \in (0, 1)$ . The second term in (1) is the extrinsic payoff that arises from conformist consumption. The extrinsic payoff from buying in period t is increasing in aggregate demand in that period,  $d_t$ , where  $\lambda > 0$  captures the strength of consumers' taste for conformity. I assume  $\lambda < 1 - A$  to rule out corner solutions where all consumers buy, rather than take their outside option, which has a value of zero.

The timing of the game is as follows. At t = 0, nature draws the value of m, which is observed by the firm. This value is also observed by each consumer with probability  $(1 - \alpha) \in [0, 1)$ . I say that consumers who observe m are *informed* and that consumers who do not observe m are *uninformed*. At t = 1, the firm sets capacity  $K \in \mathbb{R}_+$  and price  $p_1 \in \mathbb{R}_+$ , and both are publicly revealed. Period 1 consumers then simultaneously choose whether to buy, after which they leave the market. The resulting aggregate demand is  $d_1$  and aggregate sales are  $q_1 = min\{d_1, K\}$ . The value of  $q_1$ , but not of  $d_1$ , is then publicly revealed, and the period ends. At t = 2, the firm sets price  $p_2 \in \mathbb{R}_+$  and period 2 consumers simultaneously choose whether to buy. The resulting demand is  $d_2$  and sales are  $q_2 = min\{d_2, K\}$ . Payoffs are then realized and the game ends.

Production costs are zero, so profits are the difference between total discounted revenue and the cost of capacity. Total discounted revenue is  $p_1q_1 + \delta p_2q_2$ . The cost of capacity can be written as  $\beta C(K)$ , with  $C(K) \geq 0$ , and C'(K) > 0 for all  $K \geq 0$ , and where  $\beta$ is a strictly positive constant. For the analysis, I take the limit as  $\beta$  approaches zero, so that costs are strictly positive but of second order compared to revenue. This assumption is useful to simplify calculations for the optimal price and capacity but is not crucial for the qualitative results.

A strategy for the firm is a rule specifying, for each  $m \in [0, 1]$ , a choice of K and  $p_1$ , along with a choice of  $p_2$  conditional on  $q_1 \in [0, K]$ . A strategy for an uninformed period 1 consumer is a rule specifying whether to buy given his type,  $\theta$ , and given K and  $p_1$ . A strategy for an informed period 1 consumer consists of such a decision rule for each  $m \in [0, 1]$ . Similarly, a strategy for an uninformed period 2 consumer is a rule specifying whether to buy given his type  $\theta$ , and given K,  $p_1$ ,  $q_1$  and  $p_2$ . A strategy for an informed period 2 consumer consists of such a decision rule for each  $m \in [0, 1]$ .

I look for a *cursed equilibrium* where the firm's strategy is optimal given the strategies of consumers, each consumer's strategy is optimal given other consumers' strategies and the strategy of the firm, and where uninformed consumers' beliefs about the market size follow from Bayes' rule in all respects save one: they neglect any correlation between the realized market size and the firm's equilibrium actions. Specifically, let  $\mathbb{P}(K, p_1|m)$  denote the conditional probability that the firm sets K and  $p_1$  in equilibrium, given market size m. Consumers believe this probability actually equals  $\mathbb{P}(K, p_1) = \int_0^1 \mathbb{P}(K, p_1|m) dF(m)$ , the unconditional equilibrium probability of observing K and  $p_1$ . Similarly, consumers believe that  $\mathbb{P}(p_2|K, p_1, q_1, m)$  actually equals  $\mathbb{P}(p_2|K, p_1, q_1)$ , the equilibrium probability of observing  $p_2$ , conditional only on K,  $p_1$  and  $q_1$ .

#### 3 Analysis

I begin the analysis by describing how conformity creates a link between aggregate demand and consumer beliefs about the market size. For any given beliefs, aggregate demand is directly increasing in the realized market size, simply because a large market has more consumers. In particular, a large market has more high-type consumers whose intrinsic payoff from buying exceeds the price. Consumers understand this direct link between demand and market size and expect a higher extrinsic payoff from buying in a large market. When consumers believe the market is large, they find buying more attractive, which itself increases willingness to pay and aggregate demand.<sup>8</sup>

To denote the beliefs of uninformed period t consumers, I introduce the distribution function  $F_t$ , defined on [0, 1].<sup>9</sup> It turns out that aggregate demand depends on  $F_t$  through the following *belief multiplier*,  $X(F_t)$ :

$$X(F_t) \equiv \frac{1 + \int_0^1 \left(\frac{(1-\alpha)\lambda m'}{1-(1-\alpha)\lambda m'}\right) dF_t(m')}{1 - \alpha\lambda \int_0^1 m' dF_t(m') - \alpha\lambda \int_0^1 \left(\frac{(1-\alpha)\lambda m'^2}{1-(1-\alpha)\lambda m'}\right) dF_t(m')}.$$
(2)

Each integral in (2) represents uninformed consumers' expectation of some function of the market size, given their beliefs. For now, notice that  $X(F_t) = 1$  if each consumer believes he is alone in the market, so if  $F_t$  places probability one on m = 0. The multiplier increases as consumers become more optimistic about the market size, as represented by beliefs that place more weight on higher values of m. Specifically, consider beliefs  $F_t$  and  $F'_t$  with a relation of first-order stochastic dominance:  $F'_t(m) \leq F_t(m)$  for all  $m \in [0, 1]$ , with a strict inequality for some m. Then the multiplier is larger under  $F'_t$  than under  $F_t$ :  $X(F'_t) > X(F_t)$ .

Lemma 1 describes the precise relationship between consumer beliefs and aggregate de-

<sup>&</sup>lt;sup>8</sup>For any given market size m, price  $p_t$ , and consumer beliefs  $F_t$ , there will be a unique level of aggregate demand, because the extrinsic payoff from buying is linear in demand.

<sup>&</sup>lt;sup>9</sup>Throughout the analysis, the term *beliefs* will refer to beliefs about market size, rather than beliefs about consumer type or strategies.

mand.

**Lemma 1.** Given price  $p_t$  and beliefs  $F_t$ , demand from uninformed consumers is

$$d_t^u(p_t, F_t) = \left(\alpha m(A - p_t)\right)[X(F_t)],$$

and demand from informed consumers is

$$d_t^i(p_t, F_t) = \left( (1 - \alpha)m(A - p_t) \right) \left[ \frac{1 + \alpha m \lambda X(F_t)}{1 - (1 - \alpha)\lambda m} \right],$$

with  $X(F_t)$  given by (2).

Demand from both informed and uninformed consumers is increasing in  $X(F_t)$  and is the product of two terms. The first term is the measure of consumers whose intrinsic payoff from buying exceeds the price. The second term, written in square brackets, is a multiplier that captures the impact of consumer conformity. The multiplier for uninformed consumers is exactly  $X_t$ , which is increasing in their optimism about the market size. The multiplier for informed consumers is increasing both in uninformed consumers' optimism and in the realized value of the market size, which informed consumers observe.

Lemma 1 captures in particular how informed consumers' demand depends on uninformed consumers' beliefs. If the firm can convince uninformed consumers that the market is large, then they will increase their demand, which informed consumers realize and so demand more themselves. Uninformed consumers in turn understand that informed consumers demand more, which makes them increase their own demand further still, and so on, where each step increases the extrinsic payoff from buying. In this way, consumer conformity creates a feedback mechanism through which a small change in beliefs can trigger a relatively large change in aggregate demand through bandwagon behavior. Summing over demand from both informed and uninformed consumers gives

$$d_t(p_t, F_t) = \left(\alpha X(F_t) + (1 - \alpha) \left(\frac{1 + \alpha m \lambda X(F_t)}{1 - (1 - \alpha)\lambda m}\right)\right) m(A - p_t).$$
(3)

If  $\lambda = 0$ , then  $X_t = 1$ , so that (3) reduces to  $d_t = m(A - p_t)$ , the level of aggregate demand if consumers were not conformists. Quantity demanded is then simply the measure of consumers whose intrinsic payoff from buying exceeds the price. If  $\alpha = 0$ , then (3) reduces to  $d_t = m(A - p_t)/(1 - \lambda m)$ , the level of aggregate demand if all consumers were informed. The denominator  $(1 - \lambda m)$  is less than one and decreasing in  $\lambda$ , reflecting how consumer conformity increases willingness to pay.

Given how aggregate demand depends on beliefs, the firm would like uninformed consumers to be as optimistic about the market size as possible, so as to generate large bandwagon effects. The problem is that the firm cannot directly influence beliefs through its choice of price and capacity. Recall that the firm sets price and capacity after observing the realized market size. Uninformed period 1 consumers then observe price and capacity but neglect how the market size may be correlated with the firm's actions. As a result, the firm cannot signal any information to period 1 consumers, who maintain prior beliefs  $F_1 = F$ when deciding whether to buy.

In contrast, the firm can indirectly influence period 2 consumers, who update their beliefs  $F_2$  from the prior after observing period 1 sales. For a given amount of sales, consumers' inference about the market size naturally depends on the price at which these sales occured. I show below that their inference may actually take into account the combination of price and capacity. In terms of notation, let  $F_m$  denote the beliefs that place probability one on some  $m \in [0, 1]$ . Let  $F_{m+}$  denote the prior beliefs F but left-truncated at m. Formally,

$$F_m(m') = 0 \text{ if } m' \in [0, m)$$
  
= 1 if  $m' \in [m, 1],$  (4)

and

$$F_{m+}(m') = 0 \quad \text{if } m' \in [0,m) \\ = \frac{F(m') - F(m)}{1 - F(m)} \quad \text{if } m' \in [m,1].$$
(5)

The following result shows that period 2 beliefs depend in large part on whether the firm's chosen capacity and price generate a sellout.

**Lemma 2.** If the firm does not sell out in period 1, then period 2 consumers believe the market size takes on its true value with probability one. That is,  $d_1(p_1, F) < K$  implies  $F_2 = F_m$ , given by (4).

If the firm sells out in period 1, then period 2 consumers believe the market size exceeds a certain threshold value. That is, for any  $(K, p_1)$ , there exists  $m(K, p_1) \in [0, m]$  such that  $d_1(p_1, F) \geq K$  implies  $F_2 = F_{m(K, p_1)+}$ , given by (5). The threshold  $m(K, p_1)$  is strictly increasing in  $p_1$  for all  $d_1(p_1, F) > K$ , and satisfies  $m(d_1, p_1) = m$ .

If the firm does not sell out, then consumers realize there is a unique level of demand consistent with the sales they observe: quantity demanded must equal quantity sold. If demand were slightly lower, or slightly higher, then consumers would observe slightly lower or slightly higher sales, given the same capacity and price. Consumers also understand that demand is strictly increasing in the market size according to (3). They can invert this expression, taking into account the observed price, to infer the exact market size.

The inference of consumers is very different following a sellout, since multiple levels of demand are then consistent with the price, capacity, and sales they observe. By selling out, the firm effectively withholds information from consumers, preventing them from inferring the exact market size. In this sense, sellouts serve as a tool for obfuscation. Consumers only infer that the market is sufficiently large to yield a sellout, so for demand to weakly exceed capacity at that price. They then use (3) to rule out values of m below a certain threshold. This threshold is the value of m for which quantity demanded equals capacity, and as such, it depends on both the firm's chosen capacity and price. In particular, the

threshold is increasing in the period 1 price, since selling out at a higher price provides more compelling evidence of high demand.

Having described how aggregate demand depends on beliefs, and how period 2 beliefs depend on the period 1 market outcome, I now turn to the firm's optimal strategy. To state the main results, let  $m_0 \in (0, 1)$  denote the critical market size implicitly defined by  $X(F_{m_0}) = X(F)$ , using (2) and (5). In words, if uninformed consumers believe the market size is  $m_0$  with probability one, then the belief multiplier takes on the same value as under the prior. An equivalent definition using (3) is that beliefs  $F_{m_0}$  and F give the same level of aggregate demand.<sup>10</sup>

I first consider a baseline setting where the firm cannot manipulate beliefs because period 2 consumers directly observe period 1 demand. These consumers also observe the period 1 price, so they can use (3) to infer the exact market size, regardless of whether the firm sells out. All other aspects of the economic environment remain unchanged, as do beliefs and demand from period 1 consumers. By comparing the firm's optimal strategy in the baseline with its optimal strategy with unobserved demand, I can identify how attempts to mislead period 2 consumers affect the market outcome.

**Proposition 1.** Consider the baseline where demand  $d_1$  is publicly revealed after period 1. The firm will set prices  $p_1 = p_2 = A/2$  and capacity  $K = max\{d_1(A/2, F), d_2(A/2, F_m)\}$ . The probability of sellouts in both periods,  $\mathbb{P}(d_1 = d_2 = K)$ , is equal to zero.

In the baseline, the firm cannot use sellouts as a tool for obfuscation, because the observed price and demand reveal all information about the market size. The firm sets capacity not to influence beliefs but simply to meet all demand at the optimal period 1 and period 2 prices. These prices in fact equal one another, due to the multiplicative form of (3), even though demand varies across periods. If  $m < m_0$ , then willingness to pay drops after consumers infer the exact market size, and demand drops as well. The firm will set capacity equal to

<sup>&</sup>lt;sup>10</sup>Such a value of  $m_0$  exists and is unique, since  $X(F_m)$  is continuous and increasing in m, with  $X(F_{m=0}) < X(F) < X(F_{m=1})$ .

period 1 demand and have excess capacity in period 2. If instead  $m > m_0$ , then demand rises after consumers infer the exact market size, so the firm will set capacity equal to period 2 demand and have excess capacity in period 1.

The firm's optimal strategy differs when demand is unobservable, as it now becomes possible to profitably mislead consumers.

**Proposition 2.** Suppose that demand  $d_1$  is not revealed after period 1. Then there exist critical values  $m_1$  and  $m_2$ , with  $m_0 < m_1 < m_2 < 1$ , such that the following holds. If  $m \in [0, m_1)$ , then the firm sells out in both periods, offering a period 1 price discount and charging a period 2 price premium:  $p_1 < A/2 < p_2$ , with  $K = d_1(p_1, F) = d_2(p_2, F_{m+})$ . If  $m \in (m_2, 1]$ , then the firm sets price and capacity as in the baseline, and does not sell out in period 1:  $p_1 = p_2 = A/2$ , with  $K = d_2(A/2, F_m) > d_1(A/2, F)$ .

When the market is large,  $m > m_2$ , the firm acts just as in the baseline, setting the same price and capacity, and making the same profits. It effectively chooses not to mislead consumers, who infer the exact market size after period 1. However, the firm follows a very different strategy when the market is small,  $m < m_1$ . It then sets price and capacity to sell out in both periods. Aggregate demand is increasing in market size, meaning that sellouts tend to occur when demand is relatively low, not high. Sellouts are accompanied by introductory pricing, with a lower price than the baseline in period 1 and a higher price in period 2. When  $m_0 < m < m_1$ , the firm does not sell out in period 1 in the baseline, but it sells out now, with the express purpose of misleading consumers.

The intuition for Proposition 2 is that selling out in period 1 generates both a benefit and a cost. The benefit of selling out arises from its impact on consumer beliefs, which by Lemma 2 depends on the period 1 price. If the firm sells out at a very low price, then consumers reason that almost any market size is consistent with a sellout, and they update their beliefs very little from the prior. The firm takes this reasoning into account when selling out by setting the highest possible price for which demand weakly exceeds its chosen capacity. With this choice of price, the firm is best able to exploit consumers' partial sophistication, maximizing the extent to which they overestimate demand and later engage in bandwagon behavior. Consumers who observe a sellout infer that demand is greater than or equal to capacity, rather than simply equal to capacity, and so overestimate the market size with probability one.

Put another way, the firm ensures that a period 1 sellout discloses the optimal amount of information to consumers. Setting a very low price discloses little information about the market size because consumers can infer little from observing a sellout. Setting a very high price indirectly discloses all information to consumers, who observe that sales are below capacity and infer the true market size. In this sense, the firm maximizes the benefit from selling out by engaging in selective obfuscation. It discloses all information that will increase demand, by indirectly revealing all values of m that are below the true market size, but nothing more.

The cost of selling out is that the firm cannot increase sales over time. If demand increases after period 1, then the firm should capitalize on this situation by increasing quantity sold, but sales following a sellout are limited by capacity. The firm deals with this constraint by simply increasing its price. However, this price is distorted upwards compared to p = A/2, the optimum in the relevant counterfactual, where consumers infer the true market size and where sales are not limited by capacity. The price distortion following a sellout means that period 2 profits may actually be lower than in the baseline, even though selling out increases demand.

The firm sells out when the market is small because the benefit then exceeds the cost. When  $m \in [0, m_0)$ , the cost is actually zero, since the capacity constraint would not bind if period 2 consumers inferred the true market size. Demand would drop after period 1 if the firm did not sell out, so its inability to increase sales after a sellout is of little concern. When the market is somewhat larger but still relatively small,  $m \in (m_0, m_1)$ , the cost of selling out is positive but smaller than the benefit, since demand would only increase by a small amount if consumers inferred the true market size.

In contrast, the cost of selling out exceeds the benefit when the market is large,  $m \in (m_2, 1]$ . The benefit of selling out is small in this situation, since consumers who observe a sellout do not overestimate the market size by very much.<sup>11</sup> The cost is high, since demand would increase substantially even without a sellout, and the firm would benefit greatly by increasing sales over time. Rather than mislead consumers, the firm sets the unconstrained optimal price in each period, along with a sufficiently high capacity, and allows consumers to infer the true market size. Paradoxically, period 1 sellouts do not occur when the market is large, because the firm then has little reason to manipulate beliefs.

The preceding discussion explains why the firm increases its price following a sellout, but not why it offers a period 1 discount ( $p_1 < A/2$ ) compared with the baseline, or why it charges a period 2 premium ( $p_2 > A/2$ ). Such introductory pricing, with low initial prices followed by subsequent price hikes, is common in markets with network externalities or switching costs (see, e.g., Cabral et al. (1999), Farrell and Klemperer (2007) and the references therein). The standard rationale for a low initial price is that it allows the firm to establish an initial customer base. In the case of network externalities, the existence of this initial base directly convinces consumers who follow to pay a higher price.<sup>12</sup> In the case of switching costs, the firm can exploit its base by increasing the price over time, knowing these locked-in consumers cannot turn to another provider. Introductory pricing in the current setting occurs for an entirely different reason, related to the firm's intertemporal trade-off when choosing capacity.

To sell out in a way that maximizes period 1 profits, the firm should set a low capacity to exactly meet period 1 demand at price  $p_1 = A/2$ . The problem is that this capacity

<sup>&</sup>lt;sup>11</sup>Formally, the difference between beliefs  $F_m$  in the baseline and  $F_{m+}$  following a sellout becomes small when m is close to 1.

<sup>&</sup>lt;sup>12</sup>This effect is absent from the analysis here, since the extrinsic payoff from buying depends only on current demand.

constrains period 2 sales far below their optimal level after demand increases following the sellout. A large price distortion  $p_2 \gg A/2$  is then needed to avoid period 2 excess demand. To maximize period 2 profits, the firm should instead set a high capacity to exactly meet period 2 demand at price  $p_2 = A/2$ . However, a large price distortion  $p_1 \ll A/2$  is then required in period 1, as the firm can only sell out at a very low price. The firm balances these concerns by setting capacity between the optimal period 1 and period 2 levels, so that sellouts are accompanied by a period 1 price discount ( $p_1 < A/2$ ) and a period 2 price premium ( $p_2 > A/2$ ).

The size of the period 1 discount and period 2 premium will depend on the market size, the strength of consumers' taste for conformity, and the discount factor.

**Proposition 3.** Suppose  $m \in [0, m_1)$ , so that the firm sells out in both periods. Then the period 1 price is decreasing in the market size and in the strength of consumer conformity, whereas the period 2 price is increasing in these parameters, whenever the fraction of uninformed consumers is sufficiently large: there exists  $\bar{\alpha} \in (0, 1)$  such that  $\frac{\partial p_1}{\partial m} < 0$ ,  $\frac{\partial p_1}{\partial \lambda} < 0$ ,  $\frac{\partial p_2}{\partial m} > 0$ ,  $\frac{\partial p_2}{\partial \lambda} > 0$  for all  $\alpha \in (\bar{\alpha}, 1]$ . Both prices are decreasing in the discount factor:  $\frac{\partial p_1}{\partial \delta} < 0$ ,  $\frac{\partial p_2}{\partial \delta} < 0$  for all  $\alpha \in (0, 1]$ . Moreover, the ex-ante probability of selling out in both periods,  $\mathbb{P}(q_1 = q_2 = K)$ , is increasing in  $\delta$  and bounded below by  $F(m_o) > 0$ .

Even though sellouts tend to occur when the market is small, the pricing distortions they generate are increasing in market size. Both the period 1 discount and the period 2 premium also tend to be large if consumers are strongly conformist. Intuitively, these are precisely the sort of consumers that the firm can profitably mislead. The proof of Proposition 3 shows more generally that the derivative of period 1 and period 2 price have the opposite sign to one another, whether the derivative is taken with respect to m,  $\lambda$ , or  $\alpha$ . It follows that a large period 1 discount should often precede a large period 2 premium. It would be natural to suspect a causal relationship between the two, that a low price today stimulates demand and allows the firm to charge a higher price tomorrow, but that is not true in this setting. The size of the discount and the premium are both driven by a third factor, which is the market size or the strength of consumer conformity.<sup>13</sup>

An increase in the discount factor has a different effect, as it increases the period 1 discount but decreases the period 2 premium. This result also reflects the firm's intertemporal tradeoff when setting capacity. If the discount factor is large, then the firm places a large weight on future profits, so the optimal capacity at which to sell out is close to the profitmaximizing level for period 2. This level for period 2 is higher than for period 1, since demand increases following a sellout. It follows that the optimal capacity is increasing in the discount factor. A firm that sets a high capacity needs to offer a large initial discount to sell out, but it can then charge a price close to the unconstrained optimum  $p_2 = A/2$  in the next period, without generating excess demand.

The relationship between the discount factor and the ex-ante probability of selling out depends on a different intertemporal trade-off. The optimal way to sell out, with introductory pricing, yields period 1 profits that are lower than in the baseline. The firm can only be indifferent between selling out and following its baseline strategy if sellouts yield strictly higher profits in period 2. An increase in the discount factor increases the weight on these profits, so that the firm strictly prefers to sell out.

Finally, Proposition 3 shows that the probability of selling out in both periods is bounded away from zero. The firm continues to mislead consumers with positive probability even in the limit as the strength of consumer conformity, the fraction of uninformed consumers, or the discount factor tend to zero. The benefit of selling out becomes small in this limit but so does the cost. In particular, the firm will always mislead consumers when doing so is costless, in situations where demand would drop if consumers inferred the true market size.

To see how the firm's strategic use of sellouts affects consumer welfare, I now compare payoffs in Proposition 2 to those from the baseline in Proposition 1. Both intrinsic and

<sup>&</sup>lt;sup>13</sup>Ceteris paribus, a large period 1 discount is actually associated with a small period 2 premium, since a firm that sells out at high capacity can set a subsequent price closer to the unconstrained optimum  $p_2 = A/2$ .

extrinsic payoffs are included in the welfare calculations. For a given market size, the extrinsic payoff from buying depends on that particular value of m, with the expectation then taken over the ex-ante distribution of the market size, F. It stands to reason that some uninformed consumers will suffer a welfare loss compared to the baseline, because they systematically overestimate aggregate demand following a sellout. While this is correct, other consumers, both informed and uninformed, will actually enjoy a welfare gain.

**Proposition 4.** Consider expected payoffs from Proposition 2, in relation to baseline expected payoffs from Proposition 1. Expected profits are higher than in the baseline. All informed and some uninformed period 1 consumers earn a higher expected payoff than in the baseline, but at least some period 2 consumers earn a lower expected payoff. If  $\delta$  is sufficiently large, then all informed and some uninformed period 2 consumers earn a higher earn a higher expected payoff than in the baseline.

Equilibrium profits can be no lower than baseline profits, since the firm can always follow its baseline strategy. In fact, the firm can do better still, and earn higher profits by setting price and capacity according to Proposition 2. Period 1 consumers directly benefit when the firm offers a period 1 price discount. They also enjoy a higher extrinsic payoff because more consumers buy at this low price.

The situation is different for period 2 consumers, who face a price premium following a sellout, and who may overestimate aggregate demand. Some of these consumers are misled into buying and are left worse off. However, the fact that these consumers are misled benefits the others who buy, by increasing aggregate demand and increasing their extrinsic payoff. As the discount factor becomes large, the price premium becomes small, and its negative impact on consumers is dominated by the benefits of bandwagon behavior. All informed consumers are then better off than in the baseline, as are all uninformed consumers who still would have bought had they not been misled.

The fact that informed consumers can benefit from the firm's manipulation of uninformed

consumers echoes results from Gabaix and Laibson (2006) and the ensuing literature on strategic obfuscation. There, the presence of naive consumers in the market tends to benefit consumers who are sophisticated, who substitute away from expensive add-ons, and who pay a base price that is driven down by competition. Here, competition plays no role, but informed consumers benefit from the firm's discount pricing and the bandwagon behavior from those who are misled. Proposition 4 raises the additional possibility that the firm's manipulation of uninformed consumers can actually be to some of their benefit.

Formally, the statement of Proposition 4 allows  $\delta$  to take on arbitrarily large values, including those greater than one. Such a high discount factor can be interpreted as the cumulative weight on post-period 1 profits in an analogous setting with more than two periods. Specifically, suppose that in each period  $t = 1, 2, \ldots, T$ , a measure m of consumers enter the market, observe all previous prices and sales, decide whether to buy, and then exit. The equilibrium outcome of this new game is identical to the one analyzed here, except the period 2 market outcome is repeated in all later periods, and the firm has  $\sum_{t=2}^{T} \delta^{t-1}$  as an effective discount factor.<sup>14</sup> A discount factor close to one when T is large would then correspond to  $\delta \gg 1$  in the original model.

One feature of the analysis has been that period 1 sellouts sometime occur for reasons unrelated to belief manipulation, as they do in the baseline. Another is that sellouts are never accompanied by strictly positive excess demand. Both features follow from the assumption that the firm knows the exact level of period 1 demand, and both qualitatively change if demand is perturbed by a small amount of noise.

**Proposition 5.** Let  $\epsilon$  be an unobserved random variable with mean zero that follows an atomless distribution with full support on  $[-\Delta, \Delta]$ . Suppose that period 1 demand is  $D_1 =$  $d_1(p_1, F_1) + \epsilon$ , and period 2 demand is  $D_2 = d_2(p_2, F_2)$ , with  $d_t(p_t, F_t)$  given by (3) for

<sup>&</sup>lt;sup>14</sup>The intuition is that selling out according to Proposition 2 already misleads consumers to the maximum possible extent, since they believe the market size exceeds its true value with probability one. The best the firm can do in any period  $t \ge 2$  is to maximize period t profits and maintain these same beliefs into period t + 1, which it can accomplish by setting  $p_t = p_2 > A/2$ , so that demand equals capacity.

 $t \in \{1, 2\}.$ 

- (i) The probability of a period 1 sellout in the baseline, where D<sub>1</sub> is revealed after period
  1, is zero: P(D<sub>1</sub> ≥ K) = 0.
- (ii) Consider any  $m \in [0, m_1)$ , for which Proposition 2 shows that the firm sells out in period 1:  $d_1(p_1, F) = K$ . In the limit as  $\Delta$  tend to zero, the probability of strictly positive excess demand tends to one :  $\lim_{\Delta \to 0} \mathbb{P}(D_1 > K) = 1$ .

If the firm is uncertain about period 1 demand, then it won't sell out in the baseline, where sellouts have no impact on consumer beliefs. The firm's only concern in the baseline is setting a sufficiently high capacity to avoid any possibility of excess demand. This optimal capacity is almost surely higher than period 1 demand, so that sellouts do not occur with positive probability. The situation changes when demand is unobserved, because period 2 profits are then discontinuous in period 1 demand. The firm realizes that consumers form very different beliefs when demand is slightly below capacity than they do when it is slightly above. In the latter case, consumers observe a sellout, and their willingness to pay jumps as a result. The firm is willing to accept a high probability of a small amount of excess demand to ensure that it sells out, so it can reap the benefits in the following period.

#### 4 Conclusion

This paper explores a feature of firm behavior that appears important in practice but that has received little attention up to now: firms looking to emphasize their goods' popularity often point to past sellouts to promote future sales. It adds to the literature on conformist consumption by showing how a monopolist's joint choice of price and capacity can influence consumer beliefs about aggregate demand and encourage bandwagon behavior. Consistent with recent evidence on limited strategic reasoning, I assume that consumers do not take the firm's incentives into account when updating their beliefs, which raises the possibility that they may be mislead. Sellouts allow the firm to do just that, to manipulate consumer beliefs and increase their willingness to pay by effectively withholding negative information about aggregate demand. This strategic obfuscation through sellouts has consequences both for pricing and welfare, hurting some consumers, but benefiting others.

## Appendix

Proof of Lemma 1. By (1), an informed consumer of type  $\theta_t^i$  and an uninformed consumer of type  $\theta_t^u$  are indifferent about buying at price  $p_t$  if

$$\theta_t^i = p_t - \lambda \left( d_t^u + d_t^i \right), \tag{6}$$

$$\theta_t^u = p_t - \lambda \int_0^1 (d_t^u + d_t^i) dF_t, \tag{7}$$

where the right-hand side of (7) integrates over  $m' \in [0, 1]$ , given uninformed consumer beliefs,  $F_t$ . Notice that  $\theta_t^i > -(1 - A)$  and  $\theta_t^u > -(1 - A)$ , by  $p_t \ge 0$ ,  $\lambda < 1 - A$ , and  $d_t^i + d_t^u \le m \le 1$ . This means there is always a positive measure of both informed and uninformed consumers who do not buy:  $d_t^i < (1 - \alpha)m$ ,  $d_t^u < \alpha m$ .

Suppose for now that  $\theta_t^i < A$  and  $\theta_t^u < A$ . Demand is then  $d_t^i = (1 - \alpha)m \int_{\theta_t^i}^A d\theta > 0$  and  $d_t^u = \alpha m \int_{\theta_t^u}^A d\theta > 0$ , since type is uniformly distributed on [-(1 - A), A], and willingness to pay is increasing in type. Using (6) and (7) to substitute for  $\theta_t^i$  and  $\theta_t^u$  yields

$$d_t^i = (1 - \alpha)m \left(A + \lambda \left(d_t^i + d_t^u\right) - p_t\right), \tag{8}$$

$$d_t^u = \alpha m \left( A + \lambda \int_0^1 (d_t^i + d_t^u) dF_t - p_t \right), \tag{9}$$

where  $d_t^u$  is proportional to *m*. Rearranging (8) gives

$$d_t^i = \left(\frac{(1-\alpha)m}{1-(1-\alpha)\lambda m}\right)(A+\lambda d_t^u - p_t).$$
(10)

and substituting (10) into (9) yields

$$d_t^u = \alpha m \left( A + \lambda \int_0^1 d_t^u dF_t + \lambda \int_0^1 \left[ \left( \frac{(1-\alpha)m'}{1-(1-\alpha)\lambda m'} \right) (A + \lambda d_t^u - p_t) \right] dF_t - p_t \right).$$
(11)

Define  $X(F_t) \equiv d_t^u / (\alpha m (A - p_t))$ , which is independent of m. We have

$$d_t^u = \alpha m (A - p_t) [X(F_t)]. \tag{12}$$

Substituting (12) into (10) then gives

$$d_t^i = \left((1-\alpha)m(A-p_t)\right) \left[\frac{1+\alpha m\lambda X(F_t)}{1-(1-\alpha)\lambda m}\right].$$
(13)

Comparing (12) with (11) and grouping terms yields

$$X(F_t) \equiv \frac{1 + \int_0^1 \left(\frac{(1-\alpha)\lambda m'}{1-(1-\alpha)\lambda m'}\right) dF_t(m')}{1 - \alpha\lambda \int_0^1 m' dF_t(m') - \alpha\lambda \int_0^1 \left(\frac{(1-\alpha)\lambda m'^2}{1-(1-\alpha)\lambda m'}\right) dF_t(m')}$$

as required.

From (12) and (13), the firm will never set  $p_t \ge A$ , since demand is then zero,  $d_t^i + d_t^u = 0$ . Moreover,  $d_t^i > 0$  and  $d_t^u > 0$  for any  $p_t < A$ , confirming that  $\theta_t^i < A$  and  $\theta_t^u < A$ .

Proof of Lemma 2. Let f denote the pdf of F, and  $f_t$  the pdf of  $F_t$ , for  $t \in \{1, 2\}$ . Bayes' rule implies

$$f_1(m|K, p_1) = \frac{\mathbb{P}(K, p_1|m)}{\mathbb{P}(K, p_1)} f(m),$$
(14)

with  $\mathbb{P}(K, p_1) = \int_0^1 \mathbb{P}(K, p_1|m') dF(m')$ , where  $\mathbb{P}(K, p_1|m)$  follows from the firm's equilibrium strategy. Consumers update beliefs according to Bayes' rule, except they use  $\mathbb{P}(K, p_1|m) = \mathbb{P}(K, p_1)$ . Substituting into (14) yields  $f_1(m|K, p_1) = f(m)$ , or equivalently  $F_1(m|K, p_1) = F(m)$ , confirming that period 1 beliefs are given by the prior.

For period 2, Bayes' rule implies

$$f_{2}(m|K, p_{1}, q_{1}, p_{2}) = \frac{\mathbb{P}(q_{1}, p_{2}|m, K, p_{1})}{\mathbb{P}(q_{1}, p_{2}|K, p_{1})} f_{1}(m|K, p_{1})$$

$$= \left[\frac{\mathbb{P}(q_{1}|m, K, p_{1})\mathbb{P}(p_{2}|m, K, p_{1}, q_{1})}{\mathbb{P}(q_{1}|K, p_{1})\mathbb{P}(p_{2}|K, p_{1}, q_{1})}\right] f_{1}(m|K, p_{1}),$$
(15)

with  $\mathbb{P}(p_2|K, p_1, q_1) = \int_0^1 \mathbb{P}(p_2|m', K, p_1, q_1) dF(m')$ , where  $\mathbb{P}(p_2|m, K, p_1, q_1)$  follows from the firm's equilibrium strategy. Consumers update beliefs according to (15) except they use  $\mathbb{P}(p_2|m, K, p_1, q_1) = \mathbb{P}(p_2|K, p_1, q_1)$ . Together with  $f_1(m|K, p_1) = f(m)$ , this implies

$$f_2(m|K, p_1, q_1, p_2) = \frac{\mathbb{P}(q_1|m, K, p_1)}{\mathbb{P}(q_1|K, p_1)} f(m),$$

where integrating gives the distribution function

$$F_2(m|K, p_1, q_1, p_2) = \frac{\int_0^m \mathbb{P}(q_1|m', K, p_1) dF(m')}{\int_0^1 \mathbb{P}(q_1|m', K, p_1) dF(m')},$$
(16)

with  $\int_0^1 \mathbb{P}(q_1|m', K, p_1) dF(m') = \mathbb{P}(q_1|K, p_1)$ . The probability  $\mathbb{P}(q_1|m, K, p_1)$  follows from consumer equilibrium strategies and the firm's choice of K and  $p_1$ . Specifically, demand  $d_1(p_1, F)$  is given by (3), with sales  $q_1 = min\{d_1, K\}$ .

From (3),  $d_1$  is strictly increasing in m, for any  $p_1 < A$ . Hence, for any pair  $d_1 \in [0, 1]$ and  $p_1 < A$ , there is a unique market size  $m(d_1, p_1)$  consistent with this demand and price. Expression (3) shows that  $d_1$  is strictly decreasing in  $p_1$ , so that  $m(d_1, p_1)$  is strictly increasing in both arguments.

If  $d_1 < K$ , then consumers observe  $q_1 = min\{d_1, K\} < K$ , and they infer  $d_1 = q_1$ . This implies  $\mathbb{P}(q_1|m', K, p_1) = 1$  for  $m' = m(d_1, p_1)$  and  $\mathbb{P}(q_1|m', K, p_1) = 0$  for all  $m' \neq m(d_1, p_1)$ , where  $m(d_1, p_1) = m$ , the true market size. Expression (16) then reduces to  $F_2 = F_m$ , as required. If  $d_1 \geq K$ , then consumers observe  $q_1 = min\{d_1, K\} = K$ , and they infer  $d_1 \geq K$ . This implies  $\mathbb{P}(q_1|m', K, p_1) = 1$  for all  $m' \geq m(K, p_1)$  and  $\mathbb{P}(q_1|m', K, p_1) = 0$  for all  $m' < m(K, p_1)$ . Expression (16) then reduces to  $F_2 = F_{m(K, p_1)+}$ , as required. The threshold  $m(K, p_1)$  is strictly increasing in  $p_1$ , since  $m(d_1, p_1)$  is strictly increasing in both arguments. When  $K = d_1$ , the threshold reduces to  $m(d_1, p_1) = m$ , the true market size.

Proof of Proposition 1. Suppose  $d_1$  is revealed after period 1. By the proof of Lemma 2, period 2 beliefs are then  $F_2 = F_m$ , which are independent of K,  $p_1$ , and  $q_1$ . Demand is  $d_2(p_2, F_m)$ , which by (3) is proportional to  $(A - p_2)$ . The optimal price is therefore  $p_2 = A/2$ , provided that  $K \ge d_2(A/2, F_m)$ . Period 2 profits are  $\pi_2 = d_2(A/2, F_m)A/2$  if  $K \ge d_2(A/2, F_m)$ , and  $\pi_2 < d_2(A/2, F_m)A/2$  if  $K < d_2(A/2, F_m)$ . They are independent of  $p_1$ . Period 1 demand is  $d_1(p_1, F)$ , which by (3) is proportional to  $(A - p_1)$ . The optimal price is therefore  $p_1 = A/2$ , provided that  $K \ge d_1(A/2, F)$ . Period 1 profits are  $\pi_1 = d_1(A/2, F)A/2$  if  $K \ge d_1(A/2, F)$  and  $\pi_1 < d_1(A/2, F)A/2$  if  $K < d_1(A/2, F)$ . Hence, the firm maximizes profits by setting  $p_1 = p_2 = A/2$ , with  $K = max\{d_1(A/2, F), d_2(A/2, F_m)\}$ . The firm sells out in both periods with probability  $\mathbb{P}(m_0)$ , where  $m_0$  is the unique value of m for which  $d_1(p, F) = d_2(p, F_m)$ . This probability equals zero since m follows an atomless distribution.

Proof of Proposition 2. Let  $\pi_{so}$  denote the maximum profits the firm can earn by selling out in period 1. Let  $\pi_{no}$  denote the maximum profits it can earn by not selling out in period 1. Specifically, by  $F_1 = F$  and Lemma 2, write

$$\pi_{no} = \max_{K, p_1, p_2} \left( \min\{d_1(p_1, F), K\} p_1 + \delta \min\{d_2(p_2, F_m), K\} p_2 \right) \quad \text{s.t.} \quad d_1(p_1, F) < K, \quad (17)$$
$$\pi_{so} = \max_{K, p_1, p_2} \left( \min\{d_1(p_1, F), K\} p_1 + \delta \min\{d_2(p_2, F_{m(K, p_1)+}), K\} p_2 \right) \quad \text{s.t.} \quad d_1(p_1, F) \ge K.$$
(18)

Demand in (17) is identical to the baseline, so Proposition 1 implies

$$\pi_{no} = d_1(A/2, F)A/2 + \delta d_2(A/2, F_m)A/2, \tag{19}$$

which the firm earns by setting  $p_1 = p_2 = A/2$ .

Write  $\pi_{so} = Kp_1 + \delta min\{d_2(p_2, F_2), K\}p_2$ , where  $F_2 = F_{m(K,p_1)+}$ . If  $d_2(p_2, F_2) > K$ , then  $\pi_{so} = Kp_1 + \delta Kp_2$ . Deviating to  $p_2 + \epsilon$ , for  $\epsilon > 0$  but small, yields strictly higher profits,  $\pi = K_1p_1 + \delta K(p_2 + \epsilon) > \pi_{so}$ . It follows that the firm sets  $d_2(p_2, F_2) \leq K$ . If  $d_2(p_2, F_2) \leq K$  and  $d_1(p_1, F) > K$ , then  $\pi_{so} = Kp_1 + \delta d_2(p_2, F_{m(K,p_1)+})p_2$ . Deviating to  $p_1 + \epsilon$ , for  $\epsilon > 0$  but small, yields  $\pi = K(p_1 + \epsilon) + \delta d_2(p_2, F_{m(K,p_1+\epsilon)+})p_2$ , with  $m(K, p_1 + \epsilon) > m(K, p_1)$  by Lemma 2. To show that  $\pi > \pi_{so}$ , I now establish  $d_2(p_2, F_{m(K,p_1+\epsilon)+}) > d_2(p_2, F_{m(K,p_1)+})$ . Setting  $F_t = F_{m(K,p_1)+}$  in (2) gives

$$X(F_{m(K,p_1)+}) \equiv \frac{1 + \int_0^1 \left(\frac{(1-\alpha)\lambda m'}{1-(1-\alpha)\lambda m'}\right) dF_{m(K,p_1)+}(m')}{1 - \alpha\lambda \int_0^1 m' dF_{m(K,p_1)+}(m') - \alpha\lambda \int_0^1 \left(\frac{(1-\alpha)\lambda m'^2}{1-(1-\alpha)\lambda m'}\right) dF_{m(K,p_1)+}(m')}$$

The expression in each expectation is strictly positive, and (5) implies  $F_{m(K,p_1+\epsilon)+} \leq F_{m(K,p_1)+}$  for all  $m' \in [0,1]$ , with  $F_{m(K,p_1+\epsilon)+} < F_{m(K,p_1)+}$  for all  $m(K,p_1) \leq m' < m(K,p_1+\epsilon)$ . Thus,  $X(F_{m(K,p_1+\epsilon)+}) > X(F_{m(K,p_1)+})$ , so that (3) implies  $d_2(p_2, F_{m(K,p_1+\epsilon)+}) > d_2(p_2, F_{m(K,p_1)+})$ . It follows that the firm sets  $d_1(p_1, F) = K$ .

Hence, profits from selling out are  $\pi_{so} = d_1(p_1, F)p_1 + \delta d_2(p_2, F_{m+})p_2$ , where  $d_1(p_1, F) = K$  and  $d_2(p_2, F_{m+}) \leq K$ . I now show that  $d_2(p_2, F_{m+}) = K$ . Suppose instead that  $d_2(p_2, F_{m+}) < K$ . Then by (3), the firm must set  $p_2 = A/2$ , to maximize period 2 profits  $d_2(p_2, F_{m+})p_2$ . The inequality  $d_1(p_1, F) > d_2(A/2, F_{m+})$  then implies  $p_1 < A/2$ , since  $d(p, F) < d(p, F_{m+})$  for all m > 0, by  $F = F_{0+}$  from (5), and by  $\frac{\partial}{\partial m}d(p, F_{m+})$  shown above. Moreover,  $p_1 < A/2$  implies  $\frac{\partial}{\partial p_1}d_1(p_1, F)p_1 > 0$ , by (3). Write  $\pi_{so} = d_1(p_1, F)p_1 + \delta d_2(A/2, F_{m+})(\frac{A}{2})$ , where  $K = d_1(p_1, F)$ . Now suppose the firm deviates to period 1 price  $p_1 + \epsilon$  and capacity  $d_1(p_1 + \epsilon, F) > d_2(A/2, F_{m+})$ , for  $\epsilon > 0$  but small. This

deviation yields  $\pi = d_1(p_1 + \epsilon, F)(p_1 + \epsilon) + \delta d_2(A/2, F_{m+})(\frac{A}{2}) > \pi_{so}$ , by  $\frac{\partial}{\partial p_1} d_1(p_1, F)p_1 > 0$ . It follows that the firm will set  $d_1(p_1, F) = d_2(p_2, F_{m+}) = K$ , so that

$$\pi_{so} = d_1(p_1, F)p_1 + \delta d_2(p_2, F_{m+})p_2 \quad \text{s.t.} \quad d_1(p_1, F) = d_2(p_2, F_{m+}) = K,$$
(20)

evaluated at the optimal K. To solve for this optimal capacity, and by extension the corresponding prices, define

$$C(F) \equiv \frac{d_1(p_1, F)}{A - p_1},$$
 (21)

$$C(F_{m+}) \equiv \frac{d_2(p_2, F_{m+})}{A - p_2},$$
(22)

which by (3) are independent of price. Substituting (21) and (22) into (20) gives

$$\pi_{so} = K\left(A - \frac{K}{C(F)}\right) + \delta K\left(A - \frac{K}{C(F_{m+})}\right),$$

yielding optimal capacity

$$K = \left(\frac{A}{2}\right) \left(\frac{1+\delta}{\frac{1}{C(F)} + \frac{\delta}{C(F_{m+})}}\right).$$
(23)

Profits  $\pi_{so}$  are defined by (20) and (23). Differentiating (23) with respect to  $\delta$  and simplifying yields

$$\frac{\partial K}{\partial \delta} = \left(\frac{A}{2}\right) \frac{\left(\frac{1}{C(F)} - \frac{1}{C(F_{m+1})}\right)}{\left(\frac{1}{C(F)} + \frac{\delta}{C(F_{m+1})}\right)^2},\tag{24}$$

which is strictly positive, since  $d_2(p, F_{m+}) > d_1(p, F)$  implies  $C(F_{m+}) > C(F)$ . Hence, (23) is strictly increasing in the discount factor, tending to C(F)(A/2) as  $\delta$  tends to zero and to  $C(F_{m+})(A/2)$  as  $\delta$  tends to infinity. (21) and (22) therefore imply  $d_1(A/2, F) < K < d_2(A/2, F_{m+})$  for all  $\delta > 0$ . This is equivalent to  $p_1 < A/2 < p_2$ , by  $d_1(p_1, F) = d_2(p_2, F_{m+}) = K$ . To complete the proof, it remains to show that there exists  $m_1$  and  $m_2$ , with  $m_0 < m_1 < m_2 < 1$ , such that  $\pi_{so} > \pi_{no}$  for all  $m \in [0, m_1)$ , and  $\pi_{no} > \pi_{so}$  for all  $m \in (m_2, 1]$ . Recall  $m_0$  is defined by  $X(F_{m_0}) = X(F)$ , from (2) and (5), or equivalently by  $d_1(p, F) = d_2(p, F_{m_0})$ .

Suppose  $m \leq m_0$ . In this case,  $d_1(p, F) \geq d_2(p, F_m)$ , so that (19) implies  $\pi_{no} \leq d_1(A/2, F)(A/2) + \delta d_1(A/2, F)A/2$ . Now say the firm sets  $p_1 = p_2 = A/2$  and  $K = d_1(A/2, F)$ . Then it earns  $d_1(A/2, F)(A/2) + \delta d_1(A/2, F)A/2$ , which is strictly less than  $\pi_{so}$ , since the optimal prices when selling out satisfy  $p_1 < A/2 < p_2$ . It follows that  $\pi_{so} > \pi_{no}$ . Moreover, both  $\pi_{so}$  and  $\pi_{no}$  are continuous in m, so there exists  $m_1 > m_0$  such that  $\pi_{so} > \pi_{no}$  for all  $m \in [0, m_1)$ .

Now suppose m = 1. In this case,  $\pi_{no} = d_1(A/2, F)(A/2) + \delta d_2(A/2, F_{m=1})(A/2)$  and  $\pi_{so} = d_1(p_1, F)p_1 + \delta d_2(p_2, F_{(m=1)+})p_2$ , with  $p_1 < A/2 < p_2$ . (4) and (5) imply  $F_{m=1} = F_{(m=1)+}$ . Moreover, (3) shows that  $d_1(p_1, F)p_1$  and  $d_2(p_2, F_{m=1})p_2$  have a unique maximum at  $p_1 = A/2$  and  $p_2 = A/2$ . It follows that  $\pi_{no} > \pi_{so}$  when m = 1, where  $K = d_2(A/2, F_{m=1}) > d_1(A/2, F)$  by  $m_0 < 1$ . Both  $\pi_{so}$  and  $\pi_{no}$  are continuous in m, so there exists  $m_2 < 1$  such that  $\pi_{no} > \pi_{so}$  for all  $m \in (m_2, 1]$ .

Proof of Proposition 3. Suppose  $m \in [0, m_1)$ , so that  $\pi_{so} > \pi_{no}$ . Proposition 2 shows that  $p_1$  and  $p_2$  are defined by  $d_1(p_1, F) = d_2(p_2, F_{m+}) = K$ , given by (3) and (23). This implies  $\frac{\partial p_t}{\partial \delta} = \frac{\partial p_t}{\partial K} \frac{\partial K}{\partial \delta}$  for  $t \in \{1, 2\}$ . Clearly,  $\frac{\partial p_t}{\partial K} < 0$  for  $t \in \{1, 2\}$ , since  $d_1(p, F)$  and  $d_2(p, F_{m+})$  are both decreasing in p. Since  $\frac{\partial K}{\partial \delta} > 0$  by (24), it follows that  $\frac{\partial p_1}{\partial \delta} < 0$  and  $\frac{\partial p_2}{\partial \delta} < 0$ .

Write  $p_1 = A - \frac{K}{C(F)}$  and  $p_2 = A - \frac{K}{C(F_{m+})}$ , by (21) and (22). Using (23) to substitute for K in these expressions gives

$$p_1 = A\left(1 - \frac{(1+\delta)}{2}\left(\frac{1}{1+\delta\frac{C(F)}{C(F_{m+1})}}\right)\right),\tag{25}$$

$$p_2 = A\left(1 - \frac{(1+\delta)}{2}\left(\frac{1}{\frac{C(F_{m+1})}{C(F)} + \delta}\right)\right),\tag{26}$$

which immediately yields  $Sign(\frac{\partial p_1}{\partial x}) = Sign(\frac{\partial}{\partial x}\frac{C(F)}{C(F_{m+1})}) = -Sign(\frac{\partial p_2}{\partial x})$ , for  $x \in \{m, \lambda, \alpha\}$ . Suppose  $\alpha = 1$ . In this case, (3) and (21) imply C(F) = mX(F), whereas (3) and (22) imply  $C(F_{m+1}) = mX(F_{m+1})$ , so that

$$\frac{\partial}{\partial x} \left( \frac{C(F)}{C(F_{m+})} \right) = \frac{\left( \frac{\partial X(F)}{\partial x} \right) X(F_{m+}) - \left( \frac{\partial X(F_{m+})}{\partial x} \right) X(F)}{X(F_{m+})^2}.$$
(27)

Setting  $\alpha = 1$  in (2) gives  $X(F) = 1/(1 - \lambda \mathbb{E}(m|F))$  and  $X(F_{m+}) = 1/(1 - \lambda \mathbb{E}(m|F_{m+}))$ , where I have written the integral  $\int_0^1 m' dF_t(m')$  as  $\mathbb{E}(m|F_t)$ . If x = m, then  $\frac{\partial X(F)}{\partial x} = 0$  and  $\frac{\partial X(F_{m+})}{\partial x} > 0$ , so that (27) is strictly negative. This implies  $\frac{\partial p_1}{\partial m} < 0$  and  $\frac{\partial p_2}{\partial m} > 0$  when evaluated at  $\alpha = 1$ . If  $x = \lambda$ , then the numerator of (27) becomes

$$\left(\frac{\mathbb{E}(m|F)}{(1-\lambda\mathbb{E}(m|F))^2}\right)\left(\frac{1}{1-\lambda\mathbb{E}(m|F_{m+})}\right) - \left(\frac{\mathbb{E}(m|F_{m+})}{(1-\lambda\mathbb{E}(m|F_{m+}))^2}\right)\left(\frac{1}{1-\lambda\mathbb{E}(m|F)}\right),$$

which is strictly negative by  $\mathbb{E}(m|F_{m+}) > \mathbb{E}(m|F)$ . It follows that  $\frac{\partial p_1}{\partial \lambda} < 0$  and  $\frac{\partial p_2}{\partial \lambda} > 0$ when evaluated at  $\alpha = 1$ . All expressions are continuous in  $\alpha$ , so there exists  $\bar{\alpha} \in (0, 1)$  such that  $\frac{\partial p_1}{\partial m} < 0$ ,  $\frac{\partial p_1}{\partial \lambda} < 0$ ,  $\frac{\partial p_2}{\partial m} > 0$ , and  $\frac{\partial p_2}{\partial \lambda} > 0$  for all  $\alpha \in (\bar{\alpha}, 1]$ .

To establish  $\frac{\partial}{\partial \delta} \mathbb{P}(q_1 = q_2 = K) > 0$ , I need to show that  $\frac{d\pi_{so}}{d\delta} > \frac{d\pi_{no}}{d\delta}$  when evaluated at m such that  $\pi_{so} = \pi_{no}$ , with  $\pi_{no}$  and  $\pi_{so}$  given by (19), (20) and (23). Such an m exists since  $\pi_{so} > \pi_{no}$  for  $m \in [0, m_1)$  and  $\pi_{so} < \pi_{no}$  for  $m \in (m_2, 1]$ , and since both  $\pi_{so}$  and  $\pi_{no}$  are continuous in  $\delta$ . Moreover,  $\mathbb{P}(q_1 = q_2 = K) \ge F(m_0)$  follows immediately from  $m_1 > m_0$ .

From (19), write

$$\frac{d\pi_{no}}{d\delta} = d_2(A/2, F_m)(A/2).$$
(28)

From (20), write

$$\frac{d\pi_{so}}{d\delta} = \frac{\partial\pi_{so}}{\partial\delta} + \frac{\partial\pi_{so}}{\partial K}\frac{dK}{\partial\delta}$$

where the envelope theorem implies  $\frac{\partial \pi_{so}}{\partial K} = 0$  at the optimal capacity (23), so that

$$\frac{d\pi_{so}}{d\delta} = \frac{\partial\pi_{so}}{\partial\delta} = Kp_2.$$
(29)

Comparing (28) and (29) shows  $\frac{d\pi_{so}}{d\delta} - \frac{d\pi_{no}}{d\delta} = Kp_2 - d_2(A/2, F_m)(A/2)$ . Rewrite  $\pi_{so} - \pi_{no} = 0$  as

$$\left(Kp_1 - d_1(A/2, F)(A/2)\right) + \delta\left(Kp_2 - d_2(A/2, F_m)(A/2)\right) = 0$$

where  $K = d_1(p_1, F) = d_2(p_2, F_{m+})$ . The first expression in large brackets is strictly negative, since  $p_1 < A/2$ , which implies  $Kp_2 - d_2(A/2, F_m)(A/2) > 0$ , as required.

Proof of Proposition 4. Expected profits from Proposition 2 are  $\int_0^1 \max\{\pi_{so}, \pi_{no}\} dF$ , defined by (19), (20), and (23). Expected profits in the baseline are  $\int_0^1 \pi_{no} dF$ . Proposition 3 shows that  $\mathbb{P}(\pi_{so} > \pi_{no}) > F(m_0) > 0$ , which implies  $\int_0^1 \max\{\pi_{so}, \pi_{no}\} dF > \int_0^1 \pi_{no} dF$ .

To establish the results for consumer payoffs, it is sufficient to consider values of m for which  $\pi_{so} > \pi_{no}$ , since otherwise the firm sets the baseline price and capacity. First consider period 1 consumers. Proposition 2 shows that  $p_1 < A/2$ , with  $d_1(p_1, F) = q_1(p_1, F) =$ K. By (1), a type  $\theta$  consumer earns  $\theta + \lambda d_1(p_1, F) - p_1$  from buying. He buys when  $\theta + \lambda d_1(p_1, F) - p_1 \ge 0$  if informed, and when  $\theta + \lambda \int_0^1 d_1(p_1, F) dF - p_1 \ge 0$  if uninformed. In the baseline, a type  $\theta$  consumer earns  $\theta + \lambda d_1(A/2, F) - A/2$  from buying. He buys when  $\theta + \lambda d_1(A/2, F) - A/2 \ge 0$  if informed, and when  $\theta + \lambda \int_0^1 d_1(A/2, F) dF - A/2 \ge 0$ if uninformed. Hence,  $p_1 < A/2$  and  $\frac{\partial d_1(p_1, F)}{\partial p} < 0$  imply that all consumers who buy in the baseline also buy according to Proposition 2, and earn a strictly higher payoff. Informed consumers who do not buy in the baseline cannot earn less according to Proposition 2, as they only buy if their payoff exceeds zero.

Now consider period 2 consumers. Proposition 2 shows that  $p_2 > A/2$ , with  $d_2(p_2, F_{m+}) = q_2(p_2, F_{m+}) = K$ . An uninformed type  $\theta$  consumer earns  $\theta + \lambda d_2(p_2, F_{m+})dF - p_2$  from buying, and he buys if  $\theta + \lambda \int_0^1 d_2(p_2, F_{m+})dF - p_2 \ge 0$ . Define  $\theta'$  as the supremum of the

set  $\theta < p_2 - \lambda \int_0^1 d_2(p_2, F_{m+}) dF$ , for all m such that  $\pi_{so} > \pi_{no}$ . An uninformed consumer of type  $\theta' + \epsilon$  buys when  $\theta' + \epsilon + \lambda \int_0^1 d_2(p_2, F_{m+}) dF - p_2 \ge 0$ . As  $\epsilon > 0$  becomes small,  $\theta' + \epsilon + \lambda \int_0^1 d_2(p_2, F_{m+}) dF - p_2$  approaches zero. Thus, by  $d_2(p_2, F_{m+}) < \int_0^1 d_2(p_2, F_{m+}) dF$ , this consumer only buys when  $\theta' + \epsilon + \lambda d_2(p_2, F_{m+}) dF - p_2 < 0$ . He therefore earns a negative expected payoff, whereas his payoff in the baseline is bounded below by zero.

An informed type  $\theta$  consumer also earns  $\theta + \lambda d_2(p_2, F_{m+}) - p_2$  from buying, and he buys when  $\theta + \lambda d_2(p_2, F_{m+}) - p_2 \ge 0$ . In the baseline, a type  $\theta$  consumer earns  $\theta + \lambda d_2(A/2, F_m) - A/2$  from buying, and he buys when  $\theta + \lambda d_2(A/2, F_m) - A/2 \ge 0$ . The proof of Proposition 2 shows that the optimal capacity (23) when  $\pi_{so} > \pi_{no}$  tends to  $K = d_2(A/2, F_{m+})$  as  $\delta$ tends to infinity. The optimal period 2 price therefore tends to  $p_2 = A/2$ . Hence, when  $\delta$  is sufficiently large,  $d_2(A/2, F_{m+}) > d_2(A/2, F_m)$  implies  $\theta + \lambda \int_0^1 d_2(p_2, F_{m+}) dF - p/2 > \theta + \lambda d_2(p_2, F_{m+}) - p_2 > \theta + \lambda d_2(A/2, F_m) - A/2$ . That is, all consumers who buy in the baseline also buy according to Proposition 2, and earn a strictly higher payoff. Informed consumers who do not buy in the baseline cannot earn less according to Proposition 2, as they only buy if their payoff exceeds zero.

Proof of Proposition 5. From the proof of Lemma 2, period 1 beliefs are  $f_1(m|K, p_1) = f(m)$ , and period 2 beliefs are

$$f_{2,\Delta}(m|K, p_1, q_1, p_2) = \frac{\mathbb{P}(q_1|m, K, p_1)f(m)}{\int_0^1 \mathbb{P}(q_1|m', K, p_1)f(m')dm'},$$
(30)

which may depend on  $\Delta$ . We now have  $q_1 = \min\{D_1, K\} = \min\{d_1 + \epsilon, K\}$ , with  $d_1$  given by (3) and  $F_1 = F$ . Let g denote the pdf of  $\epsilon$ , with  $g(\epsilon) > 0$  for all  $\epsilon \in [-\Delta, \Delta]$  and  $g(\epsilon) = 0$  otherwise. Let G denote the corresponding distribution function.

Suppose  $q_1 < K$ . Consumers then infer  $D_1 = q_1$  and  $\mathbb{P}(q_1|m, K, p_1) = \mathbb{P}(D_1|m, K, p_1)$ . For given  $m \in [0, 1]$ , define  $\epsilon_m = q_1 - d_1$ , so that  $g(\epsilon_m) = \mathbb{P}(D_1|m, K, p_1) = \mathbb{P}(q_1|m, K, p_1)$ . Substituting into (30) gives

$$f_{2,\Delta}(m|K, p_1, q_1, p_2; q_1 < K) = \frac{g(\epsilon_m)f(m)}{\int_0^1 g(\epsilon_{m'})f(m')dm'},$$
(31)

where  $g(\epsilon_m) > 0$  for all m such that  $d_1 \in [q_1 - \Delta, q_1 + \Delta]$ , and  $g(\epsilon_m) = 0$  otherwise. Expression (31) also gives period 2 beliefs in the baseline, where  $D_1$  is publicly revealed.

Fixing *m*, baseline profits are  $\pi = p_1 \min\{d_1(p_1, F) + \epsilon, K\} + \delta p_2 \min\{d_2(p_2, F_2), K\}$ . Beliefs  $F_2 = \int_0^m f_{2,\Delta}(m'|K, p_1, q_1, p_2; q_1 < K) dm'$  follow from (31) and are independent of *K* and  $p_1$ . Write  $\min\{d_1(p_1, F) + \epsilon, K\} = d_1(p_1, F) + \epsilon - \max\{d_1(p_1, F) + \epsilon - K, 0\}$ . Using  $\mathbb{E}(\epsilon) = \int_{-\Delta}^{\Delta} \epsilon g(\epsilon) d\epsilon = 0$ , expected period 1 sales are

$$\mathbb{E}(q_1) = d_1(p_1, F) - \int_{-\Delta}^{\Delta} \max\{d_1(p_1, F) + \epsilon - K, 0\}g(\epsilon)d\epsilon.$$
(32)

(32) is strictly increasing in K whenever  $K < d_1(p_1, F) + \Delta$ , since  $g(\epsilon) > 0$  for all  $\epsilon \in [-\Delta, \Delta]$ . Hence, the firm sets  $K \ge d_1(p_1, F) + \Delta$ , giving  $\mathbb{P}(D_1 < K) = 1$ .

Now suppose  $D_1$  is not revealed after period 1, and that  $q_1 = K$ . Consumers then infer  $D_1 \ge K$  and  $\mathbb{P}(q_1|m, K, p_1) = \mathbb{P}(D_1 \ge K|m, K, p_1)$ . By  $D_1 = d_1 + \epsilon$ , write  $\mathbb{P}(q_1|m, K, p_1) =$  $\mathbb{P}(\epsilon \ge K - d_1) = 1 - G(K - d_1)$ . Substituting into (30) gives,

$$f_{2,\Delta}(m|K, p_1, q_1, p_2; q_1 = K) = \frac{(1 - G(K - d_1))f(m)}{\int_0^1 (1 - G(K - d_1))f(m')dm'},$$
(33)

where  $d_1$  depends on m. Note that  $1 - G(K - d_1) = 0$  for all m such that  $K - d_1 \ge \Delta$ ,  $1 - G(K - d_1) \in (0, 1)$  for m such that  $K - d_1 \in (-\Delta, \Delta)$ , and  $1 - G(K - d_1) = 1$  for all m such that  $K - d_1 \le \Delta$ .

Let  $\Delta$  tend to zero. Period 1 demand  $D_1 = d_1(p_1, F) + \epsilon$  tends to  $d_1(p_1, F)$ , since  $\epsilon$  tends to zero with probability 1. If  $D_1 < K$ , then period 2 beliefs  $F_2$  tend in probability to  $F_m$ , where m is the true market size, since (31) tends to infinity when evaluated at this

particular m, and to zero when evaluated at any other market size. Demand is continuous with respect to  $X(F_t)$  by (3), and  $X(F_t)$  is continuous with respect to  $F_t$  by (2), so period 2 demand when  $D_1 < K$  tends to  $d_2(p_2, F_m)$ . If  $D_1 \ge K$ , then period 2 beliefs tend in probability to  $F_{m(K,p_1)+}$ , since (33) tends to zero for m such that  $d_1(p_1, F) < K$  and to  $f(m)/(1 - F(m(K, p_1)))$  for other values of m, where  $m(K, p_1)$  is the market size for which  $d_1(p_1, F) = K$ . By continuity of (2) and (3), period 2 demand when  $D_1 \ge K$  tends to  $d_2(p_2, F_{m(K,p_1)+})$ .

Since period 1 and period 2 demand tend to their level from Proposition 2, expected profits in the limit are bounded above by  $\max\{\pi_{no}, \pi_{so}\}$ . Take any m such that  $\pi_{so} > \pi_{no}$ , and suppose the firm sets  $K = d_1(p_1, F) - \Delta = d_2(p_2, F_{m+})$ , given by (23). This strategy is optimal in the limit as  $\Delta$  tend to zero, since profits tend to  $\pi_{so}$  from (20). It also yields excess demand with probability one for any  $\Delta > 0$ , since  $\mathbb{P}(D_1 > K) = \mathbb{P}(\epsilon > -\Delta) = 1 - G(-\Delta) = 1$ . Following a strategy for which  $\mathbb{P}(D_1 > K) < 1$  in the limit implies  $-\Delta < K - d_1(p_1, F)$  for sufficiently small  $\Delta$ . Expected profits in the limit are then  $d_1(p_1, F)p_1 + \delta p_2(d_2(p_2, F_{m+})\mathbb{P}(D_1 \ge K) + d_2(p_2, F_m)\mathbb{P}(D_1 < K))$ . These profits are strictly lower than  $\pi_{so}$ , by  $\mathbb{P}(D_1 < K) = G(K - d_1(p_1, F)) > 0$  and  $d_2(p_2, F_m) < d_2(p_2, F_{m+})$ , so such a strategy cannot be optimal.

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