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Perception and quality choice in vertically differentiated markets

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Abstract

Consumers are assumed to be unable to discriminate between two goods of differing qualities provided that the qualities are close enough. It is shown that in a vertically differentiated duopoly this results in multiple equilibria. Demand for each firm's good is reduced. Firms' profits may be higher or lower depending on which equilibrium is selected.

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1 Introduction

Can we always tell two goods apart? Standard theory assumes that we can: As long as the goods are not completely identical, we can perceive even the most microscopic difference. This paper explores the consequences of relaxing this assumption, allowing consumers to be boundedly rational in that their perception of goods is imperfect and limited. Boundedly rational individuals are a common target of study in behavioural economics, however this study focuses primarily not on the individuals themselves but rather on how firms and markets adapt when consumers' ability to perceive goods is bounded. Specifically it examines a vertically differentiated product market when consumers are unable to perfectly perceive the quality of a good, revealing the effect on product choice, profit and consumer welfare. It thus contributes to the growing literature on behavioural industrial organization.

That consumers' perception is limited is readily observable: Consider the recent example of the brewer Anheuser-Busch, who were accused of watering down their beer.[1] According to

[1]Source: "Tears for beers: Anheuser-Busch accused of watering down US brews", *The Guardian*, 27/02/13, http://www.theguardian.com/business/2013/feb/27/anheuser-busch-beer-watering-down-lawsuit accessed 13/08/13.

standard theory, this should have resulted in consumers perceiving the change and adjusting their willingness-to-pay accordingly. In reality, consumers were unable to detect the difference between the quality they consumed (around 4% ABV) and the quality stated on the packaging (around 5% ABV) and the result was a lawsuit.

Consumers in this paper are modeled as being unable to distinguish between the quality of two goods if they are sufficiently similar. Such a perceptual limitation could arise for many reasons. It could simply be due to a fundamental limit to the human senses. Such limitations are studied in psychophysics, a subdiscipline of psychology which seeks to discover how physical stimuli are perceived. (See for example Falmagne (2002) or Weber (2004).) There is a long tradition of studying how far apart physical stimuli (such as weight, sound or heat) must be for a difference to be detectable. This tradition is here applied to an economic context: When the ratio of the qualities of two goods is below a certain threshold, consumers cannot perceive the difference and regard them as being of the same quality.

Although the mathematical modeling approach adopted here is inspired by classical psychophysics, it is not supposed that consumers' perceptual limitations are purely sensory in nature. For example, it could be that consumers do not perceive the difference between goods due to time pressure preventing a thorough assessment of them, or due to such an assessment being prohibitively costly in terms of effort.

Another reason for perceptual limitations could be that consumers' attention is drawn to something other than the intrinsic quality of a good. The attention which individuals give to various attributes has been a topic of recent study, with Bordalo, Gennaioli, and Shleifer (2012, 2013a, 2013b, 2013c, 2013d) developing a model in which greater weight is given in decision making to particularly salient attributes of a good. Kőszegi and Szeidl (2013) also construct a similar model.

This paper takes an agnostic approach to the source of consumers' perceptual limitations: Any or all of the aforementioned explanations can motivate them. It is instead examined how, given perceptual limitations, firms and consumers interact in a market setting. It seeks to answer questions such as whether firms can benefit from perceptual limitations, whether they can exploit consumers' inability to tell an inferior product from a superior one, or whether they will lose out as they find it harder to distinguish their product in the marketplace.

The importance of consumers' perception has long been recognized in the field of marketing, for example Chandon and Ordabayeva (2009) examine how packaging shape can bias individuals' assessment of volume and Walsh and Mitchell (2005) attempt to construct a measure of how similar consumers perceive products to be. There has been investigation into the influence of labeling on consumer choice (for example Kwortnik, Creyer, and Ross (2006)). This research has also expanded into the question of whether nutritional labels induce consumers to make healthier choices (Variyam, 2008).

Several authors examine models of similarity showing, examining how individuals make decisions when presented with similar options. Rubenstein (1988), Azipurua, Ishiishi, Nieto, and Uriarte (1993), Leland (1994) and Sileo (1995) use these models to explain certain anomalies in choice under uncertainty, with Buschena and Zilberman (1999) providing experimental evidence for these models. Leland (2002) uses a similar approach with choice over time to explain deviations from the discounted utility model. All of these studies examine only individual decision making with abstract or artificial assets and lotteries, and do not consider how profit maximizing firms interact with such boundedly rational individuals.

The specific economic institution considered here is a vertically differentiated product market, in which competing firms sell a good with heterogeneous levels of some objectively measurable quality. This is an institution well suited to yield insights into the effects of perceptual limitations, since the ability of firms to distinguish the attributes of the goods they produce is instrumental in the demand for it. This model was first introduced by Mussa and Rosen (1978), who assumed that firms engaged in Cournot competition, with Shaked and Sutton (1982) adapting the model for price competition. Since then there have been a myriad of theoretical adaptations and extensions as well as empirical applications, for example international trade (Hallak & Schott, 2011), mergers (Inderst & Tommaso, 2011) and the pharmaceutical industry (Brekke, Holmas, & Straume, 2011). Here, consumers with perceptual limitations are introduced to a model similar to that in Motta (1993). This model serves as a baseline against which the effects of perceptual limitations may be judged.

Section 2 gives an exposition of the standard model. Section 3 examines the impact of introducing consumers with perceptual limitations, and the findings are discussed in section 4. Section 5 concludes.

2 Standard model

There are two identical firms, 1 and 2, which produce a good with a one dimensional objective quality $q_i \in \mathbb{R}_+$ which is sold at price p_i, $i \in \{1, 2\}$. Costs of quality are fixed with the functional form $\frac{k}{2}q_i^2$, $k \in \mathbb{R}_{++}$ for both firms. Marginal costs of production are 0.

Consumers may purchase a single unit of the good from which they obtain utility

$$u(q, p) = \alpha q - p \tag{1}$$

where $\alpha \in \mathbb{R}_{++}$ represents a consumer's taste for quality, with a higher α representing a higher willingness-to-pay (WTP) for a given level of quality. If they opt not to consume, they get utility 0. Consumers are distributed with uniform density between $\alpha = 0$ and $\alpha = \bar{\alpha}$.[2] Let h always denote the firm producing the higher quality good, $h \in \{1, 2\}$, and l the firm producing the lower quality good, $l \in \{1, 2\}$, $h \neq l$, so that $q_h \geq q_l$. This implies that in equilibrium $p_h \geq p_l$, i.e. the high quality firm charges a higher price. Demand for each firm is then

$$D_h(q_h, p_h; q_l, p_l) = \bar{\alpha} - \frac{p_h - p_l}{q_h - q_l} \qquad D_l(q_h, p_h; q_l, p_l) = \frac{p_h - p_l}{q_h - q_l} - \frac{p_l}{q_l} \tag{2}$$

and profits are

$$\pi_h(q_h, p_h; q_l, p_l) = p_h\left(\bar{\alpha} - \frac{p_h - p_l}{q_h - q_l}\right) - \frac{k}{2}q_h^2 \tag{3a}$$

$$\pi_l(q_h, p_h; q_l, p_l) = p_l\left(\frac{p_h - p_l}{q_h - q_l} - \frac{p_l}{q_l}\right) - \frac{k}{2}q_l^2. \tag{3b}$$

The timing of the game is as follows:

Period 1: Firms simultaneously choose qualities, incurring fixed costs of quality.

Period 2: Having observed the chosen qualities, firms set prices, consumers purchase and firms earn revenue.

Having defined the structure of the market, standard equilibrium prices and qualities will be stated. A full derivation of these expressions may be found in Motta (1993).

Given qualities q_h and q_l, equilibrium prices in period 2 are

$$p_h^* = 2\bar{\alpha}q_h\left(\frac{q_h - q_l}{4q_h - q_l}\right) \qquad p_l^* = \bar{\alpha}q_l\left(\frac{q_h - q_l}{4q_h - q_l}\right) \tag{4}$$

so that firms then choose qualities in the first period to maximize

$$\pi_h(q_h, q_l) = \frac{4\bar{\alpha}^2 q_h^2 (q_h - q_l)}{(4q_h - q_l)^2} - \frac{k}{2}q_h^2 \tag{5a}$$

$$\pi_l(q_h, q_l) = \frac{\bar{\alpha}^2 q_h q_l (q_h - q_l)}{(4q_h - q_l)^2} - \frac{k}{2}q_l^2. \tag{5b}$$

Standard equilibrium qualities q_h^* and q_l^* are then

$$q_h^* = \frac{\bar{\alpha}^2\mu^3}{k}\left(\frac{4\mu - 7}{(4\mu - 1)^3}\right) \qquad q_l^* = \frac{\bar{\alpha}^2\mu^2}{k}\left(\frac{4\mu - 7}{(4\mu - 1)^3}\right) \tag{6}$$

[2] The lower bound of $\alpha = 0$ implies that the market will always be uncovered except in the case where $p_1 = p_2 = 0$ when it is just covered.

where $\mu = \frac{q_h^*}{q_l^*}$ is the ratio of the high quality to the low quality in equilibrium. This ratio is constant and can be shown to be equal to the unique real solution of

$$4\mu^3 - 23\mu^2 + 12\mu - 8 = 0. \tag{7}$$

Although the absolute values of quality chosen in equilibrium depends on the parameters $\bar{\alpha}$ (the size of the market) and k (the cost of quality), the ratio μ is unchanged. The solution to equation (7) is approximately $\mu \approx 5.251$, which implies from equation (6) that $q_h^* \approx 0.253\frac{\bar{\alpha}^2}{k}$ and $q_l^* \approx 0.048\frac{\bar{\alpha}^2}{k}$.

3 Imperfect quality perception

Consumers are now assumed to have limitations to their perception of the quality of goods. If two goods are close enough to each other in quality terms, then consumers are unable to to perceive the difference between them. Effectively, the goods are similar enough that consumers perceive the goods to be homogeneous.

A threshold parameter, identical for all consumers and capturing this limitation is introduced and given the label δ, with $\delta \in [\,1, \infty\,)$. When $\frac{q_h}{q_l} \geq \delta$, so that the ratio of high to low quality is at least as big as δ, the qualities are far enough apart to be distinguishable. However, when $\frac{q_h}{q_l} < \delta$ so that the ratio of high to low quality is below the threshold, then consumers cannot tell the goods apart. That the perception threshold depends on the ratio of qualities captures the essence of the Weber-Fechner law in psychology.[3] The greater the absolute value of a good's quality, the greater the absolute difference must be in the quality of a comparison good for the difference to be detectable. There need only be a small absolute difference between two fairly low quality goods for consumers to be able to distinguish between them, but the absolute difference must be much larger for consumers to be able to tell the difference between fairly high quality goods.

In principle, there could also be a limitation introduced on consumers' perception of price as well as quality. However, it is assumed that they are still able to perfectly perceive any difference in price. This assumption captures the fact that the quality of a consumer good is generally a much more nebulous, harder to detect attribute than its price, which is easy to compare to another good's price, even when the relative price difference is minimal. When presented with two price tags side-by-side, one of £1000 and one of £1000.01, consumers can still easily see that the former is cheaper than the latter.

[3]In fact it is equivalent to a common formulation of this law, $\frac{\Delta I}{I} = K$, where I is the intensity of some stimulus, ΔI is the smallest detectable increase in intensity and K is a constant. If $\delta = K + 1$ this is exactly the formulation employed here.

Given that for $\frac{q_h}{q_l} < \delta$ consumers perceive the two goods as homogeneous, they will either purchase the good with the lower price, or not purchase. They will never purchase the good with the higher price.[4]

That consumers never purchase the good with the higher price leads to a key result.

Proposition 1*a*. *In equilibrium, firms will never choose qualities such that* $\frac{q_i}{q_j} < \delta$, *i.e., consumers will always be able to perceive a difference between the firms' equilibrium qualities.*

In period 2, quality is not changeable, and the costs of quality are sunk. Hence if $\frac{q_h}{q_l} < \delta$, Bertrand competition with effectively homogeneous goods and identical marginal costs of 0 for each firm occurs. Firms earn no revenue in period 2 and make a loss overall. Since such actions are dominated by choosing 0 quality and making 0 profit, they will not be observed in equilibrium.

A further statement about the ratio of equilibrium qualities may be made:

Proposition 1*b*. *For* $\delta > \mu$, *in equilibrium firms will never choose qualities such that* $\frac{q_h}{q_l} > \delta$.

This follows from the fact that for $\frac{q_h}{q_l} > \delta$, consumers' perceptual limitations have no effect on either firm's profit, and as such the firms' best response functions are locally the same as in the standard case, meaning that any equilibrium must also be an equilibrium in the standard case. For $\delta > \mu$, the ratio of the standard equilibrium qualities lies below the perception threshold, so any q_h and q_l such that $\frac{q_h}{q_l} > \delta$ cannot be an equilibrium in the standard case and hence will not be observed.

Together propositions $1a$ and $1b$ imply that when $\delta > \mu$ so that consumers cannot perceive the difference between the standard equilibrium qualities, the firms will choose qualities that lie right on the perception threshold. If the qualities are any closer together, each firm makes a loss, and if they are any farther apart, each firm has an incentive to deviate to some other quality.

Whilst these properties ensure that there will be a unique ratio of equilibrium qualities, there are not necessarily unique levels of equilibrium qualities; in fact there will be multiple equilibria.[5] To see why this must be so, note that for $\delta > \mu$ there will be a range of qualities satisfying $\frac{q_h}{q_l} = \delta$ such that in the standard case of $\delta = 1$ each firm's only deviation that yields

[4] This should not be interpreted as asserting that consumers will exclusively act this way in every circumstance in which perceiving quality is prohibitively costly or difficult due to time constraints. For some goods, for example they could use the heuristic of using a higher price as a signal of higher quality, or use a "satisfycing" heuristic where they purchase the first good they come across that exceeds some reservation value (Reutskaja, Nagel, Camerer, & Rangel, 2011). Whilst these are no doubt worthy of investigation, the method of treating similar goods as homogeneous and thus choosing the one with the lower price is the exclusive focus of this paper.

[5] This is in addition to the trivial multiplicity of equilibria in the standard case caused by the fact that each identical firm may either be the low or high quality producer.

a greater profit is to profit is to produce a quality closer to its rival's. Thus when consumers' perceptual limitations are such that now such a deviation yields a loss, it must be that all qualities in that range are equilibria.

To aid comparisons to the standard equilibrium, when $\delta > \mu$ the levels of quality in equilibria are written in terms of the standard equilibrium qualities. To this end, the parameter $r \in \mathbb{R}_{++}$ is introduced and defined as the ratio of firm h's new equilibrium quality to its standard equilibrium quality. Equilibrium may now be characterized as follows:

Proposition 2. *Equilibrium is as in the standard case for $\delta \leq \mu$. For $\delta > \mu$ there are multiple equilibria, with the new equilibrium qualities $\tilde{q}_h$, $\tilde{q}_l$ having the form*

$$\tilde{q}_h = r q_h^* \qquad\qquad \tilde{q}_l = \frac{r\mu}{\delta} q_l^* \tag{8}$$

where $r \in \mathbb{R}_{++}$ and q_h^* and q_l^* are the standard equilibrium qualities.

$\tilde{q}_h$ comes from the definition of r and $\tilde{q}_l$ is arrived at by imposing $\frac{\tilde{q}_h}{\tilde{q}_l} = \delta$.

The levels of quality observed in a given equilibrium are now parameterized by the ratio r. For a given r, the qualities $\tilde{q}_h$ and $\tilde{q}_l$ then constitute an equilibrium if neither firm has an incentive to deviate. The following six conditions provide an exhaustive list of the possible ways each firm can deviate, and so if all six hold, then the firms must be in equilibrium:

i	Firm h does not wish to shut down.	**(FhS)**
ii	Firm l does not wish to shut down.	**(FlS)**
iii	Firm h does not wish to undercut firm l by producing below $\tilde{q}_l$.	**(FhU)**
iv	Firm h does not wish to produce $q_h > r q_h^*$ above the perception threshold.	**(FhB)**
v	Firm l does not wish to leapfrog firm h by producing above $\tilde{q}_h$.	**(FlL)**
vi	Firm l does not wish to produce $q_l < \frac{r\mu}{\delta} q_l^*$ below the perception threshold.	**(FlB)**

In the appendix, mathematical expressions for these conditions are derived and it is demonstrated that conditions FhS, FlS and FlL are redundant. The remaining conditions are

$$\text{Condition FhU:} \qquad r \leq \frac{2\delta^3 (4\mu - 1)^2 (4\delta^2 - 1)}{\mu^3 (4\mu - 7)(4\delta - 1)^2 (1 + \delta + \delta^2 + \delta^3)} \tag{9}$$

$$\text{Condition FhB:} \qquad r \geq \frac{4\delta (4\mu - 1)^3 (4\delta^2 - 3\delta + 2)}{\mu^3 (4\mu - 7)(4\delta - 1)^3} \tag{10}$$

$$\text{Condition FlB:} \qquad r \leq \frac{\delta^3 (4\mu - 1)^3 (4\delta - 7)}{\mu^3 (4\mu - 7)(4\delta - 1)^3}. \tag{11}$$

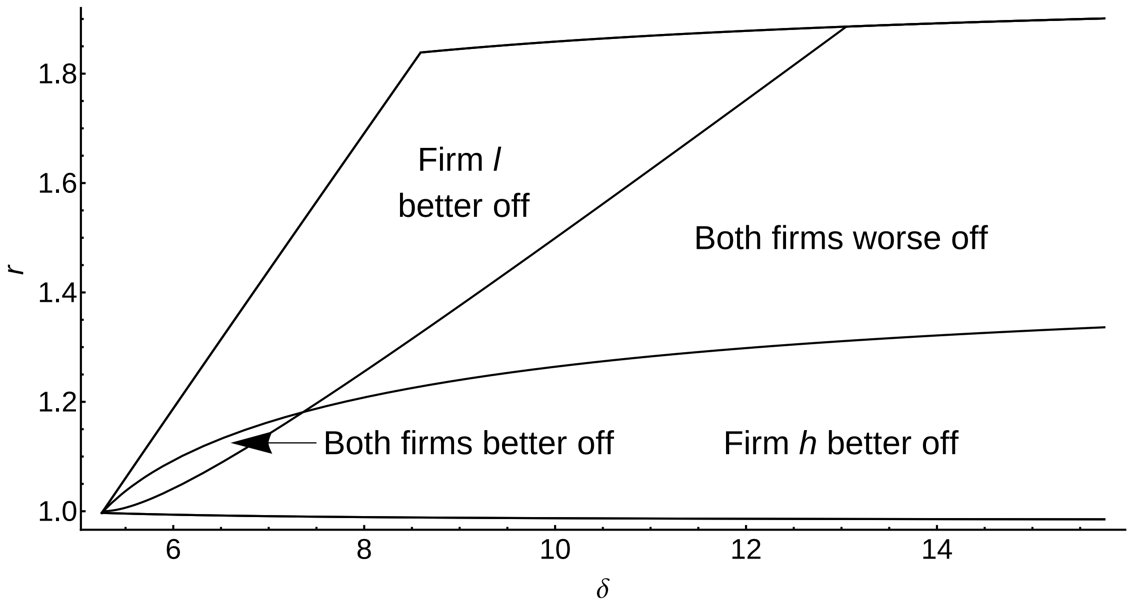

Figure 1: Equilibria of the simultaneous choice game

The lower limit on r is set by condition FhB and the upper limit is set by whichever of conditions FhU and FlB is lower for a given δ. For low δ, condition FlB is binding, and for high δ condition FhU is binding, with the switchpoint at δ_s', defined as the unique real root of equation (A.19) which is greater than 1, with $\delta_s' \approx 8.591$. Thus the qualities in equation (8) represent an equilibrium for

$$\frac{4\delta\left(4\mu-1\right)^{3}\left(4\delta^{2}-3\delta+2\right)}{\mu^{3}\left(4\mu-7\right)\left(4\delta-1\right)^{3}} \leq r$$

$$\leq \min\left\{\frac{\delta^{3}\left(4\mu-1\right)^{3}\left(4\delta-7\right)}{\mu^{3}\left(4\mu-7\right)\left(4\delta-1\right)^{3}}, \frac{2\delta^{3}\left(4\mu-1\right)^{2}\left(4\delta^{2}-1\right)}{\mu^{3}\left(4\mu-7\right)\left(4\delta-1\right)^{2}\left(1+\delta+\delta^{2}+\delta^{3}\right)}\right\}. \tag{12}$$

Having shown which qualities constitute an equilibrium in the case $\delta > \mu$, it is possible to examine the properties of these equilibria. To begin with consumers' equilibrium purchases, the demand for each firm's product is given by equation (2). Substituting in the relevant qualities and prices, demands in the standard case are

$$D_h^*\left(q_h, q_l\right) = \frac{2\bar{\alpha}\mu}{4\mu-1} \qquad D_l^*\left(q_h, q_l\right) = \frac{\bar{\alpha}\mu}{4\mu-1} \tag{13}$$

whereas if $\delta > \mu$ so that perceptual limitations influence the firms' choice of quality, demands become

$$\tilde{D}_h(q_h, q_l) = \frac{2\bar{\alpha}\delta}{4\delta - 1} \qquad \tilde{D}_l(q_h, q_l) = \frac{\bar{\alpha}\delta}{4\delta - 1}. \tag{14}$$

Comparing these expressions yields

Proposition 3. *Demand for each firm is as in the standard case for $\delta \leq \mu$. For $\delta > \mu$, demand for both the high and low quality firm's good is lower than in the standard case and is decreasing in δ.*

This proposition stems from the fact that in equilibrium demand for both the high and low quality good is increasing in the ratio of firms' qualities, and for $\delta > \mu$ consumers' perceptual limitations force firms to produce goods which are further apart in quality space.

It is possible to compare profits in equilibrium to profits in the standard case. In the appendix, the conditions under which firms h and l make greater profits than in the standard case are derived and stated in inequalities (A.22) and (A.24) respectively. The conditions are illustrated in figure (1), as are the equilibrium conditions. It can be seen that, depending on δ, there are mutually beneficial equilibria, equilibria where only one firm is better off and equilibria where both firms are worse off.

4 Discussion

The introduction of the perception threshold was inspired by the Weber-Fechner law. The fraction $\frac{\Delta I}{I}$ (known as the Weber fraction), where I is the intensity of some stimulus and ΔI is the minimum perceivable difference in intensity, equivalent here to $\delta - 1$, typically takes much smaller values than the values δ must take to have an impact on quality choice. The Weber fraction has been estimated to be 0.079 for brightness, 0.048 for loudness and 0.02 for heaviness (Techtsoonian, 1971), but here $\delta \gtrsim 5.251$ for perceptual limitations to have any relevance. However, the particular ratio of standard equilibrium qualities which specifies the minimum δ required is not general, but depends heavily on the assumption of how consumers are distributed in the economy.[6] The assumption of distribution with uniform density was made purely for reasons of tractability. A distribution with a more concentrated mass of consumers demanding medium quality and few demanding either high or low quality (such as a normal distribution) would lead to a smaller standard equilibrium quality ratio as firms compete for the concentrated demand in the middle ground. Thus a far lower perception threshold would be needed to influence quality choice.

[6] As does the fact that the ratio is independent of $\bar{\alpha}$ and k.

The perception threshold was chosen to be a fixed value for all consumers, regardless of their taste for quality. One objection to this is that it does not capture the observation that a higher taste for a good's quality is often associated with a greater ability to perceive differences in quality. The gourmet, or the wine buff, or the audiophile not only takes greater enjoyment in their consumption but can also detect minute differences to which the rest of us are oblivious. Yet this does not mean that including an inverse relationship between the perception threshold and the taste parameter is a more appropriate modeling choice. One mundane reason for homogeneous perception is that it results in a more tractable model. A better justification is that there are many consumer goods for which there are no "experts" who can perceive more clearly the differences in quality than the typical consumer: There is no such thing as a toilet paper aficionado, or a light-bulb aficionado. Even in other markets with expert consumers, they may be few enough in number that homogeneous perception is a good approximation.

Another argument against an inverse relationship between δ and α is that being a taste parameter is only one interpretation of α. The same distribution of willingness-to-pay for quality may be obtained by considering consumers with identical preferences but heterogeneous wealth. Thus a high α does not represent a consumer who benefits more from a high quality good, but rather one who is willing to pay more for higher quality due to possessing greater wealth, implying that a homogeneous perception threshold is the more appropriate choice. Nevertheless, there are no doubt markets where consumers are better modeled as having a heterogeneous perception threshold, and this represents an avenue of exploration for future research.

As was shown in proposition $1a$, in equilibrium $\frac{q_h}{q_l} < \delta$ will never be observed, so that consumers will always be able to distinguish between the products they are offered in equilibrium. This further implies that the higher δ is, the greater the ratio $\frac{q_h}{q_l}$ must be in equilibrium: The harder it is for consumers to perceive differences in quality, the further apart the qualities chosen will be. These findings are mostly driven by the fact that costs of quality are fixed and there are no marginal costs of production, so that when firms compete in prices with effectively homogeneous goods they earn 0 revenue. The implication is that for markets where fixed costs of quality dominate marginal costs, it should be that the harder consumers find it to discriminate between products, the further apart products are positioned. There are now multiple equilibria, compared to a single equilibrium in the standard case. The reason is that with the standard equilibrium unobtainable, the firms need to alter their qualities to be such that they lie on the perception threshold, yet there is no unique way of doing this. Firm h could remain producing the same quality as in the standard case and firm l only could reduce its quality until consumers perceive a difference. (This is the case $r = 1$.) On the other hand, firm l could remain producing as in the standard case and firm h could increase its quality until consumers perceive a difference. (This is the case $r = \frac{\delta}{\mu}$.) r is defined as the ratio of $\tilde{q}_h$ to q_h^*, but a more intuitive interpretation is that it measures how much of the necessary shift in qualities to reach the perception threshold is done by the high quality firm. $r = 1$ means the whole burden of change is borne by the lower firm whereas $r = \frac{\mu}{\delta}$ means the burden of change rests solely with the high quality firm.

A higher perception threshold leads to a greater range of r such that $\tilde{q}_h$ and $\tilde{q}_l$ constitute an equilibrium. The interpretation of r as a measure of each firm's share of the required change in qualities helps explain this: the greater the amount that qualities must change by to reach the perception threshold, the more ways there are of splitting the amount between the firms.

Note that equilibria are possible with $r < 1$, implying $\tilde{q}_h < q_h^*$, so that both firms produce below the standard equilibrium qualities, and $r > \frac{\delta}{\mu}$, implying $\tilde{q}_l > q_l^*$, so that both firms produce above the standard equilibrium qualities. The principle effect of perceptual limitations is to force firms to produce qualities that are further apart than in the standard case, so it is initially puzzling that equilibria exist with one firm moving closer to the other firm's standard equilibrium quality. The reason is that in standard equilibrium the high quality firm produces above the ideal monopoly quality that it would choose in the absence of competition. The presence of a competing firm drives firm h up to producing q_h^*. However, when perceptual limitations are introduced, firm l is prevented from producing higher than $\frac{r\mu}{\delta} q_h^*$ and thus driving up firm h's quality choice by the prospect of the firms' goods becoming indistinguishable. Firm h in $r < 1$ equilibria exploits the fact that firm l cannot choose a quality too close to its own to produce closer to the quality it would select were it to enjoy a monopoly. A similar argument applies to the case of $r > \frac{\delta}{\mu}$ and firm l producing above the standard equilibrium quality.

The existence of multiple equilibria allows the effect of perceptual limitations on firms' profits to be ambiguous. That the ratio of firms' qualities is greater has two contradictory effects. The first is that if qualities are further apart, the price competition in period 2 is less fierce, increasing a firm's profit. The second is that high quality is costly and low quality is unappealing to consumers, which decreases profit. Depending on which equilibrium is selected, either effect may dominate for each firm. Consider, for example, the case of $r = 1$, so that firm h's quality is unchanged from the standard case. Firm l's quality is forced further away in quality space by consumers' perceptual threshold, so that firm h is able to charge a higher price for the same quality and it earns a greater profit than in the standard case. Similarly for $r = \frac{\delta}{\mu}$, firm l's quality is unchanged, so it charges a higher price and makes a greater profit than in the standard case.

However, for sufficiently high r (implying a high $\tilde{q}_h$), the cost of high quality leads to firm h making a lower profit than in the standard case. Analogously for firm l and a sufficiently low r, consumers' low willingness-to.pay results in a lower profit than in the standard case.

The existence of equilibria in which both firms make a lower profit than in the standard case implies that they may have an incentive to lower consumers' perception threshold. Depending on the source of perceptual limitations, this may be possible in various ways. For example, firms may opt not to distinguish themselves by their goods' objective quality, but by "artificial" methods such as branding. It may also cause firms to coordinate the presentation of their goods so that consumers can perceive their attributes more easily. An example of this is milk, as a given type of milk has the same general colour packaging, regardless of the dairy it comes from, so that consumers have an easier task of perceiving the type (i.e. skimmed, semi-skimmed, etc.) of milk.

Although firms' profits may be greater than or lower than the standard equilibrium, demand for each firm is unambiguously lower. No matter how great the increase in firm h's quality, this is always offset by the increase in price, causing some consumers to switch to buying the low quality good. For firm l, the influx of former high quality purchasers is not enough to make up for the consumers who, due to a lesser quality and a relatively higher price, opt not to purchase.

5 Conclusion

It is impossible to perceive a small enough difference between two products. This paper has taken this simple observation and shown that it can have significant influence on market equilibria, in terms of the qualities and prices observed in equilibria, demand and consumer surplus, both of which are always lower, and profit, the effect on which is ambiguous.

There are many extensions that may be made to the simple framework used here. For example, it will be instructive in future work to examine whether firms could find it advantageous to try to educate consumers as to the differences between their products, or develop a simple metric which consumers could use to compare goods (as when firms print the weight/volume of their products on packaging). Repeated interactions would be another fruitful avenue of research, both if consumers' perception of quality increases with repeated consumption, and if firms have an incentive to reduce quality over time when consumers perceive the good they buy today as the same as the good they bought yesterday.

There is a growing recognition that perceptual issues should not be lightly disregarded in both empirical and theoretical work, and this has been again demonstrated in this study.

Appendix

Let $\delta > \mu$.

Condition FhS: Firm h shutdown

Firm h's profit is given by equation (5a). Substituting in the equilibrium qualities $\tilde{q}_h$ and $\tilde{q}_l$ from equation (8), then further substituting in the standard equilibrium quantities given in equation (6) results in

$$\tilde{\pi}_h\left(q_h, q_l\right) = \frac{4\bar{\alpha}^4\delta\mu^3\left(4\mu-7\right)\left(\delta-1\right)}{k\left(4\mu-1\right)^3\left(4\delta-1\right)}r - \frac{\bar{\alpha}^4\mu^6\left(4\mu-7\right)^2}{2k\left(4\mu-1\right)^6}r^2. \tag{A.1}$$

This is non-negative when

$$r \leq \frac{8\delta\left(\delta-1\right)\left(4\mu-1\right)^3}{\mu^3\left(4\mu-7\right)\left(4\delta-1\right)^2}. \tag{A.2}$$

Condition FlS: Firm l shutdown

Firm l's profit is given by equation (5b). Substituting in the equilibrium qualities $\tilde{q}_h$ and $\tilde{q}_l$ from equation (8), then further substituting in the standard equilibrium quantities given in equation (6) results in

$$\tilde{\pi}_l\left(q_h, q_l\right) = \frac{\bar{\alpha}^4\mu^3\left(4\mu-7\right)\left(\delta-1\right)}{k\left(4\mu-1\right)^3\left(4\delta-1\right)^2}r - \frac{\bar{\alpha}^4\mu^6\left(4\mu-7\right)^2}{2k\delta^2\left(4\mu-1\right)^6}r^2. \tag{A.3}$$

This is non-negative when

$$r \leq \frac{2\delta^2\left(\delta-1\right)\left(4\mu-1\right)^3}{\mu^3\left(4\delta-1\right)^2\left(4\mu-7\right)}. \tag{A.4}$$

Condition FhU: Firm h undercut

Consider the case when $h = 1$, so that firm 1 is the high quality firm with profits given by equation (5a) and firm 2 the low quality firm. It's profits are given by equation (A.1). Suppose firm 1 undercuts firm 2 to become the low quality firm with profit now given by equation (5b). Suppose also that when undercutting, it produces the highest quality below firm 2 that is distinguishable. Then substituting in $q_1 = \frac{r\mu}{\delta^2}q_2^*$ and $q_2 = \frac{r\mu}{\delta}q_2^*$ and further substituting for q_2^* results in

$$\pi_1^{uc}\left(q_1, q_2\right) = \frac{\bar{\alpha}^4\mu^3\left(4\mu-7\right)\left(\delta-1\right)}{k\delta\left(4\mu-1\right)^3\left(4\delta-1\right)^2}r - \frac{\bar{\alpha}^4\mu^6\left(4\mu-7\right)^2}{2k\delta^4\left(4\mu-1\right)^6}r^2. \tag{A.5}$$

Then from $\tilde{\pi}_1(q_1, q_2) - \pi_1^{uc}(q_1, q_2) \geq 0$, given the assumption that firm 1 when undercutting produces the highest quality distinguishable from firm 2, the condition for no undercutting is

$$r \leq \frac{2\delta^3 (4\mu - 1)^3 (4\delta^2 - 1)}{\mu^3 (4\mu - 7)(4\delta - 1)^2 (1 + \delta + \delta^2 + \delta^3)}. \tag{A.6}$$

The first and second order conditions for profit maximization if firm 1 undercuts are

$$\frac{\partial \pi_1^{uc}(q_1, q_2)}{\partial q_1} = \frac{\bar{\alpha}^2 q_2^2 (4q_2 - 7q_1)}{(4q_2 - q_1)^3} - kq_1 \qquad \frac{\partial^2 \pi_2^{uc}(q_1, q_2)}{\partial q_1^2} = -\frac{2\bar{\alpha}^2 q_2^2 (8q_2 + 7q_1)}{(4q_2 - q_1)^4} - k. \tag{A.7}$$

Thus there is a single maximum of $\pi_1^{uc}(q_1, q_2)$, with profit increasing in q_1 below this point and if at any q_1 the first order condition is positive, producing any quality below this point will result in less profit.

When firm 1 undercuts to produce on the perception threshold, the first order condition is

$$\left.\frac{\partial \pi_1^{uc}(q_1, q_2)}{\partial q_1}\right|_{q_1 = \frac{r}{\delta^2} q_2^*} = \bar{\alpha}^2 \left(\frac{\delta^4 (4\delta - 7)(4\mu - 1)^3 - r\mu^3 (4\mu - 7)(4\delta - 1)^3}{k\delta^2 (4\mu - 1)^3 (4\delta - 1)^3} \right) \tag{A.8}$$

which is linearly decreasing in r. Thus, if at the maximum r satisfying inequality (A.6) the first order condition is positive, it must be positive for all r less than this. Then any r at which firm 1's maximum profit given it undercuts is obtained by producing a quality below the maximum distinguishable level is above the maximum r at which firm 1 wishes to undercut rather than produce the equilibrium quality.

Substituting in the maximum value of r satisfying inequality (A.6) into equation (A.8) results in the condition for the first order condition being positive is reduced to

$$4\delta^5 - 3\delta^4 - 35\delta^3 + 5\delta^2 + \delta - 2 > 0. \tag{A.9}$$

Numerically, the real roots of the polynomial are found to be less than μ, the minimum value of δ such that the standard equilibrium does not hold. Substituting in $\delta = \mu$ shows the polynomial is positive at this point, and thus positive at all δ greater than this. Thus the condition for firm 1 not undercutting is given by inequality (A.6).

An analogous argument holds when firm 2, rather than firm 1 is the high quality firm.

Condition FhB: Firm h above boundary

Firm h's first and second order conditions for profit maximization are

$$\frac{\partial \pi_h (q_h, q_l)}{\partial q_h} = \frac{4\bar{\alpha}^2 q_h (4q_h^2 - 3q_h q_l + 2q_l^2)}{(4q_h - q_l)^3} - kq_h. \qquad \frac{\partial^2 \pi_h (q_h, q_l)}{\partial q_h^2} = -\frac{\bar{\alpha}^2 q_l^2 (40q_h + 8q_l)}{(4q_h - q_l)^4} - k \tag{A.10}$$

Then for firm h to wish to produce a quality above the boundary of discriminability it must be that the first order condition is positive. Substitution and rearrangement reveals that firm h does not wish to produce above the boundary when

$$r \geq \frac{4\delta (4\mu - 1)^3 (4\delta^2 - 3\delta + 2)}{\mu^3 (4\mu - 7) (4\delta - 1)^3}. \tag{A.11}$$

Condition FlL: Firm l leapfrog

Consider the case where $l = 1$, so that firm 1 is the low quality firm with profits given by equation (5b) and firm 2 the high quality firm. Its profit is given by equation (A.4). Suppose firm 1 leapfrogs firm 2 to become the high quality firm with profit now given by equation (5a). Suppose also that when leapfrogging, firm 1 produces the lowest quality above firm 2 that is distinguishable. Then substituting $q_2 = rq_2^*$ and $q_1 = r\delta q_2^*$ and further substituting for q_2^* results in

$$\pi_1^{le} (q_1, q_2) = \frac{4\bar{\alpha}^4 \mu^3 \delta^2 (4\mu - 7) (\delta - 1)}{k (4\mu - 1)^3 (4\delta - 1)^2} r - \frac{\bar{\alpha}^4 \mu^6 \delta^2 (4\mu - 7)^2}{2k (4\mu - 1)^6} r^2. \tag{A.12}$$

From $\tilde{\pi}_1 (q_1, q_2) - \pi_1^{le} (q_1, q_2) \geq 0$, the condition for firm 1 not leapfrogging is then

$$r \geq \frac{2\delta^2 (4\mu - 1)^3 (4\delta^2 - 1)}{\mu^3 (4\mu - 7) (4\delta - 1)^2 (1 + \delta + \delta^2 + \delta^3)} \tag{A.13}$$

provided that firm 1 produces on the boundary when leapfrogging.

The first and second order conditions for profit maximization given that it leapfrogs are

$$\frac{\partial \pi_1^{le} (q_1, q_2)}{\partial q_1} = \frac{4\bar{\alpha}^2 q_1 (4q_1^2 - 3q_2 q_1 + 2q_2^2)}{(4q_1 - q_2)^3} - kq_1 \qquad \frac{\partial^2 \pi_1^{le} (q_1, q_2)}{\partial q_1^2} = -\bar{\alpha}^2 \frac{q_2^2 (40q_1 + 8q_2)}{(4q_1 - q_2)^4} - k. \tag{A.14}$$

Thus for firm 1 to make a greater profit by leapfrogging above the boundary than leapfrogging to the boundary, the first order condition must be positive at the boundary.

At the boundary, the first order condition is

$$\left.\frac{\partial \pi_1^{le}(q_1,q_2)}{\partial q_1}\right|_{q_1=r\delta q_2^*} = \bar{\alpha}^2\delta\left(\frac{4(4\mu-1)^3(4\delta^2-3\delta+2)-rk\mu^3(4\mu-7)(4\delta-1)^3}{(4\mu-1)^3(4\delta-1)^3}\right) \quad (A.15)$$

which is linearly decreasing in r. Substituting the minimum r at which inequality (A.13) holds, the condition for the first order condition to be positive may be reduced to

$$-8\delta^5+6\delta^4+10\delta^3+5\delta^2-2\delta+4>0. \quad (A.16)$$

The real roots of this polynomial lie below μ, the minimum assumed value of δ. It is also negative at μ, which implies that it must be negative for all δ above this. Hence at the highest r at which firm 1 wishes to leapfrog above the boundary is below the highest r at which it wishes to leapfrog to the boundary. The condition for firm 1 not leapfrogging is given by inequality (A.13).

An analogous argument holds when firm 2, rather than firm 1 is the low quality firm.

Condition FlB: Firm l below boundary

Firm l's first and second order conditions for profit maximization are

$$\frac{\partial \pi_l(q_h,q_l)}{\partial q_l} = \frac{\bar{\alpha}^2 q_h^2(4q_h-7q_l)}{(4q_h-q_l)^3}-kq_l \qquad \frac{\partial^2 \pi_l(q_h,q_l)}{\partial q_l^2} = -\frac{2\bar{\alpha}^2 q_h^2(8q_h+7q_l)}{(4q_h-q_l)^4}-k. \quad (A.17)$$

So for firm l to produce below the boundary it must be that the first order condition is negative at this point. If it is nonnegative its profit will always decrease with decreasing quality.

Substitution and rearrangement results in the condition for firm l not wishing to produce below the boundary is

$$r \le \frac{\delta^3(4\mu-1)^3(4\delta-7)}{\mu^3(4\mu-7)(4\delta-1)^3}. \quad (A.18)$$

Condition redundancy

Of the six conditions, FhS, FlS, FhU and FlB set an upper limit on r, and FhB and FlL set a lower limit. Taking the upper limits first, condition FhS is lower than condition FlS when $\delta>4$. Since $4<\mu\approx 5.251$, when $\delta>\mu$ and by proposition 2 the standard equilibrium does not obtain, condition FhS is always lower and condition FlS is redundant. Comparing condition FhS to condition FhU, it is found that condition FhU is lower when $\delta > 2$. Again this is less than μ so condition FhU is always lower and condition FhS is redundant.

Comparing condition FhU to condition FlB and rearranging, the latter is found to be lower than the former if

$$f(\delta) = 4\delta^4 - 35\delta^3 + 5\delta^2 + 5\delta - 9 \tag{A.19}$$

is negative. If $f(\delta)$ is positive, then condition FhU is lower than condition FlB. Define the unique root of $f(\delta)$ with $\delta > 1$ as δ_s', with $\delta_s' \approx 8.591$. Below δ_s' it is negative, so the upper limit on r is set by condition FlB. Above δ_s', $f(\delta)$ is positive, and the upper limit on r is set by condition FhU.

Turning to the lower limit, comparing conditions FhB and FlL and rearranging shows that condition FhB is greater if

$$g(\delta) = 8\delta^5 - 14\delta^4 + 4\delta^3 + 5\delta^2 - 3\delta + 4 \tag{A.20}$$

is positive. $g(\delta)$ has no real positive roots, and so it may be seen that it is always positive for $\delta > 1$. Thus the lower limit on r is set by condition FhB.

Profits

Substitution reveals that in the standard case, firm h's profit is

$$\pi_h^*(q_h, q_l) = \frac{\bar{\alpha}^4 \mu^4 (4\mu - 7)(8 - 40\mu + 39\mu^2 - 4\mu^3)}{2k(4\mu - 1)^6} \tag{A.21}$$

whereas new equilibrium profits are given by equation (A.1). From $\frac{\tilde{\pi_h}(q_h,q_l)}{\pi_h^*(q_h,q_l)} > 1$, firm h makes greater profits than in the standard equilibrium when

$$\frac{(4\mu-1)^3}{\mu^3(4\mu-7)}\left(\frac{4\delta(\delta-1)}{(4\delta-1)^2} - \sqrt{\frac{16\delta^2(\delta-1)^2}{(4\delta-1)^4} - \frac{\mu^4(4\mu-7)(8-40\mu+39\mu^2-4\mu^3)}{(4\mu-1)^6}}\right) < r <$$
$$\frac{(4\mu-1)^3}{\mu^3(4\mu-7)}\left(\frac{4\delta(\delta-1)}{(4\delta-1)^2} + \sqrt{\frac{16\delta^2(\delta-1)^2}{(4\delta-1)^4} - \frac{\mu^4(4\mu-7)(8-40\mu+39\mu^2-4\mu^3)}{(4\mu-1)^6}}\right). \tag{A.22}$$

Firm l's profit in the standard case is

$$\pi_l^*(q_h, q_l) = \frac{\bar{\alpha}^4 \mu^3 (4\mu - 7)(4\mu^2 - 3\mu + 2)}{2k(4\mu - 1)^6} \tag{A.23}$$

whereas new equilibrium profits are given by equation (A.4). From $\frac{\tilde{\pi}_l(q_h,q_l)}{\pi_l^*(q_h,q_l)} > 1$, firm l makes greater profits than in the standard equilibrium when

$$\frac{\delta^2 (4\mu-1)^3}{\mu^3 (4\mu-7)} \left(\frac{(\delta-1)}{(4\delta-1)^2} - \sqrt{\frac{(\delta-1)^2}{(4\delta-1)^4} - \frac{\mu^3 (4\mu-7)(4\mu^2-3\mu+2)}{\delta^2 (4\mu-1)^6}} \right) < r <$$

$$\frac{\delta^2 (4\mu-1)^3}{\mu^3 (4\mu-7)} \left(\frac{(\delta-1)}{(4\delta-1)^2} + \sqrt{\frac{(\delta-1)^2}{(4\delta-1)^4} - \frac{\mu^3 (4\mu-7)(4\mu^2-3\mu+2)}{\delta^2 (4\mu-1)^6}} \right). \quad \text{(A.24)}$$

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