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Thomas Jensen

Øster Farimagsgade 5, Building 26, DK-1353 Copenhagen K., Denmark
Tel.: +45 35 32 30 01 – Fax: +45 35 32 30 00
http://www.econ.ku.dk

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Thomas Jensen†

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Abstract

The quality of political candidates often depends on the current state of the world, for example because their personal characteristics are more valuable in some situations than in others. We explore the implications of state-dependent candidate quality in a model of electoral competition where voters are uncertain about the state. Candidates are fully informed and completely office-motivated. With a reasonable restriction on voters' beliefs, an equilibrium where candidates' positions reveal the true state does not exist. Non-revealing equilibria always exist. Some main findings are that candidates' positions can diverge more in equilibrium when they differ more in state-dependent quality and when the electorate is less well informed.

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†Department of Economics, University of Copenhagen, Øster Farimagsgade 5, Building 26, DK-1353 Copenhagen K, Denmark. Email: Thomas.Jensen@econ.ku.dk.

1 Introduction

In representative democracy, political candidates are not evaluated solely on their policy positions. Personal characteristics such as background, experience, and skills also matter for candidate evaluation. Suppose voters aggregate these characteristics into a one-dimensional measure of candidate quality (valence). It is likely that the impact of different characteristics on the measure of quality will depend on the current political challenges. For example, a candidate for national office with a military background and primary experience in foreign policy will probably be seen as better qualified when national security is among the main concerns of voters than when it is not. In other words, *candidate quality is state-dependent*. Thus, in an election with two candidates who differ in quality it is not necessarily clear who the high quality candidate is. If voters are uncertain about the state of the world they may also well be uncertain about who is better qualified for office. Consider the 2008 US presidential election. Polls taken during the campaign show that a majority trusted McCain more on terrorism while Obama was trusted more on the economy.[1] This suggests that candidates can differ with respect to how qualified they are to handle different issues and therefore that the overall quality ranking of candidates may well depend on the relative importance of these issues in the current state of the world.[2]

To our knowledge, this is the first paper to consider the concept of state-dependent candidate quality. We explore its implications in a model of electoral competition where voters, but not candidates, are uncertain about the state of the world. Main insights from the analysis are that candidates' policy positions can diverge more in equilibrium when they differ more in state-dependent quality and when the electorate is less well informed.

In the model there are two office-motivated candidates and two states of the world. One candidate has a quality advantage in one state while the other candidate has a similar advantage in the other state. The electorate receives a noisy signal about the true state while candidates are fully informed. This informational asymmetry between voters and politicians is natural. For example, politicians have much stronger incentives than voters to be well informed since their careers depend on how they do as policy makers.[3] Furthermore, politicians often have staff and advisors to help them receive and evaluate information and sometimes have access

[1] See, for example, the Washington Post-ABC News Poll from October 8-11, 2008 (question 9): *www.washingtonpost.com/wp-srv/politics/polls/postpoll_ 101308.html*.

[2] See also the literature on issue ownership originated by Petrocik (1996).

[3] This is not necessarily the case within the model if we extend it by an information acquisition stage (because all reasonable equilibria are non-revealing). However, going beyond the model there are many potential benefits of information for a candidate. For example from starting his term in office well informed if he is elected. Therefore we find the assumption natural.

to classified information, for example information related to national security.

The policy space is one-dimensional and the state of the world matters for voters' policy preferences. Suppose the state changes. Then the preferred policy of each voter moves some fixed distance in the same direction. To give an example, the one-dimensional issue could be how much of a fixed budget to spend on national security and the two states could represent situations with different levels of real threat to national security. It is reasonable to assume that each voter prefers higher spending when the threat is high. Thus the distribution of voters' preferred policies shifts to the right (towards higher national security spending) when we go from the low threat state to the high threat state.

Before the election, the candidates simultaneously take credible policy positions. Voters observe these positions, update their beliefs about the state, and then vote for the candidate providing the highest expected utility. Note that, from the point of view of the candidates when they take positions, voting is stochastic.

A basic premise of the model is that even though both candidates know the true state of the world, neither of them can credibly communicate it directly to the electorate. For example, as is likely to be the case if the issue is national security spending, because doing so would require a leak of highly classified information to the public. In other situations it may in principle be possible, but practically infeasible, for a candidate to directly reveal his information because it would require successful communication of long and complicated lines of reasoning. In such situations, direct revelation of information is even more unlikely to be successful if one candidate prefers that the information is not revealed and therefore has an incentive to "sabotage" efforts to reveal it. This is always the case in our model. Thus, the only way candidates can possibly reveal their information is by their choices of policy positions.

We show that if we impose a well known condition on out-of-equilibrium beliefs in multi-sender signaling games then, in equilibrium, it is not possible for candidates to reveal information to voters. Non-revealing equilibria always exist. The set of equilibria depends on the parameters. Restricting attention to a subset of equilibria (we argue that all other equilibria are less reasonable), we get the results mentioned earlier: candidates' positions can diverge more when the electorate is less well informed and when they differ more in state-dependent quality.

In an equilibrium with some level of divergence, the candidate who is least likely to be of high quality given the voters' signal does not want to join the other candidate in announcing the median voter's position in the most likely state. Because doing so would make voters believe with certainty that he is the low quality candidate. At his equilibrium position he will, loosely speaking, have a moderate disadvantage with respect to both policy and quality. If he deviates to the other candidate's position then he will have a large disadvantage with respect to quality only. The equilibrium disadvantage is larger the more precise the voters' signal is and the further he is from the other candidate. The deviation disadvantage is independent of these things because the candidates then converge and voters believe with certainty in one state. Therefore, when the signal of the voters is less precise, the candidate who is least likely to be of high quality is willing to be further from the other candidate's position in order to avoid that voters believe him to be the low quality candidate with certainty. This is the reason why more divergence is possible in equilibrium the less well informed the electorate is.

The explanation why more divergence is possible when candidates differ more in quality is similar. When candidates differ more on the quality dimension, deviation is relatively less attractive for the disadvantaged candidate and therefore he is willing to be further from the median voter's position in the most likely state.

Finally, we compare the non-revealing equilibria of the model with respect to voters' welfare. By restricting attention to symmetric equilibria, we can uniquely represent each equilibrium by its level of candidate divergence (polarization), which can be anything from zero to the distance between the median voter's preferred policies in the two states ("full polarization"). We show that all voters prefer convergence over full polarization. However, increased divergence can be welfare increasing for some voters, it depends on their preferred policies. Thus there is no general welfare effect of divergence for the electorate as a whole.

All proofs can be found in the appendix or the supplemental material.

2 Related Literature

Krasa and Polborn (2010; 2012) analyze models of electoral competition where candidates' abilities are policy dependent (see also Schofield (2003) for a model with an additive, policy-dependent valence term). For example, some candidates may be better at running a small government while other candidates are better at running a big government. When candidates have different abilities it is shown that, even though they are completely office-motivated, there will typically be policy divergence in equilibrium. Either the candidates will play to their own strengths or they will compensate for their weaknesses. The idea of policy dependent candidate abilities is obviously related to the idea of state-dependent candidate quality presented here. However, the ideas are also clearly distinct. In our model, voters' evaluations of candidates' characteristics do not directly depend on their policy positions. If, for example, candidates converge and voters rely on their signal then the outcome of the election does not depend on where the candidates are in the policy space.

A substantial amount of literature has considered electoral competition when candidates are better informed than voters. This includes the contributions by Schultz (1996), Martinelli (2001), Heidhues and Lagerlöf (2003), Laslier and Van

Der Straeten (2004), and Loertscher (2012). The main questions in this literature are whether candidates' choices of policy platforms reveal their information to voters and how this depends on various assumptions about, for example, candidates' preferences and voters' information. Schultz (1996) and Martinelli (2001) consider primarily policy-motivated candidates, which is fundamentally different from our model. In Heidhues and Lagerlöf (2003) and Laslier and Van Der Straeten (2004) candidates are office-motivated and both the policy space and the state space is binary. The main difference between the two models is that in the latter the electorate receives a signal that is not observed by the candidates. In the model of Heidhues and Lagerlöf a pure revealing equilibrium does not exist while such an equilibrium always exists in the model of Laslier and Van Der Straeten. Our model is closest to that of Heidhues and Lagerlöf because the signal of the voters is observed by the candidates. Indeed, our result that a revealing equilibrium satisfying an additional condition on voters' out-of-equilibrium beliefs does not exist is closely related to their non-revelation result. Loertscher (2012) considers a set-up that is similar to that of Heidhues and Lagerlöf except that the policy space is, as in our model, continuous rather than binary.

Groseclose (2001) studies an electoral competition model where candidates are at least partially policy-motivated and one candidate has a known valence advantage.[4] One implication of the model is that the distance between candidates' policy positions is increasing in the quality difference. This finding seems related to our result that more divergence is possible when candidates differ more in (state-dependent) quality. However, the reasons behind the two findings are different. In Groseclose's model, candidates diverge more because of their policy preferences. In our model, more divergence is possible when candidates differ more in quality because of voters' uncertainty about the state.

Finally, in Kartik and McAfee (2007) a fraction of possible candidates have character, i.e., are exogenously committed to a policy platform. Character is desirable but unobservable to voters, so in this sense there is uncertainty about candidate quality.

3 The Model

We consider a one issue election. The policy space X is the real axis or some bounded interval. For example, as mentioned in the introduction, the issue could be how much of a fixed budget to spend on national security. There are two purely office-motivated candidates, i.e., they maximize the probability of winning. The candidates simultaneously announce credible policy positions before the election.

[4] Other contributions to the literature on known valence include Ansolabehere and Snyder (2000) and Aragones and Palfrey (2002; 2005). The seminal reference is Stokes (1963).

The electorate consists of an odd number of voters. The voters have utility functions over the policy space. These depend on the state of the world ω, which can be either L or H. In the national security example, think of situations with a low or high security threat. The policy utility function of voter i is

$$u_i(x|\omega) = -|x - x_i^*(\omega)|,$$

where $x_i^*(\omega)$ is the preferred policy of voter i in state ω. Each voter's preferred policy is further to the right in state H than in state L (he prefers higher national security spending when the threat is high). More precisely, for each voter i,

$$x_i^*(H) = x_i^*(L) + D,$$

where $D > 0$ is a constant. $x_m^*(\omega)$ denotes the median voter's preferred policy.

Besides policy, voters also care about candidate quality, which is state-dependent. One candidate, Candidate $\mathcal{L}$, has a quality advantage in state L while the other candidate, Candidate $\mathcal{H}$, has a similar quality advantage in state H. On top of this, the electorate may, through the course of the electoral campaign, develop a general preference for one of the candidates. For example because voters learn more about candidates' personalities. Or because of the general uncertainty always present in electoral campaigns. One candidate may respond better to an unforeseen event, a previously unknown scandal could be revealed, a candidate could make a gaffe at a public event, or a particular series of ads could resonate well with the electorate. Since policy positions are announced early in the campaign, this makes voting probabilistic from the point of view of the candidates.

Suppose Candidate $\mathcal{L}$ has announced the policy $x^{\mathcal{L}}$ and that Candidate $\mathcal{H}$ has announced $x^{\mathcal{H}}$. Then voter i's utility if Candidate $\mathcal{L}$ is elected is

$$U_i(\mathcal{L}, x^{\mathcal{L}}|\omega) = \left\{ \begin{array}{ll} u_i(x^{\mathcal{L}}|L) + \gamma & \text{if } \omega = L \\ u_i(x^{\mathcal{L}}|H) & \text{if } \omega = H \end{array} \right\} + \delta,$$

where $\gamma > 0$ is a parameter and, for some parameter $\sigma > 0$, δ is drawn from the uniform distribution on $[-\frac{1}{2\sigma}, \frac{1}{2\sigma}]$. The realized value of δ is the same for all voters, independent of the state of the world, and unknown to the candidates when they announce positions. Voter i's utility if Candidate $\mathcal{H}$ is elected is

$$U_i(\mathcal{H}, x^{\mathcal{H}}|\omega) = \left\{ \begin{array}{ll} u_i(x^{\mathcal{H}}|L) & \text{if } \omega = L \\ u_i(x^{\mathcal{H}}|H) + \gamma & \text{if } \omega = H \end{array} \right\}.$$

Each voter votes for the candidate providing the highest expected utility. So if voter i believes that the probability of state L is μ_L then he votes for Candidate $\mathcal{L}$ if and only if

$$\mu_L(u_i(x^{\mathcal{L}}|L)+\gamma+\delta)+(1-\mu_L)(u_i(x^{\mathcal{L}}|H)+\delta) > \mu_L u_i(x^{\mathcal{H}}|L)+(1-\mu_L)(u_i(x^{\mathcal{H}}|H)+\gamma).$$

By plugging in the policy utility function of the voter, we get

$$\delta > \mu_L(|x^{\mathcal{L}} - x_i^*(L)| - |x^{\mathcal{H}} - x_i^*(L)| - \gamma) \\ + (1-\mu_L)(|x^{\mathcal{L}} - x_i^*(H)| - |x^{\mathcal{H}} - x_i^*(H)| + \gamma). \quad (1)$$

With respect to information, both candidates are fully informed about the state of the world. However, as discussed in the introduction, neither of them can credibly communicate it directly to voters. The voters only receive a signal $\omega^V \in \{l, h\}$, which is distributed according to

$$\Pr(\omega^V = l|L) = \Pr(\omega^V = h|H) = \theta,$$

where $\theta \in (\frac{1}{2}, 1)$ is a parameter. All voters receive the same signal and they all have the prior belief that each state is equally likely. Thus, if voters do Bayesian updating based on their signal, their belief about the state is simply given by

$$\Pr(L|\omega^V = l) = \Pr(H|\omega^V = h) = \theta.$$

Since candidates are also voters they also receive the signal ω^V.

The timeline of the model is:

1. The candidates observe the true state of the world and voters (and candidates) receive the signal ω^V;
2. The candidates simultaneously announce policy positions;
3. The value of δ is realized and the voters cast their votes;
4. The winning candidate enacts his announced position.

Throughout the paper we assume that

$$\gamma + D < \frac{1}{2\sigma}.$$

This condition implies that if the distance between the candidates' positions is at most D then, no matter what voters believe about the state, each candidate has a strictly positive probability of winning the election.

4 Strategies, Beliefs, and Equilibrium

The candidates observe both the true state and the signal of the voters before taking positions. Therefore, a strategy for Candidate i, $i = \mathcal{L}, \mathcal{H}$, consists of four policy positions, one for each possible combination of ω and ω^V. We write the strategy as $x^i(\omega, \omega^V)$.

As mentioned above, each voter votes for the candidate providing the highest expected utility. To do this calculation, each voter forms a belief about the true state. This belief can depend on the positions of the candidates and the signal ω^V. We make the simplifying assumption that all voters share the same belief function. The belief about the probability of state L is written $\mu_L(x^{\mathcal{L}}, x^{\mathcal{H}}, \omega^V)$.

Our notion of equilibrium is the standard concept of Perfect Bayesian Equilibrium. We only consider pure strategies.

Definition 1 (Equilibrium)
An equilibrium consists of candidate strategies and a voter belief function,

$$\hat{x}^{\mathcal{L}}(\omega, \omega^V),\ \hat{x}^{\mathcal{H}}(\omega, \omega^V),\ \textit{and}\ \hat{\mu}_L(x^{\mathcal{L}}, x^{\mathcal{H}}, \omega^V),$$

such that the following two conditions hold:

1. *For each combination of ω and ω^V, each candidate's position maximizes his probability of winning given the other candidate's position, the belief function of the voters, and the distribution of δ.*

2. *The belief function is consistent with Bayes' rule on the equilibrium path. I.e., if, for some value of ω^V, at least one candidate takes different positions in the two states then voters will infer the true state. Otherwise they will do Bayesian updating based on their signal.*

In a *fully revealing equilibrium* there is always (i.e., for both values of ω^V) at least one candidate who reveals the state. In a *partially revealing equilibrium* the state is revealed for one value of the voters' signal but not for the other. Finally, in a *non-revealing equilibrium* the state is not revealed for any value of ω^V.

Our first result is that the median voter decides the outcome of the election.

Lemma 1 (The Median Voter Decides the Outcome)
Suppose that, given the candidates' positions, the voters' signal, and the realization of δ, the median voter strictly prefers Candidate $\mathcal{L}$ ($\mathcal{H}$). Then a majority of voters strictly prefers Candidate $\mathcal{L}$ ($\mathcal{H}$).

Now consider the objective of Candidate $\mathcal{L}$ given Candidate $\mathcal{H}$'s strategy and the belief function of the voters. Let x denote Candidate $\mathcal{L}$'s position. Then his probability of winning the election given ω and ω^V is (see (1))

$$\Pr_{\delta}[\delta > \mu_L(x, x^{\mathcal{H}}(\omega, \omega^V), \omega^V)(|x - x_m^*(L)| - |x^{\mathcal{H}}(\omega, \omega^V) - x_m^*(L)| - \gamma)$$
$$+ (1 - \mu_L(x, x^{\mathcal{H}}(\omega, \omega^V), \omega^V))(|x - x_m^*(H)| - |x^{\mathcal{H}}(\omega, \omega^V) - x_m^*(H)| + \gamma)].$$

His objective is to maximize this probability with respect to x. The objective of Candidate $\mathcal{H}$ follows analogously.

So far, the only assumption we have made about voters' beliefs is that they all share the same belief function. We will also assume that if one candidate moves to the right, this will not make voters believe that state L is more likely (and vice versa). More formally, the voter belief function $\mu_L(x^{\mathcal{L}}, x^{\mathcal{H}}, \omega^V)$ is, for both values of ω^V, assumed to be weakly decreasing with respect to both $x^{\mathcal{L}}$ and $x^{\mathcal{H}}$. There are no directly state-dependent costs for the candidates that can justify this assumption in a rigorous game theoretic way. Nevertheless, since each voter's preferred policy is further to the right in state H than in state L and candidates are completely office-motivated, it is intuitively reasonable that a candidate move to the right does not make voters believe more in state L. Below we will simply refer to this condition as *the monotonicity condition* on voters' beliefs.

5 Equilibrium Analysis

We first consider equilibria that are fully or partially revealing. It turns out that if we do not impose any conditions on the voter belief function other than the monotonicity condition defined above, a fully revealing equilibrium always exists.

Proposition 1 (Existence of a Fully Revealing Equilibrium)
For all parameter values there exists a fully revealing equilibrium where the strategies of the candidates are given by

$$\hat{x}^{\mathcal{L}}(\omega, \omega^V) = \left\{ \begin{array}{cc} x^*_m(L) - \gamma & \textit{if } \omega = L,\ \omega^V = l, h \\ x^*_m(H) & \textit{if } \omega = H,\ \omega^V = l, h \end{array} \right\}$$

and

$$\hat{x}^{\mathcal{H}}(\omega, \omega^V) = \left\{ \begin{array}{cc} x^*_m(L) & \textit{if } \omega = L,\ \omega^V = l, h \\ x^*_m(H) + \gamma & \textit{if } \omega = H,\ \omega^V = l, h \end{array} \right\}.$$

In such an equilibrium each candidate always wins with probability one half.

We do not find this equilibrium plausible however. Each candidate takes different positions in the two states. Therefore, voters can infer the true state by observing the position of either candidate. So if, in some state, one candidate deviates to an out-of-equilibrium position then it can be argued that voters should just ignore this deviation and infer the true state from the position of the other candidate. This is the reasoning behind a condition on out-of-equilibrium beliefs in multi-sender signaling games that was introduced by Bagwell and Ramey (1991) and has been used in models of electoral competition by Schultz (1996; 1999) and others. The condition is meant to eliminate unreasonable revealing equilibria. It is easy to see that any equilibrium where candidates' strategies are as in Proposition 1 does not satisfy the condition. Suppose we are in state L and consider a deviation by Candidate $\mathcal{L}$ to $x_m^*(L)$. This deviation will make voters believe in state H with some positive probability, otherwise it would increase Candidate $\mathcal{L}$'s probability of winning. However, if we accept the argument above, voters should just ignore the deviation and continue to infer that the state is L from the position of Candidate $\mathcal{H}$.

Even a weaker local version of the Bagwell-Ramey condition[5] (only small out-of-equilibrium deviations are assumed to be ignored by the voters, see the supplemental material for a formal definition) eliminates all fully and partially revealing equilibria in our model. The reasoning behind this result, which is formulated as a proposition below, is as follows. First, if a revealing equilibrium satisfies the local Bagwell-Ramey condition then the position of each candidate must be the median voter's position in the true state. Otherwise it would be profitable for a candidate to move slightly towards that position. Finally, given that these are the only possible revealing equilibrium strategies, it is easy to see that in at least one of the states the low quality candidate will have an incentive to deviate to the median voter's position in the false state. A formal proof of the proposition can be found in the supplemental material.

Proposition 2 (No Revelation with Local Bagwell-Ramey)
There does not exist a fully or partially revealing equilibrium satisfying the local Bagwell-Ramey condition.

We then turn attention to non-revealing equilibria. Strategies in non-revealing equilibria can depend only on the voters' signal. Therefore, equilibrium strategies for the two candidates are written $\hat{x}^{\mathcal{L}}(\omega^V)$ and $\hat{x}^{\mathcal{H}}(\omega^V)$.

Our first observation is that we can use the monotonicity of the belief function to derive restrictions on candidates' equilibrium strategies. Suppose we have an equilibrium with $\hat{x}^{\mathcal{L}}(l) > x_m^*(L)$. Then a deviation by Candidate $\mathcal{L}$ to $x_m^*(L)$ when $\omega^V = l$ will increase the median voter's utility of Candidate $\mathcal{L}$ being elected at least as much in state L as it will decrease it in state H. Furthermore, the deviation will not make voters believe that state L is less likely. And since they already (i.e., in equilibrium) believe that state L is more likely than state H, it is easy to show that the deviation is profitable. In a similar way it follows that we cannot have an equilibrium with $\hat{x}^{\mathcal{L}}(h) > x_m^*(H)$. Thus, the strategies must satisfy the following conditions (the conditions on $\hat{x}^{\mathcal{H}}(\omega^V)$ follow by symmetry):

$$\hat{x}^{\mathcal{L}}(l) \leq x_m^*(L),\ \hat{x}^{\mathcal{L}}(h) \leq x_m^*(H),\ x_m^*(L) \leq \hat{x}^{\mathcal{H}}(l),\ \text{and}\ x_m^*(H) \leq \hat{x}^{\mathcal{H}}(h). \tag{2}$$

[5] To our knowledge, the local version was first considered by Yehezkel (2006). Experiments suggest it is more reasonable than the original condition (see Müller, Spiegel, and Yehezkel 2009).

In the proposition below we find all equilibria where all candidate positions are in the closed interval between the median voter's preferred positions in the two states. Later we will explain why we restrict attention to this type of equilibria.

Proposition 3 (Non-Revealing Equilibria)
$\hat{x}^{\mathcal{L}}(\omega^V), \hat{x}^{\mathcal{H}}(\omega^V) \in [x_m^*(L), x_m^*(H)]^2$ *are the strategies of a non-revealing equilibrium if and only if*

$$\hat{x}^{\mathcal{L}}(l) = x_m^*(L),\ \hat{x}^{\mathcal{H}}(h) = x_m^*(H),$$

and

$$\theta \leq \frac{1}{2}(1 + \frac{\gamma}{\gamma + \max\{\hat{x}^{\mathcal{H}}(l) - x_m^*(L), x_m^*(H) - \hat{x}^{\mathcal{L}}(h)\}}).$$

It immediately follows that there always exists a non-revealing equilibrium with

$$\hat{x}^{\mathcal{L}}(l) = \hat{x}^{\mathcal{H}}(l) = x_m^*(L) \quad \text{and} \quad \hat{x}^{\mathcal{L}}(h) = \hat{x}^{\mathcal{H}}(h) = x_m^*(H).$$

That is, candidates converge on the median voter's preferred policy in the state that is most likely given the voters' signal. We refer to this type of candidate behavior as *pandering.*

If θ is not too high, the candidate who is most likely to be the low quality candidate given the voters' signal (Candidate $\mathcal{H}$ when $\omega^V = l$, Candidate $\mathcal{L}$ when $\omega^V = h$) need not be close to the median voter's position in the most likely state. For example, the strategies given by

$$\hat{x}^{\mathcal{L}}(l) = \hat{x}^{\mathcal{L}}(h) = x_m^*(L) \quad \text{and} \quad \hat{x}^{\mathcal{H}}(l) = \hat{x}^{\mathcal{H}}(h) = x_m^*(H)$$

are possible in equilibrium if

$$\theta \leq \frac{1}{2}(1 + \frac{\gamma}{\gamma + D}).$$

Thus, if the accuracy of the voters' signal is below this cut-off value, it is possible that each candidate plays to his own strengths by always announcing the median voter's preferred position in his high quality state. Then candidates will always be polarized by the distance between the two medians (D), so we refer to this kind of candidate behavior as *full polarization.* Note that if θ is higher than the cut-off value for full polarization then we can still have some polarization.

It is straightforward to see that the maximal level of θ for which full polarization is an equilibrium is increasing in γ. So if each candidate becomes better qualified in his high quality state then full polarization is an equilibrium outcome for better informed electorates. Also note that if D is increased, full polarization is only possible for lower levels of θ. Remember, however, that a higher D means that candidates are further apart in the full polarization outcome.

Finally, the inequality from the proposition can be rewritten as

$$P \leq \frac{2(1-\theta)}{2\theta - 1}\gamma,$$

where $P = \max\{\hat{x}^{\mathcal{H}}(l) - x_m^*(L), x_m^*(H) - \hat{x}^{\mathcal{L}}(h)\}$. Thus it follows that the maximal level of polarization (divergence) in equilibrium is increasing in γ and decreasing in θ (as long as it is below D).

A brief explanation of the insights from Proposition 3 goes as follows. Suppose $\omega^V = l$ and consider a policy position $x > x_m^*(L)$ for Candidate $\mathcal{H}$. This position is part of a non-revealing equilibrium if we can define the belief function such that Candidate $\mathcal{H}$ cannot profitably deviate to another position. It turns out that the only situation we really need to consider is for Candidate $\mathcal{H}$ to deviate to $x_m^*(L)$ when such a deviation will make voters believe with certainty that the state is L (see the proof for details). Thus, loosely speaking, Candidate $\mathcal{H}$ must choose between a moderate disadvantage with respect to both policy and quality (if he stays at x) and a larger disadvantage with respect to quality only (if he deviates to $x_m^*(L)$). Note that the sizes of these disadvantages only matter because voting is probabilistic, so this assumption is fundamental for our results. Since the moderate policy and quality disadvantage is increasing in θ it follows that x is only part of an equilibrium if θ is below some cut-off value. Furthermore, since the moderate policy disadvantage is increasing in the distance between x and $x_m^*(L)$ it follows that the cut-off value of θ is decreasing in the level of polarization. Finally, when γ increases the larger quality disadvantage increases more than the moderate disadvantage with respect to both policy and quality. Therefore, the deviation to $x_m^*(L)$ is relatively less attractive the higher γ is. This implies that we can have more polarization in equilibrium when candidates differ more in quality.

While Proposition 3 classifies all non-revealing equilibria where candidates' positions are always between the median voter's positions in the two states, other non-revealing equilibria do exist. By (2) it follows that, in equilibrium, Candidate $\mathcal{L}$ will never take a position to the right of $x_m^*(H)$ and Candidate $\mathcal{H}$ will never take a position to the left of $x_m^*(L)$. But Candidate $\mathcal{L}$ can take a position to the left of $x_m^*(L)$ and, similarly, Candidate $\mathcal{H}$ can take a position to the right of $x_m^*(H)$. For example,

$$\hat{x}^{\mathcal{L}}(l) = \hat{x}^{\mathcal{L}}(h) = x_m^*(L) - \varepsilon \quad \text{and} \quad \hat{x}^{\mathcal{H}}(l) = \hat{x}^{\mathcal{H}}(h) = x_m^*(H) + \varepsilon$$

are equilibrium strategies if, loosely speaking, ε is small and θ is sufficiently close to $\frac{1}{2}$ (the precise details can be found in the supplemental material). So D is not the highest possible level of polarization in equilibrium, for some parameter values the distance between the candidates can be higher. However, as will be discussed below, we find the equilibria with positions outside $[x_m^*(L), x_m^*(H)]$ less reasonable than the ones classified in Proposition 3.

Consider a non-revealing equilibrium where, for some realized voter signal, Candidate $\mathcal{L}$ takes a position to the left of $x_m^*(L)$. Then a deviation by Candidate $\mathcal{L}$ to (or just towards) $x_m^*(L)$ will make voters believe that state L is less likely to be the true state, otherwise the deviation would be profitable. This is of course consistent with the monotonicity condition on voters' beliefs. However, given that there is no directly state-dependent cost of taking a particular position for either candidate because they are purely office-motivated, we find it unrealistic that voters would believe less in the high quality state of Candidate $\mathcal{L}$ if he moves to a position that is better for a majority of voters no matter what the true state is. Voters have no compelling reason to believe less in state L after a deviation to $x_m^*(L)$ and the candidate can point this out explicitly. Clearly, this argument is based on a subjective judgment about realistic voter beliefs rather than a rigorous game theoretic method of equilibrium selection. Still, our view is that in equilibria with positions outside $[x_m^*(L), x_m^*(H)]$, part of the polarization is purely driven by voter beliefs that somewhat artificially force candidates not to move towards positions that are preferred by a majority of voters in both states. Therefore, we prefer that the conclusions about polarization that we draw from the model does not depend on such equilibria. And this is why we focus on the equilibria classified in Proposition 3.

6 Welfare and Polarization

Now we will compare the equilibria of the model with respect to the welfare of voters. A natural way to measure the welfare of a voter in an equilibrium is to use the *ex ante expected utility* he obtains, i.e., his expected utility before the uncertainty about ω, ω^V, and δ is resolved. As all (fully or partially) revealing equilibria are eliminated by imposing the local Bagwell-Ramey condition we restrict attention to non-revealing equilibria. Furthermore, we only consider equilibria where all positions are in $[x_m^*(L), x_m^*(H)]$ and equilibria that are symmetric in the sense that the distance between the candidates is the same for both voter signals. Thus it follows from Proposition 3 that an equilibrium can be uniquely represented by the level of polarization

$$P = \hat{x}^{\mathcal{H}}(l) - x_m^*(L) = x_m^*(H) - \hat{x}^{\mathcal{L}}(h).$$

Note that $P = 0$ represents pandering and $P = D$ represents full polarization.

We compare equilibria for fixed parameter values. The results below show that there is no general welfare effect of polarization for all voters. However, we do have that full polarization is worse than pandering for all voters.

Proposition 4 (Welfare and Polarization)
Fix a set of parameter values. Then the following statements hold:

1. *The median voter and all voters i with $|x_i^*(L) - x_m^*(L)| \geq D$ are strictly better off in an equilibrium with less polarization than in an equilibrium with more polarization.*

2. *There exists an equilibrium with $P \in (0, \frac{D}{2})$ such that all voters i with $|x_i^*(L) - x_m^*(L)| \in (P, D - P)$ are strictly better off in this equilibrium than in the pandering equilibrium ($P = 0$).*

3. *Suppose full polarization ($P = D$) is an equilibrium (see Proposition 3). Then all voters are strictly better off in the pandering equilibrium than in the full polarization equilibrium.*

To explain why we get the results above, consider the pandering equilibrium and an equilibrium with a positive level of polarization $P < \frac{D}{2}$. We want to compare each voter's welfare in these two equilibria. We split the expected utility into two parts, policy utility (coming from the voter's policy utility function) and quality utility (coming from the γ and δ terms). It is easy to see that the median voter's ex ante expected policy utility is higher in the pandering equilibrium than in the polarization equilibrium. Then consider a voter with $|x_i^*(L)-x_m^*(L)| \in (P, D-P)$. For such a voter it is the other way around, polarization is better than pandering with respect to policy utility. For some (ω, ω^V)-combinations it is a good thing that one candidate is not at the most likely median given ω^V because that candidate is closer to the voter's ideal policy. For other (ω, ω^V)-combinations it is not a good thing, but when we sum everything up the total effect of polarization is positive. Finally, voters with $|x_i^*(L) - x_m^*(L)| \geq D$ are, with respect to policy, indifferent between the two equilibria.

With respect to quality, all voters have the same preferences. When the candidates converge voters make the optimal choice with respect to quality (given their information). If the candidates do not converge then voters need to take the difference in policy positions into consideration and thus the electorate does not always make the optimal choice with respect to quality. Thus, for the median voter and all voters with $|x_i^*(L) - x_m^*(L)| \geq D$ pandering is, in total, better than the polarization equilibrium. For voters with $|x_i^*(L) - x_m^*(L)| \in (P, D - P)$ the conclusion is less straightforward, pandering is worse with respect to policy but better with respect to quality. However, the marginal effect of polarization on quality utility is equal to zero at $P = 0$ while the effect on policy utility is positive. So for P sufficiently small the positive effect on policy utility dominates, which means that a little polarization is better than pandering.

As a final remark, it follows from Proposition 4 that there does not exist a unique welfare maximizing equilibrium independent of the distribution of voters' preferred policies. More precisely, let total welfare be defined as the sum of all voters' ex ante expected utilities. If all voters have identical preferences (i.e.,

$x_i^*(L) = x_m^*(L)$ for all i), then the pandering equilibrium is obviously the optimal equilibrium. This is also the case if all voters i have either $x_i^*(L) = x_m^*(L)$ or $|x_i^*(L) - x_m^*(L)| \geq D$. On the other hand, if there are sufficiently many voters with $0 < |x_i^*(L) - x_m^*(L)| < D$, then some level of polarization $P > 0$ will result in higher welfare than $P = 0$. So the pandering equilibrium is not welfare maximizing for all electorates.

7 Discussion

We have analyzed a model of electoral competition where voters, but not candidates, are uncertain about the state of the world and candidates differ in state-dependent quality. The main results were that candidates' policy positions can diverge more in equilibrium when they differ more in state-dependent quality and when voters are less well informed about the state. An interesting observation on welfare was that while increased polarization (divergence) is always bad for some voters, it can in fact be good for others.

It is worth emphasizing the underlying reason why candidates may diverge in equilibrium. Especially since the divergence has nothing to do with policy preferences because the candidates are completely office-motivated. It is the combination of voter uncertainty and differences in candidate quality (and probabilistic voting) that leads to the possibility of divergence. The candidate who is most likely to be of low quality given the voters' signal does not want to move closer to his opponent because then voters will believe with certainty that he is the low quality candidate. If candidates did not differ in quality then they would have no incentive not to converge on the median voter's preferred policy in the true state. If voters were fully informed then they would know the identity of the high quality candidate and thus, again, the candidates would converge on the true median. Thus, neither differences in state-dependent candidate quality between the candidates nor voter uncertainty about the true state could, in isolation, lead to policy divergence.

Appendix

Proof of Lemma 1.
Let $x^{\mathcal{L}}$ and $x^{\mathcal{H}}$ be the positions of the candidates and let μ_L be the belief of the voters given these positions and the voters' signal. Suppose the median voter strictly prefers Candidate $\mathcal{L}$ (the other case is analogous), i.e.,

$$\delta > \mu_L(|x^{\mathcal{L}}-x_m^*(L)|-|x^{\mathcal{H}}-x_m^*(L)|-\gamma)+(1-\mu_L)(|x^{\mathcal{L}}-x_m^*(H)|-|x^{\mathcal{H}}-x_m^*(H)|+\gamma).$$

We then have to show that, for each voter i in a strict majority,

$$\delta > \mu_L(|x^{\mathcal{L}}-x_i^*(L)|-|x^{\mathcal{H}}-x_i^*(L)|-\gamma)+(1-\mu_L)(|x^{\mathcal{L}}-x_i^*(H)|-|x^{\mathcal{H}}-x_i^*(H)|+\gamma).$$

Suppose $x^{\mathcal{L}} \leq x^{\mathcal{H}}$ (the other case is analogous). It suffices to show that the inequality above holds for all voters i with $x_i^*(L) \leq x_m^*(L)$. This is the case if

$$\begin{aligned} |x^{\mathcal{L}}-x_i^*(L)|-|x^{\mathcal{H}}-x_i^*(L)| &\leq |x^{\mathcal{L}}-x_m^*(L)|-|x^{\mathcal{H}}-x_m^*(L)| \quad \text{and} \\ |x^{\mathcal{L}}-x_i^*(H)|-|x^{\mathcal{H}}-x_i^*(H)| &\leq |x^{\mathcal{L}}-x_m^*(H)|-|x^{\mathcal{H}}-x_m^*(H)| \end{aligned}$$

for all such voters, which is straightforward to verify. □

Proof of Proposition 1.
Let $\hat{x}^{\mathcal{L}}(\omega)$ and $\hat{x}^{\mathcal{H}}(\omega)$ be the strategies from the proposition (note that positions do not depend on ω^V). Obviously, if they are equilibrium strategies then the equilibrium is fully revealing and each candidate always wins with probability one half. Consider a monotone belief function $\hat{\mu}_L(\hat{x}^{\mathcal{L}}, \hat{x}^{\mathcal{H}}, \omega^V)$ satisfying the following conditions (it is easy to see that they do not violate monotonicity):

$$\begin{aligned}
\hat{\mu}_L(x, \hat{x}^{\mathcal{H}}(L), \omega^V) &= 1 \text{ for all } x \leq \hat{x}^{\mathcal{L}}(L), \omega^V = l, h; \\
\hat{\mu}_L(x, \hat{x}^{\mathcal{H}}(L), \omega^V) &= \frac{1}{2} \text{ for all } x > \hat{x}^{\mathcal{L}}(L), \omega^V = l, h; \\
\hat{\mu}_L(x, \hat{x}^{\mathcal{H}}(H), \omega^V) &= \frac{1}{2} \text{ for all } x \leq \hat{x}^{\mathcal{L}}(L), \omega^V = l, h; \\
\hat{\mu}_L(x, \hat{x}^{\mathcal{H}}(H), \omega^V) &= 0 \text{ for all } x > \hat{x}^{\mathcal{L}}(L), \omega^V = l, h; \\
\hat{\mu}_L(\hat{x}^{\mathcal{L}}(H), x, \omega^V) &= \frac{1}{2} \text{ for all } x < \hat{x}^{\mathcal{H}}(H), \omega^V = l, h; \\
\hat{\mu}_L(\hat{x}^{\mathcal{L}}(H), x, \omega^V) &= 0 \text{ for all } x \geq \hat{x}^{\mathcal{H}}(H), \omega^V = l, h; \\
\hat{\mu}_L(\hat{x}^{\mathcal{L}}(L), x, \omega^V) &= 1 \text{ for all } x < \hat{x}^{\mathcal{H}}(H), \omega^V = l, h; \\
\hat{\mu}_L(\hat{x}^{\mathcal{L}}(L), x, \omega^V) &= \frac{1}{2} \text{ for all } x \geq \hat{x}^{\mathcal{H}}(H), \omega^V = l, h.
\end{aligned}$$

It is straightforward to check that the following statements hold:

- In each state, a deviation by the low quality candidate to a position different from his equilibrium position in the other state will give him a probability of winning that is strictly smaller than one half.

- In each state, a deviation by the low quality candidate to his equilibrium position in the other state will give him a probability of winning equal to one half.

- In each state, a deviation by the high quality candidate to a position $x \notin [x^*_m(L), x^*_m(H)]$ will give him a probability of winning that is strictly smaller than one half.

- In each state, a deviation by the high quality candidate to a position $x \in [x^*_m(L), x^*_m(H)]$ will give him a probability of winning that is equal to one half.

From these statements it immediately follows that, in each state, neither candidate has a profitable deviation. Thus we have an equilibrium. □

Proof of Proposition 3.
First assume that $\hat{x}^{\mathcal{L}}(l) = x^*_m(L)$, $\hat{x}^{\mathcal{H}}(h) = x^*_m(H)$, and that the inequality holds. Consider a monotone belief function $\hat{\mu}_L(x^{\mathcal{L}}, x^{\mathcal{H}}, \omega^V)$ satisfying the following conditions (these conditions do not violate the monotonicity condition):

$$\begin{aligned}
\hat{\mu}_L(x^*_m(L), x, l) &= 1 \text{ for all } x < \hat{x}^{\mathcal{H}}(l); \\
\hat{\mu}_L(x^*_m(L), x, l) &= \theta \text{ for all } x \geq \hat{x}^{\mathcal{H}}(l); \\
\hat{\mu}_L(x, \hat{x}^{\mathcal{H}}(l), l) &= \theta \text{ for all } x \in X; \\
\hat{\mu}_L(x, x^*_m(H), h) &= 1-\theta \text{ for all } x \leq \hat{x}^{\mathcal{L}}(h); \\
\hat{\mu}_L(x, x^*_m(H), h) &= 0 \text{ for all } x > \hat{x}^{\mathcal{L}}(h); \\
\hat{\mu}_L(\hat{x}^{\mathcal{L}}(h), x, h) &= 1-\theta \text{ for all } x \in X.
\end{aligned}$$

We claim that $\hat{x}^{\mathcal{L}}(\omega^V)$, $\hat{x}^{\mathcal{H}}(\omega^V)$, $\hat{\mu}_L(x^{\mathcal{L}}, x^{\mathcal{H}}, \omega^V)$ is an equilibrium. It is easy to see that if Candidate $\mathcal{H}$ cannot profitably deviate to $x^*_m(L)$ when $\omega^V = l$ and Candidate $\mathcal{L}$ cannot profitably deviate to $x^*_m(H)$ when $\omega^V = h$ (assume for now that $\hat{x}^{\mathcal{H}}(l) \neq x^*_m(L)$ and $\hat{x}^{\mathcal{L}}(h) \neq x^*_m(H)$) then neither candidate has a profitable deviation. When $\omega^V = l$, Candidate $\mathcal{H}$'s probability of winning is

$$\begin{aligned}
& \Pr[\delta < \theta(-|\hat{x}^{\mathcal{H}}(l) - x^*_m(L)| - \gamma) + (1-\theta)(D - |\hat{x}^{\mathcal{H}}(l) - x^*_m(H)| + \gamma)] \\
= \ & \Pr[\delta < (1-2\theta)(\hat{x}^{\mathcal{H}}(l) - x^*_m(L) + \gamma)] = \frac{1}{2} + (1-2\theta)(\hat{x}^{\mathcal{H}}(l) - x^*_m(L) + \gamma)\sigma.
\end{aligned}$$

If he deviates to $x_m^*(L)$ then his probability of winning is

$$\Pr[\delta < -\gamma] = (-\gamma + \frac{1}{2\sigma})\sigma = \frac{1}{2} - \gamma\sigma.$$

So the deviation is not profitable precisely if

$$(1-2\theta)(\hat{x}^{\mathcal{H}}(l) - x_m^*(L) + \gamma) \geq -\gamma,$$

which is equivalent to

$$\theta \leq \frac{1}{2}(1 + \frac{\gamma}{\gamma + \hat{x}^{\mathcal{H}}(l) - x_m^*(L)}).$$

Analogously we get that Candidate $\mathcal{L}$ cannot profitably deviate to $x_m^*(H)$ when $\omega^V = h$ precisely if

$$\theta \leq \frac{1}{2}(1 + \frac{\gamma}{\gamma + x_m^*(H) - \hat{x}^{\mathcal{L}}(h)}).$$

When the inequality in the proposition holds, the two inequalities above hold. Thus $\hat{x}^{\mathcal{L}}(\omega^V)$, $\hat{x}^{\mathcal{H}}(\omega^V)$, $\hat{\mu}_L(x^{\mathcal{L}}, x^{\mathcal{H}}, \omega^V)$ is an equilibrium.

Then assume $\hat{x}^{\mathcal{L}}(\omega^V)$, $\hat{x}^{\mathcal{H}}(\omega^V)$, $\hat{\mu}_L(x^{\mathcal{L}}, x^{\mathcal{H}}, \omega^V)$ is an equilibrium. From the observations above Proposition 3 we get $\hat{x}^{\mathcal{L}}(l) = x_m^*(L)$ and $\hat{x}^{\mathcal{H}}(h) = x_m^*(H)$. Let

$$\hat{\mu}_L^l = \hat{\mu}_L(x_m^*(L), x_m^*(L), l) \quad \text{and} \quad \hat{\mu}_L^h = \hat{\mu}_L(x_m^*(H), x_m^*(H), h)$$

(as above, assume that $\hat{x}^{\mathcal{H}}(l) \neq x_m^*(L)$ and $\hat{x}^{\mathcal{H}}(h) \neq x_m^*(H)$). By the conditions that Candidate $\mathcal{H}$ cannot profitably deviate to $x_m^*(L)$ when $\omega^V = l$ and that Candidate $\mathcal{L}$ cannot profitably deviate to $x_m^*(H)$ when $\omega^V = h$ we get the following inequalities (the calculations are similar to those in the first part of the proof):

$$\theta \leq \frac{1}{2}(1 + \frac{(2\hat{\mu}_L^l - 1)\gamma}{\gamma + \hat{x}^{\mathcal{H}}(l) - x_m^*(L)}) \quad \text{and} \quad \theta \leq \frac{1}{2}(1 + \frac{(1 - 2\hat{\mu}_L^h)\gamma}{\gamma + x_m^*(H) - \hat{x}^{\mathcal{L}}(h)}).$$

From these inequalities it easily follows that the inequality in the proposition holds.

The arguments above are easily modified to cover the cases where $\hat{x}^{\mathcal{H}}(l) = x_m^*(L)$ or $\hat{x}^{\mathcal{H}}(h) = x_m^*(H)$ (or both). □

Proof of Proposition 4.
1. Fix a set of parameter values and consider an equilibrium with polarization $P \geq 0$. Let δ_P^* denote the δ-value for which the median voter switches his vote when $\omega^V = l$. I.e., the median voter is indifferent between the candidates when $\omega^V = l$ and $\delta = \delta_P^*$:

$$\theta(\gamma + \delta_P^*) + (1-\theta)(-D + \delta_P^*) = \theta(-P) + (1-\theta)(-D + P + \gamma).$$

From this equation we get

$$\delta_P^* = (1 - 2\theta)(\gamma + P).$$

The probability that Candidate $\mathcal{L}$ is elected when $\omega^V = l$ is

$$\Pr(\delta > \delta_P^*) = (\frac{1}{2\sigma} - \delta_P^*)\sigma = \frac{1}{2} + (2\theta - 1)(\gamma + P)\sigma.$$

The δ-value for which the median voter switches his vote if $\omega^V = h$ is $-\delta_P^*$. So the probability that Candidate $\mathcal{L}$ is elected when $\omega^V = h$ is

$$\Pr(\delta > -\delta_P^*) = (\frac{1}{2\sigma} + \delta_P^*)\sigma = \frac{1}{2} - (2\theta - 1)(\gamma + P)\sigma.$$

Now consider the ex ante expected *policy* utility (i.e., we disregard the quality terms γ and δ) in the equilibrium for the median voter:

$$\begin{aligned}
&\Pr(L,l)[\Pr(\delta > \delta_P^*) \cdot 0 + \Pr(\delta < \delta_P^*) \cdot (-P)] \\
&+ \Pr(H,h)[\Pr(\delta < -\delta_P^*) \cdot 0 + \Pr(\delta > -\delta_P^*) \cdot (-P)] \\
&+ \Pr(H,l)[\Pr(\delta > \delta_P^*) \cdot (-D) + \Pr(\delta < \delta_P^*) \cdot (-D + P)] \\
&+ \Pr(L,h)[\Pr(\delta < -\delta_P^*) \cdot (-D) + \Pr(\delta > -\delta_P^*) \cdot (-D + P)].
\end{aligned}$$

Note that, for example, $\Pr(L,l)$ denotes the ex ante probability of $(\omega, \omega^V) = (L, l)$, which is equal to $\frac{1}{2}\theta$. By straightforward calculations it follows that the expression above is equal to

$$-(1 - \theta)D - (2\theta - 1)(\frac{1}{2} - (2\theta - 1)(\gamma + P)\sigma)P.$$

We claim that this expression is decreasing in P. The derivative with respect to P is

$$-(2\theta - 1)(\frac{1}{2} - (2\theta - 1)(\gamma + 2P)\sigma).$$

So the derivative is negative if $(2\theta - 1)(\gamma + 2P)\sigma < \frac{1}{2}$. Thus, it suffices to show that this inequality is satisfied given that the level of polarization P is indeed possible in equilibrium, i.e., given that $\theta \le \frac{1}{2} + \frac{\gamma}{2(\gamma+P)}$ (see Proposition 3). This is straightforward to verify (use the assumption $\gamma + D < \frac{1}{2\sigma}$). So we have that the ex ante expected policy utility of the median voter is (strictly) decreasing with the level of polarization.

For a voter with $d_i = |x_i^*(L) - x_m^*(L)| \ge D$ it follows from straightforward calculations that the ex ante expected policy utility in equilibrium is simply equal to $-d_i$. Thus it is independent of P.

By what we have shown above, it suffices to show that voters' ex ante expected *quality* utility (given by the γ and δ terms) in equilibrium is strictly decreasing in P. Note that all voters have the same preferences with respect to quality, so we do not need to distinguish between voters. The ex ante expected quality utility in equilibrium can be written

$$\begin{aligned}
& \Pr(L,l)[\Pr(\delta > \delta_P^*) \cdot (\gamma + E(\delta|\delta > \delta_P^*)) + \Pr(\delta < \delta_P^*) \cdot 0] \\
+ \quad & \Pr(H,h)[\Pr(\delta < -\delta_P^*) \cdot \gamma + \Pr(\delta > -\delta_P^*) \cdot E(\delta|\delta > -\delta_P^*)] \\
+ \quad & \Pr(H,l)[\Pr(\delta > \delta_P^*) \cdot E(\delta|\delta > \delta_P^*) + \Pr(\delta < \delta_P^*) \cdot \gamma] \\
+ \quad & \Pr(L,h)[\Pr(\delta < -\delta_P^*) \cdot 0 + \Pr(\delta > -\delta_P^*) \cdot (\gamma + E(\delta|\delta > -\delta_P^*))].
\end{aligned}$$

Note that

$$E(\delta|\delta > \pm\delta_P^*) = \frac{1}{2}(\frac{1}{2\sigma} \pm \delta_P^*).$$

Straightforward calculations reveal that the ex ante expected quality utility in equilibrium reduces to

$$(\theta + (1 - 2\theta) \Pr(\delta < \delta_P^*))\gamma + \frac{1}{4}(\frac{1}{2\sigma} + \delta_P^*) - \frac{1}{2} \Pr(\delta < \delta_P^*)\delta_P^*.$$

Furthermore, by differentiation and some further calculations we get that the derivative with respect to P is

$$-(2\theta - 1)^2 P\sigma.$$

Thus, voters' ex ante expected quality utility in equilibrium is strictly decreasing in P.

2. First, consider the ex ante expected policy utility in equilibrium of a voter with $d_i = |x_i^*(L) - x_m^*(L)| \in (0, D)$ when the level of polarization is $P < d_i, D - d_i$. By symmetry we can restrict attention to the case $x_i^*(L) > x_m^*(L)$ such that $d_i = x_i^*(L) - x_m^*(L)$. The ex ante expected policy utility is

$$\begin{aligned}
& \Pr(L,l)[\Pr(\delta > \delta_P^*) \cdot (-d_i) + \Pr(\delta < \delta_P^*) \cdot (-d_i + P)] \\
+ \quad & \Pr(H,h)[\Pr(\delta < -\delta_P^*) \cdot (-d_i) + \Pr(\delta > -\delta_P^*) \cdot (-d_i - P)] \\
+ \quad & \Pr(H,l)[\Pr(\delta > \delta_P^*) \cdot (-D - d_i) + \Pr(\delta < \delta_P^*) \cdot (-D - d_i + P)] \\
+ \quad & \Pr(L,h)[\Pr(\delta < -\delta_P^*) \cdot (-D + d_i) + \Pr(\delta > -\delta_P^*) \cdot (-D + d_i + P)].
\end{aligned}$$

Since we are interested in the effect of P, we only need to consider the terms that depend on P. This part of the ex ante expected policy utility is

$$\begin{aligned}
& \frac{1}{2}\theta \Pr(\delta < \delta_P^*)P - \frac{1}{2}\theta \Pr(\delta > -\delta_P^*)P \\
& \quad + \frac{1}{2}(1 - \theta) \Pr(\delta < \delta_P^*)P + \frac{1}{2}(1 - \theta) \Pr(\delta > -\delta_P^*)P = (1 - \theta) \Pr(\delta < \delta_P^*)P.
\end{aligned}$$

So the derivative w.r.t. P of the ex ante expected policy utility is

$$(1-\theta)\Pr(\delta < \delta_P^*) + (1-\theta)P\frac{\partial}{\partial P}\Pr(\delta < \delta_P^*) = (1-\theta)(\frac{1}{2} - (2\theta-1)(\gamma+2P)\sigma).$$

Note that it is independent of d_i and strictly positive for all $P < d_i, D - d_i$ (the latter inequality implies $P < \frac{D}{2}$ and then strict positivity follows from the assumption that $\gamma + D < \frac{1}{2\sigma}$).

Then consider ex ante expected quality utility for any voter i in equilibrium. We already have from part one of the proof that the derivative with respect to P is

$$-(2\theta-1)^2 P\sigma.$$

Note that the derivative at $P = 0$ is equal to zero. Thus, for all voters i with $d_i \in (0, D)$, the derivative at $P = 0$ of the ex ante expected *total* utility is strictly positive at $P = 0$. So there exist $P > 0$ such that all voters i with $d_i \in (P, D - P)$ are better off in an equilibrium with polarization P than in the pandering equilibrium.

3. We know from part one that the median voter and voters with $d_i = |x_i^*(L) - x_m^*(L)| \geq D$ are at strictly better off in the pandering equilibrium than in the full polarization equilibrium. We also know that, for all voters, the ex ante expected quality utility is strictly higher in the pandering equilibrium than in the full polarization equilibrium. Thus it suffices to show that the ex ante expected policy utility in equilibrium is higher when $P = 0$ than when $P = D$ for voters with $d_i = |x_i^*(L) - x_m^*(L)| \in (0, D)$. From straightforward calculations it follows that the ex ante expected policy utility of the pandering equilibrium for such a voter is

$$-(1-\theta)D - d_i\theta = -D + \theta(D - d_i).$$

Similarly, the ex ante expected utility of the full polarization equilibrium is

$$\begin{aligned} & D((2\theta-1)^2(D+\gamma)\sigma - \frac{1}{2}) - d_i((2\theta-1)^2(D+\gamma)\sigma + \frac{1}{2}) \\ = \; & -D + \frac{1}{2}(D - d_i) + (2\theta-1)^2(D+\gamma)\sigma(D - d_i) \\ \leq \; & -D + \frac{1}{2}(D - d_i) + \frac{1}{2}(2\theta-1)^2(D - d_i) = -D + (2\theta^2 - 2\theta + 1)(D - d_i). \end{aligned}$$

Finally, the desired conclusion follows from the observation that $2\theta^2 - 2\theta + 1 < \theta$ for all $\theta \in (\frac{1}{2}, 1)$. $\square$

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Supplemental Material

The local Bagwell-Ramey condition and proof of Proposition 2.

Definition 2 (The Local Bagwell-Ramey Condition)
An equilibrium $\hat{x}^{\mathcal{L}}(\omega,\omega^V)$, $\hat{x}^{\mathcal{H}}(\omega,\omega^V)$, $\hat{\mu}_L(x^{\mathcal{L}},x^{\mathcal{H}},\omega^V)$ satisfies the local Bagwell-Ramey condition if and only if the following conditions hold:

1. *Suppose $\hat{x}^{\mathcal{L}}(L,\omega^V) \neq \hat{x}^{\mathcal{L}}(H,\omega^V)$ for some voter signal ω^V. Then there exists an $\varepsilon > 0$ such that*

$$\hat{\mu}_L(\hat{x}^{\mathcal{L}}(L,\omega^V),x,\omega^V) = 1 \textit{ if } x \neq \hat{x}^{\mathcal{H}}(H,\omega^V) \textit{ and } |\hat{x}^{\mathcal{H}}(L,\omega^V) - x| < \varepsilon$$

and

$$\hat{\mu}_L(\hat{x}^{\mathcal{L}}(H,\omega^V),x,\omega^V) = 0 \textit{ if } x \neq \hat{x}^{\mathcal{H}}(L,\omega^V) \textit{ and } |\hat{x}^{\mathcal{H}}(H,\omega^V) - x| < \varepsilon.$$

2. *Suppose $\hat{x}^{\mathcal{H}}(L,\omega^V) \neq \hat{x}^{\mathcal{H}}(H,\omega^V)$ for some voter signal ω^V. Then there exists an $\varepsilon > 0$ such that*

$$\hat{\mu}_L(x,\hat{x}^{\mathcal{H}}(L,\omega^V),\omega^V) = 1 \textit{ if } x \neq \hat{x}^{\mathcal{L}}(H,\omega^V) \textit{ and } |\hat{x}^{\mathcal{L}}(L,\omega^V) - x| < \varepsilon$$

and

$$\hat{\mu}_L(x,\hat{x}^{\mathcal{H}}(H,\omega^V),\omega^V) = 0 \textit{ if } x \neq \hat{x}^{\mathcal{L}}(L,\omega^V) \textit{ and } |\hat{x}^{\mathcal{L}}(H,\omega^V) - x| < \varepsilon.$$

In our view, the local Bagwell-Ramey condition is substantially more reasonable than the original condition. For example, consider again the equilibrium strategies from Proposition 1 and suppose we are in state L. Then, with the original version of the condition, a deviation by Candidate $\mathcal{L}$ to a position very close to $x_m^*(H)$ (his equilibrium position in state H) would not make voters change their belief that the state is L with certainty. This seems like a harsh assumption. It is much easier to justify that if Candidate $\mathcal{L}$ deviates slightly then voters ignore this "small tremble" by one candidate. It is also worth noting that the local condition is, contrary to the original version, consistent with recent experimental evidence on multi-sender signaling games (Müller, Spiegel, and Yehezkel 2009).

We will now prove Proposition 2. We first prove the following lemma.

Lemma 2 *Consider an equilibrium $\hat{x}^{\mathcal{L}}(\omega,\omega^V)$, $\hat{x}^{\mathcal{H}}(\omega,\omega^V)$, $\hat{\mu}_L(x^{\mathcal{L}},x^{\mathcal{H}},\omega^V)$ that satisfies the local Bagwell-Ramey condition. If $\hat{x}^i(L,\omega^V) \neq \hat{x}^i(H,\omega^V)$ for some i and some ω^V then we must have*

$$\hat{x}^{\mathcal{L}}(L,\omega^V) = \hat{x}^{\mathcal{H}}(L,\omega^V) = x_m^*(L) \textit{ and } \hat{x}^{\mathcal{L}}(H,\omega^V) = \hat{x}^{\mathcal{H}}(H,\omega^V) = x_m^*(H).$$

Proof. Suppose $\hat{x}^i(L, \omega^V) \neq \hat{x}^i(H, \omega^V)$ for some voter signal and consider the positions of Candidate j for the same voter signal. If, in some state, Candidate j's position is not the true median then he can profitably deviate by moving slightly towards this position. Because, by the local Bagwell-Ramey condition, voters will continue to infer the true state with certainty after a sufficiently small deviation and therefore a move towards the true median will increase Candidate j's probability of winning (because of the assumption that $\gamma + D < \frac{1}{2\sigma}$, each candidate always has a strictly positive probability of winning the election in equilibrium). Thus we must have

$$\hat{x}^j(L, \omega^V) = x_m^*(L) \text{ and } \hat{x}^j(H, \omega^V) = x_m^*(H).$$

Finally, interchange the roles of i and j and repeat the argument above to get the same conclusion for Candidate i. □

Using this lemma, it is easy to see that the local Bagwell-Ramey condition makes revelation impossible in equilibrium. Suppose, for example, that we have an equilibrium where the state is revealed for $\omega^V = l$. Then both candidates announce $x_m^*(L)$ in state L and $x_m^*(H)$ in state H. Two necessary conditions for equilibrium are that Candidate $\mathcal{H}$ cannot profitably deviate to $x_m^*(H)$ in state L and that Candidate $\mathcal{L}$ cannot profitably deviate to $x_m^*(L)$ in state H. It is straightforward to see that the first condition implies $\hat{\mu}_L(x_m^*(L), x_m^*(H), l) > \frac{1}{2}$ and that the second condition implies $\hat{\mu}_L(x_m^*(L), x_m^*(H), l) < \frac{1}{2}$, which is a contradiction. Thus we have proved Proposition 2.

A non-revealing equilibrium with positions outside $[x_m^*(L), x_m^*(H)]$.

CLAIM: Let $\varepsilon > 0$. There exists a non-revealing equilibrium with strategies

$$\hat{x}^{\mathcal{L}}(l) = \hat{x}^{\mathcal{L}}(h) = x_m^*(L) - \varepsilon \quad \text{and} \quad \hat{x}^{\mathcal{H}}(l) = \hat{x}^{\mathcal{H}}(h) = x_m^*(H) + \varepsilon$$

if and only if $\varepsilon < \gamma$ and

$$\theta \leq \frac{1}{2}\left(1 + \frac{\gamma - \varepsilon}{\gamma + D}\right).$$

Proof of claim: Consider a monotone belief function satisfying the following conditions:

$$\begin{aligned}
\hat{\mu}_L(x_m^*(L) - \varepsilon, x, \omega^V) &= 1 \text{ for all } x < x_m^*(H) + \varepsilon, \omega^V = l, h; \\
\hat{\mu}_L(x_m^*(L) - \varepsilon, x, \omega^V) &= \Pr(L|\omega^V) \text{ for all } x \geq x_m^*(H) + \varepsilon, \omega^V = l, h; \\
\hat{\mu}_L(x, x_m^*(H) + \varepsilon, \omega^V) &= \Pr(L|\omega^V) \text{ for all } x \leq x_m^*(L) - \varepsilon, \omega^V = l, h; \\
\hat{\mu}_L(x, x_m^*(H) + \varepsilon, \omega^V) &= 0 \text{ for all } x > x_m^*(L) - \varepsilon, \omega^V = l, h.
\end{aligned}$$

It is easy to see that if Candidate $\mathcal{H}$ cannot profitably deviate to $x_m^*(L)$ when $\omega^V = l$ then neither candidate has a profitable deviation. When $\omega^V = l$, Candidate $\mathcal{H}$'s probability of winning is

$$\frac{1}{2} + (1 - 2\theta)(D + \gamma)\sigma.$$

If he deviates to $x_m^*(L)$ then his probability of winning is

$$\frac{1}{2} - (\gamma - \varepsilon)\sigma.$$

Thus we have an equilibrium if

$$\frac{1}{2} + (1 - 2\theta)(D + \gamma)\sigma \geq \frac{1}{2} - (\gamma - \varepsilon)\sigma,$$

which is equivalent to

$$\theta \leq \frac{1}{2}(1 + \frac{\gamma - \varepsilon}{\gamma + D}).$$

If $\varepsilon \geq \gamma$ or $\theta > \frac{1}{2}(1 + \frac{\gamma-\varepsilon}{\gamma+D})$ then it is easy to see that it will be profitable for Candidate $\mathcal{H}$ to deviate to $x_m^*(L)$ when $\omega^V = l$ no matter what the voters' beliefs are after such a deviation. $\square$