Loss Aversion and the Asymmetric Transmission of Monetary Policy

Edoardo Gaffeo, Ivan Petrella, Damjan Pfajfar and Emiliano Santoro
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Edoardo Gaffeo  
*University of Trento*

Ivan Petrella  
*Birkbeck, University of London*

Damjan Pfajfar  
*University of Tilburg*

Emiliano Santoro*  
*Catholic University of Milan*  
*University of Copenhagen*

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Abstract

There is widespread evidence that monetary policy exerts asymmetric effects on output over contractions and expansions in economic activity, while price responses display no sizeable asymmetry. To rationalize these facts we develop a dynamic general equilibrium model where households’ utility depends on consumption deviations from a reference level below which loss aversion is displayed. In line with the prospect theory pioneered by Kahneman and Tversky (1979), losses in consumption loom larger than gains. State-dependent degrees of real rigidity and elasticity of intertemporal substitution in consumption generate competing effects on output and inflation. The resulting state-dependent trade-off between output and inflation stabilization recommends stronger policy activism towards inflation during expansions.

*JEL:* E32; E42; E52; D03; D11.

*Keywords:* Asymmetry, Monetary Policy, Business Cycle, Prospect Theory.
1 Introduction

Since the seminal work by Mitchell (1927), considerable effort has been devoted to examine non-linearities in macroeconomic time series. Graham (1930), Keynes (1936) and Friedman and Schwartz (1963) have then stimulated a vast debate on the asymmetric effects of monetary policy. Widespread empirical evidence has been produced in support of the view that monetary policy exerts asymmetric effects on output and prices with respect to the economic conditions as well as the direction and size of the policy action. Such effects have important implications not only for the way we think about the macroeconomy, but also for the conduct of economic policy.

Lo and Piger (2005) account for different forms of asymmetry in the monetary transmission mechanism. According to their empirical analysis, the most important form of non-linearity is represented by the asymmetric transmission of monetary policy over contractions and expansions in the business cycle.\(^1\) In this respect, the econometric evidence available to date has reported two coexisting regularities (see also Weise, 1999 and Peersman and Smets, 2005). On one hand, monetary policy innovations have greater impact on output during negative stages of the cycle. On the other hand, changes in the monetary policy stance do not induce statistically different responses of prices during different cyclical phases. Our objective is to provide a parsimonious explanation of these facts. To that effect, this paper develops a tractable macroeconomic model in which households display reference-dependent preferences of the type popularized by Kahneman and Tversky (1979) as ‘prospect theory’. The modeling strategy consists of assuming that households’ utility partly depends on the deviation of their consumption from a habit-based reference level of consumption below which loss aversion is displayed. In line with the key tenet of prospect theory, losses in consumption utility resonate more than gains.

The behavioral mechanism underlying loss-averse preferences has found wide empirical and experimental support in the literature (Thaler, Tversky, Kahneman, and Schwartz,\(^1\)Contractions (expansions) are intended as periods in which the cycle moves from its peak (trough) to the trough (peak). Zarnowitz (1992) refers to these phases of the business cycle as ‘growth cycles’. McKay and Reis (2008) have recently revived the interest in asymmetric fluctuations over different cyclical stages, considering the concept of growth cycles as opposed to classical cycles.)
1997). Benartzi and Thaler (1995) and Barberis, Huang, and Santos (2001) show that prospect theory may help at explaining the behavior of asset returns and resolving a number of quantitative asset pricing puzzles. Koszegi and Rabin (2006, 2009) assume that households care about gains and losses in consumption, an hypothesis that finds empirical support in Yogo (2008) and Rosenblatt-Wisch (2008). However, none of these approaches takes the analysis to a general equilibrium perspective.\(^2\) The novelty of this paper is to embed prospect theory in a dynamic general equilibrium framework and focus on the transmission of monetary policy to output and inflation. In this respect, two key mechanisms are characterized. First, during contractions changes in the real rate of interest exert stronger impact on output through an increase in the elasticity of intertemporal substitution between current and future consumption. This feature has been extensively examined in the literature on asset pricing (Yogo, 2008). Second, embedding loss-averse preferences in a general equilibrium setting implies a state-dependent marginal rate of substitution between consumption and leisure that can be related to firms’ real marginal cost, so that equilibrium in the labor market holds. The resulting labor supply schedule retains the key property of being flatter below the reference point, so that real wages feature downward stickiness in contractions. Both features of the model are compatible with output being more adversely affected by monetary policy innovations during contractionary phases of the cycle. Concurrently, during negative growth cycles inflation responses are attenuated through an increased degree of real rigidity in the labor market. As a result, no difference can be appreciated between inflation responses over different cyclical phases. State-dependent degrees of intertemporal substitutability in consumption and intratemporal substitutability between consumption and leisure induce empirically relevant non-linearities with respect to the economic conditions as well as the direction of the policy action. The model predicts stronger output responses when monetary policy is restrictive, as compared with expansive policy actions, while inflation displays nearly symmetric responses to monetary shocks with different signs.\(^3\) In addition, the cyclical

\(^2\)So far little effort has been made to explore the relevance of prospect theory for the dynamics of macroeconomic aggregates. Some applications to price-setting (Heidhues and Koszegi, 2005) and consumption theory (Bowman et al., 1999 and Koszegi and Rabin, 2009) have been proposed.

\(^3\)These properties are in line with the evidence reported by Cover (1992), Morgan (1993), Karras and
movements of real activity as implied by the model are manifestly asymmetric, with statistical evidence of both ‘deepness’ (troughs are deeper than peaks are tall) and ‘steepness’ (contractions are steeper than expansions). ⁴

It is important to acknowledge that the macroeconomic literature has proposed a variety of mechanisms acting from both the supply and the demand side of the economy and capable to take account of different forms of non-linearity.⁵ For instance, Peersman and Smets (2005) suggest that the financial accelerator theory may explain why the effects of money on output are stronger in contractions. However, this mechanism implies an analogous amplification (attenuation) of monetary policy innovations on both prices and real activity during contractionary (expansionary) phases. To overcome such a discrepancy with the existing empirical evidence, the balance-sheet channel needs to be complemented with a mechanism capable of producing competing effects on prices, so as to obtain the desired non-linearity in the response of output, while generating symmetric price responses. In this respect, models with inverse ‘L-shaped’ or convex aggregate supply curves that belong to the Keynesian tradition are plausible candidates. A convex aggregate supply retains the property to be steeper for price levels above expected prices (see, e.g., Ball and Mankiw, 1994), so that it ensures a stronger (lower) reaction of output (prices) in contraction. Therefore, reconciling the macroeconomic theory with the evidence of no asymmetry in the response of prices typically calls for the coexistence of multiple driving forces. This paper provides an alternative explanation based on a simple and well-established behavioral mechanism.

Once it is recognized that interest rate innovations exert asymmetric effects on output and prices over different stages of the cycle, it seems relevant to provide some guidance as to how monetary policy should be designed to cope with such non-linearities. We explore

⁴These features have been extensively documented, among others, by Neftci (1984), Hamilton (1989), Sichel (1993) and, more recently, Morley and Piger (2012).

⁵The list of mechanisms that may give rise to asymmetries in the monetary transmission mechanism includes: non-linearities in investment (Bertola and Caballero, 1994), patterns of entry and exit from a given market under uncertainty about profit perspectives (Dixit, 1989), nominal rigidities in the labor and goods market (Ball and Mankiw, 1994), learning and information aggregation (Chalkley and Lee, 1998), state-dependent pricing and convex aggregate supply (Devereux and Siu, 2007), switches in consumer sentiment (De Grauwe, 2010). However, none of these mechanisms is per se capable to take account of cyclical asymmetries in the joint reaction of output and prices to monetary policy innovations.
the state-dependent trade-off that naturally arises in our framework. In the expansionary
state the monetary authority can attain a policy frontier which is otherwise unattainable
during contractions. The key implication is that reducing inflation variability by the
same amount and from the same level in the two cyclical stages entails higher costs in
terms of output variability during negative growth cycles. In line with recent evidence
on asymmetries in the policy reaction function (e.g., Rabanal, 2004; Cukierman and
Muscatelli, 2008), the optimal policy imposes stronger reactiveness to expected inflation
during expansions.

The remainder of the paper is laid out as follows: Section 2 details the theoretical
framework we propose to account for these facts; Section 3 details the model solution
technique; Section 4 discusses the key mechanisms that generate non-linear responses
of output and inflation to monetary innovations; Section 5 discusses the main policy
implications of embedding loss-averse consumption preferences in a general equilibrium
setting; the last section concludes.

2 A Model of Loss-averse Consumption

This section sets out the structure of the model put forward to explain asymmetries in the
responses of output and prices to monetary innovations. The supply side is populated by
monopolistically competitive firms that produce intermediate goods, indexed by \( j \in [0, 1] \),
and a perfectly competitive sector of production that sells a composite of consumption
goods. As to the demand side, there is a continuum of atomistic consumers, indexed by
\( i \in [0, 1] \).

2.1 Demand Side

Households have preferences defined over leisure \((1 - N_{it})\), consumption \((C_{it})\) and gains
and losses in consumption relative to its reference level \((X_{it})\). They maximize the expected
present discounted value of their utility:

\[
W_{it} = E_t \sum_{s=0}^{\infty} \beta^s \left[ U(C_{it+s}, X_{it+s}) - \chi \frac{N_{it+s}^{1+\eta}}{1+\eta} \right]; \quad \chi > 0, \tag{1}
\]

where \( \beta \) is the intertemporal discount factor and \( \eta \) is the inverse of the Frisch elasticity of labor supply. Following Koszegi and Rabin (2006) and Yogo (2008), a general class of reference-dependent preferences is considered:

\[
U(C, X) = \alpha V(C) + (1 - \alpha) \Lambda((V(C) - V(X))); \quad \alpha \in [0, 1], \tag{2}
\]

where \( V(C) \) is a neoclassical utility function: this is assumed to be continuously differentiable, strictly increasing, and concave for all \( C > 0 \). The term \( \Lambda(\cdot) \) is a gain-loss function (Kahneman and Tversky, 1979), that is, utility derived from the deviation of consumption utility from its reference level, \( V(X) \). Preferences that depend on a reference level of consumption have psychological foundations in hedonic adaptation (see Frederick and Loewenstein, 1999). We assume that \( \Lambda(\cdot) \) satisfies certain properties. Specifically: (i) \( \Lambda(Z) \) is continuous for all \( Z \)'s, twice differentiable for \( Z \neq 0 \) and \( \Lambda(0) = 0 \); (ii) \( \Lambda(Z) \) is strictly increasing; (iii) \( -\Lambda(-Z) > \Lambda(Z) \) and \( \Lambda'(-Z) > \Lambda'(Z), \forall Z > 0 \); (iv) \( \Lambda''(Z) \leq 0 \) for \( Z > 0 \) and \( \Lambda''(Z) \geq 0 \) for \( Z < 0 \). Properties (i) and (ii) imply monotonicity, i.e. utility is strictly increasing in the magnitude of the gain. Property (iii) captures the notion of loss aversion, i.e. the impact of a loss is greater than that of an equally-sized gain. In particular, the latter inequality is the strong-form of loss aversion of Wakker and Tversky (1993). Overall, these properties imply that the representative consumer becomes more sensitive to deviations from her relative consumption when she is in a bad state, compared with a good state. Finally, property (iv) is referred to as diminishing sensitivity, i.e. the marginal effect of a gain or a loss diminishes with its magnitude. This

\footnote{For the time being, and without loss of generality, we introduce reference-dependent preferences by reporting the relevant variables without time subscripts.}
translates into a gain-loss function whose curvature approaches zero as $Z \to \pm \infty$.\(^7\)

To take account of these properties, an exponential gain-loss utility is considered (Köbberling and Wakker, 2005):

$$
\Lambda(Z) = \begin{cases} 
  \frac{1 - \exp(-\theta Z)}{\theta} & \text{iff } Z \geq 0 \\
  -\lambda \left[1 - \frac{1 - \exp\left(\frac{\theta Z}{\theta}\right)}{\theta}\right] & \text{otherwise}
\end{cases}$$

(3)

where $\theta$ governs diminishing sensitivity and $\lambda$ is a parameter that indexes the degree of loss aversion. Note that for $\theta = 0$ a linear gain-loss function is obtained. Otherwise, (3) retains the property to be smooth at the reference point.\(^8\) To gain further intuition on the structure of reference-dependent preferences over consumption, Figure 1 plots the exponential gain-loss function and its first order derivative for different values of $Z$ (x-axis) and $\lambda$. As predicted by Kahneman and Tversky (1979), loss aversion reflects the widely observed behavior that agents are more sensitive to losses than gains, resulting in a gain-loss function that is steeper in the first case (see the left-hand panel of Figure 1). Moreover, the right-hand panel of Figure 1 captures the essence of diminishing sensitivity in consumer preferences, according to which marginal departures from the reference point are more (less) important the less (more) away they are from it.

As to the reference consumption level, it is assumed that consumers evaluate the distance between consumption utility and a function of the average consumption in the previous period: $X_{it} = C_{t-1}^\gamma$, where $\gamma \in [0, 1]$ indexes the importance of external habit

\(^7\)This specification offers a parsimonious framework to think about risk aversion and loss aversion. Risk aversion refers to the curvature of consumption utility, which determines the household’s behavior for large gambles. Loss aversion refers to the magnitude of marginal utility for losses relative to gains, which determines the household’s behavior for small gambles. Tversky and Kahneman (1991) extend their treatment of choice under uncertainty (see, e.g., Kahneman and Tversky, 1979) to the problem of facing a riskless choice.

\(^8\)This property is particularly useful in the perspective of linearizing the model economy.
The $i^{th}$ consumer, whose labor is remunerated at the real wage $W_t$, enters period $t$ with cash holdings $M_{it}$, $B_{it-1}$ one-period nominal bonds that pay $R_{t-1} (= 1 + i_{t-1})$ gross interest. Moreover, she receives the flow of dividends from a continuum of monopolistically competitive producers, $\Gamma_{it}$, and a lump sum transfer from the monetary authority, $T_{it}$:

$$P_tC_{it} + B_{it} + M_{it+1} \leq M_{it} + R_{t-1}B_{it-1} + P_tW_tN_{it} + \Gamma_{it} + T_{it},$$

where $\Gamma_{it} = \int_{0}^{1} D_{ijt}dj$ and $D_{ijt}$ denotes the dividends of firm $j$ paid to the $i^{th}$ household.

Differentiating the Lagrangian with respect to individual consumption ($C_{it}$) and taking the consumption reference level as external to the $i^{th}$ household returns the following Euler equation:

$$1 = \beta E_t \left\{ \frac{R_{it}}{\Pi_{t+1}} \frac{U_C(C_{it+1}, X_{it+1} | \xi_{it+1})}{U_C(C_{it}, X_{it} | \xi_{it})} \right\},$$

where $\Pi_t = 1 + \pi_t$ denotes the gross rate of inflation and $\xi_{it+1}$ is a discrete valued random variable that equals one if consumption utility is above its reference level (i.e., $V(C_{it}) \geq V(X_{it})$) and zero otherwise. Therefore, the marginal utility of consumption depends on the gain-loss profile and its shape changes depending on whether consumption is above or below its reference level.

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9. Gill and Prowse (2012) report experimental evidence that supports the role of endogenous choice-acclimating reference points in economic decisions. In the present context, external habits allow us to establish a direct link between the empirical evidence on the transmission of monetary policy during contractionary/expansionary phases of the cycle and the state-dependent model we build up. In principle, internal habit formation could have been considered. However, along with being computationally prohibitive, such a modelling option would bring no specific insight into the problem under examination.

10. In line with Yogo (2008), we embed external habit formation in a model of reference-dependent consumption preferences. Since the work of Abel (1990), external habit formation has become known as "catching up with the Joneses." External habit formation in consumption is usually introduced to account for the empirical persistence in the consumption process (Smets and Wouters, 2007). Unlike internal habit formation, this mechanism implies that households fail to internalize the externality of their own consumption on the utility of other households.
The expected marginal rate of substitution between $C_t$ and $N_t$ reads as:

$$\frac{\chi N_t^n}{U_C(C_t, X_t | \xi_t)} = W_t.$$

Equations (5) and (6) are paramount to understand how cyclical asymmetries in the transmission of monetary policy may arise in our model. Equation (5) regulates intertemporal substitution between current and future consumption. A closer look at this relationship allows us to provide some intuition on the key mechanism governing consumption dynamics. The curvature of the gain-loss function is higher when consumption is below its reference level, implying higher elasticity of intertemporal substitution during negative growth cycles. Concurrently, equation (6) governs the intratemporal substitution between consumption and leisure. For a given $|V(C) - V(X)|$, the marginal rate of substitution between labor and consumption is lower when $V(C) < V(X)$. Under these circumstances households are more willing to cut on their leisure so as to increase consumption in the same period, as compared with what happens when $V(C) \geq V(X)$.

Section 4.1 details the key implications of this mechanism for the labor market equilibrium.

### 2.2 Supply Side

The supply side of the model conforms to the standard treatment of frameworks with nominal price rigidities. The final good is produced by perfectly competitive firms and requires the assembly of a continuum of intermediate goods via the following technology:

$$Y_t = \left( \int_0^1 (Y_{jt})^{\frac{\varepsilon - 1}{\varepsilon}} dj \right)^\frac{1}{\varepsilon - 1},$$

where $\varepsilon$ denotes the elasticity of substitution between differentiated goods in the production composite. Profit maximization leads to the demand function $Y_{jt} = (P_{jt}/P_t)^{-\varepsilon} Y_t$ for the $j^{th}$ type of good, where $P_t = \left( \int_0^1 (P_{jt})^{1-\varepsilon} dj \right)^\frac{1}{1-\varepsilon}$ is the price index consistent with the final good producer earning null profits. Total production equals aggregate consumption.

The intermediate goods sector is populated by a continuum of monopolistically com-
petitive firms, each of them employing labor under the following constant returns to scale technology: $Y_{jt} = N_{jt}$. Firms are able to reset their prices at random intervals of time (see Calvo, 1983 and Yun, 1996), with the probability that a firm can re-optimize its price in each period being $1 - \omega$. Profit maximization under staggered price-setting leads to the conventional New Keynesian supply function, according to which current inflation depends on current and expected future real marginal costs. The real marginal cost retrieved from the cost minimization problem faced by firms in the intermediate goods sector is $RMC_t = W_t$.

### 2.3 The Monetary Authority

The government sets the nominal rate of interest in accordance with a standard instrumental rule:

$$\frac{R_t}{R} = \left( \frac{\Pi_t}{\Pi} \right)^{\gamma} \exp(\mu_t) ,$$

where $\mu_t$ captures non-systematic monetary policy responses.\footnote{In the remainder variables without time subscript will denote the steady state values of their time-indexed counterparts.} We assume that the government adheres to this rule via open market operations, which are financed by means of money transfers to the households, such that any deficits are equal to zero, i.e. $T_t = B_t - R_{t-1}B_{t-1}$.

Assuming a symmetric policy function to stabilize inflation represents a convenient way to close the model and focus on the effects of introducing reference-dependent preferences into an otherwise standard framework. Section 5 examines the monetary policy implications of this modeling assumption and formulates some inherent policy prescriptions.
3 Model Solution

In the framework set out above households’ utility is reference-dependent, i.e. its functional form depends on whether individual consumption is above or below the reference level (which is itself determined by aggregate past consumption). At this stage of the analysis we need to specify a mechanism that governs switching in consumers’ preferences, which in turn depend on their consumption profile with respect to the reference level. Given the intertemporal dimension of households’ decisions, accounting for the expectations of future consumption is paramount. To solve for endogenous (consumption) regime switching we use the monotone map algorithm that finds fixed point in decision rules (Coleman, 1991).12

As a preliminary step to solve the model, we retrieve its quasi-linear representation. In the absence of sector-specific shocks or other forms of heterogeneity, households are symmetric and make identical consumption-saving decisions. Therefore, the equilibrium conditions and the policy reaction function can be linearized in the neighborhood of the steady state consistent with $C/X = 1$.13

\[ y_t = \phi_1 \left( E_t \xi_{t+1}, \xi_t \right) E_t y_{t+1} + \phi_2 \left( E_t \xi_{t+1}, \xi_t \right) y_{t-1} - \phi_3 \left( E_t \xi_{t+1}, \xi_t \right) \left( i_t - E_t \pi_{t+1} \right) \]  
(8)

\[ \pi_t = \beta E_t \pi_{t+1} + \psi_1 \left( E_t \xi_{t+1}, \xi_t \right) y_t + \psi_2 \left( E_t \xi_{t+1}, \xi_t \right) y_{t-1} + u_t, \]  
(9)

\[ i_t = r \pi_t + \mu_t, \]  
(10)

where the supply disturbance $u_t = \rho_u u_{t-1} + \epsilon_{it}^u$ is such that $\rho_u \in [0, 1)$ and $\epsilon_{it}^u \overset{i.i.d.}{\sim} N \left( 0, \sigma_u^2 \right)$. This shock is imposed so as to generate contractionary and expansionary paths which are independent from monetary innovations (see Section 4.2 for further details).

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12 The problem we tackle is isomorphic to the preemptive policy behavior examined by Davig and Leeper (2008). To initialize the algorithm, we start from the solution of the framework without loss-averse preferences, but also check that the final solution is not sensitive to initial conditions by perturbing these initial conditions. The final solution is invariant with respect to perturbations in the initial rules, suggesting that the solution is locally unique.

13 The difference between the logarithm of a generic variable $Z_t$ and that of its steady state counterpart $Z$ is denoted by $z_t$. We also assume, without loss of generality, logarithmic consumption utility. For further details, see Appendix A, where we report the linearized conditions for each of the four consumption regimes taken separately.
In every period the model can generate four states, depending on whether \( E_t \xi_{t+1} = \{1, 0\} \) and \( \xi_t = \{1, 0\} \). Therefore, we deal with a Markov Switching Rational Expectations (MSRE) model (see Davig and Leeper, 2007 and Farmer, Waggoner and Zha, 2008, 2009). As such, some of the parameters in the equations describing the private sector’s behavior depend on the state of the economy and the probability of switching across different states is endogenously determined. In addition, the process of (rational) expectation formation necessarily accounts for the presence of switching across different consumption regimes. The solution is a function that maps the minimum set of state variables, \( \Theta_t = (u_t, \mu_t, y_{t-1}) \), into values for the endogenous variables, so that the rules for output and inflation can be expressed as \( h^y (u_t, \mu_t, y_{t-1}) = y_t \) and \( h^\pi (u_t, \mu_t, y_{t-1}) = \pi_t \).\(^{14}\)

4 The Asymmetric Transmission of Monetary Policy

This section discusses the key mechanisms at work in the model and shows how different types of asymmetry can be generated in connection with the transmission of monetary policy.

4.1 Some Qualitative Insights

Embedding loss-averse preferences over consumption in a general equilibrium setting induces two major modifications in the equations accounting for the dynamics of real activity and prices. First, the elasticity of intertemporal substitution is state-dependent, being higher (lower) in contraction (expansion). Second, the state-dependent marginal rate of substitution between consumption and leisure dampens the impact of real activity on firms’ price-setting behavior during contractions. The first property has been widely explored and validated by Yogo (2008). The second property is intimately connected with the role of loss-averse preferences in a general equilibrium setting. A globally convex aggregate supply function can be envisaged in this context, which retains the property to be steeper (flatter) during expansionary (contractionary) episodes. Similar

\(^{14}\)Additional details on the solution of the state-dependent model with endogenous switching are reported in Appendix B.
functional forms have been explored in the literature on the Phillips curve, emphasizing the role of large shocks relative to small ones for firms’ price-setting behavior. In this respect, the existence of menu costs can rationalize a convex aggregate supply schedule. Nonetheless, it is interesting to note that introducing loss-averse preferences in a general equilibrium setting allows us to provide a microfoundation that emphasizes the role of state-dependent degrees of real rigidity in the labor market equilibrium allocation, rather than nominal rigidities. Equation (6) accounts for the dynamics of labor supply: its key property is to be steeper at levels of total hours above the reference consumption/production point. Figure 2 portrays this function against a perfectly elastic labor demand. The two schedules intersect at the point consistent with \( V(C) = V(X) \). Due to lower substitutability between labor and consumption when \( V(C) < V(X) \), a contraction in labor demand induces a larger (smaller) drop in equilibrium employment (wage), as compared with the responses induced by an equally-sized upward shift in the labor demand schedule. Therefore, the model generates downward real wage rigidity during contractionary episodes due to higher intratemporal substitutability between leisure and consumption opportunities, and not from asymmetric wage-setting frictions affecting the labor market (as in Kim and Ruge-Murcia, 2009 and Benigno and Ricci, 2011). This type of mechanism, which finds empirical and experimental support in Farber (2008) and Goette, Huffman, and Fehr (2004), marks the key difference between our framework and traditional Keynesian theories of macroeconomic fluctuations, which rather emphasize sources of nominal and real rigidity that affect the shape of the labor demand schedule.

To elaborate further on these intuitions, the linearized relationships describing the behavior of demand and supply in different consumption regimes are inspected. In this context, expectation formation is non-trivial, as agents in the model economy do not

\[ 15 \text{ See Laxton, Rose, and Tambakis (1999) for a review of the literature and the analysis of the monetary policy implications of assuming a convex aggregate supply.} \]

\[ 16 \text{ It should be noted that the degree of asymmetric reaction increases in the labor supply elasticity. In fact, at low levels of } 1/\eta \text{ the state-dependent marginal consumption utility exerts a weak impact on the marginal rate of substitution between consumption and leisure, which is instead dominated by the marginal utility of leisure. In the limit (i.e., } 1/\eta \rightarrow 0 \text{) the income effect tends to fully compensate the substitution effect from a wage increase and the resulting labor supply function is close to anaelastic.} \]
know the realization of all future regimes. In turn, this sequence depends on the sequences of exogenous shocks that are realized and on the serial correlation properties of those shocks.\textsuperscript{17} However, to provide a simple intuition of how output and inflation respond to a monetary policy shock over different cyclical phases, it is temporarily assumed that agents ‘naively’ expect the economy to permanently stay in either expansion or contraction. Furthermore, $\gamma$ is set to zero, so that households consider the deviation of their consumption utility from the utility accruing from a constant reference level of consumption. Thus, we implicitly look at cyclical variations in output rather than at expansions/contractions in the business cycle. These simplifying assumptions allow us to highlight the main structural differences between the state-dependent model with loss aversion and the standard New Keynesian setting. However, analogous implications carry over to the model with endogenous switching across consumption regimes, as we detail in the next section. Table 1 reports the state-dependent IS schedule and the New Keynesian Phillips curve (NKPC).

Assuming loss-averse consumers implies state-dependent degrees of real rigidity and elasticity of intertemporal substitution in consumption that generate competing effects in the responses of output and inflation to monetary innovations. Specifically, during contractions the IS schedule displays higher elasticity of current consumption to the real rate of interest, as $\lambda (\lambda - \theta (1 - \alpha))^{-1} > (1 + (1 - \alpha) \theta)^{-1}$.\textsuperscript{18} As to the state-dependent NKPC, the elasticity of inflation to output deviations from its steady state level is higher in expansions, as $1 + \eta + \theta (1 - \alpha) > \lambda^{-1} (\lambda (1 + \eta) - \theta (1 - \alpha))$. This results from the labor supply schedule being convex, which induces a dampened response of firms’ real marginal cost with respect to consumption (output) when the latter lies below its reference point. Therefore, during contractions greater responsiveness of output to the real interest

\begin{table}[h]
\centering
\begin{tabular}{|c|c|}
\hline
State & IS Schedule & NKPC \\
\hline
Expansion & & \\
\hline
\end{tabular}
\caption{State-dependent IS schedule and New Keynesian Phillips curve (NKPC).}
\end{table}

\textsuperscript{17}Appendix B shows how to compute time-varying probabilities of regime switching.

\textsuperscript{18}According to this mechanism, loss-averse consumption preferences should also be able to generate empirically relevant asymmetries in the transmission of fiscal shocks. Auerbach and Gorodnichenko (2012, 2013) and Fazzari, Morley, and Panovska (2012) show that the effects of a government spending shock on output are significantly larger and more persistent when the economy is characterized by underutilization of resources, as compared with a situation of full capacity.
rate is counteracted by a flatter NKPC. The ultimate impact of a monetary policy shock on inflation depends on the relative magnitude of these competing forces.

4.2 Quantitative Analysis

To quantify the asymmetric impact of monetary policy over contractions and expansions, we compute the solution of the quasi-linear model economy. To this end, the model is calibrated at a quarterly frequency. The discount factor $\beta = 0.99$. The inverse of the Frisch elasticity of labor supply, $\eta$, is set to 0.25. As to households’ consumption preferences, $\alpha = 0.5$ and $\gamma = 0.9$. The literature on dynamic general equilibrium models does not provide us with any empirical reference on the coefficient that indexes the degree of loss aversion. Therefore, $\lambda$ is set to 2.25, in accordance with Tversky and Kahneman (1992). In line with Yogo (2008), $\theta = 1$. Nominal rigidity in price-setting is such that $\omega = 0.66$. As to the policy reaction function, $r_{\Pi} = 1.5$, while the non-systematic component $\mu_t = \rho_\mu \mu_{t-1} + \epsilon_\mu^t$, where $\rho_\mu = 0.5$ and $\epsilon_\mu^t \sim N(0,0.02)$. The process governing the supply shock is such that $\rho_u = 0.9$ and $\sigma_u^2 = 0.06$.20

A quantitative assessment of the asymmetric impact of monetary innovations is an obvious goal for this study. To this end, contractions and expansions are generated by perturbing the system with a supply shock of the appropriate sign. Concurrently a monetary policy shock is induced whose magnitude is not large enough to reverse the cyclical movement in output, so that it is not the Central Bank to determine the regime in place at any given point in time. Therefore, policy interventions are only "modest" in their scope (Leeper and Zha, 2003). Finally, the response to the supply shock is subtracted from the overall response of the system to both sources of exogenous perturbation, so as

\begin{footnotesize}
\footnote{Recall that $\eta$ measures the elasticity of the marginal disutility of labor with respect to hours worked. Rotemberg and Woodford (1998) report evidence of low values of this elasticity, generally between 0.25 and 0.4, while McCallum (2001) suggests values closer to the lower bound.}
\footnote{We assume higher volatility in supply innovations, so as to make it possible to generate contractionary and expansionary episodes that are predominantly driven by real rather than monetary innovations.}
\footnote{Monetary innovations are generated so as to induce a monetary tightening (loosening) during expansions (contractions). This is coherent with the empirical evidence of Romer and Romer (1994), who show that the Fed funds rate declines (increases) right after the cyclical peak (trough).}
\end{footnotesize}
to isolate the effect of monetary innovations. Figure 3 displays greater reactiveness of output to monetary shocks during contractions as opposed to expansions – a result in line with the arguments of Section 4.1 – which confirms the robustness of the mechanism at work in generating asymmetric responses of real activity over positive and negative growth cycles. Otherwise, the difference between inflation responses over different stages of the cycle is negligible, suggesting that higher (lower) responsiveness of real activity in contractions (expansions) is attenuated (amplified) once output movements are passed through onto prices.

Insert Figure 4 here

It is certainly important to assess the behavior of the model in response to sources of fluctuations other than monetary shocks, so as to highlight the distinctive features of contractionary and expansionary output movements. In this respect, the model displays elements of business cycle asymmetry that have been extensively documented, among others, by Neftci (1984), Hamilton (1989), Sichel (1993) and, more recently, Morley and Piger (2012). Figure 4 portrays equilibrium dynamics in response to positive and negative supply shocks of differing signs and magnitudes. In line with the evidence on the effects of monetary innovations, an adverse supply shock induces a deeper contraction, as compared with the expansion that follows from an equally-sized positive shock (i.e., troughs are deeper than peaks are tall). Concurrently, contractionary movements tend to display higher steepness as compared with expansionary movements, given that in the first case output deviations from the steady state take longer to peter out. To investigate these properties in further detail, we implement a battery of asymmetry tests that aim at quantifying the statistical significance of deepness and steepness in the simulated business cycle series, conditional on different sources of exogenous disturbance. The results reported in Table 2 confirm that cyclical movements in the output series are manifestly asymmetric, with statistical evidence of both deepness and steepness. Notably, both types of asymmetry are primarily driven by monetary policy innovations, rather than by supply shocks. The intuition for this result is that monetary innovations induce greater deviations of the ex-ante real rate of interest from its steady state level, as compared
with supply shocks, which make the nominal rate of interest and the rate of inflation move in the same direction. In turn, greater deviations of the real rate of interest from its steady state level necessarily induce a more marked reaction of real output, so that the non-linear mechanism of switching across different consumption regimes is magnified in the face of monetary policy innovations.

Unlike models based on exogenous mechanisms of switching across different states, our framework can generate non-linear responses to shocks with different signs and magnitudes. A vast empirical evidence has shown that monetary policy induces asymmetric responses of output and prices not only with respect to the economic conditions, but also depending on the size and direction of the policy action. A wide consensus has been reached on the view that money affects output strongly when monetary policy is restrictive, whereas it exerts little or no effect when it is expansive (Cover, 1992; Morgan, 1993; Dufrenot et al., 2004), while the effect on prices is nearly symmetric (Karras and Stokes, 1999; Weise, 1999). There is also some evidence that shocks of different magnitudes have asymmetric effects on output. Weise (1999) shows that if the economy starts in a low-growth state, large negative shocks induce substantially larger contractionary (on impact) responses in output, though on a longer time horizon no asymmetry can be appreciated with respect to the size of the shock. It is possible to show that our model can account for empirically relevant non-linearities in the transmission of monetary shocks with different signs. Figure 5 shows how monetary contractions cause greater effects on output, as compared with the impact induced by positive monetary shocks of the same absolute size. By contrast, inflation displays nearly symmetric responses. This can be intuitively explained based on the fact that a rise in the nominal rate of interest increases the chances to trigger or deepen contractionary output movements, as compared with a loose monetary stance.\textsuperscript{22} This is consistent with the evidence reported by Garcia and

\textsuperscript{22}Concurrently, non-linearity in the labor supply schedule implies an attenuation of the pass-through from output to inflation during a contractionary stage of the cycle.
Schaller (2002), who show that an increase in the nominal rate of interest raises the probability of moving from an expansion to a recession. Similarly, an interest rate cut is typically associated with a higher probability of getting out of a recession.

The responses to monetary innovations reported in Figure 5 are conditional on the realization of no supply shocks. To widen our perspective, we generate contour maps of output and inflation responses to monetary innovations with different signs and magnitudes, conditional on both contractionary and expansionary output movements (see Figure 6). Once again, we filter the system’s overall response to the supply innovations, so as to isolate its reaction to monetary policy shocks. Notably, negative monetary innovations exert greater effects on output during contractions, as compared with the impact of policy innovations of the same size that hit the system during expansions. Yet, the size of the shock has no role in stimulating asymmetric responses. Moreover, monetary innovations have limited capability to enhance expansionary movements in output that originate from positive supply innovations. This is evident from inspecting the top panels of Figure 6, and specifically the top-left quadrant of each panel: here output responses are lower than the average (absolute) responses in other quadrants. On the other hand, monetary innovations have greater capability to affect output in contractions, as compared with expansions. This result is in line with Romer and Romer (1994), whose evidence shows that monetary policy alone is a powerful tool to end recessions and it has represented the source of most US postwar recoveries.

Insert Figure 6 here

5 Monetary Policy Implications

Once it is recognized that monetary policy exerts an asymmetric impact on output and inflation, it is of obvious importance to explore how the policy maker should take account of these facts. The purpose of this section is to sketch the policy implications of embedding loss-averse preferences into a dynamic general equilibrium context. To this end, it is useful to think about a scenario in which the monetary authority acts discretionally and takes the
perceived expansionary and contractionary stages as given. This is done for two main reasons. First, from a practical viewpoint, working under discretion allows us to envisage a sequence of static optimization problems. In this perspective, the Central Bank does not need to consider the probability of switching across different states. Second, from an institutional viewpoint it is hard to think about a Central Bank that makes any strictly binding commitment on its future policy action (Clarida, Gali, and Gertler, 1999).

To simplify the analysis, a purely forward-looking system is examined. In each period the monetary authority chooses $y_t$ and $\pi_t$ to maximize the welfare criterion

$$L_t = -\frac{1}{2}E_t \sum_{i=0}^{\infty} \beta^i [\pi_{t+i}^2 + \varrho y_{t+i}^2],$$

subject to the state-dependent supply schedule, whose slope $\psi(E_t, \xi_{t+1}, \xi_t)$ changes depending on the (sign of the) deviation of consumption from its reference level (see Table 1). The solution to this problem returns the well known relationship:

$$y_t = -\frac{\psi(E_t, \xi_{t+1}, \xi_t)}{\varrho} \pi_t,$$

which means that whenever inflation is above the target the Central Bank should contract output below capacity (thus implementing a "leaning against the wind" policy). However, allowing for loss-averse preferences determines state-dependent degrees of real rigidity that alter the nature of the trade-off between output and inflation stabilization depending on the deviation of consumption from its reference level. Intuitively, for a given level of above-the-target inflation, the Central Bank does not need to contract

\footnote{Recall that in this setup the Central Bank does not exert any control on which regime is in place at each point in time. In this respect, policy interventions are only "modest" in their scope (see Leeper and Zha, 2003).}

\footnote{Once again, this amounts to set $\gamma = 0$, thus allowing for a gain-loss function in which deviations of consumption from a constant reference level ($X = 1$) are weighed.}

\footnote{This welfare criterion is widely used to capture the stabilization objective of the Central Bank as a function of $\pi_t$ and $y_t$ (Clarida et al., 1999). Deriving a state-dependent welfare metric consistent with households’ preferences is beyond the scope of the present study. We leave this task for future research.}
output during a negative growth-cycle as much as it should do during a positive one. Moreover, given that in contractionary states the real interest rate has stronger effects on real activity (while inducing only moderate effects on inflation), the discretionary rule

\[ i_t = r \left( E_t \xi_{t+1}, \xi_t \right) E_t \sigma_{\tau_{t+1}} \]

imposes greater reactiveness to the one-period-ahead expected rate of inflation during expansions, as compared with contractions:

\[ r \left( E_t \xi_{t+1} = 1, \xi_t = 1 \right) > r \left( E_t \xi_{t+1} = 0, \xi_t = 0 \right). \quad (12) \]

A useful way to illustrate the trade-off between inflation and output stabilization implied by the model is to construct the corresponding efficient policy frontier. Combining the IS with the aggregate supply schedule and the optimal policy under discretion returns the locus of points that characterize how the unconditional variances of output and inflation vary with Central Bank preferences, as indexed by \( \varphi \). Figure 7 portrays the efficient policy frontiers for the two alternative scenarios under the calibration considered in the previous section.

Insert Figure 7 here

In the expansionary state monetary policy can reach a policy frontier which is instead infeasible under the contractionary one. During contractions an aggressive monetary stance on inflation, i.e. a policy that aims at completely offsetting fluctuations in the rate of inflation, incurs into relatively higher costs in terms of output volatility. However, attaching increasing importance to output volatility gradually leads to similar costs in terms of inflation volatility across different regimes. A perhaps more important observation is that decreasing output variability by the same amount in contractions and expansions entails a lower increase in inflation volatility in the first case. This can be readily noted by picking a point on both frontiers at the same level of \( \sigma_y \), thus moving down along each locus so as to attain the same reduction in output variability: the relative increase in \( \sigma_{\tau} \) is greater under expansions than contractions. Therefore, pursuing a decrease in output variability as a policy objective should rather be done during economic
slowdowns, so has to exploit the reduced pass-through from output to inflation and trigger lower pressures in terms of inflation volatility, provided that the Central Bank aims at remaining on its policy frontier.

Altogether, these results suggest an alternative interpretation of the empirical evidence showing that the response coefficients in the Taylor rules adopted by various monetary authorities have changed over time and, in particular, have varied in connection with changes in the economic conditions.26 For instance, Rabanal (2004) shows that monetary policy in the U.S. has been somewhat less active during expansions, attaching higher weight to inflation responses and reflecting stronger interest-rate inertia. By contrast, during contractions the Federal Reserve has shifted its relative focus on controlling output growth. While a natural interpretation of shifts in the policy action is to attribute them to changes in the preferences of the policy maker (Surico, 2007), our analysis shows that state-dependent behavior in monetary policy making could be explained by changes in the preferences of the public.

6 Concluding Remarks

Vast empirical evidence shows that output and prices react asymmetrically to monetary policy innovations over contractionary and expansionary phases of the business cycle. It is a well-established finding that monetary policy has stronger effects on the GDP during contractions, as compared with expansions. As to price responses, these are not statistically different across different stages of the cycle. This paper shows that embedding prospect theory into an otherwise standard dynamic general equilibrium model may rationalize these facts. Loss-averse consumption preferences imply state-dependent degrees of real rigidity and elasticity of intertemporal substitution in consumption that generate competing effects on the responses of output and inflation following a monetary innovation. The qualitative and quantitative analyses return predictions that are in line with the empirical evidence. Output responses to a monetary tightening are greater in con-

tractions as compared with expansions. Despite the amplification of output responses, downward wage rigidity induced by loss-averse preferences tends to attenuate inflation responses during negative growth cycles. As a consequence, we cannot detect statistically relevant differences in inflation responses over alternative cyclical phases. In addition, the model can successfully reproduce empirically relevant non-linearities in the reaction of output and inflation to monetary innovations with different signs. A rise in the nominal rate of interest increases the chances to trigger or deepen a contractionary movement in output, as compared with a loose monetary stance. Therefore, unexpected monetary contractions have greater effects on output, as compared with the impact induced by positive shocks of the same absolute size. By contrast, inflation displays nearly symmetric responses.

A state-dependent trade-off between inflation and output stabilization naturally arises under loss-averse consumption preferences. During contractions an aggressive monetary stance on inflation, i.e. a policy that aims at offsetting fluctuations in the rate of inflation, incurs into relatively higher costs in terms of output volatility. In line with recent evidence on asymmetries in the policy reaction function, the optimal policy under discretion imposes a stronger degree of reactiveness to the expected rate of inflation in the expansionary state.
References


APPENDIX A: Log-linear State-Dependent System (Not Intended for Publication)

This appendix reports the model linearized around the non-stochastic steady state. For clarity of exposition, we present the equations describing private sector’s behavior in each of the four possible states depending on consumption dynamics.

The IS Curve

We linearize the Euler equation in the neighborhood of the steady state consistent with $C/X = 1$, obtaining the following state-dependent system of linearized IS curves:

$$y_n = \begin{cases} 
\frac{1+(1-\alpha)\theta}{1+(1-\alpha)(1+\gamma)} E_t y_{t+1} + \frac{(1-\alpha)\theta}{1+(1-\alpha)(1+\gamma)} y_{t-1} - \frac{i_{t}-E_{t}\pi_{t+1}}{1+(1-\alpha)(1+\gamma)} & \text{iff } E_t \xi_{t+1} = \xi_t = 1 \\
\frac{1-(1-\alpha)\theta}{1+(1-\alpha)(1+\gamma)} E_t y_{t+1} + \frac{(1-\alpha)\theta}{1+(1-\alpha)(1+\gamma)} y_{t-1} - i_{t}-E_{t}\pi_{t+1} & \text{iff } E_t \xi_{t+1} = 0, \xi_t = 1 \\
\frac{1+(1-\alpha)\theta}{1+(1-\alpha)(1+\gamma)} E_t y_{t+1} - \frac{(1-\alpha)\theta}{1+(1-\alpha)(1+\gamma)} y_{t-1} - \frac{i_{t}-E_{t}\pi_{t+1}}{1+(1-\alpha)(1+\gamma)} & \text{iff } E_t \xi_{t+1} = 1, \xi_t = 0 \\
\frac{1-(1-\alpha)\theta}{1+(1-\alpha)(1+\gamma)} E_t y_{t+1} - \frac{(1-\alpha)\theta}{1+(1-\alpha)(1+\gamma)} y_{t-1} - \frac{i_{t}-E_{t}\pi_{t+1}}{1+(1-\alpha)(1+\gamma)} & \text{iff } E_t \xi_{t+1} = \xi_t = 0 
\end{cases}$$

where we have aggregated across individuals (imposing homogeneity) and used the goods market clearing condition, $Y_t = C_t$.

When it comes to linearize the model economy in the neighborhood of $C/X = 1$, it is important to note that $\Lambda'(Z)$ presents an ordinary double point at $Z = 0$. As such, $\Lambda'(Z)$ is not purely differentiable in that point, as also implied by property (i). Therefore, standard linear approximation techniques such as the Taylor expansion do not immediately apply in this case. However, we can resort to a first-order approximation of $\Lambda'(Z)$ by computing an affine global underestimator, thus determining the subgradients of the marginal utility function at $Z = 0$. A subgradient determines a support hyperplane to the graph of the function under scrutiny. In such a case the corresponding subdifferential is a direct generalization of the differentiable case. For a convex and non necessarily differentiable function $f : \mathbb{R}^n \to \mathbb{R}$, the subdifferential at $x_0$ is defined as $\partial f(x_0) = \{g \in \mathbb{R} : f(x) \geq f(x_0) + \langle g, x-x_0 \rangle \}$. Thus, $g \in f(x_0)$ is subgradient in $x_0$. In our case it is straightforward to note that at $Z = 0$ there will be a single subgradient for each branch of the function under scrutiny. To gain intuition on this, we can re-write the marginal utility as $\Lambda'(Z) = \min \{\Lambda'_A(Z), \Lambda'_B(Z)\}$ for $Z \in \mathbb{R}$, where $\Lambda'_A(Z)$ and $\Lambda'_B(Z)$ are the functions that encompass the arms of marginal utility corresponding to $Z > 0$ and $Z < 0$, respectively. These functions are both convex. It is also easy to see that $\Lambda'_B(Z) > \Lambda'_A(Z)$ for $Z \in \mathbb{R}^+$ and $\Lambda'_B(Z) < \Lambda'_A(Z)$ for $Z \in \mathbb{R}^-$. Hence, our approach amounts to a piecewise linear approximation in the neighborhood of $Z = 0$. Note also that assuming a smooth gain-loss function $\Lambda(Z)$ at $Z = 0$ allows us to obtain a continuous first derivative function, which improves the approximation around the point $Z = 0$, compared with what would happen, say, with a linear gain-loss function, which implies a discontinuity at $\Lambda'(0)$.

---

Inflation Dynamics

After applying some trivial algebra we retrieve a log-linearized expression for the real marginal cost:

\[ rmc_t = \begin{cases} 
(\eta + 1 (1 - \alpha) \theta) y_t - (1 - \alpha) \theta y_{t-1} & \text{iff } \xi_t = 1 \\
(\eta + 1 (1 - \alpha) \frac{\theta}{\lambda}) y_t + (1 - \alpha) \frac{\theta}{\lambda} y_{t-1} & \text{otherwise}
\end{cases} \]

Thus the piecewise linear NKPC reads as:

\[ \pi_t = \beta E_t \pi_{t+1} + \kappa \begin{cases} 
(\eta + 1 (1 - \alpha) \theta) y_t - (1 - \alpha) \theta y_{t-1} & \text{iff } \xi_t = 1 \\
(\eta + 1 (1 - \alpha) \frac{\theta}{\lambda}) y_t + (1 - \alpha) \frac{\theta}{\lambda} y_{t-1} & \text{otherwise}
\end{cases} \]

APPENDIX B: Model Solution (Not Intended for Publication)

The solution to the model is a function that maps the minimum set of state variables into values for the endogenous variables. Implementation of the map algorithm begins by taking the initial rules for inflation and the output gap, \( \hat{h}^\pi (u_t, \mu_t, y_{t-1}) = \pi_t \) and \( \hat{h}^y (u_t, \mu_t, y_{t-1}) = y_t \). We then substitute them, together with the interest rate rule, into the functions describing private sector behavior, yielding:

\[
y_t = \phi_1 (E_t \xi_{t+1}, \xi_t) E_t \left[ \hat{h}^y \left( u_{t+1}, \mu_{t+1}, y_t \right) \right] + \phi_2 (E_t \xi_{t+1}, \xi_t) y_{t-1}
- \phi_3 (E_t \xi_{t+1}, \xi_t) \left( r \pi_t + \mu_t - E_t \left[ \hat{h}^\pi \left( u_{t+1}, \mu_{t+1}, y_t \right) \right] \right),
\]

\[
\pi_t = \beta E_t \left[ \hat{h}^\pi \left( u_{t+1}, \mu_{t+1}, y_t \right) \right] + \psi_1 (E_t \xi_{t+1}, \xi_t) y_t + \psi_2 (E_t \xi_{t+1}, \xi_t) y_{t-1} + u_t,
\]

This system translates into:

\[
y_t = \phi_1 (E_t \xi_{t+1}, \xi_t) \int_{a_u}^{b_u} \phi \left( u; \sigma_u^2 \right) \int_{a_\mu}^{b_\mu} \phi \left( \mu; \sigma_\mu^2 \right) \hat{h}^y (u, \mu, y_t) d\mu du + \phi_2 (E_t \xi_{t+1}, \xi_t) y_{t-1}
- \phi_3 (E_t \xi_{t+1}, \xi_t) \left( r \pi_t + \mu_t - \int_{a_u}^{b_u} \phi \left( u; \sigma_u^2 \right) \int_{a_\mu}^{b_\mu} \phi \left( \mu; \sigma_\mu^2 \right) \hat{h}^\pi (u, \mu, y_t) d\mu du \right),
\]

\[
\pi_t = \beta \int_{a_u}^{b_u} \phi \left( u; \sigma_u^2 \right) \int_{a_\mu}^{b_\mu} \phi \left( \mu; \sigma_\mu^2 \right) \hat{h}^\pi (u, \mu, y_t) d\mu du
+ \psi_1 (E_t \xi_{t+1}, \xi_t) y_t + \psi_2 (E_t \xi_{t+1}, \xi_t) y_{t-1} + u_t,
\]

where \( \phi (\cdot) \) is the normal density, \( a_i = -3\sigma_i^2 \) (\( i = u, \mu \)) and \( b_i = 3\sigma_i^2 \) (\( i = u, \mu \)). Expectations are evaluated using trapezoid integration. Linear interpolation is then used to
evaluate $\hat{h}^u (u, \mu, y_{t-1})$ and $\hat{h}^\mu (u, \mu, y_{t-1})$ for $i = 1, 2, ..., N_u$ and $k = 1, 2, ..., N_\mu$, where $N_i (i = u, \mu)$ denotes the number of nodes in each shock dimension. The state vector and the decision rules are taken as given when solving the system. The system is then solved for every set of state variables over a discrete partition of the state space. This procedure is repeated until the iteration improves the current decision rules at any given state vector by less than some convergence criterion, that we set to 1e-8. Note that to initialize the algorithm, we start from the solution of the framework without loss-averse preferences, but also check that the final solution is not sensitive to initial conditions by perturbing these initial conditions. The final solution is invariant with respect to perturbations in the initial rules, suggesting the solution is locally unique.

In this setting, the probability of future regimes can be characterized. For instance, assume that we are interested in computing the probability that consumption expands above its reference level in the current period and it is expected to do so even in the next period. This amounts to compute the probability that shocks that buffet the system in the current period will not cause consumption to fall below the reference level, neither at time $t$ nor at time $t+1$, conditional on the information set available at time $t$. To provide an example, we rule out monetary policy non-systematic responses, so as to assume that endogenous switching is not influenced by monetary policy shocks. In this setting, the smallest innovation to the supply shock process necessary to induce $E_t\xi_{t+1} = \xi_t = 1$ is given by the solution to:

$$\min_{\epsilon_t^u} [f(\epsilon_t^u), \ g(\epsilon_t^u)] \ s.t. \ E_t y_{t+1} \geq \gamma y_t \ \text{and} \ y_t \geq \gamma y_{t-1},$$

(18)

where:

$$f(\epsilon_t^u) = E_t h^y (\rho_u (\rho_u u_{t-1} + \epsilon_t^u) + \epsilon_{t+1}^u, h^y (\rho_u u_{t-1} + \epsilon_t^u, y_{t-1})) - \gamma h^y (\rho_u u_{t-1} + \epsilon_t^u, h^y (u_{t-1}, y_{t-2})), \quad (19)$$

$$g(\epsilon_t^u) = h^y (\rho_u u_{t-1} + \epsilon_t^u, h^y (u_{t-1}, y_{t-2})) - \gamma h^y (u_{t-1}, y_{t-2}). \quad (20)$$

Therefore, the probability that both $E_t y_{t+1} \geq \gamma y_t$ and $y_t \geq \gamma y_{t-1}$ is:

$$\Pr [E_t \xi_{t+1} = \xi_t = 1 | \Theta_t] = \int_{\epsilon_t^u} \phi (e^u; \sigma_u^2) \, d\epsilon^u,$$

(22)

where $\epsilon_t^u$ is a positive truncation point and $\epsilon_t^{u*}$ is the solution to the minimization problem.
Tables and Figures

<table>
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<th>Expansion (E_t \xi_{t+1} = \xi_t = 1)</th>
<th>Contraction (E_t \xi_{t+1} = \xi_t = 0)</th>
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<td>(E_t y_{t+1} - (1 + (1 - \alpha) \theta)^{-1} (i_t - E_t \pi_{t+1}))</td>
<td>(E_t y_{t+1} - \lambda (\lambda - (1 - \alpha) \theta)^{-1} (i_t - E_t \pi_{t+1}))</td>
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<td>(\beta E_t \pi_{t+1} + \kappa (\eta + 1 + (1 - \alpha) \theta) y_t + u_t)</td>
<td>(\beta E_t \pi_{t+1} + \kappa \lambda^{-1} (\lambda (1 + \eta) - \theta (1 - \alpha)) y_t + u_t)</td>
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Notes. The coefficient \(\kappa\) equals \((1 - \beta \omega) (1 - \omega) \omega^{-1}\).
Table 2. Tests for asymmetry of the cycle.

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<th>$Skew (\Delta \bar{y}_t)$</th>
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</tr>
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</table>

Notes. Table 2 reports some tests for asymmetry in the cyclical behavior of the simulated output series, conditional on different shock configurations. The first column reports the skewness of detrended output (a measure of deepness), whereas the second column reports the skewness of the growth rate of detrended output (a measure of steepness). Standard deviations are reported in square brackets. For more details on the testing procedure see Psaradakis and Sola (2002). All statistics are significant at the 1% level.
Notes. Gain-loss function (LHS panel) and its first-order derivative (RHS panel), for $\theta = 1$ and different values of $\lambda$. 
Notes. The red-dashed-dotted line corresponds to the labor demand schedule; the blue-continuous line is the labor supply schedule under loss-averse preferences ($\alpha = 0.5$), while the blue-dashed line is the labor supply schedule under standard neoclassical preferences ($\alpha = 1$). We set $\eta = 0.25$, $\theta = 1$ and $\lambda = 2.25$. 
Figure 3. Impulse responses to monetary policy shocks in different cyclical phases.

Notes. The continuous line denotes the response to a (one standard deviation) negative monetary shock during a contractionary regime; the dashed line denotes the response to a (one standard deviation) positive monetary shock during an expansionary regime. To obtain these responses, we preliminary generate contractions and expansions by perturbing the system with a (one standard deviation) supply shock of the appropriate sign. Concurrently, we induce a (one standard deviation) monetary policy shock. Finally, we subtract the reaction to the supply shock from the overall response of the system to both sources of exogenous perturbation, so as to isolate the effects of monetary innovations.
Figure 4. Impulse responses to supply shocks.

Notes. We portray equilibrium dynamics in response to positive and negative supply shocks of different magnitudes, specifically: 2 standard deviations (continuous line), 1.5 standard deviations (dashed line), 0.5 standard deviation (dotted line). The green (light) line is associated with a negative supply shock, while the blue (dark) line is associated with a positive supply shock.
Figure 5. Responses to monetary policy innovations.

Notes. The blue-continuous line portrays cumulative responses to monetary policy innovations, while the green-dashed line accounts for impact responses. The magnitude of the monetary policy innovation is measured over the x-axis.
Notes. Each contour line refers to cumulative responses (LHS panel) or impact responses (RHS panel) to monetary innovations of different magnitudes under contractionary and expansionary regimes induced by supply shocks with different signs and magnitudes. The monetary policy innovation is measured on the x-axis, while the supply innovation is measured on the y-axis.
Figure 7. Efficient policy frontiers.

Notes. The dashed (continuous) line corresponds to the locus of different combinations of $\sigma_\pi$ and $\sigma_y$, for varying values of $\varrho$ in the expansionary (contractionary) state.