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Torben Schmith, Søren Johansen, Peter Thejll

Øster Farimagsgade 5, Building 26, DK-1353 Copenhagen K., Denmark
Tel.: +45 35 32 30 01 – Fax: +45 35 32 30 00
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Statistical analysis of global surface air temperature and sea level using cointegration methods

Torben Schmith[1]
Danish Meteorological Institute, Copenhagen, Denmark

Søren Johansen
University of Copenhagen, Copenhagen, Denmark
and CREATES, Aarhus University, Denmark

Peter Thejll
Danish Meteorological Institute, Copenhagen, Denmark

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[1] *Corresponding author address*: Torben Schmith, Danish Meteorological Institute, Lyngbyvej 100, DK-2100 Copenhagen Ø, Denmark.

E-mail: ts@dmi.dk

Abstract

Global sea levels are rising which is widely understood as a consequence of thermal expansion and melting of glaciers and land-based ice caps. Due to physically-based models being unable to simulate observed sea level trends, semi-empirical models have been applied as an alternative for projecting of future sea levels. There is in this, however, potential pitfalls due to the trending nature of the time series. We apply a statistical method called cointegration analysis to observed global sea level and surface air temperature, capable of handling such peculiarities. We find a relationship between sea level and temperature and find that temperature causally depends on the sea level, which can be understood as a consequence of the large heat capacity of the ocean. We further find that the warming episode in the 1940s is exceptional in the sense that sea level and warming deviates from the expected relationship. This suggests that this warming episode is mainly due to internal dynamics of the ocean rather than external radiative forcing. On the other hand, the present warming follows the expected relationship, suggesting that it is mainly due to radiative forcing. In a second step, we use the total radiative forcing as an explanatory variable, but unexpectedly find that the sea level does not depend on the forcing. We hypothesize that this is due to a long adjustment time scale of the ocean and show that the number of years of data needed to build statistical models that have the relationship expected from physics exceeds what is currently available by a factor of almost ten.

1. Introduction

Sea level rise is one of the important societal aspects of climate change but unfortunately there is large uncertainty about expected changes. The most recent assessment report from the IPCC (IPCC, 2007) specifically mentions that the published projections can not be taken as the upper limit because the full effect of Greenland ice streams and other effects are only partially included due to lack of understanding of the relevant processes.

As an alternative to process-based, physical models, Rahmstorf (2007) suggested to apply semi-empirical models and he formulated a statistical relationship between global averages and surface temperature and rate of sea level rise. The work caused some debate (Holgate et al., 2007; Schmith et al., 2007; von Storch et al., 2008; Rahmstorf, 2008), which led to a modification of the model (Vermeer and Rahmstorf, 2009) debated in Taboada and Anadón (2010) and Vermeer and Rahmstorf (2010). Parallel to this, Grinsted et al. (2010) and Jevrejeva et al. (2009, 2010) formulated similar semi-empirical models. Common to these semi-empirical methods is that they yield larger projected global sea level changes by the end of this century than stated in IPCC (2007) and therefore they have caused concern.

Our reservation about the statistical approach applied in the above work is that it does not take the possibility of random walk character and the resulting stochastic trends into account. This implies a risk of misleading and biased determination and estimation of statistical models (Yule, 1926; Granger and Newbold, 1974).

The presence of stochastic trends in climate series is well-documented (e.g. Kaufmann and Stern, 1997; Kaufmann and Stern, 2002; Kaufmann et al., 2006a; Kaufmann et al., 2006b; Richards, 1993, 1998; Stern and Kaufmann, 1999; Stern and Kaufmann, 2000, and references therein). Within the field of econometrics there exists an extensive body of literature on analysis of time series containing a combination of stochastic and deterministic trends. We find it peculiar that these analysis tools, which are applied on a routine-basis in economical analyses, are rarely applied in the analysis of climate data, with Kaufmann and Stern (1997, 2002), Kaufmann et al. (2006a, 2006b), Liu and Rodriguez (2005), Mills (2009), Stephenson et al. (2000), Stern (2006), Stern and Kaufmann (1999, 2000) as notable exceptions. A central concept in these works is cointegration, which is also the backbone in the method we will apply. The concept of cointegration and common stochastic trends is due to Clive W. J. Granger, who in 2003 was awarded the Nobel Prize in economics for this. A popular explanation of cointegration can be found in Murray (1994).

2. Data

We use annual averages of the combined land-ocean surface temperature data described in Hansen et al. (2010) and shown in Figure 1a. As primary global sea level dataset we use Church and White (2006) constructed by combining satellite altimeter data from the beginning of the 1990es with conventional sea level records to obtain a record of global average sea level back to 1880. Alternatively, we use Jevrejeva et al. (2009) who use a ‘virtual stacking method’, based solely on conventional sea level records. These are shown in Figure 1b. Radiative forcing data on an annual basis from natural and anthropogenic sources, separated into: volcanic, solar, green house gases and man-made tropospheric aerosols were taken from Crowley (2000). As an alternative forcing dataset, we used Myhre et al. (2001) with in total nine anthropogenic and natural forcing components. For each of these datasets, we calculate the total radiative forcing by simple addition of the individual components. These are shown in Figure 1c. These data are in the following analyzed in the overlapping period 1880-1998.

The sea level has been almost steadily increasing almost steadily while the temperature has a more complicated development with an increase until around 1940, then a slight decrease until around 1980, followed by an increase. In particular, we note that the increase in sea level begins before the warming (and the increase in forcing). The forcing has also been steadily increasing throughout the period with intermittent negative spikes caused by major volcanic eruptions. None of these negative forcing spikes occur in the 1930s, 1940s and 1950s.

The resasons for the temperatures' complicated behavior is still unclear: It could be due to variations in the external forcing, mainly anthropogenic and/or volcanic aerosols, and/or it could be due to internal variability in the climate system. Studies using coupled atmosphere-ocean GCMs arrive at the conclusion that the full range of anthropogenic and natural external forcings can account for the gross features in the observed global temperature changes over the 20th century while natural external forcings alone can not (e.g. Brocolli et al., 2003; Knutson et al., 2006; Stott et al., 2006). Other studies claim that a residual remains, which can not be accounted for by external forcing (Andronova and Schlesinger, 2000; Kravtsov and Spannagle, 2008). It is our aim that the present statistical analysis will shed light on this problem also.

3. Method

Since the method we intend to apply is non-standard within the climate community, we will explain it in some detail. More comprehensive descriptions can be found in e.g. Hendry and Juselius (1999, 2000) and Juselius (2006).

To begin with, we consider the bivariate time series (T_t, h_t) of the annual global anomalies of surface temperature and sea level. We model it statistically as a vector-autoregressive (VAR) model, which in its simplest form (first order) is

$$\begin{pmatrix} T_t \\ h_t \end{pmatrix} = \mathbf{A_1} \begin{pmatrix} T_{t-1} \\ h_{t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_{Tt} \\ \varepsilon_{ht} \end{pmatrix}, \qquad (1)$$

where $\mathbf{A_1}$ is a 2×2 matrix and $(\varepsilon_{Tt}, \varepsilon_{ht})$ is bivariate white noise. The model (1) is the multivariate analogue of a univariate first order autoregressive (AR) model. The properties of the matrix $\mathbf{A_1}$ determines the properties of the VAR-process, as will be demonstrated below. This is equivalent to the role which the lag-one autocorrelation coefficient plays in a univariate AR-model. In the univariate case, whenever this parameter is strictly less than one in absolute value, the process adjusts to its mean value, with longer adjustment time scale the large the parameter. When the parameter equals unity the adjustment time scale becomes infinitely long, i.e. the process is a random walk without any affinity to approaching its mean value (or any other value). Rather, such a process exhibit stochastic trends over long periods of time.

We reformulate our bivariate VAR-model as

$$\begin{pmatrix} \Delta T_t \\ \Delta h_t \end{pmatrix} = \mathbf{\Pi} \begin{pmatrix} T_{t-1} \\ h_{t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_{Tt} \\ \varepsilon_{ht} \end{pmatrix}, \tag{2}$$

where $\Delta T_t = T_t - T_{t-1}$ and $\Delta h_t = h_t - h_{t-1}$ are the differences of global temperature and sea level respectively. The matrix $\mathbf{\Pi}=\mathbf{A_1}\text{-}\mathbf{1}$ is called the impact matrix.

The rank of $\mathbf{\Pi}$ determines the properties of the process, as will be illustrated in the following. If the rank equals two (full rank) the model (2) describes two correlated and stationary processes (Figure 2a). If the rank equals zero, all elements in $\mathbf{\Pi}$ are zero and the process (2) describes two dynamically unrelated random walk processes (Figure 2b). Finally, when the rank equals one (Fig 2c) the two time series exhibits non-stationarity and stochastic trends, like the rank zero case. Unlike that case, however, the upward/downward stochastic trends of the two series appear concurrently. We say that the two series share a common stochastic trend – they are cointegrated. Cointegration can be thought of as the nonstationary analogue to correlation.

This can be clarified further. If the impact matrix has rank one, we can write

$$\mathbf{\Pi} = \begin{pmatrix} \alpha_T \\ \alpha_h \end{pmatrix} \begin{pmatrix} \beta_T & \beta_h \end{pmatrix}, \tag{3}$$

and (2) can be re-written as

$$\begin{aligned} \begin{pmatrix} \Delta T_t \\ \Delta h_t \end{pmatrix} &= \begin{pmatrix} \alpha_T \\ \alpha_h \end{pmatrix} \begin{pmatrix} \beta_T & \beta_h \end{pmatrix} \begin{pmatrix} T_{t-1} \\ h_{t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_{Tt} \\ \varepsilon_{ht} \end{pmatrix} \\ &= \begin{pmatrix} \alpha_T \\ \alpha_h \end{pmatrix} \cdot z_{t-1} + \begin{pmatrix} \varepsilon_{Tt} \\ \varepsilon_{ht} \end{pmatrix}, \end{aligned} \tag{4}$$

where

$$z_t = \beta_T T_t + \beta_h h_t . \tag{5}$$

called the disequilibrium error, can be shown to be stationary (e.g. Juselius, 2006).

The model (4)-(5) is called the error correction form of the VAR, abbreviated VECM, which we recall is only possible if the two series are cointegrated. By the transformation to the VECM-description the two coupled equations for the differences in the VAR-description separates into two equations relating the differences of the original variables, T_t and h_t to the disequilibrium error, z_t and random and uncorrelated disturbances.

Since the disequilibrium error (5) itself depends of the original variables, (4) represents adjustment to the line

$$z_t = \beta_T T_t + \beta_h h_t = 0 , \tag{6}$$

called the cointegration line. Insight into the adjustment process, determined by the adjustment coefficients α_T and α_h may give valuable information on the underlying physics of the system. These coefficients determine time scales, with which each of the two variables adjust to the cointegration line, and therefore the adjustment time will in general be different for the two variables. In practice, only estimates of the adjustment coefficients are available and therefore inference on these becomes crucial. In particular, it could be that one coefficient is virtually zero, which means that this variable does not adjust to the cointegration line.

The VECM described by (4)-(5) is the simplest one. We recall that it is developed from a VAR model where present variable values were dependent of values from the previous time step only and therefore it is of order one. The VECM of any order could be formulated. For instance, the VECM of order two, i.e. with variable values from the two previous time steps, and with deterministic drift terms reads

$$\begin{pmatrix} \Delta T_t \\ \Delta h_t \end{pmatrix} = \begin{pmatrix} \alpha_T \\ \alpha_h \end{pmatrix} \cdot z_{t-1} + \mathbf{\Gamma_1} \begin{pmatrix} \Delta T_{t-1} \\ \Delta h_{t-1} \end{pmatrix} + \begin{pmatrix} \mu_T \\ \mu_h \end{pmatrix} + \begin{pmatrix} \varepsilon_{Tt} \\ \varepsilon_{ht} \end{pmatrix}. \tag{7}$$

Here $\mathbf{\Gamma_1}$ is a 2×2-matrix of coefficients describing the direct response of the differences to their values at the previous time step, called the short run dynamics. The terms μ_T and μ_h are linear drift terms.

Certain choices must be made regarding the form of the drift terms and this requires some attention. With no restrictions (called 'unrestricted constant') on the drift terms in (7) deterministic linear trends may be present in the series and it can be shown that these trends are related to the drift terms. In the 'restricted constant' case we require that $\begin{pmatrix} \mu_T & \mu_h \end{pmatrix}$ is proportional to $\begin{pmatrix} \alpha_T & \alpha_h \end{pmatrix}$, and with the constant of proportionality being z_0 the system (7) then reduces to:

$$\begin{pmatrix} \Delta T_t \\ \Delta h_t \end{pmatrix} = \begin{pmatrix} \alpha_T \\ \alpha_h \end{pmatrix} \cdot (z_{t-1} + z_0) + \mathbf{\Gamma_1} \begin{pmatrix} \Delta T_{t-1} \\ \Delta h_{t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_{Tt} \\ \varepsilon_{ht} \end{pmatrix} \tag{8}$$

This means that the series have no deterministic trends but the disequilibrium error

$$z_t^{'} = \beta_T T_t + \beta_h h_t + z_0 \tag{9}$$

can have an arbitrary non-zero mean level determined by the drift terms. As can be seen from (9) this is equivalent to taking arbitrary non-zero offsets of the variables T_t and h_t into account.

In applications of VAR model fitting, the first step is to determine the order of the model. This is a balance between wanting a parsimonious model on one hand and retaining a sufficient number of lags to ensure normally distributed and uncorrelated residuals on the other hand. This is because having residuals with these properties ensures valid statistical inference.

The technique to ensure this is to begin with fitting a high order model. The residuals $\hat{\varepsilon}_{Tt}$ and $\hat{\varepsilon}_{ht}$ are then examined with respect to the occurrence of time trends and heteroscedasticity, their histograms are examined with respect to being normal and their auto correlation functions are examined with respect to being zero for nonzero lags. If all conditions are fulfilled, one repeats the procedure with lower order models and the adequate order is the lowest order where all conditions are fulfilled.

Next we determine the rank of the impact matrix Π, since this determines whether or not the two series are cointegrated and the VECM-formalism therefore can be applied. Since only an estimate of Π is available, the rank of Π can not be exactly determined, but will be determined by a statistical procedure described in Johansen (1988). This procedure begins by assuming rank zero and then ‘testing up’, i.e. if rank zero is rejected at the significance level used, rank one is assumed, and so on.

If cointegration is found, our VAR model is re-formulated as a VECM, as described above and can be estimated using the maximum-likelihood procedure described in Johansen and Juselius (1990). This includes the estimation of all parameters in the model and their confidence intervals and significance test statistics.

Engle and Granger (1987) pointed out that once the disequilibrium error z_t has been determined, then the estimation of (7) or (8) can be done by ordinary lest-squares regression analysis of the two equations. Here ordinary test-statistics, like t-test, can be used and makes is easy to test the significance of each independent variable and make revised estimates with insignificant variables omitted.

4. Application of the method

We apply the method described in the previous section to the global temperature series and the sea level series by Church and White (2006). We expect both series to exhibit an upward deterministic trend arising from the increase through time of the external radiative forcing and therefore we assume the 'unrestricted constant' case, see (7), in the following analysis.

Since the series of differences play a central role in this type of analysis it is illustrative to plot these two derived series (Figure 3). These appear stationary and much more like 'ordinary' time series when compared to the original series shown in Figure 1a and 1b. The fact that the difference series appear stationary is a sign that the original series have random walk character. The aim of applying the methodology described in the previous section is to reveal whether they are co-integrated.

By inspecting the residuals from fitted VAR-models of successively lower order, as described in the previous section, we find that at VAR-model of order two is a satisfactory description of the data. When applying the cointegration test we conclude that the impact matrix $\mathbf{\Pi}$ has rank one, since we obtain a p-value around 0.005 for the 'rank zero' hypothesis, which we therefore reject, while we obtain a p-values of 0.25 for the 'rank one' hypothesis, which we therefore cannot reject. Based on this, we regard the two series as cointegrated and describe their mutual time-development as a VECM of order two.

Since the two series are cointegrated, we estimate the VECM using the maximum-likelihood procedure mentioned in the previous section and find the disequilibrium error:

$$z_t = T_t - \underset{(0.4)}{3.3}\left[\mathrm{K\ m^{-1}}\right] h_t \qquad (10).$$

This relationship describes a line, to which temperature and sea level adjust over time. We note that the disequilibrium error (10) implies that increasing temperatures and increasing sea level anomalies are connected as expected. Throughout this paper, a number in parenthesis below an estimated parameter is the standard error of the estimate.

Thermal expansion of the ocean is responsible for a large portion of the observed sea level rise. This means that sea level depends on the average ocean temperature over a characteristic layer of depth H involved in the thermal expansion. Approximately, we have: $h_t \approx \kappa H T_t$, where $\kappa \approx 20 \cdot 10^{-5}$ K^{-1} is a typical value of the volumetric thermal expansion coefficient of sea water, and by comparing with the cointegration relation we see that $\beta_h = -\frac{1}{\kappa \cdot H}$, from which

$$H = -\frac{1}{\beta_h \kappa} \approx \frac{1}{3.3 \cdot 20 \cdot 10^{-5}} \approx 10^3 \text{ m} \tag{11}$$

Thus, the thermal expansion is not confined to the mixed layer, which has a thickness in the order of 100 m, but rather, the results indicates that the deeper water masses are also heating. This is in accordance with Barnett et al. (2001).

As mentioned in section 2, the disequilibrium error should be a stationary series, i.e. without any random walk character. This is confirmed by plotting z_t given by (10) against time (Fig. 4), although it admittedly contains some decadal-term deviations, e.g. around 1940. This means that the series can not be described as a simple autoregressive-type relaxation. We will discuss the temporal evolution of the disequilibrium error later in the paper.

Next we estimate the parameters in the equations (7) for the difference series. As explained in section 2, we do that by multiregression analysis and find that the statistical significance, estimated by an ordinary t-test, differs among the parameters. Retaining only significant terms, the equations reduce to:

$$\begin{aligned} \Delta T_t &= -0.4 z_{t-1} - 0.03 \text{ [K]} \\ &\quad\;\; (0.1) \quad\;\; (0.01) \end{aligned} \tag{12}$$

and

$$\Delta h_t = -0.3\Delta h_{t-1} + 0.002 \; [\mathrm{m}] \qquad (13).$$
$$\quad (0.1) \qquad (0.001)$$

In (12), the estimated significantly negative value for α_T means that the temperature reacts to disequilibrium by adjusting to the cointegration line. The other terms, short-run dynamics and drift term are non-significant. In (13), we find in (13) that α_h is not statistically different from zero which means that sea level does not seem to react to disequilibrium. We call sea level a weakly exogeneous variable. From (13) it is evident that the differences Δh_t have a positive mean value equivalent to a deterministic trend, described by the drift term. In addition to this there is a quite strong persistence of the differences.

The different properties of the adjustment coefficients in the two equations (12-13) can be illustrated by the partial residual scatterplot of ΔT_t and Δh_t ,both corrected for any linearly dependence on ΔT_{t-1} and Δh_{t-1}, against z_{t-1}, displayed in Figure 5. Beginning with Figure 5b, the estimated $\alpha_h = 0$ is evidently due to lack of any relationship between the two variables and not due to e.g a quadratic relationship. Continuing with Figure 5a, the linear relationship appears convincing and not due to single outlying and therefore influential observations.

In summary, our statistical analysis shows that the points (T_t, h_t) tend to be clustered along the cointegration line in the (T_t, h_t)-plane. The two variables, however, behave very differently. The differences in the global sea level exhibit a deterministic positive trend, representing the influence of the radiative forcing, while the differences of the global temperature have the role of keeping (T_t, h_t) near the cointegration line.

We can understand this result from a physical point of view as follows: due to the much larger heat capacity of the ocean compared to that of the atmosphere, the surface air temperature adjusts to the average temperature of the upper ocean, which, as remarked earlier, is strongly related to the sea level due to thermal expansion. The result is therefore in opposition to the physical thinking behind the model proposed by Rahmstorf (2007), where adjustment of the sea level to temperature is assumed.

To investigate whether the reason for the above was due to the specific nature of the sea level data used, we redid the whole analysis using the sea level series by Jevrejeva et al. (2009), but this left all of the above results virtually unchanged.

5. Cointegration analysis conditioned on the external radiative forcing

In the analysis presented until now, the forcing of the sea level rise comes about through a deterministic trend, determined by the drift parameter μ_h. Since we expect the changes in the external radiative forcing to be responsible for this trend, it would be satisfactory if we were able to explain this trend in the sea level directly by the external forcing rather than just *ad hoc* as a parameter estimated in the analysis.

The cointegration methods described in section 3 can be extended to take so-called strictly exogeneous variables into account. These are variables, which are external to the model. The VAR model with exogeneous variable is fitted and cointegration is tested for based on the rank of the impact matrix $\mathbf{\Pi}$, which is now a 2×3 matrix. We will regard the total external radiative forcing F_t as a strictly exogeneous variable and can formulate a second order VAR-model as:

$$\begin{pmatrix} \Delta T_t \\ \Delta h_t \end{pmatrix} = \mathbf{\Pi} \begin{pmatrix} T_{t-1} \\ h_{t-1} \\ F_{t-1} \end{pmatrix} + \boldsymbol{\omega} \cdot \Delta F_t + \mathbf{\Gamma}_1 \begin{pmatrix} \Delta T_{t-1} \\ \Delta h_{t-1} \\ \Delta F_{t-1} \end{pmatrix} + \begin{pmatrix} \mu_T \\ \mu_h \end{pmatrix} + \begin{pmatrix} \varepsilon_{Tt} \\ \varepsilon_{ht} \end{pmatrix}, \tag{14}$$

which in the case of cointegration can be transformed to

$$\begin{pmatrix} \Delta T_t \\ \Delta h_t \end{pmatrix} = \begin{pmatrix} \alpha_T \\ \alpha_h \end{pmatrix} \cdot z_{t-1} + \boldsymbol{\omega} \cdot \Delta F_t + \mathbf{\Gamma}_1 \begin{pmatrix} \Delta T_{t-1} \\ \Delta h_{t-1} \\ \Delta F_{t-1} \end{pmatrix} + \begin{pmatrix} \mu_T \\ \mu_h \end{pmatrix} + \begin{pmatrix} \varepsilon_{Tt} \\ \varepsilon_{ht} \end{pmatrix}. \tag{15}$$

As in (7), the model (15) relates the differences to a combination of adjustment terms , in which the disequilibrium error now has the external forcing included and is given by:

$$z_t = \beta_T T_t + \beta_h h_t + \beta_F F_t, \tag{16}$$

short run dynamics determined by the 2×1-matrix ω and the 2×3-matrix Γ, drift terms and random disturbances.

We conduct an analysis with the total external forcing calculated from Crowley (2000) specified as a strictly exogenous variable. We thus ignore any feedback from changes in global temperature on e.g the atmospheric CO_2 concentration.

As previously explained, our motivation for introducing the total radiative forcing as a strictly exogeneous variable in the model was to replace the deterministic trend of sea level as a driver of the model. As a consequence of this, we expect no deterministic trends in our VAR-model (14) and should test and estimate it under the 'restricted constant' assumption. However, this estimating yields residuals $\hat{\varepsilon}_{ht}$ (not shown) with non-zero average, while the residuals $\hat{\varepsilon}_{Tt}$ have close to zero average. This is a sign of that there is a trend in the sea level not explainable by the external forcing and that the model does not fit the data satisfactorily. We therefore continue our analysis using the 'unrestricted constant' assumption.

We conduct the cointegration test and find that the impact matrix Π has rank one, since we obtain a p-value of < 0.03 for the 'rank zero' hypothesis, which we therefore reject, while we obtain a p-values of 0.01 for the 'rank one' hypothesis, which we therefore cannot reject. Therefore, a reasonable description of the data is the cointegrated model (15)-(16).

Estimating (16) under the 'unrestricted constant' assumption, we get the disequilibrium error:

$$z_t = T_t - \underset{(0.6)}{1.4}\left[\mathrm{K\,m^{-1}}\right] h_t - \underset{(0.1)}{0.3}\left[\mathrm{K}\left(\mathrm{W\,m^{-2}}\right)^{-1}\right] F_t \tag{17}$$

For a fixed total forcing F_t (17) defines an equilibrium relationship between T_t and h_t and we note as in the unconditional model that increasing temperatures and increasing sea level anomalies are connected. The coefficient of h_t has been reduced from -3.3 K m^{-1} in the unconditional case to -1.4 Km^{-1} in the conditional case.

We plot the disequilibrium error for both the unconditional and the conditional model (Fig. 6) and see that this variable is almost unchanged, which tells us that the effect of introducing the external radiative forcing as exogeneous variable is small. There is still a large positive anomaly around the 1940s, which means that this anomaly can not be explained by the external forcing.

Having determined the cointegration relation and the associated disequilibrium error, let us now turn to the equations for the differences ΔT_t and Δh_t. Estimating these and retaining only significant terms gives:

$$\underset{}{\Delta T_t} = \underset{(0.1)}{-0.5 z_{t-1}} + \underset{(0.02)}{0.08} \left[\mathrm{K}\left(\mathrm{Wm}^{-2}\right)^{-1}\right] \Delta F_t + \underset{(0.02)}{0.11} \, [K] \tag{18}$$

and

$$\Delta h_t = \underset{(0.1)}{-0.3 \Delta h_{t-1}} + \underset{(0.001)}{0.002} \, [\mathrm{m}] \tag{19}$$

For the differences in temperature, the first term in (18) is an adjustment to the equilibrium cointegrating relation, while the second term represents a short run response in temperature to an increase in total forcing. Note that the drift term in (18) vanishes. This means that differences in the temperature are explained by either disequilibrium from the cointegration line or by immediate differences in the external forcing.

For the differences in sea level, we see from (19) that the sea level also in this model acts like a weakly exogeneous variable, in fact is (19) identical to (13). This unexpected result will discussed later.

We expect the immediate (within one year) difference in temperature arising from changes in the external forcing to be determined by the effective mixed layer depth of the ocean, *D*. Since we used the combined land-ocean surface temperature throughout our analysis, we can with a good approximation assume that this temperature represents the average ocean mixed layer temperature. In that case we can write the enrgy budget for the ocean mixed layer:

$$D\rho c_w \Delta T_t = \Delta F_t \cdot \Delta t\,, \tag{20}$$

with Δt being the length of one year, from which we get

$$\Delta T_t = \frac{\Delta t}{D\rho c_w}\Delta F_t\,. \tag{21}$$

From (21) it is evident that we can give an estimate of *D* by determining the total effect of ΔF_t on ΔF_t. This consist of the direct effect, determined by ω_T and the effect through z_{t-1} determined by $\alpha_T\gamma$, where $\gamma = -0.04\left[K\left(Wm^{-2}\right)^{-1}\right]$ is the regression coefficient of z_{t-1} regressed on ΔF_t.

By comparing this with (21) we get

$$\frac{\Delta t}{D\rho c_w} = \alpha_T\gamma + \omega_T = (-0.5)\cdot\left\{-0.04\left[K\left(Wm^{-2}\right)^{-1}\right]\right\} + 0.08\left[K\left(Wm^{-2}\right)^{-1}\right] = 0.10\left[K\left(Wm^{-2}\right)^{-1}\right], \tag{22}$$

and we can estimate the effective mixed layer depth of the ocean as

$$D_{ml} = \frac{\Delta t}{\left(\alpha_T\gamma + \omega_T\right)\rho c_w} = \frac{3600\cdot 24\cdot 365{,}25\ \mathrm{s}}{0.10\ \mathrm{K}\ \left(\mathrm{W\ m^{-2}}\right)^{-1}\cdot 10^3\ \mathrm{kg\ m^{-3}}\cdot 4\cdot 10^3\ \mathrm{J\ kg^{-1}\ K^{-1}}} \approx 80\ \mathrm{m}\,, \tag{23}$$

which is a realistic typical value for the ocean mixed layer and thus confirms our hypothesis.

We repeated the whole analysis using the total forcing calculated from Myhre et al. (2001) instead. This caused only minor changes to the estimation of (17) and also the form of (18)-(19) was virtually unchanged as well as estimation of the parameters. In particular we note that also here the differences of the sea level do not depend on the disequilibrium error.

Summing up for this second part of the analysis with the external forcing included, equation (19) describing the adjustment of sea level contains no term representing adjustment to the cointegration line. Thus, sea level still behaves like a weakly exogeneous variable, i.e. it is neither affected by the external forcing nor reacts to disequilibrium from the cointegration.

6. Discussion

a. Robustness of the analysis

Whereas there is more or less agreement between different estimates of global surface temperature, this is not the case for estimates of global average mean sea level. This is because the basic data, which are historical sea level records, are sparsely distributed and mainly along the coastline, and the interior ocean completely lacks information on sea level prior to the satellite era. This leaves room for variations due to the methodology applied in calculating the global average (see Christiansen et al., 2010).

There is also much uncertainty about the historical radiative forcing. This is because both the series of historical concentrations of greenhouse gases, aerosols and other radiative constituents as well as the model to transform these concentrations to net forcing differ.

We carried out the above analysis with two different global sea level time series and two different time series of external forcing and saw that the main conclusions remained unchanged, which gives confidence to our results.

b. Causes for the warming in the 1940s and the present warming

We have seen the disequilibrium error exhibit decadal-scale undulations from its average. The most prominent one is the period of above 0.1K and up to 0.2 K positive deviations around 1940. In other words, in this period surface temperatures are relatively high or, equivalently, sea level relatively low. This period, known as the Early 20th Century Warming (E20CW), is known as a period with relative high surface temperatures, averaged over the globe and in particular over the North Atlantic and the Arctic (e.g. Johannessen et al., 2004; Polyakov et al., 2003; Wood and Overland, 2009). The 1960s and 1970s are characterized by moderately negative disequilibrium anomalies, while the period from 1980 onwards have smaller positive values of the disequilibrium error. This last period is known as the Present Warming (PW). This intriguing difference in the disequilibrium error between the two warming periods of the 20th century and this difference may tell us something about the different nature of these warming periods.

Whereas the PW is generally ascribed to increased external forcing from greenhouse gases, is still unclear whether the E20CW is entirely caused by changes in the external forcing due to changes in solar or volcanic activity or the emission of man-made aerosols (Stott et al., 2000) or can in part be attributed to a multidecadal variability mode in thermohaline overturning circulation of the world ocean (Delworth & Knutson, 2000).

Polyakov et al. (2009) used hydrographic observations to identify such a variability mode in the North Atlantic with maximum amplitude around 1940 and with an out-of-phase relation between the temperature in the surface of the ocean and at depth. We would expect a quite modest signal in the sea surface height from such a mode, since expansion at the top would partly be compensated by contraction at depth and vice versa. Therefore, we would see a high disequilibrium error in such a situation and therefore our analysis supports the view that the E20CW is at least in parts attributable to an anomaly in the thermohaline overturning circulation.

If the PW is mainly cased by radiative anomalies and only to a minor degree by anomalies in the overturning circulation, we would not have the expansion/contraction described above and therefore we would see modest disequilibrium errors. This is indeed what we have found from our analysis, from which we conclude that there is a minor contribution from overturning anomalies but the main contribution is from radiative forcing.

c. Investigating the reasons behind the undetected adjustment of sea level: Monte Carlo experiments

In our last analysis with forcing included we would a priori expect that sea level changes were driven by the total external radiative forcing. Therefore it is surprising that sea level does not react to the disequilibrium error given by (17).

Could this result be an effect of having a too short period of data at hand? To investigate this we performed Monte Carlo experiments based on the model described in eq. (15) as follows. A more than 1100 years long time series of external forcing was constructed from the forcing series by Crowley et al. (2000) by repeating the series a sufficient number of times and in each repetition add an offset to the series. The offset has a magnitude so that the final series is steadily increasing. We then use this long forcing series to construct, in a Monte Carlo simulation, artificial series of T_t and h_t using the equations (17-19) with the important difference that the term $\alpha_h z_{t-1}$ is retained in eq (19) with α_h set equal to its central estimate. The white-noise series ε_{Tt} and ε_{ht} are different in each MC-realisation. The series constructed in this way are therefore per definition cointegrated. We then estimate a VECM-model described in (15). This estimation is done based on both year 101-200 (‘short’ case) and year 101-1100 (‘long’ case) of the artificial series. Based on the MC-simulation, we finally make histograms of the estimated adjustment coefficients.

Looking at these histograms, shown in Fig. 7, we note that the adjustment coefficient α_T is symmetrically distributed around its specified value of -0.49, and the histogram is clearly separated from zero both in the ’long’ and ’short’ case. For the adjustment coefficient α_h, the adjustment coefficient is symmetrically distributed around its true value of 2.4 m K^{-1} but with a large spread, so that the distribution is not clearly separated from zero for the ‘short’ case. For the ‘long’ case, the distribution is almost separable from zero. From this we conclude, that we need on the order of 1000 years of data in order to detect any reaction of the sea level to disequilibrium.

7. Summary and concluding remarks

We have introduced a statistical tool, rather unknown within geophysics but widely used within the field of econometrics, to the problem of analyzing the relationship between global surface air temperature and sea level. We have found that the time series for global surface temperature and sea level both contain stochastic trends and therefore the risk of spurious regression is present which in turn justifies the used of these methods. We applied found that the two time series shared a common stochastic trend, i.e. they cointegrate. This is in accordance with our physical expectations, since we expect both series to be driven by the external radiative forcing. Accordingly, the common time-development can be described by a VECM introduced in the beginning of the paper.

Three main conclusions can be drawn by fitting the VECM to data. First, we show that temperature adjusts to the cointegration equilibrium, while there is no detectable adjustment of sea level. We can understand this in physical terms. The ocean represents a much larger heat capacity than the atmosphere and therefore the surface air temperature adjusts to the average temperature of the upper ocean, which again is proportional to the sea level anomaly. This mechanism as also been shown in GCM modeling experiments (Hoerling et al., 2008; Dommenget, 2009).

Secondly, by studying the disequilibrium error we have learned that there is an important difference between the two warming periods during the 20th century: While the present warming is mainly caused by changes in the external radiative forcing, the warming period in the 1940s seems to be at least partly connected with multidecadal variations internal to the climate system and connected to the thermohaline overturning circulation of the ocean.

In the second part of our analysis we introduced the external radiative forcing as an external, explanatory variable. Contrary to our expectations, this did not extensively change our overall conclusions. We expected the forcing to determine the development of sea level, but could not find any sign of this in our data material. A Monte Carlo experiment revealed that we need in the order of 1000 year of data in order to detect this effect. This is in contrast with results from global climate model experiments, where a clear signal from expansion of the sea water is seen in 20th century integrations (e.g. Gregory et al., 2001) as a result of the increased forcing. At present, we can only suggest explanations for this discrepancy. As seen on Fig1b, the sea level has been rising also well before the forcing (Fig. 1c) increases in the mid-20th century. This secular sea level rise seems to have persisted for at least some centuries (Kearney, 2001) and may dominate even today, which may be the reason why we can not detect any influence from the external radiative forcing on sea level.

Another matter is data quality. As pointed out by Munk (2001) and Woodworth (2006) there is a large discrepancy between the observed sea level rise and the contributions from the different components, when seen over the whole 20th century. One explanantion for this 'enigma' could be the quality of the estimated historical sea level back in time, and this would probably also affect our analysis. Besides that, the confidence we can have in the historical forcings is unknown. One meaningful next step would therefore be to do a similar analysis on a transient coupled GCM experiment, to see whether we arrive at similar conclusions when using such consistent data.

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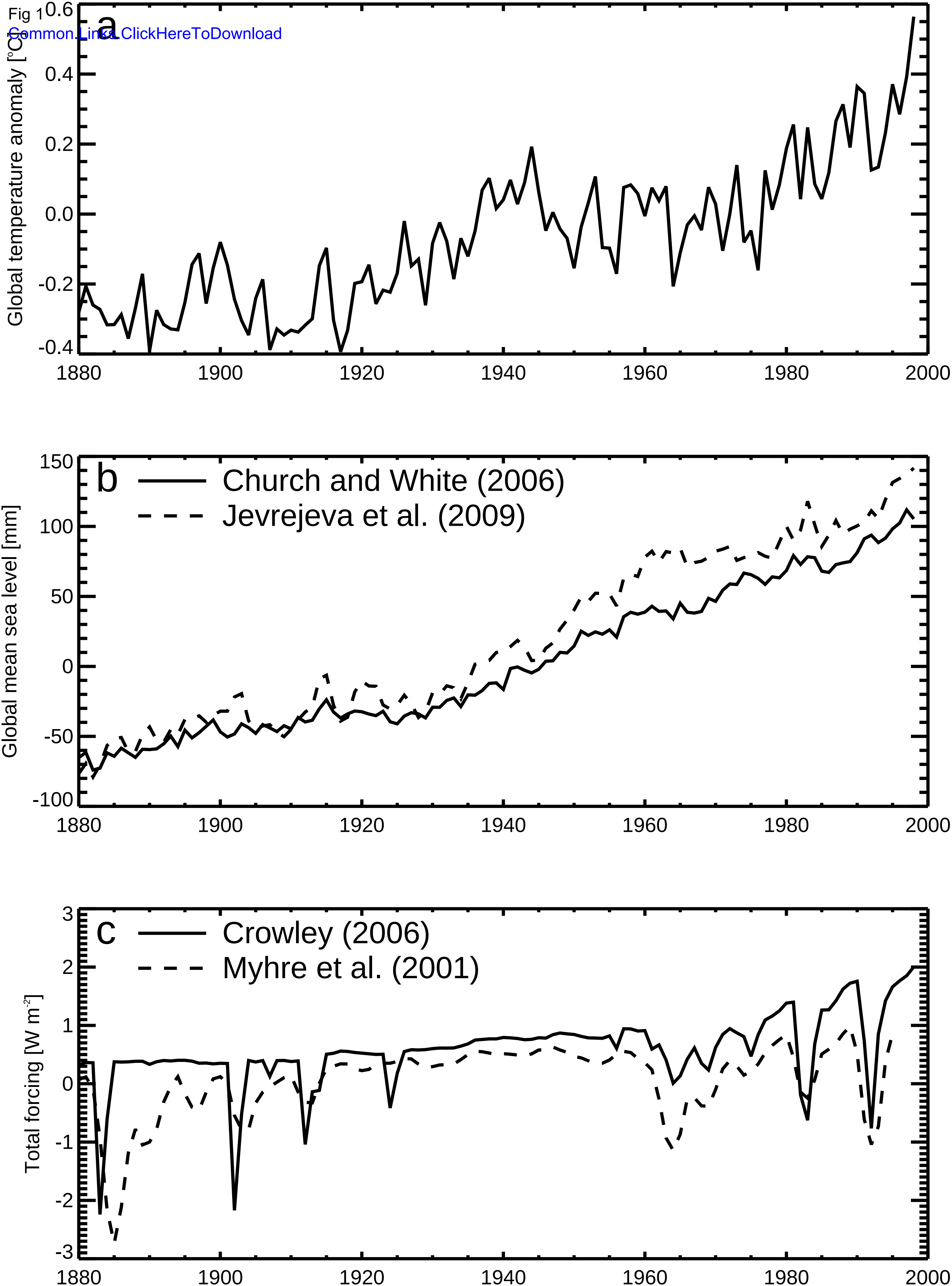

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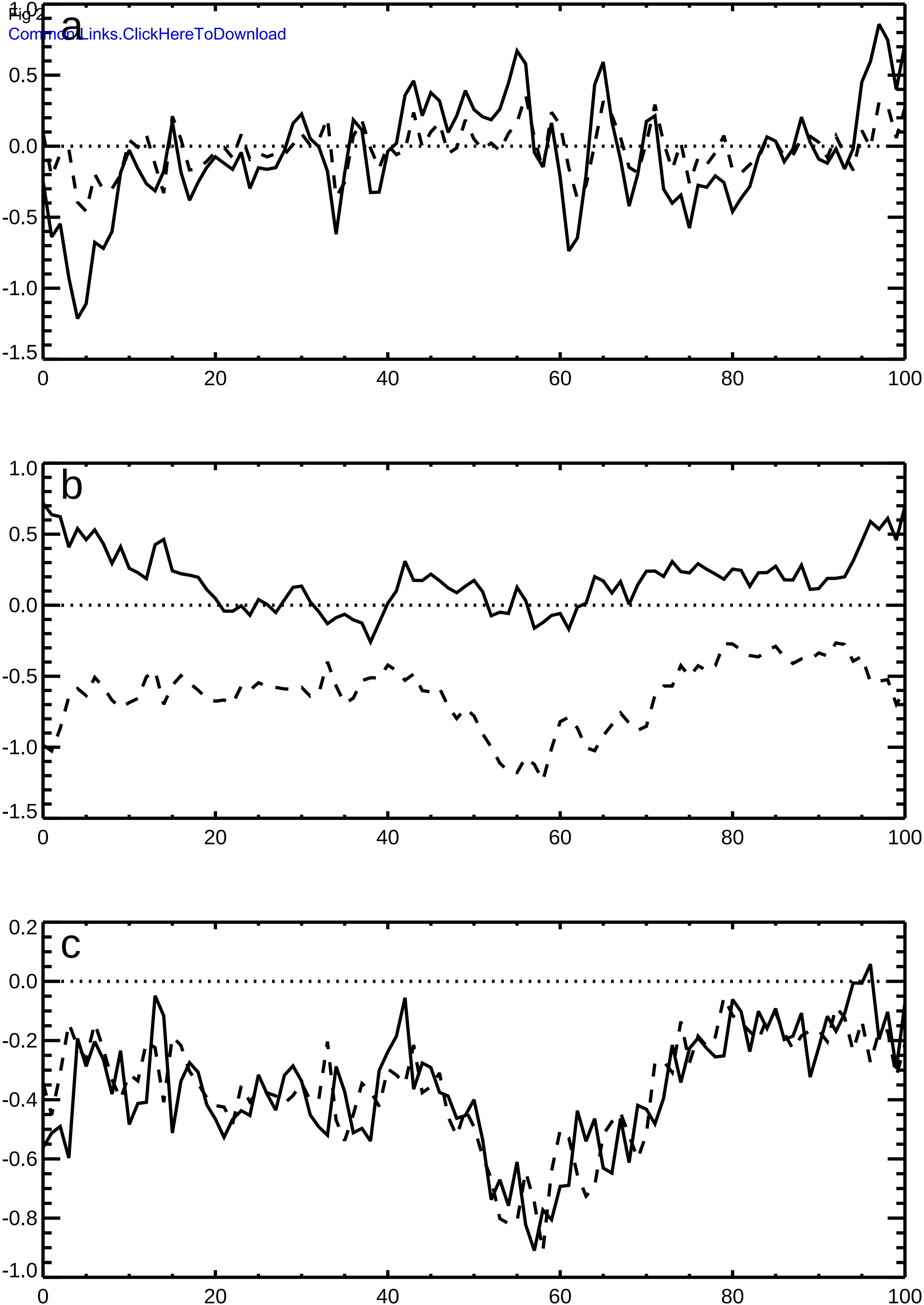

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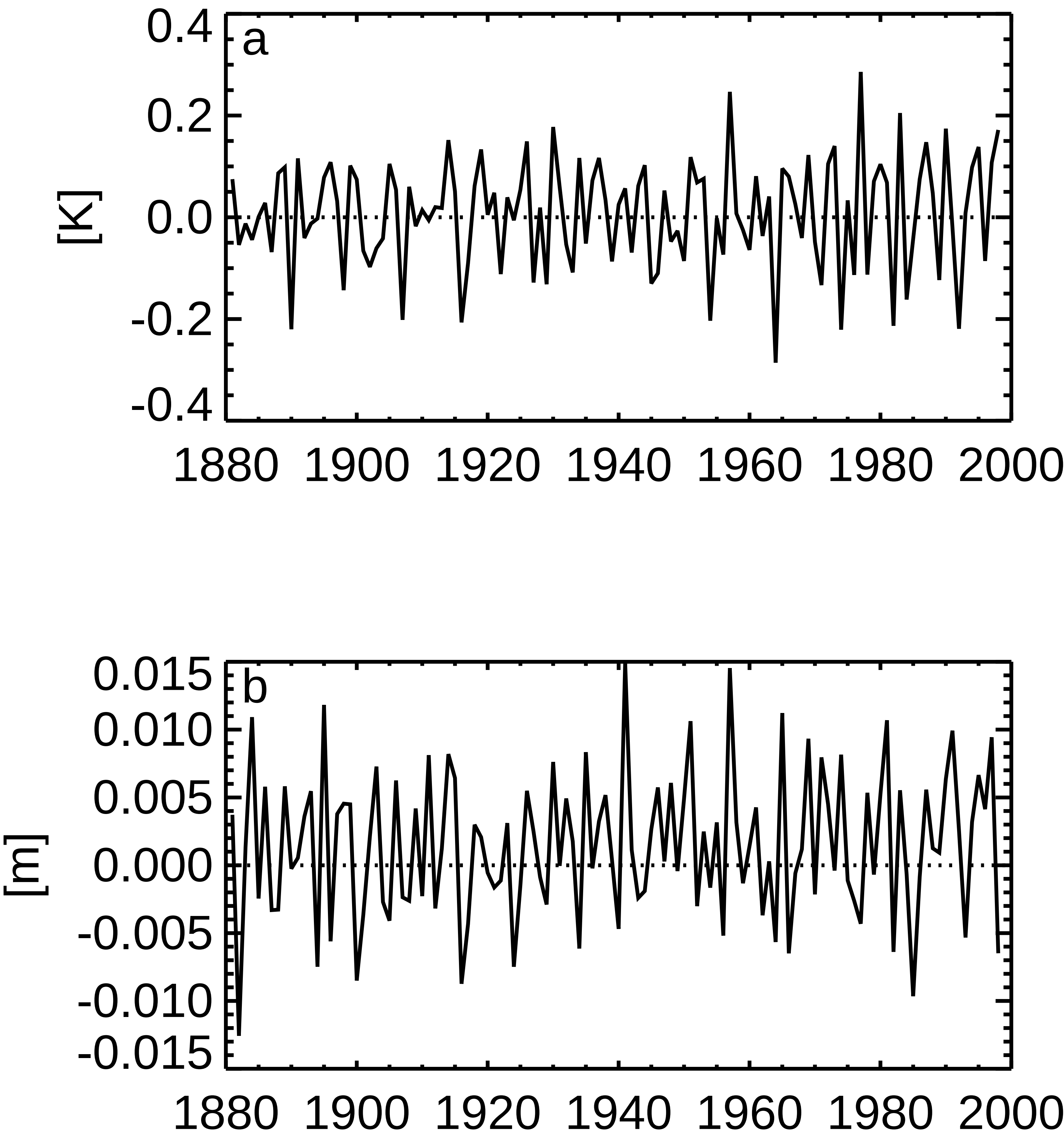

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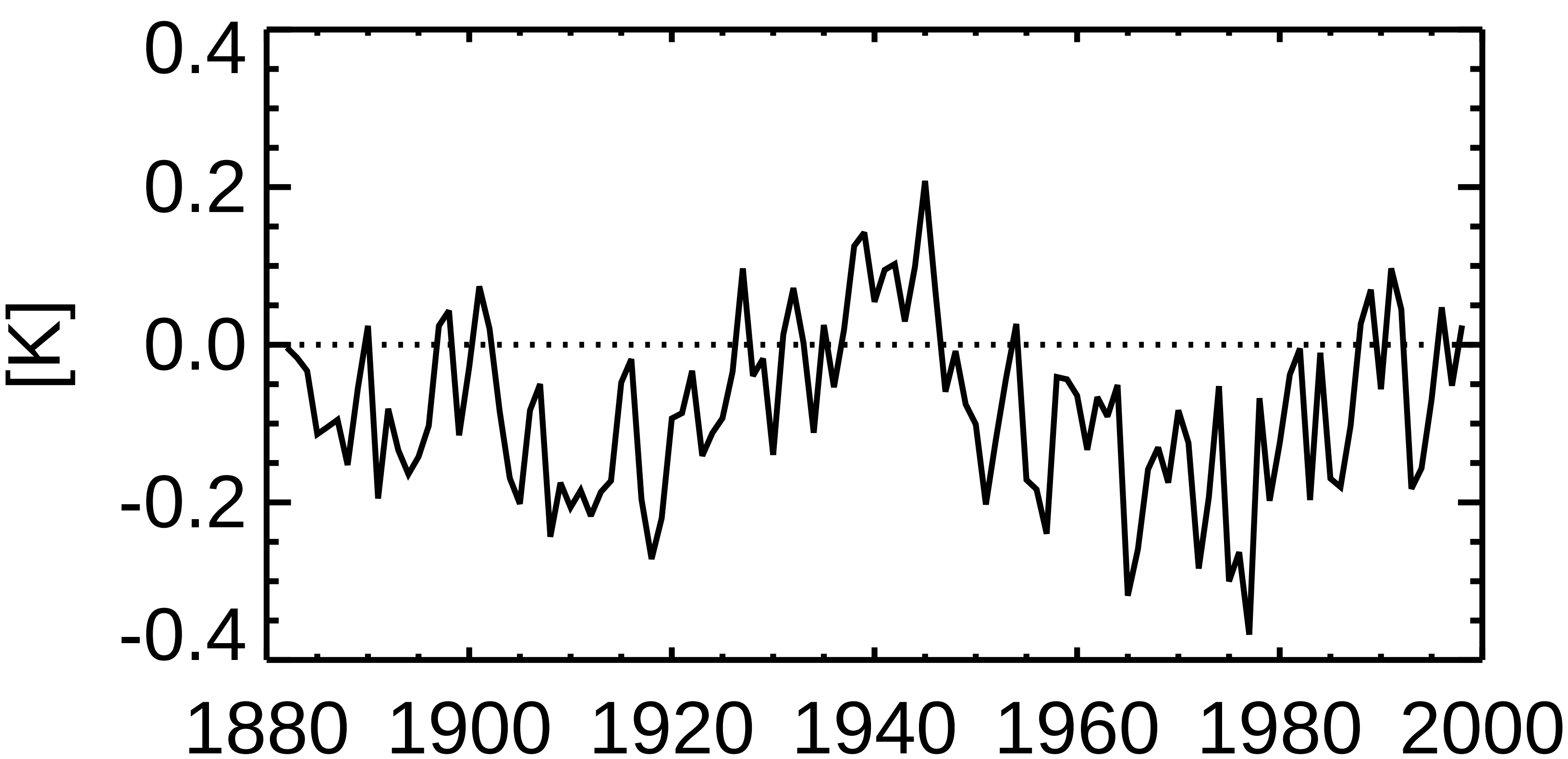

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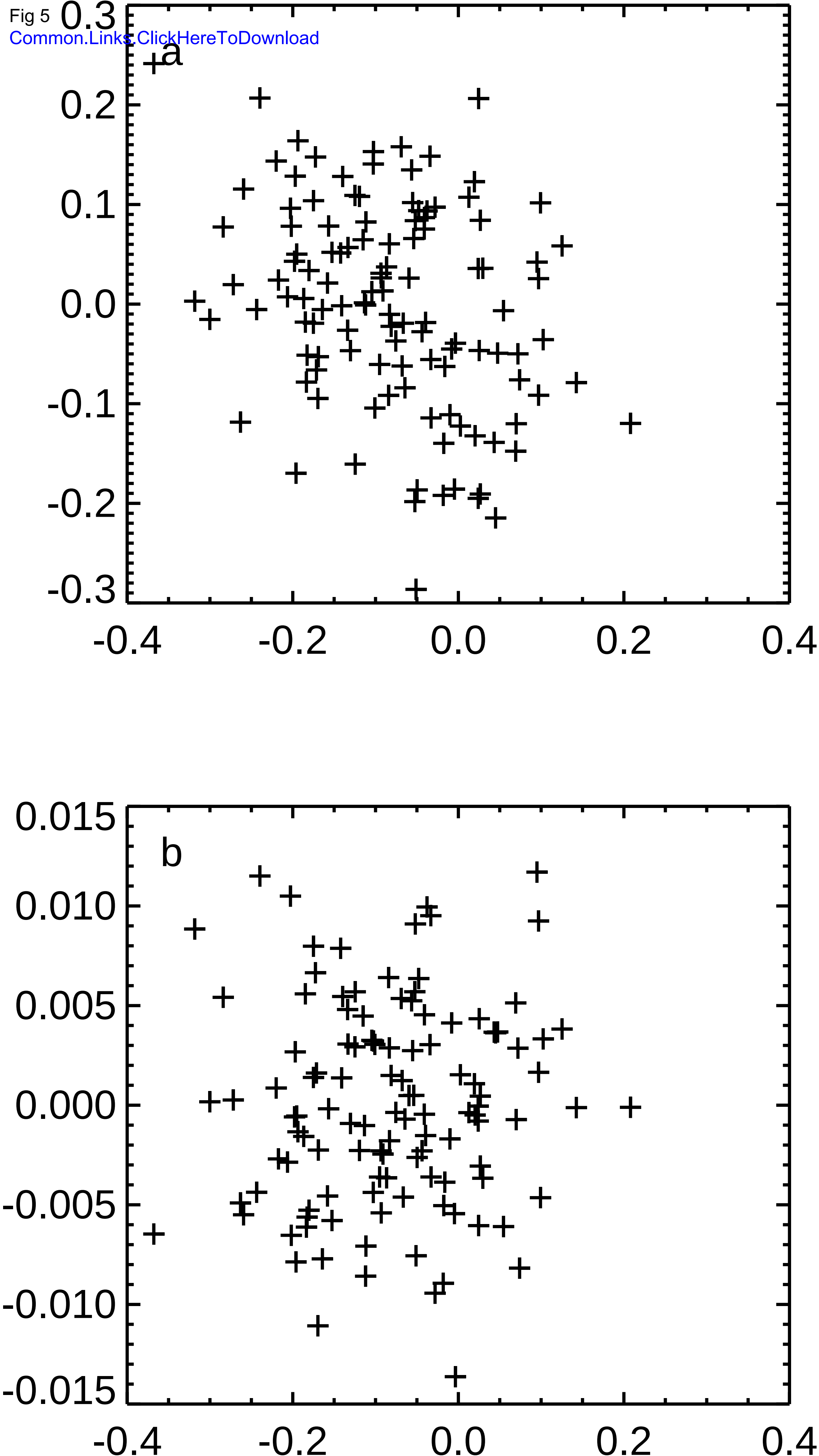

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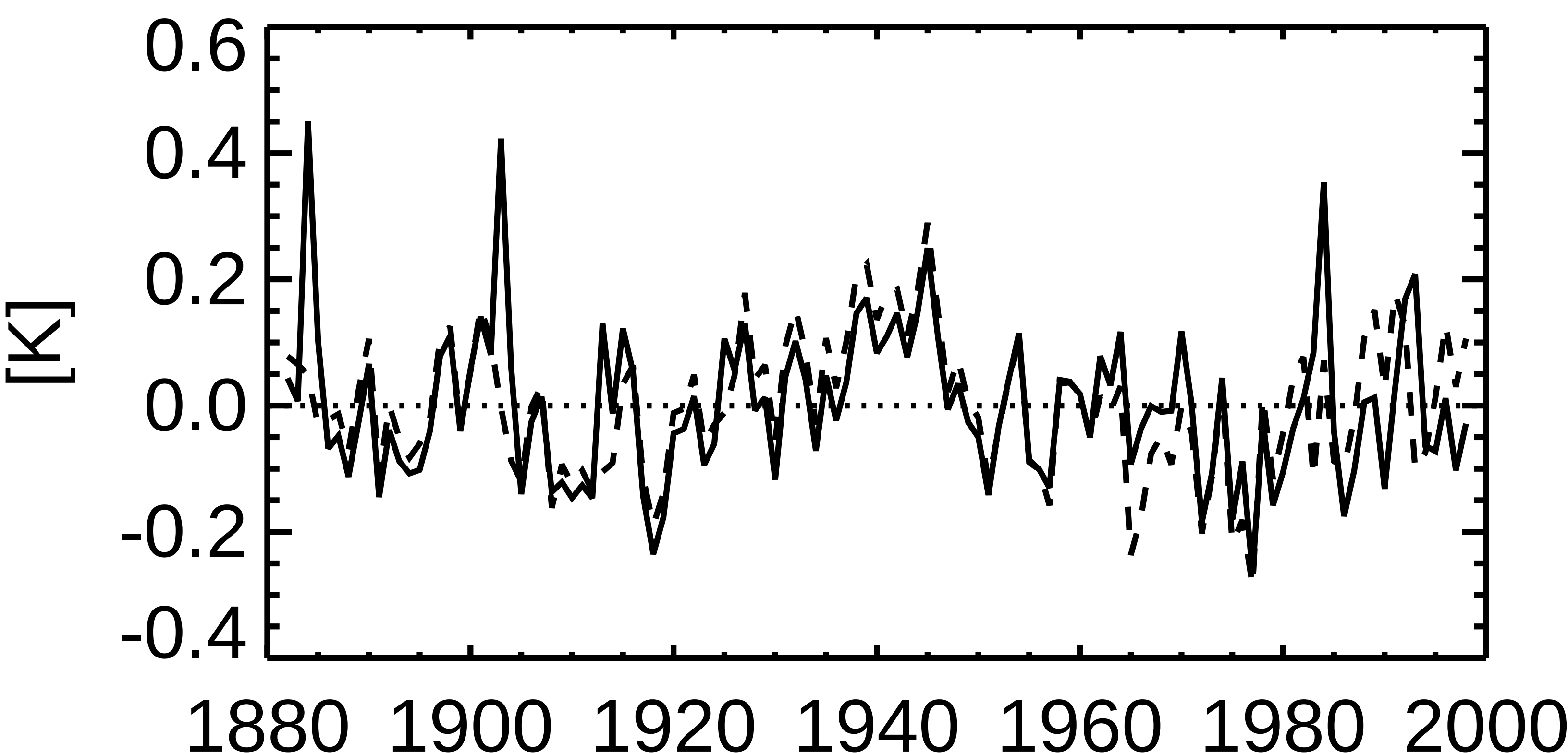

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