Discussion Papers
Department of Economics
University of Copenhagen

No. 11-24

A new axiomatic approach to the evaluation of population health

Jens Leth Hougaard, Juan D. Moreno-Ternero, Lars Peter Østerdal

Øster Farimagsgade 5, Building 26, DK-1353 Copenhagen K., Denmark
Tel.: +45 35 32 30 01 – Fax: +45 35 32 30 00
http://www.econ.ku.dk

ISSN: 1601-2461 (E)

A new axiomatic approach to the evaluation of population health*

Jens Leth Hougaard[†], Juan D. Moreno-Ternero[‡], Lars Peter Østerdal[§¶]

October 11, 2011

Abstract

We explore in this paper the implications of ethical and operational principles for the evaluation of population health. We formalize those principles as axioms for social preferences over distributions of health for a given population. We single out several focal *population health evaluation functions*, which represent social preferences, as a result of combinations of those axioms. Our results provide rationale for popular theories in health economics (such as the unweighted aggregation of QALYs or HYEs, and generalizations of the two, aimed to capture concerns for distributive justice) without resorting to controversial assumptions over individual preferences.

JEL numbers: *D63, I10.*

Keywords: *population health, QALYs, HYEs, axioms.*

*Financial support from the Spanish Ministry of Science and Innovation (ECO2008-03883, ECO2011-22919) as well as from the Andalusian Department of Economy, Innovation and Science (SEJ-4154, SEJ-5980) via the "FEDER operational program for Andalusia 2007-2013" is gratefully acknowledged.

[†]Institute of Food and Resource Economics, University of Copenhagen.

[‡]Department of Economics, Universidad Pablo de Olavide, and CORE, Université catholique de Louvain.

[§]Department of Economics, University of Copenhagen.

[¶]Address for correspondence: Lars Peter Østerdal, Department of Economics, University of Copenhagen, Øster Farimagsgade 5, DK-1353 Copenhagen K, Denmark. E-mail: lars.p.osterdal@econ.ku.dk.

1 Introduction

The goal of all health services activities and programs is to improve the health of people. Such a goal places a central role to the definition and measurement of benefit in health care, as well as how it should be distributed in a population. It is not surprising that, over the years, there has been considerable interest and activity in developing methods to measure quantitatively the health status of individuals and populations (e.g., Torrance, 1986). Economists, practitioners and social scientists alike have long been concerned with this issue. Attempts to develop appealing measures to evaluate the health of a population abound in the literature (e.g., Pliskin et al., 1980; Torrance, 1986; Mehrez and Gafni, 1989; Wagstaff, 1991; Bleichrodt, 1995, 1997; Williams, 1997; Dolan, 1998, 2000; Murray et al., 2002; Bleichrodt et al., 2004; Guerrero and Herrero, 2005; Østerdal, 2005; Fleurbaey and Schokkaert, 2009).

The purpose of this paper is to take the axiomatic approach to the evaluation of population health, a somewhat unexplored approach in the health economics literature, in contrast to many other fields in economics. An axiomatic study begins with the specification of a domain of problems, and the formulation of a list of desirable properties (axioms) of solutions for the domain, whereas it ends with (as complete as possible) descriptions of the families of solutions satisfying various combinations of the properties (e.g., Thomson, 2001). An axiomatic study often results in characterization theorems. They are theorems identifying a particular solution, or perhaps a family of solutions, as the only solution or family of solutions, satisfying a given list of axioms. This is precisely what we aim to do in this paper.

It has been frequently argued that the benefit that a patient derives from a particular health care intervention is defined according to two dimensions: quality of life and quantity of life. We endorse such assumption in our model. More precisely, we assume that the distribution of health in a population is defined by a collection of duplets, each indicating the status that an agent of the population achieves in the two dimensions: health (quality of life) and time (quantity of life). We shall refer to those distributions as *population health distributions*. We, however, departure from earlier contributions within the health-economics literature, to deal with the evaluation of health distributions, which presume an implicit relationship between the two dimensions at the individual level (e.g., Bleichrodt, 1995, 1997; Østerdal, 2005; Bleichrodt et al., 2004; Doctor et al., 2009). In other words, we do not assume from the outset the existence of an index summarizing the information of each duplet. Nevertheless, as we shall see later, we implicitly derive indices of that sort as a result of combining our axioms.

Our ultimate goal in this paper will be to single out, as a result of combining several axioms reflecting ethical and operational principles, specific measures to evaluate population health distributions. The key tool of our work to achieve that goal will be what we call a *population health evaluation function* ("PHEF" hereafter). A PHEF is a mapping that associates with each population health distribution a real number intended to perform comparisons among them and, hence, reflect social preferences over those distributions. We shall single out several PHEFs, each reflecting well-known natural views in the economic appraisal of health care programmes for resource allocation decisions. The axiomatic characterizations leading to these PHEFs will allow us to scrutinize their relative virtues by means of comparing the principles (axioms) that drive each of them.

One of the PHEFs we shall single out is the so-called *linear QALY* function, which evaluates population health by means of the unweighted aggregation of quality-adjusted life-years (QALYs). In contrast to the literature dealing with this focal and common procedure to aggregate health benefits in economic evaluations of health care, we derive it without imposing any structure on individual preferences over health. As a result, we avoid one of the main criticisms of the QALY measure, namely that it relies on restrictive assumptions over individual preferences (e.g., Loomes and McKenzie, 1989). We shall also single out the so-called *linear HYE* function, which evaluates population health by means of the unweighted aggregation of healthy years equivalents (HYEs). In doing so, we shall be able to scrutinize further the similarities and differences between these two focal concepts (HYEs and QALYs) in health economics, an aspect that has received considerable attention in the literature (e.g., Mehrez and Gafni, 1989; Culyer and Wagstaff, 1993; Gafni et al., 1993; Bleichrodt, 1995).

Both PHEFs (linear QALY and linear HYE) dismiss a concern for distributive justice in the evaluation process of population health distributions. They simply focus on the aggregate number (of QALYs or HYEs) that a distribution delivers. We also characterize in this paper PHEFs modifying the previous two in order to capture concerns for distributive justice. As we shall see later, the resulting PHEFs, which are also inspired by classical contributions in the economics literature, are closely connected to other proposals in the literature on health economics.

The rest of the paper is organized as follows. In Section 2, we introduce the model and the axioms we study. In Section 3, we present the PHEFs and their characterization results. We discuss the results and some possible extensions in Section 4. For a smooth passage, we defer the proofs and provide them in an appendix.

2 The preliminaries

We consider a policy maker who has to make a choice between distributions of health for a population of fixed size $n \geq 3$. We identify the population (society) with the set $N = \{1, ..., n\}$. The health of each individual in the population will be described by a duplet indicating the level achieved in two parameters: quality of life and quantity of life. Assume that there exists a set of possible health states, A, defined generally enough to encompass all possible health states for everybody in the population. We emphasize that A is an abstract set without any particular mathematical structure.[1] Quantity of life will simply be described by a set of nonnegative real numbers, $T \subset \mathbb{R}$. In what follows, and unless otherwise stated, we assume that $T = [0, +\infty)$. Formally, let $h_i = (a_i, t_i) \in A \times T$ denote the health duplet of individual i.[2] A population health distribution (or, simply, a health profile) $h = [h_1, \ldots, h_n] = [(a_1, t_1), ..., (a_n, t_n)]$ specifies the health duplet of each individual in society. We denote the set of all possible health profiles by H, i.e., H is the n-Cartesian product of the set $A \times T$. Even though we do not impose a specific mathematical structure on the set A, we assume that it contains a specific element, a_*, which we refer to as *perfect health* and which is univocally identified, as a "superior" state, by all agents in the population.

The policy maker's preferences (or social preferences) over health profiles are expressed by a preference relation $\succsim$, to be read as "at least as preferred as". As usual, $\succ$ denotes strict preference and $\sim$ denotes indifference. We assume that the relation $\succsim$ is a weak order, i.e., it is complete (for each health profiles h, h', either $h \succsim h'$, or $h' \succsim h$, or both) and transitive (if $h \succsim h'$ and $h' \succsim h''$ then $h \succsim h''$).

A *population health evaluation function* (PHEF) is a real-valued function $P : H \to \mathbb{R}$. We say that P represents $\succsim$ if

$$P(h) \geq P(h') \Leftrightarrow h \succsim h',$$

for each $h, h' \in H$. Note that if P represents $\succsim$ then any strictly increasing transformation of P would also do so.

The model we just outlined has at least two possible interpretations. On the one hand, the pair (a_i, t_i) is identifying an agent having a chronic (or "average") health state a_i throughout a (remaining) lifetime of length t_i. In this sense, the model can be used to express a social

[1] A could for instance refer to the resulting multidimensional health states after combining the levels of each dimension of a categorical measure, such as EQ-5D, in all possible ways.

[2] For ease of exposition, we establish the notational convention that $h_S \equiv (h_i)_{i \in S}$, for each $S \subset N$.

planner's preference over societies (of the same size) with different distributions of the citizens health states and (remaining) life years. On the other hand, we may think of the scenario in which the planner launches an intervention and this intervention results in a health state a_i for a period of time t_i for each agent i in the society, relative to some status-quo distribution. Different interventions can then be compared on the basis of their resulting distributions of health states and time periods.

2.1 Axioms

We now list the axioms for social preferences that we consider in this paper. As we shall see, each of them will reflect an ethical or an operational principle.

Our first axiom, *anonymity*, is a standard formalization of the principle of *impartiality*, which refers to the fact that ethically irrelevant information is excluded from the evaluation process. In other words, the identity of agents should not matter and the evaluation of the population health should depend only on the list of quality-quantity duplets, not on who holds them. Formally, let Π^N denote the class of bijections from N into itself. Then,

ANON: $h \sim h_\pi$ for each $h \in H$, and each $\pi \in \Pi^N$.

The next axiom, *separability*, underlies the use of incremental analysis in cost-effectiveness analysis, which implies that individuals for whom two treatments yield the same health should not influence the relative evaluation of these treatments (e.g., Gold et al., 1996; Turpcu et al., 2011). More precisely, it says that if the distribution of health in a population changes only for a subgroup of agents in the population, the relative evaluation of the two distributions should only depend on that subgroup.[3] Formally,

SEP: $\left[h_S, h_{N\setminus S}\right] \succsim \left[h'_S, h_{N\setminus S}\right] \Leftrightarrow \left[h_S, h'_{N\setminus S}\right] \succsim \left[h'_S, h'_{N\setminus S}\right]$, for each $S \subseteq N$, and $h, h' \in H$.

The next axiom, *perfect health superiority*, refers to the superiority of the state of perfect health. More precisely, it says that replacing the health status of an agent by that of perfect health, ceteris paribus, cannot worsen the evaluation of the population health. It constitutes the closest approximation in our context to the Pareto principle of optimality, and it also reflects a certain notion of *solidarity*, a principle with a long tradition of use in the theory of justice (e.g., Moreno-Ternero and Roemer, 2006).

[3]The notion of separability has a long tradition of use in models of cooperative decision making (e.g., Moulin, 1988).

PHS: For each $h = [h_1, \ldots, h_n] \in H$ and $i \in N$, let $h_i^* = (a_*, t_i)$. Then, $[h_i^*, h_{N \setminus \{i\}}] \succsim h$.

A somewhat related axiom comes next. *Time monotonicity at perfect health* says that if each agent is at perfect health, increasing the time dimension is strictly better for society. Formally,

TMPH: If, for each $i \in N$, $t_i \geq t_i'$, at least one strict, then $[(a_*, t_1), \ldots, (a_*, t_n)] \succ [(a_*, t_1'), \ldots, (a_*, t_n')]$.

We then move to an axiom, *healthy years equivalence*, which says that any population health distribution has a socially equivalent one in which the health outcome of one (and only one) agent is replaced by that of full health, for some quantity of time.[4] Formally,

HYE: For each $h \in H$ and $i \in N$ there exists t_i^* such that $h \sim [h_{N \setminus \{i\}}, (a_*, t_i^*)]$.

Note that the HYE axiom only postulates that, for each agent and health profile, a "healthy years equivalent" exists, but not how it should be determined as a function of the health profile.

Continuity is the adaptation to our context of a usual axiom. It says that for fixed distributions of health states, the population health ordering is smooth in lifetimes. In spite of being an apparently technical condition, it is also an ethically attractive axiom. It models *non-arbitrariness* of the social preferences.

CONT: Let $h, h' \in H$, and $h^{(k)}$ be a sequence in H such that, for each $i \in N$, $h_i^{(k)} = (a_i, t_i^{(k)}) \to (a_i, t_i) = h_i$. If $h^{(k)} \succsim h'$ for each k, then $h \succsim h'$.

The next axiom, *social zero condition*, is reminiscent of a widely used condition for individual utility functions on health (e.g., Bleichrodt et al., 1997; Miyamoto et al., 1998; Østerdal 2005). It says that if an agent gets zero lifetime, then his/her health state does not influence the social desirability of the health distribution. Formally,

ZERO: For each $h \in H$ and $i \in N$ such that $t_i = 0$, and $a_i' \in A \setminus \{a_i\}$, $h \sim [h_{N \setminus \{i\}}, (a_i', 0)]$.

The following axiom, *time invariance at common health*, refers to the equal value of life gains for persons with common health states. Thus, it conveys an absence of lifetime discrimination: an individual is not less worthy of treatment on the sole grounds that he/she has a longer

[4] This notion can be traced back to Mehrez and Gafni (1989) who propose it as a plausible way to reflect patient's preferences over health.

lifetime.[5] Formally,

TICH: For each $h \in H$, $c > 0$, and $i, j \in N$, such that $a_i = a_j$,

$$\left[(a_i, t_i + c), (a_j, t_j), h_{N\setminus\{i,j\}}\right] \sim \left[(a_i, t_i), (a_j, t_j + c), h_{N\setminus\{i,j\}}\right].$$

Time invariance at perfect health is the weakening of the above axiom to the case in which the common health state is precisely the perfect health state. Formally,

TIPH: For each $h \in H$, $c > 0$, and $i, j \in N$, such that $a_i = a_j = a_*$,

$$\left[(a_i, t_i + c), (a_j, t_j), h_{N\setminus\{i,j\}}\right] \sim \left[(a_i, t_i), (a_j, t_j + c), h_{N\setminus\{i,j\}}\right].$$

The last axiom we consider, *time scale independence*, says that evaluations should not depend on the variable we use to measure quantity of life (e.g., days, months, years). More precisely, it says that the ranking of a pair of population health distributions does not reverse when all lifetimes are multiplied by a common positive constant.[6] The individual level counterpart to this axiom is often referred to in the literature as *the constant proportional trade-off* assumption (e.g., Pliskin et al., 1980). Previous experimental studies have provided mixed support for such assumption (e.g., Attema and Brouwer, 2010), with mostly person and context-specific violations. Thus, we believe it may be more acceptable for a planner to assume scale independence in time for social preferences rather than for individual preferences.

TSI: For each $c > 0$, and $h = [(a_i, t_i)_{i\in N}]$, $h' = [(a'_i, t'_i)_{i\in N}]$,

$$h \succsim h' \Rightarrow [(a_i, ct_i)_{i\in N}] \succsim [(a'_i, ct'_i)_{i\in N}].$$

A plausible weakening of the previous axiom, *time scale independence at perfect health*, says that the notion only applies when restricted to perfect health. Formally,

TSIPH: For each $c > 0$, and $h = [(a_*, t_i)_{i\in N}]$, $h' = [(a_*, t'_i)_{i\in N}]$,

$$h \succsim h' \Rightarrow [(a_*, ct_i)_{i\in N}] \succsim [(a_*, ct'_i)_{i\in N}].$$

3 Axiomatic characterizations

We provide in this section the main results of this paper, which characterize a group of focal PHEFs.

[5] This axiom is very similar, although not identical, to the so-called non-age dependence axiom in Østerdal (2005).

[6] Østerdal (2005) considers a counterpart of this axiom in his context.

The most widely employed way of combining the quality of life and quantity of life derived from a particular health care intervention is by means of QALYs. The following PHEF, which we call *linear QALY*, evaluates population health distributions by means of the unweighted aggregation of individual QALYs in society, or, in other words, by the weighted (through health levels) aggregate time span the distribution yields. Formally,

$$P^q[h_1, \ldots, h_n] = P^q[(a_1, t_1), \ldots, (a_n, t_n)] = \sum_{i=1}^{n} q(a_i) t_i, \tag{1}$$

where $q : A \to [0, 1]$ is an arbitrary function satisfying $0 \le q(a_i) \le q(a_*) = 1$, for each $a_i \in A$.

Our first result says that the linear QALY PHEF is characterized by the combination of the six structural axioms ANON, SEP, PHS, TMPH, HYE and CONT plus the specific axioms of ZERO and TICH. Formally,

Theorem 1 *The following statements are equivalent:*

1. $\succsim$ *is represented by a PHEF satisfying* (1).

2. $\succsim$ *satisfies ANON, SEP, PHS, TMPH, HYE, CONT, ZERO and TICH.*

An alternative way of combining the quality of life and quantity of life derived from a particular health care intervention is by means of HYEs. The next PHEF, which we call *linear HYE*, evaluates population health distributions by means of the aggregation of individuals' HYEs. Formally,

$$P^h[h_1, \ldots, h_n] = P^h[(a_1, t_1), \ldots, (a_n, t_n)] = \sum_{i=1}^{n} f(a_i, t_i), \tag{2}$$

where $f : A \times T \to T$ is a function indicating the HYEs for each possible individual, i.e., for each $h = [h_1, \ldots, h_n] = [(a_1, t_1), \ldots, (a_n, t_n)] \in H$,

$$h \sim [(a_*, f(a_i, t_i))_{i \in N}].$$

Our second result characterizes the linear HYE PHEF by another combination of the axioms stated above. Formally,

Theorem 2 *The following statements are equivalent:*

1. $\succsim$ *is represented by a PHEF satisfying* (2).

2. $\succsim$ *satisfies ANON, SEP, PHS, TMPH, HYE, CONT and TIPH.*

As we can observe from the statements of the two theorems, disregarding ZERO and weakening TICH, so that the axiom is restricted to the case in which the common health state is the perfect health state) allows us to move from the (linear) QALY PHEF to the much broader family of (linear) HYE PHEFs.

Both (linear) PHEFs highlighted above amount unweighted aggregation across individuals, an aspect usually criticized by its lack of concern for distributive justice (e.g., Loomes and McKenzie, 1989; Wagstaff, 1991; Dolan, 1998). We introduce next two PHEFs, which generalize the above two, in order to capture such a concern. They simply introduce suitable powers in the QALYs (or HYEs) each agent in the population gets. The rationale behind this idea can be traced back to Bergson (e.g., Burk, 1936). In a health-economics context, power functions of QALYs were introduced, at an individual level, by Pliskin et al., (1980). The so-called Bergsonian approach has also been discussed in the health-economics literature by Wagstaff (1991) and Williams (1997), among others.

Formally, the *power QALY* PHEF is defined as:

$$P^{pq}[h_1, \dots, h_n] = P^{pq}[(a_1, t_1), \dots, (a_n, t_n)] = \sum_{i=1}^{n} q(a_i) t_i^{\gamma}, \tag{3}$$

where $q : A \rightarrow [0, 1]$ is an arbitrary function satisfying $0 \leq q(a_i) \leq q(a_*) = 1$, for each $a_i \in A$, and $\gamma \in \mathbb{R}_{++}$ is a positive scalar.

Likewise, the *power HYE* PHEF is defined as:

$$P^{ph}[h_1, \dots, h_n] = P^{ph}[(a_1, t_1), \dots, (a_n, t_n)] = \sum_{i=1}^{n} f(a_i, t_i)^{\gamma}, \tag{4}$$

where $\gamma \in \mathbb{R}_{++}$ is a positive scalar, and $f : A \times T \rightarrow T$ is a function indicating the HYEs for each possible individual, i.e., for each $h = [h_1, \dots, h_n] = [(a_1, t_1), \dots, (a_n, t_n)] \in H$,

$$h \sim [(a_*, f(a_i, t_i))_{i \in N}].$$

The characterizations of both families come next.[7]

Theorem 3 *The following statements are equivalent:*

[7] One might find natural to restrict both families to those PHEFs corresponding to $\gamma \leq 1$. It turns out that the following characterization results stated in Theorems 3 and 4 could be enriched to characterize the subsequent families, upon adding a *Pigou-Dalton transfer* axiom stating that a health profile in which two agents at perfect health have different time spans is dominated by the subsequent profile in which those agents keep the same perfect health status, but share a time span equal to the average of the former two.

1. $\succsim$ *is represented by a PHEF satisfying* (3)*.*

2. $\succsim$ *satisfies ANON, SEP, PHS, TMPH, HYE, CONT and TSI.*

Theorem 4 *The following statements are equivalent:*

1. $\succsim$ *is represented by a PHEF satisfying* (4)*.*

2. $\succsim$ *satisfies ANON, SEP, PHS, TMPH, HYE, CONT and TSIPH.*

It is worth noting that ZERO is not needed in Theorem 3, as opposed to the counterpart characterization of the linear QALY PHEF at Theorem 1. Other than that, the parallelism between Theorems 2 and 4 mimics the parallelism between Theorems 1 and 3.

4 Discussion

We have presented in this paper a new axiomatic approach to the evaluation of population health. Contrary to most of the previous axiomatic work within the field, we have considered a model in which no considerations about individual preferences over health have been made. The reasons for it are threefold. First, it is well known that individual preferences over health profiles are difficult to articulate/assess (e.g., Dolan, 2000). Second, and partly motivated by concerns for distributive justice, it has been argued that person trade-off information (in contrast to individual preference information) should provide the basis for making priorities.[8] Third, it has been argued elsewhere that widely used representations of individual preferences for health (such as QALYs) rely on fairly restrictive assumptions (e.g. Loomes and McKenzie, 1989; Mehrez and Gafni, 1989). Thus, the aim of this paper was to consider a framework in which information on individual preferences over health is not available, either for practical or ethical reasons, but in which sound decisions over the evaluation of population health could still be made.

Somewhat related, even though we assume (as customary) that individual health is determined by two dimensions (quality and quantity) we do not presume that individuals evaluate them in a specific given way. In other words, as opposed to earlier axiomatic contributions on the evaluation of health profiles, we do not assume that the health of an individual is summarized by a number (to be interpreted as the number of QALYs, life years, or "health utility" experienced by a person), but by a duplet referring to the two dimensions.

[8] This is, for instance, the case of the so-called Global Burden of Diseases, Injuries, and Risk Factors Study.

Our aim has been to derive specific representations of social preferences over population health distributions as a result of combining several ethical and operational axioms. We believe our axioms are compelling from a normative viewpoint (although, obviously, some to a higher extent than others). The positive appeal of our axioms has not been tested in this paper. Nevertheless, there exists a wide variety of experimental contributions testing empirically some of the principles (or related ideas) over which our axioms rely in related contexts (e.g., Spencer, 2003; Doctor et al., 2004; Spencer and Robinson, 2007; Turpcu et al., 2011). The test of the precise axioms we use in our specific context is left for further research. We, nevertheless, would remain cautious about the eventual results of such a venture. As Amos Tversky, one of the leading scholars in the positive approach to decision theory put it, "give me an axiom and I will design a questionnaire so that the majority of the people will reject it" (e.g., Gilboa, 2009).

We have characterized two focal representations of social preferences over population health distributions; namely, the linear QALY and HYE representations. They translate into our context two of the most well-known and employed techniques to measure the benefits of health interventions in cost-utility analyses. Those techniques were initially considered as polars, although some of their differences and similar aspects were addressed (e.g., Mehrez and Gafni, 1989; Culyer and Wagstaff, 1993; Gafni et al., 1993; Bleichrodt, 1995). We have seen in this paper that they share a solid common ground. To wit, their characterizations share many axioms (anonymity, separability, continuity, perfect health superiority, time monotonicity at perfect health and healthy years equivalence) and differ only in a few (the linear QALY also requires time invariance at common health, and the social zero condition, whereas the linear HYE only requires time invariance at perfect health on top of the common axioms).

Our study has focussed on distributions of populations with equal size. Nevertheless, the analysis could be extended to distributions of variable population size. A plausible way to do so would be relying on a standard axiom in welfare economics, known as *replication invariance*.[9] Adding such axiom to our theorems would allow us to characterize the counterpart variable-population versions of our PHEFs.

To conclude, it is also worth commenting on another important aspect of our model. We have not dealt with uncertainty in our analysis.[10] More precisely, we have considered a formu-

[9] See Blackorby et al. (2006) for a scrutiny of that axiom, as well as other related population issues in welfare economics.

[10] Broome (1993) argues that uncertainty is a complication rather than an essential part of the problem of valuing lives, and it ought not to be introduced into the analysis.

lation of the population health evaluation problem which contains no explicit element of risk, and in which we obtain characterizations of population health evaluation functions without assumptions on the social planner's (or individuals') risk attitudes. Rather than a limitation, we see this aspect of our analysis as an advantage, as it allows us to escape from the usual critiques of the expected utility theory that are normally considered in the health economics literature (e.g., Bleichrodt et al., 2001).

5 Appendix. Proofs

In order to prove our theorems, we introduce first an auxiliary lemma, which is interesting on its own, that characterizes a general family of PHEFs by means of combining the six structural axioms of anonymity, separability, perfect health superiority, time monotonicity at perfect health, healthy years equivalence and continuity. Formally, *separable* PHEFs are defined as follows:

$$P^{gf}[h_1,\ldots,h_n] = P^{gf}[(a_1,t_1),\ldots,(a_n,t_n)] = \sum_{i=1}^{n} g\left(f(a_i,t_i)\right), \tag{5}$$

where $f : A\times T \to \mathbb{R}_+$ is a continuous function with respect to its second variable and satisfying $0 \le f(a_i,t_i) \le t_i$, for each $(a_i,t_i) \in A\times T$, and $h \sim [(a_*, f(a_i,t_i))_{i\in N}]$, and $g : \mathbb{R}_+ \to \mathbb{R}$ is a strictly increasing and continuous function.

Lemma 1 *The following statements are equivalent:*

1. $\succsim$ *is represented by a PHEF satisfying* (5).

2. $\succsim$ *satisfies ANON, SEP, PHS, TMPH, HYE and CONT.*

Proof. We focus on the non-trivial implication. Formally, assume $\succsim$ satisfies ANON, SEP, PHS, TMPH, HYE and CONT. Let $h \in H$. By HYE, for each $i \in N$, there exists t_i^* such that $h \sim [h_{N\setminus\{i\}}, (a_*, t_i^*)]$. By SEP, t_i^* only depends on (a_i,t_i) (and, thus, is independent of the remaining duplets of the profile). Thus, for each $i = 1,\ldots,n$, let $f_i : A\times T \to \mathbb{R}$ be defined such that $f_i(a_i,t_i) = t_i^*$, for each $(a_i,t_i) \in A\times T$. By ANON, $f_i(\cdot,\cdot) \equiv f_j(\cdot,\cdot) \equiv f(\cdot,\cdot)$, for each $i,j \in N$. By TMPH and PHS, $0 \le f(a_i,t_i) \le t_i$, for each $(a_i,t_i) \in A\times T$ and, by CONT, f is a continuous function with respect to its second variable. Furthermore,

$$h \sim [(a_*, f(a_i,t_i))_{i\in N}],$$

which implicitly says that social preferences only depend on the profile of healthy years equivalents, and, by CONT, they do so continuously. It also follows that the range of f is a connected subset of $\mathbb{R}$. By Theorem 3 in Debreu (1960), there exists a strictly increasing and continuous function $g : \mathbb{R}_+ \to \mathbb{R}$ such that

$$h \succsim h' \iff \sum_{i=1}^{n} g\left(f(a_i, t_i)\right) \geq \sum_{i=1}^{n} g\left(f(a_i', t_i')\right),$$

which concludes the proof. ■

5.1 Proof of Theorem 1

We focus on the non-trivial implication, i.e., $2 \to 1$. Formally, assume $\succsim$ satisfies ANON, SEP, PHS, TMPH, HYE, CONT, ZERO and TICH. Then, by Lemma 1, there exists $f : A \times T \to \mathbb{R}_+$ and $g : \mathbb{R}_+ \to \mathbb{R}$ such that:

- f is continuous with respect to its second variable,
- $0 \leq f(a_i, t_i) \leq t_i$, for each $(a_i, t_i) \in A \times T$, and
- g is a strictly increasing and continuous function.

Furthermore, for each $h = [h_1, \ldots, h_n] = [(a_1, t_1), \ldots, (a_n, t_n)] \in H$,

$$h \sim [(a_*, f(a_i, t_i))_{i \in N}],$$

and, for each $h = [(a_1, t_1), \ldots, (a_n, t_n)] \in H$, and $h' = [(a_1', t_1'), \ldots, (a_n', t_n')] \in H$,

$$h \succsim h' \iff \sum_{i=1}^{n} g\left(f(a_i, t_i)\right) \geq \sum_{i=1}^{n} g\left(f(a_i', t_i')\right).$$

Without loss of generality, let us assume that $g(f(\hat{a}, 0)) = 0$ for some $\hat{a} \in A$.

Let $\bar{a} \in A$ be an arbitrary health state. Then, by iterated application of TICH, and the transitivity of $\succsim$,

$$\sum_{i=1}^{n} g\left(f(\bar{a}, t_i)\right) = g\left(f\left(\bar{a}, \sum_{i=1}^{n} t_i\right)\right) + (n-1) g\left(f(\bar{a}, 0)\right).$$

By ZERO, $g\left(f(\bar{a}, 0)\right) = g\left(f(\hat{a}, 0)\right) = 0$. Thus,

$$\sum_{i=1}^{n} g\left(f(\bar{a}, t_i)\right) = g\left(f\left(\bar{a}, \sum_{i=1}^{n} t_i\right)\right).$$

It then follows that $g\left(f(\bar{a},\cdot)\right)$ satisfies

$$g(f(\bar{a},t_1+t_2)) = g(f(\bar{a},t_1)) + g(f(\bar{a},t_2))$$

for any $t_1, t_2 \in T$, which is precisely one of Cauchy's canonical functional equations. As $g(f(\bar{a},\cdot))$ is a continuous function, it follows that the unique solutions to such equation are the linear functions (e.g., Aczel, 2006; page 34). More precisely, there exists a function $\hat{q} : A \to \mathbb{R}$ such that

$$g(f(\bar{a},t)) = \hat{q}(\bar{a})t,$$

for each $\bar{a} \in A$, and $t \in T$. By PHS and TMPH, it follows that $0 \leq \hat{q}(a_*)$ and $\hat{q}(a_i) \leq \hat{q}(a_*)$, for each $a_i \in A$. We now show that $\hat{q}(a_i) \geq 0$, for each $a_i \in A$. To do so, let $a_i \in A$. By HYE and SEP, there exists $t_i \in T$ such that $\hat{q}(a_i) = g(f(a_i,1)) = g(f(a_*,t_i))$. By TMPH and ZERO, $g(f(a_*,t_i)) \geq g(f(a_*,0)) = g(f(\hat{a},0)) = 0$. Altogether, it says that $\hat{q}(a_i) \geq 0$. To conclude, let $q : A \to \mathbb{R}$ be such that $q(a) = \frac{\hat{q}(a)}{\hat{q}(a_*)}$, for each $a \in A$. Then, it follows from the above that $0 \leq q(a_i) \leq q(a_*) = 1$, for each $a_i \in A$. Thus, $\succsim$ is represented by a PHEF satisfying (1), as desired. ■

5.2 Proof of Theorem 2

We focus on the non-trivial implication, i.e., $2 \to 1$. Formally, assume $\succsim$ satisfies ANON, SEP, PHS, TMPH, HYE, CONT and TIPH. Then, by Lemma 1, there exists $f : A \times T \to \mathbb{R}_+$ continuous with respect to its second variable, and satisfying that $0 \leq f(a_i,t_i) \leq t_i$, for each $(a_i,t_i) \in A \times T$, such that, for each $h = [h_1,\ldots,h_n] = [(a_1,t_1),\ldots,(a_n,t_n)] \in H$,

$$h \sim [(a_*, f(a_i,t_i))_{i \in N}].$$

Let $h = [(a_1,t_1),\ldots,(a_n,t_n)] \in H$, and $h' = [(a'_1,t'_1),\ldots,(a'_n,t'_n)] \in H$. Then, by iterated application of TIPH, and the transitivity of $\succsim$,

$$h \succsim h' \iff [(a_*, \sum_{i \in N} f(a_i,t_i)), (a_*,0)_{k \in N\setminus\{i\}}] \succsim [(a_*, \sum_{i \in N} f(a'_i,t'_i)), (a_*,0)_{k \in N\setminus\{i\}}].$$

By PHS, and the transitivity of $\succsim$,

$$h \succsim h' \iff \sum_{i \in N} f(a_i,t_i) \geq \sum_{i \in N} f(a'_i,t'_i),$$

as desired. ■

5.3 Proof of Theorem 3

We focus on the non-trivial implication, i.e., $2 \to 1$. Formally, assume $\succsim$ satisfies ANON, SEP, PHS, TMPH, HYE, CONT and TSI. Then, by Lemma 1, there exists $f : A \times T \to \mathbb{R}_+$ and $g : \mathbb{R}_+ \to \mathbb{R}$ such that:

- f is continuous with respect to its second variable,
- $0 \leq f(a_i, t_i) \leq t_i$, for each $(a_i, t_i) \in A \times T$, and
- g is a strictly increasing and continuous function.

Furthermore, for each $h = [h_1, \dots, h_n] = [(a_1, t_1), \dots, (a_n, t_n)] \in H$,

$$h \sim [(a_*, f(a_i, t_i))_{i \in N}],$$

and, for each $h = [(a_1, t_1), \dots, (a_n, t_n)] \in H$, and $h' = [(a'_1, t'_1), \dots, (a'_n, t'_n)] \in H$,

$$h \succsim h' \iff \sum_{i=1}^{n} g\left(f(a_i, t_i)\right) \geq \sum_{i=1}^{n} g\left(f(a'_i, t'_i)\right).$$

Step 1. We claim that for each $(a, t), (a', t') \in A \times T$, and $c > 0$,

$$f(a, t) \geq f(a', t') \iff f(a, ct) \geq f(a', ct').$$

Indeed, let $h = [(a_1, t_1), \dots, (a_n, t_n)] \in H$ and $c > 0$. Denote $h^c = [(a_1, ct_1), \dots, (a_n, ct_n)]$ and let $(a, t), (a', t') \in A \times T$. By (5),

$$[(a, t), h_{N \setminus \{i\}}] \succsim [(a', t'), h_{N \setminus \{i\}}] \iff f(a, t) \geq f(a', t'),$$

and

$$[(a, ct), h^c_{N \setminus \{i\}}] \succsim [(a', ct'), h^c_{N \setminus \{i\}}] \iff f(a, ct) \geq f(a', ct').$$

By TSI,

$$[(a, t), h_{N \setminus \{i\}}] \succsim [(a', t'), h_{N \setminus \{i\}}] \iff [(a, ct), h^c_{N \setminus \{i\}}] \succsim [(a', ct'), h^c_{N \setminus \{i\}}].$$

The transitivity of $\succsim$ concludes.

Step 2. We now claim the following. Let $\bar{q} : A \to \mathbb{R}$ be such that $\bar{q}(a_-) = f(a_-, 1)$, for each $a_- \in A$. Then,

$$f(a, t) \geq f(a', t') \iff \bar{q}(a)t \geq \bar{q}(a')t',$$

for each $(a,t),(a',t') \in A \times T$.

Indeed, by definition, $f(a_i,1) = f(a_*, \bar{q}(a_i))$. By Step 1,

$$f(a,t) = f(a',t') \iff f(a,ct) = f(a',ct').$$

Thus, $f(a_i,t_i) = \bar{q}(a_i)t_i$, as desired.

Step 3. There exists $\gamma > 0$ such that, for each $h = [(a_1,t_1),\ldots,(a_n,t_n)] \in H$, and $h' = [(a'_1,t'_1),\ldots,(a'_n,t'_n)] \in H$,

$$h \succsim h' \iff \sum_{i=1}^{n} (\bar{q}(a_i)t_i)^{\gamma} \geq \sum_{i=1}^{n} (\bar{q}(a'_i)t'_i)^{\gamma}.$$

Let P denote the PHEF defined by[11]

$$P[h_1,\ldots,h_n] = P[(a_1,t_1),\ldots,(a_n,t_n)] = \sum_{i=1}^{n} g\left(\bar{q}(a_i)t_i\right),$$

By Step 2, $f(\cdot,\cdot)$ is a monotonic transformation of the function $\tau : A \times T \to \mathbb{R}$ defined by $\tau(a,t) = \bar{q}(a)t$, for each $a,t \in A \times T$. Then, by (5), P represents $\succsim$. By TSI,

$$\sum_{i=1}^{n} g\left(\bar{q}(a_i)t_{-i}\right) \geq \sum_{i=1}^{n} g\left(\bar{q}(a'_i)t'_{-i}\right) \iff \sum_{i=1}^{n} g\left(\bar{q}(a_i)ct_{-i}\right) \geq \sum_{i=1}^{n} g\left(\bar{q}(a'_i)ct'_{-i}\right),$$

for each $h = [(a_1,t_1),\ldots,(a_n,t_n)] \in H$, $h' = [(a'_1,t'_1),\ldots,(a'_n,t'_n)] \in H$ and $c > 0$.

By Bergson and Samuelson (e.g., Burk, 1936; Samuelson, 1965), there are only three possible functional forms for P:

- $P[h_1,\ldots,h_n] = P[(a_1,t_1),\ldots,(a_n,t_n)] = \sum_{i=1}^{n} \alpha_i \left(\bar{q}(a_i)t_i\right)^{\gamma}$,
- $P[h_1,\ldots,h_n] = P[(a_1,t_1),\ldots,(a_n,t_n)] = -\sum_{i=1}^{n} \alpha_i \left(\bar{q}(a_i)t_i\right)^{\delta}$,
- $P[h_1,\ldots,h_n] = P[(a_1,t_1),\ldots,(a_n,t_n)] = \sum_{i=1}^{n} \alpha_i \log\left(\bar{q}(a_i)t_i\right)$,

for some $\gamma > 0$, $\delta < 0$ and sequence $\{\alpha_i\}$ such that $\alpha_i > 0$ for each $i \in N$, and for each $h = [(a_1,t_1),\ldots,(a_n,t_n)] \in H$ such that $t_i > 0$, for each $i \in N$. It is straightforward to show that the last two functional forms cannot be continuously extended to the whole domain H, in which zero time spans are allowed. By ANON, $\alpha_i = \alpha_j$ for each $i,j \in N$. Finally, let $q : A \to \mathbb{R}$ be such that $q(a) = \bar{q}(a)^{\gamma}$, for each $a \in A$. Altogether, we have that $P = P^{pq}$, as desired. ■

[11] The proof of this step follows closely the argument of the proof of Theorem 8 in Østerdal (2005).

5.4 Proof of Theorem 4

We focus on the non-trivial implication, i.e., $2 \to 1$. Formally, assume $\succsim$ satisfies ANON, SEP, PHS, TMPH, HYE, CONT and TSIPH. Then, by Lemma 1, there exists $f : A \times T \to \mathbb{R}_+$ and $g : \mathbb{R}_+ \to \mathbb{R}$ such that:

- f is continuous with respect to its second variable,
- $0 \leq f(a_i, t_i) \leq t_i$, for each $(a_i, t_i) \in A \times T$, and
- g is a strictly increasing and continuous function.

Furthermore, for each $h = [h_1, \ldots, h_n] = [(a_1, t_1), \ldots, (a_n, t_n)] \in H$,

$$h \sim [(a_*, f(a_i, t_i))_{i \in N}],$$

and, for each $h = [(a_1, t_1), \ldots, (a_n, t_n)] \in H$, and $h' = [(a'_1, t'_1), \ldots, (a'_n, t'_n)] \in H$,

$$h \succsim h' \iff \sum_{i=1}^{n} g\left(f(a_i, t_i)\right) \geq \sum_{i=1}^{n} g\left(f(a'_i, t'_i)\right).$$

By TSIPH,

$$\sum_{i=1}^{n} g\left(f(a_i, t_i)\right) \geq \sum_{i=1}^{n} g\left(f(a'_i, t'_i)\right) \iff \sum_{i=1}^{n} g\left(f(a_i, ct_i)\right) \geq \sum_{i=1}^{n} g\left(f(a'_i, ct'_i)\right),$$

for each $h = [(a_1, t_1), \ldots, (a_n, t_n)] \in H$, $h' = [(a'_1, t'_1), \ldots, (a'_n, t'_n)] \in H$ and $c > 0$.

Let P be a PHEF representing $\succsim$. As in the proof of Theorem 3, by Bergson and Samuelson, ANON, and the requirement that P represent $\succsim$ in the whole domain H, it follows that

$$P[h_1, \ldots, h_n] = P[(a_1, t_1), \ldots, (a_n, t_n)] = \sum_{i=1}^{n} \left(f(a_i, t_i)\right)^{\gamma},$$

for some $\gamma > 0$, and for each $h = [(a_1, t_1), \ldots, (a_n, t_n)] \in H$, which concludes the proof. ■

References

[1] Aczel, J., 2006. Lectures on functional equations and their applications. Dover.

[2] Attema A., Brouwer W., 2010. On the (not so) constant proportional trade-off. Quality of Life Research 19, 489-497.

[3] Bleichrodt, H., 1995. QALYs and HYEs: Under what conditions are they equivalent? Journal of Health Economics 14, 17-37.

[4] Bleichrodt, H., 1997. Health utility indices and equity considerations. Journal of Health Economics 16, 65-91.

[5] Bleichrodt, H., Pinto, J., Wakker, P., 2001. Making Descriptive Use of Prospect Theory to Improve the Prescriptive Use of Expected Utility. Management Science 47, 1498-1514.

[6] Bleichrodt, H., Wakker, P., Johannesson, M., 1997. Characterizing QALYs by risk neutrality. Journal of Risk and Uncertainty 15, 107-114.

[7] Bleichrodt, H., Diecidue, E., Quiggin, J., 2004. Equity Weights in the Allocation of Health Care: The Rank-Dependent QALY Model. Journal of Health Economics 23, 157-171.

[8] Broome, J., 1993. Qalys. Journal of Public Economics 50, 149-167.

[9] Blackorby, C., Bossert, W., Donaldson, D., 2006. Population Issues in Social Choice Theory, Welfare Economics, and Ethics. Cambridge University Press.

[10] Burk, A., 1936. Real income, expenditure proportionality, and Frisch's new methods of measuring marginal utility. Review of Economic Studies 4, 33-52.

[11] Culyer, A., Wagstaff, A., 1993. QALYs versus HYEs, Journal of Health Economics 12, 311-323.

[12] Debreu, G., 1960, Topological methods in cardinal utility theory, in K. Arrow, S. Karlin and P. Suppes, (Eds.), Mathematical methods in the social sciences. Stanford University Press.

[13] Doctor, J., Bleichrodt, H., Miyamoto, J., Temkin, N., Dikmen, S., 2004. A New and More Robust Test of QALYs. Journal of Health Economics 23, 353-367.

[14] Doctor, J., Miyamoto, J., Bleichrodt H., 2009. When are person tradeoffs valid? Journal of Health Economics 28, 1018-1027.

[15] Dolan, P., 1998. The measurement of individual utility and social welfare. Journal of Health Economics 17, 39-52.

[16] Dolan, P., 2000. The measurement of health-related quality of life, in Culyer, A.J., Newhouse, J.P. (Eds.), Handbook of Health Economics. Elsevier Science, North-Holland, Amsterdam.

[17] Fleurbaey, M., Schokkaert, E., 2009. Unfair inequalities in health and health care. Journal of Health Economics 28, 73-90.

[18] Gafni, A., Birch, S., Mehrez, A., 1993. Economics, health and health economics: HYEs versus QALYs, Journal of Health Economics 12, 325-339.

[19] Gilboa, I., 2011, Making better decisions. Wiley-Blackwell

[20] Gold, M., Siegel, J., Rusell, L., Weinstein, M., 1996. Cost-effectiveness in Health and Medicine, Oxford University Press, New York.

[21] Guerrero A., Herrero, C., 2005. A semi-separable utility function for health profiles. Journal of Health Economics 24, 33-54.

[22] Loomes, G., McKenzie, L., 1989. The use of QALYs in health care decision making. Social Science and Medicine 28, 299-308.

[23] Mehrez, A., Gafni, A., 1989, Quality-adjusted life years, utility theory, and healthy years equivalents. Medical Decision Making 9, 142-149.

[24] Miyamoto, J., Wakker, P., Bleichrodt, H., Peters, H., 1998. The zero-condition: A simplifying assumption in QALY measurement and multiattribute utility. Management Science 44, 839-849.

[25] Moreno-Ternero J., Roemer J., 2006. Impartiality, priority, and solidarity in the theory of justice. Econometrica 74, 1419-1427.

[26] Moulin, H., 1988. Axioms of cooperative decision making. Cambridge University Press.

[27] Murray C., Salomon J., Mathers C., 2002, A critical examination of summary measures of population health, in Murray C., Salomon J., Mathers C., Lopez A., (Eds.) Summary Measures of Population Health: Concepts, Ethics, Measurement and Application. World Health Organization.

[28] Pliskin, J., Shepard, D., Weinstein, M., 1980. Utility functions for life years and health status. Operations Research 28, 206-224.

[29] Samuelson, P.A., 1965. Using full duality to show that simultaneously additive direct and indirect utilities implies unitary price elasticity of demand. Econometrica 33, 781-796

[30] Spencer, A., 2003. A test of the QALY model when health varies over time. Social Science and Medicine 57, 1697-1706

[31] Spencer, A., Robinson, A., 2007, Tests of utility independence when health varies over time. Journal of Health Economics 26, 1003-1013

[32] Thomson, W., 2001. On the axiomatic method and its recent applications to game theory and resource allocation. Social Choice and Welfare 18, 327-386.

[33] Torrance, G., 1986. Measurement of Health State Utilities for Economic Appraisal. A Review. Journal of Health Economics 5, 1-30.

[34] Turpcu, A., Bleichrodt, H., Le, Q., Doctor J., 2011. How To Aggregate Health? Separability and the Effect of Framing. Medical Decision Making. Forthcoming.

[35] Wagstaff A., 1991. QALYs and the equity-efficiency trade-off. Journal of Health Economics 10, 21-41.

[36] Williams, A., 1997. Intergenerational equity: an exploration of the 'fair innings' argument. Health Economics 6, 117-132.

[37] Østerdal, L., 2005. Axioms for health care resource allocation. Journal of Health Economics 24, 679-702.