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# Multidimensional procurement auctions with unknown weights

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**Abstract**

This paper studies the consequences of holding a procurement auction when the principal chooses not to show its preferences. My paper extends the procurement auction model of Che (1993) to a situation where both the principal and the agents have private information. Thus, unknown parameters of both the principal and the agents leads to unclear reaction strategies. I show that an unknown weight on the principal's valuation of quality leads to the production of to much quality and to high informational rent. A problem that can be reduced using a revelation mechanism. Having an unknown weight on quality gives rise to an analysis of a principal that can not fully commit to the outcome induced by the scoring rule. Therefore, my result apply to contract theory and it's problems with imperfect commitment.

*Keywords*: Mechanism design, contract theory, multidimensional auctions

## 1. Introduction

An object can be sold in many ways. It can be sold through a beauty contest, a signed contract without any forms of competition or through an auction. Auctions are a used format to award contracts in many industries. A government (principal) use auctions to award defence contracts, contracts for hospitals, transport or television franchising (Che, 1993; Branco, 1997; Mougeot and Naegelen, 2003) and private use auctions to award supply chain, running costs and more (Bushnell and Oren, 1995; Milgrom, 2000). Auction is a much used format since competition for the contract is expected to result in lower costs. These kinds of auctions are typically refer to procurement auctions or reverse auctions, because the principal is the buyer and the agents are the sellers. The object being auctioned is the right to supply.

Procurement auctions rarely involves considerations of cost alone. A principal will typically pay attention to cost *and* some sort of quality of the object delivered. Therefore, formats for selling paintings or antiquities will not work well for handing a multidimensional procurement contract. In this article, I consider an auction where a principal, who cares about quality, sells a procurement contract to an agent. Contracts can involve many dimensions and very complex features. Therefore, multidimensional auctions can per se be very complex and a winner can be difficult to select. This is a well-known problem in the theory of auctions. Auctions is a well-studied problem, specially when the auction is a one-dimensional auction. However, the study of multidimensional auctions is still sparse, but increasing due to the increasing use of procurement contracts.

There is literature on multidimensional auctions. Thiel (1988) used consumer theory to show that the design of multidimensional auctions under some conditions will be equivalent to the design of one-dimensional auctions to conclude that there would be no need to investigate multidimensional auctions separately.

The most well-known multidimensional auction is a score auction. In the score auction, the principal maps the multidimensional bids into a one-dimensional score. Theory often assumes that the score represents the principal's utility or the welfare function for which the highest score represents the highest possible utility or welfare. Dasgupta and Spulber (1990) used a score in a per-unit auction. They present a variable-quantity procurement auction, where agents submit their private cost parameter $\theta$ in the light of a quantity schedule preannounced by the principal $Q(\cdot)$. The principal chooses the agent with the lowest $\theta$ who will deliver $Q(\theta)$ and will receive $B(\theta)$. Special for the Dasgupta and Spulber model is that the implementation does not work for general cost functions, but only when the function exhibits increasing returns to scale.

The work by Che (1993) is central in the litterature on multidimensional score auctions. Che considers a multidimensional auction in which agents compete in price and quality in an environment of asymmetric information (as Dasgupta and Spulber). In contrast to Dasgupta and Spulber, the model of Che does not require a special form of cost function. More interesting is that Che shows how the existing theory of auctions can be generalized to multidimensional auctions. He considers a first-score, second-score and a so-called second-offer auction. In all auctions, agents' bids are evaluated according to a scoring rule and the winner is the agent with the highest score. The variation between the three formats follow the general litterature. One result from Che is interesting from my point of view. Che shows that a principal that can not commit to a scoring rule will end up buying an excessive among of quality. The result is based in a context where the principal will find it difficult to communicate it's preferences over complicated quality specifications. Therefore, the principal can only credibly commit to a scoring rule that reflects it's own true preferences. Che compare this naive scoring rule with a optimal mechanism defined by revelation principle which results in a lower among of quality, and argue that the lack of commitment results in excessive quality, because the true utility function/true perference ordering fails to internalize the information rent associated with increasing quality.

The model presented in this paper challenge this conclusion. In the Che model the principal's valuation function reflecting the true preference ordering is public known. The model described in this paper is an extension of the Che model to include an unknown weight on the principal's valuation of quality. This means that the agents do not know the preferences of the principal. These unknown preferences give rise to uncertainty among the agents, because agents do not know what kind of consideration the principal will take into account in the final decision. Instead, the agents must search for the best strategy given the unknown $\omega$. One could ask why a principal in a procurement auction want to hide it's preferences. One explanation could be that the principal is unsure about it's own preferences and therefore can not commit to it's own scoring rule. This is interesting for the theory of contracting and in line with the Che model. In the Che model the noncommitment relates to the difficulties associated with contracting as a result of complicated quality specifications. If we assume that an unknown weight on quality is related to a principal that can not commit to it's scoring rule, the noncommitment situation in my paper comes from an unknown weight on the principal's valuation of quality $\omega$, not because of some true perference ordering. In a modified version of Che (1993), I show that under certain conditions noncommitment, not based on a scoring rule that reflects the principal's preference ordering, but through an unknown weight on quality, results in excessive quality and to high informational rent. The reason for my result is that an unknown weight on the valuation of quality and a principal indicating that quality is desirable makes the agents unclear about the expected, wherefore they bid as much as they can. Because this is the central result of the my paper, agents operate under limit capacity. Both Che (1993) and Dasgupta and Spulber (1990) consider a model where the agents have unlimited capacity.

In contrast to other papers, I present an analysis of the consequences having an unknown weight on the principal's valuation of quality. Compared to the optimal revelation principle, I find that an unknown weigth on quality entails excessive quality and too high informational rent following asymmetric information under a first-price sealed-bid auction. This suggests that there may be an incentive for the principal to deviate from a scoring rule auction and instead use a mechanism based on the revelation principle. If the principal cannot commit to any scoring rule, the principal can ask the agents to directly bid their private cost parameter.

The lowest cost parameter wins the contract. Using the revelation principle, the principal can, because of the the true cost parameter, reduce the informational rent by decreasing the quality.

A crucial assumption of my model is a principal that values marginal quality more than the agents marginal cost. The assumption reflects the feature that a principal/politician holding a procurement auction is under close media attention. So a politician with local private interest will typically have a larger marginal utility for quality compare to an agent that can produce an extra among of quality at a low cost. The reaction strategies for both the principal and the agents become unclear when this assumption is dropped.

This paper is organized as follows: Section 2 explains the structure of the model and the revelation principle. In section 3 the first-price sealed-bid auction and the main result is presented. Section 4 contains the conclusion and fields of further research.

## 2. The model

In this section, we introduce the idea of an uncertain scoring rule and consider its impact on what may be considered as the benchmark case, namely an optimal agency contract.

Consider a principal proposes a procurement contract for delivering a certain good by an agent. The contract specifies a vector of quality $q$ and a cost $C(q, \theta_i)$, where $\theta_i$ is the parameter describing the efficiency level of agent $i$, $i = 1, ..., n$. Quality is modelled as a one-dimensional attribute and $\theta_i$ is private information. The agents operate under capacity constraints and the cost function exhibits constant returns to scale. The cost function is given by

$$C(q, \theta_i) = \theta_i C_0(q), \qquad q \leq \theta_i q_c, \tag{1}$$

where $C_0(q) = K + cq$. Here, $K > 0$ is fixed cost and $c > 0$ is marginal cost. Assume that the fixed cost to perform the procurement contract is sufficiently high compared to the total cost. Assume that the principal knows this proportion. Define $C_q$ and $C_{\theta_i}$ as the marginal cost with respect to $q$ and $\theta_i$ and $C_{\theta_i q}$ as the mixed partial, first with respect to $\theta_i$ and then with respect to $q$. Given the cost function, we have that $C_q > 0$, $C_{\theta_i} > 0$ and $C_{\theta_i q} > 0$. Define $\theta_i q_c$ as the capacity constraint of an agent of type $\theta$. Hence, the cost function is affine and increasing in $q$ and $\theta_i$ for any quality level $q$ below $\theta_i q_c$. Any $q$ above $\theta_i q_c$ is not possible. The type variable $\theta$ is one-dimensional and it is identically and independently distributed over $\theta \in [\underline{\theta}, \bar{\theta}]$ according to a distribution function $F(\cdot)$ with a continously differentiable density function $f(\cdot) > 0$, $F(\underline{\theta}) = 0$ and $F(\bar{\theta}) = 1$. $F(\cdot)$ be common knowledge and satisfy the increasing monotone hazard rate, $\dfrac{F(\cdot)}{f(\cdot)} > 0$. Since the agents have the same type of cost function and only know the distribution of the types, we can analyze the model as a symmetric model. Because of symmetry among agents, the subscript $i$ is dropped.

The agents propose a contract by announcing a $\theta$. The agent awarded the contract will have its cost compensated by a transfer $t$. Therefore, the announcement can be rewritten as $(q, t)$. The principal obtains utility from quality and transfer. The principal's utility from a contract $(q, t)$, given the agent is of type $\theta$, is given by

$$U(q(\theta), t(\theta)) = \omega V(q(\theta)) - t(\theta), \tag{2}$$

where $\omega$ is an unknown weight assigned to the principal's valuation of the announced quality $V(q(\theta))$, $\omega \in [\underline{\omega}, \bar{\omega}]$, $\omega > 0$. We assume that $\omega V(q(\theta))$ is increasing and concave in $q(\theta)$, $\omega V'(q(\theta)) > 0$, $\omega V''(q(\theta)) < 0$ for every $\omega$. Given $\omega V(q(\theta))$ and $t(\theta)$, $U(q(\theta), t(\theta))$ is increasing and concave in $q(\theta)$ and $t(\theta)$. One can say that the weight camouflage the principal's utility, because the principal is unsure about it's own preferences. The unknown weight results in unclear reactions, since the agents do not know exactly how the principal will finally decide.

A agent $i$, upon winning, earns from an announcement ($q(\theta)$, $t(\theta)$) a profit of the form

$$\pi(\theta) = t(\theta) - C(q(\theta), \theta), \tag{3}$$

Assumption 1. $\dfrac{\partial U(q(\theta), t(\theta))}{\partial q} > \dfrac{\partial C(q(\theta), \theta)}{\partial q} > 0$

where $\dfrac{\partial U(q(\theta), t(\theta))}{\partial q} = \omega V'(q(\theta))$ and $\dfrac{\partial C(q(\theta), \theta)}{\partial q} = C_q(q(\theta), \theta)$. The assumption implies that the principal's marginal utility is higher than an agent's marginal utility for any $q$. Assumption 1 is related to the assumption that we have a principal who prefers quality. Assumption 1 is a strong assumption, but the intuition behind this assumption is very important for understanding an eligible politician's preferences and choices when some procurement contracts are sold. Some areas have local interest and some areas are under close media attention. Therefore, the utility of an extra unit of quality is high and in my case higher than a cost function exhibiting constant returns to scale. Of course there are areas for which there are no public interest and the aim here could be best quality for lowest price, but if there is interest in some areas, it would be an error not to assume something in the direction of assumption 1.

Assume that the difference between $\dfrac{\partial U(q(\theta), t(\theta))}{\partial q}$ and $\dfrac{\partial C(q(\theta), \theta)}{\partial q}$ is relative small. The assumption is related to the cost function. A principal knowing roughly the size of the agents costs, because of $K$, adjust it's preferences hereafter. Therefore, the principal's marginal utility comes closer to the agent's cost. Given the asymmetric information, the agent getting the contract will have a rent. Define $C_{\theta q}(q(\theta), \theta)\frac{F(\cdot)}{f(\cdot)}$ as the informational rent. Taking these properties as given, assume that the informational rent has a significant size compared to the difference between $\frac{\partial U(q(\theta), t(\theta))}{\partial q}$ and $\frac{\partial C(q(\theta), \theta)}{\partial q}$. In other words, assume that the informational rent has a suitable size for every $q$ produced. This assumption is typically in real-life when a principal/politican is under close consumer and media attention. A principal/politican under pressure will in a procurement process have it's focus on a reliable delivering and economic stability and will typically accept some informational rent in some suitable size. Also in cases where the principal/politican has an idea regarding the size of the cost.

As said in the introduction an allocation mechanism can take many forms. It can be a beauty contest, an auction or simply a signed contract without any competition. In the rest of this section I take in the set-up from above and derive the principal's optimal contract that uses the quality and cost/transfer as input in the contract terms. We can address the problem using the revelation principle developed by Myerson (1981). According to the revelation principle, there is a direct (the agent only reports it's private cost) and truthful (the agent truthfully reports it's cost) mechanism that is optimal. In the context of a direct revelation game each agent is assumed to announce to the principal a report of it's cost parameter $\theta$. At the equilibrium, truth-telling is the best response to truth-telling by all other agents. In the context of winning a contract the lowest reported $\theta$ wins the constract. The losing agents do not produce. The revelation principle has been thoroughly investigated. My treatment of this case will be resticted to investigate the revelation principle described in Laffont and Tirole (1987) to the present situation with an unknown weigth on quality. This will be the topic of the remaining part of this section. I will then analyse the set-up in a first-price sealed bid auction in the following section.

Consider a principal facing a mechanism design problem and some agents facing an unknown weight on quality. Both are restriced because of incomplete information. The principal maximizes expected welfare and agents maximize the expected value of their profits. The problem is that both are a certain type, but can announce something else. The principal has not announced it's preferences. The agents can misreport it's cost by announcing a higher cost, $\hat{\theta} > \theta$. I am looking for a mechanism where the agents announce it's truth type $\theta$ and the principal chooses $(q(\theta), t(\theta))$ which maximizes expected welfare. The principal will make his decisions based on welfare, which takes into account the informational rent that will be given up to the agent getting the contract. The following proposition is a restatement of a result from Laffont and Tirole (1987) which holds for my situation with an unknown weight on quality. A proof is given in the appendix of this paper.

Proposition 1. *Assume that $U(q(\theta), t(\theta))$, $C(q(\theta), \theta)$ and $\dfrac{F(\cdot)}{f(\cdot)}$ are increasing in $q(\theta)$ and $\theta$. Then the optimal revelation mechanism $(q^*(\theta), t^*(\theta))$ satisfies the following equations*

$$\omega V'(q^*(\theta)) = C_q(q^*(\theta), \theta) + C_{\theta q}(q^*(\theta), \theta)\frac{F(\cdot)}{f(\cdot)}, \tag{4}$$

$$t^*(\theta) = C(q^*(\theta), \theta) + \int_{\theta}^{\bar{\theta}} C_\theta(q^*(s), s)\, d(s) \tag{5}$$

Equation (4) says that the principal's marginal utility is equal to the agent's marginal cost of quality adjusted for some informational rent. So given the unknown weight on quality, a higher weight means higher valuation, higher marginal cost and/or higher informational rent. This informational rent is given by the integral in equation (5) and is greater, the lower is the agent's marginal cost.

My result is classical in the literature due to Myerson (1979), Laffont and Tirole (1987) and more and used in different context, see for example Che (1993), Dasgupta and Spulber (1989), Mougeot and Naegelen (2003). In the classical interpretation of the equations, informational rent means that the optimum is not efficient, unless the winning agent is the lowest-cost agent, $\theta = \underline{\theta}$. The ability of the low-cost agent to mimic the higher-cost agent and report higher cost forces the principal to allow for some informational rent to the agent awarded the contract. To reduce this rent, quality is distorted downwards. This distortion is represented by $C_{\theta q}(q^*(\theta), \theta)\dfrac{F(\cdot)}{f(\cdot)}$, so the higher the lowest reported cost, the higher the quality distortion. Thus, equations (4) and (5) together show that lower $\theta$ means higher informational rent and higher quality level.

In the next section I discuss the first-price sealed-bid auction. Specifically, I will discuss the bid $(q(\theta), t(\theta))$ in the multidimensional auction considered in Che (1993).

## 3. First-price sealed-bid auction

In this section I study the properties of using an auction instead of a revelation mechanism. Using the revelation principle as a benchmark, the aim in this section is to show that a first-price sealed-bid auction used in a procurement situation yields inefficient quality outcomes and too high informational rent.

The auction model used in this section is similar to the first score auction in Che (1993), but in my case the principal announces its scoring rule with an unknown weight on it's valuation of quality. Compared to Che, this means that the agents have to search for the best strategy for each weight scenario. In contrast to litterature, this paper study the agents reaction to the unknown preference rather than searching for the optimal scoring rule. I assume that the principal use a first-price sealed-bid auction where the agent with the highest score wins the contract. The first-price sealed-bid auction is an auction format which is much used in real-life and therefore it seems natural to use it in the present paper. The model presented in this paper is important to analyse not only because of it's prevalence in practice, but also because it skeds some light to the auction theory. In particular, it highlights the problems and understanding of the problems involving an uncleared principal.

Using the set-up from section 2, consider a principal announcing an unclear criteria for quality in which agents submit bids $(q, t)$, where $q$ represents the level of quality offered and $t$ is the payment asked by an agent for delivering $q$. The principal ranks the agents bid according to a scoring rule $S(q(\theta),\ t(\theta))$. Assume that the scoring rule is quasilinear. The principal then select an internal more precise scoring rule and a winner is chosen. The internal scoring rule ranking the offered $(q(\theta),\ t(\theta))$ is private information to the principal. The winner performs the offered bid. Using a known weight on quality, the agents can take the principal's preferences as given, placed a bid for which the agent's profit is maximized and known that the highest score wins the contract. Using a unknown weight on quality makes the selecting process closer to a random selection process rather than an auction where highest score wins. The uncertainty arises because the agents do not know exactly what kind of considerations the principal will take into account in the final decision. In other words, it is unclear how strongly the principal values quality in the final decision.

Figure 1 illustrates the idea behind my model. An agent is represented by it's cost function $C(q(\theta), \theta)$ and the principal by it's utility function $U(q(\theta), t(\theta))$. The cost function is above origo which indicates that the agents add a mark-up on the cost offered. By Assumption 1, the principal's utility function has a greater slope than the agent's cost function. Besides the higher marginal utility in relation to cost, the arrows indicate the preferences of the two parties. The principal prefers more quality and lower cost or points further to the south-east. An agent prefers higher transfer and less quality or points further to north-west. Agents $\theta$ and $\theta'$, $\theta < \theta'$, operate under capacity constraints which is given by $\theta q_c$ and $\theta' q_c$. The area between the two lines $\underline{\omega}$ and $\bar{\omega}$ (the two dotted lines) represents the area for which Assumption 1 is fulfilled and where the agents can rely their strategy on. Below $\underline{\omega}$, Assumption 1 will not be fulfilled. Therefore, the principal's utility function/scoring rule will lie between $\underline{\omega}$ and $\bar{\omega}$ where every points along this utility function/scoring rule gives the same probability of winning the contract.

I am interested in an equilibrium where all agents evaluate their best response and rank their strategies within the two dotted lines. Analysing this, one can prove that the unknown weight on quality entails an offered quality equals to agents production capacity. The solu-

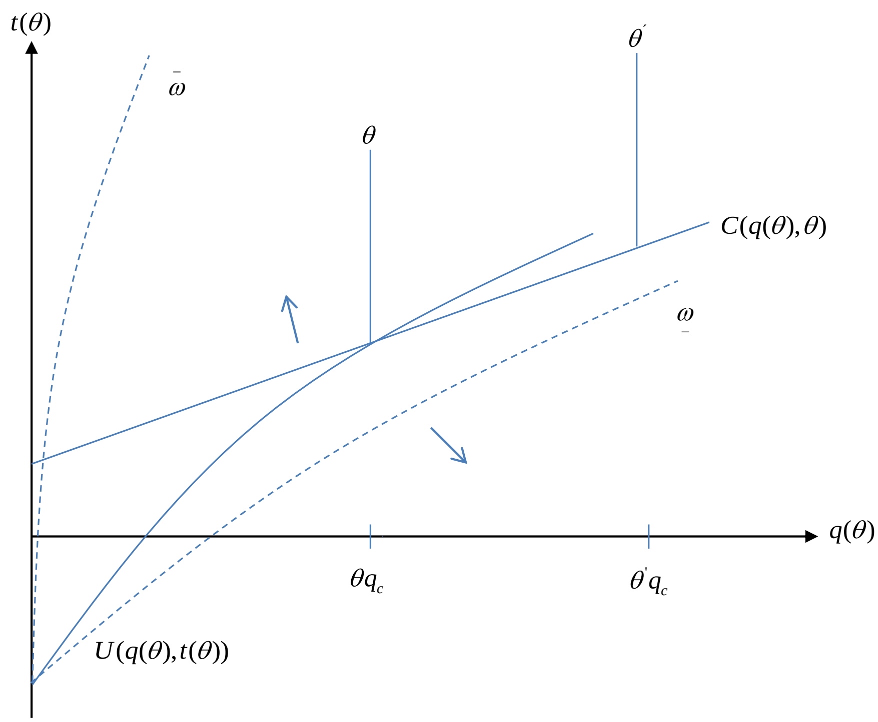


**Fig. 1.** Equilibrium with unknown weights

tion is given by Proposition 2.

Proposition 2. *For every type $\theta \in [\underline{\theta}, \bar{\theta}]$, there is an equilibrium such that an agent produces at the capacity $\theta q_c$.*

Proof. *see Appendix*

It is easy to check that the quality produced in the first-price sealed-bid auction is higher than using the revelation mechanism. The reason for this result is the following. The agents are uncertain about the principal's valuation of the quality. The agents know that the principal prefers quality, but they do not know the preferences of the principal exactly. Likewise, the agents do not know exactly what kind of considerations the principal will take into account in the final decision. Because of this, the agents produce as the point of departure as much as they can.

However, in a revelation mechanism the agents truthfully reports it's cost and the principal makes it's decision upon the agents report. The cost informed yields the information needed for the decision upon the rent given up to the agent getting the contract. In other words, the rent because of asymmetric information becomes visible. Hence, the principal using the revelation mechanism can use this information to distort the quality downward to limit the informational rent before the contract is given regardless of the unknown weight on quality. This is possible, because an agent announce it's cost directly before given the contract and producing. In an auction the principal do not get any information about the agents cost before it offers a quality and a transfer and performs. Therefore, the principal can not internalize the informational rent associated with an increase in quality before a winner shall be chosen. So given the unknown weight on quality and a principal showing interest in quality leads to the production of to much quality when using a first-price sealed-bid auction. The possibility for the principal to distort the quality or adjust for cost and rent leads to a smaller among of quality when using a revelation mechanism compared to the auction. Reducing the quality could have an effect on welfare. Reducing the quality reduce rent/cost and keeping the winner's profit unchanged generates welfare.

The main result of this article is the production of to much quality as a result of the agents uncertainty about the unknown weight. But why should a principal hide it's preferences? One explanation could be that the principal itself is unsure about it's own preferences and therefore can not commit to it's own scoring rule. If this is truth, lack of commitment give rise for to much quality and therefore to high a transfer from the principal to the winning agent. One suggestion from the point of my analysis is to deviate from the first-price sealed-bid auction for instead to use a contract design based on the revelation principle. The main advantage of using an auction is the simplicity. The drawback of using my auction model is the uncertainty. In an(/my) auction the winning agent will be selected before it's performs. If the principal is unsure about it's own preferences and therefore announce a unclear scoring rule, the outcome of the auction is unclear and may not be the wanted outcome. In addition, there is a possibility that the winning agent could perform better if the principal knew the agents private cost information. Using a contract design based on the revelation principle (that have an agent announce it's true type before producing), the principal can take advance of the information revealed to distort down the rent. Looking at (4), proposition 2 and assumption 1, one can see that the use of a revelation mechanism lowering informational rent and makes the difference between the two options bigger. So based on the results in this article, the effect from using a contract could be quality distortion, lower informational rent and welfare improvement.

# 4. Conclusion and Extensions

The contribution of this paper is an extension of the model of Che (1993) to contacting problems when both the principal and the agents have private information. More precisely, the paper makes contribution to contracting problems with imperfect commitment. I show that an unknown weight on a principal's valuation of quality in a multidimensional procurement auction leads to excessive quality and to high informational rent. This problem can be reduced by the use of a revelation mechanism. If we assume that an unknown weight on quality comes from the fact that we have principal that can not commit to its own scoring rule, my result is in line with the Che model. In contrast to Che, my model is general for the extent to study problems with limited commitment. Interesting compared to the Che model is the unknown weight on quality. In the Che model the optimum will be known to all agents and will be the tangency point between the agent's iso-profit curve and the principal's announced iso-score curve. With an unknown valuation on quality, this tangency point is unclear. An agent that do not know how much to bid to win the contract chooses to bid as much as it can. So compared to a game with a known weight on quality, an unclear principal using an auction results in an ineffective among of quality and to high a cost.

As most of the literature on auctions search for the optimal auction/mechanism design, my analysis only study the problem having a scoring rule with an unknown weight on the valuation of quality. Therefore, it remains an open question to what extent one can characterize an optimal scoring rule in settings with an unknown weight on quality or an uncommitted principal. In connection to this, other auction formats could be analysed. Che (1993) study a first-score, a second-score and a so-called second-preferred-offer auction. Likewise, one could study an open auction format.

Another issue is the assumption of a principal that values marginal quality more an agent's costs. In practice, however, a principal can have a lower utility for quality than an agent's cost. For instant for cases where there is no media attention or no political interest in the subject. In this case, the reaction strategies for both parties become unclear. Thus, dropping assumption 1 causes complications in the analysis finding an equilibrium. One could generalize this assumption.

A third issue could be the impact of bargaining. Factors like unknown preferences, uncertainty because of complicated quality specifications or other moral hazard/adverse selection-type issues may provide a rational for using bargaining. Studying the impact on the performance of bargaining on outcome of a procurement process is therefore relevant. There seems to be consensus among economists that an auction followed by negotiation is preferred. One could argue that a principal after ending the auction could influence the agents reaction in a more prefered way. There are several papers studying negotiation with auctions (Branco, 1997; Bulow and Klemperer, 1996, 2009, Asker and Cantillon, 2010). Specially Branco (1997) study multidimensional auctions and negotiation in a two-stage model. In the first stage the principal evaluates each bid according to a scoring function. In the second stage the principal and the winner of the auction bargain over the level of quality to be provided. Compared to Branco, the analysis in connection to my paper should be concentrated to an unknown weight on the principal's valuation of quality. In connection to this is noncommitment. Laffont and Tirole (1993) study negotiation and noncommitment. They too study a two-period model, but compared to my paper the noncommitment part in the Laffont-Tirole model is on the agents. Laffont and Tirole shows that the first-period contract should have some incentive or rewards if the noncommitted part is going to reveal it's information and commit to the contract. Lack of incentive makes such a revelation and commitment costly to the agents. In connection to my model, one could turn this around to think of a principal and an agent discussing the principal's preferences, the firm's performances and the details for commitment. These areas could be the topic of future research.

# 5. Appendix

Proof of proposition 1. Let $\pi(\hat{\theta}, \theta) = t(\hat{\theta}) - C(q(\hat{\theta}), \theta)$ be an agent's profit if it's private parameter is $\theta$, but it announces $\hat{\theta}$, while all other agents truthfully reveal their private parameter. Let $\pi(\theta)$ be given by (3). From (2) for $\theta, \hat{\theta} \in [\underline{\theta}, \bar{\theta}]$ let the principal's maximization problem be

$$\max_{q(\cdot),t(\cdot)} \int_{\underline{\theta}}^{\bar{\theta}} [\omega V(q(\theta)) - t(\theta)]\, f(\theta) d\theta$$

subject to

$$\pi(\theta) = t(\theta) - C(q(\theta), \theta) \geq 0, \tag{6}$$

$$t(\theta) - C(q(\theta), \theta) \geq t(\hat{\theta}) - C(q(\hat{\theta}), \theta), \tag{7}$$

where (6) is the individual rationality constraint (IR) and (7) is the incentive compatible constraint (IC). Assume that condition (6) is satisfied. Truth-telling implies that profits are maximized at $\hat{\theta} = \theta$,

$$\frac{\partial \pi(\theta)}{\partial \theta} = \frac{\partial t(\theta)}{\partial \theta} - C_q \frac{\partial q(\theta)}{\partial \theta} - C_\theta(q(\theta), \theta)$$

Since $\frac{\partial t(\theta)}{\partial \theta}$ and $C_q \frac{\partial q(\theta)}{\partial \theta}$ are 0 in optimum, we have

$$\dot{\pi}(\theta) = -C_\theta(q(\theta), \theta)$$

Then, by the envelope theorem of Milgrom and Segal (2002), we can write

$$\pi(\theta) = \pi(\bar{\theta}) + \int_\theta^{\bar{\theta}} C_\theta(q(s), s)\, ds \tag{8}$$

From (3) define

$$\pi(\bar{\theta}) = t(\bar{\theta}) - C(q(\bar{\theta}), \bar{\theta}) \tag{9}$$

Using (8) and (9), (3) can be written as

$$t(\theta) = t(\bar{\theta}) - C(q(\bar{\theta}), \bar{\theta}) + C(q(\theta), \theta) + \int_\theta^{\bar{\theta}} C_\theta(q(s), s)\, ds$$

for which the principal's problem can be rewritten as

$$\max_{q(\cdot), t(\cdot)} \int_{\underline{\theta}}^{\bar{\theta}} [\omega V(q(\theta)) - (t(\bar{\theta}) - C(q(\bar{\theta}), \bar{\theta}) + C(q(\theta), \theta) + \int_\theta^{\bar{\theta}} C_\theta(q(s), s)\, ds)]\, f(\theta) d\theta$$

or

$$\max_{q(\cdot), t(\cdot)} \int_{\underline{\theta}}^{\bar{\theta}} [\omega V(q(\theta)) - \xi - C(q(\theta), \theta)]\, f(\theta) d\theta - \int_{\underline{\theta}}^{\bar{\theta}} \int_\theta^{\bar{\theta}} [C_\theta(q(s), s)\, ds]\, f(\theta) d\theta$$

where $\xi = t(\bar{\theta}) - C(q(\bar{\theta}), \bar{\theta})$ is a constant. Changing the order of integration in the second member leads to

$$\max_{q(\cdot),t(\cdot)} \int_{\underline{\theta}}^{\bar{\theta}} [\omega V(q(\theta)) - \xi - C(q(\theta),\theta)]\, f(\theta) d\theta - \int_{\underline{\theta}}^{\bar{\theta}} \int_{\underline{\theta}}^{\theta} [C_\theta(q(s),s)\, f(\theta) d\theta] ds$$

Integrating the third integral and using that $F(\underline{\theta}) = 0$, we get

$$\max_{q(\cdot),t(\cdot)} \int_{\underline{\theta}}^{\bar{\theta}} [\omega V(q(\theta)) - \xi - C(q(\theta),\theta)]\, f(\theta) d\theta - \int_{\underline{\theta}}^{\bar{\theta}} C_\theta(q(\theta),\theta)\, F(\theta) d\theta$$

Rearranging and the problem will be

$$\max_{q(\cdot),t(\cdot)} \int_{\underline{\theta}}^{\bar{\theta}} [\omega V(q(\theta)) - \xi - C(q(\theta),\theta) - C_\theta(q(\theta),\theta)\frac{F(\cdot)}{f(\cdot)}]\, f(\theta) d\theta$$

Define

$$H = (\omega V(q(\theta)) - \xi - C(q(\theta),\theta) - C_\theta(q(\theta),\theta)\frac{F(\cdot)}{f(\cdot)})\, f(\theta)$$

as the Hamiltonian without co-state variable. The maximum principle gives

$$\omega V'(q(\theta)) = C_q(q(\theta),\theta) + C_{\theta q}(q(\theta),\theta)\frac{F(\cdot)}{f(\cdot)}$$

which is (4).

Using (3), (8) and the fact that $\pi(\bar{\theta}) = 0$ gives the optimal transfer

$$t(\theta)) = C(q(\theta),\theta) + \int_{\theta}^{\bar{\theta}} C_\theta(q(s),s)\, d(s)$$

which is (5).

□

Proof of proposition 2. From the possible interval $\omega \in [\underline{\omega}, \bar{\omega}]$, it will be enough to analyse the agents response in connection to $\underline{\omega}$. Suppose that $(q(\theta), t(\theta))$ is an equilibrium where $q(\theta) \neq \theta q_c$ for at least one agent $\theta < \bar{\theta}$. A contradiction is to show that $(q(\theta), t(\theta))$ is strictly dominated by an alternative bid $(q'(\theta), t'(\theta))$, where $q'(\theta) = \theta q_c$. Because $(q(\theta), t(\theta))$ and $(q'(\theta), t'(\theta))$ have the same probability of winning the auction, we have that $S(q(\theta), t(\theta)) = S(q'(\theta), t'(\theta))$ or $\omega V(q(\theta)) - t(\theta) = \omega V(q'(\theta)) - t'(\theta)$. Analysing for $\underline{\omega}$ gives us

$$
\begin{aligned}
\underline{\omega}V(q(\theta)) - t(\theta) &= \underline{\omega}V(q'(\theta)) - t'(\theta) \\
t'(\theta) &= t(\theta) + \underline{\omega}V(q'(\theta)) - \underline{\omega}V(q(\theta))
\end{aligned}
$$

Define $\varphi = (\underline{\omega}V(\theta q_c) - C(\theta q_c, \theta)) - (\underline{\omega}V(q(\theta)) - C(q(\theta), \theta))$.

$$
\begin{aligned}
\pi(q'(\theta), t'(\theta)) &= [t'(\theta) - C(q'(\theta), \theta)]Prob\{win|S(q'(\theta), t'(\theta))\} \\
&= [t(\theta) + \underline{\omega}V(\theta q_c) - \underline{\omega}V(q(\theta)) - C(\theta q_c, \theta)]Prob\{win|S(q(\theta), t(\theta))\} \\
&= [t(\theta) - C(q(\theta), \theta) + \varphi]Prob\{win|S(q(\theta), t(\theta))\} \\
&> [t(\theta) - C(q(\theta), \theta)]Prob\{win|S(q(\theta), t(\theta))\} \\
&= \pi(q(\theta), t(\theta))
\end{aligned}
$$

Using assumption 1, the fact that $\underline{\omega}$ is the same in $\varphi$ and add and subtract $C(q(\theta), \theta)$, we then have that $\varphi$ is a positive number, and we can conclude that $\pi(q'(\theta), t'(\theta)) > \pi(q(\theta), t(\theta))$. $\square$

# 6. References


Asker J. and E. Cantillon (2010), Procurement when price and quality matter, RAND Journal of Economics, Vol. 41(1), 1–34.

Branco F. (1997), The design of multidimensional auctions, RAND Journal of Economics, Vol. 28(1), 63 – 81.

Bulow J. and P. Klemperer (1996), Auctions Versus Negotiation, The American Economic Review, Vol. 86(1), 180 – 194.

Bulow J. and P. Klemperer (2009), Why Do sellers (Usually) Prefer Auctions, The American Economic Review, Vol. 99(4), 1544 – 1575.

Bushnell J.B (1995), Internal auctions for the efficient sourcing of intermediate products, Journal of Operations Management Vol. 12(3-4), 311 – 320.

Che Y.-K (1993), Design Competition Through Multidimensional Auctions, RAND Journal of Economics, Vol. 24(4), 668 – 680.

Dasgupta S. and D.F. Spulber (1989/1990), Managing Procurement Auctions, Information Economics and Policy, Vol. 4(1), 5 – 29

Laffont J.J. and J. Tirole (1987), Auctioning Incentive Contracts, Journal of Political Economy, Vol. 95(5), 921 – 937.

Laffont J.J. and J. Tirole (1993), A Theory of Incentive in Procurement and Regulation, M.I.T. Press, Cambridge, MA.

Milgrom P. (2000), An economist's vision of the b-to-b marketplace, An Executive White Paper, www.perfect.com.

Milgrom P. and I. Segal (2002), Envelope Theorems for Arbitrary Choice Sets, Econometrica Vol. 70(2), 583 – 601.

Mougeot M. and F. Naegelen (2003), Using Auctions for Contracting with Hospitals When Quality Matters, International Journal of Health Care Finance and Economics, Vol 3(1), 5 – 23.

Myerson R. (1979), Incentive Compatibility and the Bargaining Problem, Econometrica, Vol. 47(1), 61 – 73.

Myerson R. (1981), Optimal coordination mechanisms in generalized principal agent problems, Journal of Mathematical Economics, Vol 10(1), 67 – 81.

Thiel S.E (1988), Multidimensional auctions, Economics Letters, Vol. 28(1), 37 – 40.