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Imperfect Knowledge, Asset Price Swings and Structural Slumps: A Cointegrated VAR Analysis of their Interdependence

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Abstract

This paper is an empirically based discussion of interactions between speculative behavior in the currency markets and aggregate fluctuations in the real economy. It builds on the recent theory of Imperfect Knowledge Economics in Frydman and Goldberg (2007) and combines this with the Structural Slumps theory in Phelps (1994). The paper argues that this is likely to imcrease our understanding of the long recurrent spells of high unemployment that continue to mar our economies.

Keywords: Financial markets, Speculation, Long Swings, Imperfect Knowledge, CVAR

JEL Classification:E24, F31, F41

*The views in this paper are strongly influenced by previous and ongoing research with Roman Frydman, Michael Goldberg and Søren Johansen. I am deeply grateful to Roman and Michael for sharing with me their profound insight in international macro and how imperfect knowledge economics can resolve its many empirical puzzles. I am much indebted to Roman Frydman and Mikael Juselius for numerous comments on an early version which have significantly improved this paper.

1 Introduction

The aim of this paper is to discuss empirically interactions between speculative behavior in the currency markets and aggregate activity in the real economy. The main idea is to combine the recent theory of Imperfect Knowledge Economics (IKE) in Frydman and Goldberg (2007) with the Structural Slumps theory in Phelps (1994) to gain a more complete understanding of the two-way interdependence between persistent swings in asset markets and persistent fluctuations in the real economy.

Phelps (1994) provides a coherent theoretical framework for how non-monetary mechanisms can generate unemployment slumps in open economies connected by the world real interest rate and the real exchange rate, whereas Frydman and Goldberg (2007) provides the basis for why real exchange rates tend to fluctuate around long-run benchmark values and why the real interest differential is likely to move in similar persistent swings.

Thus, the structural slumps theory emphasize real interest rates and real exchange rates as potentially important determinants underlying the persistent fluctuations in aggregate activities and the IKE theory provides conditions under which speculative behavior generates persistence in real exchange rates and real interest rates. The combination of IKE with the structural slumps theory is, therefore, likely to give us a framework within which we can understand how and when financial markets tend to generate persistent fluctuations in the real economy.

Whether this is empirically relevant is ultimately a question of econometric testing. Macroeconomic models are notoriously difficult to take to data characterized by unit root nonstationarity and breaks. The Cointegrated VAR (CVAR) approach (Johansen, 1996 and Juselius, 2006) chosen here offers a precise way of handling unit roots, breaks and feed-back dynamics and, thus, can be used to discriminate between empirically relevant and irrelevant claims.

The organization of the paper is as follows: Sections 2 contrasts a Rational Expectations Hypothesis (REH) based model for exchange rate determination with an IKE based model and discusses their persistency implications. Section 3 suggest an empirical methodology for using persistence as a structuring device and Section 4 how to apply it based on the CVAR model. By translating the main theoretical assumptions underlying the above two models into testable hypotheses within the CVAR model, Section 5 discusses their empirical relevance. Section 6 discusses how foreign currency speculation under IKE interacts with a customer market economy where profit shares are adjusting to fluctuations in the real exchange rate and where the natural rate is a function of a nonstationary real long-term interest rate. Section 7 concludes.

2 Expectations formation and the persistence in the real exchange

In a forthcoming paper, Frydman et al. (2010) show how the REH and the IKE assumption on expectations formation lead to different hypotheses on persistence of real exchange rate and real interest rate differentials and co-movements between them. The discussion here is broadly based on this paper.

2.1 Rational expectations based models

The REH-based monetary model assumes that PPP holds as an equilibrium condition so that the real exchange rate, q_t, is a stationary process, i.e.

$$q_t = \rho q_{t-1} + \varepsilon_{1,t} \tag{1}$$

where $\rho < 1.0$. The stationarity of the real exchange rate is consistent with UIP as a market clearing mechanism:

$$i_{1,t} - i_{2,t} = \Delta s^e_{t+1} + rp_t \tag{2}$$

where rp_t is a stationary risk premium. Provided (1) and (2) holds, the Fisher parity holds as a stationary condition:

$$i_t = \bar{r} + \Delta p^e \tag{3}$$

where $\bar{r}$ is an average real interest rate. Similarly, under the above conditions the term spread is stationary and the term structure of interest rates is well described by the expectations hypothesis.

2.2 Imperfect Knowledge based models

The theory of IKE assumes that individuals (bulls and bears) in the foreign currency market recognize their imperfect knowledge about the processes driving outcomes (Frydman and Goldberg, 2007) and, therefore, use a multitude of forecasting strategies which they revise over time in a way that cannot be fully prespecified in advance. Under certain conditions on individuals' revisions of forecasting strategies and as long as their forecasting variables are persistent, nominal exchange rates will show a tendency to move away from benchmark values (see Frydman and Goldberg, 2007). In the foreign exchange market the PPP is a natural benchmark and the real exchange rate will fluctuate around this value.

Thus, IKE revision of forecasts will generate an additional persistence in nominal exchange rates, the so called long swings, which is different from the persistence implied by REH based models. As a consequence, nominal and real exchange rates will tend to exhibit similar persistent swings.

Frydman et al. (2010) showed that under the above conditions, the change in real exchange rate, Δq_t, can be approximated with the following model:

$$\Delta q_t = \zeta_t + \varepsilon_{1,t} \tag{4}$$

where

$$\zeta_t = \bar{\rho}\zeta_{t-1} + \varepsilon_{2,t},$$

and ζ_t is a measure of the change in individuals' forecast due to a change of the explanatory variables and a change in individuals' forecasting strategies and $\bar{\rho}$ is an average of ρ_t, $t = 1, ..., T$ such that $\rho_t \approx 1.0$ when q_t is not too far away from benchmark values and $\rho_t < 1.0$ when when q_t is far away from such values. Thus, $\bar{\rho}$ may vary over different sample periods but generally within a small band close to the unit circle. As long as the non-constant drift term, ζ_t, is well approximated with a near $I(1)$ process, the real exchange rate behaves as a near $I(2)$ process, i.e. it exhibits a very pronounced persistence as illustrated in Figure 4. Modelling the real exchange rate as a near $I(2)$ process is consistent with swings of shorter and longer duration which implies that the length of these swings is not predictable (Frydman and Goldberg, 2007). That the near I(2) process is a good approximation has been shown empirically in Johansen et al. (2010), thus confirming the theoretically expected results in Frydman and Goldberg (2007, 2010b) and Frydman et al (2010a,b).

When the real exchange rate is moving away from its benchmark values, the real interest rate differential has to move similarly to restore equilibrium in the product market (see Frydman and Goldberg, 2007). This implies that the IKE equilibrium relation is a cointegration relation between the real exchange rate, the nominal interest rate differential, and the inflation rate differential:

$$(p_{1,t} - p_{2,t} - s_{12,t}) = \omega\{(i_{1,t} - i_{2,t}) - (\Delta p_{1,t} - \Delta p_{2,t})\} + e_t \tag{5}$$

where e_t is a stationary equilibrium error. In (5), the real exchange rate and the nominal interest rate differential are both near I(2) and cointegrate to near I(1). Adding the inflation rate differential, being near I(1), makes the relation stationary.

Under IKE, the standard UIP needs to be replaced by the Uncertainty Adjusted Uncovered Interest Rate Parity (Frydman and Goldberg, 2007) as a market clearing mechanism:

$$i_{1,t} - i_{2,t} = \Delta s^e_{t+1} + up_t \tag{6}$$

where $up_t = f(p_{1,t} - p_{2,t} - s_{12,t})$ is an uncertainty premium measuring how far the market has moved away from PPP benchmark values. The nominal interest rate differential and the uncertainty premium, both near I(2) processes, cointegrate to near I(1) and the interest rate differential corrected for the uncertainty premium cointegrates with Δs^e_{t+1} to produce a stationary market clearing mechanism.

In the IKE world, the nominal interest rate and the CPI inflation rate are integrated of different orders and the Fisher parity does not hold as a stationary condition. Figure 6 illustrates the persistence in real long-term rates. Similarly, the term spreads are not stationary implying that the expectations hypothesis is a misleading description of the term structure of interest rates. Figure 5 similarly illustrates the persistence in short-long interest rates for USA and Germany.

3 Persistence as a structuring device

The previous section showed that REH-based models differ from IKE models in a very important aspect: the former imply no persistence in the changes of the real exchange rate whereas the latter are consistent with a marked persistence. To empirically distinguish between the two model classes we need an econometric methodology that can discriminate between different degrees of persistence. The aim of this section is to discuss some basic principles for such a framework and to provide an intuition for how it works. A reader not interested in empirical methodology can jump to the next section without loosing the track.

3.1 Time-series persistence

The notion of persistence is associated with the strength of the time dependence of a shock to a variable. If the effect of a shock dies out quickly it is considered transitory and the corresponding variable is stationary whereas if the shock has a lasting effect it is considered permanent and the variable is called unit root nonstationary. Distinguishing broadly between transitory (stationary) and persistent (nonstationary) behavior is, however, not sufficient for the purpose of classifying data according to their different persistency profiles. For example, stationary processes can be divided into highly erratic $I(-1)$ processes and $I(0)$ processes, both of which describe transitory behavior. Nonstationary unit root processes can be generated from shocks which cumulate once, dubbed $I(1)$; or from shocks which cumulate twice, dubbed $I(2)$. [1] The latter is particularly important for describing speculative behavior under IKE.

While such a classification is mathematically unambiguous, it can be more problematic from the point of view of empirical modelling. Depending on the sample size, the degree of permanence, and the relative noise ratio of $I(1)$ and $I(2)$ components, there are often grey zones where data could be said to be near $I(1)$ rather than $I(1)$ or $I(0)$, and near $I(2)$ rather than $I(1)$ or $I(2)$. For example, a random walk process, $x_t = x_{t-1} + \varepsilon_t$, i.e. an $I(1)$ process, and a strongly autoregressive AR(1) process, $x_t = 0.95x_{t-1} + \varepsilon_t$, (mathematically an $I(0)$ process) would often be difficult to distinguish from each other even based on relatively long samples. This is illustrated in Figure 1 where an AR(1) with $\rho = 0.95$ and a random walk are simulated in 200 steps. Both series are seemingly characterized by similar persistence. For a short time series, the difficulty of discriminating between near unit roots and unit roots becomes even more pronounced. This is illustrated in Figure **??** for a stationary AR(1) process with autoregressive parameter $\rho = 0.80$ and a random walk process simulated in 50 steps. In contrast, an AR(1) with $\rho = 0.99$, say, would often be found significantly different from 1.0 in a large sample of 5000 observations even though the variable/relation would exhibit very pronounced persistence. Characterizing it as type $I(0)$ would, however, imply that we give up using cointegration as a tool to identify similar persistent trends among related variables. Since cointegration between two variables implies that they are sharing the same persistent shocks, it

[1] See Johansen (1996) for a mathematically precise definition of the order of integration of stochastic processes.

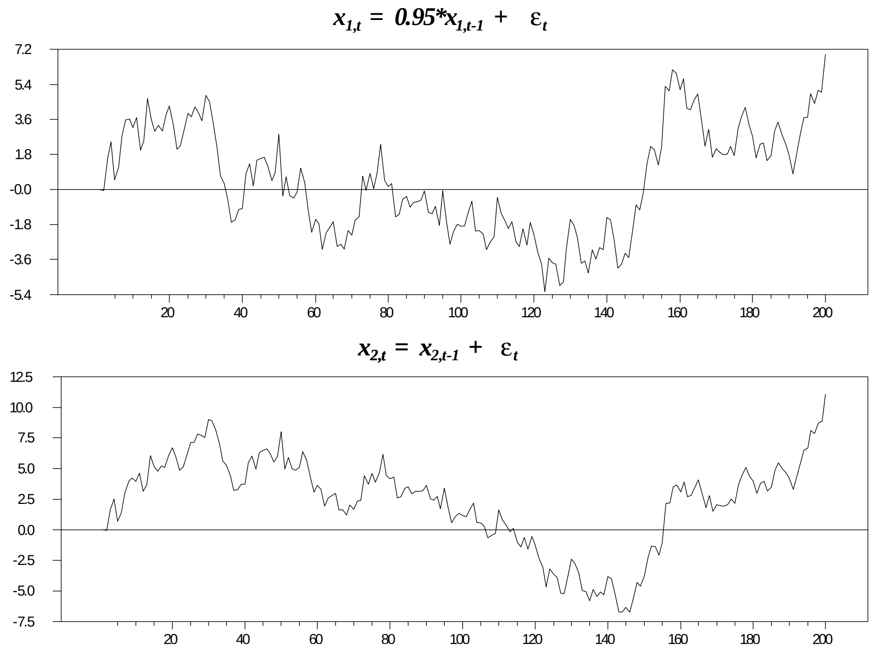

Figure 1: The simulated series of an AR(1) process with $\rho = 0.95$ (upper panel) and a random walk (lower panel).

is a powerful tool by which causal links can be detected. Thus, statistical significance alone does not seem as a good organizing principle for classifying data into different persistence profiles. See Hendry and Juselius (2000) for a discussion.

3.2 Different levels of persistence

A meaningful way to discuss persistence seems to be in terms of the (modulus of) characteristic (eigenvalue) roots of the autoregressive polynomial which for non-explosive models are defined in the interval (-1, 1). Such roots can be given a convenient interpretation as a measure of the speed of adjustment. As an example, consider the simple AR(1) model, $x_t = \rho_1 x_{t-1} + \varepsilon_t$ or equivalently $\Delta x_t = -(1-\rho_1)x_{t-1} + \varepsilon_t$. Assume a root $\rho_1 = 0.9$ corresponding roughly to an adjustment coefficient $\alpha_1 \simeq -(1- \rho_1) = -0.10$. An adjustment coefficient of -0.10 corresponds to a half life of $ln(2)/(1-0.10) = 7$ periods. With annual data this would imply an average adjustment period of 7 years, with quarterly

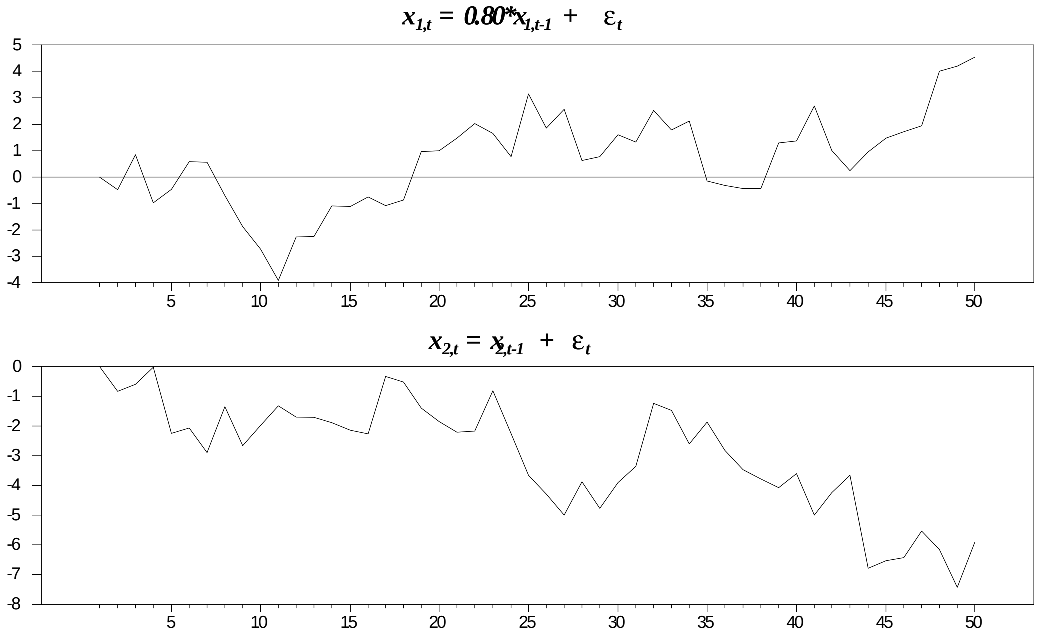

Figure 2: A simulated AR(1) process with $\rho = 0.8$ (upper panel) and a random walk (lower panel).

data it would be almost 2 years, with monthly data slightly more than half a year, with weekly data less than 2 months, etc. Whether a characteristic root can be interpreted as evidence of persistent behavior or not depends, therefore, both on the sample period and the observational frequency.

To illustrate the idea, consider a variable, x_t, which has the following autoregressive representation $(1 - \varphi_1 L - \cdots - \varphi_p L^p)x_t = \varepsilon_t$ where ε_t is $Niid$ and a threshold parameter, ρ^*, above which the process is considered persistent. The choice of ρ^* is to some extent subject to judgement. With high frequency data its value would generally be closer to the unit circle than with low frequency data. In the context of a specific theory, ρ^* could in some cases be thought of as defining the longest adjustment time for which the policy implications of the model are still useful.

The persistence of x_t could for example by defined as:

- $I(0)$ type when the modulus of the largest root, ρ_1, satisfies $\rho_1 < \rho^*$.

- $I(1)$ type when the modulus of the largest root, ρ_1, satisfies $\rho^* < \rho_1 \leq$

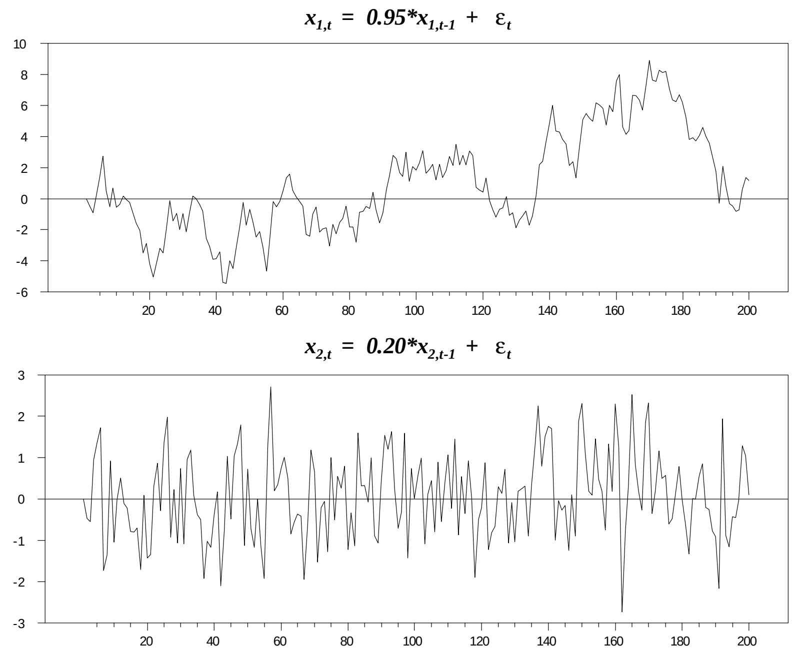

Figure 3: The graphs of an AR(1) process with $\rho = 0.95$ (upper panel) and with $\rho = 0.20$ lower panel.

1.0 and the next root $\rho_2 < \rho^*$.

- $I(2)$ type when the modulus of the largest root, $\rho_1 = 1.0$, and the next one satisfies $\rho^* < \rho_2 \leq 1.0$.

While the above classification is defined for a univariate model, the CVAR analysis is defined for a multivariate model and the persistency classification needs to be extended to this case. There are some important differences: In a univariate model a large characteristic root can be directly associated with the variable in question, $x_{i,t}$, whereas in a p-dimensional VAR model of $x_t' = [x_{1,t}, ..., x_{p,t}]$, the number of large roots in the characteristic polynomial depends on the number of common stochastic trends pushing the system, i.e. on $p - r$, where r is the number of cointegration relations. Furthermore, it also depends on whether the stochastic trends are of first order, s_1, or second order, s_2 where $s_1 + s_2 = p - r$. Consider for example a five-dimensional VAR model in which three of the characteristic roots are greater than ρ^*.

This would be consistent with three stochastic trends of first order ($p - r = 3, s_1 = 3$), or with two stochastic trends of first order and one of second order ($p - r = 2, s_1 = 1, s_2 = 1$).

Thus, to be able to determine the order of persistence of the variables and the relations, the order of integration of the vector process has to be determined as well as the division into type $I(1)$ and $I(2)$ stochastic trends. The reason for distinguishing between these two types is not just because they are frequently observed in data, but also because the most crucial difference between REH-based and IKE-based models can be formulated in terms of the number of $I(1)$ versus $I(2)$ trends in the VAR.

While it is not straightforward to distinguish between $I(1)$ and $I(2)$ type of persistence based on the characteristic roots, a simple procedure based on a combination of counting large roots and testing can be envisaged. For this purpose, a maximum likelihood test procedure is readily available (Nielsen and Rahbek, 2007), albeit the peril of relying exclusively on significance testing without considering the effect of the sample size is equally relevant for the multivariate as for the univariate case. A simple procedure could for example be to combine the likelihood ratio test procedure with the number of roots larger than ρ^* in the characteristic polynomial in the following way: start with the unrestricted VAR model ($r = p, s_1 = 0, s_2 = 0$) and determine the number of characteristic roots $\geq \rho^*$.[2] Then study those cases (r, s_1, s_2) for which the number of unit roots in the characteristic polynomial is equal to m^*. Test the relevant combinations with the trace test. An empirically relevant candidate is found when the trace test is not rejected, all unrestricted characteristic roots $\rho_i < \rho^*$, and the number of restricted unit roots is m^*.

3.3 Further issues

Another important issue is whether the probability theory for $I(1)$ and $I(2)$ models can be used for near unit root approximations. In this case, Elliot (1998) shows that the asymptotic distribution changes to some extent so that the near unit root distribution falls between the unit root (T consistency)

[2]Note that if a large modulus root corresponds to a complex pair with a significant imaginary part it is not possible to force it to become a unit root on the real line. In this case, it will be considered a stationary, albeit persistent, cyclical component. Also, Nielsen and Nielsen (2009) has shown ´that if the VAR model is estimated with many lags (for example adding lags to compensate for a structural break) the number of large, but insignificant, roots will increase. In such a case, the number becomes uninformative.

and the stationary ($\sqrt{T}$ consistency) distribution. An important question is whether this effect on the asymptotic inference is large in finite samples. For example, Johansen (2006) shows with simulations that some inference become very fragile if near unit roots are treated as stationary in moderately sized samples. Up to 5000 observations were needed for the empirical distribution to converge to Students t when the root was very close to the unit circle.

An even more important issue is whether it at all make sense to associate an $I(1)$-type process (say with a root of 0.95) with an $I(0)$-type process (say with a root less than 0.2). As illustrated in Figure 3 such processes display very different persistency profiles in contrast to the graphs in Figure 1 and 2. It seems futile from the outset to associate an $I(1)$-type persistent variable with an $I(0)$-type variable, whereas two $I(1)$-type variables may very well share a common trend and, thus, be cointegrated (which is testable). Thus, structuring the data according to their persistence profiles is likely to be helpful for uncovering empirical regularities that originate from the same kind of persistent shocks.

All empirical findings discussed in the subsequent sections have been obtained by following this very intuitive and simple principle applied to the CVAR model.

4 Structuring persistence using the CVAR

The CVAR models are inherently consistent with a world where unanticipated shocks cumulate over time to generate stochastic trends which move the economic equilibria (the pushing forces) and where the deviations from these equilibria are corrected by means of the dynamics of the adjustment mechanism (the pulling forces). Thus, the CVAR model has a good chance of nesting a multivariate, path-dependent data-generating process and relevant dynamic macroeconomic theories. See Hoover et al. (2008). This motivated Juselius (2006) and Juselius and Franchi (2007) to ask the question: Which empirical regularities would we see in the data, if the assumptions of exogenous shocks, equilibrium relations, steady-state behavior, and dynamic adjustment were correct in the theoretical model? The answer was formalized in what was called a theory consistent CVAR scenario, which essentially translates all basic assumptions about the theoretical model's shock structure, equilibrium relations and steady-state behavior as testable hypotheses on common stochastic trends, cointegration, steady-state values and dynamic adjustment[3]. Because of its ability to structure the relevant data into economically meaningful directions without subjecting them to prior restrictions, the CVAR can be thought of as providing broadly defined 'confidence intervals' within which empirically relevant models should fall.

The CVAR approach starts from an unrestricted VAR model which is essentially just a representation of the covariances of the data. By imposing (testable) reduced rank restrictions on the VAR model, it is formulated as a vector equilibrium-error-correcting model of first order, the I(1) CVAR model, or second order, the I(2) CVAR model. The former is appropriate to describe an economy where growth rates and deviations from equilibria are stationary, the latter where they are unit root nonstationary. See Appendix for a definition of the I(1) and I(2) models and an intuitive interpretation of their structural decomposition.

In the I(1) model, deviations from static equilibria are stationary, implying that a static equilibrium relation holds as a stationary relation. For example, REH based models would allow real exchange rates to move away from PPP values albeit in a stationary manner and the α coefficients in (7) (see Appendix) would describe the speed of adjustment back to equilibrium.

In the I(2) model, deviations from equilibrium can exhibit a pronounced persistence, implying that the nominal exchange rate, say, can move away from equilibrium values for extended periods of time. This, however, requires that the movements away are compensated by something else. The IKE theory predicts that a long swing in the real exchange rate requires a compensating movement in the real interest rate differential.

5 Testable empirical regularities under REH and IKE: a summary

Juselius (2010) extended the CVAR scenario idea to translate different assumptions on expectations formation and forecasting behavior in an REH and IKE based model for nominal exchange rate determination into testable hypotheses on the CVAR, the most important of which can be summarized as:

[3] For a comprehensive treatment, see Juselius (2006).

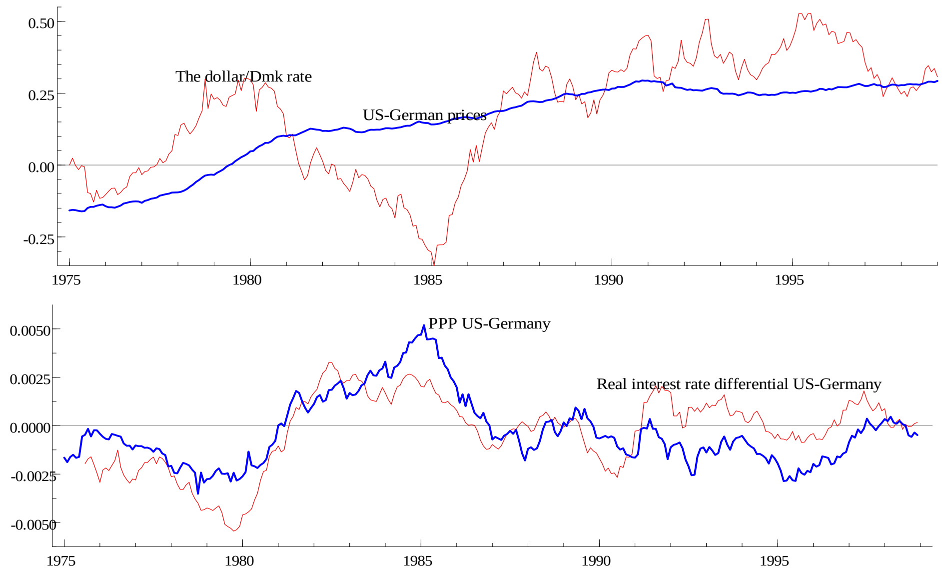

Figure 4: The graphs of the dollar/Dmk rate and the relative prices between USA and Germany (upper panel) and the real exchange rate together with the US-German real long-term interest rate differential (lower panel).

1. Under IKE, speculative behavior in the currency market tends to drive nominal exchange rates away from PPP benchmark values for extended periods of time. These persistent movements would have the property of a near $I(2)$ process so that real exchange rates are near $I(2)$. REH-based models assume that movements away from PPP values are stationary, or at most near $I(1)$.

2. Under IKE, the persistent swings in the real exchange rate are co-moving with the real interest rate differentials. Hence, the latter exhibit a similar persistence as real exchange rates, so are near $I(2)$. The REH-based theory assume that the real interest rate differential is stationary or at most near $I(1)$.

3. Under IKE, real exchange rates and real interest rate differentials cointegrate to a stationary equilibrium relation. Under REH, they are individually stationary albeit allowed to exhibit some persistence.

4. Under IKE, the nominal interest rate and inflation rate are not cointegrated so that the Fisher parity does not hold as a stationary condition. Under REH, the Fisher parity is stationary.

5. Under IKE, the term structure of interest rates is driven by two stochastic trends, one typically associated with the short end (monetary policy shocks), the other with the long end (financial market shocks). Thus, the standard expectations hypothesis does not hold under IKE. The interest rate spreads are nonstationary but cointegrated. Under REH, there is one stochastic trend, the expectations hypothesis holds and the spreads are stationary.

The test results showed that the REH-based scenario was empirically rejected on all counts[4], whereas the IKE-based scenario obtained a remarkable support for every single testable hypothesis. Detailed results are reported in Frydman et al. (2010) and Juselius (2010)[5].

An intuitive understanding of these results can be gained by studying the graphs of the time series. Figure 4 (upper panel) pictures US-German prices and the $/Dmk rate. The smooth upward sloping trend in relative prices (also visible in the nominal exchange rate) corresponds to one of the two stochastic near I(2) trends, whereas the long persistent swings of the real dollar/Dmk rates around this trend corresponds to the other. Treating them as I(1) would leave two large characteristic roots (0.96, 0.96) in the model, rendering any inference on stationarity somewhat illusory (see Juselius, 2009). The lower panel illustrates the close co-movements between the real exchange rate and the real long-term interest rate differential. Figure 5 pictures the US and German short-long interest rate spread. While the spreads are tiny in absolute value, they are nonetheless very persistent. Figure 6 pictures US and German ex post real interest rates together with a 12 months moving average to smooth out the strong seasonal price variation. Both of them are

[4]For an argument how the empirical difficulties of the REH models discussed here can be traced to their epistemologicallly flawed micro foundation, see Frydman and Goldberg (2010a, 2011).

[5]Previously Juselius (1995), Juselius and MacDonald (2004, 2007) reported empirical support for these hypotheses based on data for Danish - German, US -Japan and US - German exchange rates, prices and interest rates in the post Bretton Woods period of currency floats. The theory of imperfect knowledge economics provided the missing theoretical background.

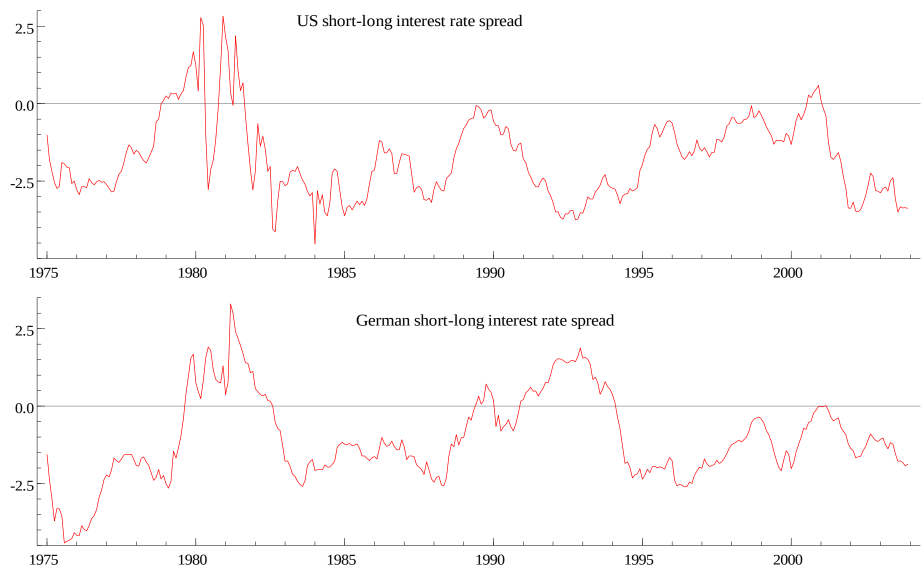

Figure 5: The graphs of the US short-long interest rate spread (upper panel) and the German one (lower panel).

characterized by a pronounced persistence (the US real rate even more so) that was found to be near I(1) by the tests.

6 Currency speculation and structural slumps: A discussion

The discussion here is based on many years of systematic investigation by myself and my students of how the persistency of the real exchange rates, real interest rates and the term spreads typical of most Western economies has influenced the real economy, in particular wage, price and unemployment dynamics. Phelps (1994) theory was an early reference to be followed by the the theory of IKE. This paper is a bold attempt to put all these bits and pieces together into something that may have the potential of becoming a coherent theoretical and empirical framework for understanding fluctuations in the macro economy.

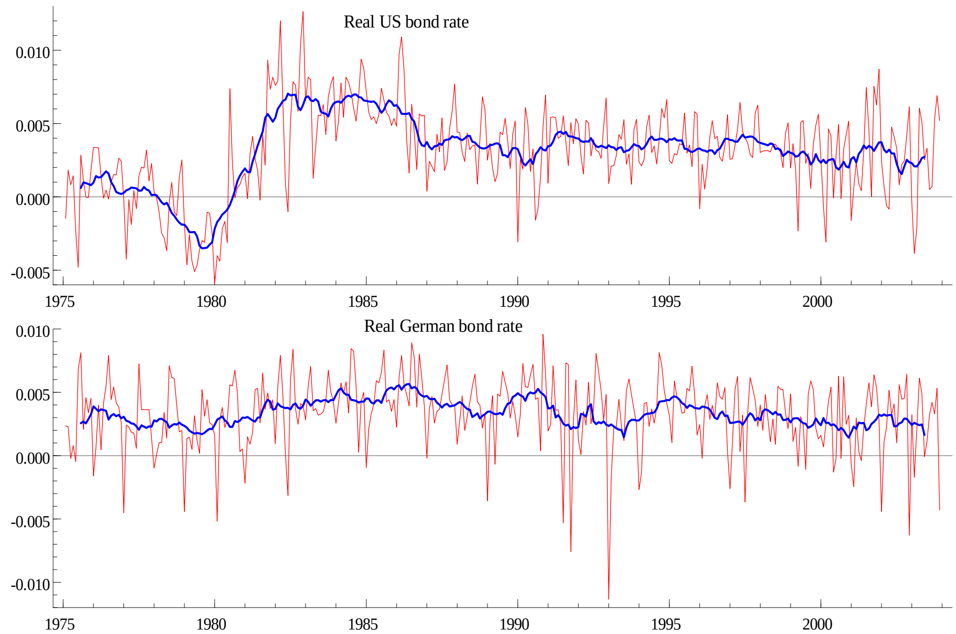

Figure 6: The graphs of the real long-term bond rates together with a 12 months moving average in USA (upper panel) and in Germany (lower panel).

The major results are given in Section 6.4. To get the logic of the discussion I first have to discuss the role of a nonstationary long-term interest rate and Fisher parity, what initiates swings and what allows them to be so long-lasting.

6.1 Preliminaries

The role of the long-term interest rate: The structural slumps theory explains how open economies connected by the world real interest rate (set in the global capital market) and by the real exchange rate (determined in a global customers market for tradables) can be hit by long episodes of unemployment. The theory predicts that an exogenous shock to the world level of public debt and/or capital stock will change the world level of interest rates, whereas an exogenous shock to the public debt of an individual open economy tend to increase its interest rate relative to the world interest rate. This has a close correspondence to the finding in Johansen et al. (2010), Frydman et al. (2010) and Juselius (2010) that shocks to the long-term US bond rate (a proxy for the world interest rate) and to the US-German interest rate differential (measuring relative debt levels between the two countries) were the main exogenous forces in a system comprising US-German prices, nominal exchange rates, and long-term interest rates.

The role of a nonstationary Fisher parity. In an IKE model, both the nominal interest rates and the nominal exchange rate are likely to exhibit a pronounced persistence due to the uncertainty premium. However, prices of tradable goods are essentially determined by supply and demand in a very competitive global world and, therefore, less susceptible to speculative movements (energy, precious metals and, recently, grain are exceptions in this respect). When nominal interest rates exhibit long persistent swings but CPI inflation rate does not, the real interest rate will also move in long persistent swings with obvious incentives for speculation. This is in contrast to REH models, where the Fisher parity holds as a stationary condition. In this case, an increase in the long-term interest rate would be associated with an expected increase in the inflation rate and speculators would have no specific incentive to invest their long-term capital in such an economy.

6.2 What is initiating a long swing

A shock to the long-term interest rate level (for example, as a result of a domestic increase in sovereign debt) without a corresponding increase in inflation rate, increases the amount of speculative capital moving into the economy, thereby causing an appreciation of the exchange rate. The latter would worsen the competitiveness of the economy thereby increasing previous imbalances and causing further increases of the interest rate. Thus, the interest rate is likely to keep increasing as long as the structural imbalance are growing in the economy, generating persistent movements in real interest rates and real exchange rates. Figure 4 illustrates.

Persistent shocks to the domestic-world long-term interest rate differential are likely to hit the economy in periods of structural imbalances (compared to a representative open world economy). In Europe such imbalances have typically been associated with a political reluctance to adequately address painful structural reforms in the labor market. In USA, they are typically associated with trade balance problems.

This circle of increasing/decreasing real interest rates and real appreciation/depreciation rates, empirically manifested in Juselius (2010) as an equilibrium error increasing behavior of the δ adjustment in (8) in the Appendix. The fact that risk averse individuals will require increasingly large risk premiums for holding the domestic currency as the imbalances grow, will sooner or later cause a reversal in the exchange rate movement (Frydman and Goldberg, 2007). In the empirical analysis this was manifested in the equilibrium error correcting behavior of the α adjustment in (8) in the Appendix.

6.3 How can swings be so long-lasting?

A persistent deviation away from benchmark values, is likely to trigger off a compensating reaction in other sectors of the economy, causing a new variable to move away from its benchmark value. For example, nonstationary movements in the real exchange rate are compensated by nonstationary movements in the nominal interest rate differential corrected for the inflation rate differential. As long as the real interest rate differential moves in a compensating way, the deviations from long-run PPP equilibria can be long-lasting. This is in essence the elements of a reflexive process (Soros, 1987) which may explain why a real exchange rate swing can be so long-lasting. It also explains for why the I(2) model plays such a crucial role in an IKE world: this model is formulated precisely to describe an economy where persistent deviations from long-run static equilibrium values are compensated by other variables, usually nominal growth rates.

As a persistent movement away from long-run benchmarks is counteracted by another similar movement, an IKE economy is still characterized by equilibrating forces but in a dynamic rather than static set-up.

6.4 Fluctuations in the real economy

This reflexive tendency of the domestic real interest rate to increase and the real exchange rate to appreciate at the same time is likely to aggravate domestic competitiveness in the tradable sector. This is because firms cannot in general count on floating exchange rates to restore competitiveness after a relative cost shock in an IKE economy where the nominal exchange rate is determined by speculation. Thus, they cannot use constant mark-up pricing without loosing market shares and to preserve them, have to adjust productivity or profits rather than the product price.

Therefore, in an IKE model, customer market pricing (Phelps, 1994) is likely to replace constant mark-up pricing as the pricing mechanism and one would expect profits to be squeezed in periods of persistent appreciation and increased during periods of depreciation. Evidence of a nonstationary profit share comoving with the real exchange rate has is reported in Juselius (2006) for Danish data.

As mentioned above, labor productivity and unemployment are also expected to rise in a period of real appreciation and increasing real long-term interest rates. For example, given an increase in the relative wage costs in a common currency, a customer market firm would be more prone to improve labor productivity than to increase its product price. The former can be achieved by new technology and/or by laying off the least productive part of the labor force, thereby producing the same output with less labor.

Evidence of unemployment comoving with trend-adjusted productivity has been found in Juselius (2006) for Danish data and for other European countries.

Increasing unemployment generally exerts a downward pressure on nominal/real wage claims and, thus, on wage inflation, Δw. Thus, wage inflation is expected to be negatively associated with unemployment, u, in an augmented Phillips Curve relation with a non-stationary natural rate, u^*: $\Delta w = -b_1(u - u^*)$, where $u^* = f(r)$ is a function of the real interest rate level, r. In Phelps (1994) the latter is a function of domestic government debt and the world real interest rate level.

Evidence of a non-stationary natural rate as a function of the long-term real interest rate is reported in Juselius (2006) and Juselius and Ordonez (2009).

Thus, there seems to be a direct link from financial market behavior causing long persistent swings in real exchange rates and real interest rates to the recurring unemployment slumps discussed in Phelps (1994).

6.5 Further remarks

Frydman and Goldberg (2007) suggests that the uncertainty premium increases with the deviation from the fundamental PPP value. But as discussed above one can also think of the unemployment rate and the profit share as alternative, but related, measures of deviations from benchmark values that eventually will put an end to the long swings movements in nominal exchange rates.

The structural slumps mechanism seems to work well in periods when the major driver underlying the fluctuations in aggregate activity is the long swings in real exchange rates, but it is not likely to work well in the aftermath of a fundamental financial crises as the present one as explained in Koo (2010). This is so because when numerous balance sheets in the economy are 'under water', savings will primarily be used for financial consolidation rather than for investment. As the Japanese experience after the collapse of the housing bubble in the nineties showed, not even a zero interest rate will have the intended effect in such a situation.

7 Concluding discussion

Macroeconomic data have a reputation for not being sufficiently informative, thereby justifying the use of 'mild force' to make them tell an economically relevant story. Based on my long experience of analyzing macroeconomic data[6] I would like to insist that macroeconomic data are surprisingly informative, but only if you let them tell the story they want to tell.

So, what do the data tell if they are allowed to speak freely? Some robust findings typical of the last three decades of capital deregulation and globalization can be summarized as follows: First, there is more persistence in the data than standard REH based theories can explain. In particular, basic parity conditions such as purchasing power parity, real interest rates, uncovered interest rate parity, and the term spread seem to exhibit a pronounced persistence untenable with I(0) type stationarity. Second, this persistence seems to originate from complex interactions between speculative financial markets and the real economy that tend to drive prices away from benchmark values.

To conclude, the stories data tell seem consistent with speculative behavior, imperfect knowledge, long swings, and strong reflexivity between the financial and the real economy. All of this has a number of important implications for how to think about economic theory and policy. Further research along these lines is likely to result in a fruitful synthesis between the theoretical framework of Phelps (1994) and Frydman and Goldberg' (2007) IKE theory, thereby improving our understanding of the long recurrent spells of high unemployment that continue to mar our economies.

[6] This includes supervising hundreds and hundreds of seminar papers, BSc, MSc, and PhD theses.

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A The I(1) and I(2) model

A.1 The I(1) model

To introduce notation and the idea of structuring the data into pulling and pushing forces, I shall use a simple 3-dimensional VAR model for $x_t' = [p_1, p_2, s_{12}]$, where the three variables describe domestic and foreign prices and the nominal exchange rate. The model is structured around $p - r$ stochastic trends (the pushing or exogenous forces) and r cointegration relations (the pulling or equilibrating forces). I shall consider the case $(r = 1, p - r = 2)$.

The pulling force is formulated as the vector equilibrium error correction model, $\Delta x_t = \alpha\beta' x_{t-1} + \varepsilon_t$, i.e. as:

$$\begin{bmatrix} \Delta p_{1,t} \\ \Delta p_{2,t} \\ \Delta s_{12,t} \end{bmatrix} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} \beta' x_{t-1} + \cdots + \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \\ \varepsilon_{3,t} \end{bmatrix} \tag{7}$$

where $\beta' x_t$ is an equilibrium error and α_i is an adjustment coefficient describing how the system adjusts back to equilibrium when it has been pushed away. For example, $\beta' x_t = p_{1,t} - p_{2,t} - s_{12,t}$ would describe an economy where purchasing power parity holds as a stationary condition. The α_i coefficients tell us whether it is prices or exchange rates or all three variables that take the adjustment when unanticipated shocks, $\varepsilon_{i,t}$, have pushed the system out of equilibrium.

The pushing forces are analyzed in the moving average form of the CVAR model, describing the cumulated effects of the exogenous shocks, $u_{i,t}$, on the variables:

$$\begin{bmatrix} p_{1,t} \\ p_{2,t} \\ s_{12,t} \end{bmatrix} = \begin{bmatrix} \beta_{\perp,11} & \beta_{\perp,21} \\ \beta_{\perp,12} & \beta_{\perp,21} \\ \beta_{\perp,13} & \beta_{\perp,21} \end{bmatrix} \begin{bmatrix} \sum_{i=1}^{t} u_{1,i} \\ \sum_{i=1}^{t} u_{2,i} \end{bmatrix} + ... + \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \\ \varepsilon_{3,t} \end{bmatrix}$$

where $u_{1,t} = \alpha'_{\perp,1}\varepsilon_t$ and $u_{2,t} = \alpha'_{\perp,2}\varepsilon_t$ are two autonomous common shocks that have a permanent effect on the system and $\alpha_\perp = [\alpha_{\perp,1}, \alpha_{\perp,2}]$, is a 3×2 matrix, orthogonal to α, defining the two common shocks as linear combination of the VAR residuals $\hat{\varepsilon}_t$ and $\beta_\perp = [\beta_{\perp,1}, \beta_{\perp,2}]$, is a 3×2 matrix orthogonal to β describing the steady-state effect of a structural shock to the system.

For example, $\alpha'_{\perp,1} = [1, -1, 0]$ and $\alpha'_{\perp,2} = [0, 0, 1]$ would describe an economy where shocks to relative prices and shocks to the nominal exchange rate are the main exogenous driving forces. The case $\beta'_{\perp,1} = [a, a, 0]$ and $\beta'_{\perp,2} = [b, c, b - c]$ would define a stationary real exchange rate:

$$\begin{bmatrix} p_{1,t} \\ p_{2,t} \\ s_{12,t} \end{bmatrix} = \begin{bmatrix} a & b \\ a & c \\ 0 & b-c \end{bmatrix} \begin{bmatrix} \sum_{i=1}^{t} u_{1,i} \\ \sum_{i=1}^{t} u_{2,i} \end{bmatrix} + ... + \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \\ \varepsilon_{3,t} \end{bmatrix}$$

A.2 The I(2) model

The I(2) model is useful to describe en economy where the persistency in the data is one degree higher than in the I(1) world. To account for this, the I(2) model is formulated in acceleration rates, medium run relations between growth rates and dynamic relations. It has a richer but also more complicated structure. The vector x_t is now integrated of order 2 and the $p-r$ stochastic trends are divided into s_1 first order and s_2 second order stochastic trends, i.e. $p-r = s_1 + s_2$. The r cointegration relations, $\beta' x_t$, are generally integrated of order 1, i.e. they cointegrate from $I(2)$ to $I(1)$ and becomes stationary by adding a linear combination of the growth rates, $\delta'\Delta x_t$. In addition there are s_1 linear combinations, $\beta'_{\perp 1} x_t \sim I(1)$, which can become stationary exclusively by differencing, i.e. $\beta'_{\perp 1}\Delta x_t \sim I(0)$. Thus, the $I(2)$ model contains $p-s_2$ relations, $\tau' x_t$, which cointegrate from I(2) to I(1), where $\tau = (\beta, \beta_{\perp 1})$.[7]

I consider here the case $(r = 1, s_1 = 1, s_2 = 1)$ implying as before one equilibrium relation and two stochastic trends. The difference is because the equilibrium relation needs to be combined with a growth rate to become stationary and one of the common stochastic trends is an I(2) trend whereas the other is an I(1) trend. The former could, for example describe price shocks and the latter exchange rate shocks.

Under this assumption, the vector equilibrium error correcting model for I(2) data can be formulated as $\Delta^2 x_t = \alpha(\beta' x_{t-1} + \delta'\Delta x_{t-1}) + \zeta\tau'\Delta x_{t-1} + \varepsilon_t$, where $\tau = [\beta, \beta_{\perp 1}]$. The system of prices and exchange rates would look like:

$$\begin{bmatrix} \Delta^2 p_{1,t} \\ \Delta^2 p_{2,t} \\ \Delta^2 s_{12,t} \end{bmatrix} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} (\beta' x_{t-1} + \delta'\Delta x_{t-1}) + \begin{bmatrix} \zeta_{11} & \zeta_{21} \\ \zeta_{12} & \zeta_{22} \\ \zeta_{13} & \zeta_{23} \end{bmatrix} \begin{bmatrix} \beta'\Delta x_{t-1} \\ \beta'_{\perp 1}\Delta x_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \\ \varepsilon_{3,t} \end{bmatrix} \tag{8}$$

where $\beta' x_{t-1} + \delta'\Delta x_{t-1}$ describes a deviation from a dynamic equilibrium relation, and $\beta'\Delta x_{t-1}$ and $\beta'_{\perp 1}\Delta x_{t-1}$ describe deviations from two medium-run equilibrium relations among growth rates. For example, if $\beta' x_{t-1} + \delta'\Delta x_{t-1} = (p_{1,t} - p_{2,t} - s_{12,t}) + \delta_1 \Delta p_{1,t}$, then this would describe an economy where deviations from PPP exhibit type I(1) persistence which is compensated by a similar persistence in country 1 inflation rate.

The common stochastic trends are analyzed in the moving average form of the CVAR model, $x_t = \beta_{\perp 2}\Sigma\Sigma u_s + B\Sigma u_i + ... + \varepsilon_t$. For the price and

[7] If $r - s_2 > 0$, then it is possible to find $r - s_2$ relations $\beta' x$ which are stationary without adding the growth rates.

exchange rate system it can be formulated as:

$$\begin{bmatrix} p_{1,t} \\ p_{2,t} \\ s_{12,t} \end{bmatrix} = \begin{bmatrix} \beta_{\perp 2,1} \\ \beta_{\perp 2,2} \\ \beta_{\perp 2,3} \end{bmatrix} \left[\sum_{i=1}^{t} \sum_{s=1}^{i} u_{1,s} \right] + \begin{bmatrix} b_{11} & b_{21} \\ b_{12} & b_{22} \\ b_{13} & b_{23} \end{bmatrix} \begin{bmatrix} \sum_{i=1}^{t} u_{1,i} \\ \sum_{i=1}^{t} u_{2,i} \end{bmatrix} + ...$$

where $u_{1,t} = \alpha'_{\perp 2}\varepsilon_t$ is an autonomous shock that cumulates twice over time, $u_{2,t} = \alpha'_{\perp 1}\varepsilon_t$ is an autonomous shocks that cumulates once over time, $\alpha_{\perp} = [\alpha_{\perp,1}, \alpha_{\perp,2}]$, is a 3×2 matrix orthogonal to α, defining the two shocks as linear combination of the VAR residuals $\hat{\varepsilon}_t$ and $\beta_{\perp 2}$ is a 3×1 vector orthogonal to $\{\beta, \beta_{\perp 1}\}$ describing the steady-state effect of a structural I(2) shock to the system. If $u_{1,t}$ is a relative price shock then $\alpha'_{\perp 2} = [1, -1, 0]$ and $u_{2,t}$ a nominal exchange rate shock, then $\alpha'_{\perp 1} = [0, 0, 1]$. Assuming that only the two prices are affected by the I(2) trend the system could be described by:

$$\begin{bmatrix} p_{1,t} \\ p_{2,t} \\ s_{12,t} \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \left[\sum_{i=1}^{t} \sum_{s=1}^{i} u_{1,s} \right] + \begin{bmatrix} b_{11} & b_{21} \\ b_{12} & b_{22} \\ b_{13} & b_{23} \end{bmatrix} \begin{bmatrix} \sum_{i=1}^{t} u_{1,i} \\ \sum_{i=1}^{t} u_{2,i} \end{bmatrix} + ...$$

Thus, prices would be type I(2), but relative prices and the nominal exchange rate type I(1). The real exchange rate would generally be I(1) unless $b_{13} = b_{11} - b_{12}$ and $b_{23} = b_{21} - b_{22}$.