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Carsten S. Nielsen, Alexander Sebald

Øster Farimagsgade 5, Building 26, DK-1353 Copenhagen K., Denmark
Tel.: +45 35 32 30 01 – Fax: +45 35 32 30 00
http://www.econ.ku.dk

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Unawareness in Dynamic Psychological Games*

Carsten S. Nielsen† Alexander Sebald‡

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Abstract

Building on Battigalli and Dufwenberg (2009)'s framework of dynamic psychological games and the recent progress in the modeling of dynamic unawareness, we provide a general framework that allows for 'unawareness' in the strategic interaction of players motivated by belief-dependent psychological preferences like reciprocity and guilt. We show that unawareness has a pervasive impact on the strategic interaction of psychologically motivated players. Intuitively, unawareness influences players' beliefs concerning, for example, the intentions and expectations of others which in turn impacts their behavior. Moreover, we highlight the strategic role of communication concerning feasible paths of play in these environments.

Keywords: Unawareness; Extensive-form games; Communication; Belief-dependent preferences; Sequential equilibrium.

JEL-Classifications: C72, C73, D80

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†Department of Economics, University of Copenhagen, Øster Farimagsgade 5, Building 26, DK-1353, Copenhagen K, Denmark. Phone: (+45) 3532-3051. Fax: (+45) 3532-3064. E-mail: carsten.nielsen@econ.ku.dk. Web: http://www.econ.ku.dk/phdstudent/nielsen.

‡Department of Economics, University of Copenhagen, Øster Farimagsgade 5, Building 26, DK-1353, Copenhagen K, Denmark. Phone: (+45) 3532-4418. Fax: (+45) 3532-3064. E-mail: alexander.sebald@econ.ku.dk. Web: http://www.econ.ku.dk/sebald.

1 Introduction

Recent lab and field evidence suggests that people not only care about the monetary consequences of their actions, but that their behavior is also driven by belief-dependent psychological preferences [see e.g. Fehr *et al.* (1993), Charness and Dufwenberg (2006), Falk *et al.* (2008), Bellemare *et al.* (2010)]. Two prominent examples of belief-dependent preferences in the hitherto existing literature are reciprocity [see e.g. Rabin (1993), Dufwenberg and Kirchsteiger (2004), Falk and Fischbacher (2006)] and guilt aversion [see e.g. Charness and Dufwenberg (2006), Battigalli and Dufwenberg (2007b)]. Departing from the strictly consequentialist tradition in economics Geanakoplos *et al.* (1989) and Battigalli and Dufwenberg (2009) present general frameworks for analyzing the strategic interaction of people with belief-dependent psychological preferences: 'psychological games'. Roughly speaking, psychological games are games in which players' preferences depend upon players' beliefs about the strategies that are being played, players' beliefs about the beliefs of others about the strategies that are being played, and so on *ad infinitum*.

A widely unspoken assumption that is underlying all psychological as well as standard (i.e. non-psychological) game-theoretic analyses is that players are aware of the complete structure of the strategic environment they are in. Bluntly speaking, it is assumed that players are aware of everything. However, in many real life situations this is not the case–people often have asymmetric awareness levels concerning their own as well as others' feasible choices although they are part of the same strategic environment. Players are frequently 'surprised' in the sense that they become aware of new strategic alternatives by e.g. observing actions they had previously been unaware of or through verifiable communication. It has been shown that, although unawareness has important implications for strategic interactions, any non-trivial notion of unawareness is precluded in the standard Bayesian framework [see e.g. Dekel *et al.* (1998), Modica and Rustichini (1999)]. In the standard framework there may be 'details' (i.e. states of the world) that players do not know, but they can identify all of them (this is known as the axiom of wisdom [see e.g. Samuelson (2004)]). In a sense players cannot be truly surprised.

However, it is not only in standard games that unawareness is important. In line with recent experimental evidence suggesting that people are more prone to selfish choices if they believe that others will remain unaware of them [see e.g. Dana *et al.* (2006), Dana *et al.* (2007), Broberg *et al.* (2007), Tadelis (2008), Andreoni and Bernheim (2009), Lazear *et al.* (2009)], we show in our analysis here that asymmetric awareness also has a profound impact on the strategic interaction of players with belief-dependent psychological preferences. To see this consider the following intuitive example: Imagine two friends, *Ann* and *Bob*. Assume it is *Bob*'s birthday, he is planning a party and would be very happy, if *Ann* could come. Unfortunately, *Ann* has an important exam the next day and therefore cannot make it. *Ann* is certain that *Bob* would feel let down, if she were to cancel his party without having a very good excuse. Quite intuitively, in this situation *Ann* might not experience any guilt towards *Bob* for not coming to his party. She knows that the important exam is a good excuse and that *Bob* is not let down as he does not expect her to come. In contrast, consider now the following variant of the same example: *Ann* is aware of the fact that the exam is postponed, meaning that it is feasible for her to attend *Bob*'s party. However, she has studied so hard for days and nights that she feels too tired to go. Quite intuitively, in this situation *Ann* might not feel guilty towards *Bob* as long as she believes that *Bob* is unaware of the fact that the exam is postponed.[1] As long as she believes that *Bob* is unaware of the fact that she actually has the possibility/time to come, she might not feel guilty towards him as she believes that he does not expect her to come and, hence, is not let down. In fact, if she were sure that *Bob* would never become aware of the fact that her exam can and is postponed, she probably had a strong emotional incentive to stick to the original story and leave him unaware in order not to raise his expectations. In other words, she had a strong incentive not to make him aware of the fact that she actually has the time to come to his party, but is too tired. Interestingly, if *Ann* were only interested in her own payoff in this strategic situation with unawareness, she would not care whether *Bob* is or will become aware of the postponement. She would simply not attend his party irrespective of his awareness. Only her belief-dependent feeling of guilt towards *Bob* creates the strong emotional incentive not to make him aware.

Bob's unawareness concerning *Ann*'s possibility to come to his party and, connectedly, *Ann*'s incentive not to tell him about the postponement of her exam intuitively highlight the focus of our analysis here. We analyze the influence and importance of asymmetric awareness and communication concerning feasible paths of play for the strategic interaction of players with belief-dependent preferences. This means, building on Battigalli and Dufwenberg (2009)'s framework of dynamic psychological games and the recent progress in the modeling of unawareness [i.e. Heifetz *et al.* (2006), Heifetz *et al.* (2008) and Heifetz

[1]Assume, for example, that *Ann* thinks that *Bob* can simply not conceive that exams can be postponed. One may wonder to what extend *Bob*'s unawareness can be modeled as zero probability events. First, *Bob* is unable to conceive the event 'the exam is postponed' and will have to assign probability zero to it. Second, if we are to take *Bob*'s limited cognition seriously, then *Bob* must also be unable to conceive the complementary event 'the exam is *not* postponed' and thus also assign probability zero to that. Because of additivity, a probability measure in the standard Bayesian framework can never assign both zero to an event and its complement. Capturing unawareness thus requires drastic modeling innovations, including a rethinking of the basic concept of the standard framework.

et al. (2010)], we first present an extensive form that allows for unawareness and communication in the strategic interaction of players motivated by belief-dependent psychological preferences like reciprocity and guilt. Second, we provide a solution concept which can be used in our class of dynamic games with unawareness, communication and belief-dependent preferences and, third, we discuss an application to exemplify the influence of unawareness and communication using a specific type of belief-dependent preference: reciprocity.

More specifically, to allow for unawareness we extend the existing multi-stage framework along two dimensions. First, we divide extensive forms into subforms consisting of paths of play. These subforms are used to define players' levels of awareness. Second, players may become aware of more by learning from choices made by others. However, as our analysis concentrates on the influence of asymmetric awareness on the strategic interactions of players with belief-dependent preferences, we abstract from the question of how players become aware. We simply assume that whenever they observe a choice that they had previously been unaware of they become aware of some 'larger' subforms which is consistent with observed choices. To model unawareness and changes in awareness levels we adopt Heifetz *et al.* (2010)'s definition of unawareness in dynamic strategic environments. Other ways of modeling unawareness that have been suggested in recent years include e.g. Fagin and Halpern (1988), Modica and Rustichini (1999), Halpern (2001), Heifetz *et al.* (2006), Halpern and Rêgo (2008), Heifetz *et al.* (2008), Feinberg (2009), Grant and Quiggin (2009), Li (2009) and Mengel *et al.* (2009).

In the spirit of our example above, we also allow for communication in our framework. We model such communication by assuming that players can choose to send verifiable 'awareness messages' containing feasible paths of play they are aware of, or they can choose not to communicate. Note that this is different from the communication allowed for in, for example, the experimental setting of Charness and Dufwenberg (2006). In their setting players are aware of everything and can send messages concerning intended play. In contrast, a message in our setting contains information concerning a set of feasible paths of play. Communicating feasible paths of play is obviously meaningless in strategic environments without unawareness. It is the asymmetric awareness of players which makes communication an important integral part of the strategic environment with unawareness. If a player observes a message containing information about paths of play that he was previously unaware of, he will update his level of awareness by taking this information into account.

Having defined our class of extensive forms with unawareness and communication, we formally characterize belief-dependent preferences. In synthesis, for each player confined to a certain awareness level, his pure strategy is defined on the extensive form he is confined to and his beliefs concerning the other players' strategies are defined on each of the extensive forms induced by all subforms he is aware of. A behavioral strategy profile is thus an independent probability distribution over these pure strategies each specifying a definite choice. Beliefs about others' pure strategies a player is aware of (first-order beliefs), beliefs about their beliefs about others' pure strategies he is aware of (second-order beliefs), and so on, are shown to exist for all possible hierarchies. We use these hierarchies of beliefs for the general specification of belief-dependent psychological preferences. As mentioned above, specific types of belief-dependent preferences that can be embedded in our general setting with unawareness and communication are among others reciprocity and guilt aversion. In contrast to Battigalli and Dufwenberg (2009), in our setting such psychological preferences will be limited by the awareness of each player who plays 'partial games'. A partial game is a description of the strategic situation a player is aware of. As players may be aware of different partial games at different stages, we define a dynamic psychological game with unawareness and communication as the 'modelers' game which entails all relevant partial games.

Given the characterization of dynamic psychological games with unawareness and communication, we propose a sequential equilibrium solution concept and prove its existence. We assume that a profile of first-order beliefs (conjectures) in a partial game is derived from a behavioral strategy profile in the same game. This implies, that in equilibrium any two players confined to the same partial game will independently hold the same first-order beliefs about any third player. An assessment in our structure, a behavioral strategy profile and a profile of infinite hierarchies of beliefs, is consistent if the profile of first-order beliefs is derived from the behavioral strategy profile and each higher-order belief assigns probability one to lower-order beliefs. Intuitively, players aware of the same must in equilibrium hold common, correct beliefs about each others infinite belief hierarchies. A consistent assessment and sequential rationality (based on belief-dependent preferences) induce a sequential equilibrium in the partial game. As players are unaware of any situation in which other players are aware of more than themselves, they believe that the game they are confined to is the most expressive. This implies that there exists an equilibrium strategy in which players confined to a partial game fix the equilibrium strategies of other players, whom they believe are confined to 'smaller' partial games, and then choose an equilibrium strategy based on this belief.

After defining our class of extended psychological games and characterizing our solution concept, we use an application to demonstrate the influence and importance of unawareness on the strategic interaction of agents with belief-dependent preferences. That is, we use the sequential prisoners dilemma also analyzed by Dufwenberg and Kirchsteiger (2004) to show the impact of unawareness and communication on the strategic interaction of reciprocal agents. As a benchmark we start from their results and subsequently discuss two scenarios in which players have asymmetric awareness levels. Importantly, the application shows how asymmetric awareness levels of players concerning feasible paths of play can give rise to equilibrium predictions that are distinct from predictions using Dufwenberg and Kirchsteiger (2004)'s setting without unawareness and a standard setting in which people are only concerned about the monetary consequences of their actions.

The organization of the paper is as follows: In section 2 we introduce a class of extensive forms with unawareness and communication. Following this, in section 3 we define hierarchies of conditional beliefs and belief-dependent preferences in our class of extensive forms. Section 4 contains the definition of our equilibrium concept: sequential equilibrium. In section 5 we discuss a specific application. Sections 6 and 7 respectively contain extensions and a discussion of some of our assumptions as well as a conclusion.

2 The framework

In this section we introduce a class of extensive forms with unawareness and communication. First, we define awareness subtrees as the basis for our analysis (2.1). Following this, we characterize the messages players can send (2.2), and introduce extensive forms with unawareness and communication (2.3).

2.1 Awareness subtrees

A multi-player decision tree with observable actions, no chance moves, and complete information is a tuple $\langle I, N \rangle$ where I is the finite set of players, and N is the finite set of decision nodes. Let A_i^N be the set of all actions player i can take in N. A decision node of length $l \in L$ is a sequence of actions $n = (a^1, \ldots, a^l)$ where each $a^t = (a_1^t, \ldots, a_{|I|}^t)$ represents the profile of actions taken at stage t $(1 \leq t \leq l)$. The decision node $\tilde{n} = (\tilde{a}^1, \ldots, \tilde{a}^k)$ precedes $n = (a^1, \ldots, a^l)$, written $\tilde{n} < n$, if $\tilde{n}$ is a prefix of n (i.e., $k < l$ and $(\tilde{a}^1, \ldots, \tilde{a}^k) = (a^1, \ldots, a^k)$). The initial empty node, denoted by n^0, is an element of N. Y denotes the set of terminal nodes.

Consider now a family $\boldsymbol{T}$ of awareness subtrees of N, partially ordered $\leq$ by the inclusion of paths of play. That is,

$$\boldsymbol{T} = \{T \subseteq N : \exists D \in 2^Y \backslash \{\varnothing\}, T = \{n : \exists y \in D : n \leq y\}\},$$

where $n \leq y$ means that n is y or a prefix of y.

Each subtree $T \in \boldsymbol{T}$ represents a set of feasible paths of play. The 'largest' of these trees is the set N itself. To further clarify the structure of each $T \in \boldsymbol{T}$ we state the following definition for awareness subtrees:

Definition 1 (Awareness subtrees)**.** A set of nodes $T \in \boldsymbol{T}$ is an awareness subtree if there is some nonempty subset of terminal nodes $D \subseteq Y$ such that

$$T = \{n \in N : n \leq y \text{ for some } y \in D\}.$$

Such a construction of subtrees ensures that any $T \in \boldsymbol{T}$ starts at the root n^0, that it is naturally ordered by proper subnodes, and implies that each terminal node of each subtree $y \in D$ is associated with a well defined terminal history in Y. We denote the set of actions of player i in the subtree T by A_i^T.

Example 1: The construction of the family $\boldsymbol{T}$ can be demonstrated by a simple example. Consider the extensive form underlying the sequential prisoners dilemma also analyzed by Dufwenberg and Kirchsteiger (2004).

[Figures 1]

Figure 1 shows an extensive form without communication $\langle I, N\rangle$ with $I = \{Ann, Bob\}$ and $N = \{n^0, n^1, n^2, n^3, n^4, n^5, n^6\}$.[2] In the initial node n^0 *Ann* can choose between *cooperate* and *defect* and *Bob* is passive. In nodes n^1 and n^2 *Bob* can respectively choose between *cooperate* and *defect* and *Ann* is passive. Histories n^3, n^4, n^5 and n^6 are terminal nodes.

The cardinality of the family of subtrees $\boldsymbol{T}$ is $|\boldsymbol{T}| = 2^{|Y|} - 1$. In the context of our example this means $|Y| = 4$ and $|\boldsymbol{T}| = 15$:

[Figures 2]

[2] We will draw on this example in the subsequent sections and develop it further along the lines of our analysis.

2.2 Messages about feasible paths of play

Verifiable communication is an integral and important part of strategic interactions in situations in which players might be unaware concerning feasible paths of play. Therefore we next define the set of verifiable messages that players can send concerning the feasible paths of play and then augment our extensive form with unawareness and communication. Assume that players can either choose to communicate some set of feasible paths of play or choose not to communicate which we denote by sending the empty message $m^{\varnothing}$. This means, the set of possible messages associated with some subtree T is defined as:

$$M^T = \left\{ \{T'\}_{T' \preceq T} \cup m^{\varnothing} \right\}.$$

The set of possible messages for all other subtrees is defined analogously. The set of messages which is associated with the largest tree in the family $\boldsymbol{T}$ is denoted by M^N.

Each of these messages only reveals information about the structure of the game, i.e. feasible paths of play. Therefore, over and above a potential role as coordination devices, our messages are irrelevant in settings with full awareness since they contain no new information. However, in settings with asymmetric awareness such messages are an important part of the strategic environment. By construction our messages can only be informative.

2.3 Extensive forms with unawareness and communication

A finite extensive form with unawareness and communication is a tuple $\langle I, H_T \rangle$ where H_T is the finite set of histories. Let $C_i^T = A_i^T \times M^T$ be the set of choices player i can make in H_T. A history of length $l \in L$ is a sequence of choices $h_T = (c^1, \ldots, c^l)$ where each $c^t = (c_1^t, \ldots, c_{|I|}^t)$ with $c_i^t \in C_{i,T}$ represents the profile of choices made at stage t $(1 < t < l)$. The finite set of feasible choices for player i at history h_T is denoted by C_{i,h_T}. Player i is active at h_T if C_{i,h_T} is not a singleton.[3] The set H_T of histories h_T will rather informally be referred to as a 'T-sub-extensive-form', or just 'subform'. Z_T denotes the set of terminal histories z_T. The 'largest' of these subforms is H_N. H_T consists of copies h_T of the histories $h_N \in H_N$. Obviously, whenever two histories $h_T \in H_T$ and $h_{T'} \in H_{T'}$ are copies of the same history $h_N \in H_N$, they are also copies of each other. Let $\boldsymbol{H} = \{H_T\}_{T \in \boldsymbol{T}}$ be the family of subforms, partially ordered $\preceq$ by the inclusion of paths of play based on choices.

Example 2: Consider again the extensive form in Figure 1. Let's concentrate on

[3] The restrictions made by observable actions, no chance moves, and complete information can be removed, at the cost of additional notational complexity. Different extensions of our general framework are discussed in section 6.

the initial node n^0. In our extensive form without communication *Ann* can take actions (C)ooperate and (D)efect. Her set of feasible choices in the initial history h_N^0 of the extensive form $\langle I, H_T\rangle$ associated with $\langle I, N\rangle$ could thus be $C_{A,h_N^0} = \{(C,T_1),\cdots,(C,T_{15}),(C,m^{\varnothing}),(D,T_1),\cdots,(D,T_{15}),(D,m^{\varnothing})\}$. On the other hand, Bob who is (P)assive in n^0 can only communicate, i.e. his set of feasible choices could be $C_{B,h_N^0} = \{(P,T_1),\cdots,(P,T_{15}),(P,m(\varnothing))\}$.

To model that players may have different views on the set of feasible paths of play in different histories we define players' perceptions concerning the strategic environment.

Definition 2. For each player $i \in I$ in our extensive form with communication there exists a perception function:[4]

$$\pi_i : \left[\bigcup_{T\in\boldsymbol{T}} H_T\right] \to \left[\bigcup_{T\in\boldsymbol{T}} H_T\right].$$

which defines for each h_T player i's perception $\pi_i(h_T)$.

The properties of this function parallel the properties in Heifetz *et al.* (2006, p. 83) and Heifetz *et al.* (2010, p. 47):[5]

(i) Confined Awareness: If $h_T \in H_T$, then $\pi_i(h_T) \in H_{T'}$, with $H_{T'} \leq H_T$.

(ii) Generalized Reflexivity: If $H_{T'} \leq H_T$, $h_T \in H_T$, $\pi_i(h_T) \in H_{T'}$ and $H_{T'}$ contains a copy $h_{T'}$ of h_T, then $h_{T'} = \pi_i(h_T)$.

(iii) Subforms Preserve Awareness: If $h_T \in H_T$, $h_T = \pi_i(h_T)$, $H_{T'} \leq H_T$ and $H_{T'}$ contains a copy $h_{T'}$ of h_T, then $h_{T'} = \pi_i(h_{T'})$

(iv) Subforms Preserve Ignorance: If $H_{T''} \leq H_{T'} \leq H_T$, $h_T \in H_T$, $\pi_i(h_T) \in H_{T''}$ and $H_{T'}$ contains the copy $h_{T'}$ of h_T, then $\pi_i(h_{T'}) = \pi_i(h_T)$.

(v) Subforms Preserve Knowledge: If $H_{T''} \leq H_{T'} \leq H_T$, $h_T \in H_T$, $\pi_i(h_T) \in H_{T'}$ and $H_{T''}$ contains a copy $h_{T''}$ of h_T, then $\pi_i(h_{T''})$ consists of the copy that exists in $H_{T''}$ of the node $\pi_i(h_T)$.

(vi) Dynamic Awareness: for any two histories $\tilde{h}_T, h_T \in H_T$ directly preceding each other (i.e. $h_T = (\tilde{h}_T, c)$) and $\pi_i(\tilde{h}_T) \in H_{T'}$, then (i) $\pi_i(h_T) \in H_{T'}$, if $H_{T'}$ contains the copy $h_{T'}$ of h_T, or (ii) $\pi_i(h_T) \in H_{T''}$ with $H_{T'} \leq H_{T''}$.

[4] We have chosen the term 'perception function' instead of possibility correspondence in order to avoid any confounding with settings of imperfect information.

[5] Note that introspection does not play a role in our setting as our setting is restricted to observable actions, i.e. singleton information sets.

The perception function and its properties describe for all possible histories the players' perceptions and change in perceptions about the strategic environment. More specifically, 'Confined Awareness' says that the players' perceptions in some history h_T are confined to subforms 'smaller or equal' to the subform h_T is in. The property of 'Generalized Reflexivity' implies that at some history h_T players know the (observable) choices that have led to history h_T. Properties $(iii)-(v)$ guarantee the coherence of the knowledge and the awareness of players down the partial order. 'Subforms Preserve Awareness' means that players that can perceive some history in some subform must also perceive copies of that history in 'smaller' subforms. 'Subforms Preserve Ignorance' implies that at histories in 'smaller' subforms players cannot perceive anything that they cannot perceive at copies of these histories in 'larger' subforms and 'Subforms Preserve Knowledge' says that players who perceive to be in some history also perceive copies of that history in all 'smaller' subforms. Finally, the property of 'Dynamic Awareness' regards the dynamic nature of the strategic interaction. It implies that at each history players perceive subforms that are consistent with the choices made. If a player observes that the choices taken by others are different from what he had foreseen, he will have an enlightening moment and discover some subform $H_{T'}$ which is consistent with the choices just taken. This kind of learning thus implies that player i, by constructing a new subform to which he is confined, updates his current awareness. He does so by aggregating information about paths of play gained from either unforeseen actions taken by others, or messages containing new information.

For extensive forms $H_T, H_{T'} \in \boldsymbol{H}$ we (abuse notation slightly and) denote $T \rightarrowtail T'$ whenever for some history $h_T \in H_T$ it is the case that the copy $\pi_i(h_T) \in H_{T'}$. Denote by $\hookrightarrow$ the transitive closure of $\rightarrowtail$. That is, $T \rightarrowtail T''$ if there is a sequence of trees $H_T, H_{T'} ..., H_{T''} \in \boldsymbol{H}$ satisfying $T \rightarrowtail T' \rightarrowtail ... \rightarrowtail T''$. If $h_T \in H_T$ but $T \not\hookrightarrow T'$, then at the history h_T a player may be interpreted as being unaware of histories in $H_{T'} \backslash H_T$. We denote by $\boldsymbol{h}_T = \{h_{T'}\}_{T \hookrightarrow T'}$ the 'historical event' that a history and copies thereof that a player is aware of obtains. $\boldsymbol{H}_T$ is the set of such events and $\boldsymbol{Z}_T$ denotes the set of terminal historical events.

For any given subform $H_T \in \boldsymbol{H}$, let $S_i^{H_T}$ denote the set of (pure) strategies of player i. A typical strategy is denoted $s_i^T = (s_{i,h_T}^T)_{h_T \in H_T \backslash Z_T}$, where s_{i,h_T}^T is the choice that would be selected by s_i^T if history h_T obtained. Define $S^{H_T} = \prod_{i \in I} S_i^{H_T}$ and $S_{-i}^{H_T} = \prod_{j \neq i} S_j^{H_T}$. The set of i's strategies that allow history h_T is denoted $S_i^{H_T}(h_T)$. Similar notation is used for strategy profiles: $S^{H_T}(h_T) = \prod_{i \in I} S_i^{H_T}(h_T)$ and $S_{-i}^{H_T}(h_T) = \prod_{j \neq i} S_j^{H_T}(h_T)$.

Strategies cannot be interpreted as an ex-ante plan of choices since players might be unaware of histories in $H_{T'}$ for which $T \not\hookrightarrow T'$. A strategy should therefore rather be viewed as a list of answers to the hypothetical question: 'what would the player do if h_T where the history he considered possible?' However, there is no guarantee that such a question is meaningful to the player at histories he is unaware of. The answer should therefore be interpreted as given by the modeler, as part of the description of the strategic situation.

This concludes the definition of our class of extensive forms with observable actions, messages and unawareness. In the next section we define dynamic psychological games in the context of our class of extensive forms.

3 Dynamic psychological games with unawareness

In this section we develop our notion of dynamic psychological games with unawareness. We start by model a universal belief space that accounts for updated beliefs (3.1), and put forth our general definition of a psychological game with unawareness (3.2).

3.1 Belief hierarchies in the unawareness structure

As the game progresses, players update and/or revise their beliefs in light of newly acquired information. To account for this process, we represent beliefs by means of conditional probability systems (see Battigalli and Siniscalchi (1999) for proofs, details, and further references).

Consider a player who is uncertain about which element in a set X is true. Assume X is a compact Polish space. Players assign probabilities to events $E, F, ...$ in the Borel sigma-algebra $\mathscr{B}_X$ of X according to some (countably additive) probability measure. Let $\Delta(X)$ denote the set of all probability measures on $(X, \mathscr{B}_X)$. As events unfold players update their beliefs. Let $\mathscr{C} \subseteq \mathscr{B}_X$ be a nonempty, finite or countable collection, such that each $\varnothing \notin \mathscr{B}_X$. The interpretation is that any given player i is uncertain about the element $x \in X$, and $\mathscr{C}$ represents a collection of 'relevant hypotheses'.

Definition 3. A conditional probability system (*cps*) on $(X, \mathscr{B}_X, \mathscr{C})$ is a mapping $\mu(\cdot|\cdot) : \mathscr{B}_X \times \mathscr{C} \to [0,1]$ such that, for all $E \in \mathscr{B}_X$ and $F', F \in \mathscr{C}$, (i) $\mu(\cdot|\cdot) \in \Delta(X)$, (ii) $\mu(F|F) = 1$, and (iii) $E \subseteq F' \subseteq F$ implies $\mu(E|F) = \mu(E|F')\,\mu(F'|F)$.

We regard the set of cps' on $(X, \mathscr{B}_X, \mathscr{C})$ as a subset of the topological space $[\Delta(X)]^{\mathscr{C}}$, where $\Delta(X)$ is endowed with the topology of weak convergence of measures (which makes it Polish) and $[\Delta(X)]^{\mathscr{C}}$ is endowed with the product topology.

Throughout this paper, we shall solely be interested in 'relevant hypotheses' corresponding to the event that a certain partial history has occurred. Fix some subform $H_T \in \boldsymbol{H}$ and player $i \in I$. Player i's first-order cps' about $-i$'s behavior in any subform he is aware of may be represented by taking $X = \left[\bigcup_{T \hookrightarrow T'} S_{-i}^{H_{T'}}\right]$ and $\mathscr{C} = \left\{F \subseteq \left[\bigcup_{T \hookrightarrow T'} S_{-i}^{H_{T'}}\right] : F = \left[\bigcup_{T \hookrightarrow T'} S_{-i}^{H_{T'}}(h_{T'})\right] \text{ for copies } h_{T'} \text{ of } h_T \in H_T\right\}$. Since each element of $\mathscr{C}$ represents the historical event that a history and copies thereof that a player is aware of obtains, we simplify our notation of cps' on $\left[\bigcup_{T \hookrightarrow T'} S_{-i}^{H_{T'}}\right]$ and replace $\mathscr{C}$ with $\boldsymbol{H}_T$. The collection of cps' on $\left[\left(\bigcup_{T \hookrightarrow T'} S_{-i}^{H_{T'}}, \mathscr{C}\right)\right]$ is thus defined by $\Delta^{\boldsymbol{H}_T}\left(\bigcup_{T \hookrightarrow T'} S_{-i}^{H_{T'}}\right)$. Since $\left[\bigcup_{T \hookrightarrow T'} S_{-i}^{H_{T'}}\right]$ and $\boldsymbol{H}_T$ are finite, $\Delta^{\boldsymbol{H}_T}\left(\bigcup_{T \hookrightarrow T'} S_{-i}^{H_{T'}}\right)$ is easily seen to be a closed subset of Euclidean $|\boldsymbol{H}_T| \cdot \left|\bigcup_{T \hookrightarrow T'} S_{-i}^{H_{T'}}\right|$-dimensional space. To present player i's higher-order beliefs, we introduce the notion of a hierarchical cps space. Hierarchies of cps' are in our unawareness structure defined recursively as follows:

$$\begin{aligned}
&X_{-i,T}^0 = S_{-i}^{H_T};\\
&\text{for all } k \geq 1,\\
&X_{-i,T}^k = X_{-i,T}^{k-1} \times \Delta^{\boldsymbol{H}_T}\left(\bigcup_{T \hookrightarrow T'} X_{-j,T'}^{k-1}\right).
\end{aligned}$$

A cps $\mu_{i,T}^k \in \Delta^{\boldsymbol{H}_T}\left(\bigcup_{T \hookrightarrow T'} X_{-i,T'}^{k-1}\right)$ is called a k-order cps. A hierarchy of cps' is a countably infinite sequence of cps' $\mu_{i,T} = (\mu_{i,T}^1, \mu_{i,T}^2, \dots) \in \prod_{k \geq 1} \Delta^{\boldsymbol{H}_T}\left(\bigcup_{T \hookrightarrow T'} X_{-i,T'}^{k-1}\right)$. If player i is assigned with the lowest level of awareness ($T \not\hookrightarrow T'$ for all $T' \in \boldsymbol{T}$ different from T), then the hierarchy will be equal to the that provided by Battigalli and Dufwenberg (2009).

Let $B_{i,T}$ be the set of hierarchies of cps' that are known with common certainty of coherency[6] at the subform i is confined to. The finite disjoint union of Polish spaces is Polish and each $X_{-i,T}^k$ is thus a cross-product of compact Polish spaces, hence $B_{i,T}$ is itself a compact Polish space. Player i has higher-order cps' about $-i$'s strategies and beliefs in any subform he is aware of. Therefore the structure $(X, \mathscr{C})$ is specified as follows: $X = \left[\bigcup_{T \hookrightarrow T'} S_{-i}^{H_{T'}} \times B_{-i,T'}\right]$ and $\mathscr{C} = \left\{F \subseteq \left[\bigcup_{T \hookrightarrow T'} S_{-i}^{H_{T'}} \times B_{-i,T'}\right] : F = \left[\bigcup_{T \hookrightarrow T'} S_{-i}^{H_{T'}}(h_{T'}) \times B_{-i,T'}\right] \text{ for copies } h_{T'} \text{ of } h_T \in H_T\right\}$. The set of cps' on $\left(\left[\bigcup_{T \hookrightarrow T'} S_{-i}^{H_{T'}} \times B_{-i,T'}\right], \mathscr{C}\right)$ will be denoted by $\Delta^{\boldsymbol{H}_T}\left(\bigcup_{T \hookrightarrow T'} S_{-i}^{H_{T'}} \times B_{-i,T'}\right)$–a compact Polish space.

The following definition establish that countably infinite hierarchies of cps' are sufficient

[6] Coherency is the condition that various orders of cps' of a player cannot contradict each other, i.e. $\mu_{i,T}^k(\cdot|\boldsymbol{h}_T) = \operatorname{marg}_{X_{-i,T}^{k-1}} \mu_{i,T}^{k+1}(\cdot|\boldsymbol{h}_T)$ for all $k \geq 1$ and $\boldsymbol{h}_T \in \boldsymbol{H}_T$.

for the strategic analysis; $B_{i,T}$ is isomorphic to $\Delta^{\boldsymbol{H}_T}\left(\bigcup_{T\hookrightarrow T'} S_{-i}^{H_{T'}} \times B_{-i,T'}\right)$, so each $\mu_{i,T} \in B_{i,T}$ corresponds to a cps on $\left[\bigcup_{T\hookrightarrow T'} S_{-i}^{H_{T'}} \times B_{-i,T'}\right]$:

Lemma 1. (cf. Battigalli and Dufwenberg (2009)) For each player $i \in I$:

$$f_{i,T} = (f_{i,\boldsymbol{h}_T})_{\boldsymbol{h}_T \in \boldsymbol{H}_T} : B_{i,T} \to \Delta^{\boldsymbol{H}_T}\left(\bigcup_{T\hookrightarrow T'} S_{-i}^{H_{T'}} \times B_{-i,T'}\right)$$

is a 1-to-1 and onto continuous mapping whose inverse is also continuous.

We let $B_{i,T}^k$ denote the set of k-order cps' consistent with common certainty of coherency, that is, the projection of $B_{i,T}$ on $\Delta^{H_T}\left(\bigcup_{T\hookrightarrow T'} X_{-i,T'}^{k-1}\right)$. For example, the set of player i's second-order beliefs $\left(B_{i,T}^2\right)$ are about $-i$'s strategies and first-order beliefs $\left(S_{-i}^{H_{T'}} \times B_{-i,T'}^1\right)$ in any subform he is confined to. One might be concerned as to why the isomorphism $f_{i,\boldsymbol{h}_T}$ is 'natural'. The reason is that the marginal probability assigned by each $f_{i,\boldsymbol{h}_T}(\mu_{i,T}^1, \mu_{i,T}^2, \dots)$ to a given event in $\left[\bigcup_{T\hookrightarrow T'} X_{-i,T'}^{k-1}\right]$ is equal to the probability that $\mu_{i,T}^k$ assigns to that same event. That is, in deriving probabilities on the product space $\left[\bigcup_{T\hookrightarrow T'} X_{i,T'}^{\infty}\right] = \left[\bigcup_{T\hookrightarrow T'} S_{-i}^{H_{T'}} \times B_{-i,T'}^1 \times B_{-i,T'}^2 \times \cdots\right]$ from $(\mu_{i,T}^1, \mu_{i,T}^2, \dots)$, the function $f_{i,\boldsymbol{h}_T}$ preserves the probabilities specified by $\mu_{i,T}^k$ on each $\left[\bigcup_{T\hookrightarrow T'} X_{-i,T'}^{k-1}\right]$.

Lemma 2. Each coordinate function $f_{i,\boldsymbol{h}_T}$ is such that for all $\mu_{i,T} = (\mu_{i,T}^1, \mu_{i,T}^2, \dots) \in B_{i,T}$, and $k \geq 1$:

$$\mu_{i,T}^k(\cdot|\boldsymbol{h}_T) = \text{marg}_{[\bigcup_{T\hookrightarrow T'} S_{-i}^{H_{T'}} \times B_{-i,T'}^1 \cdots \times B_{-i,T'}^{k-1}]} f_{i,\boldsymbol{h}_T}(\mu_{i,T}).$$

Absent in our definition of a hierarchical cps space is the description of the beliefs of a player about himself. We omit such beliefs about the opponents. Thus, beliefs about oneself do not play an explicit role. However, our analysis is consistent with the standard assumption that a player knows his beliefs and assigns probability one to the strategy he intends to carry out.

3.2 Psychological multi-stage games with unawareness

We are now ready to state our definition of a dynamic psychological game with unawareness:

Definition 4. A dynamic psychological game with unawareness and belief-dependent preferences is a tuple

$$\Gamma = \left\langle I, \left[\bigcup_{T\in\boldsymbol{T}} H_T\right], (\pi_i)_{i\in I}, (u_i)_{i\in I}\right\rangle,$$ [7]

[7] In conventional game theory payoffs are the same if the paths of actions (as opposed to choices) leads to the same terminal. If we only allow players to send 'empty messages', then our framework will be equivalent to the conventional framework.

where $u_i = (u_{i,T})_{T\in\boldsymbol{T}}$ and $u_{i,T} : Z_T \times B_{i,T} \to \mathbb{R}$ is a continuous psychological payoff function of player $i \in I$ who is confined to the subform H_T.

In a game where some players are unaware of some paths of play, other players will, in general, be aware of this possibility. A game with unawareness is therefore not common knowledge among the players, and should be interpreted as the modelers' point of view. However, if we were to make such a common knowledge assumption here, then the domain and codomain of the perception function π_i will become the same for all players. The game is therefore just a Battigalli and Dufwenberg (2009, Definition 4) game (henceforth; B&D-game). The standard assumption of common knowledge of the game must therefore be replaced with a structure in which each player assigns to the others a possible level of awareness. For this purpose we define partial games as follows:

Definition 5. For any $H_T \in \boldsymbol{H}$, a T-partial game is a tuple

$$G_T = \langle I, H_T, (u_{i,T})_{i\in I} \rangle .$$

From the modelers point of view there exists a set of T-partial games $\boldsymbol{G} = \{G_T\}_{T\in\boldsymbol{T}}$, with the partial order $\preceq$ on $\boldsymbol{G}$ defined (with slight abuse of notation) by the transitive closure $\hookrightarrow$ generated by the relational requirement $\rightarrowtail$ on subforms. Since $\boldsymbol{G}$ is a finite set of T-partial games, any 'awareness chain' in $\boldsymbol{G}$ must have both a minimal element under $\preceq$, characterized as a strategic situation in which all players think that others are aware of the same paths of play as themselves (the B&D-game), and a maximal element under $\preceq$, namely the modelers game.

To highlight the recursive nature of this structure consider the following variant of our introductory example, in which *Ann*'s exam was postponed and she could have gone to *Bob*'s party: assume now that *Ann* is aware that the exam change-of-date is posted on the instructor's website. Furthermore, assume that *Ann* imagines that *Bob* is also aware of that fact, but think that *Ann* is unaware (cannot conceive that there exist a website). That is, *Ann* thinks that *Bob* is unaware (cannot conceive) that she could be aware that he revealed her lie. This situation could be modeled by having *Ann* being confined to some partial game in which she thinks that, (i) *Bob* is confined to the same partial game as her, and (ii) *Bob* thinks that she is confined to some 'smaller' partial game which exclude paths of play with *Bob*'s action 'check instructors website'. *Bob* might in this situation be either generous enough not to reveal that he caught her lying, or reveal everything because he is furious that she lied to him.

4 Sequential psychological equilibrium

In the following we propose a sequential equilibrium concept for dynamic psychological games with unawareness. We will define and interpret consistent assessments (4.1), state the main definition of equilibrium and provide an existence theorem (4.2). Lengthy mathematical proofs are relegated to Appendix (A).

4.1 Consistent assessments

Battigalli and Dufwenberg (2009) adapt Kreps and Wilson (1982)'s concept of sequential equilibrium to their class of dynamic psychological games without unawareness. They do so by characterizing consistent assessments that do not only consist of first-, but also of higher-order beliefs and defining sequential equilibria as sequential rational consistent assessments.

In turn, we adapt Battigalli and Dufwenberg (2009) concept to our setting with unawareness and communication. As in Battigalli and Dufwenberg (2009), assessments in our setting also refer to behavioral strategies, i.e. implicit randomizations over sets of choices at each history the player is aware of. The interpretation of behavioral strategies used in psychological games exclude actual randomizations. Rather, we assume that players do not know the pure strategies of others, and the randomization represents their uncertainty, their first-order beliefs (conjectures) about others' pure strategies (Aumann and Brandenburger, 1995). Fix any T-partial game G_T. We denote a behavioral strategy of player i by $\sigma_{i,T} = (\sigma_{i,T}(\cdot|\boldsymbol{h}_T))_{\boldsymbol{h}_T \in \boldsymbol{H}_T}$. The behavioral choice $\sigma_{i,T}(\cdot|\boldsymbol{h}_T) \in \Delta(\bigcup_{T \hookrightarrow T'} C_{i,h_{T'}})$ should be understood as a stochastic independent randomization over the set of choices in histories player i is aware of.

Each behavioral strategy $\sigma_{j,T}$ induces a probability measure $\Pr_{\sigma_{j,T}}$ on the set $\left[\bigcup_{T \hookrightarrow T'} S_j^{H_{T'}}(h_{T'})\right]$ of strategies, in the continuation of play (i.e. strategies defined on histories $h'_{T'}$ that are not predecessors of $h_{T'}$–denoted $h'_{T'} \nless h_{T'}$), allowed for by a history $h_{T'}$ that player i believes j is at: for all $s_j^{T'} \in \left[\bigcup_{T \hookrightarrow T'} S_j^{H_{T'}}(h_{T'})\right]$,

$$\Pr_{\sigma_{j,T}}\left(s_j^{T'}|\boldsymbol{h}_T\right) := \prod_{h'_{T'} \in H_{T'} \setminus Z_{T'} : h'_{T'} \nless h_{T'}} \sigma_{j,T}\left(s_{j,h'_{T'}}^{T'}|\boldsymbol{h}_T\right).$$

In the original characterization Kreps and Wilson (1982) propose three conditions to ensure consistency of assessments: (i) beliefs must be derived using Bayes' rule, (ii) beliefs must reflect that players choose their strategies independently, and (iii) players with identical information have identical beliefs. In addition to these conditions, an additional requirement for consistency is needed in psychological games: (iv) players hold correct beliefs about each others beliefs.

Condition (i) holds by the definition of cps' (Definition 3). That is, cps' are defined in such a way that they are consistent with Bayes' rule. Conditions (ii)-(iii) are ensured if we assume that the profile of first-order beliefs $\mu_T^1 = \left(\left(\mu_{i,T}^1(\cdot|\boldsymbol{h}_T)\right)_{\boldsymbol{h}_T \in \boldsymbol{H}_T}\right)_{i\in I}$ is derived from the stochastic independent behavioral strategy profile $\sigma_T = (\sigma_{i,T})_{i\in I}$. That is, for all $i \in I$, $s_{-i}^{T'} \in \left[\bigcup_{T\hookrightarrow T'} S_{-i}^{H_{T'}}\right]$, and $\boldsymbol{h}_T \in \boldsymbol{H}_T$:

$$\mu_i^1(s_{-i}^{T'}|\boldsymbol{h}_T) = \prod_{j\neq i} \mathrm{Pr}_{\sigma_{j,T}}(s_j^{T'}|\boldsymbol{h}_T).$$

If a profile of first-order beliefs is derived from a profile of stochastic independent behavioral strategies, then the marginal first-order belief of any two players i, j about a third player k must coincide. That is, for all $s_k^{T'} \in \left[\bigcup_{T\hookrightarrow T'} S_k^{H_{T'}}\right]$ and $\boldsymbol{h}_T \in \boldsymbol{H}_T$:

$$\mathrm{marg}_{S_k^{H_{T'}}} \mu_i^1(s_k^{T'}|\boldsymbol{h}_T) = \mathrm{Pr}_{\sigma_{k,T}}(s_k^{T'}|\boldsymbol{h}_T) = \mathrm{marg}_{S_k^{H_{T'}}} \mu_j^1(s_k^{T'}|\boldsymbol{h}_T).$$

Finally, condition (iv) follows from the second condition in the following definition of a consistent assessment:

Definition 6 (cf. Battigalli and Dufwenberg (2009))**.** An assessment (σ_T, μ_T) in any T-partial game $G_T \in \boldsymbol{G}$ is consistent if

(i) μ_T^1 is derived from σ_T,

(ii) and higher order beliefs in μ_T assign probability 1 to the lower order beliefs, such that for all $i \in I$, $k > 1$, $\boldsymbol{h}_T \in \boldsymbol{H}_T$

$$\mu_{i,T}^k(\cdot|\boldsymbol{h}_T) = \mu_{i,T}^{k-1}(\cdot|\boldsymbol{h}_T) \times \delta_{\mu_{-i,T}^{k-1}}$$

where δ_x is the Dirac measure which assigns probability 1 to singleton $\{x\}$.

The first condition capture the assumption that beliefs should be the end-product of a transparent reasoning process of rational players. The second condition is analog to Geanakoplos *et al.* (1989)'s condition requiring that players (confined to the same T-partial game) hold common and correct beliefs about each others' beliefs.

4.2 Equilibrium concept

We now move to the section's main definition: a consistent assessment is a sequential equilibrium if it satisfies sequential rationality. Formally, fix a T-partial game, a player i, a hierarchy of cps' $\mu_{i,T}$, a (non-terminal) history h_T that i thinks that he is at, and a strategy $s_i^T \in S_i^{H_T}(h_T)$. The expectation of u_i conditional on $\boldsymbol{h}_T$, given s_i^T and $\mu_{i,T}$ is:

$$\begin{aligned}\mathbb{E}_{s_i^T,\mu_{i,T}}[u_{i,T}|\boldsymbol{h}_T] := \sum_{T\hookrightarrow T'} \mu_{i,T}^1(h_{T'}|\boldsymbol{h}_T)\times \\ \sum_{s_{-i}^{T'}\in S_{-i}^{H_{T'}}(h_{T'})} \mu_{i,T}^1\left(s_{-i}^{T'}|h_{T'}\right) u_{i,T}\left(\zeta\left(s_i^T, s_{-i}^{T'}\right), \mu_{i,T}\right),\end{aligned} \tag{1}$$

where $\zeta(s_i^T, s_{-i}^{T'}) \in Z_T$ is a path function which defines the terminal history z_T induced by $(s_i^T, s_{-i}^{T'})$.[8] This expression gives the expected payoff from the strategies of others he is aware of. However, player i confined to the T-partial game does–in general–not know the awareness and strategies of the others and thus evaluates his payoff with respect to his first-order belief. Here we first use the idea that the event $F' = h_{T'}$ is a subset of the event $F = \boldsymbol{h}_T$ (for $F', F \in \mathscr{C}$) such that $\mu_i^1(\cdot|\boldsymbol{h}_T) = \mu_i^1(\cdot|h_{T'})\mu_i^1(h_{T'}|\boldsymbol{h}_T)$, and then the fact that the sets $\left[\bigcup_{T\hookrightarrow T'} S_j^{H_{T'}}(h_{T'})\right]$ are disjoint.

Definition 7. An assessment (σ_T, μ_T) is a sequential equilibrium (SE) if it is consistent and for all players $i \in I$, non-terminal hisorical event $\boldsymbol{h}_T \in \boldsymbol{H}_T \backslash \boldsymbol{Z}_T$, and optimal strategies $s_i^{T,*} \in S_i^{H_T}(h_T)$ allowed for by the history h_T that player i thinks he is at: for all $j \neq i$,

$$\text{supp marg}_{S_i^{H_T}} \mu_{j,T}^1(\cdot|\boldsymbol{h}_T) \subseteq \arg \max_{s_i^{T,*}\in S_i^{H_T}(h_T)} \mathbb{E}_{s_i^{T,*},\mu_{i,T}}[u_{i,T}|\boldsymbol{h}_T].$$

By consistency, $\sigma_{i,T}$ represents the first-order beliefs of the other players about player i, and furthermore there is common certainty of the correct belief profile μ_T at every history in the T-partial game. This clarifies that SE is a an equilibrium in beliefs.

The next result shows that it suffices to check whether there are any histories h_T at player i's awareness level where he can gain by deviating from the choices prescribed by $s_i^{T,*}$ at h_T and conforming to $s_i^{T,*}$ thereafter. Since this 'one-stage-deviation principle' is essentially the principle of optimality in dynamic programing, which is based on backwards induction, it also establishes that one can use backwards induction to find optimal strategies in T-partial games.

First we need to define what we mean by taking the point of view of an 'agent' (i, h_T) of player i in charge of the move at h_T. In order to facilitate comparison with the existing

[8]The path function $\zeta : S_i^{H_T} \times \left[\bigcup_{T\hookrightarrow T'} S_{-i}^{H_{T'}}\right] \to Z_T$ is defined such that $z_T = (c^1, \dots, c^L) = \zeta(s_i^T, s_{-i}^{T'})$ if and only if $c^1 = (s_{i,h_T^0}^T, s_{-i,h_{T'}^0}^{T'})$ and $c^{t+1} = (s_{i,(c^1,\dots,c^t)}^T, s_{-i,(c^1,\dots,c^t)}^{T'})$ for all $t \in \{1, \dots, L-1\}$.

literature on dynamic games, we will in the following adopt the standard notation. The expectation operator using $\Pr_{\sigma_T} = \prod_{j \in I} \Pr_{\sigma_{j,T}}$ is denoted $\mathbb{E}_{\sigma_T, \mu_T}[\cdot]$; particular $\mathbb{E}_{\sigma_T, \mu_T}[u_{i,T}]$ is player i's expected payoff in the T-partial game from the assessment (σ_T, μ_T). Agent (i, h_T) considering whether he should deviate by making some choice has to consider the uncertain continuation of play following his choice (strategies defined on histories h'_T that are not predecessors or h_T itself–denoted $h'_T \not\lesssim h_T$). The induced probability measure of agent (i, h_T) following his choice $c_i \in C_{i,h_T}$ is: for all $s_i^T \in S_i^{H_T}(h_T)$,

$$\Pr_{\sigma_{i,T}}\left(s_i^T | \boldsymbol{h}_T, c_i\right) := \prod_{h'_T \in H_T \setminus Z_T : h'_T \not\lesssim h_T} \sigma_{i,T}\left(s^T_{j,h'_T} | \boldsymbol{h}_T\right).$$

The expected utility of agent (i, h_T) conditional on the copies he is aware of $\boldsymbol{h}_T$ and his choice c_i given the assessment (σ_T, μ_T) can be expressed as

$$\begin{aligned}\mathbb{E}_{\sigma_T,\mu_T}[u_{i,T} | \boldsymbol{h}_T, c_i] := \sum_{T \hookrightarrow T'} \sum_{s_{-i}^{T'} \in S_{-i}^{H_{T'}}(h_{T'})} \prod_{j \neq i} \Pr_{\sigma_{j,T}}\left(s_j^{T'} | \boldsymbol{h}_T\right) \times \\ \sum_{s_i^T \in S_i^{H_T}(h_T, c_i)} \Pr_{\sigma_{i,T}}\left(s_i^T | \boldsymbol{h}_T, c_i\right) u_{i,T}\left(\zeta\left(s_i^T, s_{-i}^{T'}\right), \mu_{i,T}\right).\end{aligned}$$

This expression gives player i's expected payoff for a given combination of continuation strategies of others, and his own continuation strategies given his choice. Remember, player i knows his own belief and assigns probability one to the strategy he intends to carry out.

The following property formalizes the intuition of the one-stage-deviation principle: For a given combination of strategies of others, a player's strategy is optimal from any stage of the T-partial game if and only if there is no stage from which the player can gain by changing his strategy there, keeping it fixed at all other stages.

Proposition 1. An optimal strategy of any player in the T-partial game $G_T \in \boldsymbol{G}$ satisfies the one-stage-deviation property since it holds for all $i \in I$, $\boldsymbol{h}_T \in \boldsymbol{H}_T \setminus \boldsymbol{Z}_T$, that

$$\max_{c_i \in C_{i,h_T}} \mathbb{E}_{\sigma_T,\mu_T}[u_{i,T} | \boldsymbol{h}_T, c_i] = \max_{s_i^T \in S_i^{\boldsymbol{H}_T}(h_T)} \mathbb{E}_{s_i^T, \mu_{i,T}}[u_{i,T} | \boldsymbol{h}_{T'}].$$

Proof. See Appendix (A).

The following existence theorem of T-partial games obtains:

Theorem 1. If belief-dependent payoffs are continuous, then there exists at least one sequential equilibrium assessment in each T-partial game $G_T \in \boldsymbol{G}$.

Proof. See Appendix (A).

The proof of existence basically relies on the trembling-hand perfect equilibrium concept [Selten (1975)]: no matter how close to being rational players are, they will never be perfectly rational. There will always be some chance that a player will make a mistake. This idea can be used to approximate a candidate equilibrium behavioral strategy profile by a nearby completely mixed strategy profile (tremble) and require that any deliberately made choices, i.e. those given positive probability in the candidate strategy profile be optimal–not only against the candidate strategy profile, but also against the nearby mixed strategy profile. More formally, any profile of behavioral strategies σ_T is a perfect equilibrium if there is a sequence of completely mixed strategy profiles $\{\epsilon^k\}$ such that at each history and for each ϵ^k, the behavior of σ_T at the history is optimal against ϵ^k, i.e. is optimal when behavior at all other histories is given by ϵ^k. It is shown by Kakutani's fixed point theorem that in each ϵ^k-perturbed game there exists at least one ϵ^k-equilibrium strategy profile σ_T^k, implying that there exist an assessment $(\sigma_T^k, \beta(\sigma_T^k))$ where $\beta(\sigma_T^k) = \mu_T$.[9] As $\epsilon^k \to 0$ the corresponding strategy σ_T^k has an accumulation point σ_T^*, such that $(\sigma_T^*, \beta(\sigma_T^*))$. For each agent (i, h_T), $\sigma_{i,T}^*(\cdot|h_T)$ assigns positive probability only to choices that are best responses to $(\sigma_T^*, \beta(\sigma_T^*))$ at h_T. By Definition 7 and Proposition 1 each $(\sigma_T^*, \beta(\sigma_T^*))$ is a sequential equilibrium assessment.

Corollary 1. Define the order l as the maximum length of an awareness chain in the partially ordered set $(\boldsymbol{G}, \leq)$. For any $l > k \geq 0$, sequential equilibria in the T_l-partial game can be found by first considering the T_0-partial game (the B&D-game), and then extend the equilibria step-by-step to the T_l-partial game by taking the equilibria of other players in the T_k-partial games as given.

Proof. See Appendix (A).

This corollary suggests a procedure for finding equilibria in our structure. First, fix the T-partial game under consideration. Start from the last stage in this game: any historical event in this partial game for which the feasible choices terminate the game. Then look for equilibria in each subgame a player is aware of, by: (i) calculating the best responses of other players at the history of the last stage in the 'smallest' partial game (the B&D-game), and (ii) extend the equilibria step-by-step to histories of the last stage in 'larger' partial games by finding a fixed point given the optimal choices of other players at the copies in 'smaller'

[9] Let $\beta^1(\sigma_T) = (\beta^1(\sigma_T))_{i \in N}$ denote the profile of first-order beliefs derived from σ_T according to condition (i) in Definition 6. The profile of infinite belief hierarchies $\mu_T = \beta(\sigma_T)$ is obtained by applying condition (ii) in the same definition.

partial games. Now go backward and look at historical events in the second-to-last stage. The best responses has already been calculated for the historical event $(\boldsymbol{h}_T, c)$, because such events correspond to the last stage of the game. We assume that each active player at the second-to-last stage makes feasible choices that maximizes his expected payoff given the best responses in the last stage, because he expects that the other players will also best respond in the last stage. Again, extend the equlibria in the second-to-last stage step-by-step from the 'smallest' partial game to the T-partial game. We continue to go backwards in this ways until we reach the initial stage. If a player at some history becomes aware of more (a new chain of partial games in $\boldsymbol{G}$), then he re-evaluates the strategic situation and starts over by backward inducting from the last stage.

5 Application

In the following we will use a sequential prisoners dilemma to highlight the impact and importance of unawareness in the strategic interactions of players with belief-dependent preferences. The specific belief-dependent motivation that we concentrate on is a modified version of Dufwenberg and Kirchsteiger (2004)'s 'theory of sequential reciprocity' (5.1). A full description of the strategic interaction with all possible awareness levels and equilibria is beyond the scope of this paper. Therefore, we limit the analysis to three different awareness scenarios and the respective characterization of only one equilibrium (5.2). Results and intuitions are presented in this section, lengthy mathematical proofs are relegated to the Appendix (B).

5.1 A sequential prisoners dilemma with reciprocity

Consider the following sequential prisoners dilemma also analyzed by Dufwenberg and Kirchsteiger (2004):

[Figure 3]

Figure 3 is a multi-player decision tree, where $I = \{Ann, Bob\}$, $N = \{n^0, n^1, n^2, n^3, n^4, n^5, n^6\}$, augmented with material payoffs associated with each joint strategy profile. Ann can in the initial node n^0 choose between $Cooperate$ and $Defect$ and Bob is passive. While in node n^1 and n^2 Bob can choose between $cooperate$ and $defect$, respectively, and Ann is passive. If the path $(Cooperate, cooperate)$ is chosen both players get a material payoff of 1, if $(Cooperate, defect)$ is chosen Ann gets -1 and Bob gets 2.

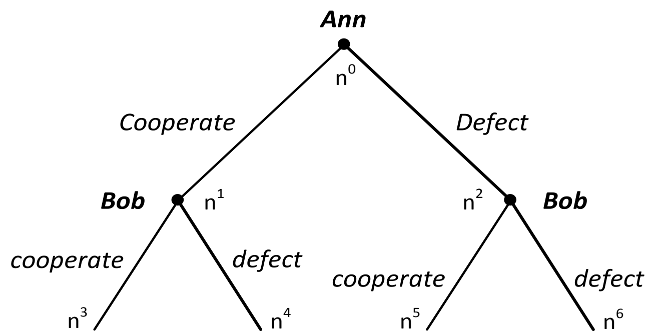

Figure 1: A Multi-Player Decision Tree

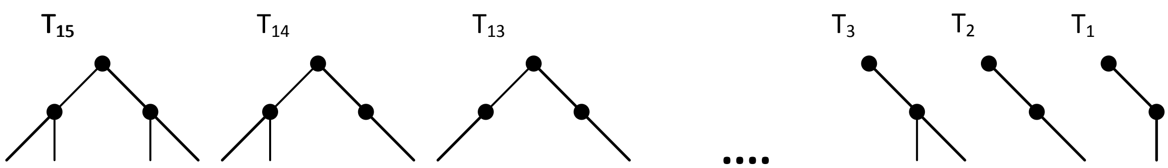

Figure 2: The Family of Subtrees $\boldsymbol{T}$

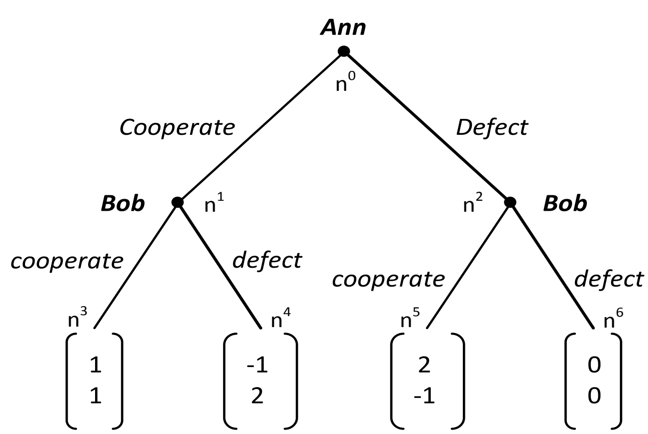

Figure 3: 'Sequential Prisoners Dilemma'

Furthermore, if path $(Defect, defect)$ is chosen both players get a material payoff of 0 and if $(Defect, cooperate)$ is chosen Ann gets 2 and Bob gets -1.

Following Section 2, we now consider the extensive form with unawareness and communication associated with the just described game.

For simplicity we assume that only *Bob* is motivated by belief-dependent reciprocity.[10] More specifically, for any $T \in \boldsymbol{T}$ *Bob*'s utility is given by:

$$u_{B,T}(\zeta(s_B^T, s_A^{T'}), \mu_B) = \pi_B(\cdot) + Y \times \kappa_{BA}(\cdot) \times \lambda_{BAB}(\cdot),$$

where $s_B^T \in S_B^{H_T}$ and $s_A^{T'} \in \left[\bigcup_{T \hookrightarrow T'} S_A^{H_{T'}}\right]$. $\pi_B(\cdot)$ is *Bob*'s expected monetary payoff which depends on his first-order belief concerning *Ann*'s strategy $(\mu_{B,T}^1(s_A^{T'}))$ and his own strategy (s_B^T). That means, at $\boldsymbol{h}_T$ *Bob*'s expected monetary payoff is given by $\pi_B(\mu_{B,T}^1\left(s_A^{T'}|\boldsymbol{h}_T\right), s_B^T)$. $Y > 0$ is a constant that captures his sensitivity to reciprocity towards *Ann*. *Bob*'s belief about his kindness towards *Ann* is $\kappa_{BA}(\cdot)$ and *Bob*'s perception of *Ann*'s kindness towards him is $\lambda_{BAB}(\cdot)$.

Formally, *Bob*'s perception of *Ann*'s kindness towards him at $\boldsymbol{h}_T$ is:

$$\lambda_{BAB}(\cdot) = \pi_B\left(\mu_{B,T}^1\left(s_A^{T'}|h_T\right), \mu_{B,T}^2\left(\cdot|h_T\right)\right) - \pi_B^{e_A}\left(\mu_{B,T}^1\left(s_A^{T'}|h_T\right), \mu_{B,T}^2\left(\cdot|h_T\right)\right),$$

where $\mu_{B,T}^1\left(s_A^{T'}|h_T\right)$ and $\mu_{B,T}^2\left(\cdot|h_T\right)$ respectively are *Bob*'s (updated) first- and second-order beliefs conditional on h_T in $\boldsymbol{h}_T$. Of course the domain of $\lambda_{BAB}(\cdot)$ is $\boldsymbol{h}_T$. However, we assume *Bob* only cares about *Ann*'s strategies allowed for by the history h_T in his evaluation of *Ann*'s kindness towards him. That implies, in his evaluation of *Ann*'s kindness towards him, *Bob* basically assigns probability 0 to every strategy in $\left[\bigcup_{T \hookrightarrow T'} S_A^{H_{T'}}\right] \backslash S_A^{H_T}$. Intuitively, these beliefs describe what *Bob* believes *Ann* would do and believe had she the same awareness level as him. Given this, $\pi_B(\cdot)$ and $\pi_B^{e_A}(\cdot)$ respectively describe what *Bob* believes *Ann* would intend for him and the average that *Ann* would be able to give had she the same awareness level as *Bob*. The equitable payoff is formally defined as follows:

$$\begin{aligned}\pi_B^{e_A}\left(\cdot\right) = \frac{1}{2}\Bigg[&\max\left\{\pi_B\left(\mu_{B,T}^1\left(s_A^{T'}|h_T\right), \mu_{B,T}^2\left(\cdot|h_T\right)\right), s_A^{T'} \in \left[\bigcup_{T \hookrightarrow T'} S_A^{H_{T'}}\right]\right\} \\ &+ \min\left\{\pi_B\left(\mu_{B,T}^1\left(s_A^{T'}|h_T\right), \mu_{B,T}^2\left(\cdot|h_T\right)\right), s_A^{T'} \in \left[\bigcup_{T \hookrightarrow T'} S_A^{H_{T'}}\right]\right\}\Bigg]. \qquad (2)\end{aligned}$$

[10] It is assumed that *Ann* is only interested in her own monetary payoff.

The first term in the brackets, $\max\{\pi_B(\mu^1_{B,T}(s^{T'}_A|h_T), \mu^2_{B,T}(\cdot|h_T))\}$, describes *Bob*'s belief about *Ann*'s belief about the maximum that she could have given to him. On the other hand, $\min\{\pi_B(\mu^1_{B,T}(s^{T'}_A|h_T), \mu^2_{B,T}(\cdot|h_T))\}$ describes *Bob*'s belief about *Ann*'s belief concerning the minimum she could have given to him. Intuitively *Bob* does not blame *Ann* for being unaware of some paths of play. He just forms a belief about what *Ann* would and could do were she of the same awareness level as he is.

In Dufwenberg and Kirchsteiger (2004) the set of joint strategy profiles is commonly known. However, in our setting with unawareness kindness perceptions take into account the fact that others might be aware of less. Furthermore, full awareness implies, that the basis upon which the others' kindness is evaluated remains unchanged. In contrast, in our setting the basis upon which the own as well as the kindness of others is judged changes as players become aware of more feasible paths of play.

Bob's kindness towards *Ann* at $\boldsymbol{h}_T$ can be described as:

$$\kappa_{BA}(\cdot) = \pi_A\left(\mu^1_{B,T}\left(s^{T'}_A|\boldsymbol{h}_T\right), s^T_B\right) - \pi^{e_B}_A\left(\mu^1_{B,T}\left(s^{T'}_A|\boldsymbol{h}_T\right), s^T_B\right),$$

where $s^{T'}_A \in \left[\bigcup_{T \hookrightarrow T'} S^{H_{T'}}_A\right]$ and $\pi^{e_B}_A(\cdot)$ is defined in an analogous fashion to Equation 2.

Ann's expected material payoff $\pi_A(\cdot)$ describes what *Bob* believes *Ann* gets, given his beliefs concerning her strategy $s^{T'}_A \in \left[\bigcup_{T \hookrightarrow T'} S^{H_{T'}}_A\right]$ and his own strategy $s^T_B \in S^{H_T}_B$ where $S^{H_T}_B$ is the set of own strategies that *Bob* is aware of in copies $\boldsymbol{h}_T$. Furthermore, $\pi^{e_B}_A(\cdot)$ is *Bob*'s belief about the average that he can give to *Ann*.

This concludes the definition of our sequential prisoners dilemma with reciprocity.

5.2 Three Different Awareness Scenarios

As already mentioned, we will concentrate on three different awareness scenarios and the respective characterization of one equilibrium. By considering these three awareness scenarios we limit our attention to a subset of all possible equilibria. The first scenario represents the benchmark case without unawareness also analyzed in Dufwenberg and Kirchsteiger (2004). Scenarios 2 and 3, on the other hand, include asymmetric awareness. For simplicity, both have the following characteristics:

(i) one player is initially aware of more than the other,

(ii) the player that is initially aware of more is certain of the other player's awareness and about the impact of his choices on the other player's awareness,

(iii) the player that is initially aware of less is certain that the other player is of the same awareness level as himself,

These simplifying assumptions imply that we can check for equilibria in our sequential prisoners dilemma in the normal way, i.e. by looking at the second mover following all possible choices of the first mover. Analyze his optimal behavior given his awareness. Go one step backward and analyze the optimal behavior of the first mover given the optimal choices of the second mover.

Scenario 1: As a first awareness scenario consider the benchmark case in which *Ann* and *Bob* are aware of everything. That is, there is no unawareness. Obviously, in such an environment messages that contain feasible paths of play are irrelevant because everyone is aware of all feasible paths of play. Given this we can abstract from messages in our benchmark case and concentrate on the actions of *Ann* and *Bob*. From Dufwenberg and Kirchsteiger (2004) we know that:

Result 1. If *Ann* chooses $Defect$, *Bob* also chooses $defect$ in equilibrium independent of his sensitivity to reciprocity Y. Furthermore, if *Ann* chooses $Cooperate$, *Bob* chooses *cooperate* in equilibrium if his sensitivity to reciprocity is $Y \geq 1$.

Proof. See Dufwenberg and Kirchsteiger (2004), p. 293.

Given *Bob*'s behavior following *Ann*'s action, it also holds in our benchmark case that:

Result 2. If *Bob*'s sensitivity to reciprocity is $Y \geq 1$, *Ann* chooses $Cooperate$ in equilibrium.

Proof. It is easy to see that *Ann* chooses $Cooperate$ given *Bob*'s equilibrium behavior, as this gives her 1 in monetary payoffs, rather than 0 which she would get by choosing $Defect$. ■

This shows that without unawareness and a reciprocal *Bob* ($Y \geq 1$), *Ann* can trigger a cooperative reaction from *Bob* by choosing to cooperate. Note that this very intuitive result stands in contrast to the result we would obtain with traditional assumptions about human behavior, i.e. egoistic preferences. If both players are only interested in their own monetary payoff, then *Ann* and *Bob* defecting would be part of the only pure strategy sequential equilibrium.

Scenario 2: As a second simple awareness scenario consider now the following:

- *Bob* is aware of everything, i.e. $\{H_{T'}\}_{T_{15} \rightsquigarrow T'}$ with $T_{15} = \{n^0, n^1, n^2, n^3, n^4, n^5, n^6\}$.[11]

[11] Note that subtree in our application are indexed in line with the subtree in Figure 2.

- *Ann* is initially only aware of $\{H_{T'}\}_{T_3 \hookrightarrow T'}$ with $T_3 = \{n^0, n^2, n^5, n^6\}$.
- *Bob* is certain that *Ann* is initially only aware of $\{H_{T'}\}_{T_3 \hookrightarrow T'}$.
- Wherever *Ann* finds herself, she will be certain that *Bob* has the same awareness level her.

Different to the previous scenario without unawareness, in this scenario *Ann* is initially unaware of her action *Cooperate* and *Bob*'s actions *cooperate* and *defect* following it. As before, we start by looking at the optimal behavior of *Bob*. That is, we start to look at all possible partial games *Bob* can find himself in after *Ann*'s choice. We fix his optimal behavior in these worlds and then go one step back to analyze *Ann*'s optimal choice given the optimal choice of *Bob*.

Result 3. If *Ann* chooses *Defect*, then *Bob* chooses *cooperate* and sends any message if his sensitivity to reciprocity is $Y \geq 1$.

Proof. See Appendix (B).

The reason why *Bob* nevertheless cooperates even after the seemingly unkind action *Defect* of *Ann* is the following: *Bob* is aware of the fact that *Ann* is not aware of her action *Cooperate* and his actions *cooperate* and *defect* following it. However, *Bob* evaluates *Ann*'s kindness on the basis of what he is aware of. *Bob* holds the equilibrium belief that *Ann* would have cooperated had she been aware of what he is aware of. In equilibrium *Bob* believes that *Ann* would have played *Cooperate* and, hence, would have acted kind, had she been aware of what he is aware of. As he is the last to choose in this situation, his choice is independent of the specific message that he sends, i.e. any of his messages is part of this equilibrium.

Concerning the behavior of *Ann* it is easy to see that her equilibrium behavior is:

Result 4. In any sequential equilibria *Ann* chooses *Defect* and sends any message.

Obviously *Ann* chooses *Defect* in Scenario 2 because this is the only feasible action that she is initially aware of. Furthermore, as she is certain that *Bob* is aware of what she is aware of messages do not play any strategic role for her, and, therefore, any message is part of this sequential equilibrium. This completes the second awareness scenario.

Different to the setting without unawareness by Dufwenberg and Kirchsteiger (2004), in our setting with unawareness *Bob* still cooperates even after the seemingly unkind action *Defect*. *Bob* simply takes into account that *Ann* was unaware of her action *Cooperate* and his subsequent actions *defect* and *cooperate* and, hence, evaluates her kindness on what she would have done had she been aware of what he is aware of. Importantly, $(Defect, cooperate)$ is neither part of an equilibrium given classical assumptions about human behavior, nor is it part of an equilibrium given reciprocal preferences and full awareness. It is the asymmetric awareness of *Bob* and *Ann* that produces this prediction. This demonstrates how allowing for asymmetric awareness influences our equilibrium predictions.

This scenario practically demonstrates how one can solve for sequential equilibria in our class of psychological games with unawareness and communication. One first has to look at the optimal behavior of all players active in the last non-terminal histories in all their partial games and then go backward history by history repeating the same procedure until the initial history.

Scenario 3: To furthermore see the importance of messages assume now the following awareness scenario:

- *Ann* is aware of everything, i.e. $\{H_{T'}\}_{T_{15} \hookrightarrow T'}$ with $T_{15} = \{n^0, n^1, n^2, n^3, n^4, n^5, n^6\}$.
- *Bob* is initially only aware of $\{H_{T'}\}_{T_4 \hookrightarrow T'}$ with $T_4 = \{n^0, n^1, n^3, n^4\}$
- *Ann* is certain that *Bob* is initially only aware of $\{H_{T'}\}_{T_4 \hookrightarrow T'}$.
- *Ann* is certain that, wherever *Bob* finds himself, he will believe that *Ann* has the same awareness level as him.
- *Ann* is certain that *Bob* will become aware of everything, i.e. $\{H_{T'}\}_{T_{15} \hookrightarrow T'}$, if she chooses $Defect$.

We start again by analyzing this situation by looking at *Bob*'s choices in all the partial games that he can be in following all possible choices of *Ann*.

Result 5. If *Ann* chooses $Defect$ and any message, *Bob* chooses $defect$ and sends any message in all sequential equilibria.

Proof. See Appendix (B).

To see this, remember that if *Ann* chooses $Defect$, *Bob* becomes aware of everything independent of the message that *Ann* sends in addition to her action. This means, in any history following *Ann*'s action $Defect$ *Bob* re-evaluates *Ann*'s kindness towards him on the basis of $\{H_{T'}\}_{T_{15} \hookrightarrow T'}$. Doing this, *Bob* perceives *Ann*'s choice as unkind independent of the message that she sends. Therefore, *Bob* chooses $defect$ out of reciprocity as well as own monetary considerations. Note, our result 5 is analog to Dufwenberg and Kirchsteiger (2004, p. 282)'s Observation 1 in the context of their sequential prisoners dilemma.

Next, consider *Bob*'s behavior following *Ann*'s action *Cooperate*:

Result 6. If *Ann* chooses *Cooperate* and sends

(i) a message that does not contain any new information, then *Bob* chooses *defect* in equilibrium and sends any message independent of his sensitivity to reciprocity.

(ii) a message which contains $T_3 = \{n^0, n^2, n^5, n^6\}$, then *Bob* chooses *cooperate* in equilibrium and sends any message, if his sensitivity to reciprocity is $Y \geq 1$.

(iii) a message which contains only $T_2 = \{n^0, n^2, n^6\}$, then *Bob* chooses *cooperate* in equilibrium and sends any message, if his sensitivity to reciprocity is $Y \geq 1$.

(iv) a message which contains only $T_1 = \{n^0, n^2, n^5\}$, then *Bob* chooses *cooperate* in equilibrium and sends any message, if his sensitivity to reciprocity is $Y \geq \frac{1}{2}$.

Proof. See Appendix (B).

Result 6 gives a first impression of how messages about feasible paths of play influence the strategic interaction of reciprocal players. Different to Dufwenberg and Kirchsteiger (2004, p. 282) in the context of their sequential prisoners dilemma with full awareness, our result 6 depends on *Ann*'s message to *Bob*. By sending a message *Ann* can influence the basis on which *Bob* evaluates her kindness. That is, she can influence the partial game that *Bob* will find himself in. If he is unaware of *Ann*'s action *Defect* and all of his own subsequent actions, *Bob* evaluates the kindness of *Ann* following her choice *Cooperate* on the basis of $\{H_{T'}\}_{T_4 \rightsquigarrow T'}$ with $T_4 = \{n^0, n^1, n^3, n^4\}$. This implies that he perceives a kindness $\lambda_{BAB} = 0$. This in turn means that *Bob* only takes into account his own monetary payoff when optimizing his choice. Only when *Ann* sends a message that contains some new information, i.e. a subtree consistent with her action *Defect*, *Bob*'s awareness and, hence, the partial game he plays as well as the basis upon which he evaluates *Ann*'s kindness changes.

By sending a message which contains $T_1 = \{n^0, n^2, n^5\}$ as new information, *Bob* becomes aware of $\{H_{T'}\}_{T_{12} \rightsquigarrow T'}$ with $T_{12} = \{n^0, n^1, n^2, n^3, n^4, n^5\}$ (case (iv) of result 6). Hence, *Bob* finds himself in a new partial game and has a new basis upon which he evaluates the kindness of *Ann*. Now *Bob* is aware of the fact that *Ann* could have chosen *Defect* which would have implied (according to his awareness) a material payoff of -1 for him. Given this, he perceives *Ann*'s choice *Cooperate* as kind because independent of his choice following *Ann*'s choice *Cooperate*, his material payoff is higher than -1. He reciprocates this kindness in equilibrium if his sensitivity to reciprocity is $Y \geq \frac{1}{2}$. Following the same kind of reasoning in cases (ii) and (iii) implies that *Bob* reciprocates by choosing *cooperate*, if his sensitivity to reciprocity is $Y \geq 1$.

As can easily be seen, if *Ann* had no possibility to send a message to *Bob*, i.e. to make *Bob* aware of what else she could have done, *Ann* would be unable to induce *Bob* to cooperate. *Bob* would simply remain aware of what he was aware of before and continue to evaluate *Ann*'s kindness on this basis.

This brings us to the equilibrium behavior of *Ann*

Result 7. *Ann*'s equilibrium behavior depends on *Bob*'s sensitivity to reciprocity Y:

(i) If *Bob*'s sensitivity to reciprocity is $Y < \frac{1}{2}$, *Ann* chooses $Defect$ in equilibrium and sends any message.

(ii) If *Bob*'s sensitivity to reciprocity is $\frac{1}{2} \leq Y \leq 1$, *Ann* chooses $Cooperate$ in equilibrium and sends a message which contains *only* $T_1 = \{n^0, n^2, n^5\}$.

(iii) If *Bob*'s sensitivity to reciprocity is $Y \geq 1$, *Ann* chooses $Cooperate$ in equilibrium and sends a message which contains *at least* $T_1 = \{n^0, n^2, n^5\}$.

Proof. See Appendix (B).

Intuitively, if *Bob*'s sensitivity to reciprocity is low, i.e. $Y < \frac{1}{2}$, *Ann* knows that whatever she makes *Bob* aware of, he will always choose $defect$. Given this, she prefers to choose $Defect$ to get 0 in monetary payoffs, rather than $Cooperate$ which would give her -1. Now, if *Bob* has a sensitivity to reciprocity $Y \geq \frac{1}{2}$, *Ann* can induce *Bob* to cooperate by choosing $Cooperate$ and making him aware of her action $Defect$ and *Bob*'s subsequent possibility $cooperate$ (case (ii) of result 7). Making *Bob* aware changes the basis on which he evaluates the kindness of *Ann* towards him. Aware of *Ann*'s action $Defect$ and *Bob*'s action $cooperate$, *Bob* realizes that *Ann*'s action $Cooperate$ was actually kind. This is something he would not have realized had he remained unaware of $Defect$ and his subsequent action $cooperate$. By choosing action $Cooperate$ and communicating either $T_3 = \{n^0, n^2, n^5, n^6\}$ or $T_2 = \{n^0, n^2, n^6\}$ *Ann* also induces a positive perception of her action, but less than in case (ii). Hence, *Ann* only chooses $Cooperate$ and one of these messages in equilibrium if *Bob*'s sensitivity is higher $Y \geq 1$.

The bottom line: awareness messages are important in the interaction of players with reciprocal preferences as they influence their perceptions about their own as well as others' kindness.

These three simple awareness scenarios demonstrate how unawareness influences the strategic interaction of players with belief-dependent preferences. Furthermore, they show the important role of awareness messages through which players can influence other players' awareness. By influencing awareness levels players influence equilibrium behavior. To put it differently, taking into account asymmetric awareness levels of players when analyzing strategic interactions leads to new and intuitive equilibrium predictions.

6 Extensions and discussion

In this section we first consider some relevant extensions of our model, namely guilt aversion (6.1), moves by nature (6.2), initial asymmetric information (6.3), and strategic information transmission (6.4). We then go on to discuss how to interpret hierarchies of beliefs (6.5), and finally consider the relevance of non-equilibrium solution concepts in out setting (6.6).

6.1 Guilt aversion and unawareness

In Section 5 we focused on reciprocity, however our framework is general implying that it can be used to analyze how unawareness affects other forms of belief-dependent motivation such as guilt and regret. In the following we will consider a simple two player example highlighting how unawareness might influence guilt aversion.

We will say that *Ann* 'lets down' *Bob* if his actual material payoff from *Ann*'s strategy, denoted $\pi_B(s_A^T)$, is lower than the payoff *Ann* believes he expects to get, $\pi_B(\mu_A^2(\cdot|\boldsymbol{h}_T), \mu_A^1(s_B^{T'}|\boldsymbol{h}_T))$. This can be measured by the following expression:

$$\max\{0, (\pi_B(\mu_A^2(\cdot|\boldsymbol{h}_T), \mu_A^1(s_B^{T'}|\boldsymbol{h}_T)) - \pi_B(s_A^T))\}.$$

Taking *Ann*'s belief concerning *Bob*'s disappointment into account, we obtain the following utility function exhibiting guilt aversion:

$$u_A(\zeta(s_A^T, s_B^{T'}), \mu_A) = \pi_A(\boldsymbol{z}_T) - Y \times \max\{0, (\pi_B(\mu_A^2(\cdot|\boldsymbol{h}_T), \mu_A^1(s_B^{T'}|\boldsymbol{h}_T)) - \pi_B(s_A^T))\},$$

where $Y \geq 0$ is some psychological sensitivity parameter of *Ann*.

Now consider the example considered in the introduction, in which *Ann*'s exam was postponed and she could have gone to *Bob*'s party. Remember, *Ann* would rather not go to the party because she is tired. Now imagine that *Ann* correctly believes that *Bob* is unaware of the postponement: *Ann* will in equilibrium be certain that *Bob* will be certain that she cannot come, and *Ann* will therefore feel no guilt if she stays away. In a game with full awareness this would however not be a unique equilibrium. *Ann* could also be certain that *Bob* expects her to come because her exam was canceled. If *Ann*'s sensitivity to disappointing *Bob* in this situation is high enough, she would come to his party.

The two forms of belief-dependent motivation we have considered up to now (reciprocity and guilt) have relied on first- and second-order beliefs. However, our model is not restricted to only looking at these forms of beliefs. An example involving dependence on third-order beliefs is Battigalli and Dufwenberg (2007b)'s 'guilt from blame,' which assumes that a player cares about the other player's inferences regarding the extend to which he is willing to let him down. Intuitively, *Ann* experiences guilt to the extent that *Bob*'s beliefs indicate that *Ann* intended to disappoint him.

6.2 Moves by nature

Moves by nature is an important extension for applications. For example, Sebald (2010) shows that the strategic interactions of reciprocal players may be influenced by the possibility that material payoffs are affected by moves of nature rather than players. One could easily imagine that such considerations might be amplified (or mitigated) by unawareness.

Let $I^0 = \{0, 1, \dots, n\}$ where index 0 denotes nature, and $\sigma_{0,T} := \sigma_{0,T}(\cdot|\boldsymbol{h}_T) \in \prod_{\boldsymbol{h}_T \in \boldsymbol{H}_T \setminus \boldsymbol{Z}_T} \Delta^0(\bigcup_{T \rightarrow T'} C_{0,h_{T'}})$ be the awareness restricted strictly positive objective plan of moves by nature. Note that given some awareness level, a player would never think that nature would send messages from which he could learn. We do therefore not consider messages send by nature.

An assessment $(\sigma_T, \mu_T) = (\sigma_{i,T}, \mu_{i,T})_{i \in I^0}$ is consistent if there is a sequence of strictly positive behavioral strategy profiles $\sigma^k \rightarrow \sigma$ such that for all $i \in I$, $s_{-i}^{T'} \in \left[\bigcup_{T \rightarrow T'} S_{-i}^{H_{T'}}\right]$, $\boldsymbol{h}_T \in \boldsymbol{H}_T$,

$$\mu_{i,T}^1(s_{-i}^{T'}|\boldsymbol{h}_T) = \lim_{k \rightarrow \infty} \frac{\Pr_{\sigma_{0,T}|\boldsymbol{h}_T}(s_0^{T'}) \prod_{j \neq 0,i} \Pr_{\sigma_{j,T}^k}(s_j^{T'}|\boldsymbol{h}_T)}{\sum_{\tilde{s}_{-i}^{T'} \in \left[\bigcup_{T \rightarrow T'} S_{-i}^{\boldsymbol{H}_{T'}}\right]} \Pr_{\sigma_{0,T}}(\tilde{s}_0^{T'}|\boldsymbol{h}_T) \prod_{j \neq 0,i} \Pr_{\sigma_{j,T}^k}(\tilde{s}_j^{T'}|\boldsymbol{h}_T)}.$$

Kreps and Wilson (1982, Section 5) have a similar condition that refers to cps' of histories (or nodes), and further more for all $l > 1$, $\mu_{i,T}^l$ assigns probability 1 to $\mu_{-i,T}^{l-1}$. (σ_T, μ_T) is a sequential equilibrium if it is consistent and for all $i \in I$, $\boldsymbol{h}_T \in \boldsymbol{H_T} \setminus \boldsymbol{Z_T}$,

$$\forall j \neq i, \text{supp } \text{marg}_{S_i^{H_T}} \mu_{i,T}(\cdot|\boldsymbol{h_T}) \subseteq \arg \max_{s_i^T \in S_i^{H_T}(h_T)} \mathbb{E}_{s_i^T, \mu_{i,T'}}[u_{i,T}|\boldsymbol{h}_T],$$

where $\mathbb{E}_{s_i^T, \mu_{i,T}}[u_{i,T}|\boldsymbol{h}_T]$ is the obvious modification of Equation 1. It can easily be proven that that the existence theorem also holds when we add nature as a player (if the payoff functions are continuous).

6.3 Initial asymmetric information

One might argue that it is unrealistic to assume that players know each psychological propensity, unless one models interaction within a family or amongst friends. This observation motivates the following extension.

If we want to model asymmetric information about initial moves by nature, we should assume that at the initial history h_T^0 the only active player is 0 (nature), $C_{0,h_T^0} = \Theta$, where $\Theta \subseteq \Theta_1 \times \cdots \times \Theta_n$ is a set of exogenous payoff relevant parameters. Each player i observes only coordinate θ_i of $\theta = (\theta_1, \ldots, \theta_n)$; θ may affect payoffs, or choice sets, or the probability of future moves by nature. Note that by defining asymmetric information in this way one introduces fictitious ex ante beliefs.

A full blown generalization of information in our model would also include imperfectly observable choices. However, such an extension is beyond the scope of this paper.

6.4 Strategic information transmission

Strategic information transmission has been studied in economic theory for over a quarter of a century. Traditionally this has been done via signaling, whereby a player can influence the beliefs of other players by his actions (e.g. choice of education). To highlight the difference between influencing players' perceptions through signals and awareness messages, we will focus solely on the updating of players' beliefs. The discussion is therefore relevant for, among others, costly market signaling [Spence (1973), Rothschild and Stiglitz (1976), Wilson (1977)], cheap talk [Crawford and Sobel (1982), Farrell (1993)], and observational learning [Banerjee (1992), Bikhchandani *et al.* (1992), Smith and Sørensen (2000)].

The canonical signaling game for our class of unawareness games is basically a Bayesian extensive form with observable actions. We will say that nature selects types independently for the players and refer to player i after he receives information θ_i as type θ_i and $\theta = (\theta_1 \times \cdots \times \theta_n)$ as the state of nature. We assume that there exists a common prior $p \in \Delta(\Theta)$ with the properties that for all i, θ_i and θ_{-i}, $p(\{\theta_i\} \times \Theta_{-i}) > 0$ (type θ_i has positive 'prior' probability) and $p(\theta_{-i}|\theta_i) = p((\theta_i, \theta_{-i})|\{\theta_i\} \times \Theta_{-i})$ (i.e., $p(\theta_{-i}|\theta_i)$ is the conditional probability of θ_{-i} given θ_i). Since types are independent we have that the product measures $p = (p_1 \times \cdots \times p_n)$ is a common prior, where $p_i \in \Delta(\Theta_i)$ is the marginal probability on $\Theta_1 \times \cdots \Theta_n$ for some $i \in I$; equivalently, $p(\theta_{-i}|\theta_i) = \prod_{j \neq i} p_j(\theta_j)$ for all i and θ. We can now associate a signaling game with the set of histories $\boldsymbol{H}_T \times \Theta$. Each information set of each player j takes the form $I(\boldsymbol{h}_T, \theta_j) = \{(\boldsymbol{h}_T, (\theta_j, \theta'_{-j})) : \theta'_{-j} \in \Theta_{-j}\}$ for $\theta_j \in \Theta_j$. Player j's behavioral strategies is denoted by $\sigma_{j,T'}(\cdot|(\boldsymbol{h}_T, \theta_j))) \in [\Delta(\bigcup_{T \rightarrow T'} A_{j,h_{T'}})]^{\Theta}$. We interpret $\sigma_{j,T}$ as a common array of common conditional first-order beliefs $\mu^1_{-j,T}$ held by j's opponents. As is standard in signaling we assume that beliefs are determined by actions, which implies that: (i) if player j does not have to move then the actions taken do not affect the other players' belief about player j's type and (ii) if player j is one of the players who takes an action then the other players' beliefs about j's type depend only on the action taken by j, not on the other players' actions. (This is consistent with behavioral strategies being independent.) If $p_j(\theta_j|\boldsymbol{h}^0_T) = p_j(\theta_j)$ and a_j is in the support of $\mu^1_{-j,T}(\cdot|(\boldsymbol{h}_T, \theta_j))$ then for any $\theta_j \in \Theta_j$ we have

$$p_j(\theta_j|\boldsymbol{h}_T, a) = \frac{\mu^1_{-j,T}(a_j|(\boldsymbol{h}_T, \theta_j)) \cdot p_j(\theta_j|\boldsymbol{h}_T)}{\sum_{\theta'_j \in \Theta_j} \mu^1_{-j,T}(a_j|(\boldsymbol{h}_T, \theta'_j) \cdot p_j(\theta'_j|\boldsymbol{h}_T)}.$$

Upon observing the signal from player j the other players update their beliefs about player j's exogenous type using Bayes' rule until his behavior contradicts the other players' common belief $\mu^1_{-j,T}$, at which point they form a new conjecture about player i's type that is the basis for future Bayesian updating until there is another conflict with $\mu^1_{-j,T}$. Such influencing of others' beliefs through signaling does not exists when there is perfect information (i.e., Θ is a singleton).

Taking actions or sending messages that other players are unaware of can in our class of games (with complete information) also be interpreted as strategic information transmission. Since each of these actions/messages only reveals information about the structure of the game, and not about the probability of other players being of certain exogenous types, the information transmission we allow for is different from that known from signaling. Remember, in equilibrium player i confined to some subform forms beliefs about some other player j's equilibrium beliefs at each subform he might be aware of. By strategically revealing paths of play, player i can exclude the subforms player j can be confined to which does not allow for the revealed paths. This means that our information revealing actions/messages are irrelevant in settings with full awareness. However, in games with asymmetric awareness such information transmission becomes an important part of the strategic interaction.

6.5 Hierarchy representation of beliefs

The hierarchy representation of beliefs plays a prominent role in belief-dependent preferences. The interpretation of such a representation has been discussed a great deal in the literature, and it is therefore important to clarify how one should interpret such hierarchies in our framework. By using a hierarchy representation, we implicitly assume that the game is analyzed at a 'point in time' subsequent to the player knowing his beliefs. That is, there exist no beliefs at a 'prior' point in time, nor is there any information about what the players would have believed had their information been 'less' or 'more' than what it in fact is. The hierarchy of beliefs therefore offers no meaningful argument for identifying beliefs at a prior point in time. When considering unawareness any interpretation of beliefs at a prior point in time becomes nonsensical: one would have to imagine that each player had been aware of all relevant paths of play at some prior point and then become unaware of some of the paths ex-ante, while nevertheless having received more information about the paths they are aware of. Insisting that priors be common does in this setting not reflect where differences in beliefs may come from, but rather constitutes a complex and unintuitive restriction on each hierarchy of beliefs. Even if we were to impose common priors this would not render a prior point in time relevant, nor would it render the prior distribution meaningful.[12]

6.6 Non-equilibrium solution concepts

Our solution concept ideally involves interpreting hierarchies beliefs as a rest-point of a transparent reasoning process, one could argue that it is difficult to carry over such interpretations to a setting in which every increase of awareness is by definition a shock or surprise. Once the player's view of the game itself is challenged in the course of play, some may find it difficult to justify the idea that a new set of equilibrium hierarchy beliefs for the continuation of the game are readily available. One could, for example, consider some version of extensive-form rationalizability (Battigalli, 1997) since it embodies forward inductive reasoning. If somebody makes a player aware of some relevant paths of play, it seems like a strong assumption to dismiss the increased level of awareness as an unintended consequence of others' behavior. Rather, the player should try to infer from others' choices, re-interpret others' past behavior, and try to infer from it their future moves. In psychological games payoffs are affected by hierarchical beliefs, so rationalizability has to be defined as a property of the whole structure the player is aware of rather than of strategies, and one therefore has to consider players' belief revision processes (Battigalli and Siniscalchi, 2002).

[12] The plausibility and justification of the ex-ante versus the interim view of beliefs has been extensively discussed in the literature, see Harsanyi (1967–68), Dekel and Gul (1997), Gul (1998), and Aumann (1998).

In order to facilitate comparison, and highlight common features, with the existing literature on psychological games with sequential moves, we have chosen to adopt Kreps and Wilson (1982)'s sequential equilibrium concept which has become a benchmark for the analysis of such games (see for example, Dufwenberg and Kirchsteiger, 2004 and Battigalli and Dufwenberg, 2007b).

7 Conclusion

In our analysis we have shown that unawareness has a profound impact on the strategic interaction of players with belief-dependent preferences. That means, taking account of asymmetric awareness levels leads to intuitive and distinct equilibrium predictions. Moreover, we have demonstrated that communication is an important integral part of the strategic environment when players have asymmetric awareness–a type of communication that is meaningless in environments without unawareness. In our analysis we have first formalized a general framework with unawareness, communication and belief-dependent psychological preferences. Second, we have presented a solution concept and shown that all dynamic psychological games with continuous utility functions have at least one sequential equilibrium. Third, we have analyzed a specific application to demonstrate the impact of unawareness and communication in a specific context with reciprocal agents. The application has highlighted the fact that any analysis of strategic interactions with asymmetric awareness levels has to start with a description of what players are aware of and what they become aware of when play unravels. Finally, the application has also practically demonstrated how sequential psychological equilibria can be found in specific strategic settings.

Summarizing, unawareness has a profound impact on the strategic interaction of players with belief-dependent psychological preferences. Thus, it should not be neglected and assumed away, but rather taken into account as an integral part of strategic environments.

A Appendix

A.1 Proof of Proposition 1

The Proof follows naturally from the following Lemma, which itself is essentially an adaptation of the dynamic programming approach due to Battigalli and Dufwenberg (2007a, Section 3). We want to relate the problem $\max_{s_i^T \in S_i^{H_T}(h_T)} \mathbb{E}_{s_i^T, \mu_{i,T}}[u_{i,T} | \boldsymbol{h}_T]$ to a dynamic programming problem on the decision tree induced by $\mu_{i,T}$, that is the decision tree player i thinks he is in. Important for the following analysis is our assumption that player i knows his own belief and assigns probability one the the strategy he intends to carryout. However, first we develop some notation needed for the Lemma.

Depth of the decision tree:

- For each k with $0 \le k \le l(h_T)$ (recall that $l(h_T)$ denotes the length of history h_T). Let c_i^k be the choice made by some $i \in I$ in h_T at the predecessor of h_T of length k. Thus, by definition $h_T = (c^0, c^1, \dots, c^{l(h_T)-1})$ where $c^k = (c_1^k, \dots, c_{|I|}^k)$.
- Let $d(h_T) = \max_{h_T \le z_T} [l(z_T) - l(h_T)]$ denote the depth of the decision tree with root h_T.

Strategies:

- $(s_i^T | h_T)$ denotes the strategy that takes all the choices of player i in history h_T and behaves as s_i^T otherwise:

$$(s_i^T | h_T)_{\tilde{h}_T} = \begin{cases} s_{i,\tilde{h}_T}^T & \text{if } \tilde{h}_T \nless h_T, \\ c_i^{l(\tilde{h}_T)} & \text{if } \tilde{h}_T < h_T. \end{cases}$$

  Intuitively, $(s_i^T | h_T)$ is a strategy that takes on the observed choices made prior to the history h_T, and then agrees with strategy s_i^T at h_T and in what follows.

- Now change $(s_i^T | h_T)$ at h_T so that it is the strategy obtained from $(s_i^T | h_T)$ by replacing $s_{i,\tilde{h}_T}^T$ with $c_i \in C_{i,h_T}$. The resulting strategy is denoted $(s_i^T | h_T, c_i)$. That is,

$$(s_i^T | h_T, c_i)_{\tilde{h}_T} = \begin{cases} (s_i^T | h_T)_{\tilde{h}_T} & \text{if } \tilde{h}_T \neq h_T, \\ c_i & \text{if } \tilde{h}_T = h_T. \end{cases}$$

  In words, $(s_i^T | h_T, c_i)$ is the strategy consistent with h_T that chooses c_i at h_T and behaves as $(s_i^T | h_T)$ in all other histories $\tilde{h}_T$. That is, $(s_i^T | h_T)$ takes an ex ante (before player i

makes his choice at h_T) point of view of the strategy $s_i^T \in S_i^{H_T}(h_T)$ which is consistent with h_T, while $(s_i^T | h_T, c_i)$ takes on an ex post (after player i makes his choice at h_T) view of the strategy $s_i^T \in S_i^{H_T}(h_T, c_i)$ which is consistent with h_T and the choice c_i he is about to make.

Value functions on the decision tree:

- Define the two value functions $V_{\mu_{i,T}} : \boldsymbol{H}_T \to \mathbb{R}$ and $\overline{V}_{\mu_{i,T}} : (\boldsymbol{H}_T \backslash \boldsymbol{Z}_T) \times C_{i,h_T} \to \mathbb{R}$ induced by $\mu_{i,T}$.

- For terminal copies player i is aware of $\boldsymbol{z}_T \in \boldsymbol{Z}_T$, let

$$V_{\mu_{i,T}} = \sum_{T \hookrightarrow T'} \mu_{i,T}^1 (S_{-i}^{H_{T'}}(z_{T'}) | \boldsymbol{z}_T) u_{i,T}(z_T, \mu_{i,T}).$$

- Assuming that the value function $V_{\mu_{i,T}}(\boldsymbol{h}_T, c)$ has been defined for all immediate successors $(\boldsymbol{h}_T, c)$ of copies player i is aware of, let

$$\overline{V}_{\mu_{i,T}}(\boldsymbol{h}_T, c_i) = \sum_{T \hookrightarrow T'} \sum_{c_{-i} \in C_{-i,h_{T'}}} \mu_{i,T}^1 (S_{-i}^{H_{T'}}(h_{T'}, c_{-i}) | \boldsymbol{h}_T) V_{\mu_{i,T}}(\boldsymbol{h}_T, c). \tag{i}$$

For each $c_i \in C_{i,h_T}$, $V_{\mu_{i,T}}(\boldsymbol{h}_T)$ is defined as

$$V_{\mu_{i,T}}(\boldsymbol{h}_T) = \max_{c_i \in C_{i,h_T}} \overline{V}_{\mu_{i,T}}(\boldsymbol{h}_T, c_i).$$

Next we state the dynamic programming problem:

Lemma 3 (Dynamic Programming)**.** Suppose that for all $\boldsymbol{h}_T \in \boldsymbol{H}_T \backslash \boldsymbol{Z}_T$,

$$s_{i,h_T}^{T,*} \in \arg\max_{c_i \in C_{i,h_T}} \overline{V}_{\mu_{i,T}}(\boldsymbol{h}_T, c_i).$$

Then for all $\boldsymbol{h}_T \in \boldsymbol{H}_T \backslash \boldsymbol{Z}_T$,

$$\mathbb{E}_{(s_i^{T,*}|h_T),\mu_{i,T}}[u_{i,T} | \boldsymbol{h}_T] = V_{\mu_{i,T}}(\boldsymbol{h}_T) = \max_{s_i^T \in S_i^{\boldsymbol{H}_T}(h_T)} \mathbb{E}_{s_i^T,\mu_{i,T}}[u_{i,T} | \boldsymbol{h}_T]. \tag{DP}$$

Proof of Lemma. The proof is by induction on $d(h_T)$.

Basic step: We start from the last stage of any T-partial game: h_T is such that all feasible choices following copies h_T terminate the game, i.e. $d(h_T) = 1$. Clearly (DP) holds for all h_T for which $d(h_T) = 1$.

Inductive step: We now fix some stage $k \geq 1$, which is not the last stage, and look at the stage just preceding it. Suppose (DP) holds for all h_T such that $1 \leq d(h_T) \leq k$. Let $d(h_T) = k+1$.

By the law of iterated expectations for all $c_i \in C_{i,h_T}$:

$$\mathbb{E}_{(s_i^{T,*}|h_T,c_i),\mu_{i,T}}[u_{i,T}|\boldsymbol{h}_T] = \sum_{T \hookrightarrow T'} \sum_{c_{-i} \in C_{-i,h_{T'}}} \mu_{i,T}^1(S_{-i}^{H_{T'}}(h_{T'}, c_{-i})|\boldsymbol{h}_T)\mathbb{E}_{(s_i^{T,*}|h_T,c_i),\mu_{i,T}}[u_{i,T}|\boldsymbol{h}_T, c]. \quad \text{(ii)}$$

By the inductive hypothesis, for all $c \in \prod_{i \in I} C_{i,h_T}$:

$$\mathbb{E}_{(s_i^{T,*}|h_T,c_i),\mu_{i,T}}[u_{i,T}|\boldsymbol{h}_T, c] = V_{\mu_{i,T}}(\boldsymbol{h}_T, c) = \max_{s_i^T \in S_i^{\boldsymbol{H}_T}(h_T,c)} \mathbb{E}_{s_i^T,\mu_{i,T}}[u_{i,T}|\boldsymbol{h}_T, c]. \quad \text{(iii)}$$

If we plug (iii) into (ii) and compare with (i), we get:

$$\mathbb{E}_{(s_i^{T,*}|\boldsymbol{h}_T,c_i),\mu_{i,T}}[u_{i,T}|\boldsymbol{h}_T] = \overline{V}_{\mu_{i,T}}(\boldsymbol{h}_T, c_i).$$

Therefore,

$$\begin{gathered}
\mathbb{E}_{(s_i^{T,*}|h_T),\mu_{i,T}}[u_{i,T}|\boldsymbol{h}_T] = V_{\mu_{i,T}}(\boldsymbol{h}_T) = \max_{s_i^T \in S_i^{\boldsymbol{H}_T}(h_T)} \mathbb{E}_{s_i^T,\mu_{i,T}}[u_{i,T}|\boldsymbol{h}_T] \\
\textit{if and only if} \\
s_{i,h_T}^{T,*} \in \arg\max_{c_i \in C_{i,h_T}} \mathbb{E}_{(s_i^{T,*}|h_T,c_i),\mu_{i,T}}[u_{i,T}|\boldsymbol{h}_T] \\
\textit{if and only if} \\
s_{i,h_T}^{T,*} \in \arg\max_{c_i \in C_{i,h_T}} \overline{V}_{\mu_{i,T}}(\boldsymbol{h}_T, c_i).
\end{gathered}$$

The latter condition holds by assumption and the inductive step is hereby proven. ■

Proof of Proposition. Let (σ_T, μ_T) be consistent. Then for each $\boldsymbol{z}_T \in \boldsymbol{Z}_T$,

$$V_{\mu_{i,T}}(\boldsymbol{z}_T) = \mathbb{E}_{\sigma_T,\mu_T}[u_{i,T}|\boldsymbol{z}_T],$$

and for all h_T with $d(h_T) = 1$ we have

$$V_{\mu_{i,T}}(\boldsymbol{h}_T) = \max_{c_i \in C_{i,h_T}} \mathbb{E}_{\sigma_T,\mu_T}[u_{i,T}|\boldsymbol{h}_T, c_i]. \quad \text{(BI)}$$

Then a straightforward backwards induction argument shows (BI) holds for all $\boldsymbol{h}_T \in \boldsymbol{H}_T \backslash \boldsymbol{Z}_T$. Therefore the Lemma implies that the Proposition holds.

■

A.2 Proof of Theorem 1

First let $\beta^1(\sigma_T) = (\beta^1(\sigma_T))_{i\in I}$ denote the profile of first-order beliefs derived from σ_T according to condition (i) in Definition 6. The profile of infinite belief hierarchies $\mu_T = \beta(\sigma_T)$ is obtained by condition (ii) in Definition 6. By construction, the assessment $(\sigma_T, \beta(\sigma_T))$ is consistent. It follows that $\beta(\cdot)$ is a continuous function.

Suppose that each player i is subject to a slight imperfection of rationality (tremble) of the following kind. At every history h_T there is a small positive probability ϵ_{i,h_T} for the breakdown of rationality. Whenever rationality breaks down, every choice c_i will be selected with some positive probability $\sigma_{i,T}(c_i|\boldsymbol{h}_T) = \epsilon_{i,h_T}(c_i)$. Formally, fix a strictly positive vector $\epsilon = (\epsilon_{i,h_T}(c_i)_{c_i\in C_{i,h_T}})_{i\in I, h_T\in H_T\backslash Z_T}$ such that for all $h_T \in H_T\backslash Z_T$, $\sum_{c_i\in C_{i,h_T}} \epsilon_{i,h_T}(c_i) < 1$. Now define an (agent-form, psychological) ϵ-constrained equilibrium in a T-partial game:

Definition 8 (ϵ-constrained equilibrium)**.** An ϵ-constrained equilibrium in a T-partial game $G_T \in \boldsymbol{G}$ is a set of behavioral strategies profiles σ_T such that for all $i \in T$, $\boldsymbol{h}_T \in \boldsymbol{H}_T$, $c_i \in C_{i,h_T}$:

(i) $\sigma_{i,T}(c_i|\boldsymbol{h}_T) \geq \epsilon_{i,h_T}(c_i)$,

(ii) $c_i \notin \arg\max_{c_i\in C_{i,h_T}} \mathbb{E}_{\sigma_T,\beta(\sigma_T)}[u_{i,T}|\boldsymbol{h}_T, c_i] \Rightarrow \sigma_{i,T}(c_i|h_T) = \epsilon_{i,h_T}(c_i)$.

Let $\Sigma_\epsilon = \prod_{i\in I} \Sigma_{\epsilon,i}$ be the set of behavioral strategy profiles satisfying condition (i) in Definition 8, and let $\mathrm{BR}_\epsilon : \Sigma_\epsilon \to \Sigma_\epsilon$ be the ϵ-best response correspondence that assigns to each profile σ_T the subset of profiles in Σ_ϵ satisfying condition (ii) of the definition,

$$\mathrm{BR}_{\epsilon,i}(\sigma_T) = \{\sigma_{i,T} \in \Sigma_{\epsilon,i} : c_i \notin \arg\max_{\tilde{c}_i\in C_{i,h_T}} \mathbb{E}_{\sigma_T,\beta(\sigma_T)}[u_{i,T'}|\boldsymbol{h}_T, \tilde{c}_i]$$
$$\Rightarrow \sigma_{i,T}(c_i|\boldsymbol{h}_T) = \epsilon_{i,h_T}(c_i), \forall \boldsymbol{h}_T \in \boldsymbol{H}_T, \forall c_i \in C_{i,h_T}\},$$

$$\mathrm{BR}_\epsilon(\sigma_T) = \prod_{i\in I} \mathrm{BR}_{\epsilon,i}(\sigma_T).$$

$\mathrm{BR}_{\epsilon,i}(\sigma_T)$ is a nonempty convex subset of Euclidean space $\Delta(C_{i,h_T})$. Since $\mathbb{E}_{\sigma_T,\mu_T}[u_{i,T}|\boldsymbol{h}_T, c_i]$ is continuous in (σ_T, μ_T) and $\mu_T = \beta(\sigma_T)$ is a continuous function, $\mathbb{E}_{\sigma_T,\beta(\sigma_T)}[u_{i,T}|\boldsymbol{h}_T, c_i]$ is continuous in σ_T.

We now have enough structure to apply Kakutani's fixed point theorem to the best response correspondence. $\mathrm{BR}_\epsilon(\sigma_T)$ is upper hemicontinuous because $\mathbb{E}_{\sigma_T,\beta(\sigma_T)}[u_{i,T}|\boldsymbol{h}_T, c_i]$ is continuous for each (finite) $\boldsymbol{h}_T \in \boldsymbol{H}_T$ and $c_i \in C_{i,h_T}$, nonempty since each $\mathbb{E}_{\sigma_T,\beta(\sigma_T)}[u_{i,T}|\boldsymbol{h}_T, c_i]$ is continuous and Σ_ϵ is compact, and convex valued because each $\mathbb{E}_{\sigma_T,\beta(\sigma_T)}[u_{i,T}|\boldsymbol{h}_T, c_i]$ is quasi-concave on Σ_ϵ. Therefore $\mathrm{BR}_\epsilon(\sigma_T)$ has a fixed point, which is an ϵ-constrained equilibrium.

Fix a sequence $\epsilon^k \to 0$ and a corresponding sequence of ϵ^k-constraint equilibrium strategies σ_T^k. By compactness, the sequence (σ_T^k) has a limit point σ_T^*. A trembling-hand perfect equilibrium is any limit of ϵ-constraint equilibria as $\epsilon^k \to 0$. We will now prove that the trembling-hand perfect equilibrium $(\sigma_T^*, \beta(\sigma_T^*))$ is a sequential equilibrium.

Assessment $(\sigma_T^*, \beta(\sigma_T^*))$ is continuous: to see this note that, by continuity, $\beta(\sigma_T^*)$ is a limit point of $\beta(\sigma_T^k)$, and that the set of consistent assessment is closed. By continuity of $\mathbb{E}_{\sigma_T,\beta(\sigma_T)}[u_{i,T}|\boldsymbol{h}_T, c_i]$ in σ_T (and fitness of C_{i,h_T}), for k sufficiently large

$$\arg\max_{c_i \in C_{i,h_T}} \mathbb{E}_{\sigma_T^*,\beta(\sigma_T^*)}[u_{i,T}|\boldsymbol{h}_T, c_i] = \arg\max_{c_i \in C_{i,h_T}} \mathbb{E}_{\sigma_T^k,\beta(\sigma_T^k)}[u_{i,T}|\boldsymbol{h}_T, c_i].$$

By Definition 7 and Proposition 1 each $(\sigma_T^*, \beta(\sigma_T^*))$ is a sequential equilibrium assessment. ■

A.3 Proof of Corollary 1

Define the order l as the maximum length of a chain in the set $\boldsymbol{G}$ of T-partial games. We can now derive the Corollary by induction, staring with the observation that any maximal chain in $\boldsymbol{G}$ must have, as its minimal element under $\leq$, a game with common knowledge of the structure.

For the case $l = 1$, the T_1-partial game corresponds to a standard dynamic psychological game [Battigalli and Dufwenberg (2009)] and therefore the standard computation of sequential equilibria apply. For any $l > k \geq 0$, the T_l-partial game is such that players thinking that they are in T_k-partial (due to unawareness) play their sequential equilibria in the T_k-partial game.

The recursive nature of the chain in $\boldsymbol{G}$ ensures that we can solve for sequential equilibria by first considering the T_0-partial game (with common knowledge of the game), and then extend the equilibria step-by-step to the T_l-partial game by taking the equilibria of other players in T_k-partial games as given. ■

B Appendix

B.1 Proof of Result 3

Remember in Scenario 2 *Bob* is aware of everything. Hence, if *Ann* chooses *Defect*, *Bob* evaluates *Ann*'s kindness on the basis of $\{H_{T'}\}_{T_{15} \hookrightarrow T'}$ with $T_{15} = \{n^0, n^1, n^2, n^3, n^4, n^5, n^6\}$ in the history that he finds himself in. In Result 1 we have shown that full awareness would imply that *Bob* chooses *defect* out of monetary and reciprocity reasons. Although *Bob* is aware of everything and observes *Ann*'s choice *Defect*, he knows that *Ann* is unaware of her action *Cooperate* and his subsequent actions. *Bob*, hence, forms an equilibrium belief about what *Ann* would have done had she been of the same awareness level as he is. From Scenario 1 we know that the only sequential equilibrium given full awareness and $Y \geq 1$ involves *Ann* playing *Cooperate* and *Bob* playing *cooperate*. This means, *Bob* holds the equilibrium belief given his awareness level that $(Cooperate, (cooperate, defect))$ would have been the actions in the joint equilibrium strategy, if *Ann* had been of the same awareness level as he is. Given this, *Bob*'s evaluation of *Ann*'s kindness even following *Ann*'s choice *Defect* is:

$$\lambda_{BAB} = 1 - \frac{1}{2}[1 + 0] = 0.5.$$

$\lambda_{BAB} = 0.5$ is Bob's perception about *Ann*'s kindness after *Ann*'s action *Cooperate* in the equilibrium they would have played had both been aware of everything. As *Bob* does not hold her responsible for being unaware, this is also his perception concerning *Ann*'s kindness following her choice *Defect* and $T_{15} = \{n^0, n^2, n^5, n^6\}$. In other words, this is *Bob*'s equilibrium belief about *Ann*'s kindness given $T = \{n^0, n^2, n^5, n^6\}$ and following her choice of action *Defect*. On the other hand, the kindness that *Bob* can show to *Ann* is given by

$$\kappa_{BA} = 2 - \frac{1}{2}(2 + 0) = 1$$

by choosing *cooperate* and

$$\kappa_{21} = 0 - \frac{1}{2}(2 + 0) = -1$$

by choosing *defect*. Bringing things together, *Bob* chooses *cooperate* if the utility from choosing *cooperate*, i.e. $-1 + Y \cdot (0.5) \cdot (1)$, is higher than the utility from choosing *defect*, i.e. $0 + Y \cdot (0.5) \cdot (-1)$. This is the case when $Y \geq 1$. In other words, *Bob* chooses to accept -1 in order not to be unkind to *Ann* who he believes would have been kind to him if she had been aware of everything that he is aware of. ■

B.2 Proof of Result 5

To understand Result 5 it is important to see that whatever *Ann* beliefs about *Bob*'s strategy following her choice *Defect*, *Bob* is worse of than if she would have chosen *Cooperate* (see also Result 1 and the proof to observation 1 in Dufwenberg and Kirchsteiger (2004)). This means it is sure that *Bob* who becomes aware of everything when *Ann* chooses *Defect* considers *Defect* as an unkind choice. Given this, his belief-dependent reciprocity preferences plus his own monetary payoff makes him to choose his action *defect*. Furthermore, as *Bob* correctly believes that *Ann* is also aware of everything, messages do not play any strategic role for him and, hence, he chooses any message. ■

B.3 Proof of Result 6

Consider first part (i): By sending a message which does not contain any new information *Bob* does not become aware of any new feasible path of play. This implies that *Bob* will continue to evaluate *Ann*'s kindness on the basis of $\{H_{T'}\}_{T_4 \hookrightarrow T'}$ with $T_4 = \{n^0, n^1, n^3, n^4\}$. As $T_4 = \{n^0, n^1, n^3, n^4\}$ only entails one action for *Ann*, *Bob*'s belief about the intentions of *Ann* towards him as well as *Bob*'s belief about the maximum and minimum that *Ann* could have given to him coincide. Hence, $\lambda_{BAB} = 0$. Given this, *Bob*'s psychological utility from reciprocity is $Y \cdot \kappa_{BA} \cdot \lambda_{BAB} = 0$ and he consequently maximizes his own monetary payoff, i.e. *Bob* chooses action *defect*. Consider now part (ii) and (iii): if *Ann* chooses *Cooperate* and a message that contains at least $T_2 = \{n^0, n^2, n^6\}$ as new information, then *Bob* evaluates *Ann*'s kindness either on $\{H_{T'}\}_{T_{15} \hookrightarrow T'}$ with $T_{15} = \{n^0, n^1, n^2, n^3, n^4, n^5, n^6\}$ or $\{H_{T'}\}_{T_{13} \hookrightarrow T'}$ with $T_{13} = \{n^0, n^1, n^2, n^3, n^4, n^6\}$ depending on *Ann*'s message. To evaluate *Bob*'s perception concerning *Ann*'s kindness in this case we have to specify his belief concerning *Ann*'s belief regarding his choice following *Ann*'s action *Cooperate*. Denote *Bob*'s belief concerning *Ann*'s belief concerning the likelihood with which he plays *cooperate* following her action *Cooperate* by β. This implies that he believes that *Ann* believes that he plays *defect* following her choice of *Cooperate* with probability $(1-\beta)$. Furthermore, note that in this situation *Bob* believes that in equilibrium he would have chosen *defect* following *Ann*'s choice *Defect* giving him a payoff of 0. Given this, *Bob* perceives *Ann*'s choice *Cooperate* and the message which contains *at least* $T_2 = \{n^0, n^2, n^6\}$ as

$$\lambda_{BAB} = \beta + (1-\beta)2 - \frac{1}{2}[\beta + (1-\beta)2 + 0]$$

where $\frac{1}{2}[\beta + (1-\beta)2 + 0]$ is *Bob*'s perception given his awareness level concerning the average that *Ann* could have given him. λ_{BAB} reduces to $1 - \frac{1}{2}\beta$. In equilibrium beliefs have to be correct! Hence, *Bob*'s perception of *Ann*'s kindness in an equilibrium involving his action *cooperate* following *Ann*'s action *Cooperate* ($\beta = 1$) is $\frac{1}{2}$. On the other hand, in this situation *Bob*'s kindness towards *Ann* by choosing *cooperate* and any message is $\kappa_{BA} = 1 - \frac{1}{2}[1 + (-1)] = 1$ and his kindness from choosing *defect* and any message is $\kappa_{BA} = -1 - \frac{1}{2}[1 + (-1)] = -1$. This means he chooses *cooperate* in equilibrium if:

$$1 + Y(\frac{1}{2})(1) \geq 2 + Y(\frac{1}{2})(-1)$$

which holds if $Y \geq 1$. Consider now part (iv). We follow the same reasoning as before: if *Ann* chooses *Cooperate* and a message that contains only $T_1 = \{n^0, n^2, n^5\}$ as new information, then *Bob* evaluates *Ann*'s kindness on $\{H_{T'}\}_{T_{12} \hookrightarrow T'}$ with $T_{12} = \{n^0, n^1, n^2, n^3, n^4, n^5\}$. In this case *Bob* believes that he would have chosen *cooperate* following *Ann*'s choice *Defect* as this is the only of his actions following *Ann*'s choice *Defect* that he has become aware of by *Ann*'s message. Again, denote *Bob*'s belief concerning *Ann*'s belief concerning the likelihood with which he plays *cooperate* following *Ann*'s action *Cooperate* by β. This means that *Bob* perceives *Ann*'s choice *Cooperate* and the message which contains only $T_1 = \{n^0, n^2, n^5\}$ as new information as:

$$\lambda_{BAB} = \beta + (1 - \beta)2 - \frac{1}{2}[\beta + (1 - \beta)2 + (-1)]$$

which reduces to $1\frac{1}{2} - \frac{1}{2}\beta$. As before, in equilibrium beliefs have to be correct. Hence, *Bob*'s perception of *Ann*'s kindness in an equilibrium involving his action *cooperate* following *Ann*'s choice *Cooperate* ($\beta = 1$) is 1. As in the cases (ii) and (iii), in this situation *Bob*'s kindness towards *Ann* by choosing *cooperate* and any message is $\kappa_{BA} = 1 - \frac{1}{2}[1 + (-1)] = 1$ and his kindness from choosing *defect* and any message is $\kappa_{BA} = -1 - \frac{1}{2}[1 + (-1)] = -1$. This means he chooses *cooperate* in equilibrium if:

$$1 + Y(1)(1) \geq 2 + Y(1)(-1)$$

which holds if $Y \geq \frac{1}{2}$. ■

B.4 Proof of Result 7

Case (i): If *Bob*'s sensitivity to reciprocity is $Y < \frac{1}{2}$, *Ann* knows that *Bob* will defect no matter what she does and which messages she sends. Hence, she chooses *Defect* to get in equilibrium 0, rather than −1 which she would get by choosing *Cooperate*. Case (ii): If *Bob*'s sensitivity to reciprocity is $\frac{1}{2} \leq Y \leq 1$, *Ann* knows that *Bob* will *cooperate* when she chooses $Cooperate$ and a message which contains only $T_1 = \{n^0, n^2, n^5\}$ as new information. As this gives her 1 in monetary payoffs which is more than with any of her other actions and messages, she chooses to cooperate and send a message which contains only $T_1 = \{n^0, n^2, n^5\}$ as new information. Case (iii): In case (iii) we can apply the same reasoning as in case (ii). But, as a message that contains either $T_3 = \{n^0, n^2, n^5, n^6\}$ or $T_2 = \{n^0, n^2, n^6\}$ implies a lower kindness perception in *Bob*'s eyes about *Ann*'s action $Cooperate$, *Bob* chooses to *cooperate* in equilibrium only if $Y \geq 1$. Hence, *Ann* chooses this action and message only if $Y \geq 1$.

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