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Ricardo Andrés Guzmán, Jacob Louis Weisdorf

Øster Farimagsgade 5, Building 26, DK-1353 Copenhagen K., Denmark
Tel.: +45 35 32 30 01 – Fax: +45 35 32 30 00
http://www.econ.ku.dk

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Product variety and the demographic transition

Ricardo Andrés Guzmán* Jacob Louis Weisdorf†

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Abstract

Why does the rate of population growth decline in the face of economic growth? We show that growing product variety may induce a permanent reduction in the demand for children and a continuous rise in income and consumption.

Keywords: Demographic transition; demand for children; product variety; Malthus.
JEL classification: J13; N30; O10.

1 Introduction

Starting in the late 19th century, net reproduction rates in western Europe dropped from an average of three surviving children per woman to just below two children in the early 21st century (Maddison, 2001). This is known as the *demographic transition*. Yet, over the same period, income per capita has increased ninefold (*ibid.*). If we believe that children are normal goods, then the fall in the demand for children must be explained by negative price effects that overrode the positive income effect. While more expensive children are certainly part of the explanation (e.g., Bergstrom, 2007; Galor, 2005; Galor and Weil, 1999, 2000; de la Croix and Licandro, 2009), we show that a continuous increase in the consumption goods variety may also depress the demand for children and speed up the growth of income and consumption. Two conditions are needed for this: children and other consumption goods must be normal goods, and they must be substitutes for each other.

*Facultad de Gobierno and Facultad de Economía y Negocios, Universidad del Desarrollo. Address: Av. Plaza 700, San Carlos de Apoquindo, Las Condes, Santiago, Chile. E-mail: rnguzman@uc.cl.

†Department of Economic History, Lund University, and Department of Economics, University of Copenhagen. Address: Öster Farimagsgade 5, DK-1353, Copenhagen, Denmark. Phone: +45 35323033. Fax: +45 35323000. E-Mail: jacob.weisdorf@econ.ku.dk. (Corresponding author)

2 The model

2.1 Setup

Consider a small, open economy. Time is continuous, indexed by $t \geq 0$. The number of adults alive in time t is $N(t) > 0$. Adults live for one period: those alive in time t will be dead in time $t + \Delta$, where $\Delta \in \mathbb{R}_+$ is any positive real number. When the adults die, they are replaced by their children. All adults are identical.

A typical time-t adult maximizes a CES utility function:

$$u(t) = \left(\phi c(t)^{\frac{\sigma-1}{\sigma}} + (1-\phi)\, n(t)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}, \tag{1}$$

where $c(t)$ is his consumption of a composite good, and $n(t) > 0$ is the number of his children. Parameter $\phi \in (0,1)$ is the weight of children on the utility, and $\sigma > 1$ is the elasticity of substitution between consumption and children. Because $\sigma > 1$, consumption and children are gross substitutes.

The composite encompasses $G(t) \in \mathbb{R}_+$ different consumption goods in time t:

$$c(t) = \left(\int_0^{G(t)} x(g,t)^{\frac{\rho-1}{\rho}}\, \mathrm{d}\, g\right)^{\frac{\rho}{\rho-1}}, \tag{2}$$

where $x(g,t)$ is the adult's consumption of good $g \in [0, G(t)]$. We call $G(t)$ the *variety*. Parameter $\rho > 1$ represents the elasticity of substitution between the different types of goods. The fact that $\rho > 1$ implies that adults will want to diversify consumption.

Each time-t adult earns a nominal wage $w(t) > 0$. The typical adult faces the following budget constraint:

$$w(t) \geq p_c(t)c(t) + p_n(t)\, n(t)\,,$$

where $p_c(t)$ denotes the price of the composite in time t, and $p_n(t) > 0$ denotes the price of a child. The economy is small and open, so all prices are exogenous.

Standard calculations yield the following Marshallian demands for consumption and children:

$$c(t) = \frac{\phi^\sigma p_c(t)^{-\sigma} w(t)}{\phi^\sigma p_c(t)^{1-\sigma} + (1-\phi)^\sigma p_n(t)^{1-\sigma}}, \tag{3}$$

$$n(t) = \frac{(1-\phi)^\sigma p_n(t)^{-\sigma} w(t)}{\phi^\sigma p_c(t)^{1-\sigma} + (1-\phi)^\sigma p_n(t)^{1-\sigma}}. \tag{4}$$

Since all goods cost the same, they will be consumed on equal amounts:

$$x(0,t) = x(g,t), \text{ for all } g \in [0, G(t)]. \tag{5}$$

It follows that the total expenditure in the composite is

$$p_c(t)c(t) = \int_0^{G(t)} p_g x(g,t) \, \mathrm{d}\, g = p_g x(0,t) G(t), \tag{6}$$

where p_g is the price of each individual good type. Using equations (2) and (5), we obtain the following:

$$c(t) = x(0,t) G(t)^{\frac{\rho}{\rho-1}}. \tag{7}$$

And combining equations (6) and (7), we get the price of the composite:

$$p_c(t) = p_g G(t)^{-\frac{1}{\rho-1}}. \tag{8}$$

The economy produces goods of one type, while the remaining types of goods are imported from abroad. Labor is immobile, and the domestic labor supply is inelastic and equal to $N(t)$. The nominal wage is given by

$$w(t) = p_g A(t) N(t)^{-\alpha}, \tag{9}$$

where $A(t) > 0$ is the total factor productivity (TFP) in time t, and $\alpha \in (0,1)$. Because $\alpha \in (0,1)$, the wage falls as population rises.

Finally, the following equation governs population dynamics:

$$\frac{\mathrm{d} \ln N(t)}{\mathrm{d}\, t} = n(t) - \bar{n}, \tag{10}$$

where $\bar{n}$ is the replacement fertility rate. Equations (9) and (10) constitute the classical Malthusian assumptions.

2.2 Equilibrium

Assume that TFP, the price of children, and variety change at constant, non-negative rates:

$$\frac{\mathrm{d} \ln A(t)}{\mathrm{d}\, t} = \gamma_A, \tag{11}$$

$$\frac{\mathrm{d} \ln p_n(t)}{\mathrm{d}\, t} = \gamma_{p_n}, \tag{12}$$

$$\frac{\mathrm{d} \ln G(t)}{\mathrm{d}\, t} = \gamma_G, \tag{13}$$

where $\gamma_A, \gamma_{p_n}, \gamma_G \geq 0$.

Combining equations (4), (8), and (9), we obtain the demand for children:

$$n(t) = \frac{(1-\phi)^{\sigma}\, p_n(t)^{-\sigma} p_g A(t) N(t)^{-\alpha}}{\phi^{\sigma} p_g^{1-\sigma} G(t)^{-\frac{1-\sigma}{\rho-1}} + (1-\phi)^{\sigma}\, p_n(t)^{1-\sigma}}.$$

Log-differentiating the above equation with respect to t, taking limits and rearranging, we get:

$$\begin{aligned} \lim_{t\to\infty} \alpha\,[n(t) - \bar{n}] &= \gamma_A - \lim_{t\to\infty} \frac{\mathrm{d}\ln n(t)}{\mathrm{d}\,t} - \sigma\gamma_{p_n} \\ &\quad - \lim_{t\to\infty} \frac{\mathrm{d}\ln[\phi^{\sigma} p_g^{1-\sigma} G(t)^{-\frac{1-\sigma}{\rho-1}} + (1-\phi)^{\sigma}\, p_n(t)^{1-\sigma}]}{\mathrm{d}\,t}, \end{aligned} \tag{14}$$

where we have used equations (10)–(13).

In the long run, the demand for children is constant:

$$\lim_{t\to\infty} n(t) = n_{\mathrm{LR}}, \tag{15}$$

$$\lim_{t\to\infty} \frac{\mathrm{d}\ln n(t)}{\mathrm{d}\,t} = 0, \tag{16}$$

where n_{LR} denotes the long-run demand for children. Inserting (15) and (16) into equation (14), we get an expression for the long-run demand for children:

$$n_{\mathrm{LR}} = \bar{n} + \frac{1}{\alpha}\left(\gamma_A - \sigma\gamma_{p_n} - \lim_{t\to\infty} \frac{\mathrm{d}\ln[\phi^{\sigma} p_g^{1-\sigma} G(t)^{-\frac{1-\sigma}{\rho-1}} + (1-\phi)^{\sigma}\, p_n(t)^{1-\sigma}]}{\mathrm{d}\,t}\right).$$

It is straightforward to show that

$$\lim_{t\to\infty} \frac{\mathrm{d}\ln[\phi^{\sigma} p_g^{1-\sigma} G(t)^{-\frac{1-\sigma}{\rho-1}} + (1-\phi)^{\sigma}\, p_n(t)^{1-\sigma}]}{\mathrm{d}\,t} = \frac{\sigma-1}{\rho-1}\gamma_G. \tag{17}$$

Hence, the long-run demand for children can be expressed as

$$n_{\mathrm{LR}} = \bar{n} + \frac{1}{\alpha}\left(\gamma_A - \sigma\gamma_{p_n} - \frac{\sigma-1}{\rho-1}\gamma_G\right). \tag{18}$$

Once we know n_{LR}, the rates of change in the real wage (w/p_g) and in consumption are easily obtained:

$$\gamma_{w/p_g} = \sigma\gamma_{p_n} + \frac{\sigma-1}{\rho-1}\gamma_G, \tag{19}$$

$$\gamma_c = \sigma\left(\gamma_{p_n} + \frac{1}{\rho-1}\gamma_G\right). \tag{20}$$

Two main results emerge from equations (18) and (19). First, consistent with previous results, equation (18) shows that a rising cost of children ($\gamma_{p_n} > 0$) will dampen the positive effect of technological progress ($\gamma_A > 0$) on the demand for children. Since more expensive children moderate the growth of population, this helps to generate growth in real income per capita and consumption. Implicitly, this is caused by diminishing returns to labor in production. Second, if children and consumption goods are gross substitutes (i.e. if $\sigma, \rho > 1$), then, in response to more product variety, adults will reduce their demand for children. Since more product variety moderates the growth of population, this, too, helps to generate growth in real income per capita.

Note that the existing literature overlooks the effect of more product variety on the demand for children because of the widespread use of Cobb-Douglas preferences. In the Cobb-Douglas case, the elasticity of substitution between consumption goods and children equals one ($\sigma = 1$), which eliminates the product variety from equations (18) and (19).

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