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Determinacy, Stock Market Dynamics and Monetary Policy Inertia

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Abstract. This note deals with the stability properties of an economy where the central bank is concerned with stock market developments. We introduce a Taylor rule reacting to stock price growth rates along with inflation and output gap in a New-Keynesian setup. We explore the performance of this rule from the vantage of equilibrium uniqueness. We show that this reaction function is isomorphic to a rule with an interest rate smoothing term, whose magnitude increases in the degree of aggressiveness towards asset prices growth. As shown by Bullard and Mitra (2007, Determinacy, learnability, and monetary policy inertia, *Journal of Money, Credit and Banking* 39, 1177–1212) this feature of monetary policy inertia can help at alleviating problems of indeterminacy.

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1. Introduction

The increasingly frequent episodes of financial turmoil in the last two decades have drawn considerable attention on stock markets developments and on their interdependencies with the real economy. Both policy makers and researchers have debated around the opportunity to design policies capable to affect stock price dynamics in order to improve the macroeconomic performance of both industrialized economies and emerging markets. At the same time, since the seminal work by Taylor (1993) it has become common practice to think about monetary policy in terms of interest rate rules whereby the monetary authority controls the nominal rate of interest in response to inflation and output deviations from their equilibrium level. These parallel developments have stimulated a long-standing debate on the role and scope of central banks to implement interest rate rules where the policy instrument responds to asset prices deviations from their equilibrium level, along with reacting to changes in economic conditions. Bullard and Schaling (2002) show that responding to equity prices misalignments from their equilibrium level does not improve the economic performance, and might possibly harm real and financial stability. Including equity prices misalignments into a Taylor-type policy rule potentially introduces a root of indeterminacy of the rational expectations equilibrium. Our study builds on this framework.

We show that an explicit response to stock price growth rates translates into a policy rule featuring an interest rate smoothing term,[1] whose magnitude increases in the degree of aggressiveness towards asset prices growth. Thus the central bank will smooth out changes in the nominal interest rate in response to changes in economic conditions. Conversely, the structural response coefficients to output gap and inflation are weakened by an increase in the response to stock prices growth. As shown by Woodford (2003) and Bullard and Mitra (2007), monetary policy inertia can help at alleviating problems of indeterminacy and non-existence of stationary equilibrium observed for some commonly-studied monetary policy rules. Our results suggest that the reaction parameters in the inertial rule can be obtained through the re-parameterization of an original rule where the central bank responds to equity rates of return. In turn, this could indeed reflect an interest in stock market developments from the policy maker's perspective. Also Rudebusch (2006) suggests that policy gradualism could reflect some desire on the part of the central bank to reduce the volatility in interest rates and, more generally, in asset prices.

A substantial body of theoretical and empirical research has explored the potential role of monetary authorities in enhancing financial stability and preventing non-fundamental movements in the stock market. However, broad consensus has so far not emerged. Bernanke and Gertler (1999, 2001) design a framework where financial frictions give rise to a financial accelerator mechanism that magnifies the effects of both exogenous and policy shocks. In their framework a shock to asset prices increases aggregate demand, hence driving up the price level. They conclude that there is no need for a direct response to asset prices, as a central bank that responds to general price inflation is

[1] Inertia is a well-documented feature of central bank behavior in industrialized countries. Rudebusch (1995, 2006) provides insightful statistical analysis of this fact.

implicitly responding to asset price movements. They argue in favor of a monetary policy that does not respond to asset prices, except insofar as they signal changes in expected inflation. Conversely, Genberg et al. (2000) follow the modelling strategy of Bernanke and Gertler (1999, 2001), and argue that central banks should respond to asset prices to stabilize the economy and to prevent from the rise of bubbles.[2] Carlstrom and Fuerst (2007) emphasize the link between profitability and output gap in a sticky price environment. They show how a central bank trying to avoid bubbles can inadvertently introduce non-fundamental movements into both asset prices and real activity by reacting to asset prices misalignments. It is a well-established fact that in sticky price models marginal costs are proportional to the output gap. An interest rate rule that responds positively to (expected or current) values of stock prices is a rule that responds positively to dividends. This creates a potential problem from the perspective of equilibrium determinacy.

Di Giorgio and Nisticò (2007) study monetary policy design in a two-country model where agents can invest their wealth in stock and bond markets. They show that central banks reacting to stock price growth help at eliminating risks of endogenous instability. In this case, the simple commitment to the Taylor Principle is sufficient to restore equilibrium determinacy. Nisticò (2006) discusses a structural model with stock-wealth effects. He finds that adopting an instrument rule that responds to the stock-price gap incurs risks of endogenous instability that depend on the average price markup in the economy, while reacting to the stock-price growth can achieve substantial stability gains. These findings are in line with the analytical results presented in this paper. Pfajfar and Santoro (2008) show that when cost side effects are at work responding to asset price deviations from their frictionless level might be beneficial from the vantage of equilibrium determinacy. This result is intimately linked to the presence of nominal stickiness and the way this reflects into the relationship between firms' profitability and output gap.

The remainder of the paper is laid out as follows: Section 1 introduces the theoretical setting proposed by Bullard and Schaling (2002), while Section 2 explores the conditions for equilibrium uniqueness under a rule responding to stock prices growth; last section concludes.

2. Model

Bullard and Schaling (2002) develop their analysis on the framework put forward by Rotemberg and Woodford (1999). They consider an economy characterized by a continuum of infinitely-lived agents that derive utility from consumption and incur disutility from production. Each household produces a single differentiated good, but consumes a Dixit-Stiglitz bundle of the goods produced in the economy. Output is sold at a utility-maximizing price under the constraint that only a fraction of the goods prices may be changed in any given period and that other prices must be left at their value in the previous period. This introduces price stickiness. The solution of the

[2] Bernanke and Gertler (2001) comment on these results claiming that, although the models used are similar, Genberg et al. (2000) assume that the policymaker knows with certainty the stock price process and, most importantly, when the bubble is going to burst.

households' problem, suitably linearized and simplified as in Woodford (1999), produces equations (1) and (2) below which describe the dynamics of output and inflation in the economy. The first equation is given by:

$$x_t = E_t x_{t+1} - \sigma^{-1} \left(i_t - E_t \pi_{t+1} - i_t^n \right), \tag{1}$$

where x_t denotes output gap, π_t is the inflation rate, i_t is the the nominal (risk free) interest rate, i_t^n is a shock term that follows an AR(1) process.

Inflation is determined by:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t, \tag{2}$$

where κ relates to the degree of price stickiness and β denotes the traditional discount factor.

2.1. Equity Prices. In the Rotemberg and Woodford (1998) framework, as in many dynamic stochastic general equilibrium frameworks, arbitrage relationships can be used to price any asset that might be held by households, provided that financial markets are complete. This means that a financial claim on a random nominal quantity X_T has value $E_t \left[\delta_{t,T} X_T \right]$ at time t, where $\delta_{t,T}$ is the stochastic discount factor:

$$\delta_{t,T} = \frac{\beta U' \left(C_T \right)}{U' \left(C_t \right)},$$

where $U' \left(C_t \right)$ is the marginal utility derived from consumption at time t. The gross nominal interest rate on a nominal one-period bond is then given by:

$$R_t = E_t \left[\delta_{t,t+1} \right]^{-1},$$

as in Rotemberg and Woodford (1998). Since the stochastic discount factor prices all assets in this model, Bullard and Schaling (2002) denote the price of a share of aggregate equity by Q_t and note that $Q_t = 1/R_t$. As in Rotemberg and Woodford (1998), the short-term nominal interest rate is defined as $i_t = \ln R_t$. Therefore, as $\ln R_t = -\ln Q_t$, we conclude that:

$$i_t = -q_t, \tag{3}$$

where $q_t = \ln Q_t$.

2.2. Monetary Policy and Stock Price Dynamics. We close the model with an instrumental Taylor-type policy rule. We opt for a rule where the nominal rate of interest reacts to lagged values of inflation and output deviations from its equilibrium level. This specification is considered operational by McCallum (1999), as it does not call for the central bank to react to contemporaneous or expected future data on output gap and inflation. The interest on this rule over other alternatives derives from the consideration that it requires information that is plausibly in possess of the central bank. However, the nature of the Taylor-type rule is not crucial for the results reported in this paper.

We also assume that policy makers wish to include an explicit response to the stock price growth rate $\Delta q_t (= q_t - q_{t-1})$. Conversely, Bullard and Schaling (2002) assume a monetary authority responding to percentage deviations of the general level of equity prices from the long-run equilibrium level $(q_t - q^*)$.

The form of the policy rule we wish to study is therefore:

$$i_t = \gamma_\pi \pi_{t-1} + \gamma_x x_{t-1} + \gamma_q \Delta q_t,$$

with $\gamma_q > 0$. Given (3), this rule can be re-parameterized as:

$$i_t = \phi_i i_{t-1} + \phi_\pi \pi_{t-1} + \phi_x x_{t-1}, \tag{4}$$

where

$$\phi_i = \frac{\gamma_q}{1+\gamma_q} \quad \phi_\pi = \frac{\gamma_\pi}{1+\gamma_q} \quad \phi_x = \frac{\gamma_x}{1+\gamma_q}.$$

Thus, the resulting rule features a smoothing term. In particular, the policy instrument is set as a convex combination between lagged interest rate and a component reflecting the original response to lagged output gap and inflation. This rule is isomorphic to the one explored by Bullard and Mitra (2007). Notice that, in case the central bank responded to a term $(q_t - q^*)$, we would obtain an instrumental rule similar to the one explored by Bullard and Schaling (2002):[3]

$$i_t = \phi_\pi \pi_{t-1} + \phi_x x_{t-1}.$$

In this case, the overall response to inflation and output is weakened by the response to asset price gap $\left(\gamma_q\right)$. As the response to equity prices misalignments increases, it tends to drive the coefficients on inflation and output gap to zero. Bullard and Schaling (2002) report some results from Bullard and Mitra (2002) to discuss this implication and show that, as $\gamma_q \to \infty$, indeterminacy is inevitable.

When we implement the policy rule (4), the response to inflation and output is still weakened, but the monetary authority attaches higher importance to the smoothing term:

$$\frac{\partial \phi_i}{\partial \gamma_q} > 0; \quad \frac{\partial \phi_\pi}{\partial \gamma_q} < 0; \quad \frac{\partial \phi_x}{\partial \gamma_q} < 0.$$

This feature of rule (4) turns out to be crucial to the results reported in the remainder of the paper. Bullard and Mitra (2007) study the effect of policy inertia on the conditions for equilibrium

[3] It is worth pointing out that Bullard and Schaling (2002) employ a contemporaneous data rule.

uniqueness. They consider a policy rule similar to (4):

$$i_t = \psi_i i_{t-1} + \psi_\pi \pi_{t-1} + \psi_x x_{t-1}, \tag{5}$$

where $\psi_i,\ \psi_\pi,\ \psi_x$ are generic non-negative parameters. In order to transpose their analysis to our case, we can re-write the system under its state-space representation:

$$E_t y_{t+1} = B y_t + C i_t^n,$$

where $y_t = [x_t, \pi_t, i_t]'$ and

$$B = \begin{bmatrix} 1+\beta^{-1}\kappa\sigma & -\beta^{-1}\sigma & \sigma \\ -\beta^{-1}\kappa & \beta^{-1} & 0 \\ \psi_x & \psi_\pi & \psi_i \end{bmatrix},$$

where C is omitted since it is not needed in what follows. Since i_t is predetermined while x_t and π_t are free variables, according to Blanchard and Kahn (1980) equilibrium is determinate if and only if exactly one eigenvalue of B lies within the unit circle. Woodford (2003) provides necessary and sufficient conditions for determinacy of equilibrium in such a system. The details of these calculations are provided in Appendix A of Bullard and Mitra (2007). The following two conditions are shown to be jointly necessary for determinacy:

$$\kappa\left(\psi_\pi + \psi_i - 1\right) + (1-\beta)\,\psi_x \quad > \quad 0, \tag{6}$$

$$\left[\kappa\sigma + (2+\beta)\right]\psi_i + 2\,(1+\beta) \quad > \quad \sigma\left[\kappa\left(\psi_\pi - 1\right) + (1+\beta)\,\psi_x\right]. \tag{7}$$

Condition (6) is precisely what Woodford (2001, 2003) refers to as the Taylor principle, whereby in the event of a permanent one percent rise in inflation, the cumulative increase in the nominal interest rate is more than one percent. However, the Taylor principle is not generally sufficient for determinacy, because another necessary condition for determinacy is condition (7). This proves the following result:

Proposition 1. *Bullard and Mitra (2007).*

Assume that $\kappa\left(\psi_\pi + \psi_i - 1\right) + (1-\beta)\,\psi_x > 0$ for the inertial lagged data interest rule (5). Then a necessary condition for determinacy is:

$$\left[\kappa\sigma + (2+\beta)\right]\psi_i + 2\,(1+\beta) > \sigma\left[\kappa\left(\psi_\pi - 1\right) + (1+\beta)\,\psi_x\right].$$

Proof. *See Bullard and Mitra (2007), Appendix A.* ■

This proposition shows that the Taylor principle is no longer sufficient to guarantee determinacy, since it is also necessary that the degree of inertia ψ_i be large enough. If the central bank merely responds aggressively to inflation and output without displaying enough inertia, then the condition for determinacy may be violated. Bullard and Mitra (2007) also show that a set of necessary and sufficient conditions required for determinacy reduce to (6), (7) and:

$$\psi_i > 2 - (1 + \kappa\sigma)\,\beta^{-1}. \tag{8}$$

The right hand expression in (8) is less than 1 since $\kappa > 0$, $\sigma > 0$, and $0 < \beta < 1$. These conditions show that a large enough value of ψ_i always results in determinacy since this contributes to satisfy conditions (6), (7), and (8). A value of $\psi_i \geq 1$ always fulfills (6) and (8), so that if ψ_i also satisfies condition (7), the conditions for determinacy are met. Bullard and Mitra (2007) show that the analytical results given above provide intuition for a number of results obtained in more complicated models, such as those explored by Rotemberg and Woodford (1999) and McCallum and Nelson (1999). These studies generally confirm that large values of ψ_i tend to be associated with a unique equilibrium, provided that other conditions on the structural parameters are satisfied.

Let us now transpose this analysis to our context. In terms of our parameterization, the conditions above can be expressed as:

$$\begin{aligned} \kappa\,(\gamma_\pi - 1) + (1-\beta)\,\gamma_x &> 0, && (9)\\ [2\kappa\sigma + (2+\beta)]\,\gamma_q + 2\,(1+\beta)\left(1+\gamma_q\right) &> \sigma\,[\kappa\,(\gamma_\pi - 1) + (1+\beta)\,\gamma_x]\,. && (10) \end{aligned}$$

Again, the first condition corresponds to the Taylor principle. The introduction of an explicit response to asset rates of return only affects the second condition. Thus, we can reformulate the proposition above as follows:

Proposition 2. *Assume that $\kappa\,(\gamma_\pi - 1) + (1-\beta)\,\gamma_x > 0$ for the inertial lagged data interest rule (4). Then a necessary condition for determinacy is:*

$$[2\kappa\sigma + (2+\beta)]\,\gamma_q + 2\,(1+\beta)\left(1+\gamma_q\right) > \sigma\,[\kappa\,(\gamma_\pi - 1) + (1+\beta)\,\gamma_x]\,.$$

Proof. *See Bullard and Mitra (2007), Appendix A.* ■

It is clear that the left hand expression in (10) increases in γ_q. Therefore, provided that the Taylor principle holds, an increase in the degree of responsiveness to asset rates of return will relax the constraint. Moreover, in order to account for the full set of sufficient and necessary conditions for determinacy, according to Woodford (2003) and Bullard and Mitra (2007), the following constraint should be added to (9) and (10):

$$\gamma_q > \frac{2-\chi}{\chi - 1}, \tag{11}$$

where $\chi = (1+\kappa\sigma)\,\beta^{-1}$. Therefore, an increse in γ_q relaxes the constraint and alleviates the risk of indeterminacy.

Thus, we show that $\phi_i \left[= \gamma_q / \left(1 - \gamma_q\right)\right]$ can be obtained as a structural parameter from a Taylor rule where the monetary authority responds to stock price growth rates along with reacting to lagged inflation and output gap. In turn, this rule is isomorphic to an instrumental rule featuring policy inertia. These results suggest that the reaction parameters in the inertial rule could indeed reflect an interest in stock market developments from the policy maker's perspective. This is in line with the arguments explored by Rudebusch (2006), where it is suggested that an obvious rationale for policy gradualism would be some desire on the part of the central bank to reduce the volatility in interest rates and, more generally, in asset prices.

3. Concluding Remarks

In the last decade a number of contributions have explored the role and the scope of monetary authorities in acting to enhance financial stability along with ensuring price stability. The general wisdom is that including equity prices misalignments from their equilibrium level into a Taylor rule does not improve economic performance, and might possibly harm both real and financial stability, by introducing a root of indeterminacy of the rational expectations equilibrium.

In this note we show that an explicit response to stock price growth rates translates into a policy rule featuring an interest rate smoothing term. In this case the reponse coefficient to the lagged rate of interest increases in the degree of aggressiveness towards rates of return on equity. Conversely, the structural response coefficients to output gap and inflation are weakened by an increase in the response to stock prices growth. Therefore, as the central bank attaches higher importance to stock price dynamics, it will smooth out changes in the nominal rate of interest. This results suggest that the reaction parameters in inertial rules could indeed reflect an interest in stock market developments from the policy maker's perspective. As shown by Bullard and Mitra (2007), an increased degree of interest rate smoothing may help at alleviating problems of indeterminacy and non-existence of stationary equilibrium observed for some commonly-studied monetary policy rules.

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