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Does it Matter How to Measure Aggregates? The Case of Monetary Transmission Mechanisms in the Euro Area

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Abstract

Beyer, Doornik and Hendry (2000, 2001) show analytically that three out of four aggregation methods yield problematic results when exchange rate shifts induce relative-price changes between individual countries and found the least problematic method to be the variable weight method of growth rates. This papers shows, however, that the latter is sensitive to the choice of base year when based on real GDP weights whereas not on nominal GDP weights. A comparison of aggregates calculated with different methods shows that the differences are tiny in absolute value but highly persistent. To investigate the impact on the cointegration properties in empirical modelling, the monetary model in Coenen &Vega (2001) based on fixed weights was re-estimated using flexible real and nominal GDP weights. In general, the results remained reasonably robust to the choice of aggregation method.

JEL: C32, C42, E41

Keywords: Aggregation, Flexible weights, Eurowide money demand, Cointegration

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1 Introduction

A wide range of empirical macro models for the Euro zone including e.g. the area wide models by Fagan, Henry and Mestre (2005, henceforth FHM), Artis and Beyer (2004) or by Smets and Wouters (2003) are based on aggregated Euro zone data. However, for annual or quarterly econometric time series models the post monetary union sample after 1999 is still too short to allow for meaningful parameter estimates and hypothesis testing. Therefore, a key issue in empirical studies for the Euro zone is the creation of aggregated data for the period prior to the single currency. Because member countries previously had separate currencies susceptible to revaluations and devaluations, aggregate time-series data for the Euro area do not exist, and have to be created from the individual countries' records. In that context the European System of National and Regional Accounts (ESA) defines the accounting rules that allow for mutual and consistent comparison between economic time series accross EU countries, see ESA (1995) which is based on the "System of National Accounts (SNA), see SNA (1993) .

There are four main aggregation methods in common use: summing the levels data or the growth rates by using either fixed or variable weights, in any combination. Beyer, Doornik and Hendry (2000, 2001), henceforth "BDH" show analytically that three out of the four methods might yield problematic results in a situation when exchange rate shifts induce relative-price changes between individual countries in the Euro area. The least problematic method was found to be the variable weight method of growth rates considering the following two criteria. First, any method deserving serious consideration for aggregating across exchange-rate changes must work accurately when such exchange rate induced changes of relative prices do *not* occur. Second, if a variable measured in national currency increases (decreases) in every member state, then the aggregate should not move in the opposite direction. Although this may seem to be minimal requirements, BDH showed that levels aggregators need not perform appropriately in this respect when large currency changes occur: measured aggregates (of GDP say) can fall purely because of an exchange-rate change even though every country's GDP increases. Since fixed-weight methods deliver the same aggregates when applied to levels or changes, aggregating growth rates with flexible weights is left as the only method that survives both criteria.

Despite its superiority, the BDH method may, nevertheless, face a non-trivial decision problem when applied to real data. This is because the real

GDP weights needed for the aggregation of variables can be sensitive to the choice of base year for the implicit GDP price deflators. The paper shows that the time series of weights for the individual countries may deviate quite significantly in *absolute* value when different base years are chosen but that the mean corrected series are quite similar. However, this problem is shown to disappear altogether by choosing nominal rather than real GDP weights.

Because the mean-corrected relative BDH weights series seem fairly robust to the choice of base year, it is of some interest to investigate whether this choice is very important in practice. The second part of this paper tries to shed some light on this issue by checking whether the main empirical conclusions from a VAR analysis of the Euro area monetary transmission mechanisms reported in Coenen and Vega (2001), henceforth C&V, are robust to the choice of aggregation method. The robustness check is based on a variety of tests and estimates from a VAR analysis using the same core variables as in C&V aggregated by the FHM fixed and the BDH flexible real and nominal weights methods.

The organization of the paper is as follows: Section 2 first gives a brief review of the fixed FHM and flexible BDH aggregation methods, then provides an analytical expression for the base year effect on the flexible real GDP weights and demonstrates that nominal GDP weights are immune to the base year problem. Section 3 illustrates the various aggregation method effect with a stylized example. Section 4 compares the real GDP and the implicit GDP price deflator aggregated by the fixed FHM weights and flexible real and nominal BDH weights methods. Section 5 re-estimates the VAR analysis of the Euro area monetary transmission mechanisms reported in in C&V Vega using flexible weights aggregates and compares the results. Section 6 concludes.

2 Constructing aggregates: Fixed versus variable weights

The fixed weights (FHM) and flexible weights (BDH) methods are first briefly introduced. We then demonstrate why the BDH method based on real GDP weights is sensitive to the choice of base year for a price index and show that the BDH method based on nominal GDP weights is immune to this problem.

3

2.1 Log-level aggregation using fixed weights

The data for the ECB's area wide model in Fagan, Henry and Mestre (2005) has been aggregated by fixed weights log-level aggregation of national variables:

$$x_t = \sum_{i=1}^{n} x_{i,t} w_i, \tag{1}$$

where $x_{i,t} = \log(X_{i,t})$ in country i at time t and the weights are fixed over time, for example 30.5 % for Germany, 21.0 % for France and 20.3 % for Italy.

Compared to the "naïve" level method where variables in levels are first converted into a common currency and then aggregated, the above method has the advantage of avoiding distortions due to the influence of currency appreciations or devaluations. The disadvantage of any fixed weight approach remains: the choice of the fixed weights is to some extent arbitrary and fixed weights rule out that the constructed aggregates reflect the evolvement of countries' comparative competitiveness over time. See BDH for a discussion.

2.2 The BDH method based on flexible real GDP weights

Instead of aggregating log levels of a variable, x_t, BDH proposed to aggregate growth rates, Δx_t, with variable weights, so that the aggregated growth rate becomes a weighted average of the n individual country growth rates according to the formula:

$$\Delta x_t = \sum_{i=1}^{n} \Delta x_{i,t} w_{i,t-1}, \quad t = 1, ..., T \tag{2}$$

where the weights $w_{i,t-1}$ for country i at time t are constructed as

$$w_{i,t-1} = \frac{E_{i.c,t-1} y_{i,t-1}^r}{\sum_{i=1}^{n} E_{i.c,t-1} y_{i,t-1}^r} \tag{3}$$

and $y_{i,t}^r$ is the real income of country i at time t, and $E_{i.c,t}$ is the exchange rate of country i at time t vis-a-vis a common currency, c.

The level of the aggregate can be recovered from the formula:

$$x_t = \Delta x_t + x_{t-1} \text{ for } t = 1, \ldots, \quad \text{and } x_0 \text{ given.} \tag{4}$$

Table 1: Constructing weights: A three country example

	Germany	France	Italy
Nom. GDP	$Y_{1,t}^n$	$Y_{2,t}^n$	$Y_{3,t}^n$
Price Index =	$PI_{1,t} = P_{1,t}/P_{1,t_0}$	$PI_{2,t} = P_{2,t}/P_{2,t_0}$	$PI_{3,t} = P_{3,t}/P_{3,t_0}$
Nom. exchange	$E_{1.1,t}(Dmk/Dmk)$	$E_{2.1,t}(Fr/Dmk)$	$E_{3.1,t}(Lir/Dmk)$

When using (2)-(4) to construct aggregate nominal GDP, y_t^n, and real GDP, y_t^r, as well as the GDP price deflator, $p_{y,t}$, BDH showed that the constructed GDP deflator, $p_{y,t}$, coincides with the implied deflator, $y_t^n - y_t^r$. While this is an important advantage compared to the FHM method, the former is shown to be sensitive to the choice of a common base year for the price indices.

2.3 Real GDP weights: prices versus a price index

We shall focus the discussion on two cases: (1) the absolute prices of a basket of goods are known, (2) only a price index is known such as the CPI or the implicit GDP price deflator. As an illustration of why the choice of the base year matters for the real GDP weights in (3), we construct the aggregate real GDP for three countries, Germany, France and Italy assuming that Germany is the reference country. Table 1 provides the data.

Case 1: We know the absolute prices of a basket of goods, $P_{i,t}$, for each country $i = 1, 2, 3$. The weight for Germany, say, would be calculated as:

$$\omega_{1,t}^r = \frac{(Y_{1,t}^n/P_{1,t})}{(Y_{1,t}^n/P_{1,t}) + (Y_{2,t}^n/P_{2,t})E_{2.1,t}^{-1} + (Y_{3,t}^n/P_{3,t})E_{3.1,t}^{-1}} \quad (5)$$

or equivalently

$$\omega_{1,t}^r = \frac{Y_{1,t}^n}{Y_{1,t}^n + Y_{2,t}^n \left(\frac{P_{1,t}}{P_{2,t}}\right) E_{2.1,t}^{-1} + Y_{3,t}^n \left(\frac{P_{1,t}}{P_{3,t}}\right) E_{3.1,t}^{-1}} \quad (6)$$

It appears from (6) that using real income weighted by nominal exchange rates is the same as using nominal income weighted by real exchange rates. Because $\left(\frac{P_{1,t}}{P_{j,t}}\right) E_{j.1,t}^{-1} \simeq 1$, the country-specific nominal income is not transformed into a common currency in this case. Hence, the weights should be calculated from nominal income weighted by nominal exchange rates if

they are flexible or, alternatively, by relative prices translated into a common currency if nominal exchange rates are fixed.

Case 2: We know the prices relative to a base year. For example, we measure prices by a commodity price index, such as CPI, $PI_{i,t} = P_{i,t}/P_{i,t_0}$, where P_{i,t_0} is the price of country i in the base year or we measure prices by an implicit price deflator, such as the implicit GDP deflator, $PI_{i,t}$, with base year t_0. The weights become:

$$\tilde{\omega}_{1,t}^r = \frac{Y_{1,t}^n}{Y_{1,t}^n + Y_{2,t}^n \frac{P_{1,t}}{P_{2,t}} E_{2.1,t}^{-1} \left(\frac{P_{2,t_0}}{P_{1,t_0}}\right) + Y_{3,t}^n \frac{P_{1,t}}{P_{3,t}} E_{3.1,t}^{-1} \left(\frac{P_{3,t_0}}{P_{1,t_0}}\right)}. \tag{7}$$

From (7) it is easy to see that the choice of base year will influence the calculated weights. To illustrate the base year impact, it is useful first to express the nominal exchange rate at time t as $E_{j.i,t} = E_{j.i,t_0} + \delta_{ji,t}^E$ and then transform it into a growth factor:

$$E_{j.i,t} = (1 + \frac{\delta_{i.j,t}^E}{E_{j.i,t_0}}) E_{j.i,t_0} = g_{i.j,t}^E E_{j.i,t_0}. \tag{8}$$

Relative prices can be similarly formulated:

$$\left(\frac{P_{i,t}}{P_{j,t}}\right) \left(\frac{P_{j,t_0}}{P_{i,t_0}}\right) = \left(1 + \frac{\delta_{ij,t}^P}{100}\right) = g_{i.j,t}^P. \tag{9}$$

When an implicit deflator is used, real and nominal GDP are identical in the base year, so that $PI_{i,t_0} = 100$ and relative prices indices measure directly the relative price change from the base year for the two countries:

$$\frac{PI_{i,t}}{PI_{j,t}} = \left(1 + \frac{\delta_{i.j,t}^P}{100}\right) = g_{i.j,t}^P.$$

Using (8) and (9), (7) can be rewritten as:

$$\tilde{\omega}_{1,t}^r = \frac{Y_{1,t}^n}{Y_{1,t}^n + Y_{2,t}^n g_{2.1,t}^P g_{1.2,t}^E E_{1.2,t_0} + Y_{3,t}^n g_{3.1,t}^P g_{1.3,t}^E E_{1.3,t_0}}. \tag{10}$$

If nominal exchange rates correctly reflect relative price changes compared to the base period, then $g_{i.j,t}^P g_{j.i,t}^E \approx 1$ and the individual nominal incomes will be translated into the common currency using the nominal exchange rate of the base year. But this will not in general correctly reflect the relative
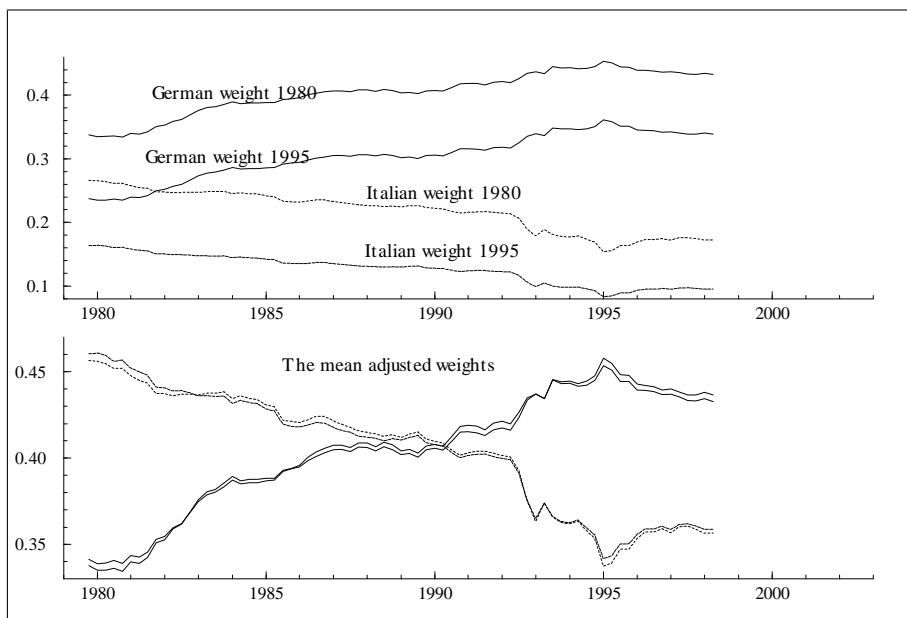
6

Figure 1: Weights for Italy and Germany, created on real GDP with base year 1980 and 1995 (upper graph) and adjusted for mean (lower graph).

strength of the two countries. For example, if at time t two countries i and j had experienced vastly different inflation rates compared to the base period and the nominal exchange rate had correctly reflected this development, then the nominal exchange rate at t_0 would no longer be representative for the economic strength of country i and j at time t.

In a fixed exchange rate regime, the weights would, instead, be based on relative price indices which implies that weights become:

$$\tilde{\omega}_{1,t}^n = \frac{Y_{1,t}^n}{Y_{1,t}^n + Y_{2,t}^n g_{1.2,t}^P E_{1.2} + Y_{3,t}^n g_{1.3,t}^P E_{1.3}}. \tag{11}$$

Because the relative price growth factor $g_{i.j,t}^P$ is not invariant to the choice of base year, the weights $\tilde{\omega}_{i.j,t}^n$ will differ depending on this choice. However, it seems quite likely that the time profile of the weights will be similar for different choice of base year, even though the absolute level of weights will differ.

This is illustrated by Figure 1. The graphs in the upper panel show that the absolute size of the real GDP weights for Germany and Italy, respectively,

7

differ a lot depending on the chosen base year, whereas the mean-adjusted graphs in the lower panel suggest that the dynamic evolvement of the weights for the two countries over time is nearly unaffected. Thus, even in a case where the nominal exchange rate have exhibited substantial variation, the weight profiles are quite similar.

Nevertheless, the calculated weight of an individual country at time t using the weights (10) or (11) may in some cases give a somewhat wrong impression of its *absolute* economic strength vis-a-vis other countries in the aggregate. For example, in 1980 comparing Germany's weight of 22% (when the base year is 1995) with Italy's weight of almost 30% (when the base year is 1980 instead), does not seem meaningful.

2.4   Proposing a solution

The ultimate task is to construct aggregation weights that adequately reflect the economic strength of the individual countries in the aggregate. In the ideal case, the weights should properly reflect a 'true' purcahsing power parity between two countries, for example measured by the absolute prices of a basket of commodities, say, in one country relative to the corresponding price in the other country, both expressed in a common currency.

Under flexible exchange rates and prices measured by a price index such as the implicit GDP deflator, there is, however, no information on absolute prices in the domestic currency. The only information we have is the nominal exchange rate that provides information on the absolute price of one unit of the currency of country i in terms of a common currency j at time t. Therefore, we propose to use nominal exchange rates as a measure of the relative price development between two countries. Expressed in terms of (3) it amounts to using nominal rather than real GDP in the calculation of the BDH weights:

$$\omega_{1,t}^n = \frac{Y_{1,t}^n}{Y_{1,t}^n + Y_{2,t}^n E_{1.2,t} + Y_{3,t}^n E_{1.3,t}}. \tag{12}$$

It is easy to see that the weights are now invariant to the choice of base year, as this is no longer an issue. In a world where purchasing power parity is satisfied in every period $E_{i.j,t} = P_{j,t}/P_{i,t}$ and the weights would indeed adequately reflect the economic strength of each country over time. However, it is well known that real exchange rates often deviate from purchasing power parity for extended periods of time (see e.g. Rogoff, 1994). Unfortunately,

8

(12) does not solve this problem, unless there is reliable information on the real depreciation/appreciation rate of country i relative to country j, $q_{i.j,t}$, at each period t. With such information, the appropriate weights would become:

$$\omega_{1,t}^n = \frac{Y_{1,t}^n}{Y_{1,t}^n + Y_{2,t}^n E_{1.2,t} q_{2.1,t} + Y_{3,t}^n E_{1.3,t} q_{3.1,t}}. \tag{13}$$

Under fixed exchange rates with known absolute prices, we propose to use the ratio of the absolute prices in one country relative to the corresponding price in the common currency, i.e. $P_{j,t}/P_{i,t}$. If prices are measured by a price index it becomes important to choose the base year such that it approximately corresponds to a period when purchasing power parity holds between the countries. Even though it is highly unlikely that such a period has existed jointly for all member states it should, nevertheless, be possible to calculate an optimal aggregate in the following way:

1. Find a base period when PPP holds for two countries, for example Germany and France, and construct a two country aggregate for GDP, $y_t^a(G+F)$, in a common currency (for example, DM) using the flexible weights method.

2. Find a new base period when PPP holds for Germany and another country, say Italy, and construct a new GDP aggregate, $y_t^a(G+F+I)$, by combining $y_t^a(G+F)$ and $y_t(I)$ using the flexible weights.

3. Continue until all relevant countries have been included in the aggregate, $y_t^a(EU)$.

3   A simple example

The two-country example in Table 2 illustrates how the real GDP weights in (2) are affected by the choice of different base years assuming either fixed or flexible exchange rates. These results are compared with the case when the weights are based on nominal rather than real GDP. To see the effect of exchange rate misalignments for the nominal GDP weights, we have calculated the weights for the case when purchasing power parity holds and when it does not. For simplicity, only three periods, $t = 0, 1, 2$, are used as this is sufficient to calculate GDP growth rates for two periods, $t = 1, 2$. In this stylized two-country, three-period, economy:

Table 2: Stylized example of BDH aggregation of two countries over three periods

	Base period $t=0$			Base period $t=2$		
	$t=0$	$t=1$	$t=2$	$t=0$	$t=1$	$t=2$
$Y_{1,t}^n$	1.0	1.0	2.0	1.0	1.0	2.0
$PI_{1,t}$	100	200	200	50	100	100
$y_{1,t}^r$	1.0	0.5	1.0	2.0	1.0	2.0
$\Delta y_{1,t}^r\ in\%$		-50%	100%		-50%	100%
$Y_{2,t}^n$	1.0	1.0	1.0	1.0	1.0	1.0
$PI_{2,t}$	100	100	100	100	100	100
$Y_{2,t}^r$	1.0	1.0	1.0	1.0	1.0	1.0
$\Delta y_{2,t}^r\ in\%$		0%	0%		0%	0%
$PI_{1,t}/PI_{2,t}$	1.0	2.0	2.0	1/2	1.0	1.0
$E_{1.2,t}(fixed\ exch.)$	1.0	1.0	1.0	1.0	1.0	1.0
$E_{1.2,t}(flex.exch.PPP)$	1.0	2.0	2.0	1.0	2.0	2.0
$E_{1.2,t}(flex.exch.NoPPP)$	1.0	1.5	1.5	1.0	1.5	1.5
Real GDP weights						
$w_{1,t}^{\mathrm{real}.y}(fixed\ exch.)$	**0.50**	**0.33**	**0.50**	0.67	0.50	0.67
$w_{1,t}^{\mathrm{real}.y}(flex.exch.PPP)$	0.50	0.50	0.67	0.67	0.33	0.80
Nominal GDP weights						
$w_{1,t}^{nom.y}(PPP\ holds)$	**0.50**	**0.33**	**0.50**	**0.50**	**0.33**	**0.50**
$w_{1,t}^{nom.y}(NoPPP)$	0.50	0.50	0.40	0.50	0.50	0.59
y_t^r aggregated with						
$w_{1,t}^{\mathrm{real}.y}(fixed\ exch.)$	**1.00**	**0.75**	**1.00**	**1.50**	**1.00**	**1.50**
$w_{1,t}^{\mathrm{real}.y}(flex.exch.PPP)$	1.00	0.75	1.12	1.50	1.00	1.33
$w_{1,t}^{nom.y}(PPP\ holds)$	**1.00**	**0.75**	**1.00**	**1.50**	**1.13**	**1.50**
$w_{1,t}^{nom.y}(NoPPP)$	1.00	0.75	1.12	1.50	1.13	1.70
Δy_t^r in %						
$w_{1,t}^{\mathrm{real}.y}(fixed\ exch.)$	-	**-25%**	**33%**	**-**	**-33%**	**50%**
$w_{1,t}^{\mathrm{real}.y}(flex.exch.PPP)$	-	-25%	50%	-	-33%	33%
$w_{1,t}^{nom.y}(PPP\ holds)$	-	**-25%**	**33%**	**-**	**-25%**	**33%**
$w_{1,t}^{nom.y}(NoPPP)$	-	-25%	50%	-	-25%	50%

1. income and prices for the two countries do not change between period zero and one,

2. nominal income in country 1 doubles from period 1 to 2, but is otherwise constant,

3. the price in country 1 doubles from period 0 to 1, and remains constant thereafter,

4. real income is identical in both countries for all periods, except for period 1 in for country 1 where real income is halved,

5. nominal exchange rate is either fixed or flexible. In the latter case it either reflects PPP or it deviates from PPP.

The first half of Table 2 reports the economic facts about the two countries. The big price increase in country one serves the purpose of illustrating the effect of choosing different base years.[1] Nominal incomes are given in absolute values, while the BDH aggregation formula (2) is based on logarithmic values. The real income growth rate is, therefore, calculated as a % change in the table. At $t = 0$, prices, nominal income and thus, real income are identical for the two countries. At $t = 1$, prices have doubled in country 1, nominal income is unchanged and thus, real income is halved compared to country 2. At $t = 2$, country 1 is back to the same real income level as $t = 2$. We note that when base year is $t = 2$, real income in country 1 is twice the size compared to base year $t = 0$. Based on these facts, a method that adequately accounts for these changes should produce a measure of real aggregate income which is identical at $t = 0, 2$ and has declined with 25% at $t = 1$. The entries in the table satisfying this criterion are in bold face.

The previous section demonstrated that the base year problem is not only associated with prices but also with exchange rates and whether these are adequately reflecting purchasing power parity or not. The exchange rates have been constructed such that at base period $t = 0$, purchasing power parity holds both when exchange rates are fixed and flexible.

According to (4) real aggregate GDP is constructed by cumulating real growth rates from an initial value. The latter has been calculated as a weighted average of the two real incomes with equal weights. Based on

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[1]This stylized example is in no way a realistic. In a real world economy, the exchange rate would, of course, not remain constant after such a dramatic price change.

this,Table 2 shows that the constructed real aggregate incomes based on the BDH fixed GDP weights and fixed exchange rates are identical at $t = 0, 2$ and drop by 25% at $t = 1$. However, the absolute level of real income is higher for base year $t = 2$. When comparing the growth rates we note that they evolve proportionally for the two base years, but the growth rates are at a higher level for base year $t = 2$. With flexible exchange rates, the real GDP weights perform less well, as predicted by (10). For base period $t = 0$, it overestimates real aggregate income at $t = 2$, and for base period $t = 2$, it underestimates its value. Furthermore, the growth rates do not evolve proportionally. This can now explain the development of the weights in Figure 1, where the relatively higher Italian weights for base year 1981 is likely to be the result of the high inflation rate in Italy relative to Germany in that period.

The BDH method based on nominal GDP weights performs well both in terms of the real aggregate income levels and their % growth rates when PPP is satisfied. When PPP is not satisfied because of a real appreciation in period 1 and 2, the aggregate becomes over-valued. Thus, if nominal GDP weights are used, one should account for any deviation of the real exchange rate from its equilibrium value according to (13).

4   Conversion with fixed and flexible weights: a comparison

The previous section demonstrated that the choice of base year matters for the BDH method when real GDP aggregation weights are used. The purpose of this section is therefore to investigate graphically how the various aggregation methods affect aggregated E11 prices, real income and interest rates in levels and changes. We have constructed aggregates based on the flexible BDH real GDP weights method with base year 1981 and 1995 (hereafter Flex81 and Flex95), the flexible BDH nominal GDP weights method (FlexNom) and the fixed FHM real GDP weights of 1995 (Fix95). Because the year 1995 was used to calculate the Euro-area aggregates in FHM used by C&V, it has been considered a reference year for the comparisons. The year 1981 represents a period when the member states were far from a common European PPP level, and has been chosen to illustrate the base year effect of the BDH method for a no-PPP period as compared to 1995 which was much
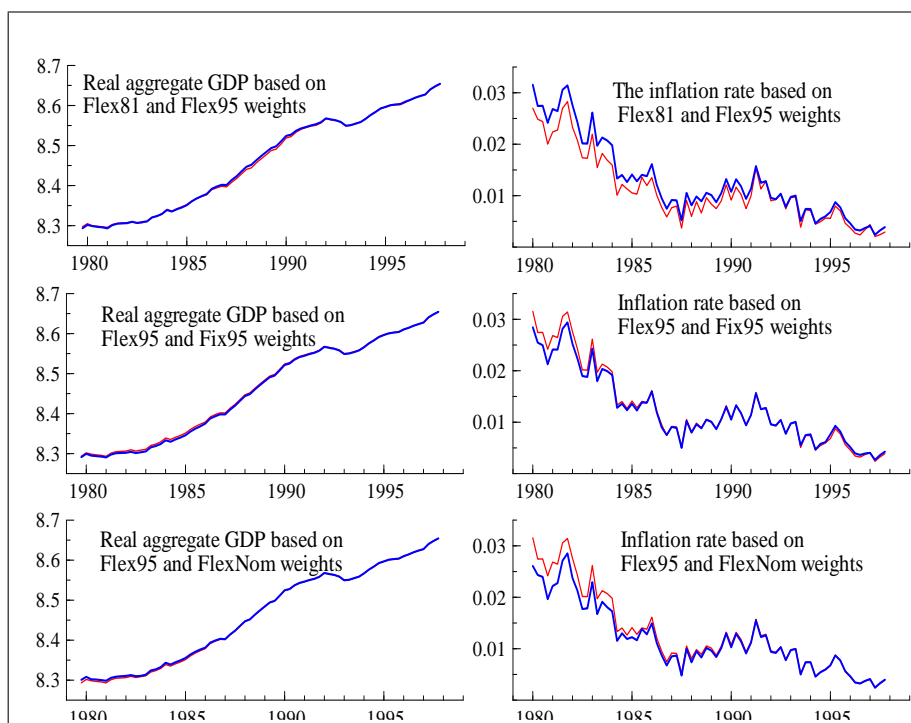
Figure 2: Comparing aggregate real GDP (left hand side) and inflation rate (right hand side) based on different aggregation weights.

closer to a PPP period.

Figure 2 shows real GDP based on Flex81 and Flex95 (upper l.h.s. panel), Flex95 and Fix95 (middle l.h.s. panel), and Flex95 and FlexNom (lower l.h.s. panel). Similar graphs are shown for inflation rates. Consistent with the results in the previous section, the absolute deviations between aggregate real GDP based on the different methods are generally small and hardly discernible in the graphs, whereas those of the inflation rate seem more significant. Generally, the first five, six years seem more affected than the more recent part of the sample period. Also, Flex95 seems to underestimate inflation rate compared to Flex81 and FlexNom.

Even though the aggregation methods produced very similar aggregates in absolute values, the deviations can be highly persistent as shown below and, therefore, may very well influence the cointegration properties of empirical models. To illustrate the persistency aspect, Figures 3 - 6 show the deviations
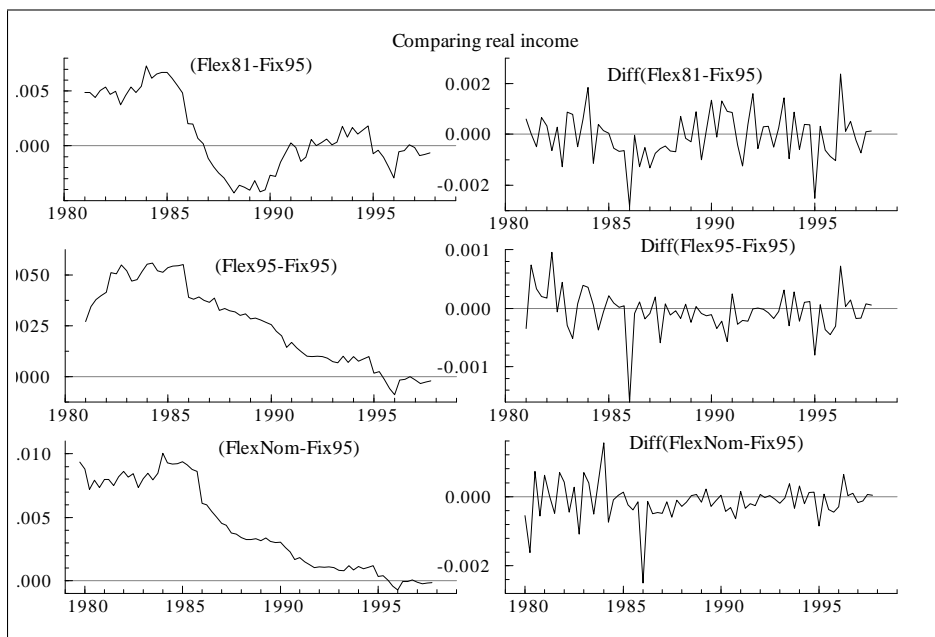
13

Figure 3: The differential between aggregate real income based on 1981, 1985, and 1995 flexible weights relative to the fixed 1995 weights (left hand sife) and its difference (right hand side).

of aggregate real GDP, prices, and the short-term interest rate[2] based on Flex81, Flex95, and FlexNom compared to Fix95. Obviously, all these tiny differentials are highly persistent. The real GDP differentials in the left hand side of Figure 4 are likely to be approximately I(1) as their differences in the right hand side of the figure look reasonably mean-reverting. The price differentials, on the other hand, in the left hand side of Figure 5 exhibit pronounced persistent behavior typical of I(2) variables. This is consistent with the I(1) behavior of the inflation rate differentials in the right hand side of the figure. We note that the fixed weights method overestimates price inflation compared to Flex81 and FlexNom. The short-term interest rate differentials shown in the left hand side of Figure 6 seem to exhibit I(1)

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[2]C&V used aggregates of income, inflation and a short and long interest rate based on fixed GDP weights to estimate a structure of three cointegration relations. To economize on space, the comparison of long-term interest rate differentials is not reported as it is very similar to the short rates.
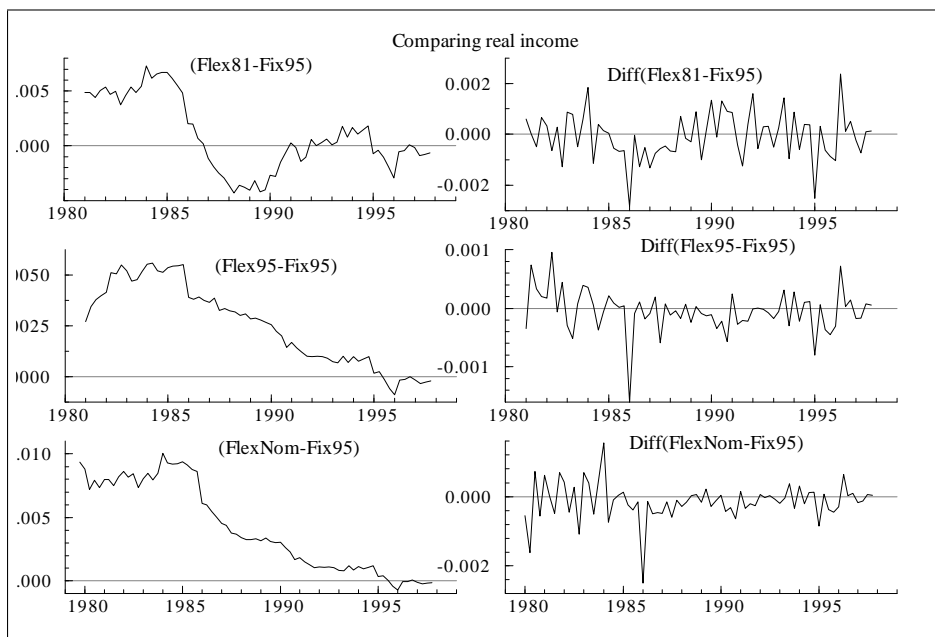
Figure 4: The differential between aggregate real income based on 1981, 1985, and 1995 flexible weights relative to the fixed 1995 weights (left hand sife) and its difference (right hand side).

behavior consistent with the mean reverting behavior of their differences.

Whatever the case, the aggregation differentials are definitely not stationary and the choice of aggregation method might, therefore, has a significant effect on the cointegration properties in empirical models. The following example illustrates such effects. Under the assumption that real GDP is unit root nonstationary, i.e. $y_t^r = y_t^n - p_t \sim I(1)$, we would expect $\{y_t^n, p_t\} \sim I(2)$ and, thus, $\{y_t^n, p_t\}$ to be cointegrated $CI(2, 1)$ with cointegration vector $[1, -1]$. As discussed in Juselius (2006, Chapters 2, 16, and 18) this would be the case when the nominal variables satisfy long-run price homogeneity, a desirable property from an economic point of view. The empirical verification of price homogeneity is, however, likely to be sensitive to measurement errors unless these errors are stationary, or at most $I(1)$. Because the price differentials in Figure 5 look $(p^{Flex} - p^{Fix})$ approximately $I(2)$, the choice of aggregation method might have an impact on the long-run price homogeneity tests.
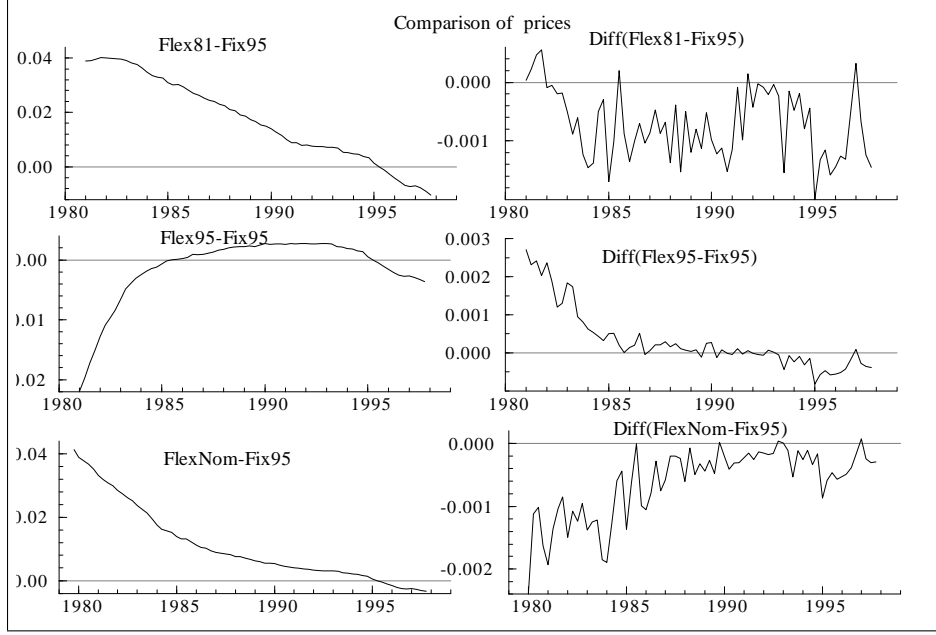
15

Figure 5: The differential between the implicit price deflator of aggregate GDP based on 1981, 1985, and 1995 flexible weights and the fixed 1995 weights (left hand sife) and its difference (right hand side).

As an illustration, assume that the flexible weights aggregation method is correct and that long-run price homogeneity is satisfied, so that $(y_t^{n,Flex} - p^{Flex}) \sim I(1)$. If, instead, the fixed weights method is used to construct $y_t^{n,Fix}$ and p^{Fix}, and both $(y_t^{n,Flex} - y_t^{n,Fix})$ and $(p^{Flex} - p^{Fix})$ are $I(2)$ then:

$$y_t^{r,Fix} = \underbrace{y_t^{n,Flex} - p_t^{Flex}}_{I(1)} - \underbrace{(y_t^{n,Flex} - y_t^{n,Fix})}_{I(2)} + \underbrace{(p^{Flex} - p^{Fix})}_{I(2)}, \qquad (14)$$

and $y_t^{r,Fix}$ would generally be I(2) unless the two $I(2)$ differentials in (14) are cointegrating $CI(2,1)$ or $CI(2,2)$.

In the comparison above, most differentials, while persistent, were tiny in absolute value. The question is whether the test for long-run price homogeneity has sufficient power to reject the null hypothesis of long-run price homogeneity when real income variable contains such a small I(2) aggregation error. In a similar set-up, Jørgensen (1998) showed by simulation experiments that such I(2) errors may not be easily detectable if they are small relative
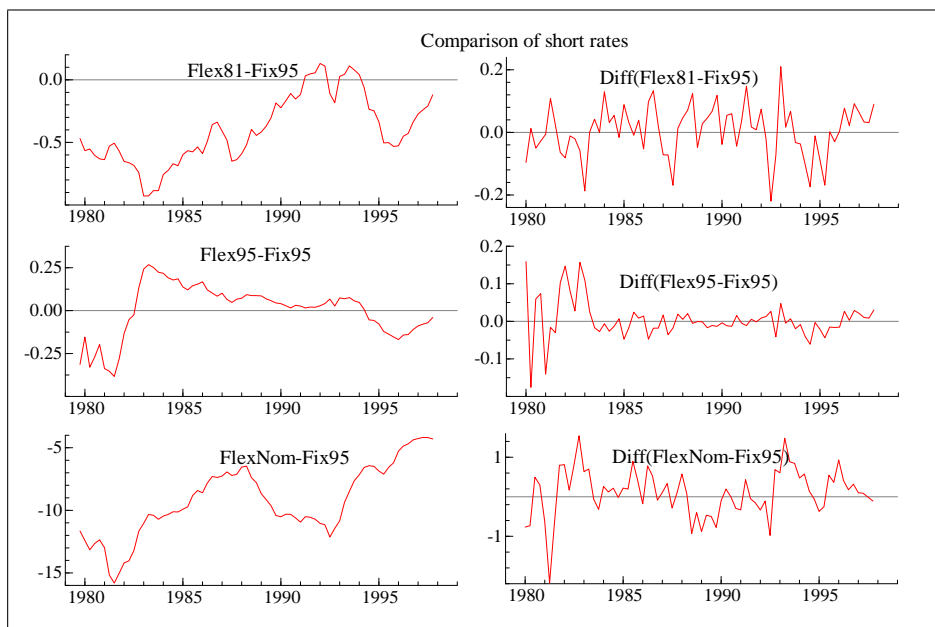
Figure 6: The differential between short-term E11 interest rate based on flexible and fixed weights (left hand side) and its difference (right hand side).

to the I(1) component. Kongsted (2005) demonstrated that even though the small I(2) component may not be found significant by testing, the trace test for cointegration rank and other inference in the cointegrated VAR model can, nevertheless, be affected in often inexplicable ways.

Thus, it is of some interest to investigate whether these tiny, but highly persistent, aggregation differentials have a significant influence on the cointegration properties of aggregate Euro area models, i.e. whether the choice of aggregation method is likely to have implications for the estimated long-run relations. In the next section we shall, therefore, take a look at the cointegration properties of the euro area model in C&V using aggregates based on the four different aggregation methods discussed above.

5 The C&V model with flexible and fixed weights

The small monetary model in C&V, one of the first empirical studies on monetary transmission mechanisms in the Euro zone, is based on aggregated fixed 1995 weights data. The paper discusses a cointegrated VAR analysis of the vector:

$$x'_t = [m3_t, y_t, \Delta p_t, R^s_t, R^l_t], \qquad t = 1981.1, ..., 1997.4,$$

where $m3$ is the log of real aggregate M3, y, is the log of real aggregate GDP, Δp is the difference of log GDP price, R^s is the short term interest rate, R^l is the long-term government bond rate. The data are aggregated over the E11 member states. Here we shall compare the results of the C&V model based on fixed 1995 weights with those obtained with flexible weights. Among the latter, we estimate the model for real GDP weights data with base year 1981 and 1995 and for nominal GDP weights. The sample covers most of the transition period from the beginning of the EMS to start of the EMU but, as in C&V, the first two years have been left out as they seem to generate instability in the VAR model. The reason why we use pre-EMU data is to exclude any influence of "proper" post EMU data on the results.

Our study follows the C&V model specification, two lags, an unrestricted constant term, and no trend in the cointegration relations. The last assumption was first checked, as one should in principle allow for a trend both in the stationary (β) and the nonstationary ($\beta_\perp$) directions when data are trending (Nielsen and Rahbek, 2000). The trend was found to be significant in the long-run relations, but the main conclusions of C&V seemed reasonably robust to this change in the model. Thus, even though the VAR model with a trend was preferable on statistical grounds, we decided to continue with the C&V model specification to preserve the comparative aspects of the study.

In the empirical analysis we examine some of the more important aspects of the VAR model: (1) the determination of cointegration rank based on the trace test, the roots in the characteristic polynomial, the largest t-ratio of α_r in the rth cointegration relation, (2) some general properties of the model describing the pulling and pushing forces by testing a zero row in α and a unit vector in α, (3) the long-run β structure described by the estimates and the p-value of the C&V long-run structure, as well as the combined long-run relation of the money and inflation equation, respectively, and (4) the

r		$r = 2$			$r = 3$	
	λ_r	$\rho_r^{\max}$	$t_{\alpha_r}^{\max}$	λ_r	$\rho_r^{\max}$	$t_{\alpha_r}^{\max}$
Fix95	0.29	0.76	3.04	0.24	0.94	3.07
Flex95	0.30	0.87	4.09	0.22	0.94	3.55
Flex81	0.29	0.66	3.54	0.16	0.76	2.23
FlexNom	0.28	0.67	3.50	0.19	0.90	3.33

Table 3: Some indicator statistics for rank determination

long-run impact of shocks on inflation and real income.

5.1 The determination of cointegration rank

The choice of rank is a crucial step in the analysis as all subsequent results are conditional on this choice. Before using the I(1) trace tests we checked whether the model showed evidence of I(2). No such evidence was detected and, following the recommendations in Juselius (2006), we report the rth eigenvalue, λ_r, the largest unrestricted characteristic root, $\rho_r^{\max}$, for a given r, and the largest t value of α_r. Based on the results reported in Table 3 for $r = 3$ (the C&V choice) and, the closest alternative, $r = 2$ it appears that the former choice leaves a fairly large unrestricted root in the model for all methods except Flex81. A large unrestricted root means that at least one of the cointegration relations is likely to exhibit a fair degree of persistence. A graphical analysis shows that it is the short-long interest rate spread, the third relation in the C&V structure, that looks rather non-stationary. Whether one should classify it as stationary can, therefore, be questioned.

Table 3 shows that all five aggregation methods give reasonably similar conclusions, possibly with the exception of Flex81, which was sticking out to some extent. Altogether, $r = 2$ seems preferable based on statistical grounds, but $r = 3$ could also be defendable. Again, to preserve comparability with the C&V results we continue with $r = 3$.

5.2 Pulling and pushing forces

The test of a zero row in α, i.e. no levels feed-back, and a unit vector in α, i.e. pure adjustment, (see Juselius, 2006) are useful as a check of whether the general dynamic properties of the model have changed as a result of the aggregation method. The results reported in Table 4 show that, for

Table 4: Pulling and pushing forces in the model

	m^r	y^r	Δp	R_s	R_l
Zero row in α $(r=3)$					
Fix95	20.50 [0.00]	8.27 [0.04]	6.89 [0.08]	**4.71** [0.19]	**4.77** [0.19]
Flex95	25.29 [0.00]	7.60 [0.06]	7.35 [0.06]	**4.29** [0.23]	**4.82** [0.19]
Flex81	7.26 [0.06]	13.68 [0.00]	12.47 [0.01]	**0.74** [0.86]	7.58 [0.06]
FlexNom	10.02 [0.02]	13.22 [0.00]	7.80 [0.05]	**1.53** [0.68]	**5.89** [0.12]
Unit vector in α $(r=3)$					
Fix95	**3.75** [0.15]	**2.87** [0.24]	**3.54** [0.17]	8.26 [0.02]	6.42 [0.04]
Flex95	**4.08** [0.13]	**1.22** [0.54]	**3.26** [0.20]	8.35 [0.02]	4.88 [0.09]
Flex81	**3.39** [0.18]	4.81 [0.09]	**1.16** [0.56]	4.41 [0.11]	**1.59** [0.45]
FlexNom	**1.48** [0.48]	**2.59** [0.27]	**2.58** [0.28]	6.37 [0.04]	**3.49** [0.17]

Entries with a p-value > 0.10 in bold face

the short rate, the zero row in α was acceptable with fairly high p-values for all aggregation methods, implying absence of levels feed-back from the other variables on the short rate. In addition, the zero row hypothesis was also accepted for the long-term bond rate except in the case of Flex81. This seems to indicate that it is the cumulated empirical shocks to the two interest rates that have broadly been pushing this system. Nonetheless, the joint hypothesis of a zero row for both interest rates was (borderline) rejected[3] and it was not possible to decompose the variables of the system into three pulling and two pushing variables. Consistent with the above results, the hypothesis of a unit vector in α seemed generally acceptable for money stock and inflation rate, and for real income. Altogether the Fix95, Flex95, and FlexNom methods seem to generate quite robust conclusions, whereas Flex 81 differed to some extent.

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[3]If accepted, it would have been inconsistent with the C&V assumption that the interest rate spread is a cointegration relation.

5.3 The long-run β structure

The three long-run relations identified in C&V consisted of:

1. A money demand relation: $m^r - \beta_{11}y^r + \beta_{12}R_s$

2. The real long-term interest rate: $R_l - \Delta p$

3. The short-long interest rate spread: $R_s - R_l$.

Since only the first relation, the money demand relation, contains free parameters to be estimated, Table 5 reports the estimates of the latter and, in the last column, the p-value for the fully identified structure. The restrictions of the C&V structure were accepted for all methods, but the p-values were much higher for Fix95 and FlexNom than for the Flex95 and Flex81 methods. The estimated coefficient to the short rate was positive for all methods, though insignificant for Flex95. A similar result was found in Bosker (2006).

We note that the interest coefficient implies a negative short-term interest rate effect on money holdings. A priori this seems surprising as R_s is likely to be strongly correlated with the interest rate on the interest yielding part of money stock. To check this finding, we also report the combined effects of all three relations given by the first row of the $\Pi = \alpha\beta'$ matrix for $r = 3$[4]. Because the C&V model is a study of monetary transmission mechanism, we also report the combined effect for the inflation equation. To improve comparability, we have normalized on money (inflation rate) and report the overall adjustment coefficient measured by the diagonal element π_m, m $(\pi_{\Delta p}, \Delta p)$ in the last column of the table.

The combined effects in the money stock equation now suggests that money demand is in fact positively related to the short rate and negatively to the long-term interest rate. The estimated coefficients to the interest rates and the real income seem quite robust, whereas those to the inflation rate less so, in particular for Flex81. The coefficients to the interest rates are similar with opposite sign, suggesting that the interest rate spread, rather than the short rate, is an appropriate measure of the alternative cost of holding money. In addition, money stock is negatively related to the inflation rate, but not very significantly so, except for Flex81.

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[4]No restrictions are imposed on α and β in this case.

Table 5: Comparison of the estimated long-run structure

	m^r	y^r	Δp	R_s	R_l	p-value
Money demand relation						
Fix95	1.0	−1.32 [−47.25]	–	2.89 [5.82]	–	0.30 [.99]
Flex95	1.0	−1.34 [−63.39]	–	0.61 [1.61]	–	7.96 [0.09]
Flex81	1.0	−1.45 [−65.21]	–	2.90 [7.29]	–	7.75 [0.10]
FlexNom	1.00 [NA]	−1.37 [−59.39]		2.59 [6.66]		3.05 [0.55]
Combined effects: Money equation						$\pi_{m,m}$
Fix95	**1.0**	**−1.34** [−6.45]	1.88 [1.25]	**−4.65** [−4.29]	**5.58** [3.24]	**−0.26** [−6.46]
Flex95	**1.0**	**−1.32** [−6.99]	0.45 [0.41]	**−4.00** [−4.79]	**4.61** [3.50]	**−0.31** [−7.06]
Flex81	**1.0**	**−1.46** [−2.66]	**7.20** [2.31]	**−7.60** [−2.78]	**3.80** [1.05]	**−0.15** [−2.84]
FlexNom	**1.0**	**−1.35** [4.95]	2.50 [−1.42]	**−5.27** [4.02]	**5.82** [−3.16]	**−0.26** [−4.97]
Combined effects: Inflation equation						$\pi_{\Delta p,\Delta p}$
Fix95	−0.01 [−0.14]	0.02 [0.39]	**1.00**	0.10 [0.55]	**−1.05** [−3.60]	**−0.58** [−3.94]
Flex95	−0.04 [−1.11]	0.06 [1.40]	**1.00**	0.24 [1.25]	**−1.22** [−4.01]	**−0.54** [−3.96]
Flex81	−0.00 [−0.13]	0.02 [0.53]	**1.00**	−0.03 [−0.14]	**−0.63** [−2.04]	**−0.59** [−3.85]
FlexNom	0.02 [−0.31]	−0.01 [0.14]	**1.00**	0.09 [−0.35]	**−0.97** [3.06]	**−0.58** [−3.59]

Coefficients with t-values > 2.0 in bold face

Altogether, the finding that the demand for M3 is primarily a function of the cost of holding money relative to bonds and real assets seems quite robust in the five aggregation methods and the basic conclusions would remain almost unchanged independently of the aggregation method.

The combined effects of the inflation rate equation are also quite similar between the models: inflation rate is essentially only related to the long-term interest rate in an approximately one to minus one relationship (except for Flex81, where the coefficient is lower); it is not significantly related to excess money, nor to the short-term interest rate.

Altogether, the comparison seems to indicate that the main conclusions are reasonably robust, but that the estimated coefficients vary to some extent.

The largest variation was found between the Flex81, the no-PPP base year method, and the other methods.

5.4   The long-run impact of empirical shocks

The C&V analysis contained a structural VAR analysis which distinguished between two permanent shocks, labelled shocks to monetary policy objective and shocks to aggregate supply, and three transitory shocks, labelled shocks to money demand, aggregate demand and an interest rate shock. Since the credibility of the labels is difficult to assess without reporting several 5×5 matrices, jeopardizing the comparative aspect of this study, we have followed a slightly different route.

The two major policy goal variables are inflation and income. The C&V study (as most structural VAR analyses) is essentially consistent with the following assumptions:

1. an aggregate supply shock, $u_{r,t}$, has no long-run impact on inflation rate,

2. an aggregate demand shock, $u_{n,t}$, has no long-run impact on real income.

The first assumption is the equivalent of saying that the inflation row of the C matrix is an estimator of a nominal shock, i.e. $u_{n,t} = \iota'_{\Delta p} C \varepsilon_t$, where ι_x is a unit vector picking out the xth variable. The second assumption that the real income row is an estimator of a real shock would then correspond to $u_{r,t} = \iota'_{y^r} C \varepsilon_t$ (Johansen, 2007). Table 6 reports these two estimates for each of the four models.

The estimates in the upper part of the table suggest that the stochastic trend in real income was positively associated with empirical shocks to real money stock (all models, but FlexNom less significantly so), negatively with empirical shocks to the short-term and positively to the long-term interest rate (all models except the Flex81 model), and positively, but not very significantly so to empirical shocks to inflation rate (all models except the Flex81 model).

The estimates in the lower half of the table suggests that the stochastic trend in inflation rate was positively associated with the residuals to the long-term interest rate and negatively (though not significantly so) to the

Table 6: Comparison of the long-run impact of shocks on inflation and real income

	$\varepsilon(m^r)$	$\varepsilon(y^r)$	$\varepsilon(\Delta p)$	$\varepsilon(R_s)$	$\varepsilon(R_l)$
The real income row in the long-run impact matrix C					
Fix95	**0.51** [2.61]	**1.38** [2.51]	1.66 [1.85]	**−6.03** [−2.29]	**5.92** [2.16]
Flex95	**0.46** [3.14]	0.50 [1.38]	0.78 [1.38]	**−5.80** [−3.29]	**2.74** [1.93]
Flex81	**0.51** [3.83]	**0.69** [2.22]	−0.65 [−1.09]	−0.82 [−0.61]	−1.33 [−0.78]
FlexNom	0.33 [1.74]	0.49 [1.54]	1.00 [1.76]	**−5.71** [−3.34]	2.55 [1.71]
The inflation row in the long-run impact matrix C					
Fix95	0.04 [1.19]	0.08 [0.83]	**0.51** [3.10]	−0.36 [−0.74]	**1.52** [3.01]
Flex95	0.05 [1.76]	0.03 [0.36]	**0.37** [3.14]	−0.48 [−1.30]	**1.41** [4.74]
Flex81	**0.03** [1.90]	0.03 [0.63]	−0.01 [−0.18]	**0.43** [2.30]	0.21 [0.89]
FlexNom	0.02 [0.66]	−0.01 [−0.09]	**0.37** [3.38]	−0.57 [−1.73]	**1.22** [4.25]

Coefficients with a t-value > 1.9 in bold face.

short-term interest rate residuals. This was the case for all models except the Flex81 model for which the long-term interest rate residual became insignificant while the short-term interest rate became significantly positive. The residuals to real money and real income do not seem to have any significant effect on the stochastic trend in the inflation rate in all five models.

Altogether, the estimates are fairly similar and the conclusions quite robust for all aggregation models with the exception of the Flex81 model. A tentative conclusion might be that the differentials between the aggregation methods are sufficiently tiny (though persistent) not to significantly change the empirical results. However, the less satisfactory performance of Flex81, suggests that this may hold as long as the base year for the fixed real GDP weights is not too far from a purchasing power parity year.

6   Summarizing the results

This paper has demonstrated that the flexible real GDP weights proposed by Beyer, Doornik, and Hendry (2001) needed for the aggregation of variables

is sensitive to the choice of base year for prices and that the time series of weights for the individual countries may deviate quite significantly in *absolute* value for different base years while the mean corrected series are likely to be more similar. The paper shows that this problem disappears altogether when nominal rather than real GDP weights are used.

A comparision of aggregates calculated with fixed and flexible weights methods, showed that the differences between the methods are not large for the aggregates in absolute value but, nevertheless, highly persistent. Thus, the choice of aggregation method, might affect the cointegration properties in empirical models. Recalculating the monetary model in Coenen and Vega (2001) for the various aggregation methods tentatively suggests that the effect on the statistical inference in the VAR model is not dramatic. The only exception was for the flexible BDH method in the case when the base year represented a period when purchasing power in the member states deviated very significantly from parity, suggesting that some caution is needed in the choice of base year. But, on the whole, most conclusions remained relatively unchanged. This is more or less in line with the conclusions in Kongsted (2005), and Jørgensen (1998) who studied a similar question.

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