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Competition in Soccer Leagues

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Abstract

In the present paper a model of competition between sports clubs in a sports league is presented. Clubs are endowed with initial players but at a cost clubs are able to sell their initial players and buy new players. The results are that: if the quality of players is one-dimensional, then equilibria in pure strategies exist, and; if the quality of players is multi-dimensional, then there need not exist equilibria in pure strategies, but equilibria in mixed strategies exist. Equilibria in mixed strategies resemblance signings on deadline day in european soccer.

Keywords: competition between sports clubs, dimension of quality of players, equilibrium in pure strategies, equilibrium in mixed strategies.

JEL-classification: C72, D21, L83.

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1 Introduction

Competition between sports clubs in a sports league has been studied for fifty years at least and many papers have pointed out several peculiarities of sports leagues. A peculiarity of sport clubs in a sports league is that there are production externalities between clubs as pointed out by El-Hodiri & Quirk (1971), Neale (1964) and Rottenberg (1956). Indeed a game takes two clubs and the quality of a game depends on the quality as well as the tactic of both clubs, but also the competitive balance in the league is of importance. Therefore without regulation the outcome of competition between sports clubs need not effecient.

Sport clubs have many players, for soccer clubs a team consists of eleven players and five substitutes, but typically clubs have more players. We stoically assume that a club has a single player to make the analysis simple. Sport clubs seem to have different objectives: profit (Liverpool and Manchester United in UK and FCK in Denmark), utility of the owner (Chelsea in UK and AC Milan and Inter Milan in Italy) and welfare of majority of club members (Barcelona and Real Madrid in Spain) just to mention a few possible objectives. We assume that clubs maximize their profits, but revenues may be interpreted as utilities of owners, welfare of majorities of members or something else.

In the present paper a model of competition between sport clubs in a league is presented. The outcome of the competition depends on the distribution of players between clubs, so the strategic variable for a club is the quality of its player and the performance of a club depends on the quality of its player as well as the quality of the players in the other clubs. Clubs are endowed with initial players, but there is a market for players, so clubs are able to sell their initial players and buy new players. Since there are externalities between clubs the decision to go on the market or not depends on the decisions of the other clubs. The cost of an initial player consists of a salary while the cost of selling an initial player and buying a new player consists of a transaction cost and a salary to the new player. Therefore there is a build-in discontinuity in the cost function of a club at the quality of the initial player.

In the paper we show that if the quality of players is one-dimensional and the revenues of clubs have increasing differences, so the change in revenue for a club of going from a player of low quality to a player of high quality is increasing in the quality of the players of the other clubs, then there exists a Nash equilibrium in pure strategies (Theorem 1). Depending on the initial players, no clubs, some clubs or all clubs sell their initial players and buy new players. More realistically, if the quality of players is multi-dimensional, then there need not exist an equilibrium in pure strategies. However we show that if the quality of players is multi-dimensional, then there exist equilibria in mixed strategies and strategies in these equilibria are rather simple as every club mixes over their initial players and one other player (Theorem 2 and Corollary 1). For a club a mixed strategy corresponds to trading players just before the transfer window closes so other clubs do not know whether the club keeps its initial player or signs a new player and therefore cannot react to the actual action of the club.

The paper is organized as follows: in Section 2 the model is introduced; in Section 3 existence of Nash equilibria in pure strategies is studied, and; in Section 4 existence and structure of Nash equilibria in mixed strategies is studied.

2 Setup and assumptions

There is a finite number n of clubs $j \in \mathcal{N} = \{1, \ldots, n\}$ and each club has one player. Clubs compete so their profits depend on the quality level of their own players as well as the players of the other clubs. Players are described by a quality vector $q \in \mathbb{R}^m_+$, where each coordinate corresponds to some capability, skill or talent.

Outside the league there is a market for players where clubs can sell and buy players. Club j is characterized by its initial player $\omega_j \in \mathbb{R}^m_+$ where

$\omega_j \neq 0$, its revenue function $r_j : (\mathbb{R}^m_+)^n \to \mathbb{R}_+$ which depends on its own player as well as the players of the other clubs, and its cost function $c_j : \mathbb{R}^m_+ \times \mathbb{R}^m_+ \to \mathbb{R}_+$ which depends on its own initial player and its new player. The cost of a new player of quality q_j consists of a cost of having the player (salary) and a transaction cost of selling the initial player and buying the new player (transfer fee).

Let $s_j(q_j)$ be the cost of having a player of quality q_j and let $t_j(q_j, \omega_j)$ be the transaction cost of selling a player of quality ω_j and buying a player of quality q_j, then the cost function takes the following form:

$$c_j(q_j, \omega_j) = \begin{cases} s_j(q_j) & \text{for } q_j = \omega_j \\ s_j(q_j) + t_j(q_j, \omega_j) & \text{for } q_j \neq \omega_j. \end{cases}$$

The cost function has a build-in discontinuity at $q_j = \omega_j$ because of the transaction cost.

The following assumptions are supposed to be satisfied

(A.1) r_j is continuous, bounded from above and strictly concave in q_j.

(A.2) s_j is continuous, convex and monotone.

(A.3) t_j is continuous, convex and monotone in q_j.

(A.4) t_j is strictly positive on the strictly positive part of the diagonal, so $t_j(q_j, \omega_j) > 0$ for all (q_j, ω_j) such that $q_j = \omega_j$ and $q_j, \omega_j > 0$.

(A.5) $s_j + t_j$ is unbounded, so $\lim_{\|q_j\| \to \infty} s_j(q_j) + t_j(q_j, \omega_j) = \infty$.

(A.1)-(A.3) are quite natural. (A.4) implies that there are transaction costs because the cost of selling a player and buying a player of identical quality is strictly positive. (A.5) implies that costs increases without bound as quality increases without bound.

3 Equilibrium in pure strategies

Let $\Pi_j(q_j, q_{-j}) = r_j(q_j, q_{-j}) - c_j(q_j, \omega_j)$ be the profit of club j. Then the problem of club j is to choose a player in order to maximize its profit given the players of the other clubs $q_{-j} = (q_1, \ldots, q_{j-1}, q_{j+1}, \ldots, q_n)$

$$\max_{q_j} \quad \Pi_j(q_j, q_{-j}) = r_j(q_j, q_{-j}) - c_j(q_j, \omega_j) \tag{1}$$

Definition 1 *A collection of players $q^* = (q_1^*, \ldots, q_n^*)$, where $q_j^* \in \mathbb{R}_+^m$ and q_j^* is a solution to Problem (1) given q_{-j}^*, is an equilibrium in pure strategies.*

Suppose that the quality of players is 1-dimensional, so the quality of a player is described by a number (or alternatively only co-linear qualities of players are available) and revenue functions has increasing differences. Then an equilibrium in pure strategies exists

Theorem 1 *Suppose that the quality of players is 1-dimensional, so $m = 1$. If $r_j(q_j, q_{-j})$ has increasing differences in (q_j, q_{-j}) (so if $q_j \geq q_j'$ and $q_{-j} \geq q_{-j}'$, then $r_j(q_j, q_{-j}) - r_j(q_j, q_{-j}') \geq r_j(q_j', q_{-j}) - r_j(q_j', q_{-j}')$), then there exists an equilibrium in pure strategies.*

Proof: Firstly it is shown that the game $(\mathcal{N}, (S_j, \Pi_j)_j)$ where $S_j = \mathbb{R}_+$ and $\Pi_j : \mathbb{R}_+^n \to \mathbb{R}$ is the profit, is a supermodular game. Secondly Theorem 4.2.1. in Topkis (1998) on existence of equilibrium in pure strategies in supermodular games is applied.

The game $(\mathcal{N}, (S_j, \Pi_j)_j)$ is supermodular: 1. S_j is a lattice; 2. Π_j is supermodular in q_j, because all functions from $\mathbb{R}$ to $\mathbb{R}$ are supermodular, and; 3. $q_{-j} \geq q_{-j}'$ implies that $\Pi_j(q_j, q_{-j}) - \Pi_j(q_j, q_{-j}')$ is non-decreasing in q_j, because $q_{-j} \geq q_{-j}'$ implies that $r_j(q_j, q_{-j}) - r_j(q_j, q_{-j}')$ is non-decreasing in q_j by assumption. Moreover, there exists $\bar{q} \in \mathbb{R}_+$, such that for all j and q_{-j}, if q_j is a solution to Problem (1), then $q_j \leq \bar{q}$ because r_j is bounded from above and $s_j + t_j$ is unbounded. Therefore the set of equilibria in the game $(\mathcal{N}, (S_j, \Pi_j)_j)$ and set of equilibria in the game $(\mathcal{N}, (q_j, \Pi_j)_j)$ where

$q_j = [0, \bar{q}]$, coincide. According to Theorem 4.2.1. in Topkis (1998), the game $(\mathcal{N}, (S_j, \Pi_j)_j)$ has an equilibrium in pure strategies.

$\square$

Let $\pi_j(q_j, q_{-j}) = r_j(q_j, q_{-j}) - s_j(q_j) - t_j(q_j, \omega_j)$ be the profit of club j given the transaction cost has to be paid even if the club keeps its initial player. Then the artificial problem of club j is

$$\max_{q_j} \quad \pi_j(q_j, q_{-j}) = r_j(q_j, q_{-j}) - s_j(q_j) - t_j(q_j, \omega_j). \tag{2}$$

Problem (2) has a solution because the revenue is bounded from above and the cost is unbounded and the solution is unique because the profit is strictly concave. Therefore let the map $F_j : \mathbb{R}_+^{n-1} \to \mathbb{R}_+$ be the solution map, so $F_j(q_{-j})$ is the solution to Problem (2) given q_{-j}, then F_j is continuous according to Berge's Maximum Theorem and F_j is bounded from above because the revenue is bounded from above and the cost is unbounded.

Clearly for all $q_{-j} \in \mathbb{R}_+^{n-1}$ there exist a pair of functions $F_j^L, F_j^U : \mathbb{R}_+^{n-1} \to \mathbb{R}_+$, such that if $\omega_j < F_j^L(q_{-j})$ or $\omega_j > F_j^U(q_{-j})$, then $F_j(q_{-j})$ is the solution to Problem (1), and if $F_j^L(q_{-j}) < \omega_j < F_j^U(q_{-j})$, then ω_j is the solution to Problem (1).

Suppose that $\pi_j(q_j, q_{-j}) = r_j(q_j, q_{-j}) - s_j(q_j) - t_j(q_j, \omega_j)$ is twice differentiable and differentiable strictly concave, so $D^2_{q_j q_j}\pi_j(q_j, q_{-j}) < 0$, then if q_j and q_k where $k \neq j$ are strategic complementarities for all k, so $D^2_{q_j q_k}\pi_j(q_j, q_{-j}) > 0$ for all $k \neq j$, then π_j satisfies increasing differences. Indeed according to the Implicit Function Theorem

$$D_{q_k} F_j(q_{-j}) \;=\; -\frac{D^2_{q_j q_k}\pi_j(q_j, q_{-j})}{D^2_{q_j q_j}\pi_j(q_j, q_{-j})}$$

so if π_j is twice differentiable strictly concave and differentiable strictly concave and q_j and q_k are strategic complementarities, then F_j is strictly monotone as illustrated in the first diagram in Figure 1. However if q_j and q_k are not strategic complementarities, then F_j need not be monotone as illustrated in the second diagram in Figure 1.

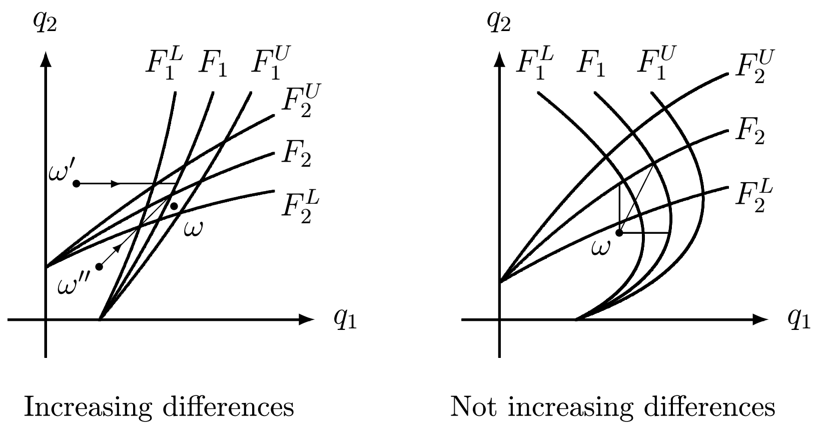

Figure 1: *The importance of increasing differences.*

In the first diagram: 1. if the initial players are ω, then ω is an equilibrium, because the clubs' initial players are sufficiently close to the optimal players; 2. if the initial players are ω', then in equilibrium club 1 trade players, while club 2 keeps its player, and; 3. if the initial players are ω'', then in equilibrium both clubs trade players, because both clubs' initial players are too far away from the optimal players. In the second diagram: if the initial players are ω, then: 1. ω is not an equilibrium; 2. if only one club trades players, then the other club also wants to trade players, and; 3. if both clubs trade players, then club 1 regrets that it traded players. Therefore in the second diagram where the profit function of club 1 does not have increasing differences, if the initial players are ω, then there does not exist an equilibrium in pure strategies.

4 Equilibrium in mixed strategies

Unfortunately Theorem 1 does not generalize to multi-dimensional players, because if q_j is multi-dimensional, then the function Π_j is not supermodular: For $q_j, q_j' \in \mathbb{R}_+^m$, let $q_j \vee q_j'$ ($q_j \wedge q_j'$) be their join (meet), so the i'th coordinate of $q_j \vee q_j'$ ($q_j \wedge q_j'$) is the maximum (minimum) of the i'th coordinate of q_j

and the i'th coordinate of q'_j. Π_j is supermodular in q_j if and only if

$$\Pi_j(q_j, q_{-j}) + \Pi_j(q'_j, q_{-j}) \ \leq \ \Pi_j(q_j \vee q'_j, q_{-j}) + \Pi_j(q_j \wedge q'_j, q_{-j}).$$

Suppose that $q_j = \omega_j$ and that q'_j converges to q_j such that $q_j \vee q'_j, q_j \wedge q'_j \neq q_j$[1]. Then $\Pi_j(q_j, q_{-j}) = \Pi_j(\omega_j, q_{-j}) > \pi_j(\omega_j, q_{-j})$ and $\Pi_j(q'_j, q_{-j})$, $\Pi_j(q_j \vee q'_j, q_{-j})$ and $\Pi_j(q_j \wedge q'_j, q_{-j})$ converges to $\pi_j(\omega_j, q_{-j})$, so Π_j is not supermodular in q_j. Note that for one-dimensional players $q_j \vee q'_j = \max\{q_j, q'_j\}$ and $q_j \wedge q'_j = \min\{q_j, q'_j\}$, so $q_j \vee q'_j = q_j$ and $q_j \wedge q'_j = q'_j$ or $q_j \vee q'_j = q'_j$ and $q_j \wedge q'_j = q_j$. Therefore it is impossible that both $q_j \vee q'_j \neq q_j$ and $q_j \wedge q'_j \neq q_j$ are satisfied for one-dimensional players.

Since an equilibrium in pure strategies need not exist for multi-dimensional players, let $\mathbb{P}$ be the set of probability measures on $\mathbb{R}^m_+$ and let $\mu \in \mathbb{P}$ be denoted a random player. The problem of club j is to choose a (random) player in order to maximize its expected profit given the random players of the other clubs $\mu_{-j} = (\mu_1, \ldots, \mu_{j-1}, \mu_{j+1}, \ldots, \mu_n)$

$$\max_{\mu_j} \int_{q_j} \left(\int_{q_{-j}} \Pi_j(q_j, q_{-j}) \, d\mu_{-j}(q_{-j}) \right) d\mu_j(q_j) \tag{3}$$

Definition 2 *A collection of random players $\mu^* = (\mu^*_1, \ldots, \mu^*_n)$ where $\mu^*_j \in \mathbb{P}$, such that μ^*_j is a solution to Problem (3) given μ^*_{-j} is an equilibrium in mixed strategies.*

Theorem 2 *There exists an equilibrium in mixed strategies.*

Proof: For all $\mu_{-j} \in \mathbb{P}^{n-1}$, there exists a solution to Problem (3). Let $\Gamma : \mathbb{P}^n \to \mathbb{P}^n$ be defined by $\mu'_j \in \Gamma_j(\mu)$ if and only if μ'_j is a solution to Problem (3) given μ_{-j}, then Γ is convex valued and upper hemi-continuous according to Berge's Maximum Theorem. Moreover, there exists $\bar{q} \in \mathbb{R}^{nm}_+$, such that for all μ if $\mu' \in \Gamma(\mu)$, then $\mu'([0, \bar{q}]) = 1$. Therefore there exists μ^* such that $\mu^* \in \Gamma(\mu^*)$ according to Kakutani's Fixed Point Theorem.

[1]If $q_j = \omega_j = (1, 1)$ and $q'_j = (1 + \varepsilon, 1 - \varepsilon)$ where $\varepsilon > 0$, then $q_j \vee q'_j = (1 + \varepsilon, 1)$ and $q_j \wedge q'_j = (1, 1 - \varepsilon)$.

□

The random player version of the artificial problem of club j is to maximize its profit given the transaction cost has to be paid even if the club keeps its initial player

$$\max_{q_j} \quad \pi_j(q_j, \mu_{-j}), \tag{4}$$

Problem (4) has a unique solution. Therefore let the map $F_j : \mathbb{P}^{n-1} \to \mathbb{R}^m_+$ be the solution map, so $F_j(\mu_{-j})$ is the solution to Problem (4) given μ_{-j}, then F_j is continuous according to Berge's Maximum Theorem and F_j is bounded from above according to (A.2).

Corollary 1 *Suppose that μ^* is an equilibrium in mixed strategies. Then for all j*

$$\mu_j^*(\omega_j) + \mu_j^*(F_j(\mu_{-j}^*)) \;=\; 1.$$

Proof: Clearly for all μ_{-j} Problem (3) has either one pure solution: ω_j or $F_j(\mu_{-j})$, or two pure solutions: ω_j and $F_j(\mu_{-j})$. Let the correspondence $G_j : \mathbb{P}^{n-1} \to \mathbb{R}^m_+$ be the solution correspondence, so $q_j \in G_j(\mu_{-j})$ if and only if q_j is a solution to Problem (3).

Let the correspondence $\Gamma : \mathbb{P}^n \to \mathbb{P}^n$ be defined by $\mu'_j \in \Gamma_j(\mu)$ if and only if μ'_j has support on $G_j(\mu_{-j})$, then $\mu'_j \in \Gamma(\mu)$ if and only if μ'_j is a solution to Problem (3) given μ_{-j}. Therefore there exists μ^* such that $\mu^* \in \Gamma(\mu^*)$ according to Theorem 2 and for all μ'_j in $\Gamma(\mu)$

$$\mu'_j(\omega_j) + \mu'_j(F_j(\mu_{-j})) \;=\; 1$$

by construction.

□

According to Corollary 1 if a club uses a mixed strategy then it mixes between keeping its initial player and selling its initial player and buying the optimal player given the transaction cost has to be paid even if the club keeps its initial player.

Signings well before the transfer window closes resemblance pure strategies as other clubs are able to react to the signings. In the real signings just before the transfer window closes world resemblance mixed strategies as other clubs are not able to react because they do not know about the signings before after the transfer window closes. Deadline day is usually one the busiest days in the the transfer window: it attracts a lot of attention from the media and some very big moves have occurred on this day.

In the present paper every club is a firm and there are production externalities between firms. Therefore the outcome should not be expected to be efficient. Indeed generally if $\mu^* = (\mu_1^*, \ldots, \mu_n^*)$ is an equilibrium collection of random players, then there exists another collection of players $q^* = (q_1^*, \ldots, q_n^*)$ such that the aggregate profit is larger $\sum_i (\pi_(q^*) - c_i(q_i^*)) > \int \sum_i (\pi_i(q) - c_i(q_i))\, d\mu^*(q)$.

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