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Life Cycle Savings, Bequest, and the Diminishing Impact of Scale on Growth*

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Abstract

There appears to be ample evidence that the size of population acted as a stimulus to growth in historical times; scale mattered. In the post World War II era, however, there is little evidence of such scale effects on growth. Where did the scale effect go? The present paper shows that the savings motive critically affects the size and sign of scale effects in standard endogenous growth models. If the bequest motive dominates, the scale effect is positive. If the life cycle motive dominates, the scale effect is ambiguous and may be negative. A declining importance of bequest in capital accumulation could therefore be one reason why scale seems to matter less today than in historical times.

Keywords: Overlapping Generations, Endogenous Growth, Scale Effects

JEL classification: O41, O57

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1 Introduction

An important difference between the two workhorse models in macroeconomics, the Diamond model and the Ramsey-Cass-Koopmans (RCK) model, is that they emphasize different motives for saving. In the Diamond model the bequest motive is absent, whereas life-cycle considerations play no role in the RCK framework. If we turn to data for current day developed economies, the relative importance of bequests and life-cycles savings in capital accumulation remains unresolved (Dynan et al., 2002). Hence, from this perspective it is not obvious which framework is a better stylized representation of the process of capital accumulation. But the fact remains that this difference is far from trivial, as it translates into different links between wage and capital income on the one hand, and the rate of capital accumulation on the other. In an RCK framework all wage income is consumed (along a steady state trajectory), whereas all capital income is consumed in the Diamond model (Bertola, 1993, 1996).

From this emanates radically different answers to questions of first order importance. Consider the impact of taxes on growth: Whereas a capital income tax reduces growth (or long-run income) in an RCK model, it can raise growth in the Diamond model (Uhlig and Yanagawa, 1996; Caballe, 1998). Likewise, the two models hold different predictions with respect to the prospect for cross country income equalization: Whereas the steady state is unique in the RCK model, supporting the Conditional Convergence hypothesis, multiple steady states may arise in the Diamond model, supporting the Club Convergence hypothesis (Galor, 1996). Finally, whereas endogenous growth is feasible in convex RCK growth models (Jones and Manuelli, 1990), the same is not feasible in a Diamond environment (Jones and Manuelli, 1992).

The present paper demonstrates that the relative importance of bequests and life cycle savings is crucial for another important issue: The impact of scale on growth. Specifically, within a standard endogenous growth framework we show that if the bequest motive for saving is paramount, then the scale effect is positive. However, if life cycle considerations dominate, the scale effect is ambiguous. Under plausible conditions it may be absent and even negative.

This observation could be of some practical relevance since the importance of bequest in capital accumulation seems to have declined substantially in the Western world over the preceding centuries. DeLong (2003) calculates that in preindustrial Eurasia bequest likely accounted for 90 percent of total wealth, compared with 43% today. While the exact numbers may be debated, it seems reasonable to assert a diminished role of bequest in the very long run. If so, our analysis would imply that the impact of scale on growth should change also. Specifically, if the importance of bequest declines, and life-cycle savings increases, the scale effect is dampened. Indeed, it could be eliminated altogether. This prediction is consistent with what appears to be known about the role of scale in the very long run.

A positive impact on living standards from an increasing population has received considerable historical support. The study by Kremer (1993) provides an ingenious test of scale effects. Assuming fertility is determined along Malthusian lines and that growth in income (technology) is subject to a positive scale effect from the size of population, Kremer demonstrates that one should expect a positive association between population growth and the size of population. Using data for world population this association in confirmed for most of human history. Likewise, the study by Diamond (1997) argues that increasing population density was a leading (proximate) cause of economic development in historical times, and the work of Hoffman (1996) provides evidence that productivity was spurred by an expanding population in various regions of France during the middle ages. The influential work of Boserup (1965) also points to a positive impact from a larger population on productivity (in agrarian societies). At the same time, the *lack* of evidence in favor of scale effects in modern day societies is well recognized. Jones (1995) refute scale effects on the growth rate on the basis of time series tests for selected OECD countries. The survey by Dinopoulos and Thompsen (1999) reaches a similar conclusion, and, employing panel data techniques Rose (2006) also reject an impact from scale (population) on growth.[1]

Accordingly, if we accept both the historical evidence, and the evidence which speaks to the contemporary growth record, one is led to the question of why scale seemingly matters less today. While our analysis does not attempt to explain *why* the importance of bequest declines, it serves to demonstrate that a changing importance of bequest and life-cycle savings in capital accumulation could be part of an answer.

The analysis makes use of a one sector overlapping generations model fea-

[1] Alcalá and Ciccone (2004), however, do find evidence in favor of scale effects (from the size of population) on the *level* of GDP per capita, using cross section data. This finding may be reconciled with the conflicting historical and contemporary evidence in the following way. Suppose indeed scale spurred growth historically, but has ceased to do so today. In that case one would still expect, *ceteris paribus,* that more populous places are more prosperous when evaluated in a cross-section. But this would simply reflect past influence from scale; the size of the population is, after all, a rather persistent variable.

turing endogenous growth. For ease of exposition the (baseline) analysis invokes an externality from the stock of capital yielding a simple "AK" production technology in reduced form.[2] People live for two periods. They derive utility from consumption in both periods, and from passing on bequest (i.e., "joy-of-giving"). This specification allows us to parameterize the strength of the bequest motive, relative to the life-cycle motive. Within this set-up we demonstrate that unless the bequest motive is sufficiently strong, the impact from scale on growth is ambiguous. The intuition for this result is most readily explained in a special case of the model developed below: The Diamond model where preferences are Cobb-Douglas.

In this setting bequests are absent by construction, and saving are only made to fund retirement (i.e., only the life-cycle motive is operative). As a result, the all-important driving force behind capital accumulation becomes wage income; the return to capital does not matter due to the absence of a bequest motive and because preferences are Cobb-Douglas. Increasing the labor force entails diminishing marginal returns to labor input, which works so as to reduce the wage rate, savings and capital accumulation. At the same time, however, increasing the labor force implies that more individuals are saving resources for old age which stimulates capital accumulation. The impact on growth from an increasing population therefore depends on which of these two effects dominate. Formally, a larger labor force leaves growth *unaffected* if the elasticity of labor demand is equal to 1, or equivalently, if the elasticity of substitution between capital and labor equals the capital share of output.

More generally, we provide numerical experiments which serves to quantify the significance of scale effects, when both motives for savings are operative. We find that under reasonable assumptions the scale effect is quantitatively small and diminishes as the size of the labor force increases.

It is important to stress that, in general, the correct measure of "scale" is not the labor force *per se*, but rather, the labor force in efficiency units. This is demonstrated in an extension to the baseline model, where growth is fueled by capital accumulation and (government financed) R&D effort. However, the general point remains: Scale (appropriately measured) matters much less when the life-cycle motive dominates.

[2] More generally, however, our argument pertains to a larger class of endogenous growth models that have the AK-structure as their ultimate form. For example, it is straight forward to show that a Romer (1987) model, featuring growth due to increasing specialization, can be reduced to an AK-model. See also the R&D driven endogenous growth model developed in Barro and Sala-i-Martin (1995, Ch. 6).

This paper is related to the, by now, large literature on scale effects (see e.g., Jones, 1995; Young, 1998; Dinopoulos and Thompson, 1998; Peretto, 1998; Howitt, 1999; Dalgaard and Kreiner, 2001; Peretto and Smulders, 2002 and Strulik, 2005). A common feature of all these contributions is that they are cast within an RCK framework, where the life-cycle motive for saving is absent. The importance of this modelling choice appears to be under appreciated in the literature. In fact, it would appear that the scale issue has not been explored systematically within a framework where the life-cycle motive is operative. The present paper corrects this error of omission, and proceeds to show how the motive for savings have bearing on the issue at hand.

The paper is also related to a smaller literature, cited above, which highlights the importance of the savings motive for macroeconomic outcomes (Bertola, 1993, 1996; Caballe, 1998; Galor, 1996; Jones and Manuelli, 1992; Uhlig and Yanagawa, 1996). Whereas previous results have revealed its importance for growth and taxes, convergence and the sustainability of growth through capital accumulation, the present paper demonstrates that it is also central to the scale effect property.

2 The Model

Consider a closed economy where activity extends infinitely into the future, but where each individual lives for only two periods. Time is discrete, and denoted by $t = 1, 2 \ldots.$ The economy produces a homogenous good that is either consumed or saved/invested. The markets for output and factors of production, labor and capital, are competitive. The size of the population is assumed to be exogenously given and constant.

2.1 Firms

The representative firm produces output, Y_t, by combining capital, K_t, and labor, L:

$$Y_t = F\left(K_t, \bar{K}_t L\right), \tag{1}$$

$\bar{K}_t$ is an externality which will equal the aggregate stock of capital in equilibrium.[3] When optimizing producers take $\bar{K}$ as given. $F(\cdot)$ exhibits constant

[3] See Barro and Sala-i-Martin (1995, ch. 4) for an analysis involving this exact technology, though embedded in an RCK model.

returns to rival inputs: K_t and L. In addition, we assume that F is twice differentiable in both arguments, and exhibits diminishing returns to capital and labor. If we define $F\left(1, \frac{\bar{K}_t L}{K_t}\right) \equiv f\left(\frac{K_t L_i}{K_t}\right)$ total output of the representative firm can be written

$$Y_t = K_t f\left(\frac{\bar{K}_t L}{K_t}\right).$$

The producers will acquire capital and hire labor until the (private) marginal products equals the user cost of capital, $r_t + \delta$, and the real wage, w_t:

$$r_t + \delta = f\left(\frac{\bar{K}_t L}{K_t}\right) - f'\left(\frac{\bar{K}_t L}{K_t}\right)\frac{\bar{K}_t}{K_t/L} = \partial Y/\partial K,$$

$$w_t = f'\left(\frac{\bar{K}_t L}{K_t}\right)\bar{K}_t = \partial Y/\partial L.$$

For the sake of brevity, we will assume that capital depreciates fully during a period: $\delta = 1$.

In equilibrium, where $K_t = \bar{K}_t$, it follows that

$$1 + r = f(L) - Lf'(L), \tag{2}$$

and

$$w_t = f'(L) K_t. \tag{3}$$

In addition, the aggregate production function simplifies to $Y_t = f(L) K_t$.

Notice the impact from scale on equilibrium factor prices. The real rate of interest is increasing in the size of the labor force. This follows from the fact that capital and labor are complements in the production function. In contrast, the wage is decreasing in L, for K_t given ($f''(L) < 0$). This follows from diminishing returns to labor input.

2.2 Consumers

In their first period of life, individuals supply one unit of labor in-elastically for which they receive a wage, w_t. They also receive bequest from their parent, b_t. On this basis the consumers divide their first period income between consumption today, c_t^1, and savings s_t. In the second period of life, individuals divide their capital income, $(1+r)s_t$, between consumption and bequest for the off spring. That is $c_{t+1}^2 = (1+r)s_t - b_{t+1}$.

We assume preferences are CES, and that individuals derive utility from own consumption during their life, and from passing on bequest:

$$U(c_t^1, c_{t+1}^2, b_{t+1}) = \frac{(c_t^1)^{1-\theta} - 1}{1-\theta} + \frac{1}{1+\rho}\left[\frac{(c_{t+1}^2)^{1-\theta} - 1}{1-\theta} + \kappa \frac{b_{t+1}^{1-\theta} - 1}{1-\theta}\right]. \quad (4)$$

The parameters fulfill: $\theta > 0$, $\rho > 0$ and $\kappa \geq 0$.

An alternative to the above approach, would be to assume households are altruistic. That is, individuals care about the utility from own consumption, and about the *utility* of descendents. In this environment it can be shown that if the weight placed on future generations is sufficiently small bequests are zero. In this case the economy behaves as a Diamond model. However, if utility within and across generations are discounted at (say) the same rate, bequests will be passed on. In this case the economy behaves as described by the RCK model, and the influence from life-cycle considerations washes out (See Blanchard and Fischer, 1989, Ch. 3).

We adopt the joy-of-giving specification for two reasons. First, we wish to study the influence from the *relative* strength of the bequest motive on scale effects. We can accomplish this in a simple way by varying κ: The utility value from passing on bequest relative to own old-age consumption. In the altruistic household model bequests are either "on" or "off"; only the "corner solutions" can be studied. Second, one may argue the joy-of-giving specification fits the data better than the altruistic household version (Altonji et al., 1997).

The two first order conditions are

$$b_{t+1} = \kappa^{\frac{1}{\theta}} c_{2t+1}$$

and

$$c_{2t+1} = \left(\frac{1 + r_{t+1}}{1+\rho}\right)^{1/\theta} c_{1t}.$$

Standard computations then lead to the following closed form solution for the savings of the young, and thus total savings

$$S_t = s_t L = s^w w_t L + s^r (1+r) K_t \quad (5)$$

where the marginal savings rates from labor income (s^w) and capital income (s^r) are given by:

$$s^r \equiv \frac{\kappa^{1/\theta} s^w}{1 + \kappa^{1/\theta}}, \; s^w \equiv \frac{\left(1 + \kappa^{1/\theta}\right)(1+r)^{\frac{1-\theta}{\theta}}}{(1+\rho)^{1/\theta} + (1+r)^{\frac{1-\theta}{\theta}}\left(1 + \kappa^{1/\theta}\right)}. \quad (6)$$

It is easy to see that $\partial s^w / \partial r, \partial s^r / \partial r \gtreqless 0$ when $\theta \lesseqgtr 1$.

The key thing to observe from equations (5) and (6) is the relationship between κ, factor incomes and savings. The marginal propensity to save from the two sources of income is determined by κ; the utility weight on bequest. As κ rises s^r / s^w increases, which shows that as the bequest motive is strengthened, capital income becomes a more important determinant of savings. This shows the close links between the motive for saving, the distribution of factor income and aggregate savings.

2.3 Steady State Growth and Scale Effects

The capital stock at time $t+1$ is given by the total savings of the young in period t. Thus, inserting equations (2) and (3) (along with the assumption that $\delta = 1$) into equation (5), we obtain – after some rearrangements – the following expression for the growth rate of the capital stock (and therefore output)

$$g_Y = g_K = s^w \left[f(L) \frac{\kappa^{1/\theta}}{1+\kappa^{1/\theta}} + \frac{1}{1+\kappa^{1/\theta}} f'(L) L \right], \tag{7}$$

where $g_X \equiv (X_{t+1} - X_t) / X_t$.

Notice that whereas $f(L)$ is increasing in L, $f'(L) L$ is not unambiguously increasing in the labor force. Moreover, as κ is lowered, greater relative weight is attached to the second term. We have the following result:

Theorem Scale and Growth when Households Save for Retirement and Bequest. *The effect on the long-run growth rate from an increase in the labor force will be positive, zero, or negative, depending on whether*

$$\varepsilon_{s,L} \cdot \left(\kappa^{1/\theta} + 1 - \alpha_K \right) + (1 - \alpha_K) \left[\left(1 + \kappa^{1/\theta} \right) - \alpha_K \sigma^{-1} \right] \gtreqless 0. \tag{8}$$

$\varepsilon_{s,L} \equiv (\partial s^w / \partial L)(L/s) \gtreqless 0$ *for* $\theta \lesseqgtr 1$, $\sigma \equiv \frac{-f'(L) \cdot \left(f(L) - L f'(L) \right)}{f''(L) \cdot L f(L)}$ *is the elasticity of substitution, while* $\alpha_K \equiv \frac{f(L) - L f'(L)}{f(L)}$ *is capital's share of total income.*

Proof. Differentiation of equation (7) yields

$$\frac{\partial s^w}{\partial L} \cdot \left[f(L) \frac{\kappa^{1/\theta}}{1+\kappa^{1/\theta}} + \frac{1}{1+\kappa^{1/\theta}} f'(L) L \right] - s^w f'(L) \left[\frac{1}{1+\kappa^{1/\theta}} \frac{-f''(L) L}{f(L)} - 1 \right] \gtreqless 0$$

which is equal to the condition

$$s^w f'(L) \left\{ \frac{\partial s^w}{\partial L} \frac{L}{s^w} \cdot \left[\frac{f(L)}{f'(L) L} \frac{\kappa^{1/\theta}}{1+\kappa^{1/\theta}} + \frac{1}{1+\kappa^{1/\theta}} \right] - \left[\frac{1}{1+\kappa^{1/\theta}} \frac{-f''(L) L}{f(L)} - 1 \right] \right\} \gtreqless 0$$

Now, notice that $:\frac{f'}{f''L} = \frac{f'(L)\cdot\left(f(L)-Lf'(L)\right)}{f''(L)\cdot Lf(L)} \cdot \frac{f(L)}{f(L)-Lf'(L)} = -\frac{\sigma}{\alpha_K}$, since $\sigma \equiv \frac{-f'(L)\cdot\left(f(L)-Lf'(L)\right)}{f''(L)\cdot Lf(L)}$ and $\alpha_K \equiv \frac{rK}{Y} = \frac{f(L)}{f(L)-Lf'(L)}$. Thus the condition can be restated to yield

$$\frac{s^w}{1+\kappa^{1/\theta}}\left\{\varepsilon_{s,L}\left[\frac{\kappa^{1/\theta}}{1-\alpha_K}+1\right]-\left[\frac{\alpha_K}{\sigma}-\left(1+\kappa^{1/\theta}\right)\right]\right\} \gtreqless 0,$$

from which the above stated condition is easily obtained. ∎

The following special case (Cobb-Douglas preferences) is a convenient starting point for an interpretation of the condition stated in the Theorem.

Corollary *If $\theta = 1$ the effect on the long-run growth rate from an increase in the labor force will be positive, zero, or negative, depending on whether*

$$1+\kappa \gtreqless \frac{-f''L}{f'} = \frac{\alpha_K}{\sigma} \tag{9}$$

As is apparent from the corollary, the scale effect is positive, *ceteris paribus*, if κ is sufficiently high. In the special case where the bequest motive is absent ($\kappa = 0$), scale ceases to matter if $\alpha_K = \sigma$, which is the same as saying that the elasticity of labor demand equals 1. The intuition for this result should now be clear. If the bequest motive is absent, savings are based on the life-cycle motive alone. Consequently, savings are funded solely by wage income. While a larger labor force in itself spurs growth as the number of saving individuals rise, diminishing returns to labor input provides a powerful countervailing force. As a result, the sign of the impact from L on growth is generally ambiguous.

If, however, the bequest motive becomes operative (κ rises), savings are influenced by capital income as well. Since the real rate of interest is unambiguously increasing in L, due to capital-labor complementarity, the tendency for diminishing returns to set in needs to be strengthened so as to to leave growth unaffected if L rises; $\frac{-f''L}{f'} > 1$ is required for zero scale effects, or, $(1+\kappa)\,\sigma = \alpha_K$.

When moving to the more general case (i.e., $\theta \neq 1, \kappa > 0$), the condition becomes more complex, since changes in L affects the real rate of interest, and therefore the savings *rate*. Since the real interest rate, is increasing in the size of the labor force ($\partial r/\partial L = -f''(L)L > 0$), the sign of $\partial s/\partial r$ will determine whether the first term in (8) works so as to promote or retard the scale effect. In general the scale effect is dampened when $\theta > 1$.

While assuming $\theta > 1$ is often standard in the literature on economic growth, available evidence suggest that changes in real rates of interest lead – at best –

to only minor changes in the savings rate.[4] The finding that $\frac{\partial s}{\partial r}\frac{r}{s} \approx 0$ is (in the present case) consistent with Cobb-Douglas preferences: $\theta = 1$. As a result, in what follows we shall focus on the Cobb-Douglas setting which leads to the "no scale-effect condition" stated in the corollary.

2.4 Numerical Experiments

The theorem establishes that scale may not matter to growth at all, if the life-cycle motive is operative. However, the derived condition is unlikely to be fulfilled generally. For example, in the case where bequests are absent, we need $\sigma = \alpha_K$. But both σ and capital's share are endogenous (in general), and therefore changes over time as factor inputs change. There is no particular reason why $\sigma = \alpha_K$ should be maintained over time.

In this respect it is important to stress that the general point of the analysis is that the life-cycle motive, if operative, *dampens* the influence from scale on growth. But how important is this effect? This is the question to which we now turn.

In order to proceed we need to specify the production function, and choose a set of parameter values. In terms of technology, we assume output is produced by the following CES production function

$$Y = F\left(K, \bar{K}L\right) = \left(\beta K^{\frac{\sigma-1}{\sigma}} + (1-\beta)\left(\bar{K}L\right)^{\frac{\sigma-1}{\sigma}}\right)^{\sigma/(\sigma-1)}.$$

Given this technology, and maintaining $\theta = 1$, we get the following expression for the growth rate

$$g_K = \frac{(1+\kappa)\left[\beta + (1-\beta)L^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} - \left[\beta + (1-\beta)L^{\frac{\sigma-1}{\sigma}}\right]^{\frac{1}{\sigma-1}}\beta}{2+\rho+\kappa}.$$

This is the growth rate of capital (and output) from one generation to the next. In the numerical experiments below we will focus on annual growth rates. Accordingly, we need to pin down the length of a generation; we choose 30 years. In addition we need parameter values for κ, β, σ and ρ.

Starting in reverse order, we pick 0.8 for ρ, which is equivalent to a 2 percent annual discount rate over a 30 year period. Based on the estimations of Chirinko el al. (2002) we put $\sigma = 0.4$. The weight parameter in the production function,

[4] Modigliani (1986) states in his Nobel lecture that "... despite a hot debate, no convincing general evidence either way have been produced, *which leads me to the provisional view that s* [the savings rate] *is largely independent of the interest rate.* (p. 304. emphasis in original). Arestis and Demetriades (1997) surveys the literature and reaches a similar conclusion.

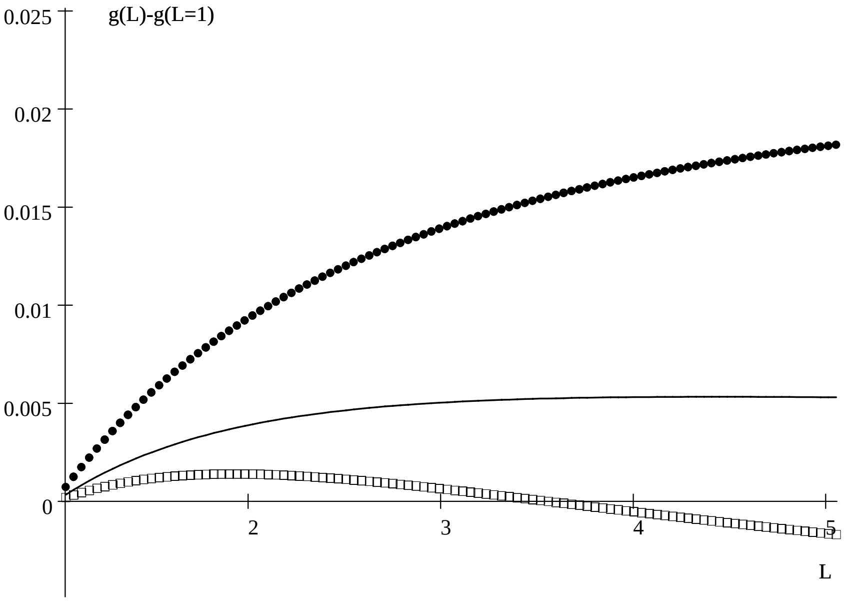

Figure 1: **The Figure shows the impact on growth from varying the size of the labor force.** *Notes*: The assumed production technology is CES. The solid line assumes κ =3/4, the diamond line assumes κ =0, whereas the dotted line assumes κ =1000. The other parametervalues are: ρ =0.8, σ =0.4, β =0.2, and θ =1. 1 generation is assumed to be 30 years.

β, is put at 0.2. This value is chosen so as to be able to generate reasonable values for capital's share of national accounts, which in the present case is given by $\alpha_K = \beta / \left(\beta + (1 - \beta) L^{\frac{\sigma-1}{\sigma}} \right)$. This leaves κ. In order to pick a realistic value for the importance of the bequest motive, we rely on DeLong (2003, Figure 2-1) who estimate that the contemporary share of bequest in total wealth is 43%. In the present model it is easy to show that

$$\frac{b_t}{k_t} \left(\frac{1}{1+r} \right) = \frac{\kappa}{1+\kappa}.$$

Assuming $b_t / [(1+r) k_t]$ is 0.43, we get a value for κ of (roughly) 0.75.

Figure 1 shows the impact on annual growth from varying the labor force by a factor of 5, for three different values of κ: 0 (i.e., no bequest), 3/4 and 1000. The latter is thought to approximate "perfect altruism"; the scenario where the bequest motive completely dominates the life-cycle motive.

Several features of the figure are worth noting. First, with a high value for κ the growth rate is monotonically rising in L. The effect is substantial: Doubling the labor force (from 1 to 2) increases the growth rate by 0.9 percentage points. Second, however, when the bequest motive is absent ($\kappa = 0$) the scale effect has a much smaller impact on growth. Repeating the exercise from before, in the Diamond environment, leads to the conclusion that growth only rises by 2/10th of a percentage point. As can be seen the scale effect turns negative at some point. This occurs when L hits 1.84; at this point $\alpha_K = \sigma = 0.4$. Finally, the "intermediate" case in the figure is where *both* motives for saving are present. The outcome is basically a convex combination of the two extremes we just described. For $\kappa = 3/4$ growth increases by 0.4 percentage points, when L rises from 1 to 2. Increasing the labor force to 4, however, only raises growth by additional 2/10th of a percentage point. Increasing L further still, to 8, *lowers* the growth rate by 0.1 percentage points. Hence, the scale effect peters out, as the size of the labor force rises.[5]

How sensitive are these results to changes in the underlying parameter values? The assumption $\theta = 1$ is clearly not driving the results of a modest scale effect. On the contrary, as established above: If $\theta > 1$ (the realistic alternative) the scale effect is further dampened. A larger labor force increases the real rate of return. But when $\theta > 1$ the income effect is dominating the substitution effect, and so savings fall. From this perspective Figure 1 *overestimates* the impact from scale on growth.

The assumption that $\sigma < 1$ is important to the quantitative nature of the results, but not the qualitative point made. Suppose, for example, that we chose $\sigma = 1.2$. This is admittedly a bit extreme, from an empirical standpoint. Estimates for σ generally tend to come out smaller than 1. But using a $\sigma > 1$ implies that scale always increases growth in the model developed above (cf. the Corollary, noting $\alpha_K < 1$). Accordingly, with a much higher elasticity of substitution, and with $\kappa = 3/4$, an increase in L from 1 to 2 will induce the growth rate to increase by 0.7 percentage points; nearly twice as large as that obtained for $\sigma = 0.4$. However, with $\kappa = 1000$ the same increase in L would instigate a growth acceleration of 1.1 percent per year. Accordingly, it remains true that the life cycle motive dampens the size of the scale effect, regardless of the value chosen for σ.

In sum, even though the model admits scale to influence the growth rate, the size of the effect is small under reasonable parameter values. That is, if both the bequest and life-cycle motive are operative. If the bequest motive is the sole

[5] Capital's share is 0.2, 0.4 and 2/3 for L = 1,2 and 4, respectively.

force behind savings the situation is rather different. Scale spurs growth, and the impact is substantial.

However, there are two reasons why an extension of the model above seems warranted. First, the notion of scale is overly simplistic: The size of the labor force, L. In a more general framework where growth is not *only* driven by capital accumulation, the appropriate measure of "scale" is modified. Second, the existing literature on scale effects have focused on R&D driven growth. It is therefore worthwhile to introduce endogenous R&D, so as to examine the robustness of the results obtained above. In the model developed in the next section growth is therefore fueled by capital accumulation and (government funded) R&D.

3 Endogenous R&D

In this section we maintain the assumption that consumers derive utility from consumption during youth, old age, and from bequest. Moreover, we also maintain $\theta = 1$; Cobb-Douglas preferences. As a result, savings of the young – determined by wages *and* bequest – fuel capital accumulation. In the absence of technological progress, however, growth will eventually cease. To sustain growth, technology therefore needs to progress.

Accordingly, the first new element to the model above is that we modify the production side of the economy. Specifically, we assume the production function of the representative firm is

$$Y_t = F\left(K_t, A_t L\right).$$

Hence, the externality is replaced by A, technological knowledge. When maximizing profits the firm takes A_t as given. This leaves us with two factor demand equations

$$r_t = f\left(x_t L\right) - x_t L f'\left(x_t L\right)$$

$$w_t = f'\left(x_t L\right) A_t,$$

where $x_t \equiv A_t / K_t$.

The second new element relates to the evolution of A_t over time. Following Antinolfi, Keister and Shell (2001), we assume that A_t expands as the result of public investments in R&D.[6] Antinolfi et al. (2001) assume that these investments are funded by a tax on the income of the young. In their model this is equivalent to a tax on wage income. In our model, in contrast, period 1 income also comprises bequests. As a result, budget balance then implies that

$$A_{t+1} = I_{A,t} = \tau \left(w_t + b_t\right) L,$$

where $\tau > 0$ is the (constant) tax rate on total period 1 income. This specification is consistent with a "fishing out" view of the research process; a given relative increase in the stock of knowledge gradually becomes more expensive as the stock of knowledge, A_t, expands. This can readily be seen by dividing through by A_t in the technology above. Unless $I_{A,t}$ rises growth in A_t will come to a halt.[7]

Given Cobb-Douglas preferences, and the presence of a tax on first period income, aggregate savings (and thereby capital in period $t+1$) is given by

$$K_{t+1} = S_t = s_t L = \left(\frac{1+\kappa}{2+\kappa+\rho}\right)(1-\tau)\left(w_t + b_t\right) L.$$

As should be clear, under this set of assumptions the ratio of the stock of knowledge to that of physical capital, is constant at all points in time. Specifically $x_t = \bar{x} \equiv \frac{\tau}{1-\tau}\frac{2+\kappa+\rho}{1+\kappa}$.

Consequently, it can be shown (by substituting for $b_t L = \frac{\kappa}{1+\kappa}\left(1+r_{t+1}\right) s_{t-1} L = \frac{\kappa}{1+\kappa}\left(1+r_{t+1}\right) K_t$ and using equilibrium factor prices) that the growth rate of the economy will be given by

$$g_A = g_K = (1-\tau)\, s \left[\frac{\kappa}{1+\kappa} f\left(\bar{x}L\right) + f'\left(\bar{x}L\right)\bar{x}L\frac{1}{1+\kappa}\right],$$

where $s \equiv \frac{1+\kappa}{2+\rho+\kappa}$.

Comparing this equation with equation (7) (for $\theta = 1$) reveal that they are identical save for the presence of $\bar{x}$ – the constant A/K ratio. Observing that $\bar{x}$ is independent of L it is clear that the Corollary carries over to the present model featuring endogenous R&D.

From a practical perspective the extended model shows that the relevant scale variable is efficiency units of labor, $\bar{x}L$, and not just raw labor, L. Still,

[6] In their paper, Antinolfi et al. (2001) immediately normalize the size of the labor force to one, for which reason their analysis is silent about the scale issue. In addition, their analysis does not allow for bequest. Hence, the present model can be seen as a generalization of their framework.

[7] The R&D specification we adopt here is essentially what is labelled "the lap equipment formulation" in the literature. See Rivera-Batiz and Romer (1991).

if we adopt a CES production function, as in the last section, the quantitative significance of increasing $\bar{x}L$ would clearly be identical to the impact recovered from changing L. Adding endogenous R&D therefore does not overturn the fundamental point: The savings motive matters for the impact of scale on growth.

4 Concluding Remarks

According to UN projections global population growth will continue to decline in the years to come. Indeed, according to some projections global de-population can be expected after 2040. What will be the implications for economic growth? Scale seems to have had a substantial impact on growth historically, but appears to be of second order importance today. Why is that?

Our analysis suggests that the impact of scale on growth depends on the savings motive. If the bequest motive is paramount, savings will be funded mainly by capital income. In endogenous growth models the central assumption is that of a constant marginal product of capital in the long run. As labor is complementary to capital in the production function, a larger labor force will increase the real rate of return, savings and growth. In models where the bequest motive is all-important, a positive scale effect is therefore an inherent feature, unless the model is somehow modified. However, if the life-cycle motive dominates, savings will depend mainly on wages. Due to diminishing returns to labor, an increasing labor force will not necessarily increase savings and growth. Numerical experiments show that the scale effect is likely to be quantitatively small, and peters out when the size of the labor force rises, if the bequest motive and the life-cycle motive are about equally important for capital accumulation. In a set-up where growth is driven by capital accumulation and R&D, this result carries over. However, the notion of scale changes from the pure size of the labor force, to the labor force in efficiency units.

The model would predict a modest impact on world growth from a declining world population. Indeed, global growth could even accelerate if the global population starts to decline. Moreover, if the importance of bequest in capital accumulation declines during development, as it arguably did in the Western world during the last few centuries, the scale effect is dampened and may disappear altogether. This could account (at least in part) for the tension between historical evidence in favor of scale effects, and the lack of the same in contemporary developed economies.

The analysis emphasizes the need for a clearer understanding of the income sources of aggregate savings. Previous research has shown that the relative importance of bequest and life-cycle savings is central to a variety of macroeconomic outcomes as it implies different links between wage and capital income on the one hand, and aggregate savings on the other. The present paper adds the impact of scale on growth to the list. Hence, understanding the sources of aggregate savings might yield important insights into the growth process. In addition, the present analysis does not explain why the importance of bequest in capital accumulation declines during development. Providing a fully articulated account for the declining importance of scale, where bequests endogenously becomes relatively less important in capital accumulation, could be an interesting topic for future research.

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