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Abstract

In this paper we contribute to the study of how democracy works when politicians are better informed than the electorate about conditions relevant for policy choice. We do so by setting up and analyzing a game theoretic model of electoral competition. An important feature of the model is that candidate quality is state-dependent. Our main insight is that if the electorate is sufficiently well informed then there exists an equilibrium where the candidates' policy positions reveal their information and the policy outcome is the same as it would be if voters were fully informed (the median policy in the true state of the world).

Keywords: Electoral Competition, Uncertainty, Private Information, Candidate Quality, Revealing Equilibria.

JEL Classification: D72, D82.

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1 Introduction

It is a reasonable assumption that politicians are generally better informed than the electorate about conditions relevant for policy choice. They usually have staff to help them receive and process information and sometimes have access to information that is not public, for example information related to national security. Furthermore they have much stronger incentives than voters to be well informed because their carreers depend on how they do as policy makers. In this paper we contribute to the study of how democracy works when politicians are better informed than voters. We do so by setting up and analyzing a game theoretic model of electoral competition.

We consider an election with two candidates and one issue. The candidates are purely office-motivated, i.e. their only objective is to maximize the probability of winning. Before the election the candidates announce credible policy positions. There are two states of the world. Both candidates are informed about the true state when they announce their positions. Voters are only partially informed about the state, they receive a signal that is correlated with the true state. This signal is private information, i.e. it is unknown to the candidates when they announce positions. Each voter has a single peaked policy utility function in each state and the preferred policy is different in the two states.

The voters do not only care about policy, they also care about candidate quality. One candidate has a quality advantage in one state and the other candidate has a quality advantage in the other state. Furthermore there is a stochastic element in voter evaluation of candidates. Suppose for example that the two candidates have announced the same position and that the voters have inferred the true state. Then the candidate with a quality advantage wins with a probability that is greater than one half (because of the quality advantage) but smaller than one (because of the stochastic element of voter evaluation).

A revealing equilibrium is one where at least one of the candidates announces different policies in the two states. Thus voters can infer the true state. Our first main result is that in any such equilibrium (satisfying a known refinement condition) the candidates converge to the median position of the true state. Our second main result is that a revealing equilibrium exists when the electorate is sufficiently well informed about the state of the world. So when voters are sufficiently well informed then it is at least a possibility (there could exist non-revealing equilibria) that electoral competition works as if the voters were fully informed.

Our first result on non-revealing equilibria show that many of these exist. Furthermore, we see that no matter how well informed the electorate is there always exists a non-revealing equilibrium, even with a symmetry restriction. None of these equilibria can be eliminated by the Intuitive Criterion (Cho and Kreps (1987)) which is the most commonly used refinement condition in signalling games. Instead we show that a monotonicity condition on voters' beliefs does eliminate many of the non-revealing equilibria. With that condition a non-revealing equilibrium only exists if the voters are not too well informed. We also see that the candidates diverge by at least the distance between the medians in the two states.

Before we move on we will present a stylized example of a real world situation where our model applies. Suppose a retired general is running against a succesful governor for the US presidency. Both of them primarily care about getting elected, policy preferences are secondary. The main issue is how much of a fixed tax revenue to spend on national security related public goods (e.g. military services, anti-terrorism, a missile defense system). The rest of the budget is spend on other public goods (e.g. health care, education, infrastructure). The candidates know more about the security threat to the country than the voters because they get national security briefings while voters only get information from the media. When the threat is high then each voter wants to spend more on security related public goods than when it is low. Thus the median preferred level of national secuity spending is higher when the threat is high. Furthermore, when the security threat is high then the general has a quality advantage (national security issues are more important) and when the threat is low then the governor has a quality advantage (domestic issues are more important). The possibility of unforeseen events, scandals, campaign mistakes etc. makes voting stochastic.

The paper is organized as follows. In Section 2 we review related literature. Then, in Section 3 and 4, we set up the model and define our notion of equilibrium. Section 5 and 6 contain our results on revealing and non-revealing equilibria. Finally we discuss and conclude in Section 7.

2 Related Literature

The two most immediately related papers are Schultz (1996) and Martinelli (2001). They ask the same general question as we do but they both assume that candidates are policy-motivated. This is fundamentally different from our assumption about completely office-motivated candidates. In Schultz (1996) candidates are fully informed about the state of the economy while voters are uninformed. Thus voters only receive information from the candidates' credible positions. There is revelation in (refined) equilibrium if at least one of the candidates have policy preferences that are sufficiently similar to the preferences of the median voter. In any revealing equilibrium there is convergence to the median policy of the true state of the world.

Martinelli (2001) considers a model where both candidates and voters receive private information about the state of the world but candidates are better informed than voters. The main result is that a revealing equilibrium always exists. This depends crucially on the assumption that voters have private information. The candidates do not converge in revealing equilibria.

Several other papers study models in which politicians are better informed than the electorate. In both Alesina and Cukierman (1990) and Harrington (1993) policy is decided after the election and voters are uncertain about the candidates' policy preferences. Therefore earlier policy decisions by the incumbent reveal information to the voters about what he will do if reelected. That induces the incumbent (who wants to be reelected) to distort his policy choice. In Alesina and Cukierman he does so by choosing a noisy policy instrument, in Harrington it is done by choosing a policy that is more likely to be well received.

Roemer (1994) considers a model where two policy motivated candidates (parties) are better informed about how the economy works than the electorate. Candidates announce both policies and theories of the economy, voters update their beliefs based only on announced theories. In equilibrium there is convergence to the median with respect to policy but divergence with respect to theory.

In Cukierman and Tommasi (1998) the incumbent is better informed than voters about how different policies map into outcomes. Voters update beliefs based on the incumbents (credible) policy announcement and votes for reelection if his announcement is preferred to the expected policy of the challenger. The main insight is that relatively extreme right wing policies are more likely to be implemented by a left wing incumbent (and vice versa) because of credibility issues.

Our model is also related to the literature on candidate quality/valence advantage. Recent contributions to this literature are Ansolabehere and Snyder (2000), Groseclose (2001) and Aragones and Palfrey (2002, 2005). These papers all analyze models of electoral competition where candidates differ in quality such that if they announce sufficiently similar policy positions then each voter votes for the candidate of highest quality. There is no uncertainty about who the high quality candidate is. This is fundamentally different from our paper where no candidate has an a priori quality advantage because quality is state-dependent. We are not aware of other models with uncertainty about candidate quality.

3 The Model

We consider a one issue election. The policy space X is some closed interval (bounded or unbounded) on the real axis. There are two purely office-motivated candidates, i.e. their only objective is to maximize the probability of winning.

The electorate consists of a continuum of voters (indexed by i). The voters have utility functions over the policy space. The utility functions depend on the state of the world ω which can be either L or H. The utility function of voter i is

$$u_i(x|\omega) = -|x - x_i^*(\omega)|, \quad x \in X, \omega \in \{L, H\},$$

where $x_i^*(\omega)$ is the preferred policy of voter i in state ω. The preferred policies of the voters in each state are distributed according to some distribution functions F_L, F_H. In each state there are unique median positions, i.e. unique $x_{m_L}^*, x_{m_H}^* \in X$ such that

$$F_L(x_{m_L}^*) = F_H(x_{m_H}^*) = \frac{1}{2}.$$

We assume that the median is further to the right in state H than in state L, i.e.

$$x_{m_L}^* < x_{m_H}^*.$$

Furthermore, we assume that the ordering of voters by preferred positions is the same in the two states. Formally, for all voters i, j,

$$x_i^*(L) \leq x_j^*(L) \iff x_i^*(H) \leq x_j^*(H)$$

This implies that, for all voters i,

$$x_i^*(L) = x_{m_L}^* \iff x_i^*(H) = x_{m_H}^*$$

Thus a voter with preferred policy equal to the median in one state also has preferred policy equal to the median in the other state. So (with only a slight abuse of language) it makes sense to speak about *the* median voter.

The candidates are fully informed about the state of the world. The voters only receive a signal

$$\omega^V \in \{l, h\}.$$

All voters receive the same signal and the signal is unknown to the candidates when they announce positions. The signal is distributed according to

$$\begin{aligned} \Pr(l|L) &= \Pr(h|H) = \theta, \\ \Pr(l|H) &= \Pr(h|L) = 1 - \theta, \end{aligned}$$

where $\theta \in (\frac{1}{2}, 1)$ is a parameter. Each voter has the prior $\Pr(L) = \Pr(H) = \frac{1}{2}$. So if voters update based on their signal then their belief is given by

$$\begin{aligned} \Pr(L|l) &= \Pr(H|h) = \theta, \\ \Pr(H|l) &= \Pr(L|h) = 1 - \theta. \end{aligned}$$

Candidate quality is state-dependent. One candidate ("Candidate L") has a quality advantage in the L state while the other candidate ("Candidate H") has a quality advantage in the H state. Furthermore there is a symmetric stochastic element to each voter's candidate preference. These two features are modelled the following way. Suppose Candidate L has announced the policy x^L and that

Candidate H has announced x^H. Then voter i's utility of voting for Candidate L is

$$U_i^L(x^L|\omega) = u_i(x^L|\omega) + \Gamma_L(\omega) + \delta,$$

where, for some parameter $\gamma > 0$,

$$\Gamma_L(\omega) = \left\{ \begin{array}{c} \gamma \text{ if } \omega = L \\ 0 \text{ if } \omega = H \end{array} \right\}$$

and, for some parameter $\sigma > 0$, δ is drawn from a uniform distribution on the interval $[-\frac{1}{2\sigma}, \frac{1}{2\sigma}]$. Note that the realized value of δ is the same for all voters. Voter i's utility of voting for Candidate H is

$$U_i^H(x^H|\omega) = u_i(x^H|\omega) + \Gamma_H(\omega).$$

where

$$\Gamma_H(\omega) = \left\{ \begin{array}{c} 0 \text{ if } \omega = L \\ \gamma \text{ if } \omega = H \end{array} \right\}.$$

Each voter votes for the candidate giving him the highest expected utility based on his belief about the state of the world. So if voter i believes that the probability of state L is μ_L then he votes for Candidate L if

$$\mu_L(u_i(x^L|L)+\gamma+\delta)+(1-\mu_L)(u_i(x^L|H)+\delta) > \mu_L u_i(x^H|L)+(1-\mu_L)(u_i(x^H|H)+\gamma).$$

This is equivalent to

$$\delta > \mu_L(u_i(x^H|L) - u_i(x^L|L) - \gamma) + (1 - \mu_L)(u_i(x^H|H) - u_i(x^L|H) + \gamma).$$

If we plug in the policy utility function of the voter then this inequality becomes

$$\delta > \mu_L(|x^L - x_i^*(L)| - |x^H - x_i^*(L)| - \gamma) + (1 - \mu_L)((|x^L - x_i^*(H)| - |x^H - x_i^*(H)| + \gamma).$$

The timeline of the election game is as follows:

1. The candidates observe the state of the world and then simultaneously announce policy positions.

2. The voters observe the candidates' positions and receive a signal about the state of the world. The value of δ is realized. The voters cast their votes.

3. The winning candidate enacts his announced position (positions are credible).

4 Equilibrium

A strategy profile for the candidates consists of a policy announcement in each state of the world for each candidate and can therefore be written

$$(x^L(L), x^L(H)), (x^H(L), x^H(H)).$$

The belief functions of the voters depend on the two candidates' announcements and the voters' signal. We make the assumption that all voters have the same belief function. The voters' belief about the probability of state L is written

$$\mu_L(x^L, x^H, \omega^V).$$

Each candidate's objective is to maximize the probability of winning in each state given the other candidate's strategy, the belief function of the voters, the distribution of the voters' signal and the distribution of δ. The following lemma shows that the median voter decides the election.

Lemma 4.1 *Suppose that, given the candidates' announcements, the voters' signal, and the realization of δ, the median voter strictly prefers Candidate L (H). Then a strict majority of voters prefers Candidate L (H).*

Proof. Suppose the median voter strictly prefers Candidate L, i.e.

$$\delta > \mu_L(|x^L - x^*_{m_L}| - |x^H - x^*_{m_L}| - \gamma) + (1 - \mu_L)(((|x^L - x^*_{m_H}| - |x^H - x^*_{m_H}| + \gamma).$$

We then have to show that for each voter i in a strict majority,

$$\delta > \mu_L(|x^L - x^*_i(L)| - |x^H - x^*_i(L)| - \gamma) + (1 - \mu_L)(((|x^L - x^*_i(H)| - |x^H - x^*_i(H)| + \gamma).$$

Suppose $x^L \leq x^H$ (the other case is analogous). It then suffices to show that the inequality above holds for all voters i with $x^*_i(L) \leq x^*_{m_L}$ (that is only a weak majority but a simple continuity argument shows that the inequality also holds for voters with a preferred point slightly to the right of the median).

Pick a voter i with $x^*_i(L) \leq x^*_{m_L}$. The inequality is satisfied if

$$|x^L - x^*_i(L)| - |x^H - x^*_i(L)| \leq |x^L - x^*_{m_L}| - |x^H - x^*_{m_L}|$$

and

$$|x^L - x^*_i(H)| - |x^H - x^*_i(H)| \leq |x^L - x^*_{m_H}| - |x^H - x^*_{m_H}|.$$

These inequalities are straightforward to verify.

The proof of the statement when the median voter strictly prefers Candidate H is analogous.□

By the lemma we see that if $(x^H(L), x^H(H))$ is the strategy of Candidate H then the problem of Candidate L in state ω is

$$\max_{x} \Pr_{\delta,(\omega^V|\omega)}[\delta > \mu_L(x, x^H(\omega), \omega^V)((|x - x^*_{m_L}| - |x^H(\omega) - x^*_{m_L}| - \gamma) + \\ (1 - \mu_L(x, x^H(\omega), \omega^V))(|x - x^*_{m_H}| - |x^H(\omega) - x^*_{m_H}| + \gamma)].$$

And if $(x^L(L), x^L(H))$ is the strategy of Candidate L then the problem of Candidate H in state ω is

$$\max_{x} \Pr_{\delta,(\omega^V|\omega)}[\delta < \mu_L(x^L(\omega), x, \omega^V)((|x^L(\omega) - x^*_{m_L}| - |x - x^*_{m_L}| - \gamma) + \\ (1 - \mu_L(x^L(\omega), x, \omega^V))(|x^L(\omega) - x^*_{m_H}| - |x - x^*_{m_H}| + \gamma)].$$

Then we are ready to define our notion of equilibrium. It is that of Perfect Bayesian Equilibrium with the extra condition that all voters have the same belief function.

Definition 4.2 (Equilibrium) *An equilibrium consists of candidate strategies*

$$(\hat{x}^L(L), \hat{x}^L(H)), (\hat{x}^H(L), \hat{x}^H(H)),$$

and a voter belief function about the probability of state L

$$\hat{\mu}_L(x^L, x^H, \omega^V)$$

such that

1. *In each state each candidate's announcement maximizes his probability of winning given the other candidates announcement, the belief function of the voter, the distribution of the voter's signal and the distribution of δ;*

2. *The belief function is consistent with Bayes' rule on the equilibrium path. I.e. if $\hat{x}^L(L) \neq \hat{x}^L(H)$ or $\hat{x}^H(L) \neq \hat{x}^H(H)$ then*

$$\begin{aligned} \hat{\mu}_L(\hat{x}^L(L), \hat{x}^H(L), l) &= \hat{\mu}_L(\hat{x}^L(L), \hat{x}^H(L), h) = 1, \\ \hat{\mu}_L(\hat{x}^L(H), \hat{x}^H(H), l) &= \hat{\mu}_L(\hat{x}^L(H), \hat{x}^H(H), h) = 0. \end{aligned}$$

   And if $\hat{x}^L(L) = \hat{x}^L(H)$ and $\hat{x}^H(L) = \hat{x}^H(H)$ then

$$\hat{\mu}_L(\hat{x}^L(L), \hat{x}^H(L), \omega^V) = \hat{\mu}_L(\hat{x}^L(H), \hat{x}^H(H), \omega^V) = \left\{ \begin{array}{cc} \theta & \text{if } \omega^V = l \\ 1-\theta & \text{if } \omega^V = h \end{array} \right\}.$$

An equilibrium where the announcements of the candidates reveal the state to the voters, i.e. at least one of the candidates announces different positions in the two states, is called a *revealing equilibrium.* An equilibrium where each candidate announces the same position in both states is called a *non-revealing equilibrium.*

5 Revealing Equilibria

We will first introduce a refinement condition that puts restrictions on out-of-equilibrium beliefs in revealing equilibria. It has been used by Schultz (1996) in a similar setting. The content of the condition is that if one candidate's strategy reveals the state (i.e. he takes different positions in the two states) and the other candidate deviates to an out-of-equilibrium position then the voters believe the non-deviating candidate.

Definition 5.1 (Refinement Condition (R1)) *Consider a revealing equilibrium where the candidate strategies are $(\hat{x}^L(L), \hat{x}^L(H))$ and $(\hat{x}^H(L), \hat{x}^H(H))$ and the voter belief function is $\hat{\mu}_L$. It satisfies (R1) if the following two conditions are satisfied.*

1. *Suppose $\hat{x}^L(L) \neq \hat{x}^L(H)$. Then*

$$\begin{aligned}\hat{\mu}_L(\hat{x}^L(L), x, l) &= \hat{\mu}_L(\hat{x}^L(L), x, h) = 1 \text{ and} \\ \hat{\mu}_L(\hat{x}^L(H), x, l) &= \hat{\mu}_L(\hat{x}^L(H), x, h) = 0 \text{ for all } x \neq \hat{x}^H(L), \hat{x}^H(H).\end{aligned}$$

2. *Suppose $\hat{x}^H(L) \neq \hat{x}^H(H)$. Then*

$$\begin{aligned}\hat{\mu}_L(x, \hat{x}^H(L), l) &= \hat{\mu}_L(x, \hat{x}^H(L), h) = 1 \text{ and} \\ \hat{\mu}_L(x, \hat{x}^H(H), l) &= \hat{\mu}_L(x, \hat{x}^H(H), h) = 0 \text{ for all } x \neq \hat{x}^L(L), \hat{x}^L(H).\end{aligned}$$

Let D denote the distance between the median position in the two states, i.e.

$$D = x^*_{m_H} - x^*_{m_L}.$$

For all of our results in this and the following section we will assume that

$$\gamma + D < \frac{1}{2\sigma}.$$

Suppose for example that each candidate announces the median of the state where he has an advantage. Then the assumption implies that, no matter what the belief of the voters is, both candidates have a positive probability of winning the election. The assumption simplifies our analysis considerably because it ensures that in all situations we need to consider the realization of δ matters.

Our first result shows that in any revealing equilibrium satisfying (R1) the candidates converge to the median position of the true state.

Theorem 5.2 *In any revealing equilibrium satisfying (R1) the candidate strategies are*

$$(\hat{x}^L(L), \hat{x}^L(H)) = (\hat{x}^H(L), \hat{x}^H(H)) = (x^*_{m_L}, x^*_{m_H}).$$

Proof. Let $(\hat{x}^L(L), \hat{x}^L(H)), (\hat{x}^H(L), \hat{x}^H(H))$ be the candidate strategies in a revealing equilibrium satisfying (R1). At least one of the candidates must announce different policies in the two states. Suppose that $\hat{x}^L(L) \neq \hat{x}^L(H)$ (the case $\hat{x}^H(L) \neq \hat{x}^H(H)$ is analogous). If $\hat{x}^H(L) \neq x^*_{m_L}$ then Candidate H can win with a higher probability in state L by deviating to a position $x \neq \hat{x}^H(H)$ that is closer to $x^*_{m_L}$ than $\hat{x}^H(L)$ (by (R1) the voter will still be sure that the state is L). Thus we must have $\hat{x}^H(L) = x^*_{m_L}$. Similarly we get $\hat{x}^H(H) = x^*_{m_H}$. And then we can use the same argument for Candidate L to get $\hat{x}^L(L) = x^*_{m_L}$ and $\hat{x}^L(H) = x^*_{m_H}$.□

Our next step is to find the set of parameter values for which a revealing equilibrium satisfying (R1) exists. The following result shows that there is a cut-off value of θ such that a revealing equilibrium satisfying (R1) exists if and only if the voter signal is at least as informative as this cut-off value.

Theorem 5.3 *There exists a revealing equilibrium satisfying (R1) if and only if*

$$\theta \geq \theta^*_R,$$

where

$$\theta^*_R = \frac{1}{2} + \frac{\gamma}{2(\gamma + D)}.$$

Proof. See the Appendix.

Note that θ^*_R is increasing in γ and

$$\lim_{\gamma \to 0} \theta^*_R = \frac{1}{2}.$$

Thus we see that if the difference-in-quality parameter γ increases then the electorate has to be better informed in order to make the candidates reveal their information. The intuition behind this observation is that the higher γ is the more costly (in terms of probability of winning) it is for the disadvantaged candidate to reveal the state relative to not revealing the state. Therefore, when γ increases the new cut-off value of θ must make it more costly for the disadvantaged candidate not to reveal, i.e. it must be higher. We also see that when the difference in candidate quality vanishes then there exists a revealing equilibrium no matter how little information the electorate has.

Also note that θ^*_R is decreasing in D. So when the median positions of the two states are further apart then a revealing equilibrium exist for less informed electorates. The reason is that a higher D makes it more costly not to reveal relative to revealing for the disadvantaged candidate. The good news from this observation is that when the state of the world really matters for policy choice (i.e.

D is high) then it takes less voter information to make the candidates reveal the true state by converging to the median.

We end this section with a remark on the equilibrium voter belief function used in the proof of Theorem 5.3.

Remark 5.4 *In the proof of Theorem 5.3 the equilibrium belief function satisfies*

$$\hat{\mu}_L(x^*_{m_L}, x^*_{m_H}, l) = 1 \quad \textit{and} \quad \hat{\mu}_L(x^*_{m_L}, x^*_{m_H}, h) = 0.$$

So if the disadvantaged candidate deviates to the median of the false state then voters overinfer from their signal. Suppose we require that voters should instead be Bayesians in this case, i.e. that

$$\hat{\mu}_L(x^*_{m_L}, x^*_{m_H}, l) = \theta \quad \textit{and} \quad \hat{\mu}_L(x^*_{m_L}, x^*_{m_H}, h) = 1 - \theta.$$

Then, by mimicking the proof of Theorem 5.3, we get that a revealing equilibrium satisfying (R1) exists if and only if

$$\theta \geq \frac{1}{2} + \frac{1}{2}\sqrt{\frac{\gamma}{\gamma + D}}.$$

Compared to θ^*_R the new cut-off value is strictly larger but has qualitatively the same dependence on γ and D.

6 Non-Revealing Equilibria

We will only consider non-revealing equilibria that are symmetric in the following sense.

Definition 6.1 (Symmetry) *Consider a non-revealing equilibrium where Candidate L announces $\hat{x}^L$ and Candidate H announces $\hat{x}^H$. It is* symmetric *if*

$$|\hat{x}^L - x^*_{m_L}| = |\hat{x}^H - x^*_{m_H}| \quad \textit{and} \quad |\hat{x}^L - x^*_{m_H}| = |\hat{x}^H - x^*_{m_L}|.$$

Note that the symmetry condition is equivalent to

$$\frac{\hat{x}^L + \hat{x}^H}{2} = \frac{x^*_{m_L} + x^*_{m_H}}{2}.$$

Also note that when $\hat{x}^L$ is specified, then so is $\hat{x}^H$.

In the following result we find all possible symmetric non-revealing equilibria and the parameter values for which they exist. Remember that we still make the assumption that $\gamma + D < \frac{1}{2\sigma}$.

Theorem 6.2 *Let $\hat{x}^L$, $\hat{x}^H$ be any pair of policy announcements satisfying the symmetry condition. Then the following statements hold.*

1. *Suppose $|\hat{x}^L - x^*_{m_L}| < \gamma$. Then:*

   (a) *If $x^*_{m_L} \leq \hat{x}^L \leq x^*_{m_H}$ then $\hat{x}^L$, $\hat{x}^H$ are equilibrium announcements if and only if*

   $$\theta \leq \frac{1}{2} + \frac{1}{2}\sqrt{\frac{\gamma - (\hat{x}^L - x^*_{m_L})}{\gamma + (\hat{x}^H - \hat{x}^L)}};$$

   (b) *If $\hat{x}^L < x^*_{m_L}$ then $\hat{x}^L$, $\hat{x}^H$ are equilibrium announcements if and only if*

   $$\theta \leq \frac{1}{2} + \frac{1}{2}\sqrt{\frac{\gamma - (x^*_{m_L} - \hat{x}^L)}{\gamma + D}};$$

   (c) *If $\hat{x}^L > x^*_{m_H}$ then $\hat{x}^L$, $\hat{x}^H$ are equilibrium announcements if and only if*

   $$\theta \leq \frac{1}{2} + \frac{1}{2}\sqrt{\frac{\gamma - (\hat{x}^L - x^*_{m_L})}{\gamma - D}}.$$

2. *Suppose $|\hat{x}^L - x^*_{m_L}| \geq \gamma$. Then $\hat{x}^L$, $\hat{x}^H$ are equilibrium announcements if and only if $\hat{x}^L - \hat{x}^H = \gamma$ and $\gamma \leq D$ (this is independent of the value of θ).*

Proof. See the Appendix.

An immediate consequence of the theorem above is that a symmetric non-revealing equilibrium always exists. If $D < \gamma$ then it follows from statement *1.(a)* that there exists an equilibrium with $\hat{x}^L = x^*_{m_H}$ and $\hat{x}^H = x^*_{m_L}$ for $\theta \leq 1$. If $\gamma \leq D$ then it follows from statement *2.* that, independent of the value of θ, there exists an equilibrium with $\hat{x}^L - \hat{x}^H = \gamma$.

The abundance of non-revealing equilibria (even with the symmetry restriction) makes it natural to ask if some of them can be eliminated by a suitable refinement condition. In signalling games the most commonly used refinement condition is the Intuitive Criterion (Cho and Kreps (1987)). For non-revealing equilibria in our model the Intuitive Criterion puts the following restrictions on out-of-equilibrium beliefs. Consider a non-revealing equilibrium $\hat{x}^L$, $\hat{x}^H$, $\hat{\mu}_L$ and a deviation by Candidate L to some x. Suppose we are allowed to change the out-of-equilibrium beliefs and that by doing so we can make the deviation profitable if and only if the state is L (H). Then we must have

$$\begin{aligned} \hat{\mu}_L(x, \hat{x}^H, l) &= \hat{\mu}_L(x, \hat{x}^H, h) = 1 \\ (\hat{\mu}_L(x, \hat{x}^H, l) &= \hat{\mu}_L(x, \hat{x}^H, h) = 0). \end{aligned}$$

Analogous restrictions are put on the belief function in out-of-equilibrium situations where Candidate H deviates.

Unfortunately, as we will now show, the Intuitive Criterion does not eliminate any of the symmetric non-revealing equilibria of our model.

Theorem 6.3 *All symmetric non-revealing equilibria satisfy the Intuitive Criterion.*

Proof. See the Appendix.

One way to eliminate many of the symmetric non-revealing equilibria is to introduce a monotonicity condition on the voter belief function. The content of the condition is that if one candidate moves to a position that is closer to $x^*_{m_L}$ ($x^*_{m_H}$) but not closer to $x^*_{m_H}$ ($x^*_{m_L}$) then μ_L $(1-\mu_L)$ does not decrease.

Definition 6.4 (Monotonicity Condition (M1)) *A voter belief function μ_L satisfies condition (M1) if the following condition holds. Suppose*

$$|x - x^*_{m_L}| \leq |y - x^*_{m_L}| \quad \textit{and} \quad |x - x^*_{m_H}| \geq |y - x^*_{m_H}|.$$

Then, for all $z \in X$, $\omega^V \in \{l, h\}$,

$$\mu_L(x, z, \omega^V) \geq \mu_L(y, z, \omega^V) \quad \textit{and} \quad \mu_L(z, x, \omega^V) \geq \mu_L(z, y, \omega^V).$$

There is no directly state-dependent cost for the candidates that can justify this condition (because they are purely office-motivated). Nevertheless it does seem appealing for voters to think that if one candidate moves closer to e.g. $x^*_{m_L}$ and not closer to $x^*_{m_H}$ then state L is not less likely to be true.

It is worth noting that only considering equilibria that satisfy (M1) (i.e. the voter belief function satisfies (M1)) does not change our result on existence of revealing equilibria. More precisely the conclusion from Theorem 5.3 still holds if we require revealing equilibria to satisfy (R1) *and* (M1). This is easily seen by checking that the equilibrium belief function used in the proof satisfies (M1).

In the following result we find the candidate announcements that are possible in symmetric non-revealing equilibria satisfying (M1).

Theorem 6.5 *The candidate announcements in any symmetric non-revealing equilibrium satisfying (M1) must satisfy*

$$\hat{x}^L \leq x^*_{m_L} \quad \textit{and} \quad x^*_{m_H} \leq \hat{x}^H.$$

Proof. See the Appendix.

It is easily checked that for $\hat{x}^L$, $\hat{x}^H$ satisfying $\hat{x}^L \leq x^*_{m_L}$ and $x^*_{m_H} \leq \hat{x}^H$ the beliefs used in the proof of Theorem 6.2 satisfy (M1). Therefore we can directly use this theorem to find out when the different symmetric non-revealing equilibria satisfying (M1) exist. The following corollary sums up the most important results.

Corollary 6.6 *There exists a symmetric non-revealing equilibrium satisfying (M1) if and only if*

$$\theta \leq \theta^*_N,$$

where

$$\theta^*_N = \frac{1}{2} + \frac{1}{2}\sqrt{\frac{\gamma}{\gamma + D}}.$$

Furthermore, for any $\theta \leq \theta^*_N$ there exists a symmetric non-revealing equilibrium satisfying (M1) with

$$\hat{x}^L = x^*_{m_L} \quad \text{and} \quad \hat{x}^H = x^*_{m_H}.$$

From Theorem 5.3 and Corollary 6.6 it follows that for all parameter values either a revealing equilibrium satisfying (R1) or a symmetric non-revealing equilibrium satisfying (M1) exists. We also see that for some parameter values both types of equilibria exist.

Corollary 6.7 *For all $\gamma, D, \sigma > 0$ with $\gamma + D < \frac{1}{2\sigma}$ we have that*

$$\theta^*_R = \frac{1}{2} + \frac{\gamma}{2(\gamma + D)} < \frac{1}{2} + \frac{1}{2}\sqrt{\frac{\gamma}{\gamma + D}} = \theta^*_N.$$

So for all parameter values there exists either a revealing equilibrium satisfying (R1) or a symmetric non-revealing equilibrium satisfying (M1). Furthermore, both types of equilibria exist for all θ's in an interval of non-zero length.

Finally note that θ^*_N is equal to the cut-off value for existence of revealing equilibria satisfying (R1) *and* the extra condition from Remark 5.4. So with that extra condition on revealing equilibria we still have the existence result from the corollary, but we only have co-existence of the two types of equilibria when $\theta = \theta^*_N$.

7 Discussion

We have analyzed how electoral competition works under the following conditions:

- Candidates are better informed than voters, but voters have some private information;

- Candidates are purely office-motivated;
- Candidate quality is state-dependent.

Our most important insight was that if the electorate is sufficiently well informed then there exists a revealing equilibrium and the policy outcome of such an equilibrium is the median position in the true state of the world. If the electorate is not sufficiently well informed then only non-revealing equilibria exist and in any such equilibrium there is a possibility that the policy outcome is not the median position in the true state. Thus our analysis emphasizes the importance of voters being well informed. It is important to note that voters do not need to be fully informed for electoral competition to function as if they were fully informed. The result that candidates will reveal the true state only if the electorate is sufficiently well informed could be called "The Matthew Principle of Information": *Those who already have good information shall know the truth, but those who do not shall be lied to*[1].

Another interesting feature of our model is that policy divergence is possible in (non-revealing) equilibrium. Thus we see that candidates being better informed than voters and state-dependent candidate quality can lead to policy divergence even when candidates are purely office-motivated. As far as we know this is a new potential explanation of policy divergence in electoral competition (see e.g. section III in the review paper by Osborne (1995) for other explanations).

8 References

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[1] This was suggested by my fellow Ph.D. student Thomas Markussen.

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9 Appendix

Proof of Theorem 5.3.

First we show that $\theta \geq \theta_R^* \Rightarrow$ existence.

Consider a belief function $\hat{\mu}_L$ satisfying

$$\begin{aligned}
\hat{\mu}_L(x^*_{m_L}, x, \omega^V) &= 1 \text{ for all } x \neq x^*_{m_H}, \omega^V = l, h; \\
\hat{\mu}_L(x, x^*_{m_H}, \omega^V) &= 0 \text{ for all } x \neq x^*_{m_L}, \omega^V = l, h; \\
\hat{\mu}_L(x^*_{m_L}, x^*_{m_H}, l) &= 1,\ \mu_L(x^*_{m_L}, x^*_{m_H}, h) = 0; \\
\hat{\mu}_L(x^*_{m_H}, x, \omega^V) &= 0 \text{ for all } x \neq x^*_{m_L}, \omega^V = l, h; \\
\hat{\mu}_L(x, x^*_{m_L}, \omega^V) &= 1 \text{ for all } x \neq x^*_{m_H}, \omega^V = l, h; \\
\hat{\mu}_L(x^*_{m_H}, x^*_{m_L}, l) &= 1,\ \mu_L(x^*_{m_H}, x^*_{m_L}, h) = 0.
\end{aligned}$$

We claim that such a belief function together with the candidate strategies

$$(\hat{x}^L(L), \hat{x}^L(H)) = (\hat{x}^H(L), \hat{x}^H(H)) = (x^*_{m_L}, x^*_{m_H})$$

satisfies the equilibrium conditions and (R1). First note that the belief function satisfies Bayes' rule on the equilibrium path and (R1). Thus we just have to check the optimality of each candidate's strategy. First consider the strategies in state L. Using (R1) it follows that none of the candidates can gain by deviating to a $x \neq x^*_{m_H}$. Thus we just have to check that neither candidate can profitably deviate to $x^*_{m_H}$.

In equilibrium Candidate L wins with probability $\frac{1}{2} + \gamma\sigma$ and Candidate H wins with probability $\frac{1}{2} - \gamma\sigma$. If Candidate L deviates to $x^*_{m_H}$ then his probability of winning is

$$\theta(\frac{1}{2\sigma} + (\gamma - D))\sigma + (1-\theta)(\frac{1}{2\sigma} - (\gamma - D))\sigma = \frac{1}{2} + (2\theta - 1)(\gamma - D)\sigma < \frac{1}{2} + \gamma\sigma.$$

So that is never a profitable deviation. If Candidate H deviates to $x^*_{m_H}$ then his probability of winning is

$$\theta(\frac{1}{2\sigma} - (\gamma + D))\sigma + (1-\theta)(\frac{1}{2\sigma} + (\gamma + D))\sigma = \frac{1}{2} - (2\theta - 1)(\gamma + D)\sigma.$$

Thus the deviation is not profitable if

$$\frac{1}{2} - (2\theta - 1)(\gamma + D)\sigma \leq \frac{1}{2} - \gamma\sigma.$$

This inequality is equivalent to

$$\theta \geq \theta^*_R.$$

By symmetry it follows that if no candidate can gain from any deviation in state L, then that is also the case in state H. Thus the equilibrium conditions are satisfied if $\theta \geq \theta^*_R$.

Finally we show that existence $\Rightarrow \theta \geq \theta^*_R$.

Suppose there exists a revealing equilibrium satisfying (R1). We know from Theorem 5.2 that the candidate strategies must be

$$(\hat{x}^L(L), \hat{x}^L(H)) = (\hat{x}^H(L), \hat{x}^H(H)) = (x^*_{m_L}, x^*_{m_H}).$$

Two necessary conditions for equilibrium are that Candidate H cannot gain by deviating to $x^*_{m_H}$ in state L and that Candidate L cannot gain by deviating to $x^*_{m_L}$ in state H. Let $\hat{\mu}_L$ be the equilibrium belief and define

$$\hat{\mu}^l_L = \hat{\mu}_L(x^*_{m_L}, x^*_{m_H}, l) \text{ and } \hat{\mu}^h_L = \hat{\mu}_L(x^*_{m_L}, x^*_{m_H}, h).$$

Then the necessary conditions can be written

$$\theta(\frac{1}{2\sigma} + \hat{\mu}^l_L(\gamma + D) - (1 - \hat{\mu}^l_L)(\gamma + D))\sigma$$
$$+ (1-\theta)(\frac{1}{2\sigma} + \hat{\mu}^h_L(\gamma + D) - (1 - \hat{\mu}^h_L)(\gamma + D))\sigma \leq \frac{1}{2} - \gamma\sigma$$

and

$$\theta(\frac{1}{2\sigma}+(1-\hat{\mu}_L^h)(\gamma+D)-\hat{\mu}_L^h(\gamma+D))\sigma$$
$$+(1-\theta)(\frac{1}{2\sigma}+(1-\hat{\mu}_L^l)(\gamma+D)-\hat{\mu}_L^l(\gamma+D))\sigma\leq\frac{1}{2}-\gamma\sigma.$$

Thus it suffices to show that if both the two inequalities above are satisfied then we have $\theta \geq \theta_R^*$. By adding the two inequalities and a bit of algebra we get

$$(\hat{\mu}_L^l-\hat{\mu}_L^h)(2\theta-1)(\gamma+D)\geq\gamma.$$

Thus we see that $\hat{\mu}_L^l > \hat{\mu}_L^h$ and then it follows that

$$(2\theta-1)(\gamma+D)\geq\frac{\gamma}{(\hat{\mu}_L^l-\hat{\mu}_L^h)}\geq\gamma.$$

Rearranging this inequality we get $\theta \geq \theta_R^*$.□

Proof of Theorem 6.2.

1.(a). First we show that $\theta \geq \frac{1}{2}+\frac{1}{2}\sqrt{\frac{\gamma-(\hat{x}^L-x^*_{m_L})}{\gamma+(\hat{x}^H-\hat{x}^L)}} \Rightarrow$ existence.

Consider a belief function $\hat{\mu}_L$ satisfying

$$\begin{aligned}\hat{\mu}_L(\hat{x}^L,\hat{x}^H,l) &= \theta \text{ and } \hat{\mu}_L(\hat{x}^L,\hat{x}^H,h)=1-\theta;\\ \hat{\mu}_L(\hat{x}^L,x,\omega^V) &= 1 \text{ for all } x\neq\hat{x}^H,\omega^V=l,h;\\ \hat{\mu}_L(x,\hat{x}^H,\omega^V) &= 0 \text{ for all } x\neq\hat{x}^L,\omega^V=l,h.\end{aligned}$$

We will show that this belief function supports $\hat{x}^L$, $\hat{x}^H$ as an equilibrium. By symmetry it suffices to show that Candidate H does not have a profitable deviation. It is easily seen that this is the case if deviating to $x^*_{m_L}$ in state L is not profitable. In state L Candidate H's equilibrium probability of winning is

$$\frac{1}{2}-(2\theta-1)^2(\gamma+(\hat{x}^H-\hat{x}^L))\sigma.$$

By deviating to $x^*_{m_L}$ in state L Candidate H wins with probability

$$\frac{1}{2}-(\gamma-(\hat{x}^L-x^*_{m_L}))\sigma.$$

Thus the deviation is not profitable if

$$\frac{1}{2}-(2\theta-1)^2(\gamma+(\hat{x}^H-\hat{x}^L))\sigma\geq\frac{1}{2}-(\gamma-(\hat{x}^L-x^*_{m_L}))\sigma.$$

This inequality is satisfied if

$$\theta \geq \frac{1}{2} + \frac{1}{2}\sqrt{\frac{\gamma - (\hat{x}^L - x^*_{m_L})}{\gamma + (\hat{x}^H - \hat{x}^L)}}.$$

Then we show that existence $\Rightarrow \theta \geq \frac{1}{2} + \frac{1}{2}\sqrt{\frac{\gamma-(\hat{x}^L - x^*_{m_L})}{\gamma+(\hat{x}^H-\hat{x}^L)}}$.

Let $\hat{\mu}_L$ be the equilibrium belief function. Define

$$\hat{\mu}^l_L = \hat{\mu}_L(\hat{x}^L, x^*_{m_L}, l) \quad \text{and} \quad \hat{\mu}^h_L = \hat{\mu}_L(\hat{x}^L, x^*_{m_L}, h).$$

If Candidate H deviates to $x^*_{m_L}$ in state L he wins with probability

$$\frac{1}{2} + \theta(1 - 2\hat{\mu}^l_L)(\gamma - (\hat{x}^L - x^*_{m_L}))\sigma + (1-\theta)(1 - 2\hat{\mu}^h_L)(\gamma - (\hat{x}^L - x^*_{m_L}))\sigma.$$

No candidate can profitably deviate so we must have

$$\frac{1}{2} + \theta(1 - 2\hat{\mu}^l_L)(\gamma - (\hat{x}^L - x^*_{m_L}))\sigma + (1-\theta)(1 - 2\hat{\mu}^h_L)(\gamma - (\hat{x}^L - x^*_{m_L}))\sigma$$
$$\leq \frac{1}{2} - (2\theta - 1)^2(\gamma + (\hat{x}^H - \hat{x}^L))\sigma.$$

Since the left hand side is decreasing in $\hat{\mu}^l_L$and $\hat{\mu}^h_L$ the inequality still holds if we replace these numbers by 1's, i.e.

$$\frac{1}{2} + (\gamma - (\hat{x}^L - x^*_{m_L}))\sigma \leq \frac{1}{2} - (2\theta - 1)^2(\gamma + (\hat{x}^H - \hat{x}^L))\sigma.$$

From this inequality we easily get

$$\theta \geq \frac{1}{2} + \frac{1}{2}\sqrt{\frac{\gamma - (\hat{x}^L - x^*_{m_L})}{\gamma + (\hat{x}^H - \hat{x}^L)}}.$$

1.(b). The proof is analogous to the proof of *1.(a)*.

1.(c). The proof is analogous to the proof of *1.(a)*.

2. First we show that if $\hat{x}^L - \hat{x}^H = \gamma$ and $\gamma \leq D$ then $\hat{x}^L$, $\hat{x}^H$ are equilibrium announcements. Consider a voter belief function $\hat{\mu}_L$ satisfying

$$\begin{aligned}
\hat{\mu}_L(\hat{x}^L, \hat{x}^H, l) &= \theta \text{ and } \hat{\mu}_L(\hat{x}^L, \hat{x}^H, h) = 1 - \theta; \\
\hat{\mu}_L(\hat{x}^L, x, \omega^V) &= 0 \text{ for all } x < \hat{x}^H, \omega^V = l, h; \\
\hat{\mu}_L(\hat{x}^L, x, \omega^V) &= 1 \text{ for all } x > \hat{x}^H, \omega^V = l, h; \\
\hat{\mu}_L(x, \hat{x}^H, \omega^V) &= 0 \text{ for all } x < \hat{x}^L, \omega^V = l, h; \\
\hat{\mu}_L(x, \hat{x}^H, \omega^V) &= 1 \text{ for all } x > \hat{x}^L, \omega^V = l, h.
\end{aligned}$$

Obviously Bayes' rule is satisfied on the equilibrium path. Thus we just have to show that no candidate can profitably deviate. In equilibrium each candidate wins with probability $\frac{1}{2}$ in each state. It is easily seen that if a candidate deviates in some state then he wins with a probability that is strictly smaller that $\frac{1}{2}$. Therefore we have an equilibrium.

Finally we show that if $\hat{x}^L - \hat{x}^H \neq \gamma$ or $\gamma > D$ then $\hat{x}^L$, $\hat{x}^H$ are not equilibrium announcements.

First suppose that $\gamma > D$. If $\hat{x}^L$, $\hat{x}^H$ are equilibrium announcements then in state L Candidate H wins with a probability strictly less than $\frac{1}{2}$ (remember that $|\hat{x}^L - x^*_{m_L}| \geq \gamma$). But no matter what voters out-of-equilibrium belief are Candidate H can win with a probability of at least $\frac{1}{2}$ by deviating to $x^*_{m_L}$. Thus $\hat{x}^L$, $\hat{x}^H$ are not equilibrium announcements.

Then suppose that $\hat{x}^L - \hat{x}^H \neq \gamma$ and $\gamma \leq D$, but for now disregard the special case $\hat{x}^L - \hat{x}^H > \gamma$ and $\gamma = D$ which is handled later. If $\hat{x}^L$, $\hat{x}^H$ are equilibrium announcements then we have that in each state one of the candidates wins with probability strictly greater than $\frac{1}{2}$. Consider the state where Candidate L wins with probability greater than $\frac{1}{2}$. It is straightforward to check that if Candidate H deviates to the position x given by

$$\begin{aligned} \hat{x}^L - x = \gamma \quad &\text{if} \quad \hat{x}^L > x^*_{m_L}, \\ x = x^*_{m_L} \quad &\text{if} \quad \hat{x}^L < x^*_{m_L} \end{aligned}$$

then he wins with a probability of at least $\frac{1}{2}$ no matter what the voters' out-of-equilibrium beliefs are. Thus $\hat{x}^L$, $\hat{x}^H$ are not equilibrium announcements.

Finally consider the special case $\hat{x}^L - \hat{x}^H > \gamma$ and $\gamma = D$. Each candidate wins with probability $\frac{1}{2}$ in each state. But, in each state, by deviating to $x^*_{m_L}$ Candidate H can win with a probability stricly greater than $\frac{1}{2}$ no matter what the voters' out-of-equilibrium beliefs are. Thus $\hat{x}^L$, $\hat{x}^H$ are not equilibrium announcements. $\square$

Proof of Theorem 6.3.

Let $\hat{x}^L$, $\hat{x}^H$, $\hat{\mu}_L$ be a symmetric non-revealing equilibrium. Consider a deviation by Candidate L to a position x. If we want to change the out-of-equilibrium beliefs such that the probability of winning for Candidate L after the deviation is maximal then we should choose μ'_L such that

$$\mu'_L(x, \hat{x}^H, l) = \mu'_L(x, \hat{x}^H, h) = 1$$

or

$$\mu'_L(x, \hat{x}^H, l) = \mu'_L(x, \hat{x}^H, h) = 0.$$

Which of these equations that should be satisfied depends on $\hat{x}^H$ and x but not on the state. Hence we see that Candidate L's maximal probability of winning after the deviation is independent of the state. In equilibrium Candidate L wins with a probability $p \geq \frac{1}{2}$ if the state is L and $1-p$ if the state is H. So if x can (by changes in the belief function) be made a profitable deviation for Candidate L in state L then the same is true in state H. Similarly we get that if x can be made a profitable deviation for Candidate H in state H then the same is true in state L. Therefore the Intuitive Criterion does not eliminate the equilibrium if

$$\hat{\mu}_L(x, \hat{x}^H, l) = \hat{\mu}_L(x, \hat{x}^H, h) = 0 \quad \text{for all} \quad x \neq \hat{x}^L$$

and

$$\hat{\mu}_L(\hat{x}^L, x, l) = \hat{\mu}_L(\hat{x}^L, x, h) = 1 \quad \text{for all} \quad x \neq \hat{x}^H.$$

All equilibrium announcements in part *1.* of Theorem 6.2 are supported by such beliefs (see the proof). That is not true for the equilibrium announcements in part *2.* of the theorem. But for those equilibrium announcements each candidate wins with probability $\frac{1}{2}$ in both states. Therefore the Intuitive Criterion does not put any restrictions at all on out-of-equilibrium beliefs. □

Proof of Theorem 6.5.

Suppose $\hat{x}^L$, $\hat{x}^H$, $\hat{\mu}_L$ is a symmetric non-revealing equilibrium satisfying (M1) with $\hat{x}^L > x^*_{m_L}$. We split the proof into two cases, $\hat{x}^L \leq x^*_{m_H}$ and $\hat{x}^L > x^*_{m_H}$.

If $\hat{x}^L \leq x^*_{m_H}$ then in state L Candidate L wins with probability

$$\frac{1}{2} + (2\theta - 1)^2(\gamma + (\hat{x}^H - \hat{x}^L))\sigma.$$

If Candidate L deviates to $x^*_{m_L}$ then, by using that $\hat{\mu}_L$ satisfies (M1), we get that he wins with a probability of at least

$$\frac{1}{2} + (2\theta - 1)^2(\gamma + (\hat{x}^H - x^*_{m_L}))\sigma.$$

Since $\hat{x}^L > x^*_{m_L}$ we have $\hat{x}^H - x^*_{m_L} > \hat{x}^H - \hat{x}^L$ and thus the deviation is profitable. That is a contradiction.

If $\hat{x}^L > x^*_{m_H}$ then in state L Candidate L wins with probability

$$\frac{1}{2} + (2\theta - 1)^2(\gamma - D)\sigma.$$

Suppose Candidate L deviates to the position x satisfying $x^*_{m_H} - x = \hat{x}^L - x^*_{m_H}$. Then Candidate L has moved closer to $x^*_{m_L}$ and not closer to $x^*_{m_H}$. And then it follows by (M1) that his probability of winning has increased. That is a contradiction. □