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Rediscovering the Solow Model: An Energy Network Approach*

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Abstract. The present paper provides a new theory of capital accumulation and growth. While the law of motion for capital per worker is structurally identical to that of the neoclassical growth model (Solow, 1956), the underlying foundation is very different. In contrast to the Solow model, the purposed theory is based on thermodynamical principles and associations reflecting the geometrical properties of energy transporting networks. The theory predicts that in the absence of technological progress growth is ultimately limited by the capacity of networks to supply sufficient energy to support continual increases in the per capita stock of capital. We also examine the theory empirically, and find that cross country data supports its key predictions.

Keywords: Economic Growth, Energy, Networks.

JEL: O11, I12, J13.

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1. Introduction

The Solow (1956) model is an enduring contribution to economic growth theory. Half a century after its publication the model remains an important tool in academic work. This is scarcely surprising since the model has proven to be empirically relevant in the context of explaining cross-country differences in GDP per worker.

The version of the Solow model which has been confronted with data comprises in essence three equations: An aggregate Cobb-Douglas production function, the national accounts identity for a closed economy, and, the behavioral assumption that savings come out as a constant fraction of total income. In addition, one may allow population and technology to expand at constant exponential rates. Key predictions of the model are confirmed when confronted with data: The savings/investment share is positively associated with GDP per worker, labor force growth is negatively associated with GDP per worker, and the numerical size of these two opposing influences on labor productivity is found to be the same. The latter finding is particularly significant in that it suggest that data support the *structure* of the Solow model when production is Cobb-Douglas (C-D).

As is well known, however, there is also an anomaly. The size of the estimated elasticity from the C-D production function appears to be off. Under standard assumptions the elasticity should coincide with capitals share in national accounts. But whereas capital's share in national accounts takes on a value of around 1/3, estimations yield a significantly higher number: 0.6. There are several ways of dealing with this problem.

One approach, pursued by Mankiw et al. (1992) (and by many since then), is to argue the estimate is biased due to the omission of an important input in the production function. Upon including additional investment rates in the regression the relevant parameter estimate should fall to a level consistent with the prior of 1/3.[1]

An alternative take on the empirical problem is that data may be rejecting the assumption required to derive the prior of 1/3. In this respect it is important to recognize that the finding of a capital-output elasticity of 0.6 only (necessarily) constitutes an anomaly when we assume that an aggregate production function exist. That is, the postulate of a functional relationship between GDP and inputs (appropriately aggregated), on which basis marginal products can be

[1] Mankiw et al. (1992) suggested that it could be human capital. Further augmentations have involved e.g. intangible capital (Nonnemann and Vanhoudt, 1996) and public infrastructure (Hulten, 1996).

derived and serve to determine factor prices in general equilibrium. On theoretical grounds there is good reason to be sceptical about such an aggregate production function, as a crucial part of the model, since the assumptions under which it exists are very strong.[2]

This perspective on the empirical problem would suggest a rather different theoretical challenge. In contrast to pursuing an augmentation of the Solow model (i.e. by adding inputs in the production function), the challenge would be to provide a different foundation for the *structure* of the Solow model that does not involve the aggregate production function, in the above sense. This is the challenge taken on in the present paper.

Accordingly, below we derive the law of motion for capital without appealing to the existence of an aggregate production function. Instead the *baseline* model delivers a law of motion for capital per worker which is founded on two principles that originate from physics and biology.[3]

The first principle is the law of energy conservation from thermodynamics; energy input and use equal each other. Specifically, we postulate that energy (human and non-human) is required to build, run and maintain capital.[4] This assumption delivers a law of motion for capital; the capital stock increases if total energy expenditure exceeds the energy costs required to run and maintain the existing stock. Two remarks on this element of the model are necessary to avoid confusion.

First, the notion of "capital" is broader than the national accounts' definition. For present purposes everything ranging from factory robots to flat screen television sets, will be considered to be capital. In national accounts, only the former would qualify - the latter being a durable consumption good. Second, economic behavior is suppressed; in the baseline model we assume that *all* energy is used to fuel capital or capital accumulation. Consequently, the baseline model will speak only to feasibility. That is, the way to think about the steady state of the baseline model is as delivering an upper boundary for the per capita stock of capital, given certain physical and technological constraints. The model suggests that this upper boundary varies from one country to the next. However, since actual economies likely fall short of this boundary the model's steady state predictions will not be directly informative about observed differences in capital per worker, as opposed to potential differences. To model the "distance" to

[2]See Fisher (1969). For some recent expositions, see Felipe and Fisher (2003) and Temple (2006).

[3]The ideas advanced in this paper owes a great intellectual debt to a set of papers by West et al. (1997, 1999, 2001) and recent work of Banavar et al. (1999, 2002), as pointed out below.

[4]West et al. (2001) similarly uses this kind of an assumption to build a growth model for living tissue; energy is required for the cells to function, be maintained, and for new cells to be created.

the boundary the framework would need to be extended to include assumptions of a behavioral nature. That is, assumptions about how energy supply is allocated between investment and consumption. Such an extension would also realign the capital concept in the model with that of national accounts, as demonstrated in Section 4 of the paper.

The second principle draws on recent advances in the field of biology. Ever since Kleiber (1932) it has been known to biologists that a strong correlation, referred to as "Kleiber's law", is found between energy intake of biological organisms (basal metabolism) and their energy requirements (body mass). Specifically, the two are related (are scaled) as follows: $B = B_0 \cdot m^b$, where B is basal metabolism, m is mass, B_0 is a constant, and $b = 3/4$. Remarkably, this association holds across biological systems spanning 27 orders of magnitude in mass; from the molecular level up to whales (West and Brown, 2005).[5] Recently, biologists and physicists in collaboration have started to provide microfoundations for this scaling law (West et al., 1997, 1999; Banavar et al., 1999, 2002). The common denominator of these theories is that they fundamentally seek to explain Kleiber's law as a manifestation of how energy is diffused and absorbed in biological systems, viewed as energy transporting networks.

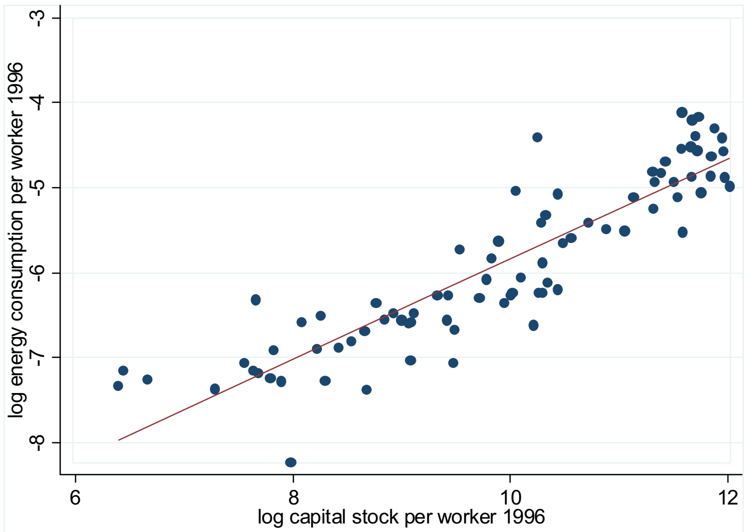
Inspired by these theories one might hypothesize that something similar holds for man-made networks. In the next section we develop a model of an economy viewed as a transport network for energy, following the work of Banavar et al. (1999). The model predicts that in economic systems energy consumption per worker (e) can, loosely speaking, be seen as the counterpart to metabolism, and capital per worker (k) as the counterpart to "body size". Accordingly, $\log(e)$ and $\log(k)$ should be linearly related. As evidenced by Figure 1, this association is clearly visible in cross-country data when cumulated investment effort is used to proxy k.

On the basis of these two principles we derive a law of motion for capital, which is *structurally* identical to the one implied by the standard Solow model when production is C-D. It is therefore less surprising that the Solow structure seems to do well when fitted to a large and heterogenous country sample; the principles underlying the alternative derivation can be expected to hold for all countries, which provides the missing justification for fitting the same structural equation to all countries on the globe.

An innovation of the present theory, compared to the standard Solow framework, is that it brings energy explicitly into the growth process. Indeed, the theory predicts that in the absence

[5] "Power laws", such as the proportionality between body mass and basal metabolism, are of course not unfamiliar to economists; see Gabaix (2006) for a succinct overview and discussion.

Figure 1: Log energy consumption per worker vs. Log capital stock per worker , 1996. 83 countries. Data: World Development Indicators 2005 & Caselli (2006).



of technological progress growth is ultimately limited by the capacity of networks to supply sufficient energy to support an ever increasing stock of capital per capita. Hence, the inability to perpetually pump sufficient amounts of energy into the network in order to sustain growth is the present model's counterpart to the classical result that diminishing returns to capital input limits growth.

After deriving our central equation, and examining its theoretical implications, we proceed by investigating key predictions of the framework empirically. In particular, we estimate the (log-)linear equation predicted by the network vision of the economy, i.e. $e \propto k^a$. As demonstrated below, the network model delivers this functional association. In addition, however, it also provides a prior for the numerical size of the slope parameter, a. If the network is efficient, and can be viewed as 3 dimensional (both in a sense to be defined below), the parameter a should equal 3/4. If, on the other hand, the network is inefficient the coefficient can go as low as 1/2. Accordingly, the theory predicts that the slope parameter should fall in an interval from 1/2 to 3/4. We find strong support for this prediction.

Over-all we detect considerable cross-country heterogeneity with respect to a, within the boundaries predicted by the model. For example, when we confine attention to the OECD area we obtain an estimate of a, which is very close to 3/4; outside the OECD the coefficient, is significantly smaller. Likewise, if we focus on Asia we obtain a slope coefficient of 0.74; if we focus on e.g. the African continent we find a coefficient near the lower boundary. Taken at face value, these results indicate that a reason for the comparative lack of economic development on the African continent is an inefficient use of energy resources.

We also look into whether the theory potentially can motivate the anomaly mentioned above. In this context we begin by re-estimating the Solow model on cross-section data for 1996; we confirm (all) the findings of Mankiw et al., which pertained to income per worker levels in 1985. Observing that the network coefficient enters the law of motion for capital in exactly the same way the production function elasticity would in a standard Solow model, we hypothesize that the former might motivate what is usually interpreted as a puzzlingly high estimate for capitals share. When estimating the network equation on the exact same selection of countries underlying the estimation of the Solow model we indeed find an estimate for a close to 0.6. In light of the findings mentioned above, this intermediate value for a can be viewed as an average for the country selection, which therefore is subject to change if the underlying sample is perturbed sufficiently.[6]

The remaining part of the paper proceeds as follows. The next section lays out a model of the economy viewed as an energy transporting network. Section 3 then combines the resulting association between energy and capital with energy conservation and proceeds to derive the transition equation for capital. This section also discuss technological change, viewed through the lenses of the present theory. Section 4 discusses the determination of long-run GDP per worker, after which we turn to the empirical testing of the model in Section 5. Finally, Section 6 concludes.

2. The Economy as a Network

While the model of Banavar et al. (1999) was developed with the purpose of explaining Kleiber's law, the authors point out that the framework is applicable to a range of networks (aside from the cardiovascular network), including those involving flow of water, air and electrical

[6] As is well known, if the regression coefficient varies in a pure cross-section regression, OLS can identify its mean value in the sample (Zellner, 1969). Durlauf and Johnson (1995) demonstrate how the structural estimate for capital's share changes when the sample is subdivided by way of regression tree techniques. The network based theory of growth would be able to explain why the estimate falls when attention is confined to the poorest countries on the globe.

currents (Banavar et al., 1999, p. 132). In this section we follow up on the latter, and consider the economy as an energy consuming network.[7]

Fundamentally, energy is assumed to originate from a source (a power plant), and is diffused across the economy to the sites at which it is used via a power grid. In keeping with the terminology of Banavar et al. (1999), we shall refer to each site as "a transfer site"; this is where energy (or "nutrients" in biology) is converted into work effort. All transfer sites are locally connected, and thus linked to the source either directly or indirectly via the transmission lines.

Technically, the network is assumed to be contained in a geometrical object; a cube, a sphere or some other shape. The precise nature of the object which engulfs the network is not restricted, except that it can be characterized by way of Euclidian geometry. More specifically, the network is space-filling, so that it is exactly contained within the geometrical object at each instant in time. In biology the geometrical object has a very tangible interpretation: the body. In the present context the surrounding object is an abstract concept: the geometrical shape which *would* be able to engulf the observed transport network. The size of the energy network is then defined as the geometric size of the shape which defines its outer contours. Specifically, the linear size of the network (thus defined) is denoted by L; the total size of the network is proportional to L^D, where D is the dimension of the network.[8]

The mean distance between the transfer sites and the source is denoted by l. The notion of "distance" between a transfer site i and the source is that of the number of transfer sites one will have to pass to get from i to the source. Now, the network expands (if it expands) in an outward direction. Hence, it is inevitable that mean distance between the transfer sites and the source rises as the network becomes larger. However, the nature of the network matters for how large the increase is. To see this, consider Figure 2 which shows (2 dimensional) sketches of networks, and summarizes some terminology.[9] Bearing the distance concept in mind, it should be clear from the sketches that mean distance will not rise to the same extend in the two cases illustrated if we add an additional transfer site, thereby increasing the size of the network incrementally. In particular, mean distance - as defined above - will rise more quickly as the network expands in the case depicted in panel B. This will turn out to be important when thinking about how

[7]For applications to drainage basins of rivers, see Maritan et al. (2002) and Rinaldo et al. (2006).

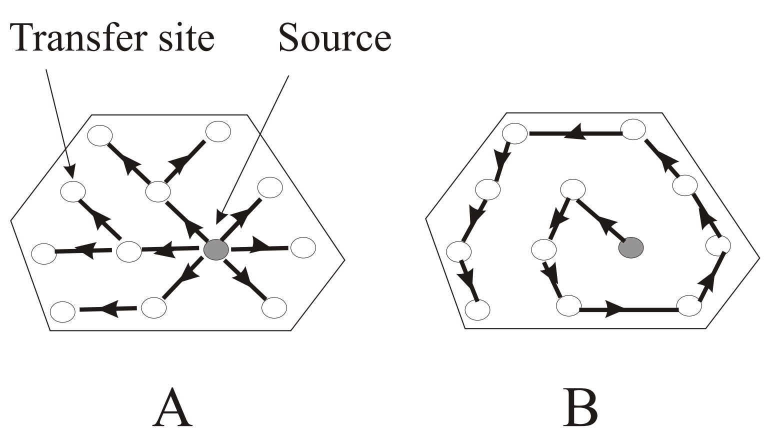
[8]Recall that the volume of a sphere (i.e. a 3 dimensional object) with radius $(1/2)\cdot L$, is $(4/3)\cdot\pi\cdot((1/2)\cdot L)^3$, which therefore is *proportional* to L^3.

[9]See Banavar et al. (1999) for further examples.

the energy requirements of the network changes in the process of capital accumulation, as will be clear below.

The fact that different networks (or the same network at different points in time) differ in total size is reflected by the total number of transfer sites it nests. To drive the link between the geometrical size of the network and the *number* of transfer sites, we impose the restriction that the *size* of each transfer site is independent of the size of the entire network. In biology, transfer sites would refer to the capillaries of the body which can be regarded as scale-invariant. In an economics context, a different interpretation is needed of course. In the present context we may think of electricity outlets as the counterpart of the capillaries (West and Brown, 2004). The size of electricity outlets are independent of the size of the associated building, and the surrounding network. They also tend to have the same size across countries; electricity outlets are no bigger in rich countries than in poor countries (though they surely are more plentiful). As a result, maintaining scale invariance of the transfer site may not be unreasonable in an economic context.

Figure 2: Sketches of networks. Source: After Banavar et al. (1999).



Assuming the network is space-filling, and that the transfer sites are scale-invariant, implies that the number of transfer sites, N, must rise with the "volume" of the network (or organism in biology): $N \propto L^D$. As each transfer site uses energy, one may anticipate an association between the size of the network, and energy consumed at (all) the transfer sites, E. More specifically, we assume

$$E \propto L^D \cdot P, \tag{1}$$

where P is the size of the population. Equation (1) says that for a given size of the network, L^D, total energy consumption should be (log-)linearly related to population size, P. As a result, changes in *per capita* energy consumption requires changes in the size of the network:

$$e \propto L^D. \tag{2}$$

That is, per capita energy consumption, e, is ultimately attributable to the number of devices (e.g. television sets, washing machines, computers and so on), which a given population utilizes. The notion is that every time a new piece of equipment is connected to an electricity outlet, a new transfer site emerges, and the network expands allowing for more energy consumption per capita.

Empirically, Kühnert et al. (2006) find strong support for a log-linear association between E and P. Using cross section energy delivery data and inhabitants of German cities, they estimate an elasticity of 1.0, with a 95% confidence interval of $[0.96; 1.06]$. In light of equation (1), an interpretation of this finding is that, at the time of data collection (year 2002), a typical city-based household (or firm) in Germany had connected an equal amount of machines and appliances with the network, irrespective of the size of city.

A key result in Banavar et al. (1999) is a mathematical theorem, which speaks to the association between *total* flow of energy in a network, F, and the linear size of the network. The theorem establishes that the following holds:

$$F \propto E \cdot l \propto E \cdot L^x, \tag{3}$$

where x depends on the characteristics of the network, as explained below.

The first part of the equation, $F \propto E \cdot l$, says that the total flow of energy in the network is proportional to energy used at the transfer sites, but with a proportionality constant that is given by the mean distance from the sites to the source. Energy is located (at a given instant in time) either at transfer sites or in the transmission lines which connect the transfer sites. Thus the total amount of energy in the network can be calculated as energy used at the sites, multiplied by the (average) distance energy has to "travel" so as to fill the entire network.

The second part (which stipulates that $l \propto L^x$) concerns how the mean distance changes as the size of the network changes. Specifically, in the most efficient class of networks - "directed networks" - $x = 1$. This type of network minimizes total energy requirements needed to fuel the economy (or organism in biology), subject to the requirement that all sites are served.[10] Figure 2, panel A, depicts an example of a directed network. At most, mean distance rises in proportion to L^D, i.e. $x = D$. This happens if the network can be seen as a space-filling spiral (see Figure 2, panel B). Accordingly, l rises as the size of the network expands, and the magnitude of the increase depends on the nature (or, efficiency) of the network in question.

Inserting equation (1) into equation (3) yields:

$$F \propto L^{D+x} \cdot P, \tag{4}$$

which implies that total energy flow per capita (F/P) rises at least with L^{D+1}, and at most by L^{2D}.

Finally, we assume proportionality between the total capital stock and total energy flow in the system:

$$F \propto K. \tag{5}$$

This assumption is thought to capture that capital is nested at the transfer sites, *and* in the network itself, i.e. in the form of the transmission lines. Hence capital is needed to transfer energy (and "hosts" energy in the process) to the sites where capital uses energy. Energy conservation in the system at large (at any given instant in time) would then suggest proportionality between K and F.

We impose exact proportionality (unit elasticity) based on the following long run consideration. Suppose we instead assumed $F \propto K^{\phi}$, where ϕ may differ from 1. If $\phi < 1$ this would mean that as the capital stock gets larger, F/K drops, which implies that capital in the limit can be applied without the use of energy. This seems like an undesirable property. Conversely, if $\phi > 1$ it implies that energy can be diffused throughout the system, in the limit, without the use of capital. This does not seem plausible either (currently at least). That is not to deny that there could be periods during which F/K rises or declines, which could be captured by allowing for a specification such as $F \propto K^{\phi}$, where ϕ differs from 1. However, we doubt such a state of affairs could be maintained in the long run. As a result, we opt for the specification where $\phi = 1$.

[10] A network is "directed" if, starting with the source, energy spreads throughout the network, away from the source, without "backtracking".

To work out the implied association between energy use per capita, and the amount of capital per capita in the economy, we need to reduce the model, which is given by equations (2), (4) and (5). Substituting equation (5) into equation (4), isolating L, and substituting the result into equation (2) yields

$$e \propto k^{\frac{D}{D+x}}, \tag{6}$$

where $k \equiv K/P$ is the per capita capital stock.

The intuition for the concave association is the following. When K rises new transfer sites emerge, and the total size of the network L^D expands. As a result, the mean distance (l) between the source and the transfer sites increase. A greater mean distance between the sites and the source implies that a greater fraction of total energy supply F is used to "fuel" the system, as opposed to being available for consumption at the sites (E).[11] Hence, while total energy supply rises in exact proportion to the expansion of the network (and thus in proportion to the capital stock), the amount of energy available at the transfer sites rises by a smaller percentage ($D/(D+x) < 1$).

Accordingly, the concavity reflects the difficulty in delivering increasing amounts of energy to machines connected to the power grid, when the size of the network expands. Moreover, notice that the coefficient would fall in a $(1/2, D/(D+1))$ interval. For a more precise prior, we need to pin down D. The most natural notion is probably that $D = 3$, i.e. a three-dimensional network. In this case the coefficient should fall in an interval from $(1/2, 3/4)$, depending on the efficiency of the network; the more efficient the higher the coefficient.

3. A Theory of Capital Accumulation and Growth

3.1. **The Model.** Consider a closed economy, described in continuous time. The first element of the growth model is the "network equation" derived above, which can be restated as

$$e(t) = \epsilon k(t)^{a}, \tag{7}$$

where $0 < a < 1$, and $\epsilon > 0$ is a constant in the sense that it is independent of capital per worker. In order to derive the fundamental law of motion for capital, we need to add an element. The additional element is the law of energy conservation.

[11] To see this formally, notice that $E/F \propto l^{-1} \propto L^{-x}$ (cf. equation (3)).

To be specific, we distinguish between two forms of energy requirements: Non-human energy $E_K(t)$ and energy supplied through humans, $E_H(t)$, i.e. human metabolism. The total energy supply, we assume, is used to run, maintain, and create capital. Supposing the energy costs of maintaining and running the "characteristic" machine is μ whereas the energy requirements to create a new machine is ν, one may express the energy balance as:

$$E_H(t) + E_K(t) = \mu K(t) + \nu \dot{K}(t). \tag{8}$$

For future reference, notice that if we were to shut off energy supply entirely, $E_H = E_K = 0$, the capital stock would shrink over time, due to lack of maintenance and replacement. The rate at which the stock shrinks is

$$\dot{K}(t)/K(t) = -\frac{\mu}{\nu},$$

which therefore can be viewed as the mirror image of the depreciation rate, commonly introduced in models of growth and capital accumulation.

Next, we divide through by the size of population, $P(t)$, in equation (8):

$$e_h(t) + e(t) = \mu k + \nu \frac{\dot{K}(t)}{P(t)},$$

where $e(t) \equiv E_K(t)/P(t)$ and $e_h \equiv E_H(t)/P(t)$. Observe that since $k(t) \equiv K(t)/P(t)$, it follows that $\dot{K}(t)/P(t) = \dot{k}(t) + nk(t)$, in so far as population expands at a constant rate $n \equiv \dot{P}(t)/P(t)$. Consequently

$$e_h(t) + e(t) = \mu k(t) + \nu \left[\dot{k}(t) + nk(t)\right],$$

which we can restate so as to yield

$$\dot{k}(t) = \left(1 + \frac{e_h}{e}\right)\frac{1}{\nu}e(t) - \left(\frac{\mu}{\nu} + n\right)k(t).$$

This expression can be simplified a bit. As pointed out by Moses and Brown (2003) human metabolism only makes out for a tiny fraction of average energy consumption in modern day industrial societies, which suggest the approximation $\frac{e_h}{e} \approx 0$.[12] Needless to say, this approximation is not entirely reasonable if one thinks about the growth process in the very long run, i.e.

[12] Moses and Brown puts it nicely into perspective (p. 296): : "*The per capita energy consumption in the United States is 11.000 W ... which is approximately 100 times the rate of biological metabolism and, ...* [it] *is the estimated rate of energy consumption of a 30.000-kg primate*"'.

the period preceding the industrial revolution where non-human energy supply was relatively scarce. But for present purposes we will work with $\frac{e_h}{e} = 0$. Accordingly, given $\frac{e_h}{e} = 0$ and in light of equation (7) we obtain

$$\dot{k}(t) = \frac{\epsilon}{\nu} k(t)^a - \left(\frac{\mu}{\nu} + n\right) k(t). \tag{9}$$

As is visually obvious, the above equation is structurally identical to the law of motion for capital per capita, as predicted by the Solow model. In standard notation, the latter would be $\dot{k}(t) = sAk(t)^\alpha - (\delta + n) k(t)$. From the perspective of the assumptions made in the present analysis, thus far, the relevant comparison would be to that of a Solow model where $s = 1$. In any case, it is unsurprising that the dynamics (including stability properties) are formally the same as in the Solow model.

The adjustment process works as follows. At any given instant in time k is predetermined. Given k a size of the underlying network is implied, and consequently the supply of energy (which is assumed to adjust), e, is determined. If e is sufficiently large, i.e. it exceeds the energy needs required to maintain and run existing capital, the stock of capital can expand further. However, as the network expands the amount of additional energy which can be made available for direct use starts to diminish (i.e. E/F declines, as noted in Section 2). Eventually, therefore, the system settles down at a steady state level of k.

As explained in the Introduction, this steady state level is to be viewed as an upper boundary to what can be achieved by way of capital accumulation, since the model (so far) ignores the national accounts dichotomy between capital and (energy consuming) durable consumption goods. Separating investment and consumption requires us to add behavioral elements to the model, and we return to this issue in Section 4.

The steady state predictions of the model are summarized in Proposition 1:

Proposition 1. *(i) In the absence of changes to the parameters of the model, the capital stock per capita converges to*

$$k^* = \left(\frac{\epsilon}{v(\mu/v + n)}\right)^{1/(1-a)}.$$

(ii) Energy consumption per capita is, in the steady state, given by

$$e^* = \epsilon \left(\frac{\epsilon}{v(\mu/v + n)}\right)^{a/(1-a)}.$$

(iii) Increases in the supply of energy per machine, ϵ, reductions in the energy costs of creating, running and maintaining capital (v and μ), and reductions in population growth, will increase capital per worker and energy consumption per worker, in the long run.

Implicit in the proposition is the assumption that energy supply can be increased to support accumulation. In this sense equation (9) pertains to *unconstrained* growth.[13] Now, it is sometimes argued that (world) growth is ultimately limited from above, by energy availability (e.g. Daly, 1977). Interestingly, however, the present analysis demonstrate that absent technological progress growth is limited *even if* energy supply were unlimited. This result is proved by the obvious existence of the steady state k^*. This brings us to the issue of how "technology" is said to be present in the model above.

3.2. **Technological Change.** Although isomorphic to the Solow model our theory proposes quite a different interpretation of adjustment dynamics and steady-states. Holding the technology of energy distribution and consumption constant any growth is conceived as adjustment towards a steady-state for capital. Only if the structure changes in terms of energy costs (e.g. ν) or energy supply (e.g. ϵ), the system restarts and converges towards a new and hopefully higher steady-state. In other words, any technological progress with permanent impact on economic performance originates from an improvement in use and distribution of energy.

At first sight the view that all lasting progress requires improvements in using or distributing energy may seem odd. It is, however, quite intuitive at closer inspection. If we accept the fact that there is little utility gained from the mere existence at subsistence level (beyond the utility received from own reproduction, see Dalgaard and Strulik, 2006), i.e. by existing merely to sustain one's own metabolism, any progress has to be fueled by non-human energy and is thus limited by the current technology to use this energy. Human *economic* development can then be understood as a perpetual series of efficiency gains in appropriating non-human energy. This way we can think of, for example, the wheel (the wheeled plow) as a device that exploits kinetic energy more efficiently and of the system of three field crop rotation as a device that exploits solar energy more efficiently than previously available methods. Indeed Jared Diamond (1997) argues that the superiority of the Eurasian vs. the Latin-American technology was caused to a large extent by the Eurasian knowledge of how to employ animal energy for various purposes

[13]Indeed, Proposition 1 implies that total energy consumption grows at the rate n in the steady state.

(and closely related, by the difference across continents in the availability of large mammals, which allowed themselves to be utilized by humans in this manner).

The emphasis on energy-use improving technologies is not meant to belittle the importance of several other pathbreaking inventions of mankind like, for example, the clock, eyeglasses, letter printing, or gunpowder. These and several other inventions have without question brought forward human *social* and *political* development a great deal. The point is that they are themselves, in scale and *economic* impact, limited by the currently available technology of energy use; think, for example, of the energy needed to print a book.

The notion of the importance of non-human energy might be helpful to understand why the introduction of some technologies did *not* initiate long-run growth. For instance, reading how Landes (1998) marvels at the efficiency benefits gained by the invention of the clock one wonders why the clock did not initiate industrialization, which instead seems to have awaited the arrival of the steam engine. Maybe the answer is that the clock predominantly improved the efficiency of using human energy? If so, the impact of the invention would be limited by the availability of human energy, which is somewhat modest in scope (see Section 3). Rather, for a *fundamental* improvement of economic development, a fundamental improvement of the use of non-human energy, like that brought forth by the steam engine, was required.

The model also sheds a new light on the general purpose technology (GPT) phenomenon. GPT innovations are viewed as "fundamental" innovations which tend to "reset" the economy, and instigate (ultimately) a growth "spurt". The process, however, may involve a non-monotonous adjustment process, with an initial slump of productivity levels while the GPT forces a replacement of old machines with new ones that employ the new basic technology.

Bresnahan and Trajtenberg (1996) who where among those who initiated GPT research asked (p. 84): "Could it be that a handful of technologies had a dramatic impact on growth over extended periods of time? What is it in the nature of the steam engine, the electric motor, or the silicon wafer, that make them prime suspects of having played such a role?" They gave a very broad answer which is still used in the literature (see e.g., Jovanovich and Roussseau, 2005): The technology must be pervasive (spread to most sectors), there must be scope for improvement over time (lowering the costs of its use) and it must be innovation spawning, i.e. it enables the production of new products. The following "handful" of technologies are usually referred to as GPT's: the waterwheel, the steam engine, electricity, railways, motor vehicles, and IT.

Based on the theory developed above we can suggest a more precise answer to Bresnahan and Trajtenberg's question. A GPT must improve either the use of energy (waterwheel, steam), its delivery through a network (railways, cars) or both (electricity, IT). Interestingly, while not all proposals of GPT candidates available in the literature coincide perfectly, electricity and IT, the technologies that revolutionized both the use and distribution of energy, are always on the lists. Speculating about what could possibly be the next GPT experts usually come up with nano-technology; again a new system for distributing energy at a new (finer) level of network. With our theory at hand it becomes intuitive why other seemingly equally fundamental innovations (e.g. the decoding of the DNA) are not GPT's: they do not (much) improve the distribution and use of non-human energy.

An ad hoc way to mimic a GPT within the standard Solow model is to simultaneously vary general productivity (A) and the depreciation rate (δ). The first parameter change is meant to capture the long-run increase in productivity, and the second captures the initial slump, originating from obsolescence of machines embodying the old technology (see Aghion and Howitt, 1998, Ch. 8.4). The problem is that both measures move the steady-state in opposite directions and some fine-tuning is needed to create the desired transitional and long-run effects.

In the current model, we have an – while admittedly equally ad hoc – more elegant way to produce the desired growth trajectory: a decrease of ν. A lower value for ν means that machines can be produced at lower energy costs, which, for example, could have been initiated through the transistor replacing the energy-intensive vacuum tube in electronic devices. From inspection of the reduced form of the model (9) we see that a lower ν has a double effect. It raises both the first term, "productivity", and the second term, "depreciation". From Proposition 1 we also see that it unambiguously raises the steady-state level. Starting at the original steady-state (using tube technology) $\dot{k}$ equals zero initially. Evaluating the RHS of (9) after the fall of ν, we see (since $a < 1$) that initially the negative effect through the depreciation channel dominates. In conclusion, energy saving technological progress causes GPT-like adjustment dynamics with an initial slump, recovery, and convergence towards a higher steady-state level.

4. Consumption, Investment and GDP per worker

The model developed in Section 3 has two drawbacks. For one thing, there is no behavioral elements, implying that the steady state of the model only pins down an upper boundary on capital accumulation. Moreover, k does not coincide with capital in the national accounts sense of the word. In this section we remedy these omissions by adding a theory of consumption/savings choice to the model developed above. This extension also allows us to study long run GDP per worker.

Following Becker (1965) we adopt the "home production" view of consumption.[14] Accordingly, consumption *services*, $\tilde{C}$, are produced using inputs of human energy (by analog to "time" in the Becker framework), $E_H = Pe_h$, and marketed goods (machines), K_c:

$$\tilde{C} = \min\left(Ze_h P, K_c\right), \tag{10}$$

where $Z > 0$ is a constant technological parameter. K_c represents capital equipment used in the context of consumption, and therefore constitutes part of the aggregate capital stock available in the economy, at a given point in time.

Assuming constant returns to scale in the home production function above, we find that per capita demand for marketed goods used in consumption fulfill

$$Ze_h = k_c.$$

On this basis we may determine the share of k^*, $1-s$, which is used for consumption purposes, in the steady state, as

$$1 - s^* = Ze_h\left(k^*\right)^{-1} = Ze_h \cdot \left[\frac{v\left(\frac{\mu}{v}+n\right)}{\epsilon}\right]^{1/(1-a)}.$$

The remaining part, s^*, is therefore used in the context of investment. Hence, if *total* per capita capital expenditures in the steady state is given by replacement, maintenance and compensation for the capital diluting effect of population growth, $(\mu/\nu+n)k^*$, then total per capita investment expenditures, *in the national accounts sense*, is given by

$$i^* = (\mu/\nu + n) \cdot s^* k^*.$$

[14] Other approaches to pinning down consumption is feasible of course. We find the Becker (1965) framework appealing in the present context, however, because it can allow human energy supply to play a role in consumption in a fairly natural way, as will be clear.

All other capital *expenditures* must therefore pertain to consumption, i.e.

$$c^* = (\mu/\nu + n) \cdot (1 - s^*) \cdot k^*.$$

Several result now follow.

First, in terms of the long run stock of capital per capita (now, in a national accounts sense), k_I^*, we have that

$$k_I^* = s^* k^*.$$

Hence, a larger propensity to invest (as captured by s^*) implies that the country gets closer to the "boundary" as given by k^*. In the limit, where $s = 1$, all machines are used for investment purposes.

Second, the level of output is determined as the sum of expenditures for consumption and investment, by virtue of the national accounts identity

$$y^* = i^* + c^* = (\mu/\nu + n)\, k^*.$$

Consequently, we have the following result

Proposition 2. *(i) In the absence of changes to the parameters of the model, output per capita converges to*

$$y^* = \frac{e^*}{\nu},$$

where e^* is the steady state level of energy consumption per capita, as given in Proposition 1.

Third, using the networks association, equation (7), we may now write labor productivity in the steady state as

$$y^* = \frac{\epsilon}{\nu}\,(k^*)^a \equiv A \cdot (k^*)^a.$$

Accordingly, the augmented model admits a reduced form log-linear association between y and k, structurally identical to a Cobb-Douglas production function. From the perspective of interpretation there are three major differences. (i) In the present case a should not be parameterized using national accounts data; (ii) A, i.e. total factor productivity in standard terminology, reflects the underlying parameters of the model (including aspects of the energy transporting network and thereby technology), and, (iii), the above association is not technological per se, but reflects a steady state association.

5. Evidence

In this section we test the framework developed above. In addition, we take a first pass look at whether the network-based approach can account for the anomaly discussed in the Introduction.

5.1. **Specifications and Estimation Strategy.** The framework developed above yields two central predictions which are amendable to direct testing by way of cross country regression analysis.

The first concerns the network equation. For empirical purposes it can be expressed as:

$$\log(e) = \beta_{0e} + \beta_{1e}\log(k) + \varepsilon_e, \tag{11}$$

where $\beta_{1e} = a$. The theoretical prediction is that β_{1e} should fall in an interval ranging from 1/2 to 3/4; one would a priori expect the latter outcome in countries where the energy transporting network is efficient, which likely is the case in advanced industrial economies.

The second prediction which admits a test to be constructed, comes from the "consumption augmented" model (Section 4), and concerns the steady state association between energy consumption and GDP. To test it we take the following equation to the data:

$$\log(Y) = \beta_{0y} + \beta_{1y}\log(E) + \varepsilon_y, \tag{12}$$

where $\log(Y)$ and $\log(E)$ refers to GDP and total energy consumption, respectively. The testable steady state prediction is that $\beta_{1y} = 1$. (cf. Proposition 2).

There is a problem with estimating equation (12), however, in that we cannot measure $1/v$. As a result, the OLS estimator will be biased, since $1/v$ determines e (cf. Proposition 1). Consquently, we invoke an IV solution. As an instrument for E we use the size of population. As should be clear from Propositions 1 and 2, P will affect E, but should not affect output for the energy supply given.[15]

5.2. **Data.** The regressions reported below pertain to 1996. Using 1996 as our year of choice allows us to use data on capital stocks calculated by Caselli (2006); PPP GDP is also taken from Caselli (2006). The size of the population comes from World Development Indicators (2005)

[15]Estimating equation (11) is not entirely problem-free either. Our empirical measure of k does not coinside perfectly with the theoretical notion, in that it does not include durable consumption goods like television sets, microwave ovens and the like. Consequently, one should expect an attenuation bias arising from the resulting measurement error. In light of the large variation in $\log k$, the hope is that this bias is "small".

(WDI). WDI is also our source for energy consumption, which is calculated as total production plus net imports of energy.

Below we also re-estimate the baseline Solow model. In this context, the investment share is calculated as an average for the 1960-96 period, and is taken from the data of Bernanke and Gurkaynak (2001). They in turn draw on Penn World Tables 6.1 (Heston, Summers and Aten, 2002). Finally, labor force growth (1960-96) derive from WDI.

5.3. **Results.** Table 1 reports our first set of results from estimating equation (11) by OLS. In the first column we run the regression on our full sample, which includes 83 countries. The fit is rather good as is also visually clear from inspecting Figure 1, which shows the full data set and the fitted regression line. As seen, the point estimate falls squarely within the predicted range at about 0.6. The next 9 columns then report the results from splitting the sample in various ways to examine the scope for parameter heterogeneity.

TABLE 1

The first split consists of testing the model on an OECD vs. Non-OECD sample. Interestingly, when we confine attention to the OECD sample we come very close to the 3/4 prediction of the model, for a 3 dimensional and efficient network. In contrast the coefficient is lower outside the OECD, suggesting inefficiencies. The next 4 columns then split the sample in a different way, by estimating equation (11) on a continent-by-continent basis. When we confine attention to Asia and Oceania, and the Americas, we once again recover a point estimate of very nearly 3/4. Results differ markedly, however, for the European and African subsamples.

As it turns out, a single country influences the estimate for the European sample considerably: Romania. If Romania is omitted the point estimate rises to 0.66, with 0.3 for standard error (not reported). Therefore, when focusing on the first three continents, a falls in a range where the hypothesis of 3/4 cannot be rejected. In contrast, the elasticity is disturbingly low in Africa, where a 3/4 hypothesis is rejected at a 5% level of significance. Instead, the elasticity is insignificantly different from 0.5, which is the lower-end prior for its size (see Section 2). These results would suggest that the low non-OECD estimate might be ascribed mainly to the influence from the African countries in the sample.

This conclusion is supported by the third split (column 7-9). In this case we split the data according to levels of GDP per worker; above and below average in the sample. For the high income sample, the coefficient is once again near 0.75, whereas it is considerably lower for the low income sample. However, if African countries are excluded (last column), we can no longer reject (at a 5 percent level of significance) that the elasticity is 3/4, though the point estimate remains low.

Table 2 reports the results from estimating the steady state prediction of the "consumption augmented" model, equation (12), by OLS. The approach is the same, in that we begin by examining the full sample and then split the sample to examine whether the association fail in some instances or not. Over-all, as should be clear, the regression results conform with priors; the slope coefficient is very close to 1, and the amount of variation in $\log(Y)$ which $\log(E)$ can explain is very high, as is visually obvious from Figure 3, which provides a visual illustration of the fit pertaining to the OECD sample. Nevertheless, omitted variable bias is a concern, as we are unable to control for $1/v$.

Figure 3: Log energy consumption vs. Log PPP GDP, 1996. 21 OECD countries. Data: World Development Indicators 2005 & Caselli (2006).

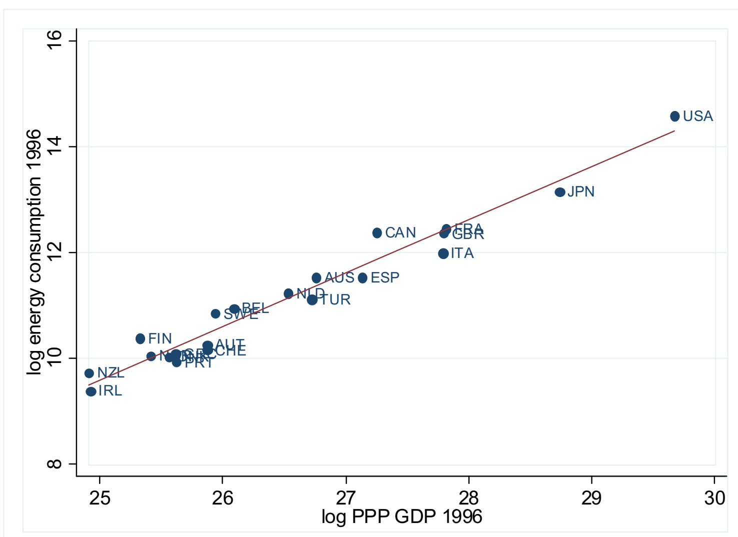


TABLE 2

Table 3 therefore re-estimates the model by 2SLS invoking $\log(P)$ (in 1960) as an instrument for $\log(E)$. Comparing the results reported in Table 2 with those of Table 3 reveal modest changes in the size of the point estimate in most cases. In general, the model does well, and the prediction of a slope coefficient of 1 cannot be rejected in the full sample and in most of the subsamples. Once again, however, Europe and Africa stands out.

TABLE 3

On the African continent the slope estimate is merely 0.88. While this estimate is insignificantly different from 1, at standard levels of significance, it reinforces the impression from Tables 1 and 2 that Africa perform considerably worse than the other continents on the globe in the context of utilizing (non human) energy.

On the European continent, in contrast, the slope estimate is significantly larger than 1. Hence, in this case the model fails to be a good description of the data, under the identifying assumption of steady state behavior. Accordingly, much like when we tested the networks theory we are forced to conclude that the European sample behaves differently from other highly developed areas.

5.4. **Revisiting the MRW "Puzzle".** The evidence presented in the last section might already be enough to suggest that the present model may account for the "anomaly" mentioned in the Introduction: the implausibly high coefficient for capital's share. Nevertheless, in this subsection we briefly examine the issue in some detail.

The Solow model evaluated at the steady-state would lead to the following solution for log capital per worker (see Mankiw et al., 1992):

$$\log(k) = \frac{1}{1-\alpha}\log(A) + \frac{1}{1-\alpha}\log(s) - \frac{1}{1-\alpha}\log(n+\delta).$$

The notation is standard: A is an index of technological sophistication which derives from the production function, s represents the investment rate, δ is the rate of capital depreciation, and α is the capital-output elasticity from the aggregate Cobb- Douglas production function.

The above equation can be implemented as the regression model:

$$\log(k) = \beta_{0k} + \beta_{1k}\log(s) + \beta_{2k}\log(n+\delta) + \varepsilon_k. \tag{13}$$

Accordingly, the model predicts that $\beta_{1k} > 0, \beta_{2k} < 0$ and that $\beta_{1k} = -\beta_{2k}$. Moreover, an implied estimate for α can be backed out as $(\beta_{1k} - 1)/\beta_{1k}$. It should be recognized that data on capital stocks are generated by the perpetual inventory method, i.e. as cumulated investments. As a result, a *significant* impact from s on k is all but guaranteed, *i.e.* $\beta_{1k} > 0$. However, neither the *size* of β_{1k} nor the sign and size of β_{2k} is a given.

The more familiar approach to getting at α is to test the Solow model's predictions regarding long-run GDP per worker:

$$\log(y) = \beta_{0y} + \beta_{1y}\log(s) + \beta_{2y}\log(n+\delta) + \varepsilon_y \tag{14}$$

where $\beta_{1y} = -\beta_{2y} = \alpha/(1-\alpha)$. This is a second specification we can employ to get an estimate for α, which we can compare with the estimate obtained using equation (13).

As should be clear, the implied capital-output elasticity (i.e. α) enters the Solow model in exactly the same way a enters the differential equation derived in Section 3. Consequently, the simple idea we are pursuing is that α and a may be conveying the same information.

TABLE 4

Table 4 reports the results from re-estimating the baseline Solow model for 1996. We report both the results from OLS, and those obtained by implementing an outlier robust estimator (the least absolute deviation - LAD - estimator). The general message from the table is that the Solow model does reasonably well, especially when focusing on the robust estimates. Whether we test the steady state predictions with respect to GDP per worker or capital per worker, the following can be concluded: The model accounts for more than half the variation in the data; the predicted determinants of long-run prosperity are significant; the prediction that $\beta_{1i} = -\beta_{2i}$, $i = k, y$ is supported and finally, the implied "share of capital", α, is about 0.6. These results conform with those of Mankiw et al. (1992).

The final two columns re-estimates the networks equation (equation (11)) on the exact same samples of countries. The key finding is that β_{1e} and the implied α from estimating the Solow model are basically coinciding - and so are their estimated standard deviations. This result hold in both samples of countries. This is encouraging in that it suggest the network theory may in fact explain the size of what is usually interpreted as the capital-output elasticity. Under the present model the magnitude of a is determined by network efficiency, and, as should be clear from the theoretical discussion above, estimates of around 0.6 are less easily refuted by the present theory, compared with the standard model where the exponent should be around 1/3.

While encouraging, these findings are admittedly only suggestive of a concordance between α and a. A stronger test would require us to retrieve data which are currently unavailable. That is, the variables v, μ and ϵ. Measuring these variables empirically and providing a full test of the model (and thereby obtaining estimates for a on this basis) will be the burden of future research.

6. Conclusion

The fundamental notion that economic growth originates from (and is limited by) energy has a long intellectual history, going back to Herbert Spencer's (1862) *First Principles.* According to Spencer the evolution of societies depends on their ability to harness increasing amounts of energy for the purpose of production. Differences in stages of development can be accounted for by energy: the more energy a society consumes the more advanced it is. Chemist and Nobel prize winner Wilhelm Ostwald (1907) developed the Spencerian ideas further. Ostwald emphasized that it is not the sheer use of energy, but the degree of efficiency by which raw energy is made available for human purposes that defines the stage of economic (and according to Ostwald also cultural) development of society.[16]

The theory developed above demonstrates that this notion of development, when given a modern network interpretation, is compatible with neoclassical growth theory. Indeed, it coincides with the structural form of the economist's core model of economic growth, the Solow growth model. At the same time, the theory developed above does not require the existence of an aggregate production function. Rather, the theory relies fundamentally on thermodynamics and efficiency laws pertaining to the distribution of energy through networks.

Empirically, we find that cross-country data support the network vision. Specifically, when we confine attention to rich areas (such as the OECD) we find support for a log-linear association between energy use and capital, with a coefficient of about 3/4. The latter is predicted by the theory, provided the network is three-dimensional and efficient. With an inefficient network, the theory predicts a lower coefficient, which is detected on the African continent, or more generally among the poorest countries in the world. Contingent on the collection of data on energy costs of production, use and maintenance of capital, a fully fledged test of the steady-state predictions of the model will be possible. This next step is left open to future research.

The proposed theory allows for some reconciliation between neoclassical growth theory and the work of some of its staunchest critics. Ecological and biophysical economists, most prominently Nicolas Georgescu-Roegen (1976) and Herman Daly (1977), reject neoclassical growth theory

[16]Further refinements were made by several natural and social scientists, among them Frederick Soddy, Alfred Lottka, and Fred Cottrell.

for not taking the laws of thermodynamics into account. The central charge is that energy, or “natural capital” in the terminology of the authors, is introduced into the models in an unsatisfactory way (if not ignored altogether). That is, by including energy in the aggregate production function as a separate input, which can be substituted for by capital. This approach is fundamentally flawed, the argument goes, because it does not take into account that any capital good is itself produced by means of energy. Moreover, this energy use cannot be avoided because any transformation of material of a low degree of order (raw material) to a high degree of order (capital goods) needs a certain amount of energy for thermodynamic reasons. Accordingly, sceptics maintain that it is difficult to see how energy can be substituted for by capital.

Here, we have explicitly taken the thermodynamic argument into account, by assuming that all (capital) goods are created, employed, and maintained through human and non-human energy use. Interestingly, however, after performing the network calculations we come up with a law of motion for capital which is structurally identical to that implied by the Solow model.

The theory also have bearing on the fundamental “limits to growth” debate. In particular, while conceding the importance of energy for growth, the theory also highlights the crucial importance of human ingenuity. As shown above, absent technological change, growth will come to a halt even with unlimited supplies of energy, since energy dissipation increases as the economic network (appliances and machines connected) becomes larger. This result therefore implies that technology, associated with the harnessing and use of energy, is as important for growth prospects as the supply of energy itself; energy and technology are equal partners in development. Indeed, as argued above, “major” innovations (which usually are referred to as GPTs) can be seen as rare instances of progress, which in a profound way improves the harnessing, transformation, or distribution of energy. Integrating the literature on endogenous technological change, with the present model of capital accumulation, would therefore seem like another useful topic for future research.

Finally, the framework could also be adapted to the study of growth in the very long run. It seems widely conceded that human societies at large enjoy income and consumption levels of historically unprecedented magnitudes (*e.g.* Galor, 2006). A key implication of the model above is that such increases is inescapably linked to the ability of human societies to expand energy supply, which requires technological innovations. In particular then, such a long-run growth model would suggest that the steam engine and the recent harnessing of electricity during the

19th century should sow the seeds of a dramatic change in human societies. These innovations allowed for investment growth, and thus income growth, of unprecedented scale, by removing the constraint on capital accumulation previously imposed by energy supply in ways of the metabolism of humans and animals. Accordingly, integrating the framework above with the existing literature on very long-run growth also seems like a fruitful avenue for future research.

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Table 1. Energy Consumption and Capital

Dep. variable:	log(e)	log(e)	log(e)	log(e)	log(e)	log(e)	log(e)	log(e)	log(e)	log(e)
Constant	-11.7^a	-13.7^a	-11.3^a	-13.4^a	-11.1^a	-13.0^a	-10.2^a	-13.0^a	-9.8^a	-10.8^a
	(0.34)	(1.37)	(0.46)	(0.74)	(1.59)	(0.94)	(0.71)	(0.98)	(0.59)	(0.07)
log(k)	0.59^a	0.77^a	0.53^a	0.74^a	0.54^a	0.71^a	0.43^a	0.71^a	0.36^a	0.47^a
	(0.03)	(0.12)	(0.05)	(0.07)	(0.14)	(0.10)	(0.09)	(0.09)	(0.07)	(0.15)
R^2	0.80	0.53	0.69	0.87	0.34	0.68	0.66	0.61	0.44	0.42
Observations	83	21	62	22	17	23	16	47	36	23
Sample	FULL	OECD	NOECD	ASIA	EUR	AME	AFR	RICH	POOR	PRNAFR

Notes: (a),(b) and (c) refer to 1, 5 and 10 percent levels of significance. The samples are (left to right): All available countries, OECD, non-OECD, Asia and Oceania, Europe,Americas, Africa, rich countries, poor countries, and poor countries without Africa. “Rich” vs. “poor” refer to above vs. below mean GDP per capita in full sample

Table 2. Energy Consumption and PPP GDP: OLS regressions

Dep. variable:	log(Y)	log(Y)	log(Y)	log(Y)	log(Y)	log(Y)	log(Y)	log(Y)	log(Y)
Constant	15.4^a	15.9^a	15.5^a	16.1^a	14.4^a	16.0^a	15.5^a	15.9^a	15.5^a
	(0.28)	(0.43)	(0.33)	(0.52)	(0.83)	(0.38)	(0.77)	(0.43)	(0.41)
log(E)	0.99^a	0.96^a	0.98^a	0.94^a	1.09^a	0.95^a	0.94^a	0.95^a	0.96^a
	(0.03)	(0.04)	(0.03)	(0.05)	(0.05)	(0.04)	(0.09)	(0.04)	(0.04)
R^2	0.91	0.96	0.87	0.95	0.93	0.95	0.82	0.94	0.86
Observations	84	21	63	22	17	23	16	48	36
Sample	FULL	OECD	NOECD	ASIA	EUR	AME	AFR	RICH	POOR

Notes: (a),(b) and (c) refer to 1, 5 and 10 percent levels of significance. The samples are (left to right): All available countries, OECD, non-OECD, Asia and Oceania, Europe,Americas, Africa, rich countries, poor countries, and poor countries without Africa. "Rich" vs. "poor" refer to above vs. below mean GDP per capita in full sample

Table 3. Energy Consumption and PPP GDP: IV regressions

Dep. variable:	log(Y)	log(Y)	log(Y)	log(Y)	log(Y)	log(Y)	log(Y)	log(Y)	log(Y)
Constant	15.2^a	15.2^a	15.2^a	15.4^a	13.9^a	15.3^a	16.0^a	15.1^a	15.1^a
	(0.36)	(0.57)	(0.45)	(0.61)	(0.77)	(0.60)	(1.14)	(0.52)	(0.45)
log(E)	1.02^a	1.02^a	1.01^a	1.00^a	1.19^a	1.02^a	0.88^a	1.03^a	1.00^a
	(0.04)	(0.05)	(0.05)	(0.05)	(0.07)	(0.06)	(0.13)	(0.05)	(0.05)
R^2	0.91	0.95	0.87	0.95	0.93	0.94	0.82	0.93	0.86
F-value (First Stage)	225.9	124.1	182.8	51.83	139.25	122.51	59.98	228.19	361.93
Observations	84	21	63	22	17	23	16	48	36
Sample	FULL	OECD	NOECD	ASIA	EUR	AME	AFR	RICH	POOR

Notes: (a),(b) and (c) refer to 1, 5 and 10 percent levels of significance. The samples are (left to right): All available countries, OECD, non-OECD, Asia and Oceania, Europe,Americas, Africa, rich countries, poor countries, and poor countries without Africa. "Rich" vs. "poor" refer to above vs. below mean GDP per capita in full sample. The instrument for $\log(E)$ is the log of population in 1960. R^2 refers to the second stage.

Table 4. Revisiting MRW

Dep. variable:	log(y)	log(y)	log(y)	log(y)	log(k)	log(k)	log(k)	log(k)	log(e)	log(e)
log(s)	1.28[a]	1.22[a]	1.55[a]	1.51[a]	2.07[a]	2.07[a]	2.46[a]	2.25[a]		
	(0.17)	(0.22)	(0.23)	(0.26)	(0.18)	(0.22)	(0.26)	(0.27)		
log (n+0.05)	−2.30[a]	−2.29[a]	−1.69[b]	−1.65[b]	−2.71[a]	−2.57[a]	−1.92[a]	−1.98[a]		
	(0.60)	(0.62)	(0.67)	(0.64)	(0.65)	(0.66)	(0.68)	(0.65)		
log(k)									0.59	0.62
									(0.04)	(0.05)
implied α/a	0.56	0.55	0.61	0.60	0.52	0.52	0.59	0.56	0.59	0.62
	(0.03)	(0.04)	(0.04)	(0.04)	(0.04)	(0.05)	(0.04)	(0.05)	(0.04)	(0.05)
$\beta_1 = -\beta_2$ (p-value)	0.16	0.18	0.87	0.86	0.40	0.55	0.54	0.74		
R^2	0.61	0.57	.	.	0.77	0.74	.	.	0.80	0.79
Observations	74	65	74	65	74	65	74	65	74	65
Estimator	OLS	OLS	LAD	LAD	OLS	OLS	LAD	LAD	OLS	OLS

Notes: All regressions include a constant. (a), (b) and (c) refer to 1, 5 and 10 percent levels of significance. Robust standard errors in parenthesis. In LAD regressions standard errors are bootstrapped with 1000 repetitions. Implied α calculated using estimates for log(s).

Countries in the non-oil (74) sample: Angola (AGO), Argentina (ARG), Australia (AUS), Austria (AUT), Belgium (BEL), Benin (BEN), Bangladesh (BGD), Bolivia (BOL), Brazil (BRA), Canada (CAN), Switzerland (CHE), Chile (CHE), Ivory Coast (CIV), Cameroon (CMR), Congo (COG), Colombia (COL), Costa Rica (CRI), Denmark (DNK), Dominican Rep. (DOM), Algeria (DZA), Ecuador (ECU), Egypt (EGY), Spain (ESP), Ethiopia (ETH), Finland (FIN), France (FRA), United Kingdom (GBR), Ghana (GHA), Greece (GRC), Guatemala (GTM), Hong Kong (HKG), Honduras (HND), Indonesia (IDN), India (IND), Ireland (IRL), Israel (ISR), Italy (ITA), Jamaica (JAM), Jordan (JOR), Japan (JPN), Kenya (KEN), Korean Republic (KOR), Sri Lanka (LKA), Morocco (MAR), Mexico (MEX), Mozambique (MOZ), Malaysia (MYS), Nigeria (NGA), Nicaragua (NIC), Netherlands (NLD), Norway (NOR), Nepal (NPL), New Zealand (NZL), Panama (PAN), Pakistan (PAK), Peru (PER), Philippines (PHL), Portugal (PRT), Paraguay (PRY), Senegal (SEN), El Salvador (SLV), Sweden (SWE), Syria (SYR), Togo (TGO), Thailand (THA), Trinidad and Tobago (TTO), Tanzania (TZA), Uruguay (URY), United States (USA), Venezuela (VEN), South Africa (ZAF), Democratic republic of Congo (ZAR), Zambia (ZMB), Zimbabwe (ZMB)