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Can Ambiguity in Electoral Competition be Explained by Projection Effects in Voters' Perceptions?*

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Abstract

Studies in political science and psychology suggest that voters' perceptions of political positions depend on their personal views of the candidates. A voter who likes/dislikes a candidate will perceive his position as closer to/further from his own than it really is (*projection*). Clearly these effects should be most pronounced when candidate positions are ambiguous. Thus a generally well liked candidate will have an incentive to take an ambiguous position. In this paper we construct a simple model to see under which conditions this incentive survives in the strategic setting of electoral competition, even if voters dislike ambiguity per se.

Keywords: Electoral competition, Ambiguity, Voter perception, Cognitive balance, Projection.

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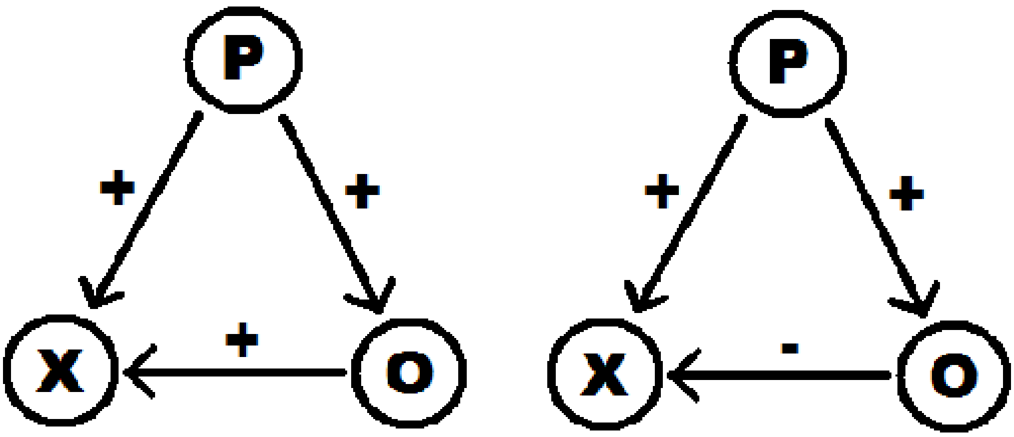
1 Introduction

Ambiguous statements are prevalent in electoral competition. Assuming politicians are rational that implies that at least some politicians have incentives to be ambiguous and that these incentives survive in the strategic setting of an election. The question then is where these incentives come from. In this paper we formulate a model that enables us to examine one possible answer to that question. The model is a standard Downsian model extended by the possibility of taking ambiguous positions and some behavioral assumptions on voter perception. In the following we will give a non-formal introduction to the model and its background.

We take the view that ambiguous policy positions should not be modelled by objective probability distributions on the policy space. This is in line with Page (1976) where it is argued that ambiguous candidates do not express their stands in a way that is properly characterized in terms of probability distributions/lotteries. Instead they give vaguely conflicting hints about their stands and only precise statements about what they do not stand for or they do not give any impression of their positions at all (p. 744-5). We claim that these observations support modelling ambiguous strategies as subsets of the policy space which will be the approach in this paper.

When ambiguous positions are represented by sets instead of probability distributions it is not obvious how to model the voters' evaluation of positions because the application of expected utility is not straightforward. For the voters to use expected utility they need to somehow associate probability distributions with the ambiguous strategies they have to rank in order to decide which candidate to vote for. This approach allows for subjectivity in voter perception which is a central aspect of our model.

Thus we need a theory for how the perception of each voter is formed, i.e. how a probability distribution is associated with a subset of policies representing an ambiguous position. For that we get inspiration from psychology, more specifically from *Balance Theory* which is a theory claiming that the attitudes and beliefs of a person (voter) will be formed or changed to achieve "cognitive balance" (see Granberg (1993), p. 75-83). Suppose we consider a person (voter) P who has attitudes towards another person (political candidate) O and an issue X (e.g. more or less redistribution) and a belief about O's attitude towards X. These are denoted P-O, P-X and O-X. Then there is cognitive balance if P agrees with and likes O or disagrees with and dislikes O. For example, if P likes O, favors more redistribution and believes that O has the same opinion, then there is cognitive balance and the set of attitudes and belief is stable. But if P likes O, favors more redistribution and thinks that O favors less redistribution then there is cognitive imbalance and the theory predicts that either P-O, P-X or X-O will change. These two situations can be illustrated by the following diagrams.



A balanced and an imbalanced set of attitudes and beliefs.

Here we will assume that before the campaign voters have formed non-policy related attitudes towards the candidates (P-O) and that they have preferences over the policy space (P-X). We also assume that these will not change during the campaign. Then, by Balance Theory, each voter's perception of the positions of the candidates (O-X) should depend on his attitudes towards the candidates and his policy preferences. Roughly speaking a voter will, in his mind, "pull" a well liked candidate towards his own preferred position and "push" a disliked candidate away from that position. This phenomenon is called *projection* or, if we want to distinguish between pulling and pushing, *assimilation* and *contrast*. Originally these terms come from *Social Judgment Theory*, but the idea that projection of some communicated message depends on the receivers attitude towards the communicator comes from Balance Theory (see Granberg (1993), p. 83-88).

Granberg (1993) cites a lot of empirical studies on projection effects in elections. Generally they support Balance Theory in the sense that voters' attitudes and beliefs are balanced although it seems to be more important to agree with a favored candidate than to disagree with a non-favored candidate. But it is hard empirically to identify how balance is achieved. Is balance reached by projection (adjusting O-X), rational selection (adjusting P-O) or persuation (adjusting P-X)? According to Granberg there is not yet a satisfactory empirical answer to that question, but some work on separating projection from rational selection and persuation suggest that projection is really happening (and so is some combination of rational selection and persuation).

A more recent empirical study is Merrill, Grofman & Adams (2001) where data from elections in Norway, France and the US are used. They also find results that can be interpreted as assimilation and contrast effects. There seems to be more assimilation than contrast. However they show that rational selection (choosing the closest candidate) together with random variation in voters' placement of candidates and/or variable voter perception of the scale can generate effects that look like assimilation. Thus there may not really be less contrast than assimilation, since part of what looks like assimilation could be rational selection (together with random variation/scale perception effects). And in some cases (US elections) there might even be more contrast than assimilation. But, even taking the rational selection caveat into consideration, "...some real assimilation is still likely to be present." (footnote 8, p. 219).

If there were no limits to projection then the positions of the candidates would become irrelevant and only non-policy related attitudes would matter for the outcome of an election. Therefore we assume that there can be no "counter factual" projection in the following sense. A certain position (a single point in the policy space) is perceived correctly by all voters. And each voter will associate an ambiguous position with some probability distribution that does not put any probability mass on policies outside the subset given by the position. In that way we ensure that voting is still policy dependent to a large extend. But projection does matter if at least one of the candidates chooses to be ambiguous. This approach to explaining political ambiguity is briefly considered in the following quote from Page (1976): "A different theoretical approach might recognize that perceptions vary, and seek incentives for ambiguity in the fact that it permits citizens to project or selectively perceive that candidates stand for whatever they want them to." (p. 748, footnote 38). Also, Social Judgment Theory claims that ambiguous messages leaves more room for projection (Granberg (1993), p. 83-84).

We will use our formal model to analyze under which conditions projection can in fact lead to ambiguity in electoral competition, even when voters are risk averse. We will save the conclusions for later, but it should already now be clear that when voters are risk averse only a generally well liked candidate can have an incentive to be ambiguous.

A number of theoretical models on ambiguity in electoral competition exist in the literature. The seminal paper is Shepsle (1972). He extends the standard Downsian model by forcing one of the candidates (the challenger) to take a lottery position. The voters are expected utility maximizers. The main result is that if a majority of voters are risk loving on an interval containing the median, then the challenger can beat an incumbent at the median by taking a lottery position with mean equal to the median. However both existence and non-existence of a winning (lottery) position for the challenger can occur.

Page (1976) is critical of Shepsle's theory of ambiguity. He notes that the prediction of ambiguity is not very strong because the challenger may not have a winning strategy. Also he questions whether (a majority of) voters are really risk loving. And, as mentioned earlier, he argues that lottery positions are not a good way of modelling ambiguous political positions because candidates do not express their positions in ways that can easily be perceived as objective probability distributions.

Page also presents his own theory of political ambiguity, *emphasis allocation theory*. He considers a multidimensional space of policy and valence dimensions. Candidates choose which dimensions (issues) to emphasize and choose positions in these dimensions. They are vague/ambiguous on issues they do not put any emphasis on. Voters evaluate a candidate by summing the utilities of the candidate's positions on the issues, weigthed by the candidate's emphasis on each issue. In an example it is shown that this leads to emphasis on consensus issues and ambiguity on issues of conflict, no matter what the risk preferences of the voters are.

Later models of ambiguity include McKelvey (1980), Glazer (1990), Alesina and Cukierman (1990), Aragones and Neeman (2000), Aragones and Postlewaite (2002) and Meirowitz (2005). Among the explanations of ambiguity are uncertainty about candidate and median voter preferences (Glazer (1990)) and sufficiently strong candidate preference for flexibility in office (Aragones and Neeman (2000)). None of the explanations offered are similar to the one we suggest in this paper.

2 The Model

Our starting point is a standard one-dimensional spatial model with two candidates. As mentioned above we will add to that model the possibility for candidates to announce ambiguous policy positions and introduce projection effects in the voters' perceptions of such positions. In the following we describe the model in details.

2.1 The Candidates

Before the election the two candidates announce policy positions. Each candidate can announce either a certain position or an ambiguous position. A certain position is simply represented by a point in the policy space $\mathbb{R}$. An ambiguous position is represented by a compact interval of non-zero length. We will assume that the maximum length of an interval representing an ambiguous position is 2 (an innocent normalization). Thus the strategy space for both candidates can be written as

$$S = \{[A - a, A + a] \mid A \in \mathbb{R},\ 0 \leq a \leq 1\}. \tag{1}$$

Announced positions are credible in the sense that the candidate who wins the election must enact a policy in his announced interval.

The candidates care only about winning or loosing, they have no policy preferences. Thus their preference relation over the outcome of the election is given by

$$win \succ draw \succ loose. \tag{2}$$

The outcome of the election is, of course, decided by how the voters evaluate the policy positions of the candidates. That will be described in the following subsection. All the information about the voters is known by the candidates.

2.2 The Voters

There is a continuum of voters and each of them has a preferred point in the policy space $\mathbb{R}$. The distribution of their preferred points is given by a density function $v : \mathbb{R} \to \mathbb{R}$. We make the rather innocent assumptions that v is continuous and that the support of v is an interval. Without loss of generality we assume that the median voter is located at $x_0 = 0$. Thus we have

$$\int_{-\infty}^{0} v(x)dx = \int_{0}^{\infty} v(x)dx = \frac{1}{2}. \tag{3}$$

Each voter has a utility function over policies. Let the utility function of the median voter be $u_0 : \mathbb{R} \to \mathbb{R}$. Then the utility function u_{x_0} of a voter with preferred point at $x_0 \in \mathbb{R}$ is defined by

$$u_{x_0}(x) = u_0(x - x_0) \quad \text{for all} \quad x \in \mathbb{R}. \tag{4}$$

We assume that u_0 is symmetric around 0 and C^2 with

$$\begin{aligned} u_0'(x) &\lesseqgtr 0 \quad \text{for all} \quad x \gtreqless 0, \\ u_0''(x) &< 0 \quad \text{for all} \quad x \in \mathbb{R}. \end{aligned} \tag{5}$$

Thus all voters are strictly risk averse.

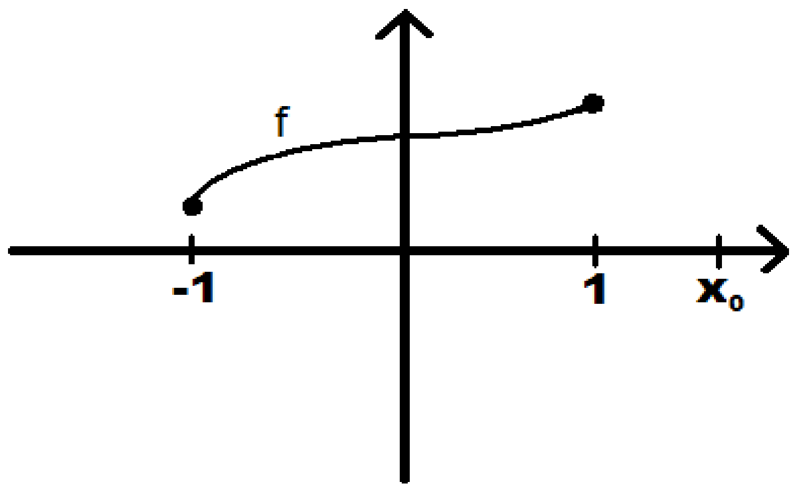
We are now ready to describe how voters decide on which candidate to vote for. If both candidates announce a certain position then each voter simply votes for the candidate announcing the position that gives the highest utility. If at least one of the candidates announces an ambiguous position then it is less obvious how the voter should decide on who to cast his vote for. We would like voters to use expected utility, but that is not straightforward since an ambiguous position is represented by an interval of policies rather than a probability distribution over policies. Thus, for a voter to use expected utility to evaluate an ambiguous position he has to somehow associate a probability distribution with the interval representing the position. The distribution represents the voter's *perception* of the ambiguous position. Or, to put it differently, the voter's *belief* about which policy the candidate will enact if elected. How voters perceive ambiguous positions is the central part of our model and we will use the rest of this section to describe it.

As described in the introduction, the main idea is that a voter's perception of an ambiguous position depends on whether he has a positive or negative (non-policy related) attitude towards the candidate announcing it and where his preferred policy is placed relative to the interval representing the position. If the voter likes the candidate then he will put most of the probability mass on the points in the interval that are closest to his preferred policy (*assimilation*). And if the voter dislikes the candidate then he will do the opposite (*contrast*). We will make the assumption that all voters have a positive attitude towards candidate 1 and a non-positive (i.e. neutral or negative) attitude towards candidate 2. It is clearly not realistic that all voters have identical attitudes towards the candidates, but it is the obvious starting point and an interesting benchmark case.

Now we are ready to describe our model of voter perception in details. First consider a voter with preferred point $x_0 \geq 1$ and suppose that candidate 1 announces the ambiguous position $[-1, 1]$ (thus we are modelling assimilation from the right in a case with maximal ambiguity around the median). Then the probability distribution that the voter associates with the ambiguous position is given by some density function $f : [-1, 1] \to \mathbb{R}$ that is weakly increasing and non-constant, i.e.

$$x \leq y \;\Rightarrow\; f(x) \leq f(y) \quad \text{and} \quad f(-1) < f(1). \tag{6}$$

We will also assume that f is continuous at the end points -1 and 1. This implies that the cumulative distribution function F strictly first order stochastically dominates that of the uniform distribution on $[-1, 1]$. That is our mathematical way of saying that the voter is assimilating $[-1, 1]$ from the right.



An example of the density function f.

Next, consider the case where, for some $0 < a < 1$, the voter has preferred point $x_0 \geq a$ and the candidate chooses the ambiguous position $[-a, a]$. Then a simple way to model the assimilation is to assume that the voters perception is given by the density function $f_a : [-a, a] \to \mathbb{R}$ defined by

$$f_a(x) = \frac{1}{a} f(\frac{x}{a}) \quad \text{for all} \quad x \in [-a, a]. \tag{7}$$

By modelling assimilation of $[-a, a]$ this way we assume that, loosely speaking, the strength of the assimilation effect does not decrease with the level of ambiguity. That will be made more precise later on (section 4) where we will present a more general way of modelling assimilation for $a < 1$ that allows for decreasing assimilation. But for now we will stick to the simple model so that we do not bury the idea of the paper in technical details.

Until now we have only considered ambiguity around the median. Suppose that candidate 1 announces an interval $[A - a, A + a]$ with $A \neq 0$. The perception of a voter with $x_0 \geq A + a$ will then be represented by the translated density function $f_{a,A} : [A - a, A + a] \to \mathbb{R}$ defined by

$$f_{a,A}(x) = f_a(x - A) \quad \text{for all} \quad x \in [-a + A, a + A]. \tag{8}$$

Thus we have modelled assimilation of any ambiguous position by voters to the right of this position. The modelling of assimilation for voters to the left of an ambiguous position follows by symmetry (reflect the density function in the midpoint of the interval).

Suppose now that candidate 2 announces an ambiguous position. Then the perceptions of the voters having a negative attitude towards him will display a contrast effect. Having modelled assimilation for voters outside the interval, it is easy to model contrast for these voters. We simply define the contrast perception of a voter to the right of the interval as the assimilation perception of a voter to the left of the interval and vice versa. The perceptions of the voters having a neutral attitude towards the candidate are given by the uniform distribution on the interval. So neutral voters all have the same (unbiased) perception of the ambiguous strategy.

It is difficult, for a general f, to make a tractable model of assimilation and contrast for voters with preferred points in the interior of an ambiguous position. So we will leave the modelling of the perceptions of "interior voters" to a special case where f has a very simple functional form. That special case will be treated in detail in the next section.

3 Results

In this section we will answer two questions. The first one is whether candidate 1 can, by being ambiguous, win the election when candidate 2 announces the median. The second question is whether candidate 1 has a winning strategy, i.e. whether he can announce an ambiguous position such that he wins no matter what candidate 2 does.

The first question we can answer without imposing any additional assumptions. To answer the second question we need to model the perceptions of interior voters, so we will only consider a special case of the model where this can be done in a fairly straightforward way.

3.1 Candidate 2 announces the median

In the standard model, a candidate announcing the median will always get at least a draw. Therefore it is interesting to see whether that is also the case in our model. It is obvious that candidate 1 will always get at least a draw by positioning himself at the median. But because of the assimilation effect that may not be the case for candidate 2. In fact the following result shows that it is not - candidate 1 can "beat the median" by being ambiguous. Considering that voters are locally risk neutral the result is actually not that surprising.

Theorem 3.1 *Suppose candidate 2 announces the median. Then there exists some* $a' > 0$ *such that, for any* $0 < a \leq a'$*, candidate 1 wins the election by announcing* $[-a, a]$ *.*

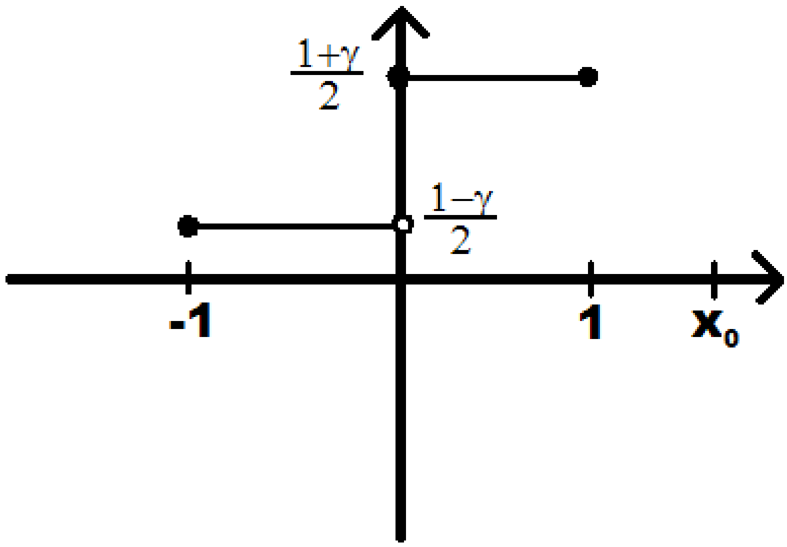
The theorem is a special case of theorem 4.1. Note that it follows immediately from the theorem that in the sequential game where candidate 2 moves first, candidate 1 can always win the election.

3.2 A winning strategy for candidate 1?

As mentioned above we will only consider a special case of the model. Let f have the following simple form

$$f(x) = \begin{cases} \frac{1-\gamma}{2} & \text{if} \quad x \in [-1, 0) \\ \\ \frac{1+\gamma}{2} & \text{if} \quad x \in [0, 1], \end{cases} \tag{9}$$

where $0 < \gamma \leq 1$ is a parameter. Obviously f is weakly increasing, non-constant and continuous at the end points.



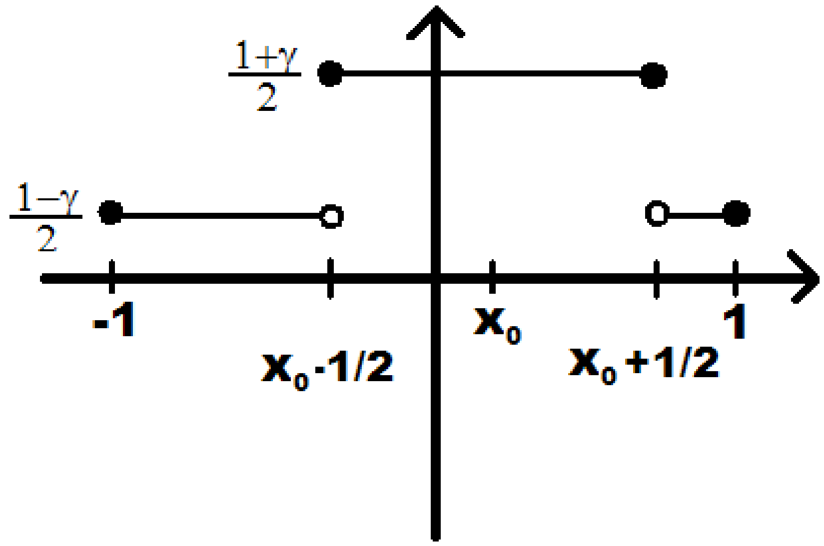
The special form of f.

As described earlier specifying f defines assimilation (and contrast) of any ambiguous strategy for "exterior voters", i.e. voters with preferred points outside the interval. For example, if candidate 1 announces $[-a, a]$ then the perception of a voter with $x_0 \geq a$ is given by

$$f_a(x) = \begin{cases} \frac{1-\gamma}{2a} & \text{if} \quad x \in [-a, 0) \\ \frac{1+\gamma}{2a} & \text{if} \quad x \in [0, a]. \end{cases} \tag{10}$$

With this simple type of projection for exterior voters there is a fairly straightforward way to extend it to interior voters. Suppose candidate 1 announces $[-1, 1]$. Then the perception of a voter with $-1 < x_0 < 1$ is given by the density function f^{x_0} defined by

$$\begin{aligned} f^{x_0}(x) &= f(x) \quad \text{if} \quad x_0 \in (\frac{1}{2}, 1), \\ f^{x_0}(x) &= \left\{ \begin{array}{ll} \frac{1-\gamma}{2} & \text{if} \quad x \notin [x_0 - \frac{1}{2}, x_0 + \frac{1}{2}] \\ \frac{1+\gamma}{2} & \text{if} \quad x \in [x_0 - \frac{1}{2}, x_0 + \frac{1}{2}] \end{array} \right\} \quad \text{if} \quad x_0 \in [-\frac{1}{2}, \frac{1}{2}] \quad \text{and} \\ f^{x_0}(x) &= f(-x) \quad \text{if} \quad x_0 \in (-1, -\frac{1}{2}). \end{aligned} \tag{11}$$



f^{x_0} for an $x_0 \in (0, \frac{1}{2})$.

This is infact all we need to model assimilation (and contrast) for interior voters, the extension to all intervals is obvious. For example, if candidate 1 announces $[-a, a]$ then the perception of a voter with $-\frac{a}{2} \leq x_0 \leq \frac{a}{2}$ is given by

$$f_a^{x_0}(x) = \begin{cases} \frac{1-\gamma}{2a} & \text{if} \quad x \notin [x_0 - \frac{a}{2}, x_0 + \frac{a}{2}] \\ \frac{1+\gamma}{2a} & \text{if} \quad x \in [x_0 - \frac{a}{2}, x_0 + \frac{a}{2}]. \end{cases} \tag{12}$$

The final assumption of this special case is that voter utility is quadratic, i.e. that $u_0(x) = -x^2$.

This special case enables us to answer the question of whether candidate 1 has a winning strategy or not, i.e. if he can announce a position (which obviously must be ambiguous) such that he will win the election no matter what candidate 2 does.

It turns out that the answer depends on γ. There exists a $\gamma^* \in (0, 1)$ such that for $\gamma \leq \gamma^*$ the answer is negative and for $\gamma > \gamma^*$ the answer is positive.

Theorem 3.2 *Consider the special case of the model described above and let* $\gamma^* = \frac{1}{2}(\sqrt{\frac{19}{3}} - 1)$ *(*≈ 0.758*). Then the following two statements hold.*

1. *Suppose* $\gamma \leq \gamma^*$*. Then candidate 1 does not have a winning strategy. More specifically, if candidate 1 announces the ambiguous position* $[A - a, A + a]$ *then candidate 2 can either win (if* $\gamma < \gamma^*$*) or get at least a draw (if* $\gamma = \gamma^*$*)*

by announcing the certain position x^* *given by*

$$x^* = \begin{cases} A + \frac{\gamma a}{2} & \textit{if} \quad A \leq 0 \\ \\ A - \frac{\gamma a}{2} & \textit{if} \quad A > 0. \end{cases} \tag{13}$$

(And if candidate 1 announces a certain position then candidate 2 can win or get a draw by announcing the median.)

2. *Suppose* $\gamma > \gamma^*$. *Then there exists an* $a' > 0$ *such that, for any* $0 < a \leq a'$, $[-a, a]$ *is a winning strategy for candidate 1.*

The theorem is a special case of theorem 4.2. Note that it follows from the two theorems in this section that if $\gamma \leq \gamma^*$ then the simultaneous move game has no Nash equilibrium. Because given any strategy of candidate 2, candidate 1 can win and given any strategy of candidate 1, candidate 2 can get at least a draw. Also note that from theorem 3.2 it follows that if $\gamma^* < \gamma$ then any pair of announcements where candidate 1 announces $[-a, a]$ for some $0 < a \leq a'$ is a Nash equilibrium (because the candidates do not care about their share of votes, only whether they win, draw or loose).

4 Extensions of the Model

In this section we will generalize our model of projection such that it allows for decreasing assimilation as the level of ambiguity decreases (in a sense that will be defined below). And then we will see how that affects our results from the previous section.

Let $(f_a)_{a\in(0,1]}$ be a family of functions such that each f_a is a density function on $[-a, a]$. Assume that each f_a is weakly increasing, non-constant and continuous at the end points. Then the f_a's define assimilation and contrast of all ambiguous positions for exterior voters (as described in section 2). For $0 < a \leq 1$ define

$$E_a = \frac{E(f_a)}{aE(f_1)}. \tag{14}$$

E_a is a first order measure of the strength of the assimilation effect. If $E_a < 1$ then the assimilation is weaker for a than for the maximal level of ambiguity in the sense that the mean of the perception has decreased proportionally faster than the level of ambiguity. We will assume that E_a weakly decreases as a decreases (so that assimilation does not increase as the level of ambiguity decreases).

It is important how strong the assimilation effect is as $a \to 0$. Therefore define

$$E_0 = \lim_{a \to 0} E_a. \tag{15}$$

If $E_0 = 0$ then the first order assimilation effect disappears as $a \to 0$. Otherwise it survives to some extend as the level of ambiguity goes to zero. Note that for the model used in the previous sections ($f_a(x) = \frac{1}{a}f(\frac{x}{a})$) we have $E_a = 1$ for all $0 < a \leq 1$ and therefore $E_0 = 1$. Thus it represents a case where the first order assimilation effect does not decrease at all as the level of ambiguity decreases. While that does make things nice and simple it is not an obviously true or innocent assumption. So it makes sense to do a robustness check by using the more general model.

The following result is a generalized version of theorem 3.1. The proof is left to the appendix.

Theorem 4.1 *Suppose $E_0 > 0$ and that candidate 2 announces the median. Then there exists an $a' > 0$ such that, for any $0 < a \leq a'$, candidate 1 wins the election by announcing $[-a, a]$.*

So as long as some of the first order assimilation effect is preserved as the level of ambiguity goes to zero, candidate 1 can "beat the median".

The general model of assimilation presented above is too general for us to answer the question of whether candidate 1 has a winning strategy. But we can generalize the result in theorem 3.2 to a situation where assimilation decreases with the level of ambiguity. Given some f we can get a model allowing for decreasing assimilation by defining

$$f_a(x) = h(a)\frac{1}{a}f(\frac{x}{a}) + (1 - h(a))\frac{1}{2a}, \tag{16}$$

where $h : (0, 1] \to [0, 1]$ is continuous, weakly increasing and satisfies $h(1) = 1$. If we let $h(a) = 1$ for all $a \in (0, 1]$ then we get the simple model. If h is strictly increasing then we have decreasing assimilation as the level of ambiguity decreases. Because by noting that

$$E(f_a) = \int_{-a}^{a} x(h(a)\frac{1}{a}f(\frac{x}{a}) + (1 - h(a))\frac{1}{2a})dx = h(a)E(f_a) = h(a)aE(f) \tag{17}$$

we see that

$$E_a = h(a) \text{ for all } 0 < a \leq 1 \tag{18}$$

and

$$E_0 = \lim_{a \to 0} h(a). \tag{19}$$

If f is the piecewise constant function considered in section 3.2 then we have that, for all $0 < a \leq 1$,

$$\begin{aligned} f_a(x) &= \begin{cases} h(a)\frac{1-\gamma}{2a} + (1-h(a))\frac{1}{2a} & \text{if} \quad x \in [-a, 0) \\ h(a)\frac{1+\gamma}{2a} + (1-h(a))\frac{1}{2a} & \text{if} \quad x \in [0, a] \end{cases} \\ &= \begin{cases} \frac{1-\gamma h(a)}{2a} & \text{if} \quad x \in [-a, 0) \\ \frac{1+\gamma h(a)}{2a} & \text{if} \quad x \in [0, a]. \end{cases} \end{aligned} \tag{20}$$

The definition of the density functions representing the perceptions of interior voters is then straightforward. With this more general model of projection for $a < 1$ we have the following generalized version of theorem 3.2. Note that we still assume that voters' utility functions are quadratic. The proof is left to the appendix.

Theorem 4.2 *Consider the special case of the model described above and let* $\gamma^* = \frac{1}{2}(\sqrt{\frac{19}{3}} - 1)$*. Then the following two statements hold.*

1. *Suppose* $\gamma \leq \gamma^*$*. Then candidate 1 does not have a winning strategy. More specifically, if candidate 1 announces the ambiguous position* $[A - a, A + a]$ *then candidate 2 can either win (if* $\gamma < \gamma^*$*) or get at least a draw (if* $\gamma = \gamma^*$*) by announcing the certain position* x^* *given by*

$$x^* = \begin{cases} A + \frac{\gamma a h(a)}{2} & \textit{if} \quad A \leq 0 \\ A - \frac{\gamma a h(a)}{2} & \textit{if} \quad A > 0. \end{cases} \tag{21}$$

   (And if candidate 1 announces a certain position then candidate 2 can win or get a draw by announcing the median.)

2. *Suppose* $\gamma E_0 > \gamma^*$*. Then there exists an* $a' > 0$ *such that, for any* $0 < a \leq a'$*,* $[-a, a]$ *is a winning strategy for candidate 1.*

The remaining question is what happens when $\gamma^* < \gamma \leq \max\{\frac{\gamma^*}{E_0}, 1\}$. In that parameter interval there is no general result, whether winning strategies for candidate 1 exist or not depends on the function h.

5 Discussion

We have extended the standard Downsian model of electoral competition by allowing ambiguous candidate positions and projection effects in voters' perceptions of such positions. The model was constructed to help us answer the question of whether projection (assimilation) can make a generally well liked candidate take an ambiguous position, even when voters dislike ambiguity per se. To put things on the edge we assumed that all voters like candidate 1 and that no voters like candidate 2.

We have seen that announcing the median does not guarantee candidate 2 a draw because candidate 1 can "beat the median" by being ambiguous. That result is quite robust. We have also seen that ambiguity can be a winning strategy for candidate 1. But that result relies on a strong assimilation effect. If the assimilation effect is not sufficiently strong then candidate 2 can beat any ambiguous position of candidate 1. This is a quite interesting result because it shows that even a substantial advantage due to assimilation is not enough to ensure candidate 1 the victory.

So assimilation by itself is not necessarily enough to explain ambiguity in electoral competition. But even when it is not it could still be an important contributing factor. For example if it is combined with the assumption that candidates have a preference for flexibility in office (see for example Aragones and Neeman (2000)).

If assimilation is a significant part of the explanation of ambiguity in electoral competition we would expect to see candidates that are generally well liked on a personal level to be more ambiguous than candidates that are not. It would be interesting to see some empirical or experimental work on that (although we recognize that there are lots of problems in doing such work). It would also be interesting to see some empirical or experimental work on the relationship between ambiguity and assimilation/contrast.

There are also possibilities for further theoretical research. As mentioned above one possibility is to include candidate preferences for flexibility in office into the model. That may give stronger predictions of ambiguity. Other possibilities are to look at candidates with policy preferences or candidates that are uncertain about the distribution of the voters' preferred points.

6 References

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7 Appendix

Here we present proofs of the theorems in section 4.

Proof of theorem 4.1.
For each $0 < a \leq b \leq 1$ define

$$B_b = \{x_0 \in \mathbb{R} \setminus \{0\} \mid |\frac{u''_{x_0}(0)}{u'_{x_0}(0)}| < \frac{2E(f_1)}{b} E_0\} \tag{22}$$

and

$$B_{b,a}^{\max} = \{x_0 \in \mathbb{R} \setminus [-a, a] \mid \max_{-a \leq y \leq a} -u''_{x_0}(y) < \frac{2E(f_1)}{b} E_0 |u'_{x_0}(0)|\}. \tag{23}$$

Then the theorem follows from the following three claims.

- Claim 1: Suppose $E_0 > 0$. Then there exists a $b' > 0$ such that a strict majority of voters have preferred points in $B_{b'}$.
- Claim 2: Suppose there exists a $b' > 0$ such that a strict majority of voters have preferred points in $B_{b'}$. Then there exists an $0 < a' \leq b'$ such that a strict majority of voters have preferred points in $B^{\max}_{b',a}$ for any $0 < a \leq a'$.
- Claim 3: Suppose there exist $0 < a' < b'$ such that a strict majority of voters have preferred points in $B^{\max}_{b',a}$ for any $0 < a \leq a'$. Also suppose that candidate 1 announces $[-a, a]$ for some $0 < a \leq a'$ and candidate 2 announces 0 (the median). Then any voter with a preferred point in $B^{\max}_{b',a}$ will vote for candidate 1.

Proof of claim 1: $|\frac{u''_{x_0}(0)}{u'_{x_0}(0)}| = |\frac{u''_0(-x_0)}{u'_0(-x_0)}|$ is a continuous function (of x_0) on $\mathbb{R}\backslash\{0\}$. Therefore it is bounded on compact subsets of $\mathbb{R}\backslash\{0\}$. And thus it follows that for any $n \in \mathbb{N}$ there exists a $b > 0$ such that

$$\{x_0 \in \mathbb{R} \mid \frac{1}{n} \leq |x_0| \leq n\} \subseteq B_b. \tag{24}$$

Since there exists an N such that a strict majority has preferred points in $\{x_0 \in \mathbb{R} \mid \frac{1}{N} \leq |x_0| \leq N\}$ the claim follows immediately.

Proof of claim 2: For any (Lebesgue measurable) $B \subseteq \mathbb{R}$ let $v(B)$ denote the share of voters with preferred points in B. Using the continuity of u'' it is easily seen that

$$\bigcup_{0<a\leq b'} B^{\max}_{b',a} = B_{b'}. \tag{25}$$

So we have

$$v(\bigcup_{0<a\leq b'} B^{\max}_{b',a}) = v(B_{b'}) > \frac{1}{2}. \tag{26}$$

From the fact that $a_1 \leq a_2 \Rightarrow B^{\max}_{b',a_2} \subseteq B^{\max}_{b',a_1}$ it follows that

$$v(B^{\max}_{b',a}) \to v(\bigcup_{0<a\leq b'} B^{\max}_{b',a}) \quad \text{for} \quad a \to 0. \tag{27}$$

Thus we must have $v(B^{\max}_{b',a}) > \frac{1}{2}$ for a sufficiently close to zero.

Proof of claim 3: Let $0 < a \leq a'$ and consider a voter with preferred point $x_0 \in B^{\max}_{b',a}$. If the voter votes for candidate 1, then so does a voter with preferred point $-x_0$. Therefore it suffices to look at the case $x_0 > 0$. The voter's perceived expected utility of the position of candidate 1 is

$$\int_{-a}^{a} u_{x_0}(x) f_a(x) dx. \tag{28}$$

Thus we have to show that

$$\int_{-a}^{a} u_{x_0}(x) f_a(x) dx > u_{x_0}(0). \tag{29}$$

By Taylors Theorem we get that for all $x \in \mathbb{R}$ there exists a ξ between 0 and x such that

$$u_{x_0}(x) = u_{x_0}(0) + u'_{x_0}(0)x + \frac{u''_{x_0}(\xi)}{2} x^2. \tag{30}$$

By the definition of $B^{\max}_{b',a}$ it then follows that for all $x \in [-a, a]$,

$$u_{x_0}(x) \geq u_{x_0}(0) + u'_{x_0}(0)x - u'_{x_0}(0)\frac{E(f_1)}{b'} E_0 x^2. \tag{31}$$

Using that inequality we get

$$\begin{aligned}
& \int_{-a}^{a} u_{x_0}(x) f_a(x) dx \\
\geq\ & \int_{-a}^{a} (u_{x_0}(0) + u'_{x_0}(0)x - u'_{x_0}(0)\frac{E(f_1)}{b'} E_0 x^2) f_a(x) dx \\
=\ & u_{x_0}(0) + u'_{x_0}(0)(\int_{-a}^{a} x f_a(x) dx - \frac{E(f_1)}{b'} E_0 \int_{-a}^{a} x^2 f_a(x) dx) \\
>\ & u_{x_0}(0) + u'_{x_0}(0)(\int_{-a}^{a} x f_a(x) dx - \frac{E(f_1)}{b'} E_0 a^2) \\
\geq\ & u_{x_0}(0) + u'_{x_0}(0)(E(f_a) - E(f_1)E_0 a) \\
=\ & u_{x_0}(0) + u'_{x_0}(0)E(f_1)a(E_a - E_0) \\
\geq\ & u_{x_0}(0).
\end{aligned} \tag{32}$$

□

Proof of theorem 4.2.

1. We will only do the proof for $A = 0$. The extension to $A \neq 0$ is straightforward.

First let $\gamma < \gamma^*$, $0 < a \leq 1$ and $x^* = \frac{\gamma a h(a)}{2}$. Candidate 2 wins the election if, for some $\varepsilon > 0$,

$$U_{x_0}([-a, a]) < u_{x_0}(x^*) \quad \text{for all} \quad x_0 > -\varepsilon, \tag{33}$$

where $U_{x_0}([-a, a])$ denotes the perceived expected utility of $[-a, a]$ (announced by candidate 1) for a voter with preferred point x_0.

First consider voters with $x_0 \geq \frac{a}{2}$. Since $E(f_a) = \frac{\gamma a h(a)}{2} = x^*$ and voters are risk averse these voters will all vote for candidate 2.

Then consider voters with $-\frac{a}{2} < x_0 < \frac{a}{2}$. In this case we have

$$\begin{aligned} U_{x_0}([-a,a]) &= \frac{1-\gamma h(a)}{2a}\int_{-a}^{x_0-\frac{a}{2}} -(x-x_0)^2 dx \\ &+ \frac{1+\gamma h(a)}{2a}\int_{x_0-\frac{a}{2}}^{x_0+\frac{a}{2}} -(x-x_0)^2 dx \\ &+ \frac{1-\gamma h(a)}{2a}\int_{x_0+\frac{a}{2}}^{a} -(x-x_0)^2 dx \\ &= (1-\gamma h(a))x_0^2 - a^2(\frac{1}{3} - \frac{\gamma h(a)}{4}). \end{aligned} \tag{34}$$

The expression for $u_{x_0}(x^*)$ is

$$u_{x_0}(x^*) = -(\frac{\gamma a h(a)}{2} - x_0)^2 = -x_0^2 + \gamma a h(a) x_0 - \frac{\gamma^2 a^2 h(a)^2}{4}. \tag{35}$$

Thus we just have to check whether there exists an $\varepsilon > 0$ such that, for all $-\varepsilon < x_0 < \frac{a}{2}$,

$$\gamma h(a) x_0^2 - \gamma a h(a) x_0 + a^2(\frac{\gamma^2 h(a)^2}{4} + \frac{\gamma h(a)}{4} - \frac{1}{3}) < 0. \tag{36}$$

The inequality holds for all x_0 between the roots of the polynomial on the left hand side. These are

$$x_0^{\pm} = \frac{a}{2} \pm \frac{a}{2}\sqrt{\frac{4}{3\gamma h(a)} - \gamma h(a)}. \tag{37}$$

So we just have to show that $\frac{4}{3\gamma h(a)} - \gamma h(a) > 1$. That is easily checked.

Then let $\gamma = \gamma^*$. It follows easily from what we did above that in this case

$$U_{x_0}([-a,a]) < u_{x_0}(x^*) \quad \text{for all} \quad x_0 > 0. \tag{38}$$

Thus, if candidate 1 announces $[-a, a]$, then candidate 2 can always get a draw by announcing x^*.

2. Let $\gamma E_0 > \gamma^*$ and define ξ by

$$\xi = \frac{\gamma}{4(\gamma E_0 - \sqrt{\frac{4-3\gamma E_0}{3}})}\ . \tag{39}$$

Note that since $\gamma E_0 > \gamma^*$, ξ is well-defined and positive. Then pick an $a' > 0$ such that there are strictly less than 50% of voters in the interval $[-a'\xi, a'\xi]$, i.e. such that

$$\int_{-a'\xi}^{a'\xi} v(x)dx < \frac{1}{2} \ . \tag{40}$$

We will show that this a' "does the job". So let $0 < a \leq a'$ and $x^* \in \mathbb{R}$ and assume that candidate 1 announces $[-a, a]$ and candidate 2 announces x^*. We have to show that a strict majority of voters will vote for candidate 1. We will only do it for $x^* \geq 0$. The proof for $x^* < 0$ is completely analogous.

First consider the case $x^* > \frac{a}{2}\sqrt{\frac{4-3\gamma h(a)}{3}}$. It is easily seen that

$$u_{x_0}(x^*) \quad = \quad -x_0^2 + 2x^* x_0 - x^{*2} \tag{41}$$

and

$$U_{x_0}([-a,a]) \quad = \quad \begin{cases} -x_0^2 - \gamma a h(a) x_0 - \frac{a^2}{3} & \text{if} \quad x_0 \leq -\frac{a}{2} \\ \\ -(1-\gamma h(a))x_0^2 - a^2(\frac{1}{3} - \frac{\gamma h(a)}{4}) & \text{if} \quad -\frac{a}{2} < x_0 < \frac{a}{2}. \end{cases} \tag{42}$$

Using these expressions it is straightforward to check that

$$U_{x_0}([-a,a]) > u_{x_0}(x^*) \quad \text{for all} \quad x_0 \leq 0. \tag{43}$$

Since both sides of the inequality are continuous in x_0 the inequality also holds for positive values of x_0 sufficiently close to 0. Thus a strict majority will vote for candidate 1.

Finally consider the remaining case, i.e. $0 \leq x^* \leq \frac{a}{2}\sqrt{\frac{4-3\gamma h(a)}{3}}$. We claim that for these x^*,

$$\{ \ x_0 \in \mathbb{R} \mid U_{x_0}([-a,a]) \leq u_{x_0}(x^*) \ \} \subseteq [-a'\xi, a'\xi] \ , \tag{44}$$

which means that a strict majority of voters will vote for candidate 1. Thus we have to show that

$$|x_0| > a'\xi \quad \Rightarrow \quad U_{x_0}([-a,a]) > u_{x_0}(x^*). \tag{45}$$

Since $x^* \geq 0$ it suffices to show that

$$x_0 > a'\xi \quad \Rightarrow \quad U_{x_0}([-a,a]) > u_{x_0}(x^*). \tag{46}$$

And since $a'\xi > a\xi > \frac{a}{2} > \frac{a}{2}\gamma h(a) > \frac{a}{2}\sqrt{\frac{4-3\gamma h(a)}{3}}$ that holds true if

$$x_0 > a'\xi \quad \Rightarrow \int_{-a}^{a} -(x-x_0)^2 f_a(x)dx > u_{x_0}(\frac{a}{2}\sqrt{\frac{4-3\gamma h(a)}{3}}), \tag{47}$$

i.e. if, for all $x_0 > a'\xi$,

$$-x_0^2 + \gamma a h(a) x_0 - \frac{a^2}{3} > -x_0^2 + a\sqrt{\frac{4-3\gamma h(a)}{3}}x_0 - \frac{a^2}{4}(\frac{4-3\gamma h(a)}{3}). \tag{48}$$

By straightforward calculations we see that the inequality above is satisfied for all

$$x_0 > a\frac{\gamma h(a)}{4(\gamma h(a) - \sqrt{\frac{4-3\gamma h(a)}{3}})}. \tag{49}$$

Since $h(a) \geq E_0$ it follows that

$$\xi \geq \frac{\gamma h(a)}{4(\gamma h(a) - \sqrt{\frac{4-3\gamma h(a)}{3}})} \tag{50}$$

and thus we are done. $\square$