Abstract. This paper investigates the interactive impact of subsistence consumption and child mortality on fertility choice and child expenditure. It offers an explanation for why mankind multiplies at higher rates at geographically unfavorable, tropical locations. In a macro-economic framework it proposes an indirect channel of geography’s influence on economic performance. It explains why it are the world’s unfavorably located regions where we observe exceedingly slow (if not stalled) economic development and demographic transition.

Keywords: Demographic Transition, Geography, Health, Cross-Country Divergence

JEL: J10, J13, O11, O12
1. Introduction

As for every species, survival of men is easier in some regions of the world and harder in others. Figure 1 shows for 137 countries average absolute latitude against the probability for a child to survive its fifth birthday. In particular tropical regions – defined by an absolute latitude below 23.5° – provide an unfavorable location for a child to survive whereas survival is almost certain at latitudes of 40° and higher.¹

![Figure 1: Absolute Latitude and Child Survival](image)

Child Survival Rate=1- Under-5-Mortality Rate, Year 2000, Data from World Bank (2004) and Masters and McMillan (2001).

Recently, following Acemoglu et al. (2001), a number of studies has found a predominantly indirect influence of geographic location on income through settler mortality and institution building. At the same time child survival is to a large extent explained by income per capita (Pritchett and Summers, 1996), so that the correlation between geography and mortality displayed in Figure 1 arises perhaps mainly indirectly. This paper will not deny this hypothesis. In fact, it will employ an income channel to child survival and argue that additionally geographic location matters. It will argue that (i) it is *ceteris paribus* easier for a child to survive in geographically favorable location (e.g. without malaria prevalence) and (ii) that a unit of

¹We focus on child survival rates, which will be the crucial variable in the theoretical model. Similar figures can be drawn for infant survival and longevity. As Schultz (1999) notes, intercountry differences in life expectancy are dominated by rates of infant and child survival.
parental income spend on child health care is more effective (in the sense of preventing death) in a geographically unfavorable location.

The debate whether geography matters directly for economic performance (e.g. through labor- and land productivity) or just indirectly through its effect on institutions is still open. The present article does not contribute to this debate. Instead, it proposes a second indirect channel through which geographic location may matter for economic development. This channel operates through child mortality, health expenditure, fertility, and population growth.

Figure 2: Absolute Latitude and Population Growth, Year 2000

\[ \text{Pop. Growth} = 2.53 - 0.04 \text{ Latitude} \]

\( R^2 = 0.506, \) data for 132 countries from World Bank (2004) and Masters and McMillan (2001).

Regression for 1960 (not shown): \( \text{Pop. Growth} = 2.96 - 0.03 \text{ Latitude}. \)

A striking demographic fact – shown in Figure 2 – is that population growth tends to be higher in geographically unfavorable regions. While biologists might be puzzled by the observation that a species multiplies at higher rates in environments for which it is less fit to live in, demographers and economists provide answers. A seemingly obvious explanation is the demographic transition according to which fertility rates follow a decline of mortality with delay so that population growth rises temporarily. Given that the demographic transition started earlier at geographically favorable locations, a negative correlation between latitude and population growth follows

automatically. High-latitude countries have accomplished the transition already, resting at low mortality and fertility rates. Low-latitude countries have experienced some reduction of mortality (which is, however, still high, cf. Figure 1) but this has not yet triggered a comparable decline of fertility rates. The problem with this argument is that a picture similar to Figure 2 could have been drawn already in 1960. Over the years the negative latitude–population growth correlation became even somewhat stronger mainly because some countries at medium latitudes (e.g. Japan, Korea, Portugal, Spain) accomplished the second step of the demographic transition and simultaneously little has changed in tropical regions.

One interpretation of the result is that the demographic transition is extraordinary slow and possibly stagnant in many tropical regions of high mortality (Bloom and Sachs, 1998). A second possible interpretation is that although a demographic transition is taking place almost everywhere (albeit at different speed) population grows at higher rates along the transition path for geographically unfavorable regions. For example, maximum population growth rates attained along the transition path of Western Europe’s countries were around 2 percent per year or lower. They are now around 4 percent for tropical countries. Both effects of geography and mortality, slower demographic transition and higher population growth along the transition path, and their consequences on macroeconomic performance will be explained and investigated in the present paper.

The standard economic model on optimal fertility choice (Barro and Becker, 1989) supports the biologist’s viewpoint: Population growth should not be lower and possibly higher when child survival probabilities improve i.e. ceteris paribus at geographically favorable locations. The simple reason is that parents are not interested in births as such but in surviving family members and time costs of producing a surviving children decrease with child mortality. This price effect occurs irrespective of aspects of human capital accumulation and a child quality-quantity trade-off.

To explain why population growth is higher at high mortality rates the literature has therefore referred to precautionary child bearing (Kalemli-Oscans, 2002, 2003). Given uncertainty about the number of surviving children at a unique date of a discrete fertility decision, it depends on curvature of the utility function whether parents react on improving child survival with higher

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3See Lagerlöf (2003) for a model on the onset of the demographic transition in the Western world.
4This conclusion is mainly driven by the countries from the African continent. Important successful outliers are Singapore (at 2°), Costa Rica (at 9°), and Thailand (at 15°).
or lower fertility rates. Yet, even if parents are sufficiently risk-avers to generate a positive correlation between mortality and population growth, the effect disappears entirely if one allows sequential child bearing i.e. the possibility to replace children who die early. This has recently been shown by Doepke (2005) who therefore argues in favor of the easier to handle deterministic model because it “leads to virtually the same conclusions as the stochastic model with sequential fertility choice.” Using a deterministic model, the present paper offers an alternative explanation for a positive link between mortality and population growth that operates through child health care expenditure.

The paper is also loosely related to a series of articles investigating the influence of longevity on human capital accumulation and economic growth. Sharing some of the demo-economic mechanisms at work, the present paper deviates from these articles by its focus on geography, child mortality and child expenditure. Ehrlich and Lui (1992) and Soares (2005) connect both literatures by investigating fertility and educational choice when both adult and child survival are uncertain. The main difference to these articles is the role of health expenditure in the present paper. A recent complementary study is provided by Corrigan et al. (2005) who investigate the influence of the AIDS epidemic on human capital accumulation and growth through the creation of orphans.

Some ideas developed in this paper were already apparent in Blackburn and Cipriani (1998) and Strulik (2004). Blackburn and Cipriani investigate a Barro-Becker (1989) model with endogenous health and mortality and focus on transitional dynamics that are consistent with the historic successful development of the Western world, while the current paper investigates the impact of geography on delayed and possibly stalled demographic transition in today least developed countries. In Strulik (2004) the central message was obscured by some non-standard assumptions about the utility function leading to an awkward differentiation between interior and corner solutions. Here, the paper employs the incidence of subsistence consumption and derives its central results much more elegantly and stringently without recurring on corner solutions. As a co-product of subsistence consumption the model can explain why savings rates and the intertemporal elasticity of substitution rise along a path of successful development. This relates the paper to general research on growth with subsistence consumption, notably Easterly (1994), Ben-David (1998), and Steger (2000).

2. Geography, Child Survival, and Health Expenditure

More than 10 Million children younger than 5 years die every year. Most frequent causes of death are neonatal disorder (33%), diarrhoea (22%), pneumonia (21%) and malaria (9%). They are mainly observed in the tropical zones of Sub-Saharan Africa (41%) and South Asia (34%). About 53% of all child deaths can be attributed to being underweight. Black et al. (2003) – the survey from which these facts are taken – identifies undernutrition and vitamin A- and zinc-deficiencies as the underlying cause of a substantial proportion of all child deaths.

Most child deaths could have been avoided if more money were spent on nutrition and health care. Not surprisingly, empirical cross-country studies usually find a strong correlation of child mortality or, more generally, life expectancy and health with income per capita and causality running from income to health (see Pritchett and Summers, 1996, and the literature cited therein). Although income per capita is already to some extend predetermined indirectly through geography (see Introduction) some authors (among others Bloom and Sachs, 1998, Schultz, 1999, Sachs, 2003) argue in favor of an independent influence of geography that makes some regions inherently less healthy than others. For example, falciporom malaria needs an ambient temperature of $22^\circ$ C or above during the incubation period. Winter frost eliminates the prevalence of many pathogens and parasites (Masters and McMillan, 2001).

In this paper we will therefore analyze the choice of parents who are able to control the number of births ($n$) but only partly the survival of their children. Survival probability ($\pi$) consists of a fundamental part and a controllable part. The fundamental survival rate ($\bar{\pi}$) depends on geographic location and the state of economic development (summarized by average income per capita). The controllable part depends on the fraction of parental income devoted to child expenditure ($h$).

We assume that an additional unit child expenditure is more effective in preventing child death when fundamental child survival probability is low. In other words, at low fundamental survival probabilities child expenditure is mainly motivated by the desire to have surviving offsprings and consists therefore to a large extent of nutrition and health care expenditure. When fundamental survival probabilities are already very high, an additional unit of $h$ is relatively ineffective in terms of increasing child survival and is therefore mainly motivated by the utility derived directly from child expenditure. This utility could originate either from a “warm glow of giving” (Andreoni, 1989) or a preference for having higher quality children (Becker, 1960). It helps to
explain why parents increase child expenditure with economic development despite decreasing
effectivity of child expenditure on child survival. Thus, the variable $h$ can be thought of human
capital in a broad sense, consisting mainly of health and nutrition when income is low and of
schooling expenditure at high incomes.

Table 1: Independent Variable: Child Survival Rate

<table>
<thead>
<tr>
<th>Variable</th>
<th>Const.</th>
<th>Income</th>
<th>Latitude</th>
<th>Health</th>
<th>(Latitude × Health)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coefficient</td>
<td>0.456</td>
<td>0.118</td>
<td>0.00206</td>
<td>0.00665</td>
<td>-0.000312</td>
</tr>
<tr>
<td>$t$-value</td>
<td>13.69</td>
<td>13.04</td>
<td>3.76</td>
<td>2.31</td>
<td>-3.51</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.73</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Variable</th>
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<th>(Income × Health)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coefficient</td>
<td>0.252</td>
<td>0.183</td>
<td>0.00038</td>
<td>0.0473</td>
<td>-0.0127</td>
</tr>
<tr>
<td>$t$-value</td>
<td>3.77</td>
<td>9.90</td>
<td>1.66</td>
<td>4.18</td>
<td>-4.36</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.74</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Child survival rate: $1−$ under five mortality rate, income: GDP per capita in current
international dollars, health: total health expenditure as fraction of GDP, all taken
from World Bank (2004). Latitude: average absolute latitude taken from Masters and

The proposed model therefore suggests that child survival depends positively on income per
capita and latitude (summarizing fundamental factors), positively on the fraction of income
spent on health (the controllable factor), and negatively on an interaction term between health
expenditure and latitude indicating that health expenditure is less effective in favorable geo-
graphic environments. A cross-country regression performed with data from World Bank (2004)
and Masters and McMillan (2001) cannot reject our hypothesis. As shown in Table 1 all coeffi-
cients assume the predicted sign on a significance level of 95 % or higher. The table also shows
an alternative specification testing whether health expenditure is less effective at high income
levels. Again, all coefficient assume the predicted signs although latitude is now significant only
at the 90% level.

Summarizing, child survival is given by

$$\pi = \bar{\pi} + (1 - \bar{\pi})\lambda h,$$

where the parameter $\lambda > 0$ measures effectivity of child expenditure. Note that the marginal
effect of child expenditure on survival, $(1 - \bar{\pi})\lambda$, is large when fundamental survival probability,
$\bar{\pi}$, is low. Fundamental child survival depends on the state of development proxied by income
\[ \bar{\pi} = \bar{\pi}(y) \quad \text{with} \quad \partial \bar{\pi} \partial y > 0, \left. \lim_{y \to \infty} \right. \partial \bar{\pi} \partial y = 0, \left. \lim_{y \to \infty} \bar{\pi} = a < 1 \right. . \tag{2} \]

In accordance with the empirical evidence the marginal income effect on child survival vanishes as income goes to infinity. For later reference we introduce geographic location by the following definition.

**Definition 1.** Consider two regions, 1 and 2, for which (2) holds. We say that region 1 is a geographically unfavorable location if \( \bar{\pi}_1(y) < \bar{\pi}_2(y) \) for all income levels. If the opposite holds, region 1 constitutes a favorable location.

### 3. Households

Consider the decision problem of young adults who derive utility from consumption now \( (c_1) \) and in old age \( (c_2) \), from having a family (of size \( \tilde{n} \)), and from child expenditure \( (h) \). Abstracting from gender differences any adult is allowed to reproduce without matching. Actual family size, \( \tilde{n} \equiv n \cdot \pi \), differs from fertility, \( n \), because child survival is uncertain. To keep the analysis tractable we consider \( n \) as continuous variable. Thus, the parent under investigation can be regarded as an economy’s average adult who bears \( n \) children, spends a fraction \( h \) of his income on each child, and observes a fraction \( \pi \) of them surviving childhood. Survival during adulthood is assumed to be certain. Summarizing, his or her utility is given by

\[ u(c_1, c_2, \tilde{n}, h) = \beta_1 \log(c_1 - \bar{c}) + \beta_2 \log(c_2) + \beta_3 \log(n\pi) + \beta_4 \log(h) \tag{3} \]

for \( y > \bar{c} \), and \( c_1 = y \leq \bar{c} \) otherwise.

In order to obtain a closed form solution we consider a logarithmic formulation of the utility function. Yet, because of the incidence of subsistence consumption, \( \bar{c} \), this assumption is less restrictive than usually. The intertemporal elasticity of substitution is – in deviation to standard models – not constant during the process of economic development. It assumes a value of zero at subsistence level and converges towards one as income and consumption go to infinity. See, among others, Atkeson and Ogaki (1996) for evidence that the elasticity of intertemporal substitution rises with increasing income and wealth accumulation.

The incidence of subsistence consumption generates a hierarchy of needs. If income raises from subsistence level, current consumption becomes less important and young adults increasingly care for future consumption and about the size of their family. This positive income effect on
savings and fertility is largest close to subsistence level and vanishes as income goes to infinity and \( \bar{c} \) becomes negligible small relative to income.

Adults supply one unit of labor and receive labor income \( y \). In old age they expect to consume capital income, \( c_2 = (1 + r)sy \), where \( r \) denotes the expected interest rate and \( s \) is the savings rate. Thus, their budget constraint is

\[
y = c_1 + sy + nhy = c_1 + c_2/(1 + r) + nhy.
\]

Maximization of (3) subject to (1) and (4) provides the following first order conditions for an interior solution.

\[
\frac{\partial u}{\partial c_1} = (1 + r)\frac{\partial u}{\partial c_2} \implies \frac{\beta_1}{c_1 - \bar{c}} = (1 + r)\frac{\beta_2}{c_2}, \tag{5a}
\]

\[
\frac{\partial u}{\partial \tilde{n}} \cdot \frac{\partial \pi}{\partial c_1} + \frac{\partial u}{\partial c_1} n \frac{\partial \pi}{\partial n} + \frac{\partial u}{\partial \tilde{h}} \frac{\partial \pi}{\partial h} = \frac{\beta_3}{\pi} \left(1 - \tilde{\pi}\right)\lambda + \frac{\beta_4}{h} = \frac{\beta_1}{c_1 - \bar{c}} ny. \tag{5c}
\]

Condition (5a) equates marginal utility from current and future consumption. Condition (5b) requires that utility from having another child equates child costs in terms of foregone utility from consumption. According to condition (5c) utility from spending an additional unit of income on children – derived directly through higher quality children (or the warm glow of giving) and indirectly through the impact of health expenditure on family size – equates marginal child costs in terms of foregone utility from consumption. Holding consumption constant we see from (5b) that higher child expenditure \((h \cdot y)\) is observed together with the desire for a smaller family (lower \( n \) and higher \( \partial u/\partial \tilde{n} \)). This is the Beckerian child quantity-quality trade-off.

The presence of mortality and health expenditure generates a second quantity-quality trade-off visible in (5c). To understand its consequence consider an increase of fundamental survival probability \( \tilde{\pi} \). This has a twofold negative effect on the first term on the left hand side: First, marginal utility from health expenditure driven by the wish for a large family \((\partial u/\partial \tilde{n} \cdot n = \beta_3/\pi)\) decreases, because more children survive anyway. Second, marginal returns of health expenditure \((\partial \pi/\partial h = [1 - \tilde{\pi}]\lambda)\) are lower because child expenditure is less effective in preventing death under the generally improved survival conditions. As a consequence parents reduce their fertility rate and the right hand side of (5c) decreases.
Now, with decreasing fertility, the familiar Beckerian trade-off in (5b) becomes operative and parents want to spend more on their children. The increase of \( h \) has a positive feedback effect on the left-hand side of (5c) so that we have to assume \( \beta_3 > \beta_4 \) for a consistent solution to exist. In other words, having a family must be more important than child quality expenditure. This parameter restriction is assumed to hold henceforth.

Note that the wish to spend more on children when \( \bar{\pi} \) rises cannot be driven by the motive to improve child survival because the starting point of the whole chain of effects was that child health expenditure became less effective with rising \( \bar{\pi} \). Thus, it must be driven by the child quality motive. This lets us conclude that child expenditure changes its character with improving fundamental survival probabilities. At low \( \bar{\pi} \)’s expenditure is driven by its \( \beta_3 \)-component, i.e. health and nutrition, whereas at high \( \bar{\pi} \)’s it is mainly motivated by the \( \beta_4 \)-component, i.e. schooling and education. The downside of this effect is that it holds also vice versa: parents react to deteriorating child survival by substituting child expenditure with increasing fertility. Thus, with endogenous health, parents in high mortality environments have a comparative advantage in child bearing i.e. in producing cheap children.\(^6\)

From the first order conditions we obtain the following solution.

\[
c_1 = \frac{\beta_1 y + (\beta_2 + \beta_3) \bar{c}}{\phi} \quad (6a)
\]

\[
s = \frac{\beta_2 (1 - \bar{c}/y)}{\phi} \quad (6b)
\]

\[
n = \frac{\beta_3 \beta_4 \lambda (1 - \bar{\pi})(1 - \bar{c}/y)}{\phi (\beta_3 - \beta_4) \bar{\pi}} \quad (6c)
\]

\[
h = \frac{(\beta_3 - \beta_4) \bar{\pi}}{\beta_4 \lambda (1 - \bar{\pi})}, \quad (6d)
\]

where \( \phi \equiv \beta_1 + \beta_2 + \beta_3 \). From (6b) we see that the savings rate is increasing with economic development and converges towards a constant as income gets large. As derived intuitively above, fertility is lower and child expenditure higher under better fundamental survival conditions:

\[
\frac{\partial n}{\partial \bar{\pi}} = -\frac{\beta_3 \beta_4 \lambda (1 - \bar{c}/y)}{\phi (\beta_3 - \beta_4) \bar{\pi}^2} < 0 \quad (7a)
\]

\[
\frac{\partial h}{\partial \bar{\pi}} = \frac{\beta_3 - \beta_4}{\beta_4 \lambda (1 - \bar{\pi})^2} > 0. \quad (7b)
\]

\(^{6}\)I borrowed this expression from Moav (2005) who derives a similar result in a very different setting where well educated mothers have a comparative advantage in teaching and poorly educated mothers in child bearing.
The incidence of subsistence consumption causes a positive effect of income on fertility. As income rises above subsistence level, current consumption becomes less essential and the desire to have children becomes more important for young adults.

\[ \frac{\partial n}{\partial y} = \frac{\beta_3 \beta_4 \lambda (1 - \bar{\pi}) \bar{c}}{\phi (\beta_3 - \beta_4) \bar{\pi} y^2} > 0. \]  \hspace{1cm} (8)

The income elasticity of child demand, \((\partial n/\partial y) = \bar{c}/(y - \bar{c})\), is infinite at subsistence level and decreases towards zero as income goes to infinity. Later, in the macroeconomic model the survival-effects in (7) are sufficient to explain a delayed demographic transition and slow growth for geographically unfavorable environments. The subsistence-effect in (8), however, is necessary to generate a stable equilibrium of stagnation.

4. Correlations of Demo-Economic Development

The rate of population growth, \(g_L := n(y, \bar{\pi}(y)) \cdot \pi(y) - 1\), is affected by income through its impact on fertility and, indirectly, through its impact on health expenditure and child survival. Thus, the total effect of a marginal increase of income on population growth consists of three parts:

\[ \frac{\partial g_L}{\partial y} = \frac{\partial n}{\partial y} \pi + \frac{\partial n}{\partial \bar{\pi}} \frac{\partial \bar{\pi}}{\partial y} + \frac{\partial \pi}{\partial y} n. \]  \hspace{1cm} (9)

The first term on the right hand side is the subsistence effect originating from the hierarchy of needs. It is positive because parents want to support a larger family as income rises from subsistence level. The second term is the quality-quantity substitution effect. It is negative because higher average income in an economy increases the probability that children survive and causes parents to substitute fertility with child expenditure. The third term is the demographic effect. It is positive because more children survive when income rises, both because increasing average income in the economy improves survival and because parents spend more on each child.

After inserting (1), (6c), (6d), and their respective derivatives into (9) and applying some algebra we get an easily analyzable expression:

\[ \frac{\partial g_L}{\partial y} = n \pi \left(\frac{c}{y(y - \bar{c})} - \frac{1}{1 - \bar{\pi} \frac{\partial \bar{\pi}}{\partial y}}\right). \]  \hspace{1cm} (10)

The first term in parentheses results from the subsistence effect. It is infinitely large at subsistence level and vanishes quickly with rising income. The second term comprises the demographic effect and the quantity-quality substitution effect. Eventually, this term also vanishes as income
gets large because of the declining impact of average income on fundamental child survival. At middle stages of development, however, the second term increases because of improving fundamental rates of child survival, i.e. decreasing \((1 - \bar{\pi})\). Thus, we observe a hump-shaped correlation between population growth and income. At the lowest stages of development the subsistence effect dominates and population growth is positively correlated with per capita income. First, when income rises from subsistence level parents want to have larger families. This effect, however, vanishes quickly and the quantity-quality substitution effect becomes dominating. When more children survive and income is well above subsistence level, parents want to spend more on each child and prefer a smaller family.

**Figure 3: Patterns of Population Growth**

At the same time, however, the *partial* effect of fundamental child survival on population growth is negative for any *given* income level, i.e. using (6c) and (7a)

\[
\frac{\partial g_L}{\partial \bar{\pi}} = \frac{\partial n}{\partial \bar{\pi}} \cdot \bar{\pi} + n \cdot \frac{\partial \bar{\pi}}{\partial \bar{\pi}} = -\frac{n \bar{\pi}}{1 - \bar{\pi}} + n \left( -\lambda h + (1 - \bar{\pi}) \lambda \frac{\partial h}{\partial \bar{\pi}} \right) = -\frac{n \bar{\pi}}{1 - \bar{\pi}} < 0. \tag{11}
\]

Consider two regions 1 and 2 with \(\bar{\pi}_1(y) < \bar{\pi}_2(y)\) implying that for any given income population grows at a higher rate in the unfavorable location 1. Irrespective of whether the demo-economic system is in its first phase of positive correlation between income and population growth or in its second phase of negative correlation, parents in the unfavorable region always have a comparative advantage in fertility. In other words, at high fundamental survival rates child expenditure consists to a lesser degree of \(\beta_3\)-components motivated by child survival (i.e. nutrition and health expenditure) and to a higher degree of \(\beta_4\)-components motivated by child quality (i.e. schooling). Thus, the Beckerian child-quality trade-off is stronger; child expenditure substitutes
family size more easily in favorable locations. Figure 1 summarizes the results. The equilibria \( y^* \) and \( \tilde{y} \) will be explained later.

In order to investigate hump-shapedness of population growth and other correlations of demo-economic development quantitatively, we consider a calibration of the model. For better interpretation generational growth rates are transformed into annual ones. Thus, \( y \) is measured as adult income per year. Let \( \psi \) denote the length of adulthood measured by the fecundity period. Annual population growth is then \( \gamma_L \equiv (1 + g_L)^{1/\psi} - 1 \). We set \( \psi \) to 25.

Fundamental child survival is parameterized as \( \bar{\pi} = a \cdot (1 - e^{-b \cdot y}) \) so that mortality decays exponentially at rate \( b \) when income rises. In other words, survival \( \bar{\pi} \) is a concave function of income, reaching a maximum at \( a \). The functional form is taken from Kalemli-Ozcan’s (2002) empirical work. Yet, we cannot adopt his parameter estimates one-to-one because now \( \bar{\pi} \) is only the first of two parts of total child survival. Survival is also determined by individual health expenditure i.e. parameters of the utility function. Therefore \( a \) and \( b \) are determined in an iterative way together with preference parameters so that the endogenously generated total survival rate corresponds with the actually observed data. This leads to an estimate of \( a = 0.72 \) and \( b = 0.004 \).

Preference parameters are set so that parents in a fully developed country (where \( \bar{c}/y \) is negligible small and fundamental survival is at the highest level) show the following behavior: a savings rate of 0.16, a total child survival rate close to one hundred percent, a child expenditure share of 0.2 per child per parent, and families consisting of 1.13 children per parent (implying a population growth rate of 0.5 percent). These values are chosen to reflect approximately the demo-economic performance of the United States. They lead to an estimate of \( \beta_1 = 0.32, \beta_2 = 0.09, \beta_3 = 0.12, \beta_4 = 0.087, \) and \( \lambda = 5.7 \).

The subsistence level \( \bar{c} \) is calibrated as a parameter that shapes the income elasticity of child demand according to the demo-economic history. We set \( \bar{c} \) so that population growth peaks at a value of 2.0 percent per year (see Lucas, 2002). The resulting income correlations are displayed by solid lines in Figure 4. Parental behavior generates a positive correlation of income with the rates of human capital expenditure and savings and an inverted-u shaped correlation of income and population growth. The dotted line in the \( y – \pi \)–diagram represents Kalemli-Ozcan’s estimate of the survival function (2002, Table 3, survival probability to age 5 in 1997 for

\footnote{Data Sources for calibration are USDA (2004), World Bank (2004).}
86 countries). One sees that the endogenously generated child survival function approximates the estimate quite well.

Figure 4: Correlations of Demo-Economic Development

\[ \beta_1 = 0.32, \beta_2 = 0.09, \beta_3 = 0.12, \beta_4 = 0.87, \lambda = 5, \bar{c} = 67, a = 0.72, \text{ and } b = 0.004 \text{ (solid lines) and } 0.0025 \text{ (dashed lines). Dotted line: estimate of the child survival function in Kalemli-Ozcan (2002, Table 3).} \]

Dashed lines in Figure 4 show behavior of families living in an unfavorable environment (e.g. a tropical region) where fundamental child survival improves less quickly when income per capita rises. For that purpose we set \( b = 0.0025 \) and keep everything else from the U.S. calibration. Population growth peaks now at a higher level and decays less quickly. At the peak a couple of adults gives birth to 5 children generating a population growth rate of 2.8 percent. These values correspond approximately with the facts observed for the poorest countries in Sub Saharan Africa.

The model suggests that the income correlations in Figure 1 will be observed along a path of successful demo-economic development. That households generate these correlations does, however, not imply that successful development will actually happen. To answer the questions of if and when a demographic transition occurs and how long it will then take to arrive at the stage of a fully developed economy we place households in a macro-economy and investigate the dynamic consequences of their actions.
5. Firms and the Macro-Economy

A large number of firms produces with a constant returns to scale technology using capital \( K \), human capital, and land, \( X \). Human capital is given by the number of young adults, \( L \), times their human capital endowment, \( \tilde{h} \). Supply of land is fixed, technological progress is exogenous, and the production function is of Cobb-Douglas type and the same as in Galor and Weil (1998).

\[
Y_t = \tilde{A}_t K_t^\beta (L_t \tilde{h}_t)^{\alpha(1-\beta)} X^{(1-\alpha)(1-\beta)}, \quad 0 < \alpha \leq 1, 0 < \beta < 1 ,
\]  

(12)

where \( \tilde{A}_t \) denotes the level of general productivity. The parameter \( \alpha \) measures the importance of arable land. If \( \alpha = 1 \), land plays no role in production. According with the empirical evidence we assume that arable land per capita is of limited supply (particularly in geographically unfavorable regions) and decreases with population growth (see the evidence cited in Sachs et al., 2004).

Factors are paid according to their marginal product. The interest rate is given by the world market. Substituting (12) into \( r = \beta Y_t/K \), solving for \( K \), and re-substituting into (12), provides \( Y_t = A_t (L_t h_t)^{\alpha X^{1-\alpha}} \) where \( A_t \equiv \tilde{A}_t^{1/(1-\beta)}(\beta/\rho)^{(1-\beta)/(1-\beta)} \). Labor income per adult is obtained as \( y_t = \alpha (1 - \beta) Y_t/L_t \), or after substituting \( Y_t \) and normalizing land to one as

\[
y_t = \alpha (1 - \beta) A_t L_t^{\alpha - 1} \tilde{h}_t^{\alpha} .
\]  

(13)

Assume now that human capital endowment of the current work force depends positively on child expenditure of their parents, \( \tilde{h}_{t+1} = f(h_t) \). Because the number of workers depends on fertility of their parents (and child survival rates at that time), and both fertility and child expenditure depend on income, equation (13) provides the link between generational income levels. Next generation’s income is given by

\[
y_{t+1} = \alpha (1 - \beta)(1 + g_A) A_t [(1 + g_L(y_t)) \cdot L_t]^{\alpha - 1} \cdot [f(h(\tilde{\pi}(y_t)))]^{\alpha} .
\]

Justified by the focus of analysis on developing economies we assume that productivity grows at an exogenously given rate \( g_A \). Let \( x_t \equiv A_t / L_t^{1-\alpha} \) define a labor supply adjusted measure of productivity and let us for notational convenience assume that human capital expenditure of parents relates one-to-one to human capital endowment of their children, \( \tilde{h}_{t+1} = h_t \). Macroeconomic
dynamics are then determined by the following two-dimensional system.\(^8\)

\[
y_{t+1} = \alpha (1 - \beta) (1 + g_A) \left[1 + g_L(y_t)\right]^{\alpha - 1} h(y_t)^{\alpha} x_t, \tag{14a}
\]
\[
x_{t+1} = (1 + g_A) \left[1 + g_L(y_t)\right]^{\alpha - 1} x_t. \tag{14b}
\]

6. Stagnation

From (14) follows that any equilibrium fulfils

\[
g_L(y) = g_L^* \equiv (1 + g_A)^{1/(1-\alpha)} - 1. \tag{15}
\]

At an equilibrium, the positive impact of technological progress is neutralized by the negative impact of population growth (through decreasing returns to scale with respect to the reproducible factors). For existence of this Malthusian-like equilibrium the fertility rate supporting \(g_L^*\) has to lie in the feasible range of parental preferences, i.e. \(g_L^* \leq \max g_L(y)\). Inspection of (15) shows that existence of an equilibrium becomes increasingly unlikely for any set of preference parameters when technological progress grows faster or when arable land becomes a less essential factor in production (i.e. \(\alpha\) rises).

Consider, for example, an economy populated by parents with preferences as those underlying the income correlations in Figure 4 (dotted lines). These parents generate a maximum population growth rate of 2.8 percent annually implying that an equilibrium exists if \((1 + g_A)^{1/(1-\alpha)} - 1 \leq 0.028\). For example, if agricultural progress grows at 0.5 percent p.a., an equilibrium exists for \(\alpha < 0.82\). If \(g_A\) is rises to one percent per year, existence requires \(\alpha < 0.64\). In other words, economies particularly susceptible for stagnation are characterized by low agricultural productivity growth and high dependency on arable land.

Recall from the analysis of Section 3 and Figures 3 and 4 that population growth is higher in unfavorable locations. Thus, for given parameters of preference and technology, existence of equilibrium is more likely for unfavorably located economies. For example, with \(g_A = 0.5\) percent and \(\alpha < 0.82\) the equilibrium does not exist for the U.S. calibration of Figure 4 (solid lines) while it exists for the tropically located country (dashed lines).

\(^8\)Because \(h\) is bounded from above and below through household behavior, subsequent results are robust for any positive function \(f(h)\). Furthermore, we could introduce technological progress as being determined by the skill level of the working population –as in Galor and Weil (2000) and Strulik (2004) – without change in qualitative results.
If an equilibrium exists, it may be unstable. Stability requires that a “Malthusian mechanism” operates according to which \( \partial g_L / \partial y > 0 \) implying that there exists at most one stable equilibrium. In order to prove this claim we exclude the degenerate case where \( g_L^* \) is exactly at \( \max(g_L) \). Then, the hump-shaped curvature of the \( g_L(y) \) curve ensures that either none or two equilibria exist, and – if two equilibria exist – that we observe \( \partial g_L / \partial y > 0 \) at the first one (labelled \( y^* \)) and \( \partial g_L / \partial y < 0 \) at the second one (labelled \( \tilde{y} \)), see Figure 3.

The elements of the Jacobian matrix \( J \) of system (14) evaluated at an equilibrium (15) are

\[
\begin{align*}
\frac{\partial y_{t+1}}{\partial y_t} &= \alpha(1 - \beta)xh^\alpha \left( (\alpha - 1)(1 + g_L)^{-1} \frac{\partial g_L}{\partial y} + \frac{\alpha h}{h \partial y} \right) \equiv J_1 \quad (16a) \\
\frac{\partial y_{t+1}}{\partial x_t} &= \alpha(1 - \beta)h^\alpha \equiv J_2 > 0 \quad (16b) \\
\frac{\partial x_{t+1}}{\partial y_t} &= -(1 - \alpha)x(1 + g_L)^{-1} \frac{\partial g_L}{\partial y} \equiv J_3 \quad (16c) \\
\frac{\partial x_{t+1}}{\partial x_t} &= 1. \quad (16d)
\end{align*}
\]

Local stability requires that both eigenvalues are smaller than one in absolute terms which translates into the condition \(|1 + J_1| < (1 + J_1) - J_2J_3\). Because \( J_2 > 0 \), this necessarily requires \( J_3 < 0 \) i.e. \( \partial g_L / \partial y > 0 \). Thus, the equilibrium at \( \tilde{y} \) is never stable.

Only the equilibrium at \( y^* \) – where the Malthusian mechanism operates – is a potential candidate for a stable poverty trap. Substituting (14a) evaluated at the equilibrium into (16) we observe that the condition \( \partial g_L / \partial y > 0 \) becomes sufficient together with

\[
1 + \alpha \frac{\partial h y}{\partial y h} > \frac{1}{2}(1 - \alpha) \frac{\partial(1 + g_L)}{\partial y} \frac{y}{1 + g_L}. \quad (17)
\]

Condition (17) requires that the income elasticity of population growth is not too large. If it is too large, equilibrium (15) becomes unstable and low incomes economies converge towards subsistence level. It can be verified numerically that (17) is not restrictive and \( y^* \) is – if it exists – stable for any reasonable parameterization of the model. Note, however, that \( y^* \) is only locally stable. Any big push or series of small positive shocks that drives \( y \) in regions where \( \partial g_L / \partial y < 0 \) enables an escape towards successful demo-economic development. The following theorem summarizes the results.
Theorem 1. There exists a non-empty set of preference and technology parameters for which a locally stable equilibrium of low income and high population growth exists in a geographically unfavorable environment but not in a geographically favorable environment.

Example: The calibration for the U.S. shown in Figure 4 and $g_A = 1/2$ percent, $\beta = 1/3$, and $\alpha = 0.8$ renders the equilibrium unstable. Keeping U.S. parameters of preferences and technology the equilibrium becomes locally stable for location parameters $b \leq 0.003$.

7. Delayed Demographic Transition

Given limited space on earth, any equilibrium with perpetually high population growth appears to be implausible from a very long-run perspective. Yet, for unfavorably located regions a successful demographic transition may be so exceedingly slow that it is almost invisible within a time-window of, for example, 50 or 100 years. Because it is visually indistinguishable, the situation may mistakenly be identified as poverty trap. Furthermore, even if demo-economic development is observable its pattern can look very differently for unfavorably located regions compared to the experiences of the Western world.

In order to demonstrate these results, we consider the following experiment. Suppose there are three countries sharing the parameters of preference for consumption, family size, and child expenditure with the U.S. calibration from Figure 4. Assume identical technologies for all three countries: $\alpha = 0.84$, $\beta = 1/3$ and $g_A = 0.001$. Given the low rate of technological progress, population grows initially at a low equilibrium rate of 0.6 percent everywhere. The three countries differ in their fundamental child survival rates, parameterized by $b$.

For purpose of comparison we normalize time so that all countries share a common onset of the demo-economic transition. At time $t = 0$ the countries experience a permanent shock of productivity growth towards $g_A = 0.005$. The new steady state of stagnation would be at $g^*_L = 0.0317$ and does not exists for all three countries. Thus, every country undergoes a demographic transition with fundamental child survival converging towards $\bar{\pi} = a$. Eventually all countries display the fertility rate, child expenditure, savings rate, and income growth rate implied by the U.S. calibration. Their ways taken towards this balanced growth path, however, are different.

Consider first the geographically favorably located country represented by $b = 0.004$ and solid lines in Figure 5. Driven by technological progress and human capital accumulation the economy
expands in the first phase at relatively high rates above 2.5 percent. Because of the subsistence effect parents want to have larger families initially and population growth rises. Yet, after about 30 years survival probabilities are sufficiently good so that the effect of quality-quantity substitution becomes dominating and population growth begins to decline. This phase is also characterized by increasing but eventually stabilizing savings rates and education levels. After about 100 years the demo-economic transition comes to its end and the system stabilizes along a balanced growth path.

Figure 5: Patterns of Development: Hills, Plateaus, and Valleys

Parameters as for Figure 4 and $\gamma_A = 0.005$, $\beta = 1/3$, and $\alpha(1-\beta) = 0.56$.

Now consider an unfavorably located country described by $b = 0.0025$ and represented by dashed lines. This country begins also to grow economically after the technology shock but at a lower rate than the favorably located one. Income growth prospects are increasingly absorbed by population growth and after about 20 years of promising development economic growth begins to decline. After reaching a low level of economic growth below one percent – observed together with perpetually high population growth – almost nothing happens for about 50 years and some observers (of only this time window) may think that the country has stabilized at a low-level equilibrium trap. In fact, the economy develops, albeit exceedingly slow. With slightly but continuously improving child survival rates and thus increasing human capital expenditure,
the demo-economic system finally reaches its turning point where population growth begins to decline and the second phase of transition ignites.

Finally, to really make the point, consider an even worse scenario where \( b=0.002 \), represented by dotted lines. Here, the reaction of population growth lets the country almost balance at stagnation level and no positive development is visible within the given time frame. Yet, we know from theoretical analysis that even this country will finally undergo the demographic transition and converge towards the balanced growth path. More generally, although every country finally manages the demographic transition, the pattern of demo-economic development may look very different at unfavorable, tropical locations compared to the successful example delivered by the Western world. In words of Pritchett (2000), we observe not only hills, but also plateaus, valleys, and plains.

Putting the patterns together our analysis suggests that we observe economic divergence from cross-country perspective during the phase of demographic transition. This relates the present paper to Lucas (2002) who argues that ultimately “we will see a world that, economically, looks more and more like the United States today” but that different speeds of transition are observed based on the heterogeneity of human capital accumulation and the child quality-quantity trade-off.\(^9\) Here we have presented a theory that explains who are the leaders and laggars in this process i.e. why it is no coincidence that the geographically unfavorably located regions are lagging behind.

\[ \text{8. Final Remarks} \]

This article has offered a theory that explains why population growth is high at geographically unfavorable (tropical) locations and how this may cause a particularly poor demo-economic performance of these regions. Thereby it has been become obvious that the question whether a country is stuck in a population- or poverty-trap or whether no such thing exists may be of second-order practical relevance.\(^{10}\) For an unfavorably located region a demographic transition (albeit perpetually ongoing) may be so exceedingly slow that it becomes indistinguishable from actual stagnation within a medium time frame.

\(^9\)See also Ngai (2004) for a related quantitative experiment within the Hansen–Prescott (2002) framework.\(^{10}\)Conceding, of course, that it can be an interesting theoretical issue. See Bloom et al. (2003) for a special investigation on geographic determinism versus poverty traps.
The proposed economic channel based on child survival and health costs complements other, socio-cultural explanations of high population growth in Sub-Saharan Africa and other tropical regions (e.g. Chesnais, 1992). The main advantage compared to explanations built on differences of preferences is that policy recommendations are straightforward to be drawn from the cost channel. In short, a health policy accomplishing that all diamonds in Figure 1 are located on a horizontal line at survival rates of today’s fully developed economies disables the proposed mechanism for slow or stagnant economic development. Thus, geography’s indirect impact on development could, at least theoretically, be overcome by improvements of nutrition and by disease eradication. This slightly optimistic outlook distinguishes the theory from the complementing institutions-based approach where the indirect influence of geography originates from mortality 200 years ago.

Technological progress has deliberately been modelled as being exogenous because the paper does not claim to contribute to the new field of “unified growth theories” that explains development of countries at the technological frontier (see Galor, 2005). Conversely, the focus on today’s (non-) developing countries has emphasized that there may exist non-unifiable patterns of demo-economic development. Depending on geographic location we observe plains and valleys where there were only hills visible at times of the Western world’s demographic transition and (unified) growth process.

Nevertheless, interesting future extensions of the model are conceivable. In particular, an extension towards a two-sector model (as in Hansen and Prescott, 2002) could explain why decreasing returns to scale are at work at underdeveloped, geographically unfavorable locations (where demand consists mainly of food) and are insignificant when a demographic transition has been successfully accomplished and income is high (and demand of industrial products and services dominates). As a first guess, it can be expected that such an extension would amplify rather than curtail geography’s indirect influence on economic development.
References


