On the Political Economy of Adverse Selection

Hervé Crès
HEC School of Management [*]

Mich Tvede
HEC School of Management

University of Copenhagen [†]

Abstract

We consider a standard insurance economy where consumers are supposed to vote over menus of insurance contracts: A menu of contracts is majority stable if there does not exist another menu which is supported by an appropriate majority of consumers. We compute the smallest level of super majority for which there always exists a stable menu of contracts, and such that all stable menus of contracts are Pareto optimal.

Keywords: Adverse selection, super majority voting.
JEL-classification: D7, D8.

1 Introduction

We consider a standard insurance economy à la Rothschild & Stiglitz (1976) with S individual states and I types of consumers, where types are characterized by their probability distributions over the individual states. The type of a consumer is private information. Since Rothschild & Stiglitz it is known that there need not exist competitive equilibria where: (i) consumers choose between insurance contracts in order to maximize utilities, and; (ii) firms produce insurance contracts in order to maximize profits. Moreover if equilibria exist then they need not be Pareto optimal relative to the available information (constrained Pareto optimal for short). The source of the market failure is discussed in Rothschild & Stiglitz and Prescott & Townsend (1984), where economies with adverse selection as well as other environments are explored in a simple, unified structure.

In order to get existence and optimality of equilibrium allocations different modifications of the market structure have been considered by Bisin & Gottardi (2000) and Rustichini & Siconolfi (2003). In both papers consumers declare their type (they may lie) and trade state-contingent goods *at type-dependent prices*. In Rustichini & Siconolfi there are no firms and a notion of weak equilibrium is introduced for which existence is proven; but weak equilibria need not be constrained Pareto optimal.

In Bisin & Gottardi the problem is modeled as a consumption externality that comes through the admissible consumption set: the set of feasible net-trades for consumers is constrained by incentive compatibility conditions and therefore by the net-trades of all types. The externalities are internalized through an expansion of the commodity space in the spirit of Arrow-Lindahl : on top of state contingent commodities, agents trade 'coasian' property rights on each other's consumption. Bisin & Gottardi show that equilibria exist and are all constrained optimal. But a large number of such markets have to be created (actually $(I-1)IS$).

The present note investigates how this minimal, yet important, complexity of the market mechanism needed to decentralize constrained Pareto-optimal allocations (in the presence of adverse selection) translates when the alternative route of a voting mechanism is followed. Here consumers are supposed to vote over menus of insurance contracts: A menu of contracts is stable if there does not exist another menu which

is supported by an appropriate majority of the consumers. In general super majority voting rules are needed to ensure existence of stable menus of contracts and stable menus of contracts need not be Pareto optimal. However we find that when the rate of super majority is high enough (i.e., larger than $1 - 1/I$ when I contracts with S states are being offered), then stable menues exist and are Pareto optimal.

Lower super majority voting rules may ensure existence of stable menus if individual states and/or types of consumers are aggregated (i.e., when there are $K(\leq I)$ contracts insuring $T(\leq S)$ groups of individual states, the rate decreases to $1-1/KT$ for $KT < I$), but at the expense of Pareto optimality. Hence the tradeoff between Pareto optimality and trading possibilities exhibited in Bisin & Gottardi and Rustichini & Siconolfi is reflected here in a tradeoff between Pareto optimality and the conservativeness of the needed super majority voting rule: the 'price' to pay for the first welfare theorem is either the construction of $(I - 1)IS$ new markets for trading external consumptions or the establishment of a conservative rate of super majority. The higher the number of missing markets, the higher the needed rate of super majority, a finding that was already made in Tvede & Crès (2005) in the case of incomplete markets.

The paper is organized as follows: In Section 2 the model is outlined; in Section 3 the notion of equilibrium is introduced and the lowest possible level of super majority needed to ensure existence of equilibrium is found; in Section 4 optimality properties of equilibria are discussed, and; finally in Section 5 a property of equilibrium contracts in case types of consumers are aggregated is established.

2   Setup

Consider a standard insurance economy: There is a finite set of individual states $\mathcal{S} = \{1, \ldots, S\}$; a consumer in state s has the endowment ω^s. There is a finite number of types of consumers $\mathcal{I} = \{1, \ldots, I\}$ and a continuum of each type. The fraction of consumers of type i is e_i, where $e_i > 0$ and $\sum_i e_i = 1$. The type of a consumer is private information.

Consumers have the same utility function $u : \mathbb{R}_+ \rightarrow \mathbb{R}$ over the set of indi-

vidual states. The state utility function $u : \mathbb{R}_+ \to \mathbb{R}$ is assumed to be concave and non-decreasing on $\mathbb{R}_+$ and either continuous on $\mathbb{R}_+$ or continuous on $\mathbb{R}_{++}$ with $\lim_{c\to 0} u(c) = -\infty$. Consumers of type i are characterized by their probability distribution $\pi_i = (\pi_i^1, \ldots, \pi_i^S)$ on the set of individual states, where $\pi_i^s > 0$ and $\sum_s \pi_i^s = 1$. The fraction of consumers of type i who are in state s is assumed to be π_i^s. The consumers are supposed to maximize their expected utility, so the utility of type i consumer choosing the insurance contract $z_i = (z_i^1, \ldots, z_i^S)$ is $\sum_s \pi_i^s u(\omega^s + z_i^s)$. Dividends of contracts z_i^s are supposed to belong to a compact set $C^s \subset [-\omega^s, \infty[$ where $0 \in C^s$.

Both states and types of consumers may be aggregated in order to reduce the complexity of menus of contracts. Let $\mathcal{P}_{\mathcal{S}} = \{\mathcal{S}_1, \ldots, \mathcal{S}_T\}$ be a partition of $\mathcal{S}$, let $\mathcal{P}_{\mathcal{I}} = \{\mathcal{I}_1, \ldots, \mathcal{I}_K\}$ be a partition of $\mathcal{I}$ and let $\mathcal{P} = (\mathcal{P}_{\mathcal{I}}, \mathcal{P}_{\mathcal{S}})$. Suppose that there is a menu z of K insurance contracts: $z = (z_1, \ldots, z_K)$, where $z_k = (z_k^1, \ldots, z_k^T)$ and $z_k^t \in \cap_{s\in\mathcal{S}_t} C^s$ for all k and t. Contracts are exclusive in the sense that consumers may hold one and only one contract. A menu of contracts is *feasible*, if the average dividend is less than zero, so

$$\sum_k \sum_t z_k^t \sum_{i\in\mathcal{I}_k} e_i \sum_{s\in\mathcal{S}_t} \pi_i^s \ \leq \ 0, \tag{1}$$

incentive compatible if consumers in $\mathcal{I}_k$ weakly prefer contract k to all other contracts, so for all $i \in \mathcal{I}_k$ and k'

$$U_i^{\mathcal{P}}(z_k) \ = \ \sum_t \sum_{s\in\mathcal{S}_t} \pi_i^s u(\omega^s + z_k^t) \ \geq \ \sum_t \sum_{s\in\mathcal{S}_t} \pi_i^s u(\omega^s + z_{k'}^t) \ = \ U_i^{\mathcal{P}}(z_{k'}) \tag{2}$$

and *individually rational* if consumers in $\mathcal{I}_k$ weakly prefer contract k to their own endowments, so for all $i \in \mathcal{I}_k$

$$U_i^{\mathcal{P}}(z_k) \ = \ \sum_t \sum_{s\in S_t} \pi_i^s u(\omega_i^s + z_k^t) \ \geq \ \sum_s \pi_i^s u_i(\omega^s) \ = \ U_i^{\mathcal{P}}(0). \tag{3}$$

For two menus of contracts z and z', let $\mathcal{I}(z, z', \mathcal{P}) \subset \mathcal{I}$ be the set of types which prefer z to z', so

$$\mathcal{I}(z, z', \mathcal{P}) \ = \ \cup_k \{i \in \mathcal{I}_k | U_i^{\mathcal{P}}(z_k) > U_i^{\mathcal{P}}(z_k')\} \ ;$$

and let $\tau(z, z', \mathcal{P}) \in [0, 1]$ denote the fraction of consumers who prefer z to z', so

$$\tau(z, z', \mathcal{P}) = \sum_{i \in \mathcal{I}(z, z', \mathcal{P})} e_i .$$

3 Equilibrium

We consider an equilibrium notion based on (super) majority voting, so if two menus of contracts z and z' are compared and the support for z' is sufficiently large compared with the total population, then z' defeats z.

Definition 1 *A ρ-**majority stable equilibrium** for $\mathcal{P}$ is a menu of contracts z such that:*

- *z is feasible, incentive compatible and individually rational, and;*

- *for all feasible, incentive compatible and individually rational menus of contracts z'*

$$\tau(z', z, \mathcal{P}) \leq \rho.$$

Menus of contracts may be described in the space of contracts or alternatively in the space of (excess) utilities. In the latter case menus of contracts are characterized by the utility in each state (minus the utility of the endowment) rather than characterized by the payment in each state. Therefore let $\mathcal{Z}^{\mathcal{P}} \subset \mathbb{R}^{IS}$ be defined by

$$\mathcal{Z}^{\mathcal{P}} = \{z \mid \sum_s \sum_i z_i^s e_i \pi_i^s \leq 0 \text{ and } \forall P \in \mathcal{P}, (i, s), (i', s') \in P : z_i^s = z_{i'}^{s'}\}.$$

Let the map $e : \mathcal{Z}^{\mathcal{P}} \to \mathbb{R}^{IS}$ be defined by

$$e(z) = (u(\omega^1 + z_1^1) - u(\omega^1), \ldots, u(\omega^S + z_I^S) - u(\omega^S))$$

and let $\mathcal{A}^{\mathcal{P}} \subset \mathbb{R}^{IS}$ be the comprehensive hull of the set of excess utilities in the state-type space that are obtainable from feasible, incentive compatible and individually rational menus of contracts that are compatible with $\mathcal{P}$:

$$\mathcal{A}^{\mathcal{P}} = (e(\mathcal{Z}^{\mathcal{P}}) - \mathbb{R}_+^{IS}) \cap \{a \mid a \text{ satisfies (4) and (5)}\}$$

5

where

$$\pi_i \cdot (a_i - a_{i'}) \geq 0 \text{ for all } i, i' \tag{4}$$

$$\pi_i \cdot a_i \geq 0 \text{ for all } i. \tag{5}$$

Lemma 1 $\mathcal{A}^{\mathcal{P}}$ *is non-empty, closed, convex and bounded.*

Proof: Firstly, $\mathcal{A}^{\mathcal{P}}$ is non-empty, because $0 \in \mathcal{A}^{\mathcal{P}}$. Secondly $\mathcal{A}^{\mathcal{P}}$ is closed and convex, because $\mathcal{Z}^{\mathcal{P}}$ is closed and convex, the map $e : \mathcal{Z}^{\mathcal{P}} \to \mathbb{R}^{IS}$ is concave and $\mathcal{A}^{\mathcal{P}}$ is the intersection of $e(\mathcal{Z}^{\mathcal{P}}) - \mathbb{R}_+^{IS}$ and some closed and convex sets defined by (4) and (5). Thirdly $\mathcal{A}^{\mathcal{P}}$ is bounded from above, because if $\bar{a}$ is defined by $\bar{a}_i^s = u(\sum_s \omega^s \sum_i e_i \pi_i^s) - u(\omega^s)$, then $\mathcal{A}^{\mathcal{P}}$ is bounded from above by $\bar{a}$. Fourthly, $\mathcal{A}^{\mathcal{P}}$ is bounded from below, because $\mathcal{A}^{\mathcal{P}}$ is bounded from above and if $a \in \mathcal{A}^{\mathcal{P}}$ then $\pi_i \cdot a_i \geq 0$ for all i.

Q.E.D

Let the map $\Pi : \mathbb{R}^{IS} \to \mathbb{R}^I$ be defined by

$$\Pi(a) = \left(\sum_s \pi_1^s a_1^s, \ldots, \sum_s \pi_I^s a_I^s \right)$$

and let $\mathcal{B}^{\mathcal{P}} \subset \mathbb{R}^I$ be defined by $\mathcal{B}^{\mathcal{P}} = \Pi(\mathcal{A}^{\mathcal{P}})$. Then $\mathcal{B}^{\mathcal{P}}$ is the set of excess utilities in the type space that are obtainable from feasible, incentive compatible and individually rational menus of contracts compatible with $\mathcal{P}$.

Corollary 1 $\mathcal{B}^{\mathcal{P}}$ *is non-empty, closed, convex and bounded.*

The rate of majority ρ needed to ensure existence of a ρ-majority stable equilibrium depends on the level of aggregation KT and the number of types I.

Theorem 1 *Suppose that*

$$\rho \geq 1 - \max\left\{ \frac{1}{KT}, \frac{1}{I} \right\}.$$

Then there exists a ρ-majority stable equilibrium for $\mathcal{P}$.

6

Proof: "$\rho \geq 1 - 1/KT$" Menus of contracts are in $\mathbb{R}^{KT}$, but clearly if a menu of incentive compatible contracts satisfies the feasibility constraint with ">", then there exists another menu of incentive compatible contracts such that all types of consumers are better off. Therefore the relevant set of contracts $\mathcal{F}^{\mathcal{P}} \subset \mathbb{R}^{KT}$ is

$$\mathcal{F}^{\mathcal{P}} \;=\; \{z \in \mathbb{R}^{KT} | \textstyle\sum_k \sum_t z_k^t \sum_{i \in \mathcal{I}_k} e_i \sum_{s \in \mathcal{S}_t} \pi_i^s \;=\; 0 \text{ and } z_k^t \in \cap_{s \in \mathcal{S}_t} C^s\}.$$

The dimension of $\mathcal{F}^{\mathcal{P}}$ is equal to or less than $KT - 1$.

Artificial agents are introduced: incentive compatibility agents (*ic*-agents for short) and individual rationality agents (*ir*-agents for short). Let the preference correspondence of the identical *ic*-agents $P_{ic} : \mathcal{F}^{\mathcal{P}} \to \mathcal{F}^{\mathcal{P}}$ be defined as follows:

$$P_{ic}(z) \;=\; \begin{cases} \emptyset & \text{for } z \text{ incentive compatible} \\[2mm] B(0, \|z\|) \cap \mathcal{F}^{\mathcal{P}} & \text{otherwise,} \end{cases}$$

where $B(0, \|z\|)$ is the open ball in $\mathcal{F}^{\mathcal{P}}$ with center 0 and radius $\|z\|$. Since the set of incentive compatible contracts is closed, the graph of P_{ic} is open. Let the preference correspondence of the identical *ir*-agents $P_{ir} : \mathcal{F}^{\mathcal{P}} \to \mathcal{F}^{\mathcal{P}}$ be defined as follows:

$$P_{ir}(z) \;=\; \begin{cases} \emptyset & \text{for } z \text{ individually rational} \\[2mm] B(0, \|z\|) \cap \mathcal{F}^{\mathcal{P}} & \text{otherwise.} \end{cases}$$

Since the set of incentive compatible contracts is closed, the graph of P_{ir} is open. Therefore according to Theorem 3 in Greenberg (1979), there exists a δ-relative equilibrium for the extended economy as soon as $\delta \geq KT - 1$, where a δ-relative equilibrium is a menu of contracts z, such that the 'number' of agents who support z' against z is less than or equal to δ times the 'number' of agents who support z against z' (see Greenberg (1979) for a formal definition of a δ-relative equilibrium).

Let e_{ic} resp. e_{ir} be the 'number' of *ic*-agents resp. *ir*-agents. Consider the extended economy with consumers and artificial agents, so the total 'number' of agents in the extended economy is $\sum_i e_i + e_{ic} + e_{ir} = 1 + e_{ic} + e_{ir}$. Take $e_{ic}, e_{ir} > KT - 1$, then the δ-relative equilibrium for the extended economy is incentive compatible and individually rational, so

$$\sum_{i \in \tau(z, z', \mathcal{P})} e_i \;\leq\; \delta \sum_{i \in \tau(z', z, \mathcal{P})} e_i.$$

Clearly if z is a δ-relative equilibrium for $\delta \geq KT - 1$, then z is ρ-majority stable equilibrium for $\rho \geq 1 - 1/KT$.

"$\rho \geq 1 - 1/I$" Let the map $\mathrm{pr} : \mathbb{R}^I \to \mathbb{R}^{I-1}$ be defined by

$$\mathrm{pr}(b) \;=\; (b_1, \ldots, b_{I-1}).$$

and let $\mathcal{C}^{\mathcal{P}} \subset \mathbb{R}^{I-1}$ be defined by $\mathcal{C}^{\mathcal{P}} = \mathrm{pr}(\mathcal{B}^{\mathcal{P}})$. Then $\mathcal{C}^{\mathcal{P}}$ is non-empty, closed, convex and bounded, because $\mathcal{B}^{\mathcal{P}}$ is non-empty, closed, convex and bounded according to Corollary 1 and $\mathrm{pr} : \mathbb{R}^I \to \mathbb{R}^{I-1}$ is a linear map. Let the preferences of consumers of type i on $\mathcal{C}^{\mathcal{P}}$ be defined by $v_i(c) = c_i$ for $i \leq I - 1$ and

$$v_i(c) \;=\; \sup_{b \in \mathrm{pr}^{-1}(c) \cap \mathcal{B}^{\mathcal{P}}} b_i.$$

for $i = I$. By construction v_i is continuous for $i \leq I - 1$ and if the correspondence $\mathrm{pr}^{-1} : \mathcal{C}^{\mathcal{P}} \to \mathcal{B}^{\mathcal{P}}$ is continuous, then it follows from Berge's Maximum Theorem that $v_i : \mathcal{C}^{\mathcal{P}} \to \mathbb{R}$ is continuous for $i = I$. Therefore if the correspondence $\mathrm{pr}^{-1} : \mathcal{C}^{\mathcal{P}} \to \mathcal{B}^{\mathcal{P}}$ is continuous then according to Theorem 2 in Greenberg (1979), there exists a ρ-majority stable equilibrium for the extended economy as soon as $\rho \geq 1 - 1/I$.

The correspondence $\mathrm{pr}^{-1} : \mathcal{C}^{\mathcal{P}} \to \mathcal{B}^{\mathcal{P}}$ is upper hemi-continuous according to Proposition 11.21 in Border (1985). The correspondence is lower hemi-continuous because if $E \subset \mathcal{C}^{\mathcal{P}}$ is open in $\mathcal{C}^{\mathcal{P}}$, then the lower inverse of the correspondence $(\mathrm{pr}^{-1})^-(E) = \{c | (\mathrm{pr}^{-1})(c) \cap E \neq \emptyset\}$ is $(E \times \mathbb{R}) \cap \mathcal{B}^{\mathcal{P}}$ which is open in $\mathcal{B}^{\mathcal{P}}$. Therefore the correspondence $\mathrm{pr}^{-1} : \mathcal{C}^{\mathcal{P}} \to \mathcal{B}^{\mathcal{P}}$ is continuous.

$$Q.E.D$$

Examples showing that the bounds are tight may easily be constructed.

Suppose that $e_i > 1/I$. Then the menu of contracts that maximizes the utility of type i given feasibility, incentive compatibility and individual rationality, is a ρ-majority stable equilibrium. Indeed for any other menu of contracts the fraction of consumers who prefer the other menu of contracts is at most $\sum_{j \neq i} e_j = 1 - e_i < 1 - 1/I$.

4   Optimality of equilibria

Clearly the fact that the type of a consumer is private information should be taken into account in the notion of Pareto optimality.

Definition 2 *A menu of feasible, incentive compatible and individually rational contracts z is* **constrained Pareto optimal** *if there is no other menu of incentive compatible and feasible contracts z' where $z' = (z_1', \ldots, z_I')$ and $z_i' = (z_i'^1, \ldots, z_i'^S)$, such that $\sum_s \pi_i^s u(\omega^s + z_i'^s) \geq \sum_t \sum_{s \in \mathcal{S}_t} \pi_i^s u_i(\omega^s + z_k^t)$ for all i with ">" for at least one i.*

Intuitively: if z is not constrained Pareto optimal, then there exists a menu of contracts z' such that all types are better off with z' than with z, and agents would unanimously support the change from z to z'.

Corollary 2 *If neither states nor types of consumers are aggregated, so $T = S$ and $K = I$, then all equilibria are constrained Pareto optimal.*

5   A property of $\mathcal{A}^{\mathcal{P}}$ and aggregation of types

Suppose that types of consumers are aggregated such that π_i is in the convex hull of $(\pi_{i'})_{i' \in \mathcal{I}_{k'}}$. Then consumer i' gets the same utility from the contracts offered to i and i'. Moreover if $(\pi_{i'})_{i' \in \mathcal{I}_{k'}}$ spans $\mathbb{R}^S$, then the contracts offered to consumer i and consumers in $\mathcal{I}_{k'}$ are identical.

Recall that if $a \in e(\mathcal{Z}^{\mathcal{P}})$ and $i, i' \in \mathcal{I}_k$, then $a_{i'} = a_i$ because $z_{i'} = z_i$.

Lemma 2 *Suppose that $a \in \mathcal{A}^{\mathcal{P}}$ and that $\pi_i = \sum_{i'} \lambda_{i'} \pi_{i'}$ where $i \in \mathcal{I}_k$, $i' \in \mathcal{I}_{k'}$ for all i' and $\lambda_{i'} > 0$. Then*

$$\pi_{i'} \cdot (a_i - a_{i'}) = 0.$$

Proof: Clearly $\pi_i \cdot (a_i - a_{i'}) = (\sum_{i' \in \mathcal{I}_{k'}} \lambda_{i'} \pi_{i'}) \cdot (a_i - a_{i'}) \geq 0$ and $\pi_{i'} \cdot (a_{i'} - a_i) \geq 0$ for all i'. Therefore $\pi_{i'} \cdot (a_i - a_{i'}) = 0$.

<div align="right">Q.E.D</div>

References

Bisin, A., & P. Gottardi (2000). Efficient Competitive Equilibria with Adverse Selection, working paper.

Border, K. (1985). *Fixed point theorems with applications to economics and game theory*, Cambridge University Press, Cambridge, England.

Greenberg, J., (1979). Consistent majority rules over compact sets of alternatives, *Econometrica*, Vol. 47, 627-636.

Prescott, E., & R. Townsend (1984). Pareto Optima and Competitive Equilibria with Adverse Selection and Moral Hazard, *Econometrica*, Vol. 52, 21-45.

Rothschild, M., & J. Stiglitz (1976). Equilibrium in Competitive Insurance Markets: An Essay on the Economics of Imperfect Competition, *Quaterly Journal of Economics*, Vol. 90, 629-649.

Rustichini, A., & P. Siconolfi (2002). General Equilibrium in Economies with Adverse Selection, working paper.

Tvede, M., & H. Crès (2005). Voting in Assemblies of Shareholders and Incomplete Markets, *Economic Theory*, Vol. 26, 887-906.