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Price Volatility and Banking in Green Certificate Markets[*]

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Abstract

There is concern that prices in a market for Green Certificates (GCs) primarily based on volatile wind power will fluctuate excessively, leading to corresponding volatility of electricity prices. Applying a rational expectations simulation model of competitive storage and speculation of GCs the paper shows that the introduction of banking of GCs may reduce price volatility considerably and lead to increased social surplus. Banking lowers average prices and is therefore not necessarily to the benefit of "green producers". Proposed price bounds on GC-prices will reduce the importance of banking and even of the GC system itself.

JEL classification code: Q28, Q42, Q48

Keywords: Electricity, Environment, Commodity Speculation, Green Certificates, Marketable permits, Uncertainty

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1. Introduction

Many countries are about to design and introduce systems of Green Certificates (GCs) in order to stimulate electricity generation from renewable energy sources. Common to these systems is that they seek to replace systems of direct governmental subsidies of renewable energy by a market mechanism that allows for voluntary demand of GCs. In these systems consumers may express their willingness to pay a surcharge to cover the higher electricity generation costs of renewable energy sources. However, to the extent that voluntary demand is not judged sufficient, the system may involve various rules of mandatory demand of GCs. This is the case in the proposed Danish and Swedish systems (see e.g. Amundsen and Mortensen (2001, 2002); Unger and Ahlgren (2003)). Supply in GC markets comes from producers of electricity using renewable sources (green producers) that obtain an amount of GCs corresponding to the amount of electricity they load into the network. For each kWh generated the green producers thus receive both a wholesale price and a GC from the certificate issuing authority that they can sell in the GC market. Demand for GCs comes from consumers/distribution companies that are required by the government to buy certificates (including voluntary purchases) corresponding to at least a given percentage (the percentage rule) of their total consumption of electricity. On the basis of supply and demand the GC-market functions like any other market to determine a price within administratively determined upper and lower price bounds. Thus, the combination of a certificate system and price bounds is not unlike the mixed permit-charge system proposed by Roberts and Spence (1976) for the case of uncertain abatements costs of a damaging emission.

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Recently, however, concern has been expressed regarding the ability of such a system to provide a stable environment for additional investments in renewable energy sources. The problem originates with two features of the system. The first feature is that the supply of power from renewable energy sources such as wind power may be stochastic and extremely volatile[2]. Using empirical data for Denmark it is estimated that the maximum variation of the annual wind generated electricity is approximately +/- 20 % with a standard deviation of approximately 10 % (see Appendix 3)[3]. As the marginal cost of wind power generation is close to zero competitive wind power generators will at all times produce what is feasible and thus generate erratic and price inelastic supply.[4] Hence, the number of GCs issued and available for sale will also be highly volatile. The other feature is that demand for GCs may be highly price inelastic under a percentage system. Due to the percentage rule, demand for GCs is derived demand stemming from consumption of electricity. As explained below, this results in a price elasticity of demand for GCs that is only a fraction of the elasticity of demand for electricity. Hence, the price of GCs will be determined by the intersection of two almost vertical curves. The consequence of such a system may thus be prices erratically bouncing up and down between the upper and lower price bounds of the system.

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[2] There may of course be other renewable technologies that are not as volatile, e.g. renewable energy based on biomass. Such technologies will not cause serious problems with respect to volatility of GC prices and electricity prices. However, to focus on the volatility problem that may be serious for countries with a sizable wind power sector (such as Denmark), we have chosen not to take such alternative technologies explicitly into account.

[3] In the Swedish and the proposed Danish system of GCs firms must cover their required share of green electricity on an annual basis. It is the implication of this restriction that we are studying. A year is therefore the natural unit of time in our model. The considerable within-year variation in wind, electricity prices and marginal costs of black generation is therefore not important for the results of our model. What is important is the annual variation of wind power generation. As noted, this may be quite sizable. See Appendix 3 for a small note on the statistical properties of the Danish wind energy series supporting the claim of a sizable annual variation.

[4] This also imposes additional demands on electricity system management, but this is not an aspect we consider in this paper.

A theoretical foundation for the analysis of intertemporal trading in pollution permit markets has been created to study policy questions concerning the design of such intertemporal markets. Chronshaw and Kruse (1996) and Rubin (1996) show that one-for-one intertemporal trading schemes enable polluting firms to jointly minimize their pollution abatement costs over time. Kling and Rubin (1997) study the effects of intertemporal trading on abatement costs and damages costs from pollution. They show that one-for-one trades may not be the preferred structure for intertemporal permit trading. Yates and Cronshaw (2001) consider the same problems in an asymmetric information setting and determine the preferred permit discount rate in the intertemporal pollution permit market. They also consider whether or not the regulator should allow intertemporal trading to take place and view their results as contribution to the literature on command and control versus decentralization over time in environmental regulation. In these papers the aggregate supply of permits is determined by the authorities. None of the papers considers a system like the GC system we consider where the supply of permits is stochastic.

The main objective of this paper is to investigate to what extent price volatility is reduced if a system of banking (or storage) of GCs is allowed. Lessons from the theory of commodity markets (see Wright and Williams (1984), Williams and Wright (1991), Deaton and Laroque (1992), and Deaton and Laroque (1996)) tell us that storage and speculation may lead to less erratic prices and reduced price variance even though occasional price spikes are unavoidable. Markets for commodities such as wheat, sugar and coffee have many of the same characteristics as GC markets: output is subject to large random shocks and short-term demand and supply elasticities are low. In commodities markets the inherent short-term price risk following from these characteristics is in part pooled and reduced through trade between regions with imperfectly correlated output. However, price risk is also reduced by trade over time, i.e. by transferring some of the output from years of abundance to years with low output. Under such conditions a rational speculator would want to keep a storable commodity only if the present value of next year's expected price net of depreciation is at least as high as this year's price (due consideration paid to convenience yield). Thus, in periods of abundance (large harvest and large inventories) when prices would otherwise be very low if driven by consumption alone, speculators will buy the commodity for storage and drive up the price until the present value of next year's expected price net of depreciation is equal to this year's price. Furthermore, in periods of scarcity (small harvests and small inventories) consumption demand will drive the price to a level where it cannot possibly pay to keep the commodity in store. Therefore, there will be a "stock out" of the commodity in question and prices then usually peak.

While there are many similarities between commodities markets and markets for GCs, there are also some differences. In particular, GCs are issued by a governmental body and are not directly subject to Nature's whims. Hence, if the price of GCs should tend to rise above some upper price bound additional GCs may be issued for sale to prevent price to increase further. This is not an option in commodities markets as Nature sets a limit to current harvest and it is not possible to borrow from the future. At the other end of the price scale, however, options are similar. Just as the authorities may pay subsidies to producers of a given commodity to keep storages so as to prevent the price from falling further, the authorities may protect the producers of "green" electricity by purchasing GCs if the price tends to fall below some given level. Historically, however, price-band schemes of this kind have had a tendency to accumulate very large stocks and when they become an intolerable burden on the public budget, the system typically collapses with severe consequences for producers

(Williams and Wright, 1991). Hence, if this will be the case also for GC markets then the price dampening effect of speculation will not in itself be sufficient to guarantee stability and sustainability of GC markets. However, as pointed out above there is an important difference as the provision of GCs is in the hands of the issuing authority and not of Nature. In addition, storage costs and depreciation for GCs are not of the same order of magnitude as for storable agricultural commodities.

In the following we set out to study the stabilizing effects of competitive storage and speculation on GC prices, electricity prices and electricity consumption. Furthermore, effects on consumers', producers' and social surplus are investigated. In particular, it will be shown that while banking - as expected - leads to increased social and consumers' surplus it does not necessarily lead to increased surplus for green producers. Finally, some questions related to price bounds are dealt with.

2. Model

2.1 General assumptions

We formulate our model[5] in discrete time, t=1,2,… It is convenient to assume that each time period corresponds to one year.[6] There are two real goods in the model: green (renewable) and black (thermal) electricity. Both types of electricity are of the same utility to consumers. There is also a financial product: green certificates (GCs). These are not assumed to have any utility in consumption, but have a value due to

[5] For non dynamical short and long run analytical model of green certificates see Amundsen and Mortensen (2001, 2002). See also Jensen and Skytte (2002) and Fristrup (2003).
[6] This assumption can easily be changed to model time periods of different length. For shorter periods seasonal fluctuations in supply (wind) and demand would then have to be taken into account.

government regulation.[7] Banking of GCs, i.e. saving them for later use, may or may not be allowed.

There are several types of agents in the model:

- Producers of green electricity: Sell electricity in the wholesale market. Receive and sell GCs in direct relation to the amount of green electricity produced.
- Producers of black electricity: Sell electricity in the wholesale market.
- Electricity retailers: Purchase electricity from producers in the wholesale market and sell to final consumers. Must cover a share of their sales with the purchase of GCs.
- Speculators in GCs: Operate only when banking of GCs is allowed.
- Consumers: Purchase electricity from retailers for final use.

Electricity and GC markets are assumed to be competitive such that all agents take prices and other aggregate quantities as given. Producers, retailers and speculators are assumed to maximize their profits and consumers maximize their utility. We assume all agents of a given type to be identical so without loss of generality we can identify quantities at the agent and the aggregate levels. The government is not assumed to intervene in markets (apart from creating the market for GCs and apart from selling and buying GCs at upper and lower price bounds, respectively, when such bounds are imposed) unless this is made explicit.

Separation of agents by activities should be interpreted as a separation of roles rather than a physical separation. Thus, the same firm could engage in production of

[7] Presumably the regulation is based on a social and political valuation of the desirability of generation of electricity from renewable sources, but this does not have a bearing on the focus of this paper and is not modeled here.

both green and black electricity as well as participate in speculation and retailing. What matters is that no agent has market power and that each agent maximizes profits from each activity separately.[8]

We assume the capacity to produce renewable electricity is given and fixed.[9] However, production of renewable energy – which will be mainly from wind – and the corresponding amount of GCs issued will fluctuate with wind conditions. This is a reasonable assumption in the short term, since variable costs of running windmills are negligible. Hence, production of renewable electricity is given as a sequence of independent and identically distributed (i.i.d.) random variables, $z_1, z_2, \ldots$ which are assumed to be exogenous in our model. In particular, they are independent of prices and other model variables. We also make the technical assumption that the distribution of z_i has support in the compact interval $[\underline{z}, \overline{z}]$, where $0 < \underline{z} < \overline{z} < \infty$.

Black producers are assumed to generate electricity from thermal sources at a fixed marginal cost *c*. Since their supply is perfectly elastic and competition is assumed to be perfect, the wholesale price of electricity – irrespective of its "color" – will also equal *c*. This implies that black producers will change their production to accommodate fluctuations in the supply of green electricity (i.e. acting as "swing producers") when these would otherwise bring prices in the wholesale market above or below *c*. The simplifying assumption about fixed marginal cost reflects that the variation in marginal

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[8] Furthermore, there can be no external economies, negative or positive, of one activity on another.

[9] Our model also encompasses the case where windmill capacity grows exogenously at the same rate as demand for electricity.

cost c is likely to be of second order compared to variation caused by change in output of green electricity combined with the extremely low derived demand elasticity[10].

Consumption of electricity, denoted by *x*, depends on the retail price of electricity, *p*, i.e. $x = D(p)$, where *D*(.) is assumed to be strictly decreasing and continuous. Inverse demand is denoted by:

$$p = P(x) = D^{-1}(x) \tag{1}$$

Demand for GCs is created through a regulatory rule that retailers must purchase GCs corresponding to a certain share $\alpha \in (0,1)$ of their electricity sales[11]. Demand for GCs in excess of this ("voluntary" demand) is assumed to be zero. This implies a simple linear relationship between sales of GCs, denoted by *w* and consumption of electricity, *viz*.

$$x = \frac{w}{\alpha} \tag{2}$$

Assume, for simplicity of notation and without loss of generality, that costs of sales, transportation and distribution are incorporated in the wholesale price *c*. Denoting the price of GCs by *s*, the final (consumer) price of electricity in competitive equilibrium is

$$p = \alpha s + c\,. \tag{3}$$

Assuming free disposal of GCs the lowest possible value for *s* is zero:

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[10] Introducing a more realistic (non constant) MC curve and elastic supply, would bring about some reduction in price fluctuations. However, we are looking at long term adjustments in a stable economy. While it is true that marginal cost may vary on a short run basis (on peak/off peak), the setting of this model is such that we are considering long run and annual averages. (GCs are settled on an annual basis). Hence a stable optimal mix of technologies is considered. Also, we look at oscillations around a steady state. For smaller variations, at least, marginal costs may be considered a constant.

[11] Note that in our model α is given by the institutional framework (like in the Swedish system, the proposed Danish system and presumably also in the new Norwegian GC-system that is

$$s \geq 0\,. \tag{4}$$

2.2 No banking permitted

If banking of certificates is not allowed all green certificates must be sold in the same year they are issued, i.e. $w \equiv z$. Combining (1), (2) and (3) under this assumption and solving for *s* yields

$$s_t = \tfrac{1}{\alpha}\left[P\left(\tfrac{z_t}{\alpha}\right) - c\right]. \tag{5}$$

However, account must be taken of years with excessive supply of green electricity – i.e. of the constraint (4) – when prices of GCs hit the bottom of zero. For this purpose, let x^c be such that $P\left(x^c\right) = c$ and define

$$z^c = \alpha x^c\,. \tag{6}$$

Then,

$$s_t = \begin{cases} \tfrac{1}{\alpha}\left[P\left(\tfrac{z_t}{\alpha}\right) - c\right] & \text{if } \; z_t \leq z^c \\ 0 & \text{if } \; z_t > z^c. \end{cases} \tag{7}$$

Define the function

$$S\left(z\right) = \tfrac{1}{\alpha}\left[P\left(\tfrac{z}{\alpha}\right) - c\right]^{+}, \; z \geq 0\,. \tag{8}$$

Then *S* is the derived, inverse demand function for GCs (i.e. derived from the demand for electricity and the supply of black producers) and $s_t = S\left(z_t\right)$, $t = 1, 2, \ldots$. Hence, prices of GCs, $\mathbf{s} = \left\{s_t; t = 1, 2, \ldots\right\}$ are a sequence of i.i.d. random variables, like the sequence of green electricity output $\mathbf{z} = \left\{z_t; t = 1, 2, \ldots\right\}$. The probability distribution of s_t can easily be derived from that of z_t.

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currently being designed). The percentage requirement is a target share of green electricity and it requires a political decision to change it.

2.3 Banking permitted[12]

When banking is not allowed, events in a given period do not influence events in later periods. Therefore, the model is in principle static and rather easily analyzed. As soon as we allow banking – saving GCs in one period for use against electricity sales in later periods – a connection is created between periods and the model becomes truly dynamic.

The aggregate stock of GCs transferred from period t to period t+1, I_t, obeys the following identity

$$\begin{aligned} I_0 &= 0 \\ I_t &= (1-\delta) I_{t-1} + z_t - w_t, \quad t \geq 1 \end{aligned} \tag{9}$$

where $\delta \in [0,1]$ is the depreciation of GCs carried over between periods. As in the previous section z_t and w_t are the amounts of GCs issued and sold, respectively, in period t, but now these variables are not necessarily equal. If $\delta = 1$, then all GCs are written off at the end of a period and we are in the same situation as in the last section, i.e. there is no trade in GCs between periods. If $0 < \delta < 1$, then GCs depreciate by the corresponding proportion when transferred between periods, but here we shall assume that $\delta = 0$ such that GCs are carried intact from one period to another and keep their value forever.[13]

When banking of GCs is permitted, speculation in them becomes relevant. As indicated before, speculation is assumed to be conducted by a separate group of agents.

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[12] In this section we adapt the commodity market model with storage to a market for GCs. Our treatment will be brief; for a comprehensive treatment of the underlying model see e.g. Deaton and Laroque (1992) and Williams and Wright (1991).

[13] There are many similarities with GCs and money, but the latter depreciate in real terms over time, e.g. because of inflation.

We assume these agents finance their operations in an efficient financial market at an interest rate $r > 0$. Speculators will therefore use the discount factor

$$\beta = \frac{1}{1+r} \in (0,1) \tag{10}$$

Speculators are assumed to entertain rational expectations, in the sense that their decisions are based on the correct model of the market. They do not have perfect foresight, so they do not know what realizations of random variables will occur, but they do know the probability distribution of random variables. Therefore, it is clear that for a speculator to be willing to hold GCs from period *t* to period *t*+1 the expected return on investment in GCs must at least equal the interest rate *r*:

$$\begin{aligned} I_t &= 0 \quad \text{if} \quad \beta E_t s_{t+1} < s_t \\ I_t &> 0 \quad \text{if} \quad \beta E_t s_{t+1} \geq s_t. \end{aligned} \tag{11}$$

Here E_t denotes the expectation conditional on (given) events occurring up to time *t*. Hence, in a given period, all GCs are sold unless the expected, discounted price in next period is at least equal to the price in the current period. Since we assume perfect competition in all markets – including the speculative market for GCs – speculation will raise prices until pure profits have vanished. Therefore, in competitive equilibrium the inequality $\beta E_t s_{t+1} \geq s_t$ of (11) turns into an exact equality and we have

$$\begin{aligned} I_t &= 0 \quad \text{if} \quad \beta E_t s_{t+1} < s_t \\ I_t &\geq 0 \quad \text{if} \quad \beta E_t s_{t+1} = s_t. \end{aligned} \tag{12}$$

In equilibrium total supply, including stocks carried over from the last period, must equal total demand, including banking demand, i.e.

$$z_t + I_{t-1} = w_t + I_t. \tag{13}$$

Recall that $s_t = S(w_t)$ and therefore we can rewrite (13) in the following way

$$z_t + I_{t-1} = S^{-1}\left(s_t\right) + I_t. \qquad (14)$$

When (12) and (14) are combined we get

$$s_t = \max\left\{\beta E_t s_{t+1}, S\left(z_t + I_{t-1}\right)\right\}. \qquad (15)$$

Given expectations for the GC price in period t+1, equation (15) determines the equilibrium price in period t and (14) determines I_t, the aggregate stock of GCs held from period t to period t+1. To close the model, it must be determined what information agents use for making decisions at time t. As noted before, we make the general assumption that all agents use the same (correct) model, they know functional forms and parameters, probability distributions etc., but as far as dynamic information is concerned, in period t agents know the total supply of green certificates, y_t:

$$y_t = z_t + I_{t-1} \qquad (16)$$

Given this variable, price of GCs, demand for GCs for current use and speculative demand are determined for period t. It is possible to show that information on other variables up to time t, say z_t, would not change the equilibrium of the model, so this assumption is not as restrictive as it appears at first sight.

Since y_t is the state variable of our model, it is natural to define the *competitive equilibrium price function* $f:\left[\underline{z},\infty\right)\rightarrow\left[0,\infty\right)$ that determines the equilibrium price of GCs as a function of the state variable. By (15) it must satisfy the equation

$$f\left(y_t\right) = \max\left\{\beta E_t f\left(y_{t+1}\right), S\left(y_t\right)\right\} \qquad (17)$$

where

$$\begin{aligned} y_{t+1} &= z_{t+1} + I_t \\ &= z_{t+1} + y_t - S^{-1}\left(f\left(y_t\right)\right) \end{aligned} \qquad (18)$$

cf. equations (16) and (14). Equation (17) holds for much more general assumptions on the distribution of the wind sequence $z_1, z_2, \ldots$ than we use here, i.e. that these random variables are i.i.d., but when this assumption is applied to (17) we get

$$f(y) = \max\left\{\beta E f\left(z + \left[y - S^{-1}\left(f(y)\right)\right]\right), S(y)\right\} \text{ for all } y \geq \underline{z}, \qquad (19)$$

where z is a generic random variable distributed like z_i. Given the regularity assumptions already made and under some additional technical conditions on S it is possible to establish existence of a solution to (19) as well as its uniqueness (see Deaton and Laroque, 1992). It can also be established that if

$$s^* = \beta E f(z) \qquad (20)$$

then we have

$$\begin{aligned} f(y) &> S(y) \quad \text{for } y \text{ such that } S(y) < s^* \\ f(y) &= S(y) \quad \text{for } y \text{ such that } S(y) \geq s^* \end{aligned} \qquad (21)$$

The "critical price" s^* therefore separates two states or regimes in the model:

1. If $s_t \geq s^*$ then $I_t = 0$ (there is no speculative demand for GCs) and $s_t = S(y_t)$ is the price which equates demand for GCs arising from current consumption of electricity and total supply, including GCs carried over from last period. In this case s_{t+1} will only depend on z_{t+1} and will be independent of the current price s_t and therefore

$$s_{t+1} = f(z_{t+1}) \quad \text{when} \quad s_t \geq s^*. \qquad (22)$$

2. If $s_t < s^*$ then $I_t = y_t - S^{-1}\left(f(y_t)\right) > 0$ (there is positive speculative demand for GCs) and total demand exceeds demand for GCs arising from current consumption of electricity. Furthermore, we have $s_t = \beta E_t s_{t+1}$.

It can be shown that the price process, $\boldsymbol{s} = \{s_t; t = 1, 2, ...\}$, is a renewal process (in the statistical sense) with a stationary (stable) long-term distribution. Furthermore, it can be shown that the process of GCs banked, ***I***, has a stationary limit distribution with compact support. Between stock-outs (i.e. between periods when $I_t=0$) the discounted price process is a martingale and events in mutually exclusive periods between stock-outs are statistically independent.

2.4 Numerical model

Solving the functional equation (19) is key to understanding properties of the competitive equilibrium with banking. However, it is not in general possible to solve the equation by analytic methods and one must resort to numerical analysis (see e.g. Gustafson, 1958). An iterative algorithm that yields a numerical solution to (19) is provided in Appendix 1. Making use of this algorithm we shall here present a numerical example to illustrate the functioning of GC-market with and without banking. Parameters were chosen with reference to the Danish electricity market and it is assumed that fluctuations in renewable energy stem exclusively from wind. Demand for electricity is assumed to be linear. A constant elastic demand function is hardly credible at very high prices. At very low prices the functional form doesn't matter in the banking case since then the speculative demand function takes over.

Even if our model is stylized, we have chosen parameters for numerical simulations in rough correspondence to the Danish electricity market. First, the parameter α is set to reflect the proposed policy for the volume share of green electricity in Denmark. Second, the model is calibrated such that the ratio of the median price of GCs relative to electricity prices (including transportation, distribution and sales costs) is in line with the corresponding ratio in Denmark. Finally, the price elasticity of electricity demand is set to 0.2 at median prices, which is on the high side, but probably of the right order of magnitude. Note that a lower imposed share of green electricity (α) would lead to more fluctuations than our results indicate; the same holds for a higher price elasticity of demand. An illustration of the model is provided in Fig. 1.

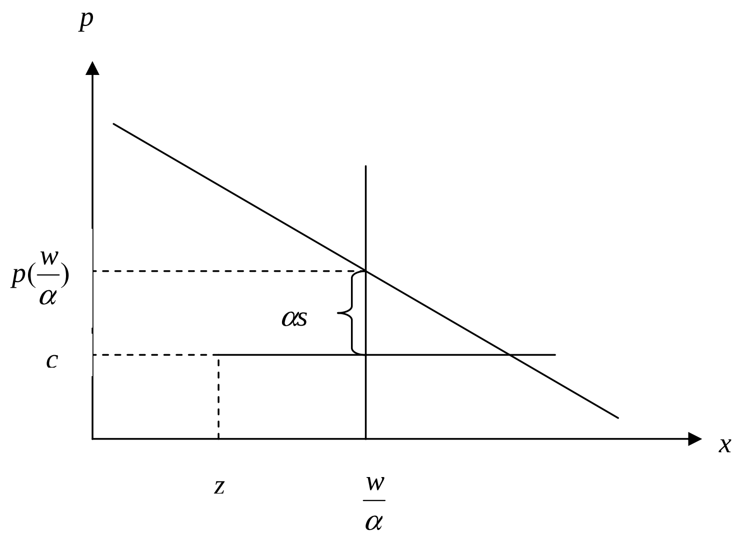

Figure 1. The relationship between green electricity generated, *z*; GCs sold, *w*; GC-price, *s*; electricity price, *p*, electricity consumption, $x = w/\alpha$; and the percentage requirement, α.

The detailed assumptions of the numerical model are as follows:

1. The inverse demand for electricity is $P(x) = a + bx$ where a=6 and b=-5. This implies that in the neighborhood of x=1 the price elasticity of electricity is 0.2.
2. The share of renewables in total electricity consumption is set at α=0.2
3. The generation of renewable electricity in successive time periods is a sequence of i.i.d. random variables $z_1, z_2, ...$ where each z_t is normally distributed with mean

$\mu_z = 0.2$ and standard deviation $\sigma_z = 0.02$ (i.e. the coefficient of variation is 10%)[14]. However, the distribution is truncated at $\mu_z \pm 2.5758 \cdot \sigma_z$ (i.e. 99% of the probability mass of the original normal distribution is retained) and the support of the distribution is therefore given by the interval [0.1485,0.2515].

4. The cost of generating black (thermal) electricity is c=0.9 per unit of electricity

It follows from the above assumptions that derived inverse demand for GCs is given by

$$S(w) = \tfrac{1}{\alpha}\left(a + b\left(\tfrac{w}{\alpha}\right) - c\right)^{+} \qquad (23)$$

Given the above assumptions, the competitive equilibrium price function *f* was calculated as described in Appendix 1.[15] The resulting approximation is displayed in Figure 2. The critical price s^* turned out to be approximately 1.6. Above this price there is no banking demand, a stockout will occur and prices – which are in this case determined by (23) – will typically peak. Below the critical price, there is positive speculative demand and prices – which are now determined by the curved part of the demand function – are relatively stable and follow the “Hotelling” relation $s_t = \beta E_t s_{t+1}$.

4. Effects on price profiles and quantities

If banking is not allowed, GC supply in period *t* will equal z_t which implies that supply is distributed in the interval [$\underline{z} = 0.1485$, $\bar{z} = 0.2515$] and prices are determined by the piecewise linear inverse demand curve for GCs *S* in Figure 2. Hence, simulating the development of prices and other variables in the non-banking case is simply done

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[14] Support for these assumptions is given in Appendix 3.

[15] An equally spaced grid of 4001 points for the values of *y* was used which was assumed – after some experimentation to ensure that the upper bound was non-binding – to take values in the interval [0.1485, 0.66]. Expectation with respect to *z*, was calculated by approximating the

by generating realizations of $z_1, z_2, ...$ from the truncated normal distribution described in Section 2.4 above and evaluating S at these values to get $s_1, s_2, ...$.

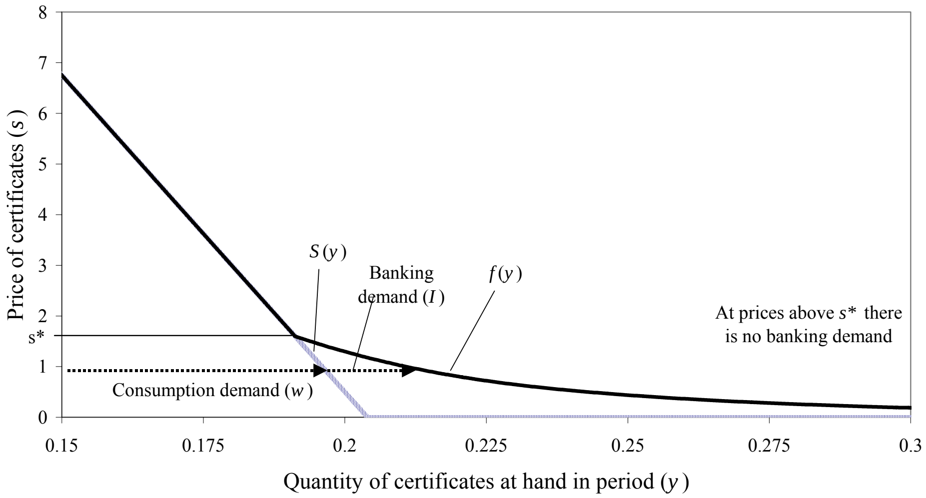

Figure 2. The effect of speculation on GC prices

If banking is allowed, speculative demand will have to be taken into account. For this case total demand is the sum of consumption demand (w) and speculative demand (I). Speculative demand vanishes when prices rise above the critical value s^*. Furthermore, when banking is allowed, total supply in each period is given by the stock of GCs carried over from the last period, I_{t-1}, plus the issued GCs in the current period, z_t and prices are determined by the competitive equilibrium price function f. Given a sequence of realizations of $z_1, z_2, ...$, other variables are generated as follows:

truncated normal distribution by a discrete distribution on an equally spaced grid over the interval [0.1485, 0.2515].

$$
\begin{aligned}
&y_1 = z_1 \\
&s_1 = f\left(y_1\right) \\
&I_1 = y_1 - S^{-1}\left(s_1\right) \\
&y_2 = I_1 + z_2 \\
&s_2 = f\left(y_2\right) \\
&I_2 = y_2 - S^{-1}\left(s_2\right) \\
&\text{etc. for } t = 3,4,...
\end{aligned}
\tag{24}
$$

Other variables, such as electricity demand and generation of black electricity, are easily derived from the above variables.

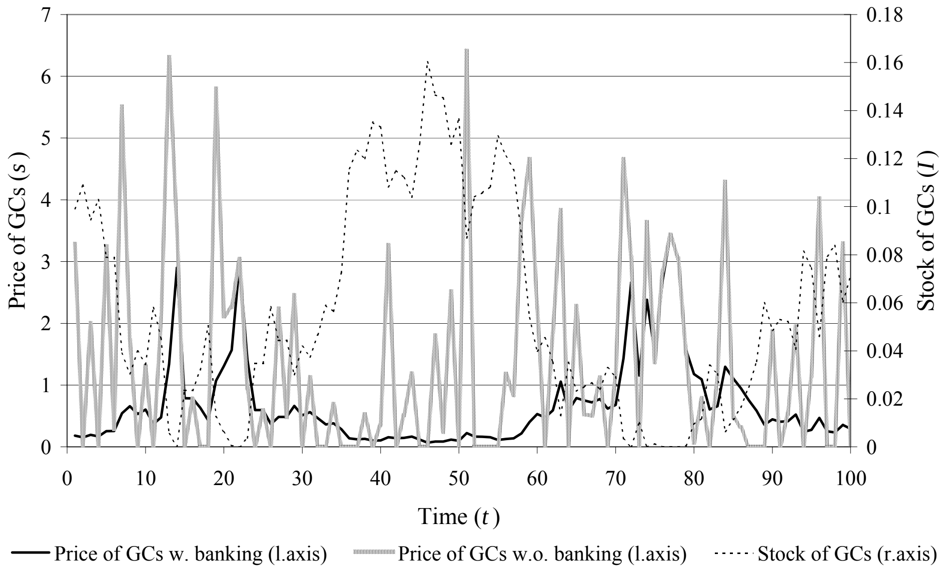

Figure 3. Simulated values of prices and stocks of green certificates

In Figure 3 an example is given of a simulation of GC prices, with and without banking. The same realizations of z_t are used in both cases and the values are taken from a simulation after some 200 periods have passed. Stocks carried between periods are also shown in the Figure 3.

Reduced fluctuations in GC prices due to speculative banking stand out clearly in Figure 2. Banking transforms the price series from fluctuating completely randomly from period to period into a strongly positively correlated series. Maximum values of stocks (approx. 0.15) in the figure are substantial and correspond to about ¾ of average annual production. A negative relation between prices with banking and the amount of GCs carried between periods is obvious. High prices and lower stocks go together. When stocks fall to zero (e.g. after period 20 in the figure), prices with and without banking coincide and in this case prices may become very high although extreme peaks are much rarer with banking then when it is ruled out. Estimates of key time series parameters of these series as well as for prices of electricity (which is a simple linear transformation of GC prices, c.f. equation (3)) are given in Table 1. The estimates were calculated from a simulation of 9000 periods.

Table 1. Time series properties estimated from simulation of 9000 periods

	GC price with banking	GC price w.o. banking	Stock of GCs	El. price with banking	El. price w.o. banking
Average	0.50	1.23	0.08	1.00	1.15
Std. deviation	0.61	1.57	0.06	0.12	0.31
Coeff. of var.	1.22	1.27	0.81	0.12	0.27
Minimum	0.00	0.00	0.00	0.90	0.90
Maximum	6.63	6.93	0.39	2.23	2.29
Serial corr. 1	0.63	0.03	0.96	0.63	0.03
Serial corr. 2	0.52	-0.01	0.92	0.52	-0.01
Serial corr. 3	0.43	0.00	0.89	0.43	0.00
Skewness	3.84	1.25	1.42	3.84	1.25
Kurtosis	20.99	0.73	2.49	20.99	0.73

It may seem surprising that the average price of GCs without banking (2nd numbers column) is over double the average price when banking is permitted (1st numbers column). The economic reason for this is that the GC price cannot turn negative even if a large amount of green electricity is generated and sold in the market (i.e. owners of certificates will not sell at a negative price). As can be seen in Figure 3, the non-negative price condition is much more binding without banking than with banking. The consequence of this is to raise the expected value of the GC price. Without the non-negativity constraint, the price of GCs without banking would be approximately normally distributed with mean 0.5 and standard deviation 1.58. Indeed, it may easily be calculated that if negative values for S were allowed, i.e. the $(.)^+$ operator were ignored in equation (23), then we would have

$$\begin{aligned} Ep &= 1 \\ \sigma_p &\approx 0.5 \\ Es &= 0.5 \\ \sigma_s &\approx 2.5 \end{aligned} \qquad (25)^{16}$$

Clearly, when the $(.)^+$ operator is applied the expectation of s and p rises substantially while their volatility falls. With banking, prices will be less volatile and, due to speculative demand, never fall as low as zero. Hence, the expected GC price falls from 1.23 to 0.5 and the standard deviation of prices is lowered from 1.57 to 0.61. Banking therefore leads to a lower expected value of the GC price and a higher and less variable electricity consumption.

In the case of banking, GC prices generally fluctuate between zero and one (90% of simulated values are lower than one) and the distribution is much more concentrated than when banking is not allowed (see Fig. 4). In calm years, however,

[16] Truncation of z is ignored in these calculations. Taking account of truncation would lower the values for the variance of p and s somewhat.

prices may peak, which leads to a skewed distribution with a heavy upper tail and high kurtosis. As evident from Table 1, the introduction of banking also transforms an i.i.d. price series to one that is highly serially correlated. Downward pressure on prices in good wind-years is lowered and price peaks are evened out by using stocks from previous years in bad wind-years. Occasionally, however, stocks may run out – there is a stock-out – and prices may multiply from normal values.

These results are qualitatively typical of those observed in commodities time series models, but because of the low price elasticity of derived demand for GCs and fluctuating, inelastic supply the effects of banking are very strong compared to results in models that are scaled to simulate commodities markets. For example, the proportion of years where a stock-out occurs is only 3% compared to a proportion on the order of 20% in models scaled for typical commodities (see e.g. Table 2 in Deaton and Laroque (1992)). Stocks of GCs carried between periods are on average 40% of annual production with a maximum of double the annual average amount issued. This implies that trade and speculation in GCs would very likely be quite lively in a real market.

The price of electricity is a linear function of the GC price ($p = \alpha s + c$), which is reflected in the statistical properties of that series.

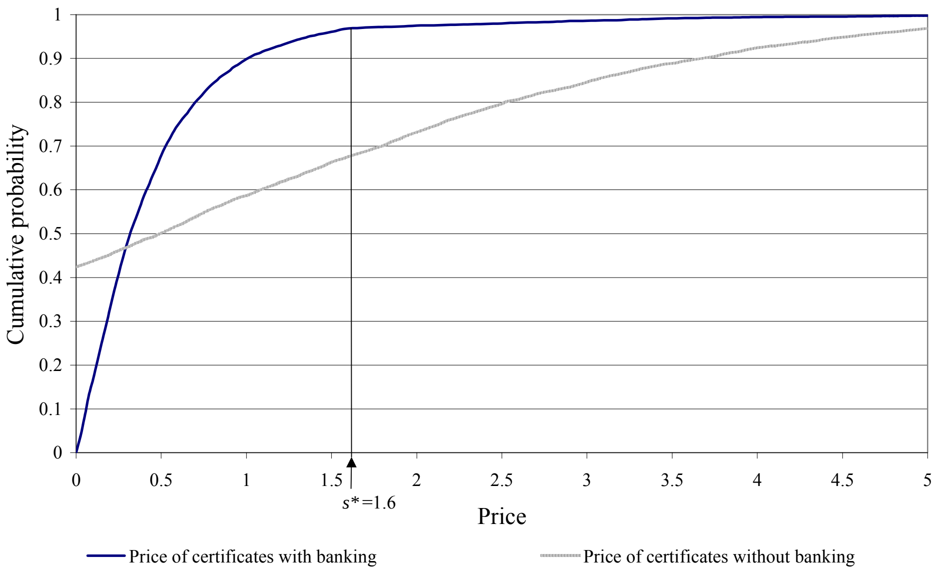

Figure 4. Cumulative distribution of green certificates price (based on a simulation of 9000 consecutive periods)

At first sight it appears contradictory that speculators are willing to hold a positive amount of GCs into the next period up to a current price of $s^* = 1.6$, which is substantially higher than the average price $\bar{s} = 0.5$. Since the price process is stationary in a probabilistic sense and therefore has a stable long-term distribution (shown in Figure 4) it seems reasonable to expect that when the current price is so far above the long-term average – almost two standard deviations – lower prices are to be expected in the next period. However, speculators will only hold GCs if they expect a positive yield r, here set to 10%. In equilibrium we have

$$E\left[s_{t+1} \middle| s_t = s^*\right] = (1+r)s^* \tag{26}$$

cf. (12). The explanation for this behavior is that due to the non-linearity of the price dynamics of $\boldsymbol{s}$ not only the long-term distribution, but also the distribution of s_{t+1} conditioned on $s_t = s^*$ is highly asymmetric with a heavy upper tail. The median of such a distribution is lower than its expected value. Therefore, the expected price in the next period may well be higher than that in the current period even if the price tends to be lowered in a probabilistic sense, e.g. going down with more than 50% probability.

5. Welfare effects

Table 2 shows effects on consumers', producers' and social surplus of going from a GC system without banking to a system with banking. As can be seen from this table the variability of all surpluses is reduced. Furthermore both consumers' surplus and social surplus increase in expected values while the expected value of producers' surplus falls.[17] However, the negative effect on the producers' surplus and the positive effect on consumers' surplus are related to the higher expected price of electricity and of GCs in the case without banking as compared to the case with banking which - as explained above - is linked to the (highly probable) event that the certificate price without banking every now and then may drop to the lower price bound of zero. Indeed, it turns out that if this lower price bound is not present, effects of banking on consumers' and producers' surpluses are reversed: consumers lose while producers gain under the assumption of a linear demand function. This is shown in Appendix 2 and illustrated in Table 2 where a very low unit cost of black production is applied. In this case the certificate price is positive with overwhelming probability (97%) and hence the expected price-quantity pair is approximately the same with and without banking. The consequence is that expected value of consumers' surplus increases and expected value of producers' surplus drops going from a GC system without banking to a system with

[17] It should also be noted that the producers' surplus in this model is identical to the producers' surplus of the green producers i.e. the black producers have no net surplus as their unit cost is constant and equal to the wholesale price of electricity.

banking.[18] However, for all cases considered the expected value of social surplus increases as banking is introduced. This last result is natural, since allowing banking represents a relaxation of a constraint on intertemporal trades which can only raise social surplus in the absence of externalities. However, as shown in Appendix 2, this result is no longer unequivocal when a lower bound on GC prices is introduced and it is possible to have the result we observe in the simulations, *viz.* that consumers' surplus increases and producers' surplus is lowered by the introduction of banking. It should be emphasised that the lower bound $s \geq 0$ is an endogenous consequence of the assumptions made regarding production technologies in our model.

[18] See Wright and Williams (1984) for a similar result for linear demand functions. With constant elasticity demand results are the same as we obtain for the derived demand function *S*, i.e. consumers' and social surpluses rise but producers' surplus falls when banking is introduced.

Table 2. Expected values with and without banking (standard deviations in parentheses)

		Expected GC-price	Expected el.price	Expected el.quantity	Expected cons.s.pl.	Expected prod.s.pl.	Expected Soc.s.pl.
Base case	With banking	0.50 (0.61)	1.00 (0.12)	1.00 (0.02)	2.50 (0.12)	0.28 (0.10)	2.78 (0.03)
	Without banking	1.23 (1.57)	1.15 (0.31)	0.97 (0.06)	2.37 (0.29)	0.40 (0.25)	2.77 (0.04)
Non-zero GC-price*	With banking	4.50 (1.39)	1.00 (0.28)	1.00 (0.06)	2.51 (0.27)	0.90 (0.20)	3.41 (0.06)
	Without banking	4.52 (2.35)	1.00 (0.47)	1.00 (0.09)	2.52 (0.47)	0.88 (0.38)	3.40 (0.09)

* Non-zero GC-prices are achieved by assuming very low marginal cost ($c=0.1$) for generation of black electricity. Observe that the expected values of this case and the base case are not directly commensurable. The essential point is the comparison between banking and no banking for each case separately.

6. Effects of price bounds

Introduction of lower and upper price bounds on GC prices involves governmental intervention in the market as prices tend to fall below the lower price bound or rise above the upper price bound. At the lower bound the governmental authority purchases GCs from green producers and thus takes GCs out of the market. The consequence is that expected electricity prices rise and expected electricity consumption falls. At the upper price bound the authority issues and sells new GCs that are added to market supply. This lowers expected electricity prices and increases expected electricity consumption.

It should be noted that in the banking case the equilibrium price function f is changed by the introduction of price bounds; i.e. bounds are not simply imposed on values already simulated, but rather we calculate and simulate a new equilibrium where agents – assumed to entertain rational expectations – take the price bounds into consideration. With banking a lower price bound raises the critical price for storage, $s*$, while an upper bound lowers the critical price for storage, $s*$. With both bounds imposed simultaneously the effect on $s*$ and average prices is ambiguous.

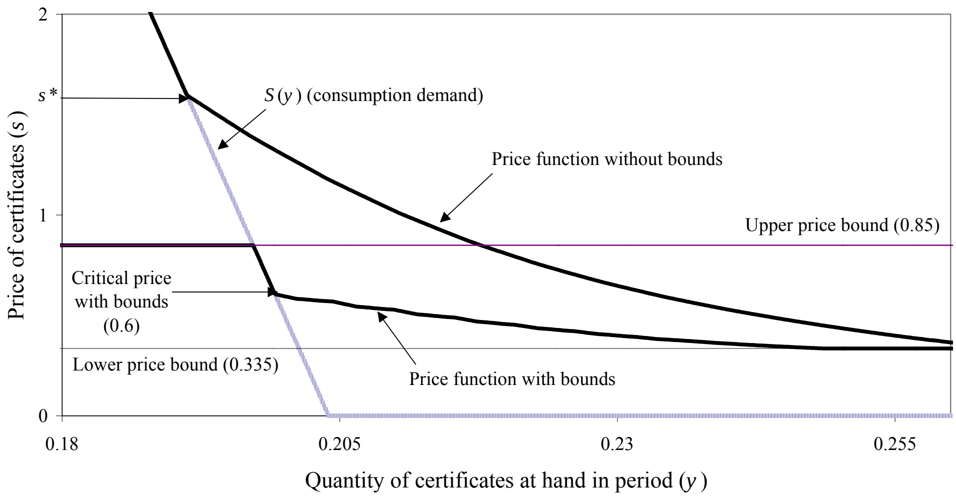

Figure 5. The effect of price bounds on demand for green certificates

The effect of price bounds on demand for GCs is shown in Figure 5, where equilibrium price functions in the banking case are shown with and without lower and upper price bounds; the values used in the figure are 0.335 and 0.85, respectively. The function is affected by governmental supply at the upper price bound and governmental demand at the lower bound. Clearly, there is much less incentive to speculate in GCs when the bounds are imposed: the critical price is lowered from 1.6 in the case without bounds to 0.6 when the bounds are imposed. This implies that there is no private banking demand for GCs at prices above 0.6 and, in general, banking demand for GCs is rather limited at "moderate" prices. When prices reach the lower bound, private banking demand peaks and governmental demand replaces it. Usually, for a given value of "amount on hand" (y: the sum of GCs issued in the present period and banking supply from last period) prices without bounds imposed exceed controlled prices. However, for high values of quantity at hand (i.e. for values of y exceeding 0.265 in this example) the lower price bound binds and "free" prices then fall below those prevailing under governmental intervention. As shown in Table 3, the resulting equilibrium distribution of prices in the two cases turns out to have the same expected value.

The governmental purchase of certificates at the lower price bound amounts to an additional subsidy to the green producers (i.e. additional to the consumer based subsidy implied by the GC system). Hence, expected value of producers' surplus increases while expected value of consumers' surplus and social surplus decreases. Sale of additional GCs at the upper price bound amounts to a tax on the green producers. Thus, this has a negative effect on producers' surplus and positive effects on consumers' surplus and social surplus. In general, directions of the various effects of price bounds are the same with and without banking. The separate effects of introducing a lower price bound and an upper price bound are illustrated in Table 3. Table 3 also shows the joint effect of a lower and an upper price bound.[19] Comparing

[19] Setting the lower and upper price bounds the governmental body may function as an insurance company partly taking over the risk of low and high GC prices. Also, a lower price bound effectively raises the subsidy to the green producers thus inducing entry of new green producers and raising green generation capacity (see, Spulber, 1985). On the other hand, an upper price bound will have the opposite effect i.e. a reduction of the subsidy. In our paper these two price bounds are set so as to balance each other, with a net expected subsidy stemming from the price bounds equal to zero (though the general subsidy implied by the GC system is still positive). This is the case for the model reported in Table 3.

Table 3 and Table 2 it is clear that the introduction of price bounds reduces the effects of banking as compared to no banking. This is the result of the reduced motive for speculation implied by the price bounds. Indeed, as the price bounds get closer the private and social benefit of banking (as compared to no banking) disappears. Clearly, if the price bounds merge completely the effects of the GC system becomes identical to the effects of a consumer financed constant unit subsidy system for green producers.

Table 3 also illustrates how the introduction of price bounds reduces the variability of the various measures considered. For example, the variability of all measures with price bounds, but without banking is lower than that of corresponding measures in the base case with banking.

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Table 3. Expected values and price bounds (standard deviation in parenthesis)

		Expected GC-price	Expected el.price	Expected el.quantity	Expected cons.s.pl.	Expected prod.s.pl.	Expected Soc.s.pl.
Base case	With banking	0.50 (0.61)	1.00 (0.12)	1.00 (0.02)	2.50 (0.12)	0.28 (0.10)	2.779 (0.01)
Lower price bound	With banking	0.72 (0.64)	1.04 (0.13)	0.99 (0.03)	2.46 (0.12)	0.32 (0.11)	2.776 (0.02)
Upper price bound	With banking	0.32 (0.24)	0.96 (0.05)	1.01 (0.01)	2.54 (0.05)	0.25 (0.05)	2.782 (0.003)
Lower and upper price bounds	With banking	0.50 (0.18)	1.00 (0.04)	1.00 (0.01)	2.50 (0.04)	0.28 (0.03)	2.780 (0.002)
	Without banking	0.58 (0.25)	1.02 (0.05)	1.00 (0.01)	2.48 (0.05)	0.29 (0.03)	2.776 (0.03)

* The lower price and upper price bounds are set at 0.335 and 0.85, respectively.

7. Summary and concluding remarks

There is concern that a GC market primarily based on volatile wind power may lead to erratic green certificate and electricity prices. The supply of GCs is determined by the generation of green electricity. The demand for GCs is determined by a percentage requirement (α) i.e. consumers should hold at least a given percentage of GCs out of total electricity consumption. The percentage requirement is given by the institutional framework and represents a target share of green electricity. It requires a political decision to change it. The problematic feature of an annually clearing GC market is that volatile generation of green electricity (and therefore GCs) multiply into the electricity market giving rise to erratic GC prices as well as electricity prices. These effects are thus driven by the volatility of green electricity from wind and the fixed α. Hence, an annually clearing GC market may be rather unstable and not very credible in the long run.

Applying a rational expectations simulation model of competitive storage and speculation of GCs, the paper shows that introduction of banking of GCs may reduce price volatility considerably and, furthermore, as expected lead to increased social surplus. Surplus of green producers, however, will not necessarily increase going from a system without banking to one with banking. The reason for this is that the GC price without banking every now and then will drop abruptly. In these cases the black producers will act as swing producers and reduce their electricity generation so as to prevent electricity price from dropping below marginal cost. This market mechanism also prevents the GC price from dropping below zero. A consequence of this is to increase expected value of GC prices as compared to a situation where black producers would not (indirectly) provide a price floor and the GC price would fluctuate more-or-less freely. When banking and speculation is introduced, prices will not drop to zero as frequently wherefore the expected value of the GC price actually becomes smaller.

The paper also considers the effects of upper and lower price bounds as typically proposed for GC systems. The separate effect of a lower price bound is equivalent to an additional subsidy to green producers and leads to increased green producers' surplus while consumers' surplus and social surplus are reduced. The reverse effect results if an upper price bound is introduced. The joint effect of lower *and* upper price bounds is to further reduce volatility of the market and a point may be reached where the additional volatility reducing effect of banking becomes negligible. Also, with narrow price bounds there may not be much point in constructing a separate market for GCs as it becomes comparable to an ordinary consumer financed constant unit subsidy system for green producers.

The model used builds on several simplifying assumptions. Some of these are of the "Occam-kind" i.e. assumptions that are simplifying without distorting the fundamental functioning of the model and results obtained. The assumptions of constant marginal cost for black producers (as opposed to increasing marginal cost) and of zero marginal cost (as opposed to positive marginal cost) for green producers are of this kind. However, a couple of other assumptions are not so innocent when it comes to the effect on green producers' surplus from banking. For instance, for commodities markets it has been shown (Wright and Williams, 1984) that the effect of speculation and storage on expected producers' surplus hinges on the form of the assumed demand function. In particular, with a linear demand function expected producers' surplus will increase while it may decrease with a constant elastic demand function as speculation and storage is introduced.[20] Observe, however, that our conclusion, with a linear underlying demand function – the derived demand function is piecewise linear, but strictly convex – is that expected producers' surplus actually may fall by the introduction of banking due to the correcting market mechanism as GC prices drop to zero. Hence, introducing a constant elastic demand function in our model may only reinforce our conclusion that green producers may stand to lose from banking.

Another essential point is that we have disregarded risk aversion and that risk is higher without banking. The effects of risk aversion and higher risk without banking may be taken care of by applying a larger discount rate. If reduced risk in the banking case is assumed to lead to an appreciably lower discount rate, then it turns out that the

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[20] Indeed, it boils down to the question of concavity and convexity of the stochastic producers' surplus function and the application of Jensen's inequality.

result that green producers are worse off with banking might be reversed. In our baseline case, where profits are 0.28 and 0.40 with and without banking, respectively, the interest rate – 10% in baseline – would have to rise to about 15% to make banking better for the green producers. The question then is whether market participants would value higher volatility in a market without banking by applying a risk premium of 5 percentage points or more. In any case, for a model involving risk aversion, risk reduction by banking will represent an additional social benefit. [21]

Finally, we have not allowed for borrowing of permits in our model. Indeed, borrowing is not a part of real proposed GC systems to our knowledge. It is, however, often suggested that borrowing, at least to a limited extent, may reduce price fluctuations over and above what banking does.

Borrowing can easily be implemented in our model where aggregate GC levels are allowed to fall to a certain minimum (negative) level, say $I_{\min} < 0$. "Stockouts" now occur when stocks fall to the level $I_{\min}$. Similar methods as in Rubin (1996) can be employed to show that, whenever the lower limit on borrowing is not binding, prices will still follow Hotelling's rule in expected value terms. Starting the system off with a stock level of zero in such a system is equivalent to starting off with stock equal to $-I_{\min} > 0$ in our present model.

Such limited borrowing would have a substantial impact in the medium term if $I_{\min}$ is large in terms of average annual production of green electricity, say corresponding to a year's production or more. In our present model average stocks correspond to less than half average annual production, while the maximum over 9.000

[21] Another pertinent question with respect to the risk reducing benefit from banking, is whether the agents involved already have sufficient alternative measures available in the capital market to pool and reduce price and revenue risk to a preferred level.

periods is approximately twice the average annual production. Starting the system off with two years worth of permits, say, would therefore lead to very low prices initially and it would probably take the system several years to work off the excess stocks. Prices would be stable, but low, in the meantime. In the long run, however, level and variation of GC prices would be unaffected; in particular prices would peak sharply whenever the lower limit on borrowing is reached just as they do when there is a stockout in our current model. Average stock levels will, however, be shifted down by an amount corresponding to the lower level on stocks.

Unlimited borrowing would on the other hand have a substantial impact on price volatility, since price spikes due to stockouts would then be eliminated. The price series would then be substantially smoother. Such a system is usually ruled out in discussion on GC systems, e.g. due to credibility issues.

References

Amundsen, E.S and J.B. Mortensen (2001): The Danish Green Certificate Market: Some simple analytical results, *Energy Economics*, 23, 489-509.

Amundsen, E.S and J.B. Mortensen (2002): The Danish Green Certificate Market: Some simple analytical results; *Erratum*, *Energy Economics*, 24, 523-524

Cronshaw M.B. and J.B. Kruse(1996) "Regulated firms in pollution permit markets with banking." *Journal of Regulatory Economics*, 9. 179-189.

Deaton, A. and G. Laroque (1992): On the behavior of commodity prices, *Review of Economic Studies* 59, 1-23.

Deaton, A. and G. Laroque (1996): Competitive storage and commodity price dynamics, *Journal of Political Economy*, 104, 896-957.

EMD International (2004): "Monthly wind energy index for Denmark", www.vindstat.dk

Fristrup, F. (2003): "Some challenges related to introducing tradable green certificates", *Energy Policy,* 31, 15-19.

Gustafson, R. L. (1958) "Carryover Levels for Grains" (US Department of Agriculture, Technical Bulletin 1178)

Jensen, G. and K. Skytte (2002) "Interactions between the power and green certificate markets", *Energy Policy,* 30, 425-435.

Kling, C. and J. Rubin (1997). "Bankable permits for the control of environmental pollution". *Journal of Public Economics,* 64. 101-115

Morthorst, P.E. (2000) "The development of a green certificate market". *Energy Policy*, 28 1085-1094.

Roberts, M.J. and M. Spence (1976). “Effluent Charges and Licenses Under Uncertainty”, *Journal of Public Economics,* 5, 193-208

Rubin. J.D. and C. Kling (1993). “An emission saved is an emission earned: an empirical study of emission banking for light-duty vehicle manufactures”, *Journal of Environmental Economics and Management,* 25, 257-274.

Rubin. J.D. (1996). “A model of intertemporal emission trading, banking and borrowing”, *Journal of Environmental Economics and Management,* 31, 269-286.

Spulber, D.F. (1985) “Effluent Regulations and Long-Run Optimality”, *Journal of Environmental Economics and Management*, 12, 103-116.

Unger, T. and E.O. Ahlgren (2003) “Impacts of a green certificate market on electricity and CO2-emission markets in the Nordic countries”, in T. Unger, Common energy and climate strategies for the Nordic countries – A model analysis, Dissertation, Chalmers University of Technology, Gothenburg, Sweden.

Williams, J. C. and B.D. Wright (1991): *Storage and Commodity markets*, Cambridge, Cambridge University Press.

Wright, B. D. and J.C. Williams (1984) “The Welfare Effects of the Introduction of Storage”, *Quarterly Journal of Economics*, 98, 169-192

Yates. A. and M.B.Cronshaw (2001). “Pollution permits markets with intertemporal trading and asymmetric information”, *Journal of Environmental Economics and Management,* 42, 104-118.

Appendix 1

Iterative algorithm

The iterative algorithm used to obtain solutions for the numerical model:

1. Establish an interval $\left[\underline{y},\overline{y}\right]$ for the state variable *y*. The lower bound is (naturally) $\underline{y}=\underline{z}$, but the upper bound must be established by experimentation. It should be high enough so that the probability of hitting it is negligible.
2. Set up a grid *Y* over $\left[\underline{y},\overline{y}\right]$ defining permissible values of *y*.
3. Define $f_0(y)=S(y),\ \ y\in Y$.
4. For *n*=1,2,… calculate a new estimate of *f* by the equation

$$f_n(y)=\max\left\{\beta E\left[f_{n-1}\left(z+\left[y-S^{-1}\left(f_n(y)\right)\right]\right)\right],S(y)\right\},\ \ y\in Y. \tag{27}$$

Stop when $\|f_n-f_{n-1}\|<\varepsilon$, where ε is a “small” number. The final f_n is the numerical solution to (19).

When a numerical estimate of *f* has been determined it is easy to simulate time series for price, demand, the stock of certificates and other model variables by generating random numbers from the distribution of *z*. Simple MATLAB programs were written which perform the above calculations.

Appendix 2

Consumers’, producers’ and social surplus with a linear demand function

The inverse demand function for electricity is assumed given by: $p(x)=a+bx$, with constants $a>0$ and $b<0$. We denote the stochastic green electricity generation by $\tilde{z}$ and its expected value by $E\left[\tilde{z}\right]=\mu$. Furthermore, we denote the stochastic electricity consumption by $\tilde{x}$ and its expected value and variance by $E\left[\tilde{x}\right]=\lambda$ and

$V(\tilde{x})$, respectively. With linear demand we have: $E[p(\tilde{x})] = a + b\lambda$ and $V(p(\tilde{x})) = b^2 V(\tilde{x})$

Consumers' surplus (CS):

$$E\left[\int_0^{\tilde{x}} p(x)dx\right] - E[p(\tilde{x})\tilde{x}] = aE[\tilde{x}] + \frac{b}{2}E\left[\tilde{x}^2\right] - E[(a+b\tilde{x})\tilde{x}]$$
$$= -\frac{b}{2}E\left[\tilde{x}^2\right] = -\frac{b}{2}(V(\tilde{x}) + \lambda^2)$$

Producers' surplus (PS):

$$E[PS] = E[p(\tilde{x})\tilde{x}] - cE[\tilde{x}] + cE[\tilde{z}] = aE[\tilde{x}] + bE\left[\tilde{x}^2\right] - cE[\tilde{x}] + cE[\tilde{z}]$$
$$= a\lambda + b(V(\tilde{x}) + \lambda^2) - c(\lambda - \mu)$$

Social surplus (SS):

$$E[SS] = E\left[\int_0^{\tilde{x}} p(x)dx\right] - cE[\tilde{x}] + cE[\tilde{z}] = aE[\tilde{x}] + \frac{b}{2}E\left[\tilde{x}^2\right] - cE[\tilde{x}] + cE[\tilde{z}]$$
$$= a\lambda + b\frac{V(\tilde{x}) + \lambda^2}{2} - c(\lambda - \mu)$$

Comparison with and without banking

We compare expected values generated from a stationary price process without banking to a stationary price process with banking. Denote the expected value and variance of the electricity price with banking by $E_b[\tilde{p}]$ and $V_b(\tilde{p})$ and without banking by $E_{nb}[\tilde{p}]$ and $V_{nb}(\tilde{p})$, respectively. We know the two processes generate the same values of expected price and that the price process for the case with banking has a lower variance than the price process for the case without banking i.e.

$E_b[\tilde{p}] = E_{nb}[\tilde{p}] = a + b\lambda$, and

$V_b(\tilde{p}) = b^2 V_b(\tilde{x}) < V_{nb}(\tilde{p}) = b^2 V_{nb}(\tilde{x})$.

Consequently: $V_b(\tilde{x}) < V_{nb}(\tilde{x})$. Inspection of expressions then shows:

$$E_b[CS] < E_{nb}[CS], \quad E_b[PS] > E_{nb}[PS], \quad E_b[SS] > E_{nb}[SS]$$

Hence, going from a situation without banking to a situation with banking consumers' surplus will fall, while producers' surplus and social surplus will increase.

The effects of a lower price bound

Next assume that a lower bound $\underline{s}$ on prices of GCs is introduced for the case without banking. The effect of this is to reduce the upper range of values for electricity consumption (and correspondingly the lower range of values for electricity prices). Consequently, the expected electricity price will increase, while expected electricity consumption will fall. Both variances will fall. Identifying quantities in the lower price bound case by $\underline{s}$ we thus have $E_{nb}^{\underline{s}}[\tilde{p}] > E_{nb}[\tilde{p}]$, $\lambda^{\underline{s}} < \lambda$, $V_{nb}^{\underline{s}}(\tilde{p}) < V_{nb}(\tilde{p})$ and $V_{nb}^{\underline{s}}(\tilde{x}) < V_{nb}(\tilde{x})$. Consulting the expression for consumers' surplus above, we can immediately conclude that $E_{nb}^{\underline{s}}[CS] < E_{nb}[CS]$.

Note that expected production is larger than the production level that maximizes producers' surplus (i.e. where expected marginal revenue is equal to marginal cost) provided $\underline{s} \le (2\alpha)^{-1}(a-c)$ (note that we must have $a > c$ so $(2\alpha)^{-1}(a-c) > 0$). In what follows we assume this condition on $\underline{s}$ is satisfied. We can then show that $E_{nb}^{\underline{s}}[PS] > E_{nb}[PS]$. For this purpose, let $F(x)$ be the cumulative distribution function of $\tilde{x}$ in the absence of the constraint $s \ge \underline{s}$. Let $\overline{x} = (\alpha\underline{s} + c - a)/b > 0$ be the upper bound on demand corresponding to $\underline{s}$ and let $\overline{\pi} = P\{\tilde{x} \ge \overline{x}\} = 1 - F(\overline{x})$. Writing $F^{\underline{s}}(x)$ for the c.d.f. of $\tilde{x}$ in the presence of the constraint $s \ge \underline{s}$, we get

$$F^{\underline{s}}(x)=\begin{cases}F(x) & x\le\overline{x}\\ 1 & x>\overline{x},\end{cases}\quad \text{and}$$

$$\overline{\pi}=P^{\underline{s}}\{\tilde{x}=\overline{x}\}.$$

Hence, we have,

$$\begin{aligned}E_{nb}[PS]&=\int_0^\infty\left[p(x)x-cx\right]dF(x)+c\mu\\&=\int_0^{\overline{x}}\left[p(x)x-cx\right]dF(x)+\overline{\pi}\left[p(\overline{x})\overline{x}-c\overline{x}\right]+c\mu\\&\quad+\int_{\overline{x}}^\infty\left[p(x)x-cx\right]dF(x)-\overline{\pi}\left[p(\overline{x})\overline{x}-c\overline{x}\right]\\&=\int_0^{\overline{x}}\left[p(x)x-cx\right]dF^{\underline{s}}(x)+c\mu\\&\quad+\int_{\overline{x}}^\infty\left[\left(p(x)x-cx\right)-\left(p(\overline{x})\overline{x}-c\overline{x}\right)\right]dF(x)\\&=E_{nb}^{\underline{s}}[PS]+\int_{\overline{x}}^\infty\left[\left(p(x)x-cx\right)-\left(p(\overline{x})\overline{x}-c\overline{x}\right)\right]dF(x)\end{aligned}$$

It is easily checked that the expression $p(x)x-cx$ is decreasing in x for $x\ge\overline{x}$ provided $\underline{s}\le(2\alpha)^{-1}(a-c)$. Therefore, the integrand in the bottom integral above is strictly negative for $x>\overline{x}$. It follows that we have $E_{nb}^{\underline{s}}[PS]>E_{nb}[PS]$.

Comparing these results – i.e. that $E_{nb}^{\underline{s}}[CS]<E_{nb}[CS]$ and $E_{nb}^{\underline{s}}[PS]>E_{nb}[PS]$ – to the conclusions drawn above with respect to expected surpluses with banking, that comparison now becomes ambiguous and we may have $E_{nb}^{\underline{s}}[PS]>E_{b}[PS]$ and $E_{nb}^{\underline{s}}[CS]<E_{b}[CS]$ i.e. the introduction of banking under a GC-system with a lower price bound $\underline{s}\le(2\alpha)^{-1}(a-c)$ may lead to a reduction of expected producers' surplus and an increase of expected consumers' surplus as indeed observed in the simulation results reported in the paper, where $\underline{s}=0$.

Appendix 3

Volatility of the Danish Wind Energy Content Series

This appendix briefly describes the statistical properties of the series on energy content of wind in Denmark in the years 1979 - 2004.[22] The data is depicted in Fig. A3.1. The index average is 97.95 for the entire period 1979 – 2004; the standard deviation of the series is 9.77 so the coefficient of variation is approximately 10%. The null hypothesis that the data are normally distributed cannot be rejected ($\chi_2^2 = 1.487$, p-value 0.48); this of course assumes independent and identically distributed observations. Autocorrelation is not significant in the series.

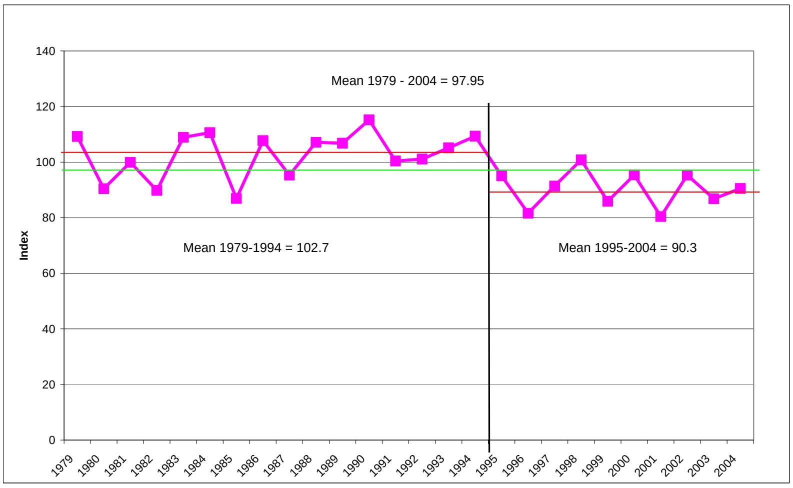

Figure A3.1

Wind Energy Content in Denmark

Looking at Fig. A3.1 it seems that there is a shift in the series from 1995 onwards: the averages for 1979 – 1994 and 1995 – 2004 are 102.7 and 90.3, respectively. A Cusum test of stability of the mean, however, does not reject a stable mean (Harvey-Collier

[22] For the data and a description of measurement methology see EMD International (2004).

t(24) = -1.6, p-value 0.12). A Chow test of a structural break in 1995 on the other hand rejects the null hypothesis of no change (t-value of -3.98, p-value 0.0006). This should, however, be taken with some caution since we decide on the the timing of the structural break by looking at the data which makes p-values of Chow tests suspect. Testing for stationarity with the Augmented Dicky-Fuller (ADF) test, however, reveals that the null hypothesis of a unit root cannot be rejected (p-value of 0.27) indicating that the series is non-stationary.

The evidence as to whether the series can be described by a series of independent and identically distributed observations is therefore mixed, but in light of the ADF and Chow-test results it was decided to take another look at the data and transform it by dividing it into two periods, 1979 – 1994 and 1995-2004, and calculating the relative deviation from the mean in each period. The result is a data series, "dev", defined by $dev_t = index_t / 102.7 - 1$ in the former period and by $dev_t = index_t / 90.3 - 1$ in the latter one, where "index" is the wind index. After this transformation, the mean of the series is obviously zero, and the standard deviation (adjusting for an extra degree of freedom lost due to the estimation of two mean parameters) is 0.078. An ADF test indicates that the series is stationary (p-value of 0.002). There is marginally significant negative autocorrelation (p-value of 10% for first lag) which indicates some overfitting. A Cusum test now reveals a very stable mean of the series. (The Chow test is now meaningless since we have adjusted for a shift of means.) Finally, the results of a Chi-Square test of normality, indicate that normality cannot be rejected (p – value of 0.26).

This analysis indicates that the wind energy series may for our purposes be modelled as a series of normally distributed i.i.d. observations and regardless of whether one assumes that a shift occured in the data the coefficient of variation is of the order of magnitude of 10%.