Rational Fear of Floating:  
A Simple Model of Exchange Rates and Income Distribution

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Abstract
We consider a simple two-country model, where each country produces a consumption good from a single input. Production takes time, and the model is considered over two consecutive periods. There are three categories of economic agents, namely factor owners, entrepreneurs, and financial intermediaries. The latter offers credits to entrepreneurs and are funded by sale internationally transferable bonds. We assume that the national credit markets are monopolistic but that other markets are competitive. Exchange rate policy is introduced in two different ways, either as a market intervention by a government, sustained by intervention in the commodity market, and, more realistically, as a policy commitment by the monetary authorities, which in equilibrium is taken into consideration by the financial intermediary.

The results of the simple model show that an increase in the value of the domestic currency from an equilibrium position will in most cases decrease aggregate welfare of the country, but it will improve welfare of the financial intermediaries. Thus, in the simple framework of our model, a specific sector – and one with a considerable influence on policy choices – stands to gain from this exchange rate policy.

Keywords: fear of floating, income distribution, financial intermediaries.
JEL classification: F37, F41

1. Introduction
In recent years, there has been increasing attention on the role of exchange rate volatility in the economic performance of developing countries, and there is some evidence which suggests that these countries are more concerned about stabilizing the exchange rate than developed countries. There are several papers which show that exchange rates are more volatile in developing than in developed countries (Calvo and Reinhart

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behavior, which after Calvo and Reinhart (2002) has been known as “fear of floating”, does not at a first glance seem to be founded in economic reasoning, and there has been much discussion on how to interpret and explain the observed phenomena.

Several models have been proposed with this purpose. The first attempt at an explanation, by Calvo and Reinhart (2002) points to the trading off of seignorage benefits from inflation against the drawbacks of unexpected inflation. Other authors point to balance sheet effects, (see e.g. Aghion, Banerjee and Bacchetta (2003)), which are important since dollar liabilities in the country may lead to bankruptcy; as pointed out by Honig (2004), many of these dollar liabilities may be domestically contracted in the foreign currency. In Lahiri and Vegh (2001), fear of floating emerges as an optimal policy given that there are costs connected with exchange rate fluctuations. Another line of argument is proposed by Caballero and Krishnamurthy (2004), which explains fear of floating as a problem of time consistency; while floating exchange rates may be the right policy to induce the firms to insure against shocks, the floating exchange rate is not optimal when a shock occurs. Subsequent contributions (Gallego and Jones (2004)) have developed this rational insurance argument further.

While the theoretical discussion, offering a consistent theoretical framework for the empirically observed phenomenon, has moved in the direction of rationalizing fear of floating in the sense of letting it emerge as optimal for society under certain assumptions on the institutional framework, but has shown a great variety in the type of explanations put forward, there seems to have been a certain consensus on the basic theoretical framework, which in most cases is that of international macroeconomics, much in the style of the exposition in Obstfeld and Rogoff (1986). In particular, there has been no attempts to go beyond the rather simplistic view of a country as an aggregate entity, consisting of producers and consumers, for which it makes sense to speak of optimal policies without further comments on the criteria of optimality.

On the other hand, it is well-known that developing countries, and Latin American countries in particular, are characterized by a very high degree of income inequality, and that indeed these countries tend to fall apart in a small upper middle class with a way of life and an outlook not differing particularly from their counterparts in United States and Western Europe, and the rest of the population living under very different conditions. It might therefore be worthwhile to treat specific phenomena of these countries with a much more direct recourse to their specific characteristics, and this is what we intend to do in this paper. Incidentally, the paper may also be seen as an attempt to reintroduce the classical modelling of international trade theory, which have had its impact even on the monetary aspects of trade, into the current debate on exchange rate regimes.

In order to present the approach in as simple a way as possible, we have chosen to keep the model as simple as possible without losing its basic features of following different types of income receivers. Thus, we consider a world with two countries, each
producing a single commodity from a single production factor inserted one period earlier. Domestic markets for factors and for commodities are competitive, and so is the market for bonds issued by the financial intermediaries in each country, which can be traded in both countries. However, the financial intermediaries of each country are assumed to have a monopoly position in the credit markets. We investigate in this model the consequences of a change in (real) exchange rates in the direction of appreciation of the home currency. The effects depend on the way in which this policy is carried out, but in the most realistic of the cases considered, where the declaration of the exchange policy is followed by the credit evaluation of the financial intermediaries, an exchange rate appreciation will lead to an increase in the income of the financial intermediaries whereas the position of the country as a whole will deteriorate.

The paper is structured as follows: In Section 2, we present the basic model and described the equilibrium, and we consider an example illustrating the situation where there is government intervention in the domestic commodity market. The more realistic case of no market intervention is considered in Section 3, where we assume first that factor markets need not balance, possible due to the possibility of self-employment, and in Section 4 where we allow for international trade in factors, so that domestic factors may be reallocated to employment in the foreign country. Section 5 contains conclusions and suggestions for further research. Some of the details of proofs are collected in a mathematical appendix.

2. The simple model: first version

In this section we introduce a simple base model of trade over time, where exchange rates matter. To keep the complexity down to a minimum, we are not considering problems of comparative advantages and patterns of trade; instead we deal with a two-country, two commodity world where each country is fully specialized. There is one non-tradable production factor in each country, which has to be inserted at \( t = 0 \), and output \( x \) is available at \( t = 1 \) according to a (neoclassical) production function \( x = f(z) \) where \( z \) is input; both input and output may be interpreted in per capita terms. The foreign country produces output \( \hat{x} \) from its input \( \hat{z} \) according to production function \( \hat{f} \); as mentioned above, we assume for simplicity that the countries produce different commodities.

Consumers in each country are divided into three distinct categories, namely (i) factor owners, (ii) producers, and (iii) financial intermediaries. All consumers carry out consumption of home and foreign country commodities at \( t = 1 \) only, and they have identical utility functions \( U(x, \hat{x}) \). Factor owners have an endowment of the relevant input commodity, to be inserted in production at \( t = 0 \). The sale of this endowment is saved by purchase of bonds which are internationally marketable, and the return is for consumption purchases at \( t = 1 \).
The firms of each country buy input at \( t = 0 \), at a price \( q \), and sell output at \( t = 1 \), at a price \( p \). Since the firms have no initial capital, they need to obtain credit with the financial intermediaries. If the loan rate of interest is \( r_L \), then firms’ profit is

\[
\pi(q, r_L) = pf(z) - (1 + r_L)qz.
\]

If we assume that factors cannot be traded, and that firms maximize profits at the given prices and loan rates, we have from the first order conditions that the optimal input \( z \) must satisfy \( pf'(z) = (1 + r_L)q \) or, equivalently

\[
r_L = \frac{p}{q} f'(z) - 1.
\]  

(1)

The expression (1) may also be considered as expressing the demand for loans of the enterprises.

We assume that in each country, credits can be obtained only from financial intermediaries of this country. The funding of these credits is obtained by taking deposits from the public, at an interest rate \( r_D \). We assume that the savers of each country have access not only to the financial intermediaries of its own country but also to those of the other country, so that the deposit rate \( r_D \) may be assumed to be identical in the two countries.

Since our ultimate goal is to study exchange rate policy, we need to introduce the exchange rate explicitly. Needless to say, the model as outlined is so simple that it does not provide much rationale for exchange rate policies, but, as we mentioned in the introduction, the purpose of this section is to exhibit a particular reason for some countries to pursue a fixed exchange rate policy; we shall later insert our basic model into a more realistic framework. Thus, at present we let the exchange rate at date \( t, t = 0, 1 \) be \( e_t \) (measured as units of home currency per unit of foreign currency).

Assume from now that \( e_t \) is a policy variable determined by the home country (the foreign country is passive in this respect, it may be assumed to follow a regime of flexible exchange rates). Given the exchange rates \( e_0 \) and \( e_1 \), consumers \( i \) in the home country with income \( I \) choose consumption \( (x_{i1}, x_{i2}) \) so as to maximize utility subject to the budget constraint

\[
px_{i1} + e_1 \hat{p}x_{i2} = (1 - \tau)I_i,
\]  

(2)

where \( \tau \) is the income tax rate. The income \( I_i \) has the form

\[
I_i(p, q, r_L, r_D) = \begin{cases} 
q \omega_i (1 + r_D) & \text{if } i \text{ is factor owner,} \\
\pi(q, r_L) & \text{if } i \text{ is entrepreneur,} \\
(r_L - r_D)qz & \text{if } i \text{ is financial intermediary.} 
\end{cases}
\]  

(3)

It should be noticed that due to interest rate arbitrage we have

\[
1 + r_D = e_0^{-1} (1 + \hat{r}_D) e_1,
\]  

(4)
that is return of deposits in home country equals return of placing savings with foreign country intermediaries.

We have included the possibility of taxation (which will only be used in an example) to illustrate the workings of exchange rate policy under perfect competition; if all prices are sufficiently flexible so as to balance supply and demands of goods, factors, and bonds, then the only way of controlling exchange rates is by operating in the market (sustaining the home exchange rate by government purchases of home products). We are not going to investigate this in details, since it is anyway not a realistic policy option, but we have chosen to include the possibility of such stabilization by allowing for income taxation. The proceeds of taxation, $\tau(I_1 + I_2 + I_3)$, are used for stabilization purposes, purchasing commodities for storage (which in this simple version just means that they are not consumed).

An equilibrium (given the exchange rate policy $(e_0, e_1)$) is an array

$$((x^0_1, p, q, r_L, r_D, e_0, e_1, \tau, \bar{x}_1, \bar{x}_2), (\hat{x}^0_1, \hat{p}, \hat{q}, \hat{r}_L, \hat{r}_D))$$

consisting of consumption, factor prices, and loan rates in each country, such that (here, only the conditions pertaining to country 1 are stated, those of the other country are similar)

(i) $(x^0_{i1}, x^0_{i2})$ maximizes $U(x_{i1}, x_{i2})$ under the constraints (2) and (3), $i = 1, 2, 3$.

(ii) financial intermediaries choose $r_L$ so as to maximize profits $(r_L - r_D)qz$ for given prices and deposit, where the demand for loans is given by (1); first order conditions for maximum are

$$f'(z) + zf''(z) = \frac{q}{p}(1 + r_D), \quad (5)$$

where $z$ is factor input.

(iii) commodity prices satisfy the arbitrage condition $\hat{p} = e_1\hat{p}$, and deposit interest rates satisfy arbitrage condition (4),

(iv) supply equals demand in all markets,

$$\sum_{i=1}^{3} x_{i1} + \sum_{i=1}^{3} \hat{x}_{i1} = f(\omega),$$

$$\sum_{i=1}^{3} x_{i2} + \sum_{i=1}^{3} \hat{x}_{i2} = \hat{f}(\hat{\omega}),$$

(v) income tax revenue covers government purchases: $\tau(I_1 + I_2 + I_3) \geq p\bar{x}_1 + e_1\hat{p}\bar{x}_2$.

Although the model is deliberately kept simple, it is not obvious that an equilibrium exists, so we insert the following result:
Theorem 1. In the model considered, assume that:

(a) the utility function is continuous, monotonic and strictly quasi-concave,

(b) the production functions \( f, \hat{f} \) are twice differentiable and concave, and in addition satisfy neoclassical conditions: \( f'(z) \to \infty \) for \( z \to 0 \), \( f(z) \to 0 \) for \( z \to \infty \).

Then for each value of \( e_0, e_1 \) such that \( e_1 \) does not exceed the marginal rate of substitution at the point \( (f(\omega), \hat{f}(\hat{\omega})) \), there exists an equilibrium.

The proof of Theorem 1 is given in an appendix. At this point, we illustrate the workings of the model by an example:

Example 1. Assume that \( \omega = \hat{\omega} = 1 \), that the utility function is \( U(x_1, x_2) = x_1 x_2 \), and the production functions are

\[ f(z) = z^{1/2}, \quad \hat{f}(\hat{z}) = \hat{z}^{1/2}. \]

If we choose \( e_0 = e_1 = 1 \), then obvious candidates for equilibrium prices might be \( p = \hat{p} \) by condition (iii), and utility maximization gives that total consumption in each country of each commodity must equal \( 1/2 \). We need to find a tax rate \( \tau \), factor prices \( q, \hat{q} \), and interest rates \( r_L, \hat{r}_L \) and \( r_D \) (equal to \( \hat{r}_D \) by (iii)) such that this allocation is sustained in equilibrium.

We start with a situation where \( \tau = 0 \). Normalizing by \( q = \hat{q} = 1 \), and setting \( r_D = 0 \), we get that the loan rates \( r_D = \hat{r}_D \) can be found from (5), which in this case becomes

\[ f'(1) + f''(1) = \frac{1}{p} \]

or \( p = 4 \), and using (1) we obtain that

\[ r_L = \frac{4}{\frac{1}{2}} - 1 = 1. \]

The incomes of the three different types of consumers can be found from (3) as \( I_1 = 1 \), \( I_2 = 2 \), and \( I_3 = 1 \), and the resulting levels of utility achieved are \( U(x_{11}, x_{12}) = \frac{1}{64} \), \( U(x_{21}, x_{22}) = \frac{1}{16} \), \( U(x_{31}, x_{32}) = \frac{1}{64} \).

In the example, the deposit rate was set rather arbitrarily to 0. If we instead have \( r_D = \hat{r}_D = 1 \), then we get \( p = 8 \) and \( r_L = 3 \); the incomes are now \( I_1 = 2 \), \( I_2 = 4 \), \( I_3 = 2 \), meaning that allocation is unaffected by the change.

Since there are equilibria for any choice of exchange rates, we may now try another case, namely \( e_0 = 1, e_1 = \frac{1}{2} \). In this situation, consumers’ choices will be such that \( x_{i2} = 2x_{i1}, \hat{x}_{i2} = 2\hat{x}_{i1} \). We try the interest rate \( r_D = 0 \) and again let factor price in country 1 be \( q = 1 \), but let \( \hat{q} \) be determined endogenously, since we want \( p = \hat{p} \) (so that \( e_1 \) is the real exchange rate). Since \( e_0 = 2e_1 \), we have by (4) that \( 2(1 + r_D) = (1 + \hat{r}_D) \), so that \( r_D = 0 \) entails \( 1 + \hat{r}_D = 2 \). As previously, we can find the loan rate \( r_L \) and the commodity price \( p \) from (5) and (1), giving \( p = 4 \) and \( r_L = 1 \), as above, leaving \( I_1, I_2 \) and
I_3 \text{ unchanged. Using } \hat{p} = 4 \text{ and } \hat{r}_D = 1, \text{ we get that } \hat{q} = 1/2 \text{ and } \hat{r}_L = 3 \text{ (note that factor prices are lower but factor owners are compensated by higher interest rates, and similarly, entrepreneurs and financial intermediaries in the foreign country end up with unchanged incomes).}

Clearly, since consumers demand the commodities in the ratio 1 : 2, this cannot be an equilibrium unless the incomes in the home country are suitably restricted. In the foreign country, consumers facing prices (8, 4) by the aggregate bundle \((\frac{1}{4}, \frac{1}{2})\), and in the home country, without tax they would buy the bundle \((\frac{1}{2}, 1)\) (at prices \((4, 2)\) ). To balance the markets, we need to extract income from home consumers at the tax rate \(\tau = \frac{1}{2}\). Since consumer demand for commodity 1 is then 1/4 in both countries, the remaining demand must come from the tax revenue \(\frac{1}{2} \cdot 4 = 2\), which exactly matches the remaining production of commodity 1 at price \(p = 4\). Thus, we have found an equilibrium at the stated exchange rates.

Due to the (rather large) public intervention needed to reestablish equilibrium, and the fact that income distribution remains unchanged, the final consumption bundles under the new exchange rates are inferior than under the old ones. In other cases, however, the gain from the change in relative prices together with changes in income distribution may result in improvement of the position of some consumers, even if the overall situation is one of a welfare loss. This is shown in the next example.

3. The simple model with anticipation of default

In this section we retain most of the features of the simple model considered in the previous section. However, we change the assumptions on the financial intermediaries of each country, who now take the default risk into account when offering credits to the entrepreneurs of their country. To keep the model simple, we assume that financial intermediaries have a forecast \(d\) (respectively, \(\hat{d}\)) of the total demand for the produced commodity, and that the profits from financial intermediation takes the form

\[ r_Ld - r_Dz = (r_L \frac{d}{z} - r_D)z, \]

so that the effective loan rate (giving the return on the total sum of money forwarded to the entrepreneurs) will increase in the case that production surpasses expected demand.

This new feature of the model is of course a departure from the strictly competitive framework which characterized the initial version of the model, at least when abstracting from the monopolistic behaviour in the financial sector. On the other hand, the change goes in the direction of increased realism, since financial intermediation, and in particular the credit side of it, involves uncertainty and anticipations in a crucial way (for a discussion of this, cf. e.g., Freixas and Rochet (1997)).
Assuming as before that the financial intermediaries choose the loan rate by maximizing profits subject to the reaction patterns of the entrepreneurs, which are subsumed in (1), we immediately get that optimal choice of loan rate at the level of anticipated demand is found as before when the profit maximizing level of \( r_L \) does not induce a production greater than anticipated demand, and

\[ r_L = \frac{p}{q} f'(d) - 1 \]  

otherwise.

For the equilibrium, we demand that anticipated demand equals actual demand – thus, anticipations should be correct in the sense that they are not contradicted by observations when the latter are available. It is straightforward to check that Theorem 1 carries over to the present version of the model.

Example 2. We now consider the economy considered in Example 1 under the new framework, where financial intermediaries anticipate future demand are act accordingly when granting credits. The usefulness of this behaviour comes to the day when we consider the case of forced exchange rate change, that is \( e_0 = 1, e_1 < 1 \). We saw in Example 1 that this change in real exchange rates which makes the products of the foreign country relatively cheaper, will result in an excess supply of the commodity produced in country 1. In the previous version of the model, this could be solved only by government purchases, something which is quite far from real economic life. Now we are able to treat the case without resort to government intervention in the market, using only the anticipations of the financial intermediaries.

Let us check the equilibrium belonging to an exchange rate \( e_1 = 0.94868 \), determined in such a way that consumers will demand the two commodities in the ratio \( 1 : \sqrt{0.9} \), and in equilibrium, there will be produced at full capacity in country 2 but only 0.9 units of the commodity in the home country. Setting \( p = \hat{p} = 4 \) and \( q = \hat{q} = 1 \), we get that for this level of anticipated demand, the loan rate is

\[ r_L = 4 \cdot \frac{1}{\sqrt{0.9}} - 1 = 1.180819. \]

Using the arbitrage condition and assuming \( \hat{r}_D = 0 \), we get that \( r_D = -0.05132 \) (thus, in this example we must have a negative rate of interests to avoid arbitrage given that the domestic currency has been appreciated). The incomes in home country are then

\[ I_1 = 3.6 - 0.9 \cdot 2.180819 = 1.63726, \quad I_2 = 0.9 \cdot 0.94868 = 0.85381 \]
\[ I_3 = (1.180819 + 0.05132) \cdot 0.9 = 1.10899. \]

Comparing with the situation prior to the exchange rate intervention, we first notice that the terms of trade of the home country have improved; on the other hand the production
is reduced. With the simple structure of consumer demand chosen in the example – and what matters here is that all have Cobb-Douglas utility preferences so that there is a unique period 1 equilibrium for any aggregate production vector – the improvement of home country welfare due to improved terms of trade is too small compared with the loss derived from reduced production. Thus, taken as a whole, the home country is worse off. On the other hand, it is seen that the financiary intermediaries are decidedly better off; they have a larger nominal income than before, and in addition, the cost of consumption has decreased, so that real income has increased more than the 45% which is the increase in nominal income. The conclusion is that the exchange rate policy, which on the face of it was detrimental to the welfare of the home country, has benefitted at least one group of income receivers in society.

The case considered in the above example strongly suggests that whereas the gains for society as a whole from an exchange rate intervention of the type considered is ambiguous and in many cases a loss rather than a gain, the financial intermediaries stand to gain from this policy, since the loss of profits due to reduced demand is compensated both by the terms-of-trade effect and from the lowering of the domestic deposit rates. The following theorem shows that this feature is not a specific property of the example but holds more generally.

**Theorem 2.** Let \( e_0 = 1, e_1 = 1 \) initially, and suppose that \( e_1 \) is changed to a value \( e'_1 < 1 \) by exchange rate intervention in the home country. Then the following holds:

(i) If in period 1, there is a unique equilibrium in the market at \( t = 1 \) for any initial allocation of the goods among the consumers in each country, then the country as a whole is worse off after the intervention.

(ii) Assume that the mapping \( z \mapsto f'(z)z \) is concave. Then the income of the financial intermediary in the home country will increase as a result of the intervention.

**Proof:** (i) Assume to the contrary that there is an exchange rate intervention with \( e_1 < 1 \) which results in a Pareto improvement for the home country. Then aggregate production is \( \hat{f}(\hat{\omega}) \) in the foreign country and \( f(z) \) in the home country, for \( z \leq \omega \), and this aggregate production corresponds to aggregate demand in the period 1 market. If the home country is better off, then it would be welfare improving to dispose of part of \( f(\omega) \) before the market opens. For this to be true, the support of the set of preferred bundles at \( (f(z), \hat{f}(\hat{\omega})) \) must intersect the support of the set of preferred bundles at \( (f(\omega), \hat{f}(\hat{\omega})) \), and the intersection point would be an initial endowment for which there are multiple equilibria, a contradiction.

(ii) Assume that input is changed by the small amount \( dz < 0 \) as a result of the intervention. Then marginal product changes by \( f''(\omega) > 0 \), and the income of the financial intermediaries is changed by the amount

\[
\frac{\partial I_3}{\partial z} = \frac{\partial}{\partial z}[(pf'(z) - q(1 + r_D))z] = [pf''(z)z + pf'(z) - q(1 + r_D)] < 0
\]
by our assumptions. Also, the income of the financial intermediaries is increased by the reduction of the effective deposit rate by

$$\frac{\partial I_3}{\partial e_1} = \frac{\partial I_3}{\partial r_D} \frac{\partial r_D}{\partial e_1} = -z e_0^{-1} (1 + \hat{r}_D) < 0.$$ 

If follows that

$$dI_3 = \frac{\partial I_3}{\partial z} dz + \frac{\partial I_3}{\partial e_1} de_1 > 0$$

since $dz < 0$, $de_1 < 0$.

It might be noticed that the condition in the statement of part (i) of the theorem rules out the so-called transfer paradox of international trade theory, arising when a transfer of commodities from one country to another before trading takes place will result in greater welfare after trade to the country which gave up the commodities. In our simple model this transfer paradox is closely connected to the problem of uniqueness of equilibria.

As mentioned in the beginning of the section, the way in which we treat the factor markets in the present version of our model may not be quite satisfactory, since we assume competitive behavior in the market but still accept – and even need – that not all the endowment is sold. We therefore consider a slightly different, and theoretically more coherent, version of the model in the next section.

4. Allowing for trade in factors

It might be interesting to investigate whether this conclusion can be upheld if we allow for trade in factors, assuming the latter to be physically identical in the two countries. In this case we loosen the connection between factor use and final production in each separate country; factors unemployed in one country may be used by the producers of the other country.

Assuming that the factors of the two countries are physically identical, the possibility of trade means that there will be a common price (except for exchange rate differences) and only one market to be cleared at $t = 0$. Though technically simpler, the change in the model means that the market balancing conditions in (iv) above needs a reformulation; introducing notation $z$, $\hat{z}$ for factor input in the two countries, the new equilibrium condition becomes

$$\sum_{i=1}^{3} x_{i1} + \sum_{i=1}^{3} \hat{x}_{i1} = f(z), \quad \sum_{i=1}^{3} x_{i2} + \sum_{i=1}^{3} \hat{x}_{i2} = \hat{f}(\hat{z}), \quad z + \hat{z} = \omega + \hat{\omega}.$$ 

We refer to the appendix for a discussion of existence of equilibrium.

**Example 3.** Let us examine the workings of the model specified in Example 2 under the new conditions. We investigate a situation where factor use $z$ in the home country
is down at 0.9 (as in Example 2) but where the remaining amount 0.1 is inserted in the foreign country, so that $\bar{z} = 1.1$. This may be equilibrium production levels in the case where $e_0 = 1$ and

$$e_1 = \frac{\sqrt{0.9}}{\sqrt{1.1}} = 0.90453.$$  

We need to analyse the situation in the two countries separately depending on whether anticipated demand is greater or smaller than production at the profit maximizing loan rate. Starting with the foreign country and assuming that the common factor price at $t = 0$ is $q = 1$, we can find the commodity price $\hat{p}$ from (5), which now becomes

$$\frac{1}{4} \bar{z}^{-\frac{1}{2}} = \frac{e_1^{-1}}{\hat{p}},$$

which gives $\hat{p} = 3.79472$.

Now we return to the home country; since $e_1$ is a real exchange rate, we have $p = \hat{p} = 3.7947$, and as in Example 2, we find the loan rate $r_L$ from (1), which gives

$$r_L = 3.7947 \cdot (2\sqrt{0.9})^{-1} - 1 = 1.$$  

From the arbitrage equation (4) we have that $r_D = -0.09547$, and we can now find the incomes as

$$I_1 = 3.7947 \cdot \sqrt{0.9} - 2 \cdot 0.9 = 3.6 - 1.8 = 1.8, \quad I_2 = 0.90453, \quad I_3 = 0.985887.$$  

Notice that in the present case, the sum of the incomes received in the three sectors exceeds the value of the production in period 1; this is a consequence of the trade in factors, whereby home country factor owners earn part of their incomes from selling factors abroad.

We see that although all incomes have fallen as compared with the benchmark case in Example 1, they have not fallen in equal proportions. Indeed, the income of the financial intermediaries have remained almost unchanged. Taken together with the fact that prices of consumption goods have fallen, we see that this sector of the economy has experienced an increase in real incomes.

It should come as no big surprise that we get a result close to that of the previous section, and again it is not connected with the specific example but holds under rather weak assumptions on preferences and technology. Indeed, the same mechanism is at work here as in the previous section; financial intermediaries gain from the lower real interest rate even though the reduction in demand prevent full exploitation of the monopoly position in the domestic credit market. The new feature here is that factor owners are held free of losses since the factor demand is unchanged.

The result stated in Theorem 2(ii) carries over to the present case with identical formulation and proof.
Theorem 3. Let $e_0 = 1$, $e_1 = 1$ initially, and suppose that $e_1$ is changed to a value $e'_1 < 1$ by exchange rate intervention in the home country. If the mapping $z \mapsto f'(z)z$ is concave, then the income of the financial intermediary in the home country will increase as a result of the intervention.

For the first part, the situation is less clear, since trade in factors means that the country derives income from other sources than production of the commodity. For this reason the counterpart of Theorem 2(i) is left out in the statement below.

5. Concluding remarks

In the previous sections, we have considered a simple model of international trade and finance, where real exchange rate appreciation may be advantageous for some income receivers even if it is disadvantageous for the country as a whole. The fact that the agents in society who stand to gain from the policy are those belonging to the financial sector may shed light on a reason that such policies – or rather policies which in our simplistic model takes this form – seems to be advocated for developing countries both by influential counsellors, domestic as well as foreign.

It goes without saying that what has been presented is at most a collection of thought-provoking examples, not a finished model. For this we would need to extend the model from the two-period framework considered here to an open-ended model, presumably with overlapping generations, in which the different generations will experience the exchange rate appreciation in different ways. We have preferred to present the simple story here in order to have as few disturbing side effects as possible. This is also the reason that uncertainty has been left out, although uncertainty in some form seems to be the necessary precondition for existence of financial intermediaries of the form considered here rather than ordinary competitive bond markets.

Clearly, the key to our results is that the financial intermediaries are not organized as a perfectly competitive industry but exhibit some kind of imperfect competition, here taken in the extreme form of monopoly. Thus, what really distinguishes the present approach is our insistence on analyzing trade under conditions which are not the usual competitive markets that typically are underlying the macroeconomic approach to international finance. A more detailed exploration of the implications following from the explicit consideration of such imperfections will be a task for future research.


In this section we present a proof of the existence result which appears as Theorem 1. Although the model is an extremely simple one, existence of equilibria is not entirely
trivial due to the presence of monopolistic behavior by financial intermediaries; as is well known (see, e.g., Gabszewicz (1999)), such behavior may occasionally cause problems in models of general equilibrium. This is however not the case in our model.

**Proof of Theorem 1:** Let \( e_0, e_1 \) be given; from the assumptions on the technologies we get that there is a (large) number \( M > 0 \) large enough so that if we find an equilibrium then all prices and interest rates must be numerically smaller than \( M \).

Define the correspondence \( \gamma : [0, M]^3 \to \mathbb{R}_+^2 \) and the mapping \( \xi : [0, M]^3 \to \mathbb{R}_+^2 \) by

\[
\gamma(p, I, \hat{I}) = \{(x, \hat{x}) \in \mathbb{R}_+^2 \mid px + e_1 p \hat{x} \leq I + \hat{I}\},
\]

\[
\xi(p, I, \hat{I}) = \{(x, \hat{x}) \in \mathbb{R}_+^2 \mid (x, \hat{x}) \text{ maximizes } U \text{ on } \gamma(p, I, \hat{I})\}.
\]

By standard results of equilibrium analysis, \( xi \) is continuous. We denote by \( \hat{xi} \) the second component of \( \xi \).

Next, define the mapping \( \rho : [0, M]^2 \to [-M, M] \) by

\[
\delta(p, q) = \min\{M, \frac{p}{q} (f'(\omega) + \omega f''(\omega)) - 1\}
\]

that is \( \delta(p, q) \) is the deposit rate solving (5) for \( z = \omega \). By our assumptions, \( \rho \) is continuous. Also, we get \( \hat{r}_D \) from \( r_D = \delta(p, q) \) by the arbitrage equation. Also, define the continuous maps \( \lambda : [0, M]^2 \to [-M, M] \) by

\[
\lambda(p, q) = \frac{p}{q} f'(\omega) - 1.
\]

Finally, we introduce the mappings \( I : [0, M]^2 \times [-M, M]^2 \) given by (3) for income determination in the home country, let \( \hat{I} = p \hat{f}(\hat{\omega}) \) and \( I(p, q, r_L, r_D) = \sum_{i=1}^3 (1 - \tau) I_i(p, q, r_L, r_D) \) for \( \tau \in [0, 1] \).

The correspondence \( \eta : [0, 1] \times [-M, M] \to [0, 1] \) is now defined by

\[
\eta(\tau) = \begin{cases} 
\{\tau' \mid \tau' > \tau\} & \text{if } \hat{\xi} > \hat{\omega}, \\
\{\tau' \mid \tau' < \tau\} & \text{if } \hat{\xi} < \hat{\omega};
\end{cases}
\]

this correspondence has an open graph and convex values. By our assumptions on \( e_1 \), we have that \( \eta(0) \) consists of all \( \tau' > 0 \), and conversely, if \( \tau = 1 \), then \( \hat{xi} < \hat{\omega} \) so that \( \eta(1) = [0, 1] \). Now, a connectedness argument assures the existence of a value \( \tau^0 \) for which \( \eta(\tau^0) = \emptyset \). It is easily seen that at this value of \( \tau^0 \), we have \( \hat{\xi} = \hat{\omega} \), and the support purchases will secure market balance in the home country as well.

It may be noticed that we prove something slightly stronger than what is formulated in the theorem, namely that there exist equilibria for any specified fixed values of factor and commodity prices; this is of course due to the very simplicity of the model, where consumption takes place only at \( t = 0 \) so that time preferences play no role. Also, the
simple structure of the model is illustrated by the fact that existence follows already by connectedness arguments, without use of fixed-point theorems. The latter feature is of course changed if the model is extended to allow for more than one commodity in each country.

Since existence of equilibria in our model is a rather straightforward consequence of the assumptions, we shall leave it to the reader to check that the reasoning carries over with only minor changes to the versions considered in the sections 3 and 4.

7. References
