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04-21

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UK MONEY DEMAND 1873–2001: A COINTEGRATED VAR ANALYSIS WITH ADDITIVE DATA CORRECTIONS

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October 20, 2004

Abstract: This paper performs a system cointegration analysis of UK money demand based on real money, real income, the opportunity cost of holding money, and inflation for the period 1873 − 2001. As a novelty we account for the effect of the world wars by estimating additive data corrections, allowing observations during the two world wars to be fundamentally different from peace-time observations. We find a single long-run relation, which links velocity to opportunity costs, and a strong link from excess money to inflation. The long-run structures are reasonably stable, although the information in the data is not evenly distributed over time. In particular, it seems important to include information from the episodes of large variations in velocity and interest rates around 1960 − 1980.

Keywords: The UK; Money Demand; Cointegration; Additive Outlier; Level Shift.
JEL Classification: E41; E31; C32

Comments from Christopher Bowdler are gratefully acknowledged. The empirical analysis was carried out using Ox, see Doornik (2001).

1 Introduction

The relation between money, income and inflation plays a key role in the economic debate and a vast amount of research on the demand for money has been carried out the past decades, see *inter alia* Goldfeld and Sichel (1990) or Sriram (2001) for reviews. Of particular importance in this line of research is the stability of money demand over time and the existence of a link from excess money to income and inflation.

In the majority of recent studies, the money demand relation is interpreted as a long-run entity and the primary econometric tool is cointegration, defining certain linear combinations that cancel the unit root non-stationarity in individual variables. In the design of these studies there is typically a trade-off between sample length and structural stability. On the one hand we want a sample sufficiently long to establish cointegration, but on the other hand a very long span of data increases the exposure to institutional changes, effects of wars and other radical events, and changes in the contents and interpretation of the variables in the analysis. For economic interpretations and for deriving relevant policy implications it is important, however, to distinguish non-stationarity related to unit roots from non-stationarity induced by external factors and institutional changes.

Several authors have analyzed the demand for money over long spans of data, see *inter alia* Friedman and Swartz (1982), Lucas (1988), and Stock and Watson (1993) for the United States; and Funke and Thornton (1999), Sarno (1999), and Muscatelli and Spinelli (2000) for Italy. For the case of the United Kingdom, Friedman and Swartz (1982) estimate a demand relation for broad money in a series of regressions using averages over phases of business cycles for the sample $1867-1975$. Hendry and Ericsson (1991) question the phase-average approach and analyze the annual observations directly, while Ericsson, Hendry, and Prestwich (1998) analyze a sample extended to 1993. Both studies estimate somewhat rudimentary long-run relations using Engle and Granger (1987) single equation cointegration regressions; and they analyze the stability of the short-run adjustment conditional on the long-run relation.

The present paper reconsiders historical UK money demand based on a set of data containing real money, real income, the opportunity cost of holding money, and the inflation rate. Compared to the existing literature on long-run money demand, there are three main contributions of the paper.

First, we take a full system approach to model the historical UK money demand by applying the cointegrated vector autoregressive framework of Johansen (1991) and Johansen (1996). This approach yields efficient estimation of the long-run demand for money, and, more importantly, it allows a characterization of the interdependencies between the variables, in particular the feedback from excess money to income and inflation.

Secondly, we deal explicitly with possible structural breaks and the effects of special events in the vector autoregressive model for the 128 years of data. The approach taken is to apply additive corrections in the variables for extreme periods and estimate the obtained model with maximum likelihood. This is equivalent to replacing observations during, for example, the war periods with artificial observations interpolated using the information in the rest of the sample, see also Nielsen (2004). Furthermore, we allow for permanent effects induced by the two world wars via additive level shifts in the variables. This is closely related to the GLS detrending approach in Saikkonen and Lütkepohl (2000a), Saikkonen and Lütkepohl (2000b), and Lütkepohl and Saikkonen (2000).

Thirdly, compared to earlier analyses of long-run UK money demand we update the data to 2001, thereby addressing the stability of money demand also for the recent years.

The rest of the paper is organized as follows. Section 2 presents the econometric tools, while Section 3 discusses the theoretical framework and measurements. The empirical analysis of the long-run structure is then presented in Section 4, while Section 5 concludes.

2 Econometric Approach

In econometric modelling of the long-run, the key point is to account for the possible non-stationarities in the data and to identify potential combinations of the variables that cancel the non-stationarity and appear as stable equilibrium relations.

Hendry (2000) notes three kinds of non-stationarity most pertinent to long, low frequency time series: First, a stochastic non-stationarity related to unit roots. Secondly, structural breaks induced by e.g. institutional changes and wars. And thirdly, changes in the measurement system and in the content and interpretation of the variables in the analysis. The sample of the present analysis, $1873 - 2001$, is characterized by numerous external shocks and institutional changes that may induce non-stationarities, including two world wars, a major depression, two large oil price shocks, financial deregulations, shifts in the exchange rate regimes, as well as gradual changes related to the monetarization of the economy, innovations in financial technology, and changes in the currency convertibility during the period.

To model the interdependencies between the variables and to allow for non-stationarity related to unit roots we use the cointegrated vector autoregressive (VAR) model as the statistical framework, see Johansen (1991) and Johansen (1996). Normally in cointegration models, the effects of external shocks are modelled by including indicator functions as unrestricted dummy variables. This strategy implies that external shocks are treated as large innovations to the VAR system and there is an implicit assumption that the transmission of the extreme shocks through the autoregressive system of equations is identical to the transmission of the normal shocks, see also the discussion in Nielsen (2004). In some cases, e.g. for large economic shocks such as the oil price increases in the 70'ties, this may be a reasonable assumption. In other cases, however, the equilibrating forces may be affected by institutional changes related to the shocks, and the transmission of the extreme shocks may be fundamentally different from the usual transmission mechanism. This could for example apply during wars with rationing and price controls.

In this paper we suggest an alternative approach to model the effects of such events. Besides considering innovations to the *economic system*, we also consider additive distortions of the *variables* directly, with no transmission through the autoregressive system. This allows the war-time observations to be fundamentally different from the rest of the observations. Technically, we insert indicator functions as additive dummy variables for the war-time observations. This is parallel to the additive outlier model discussed in Nielsen (2004), and as a by-product we obtain estimates of the effects of the external shocks, and we obtain an *adjusted* time series where the war-time observations are replaced by interpolated values. An additive outlier can be interpreted as the natural parallel to a dummy in a static regression model, where the effect of a particular observation is removed from the likelihood function.

To model potential permanent effects of the external shocks we allow for additive level shifts in the variables. This is closely related to the GLS detrending applied in Saikkonen and Lütkepohl (2000a), Saikkonen and Lütkepohl (2000b), and Lütkepohl and Saikkonen (2000). By modelling the additive effects in individual variables, we can treat the variables asymmetrically so that the shifts only affect some variables.

2.1 The Statistical Model

The starting point is a $p-$dimensional cointegrated VAR model given by

$$H(r): \Delta Z_t = \alpha\beta' Z_{t-1} + \sum_{i=1}^{k-1} \Gamma_i \Delta Z_{t-i} + \alpha\beta_0' t + \mu_0 + \phi d_t + \epsilon_t, \quad t = 1, 2, ..., T. \tag{1}$$

If the levels Z_t are cointegrated with r long-run relations then α and β are of dimension $p \times r$ such that the rank of $\Pi = \alpha\beta'$ is $r \leq p$. The remaining autoregressive parameters, $\Gamma_1, ..., \Gamma_{k-1}$, are of dimension $p \times p$. Throughout we condition on k initial values, $Z_{-k+1}, ..., Z_0$, and we assume that the innovations, ϵ_t, are independently and identically distributed. Estimation is based on the likelihood function corresponding to the case of Gaussian innovations, $\epsilon_t \sim N(0, \Omega)$.

The deterministic specification in (1) includes a linear term with a coefficient proportional to α and an unrestricted constant. That produces deterministic linear trends in all linear combinations of the data, including the stationary cointegrating relations, $\beta' Z_t$. Furthermore, the model includes a set of dummy variables, d_t, with unrestricted coefficients ϕ. Dummies in d_t are interpretable as large shocks to the system and they will follow the underlying autoregressive transmission.

In addition to the linear trend and the system shocks in d_t, we also want to allow for level shifts induced by the two world wars and we want to allow the data observed during the world wars to be fundamentally different from the peace-time observations. To do this we assume that there exist an underlying mechanism (1) generating a set of data Z_t. On top of this we add perturbations related to external shocks but unrelated to the autoregressive structure. The observed time series, X_t, is therefore given by

$$X_t = Z_t + \theta D_t, \tag{2}$$

where D_t is a $n-$dimensional vector of dummy variables, and θ is a $p \times n$ matrix of coefficients. If $D_t = 1\{t = T_0\}$ is an impulse dummy taking the value 1 at T_0 and zero otherwise, the specification (1) and (2) is the cointegrated VAR model with an additive outlier, see Nielsen (2004). It is worth noting that this specification makes the values of the likelihood function invariant to the observation X_{T_0}, because it is replaced by a value interpolated from the information in the rest of the data. This is closely related to the interpolation of missing values, see Gomez, Maravall, and Peña (1999). If $D_t = 1\{t \geq T_0\}$ is a step function the estimated parameters in θ are level shifts in the variables.

Solving (1) and (2) for the observed variables, X_t, yields the representation

$$\Delta X_t = \alpha \left(\beta' : \beta_0' : \beta_1'\right) \begin{pmatrix} X_{t-1} \\ t \\ D_{t-1} \end{pmatrix} + \sum_{i=1}^{k-1} \Gamma_i \Delta X_{t-i} + \theta \Delta D_t + \sum_{i=1}^{k-1} \theta_i \Delta D_{t-i} + \mu_0 + \phi d_t + \epsilon_t, \quad (3)$$

subject to the k sets of restrictions

$$\beta_1 = -\theta'\beta \quad (4)$$
$$\theta_i = -\Gamma_i \theta, \quad i = 1, ..., k-1. \quad (5)$$

To model level shifts Johansen, Mosconi, and Nielsen (2000) consider the representation in (3) without imposing the non-linear restrictions in (4) and (5). Thereby, the transition to the level shift is not instantaneous but will approximated by the unrestricted impulse dummies, $\Delta D_t, \Delta D_{t-1}, ..., \Delta D_{t-k+1}$. In this framework, the asymptotic distributions of the likelihood ratio (LR) tests for the cointegration rank depend on the number of level shifts, i.e. the dimension of D_t, and also on the location of the level shifts on the time axis.

In this paper we use the factor representation in (2) directly, corresponding to imposing the restrictions in (4) and (5). This is related to the specification in Saikkonen and Lütkepohl (2000a), Saikkonen and Lütkepohl (2000b), and Lütkepohl and Saikkonen (2000). These authors consider a factor representation of the form $X_t = Z_t + \tau_0 + \tau_1 t + \theta D_t$ and use a two step approach to detrend the variables before performing a cointegration analysis based on a VAR model with no deterministic terms. The detrending approach has the advantage that the presence of the shift dummies in D_t (and the locations of the shifts on the time axis) does not affect the asymptotic distributions of the rank tests. In (the majority of) the present paper we use the detrending approach based on the factor representation (2) to estimate only θ while τ_0 and τ_1 are estimated in the cointegration model (1). To estimate the cointegration model with additive corrections we use the algorithm in Nielsen (2004), see Appendix A for details.

It is worth noting that if d_t or D_t contain dummy variables referring to single observations, e.g. indicator variables $1\{t = T_0\}$, then the corresponding columns in $\widehat{\theta}$ cannot be consistent as the parameters are estimated from a finite number of observations even when $T \to \infty$, see also Davidson (2001, p. 147). If D_t contains a level shift, $1\{t \geq T_0\}$, then the corresponding estimator in $\widehat{\theta}$ is only consistent in the stationary directions $\beta'\widehat{\theta}$, while the non-stationary directions $\beta'_{\perp}\widehat{\theta}$ are not consistent, see Saikkonen and Lütkepohl (2000a) and Lütkepohl, Saikkonen, and Trenkler (2003). The intuitive reason is that the expectation is only well defined in the stationary directions and level shifts, describing changes in the expectation, can only be estimated consistently in these directions. Another way to see this point is to consider the specification in (3). Here the lagged levels, D_{t-1}, appear with the coefficient $\alpha\beta'\theta$; and since D_t refers to infinitely many observations as $T \to \infty$, we can consistently estimate $\beta'\theta$. In the non-stationary directions, on the other hand, the level shifts are essentially obtained as the accumulated effects of the first differences, ΔD_t, cf. the Granger representation theorem of Johansen (1996, theorem 4.2). But since ΔD_t is an impulse dummy referring to a single observation, the corresponding parameter, containing information on θ in the non-stationary directions, cannot be consistently estimated.

The inconsistency has consequences for the distribution of test statistics. Since the information on the parameters is limited, even asymptotically, the distribution of the test does not follow from a central limit theorem. Instead it has to be derived from the properties of individual residuals under the null. Under the assumption of Gaussian innovations a Wald test on θ will still have a standard normal or a χ^2-distribution under the null, and throughout the paper we will compare test statistics to standard distributions.

3 Theoretical Framework and Data Measurements

In this section we present the theoretical framework and the corresponding measurements used in the empirical analysis in Section 4.

A common starting point for modelling money demand is the following log-linear specification

$$m_t - p_t = \gamma_0 + \gamma_1 y_t + \gamma_2 R_t^{own} - \gamma_3 R_t^{alt} - \gamma_4 \Delta p_t + u_t, \tag{6}$$

where $m_t - p_t$ is the log of real money balances; y_t is the log of a real scale variable; R_t^{own} is the return on components inside the measure of money; while the return on alternatives to money is represented by the alternative rate, R_t^{alt}, and the inflation rate, Δp_t (interpretable as the return on goods). Finally, γ_i, $i = 0, \ldots, 4$, are coefficients with expected positive signs, and u_t measures the deviation from the proposed relation.

We note that by imposing linear homogeneity on (6) through the unit coefficient to p_t, long-run permanent money illusion is excluded. This is in line with most economic theories and from a practical point of view it implies that the nominal variables, p_t and m_t, which are likely to be driven by second order stochastic trends, does not have to analyzed separately. We can add that the presence of the inflation term allows for deviations from homogeneity in the short run. If the relation is also homogeneous in income, $\gamma_1 = 1$, then velocity appears directly.

If $\gamma_2 = \gamma_3$ then the interest rates enter only through the opportunity cost of holding money, $R_t = R_t^{alt} - R_t^{own}$. Given the difficulties in precisely defining the money stock and properly measure the corresponding rates of return of inside and outside assets over the long sample, we follow *inter alia* Hendry and Ericsson (1991) and impose this restriction *a priori*. That gives us the vector of variables, $X_t = (m_t - p_t : y_t : R_t : \Delta p_t)'$, which will form the basis for the empirical analysis.

In the empirical analysis the above relation is a candidate to a cointegrating relationship. In this case u_t is a stationary process–at least after correcting for the non-stationarity related to external shocks. If the number of long-run relations, r, is found to be larger than one, a second long-run equilibria may represent an IS curve relation between detrended income and the interest rate, or a Fisher-type relation between the interest rate and inflation, see further in Section 4.4.

3.1 Data Measurements

To quantify the theoretical concepts for the empirical modelling we consider an annual UK data set covering $1873 - 2001$. Data for the period $1873 - 1991$ are taken from Hendry (2001), and the variables are mechanically updated to 2001 from the UK Statistical Office.

The nominal money stock is a broad measure, defined as M2, M3 and M4 in different sub-periods and spliced. As the scale variable we use real GDP in constant 1985 prices. As the price level we use the deflator of GDP indexed with $1985 = 1$. As the opportunity cost of holding money we follow Friedman and Swartz (1982), Ericsson, Hendry, and Prestwich (1998) and Hendry (2001) and use a transformed short rate, $(H_t/M_t)\, R_t^s$, where R_t^s is the short interest rate and H_t/M_t is the fraction of high powered money to the broad money stock. The interpretation is that the outside rate is $R_t^{alt} = R_t^s$, while the inside rate is zero for the high powered money and R_t^s for the interest bearing part of M_t. The opportunity cost is therefore $R_t = R_t^{alt} - R_t^{own} = R_t^s - (1 - H_t/M_t)\, R_t^s$.

The time series are illustrated in Figure 1. Graph (A) depicts the log of population as well as the population growth. Over the long time period population growth has varied, leading to approximately three segmented linear trend in the population. In the variables in the empirical analysis we eliminate the trend induced by population growth by considering real money and income *per capita*, depicted in graph (B). For long sub-periods the two time series have moved relatively parallel. After the first world war (WWI) both real money and income fall considerably, but whereas real money seems to return to the pre-war trend, real income seems to continue at a permanently lower level. In the empirical analysis we want to allow level shift in the variables following the two world wars, and we test whether the shifts are significant in individual variables.

The rate of inflation and the change in nominal money are depicted in graph (C). Before WWI average inflation is close to zero, but WWI and the aftermath induce significant bouts in the inflation rates. After the second world war (WWII) the average inflation rate has been positive, with a hump following the oil crises in the 1970s. Overall there is a positive correlation between changes in money and prices. In some periods, however, the

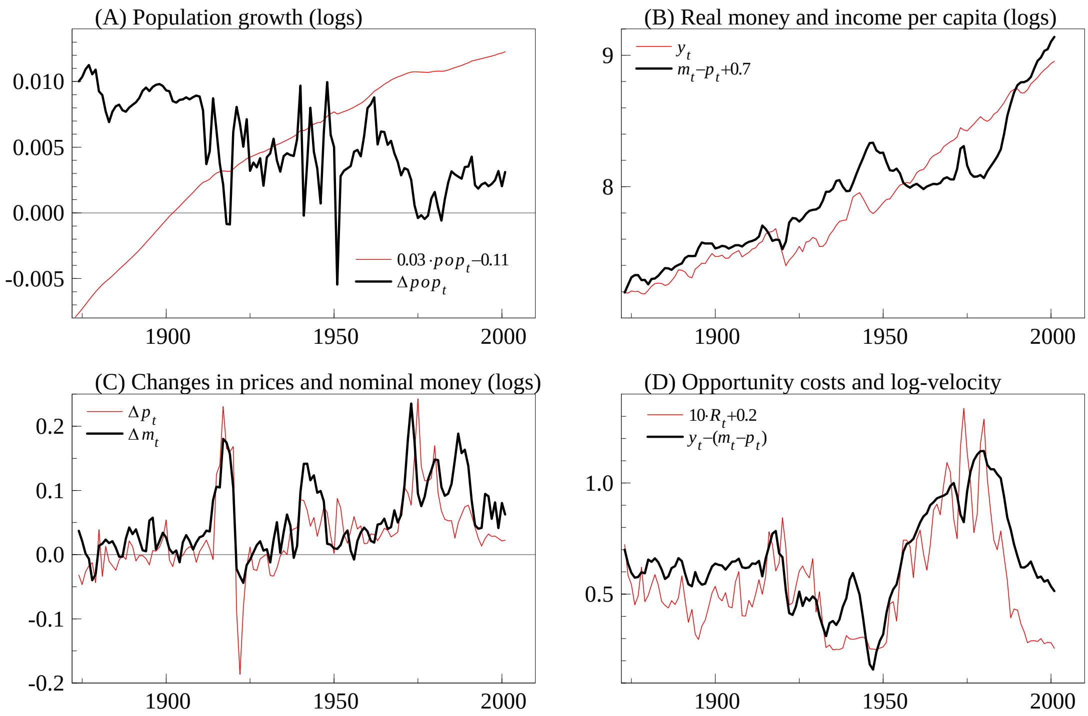

Figure 1: *Data and certain linear combinations.*

relation is weaker; most notably in the aftermath to WWI and during WWII. Finally graph (D) depicts the opportunity cost of holding money and the log of velocity, $y_t - (m_t - p_t)$. Overall the correlation is clearly positive as suggested by the relation in (6).

4 Empirical Analysis

To model the interaction between real money, real income, the interest rate, and inflation we consider in the empirical analysis the data vector $X_t = (m_t - p_t : y_t : R_t : \Delta p_t)'$ and first step is to set up an unrestricted VAR model describing the data.

As a starting point we include additive dummies for WWI and its aftermath, $1914-1923$, and for WWII, $1940-1945$. Besides these radical events we also include an additive dummy for a gross measurement error in 1880; an additive dummy for the year 1931, possibly related to the international depression or the abandoning of the gold standard; and, finally, an unrestricted dummy for 1973 to take account of the effect of the first oil crisis. The precise specification of these dummies are based on an initial test procedure along the lines of Nielsen (2004).

In a first attempt to model the data, additive level shifts were included in all variables for 1921 and 1945, to allow the two world wars to have permanent effects on all the variables. To avoid the danger of over-fitting the data we only want to maintain clearly significant level shift in the analysis, however, and in an initial analysis the shift in 1945 was insignificant in all variables[1]. Moreover, the shift in 1921 was only clearly significant in the income variable; and the joint test statistic for the 7 restrictions of 11.23 is not rejectable according to a $\chi^2(7)$ distributions. In the empirical results presented below we therefore include only a level shift in 1921 and restrict this to have effects only on real income, y_t.[2] Using this specification we estimate in total 73 parameters to additive dummies, D_t, and 4 parameters to unrestricted dummies, d_t.

To model the stochastic variation in the data we consider a third order vector autoregression. Table 1 reports the results of a number of misspecification tests applied to the unrestricted system. The null hypotheses of no autocorrelation is in general accepted, although there is some indication of autocorrelation in the residuals for opportunity costs. Since the measure for opportunity costs turns out to be a weakly exogenous variable in the system we do not consider this to be a too serious problem. Also the null of no ARCH effects is accepted, and the residuals look Gaussian in the equations for real money and opportunity costs. In the equations for income and inflation normality is rejected. This rejection is due to excess kurtosis induced by a few remaining moderate outliers as well as an excess number of small residuals implied by the large number of dummies. Rejected normality may imply that the estimation is not fully efficient, but since the properties of the misspecification tests in the presence of dummy variables are yet unknown, we have chosen to continue the analysis with the present model.

Alternatively, we have tried to include innovational and additive dummy variables for the remaining moderate outliers in the model, and have also tried to extend the lag length to remedy the mild signs of autocorrelation in the residuals from ΔR_t. In both cases we obtain by and large identical results to those presented below.

4.1 Long-Run Analysis

To determine the cointegration rank, r, we estimate the models in the nested sequence

$$H(0) \subset \cdots \subset H(r) \subset \cdots \subset H(p),$$

and calculate LR tests for the hypotheses $H(r) \mid H(4)$ and $H(r) \mid H(r+1)$, parallel to the well-known *trace tests* and the *maximum eigenvalue tests*, respectively. The asymptotic distributions of the rank tests depend in general on the deterministic specification and are functionals of Brownian motions, see Johansen (1996, chapter 11). Note, however, that the dummies for additive and innovational outliers refer to single observations, and they do not affect the asymptotic distribution. The level shift for real income, y_t, in

[1] One reason for the insignificance of the level shift in 1945 could be that ones the influence of the observations $1921-1923$, 1931, and $1940-1945$ are removed from the analysis, the inter-war period is too short to precisely estimate a separate level. In this case the level shift estimated for 1921 may include also a component from the effect of WWII.

[2] An additional reason for the significance of a level shift in 1921 could be that the original variables are not appropriately corrected for the independence of Southern Ireland in 1919.

	AR(1-2)	ARCH(1-2)	Normality
Δm_t	0.96 [0.39]	0.33 [0.72]	0.21 [0.90]
Δy_t	2.18 [0.12]	0.63 [0.53]	17.83 [0.00]
ΔR_t	4.47 [0.01]	0.60 [0.55]	5.65 [0.06]
$\Delta^2 p_t$	1.49 [0.23]	1.47 [0.24]	10.02 [0.01]
Multivariate tests:	1.42 [0.07]	...	36.50 [0.00]

Table 1: *Tests for misspecification of the unrestricted VAR(3). Figures in square brackets are $p-$values. AR(1-2) are the F-tests for autocorrelation up to second order and are distributed as F(2,109) and F(32,370) for the single equation and multivariate tests respectively. ARCH (1-2) tests for ARCH effects up to second order and is distributed as F(2,107). The last column reports results of the Doornik and Hansen (1994) test for normality, distributed as $\chi^2(2)$ and $\chi^2(8)$ respectively.*

	$H(0)$	$H(1)$	$H(2)$	$H(3)$	$H(4)$
Log-likelihood	2096.99	2121.67	2128.77	2134.81	2137.61
LR: $H(r) \mid H(4)$	81.24 [0.00]	31.86 [0.40]	17.67 [0.37]	5.60 [0.52]	...
LR: $H(r) \mid H(r+1)$	49.37 [0.00]	14.20 [0.71]	12.07 [0.42]	5.60 [0.52]	...

Table 2: *Rank determination. The asymptotic p-values in square brackets are based on the approximate critical values derived from $\Gamma-$distributions by Doornik (1998).*

1921 is estimated by the GLS procedure in the iterative algorithm, and do not affect the asymptotic distributions. This implies that we can use the conventional distributions for the case of linear term restricted to the cointegration space, published *inter alia* in Johansen (1996), Doornik (1998), and Mackinnon, Haug, and Michelis (1999).

The results are reported in Table 2. Both tests point towards a cointegration rank of $r = 1$. The null of no cointegration is clearly rejected while the model $H(1)$ has $p-$values of 0.40 and 0.71 respectively. This choice is also consistent with the eigenvalues of the companion matrices. For different values of the cointegration rank, the moduli of the roots are given in Table 3. We see that the choice of $r = 2$ induce a large unrestricted root of 0.90 in the model. This reflects that the error correction to a potential second long-run relation is slow.

Taking the model $H(1)$ as the preferred and normalizing the long-run relation on real money give the results reported under $\mathcal{H}_0$ in Table 4. In the long-run relation there is a large and significant coefficient to real income, although the unrestricted point estimate is below unity. There is also a large and significant coefficient to the interest rate, with a semi-elasticity in a money relation of -7.7. Inflation and the linear term, on the other hand, do not look too important in the long-run structure. In the adjustment matrix, α, real money is clearly endogenous emphasizing the interpretation of the cointegrating relation as an equilibrium for money demand. There is also a significant adjustment in

Model	Moduli of the eigenvalues of the companion matrix											
$H(4)$	0.996	0.918	0.766	0.717	0.717	0.646	0.646	0.599	0.599	0.344	0.344	0.283
$H(3)$	1	0.997	0.729	0.711	0.711	0.645	0.645	0.603	0.603	0.351	0.351	0.273
$H(2)$	1	1	0.899	0.673	0.673	0.649	0.649	0.607	0.607	0.317	0.317	0.270
$H(1)$	1	1	1	0.648	0.648	0.613	0.613	0.566	0.566	0.543	0.265	0.181

Table 3: *Moduli of the eigenvalues of the companion matrix corresponding to the models $H(4)$, $H(3)$, $H(2)$, and $H(1)$.*

inflation, so that excess money exert an upward pressure on inflation. Real income and the interest rate look more exogenous to the long-run relation

First, it is natural to test whether the linear term can be excluded from the cointegration space, and the results under this restriction are reported under $\mathcal{H}_1$. The restriction increases the coefficient to real income from 0.78 to 0.86, but with an asymptotic standard error of 0.023 it is still significantly smaller than unity. It is worth noting that the standard error to the coefficient decreases markedly, reflecting a collinearity and an implied trade-off in the relation between income and the linear term. The restriction is statistically accepted with a $p-$value of 0.70 obtained from the asymptotic $\chi^2(1)$ distribution.

An alternative specification is to impose a unit coefficient to income, leaving the linear term unrestricted, cf. the results under $\mathcal{H}_2$. This exploits the collinearity between income and the linear trend term and this restriction is also accepted against the data with a $p-$value of 0.25. The model under $\mathcal{H}_2$ has the property that velocity appear directly, at the cost of having a linear trend present. It should be noted, that the estimated semi-elasticities to opportunity costs under $\mathcal{H}_1$ and $\mathcal{H}_2$ are by and large identical. In the following we prefer to impose the unit coefficient, emphasizing the role of velocity. We interpret the linear trend as a proxy for changes in the measurements over the sample span and the effects of developments in financial technology.

Under $\mathcal{H}_2$ the impact of the inflation term is rather weak, both numerically and statistically. Restricting the coefficient to zero produces the simple structure reported under $\mathcal{H}_3$; with the marginal restriction accepted with a $p-$value of 0.27 according to the LR test. In this model, the feedback to income and the interest rate is weak, and imposing the additional restrictions on α gives the preferred specification reported under $\mathcal{H}_4$. In this model there is a long-run relation between velocity and the interest rate, with a semi-elasticity of -8.20. This relation explains developments in real money balances and in inflation. The structure under $\mathcal{H}_4$ is accepted with a test statistic of 6.15, which is not significant according to the asymptotic $\chi^2(4)$ distribution.

4.2 Deterministic Specification and Additive Corrections

In the model $\mathcal{H}_4$ there are deterministic linear trends in the data, and due to the imposed homogeneity restriction this linear trend is also significant in the long-run relation. From

	$\mathcal{H}_0$		$\mathcal{H}_1$		$\mathcal{H}_2$		$\mathcal{H}_3$		$\mathcal{H}_4$	
	α	β^*	α	β^*	α	β^*	α	β^*	α	β^*
$m_t - p_t$	-0.196 (6.27)	1	-0.198 (6.33)	1	-0.192 (6.41)	1	-0.197 (6.10)	1	-0.226 (7.70)	1
y_t	-0.045 (1.71)	-0.777 (-7.44)	-0.040 (1.48)	-0.857 (36.94)	-0.031 (1.23)	-1	-0.023 (0.82)	-1	0	-1
R_t	-0.018 (1.80)	7.722 (17.18)	-0.018 (1.80)	7.814 (18.08)	-0.017 (1.69)	7.946 (17.36)	-0.021 (1.98)	8.464 (22.44)	0	8.204 (21.54)
Δp_t	0.064 (2.62)	0.757 (2.13)	0.066 (2.71)	0.689 (1.95)	0.067 (2.84)	0.733 (1.98)	0.083 (3.34)	0	0.099 (4.16)	0
t		$-.147$ (.91)		0		0.232 (6.05)		0.278 (11.64)		0.290 (11.18)
LR statistic	...		0.149 [0.70]		1.347 [0.25]		2.540 [0.28]		6.152 [0.19]	
	...		$\chi^2(1)$		$\chi^2(1)$		$\chi^2(2)$		$\chi^2(4)$	

Table 4: *Testing hypotheses on the long-run structure.* $\beta^* = (\beta' : \beta_0')'$ *denotes the extended cointegration vector, and* $t-$*ratios based on asymptotic standard errors are in parentheses. The linear trend term is scaled to have increments of 0.01.*

an empirical point of view the linear trend can easily be given and interpretation in real money and income, while it is harder to interpret a deterministic linear trend in inflation and the opportunity cost over the very long run.

To analyze if the trend term is important in the individual variables, we reparameterize the model under $\mathcal{H}_4$ as

$$\Delta Z_t = \alpha\left(\beta' : \rho_1'\right)\begin{pmatrix} Z_{t-1} \\ 1 \end{pmatrix} + \sum_{i=1}^{k-1} \Gamma_i \Delta Z_{t-i} + \phi d_t + \epsilon_t \quad (7)$$

$$X = Z + \theta^* D_t^*, \quad (8)$$

where the vector of additive variables in (8), $D_t^* = (D_t' : t)'$, is extended to include the trend term. The restricted constant term in (7) will allow for non-zero means in the stationary relation, the initial values will give non-zero levels in the non-stationary relations, and the linear trends included in D_t^* will allow for linear trends in all four variables. With this parametrization we can test whether the linear trends are significant in individual variables, and impose restrictions to avoid linear trends in some of the variables. The estimated coefficients to the linear trends are given by 1.623 (3.19) for $m_t - p_t$, 1.588 (7.66) for y_t, -0.039 (-0.62) for R_t, and 0.040 (0.33) for Δp_t, respectively, where the numbers in parentheses are $t-$ratios. Based on theoretical arguments and the estimated parameters it is natural to impose the restriction that only real money and real income contain deterministic linear trends.

As discussed above the estimated corrections implied by the additive dummies amount to replacing the observed variables with interpolated values, where the interpolation is based on the estimated model. If a correction is not significant in a particular variable, one interpretation is that the observation is not significantly changed by the radical event, and information in that variable can be used to interpolate the remaining variables for that particular observation.

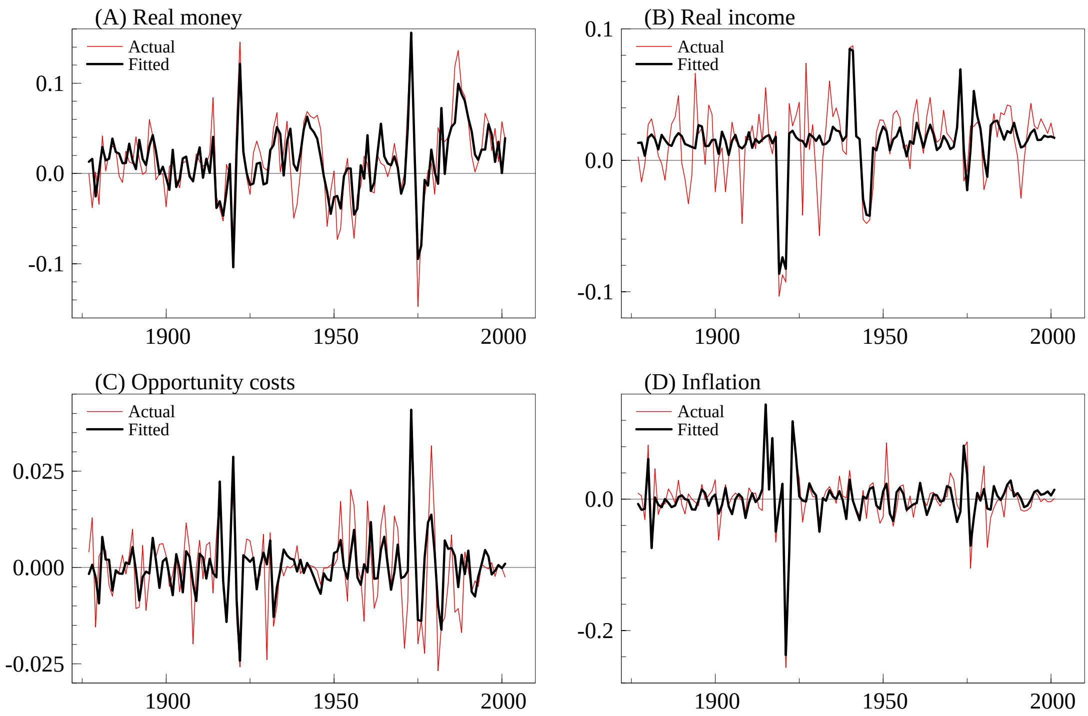

Figure 2: *Actual and fitted values for the preferred model, $\mathcal{H}_5$.*

Imposing the restrictions that the trend term only affects real money and real income, and restricting insignificant parameters to the additive components, θ, to zero, yields a final preferred model, $\mathcal{H}_5$. Apart from the deterministic terms the long-run part of the model is given by

$$\begin{pmatrix} \Delta(m_t - p_t) \\ \Delta y_t \\ \Delta R_t \\ \Delta^2 p_t \end{pmatrix} = \begin{pmatrix} \underset{(6.82)}{-0.205} \\ 0 \\ 0 \\ \underset{(3.01)}{0.072} \end{pmatrix} \left(m_{t-1} - p_{t-1} - y_{t-1} + \underset{(18.88)}{8.125} \cdot R_{t-1} \right) + \ldots$$

Based on these corrections the identifying structure is acceptable with a $p-$value of 0.19, and for this model the actual and fitted values are reported in Figure 2.

The significant parameters to the additive terms are reported in Table 5, and the observed and corrected data are graphed in Figure 3. It is apparent that the number of corrections is limited, but that the magnitudes are sometimes very large. First, the additive outlier in 1880 affects real money and inflation, with opposite signs. In 1914, in beginning of WWI, money increased a lot relative to prices. One reason for this could be that the government may have resorted to seigniorage in order to finance the start of the war, and agents did not respond by raising prices because they realized that the revenues were intended for the war effort. For WWI and its aftermath, large corrections (up to 20%)

are needed in the inflation rate. The persistence of the inflation effect, requiring positive corrections from 1915 to 1920 may reflect the fact that sterling left the Gold Standard at the end of WWI, which may have signalled to agents that more accommodating monetary policy would be adopted after the war, so some price increases that were delayed during the war, e.g. for food, were implemented rather than being delayed further (as they may have been if there had been a commitment to tough monetary policy). In real income we allow for a level shift in 1921 with a magnitude of -29%, and the dummies for 1919 and 1920 describe a gradual convergence to the new level. In real money corrections of around 8% are needed in 1920 and 1921; whereas the corrections to opportunity costs are smaller and less significant. The corrections for WWII describes a boost in real income over the period, and a smaller hike in inflation in $1940-1942$.

The long-run relation, calculated in terms of the observed data, $\beta' X_t$, is depicted in graph (A) of Figure 4 together with the expected value given by the total effect of the deterministic terms, i.e. the initial values, the intercepts, the linear term, and the dummies in d_t and D_t. There is a clear downward trend in velocity even after correcting for the interest rate. The deterministic trend is broken by a large level shift in 1921 and minor correction during WWI, WWII, 1880 and 1931. Finally there is a large effect of the oil price shock in 1973. This is a large economic shock to the system, with permanent effects on the variables in the system, but only transitory effects on the long-run relation; meaning that the system adjusts to the same equilibrium level as before the shock.

In this final specification, $\mathcal{H}_5$, a level shift for the permanent effects of WWII is still insignificant in all variables, with estimated coefficients and $t-$ratios given by -0.007 (0.44) for $m_t - p_t$, -0.024 (0.53) for y_t, -0.007 (1.23) for r_t, and -0.023 (1.46) for Δp_t. One possible interpretation os this fact could be that Britain took many long-term loans from the US during and after the war, so that macroeconomic imbalances created by the war spilled over into the current account rather than prices and output.

4.3 Stability of the Long-Run Relation

To analyze the stability of the long-run relation, we correct the data based on the estimates in the final specification, $\mathcal{H}_5$, i.e. considering $Z_t = X_t - \widehat{\theta} D_t$, and perform a recursive estimation on the corrected data. Graph (C) in Figure 4 reports the recursively estimated semi-elasticity in the long-run relation. The recursively estimated parameter is without drift but is affected by the shocks to the system over the long time period. One possible explanation for the changes in the responsiveness of money to the interest rate could be the changes in the exchange rate regime. During the inter-war years the responsiveness decreases, which could reflect a correction in money balances after sterling came off gold. Conversely, the increased responsiveness observed from the 1940s onwards could be the result of the inception of Bretton Woods.

Graph (D) illustrates the backward recursively estimated parameter. Together (C) and (D) illustrates that although the system may be stable over time, the amount of

	$m_t - p_t$	y_t	R_t	Δp_t
1880	-0.033 (-3.46)	0	0	0.078 (6.14)
1914	0.041 (4.12)	0	0	0
1915	0	0	0	0.123 (7.56)
1916	0	0	0.016 (2.71)	0.111 (6.46)
1917	0	0	0.013 (2.05)	0.206 (11.09)
1918	0	0	0	0.177 (8.99)
1919	0	-0.109 (-6.08)	0	0.159 (7.93)
1920	-0.083 (-4.63)	-0.192 (-8.10)	0.026 (4.12)	0.174 (8.10)
1921	-0.081 (-4.50)	0	0.022 (3.57)	-0.053 (-2.71)
1922	0	0	0	-0.143 (-7.33)
1923	0	0	0	-0.050 (-3.75)
1931	0	0	0.014 (3.21)	-0.030 (-2.93)
1940	0	0.072 (4.14)	0	0.054 (4.40)
1941	0	0.140 (6.05)	0	0.068 (4.68)
1942	0	0.141 (5.58)	0	0.044 (3.50)
1943	0	0.145 (5.75)	0	0
1944	0	0.105 (4.70)	0	0
1945	0	0.056 (3.32)	0	0
t	0.012 (7.02)	0.016 (8.66)	0	0
Shift 1921	0	-0.289 (-10.84)	0	0

Table 5: *Additive corrections in the preferred model, $\mathcal{H}_5$. $t-$ratios in parentheses.*

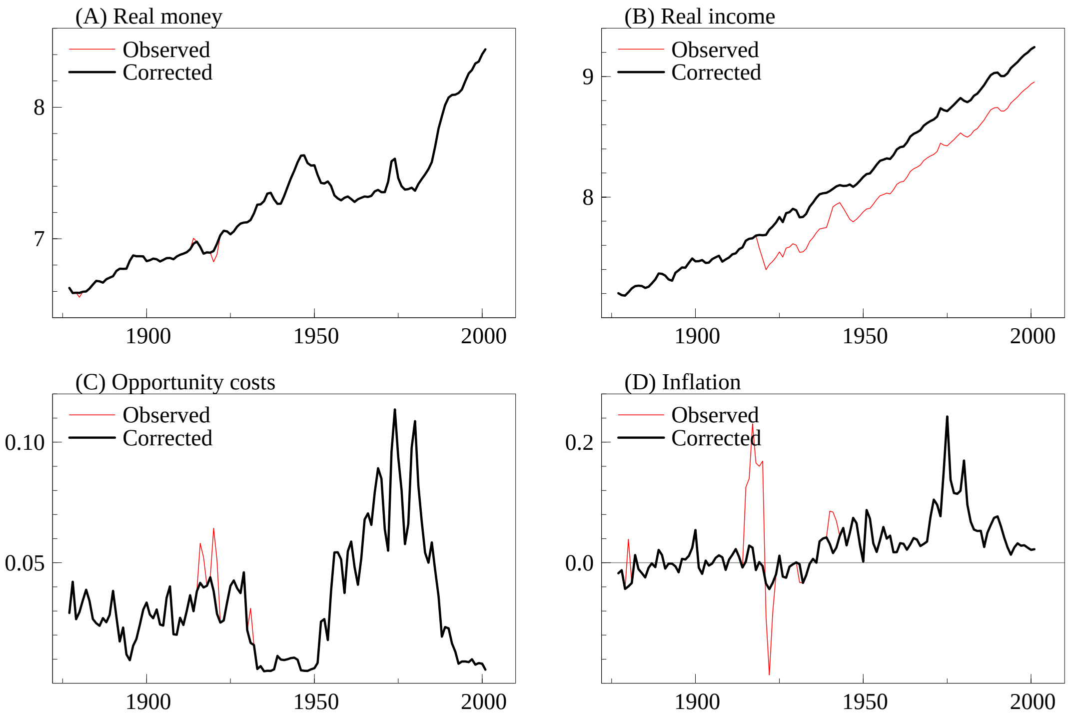

Figure 3: *Observed data and the data corrected for the additive components in D_t (not including the linear trend). The corrections are calculated from the preferred model, $\mathcal{H}_5$.*

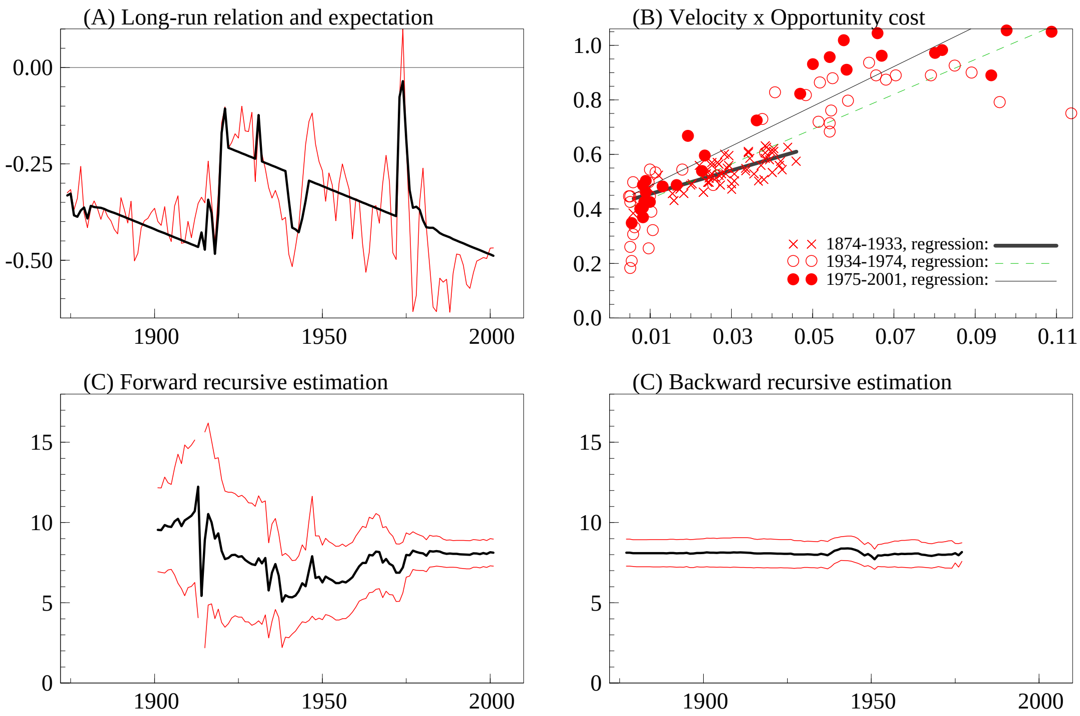

Figure 4: *(A) is the long-run relation and the expectation, both calculated from the preferred model, $\mathcal{H}_5$. (B) is a cross-plot of velocity and opportunity costs for three sub-samples and associated linear regression lines. (C) and (D) are results from a recursive estimation. The forward recursive estimation is performed for effective samples* $(1877 : ... : T_0)$ *where* T_0 *varies from* 1901 *onwards. The backward recursive estimation is performed on effective samples* $(T_0 : ... : 2001)$ *where* T_0 *varies from* 1977 *backwards. The estimation is performed on the corrected data set,* $Z_t = X_t - \widehat{\theta}^* D_t^*$.

information contained in the data need not be constant. The cross-plot of velocity and opportunity cost in graph (B) further illustrates this point. The cross-plot for the two sub-period $1934 - 1974$ and $1975 - 2001$ spans the variation of the entire data set, and the slopes of simple linear regressions are almost identical for these two sub-periods. The recursive results reflects, that ones the coefficient to opportunity cost is pinned down by the large variation in the interest rate and velocity, the additional information obtained by extending the sample is very limited. The time period for $1874 - 1933$, on the other hand, is characterized by low interest rates and almost constant velocity. This limited variation implies that the parameter estimate is relatively uncertain, and it will be very responsive when more informative observations are added. This illustrates that the properties of the estimated long-run semi-elasticity, and more generally the cointegrated VAR model, depend on the information in the common trends. In periods where the variability of the random walks is limited, the information in the data on the cointegrating relations is also limited, and the recursive estimates may look unstable.

	$\mathcal{H}_0^*$				$\mathcal{H}_1^*$			
	α		β^*		α		β^*	
$m_t - p_t$	-0.197 (-6.38)	-0.101 (-1.47)	1	0	-0.224 (-7.67)	0	1	0
y_t	-0.044 (-1.66)	-0.058 (-0.98)	-0.812 (6.66)	0.082 (0.89)	0	0	-1	0
R_t	-0.018 (-1.74)	-0.028 (-1.26)	8.422 (17.43)	-0.887 (-2.42)	0	0	8.220 (21.37)	-0.980 (-2.68)
Δp_t	0.077 (3.35)	-0.123 (-2.39)	0	1	0.093 (4.00)	-0.146 (-3.81)	0	1
t			-0.114 (-0.61)	-0.135 (-0.96)			0.280 (10.71)	0
LR statistic	...				7.889 [0.44]			
	...				$\chi^2(8)$			

Table 6: *Testing hypotheses on the long-run structure with $r = 2$. $\beta^* = (\beta' : \beta_0')'$ denotes the extended cointegration vector, and $t-$ratios based on asymptotic standard errors are in parentheses. The linear trend term is scaled to have increments of 0.01.*

4.4 A Second Long–Run Relation and the Role of Inflation

The rank determination in Table 2 indicates that adjustment to a potential second long-run relationship, $\beta_2' X_t$, was relatively slow, and there was not much support for the stationarity of the second relation. Even in this case, it may still be informative to have a look at the second relation, to see what kind of tentative structures that may suggest for the economy; in particular it may shed some light on the role of inflation in the system.

Imposing $r = 2$ and identifying the first relation with a zero coefficient on inflation and a normalization on real money, and with the second relation normalized on inflation and identified by a zero restriction on real money, yield the structure presented under $\mathcal{H}_0^*$ in Table 6. As expected, the first long-run relation mirrors the results found for the case $r = 1$. The second relation has a significant coefficient to the interest rate while real income is insignificant. The adjustment coefficients in α indicate that only inflation adjust to deviations from this relation; and the adjustment is very slow as expected from the rejected stationarity of this relation.

Imposing homogeneity on the money demand relation, to reproduce the results from $r = 1$, imposing a zero coefficient to income in the second relation, and restricting insignificant coefficient in α to zero produces the structure reported under $\mathcal{H}_1^*$. In this model the first relation is identical to the long-run relation under $r = 1$, while the second relation is close to being $R_t - \Delta p_t$. Since the opportunity cost is itself an interest rate spread, $R_t = R_t^{alt} - R_t^{own}$, and therefore in principle a real magnitude, the interpretation is not straightforward. In practice, however, opportunity cost is measured as a fraction of the short rate, $R_t = (H_t/M_t)\, R_t^s$, such that the second long-run relation just reflects stationarity of the real interest rate. According to the adjustment coefficients, inflation error corrects to the real interest rate, while the remaining variables are exogenous for this relation.

The fact that R_t and Δp_t are close to being cointegrated may explain why inflation was not relevant in the long-run structure of the preferred model. Being almost cointegrated, the two variables contain more or less the same stochastic trends and it is difficult to identify separate coefficients in the cointegrating relation.

5 Concluding Remarks

In this paper we considered UK money demand, based on real money, real income, opportunity costs, and inflation, for the period $1873-2001$. Using a cointegrated VAR approach and accounting for the effects of extreme episodes related to the world wars and the oil price shock we find clear evidence of a single long-run relation, which links velocity to opportunity costs, with a semi-elasticity of minus eight. According to the adjustment coefficients, excess money will have a clearly significant impact in inflation. Inflation is not present in the long-run relation of the preferred specification, but inflation could appear in a second tentative relation interpretable as a stationary real interest rate. The price adjustment is to slow, however, to establish the second relation as a genuine cointegrating relation.

The recursive results suggests that the long-run structures underlying money demand may be stable, but the information in the data on the parameters is not evenly distributed. In particular, it is important to have information from the episodes of large variations in velocity and interest rates around $1960-1980$ in order to identify the structures of money demand and to precisely pin down the estimated semi-elasticity. Constructively, that suggests that to efficiently model UK money demand it is sufficient to have observations covering the last few decades, while the gain in terms of information on the underlying parameters from extending the period backwards, taking into account observations before WWII, is limited.

A Estimation of the Additive Model

To obtain full information ML estimates of the parameters in (1) and (2) we use the switching algorithm in Nielsen (2004). The varying part of the log-likelihood function for the model is given by

$$\log L\left(\alpha,\beta,\Gamma_1,...,\Gamma_{k-1},\beta_0,\mu_0,\phi,\Omega,\theta\right)=-\frac{T}{2}\log|\Omega|-\frac{1}{2}\sum_{t=1}^{T}\left(\epsilon_t'\Omega^{-1}\epsilon_t\right),$$

where

$$\epsilon_t=A\left(L\right)\left(X_t-\theta D_t\right)-\alpha\beta_0't-\mu_0-\phi d_t$$

are the residuals formulated in terms of the observed variables, X_t, and

$$A\left(L\right)=\left(1-L\right)I-\alpha\beta'L-\sum_{i=1}^{k-1}\Gamma_i\left(1-L\right)L^i,$$

denotes the characteristic polynomial to the model in (1).

The likelihood function can be maximized by iterating between two conditional ML estimations. In the first step of iteration j, we condition on the estimate $\widehat{\theta}_{j-1}$ of θ from the previous iteration. Then the conditional ML estimates of the parameters in (1) can be found from a standard cointegrating analysis for the corrected data $Z_t=X_t-\widehat{\theta}_{j-1}D_t$. In the second step we can find the ML estimate $\widehat{\theta}_j$ of θ conditional on the remaining parameters from the estimated residuals for the uncorrected data, $\widehat{e}_t=\widehat{A}\left(L\right)X_t-\widehat{\alpha}\widehat{\beta}_0't-\widehat{\mu}_0-\widehat{\phi}d_t$, which under the model are given by

$$\widehat{e}_t=\widehat{A}\left(L\right)\theta D_t+\epsilon_t=\widehat{H}_t\mathrm{vec}\left(\theta\right)+\epsilon_t,$$

where vec(θ) stacks the columns of θ, and $\widehat{H}_t=D_t'\otimes\widehat{A}\left(L\right)=(\widehat{A}\left(L\right)D_{1t}:\widehat{A}\left(L\right)D_{2t}:...:\widehat{A}\left(L\right)D_{nt})$. The conditional likelihood function is maximized over θ by the GLS type estimator

$$\mathrm{vec}\left(\widehat{\theta}_j\right)=\left(\sum_{i=1}^{T}\left(\widehat{H}_t'\widehat{\Omega}^{-1}\widehat{H}_t\right)\right)^{-1}\left(\sum_{i=1}^{T}\left(\widehat{H}_t'\widehat{\Omega}^{-1}\widehat{e}_t\right)\right),\tag{9}$$

see also Tsay, Peña, and Pankratz (2000) and Saikkonen and Lütkepohl (2000b). In all the cases considered in the paper a starting value of $\theta=0$ can be used, and full information ML estimates are obtained by iterating between the two steps until convergence.

The covariance matrix of vec($\widehat{\theta}$) can be estimated by $\left(\sum_{t=1}^{T}\widehat{H}_t'\widehat{\Omega}^{-1}\widehat{H}_t\right)^{-1}$ and Wald-type tests for hypotheses on θ can easily be constructed, see also Tsay, Peña, and Pankratz (2000). It is also straightforward to impose restrictions on θ in the GLS step (9), e.g. of the form

$$\mathrm{vec}\left(\theta\right)=M\kappa,$$

where M is a $pn\times f$ dimensional design matrix, and κ contains the f free parameters.

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