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## Survival of the Fittest on Wall Street

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# Survival of the Fittest on Wall Street[*]

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## Abstract

This paper studies an application of a Darwinian theory of portfolio selection to stocks listed in the Dow Jones Industrial Average (DJIA). We analyze numerically the long-run outcome of the competition of fix-mix portfolio rules in a stock market with actual DJIA dividends. In the model seemingly rational strategies can do very poorly against seemingly irrational strategies. Moreover, the interaction of strategies can lead to stochastic time series of asset prices that do not converge. The simulations also show that the evolutionary portfolio rule discovered in Hens and Schenk-Hoppé (2004) will eventually hold total market wealth in competition with fix-mix portfolio rules derived from mean-variance optimization, maximum growth theory and behavioral finance. According to this evolutionary rule, portfolio weights should be proportional to the expected relative dividends of the assets. As an implication asset prices converge to expected relative dividends.

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# 1 Introduction

Recent empirical and experimental work has done a lot to undermine the long sustained belief in market rationality (see e.g. the surveys by Campbell (2000), Hirshleifer (2001) and De Bondt (1999)). These important findings have initiated a new behavioral paradigm for finance that—according to many researchers in the field—might replace or at least complement traditional finance. It is currently believed that thinking in terms of *excess volatility*, *irrational exuberance*, *market risk* and *loss aversion* will soon substitute the cornerstones of traditional finance, *mean-variance analysis*, *arbitrage pricing* and the *efficient market hypothesis*. While the fight of the rational and the behavioral finance paradigm is in its decisive stage, we argue that a third paradigm, evolutionary finance, should not be omitted. Thinking in terms of *strategies*, *market selection* and *mutation* seems to be very appropriate for finance. In this view, for example, a stock market is understood as a heterogeneous population of frequently interacting portfolio strategies in competition for market capital. Market selection is perhaps most severe in these markets and innovations, respectively mutations, occur frequently.

The aim of our paper is to contribute to a Darwinian theory of portfolio selection. This theory views asset markets as being stratified according to the portfolio rules that investors use to manage wealth. The building blocks of the model are therefore strategies but not the individual investor, i.e. for each strategy all wealth being managed by that strategy is added up. This is analogous to Darwin's view according to which the species but not the individual animal counts for evolution. The strategies considered in this paper are the mean-variance rule, the growth-optimal rule, the CAPM rule, naïve diversification, prospect theory based rules and a relative-dividends rule. In our model the impact of any such rule on market prices is proportional to the amount of wealth managed by the rule.

In a Darwinian model two forces are at work: one reducing the variety of species and one increasing it. In our model the first such force is the endogenous return process acting as a market selection mechanism that determines the evolution of wealth managed by the portfolio rules. That is to say, if some rule has gained wealth because it has managed to buy low and to sell high then other rules must have lost an equal amount of wealth. Secondly, any system of portfolio rules that is selected by the market selection process is checked for its evolutionary stability, i.e. it is checked whether the innovation of a new portfolio rule with very little initial wealth can grow against the incumbent rule.

The Darwinian theory of asset markets seems to describe very well a modern asset market in which most of the available capital is invested by dele-

gated management. Indeed investors typically choose funds by the portfolio rules, also called "styles," according to which the money is invested. Style consistency appears nowadays to be one of the most important features in monitoring fund managers.

A long time ago Friedman (1953) and Fama (1965) have already recognized the power of evolutionary ideas in finance. Using these ideas they argued that the market naturally selects for the rational strategies. As an effect market selection would lead to market efficiency. This specific outcome of the market selection process could not be sustained in general. For example De Long, Shleifer, Summers, and Waldmann (1990) show that under specific circumstances noise traders can earn a higher average rate of return than rational arbitrageurs. While this example has been very influential in the debate for behavioral finance it has some shortcomings that should be removed. In particular it is based on the model of overlapping generations which implies that market selection has no bite. Indeed in De Long, Shleifer, Summers, and Waldmann (1990)'s example every strategy is renewed with fresh capital in every period. In our model every strategy will have to continue investing with the wealth it has generated in the previous period so that the market selection process has more bite. Still we are able to show that seemingly rational strategies, like mean-variance optimization, can do very poorly against seemingly irrational strategies, like naïve diversification according to which wealth is distributed equally among the investment opportunities.

Some considerable progress in the field of evolutionary finance has been made since Friedman (1953) and Fama (1965). This progress was made possible due to a formalization of evolutionary reasoning based on new decision models like quantifier systems, for example, using computer simulations and advanced mathematical techniques. Many time series properties of asset prices have found an explanation by evolutionary reasoning (Arthur, Holland, LeBaron, Palmer, and Taylor (1997), LeBaron, Arthur, and Palmer (1999), Brock and Hommes (1997), and Lux (1994) among others). These results have also received very good recognition in practitioners' news letters (Mauboussin (1997)). Our paper contributes to this evolutionary asset pricing theories by showing that the market selection process studied here can also generate the phenomenon of stochastic time series of asset prices that do not converge. The interaction of strategies can lead to endogenous volatility in returns without any convergence of asset prices. While the asset price dynamics in the standard evolutionary asset pricing models is complex and hence not easy to interpret, the explanation of the time series properties of asset prices arising in our model is quite simple. In our model it is the endogenous change in the wealth shares that generates and amplifies fluctu-

ations in asset prices. A particular example with two strategies is given in which for both strategies market prices turn to the disadvantage of a strategy as its market share becomes large. Thus the more wealth is managed by a strategy, the higher is the growth potential of the other strategy—eventually leading to a reversed evolution of market shares.

In the area of portfolio theory the seminal work of Blume and Easley (1992) has laid the foundations for a series of papers (Sandroni (2000), Blume and Easley (2001), Sciubba (1999), Hens and Schenk-Hoppé (2004), Evstigneev, Hens, and Schenk-Hoppé (2002), Amir, Evstigneev, Hens, and Schenk-Hoppé (2004), Evstigneev, Hens, and Schenk-Hoppé (2003)) developing a variety of evolutionary portfolio models. This theory provides a framework in which the market selection hypothesis put forward by Friedman and Fama can be studied. It turns out that as long as there are excess returns there still exist strategies that can gain market wealth at the expense of the existing strategies. Moreover, there is one strategy, the evolutionary portfolio rule discovered in Hens and Schenk-Hoppé (2004), that eliminates all excess returns and that cannot be driven out of the market by any other strategy that is adapted to the information revealed by the history of the states.

While the theoretical papers on evolutionary portfolio selection derive asymptotic results in idealized markets, the point of this paper is to apply the evolutionary ideas to stocks from the Dow. The portfolio choice considered in our paper is the decision how to allocate wealth among shares yielding a dividend as observed in the Dow data. We focus attention on dynamic portfolio strategies. That is to say we view a modern capital market, like the stocks in the Dow, as a heterogenous population of dynamic portfolio strategies. These strategies interact repeatedly via the market mechanism and are thereby competing for market capital. Instead of considering all theoretically possible dynamic portfolio rules we take a more pragmatic point of view here and restrict attention to *fix-mix rules*. A fix-mix rule holds certain portfolio weights constant for a long period. Hence if market prices fluctuate a fix-mix rule has to adjust the number of shares it holds so as to keep the proportions of wealth in its portfolio constant. Many institutional investors follow simple fix-mix rules. Some of them because they have committed to manage third parties' money according to a certain asset allocation[1], some because they

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[1]In many prospects of mutual funds some asset allocation, e.g. 60% technology stock and 40% bricks-and-mortar stocks, is proposed as an optimal investment rule so that the investors would feel cheated if these proportions fall out of balance. Also hedge funds for example commit to certain strategies in order to increase credibility and to reduce monitoring costs.

believe that fix-mix is an optimal behavior in volatile markets[2] and some because they use trading strategies derived from some clever reasoning like contrarian behavior that in essence are fix-mix rules[3]. As Evstigneev, Hens, and Schenk-Hoppé (2002) have demonstrated in the case of short-lived assets, the simple evolutionary fix-mix rule considered in this paper is not only able to outperform any other simple portfolio strategy but it will also dominate any general portfolio strategy given it is adapted to the price process. Hence, even though this paper considers the more general case of long-lived assets, it may be argued that the restriction to simple fix-mix rules does not restrict the outcome of the market selection process.

Since from the market selection point of view the market interaction of the various portfolio rules is decisive, we cannot simply do an empirical study of the relative performance of fix-mix rules on a given return path. This would ignore the impact one strategy has on its competitors[4]. Hence we have to rely on simulations in order to show the would-be performance of various portfolio rules that are interacting in a market with Dow dividends. It turns out that the best fix-mix strategy for exogenous returns, the growth optimal portfolio, also called the maximum growth strategy (Hakansson 1970), is no longer the best performing strategy once market interaction is taken into account. Our simulations show that in competition with fix-mix rules derived from mean-variance-optimization, maximum growth theory and from behavioral finance the evolutionary finance rule discovered in Hens and Schenk-Hoppé (2004) will eventually hold total market wealth. According to this simple rule, hereafter denoted by $\lambda^*$, the portfolio weights should be proportional to the expected relative dividends of the assets. Note that the portfolio weights used by the evolutionary strategy are based solely on fundamentals, ignoring any price fluctuations! As it turns out, in the long run as its wealth share grows, prices will stop fluctuating and settle down on the relative expected dividends of the assets because eventually only the single surviving evolutionary rule will determine market prices. This is of course a very strong

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[2]Suppose for example that prices follow a random walk, an often hold assertion, then fix-mix means that on average one buys cheap and sells high. Indeed it can be shown that with idealized returns expected utility maximizers with constant relative risk aversion will choose fix-mix strategies (see for example Campbell and Viceira (2002)).

[3]Following a contrarian behavioral strategy, like that of De Bondt and Thaler (1985) for example, one sells those stocks that have gone up and buys those that went down, which is also the main feature of fix-mix rules.

[4]Note that we are not claiming that individual traders have a huge impact on market prices. From an evolutionary point of view the strategy according to which market capital is managed is crucial, it is not important whether fund X or fund Y or both are using this strategy. Hence it may be that a certain commonly used strategy like the mean-variance-rule has a huge impact while no individual fund has any impact on market prices.

prediction. Our simulations show that even though the final proceeds of the evolutionary process are huge (one gathers total market wealth), one might have to wait very long before this happens. However, even a very modest investor will be pleased with the growth of the market share of the evolutionary rule. Starting from equal grounds, after 8 periods the evolutionary rule has doubled its market share for the first time and it takes another 50 periods to double it once more. And starting from a market share of only 0.1% which is 1% of the others' market shares, the first doubling of the evolutionary rule's market shares happens after only 4 periods, the second doubling after 17 periods and the third doubling after 40 periods and after 50 periods it has reached 10 times its initial market share. Hence in contrast to the well known critique on the maximum growth literature, put forward for example by Rubinstein (1991), the convergence of the process is much faster when prices are endogenous. It should also be noted that every run of the simulation looks pretty much the same. Indeed, the variance over the different runs of the simulations is negligible. Based on 30 runs we found that the variance of the market shares averaged over all periods is only 0.36%.

In passing we would like to mention that for the case of long-lived assets considered here, so far the theoretical literature has not been able to prove what our simulations show: the global convergence of the evolutionary process towards a situation in which all wealth is managed by the evolutionary rule $\lambda^*$.[5] Hence our "application" of the evolutionary portfolio theory also hints at new theoretical results. Recently, Evstigneev, Hens, and Schenk-Hoppé (2003) have shown that in the set of all adapted strategies $\lambda^*$ is the unique evolutionary stable strategy. This is to say if $\lambda^*$ holds all market wealth then every mutant strategy that enters the market with a small fraction of wealth will driven out of the market. Moreover only $\lambda^*$ has this property, i.e. every strategy different from $\lambda^*$ can be driven out by some other strategy. This results may help to explain why rational markets like those in which prices are equal to relative dividends are more stable than irrational markets in which prices depart from their fundamental values. Also this result shows that if the market selection process converges then it has to converge to $\lambda^*$, giving some theoretical foundation for the simulations in which $\lambda^*$ is the single survivor.

In the next section we briefly recall the evolutionary portfolio model of this paper. Section 3 shows how to apply this model to a market with dividends taken from the Dow. Section 4 presents the results and section 5 tries

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[5]For the case of short-lived assets, global convergence to the evolutionary rule has been demonstrated under very general conditions (Amir, Evstigneev, Hens, and Schenk-Hoppé 2004).

to provide some intuitive explanation of the observed phenomena. Section 6 concludes.

# 2  An Evolutionary Stock Market Model

We consider a financial market with $K \geq 1$ long-lived assets $k = 1, ..., K$ in unit supply, each paying an uncertain dividend $D_t^k \geq 0$ at any period in time $t = 0, 1, ....$. Dividends pay off a perishable consumption good, as in the seminal paper by Lucas (1978).

Normalizing the price of the consumption good to one in all periods in time, an investor's wealth in terms of the numeraire is given by

$$w_{t+1}^i = \sum_{k=1}^{K} \left( D_{t+1}^k + p_{t+1}^k \right) \theta_{t,k}^i \tag{1}$$

$(\theta_{t,1}^i, ..., \theta_{t,K}^i)$ denotes investor $i$'s portfolio and $p_t^k$ is asset $k$'s price in period $t$. They are determined by

$$\theta_{t,k}^i = \frac{\lambda_{t,k}^i w_t^i}{p_t^k} \quad \text{and} \quad p_t^k = \sum_{i=1}^{I} \lambda_{t,k}^i w_t^i = \lambda_{t,k} w_t \tag{2}$$

where $\lambda_{t,k}^i$ is investor $i$'s budget share assigned to the purchase of asset $k$. Prices are determined by equating each asset's market value with the investment in that asset (supply is normalized to one).

Assume all investors consume the same fraction of their wealth in all periods in time. Denoting the budget share allocated to consumption by $\lambda_0 > 0$, one has

$$D_t = \sum_{k=1}^{K} D_t^k = \lambda_0 \sum_{i=1}^{I} w_t^i = \lambda_0 W_t \tag{3}$$

Then (1) defines an equation for investors' market shares $r_t^i = w_t^i / W_t$:

$$r_{t+1}^i = \sum_{k=1}^{K} \left( \lambda_0 d_{t+1}^k + \sum_{j=1}^{I} \lambda_{t+1,k}^j r_{t+1}^j \right) \frac{\lambda_{t,k}^i r_t^i}{\sum_{j=1}^{I} \lambda_{t,k}^j r_t^j} \tag{4}$$

where $d_{t+1}^k = D_{t+1}^k / D_{t+1}$ denotes asset $k$'s relative dividend payoff. It is assumed that at least one asset pays a dividend, $D_{t+1} > 0$. The last equation is linear in $r_{t+1} = (r_{t+1}^1, ..., r_{t+1}^I)$. Its solution is given by

$$r_{t+1} = \lambda_0 \left( \mathrm{Id} - \left[ \frac{\lambda_{t,k}^i r_t^i}{\lambda_{t,k} r_t} \right]_i^k \Lambda_{t+1} \right)^{-1} \left[ \sum_{k=1}^{K} d_{t+1}^k \frac{\lambda_{t,k}^i r_t^i}{\lambda_{t,k} r_t} \right]_i \tag{5}$$

where $\Lambda_{t+1}^T = (\lambda_{t+1,1}^T, ..., \lambda_{t+1,K}^T) \in \mathbb{R}^{I \times K}$ denotes the matrix of budget shares in period $t+1$.

Equation (5) governs the evolution of market shares for given trading strategies of investors. It is referred to as the *market selection process.*

Dividend payoffs are determined by the states of nature revealed up to and including time $t+1$. The state of nature $\omega_t \in S$ (where $S$ is a finite set) is governed by a stationary stochastic process. The relative dividend $d_t^k = d_t^k(\omega^t)$, where the observed history of states is denoted by $\omega^t = (\omega_0, ..., \omega_t)$.

A trading strategy is a sequence of budget shares $\lambda_t^i = (\lambda_0, \lambda_{t,1}^i, ..., \lambda_{t,K}^i)$ with $\lambda_0 + \sum_{k=1}^K \lambda_{t,k}^i = 1$. $\lambda_t^i$ can depend on all past observations but neither on current market-clearing prices nor on other investors' current strategies.

The evolution of market shares is well-defined if no bankruptcy occurs and markets always clear. If short sales are allowed, bankruptcy would be prevalent because equilibrium is only temporary in our approach.

The following conditions, which include the absence of short selling, ensure (5) to be well-defined. Suppose that for all $t$, $\lambda_{t,k}^i \geq 0$ (for all $i, k$) and there is an investor with $r_t^j > 0$ such that $\lambda_{t,k}^j > 0$ for all $k$. Then (5) is a well-defined map on the simplex $\Delta^I = \{r \in \mathbb{R}^I \mid r^i \geq 0, \sum_i r^i = 1\}$. For a proof see Evstigneev, Hens, and Schenk-Hoppé (2003, Proposition 1). Eq. (5) generates a (non-autonomous) random dynamical system on $\Delta^I$. For any initial distribution of wealth $w_0 \in \mathbb{R}_+^I$, (5) defines the path of market shares on the event tree with branches $\omega^t$. The initial distribution of market shares is given by $(r_0^i)_i = (w_0^i/W_0)_i$.

The wealth of a strategy $i$ in any period in time can be derived from her market share and the aggregate wealth, defined by (3), as

$$w_{t+1}^i = \frac{D_{t+1}(\omega^{t+1})}{\lambda_0} r_{t+1}^i \tag{6}$$

The further analysis is restricted to the case of simple strategies and i.i.d. dividends. We make the following assumptions.

(B.1) *Simple strategies, i.e.* $\lambda^i \in \Delta^{K+1}$ *for all* $i = 1, ..., I$ *and* $\lambda_0^i = \lambda_0$.

(B.2) *I.i.d. dividend payments* $d_t^k(\omega^t) = d^k(\omega_t)$, *for all* $k = 1, ..., K$ *and the state of nature* $\omega_t$ *follows an i.i.d. process.*

## 3   Evolutionary Investment

In this section we derive and motivate an evolutionary investment rule $\lambda^*$ which was first discovered in Hens and Schenk-Hoppé (2004) in a simpler model. This portfolio rule is the only candidate for a rule that can attract

all market wealth. That is to say, supposing the market selection process (5) converges, then the portfolio rule that conquers the whole market has to be the evolutionary investment rule $\lambda^*$. It is important to point out that in all our simulations convergence of the process was obtained if $\lambda^*$ is among the set of strategies. We also give an interpretation of the evolutionary investment rule $\lambda^*$ in terms of the well-known growth optimal portfolio rule (Hakansson 1970). It turns out that $\lambda^*$ is the growth optimal portfolio rule in a population of rules which generates prices equal to $\lambda^*$.

To this end we analyze the market selection process close to the one-owns-all states, i.e. we investigate the local dynamics close to the vertices of the simplex of market shares. We make the non-redundancy assumption

*(C) Absence of redundant assets, i.e. the matrix of relative dividend payments $(d^k(s))_{s\in S}^{k=1,\ldots,K}$ has full rank.*

Under this assumption one-owns-all states are the only deterministic steady states of the market selection process (Evstigneev, Hens, and Schenk-Hoppé 2003). The local dynamics close to a one-owns-all state is governed by the linearization of the original dynamics. We give a heuristic derivation here and refer the reader to Evstigneev, Hens, and Schenk-Hoppé (2003) for the correct mathematical approach.

Suppose one strategy, say strategy $j$, owns the market wealth. The investment strategy, $\lambda^j$ then determines prices in this case. One has $p_t^k = \lambda_k^j W_t$, and $p_{t+1}^k = \lambda_k^j W_{t+1}$. Under this assumption, (1) and (2) yield

$$
\begin{aligned}
r_{t+1}^i &= \sum_{k=1}^{K} \frac{D_{t+1}^k(\omega^{t+1}) + p_{t+1}^k}{W_{t+1}} \frac{\lambda_k^i w_t^i}{p_t^k} = \sum_{k=1}^{K} \left( \frac{D_{t+1}^k(\omega^{t+1})}{W_{t+1}} + \lambda_k^j \right) \frac{\lambda_k^i w_t^i}{\lambda_k^j W_t} \\
&= \sum_{k=1}^{K} \left( \lambda_0\, d^k(\omega_{t+1}) \frac{\lambda_k^i}{\lambda_k^j} + \lambda_k^i \right) r_t^i = \left( 1 - \lambda_0 + \lambda_0 \sum_{k=1}^{K} d^k(\omega_{t+1}) \frac{\lambda_k^i}{\lambda_k^j} \right) r_t^i
\end{aligned}
$$

where (6) implies that $\lambda_0\, d^k(\omega_{t+1}) = D_{t+1}^k(\omega^{t+1})/w_{t+1}^j = D_{t+1}^k(\omega^{t+1})/W_{t+1}$.

The exponential growth rate of strategy $i$'s market share at $\lambda^j$-prices can be inferred from this equation. It is given by

$$
g_{\lambda^j}(\lambda^i) = \mathbb{E}\ln\left[ 1 - \lambda_0 + \lambda_0 \sum_{k=1}^{K} d^k(s) \frac{\lambda_k^i}{\lambda_k^j} \right] \tag{7}
$$

where $\mathbb{E}$ denotes expected value with respect to the distribution on the set of states of nature $S$.

Evstigneev, Hens, and Schenk-Hoppé (2003) show the following result.

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**Theorem 1** *The portfolio rule $\lambda^*$, defined by*

$$\lambda_k^* = (1 - \lambda_0)\, \mathbb{E}d^k(s), \quad k = 1, ..., K \tag{8}$$

*is the only investment strategy that is locally stable against any other portfolio rule. More precisely, $g_{\lambda^*}(\lambda) < 0$ and $g_\lambda(\lambda^*) > 0$ for all $\lambda \neq \lambda^*$.*

Hence supposing the evolutionary process of wealth converges, it can only converge to $\lambda^*$. The above result assumes that investment strategies are distinct across investors. How can one analyze the case in which, for instance, more than one investor adopts the $\lambda^*$ strategy? Fortunately, even the general case of investors pursuing the same portfolio rule is straightforward: Since the relative wealth shares of two investors with the same investment rule is fixed over time, it is equivalent to assume that investors with the same strategy set up a fund with claims equal to their initial share.

The implications on the asset prices are immediate from Theorem 1. According to the strategy $\lambda^*$ one has to divide wealth across assets proportional to the present expected value of their (relative) future dividend payoffs. The discounting rate is the inverse of the saving rate $1 - \lambda_0$. If the $\lambda^*$ portfolio rule manages all market wealth then all asset prices are given by this vector of fundamental values.

In the long run only the $\lambda^*$ strategy is present in the market. Thus the $\lambda^*$-investors hold all wealth and asset prices are given by their fundamental values. Since only $\lambda^*$-investors survive, all surviving investors hold the market portfolio.

The following corollary shows the relation of the strategy $\lambda^*$ to the growth optimal portfolio.

**Corollary 1** *The portfolio rule $\lambda_k^* = (1 - \lambda_0)\, \mathbb{E}d^k(s)$ is the growth optimal investment strategy in a population where itself determines the asset prices, i.e.*

$$\lambda^* = \arg \max_{\mu \in \Delta^{K+1}:\mu_0 = \lambda_0} \mathbb{E} \ln \left( \sum_k \frac{d^k(s) + \lambda_k^*}{\lambda_k^*} \mu_k \right).$$

The proof of Corollary 1 is analogously to Theorem 1.

Before turning to the application we point out that one implication of equation (7) is that under-diversification is fatal for investment. Supposing some strategy does not use all assets it can easily be driven out by any completely diversified strategy. In particular the illusionary diversification rule according to which one puts equal weights on all assets can drive out sophisticated rules based on some optimization criterion like for example the mean-variance rule.

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**Corollary 2** *Suppose some incumbent rule $\lambda^j$ with $\lambda_k^j = 0$ for some asset $k$ has conquered the market. Then any portfolio rule $\lambda^i$ with $\lambda_k^i > 0$ for all $k$ grows against $\lambda^j$, i.e. $g_{\lambda^j}(\lambda^i) > 0$. (In fact the growth rate is arbitrarily large.)*

# 4 Application to the Dow

In this section we apply the general evolutionary portfolio theory model outlined above to dividend data from the Dow Jones Industrial Average. To this purpose we consider the total dividends paid by 21 stocks from the Dow in the years 1981 to 2001 (Appendix A lists the data). Those years and stocks have been selected in order to obtain a complete data set. For other stocks and years, the necessary data were not available. We interpret the data in terms of our model as follows.

First we assume that the 21 years are 21 realizations of a stochastic dividend process. The data reveal that the total dividends of each stock follow some growth path. Each date $t = 1, ..., 21$ is thus identified with a state $s = 1, ..., 21$. Each row of the matrix in Appendix A collects the total dividend payoff $D^k(s)$ of all assets $k = 1, ..., 21$ for one realization of the exogenous state of nature. Each column contains the dividend payoff of the respective asset across different states. In the simulation relative dividends are applied, i.e. according to the model every entry in a particular row is normalized by the sum of that row.

All portfolio strategies considered have to devote the same proportion of wealth to cash holdings. It is assumed to be 1% (i.e. $\lambda_0 = 0.01$).

We consider two types of portfolio rules. Those based solely on (exogenous) dividends and those based on (endogenous) returns. Of the first type is the behavioral finance rule which Benartzi and Thaler (1998) have called *illusionary diversification*

$$\lambda_k^{illu} = (1 - \lambda_0) \, \frac{1}{K}, \quad k = 1, ..., K.$$

According to this rule budget shares are set equal for all risky assets. Benartzi and Thaler (1998) found that surprisingly many investors use this naïve rule. Of the first type is also the evolutionary portfolio rule discovered in Hens and Schenk-Hoppé (2004):

$$\lambda_k^* = (1 - \lambda_0) \, \mathbb{E}\left(\frac{D^k}{\sum_{j=1}^K D^j}\right) = (1 - \lambda_0) \sum_{s \in S} p_s \, \frac{D^k(s)}{\sum_{j=1}^K D^j(s)}, \quad k = 1, ..., K.$$

The evolutionary rule presumes that agents calculate expected values correctly. It is however well known from behavioral finance that actual decisions of investors are based on perceived probabilities that may not coincide with the probabilities governing the relevant stochastic process. To allow for this behavioral distortion, Tversky and Kahnemann (1992) have suggested a certain transformation function $\alpha : [0,1] \rightarrow [0,1]$ that overstates small probabilities and understates high probabilities. This function is known as the *cumulative prospect theory*. We have used the cumulative prospect theory function as suggested by Tversky and Kahnemann (1992) to create a second behavioral finance rule based on the portfolio rule $\lambda^*$. That is to say the portfolio rule based on cumulative prospect theory is given by

$$\lambda_k^{cpt} = (1 - \lambda_0) \sum_{s \in S} \alpha(p_s) \frac{D^k(s)}{\sum_{j=1}^{K} D^j(s)}, \quad k = 1, ..., K.$$

The second type of portfolio rules that we consider are those based on returns. Since we only want to consider simple portfolio rules, i.e. those with time independent budget shares, we have to choose some prices that remain constant in the computation of the returns. In order not to base the second type of portfolio rules on some unreasonable prices we give them the advantage of allowing them to use the prices that eventually will emerge in the evolutionary process. Since we are mainly interested in the long run behavior of the process, these are the prices that determine the long run returns. As proven for the case of short-lived assets in Evstigneev, Hens, and Schenk-Hoppé (2002), our simulations in the case of long-lived assets show that those prices are given by $\lambda^*$. Hence we give the return based portfolio rules the advantage of knowing the $\lambda^*$-prices.

One of the most prominent examples of return based portfolio rules is the mean-variance rule suggested by Markowitz (1952). This rule is certainly one of the cornerstones of traditional finance. The interest rate is set to zero because the price of the consumption good is identical in all periods. We denote it by

$$\lambda^{\mu-\sigma}(\lambda^*).$$

The mean-variance model has always been criticized for using a risk measure, the variance, that has too many undesirable features. For example, it is well known that mean-variance optimization does not necessarily agree with first order stochastic dominance. Several alternatives, like semi-variance and Value-at-Risk for example, have been suggested in the course of this discussion. A recent concept is conditional Value-at-Risk, CVaR, which is one possible *coherent* risk measure as Artzner, Delbaen, Eber, and Heath (1997) have argued. The CVaR takes the expectation of the returns below some

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quantile of the return distribution. The quantile is usually chosen to the 5%-level. Based on this idea and the general assumptions made above, we generate the portfolio rule

$$\lambda^{CVaR}(\lambda^*).$$

The third portfolio rule based on returns is the growth optimal portfolio (Hakansson 1970). This portfolio strategy maximizes the expected growth rate of wealth on a given return process. In its most general form this portfolio strategy is allowed to adapt to the endogenous fluctuations of the returns. It then maximizes the expected logarithm of the returns, which is also known as the Kelly rule, Kelly (1956). In this most general from it is clearly unbeatable in view of the long run perspective taken here. However, in this general form it is quite difficult to actually compute this rule. One way of interpreting the results of this paper is to say that with endogenous returns there is a simple short-cut to determine a simple portfolio rule that eventually coincides with the Kelly rule: Using $\lambda^*$ and thus simply dividing wealth proportional to the expected relative dividends. That is to say (cf. Corollary 1)

$$\lambda^{gop}(\lambda^*) = \lambda^*.$$

The alternative growth optimal strategy is then the one based on equal prices

$$\lambda^{gop}(\mathbf{1}).$$

Appendix B collects the portfolio rules that have been computed according to the various strategies outlined so far. One apparent observation is that the portfolio strategies based on endogenous returns are under-diversified. The mean-variance-strategy only uses 8, the CVaR-strategy only uses 6 and the growth optimal portfolio only uses 1 out of the 21 assets! As Hens and Schenk-Hoppé (2004) have shown in the case of short lived assets, under-diversification is fatal for survival in the market selection process, see Corollary 2. Therefore, we do the mean-variance rule yet another favor and make it completely diversified by devoting to any asset at least the smallest positive budget share occurring in the under-diversified portfolio[6]. This ad hoc diversification rule is often used in praxis:

$$\lambda_\epsilon^{\mu-\sigma}(\lambda^*)$$

Thus all together we consider the market selection process given by equation (5) when it is run by these 8 portfolio rules.

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[6]Note that our notion of being completely diversified does not coincide with an intuitive notion of complete diversification, like naive diversification, according to which your portfolio has equal shares in every strategy.

# 5    Simulation Results

## 5.1    Rational Strategies can be driven out by Irrational Strategies

Friedman (1953) and Fama (1965) argued that the market naturally selects for the rational strategies. On the other hand De Long, Shleifer, Summers, and Waldmann (1990) showed that under specific circumstances noise traders can earn a higher average rate of return than rational arbitrageurs. In this section we demonstrate that at a first approximation the claim of De Long, Shleifer, Summers, and Waldmann (1990) can be given good support in our model. We consider the fix-mix mean-variance portfolio rule $\lambda^{\mu-\sigma}(\mathbf{1})$ in competition with the illusionary diversification rule $\lambda_k^{illu} = (1 - \lambda_0)/K$, $k = 1, .., K$.

Figure 1 shows a typical run of the evolution of market shares over time for a sample path of the dividend process. Starting with an initial distribution of wealth in which 90% of the wealth is in the hands of the mean-variance rule, after only 10 periods the illusionary portfolio rule has conquered more than 90% of the market wealth. Thus the illusionary diversification rule quickly drives out the mean-variance rule. Moreover, after only 15 periods the illusionary portfolio rule will hold almost all market wealth. Note that we have even given the mean-variance rule rational expectations because we allowed it to be based on the prices that will eventually prevail in the market selection process. An intuition for this result is that the mean-variance rule is under-diversified while the illusionary diversification rule is completely diversified. Evstigneev, Hens, and Schenk-Hoppé (2003) have shown that under-diversified rules cannot be evolutionary stable, but that they can perform as badly as in this example is still surprising.

## 5.2    Stochastic Time Series of Asset Prices

Our next example shows that the market selection process studied here can also generate stochastic time series of asset prices that do not converge. While the asset price dynamics in the standard evolutionary asset pricing models is complex and therefore not straightforward to interpret, the explanation of excess volatility arising in our model is quite simple. In our model it is the endogenous change in the wealth shares that generates the fluctuations in asset prices. In an example with two strategies—the mean-conditional-value at risk strategy $\lambda^{CVaR}(\lambda^*)$ and the illusionary diversification rule $\lambda^{illu}$— the following phenomenon occurs. As the wealth share of the first strategy increases, it turns market prices to its disadvantage giving the second strategy
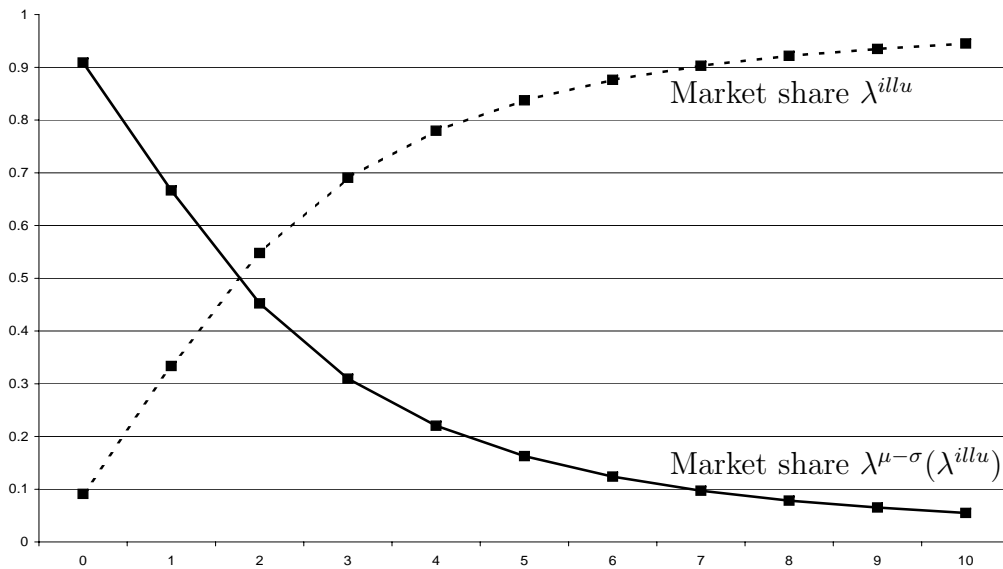
Figure 1: An irrational rule driving out a seemingly rational rule. Evolution of market shares: illusionary diversification rule (broken line) and mean-variance rule for $\lambda^{illu}$-prices (bold line).

a higher potential for growth. And the same holds true when the second strategy's wealth share increases. Note that in our model relative asset prices $q_t^k$ are the wealth average of the strategies in the market:

$$q_t^k = \sum_{i=1}^{I} \lambda_k^i r_t^i.$$

Hence with time independent strategies $\lambda^i$ price fluctuations can only result from wealth fluctuations.

Figure 2 shows the evolution of market shares over time for one sample path of the dividend process. Starting with initial wealth at 40% wealth in the hands of the mean-conditional-value at risk rule, the wealth process cycles irregularly between 35% and 48% of wealth for this rule and has not yet converged after 1000 periods.
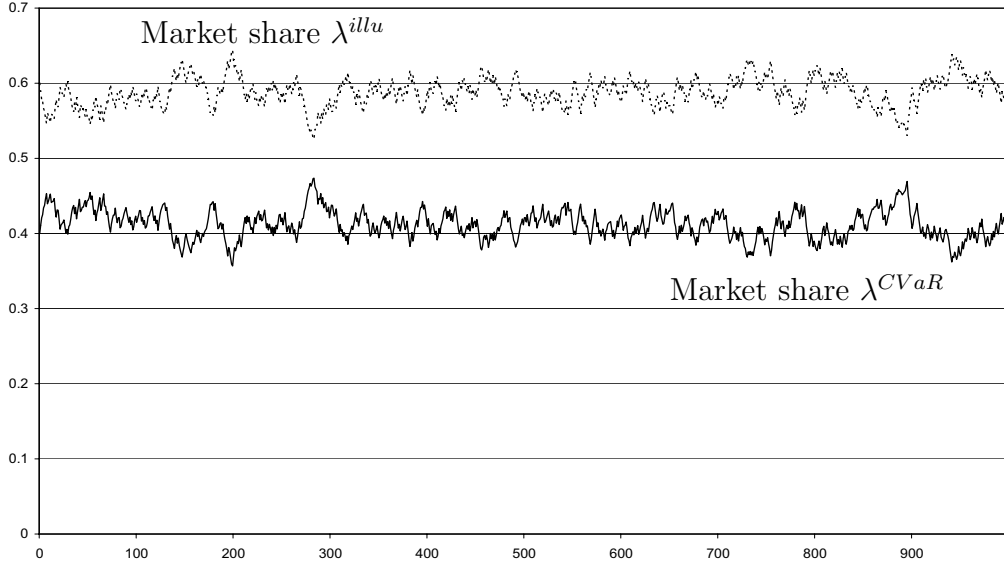
15

Figure 2: Non-convergence of the market selection process. Evolution of market shares: illusionary diversification rule (broken line) and CVaR rule (bold line).

## 5.3 Single Survivor Hypothesis

In this subsection we now include $\lambda^*$ in the simulations. Given the dividend matrix and the strategies described above, we have carried out simulations of the market selection process with different initial wealth shares for $\lambda^*$ and different number of periods. It turns out that $\lambda^*$ satisfies the single survivor property first defined in Blume and Easley (1992). On almost all paths the wealth of $\lambda^*$ grows at a faster rate than the wealth of any other strategy that we considered.

Figure 3 shows the evolution of market shares over time for one sample path of the dividend process. Starting with equal initial wealth, after 100 periods the evolutionary portfolio rule $\lambda^*$ has conquered 50% of the market wealth. We can see from Figure 3 that the market share of $\lambda^*$ increases steadily from 10% to 50% while the other strategies' market shares have a clear downward trend although some of them initially increase. Note that after less than 10 periods $\lambda^*$ has doubled its market share for the first time and that after less than 60 periods it has doubled it again.

Figure 4 shows the mean market shares, averaged over 30 runs. Equal initial wealth is given to each of the 10 strategies described above. Each run was conducted for 100 periods.
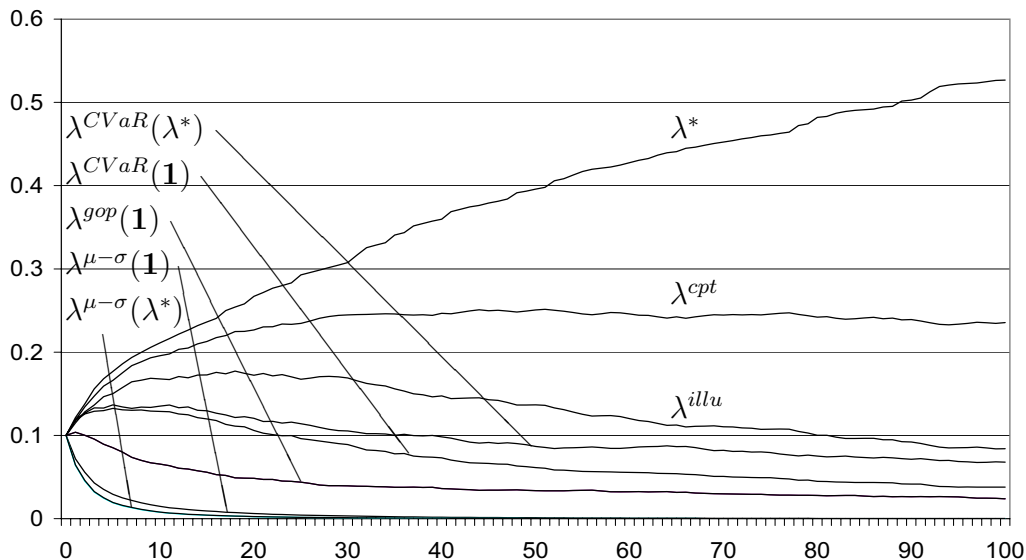
16

Figure 3: Evolution of market shares: Typical sample path.

Numerical studies show that the standard deviation of each strategies market share from the mean in anyone period is quite small with a maximum value of 3% and an average value of only 0.36%.

Figure 5 reports the prices that are implied by the evolution of market shares. It is astonishing to see that prices converge quite rapidly to their rational values which are determined by $\lambda^*$. Exxon Mobil Corp. is the company with the highest price as it pays out the highest relative dividend on average.

Figure 6 depicts the evolution of market shares when $\lambda^*$ starts with a comparative disadvantage. Initially it has only 0.1% of total wealth. This figure displays an interesting population dynamics. As long as $\lambda^*$ is small, its behavioral finance variation $\lambda^{cpt}$ drives out the other strategies. However, $\lambda^*$ grows steadily and eventually drives out and replaces $\lambda^{cpt}$. Note that the chart of $\lambda^*$ is S-shaped. While $\lambda^*$ is small it grows slowly, then it has a rapid take off and eventually—when more and more competitors get close to extinction—it slows down again. Even though $\lambda^*$ needs some time to conquer a considerable share of the market, starting from the 0.1% level it is able to double its share more rapidly than starting from the 10% level. After 4 periods is has doubled for the first time, after 17 periods it has doubled for the second time and after 40 periods it has doubled for the third time. All other strategies only play a minor role in this dynamics.

This subsection has demonstrated that a rational investor should choose
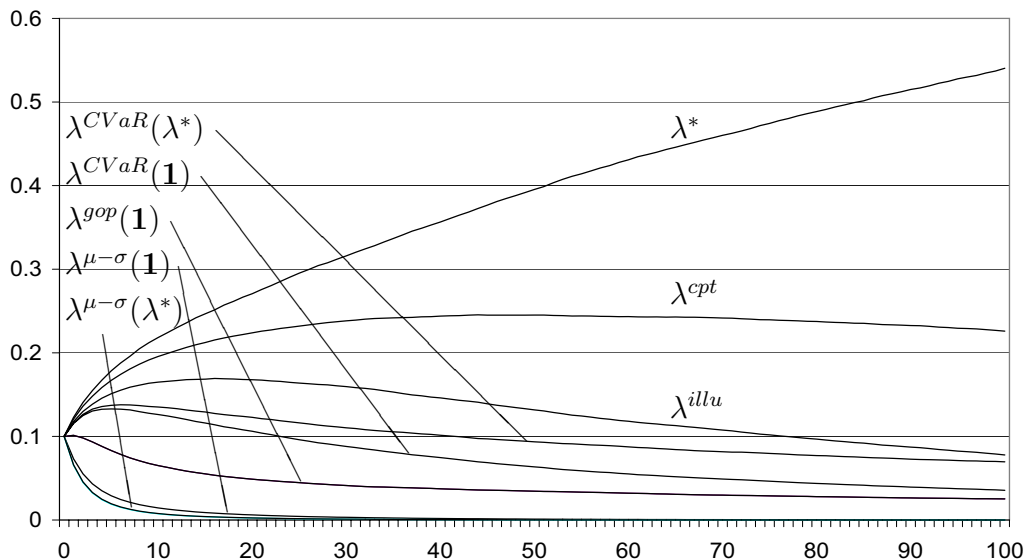
17

Figure 4: Evolution of average market shares: Average taken over 30 runs.

The figure contains the following labels: $\lambda^{CVaR}(\lambda^*)$, $\lambda^{CVaR}(\mathbf{1})$, $\lambda^{gop}(\mathbf{1})$, $\lambda^{\mu-\sigma}(\mathbf{1})$, $\lambda^{\mu-\sigma}(\lambda^*)$, $\lambda^*$, $\lambda^{cpt}$, $\lambda^{illu}$.

the strategy $\lambda^*$. He will then drive out any other strategy. Hence, even though some seemingly rational strategies may do worse than some irrational strategies, the true rational strategy will always do better than any irrational strategy. In this sense Friedman (1953) and Fama (1965) are right—eventually asset prices are as in an efficient market. They are determined only by expected relative dividends.

# 6    Some Intuition for the Results

To provide some intuition for the striking results obtained in the previous section, it is instructive to recall the results of the theoretical literature. This literature considers an investment problem with stationary dividends in which the returns at any point in time are completely re-invested for the next period. The starting point was Breiman (1961)'s observation that the best strategy for repeatedly betting on the occurrence of a finite number of states is to divide the wealth placed on these bets proportional to the probabilities of occurrence of the states. This rule has thus been called *betting your beliefs*. That is to say, if one holds fixed these proportions then, by the Law of Large Numbers, you will maximize the expected growth rate of your wealth. Note that taking the long run perspective, risk does not matter because any short run under-performance can still be recovered in
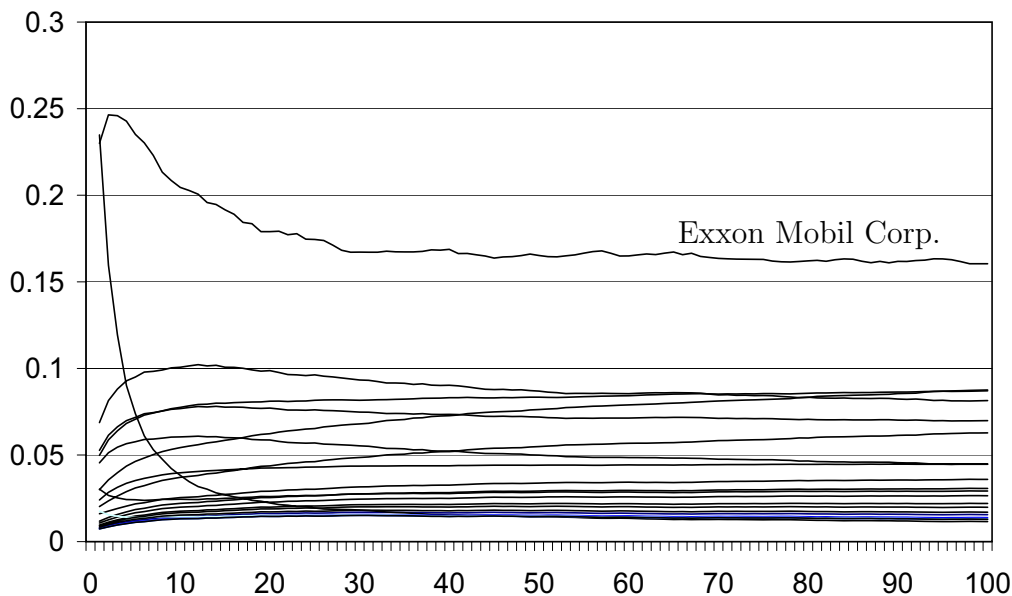
18

Figure 5: Convergence of relative stock prices as time elapses for one run. Each sample path is the time series of one firm's stock price.

the long run. This point of view on the risk involved in portfolio formation is common to all papers on evolutionary portfolio theory. The next step in this literature was to consider a market for the bets on the various states. Thus if demand for any one bet were high then the price for this bet will be high and one might argue that one should rather go for the other bets that offer a more attractive return. However, as Blume and Easley (1992) have shown, this is not true. The best portfolio rule is still to bet your beliefs. In Breiman (1961) (as well as in Blume and Easley (1992)) bets can be identified with states because they consider a complete set of Arrow-securities. Evstigneev, Hens, and Schenk-Hoppé (2002) have generalized the set up of Blume and Easley (1992) to allow for any complete or incomplete asset structure. As these authors show, the correct generalization of betting your beliefs is then to divide income proportionally to the expected payoffs of the securities. A major shortcoming of the literature so far was the assumption of short-lived assets. According to this assumption the asset is liquidated after having paid off and an identical asset is born. Wealth is assumed to be perishable so that it can only be transferred to later periods by investing once more in the exogenously supplied assets. Sandroni (2000) and Blume and Easley (2001) present the first models of this literature with long-lived assets allowing the important feature of capital gains. However, these authors assume a complete
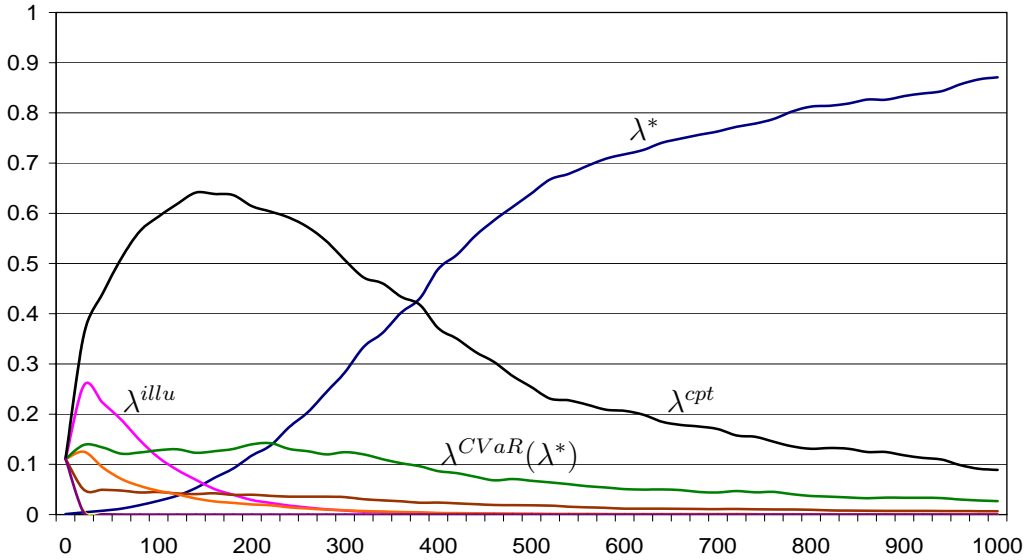
19

Figure 6: Evolution of market shares: Initial market share of $\lambda^*$ is 0.1%. Initially the behavioral finance rule $\lambda^{cpt}$ performs best. $\lambda^*$'s market share grows steadily. Eventually $\lambda^*$ drives out and replaces $\lambda^{cpt}$. All other rules play a minor role.

security market and moreover they restrict attention to portfolio rules being generated by expected utility maximizers. This paper has a model with long-lived assets and a general security market. Moreover, portfolio rules need not be generated by expected utility maximization. As it turns out, the wealth process converges to the evolutionary portfolio rule $\lambda^*$ and therefore capital gains converge to the dividend payoffs. Hence the strategy being best suited to the dividends will eventually also profit most from the capital gains. Note that only the evolutionary rule $\lambda^*$ found in Hens and Schenk-Hoppé (2004) has this property so that in the long run this strategy has the highest expected growth rate.

Let us finally compare the evolutionary portfolio rule $\lambda^*$ with the CAPM rule, $\lambda^{CAPM}$. According to the CAPM, a passive buy and hold strategy, one should hold a fixed fraction of the market portfolio. In the notation of this paper, this would translate to having the demand $a_{t,k}^{CAPM} = \gamma_t$, where $\gamma_t = (\sum_k p_t^k)^{-1}$ is some positive scalar. In terms of budget shares the CAPM strategy is given by $\lambda_{t,k}^{CAPM} = \gamma_t p_t^k$, $k = 1, ...K$.

The first observation is that in a rational and risk neutral market, $\lambda^*$ would actually coincide with the CAPM rule because in such a market asset

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prices are determined by discounted expected dividends, i.e. $p^k = \frac{1}{r_f}\mathbb{E}d^k$, $k = 1, ..., K$, where $r_f$ denotes the risk free rate of interest. As $\lambda^*$ gains total market wealth, prices converge to the rational and risk neutral valuation and thus $\lambda^*$ and the CAPM rule will eventually coincide. Hence while $\lambda^*$ exploits the wealth of other strategies it will never be able to drive out the CAPM rule. In a sense, the CAPM rule is a imitation strategy that mimics the best performing strategy in the long run.

It is noteworthy that, similar to contrarian strategies from behavioral finance, the evolutionary portfolio rule eventually eliminates the market anomaly from which it lives. As long as $p^k \neq \frac{1}{r_f}\mathbb{E}d^k, k = 1, ..., K$ there are excess returns and hence $\lambda^*$ can grow at the expense of the existing ones. In the limit, as the distribution of wealth concentrates on $\lambda^*$, these excess returns are removed.

# 7    Conclusions

Our simulations have shown that in competition with fix-mix rules derived from mean-variance-optimization, from maximum growth theory and from behavioral finance, the evolutionary portfolio rule discovered in Hens and Schenk-Hoppé (2004) will eventually hold total market wealth. According to this simple rule the portfolio weights should be proportional to the expected relative dividends of the assets. This rule may be interpreted as a CAPM rule which fixes budget shares according to the expected market capitalization and then rebalances according to these fixed weights as prices fluctuate. For sufficiently patient investors, like pension funds or insurance companies for example, this rule promises very high proceeds. On a long horizon risk is limited – the standard deviations of the average position of the market share are very small and almost constant over time. Risk will however matter if the investor is subject to shocks that require to liquidate some of its wealth at unforeseen periods. An important and also very interesting extension of this work is to introduce such liquidity shocks in the evolutionary process of market selection.

# A  The Dividend Process

Figure 7 depicts the evolution of relative dividends-per-share over time. Some firms apply dividend smoothing and distribute an almost constant stream of dividends while other firms' dividend payments vary considerably. The dividends are adjusted for buy backs. In the simulations we have identified each year with a state of the world and then we have drawn such states independently and identically distributed according to a uniform distribution.
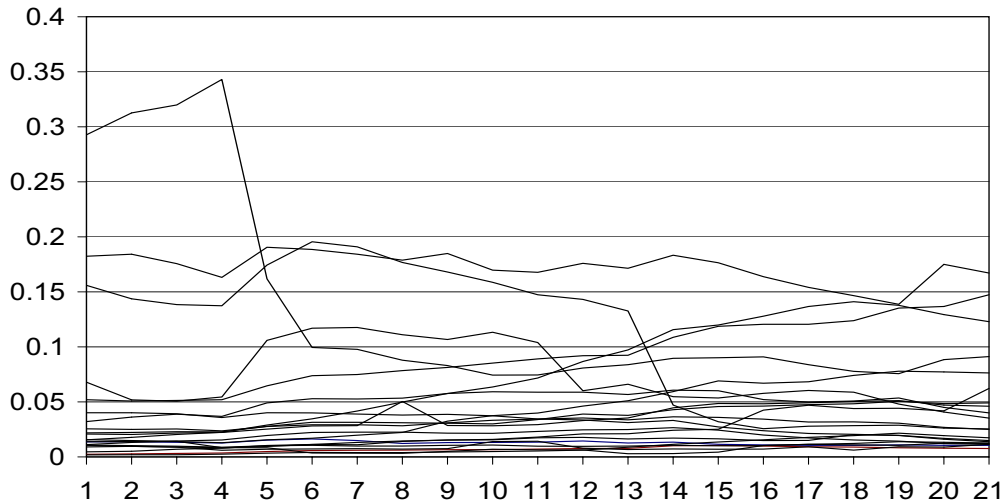


Figure 7: The relative dividend process for the DJIA (21 states).

AT&T roughly pays about one third of the total dividends in 1981-1984 (states 1-4). Exxon Mobil Corp. has a roughly constant share of 15-20% of the total dividend payments over the entire period.

Company Name (Ticker Symbol): ALCOA Inc. (AA), American Express Co. (AXP), AT&T Corp. (T), Boeing Co. (BA), Caterpillar Inc. (CAT), Coca-Cola Co. (KO), Dupont Co. (DD), Eastman Kodak Co. (EK), Exxon Mobil Corp. (XOM), General Electric Co. (GE), General Motors Corp. (GM), Hewlett Packard Co. (HPQ), International Business Machines Corp. (IBM), International Paper Co. (IP), J.P. Morgan Chase & Co. (JPM), McDonalds Corp. (MCD), Merck & Co. (MRK), Minnesota Mining & Manufacturing Co. (MMM), Phillip Morris Co. (MO), Procter & Gamble Co. (PG), United Technologies Corp. (UTX).

|      | AA     | AXP    | T      | BA     | CAT    | KO     | DD     |
|------|--------|--------|--------|--------|--------|--------|--------|
| 1981 | 0.1168 | 0.1440 | 3.7699 | 0.1349 | 0.2010 | 0.2669 | 0.4130 |
| 1982 | 0.1304 | 0.1777 | 4.2293 | 0.1298 | 0.2011 | 0.2755 | 0.4890 |
| 1983 | 0.1152 | 0.1905 | 4.2011 | 0.1196 | 0.1869 | 0.2848 | 0.5102 |
| 1984 | 0.0783 | 0.2014 | 4.4768 | 0.1083 | 0.1099 | 0.2905 | 0.4817 |
| 1985 | 0.0741 | 0.2081 | 1.7295 | 0.1011 | 0.0892 | 0.2706 | 0.5240 |
| 1986 | 0.0703 | 0.2166 | 0.9664 | 0.1104 | 0.0345 | 0.2735 | 0.5140 |
| 1987 | 0.0695 | 0.2100 | 0.9118 | 0.1228 | 0.0324 | 0.2658 | 0.4912 |
| 1988 | 0.0660 | 0.2072 | 0.8091 | 0.1330 | 0.0306 | 0.4638 | 0.4916 |
| 1989 | 0.0669 | 0.1917 | 0.7359 | 0.1352 | 0.0439 | 0.2528 | 0.5106 |
| 1990 | 0.1311 | 0.2013 | 0.6961 | 0.1451 | 0.0653 | 0.2647 | 0.5542 |
| 1991 | 0.1397 | 0.2275 | 0.7334 | 0.1716 | 0.0638 | 0.2891 | 0.5785 |
| 1992 | 0.0760 | 0.2364 | 0.7745 | 0.1700 | 0.0600 | 0.3172 | 0.5634 |
| 1993 | 0.0655 | 0.2442 | 0.8240 | 0.1603 | 0.0283 | 0.3479 | 0.5572 |
| 1994 | 0.0632 | 0.2335 | 0.7875 | 0.1509 | 0.0271 | 0.3920 | 0.5331 |
| 1995 | 0.0611 | 0.2133 | 0.7912 | 0.1439 | 0.0385 | 0.4257 | 0.5276 |
| 1996 | 0.0651 | 0.1836 | 0.8368 | 0.1371 | 0.0958 | 0.4449 | 0.4793 |
| 1997 | 0.0883 | 0.1645 | 0.8005 | 0.1430 | 0.1090 | 0.4704 | 0.4757 |
| 1998 | 0.0609 | 0.1512 | 0.7657 | 0.1991 | 0.1208 | 0.4958 | 0.5008 |
| 1999 | 0.0897 | 0.1401 | 0.7399 | 0.1908 | 0.1353 | 0.5007 | 0.5240 |
| 2000 | 0.0944 | 0.1280 | 0.9398 | 0.1701 | 0.1410 | 0.5006 | 0.4788 |
| 2001 | 0.1276 | 0.1285 | 1.0200 | 0.1539 | 0.1411 | 0.5144 | 0.4473 |

|      | EK     | XOM    | GE     | GM     | HPQ    | IBM    | IP     |
|------|--------|--------|--------|--------|--------|--------|--------|
| 1981 | 0.5164 | 2.3484 | 0.6700 | 0.8741 | 0.0240 | 2.0080 | 0.1398 |
| 1982 | 0.5438 | 2.4925 | 0.6869 | 0.7018 | 0.0259 | 1.9436 | 0.1395 |
| 1983 | 0.5147 | 2.3071 | 0.6732 | 0.6645 | 0.0266 | 1.8186 | 0.1285 |
| 1984 | 0.4674 | 2.1290 | 0.6784 | 0.7104 | 0.0318 | 1.7923 | 0.1156 |
| 1985 | 0.4296 | 2.0372 | 0.6912 | 1.1325 | 0.0364 | 1.8633 | 0.1085 |
| 1986 | 0.3889 | 1.8333 | 0.7173 | 1.1370 | 0.0401 | 1.9008 | 0.1020 |
| 1987 | 0.3638 | 1.7187 | 0.6986 | 1.0981 | 0.0370 | 1.7814 | 0.0918 |
| 1988 | 0.3506 | 1.6465 | 0.7215 | 1.0224 | 0.0368 | 1.6269 | 0.0907 |
| 1989 | 0.3423 | 1.6378 | 0.7205 | 0.9455 | 0.0394 | 1.4883 | 0.0936 |
| 1990 | 0.3502 | 1.5874 | 0.7980 | 1.0598 | 0.0459 | 1.4849 | 0.1009 |
| 1991 | 0.3395 | 1.6514 | 0.8777 | 1.0234 | 0.0534 | 1.4510 | 0.0957 |
| 1992 | 0.3216 | 1.6863 | 0.8821 | 0.5760 | 0.0595 | 1.3732 | 0.0922 |
| 1993 | 0.3064 | 1.6853 | 0.9075 | 0.6490 | 0.0863 | 1.3035 | 0.0971 |
| 1994 | 0.2916 | 1.6113 | 0.9557 | 0.4811 | 0.1012 | 0.4141 | 0.0923 |
| 1995 | 0.2395 | 1.5482 | 1.0417 | 0.4705 | 0.1185 | 0.2801 | 0.0889 |
| 1996 | 0.2192 | 1.5089 | 1.1101 | 0.5321 | 0.1435 | 0.2369 | 0.0950 |
| 1997 | 0.2033 | 1.4719 | 1.1505 | 0.5772 | 0.1698 | 0.2663 | 0.1098 |
| 1998 | 0.2027 | 1.4434 | 1.2193 | 0.5791 | 0.1902 | 0.2799 | 0.1080 |
| 1999 | 0.1925 | 1.3572 | 1.3238 | 0.4696 | 0.2114 | 0.2821 | 0.1035 |
| 2000 | 0.1784 | 1.8605 | 1.4534 | 0.4331 | 0.2059 | 0.2785 | 0.1324 |
| 2001 | 0.1664 | 1.8694 | 1.6490 | 0.3951 | 0.1948 | 0.2836 | 0.1365 |

|      | JPM    | MCD    | MRK    | MMM    | MO     | PG     | UTX    |
|------|--------|--------|--------|--------|--------|--------|--------|
| 1981 | 0.0595 | 0.0297 | 0.1781 | 0.3288 | 0.1995 | 0.2811 | 0.1757 |
| 1982 | 0.0695 | 0.0368 | 0.1891 | 0.3382 | 0.2402 | 0.3017 | 0.1873 |
| 1983 | 0.0919 | 0.0419 | 0.1835 | 0.3331 | 0.2671 | 0.3012 | 0.1744 |
| 1984 | 0.1033 | 0.0462 | 0.1678 | 0.3089 | 0.2913 | 0.2970 | 0.1657 |
| 1985 | 0.1115 | 0.0520 | 0.1665 | 0.2958 | 0.3093 | 0.2966 | 0.1633 |
| 1986 | 0.1095 | 0.0565 | 0.1653 | 0.2834 | 0.3368 | 0.3059 | 0.1583 |
| 1987 | 0.1029 | 0.0568 | 0.1839 | 0.2720 | 0.3896 | 0.2938 | 0.1388 |
| 1988 | 0.1300 | 0.0574 | 0.2053 | 0.2605 | 0.4591 | 0.2856 | 0.1125 |
| 1989 | 0.1369 | 0.0585 | 0.2879 | 0.2744 | 0.5106 | 0.2721 | 0.1155 |
| 1990 | 0.1484 | 0.0621 | 0.3509 | 0.3119 | 0.5941 | 0.2806 | 0.1227 |
| 1991 | 0.1778 | 0.0697 | 0.3920 | 0.3384 | 0.7067 | 0.3343 | 0.1326 |
| 1992 | 0.2007 | 0.0735 | 0.4426 | 0.3394 | 0.8315 | 0.3731 | 0.1392 |
| 1993 | 0.2065 | 0.0757 | 0.5017 | 0.3305 | 0.9560 | 0.3715 | 0.1245 |
| 1994 | 0.2131 | 0.0893 | 0.5213 | 0.3200 | 1.0169 | 0.3773 | 0.1185 |
| 1995 | 0.2204 | 0.0913 | 0.6068 | 0.3148 | 1.0523 | 0.4015 | 0.1007 |
| 1996 | 0.3920 | 0.0908 | 0.6171 | 0.3166 | 1.1779 | 0.4256 | 0.1010 |
| 1997 | 0.4481 | 0.0875 | 0.6522 | 0.3029 | 1.3060 | 0.4534 | 0.1000 |
| 1998 | 0.4332 | 0.0885 | 0.7292 | 0.3131 | 1.3887 | 0.4751 | 0.1040 |
| 1999 | 0.4323 | 0.0814 | 0.7622 | 0.3001 | 1.3478 | 0.4946 | 0.1069 |
| 2000 | 0.4442 | 0.0839 | 0.8205 | 0.2855 | 1.3745 | 0.5152 | 0.1118 |
| 2001 | 0.6967 | 0.0857 | 0.8542 | 0.2803 | 1.3739 | 0.5483 | 0.1182 |

# B  Portfolio Rules

The following table reports the budget shares for the investment strategies applied in this paper. The budget shares are normalized with $(1 - \lambda_0)$ for convenience. Rounding errors may prevent shares from adding up to one.

| Weights (%) | $\lambda^{\mu-\sigma}(\lambda^*)$ | $\lambda^{CVar}(\lambda^*)$ | $\lambda^*$ | $\lambda^{gop}(\mathbf{1})$ | $\lambda^{illu}$ | $\lambda^{cpt}(\lambda^*)$ |
|---|---|---|---|---|---|---|
| UTX | 0.00 | 0.00 | 1.28 | 0.00 | 4.76 | 2.82 |
| PG | 0.00 | 0.00 | 3.63 | 0.00 | 4.76 | 4.70 |
| MO | 0.00 | 27.45 | 7.81 | 0.00 | 4.76 | 6.58 |
| MMM | 0.00 | 0.00 | 3.02 | 0.00 | 4.76 | 4.08 |
| MRK | 0.46 | 0.00 | 4.31 | 0.00 | 4.76 | 5.64 |
| MCD | 91.01 | 0.00 | 0.68 | 0.00 | 4.76 | 2.19 |
| JPM | 0.00 | 0.00 | 2.35 | 0.00 | 4.76 | 3.45 |
| IP | 0.00 | 0.00 | 1.04 | 0.00 | 4.76 | 2.51 |
| IBM | 0.07 | 18.56 | 10.97 | 0.00 | 4.76 | 7.52 |
| HPQ | 3.79 | 0.00 | 0.90 | 0.00 | 4.76 | 2.51 |
| GM | 0.22 | 18.22 | 7.31 | 0.00 | 4.76 | 6.58 |
| GE | 0.10 | 0.00 | 9.28 | 0.00 | 4.76 | 7.21 |
| XOM | 0.00 | 17.26 | 17.25 | 99.99 | 4.76 | 8.46 |
| EK | 3.44 | 0.00 | 3.17 | 0.00 | 4.76 | 5.02 |
| DD | 0.00 | 4.26 | 5.02 | 0.00 | 4.76 | 5.96 |
| KO | 0.91 | 9.71 | 3.61 | 0.00 | 4.76 | 5.02 |
| CAT | 0.00 | 0.00 | 0.86 | 0.00 | 4.76 | 2.51 |
| BA | 0.00 | 0.00 | 1.43 | 0.00 | 4.76 | 3.13 |
| T | 0.00 | 4.54 | 13.33 | 0.01 | 4.76 | 8.15 |
| AXP | 0.00 | 0.00 | 1.90 | 0.00 | 4.76 | 3.45 |
| AA | 0.00 | 0.00 | 0.85 | 0.00 | 4.76 | 2.51 |

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