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More Lessons from Taking an AK Model to the Data*

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Abstract

We take an AK model to the PWT data. In the model both technology (intratemporal) and investment (intertemporal) shocks determine the variation of the growth rate. In earlier work we looked at singular models where we extracted only the technology shock using the policy functions from dynamic optimality. Here we recover time series for *both* shocks for a panel of countries and we isolate what we believe are pervasive patterns in macroeconomic models and postwar data: a negative correlation between intra and intertemporal shocks, and a somewhat lesser role for the intertemporal shock.

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1 Introduction

This paper takes an AK model with technology and investment shocks to the data. Our aim is to see what can be learned about the economic and statistical nature of these shocks.

The usefulness of the AK model to study these questions relies on its simplicity and on its ability to reproduce important features of the data. This last aspect has been strengthened by recent research by McGrattan (1998), who presents new evidence "on defense of AK growth models" from Jones (1995) critiques. To this purpose she uses long time series and takes into account that the simplest AK formulation is just a reduced form of more complete models that may consider several sectors of production and the optimal labor/leisure choice. Moreover, Fatás (2000) has recently shown that a stochastic AK model is able to reproduce the empirical evidence about the positive correlation between long term growth rates and the persistence of output fluctuations. This empirical evidence is not compatible with a concave model with exogenous productivity shocks.

Our prior is that if endogenous growth theory is correct, then a very stylized linear model should do well against the raw data, just as the early stylized concave models did against log detrended data. This, along with the existence of a vast literature on endogenous growth, is the reason we use the AK model. We assume that countries are always sufficiently close to the balanced growth path to make transitional dynamics of second order in explaining movements in growth rates. We restrict our analysis to a few countries, but Chari, Kehoe and McGrattan (2001) take this reasoning to all countries in a concave model.[1]

We acknowledge that the linearity of the AK model implies that capital is a broad investment good that may include components such as human and organizational capital. That, to aline theory and data, capital should be understood in this more general way is well established in the literature and has been shown for both concave and linear models.[2] The price of this broader stock is therefore not necessarily the same as the price of physical capital.

Our model economy contains two stylized mechanisms that affect growth

[1] "The richest countries are typically thought of as being approximately on a balanced growth path. Since the poorest countries grow approximately at the same rate as the richest, this suggest that the poorest countries are as well." [Chari, Kehoe, and McGrattan (2001), p 3]

[2] See among others Mankiw, Romer and Weil (1992), Barro and Sala-i-Martin (1995), Parente and Prescott (1994) on concave models with broader capital, and Rebelo (1991) for an AK model.

outcomes which are summarized by two different shocks: an intratemporal technology shock and an intertemporal technology shock. These two shocks have a long tradition in the literature. The intratemporal shock is a technology shock as in the RBC literature, and integrates the mechanism of growth and cycles in the spirit of Jones, Manuelli and Siu (2000). The intertemporal shock is a shock that affects the technology that transforms current savings into future productive capital.[3] Greenwood, Hercovitz and Huffman (1988) is a classic reference for the introduction of this type of shock.[4] Chari, Kehoe and McGrattan (2001) consider an intertemporal shock in the same spirit, interpret it as "investment distortions", and claim that it is an important determinant of the variability of relative income levels across countries.[5]

There is an extensive literature that looks at the contribution of these shocks, mostly in isolation, to output variation. Prescott (1986), considers only technology shocks and claims that this shock explains a large fraction of USA output variation. This line of research has been reviewed by King and Rebelo (1999). Chari, Kehoe and McGrattan (2002) consider simultaneously three types of shocks, an efficiency shock, a labor shock and an investment shock in a neoclassical growth model and conclude that the first two types of shocks explain most of the output variation.

However, Ingram, Kocherlakota and Savin (1994) show that it is impossible to measure the contribution of any individual shock to the variance of output as "The presence of correlated shocks means that it is impossible to sort out their separate effects upon a single variable such as output."[6] We show with the AK model that their theoretical result has significant empirical consequences on the PWT data.

These authors also show that it is not possible to extract from the data an unobserved shock using a model with more endogenous variables than exogenous unobservable shocks, a "singular model".

Accordingly, we begin by using data on the relative price of investment as the (observable) measure of the investment shock, in the line of Chari, Kehoe

[3]It is a proxy for a random financial intermediation technology, a random production function for capital in a multisector economy, or for technological progress embodied in capital goods.

[4]See references in their paper for earlier work.

[5]In a related paper Restuccia and Urrutia (2001) look at the cross country patterns of the relationship between the investment shock and the investment to output ratio. Both of these papers assume diminishing returns to broad capital accumulation.

[6]Ingram, Kocherlakota and Savin (1994), page 416. This argument is correct as far as finding an exact separation of the impact of the different shocks. However, Cooper and Ejarque (2000) use a model with two shocks and explore the property that investment shocks induce negative correlations to claim that these shocks cannot have been significant contributors to postwar business cycles.

and McGrattan (2001). This implies the model is singular. Our methodology takes up the challenge of Klenow and Rodriguez-Clare (1997) by being very explicit in taking model implications to the data: we use the exact policy functions implied by dynamic optimality to extract the technology shock from the data, indeed to extract two different but equally legitimate series for the technology shock. Not surprisingly, in this (singular) context the model is strongly rejected. Even some basic characteristics of the shock, such as the sign of the correlation with the intertemporal shock, may depend dramatically on the way we extract it from the data.

Next we take into account that the intertemporal shock does not refer only to the price of capital but to the price of a more general investment good. In this case the intertemporal shock becomes also unobservable. So, we use again the policy functions implied by dynamic optimality to extract from the data *both* the technology shock and the investment shock. Then we study the relationship between the two shocks, and provide a quantitative illustration of the bias when one tries to assign explanatory power to the different shocks.

The paper proceeds with the presentation of the model and a description of the data. We revisit the problems in judging which of the two mechanisms generating growth is dominant using a singular model, and then using a non singular model we obtain a negative correlation between the intertemporal and the intratemporal shock, as well as a somewhat lesser role for the intertemporal shock in the behaviour of growth rates. We believe these are pervasive features of macroeconomics in postwar data. The appendix explores variations on the model and on data treatment which show that our results are robust.

2 Model

We consider a stylized AK model and look at the planner's problem. Utility of the representative agent is maximized subject to a budget constraint where aggregate output is divided between consumption and savings, $y_t = A_t k_t = c_t + s_t$. Production is of the AK form, where A_t is the intratemporal technology shock. There is also an intertemporal technology that transforms current savings into investment, $I_t = \theta_t s_t$, and is summarized by the shock (θ_t). If we consider K as physical capital then the data counterpart of θ is $\frac{p_c}{p_I}$ as may be seen from writing $y_t = c_t + I_t / \theta_t$.[7] However, if we want to consider a broader measure of capital then we cannot obtain θ in the data. In any case, an increase in θ constitutes an increase in the efficiency of the intertemporal technology or a decrease in investment "distortions". Finally, capital depreciates at rate δ. Capital accumulation is given by, $k_{t+1} = (1-\delta) k_t + I_t$.

The problem of the planner is

$$\begin{aligned} & Max \left\{ E_{t=0} \sum_{t=0}^{\infty} \beta^t u(c_t) \right\} \\ s.t.\ k_{t+1} &= (\theta_t A_t + 1 - \delta) k_t - \theta_t c_t \end{aligned}$$

Solving with respect to k_{t+1} we obtain the Euler equation of this economy, where β is the discount factor,

$$u'(c_t) = \theta_t \beta E_t \left\{ u'(c_{t+1}) \left[A_{t+1} + \frac{1-\delta}{\theta_{t+1}} \right] \right\}$$

The solution to this model presents a balanced growth path. With a utility function given by $u(c) = \frac{1}{1-\gamma} c^{1-\gamma}$, and using $\frac{u'(c_{t+1})}{u'(c_t)} = \left(\frac{c_{t+1}}{c_t}\right)^{-\gamma} = (1+g)^{-\gamma}$, the equivalent to the unconditional steady state here is (where (A, θ) denote unconditional expectations of these variables),

$$\frac{1}{\beta\theta} = (1+g_c)^{-\gamma} \left[A + \frac{1-\delta}{\theta} \right]$$

Typically, the long run growth rate $g_c(A, \theta)$ is obtained and comparative statics are performed on this variable. For example, here the more inneficient the intertemporal technology (lower θ), the lower the growth rate of

[7] Chari, Kehoe and McGrattan (2001) use the same time series to represent their investment distortion shock. Their notation is $1 + \theta_t = p_I / p_c$. They consider a concave model and estimate a time-varying switching process for the relative price process, using the entire PWT dataset.

consumption, g_c. In this paper we want to do the inverse inference: from knowledge of g_c we want to infer the properties of (A, θ).

Furthermore, we are not interested in comparative statics, but in understanding the nature of variations in the growth rate. Therefore we now impose logarithmic utility as it allows us to solve the dynamic programming problem analitically. To the defense of this shortcut we put forth that logarithmic utility is widely used, and also that here only the particular utility shape is a strong assumption as the linearity of the model is a building block of the entire exercise. Our model therefore retains some generality and the explicit policy functions we derive will prove extraordinarily useful.[8]

Before writing the dynamic programming problem, we define our random variables as Markov processes. The state space for the two shocks is a vector index and the state is defined as one realization for the pair (A, θ), and there are n possible pairs. The Markov transition matrix for this composite random variable is defined as $\Pi = [\pi_{ji}]$, where the current state is j and the future state is i.[9] The dynamic programming problem is then

$$V(k, j) = \max_{k'} \left\{ \log \left(B_j k - \frac{1}{\theta_j} k' \right) + \beta \sum_{i=1}^{n} \pi_{ji} V(k', i) \right\}$$

where $B_j = \left[A_j + \frac{(1-\delta)}{\theta_j} \right]$. This problem has a solution for the value function:

$$V(k, j) = a_j + b_j \log(k)$$

which implies a policy function of the type $k'(k, j) = \lambda_j k$, where the b_j are functions only of (β, Π), and the λ_j are functions of $\left(\beta, \Pi, A_j^j, \theta_j^j\right)$. The slope (b_j) of the value function is the solution to $[I(n) - \beta\Pi] \times [b] = [1]$, and $b(\beta, \Pi)$ is the same for all states j, and in fact it is simply $b = 1/(1-\beta)$. Using the first order condition in the above problem, $u_c \frac{1}{\theta_t} = \beta E V_{k_{t+1}}$, the policy function is given by

$$k_{t+1} = [B_t \theta_t] \left[\frac{\beta b}{1 + \beta b} \right] k_t = \beta \theta_t \left[A_t + \frac{(1-\delta)}{\theta_t} \right] k_t$$

where $\frac{\beta b}{1+\beta b} \equiv \beta$. This policy function has a significant property: it contains no parameters of the Markov probability matrix $\Pi = [\pi_{ji}]$. This will be

[8] Recently Jovanovic (2002) uses also log utility in an AK model to derive the negative correlation between growth and volatility found by Ramey and Ramey (1995).

[9] This specification captures the process estimated by Chari, Kehoe and McGrattan (2001), also a process with a trend and a random shock around it, and finally a unit root process. For existence of solution in a linear problem with a unit root process see Deaton (1991).

important below. We can now write current optimal consumption as

$$c_t = \left[A_t + \frac{(1-\delta)}{\theta_t}\right] k_t - \left[\frac{1}{\theta_t}\right] k_{t+1} = \left[A_t + \frac{(1-\delta)}{\theta_t}\right] (1-\beta)\, k_t$$

and this is completely determined by the current values of the state variables. It is therefore not necessary to have information on $c_t,\ A_t,\ \theta_t,\ and\ k_t$, to analize this economy. One variable is redundant, and we choose to eliminate capital. Even if we want to look at the stock of capital as strictly physical capital we know that measures of capital are the least reliable. As we will later consider a broader measure of capital we must eliminate k.[10]

We can further work this expression to get consumption growth only as a function of the stochastic processes[11]

$$\frac{c_{t+1}}{c_t} = \left[A_{t+1} + \frac{(1-\delta)}{\theta_{t+1}}\right] \theta_t \beta$$

We can also eliminate capital by using the consumption to output ratio:

$$\frac{c_t}{y_t} = (1-\beta) + (1-\beta)(1-\delta)\left[\frac{1}{A_t \theta_t}\right]$$

where we know all of (θ_t, c_t, y_t) separately. Other equations can be derived from the policy function:

$$\begin{aligned} \frac{y_{t+1}}{y_t} &= \frac{A_{t+1}}{A_t}\beta\left[\theta_t A_t + (1-\delta)\right] \\ \frac{y_{t+1}}{y_t} - \frac{c_{t+1}}{c_t} &= \beta(1-\delta)\left[\frac{A_{t+1}}{A_t} - \frac{\theta_t}{\theta_{t+1}}\right] \end{aligned}$$

This last equation states that output grows faster than consumption if the growth rate of the technology shock is higher than the growth rate of the relative price of investment $(1/\theta_t)$, as in that case the relative price of consumption is rising so agents respond by consuming less and investing more. Furthermore, it clearly shows that the certainty model would have

[10] There is one further reason to eliminate capital. The NIPA procedure and perpetual inventory methods to construct capital do not correct for our θ shock. It could be that it is implicit in the relative price measure from the data but that is unclear. By focusing on the consumption share we leave this problem buried under the recomputation of the share that we use, and will try to adress it later.

[11] If we have $E(\theta_t) \approx 1$ (roughly what we see in the data for physical capital) to match models where this technology is absent, the mean of A_t will have to be somewhat bigger than δ. The *size* of the shocks (their mean) matters to get the model in line with the observed magnitudes of both $\frac{c_t}{y_t}$ and $\frac{c_{t+1}}{c_t}$, unlike in concave models.

the same growth rates for consumption and output but that in a stochastic environment this need not happen at any point.

Using the data on consumption, output, and also on relative prices, and the equations above we back out time series for the technology shock for each country. However, despite the introduction of the intertemporal shock, when we use data on $\frac{p_c}{p_I}$ to measure θ we still have the problem exposed by Ingram, Kocherlakota and Savin (1994): the different equations we derive allow us to use data on $\frac{c_t}{y_t}$ and $\frac{c_{t+1}}{c_t}$ to recover two time series for (A_t) which may not be identical. We first want to see how close these two series are to each other, and also study their relationship with θ_t. If the two series were similar we would be able to use any of them to study the characteristics of the shocks. However, this is not the case and so, we will look for a solution to overcome this problem.

3 Data

All the data used in the paper come from the PWT 6.1. The data are in real terms, in 1996 prices. We use data for 24 countries and for the years 1950 trough 2000 in our analysis. Since some countries in our restricted sample lack the observation for 1950, we actually used the sample only from 1951 to 2000, resulting in 50 observations for each of the 24 countries. The countries are chosen based on availability of the long time series, and also on a reasonable level of development measured by income per capita. In what follows in parenthesis are the labels in the PWT dataset. As a measure of the intertemporal shock we first use the price of investment goods (PI) and the price of consumption goods (PC). This is the PPP index for consumption and for investment divided by the exchange rate. Their ratio $\frac{p_c}{p_i}$ is our first time series proxy for the intertemporal shock. We also extract the consumption and investment shares of GDP (KC,KI), and a GDP measure (RGDPL) to go with them.

Government

Because the model does not have government, we must remove government expenditure from our data. The cleanest procedure is to impose balanced budget with income taxes and assume expenditure is an exogenous additive shock. This is common in the RBC literature.[12] Given $G = \tau Y$, we

[12] It is done for instance by Chari, Kehoe and McGrattan (2001). The data, however, suggest this may not be the best reduced form: only 11 out 24 countries have the same sign on the correlations $\rho(c, g)$ and $\rho(i, g)$, which is what we expect if government expenditure works as an additive shock.

have:[13]

$$Y - G = Y - \tau Y = (1-\tau)Y = (1-\tau)Ak = \tilde{A}k$$

Prices

According to the PWT we can write $pY = p_c C + p_I I + p_G G$, and if we interpret the investment data as showing $k' - (1-\delta)k = I$, the shock θ described below in the model is actually $\frac{p_c}{p_I}$. The technology shock includes more terms now:

$$\begin{aligned} \frac{p}{p_c}Y - \frac{p_G}{p_c}G &= \left(\frac{p}{p_c} - \frac{p_G}{p_c}\tau\right) Y = \left(\frac{p}{p_c} - \frac{p_G}{p_c}\tau\right) Ak = \tilde{A}k \\ \tilde{A}k &= C + \frac{p_I}{p_c}\left[k' - (1-\delta)k\right] \end{aligned}$$

External Balance

Finally, in the data the three shares of consumption (KC), investment (KI) and government expenditure (KG) do not add up to 1. The missing element is the difference between exports and imports (E=X-M). Here we assume this object is an independent component of aggregate expenditure proportional to output (at a random factor e) so that we can subtract it as another shock. We have

$$\frac{p}{p_c}Y - \frac{p_G}{p_c}G - \frac{p_E}{p_c}E = \left(\frac{p}{p_c} - \frac{p_G}{p_c}\tau - \frac{p_E}{p_c}e\right) Ak = \tilde{A}k$$

and this completes our procedure to get the model in line with the data.

Shares

The consumption and investment shares must be recomputed. New output is now equal to $\tilde{Y} = pY\left[\frac{p_c C}{pY} + \frac{p_I I}{pY}\right]$, which removes the government component and the external balance. The corresponding consumption share of this measure of output is $\left[\frac{p_c C}{pY}\right] / \left[\frac{p_c C}{pY} + \frac{p_I I}{pY}\right]$.

Data facts

The short sample characteristics of our data are important. We performed a variety of unit root tests and their outcome points to stationarity in consumption (and output) growth (c_{t+1}/c_t), and to a unit root in the consumption share (c_t/y_t) and the relative price (p_c/p_I). Our unit root testing and data treatment follows Baxter, Jermann and King (1998) who also investigate the stationarity of some NIPA ratios for eleven countries and find

[13] Canton (2001) explores an RBC model with random tax rates as a driving mechanism, very much in this spirit. However, the microeconomic structure of the intratemporal shock is not an issue in this paper.

mixed evidence of non stationarity.[14] This feature would clearly condition the inference regarding the relative importance of the two shocks, and their relationship, but we again follow the reasoning of Baxter, Jermann and King (1998) and proceed with our analysis assuming the data are draws from stationary distributions. We will explore the implications of the presence of unit roots in future work. With the data we can now examine a first implication of our model.

4 A preliminary exercise

We use the expressions for the rate of growth of consumption and income and the expression for the share of consumption to obtain a simple implication we can take to the data. After some algebra we are able to eliminate first the technology shocks and then β and δ, obtaining the following identity (which the model implies is verified at every point):

$$Z_t \equiv \frac{y_{t+1}}{y_t} - \frac{c_t}{c_{t-1}} \frac{\theta_t}{\theta_{t+1}} \frac{\theta_t}{\theta_{t-1}} \equiv 0$$

We construct a time series of the left hand side of this equation (Z_t) for each country in our panel. If the model is correct any divergences between the data and zero are due to measurement error. We assume this measurement error is iid normally distributed with mean zero, and test whether the sample means for each country are significantly different from zero. As it turns out, they are not, and the test statistic (the sample mean over its standard deviation) lies comfortably inside the usual confidence intervals for the normal distribution. Table 1 below contains the statistic to test whether a sample mean differs from zero, assuming each observation of the statistic (Z_t) is iid normally distributed with mean zero and some standard deviation:

Table1	MET		MET		MET
AUS	0.14	FRA	0.13	MEX	−0.14
AUT	−0.07	GBR	0.37	NLD	0.26
BEL	0.48	GRC	−0.03	NOR	−0.22
CAN	0.24	IRL^*	0.70	NZL	−0.19
CHE^*	0.23	ISL	−0.06	PRT^*	0.13
DNK	0.09	ITA	−0.70	SWE	−0.52
ESP	−0.03	JPN^*	−0.02	TUR	−0.82
FIN	−0.16	LUX	−0.56	USA^*	0.31

[14]The tests were a variety of univariate Dickey-Fuller tests. We computed also 95% confidence intervals for the autoregressive root, following Stock (1991).

All countries have a statistic well inside any usual (± 1.96) confidence interval of the standard normal distribution, implying a non rejection of the null hypothesis that Z_t is not statistically different from zero. An average, however, tells us only so much. We could have a zero mean with a trend intercepting zero at the sample midpoint. This would be troublesome. But it is not the case. A plot of the time series of the Z_t for all countries in Figure 1 reveals a clear noise around an almost perfect zero. The one caveat is that a regression of Z_t against a constant and Z_{t-1} shows that 19 countries in Table 1 have significant negative autocorrelation. In Table 1, the countries marked with an asterisk have a T statistic on the first lag below 1.96.

This is an interesting outcome since this test largely does not reject the model. Given the well known fragility of this model when confronted with the data, this outcome and its robustness is surprising: this test yields the same qualitative results if we use the data straight from the PWT without extracting government expenditure or external balance and also yields the same results with the previous versions of the PWT data. In addition, and to antecipate a topic to be treated in the appendix, if we test the above equation where all growth rates are divided by their respective USA counterpart, we also get the same results. However, and significantly, if we simply test the difference between consumption and output growth, we find largely the same result, which is also true in ratios relative to the USA.[15]

5 Technology shocks

We now take the relative price we find in the data as the true measure of θ, and proceed to recover *only* the A shocks and examine the coherence of the different series we are able to obtain. Then we look at the correlation between A and θ. In order to do so we need values for our parameters. To this effect we follow the standard procedure in the literature and impose a common value of (β, δ), for all countries. These values are 0.94 and 0.1 respectively, and are not estimated but rather follow a common benchmark in macroeconomic models.[16][17]

[15] This suggests the relative price shock is not playing a very active role in the data and we will return to this issue later. Also, the very pervasiveness of the test outcomes, suggests this test is not very informative. But since the above equation (as all equations shown here) is a restatement of the Euler equation, it is nevertheless a test of the model.

[16] The value of β in a concave model is related to the marginal product of capital. Here that is not the case. We need $\beta < 1$ to have a bounded problem but it is not clear we can pin down its value in the same way.

[17] We tried structural estimation by choosing parameters (β, δ) for each country to minimize the distance between the two A series. We did not obtain acceptable values for (β, δ),

We invert two of the equations derived to obtain two series for the intratemporal shock. From the consumption growth expression we obtain

$$A_{t+1} = \frac{c_{t+1}}{c_t}\frac{1}{\theta_t \beta} - \frac{(1-\delta)}{\theta_{t+1}} \Rightarrow A_t\left(\frac{c_t}{c_{t-1}}, \theta_{t-1}, \theta_t\right)$$

and from the consumption output ratio we get:

$$A_t = \frac{1}{\theta_t}\frac{(1-\beta)(1-\delta)}{\frac{c_t}{y_t} - (1-\beta)} \equiv A_t\left(\frac{c_t}{y_t}, \theta_t\right)$$

We note that from this algebra we recover the *exact* time series of A. This is conditional on the parameter values (β, δ) and on having $\frac{c_{t+1}}{c_t}$, $\frac{c_t}{y_t}$, and θ_t, measured without error.

The two equations above yield two different time series for A, meaning that our approach mantains the singularity problem exposed by Ingram, Kocherlakota and Savin (1994). If the two series were similar this would be only a theoretical problem with no empirical implications. So, we now verify if these two series are significantly different from each other. According to the model the difference

$$DA_t = A_t\left(\frac{c_t}{c_{t-1}}, \theta_{t-1}, \theta_t\right) - A_t\left(\frac{c_t}{y_t}, \theta_t\right) \equiv 0$$

should be identically zero, and therefore we assume as before that any divergences between the two series are due to iid measurement error normally distributed with mean zero.

Table2	*Test*		*Test*		*Test*
AUS	18	*FRA*	20	*MEX*	5
AUT	16	*GBR*	28	*NLD*	17
BEL	25	*GRC*	10	*NOR*	20
CAN	24	*IRL*	20	*NZL*	9
CHE	21	*ISL*	9	*PRT*	12
DNK	11	*ITA*	14	*SWE*	11
ESP	16	*JPN*	14	*TUR*	4
FIN	15	*LUX*	7	*USA*	26

and clearly the test on whether DA is different from zero shown in Table 2

and even at the optimal estimates the two A series remained statistically very different. A more detailed discussion of our exercises with singular models is contained our working paper 2003-06.

emphatically rejects it (the tests are above 1.96) for every country.[18] The difference between the two series implies that we should not use any of them as a good estimate of the true technology shock and also means a bad performance of this simple model. However, if the two series had the same characteristics implying the same qualitative behavior for the technology shock for instance in what refers to the correlation with the intertemporal shock, this could still be a useful exercice. We now show that this is not the case.

Testing the correlation coefficient

The time series $\theta_{j,t}$ for country j is treated as an exogenous shock in the model, so that we do not need to investigate its relationship with the endogenous variables. We test whether there is significant linear correlation between θ_t and A_t. Assuming both variables are stationary, the estimated correlation coefficient is given by $\hat{\rho} = \frac{Cov(A,\theta)}{\sigma(A)\sigma(\theta)}$, and this variable is distributed with mean ρ, and standard deviation $\sqrt{\left(1-\hat{\rho}^2\right)/(T-2)}$, leading to the test statistic:

$$t_{(T-2)} = (\hat{\rho}-\rho)\sqrt{\frac{T-2}{1-\hat{\rho}^2}}$$

and under the null hypothesis that they are uncorrelated we just set the true correlation at $\rho = 0$.

Table 3 below shows the correlation between the series (A, θ). There are two sets of correlations, the first one (first column) uses the A series generated using the $\frac{c_t}{y_t}$ equation, and the second one uses the A generated by the $\frac{c_{t+1}}{c_t}$ equation.. As a rule of thumb, a correlation with an absolute value above 0.28 is statistically significant at 95%.

[18] This was predictable given that the mean of $A_t\left(\frac{c_t}{y_t}, \theta_t\right)$ is 0.409 times the mean of $A_t\left(\frac{c_t}{c_{t-1}}, \theta_{t-1}, \theta_t\right)$, and the factor for the standard deviation is 0.389. These numbers are computed as cross section mean of (mean(A1)/mean(A2)) for each country and cross section mean of (std(log(A1))/std(log(A2))). Curiously, only three countries display significat trend or autocorrelation in this difference indicator. Japan is the most significant of those.

Table3 $\rho(A,\theta)$	$\frac{c_t}{y_t}$	$\frac{c_{t+1}}{c_t}$	$\rho(A,\theta)$	$\frac{c_t}{y_t}$	$\frac{c_{t+1}}{c_t}$	$\rho(A,\theta)$	$\frac{c_t}{y_t}$	$\frac{c_{t+1}}{c_t}$
AUS	-0.01^*	-0.05^*	FRA	0.49	-0.56	MEX	0.82	0.18^*
AUT	0.49	0.14^*	GBR	0.17^*	-0.06^*	NLD	-0.07^*	-0.16^*
BEL	0.65	-0.13^*	GRC	0.47	0.31	NOR	0.49	-0.13^*
CAN	0.16^*	-0.27^*	IRL	0.14^*	0.17^*	NZL	0.63	-0.01^*
CHE	0.60	0.02^*	ISL	0.30	-0.17^*	PRT	0.18^*	0.04^*
DNK	-0.65	0.07^*	ITA	-0.43	0.22^*	SWE	0.67	-0.01^*
ESP	0.29	-0.29	JPN^{**}	-0.00^*	-0.67	TUR	-0.18^*	-0.07^*
FIN	0.62	-0.32	LUX	0.21^*	0.05^*	USA	-0.33	-0.09^*

The results show that the choice of data $\left(\frac{c_t}{y_t}, \frac{c_{t+1}}{c_t}\right)$ used to generate the technology shock is not inoccuous. A test on the null that the mean of the difference between the values in the two columns is zero yields the value 3.3191 which is a rejection. Moreover, when correlations are significant using both data sources (four countries), they often come with opposite signs (Spain, Finland and France, the exception being Greece), and often the correlations are not significantly different from zero (when marked with an asterisk). Finally, the correlations are mainly positive if we use c_t/y_t but mainly negative if we use c_{t+1}/c_t.

6 Two Shocks: a broad measure of capital

Here we take into account that the capital stock should be understood as a broad measure that includes physical, human and organizational capital. This implies that the true intertemporal shock in the model has two components, only one of which we observe in the data as the relative price of consumption to investment. Consider for instance that:

$$\theta = \phi_h \theta_k$$

where we use *for illustration purposes* a subscript labelling human and physical capital.[19] The task then is to recover two shocks from the data, rather

[19] Several colleagues have noted that there is an inconsistency betwen our aim of being very specific in taking the model implications to the data, and then sweeping under the rug the obvious need for the disaggregation of the broad capital stock of the AK model. After trying hard we have not been able to come up with a model that will allow for an analytic solution with more than one stock. For such an exercise a different approach is needed and we are working on it.

than just one. Simultaneously this allows us to solve the Ingram, Kocherlakota, and Savin (1994) problem of singularity. We have again:

$$
\begin{aligned}
\frac{c_{t+1}}{c_t} &= \left[A_{t+1} + \frac{(1-\delta)}{\theta_{t+1}}\right]\theta_t\beta \\
\frac{c_t}{y_t} &= (1-\beta) + (1-\beta)(1-\delta)\left[\frac{1}{A_t\theta_t}\right] \\
\frac{y_{t+1}}{y_t} &= \frac{A_{t+1}}{A_t}\beta\left[\theta_t A_t + (1-\delta)\right]
\end{aligned}
$$

and note that we have several equations and two shocks, but that by the data construction only two equations are independent. The economic question is what we can learn from the shocks we are backing out. After some algebra we obtain:

$$
\begin{aligned}
\frac{\theta_t}{\theta_{t-1}} &= \frac{\frac{c_t}{y_t}}{\frac{c_t}{y_t} - (1-\beta)} \frac{\beta(1-\delta)}{\frac{c_t}{c_{t-1}}} \\
A_t &= \frac{1}{\theta_t}\frac{(1-\delta)}{\frac{c_t}{y_t}\frac{1}{1-\beta} - 1}
\end{aligned}
$$

and to pin down the θ_t series we assume that the initial values equal the observed relative price for each country for all countries.

The outcome, shown in Figure 2, is stunning. The shocks we back out show that A_t is growing exponentially, and second, that the true intertemporal price ($\theta_t = \phi_{ht}\theta_{kt}$) is falling also exponentially ($\frac{Pc_t}{Pi_t}$, despite some evidence of unit roots is a model of stability by comparison with the recovered composite θ_t).

Basically, consumption goods become ever cheaper, and therefore the relative price of consumption is fast approaching zero.[20] This is triggered by fast growth in technology. In order to have stable growth it is necessary for the relative price of investment to *rise* quickly. For every country. It is interesting to note that this runs counter the idea in the investment specific technological progress literature - a by now classic reference being Greenwood, Hercovitz, and Krusel (1997) - which looks at a *falling* relative price of equipment goods.

[20] This exercise is done with a common β (0.94) and a common δ (0.1). But carefully adjusting δ for each country also does not yield white noise series for A and ϕ.

6.1 Characteristics of the shocks

What can we say about the characteristics of the shocks and their relative impact on the economy? We know already from the literature that whatever inference one tries to make, depends crucially on the order of orthogonalization of the two shocks. The one case in which inference is valid is when the different shocks we obtain come out uncorrelated. So, we begin by obtaining the coeficient of correlation between the shocks. But first, since the time series of both A and composite θ, are exponential, *we take logs and remove a linear trend.*[21] Then we compute the correlation coefficient for each country of the log detrended data. The outcome is:

	$\rho(A_t, \theta_t)$		$\rho(A_t, \theta_t)$		$\rho(A_t, \theta_t)$
AUS^*	-0.744	FRA	-0.865	MEX	-0.959
AUT	-0.925	GBR	-0.754	NLD	-0.821
BEL^*	-0.682	GRC	-0.929	NOR	-0.889
CAN^*	-0.938	IRL	-0.943	NZL	-0.929
CHE^*	-0.798	ISL	-0.758	PRT	-0.941
DNK	-0.796	ITA	-0.736	SWE	-0.755
ESP	-0.986	JPN	-0.986	TUR	-0.748
FIN	-0.961	LUX	-0.647	USA	-0.913

We can see that all countries have strongly significant negative correlations between the two shocks.[22] It is also a robust outcome, as it arises often in the singular model exercises and it arises also in the model with taste shocks explored in the Appendix. This is an interesting result which has implications about the economic meaning of these shocks. It makes it difficult to accept an interpretation of these shocks based on the quality of institutions. In principle a positive shock on institutions should simultaneously help the productivity of both the final output sector and the investment sector. So we should look for alternative explanations as the investment shock we obtain (the cost of the broad measure of capital) behaves very differently from the price of physical capital directly available in the data. Looking at extensions of the AK model that include explicitly human capital as Rebelo

[21] The shocks we obtain in this way are non stationary and cannot be realizations of a stationary Markov process. However, the expressions derived are functions of the product $\phi_t A_t$, and this product is stationary, so that there is no incompatibility with the model. In the derivation of the policy functions a joint Markov process is assumed.

[22] The mean correlation is -0.8503 with a standard deviation of 0.1035. Interestingly, this is also true across countries, holding time constant. For this "transposed" exercise the mean of the 50 - time constant - cross section correlations between the shocks is -0.8528 with a standard deviation of 0.0696.

(1991) or the optimal leisure/labor choice and taxes on labor income as in McGrattan (1998) may be important.

This strong correlation also implies that inference about their relative importance depends on the order of orthogonalization. But some lessons may nevertheless be extracted from such an exercise which we perform below.

6.1.1 Which shock is more important?

To answer this question we use several measures. We use both the raw series of shocks that we extracted from the data and also transformed series that we obtain after orthogonalizing the shocks to get an independent impact of each shock

Part 1. Orthogonalizing

We conduct two different orthogonalizations because we have no prior on how the two shocks are related, and because we want to isolate the individual contribution of each shock. In one case we regress by OLS

$$\theta_t = a + bA_t + \epsilon_t$$

and then use the pair $(\hat{\theta}_t = \hat{a} + \hat{b}\bar{A} + \hat{\epsilon}_t, A_t)$ where $\bar{A}$ is the mean of A, thereby removing from θ_t the component that can be explained by A_t. In the other case we just switch the shocks. We do this for every country. *Note that this filtering is applied to the log detrended component only.* It is this noise (orthogonalized or not) that is added to the trend exponential.

Part 2. Evaluating the impact of each shock

We are interested in the impact of each shock on the movement of the different data series. We evaluate it by comparing the true data with an adequately generated artificial series. This artificial data is produced by shutting down one of the shocks at its country average (keeping only the exponential trend). We do this for the output growth equation: we compare the true $\frac{yt+1}{yt}$ data to the following two alternatives:[23]

$$\begin{aligned} \frac{y_{t+1}}{y_t}|_{\bar{A}} &= \beta\left[\theta_t\bar{A} + (1-\delta)\right] \\ \frac{y_{t+1}}{y_t}|_{\bar{\theta}} &= \frac{A_{t+1}}{A_t}\beta\left[\bar{\theta}A_t + (1-\delta)\right] \end{aligned}$$

where here $\overline{x}$ stands for the exponential trend of x with no noise.

We run an OLS regression of the true data, against the artificial series generated with only one shock and the other shock set to its exponential

[23] Chari, Kehoe and McGrattan (2001) and Restuccia and Urrutia (2001) effectively shut down the technology shock.

trend. So we run the following estimations and use the R squared as a measure of the ability of the shocks to explain the variance in the output rate of growth:[24]

$$\frac{y_{t+1}}{y_t} = \alpha_0 + \alpha_1 \left[\frac{y_{t+1}}{y_t}|_{\bar{\theta}} \right] + \epsilon_t$$

$$\frac{y_{t+1}}{y_t} = \alpha_0 + \alpha_1 \left[\frac{y_{t+1}}{y_t}|_{\bar{A}} \right] + \epsilon_t$$

Part 3. Results

Table 4 shows the R squared of a series of regressions. Column 1 shows the regression of the true $\frac{y_{t+1}}{y_t}$ data against $\frac{y_{t+1}}{y_t}|_{\bar{A}}$ as defined above, where the technology shock (A) is set at the country specific trend with no noise. This produces $R_1^2(\theta_t)$, which is a measure of the explanatory power of θ_t where θ_t is taken from the raw data.

Column 2 regresses the true $\frac{y_{t+1}}{y_t}$ data against data constructed using the projection of the residual of $\hat{\theta}_t$, that gives $\hat{\theta}$ the least explanatory power (removing from θ_t the component that can be explained by A_t which biases the explanatory away from θ). There is not much much difference between the two columns. Note that in columns 1 and 2, the artificial $\left[\frac{y_{t+1}}{y_t}|_{\bar{A}}\right]$ is always constructed using the true trend of A, and the respective θ series for each country.

Column 3 fixes θ to its country specific trend and uses the raw A series, while column 4 uses the orthogonalized series for the residual of A, that gives A the least explanatory power.

[24] There are a variety of ways to do this experiment. We could simply regress the actual data against θ, or against A, directly. But then we would have problems with mispecification of the regression due to the non linear relationship between the data and the shocks. We choose to run the regression of the actual data against the artificial series because we believe the R squared of this regression is a better measure of what is missing when we use only one shock.

Table4 y_{t+1}/y_t	$R_1^2(\theta_t)$	$R_2^2\left(\hat{\theta}_t\right)$	$R_3^2(A_t)$	$R_4^2\left(\hat{A}_t\right)$
AUS	0.13	0.00	0.99	0.66
AUT	0.01	0.02	0.98	0.09
BEL	0.09	0.06	0.98	0.75
CAN	0.00	0.06	0.98	0.76
CHE	0.01	0.00	0.99	0.79
DNK	0.03	0.00	0.98	0.18
ESP	0.05	0.03	0.95	0.01
FIN	0.04	0.00	0.98	0.10
FRA	0.02	0.03	0.96	0.03
GBR	0.03	0.00	0.99	0.52
GRC	0.01	0.02	0.98	0.04
IRL	0.06	0.02	0.99	0.33
ISL	0.11	0.00	0.99	0.22
ITA	0.16	0.10	0.99	0.45
JPN	0.06	0.38	0.89	0.01
LUX	0.00	0.00	0.99	0.27
MEX	0.07	0.00	0.99	0.34
NLD	0.09	0.03	0.99	0.08
NOR	0.00	0.01	0.98	0.18
NZL	0.06	0.04	0.99	0.43
PRT	0.03	0.00	0.97	0.10
SWE	0.09	0.01	0.99	0.06
TUR	0.11	0.00	0.99	0.47
USA	0.04	0.00	0.99	0.66
mean	0.055	0.035	0.980	0.315

Although we can't say how much each shock explains of total output variance, one pattern emerges: shutting down θ has less of an impact than shutting down A. Reducing theta to its minimal contribution yields an R squared of 3.5%, whereas reducing A to its minimal contribution yields an R squared of 31%.[25]

[25] This is in line with the results of Chari, Kehoe and McGrattan (2002), but contrary to the conclusions of Chari, Kehoe and McGrattan (2001).

7 Conclusion

In this paper we take an AK model to the data. Our prior is that if endogenous growth theory is correct, then a very stylized linear model should do well against the raw data, just as the early stylized concave models did against log detrended data. This, along with the existence of a vast literature on endogenous growth, is the reason we use the AK model.

We consider two sources of fluctuations in our model, an intratemporal shock and an intertemporal shock. Once we take into account that capital should be understood as a broad measure that also includes human and organizational capital, both shocks are unobserved. We solve explicitly for the optimal investment decision in the model and take the exact implications of this optimal decision to the data. This allows us to recover the *exact* time series for the technology shock and the investment shock. We are then able to investigate the properties of the two shocks and their impact on the growth rate.

When we extract both shocks from the data we obtain that they are negatively correlated, and that the investment shock plays a somewhat lesser role in the variation of output growth. We believe these are pervasive features of macroeconomics in postwar data as illustrated by our experiments and by other work in the literature that uses concave models.

The correlation is strong which makes it impossible to make definitive statements about the contribution of each shock to output variance. By construction the presence of the intertemporal shock (of both shocks) is a necessary condition for the model to fit the data. Therefore, even though our experiments suggest that this shock has a somewhat lesser role in the variation of output growth, this does not imply the shock itself is not "significant".

The negative sign of the correlation makes it difficult to accept an interpretation of these shocks based on the quality of institutions. In principle a positive shock on institutions should simultaneously help the productivity of both the final output sector and the investment sector. So future research should look for alternative explanations as the investment shock we obtain (the cost of the broad measure of capital) behaves very differently from the price of physical capital directly available in the data.

Finally, we emphasize here that the contribution of our exercise is on what the shocks we recover suggest regarding the underlying economics at work in the countries studied. This is so because the model with two shocks cannot be rejected by the data and all we can obtain is a measure of our ignorance that we then try to make sense of. On the other hand, as pointed out earlier in the literature and quantitatively illustrated here, a one shock model is strongly rejected by the data, so that any inference based on it is not valid. So, we do not "test" endogenous growth, but rather use the AK model as a tool to uncover characteristics of intertemporal and intratemporal shocks that we believe are common in postwar Macroeconomic data.

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8 Appendix 1: Country Ratios

Restuccia and Urrutia (2001) raise an important concern about the use of the relative price data for an individual country. They note that given the way the PWT price data is constructed all the error free information on θ we have is the ratio of thetas between two countries, or as they do, the ratio relative to the United States.[26] We explore what their constraint on using the data implies for the AK model to see if that leads to different implications from what we get using individual country price data. Specifically we want to see if our two main conclusions - the negative correlation between the two shocks, and the relative irrelevance of the theta shock - are maintained. We will do it somewhat indirectly and discuss our experiments as we go along.

8.1 Redoing the preliminary test of the model

We had for each country:

$$\frac{y_{t+1}}{y_t} \equiv \frac{c_t}{c_{t-1}} \frac{\theta_t}{\theta_{t+1}} \frac{\theta_t}{\theta_{t-1}}$$

and now for ratios

$$\frac{\frac{y_{i,t+1}}{y_{i,t}}}{\frac{y_{t+1}}{y_t}} \equiv \frac{\frac{c_{i,t}}{c_{i,t-1}} \frac{\theta_{i,t}}{\theta_{i,t+1}} \frac{\theta_{i,t}}{\theta_{i,t-1}}}{\frac{c_t}{c_{t-1}} \frac{\theta_t}{\theta_{t+1}} \frac{\theta_t}{\theta_{t-1}}} \Longleftrightarrow Z_t \equiv \frac{\frac{y_{i,t+1}}{y_{i,t}}}{\frac{y_{t+1}}{y_t}} \frac{\frac{c_t}{c_{t-1}}}{\frac{c_{i,t}}{c_{i,t-1}}} - \frac{\frac{\theta_{i,t}}{\theta_{i,t-1}}}{\frac{\theta_t}{\theta_{t-1}}} \left[\frac{\frac{\theta_{i,t+1}}{\theta_{i,t}}}{\frac{\theta_{t+1}}{\theta_t}} \right]^{-1} \equiv 0$$

where this information will be error free according to the above authors. We construct a time series of Z_t for each country in our panel. If the model is correct any divergences between Z_t and zero are due to measurement error - assumed to be iid normally distributed with mean zero. We again test whether this is true on average, and once again all values for the measurement error test are well inside the usual 1.96 confidence interval. The time series for (Z_t) is flat around zero. It is zero on average. There is significant (tstatistic >1.96) negative serial correlation in 18 countries in the Table1* experiment.

[26]Because the price measure includes an international component, measurement error in this component induces a spurious correlation between relative prices and income (or income growth) or investment rates (RU, page 119).

Table1 $*$	MET		MET		MET
AUS	-0.05	FRA	-0.23	MEX	-0.12
AUT	-0.19	GBR	-0.09	NLD	0.03
BEL	0.06	GRC	-0.09	NOR	-0.40
CAN	-0.16	IRL	0.32	NZL	-0.33
CHE	-0.12	ISL	-0.09	PRT	0.11
DNK	0.10	ITA	-0.69	SWE	-0.45
ESP	-0.13	JPN	-0.25	TUR	-0.84
FIN	-0.30	LUX	-0.59	USA	$**$

Thus, using the data in ratios or using the data for each country individually does not affect the performance of this test.

8.2 Investment rates and relative prices

Here we compare the Restuccia and Urrutia model to our model. For a given country the steady state of their (concave) model implies the investment share[27]

$$\frac{I}{Y} = \theta \frac{\alpha\left[(1+g)(1+n) - (1-\delta)\right]}{\frac{(1+g)^{\sigma}}{\beta} - (1-\delta)}$$

and under the assumption that all countries share the same parameter values except for independent draws of $\theta = \frac{p_c}{p_I}$ from a common distribution[28], the ratio between two countries yields

$$\frac{I^j/Y^j}{I^i/Y^i} = \frac{\theta^j}{\theta^i}$$

which then implies that if country j has a higher θ on average (in steady state), and thus a cheaper investment, its investment rate is correspondingly higher. They show this is approximately true in the data.[29] However, the

[27] Here g is the exogenous growth rate of technology. α is the exponent on capital in production. n is the exogenous growth rate of the labor force. δ is capital depreciation. σ is the concavity of CRRA utility, and β is the intertemporal discount factor.

[28] RU estimate a matrix that implies the volatility of the distribution falls with time if we start the time series at the initial distribution. But this is not true for every subsample of countries. Their matrix implies *any* subsample should see the cross section volatility fall secularly.

[29] Note here that RU run a cross section OLS regression of $\frac{I^j/Y^j}{I^{usa}/Y^{usa}}$ against a constant and $\frac{\theta^j}{\theta^{usa}}$, and obtain a coefficient on the theta ratio very close to 1. The R squared however is small. The year that delivers a higher R squared is also the year with the constant further from 0 and slope further from 1. The constant is also significant in most

apparent success of the RU analysis hides some caveats. If we take the variable $Z_1 = \frac{I^j/Y^j}{I^i/Y^i} - \frac{\theta^j}{\theta^i}$ for the countries in our sample, we obtain a test statistics that: i) reject that on average Z is zero, ii) reject that Z is not autocorrelated, and, iii) reject that a linear trend is not significant (for most countries).

But suppose the true model is a stochastic AK model. Then in our framework we have in any given period

$$\frac{I_t^j/Y_t^j}{I_t^i/Y_t^i} = \frac{\theta_t^j\beta - (1-\beta)(1-\delta_j)/A_t^j}{\theta_t^i\beta - (1-\beta)(1-\delta_i)/A_t^i}$$

If we perform the same tests for this expression, with A shut down to 1,[30]

$$Z_2 = \frac{I_t^j/Y_t^j}{I_t^{usa}/Y_t^{usa}} - \frac{\theta_t^j\beta - (1-\beta)(1-\delta)}{\theta_t^{usa}\beta - (1-\beta)(1-\delta)}$$

the characteristics of Z_2 are, for every country, virtually identical to those of Z_1. Again, we just set β equal to 0.94 and δ equal to 0.1 for all countries.

What does this imply for the two models? For the concave model (Z1) this implies that at the very least we reject that all countries have the same parameters. Their model in its simple form fails this test. For our model that is not quite the case. What happens is that Z2 will be captured in the implied (ϕ, A) shocks.

Regressions

Now we mimic the exercise in RU, and run a regression of the log of the left hand side against the log of the right hand side of Z1 and Z2.[31] We do this over the time series and over the cross section. Here we show the averages of the time series (for each country) and cross section (for each date) exercises:

Z1	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$T(\hat{\alpha}_0)$	$T(\hat{\alpha}_1)$	R^2
TS	0.131	1.038	5.423	7.406	0.485
CS	0.149	0.840	3.244	3.842	0.384
Z2	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$T(\hat{\alpha}_0)$	$T(\hat{\alpha}_1)$	R^2
TS	0.131	0.979	5.428	7.423	0.486
CS	0.149	0.792	3.254	3.857	0.386

cases whereas it should not be. So, even though their regression is reasonably successfull, it also can be viewed in the opposite way as showing that a substantial amount is left to be explained in the behaviour of investmnet rates.

[30] If we use the time series for the A shock derived using C/Y we would set the D2 below identically equal to zero. Also, this is what makes the exercise here equivalent to theirs.

[31] The reader is encouraged to consult figure 3 on page 106, and also table 3 on page 107. Restuccia and Urrutia (2001). If we reproduce fig 3 in their paper for our 22 countries, we obtain a clear positive trend (as our theta variable is the inverse of theirs). The regressions are produced in Octob2002.m for early PWT data and july2003.m for the PWT61 data.

The results are virtually identical for the two equations (not just on average) suggesting that this exercise cannot be a test of the model. In addition, the R squared is quite low suggesting the fit of the model is not very good.

Does it matter doing it in ratios? We redo this exercise without defining the variables as ratios relative to the USA. We get:

$Z1$	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$T(\hat{\alpha}_0)$	$T(\hat{\alpha}_1)$	R^2
TS	-1.489	0.235	-50.8	1.997	0.202
CS	-1.553	0.840	-38.7	3.842	0.385
$Z2$	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$T(\hat{\alpha}_0)$	$T(\hat{\alpha}_1)$	R^2
TS	-1.462	0.220	-59.6	1.998	0.202
CS	-1.456	0.792	-36.8	3.857	0.386

and we see that the cross section exercise is unchanged by the fact that we are looking at individual countries. The time series exercise in now quite different, in particular the R squared is much lower and the coefficient on the price variable is also much lower (further away from 1).

Using single country information generates a worse relationship between investment rates and the measured relative prices over the time series. This may put more explanatory power in the shocks (ϕ, A) that we recover using the model, than we would otherwise obtain if we could use information in country ratios. Nevertheless note that the R squared is at best 50% suggesting that most of the explanatory power we obtain from (ϕ, A) is likely to be independent of this.

8.3 Recovering the shocks using ratios

Can we generalize the ratio exercise? For example, can we recover ratios of A shocks using the model equations and information on ratios only? The equations we derive are non linear relationships between variables. So, we cannot recover the shocks. The little we can do does not seem to make much difference.

We observe without error only

$$R_{i,t} = \frac{\theta_{i,t}}{\theta_{1,t}}$$

where $\theta_{1,t}$ denotes the USA true relative price (unobserved), and $\theta_{i,t}$ denotes the true relative price of country i (also unobserved).

If we use $\frac{c_{t+1}}{c_t}$ to infer A we obtain a non tractable nonlinear expression. Using, for each country, the consumption to output ratio equation to generate

A we obtain

$$\left[\frac{c_{i,t}}{y_{i,t}}\frac{1}{(1-\beta)}-1\right]\frac{1}{(1-\delta)} \equiv X_{i,t} = \frac{1}{A_{i,t}\theta_{i,t}} = \frac{1}{A_{i,t}R_{i,t}\theta_t}$$

where we drop the subscript 1 for the USA. We can construct from the observables,

$$\frac{A_{i,t}}{A_t} = \frac{X_t\theta_t}{X_{i,t}R_{i,t}\theta_t} = \frac{X_t}{X_{i,t}}\frac{1}{R_{i,t}}$$

which is the relative technology ratio for each country at any moment. However, by constructing $X_{i,t}$ individually for each country, we are simply using the same information as before. So we cannot expect different results.

The same is true for the output growth equation:

$$\frac{\left[\frac{y_{t+1}}{y_t}\right]_j}{\left[\frac{y_{t+1}}{y_t}\right]_{usa}} = \frac{\left[\frac{A_{t+1}}{A_t}\right]_j}{\left[\frac{A_{t+1}}{A_t}\right]_{usa}}\frac{[\theta_{tj}A_{tj}+(1-\delta)]}{[\theta_{t,usa}A_{t,usa}+(1-\delta)]}\frac{\beta}{\beta}$$

so that we do not have a way of recovering the shocks using only country ratio information.

There is a simpler way to think about this problem: if we could manipulate the policy function such that we could use only country ratio information to recover the shocks, then this would imply that there would be no difference whatsoever in using country ratios relative to the USA or individual country data. But we cannot do it due to the nonlinearity of the equations. Now, it is exactly because we cannot do it that it may make a difference. And it is exactly because of that, that we cannot know what difference it makes. Note that a regression against the ratio $\frac{\theta_{tj}A_{tj}}{\theta_{t,usa}A_{t,usa}}$ is mispecified.[32]

So, does the concern raised by RU matter for our outcomes? The honest answer is that we cannot know for sure, but the experiments that we can make suggest that it may not be too serious a problem given that the relationship between the investment ratio and the price ratio is largely unexplained anyway.

[32] There is one way of adressing this problem and finding out its empirical significance. It is to construct an entirely artificial panel where we construct the data with a common component at our choice, and then run these experiments. This is something we will do in the future.

9 Appendix 2: Taste Shocks

Another extension introduces a preference shock (which amounts here to a stochastic discount factor). Consider the utility function $U(c) = \phi \log(c)$, where ϕ is a random variable. This exercise serves the purpose of showing that our results from the main text have robustness regarding model specification. Using the same approach as above we can derive the policy function:

$$k_{t+1} = \beta \left[\frac{\theta_t A_t + (1-\delta)}{\beta + \phi_t (1-\beta)} \right] k_t$$

which reduces to the previous function when $\phi = 1$. This produces the expression for the consumption income ratio:

$$\frac{c_t}{y_t} = 1 + \frac{(1-\delta)}{\theta_t A_t} - \frac{\beta}{\theta_t A_t} \left[\frac{\theta_t A_t + (1-\delta)}{\beta + \phi_t (1-\beta)} \right]$$

and one can use this model to see what the shocks we rocover look like. We can rearrange $\frac{c_t}{y_t}$, to get

$$\phi_t = \frac{\beta}{1-\beta} \left[\frac{\theta_t A_t \frac{c_t}{y_t}}{(1-\delta) + \theta_t A_t \left(1 - \frac{c_t}{y_t}\right)} \right]$$

and then use $\frac{y_{t+1}}{y_t}$, to derive recursively

$$A_{t+1} = \frac{y_{t+1}}{y_t} \frac{A_t}{(1-\delta) + \theta_t A_t \left(1 - \frac{c_t}{y_t}\right)}$$

setting an initial value for ϕ_t, and assuming all other variables, including θ, are measured without error.

There is one fundamental difference to the previous two shock model. We can see in Figure 3 that we back out a technology shock that does not grow exponentially. It is basically flat, and we can correlate it directly with the relative price (Pc/Pi) we find in the data. Then we compute the correlation coefficient for each country. This is the outcome:

	$\rho(A_t, \theta_t)$		$\rho(A_t, \theta_t)$		$\rho(A_t, \theta_t)$
AUS^*	-0.016	FRA	-0.552	MEX	-0.602
AUT	-0.303	GBR	0.025	NLD	-0.755
BEL^*	-0.785	GRC	-0.504	NOR	-0.662
CAN^*	-0.910	IRL	-0.661	NZL	-0.642
CHE^*	-0.518	ISL	-0.508	PRT	-0.185
DNK	-0.295	ITA	-0.607	SWE	-0.042
ESP	-0.084	JPN	-0.916	TUR	-0.094
FIN	-0.519	LUX	-0.737	USA	-0.850

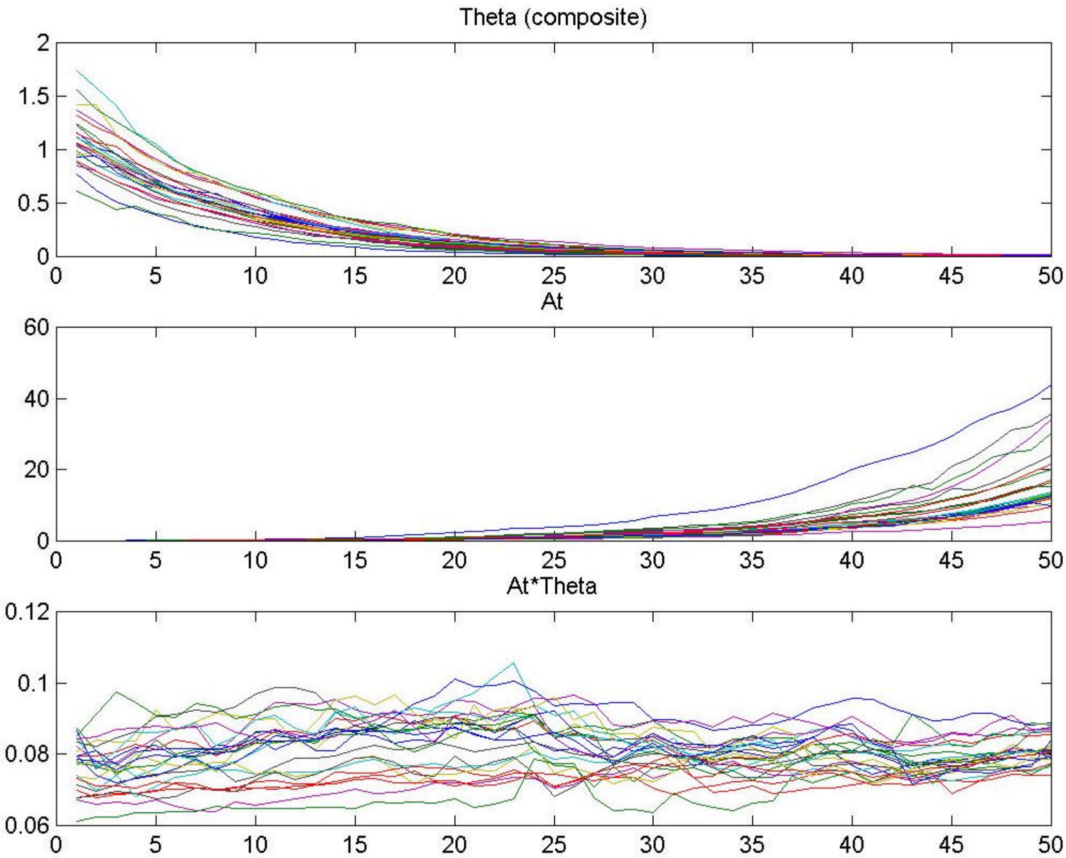
and despite the fact that a few countries have near zero correlations, most countries in the sample still display signifcant negative correlation, invalidating inference about the relative impact of the two shocks. Regarding the relative impact of the two shocks, and without any orthogonalization we get:

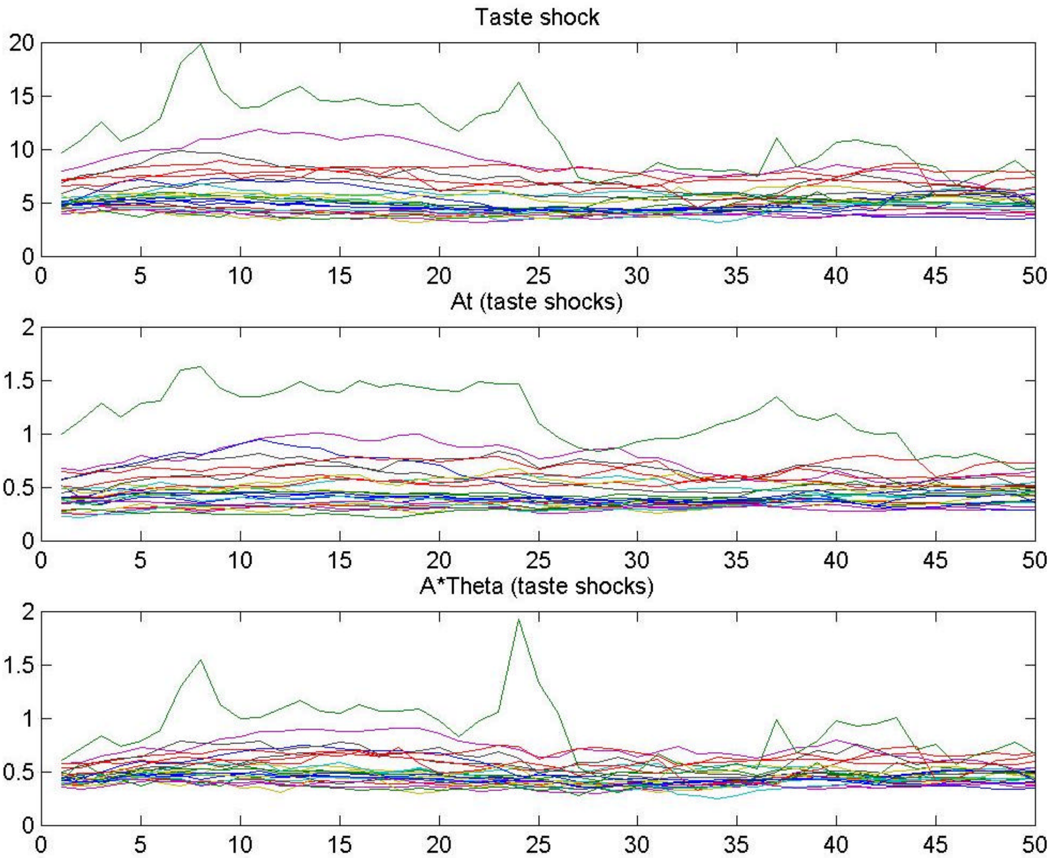
Table4 y_{t+1}/y_t	$R_1^2(\theta_t)$	$R_2^2\left(\hat{\theta}_t\right)$	$R_3^2(A_t)$	$R_4^2\left(\hat{A}_t\right)$
mean	0.043	0.043	0.600	0.528

and again we observe that shutting down the A shock drastically reduces the ability of the model to fit the data (down to 4.3%), but here shutting down the intertemporal shock (θ, not the taste shock) also reduces significantly the ability of the model to match the data (down to 52.8%) but again not at the same level. Thus, this version of the model, although very different, mostly reproduces the qualitative results of the previous one.[33]

[33] Note that if we included all the variables in the construction of the right hand side variable, the R squared would be one always and therefore there is no real sense in which we can do goodness of fit in this type of exercise.


First Measurement Error Test
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1.2
0
5
10
15
20
25
30
35
40
45
50


Theta (composite)
2
1.5
1
0.5
0
0
5
10
15
20
25
30
35
40
45
50
At
60
40
20
0
At*Theta
0.12
0.1
0.08
0.06


Taste shock
At (taste shocks)
A*Theta (taste shocks)