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Flexible Price Formation: The Institutional and Behavioural Foundations

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Flexible Price Formation; the Institutional and Behavioural Foundations

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Abstract
The present paper concerns the microeconomic basis for the macroeconomic demand constraint in the form of the Keynesian demand price constraint; taking Say into account denoted by *the extended Keynesian demand price constraint*.
Price flexibility at a scale sufficient for the working of the demand price constraint is denoted by *workable price flexibility*.
Price flexibility is not a question of historical price movements but a question of firms' practice of price setting. Price flexibility may be workable even if not every single price is flexible. Workable price flexibility depends on stability of the price formation process. In turn, this depends on *systematic economic behaviour* by consumers and by firms, in the paper discussed under the headings of *a generalized law of demand based on economizing behaviour*, and *experienced suppliers' behaviour.*

Key Words
Keynesian Demand Price Constraint. Say's Law. Representative Firm. Workable Price Flexibility. Composite Prices. Stochastic Consumer Theory. Generalized Law of Demand. Empirical Testing. Aggregated Consumption Vectors. Experienced Suppliers' Behaviour. Stability of Price Formation.

JEL Classification
E0, D1, D4

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1 Introduction; the Inspiration from Keynes, Marshall and Say

This paper is part of a more general research project with the Keynesian Demand Price Constraint and Marshall's price theory as *points of departure*. The concepts of Aggregate Demand Function and Aggregate Supply Function in nominal terms (that is why, in fact, they are aggregate prices) are found in chapter 3 of General Theory. In this chapter Keynes explains his ideas in terms of Marshall. However, to most readers, chapter 3 may not have been seen as the central contribution of General Theory.

Taking chapter 3 as the basis, General Theory can be interpreted as follows: firms decide to employ a certain number of workers on the expectation that the corresponding production of commodities and services can be sold at a stipulated aggregated *supply* price. *In the short run*, in fact, suppliers obtain the aggregated *demand* price consumers and other buyers are willing to pay for the amount of production in question. *Suppliers meet the macroeconomic demand constraint in the form of the demand price buyers are willing to pay.*

In the medium run, according to Keynes, suppliers adapt to the employment and production which can obtain an aggregated demand price equal to the stipulated aggregated supply price. In the medium run, not only prices but also wages may change.

However, the medium run equilibrium does not develop into a full employment equilibrium because of Keynes' assumptions with respect to the financial investors and the real investors; these are strongly influenced by factors like common beliefs acquired in order to overcome uncertainty. These factors are strong enough to counteract any tendencies towards long run full employment.

This interpretation of General Theory could be said to be Marshallian compared to the fixed price macro model interpretation (Samuelson's consumption function model) and to Hicks' Walras inspired interpretation (in Mr. Keynes and the Classics).

Just as inspiration can be found in the works of *Marshall* and *Keynes*, inspiration can be found in the original work of *Say*. In a contemporary setting, quit obviously, suppliers do influence demand. Say's law, that is, supply creates it's own demand, is *partly* an empirical fact. This fact is included in the theory of consumer behaviour developed below.

The Keynesian Demand Price Constraint, taking Say's insight into account, is named: *The Extended Keynesian Demand Price Constraint*.

Keynes treated consumers' demand and firms' investment very differently. In a contemporary setting, in the short run, consumers' real demand, like real investment, is not closely related to real income. This fact is included in the theory below.

The present state of an observed economy is seen as a short run equilibrium with a tendency towards a medium run equilibrium. Some unemployment is the general case. However, due to e.g. a shift of the entrepreneurial efforts or e.g. to a shift of "the state of confidence", the economy may shift to a phase of full employment.

With respect to theoretical method: Stuart Mill stated that only through the principle of competition, political economy has any pretension to the character of a science[(2)].
In this line of thought, two basic principles are employed:
(a) A principle of competitive behaviour.
(b) A principle of economizing behaviour.
However, the possibility of obtaining theoretical conclusions with respect to the working of the economy depends on the specific character of the institutions and the behaviour of the actors of the economy. This leads to a third principle:
(c) A principle of suitable institutions and suitable economic behaviour.

Among the results are:
(i) The concept of the extended Keynesian demand price constraint.
(ii) The concept of workable price flexibility.
(iii) A Generalized Law of Demand based on economizing behaviour.
(iv) Room in the theory of consumer behaviour for influence of entrepreneurial efforts.
(v) Stability of the short run price formation process conditional on experienced suppliers' behaviour.

(2) John Stuart Mill, Principles of Political Economy, p. 242.

2 Supply Prices, Demand Prices, and Macroeconomic Constraints

A basic macroeconomic model

Macroeconomic variables are indicated by capital letters. The aggregate of firms chooses the supply of economic goods Y^S and the price of the goods P^S.

The decisions are based on the general state of the economy denoted by *GSE*. This expression covers the known state of the economy including the level of prices and wages up to and just before the present date. *GSE* is influenced by the actions of monetary and political authorities. *GSE* also includes individual information on financial balances etc. Furthermore, the choice of (Y^S , P^S) depends on the entrepreneurial efforts denoted by Z. *GSE* does not include precise knowledge of Z, and the variable B to follow. Thus

(2.1) $P^S = P^S(\,GSE, Z\,)$ and

(2.2) $Y^S = Y^S(\,GSE, Z\,)$

The demand for economic goods is denoted by X^D. The demand for goods depends on the general state of confidence denoted by B (for *beliefs*). This expression covers the consumers' confidence in their financial situation, and real investors' confidence in their investment projects. Further, the demand is influenced by the degree of entrepreneurial efforts Z, and depends on the demand price P^D actually payed by the buyers. Thus

(2.3) $X^D = X^D(\,GSE, B, Z, P^D\,)$

Equilibrium between stipulated supply of and actual demand for economic goods may be obtained by short run price adjustment to the demand price determined by

(2.4) $Y^S(\,GSE, Z\,) = X^D(\,GSC, B, Z, P^D\,) \Rightarrow P^D$

In case of *rational expectations* $P^D = P^S$. That is, the realized demand price equals the price stipulated by the firms[3]. If variables B and Z are stable, it can be reasonable to assume rational expectations.

However, *on grounds of principle*, economic agents can not anticipate *shifts in their own understanding*, that is, economic agents can not foresee shifts in variables B and Z.

Because of that, in general $P^D \neq P^S$. In case of $P^D > P^S$, the firms have underestimated macroeconomic demand. In case of $P^D < P^S$, the firms have overestimated macroeconomic demand. In both cases, *the firms meet the macroeconomic demand constraint in the form of the demand price* P^D. The price adjustment to P^D is the result of a competitive process. The individual firms compete for sale of the stipulated production. The individual consumers and other buyers compete for delivery of goods.

The Keynes aspects of the macroeconomic model

Keynes' main contribution is the macroeconomic demand constraint as the fundamental determinant of the level of economic activity. Unemployment is the general case. The macroeconomic demand constraint shows up in the possibility of $P^D \neq P^S$.

One determinant of the macroeconomic demand constraint is the state of confidence denoted by B. "State of confidence" is a Keynesian concept.

The demand function (2.3) does not take feed back from current income to demand into account. With respect to contemporary economies, the demand components of consumer demand and real investor's demand are seen as having some similarity. Nowadays, in developed economies, consumer demand is less bounded by current income than in the time of Keynes.

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(3) The concept of rational expectations, the term introduced by Muth, is an extra equilibrium concept; the realized value of a variable *equals* the expected value of the same variable. In some contexts, this is an assumption not more problematic than the traditional concept of market equilibrium. Also see footnote no. 8.

The Say aspects of the macroeconomic model

Say's main contribution is demonstrating the entrepreneurs' choice of active production as the fundamental determinant of the level of economic activity. As in Keynes' theory, unemployment is the general case. In the present presentation, the entrepreneurial efforts are denoted by Z.

Say's law implies an identity $P^D \equiv P^S$. Say's law implies that the stipulated supply can be sold at the stipulated price. The incompatibility of Keynes' demand constraint and Say' law is shown by on the one hand the possibility of $P^D \neq P^S$, and on the other hand the identity $P^D \equiv P^S$.

Fundamentally, the basic macroeconomic model is a Keynesian model with the macroeconomic demand constraint as the cornerstone. However, the importance for the model of the entrepreneurial efforts expressed by the variable Z is a marked Say feature.

The representative firm's decisions

The firm chooses a *relative position* y^S, p^S, ζ in the quantity, price, quality space. The variable ζ stands for specification of the firm's chosen product. The space comprises the similar choices of all firms. The choice of such a relative position is a strategic decision. In principle, all choices are competitive moves.

In the following, supscripts S are omitted. Only one variable input is assumed, the number of employed persons denoted by q. The wage of one employed person is w. Average productivity is

(2.5) $\quad a = y \,/\, q$

In fine-tuning the relative position, the firm assumes the following three elasticities:
- *the elasticity of the price p with respect to the stipulated sales y*

(2.6) $\quad \varepsilon_{p,\,y} = dp/dy \cdot (\, y \,/\, p \,)$

- *the elasticity of the wage w with respect to the firm's employment q*

(2.7) $\quad \varepsilon_{w,\,q} = dw/dq \cdot (\, q \,/\, w \,)$

- *the elasticity of the average productivity a with respect to the firm's employment q*

(2.8) $\varepsilon_{a,q} = da/dq \cdot (q \,/\, a)$

Productivity is a question of organizing the production. Organization of the production may change from one stategic relative position to another. That is why, we do not assume a global production function. However, the elasticity assumption (2.8) implies a *local* production function, ζ given. From (2.5) follows

(2.9) $da = (dy \cdot q - dq \cdot y) \,/\, q^2$

By use of (2.9) and (2.5) we get from (2.8)

(2.10) $dy \,/\, y = (1 + \varepsilon_{a,q}) \cdot dq \,/\, q$

The general solution to the differential equation (2.10) is

(2.11) $\ln y = (1 + \varepsilon_{a,q}) \cdot \ln q + \ln c$

In (2.11) ln denotes the natural logaritm and c is an arbitrary constant. From (2.11) follows

(2.12) $y = c \cdot q^{1 + \varepsilon a, q}$

In an interval where the elasticity of productivity with respect to employment is constant, the local production function is defined by (2.12). The constant c is determined by the reference point q, y.

In fine-tuning its relative position, the firm maximizes its stipulated profit

(2.13) $\pi = y \cdot p - q \cdot w = a \cdot q \cdot p - q \cdot w$

Theorem
The local optimum of the firm is characterized by the condition (2.14)

(2.14) $a \cdot p \,/\, w = (1 + \varepsilon_{w,q}) \,/\, [(1 + \varepsilon_{a,q}) \cdot (1 + \varepsilon_{p,y})]$

$a \cdot p$ is the revenue from the employment of one person. Thus, if production should be profitable, the optimum must satisfy $a \cdot p \,/\, w > 1$. As a typical example, that will be the case if $0 < \varepsilon_{w,q}$ and $-1 < \varepsilon_{a,q} < 0$ and $-1 < \varepsilon_{p,y} < 0$. Other combinations of values may do.

Proof

Max $_{(q)}$ π implies $d\pi/dq = 0$. From (2.13)

(2.15) $d\pi/dq = da/dq \cdot q \cdot p + a \cdot (p + dp/dq \cdot q) - w - q \cdot dw/dq = 0$

$\Rightarrow$

(2.16) $a \cdot p \cdot [\, 1 + da/dq \cdot q \,/\, a + dp/dq \cdot q \,/\, p \,]$
$= w \cdot [\, 1 + dw/dq \cdot q \,/\, w \,]$

$\Rightarrow$

(2.17) $a \cdot p \cdot (1 + \varepsilon_{a,q}) \cdot [\, 1 + (dp/dq \cdot q \,/\, p) \,/\, (1 + da/dq \cdot q \,/\, a) \,]$
$= w \cdot (1 + \varepsilon_{w,q})$

$\Rightarrow$

(2.18) $a \cdot p \cdot (1 + \varepsilon_{a,q}) \cdot [\, 1 + (dp \cdot a \cdot q \,/\, p) \,/\, (dq \cdot a + da \cdot q) \,]$
$= w \cdot (1 + \varepsilon_{w,q})$

and as $y = a \cdot q$ and $dy = dq \cdot a + da \cdot q$

(2.19) $a \cdot p \cdot (1 + \varepsilon_{a,q}) \cdot [\, 1 + (dp/dy \cdot y \,/\, p) \,]$
$= w \cdot (1 + \varepsilon_{w,q})$

$\Rightarrow$

(2.20) $a \cdot p \cdot (1 + \varepsilon_{a,q}) \cdot (1 + \varepsilon_{p,y}) \; = \; w \cdot (1 + \varepsilon_{w,q})$

$\Rightarrow$

(2.21) $a \cdot p \,/\, w = (1 + \varepsilon_{w,q}) \,/\, [\, (1 + \varepsilon_{a,q}) \cdot (1 + \varepsilon_{p,y}) \,]$

That concludes the proof.

The representative firm's Pseudo Supply Function

In the following, supscripts S and D are used again. After fine-tuning, the firm's stipulated quantity/price position y^S, p^S is in accordance with the condition (2.14).

We assume the three elasticities of (2.14) to be constant in a neighbourhood of the reference point. Further, $\pi > 0$ is assumed. For the neighbourhood we make the following assumptions:

$w = w(q)$ is a monotonic increasing function,
$a = a(q)$ is a monotonic decreasing function,
$q = q(y^S)$ is a monotonic increasing function.

(2.14) can be written, γ being a constant referring to the right side of (2.14)

(2.22) $a \cdot p^S / w = \gamma > 1$
$\Rightarrow$
(2.23) $p^S = \gamma \cdot w / a$

By the monotony assumptions, a function defined by

(2.24) $p^S = f(y^S) = \gamma \cdot w / a = \gamma \cdot w(q(y^S)) / a(q(y^S))$

is a monotonic increasing function.

(2.24) is a *pseudo supply function*. It describes the relation between prices p^S and quantities y^S chosen optimally on basis of the above assumptions.

With respect to the *demand schedule expected by the firm*, the only assumption is the constancy of the *elasticity* $\varepsilon_{p,y}$. Thus, the pseudo supply function (2.24) describes the relation between prices p^S and quantities y^S chosen according to varying *positions* of the demand schedule expected by the firm.

The adjustment of the stipulated supply price to the demand price

Step 1, moment: Normally, actual demand x^{D1} at the supply price p^{S1} differs from the stipulated supply y^{S1}. The firm may have misjudged the macroeconomic conditions or the firm may have misjudged its microeconomic position vis-à-vis its competitors.

Step 2, short run: The firm adjusts its supply price to p^{S2} equal to the demand price p^{D2} corresponding to actual sales $x^{D2} = y^{S1}$. The adjustment process involves both sellers and buyers in a competitive process. Firms are competing for the sales of their stipulated production y^{S1}. Consumers and other buyers are competing for delivery of goods.

Step 3, medium run: The adjustment of the firm's supply prices to demand prices is the topic of *sections 3 and 7.* Here, assuming the formation of supply prices p^{S2} =

$p^{D\,2}$, the firm's next step is adaption to demand conditions expressed by the demand price $p^{D\,2}$.

The firm may choose a price *between* $p^{S\,1}$ and the experienced demand price $p^{D\,2}$ as its next stipulated supply price $p^{S\,3}$. From the monotonic increasing *pseudo supply function* (2.24) follows:

(2.25) $p^{D\,2} > p^{S\,1} \Rightarrow y^{S\,3} > y^{S\,1}$

(2.26) $p^{D\,2} = p^{S\,1} \Rightarrow y^{S\,3} = y^{S\,1}$

(2.27) $p^{D\,2} < p^{S\,1} \Rightarrow y^{S\,3} < y^{S\,1}$

The three steps are illustrated by *figure 1*. Empirically *observed prices* may be assumed to be close to the prices according to step 2. With respect to the representative firm, the *observed price* is assumed to be close to $p^{S\,2} = p^{D\,2}$.

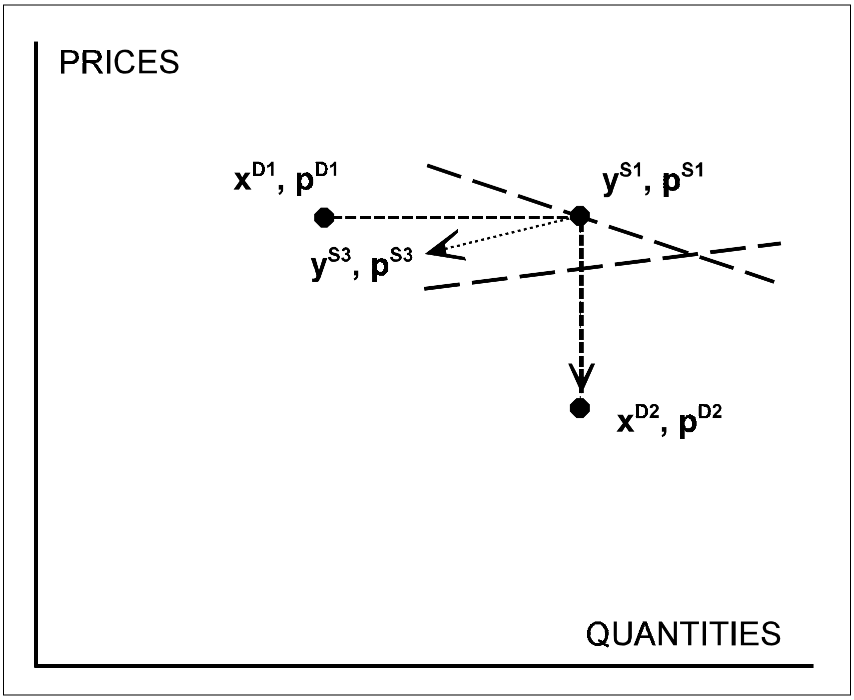

Figure 1 (referring to section 2). A representative firm chooses a position $y^{S\,1}$, $p^{S\,1}$ in the quantity, price space. The firm experiences a tendency towards a lower sale $x^{D\,1}$ than expected and, in the short run, adjusts the supply price to $p^{S\,2}$ equal to the demand price $p^{D\,2}$ corresponding to $y^{S\,1} = x^{D\,2}$. The difference between the supply price $p^{S\,1}$ and the demand price $p^{D\,2}$ prompts the firm to choose a revised position involving a reduced $y^{S\,3}$. The firm's expected demand schedule and the firm's unit costs are indicated by stippled lines.

3 Workable Price Flexibility

Historical price fluctuations

Assume a series of n + 1 observations of the price of a defined commodity or defined service, or a defined composite of economic goods; in the latter case the price is an index. The observations cover a certain period of time, e.g. one year.

$p_0,\ p_1, \cdots p_{k-1}, p_k, p_{k+1}, \cdots p_n$

The average price of the period is

(3.1) $\quad p_a = \sum_{(0,n)} p_k \,/\, (n+1)$

An indicator of price fluctuations is the sum of ***n***umerical price change ***d***eviations from the "trend" $(p_n - p_0) / n$ standardized by division by the average price p_a

(3.2) $ND = 1 / p_a \cdot \sum_{(1,n)} | (p_k - p_{k-1}) - (p_n - p_0) / n |$

The value of $ND \geq 0$ depends on the number of observations in the certain period of time. More observations *may* give a higher value.

Actual example The Standard&Poors composite index of 500 US share prices; 13 observations from ultimo December 2001 to ultimo December 2002 [(4)]. The value of the indicator *ND* is calculated to 0,54.

Financial assets' prices, and exchange rates (e.g. between the euro and the dollar) are determined in speculative markets. One expects irregular fluctuations. On the other hand, wages and salaries are expected to behave more stable. Assume e.g. the monthly salary of a specific person to be 100 in the first quarter of a year (and in last year's December), 103 in the next two quarters, and 106 in the year's last quarter. This gives a value of the indicator *ND* of only 0,10. However, due to e.g. bonus payments, in fact wages and salaries may be more variable than suggested by the example.

Turning to the prices of consumer goods, an example of a year's price observations could be: 100 in even month, and 90 in uneven month. This pattern could be the result of the widespread regular use of discounts and offers. The indicator value *ND* is 1,26.

These examples are chosen to sustain a general impression that historical price series show very different patterns according to the specific characteristics of the goods (in the broadest sense) and the specific market's institutions.

Full scale price flexibility

The extent of historical price fluctuations does not solve the problem of price flexibility in the working of the economy. Price flexibility in the working of the economy is the question whether supply prices are adjusted to the demand prices corresponding to the demand for the stipulated production of goods. In case of small

(4) The actual observations from ultimo December 2001 to ultimo December 2002 are: 1148, 1130, 1107, 1147, 1077, 1067, 990, 911, 916, 815, 886, 936, 880. Source: Yahoo Finance, Historical Prices differences between stipulated supply prices and demand prices, the extent of price fluctuations created by the adjustment process is small.

Price flexibility in the working of the economy is an empirical problem as well as a theoretical problem. In case of full scale price flexibility, observed prices are changing fast to the demand prices corresponding to the suppliers' chosen level of production. This should apply to the price of every specific commodity and service. Prices are changed by suppliers, reacting fast on indications of deviations of actual from stipulated sales of the specific economic goods.

Price flexibility depends on the competitive interaction of all suppliers and all consumers and other buyers. The short run formation of demand prices is the question of *section 7*.

Workable price flexibility

In the real market economy, we can not assume *full scale* price flexibility. It may be optimal for the firms to keep some prices constant even if expected demand deviates from stipulated demand. On the other hand, a number of pricing instruments are developed making price flexibility an optimal choice in many cases, even if it is not optimal for the firm to change the list price. Among the pricing instruments are general discounts on the list price, special offers, discounts to varying specified groups of customers, seasonal sales, spot market sales, and periodical bargain prices.

Any setting of a price implies the choice of a relative position in the structure of all prices. The price structure might be seen as having two aspects:
- *a factual price structure* of all prices, both stipulated and actual,
- *a virtual price structure* of the economic agents' comprehension of what prices ought to be.

The choice of price positions and setting up a list of many prices (in some branches of trade, firms sell maybe 50.000 different items) demand administrative efforts and rather careful considerations with respect to the reactions of competitors and consumers. This may be good reasons for not changing the list prices frequently.

Often, the kiosk prices of newspapers are seen as examples of non-flexible prices. It may be less optimal to have a sale of unsold morning papers in the late afternoon. Non-flexibility of the kiosk prices seems to be an optimal choice. On the other hand, it would be possible to distribute discount vouchers and to offer discounted kiosk prices in certain periods. So the kiosk prices of newspapers are not necessarily non-flexible. Even if the kiosk prices are non-flexible, subscription prices are flexible due to a widespread use of discounts.

However, in the eyes of the publisher of a newspaper the most relevant consideration is the aggregated revenue of the paper as a whole. This revenue comes from kiosk sales, subscriptions and sales of advertisements. Taking the newspaper as a whole as the unit relevant to the publisher, the revenue may vary even if all prices at the copy level are kept constant. That form of price flexibility is not price flexibility at the consumer level. However, from the point of view of the publisher, the revenue can be seen as an aggregated demand price. Similarly, for the management of an airline the supply of flights and the revenue of flights is the most relevant level of decisions. The aggregated demand price for one flight may vary even if the prices of tickets for seats in the different categories are kept constant.

In case of *full scale price flexibility*, conditional on stability of the price formation process, the set of demand prices are equilibrium prices corresponding to equilibrium between stipulated production and actual demand for each of the number of commodities and services. Consumers' demand decisions are based on this set of equilibrium demand prices. On the supply side, the firms' medium term decisions are based on the set of demand prices compared to the stipulated supply prices.

Even in case of some inflexible prices and maybe generally inflexible list prices, the working of the economy in effect may be close to that of a state of full price flexibility. Then we can speak of *workable price flexibility*. Whether we actually have workable price flexibility depends on the pricing instruments developed in the competitive process in the economy in question. If the prices in general are close to the equilibrium demand prices, the term workable price flexibility is justified.

Composite prices

One specific pricing practice is of special interest. A certain commodity or service may be sold at two different prices, an ordinary price p^* and a bargain price (or sales or discounted price) p^{**}. Either the normal price is used in a part λ of a time period, or to a section λ of the consumers. In the rest of the period $1 - \lambda$, or to the other section of the consumers $1 - \lambda$, the bargain price is used. The firm stipulates $\lambda = \lambda_i$. Thus, the stipulated weighted price is $p^{(i)} = \lambda_i \cdot p^* + (1 - \lambda_i) \cdot p^{**}$. The point is that the firm can adjust the weighted price by changing λ_i without changing neither the ordinary price nor the bargain price. This widely used pricing practice supports the arguments for workable price flexibility.

4 A Theory of Consumer's Behaviour; the Generalized Law of Demand Based on Economizing Behaviour

Marshall's Law of Demand

Marshall explains the Law of Demand by: "-The greater the amount to be sold, the smaller must be the price at which it is offered in order that it may find purchasers; or, in other words, the amount demanded increases with a fall in price, and diminishes with a rise in price."[(5)] According to Marshall, the law refers to the sum of demands of a number of individuals for one specific commodity, say, tea. To each individual purchaser, the price will measure the marginal (cardinal) utility of the commodity in question. Marshall knows R. Giffen's observation that a rise in the price of bread may force the poorer labouring families to consume more bread not less of it. Such cases are rare and must be treated on their own merits.[(6)] The law of demand is a statement of the normal tendency, but there may be rare exceptions.

Contrary to the consumer theory developed later by Hicks and others, fully based on ordinal utility, Marshall sees consumer demand resulting from *choices in the market between marginal units of a commodity and marginal amounts of money* (the price of one unit).

A theory of consumer behaviour

The consumer theory developed below is cognate with Marshall's demand theory in the sense, a consumer's final decision to purchase is seen as a choice in the market between a bundle of goods and an amount of money. By this, we accentuate *the importance of nominal money* in the working of a market economy.

In line with Marshall, we see social laws as statements of general tendencies; there

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(5) Marshall's Principles of Economics p. 99.

(6) Principles p. 132.

may be exceptions which must be treated on their own merits.

Consumption vectors in the n dimensional economic goods space χ

There are n different consumer commodities and services in the economy. The consumption of commodity or service no. i by an individual consumer in a defined period of time is measured by x_i. Normally, x_i is a discrete quantity and the single consumer's consumption of one of the many commodities is none or a few units. [(7)]

A bundle of goods is described by a consumption vector $x = (x_1,...x_i,...x_n)$.

Different consumption vectors are distinguished by supscripts. Thus $x^{(j)}$ denotes vector no. j.

The consumption vectors are elements of the n dimensional economic goods space χ, thus $x \in \chi$. Innovations of new goods imply a changed goods space.

Prices are denoted correspondently. The price of commodity or service no. i is p_i. The price vector is $p = (p_1,...p_i,...p_n)$. Different price vectors are distinguished by supscripts, e.g. $p^{(j)}$.

Attraction probabilities a{ x }

An individual consumer has full knowledge of only a small subset of all economic goods. The consumer has some knowledge of a somewhat larger subset of all goods. Consumers have been socialized to their roles as consumers. A consumer's general knowledge is formed by education and own experience in practise, by communication with other consumers and by presentations and advertisements by producers and firms.

To a large degree, goods are brought into consumers' attention by marketing efforts by producers. Some of the goods well known by consumers are not at all considered for purchase because they are obviously out of reach for financial reasons, e.g. prestige cars and watches. Similarly, a consumer may be well informed about goods which for other reasons are not taken into consideration.

(7) The number of different consumer commodities and services in a developed market economy are many hundred thousand, even in one region.

The probability that a bundle of economic goods is actively considered for purchase is denoted by $a\{ x \mid x \in \chi \}$ or shorter $a\{ x \}$.

(4.1) $0 \leq a\{ x \} \leq 1$ $\sum_{(x)} a\{ x \} = 1$

The probability $a\{ 0 \}$ of considering the zero vector $0 = (0,...0)$ is assumed to be positive

(4.2) $a\{ 0 \} = 1 - \sum_{(x \neq 0)} a\{ x \} > 0$

A bundle of economic goods is a solution to the consumer's problem to determine what to consume. The probabilities $a\{ x \}$ are named *attraction probabilities*, because the probabilities $a\{ x \}$ can be said to represent *the relative attractions of different available solutions* in form of consumption vectors as such.

The attraction probabilities are based on a consumer's situation, general knowledge and experience, and are influenced by the marketing efforts by the producers.

The probabilities $a\{ x \}$ are *individual characteristics* of a consumer. Typically, a consumer acts in *different roles*, e.g. on behalf of a household, or alone on a holiday trip. To each role, a set of attraction probabilities may be assigned.

Decision to purchase probability function b_x

Already the attraction probabilities are results of some economizing. As noted above, some goods are not considered because they the are thought financially to be out of reach.

However, the real economizing involves a choice between a bundle of goods and an amount of money. The probability that an actively considered consumption vector x actually is purchased is denoted by $b_x\{ p \cdot x \mid p^{(N)}, \phi \}$. This is the decision to purchase probability function.

(4.3) $0 \leq b_x\{ p \cdot x \mid p^{(N)}, \phi \} \leq 1$

The probability function b_x is specific for the individual consumer, and b_x is specific for the consumption vector x in question. The probability is a function of the aggregated price $p \cdot x$ of the consumption vector, and is conditional on $p^{(N)}$ which is a vector of what the consumer sees as the normal prices, and conditional on the consumer's financial situation expressed by the parameter ϕ. This parameter in not a budget limit but a more general measure of the consumer's financial resources and possibilities to borrow. In phases of stable inflation rates, $p^{(N)}$ and ϕ may grow at the same rate and the consumer may accept proportional increases in $p \cdot x$ without changes of b_x.

In the interval (4.3) the probability of decision to purchase is assumed to be a monotonic decreasing function of $p \cdot x$

(4.4) $\quad p^{(1)} \cdot x > p^{(2)} \cdot x \Rightarrow b_x\{ p^{(1)} \cdot x \} < b_x\{ p^{(2)} \cdot x \}$

See *figure 2.*

In applications, more specific forms of b_x may be used, see *section 5.*

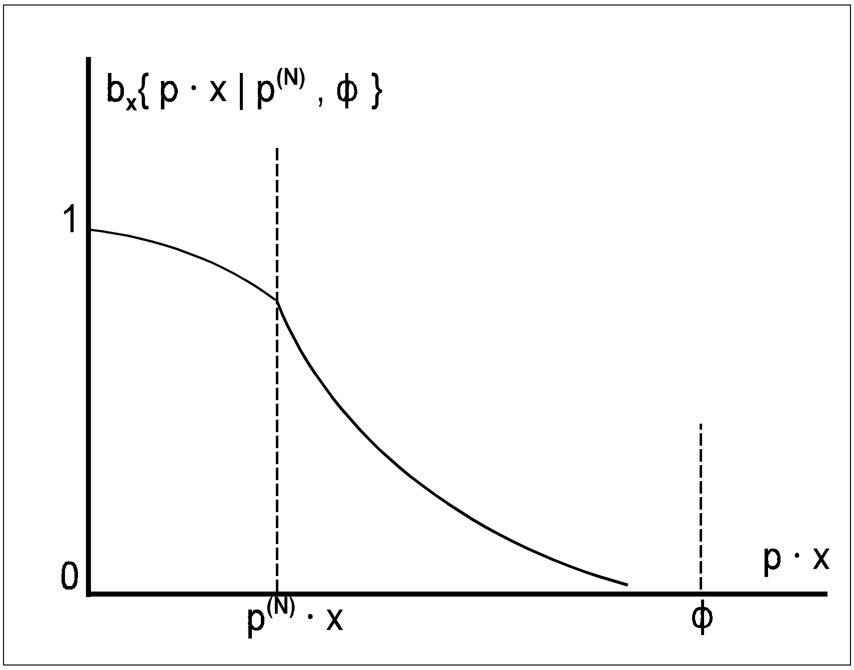

Figure 2 (referring to section 4). The decision to purchase probability function b_x. The function is shown as a continuous function of $p \cdot x$, however, with a kink for $p^{(N)} \cdot x$, which is the "normal" aggregated price of the consumption vector x. Thus, the function shown is not differentiable at that point. In section 7, b_x is assumed differentiable. The parameter ϕ is an expression of the consumer's financial situation.

Probability of choice of a consumption vector x conditional on a price vector p

The choice of a consumption vector x depends on two steps: It must be actively considered, and the consumer must decide to purchase it. It follows from the concepts introduced above that the probability of choice of x conditional on the price vector p is

$$(4.5) \qquad \mathrm{pr}\{ x \mid p \} = a\{ x \} \cdot b_x\{ p \cdot x \mid p^{(N)}, \phi \}$$

The attraction probability a{ x } represents the first step, the decision to purchase probability b_x represents the second step.

$$(4.6) \qquad \textstyle\sum_{(x)} \mathrm{pr}\{ x \mid p \} = 1$$

Typically, the probability of choice of the zero consumption vector is positive

$$(4.7) \qquad \mathrm{pr}\{ 0 \mid p \} = a\{ 0 \} \cdot b_0\{ p \} = 1 - \textstyle\sum_{(x \neq 0)} \mathrm{pr}\{ x \mid p \} > 0$$

In (4.7), $b_0\{ p \}$ is an adaptive parameter depending on p. By assumption a{ 0 } > 0, see (4.2).

Rounds of consideration and decision to purchase

The two steps mentioned may be seen as just one round of consideration and decision to purchase. In a specified period of time, the probability of one more round following round no. r − 1 is denoted by pr{ r | r − 1 }

$$(4.8) \qquad 0 \leq \mathrm{pr}\{ r \mid r-1 \} \leq 1$$

We assume the probability of a first round in the specified period of time to be one

$$(4.9) \qquad \mathrm{pr}\{ 1 \mid 0 \} = 1$$

The probability of just r rounds is

$$(4.10) \qquad \mathrm{pr}\{ r \} = \mathrm{pr}\{ 1 \mid 0 \} \cdot \ldots \cdot \mathrm{pr}\{ r \mid r-1 \} \cdot (1 - \mathrm{pr}\{ r+1 \mid r \})$$

The *mean of the number of rounds* in a specified period of time is

(4.11) $m\{ r \} = \sum_{(r)} r \cdot pr\{ 1 \mid 0 \} \cdot ... \cdot pr\{ r \mid r-1 \} \cdot (1 - pr\{ r+1 \mid r \})$

$\Rightarrow$

(4.12) $m\{ r \} = 1 \cdot pr\{ 1 \mid 0 \} \cdot (1 - pr\{ 2 \mid 1 \})$
$+ 2 \cdot pr\{ 1 \mid 0 \} \cdot pr\{ 2 \mid 1 \} \cdot (1 - pr\{ 3 \mid 2 \})$
$+ 3 \cdot pr\{ 1 \mid 0 \} \cdot pr\{ 2 \mid 1 \} \cdot pr\{ 3 \mid 2 \} \cdot (1 - pr\{ 4 \mid 3 \})$
$+ ...$

It follows by induction

(4.13) $m\{ r \} = \sum_{(r)} pr\{ 1 \mid 0 \} \cdot ... \cdot pr\{ r \mid r-1 \}$

Postulate 1 with respect to rounds: In a specified period of time, *the mean of r is independent of p*. That is

(4.14) $m\{ r \} = c$

where c is a constant. From the assumption (4.9) follows

(4.15) $c \geq 1$

Postulate 2 with respect to rounds: The probability of choice in a round is *independent of the outcome of preceding rounds*. This postulate implies that a consumer in a series of rounds might purchase the same bundle or similar bundles of goods twice or more. Clearly, it would be more realistic to assume that the attraction probabilities $a\{ x \}$ change with the outcome of the preceding round. Postulate 2 is a simplification.

The mean consumption value m{ x | p } of x conditional on a price vector p

Taking the possibility of several rounds into account, on the basis of (4.5) and (4.10) *the mean value of x conditional on p* becomes

(4.16) $m\{ x \mid p \}$
$= \sum_{(r)} \sum_{(x)} x \cdot a\{ x \} \cdot b_x\{ p \cdot x \mid p^{(N)}, \phi \} \cdot pr\{ 1 \mid 0 \} \cdot ... \cdot pr\{ r \mid r-1 \}$

By use of (4.13) follows

(4.17) $m\{ x \mid p \} = \sum_{(x)} x \cdot a\{ x \} \cdot b_x\{ p \cdot x \mid p^{(N)}, \phi \} \cdot m\{ r \}$

and from (4.14) follows

(4.18) $m\{ x \mid p \} = \sum_{(x)} x \cdot a\{ x \} \cdot b_x\{ p \cdot x \mid p^{(N)}, \phi \} \cdot c$

The Generalized Law of Demand

Theorem

Mean consumption vectors satisfy The Generalized Law of Demand. $p^{(1)}$ and $p^{(2)}$ are two price vectors. $m^{(1)}\{ x \mid p^{(1)} \}$ and $m^{(2)}\{ x \mid p^{(2)} \}$ are the corresponding mean consumption values of x. The Generalized Law of Demand is *defined* to entail the inequality

(4.19) $(p^{(1)} - p^{(2)}) \cdot (m^{(1)}\{ x \mid p^{(1)} \} - m^{(2)}\{ x \mid p^{(2)} \}) < 0$

Proof

From (4.18) follows

(4.20) $(p^{(1)} - p^{(2)}) \cdot (m^{(1)}\{ x \mid p^{(1)} \} - m^{(2)}\{ x \mid p^{(2)} \})$
$= (p^{(1)} - p^{(2)}) \cdot (\sum_{(x)} x \cdot a\{ x \} \cdot b_x\{ p^{(1)} \cdot x \mid p^{(N)}, \phi \} \cdot c$
$- \sum_{(x)} x \cdot a\{ x \} \cdot b_x\{ p^{(2)} \cdot x \mid p^{(N)}, \phi \} \cdot c)$

$\Rightarrow$

(4.21) $(p^{(1)} - p^{(2)}) \cdot (m^{(1)}\{ x \mid p^{(1)} \} - m^{(2)}\{ x \mid p^{(2)} \})$
$= \sum_{(x)} a(x) \cdot c \cdot (p^{(1)} \cdot x - p^{(2)} \cdot x)$
$\cdot (b_x\{ p^{(1)} \cdot x \mid p^{(N)}, \phi \} - b_x\{ p^{(2)} \cdot x \mid p^{(N)}, \phi \})$

By use of (4.4)

(4.22) $(p^{(1)} - p^{(2)}) \cdot (m^{(1)}\{ x \mid p^{(1)} \} - m^{(2)}\{ x \mid p^{(2)} \}) < 0$

That concludes the proof.

Separating hyper plane

The generalized law of demand is tantamount to the existence of a hyper plane with normal $p^{(1)} - p^{(2)}$ separating $m^{(1)}$ and $m^{(2)}$. On the *negative side* of the hyper plane is $m^{(1)}$. On the *positive side* of the hyper plane is $m^{(2)}$. See *figure 3.*

The inequality (4.22) can be written

(4.23) $(p^{(1)} - p^{(2)}) \cdot m^{(1)} < (p^{(1)} - p^{(2)}) \cdot m^{(2)}$

A vector x* is chosen in such a way that

(4.24) $(p^{(1)} - p^{(2)}) \cdot m^{(1)} < (p^{(1)} - p^{(2)}) \cdot x^* < (p^{(1)} - p^{(2)}) \cdot m^{(2)}$

Then, a separating hyper plane is given by

(4.25) $(p^{(1)} - p^{(2)}) \cdot x = (p^{(1)} - p^{(2)}) \cdot x^*$

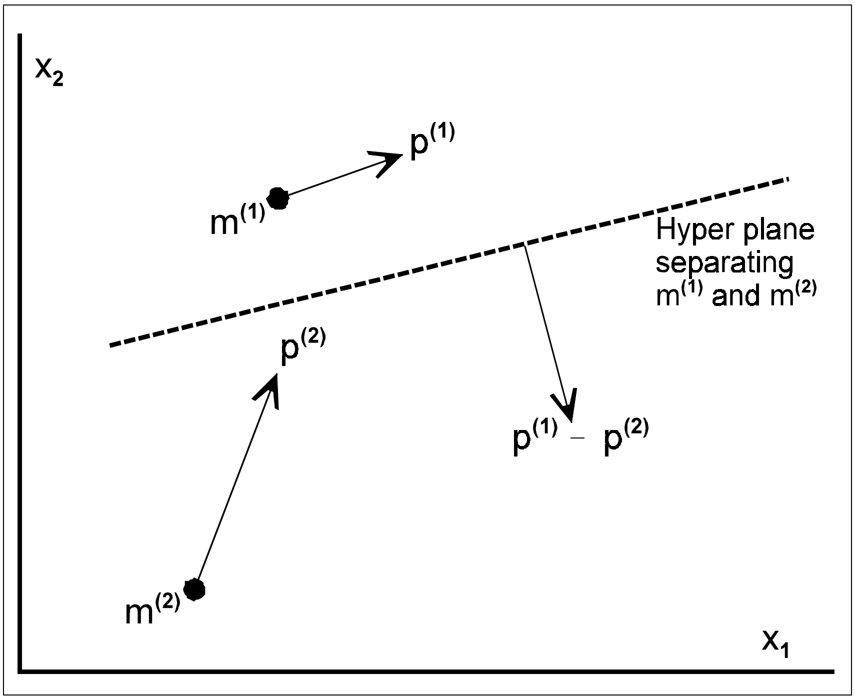

Figure 3 (referring to section 4). The two mean consumption vectors $m^{(1)}$ and $m^{(2)}$ are separated by a hyper plane with normal $p^{(1)} - p^{(2)}$. On the negative side of the hyper plane is $m^{(1)}$. On the positive side of the hyper plane is $m^{(2)}$.

Empirical testing of the generalized law of demand

Assume empirical observations of a number of corresponding mean consumption vectors and price vectors ($m^{(1)}$, $p^{(1)}$), ... ($m^{(i)}$, $p^{(i)}$), ... ($m^{(n)}$, $p^{(n)}$). The mean consumption is estimated as the average consumption of a number of consumers facing the same price vectors.

From the series of observations we can single out $n \cdot (n - 1) / 2$ different pairs $(m^{(i)}, p^{(i)})$, $(m^{(j)}, p^{(j)})$. Then it can be tested whether the location of $m^{(i)}$ and $m^{(j)}$ is according to the generalized law of demand on the correct sides of a hyper plane with normal $p^{(i)} - p^{(j)}$.

In my 1972 article (based on early versions of stochastic consumer theory), in a series of 52 observations, only 2 out of the 1326 pairs did have locations violating the weak axiom of revealed preference. On revealed preference, see below.

The generalized law of demand implies the weak axiom of revealed preference

The weak axiom of revealed preference was introduced by Samuelson in 1938 in an effort to free economic theory from traces of the utility concept. However, the word "revealed" as such vas not used in the original statement. Revealed preference refers to *observations* of price vectors and the corresponding consumption vectors. The interpretation of

$$(4.26) \quad p^{(1)} \cdot m^{(1)} \geq p^{(1)} \cdot m^{(2)}$$

is that $m^{(1)}$ is revealed preferred to $m^{(2)}$ as the consumer did choose $m^{(1)}$ even though $m^{(2)}$ is not more expensive.

According to *the weak axiom of revealed preference*, (4.26) is *inconsistent* with

$$(4.27) \quad p^{(2)} \cdot m^{(2)} \geq p^{(2)} \cdot m^{(1)}$$

the interpretation of which is that $m^{(2)}$ is revealed preferred to $m^{(1)}$.

The two inconsistent inequalities (4.26) and (4.27) can be written

$$(4.28) \quad p^{(1)} \cdot (m^{(1)} - m^{(2)}) \geq 0$$

$$(4.29) \quad p^{(2)} \cdot (m^{(1)} - m^{(2)}) \leq 0$$

By subtraction of (4.29) from ((4.28)

$$(4.30) \quad (p^{(1)} - p^{(2)}) \cdot (m^{(1)} - m^{(2)}) \geq 0$$

The derived inequality (4.30) is inconsistent with the generalized law of demand (4.19). *Thus, the weak axiom of revealed preference follows from the generalized law of demand.*

Note that (4.26) and (4.27) imply (4.30), *but (4.30) does not imply the two inequalities (4.26) and (4.27).* The generalized law of demand does not follow from the weak axiom of revealed preference.

The inequality (4.30) can be written

(4.31) $(p^{(1)} - p^{(2)}) \cdot m^{(1)} \geq (p^{(1)} - p^{(2)}) \cdot m^{(2)}$

The expression (4.31) follows from the inconsistent revealed preferences (4.26) and (4.27). The substance of (4.31) is that $m^{(1)}$ and $m^{(2)}$ are placed respectively on the positive and negative side of a hyper plane with normal $p^{(1)} - p^{(2)}$. This is just the opposite of the correct location according to the generalized law of demand, compare with (4.23) - (4.25). However, as can be demonstrated by examples, not all "incorrect" located pairs $m^{(1)}$, $m^{(2)}$ are ruled out by the weak axiom of revealed preference. The generalized law of demand does not follow from the weak axiom of revealed preference.

5 Applications; A Specific Decision to Purchase Probability Function b_x and Actual Choice from a Defined Subset χ*

In this section, we apply the decision to purchase probability function in a specified and simplified form

(5.1) $b_x\{ p \cdot x \mid \phi \} = k_x \cdot \pi^{- p \cdot x / \phi}$

The parameters k_x, π and ϕ are specific for the individual consumer and k_x is also assumed specific for x. As above, $\phi > 0$ is a general measure of the consumer's financial resources and possibilities to borrow. We assume

(5.2) $\pi > 1$

Then follows from $0 \leq b_x\{ p \cdot x \mid \phi \} \leq 1$

(5.3) $0 \le k_x \le 1$

Next, we focus on choices actually carried through from a defined subset χ^* of the economic goods space χ. In applications, the subset χ^* could be the set of all commodity bundles offered by a certain supermarket. Thus, we focus on choices of consumption vectors

(5.4) $x \in \chi^* \subset \chi$

and assume

(5.5) $0 \notin \chi^*$

The probability of choice of x given a price vector $p^{(i)}$ and conditional on choice of a vector $x \in \chi^*$ is

(5.6) $\mathrm{pr}\{ x \mid p^{(i)}, \chi^* \} = \mathrm{pr}\{ x \mid p^{(i)}, \chi \} \,/\, \sum_{x \in \chi^*} \mathrm{pr}\{ x \mid p^{(i)}, \chi \}$

We define a parameter $d_i > 0$ in such a way that

(5.7) $\pi^{di} = 1 \,/\, \sum_{x \in \chi^*} \mathrm{pr}\{ x \mid p^{(i)}, \chi \}$

The parameter d_i depends both on $p^{(i)}$ and on the defined subset χ^*. From (5.6), (5.7), (4.5) and (5.1) follows

(5.8) $\mathrm{pr}\{ x \mid p^{(i)}, \chi^* \} = \pi^{di} \cdot \mathrm{pr}\{ x \mid p^{(i)}, \chi \}$
$= \pi^{di} \cdot a\{ x \} \cdot k_x \cdot \pi^{-p(i) \cdot x / \phi} = a\{ x \} \cdot k_x \cdot \pi^{di - p(i) \cdot x / \phi}$

With respect to two price vectors $p^{(1)}$ and $p^{(2)}$, we have

(5.9) $\sum_{x \in \chi^*} a\{ x \} \cdot k_x \cdot \pi^{d1 - p(1) \cdot x / \phi} = 1$

(5.10) $\sum_{x \in \chi^*} a\{ x \} \cdot k_x \cdot \pi^{d2 - p(2) \cdot x / \phi} = 1$

A variable parameter ω is defined by:

(5.11) $\pi^{\omega} \cdot a\{ x \} \cdot k_x \cdot \pi^{d1 - p(1) \cdot x / \phi} = a\{ x \} \cdot k_x \cdot \pi^{d2 - p(2) \cdot x / \phi}$

The variable parameter ω is a measure of the relative size of the probabilities of x conditional on $p^{(1)}$ and $p^{(2)}$ respectively. ω varies with x.
From (5.11) follows

(5.12) $\quad \pi^{\omega} = \pi^{-d1 + d2 + p(1) \cdot x / \phi - p(2) \cdot x / \phi}$

$\Rightarrow$

(5.13) $\quad \omega = -d_1 + d_2 + p^{(1)} \cdot x / \phi - p^{(2)} \cdot x / \phi$

$\Rightarrow$

(5.14) $\quad (p^{(1)} - p^{(2)}) \cdot x = \phi \cdot \omega + \phi \cdot d_1 - \phi \cdot d_2$

The expression (5.14) defines a hyper plane with normal $p^{(1)} - p^{(2)}$. (5.14) can be written

(5.15) $\quad \phi \cdot \omega = (p^{(1)} - p^{(2)}) \cdot x - \phi \cdot (d_1 - d_2)$

See *figure 4.*

Since $\phi > 0$ and $\pi > 1$, it follows from (5.15) that

(5.16) $\quad (p^{(1)} - p^{(2)}) \cdot x - \phi \cdot (d_1 - d_2) > 0 \Rightarrow \omega > 0 \Rightarrow \pi^{\omega} > 1$

and

(5.17) $\quad (p^{(1)} - p^{(2)}) \cdot x - \phi \cdot (d_1 - d_2) < 0 \Rightarrow \omega < 0 \Rightarrow \pi^{\omega} < 1$

Theorem
Mean consumption vectors conditional on actual choice from a defined subset χ satisfy The Generalized Law of Demand.* Actual choice means choice of a consumption vector $x \in \chi^*$ and $0 \notin \chi^*$. The specified b_x probability function (5.1) is assumed.

Proof

It follows by use of (5.8)

(5.18) $$(p^{(1)} - p^{(2)}) \cdot (m^{(1)} - m^{(2)})$$
$$= (p^{(1)} - p^{(2)}) \cdot (\textstyle\sum_{x \in \chi^*} x \cdot pr\{ x \mid p^{(1)}, \chi^* \}$$
$$- \textstyle\sum_{x \in \chi^*} x \cdot pr\{ x \mid p^{(2)}, \chi^* \})$$
$$= (p^{(1)} - p^{(2)}) \cdot (\textstyle\sum_{x \in \chi^*} x \cdot a\{ x \} \cdot k_x \cdot \pi^{d1 - p(1) \cdot x / \phi}$$
$$- \textstyle\sum_{x \in \chi^*} x \cdot a\{ x \} \cdot k_x \cdot \pi^{d2 - p(2) \cdot x / \phi})$$

$\Rightarrow$

(5.19) $$(p^{(1)} - p^{(2)}) \cdot (m^{(1)} - m^{(2)})$$
$$= \textstyle\sum_{x \in \chi^*} (p^{(1)} - p^{(2)}) \cdot x \cdot a\{ x \} \cdot k_x$$
$$\cdot [\pi^{d1 - p(1) \cdot x / \phi} - \pi^{d2 - p(2) \cdot x / \phi}]$$

By use of (5.9) and (5.10)

(5.20) $$(p^{(1)} - p^{(2)}) \cdot (m^{(1)} - m^{(2)})$$
$$= \textstyle\sum_{x \in \chi^*} [(p^{(1)} - p^{(2)}) \cdot x - \phi \cdot (d_1 - d_2)] \cdot a\{ x \} \cdot k_x$$
$$\cdot [\pi^{d1 - p(1) \cdot x / \phi} - \pi^{d2 - p(2) \cdot x / \phi}]$$

Then, by use of (5.16) and (5.17) together with (5.11) we get the inequality

(5.21) $$(p^{(1)} - p^{(2)}) \cdot (m^{(1)} - m^{(2)}) < 0$$

Thus the mean consumption vectors satisfy the requirement of the generalized law of demand. This concludes the proof.

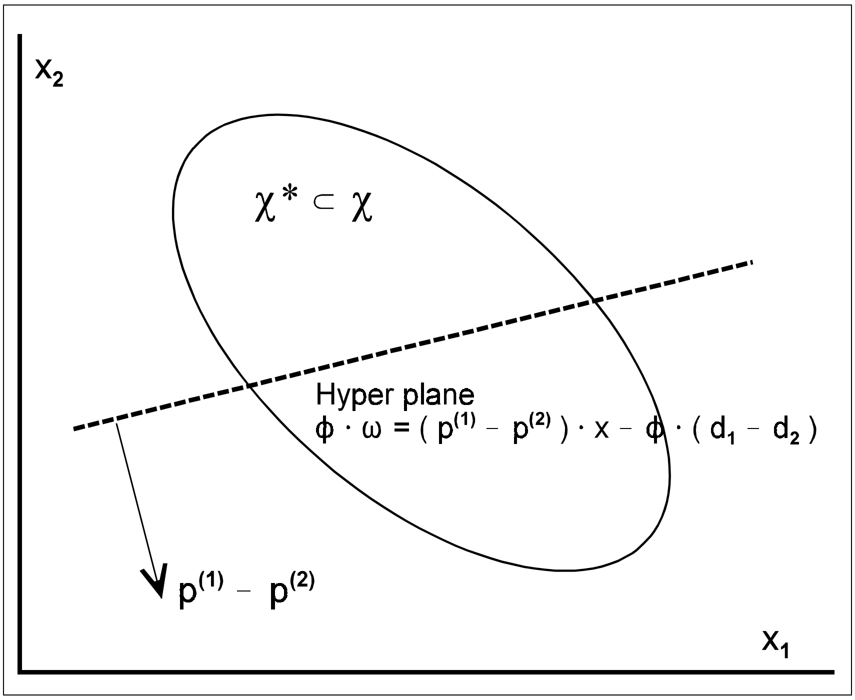

Figure 4 (referring to section 5). Actual choice from a defined subset χ^* of χ not containing the 0 vector. In the proof in section 5 of the generalized law of demand, an auxiliary hyper plane is applied. With increasing ω, the hyper plane moves in the direction of the normal $p^{(1)} - p^{(2)}$. The expression π^{ω} is a measure of the relative size of the probabilities of choice of a vector x *in* the hyper plane, conditional on $p^{(2)}$ and $p^{(1)}$ respectively.

6 The Generalized Law of Demand; Aggregates of Several Consumers and Several Consumer Roles; Aggregates With Respect to a Composite Price Vector

Typically, producers and firms are met by the aggregated response from a large number of consumers. Also, an individual consumer acts in different roles. In fact, the consumption of an individual consumer may be seen as an aggregate of the consumption decisions in the separate roles. The consumption of a household of several individuals may be seen as an aggregate of decisions on behalf of the household, and decisions in different individual roles.

Individual consumers and consumer roles are distinguished by a supscript [h] for consumer / role no. h. The mean consumption vector no. j of consumer no. h is denoted by $m^{[h],(j)}$. The aggregated mean consumption vector is denoted by a capital letter

(6.1) $M^{(j)} = \sum_{(h)} m^{[h],(j)}$

We *do not assume* choices by the individuals / roles to be *stochastically independent*. The probability of simultaneous choices of $x^{[g]}$ and $x^{[h]}$ by the two consumers no. g and no. h respectively, is denoted by $pr\{ x^{[g]}, x^{[h]} \mid p \}$. We *do assume*

(6.2) $pr\{ x^{[h]} \mid p \} = \sum_{(x[g])} pr\{ x^{[g]}, x^{[h]} \mid p \} = a\{ x^{[h]} \} \cdot b_x\{ p \cdot x^{[h]} \}$

The condition (6.2) corresponds to (4.5) and is a requirement for the proof of the generalized law of demand.

Empirical testing of the idea of separate roles of an individual consumer. In a subsection of *section 4*, we described an empirical testing of the generalized law of demand. The same testing can be done on data referring to separated roles of individual consumers. However, not all separations may work equally well with respect to the requirement (6.2).

Theorem
The aggregated mean consumption vectors satisfy the generalized law of demand.

Proof
The mean consumption vectors of consumer no. h satisfy

(6.3) $(p^{(1)} - p^{(2)}) \cdot (m^{[h],(1)} - m^{[h],(2)}) < 0$

$\Rightarrow$

(6.4) $\sum_{(h)} (p^{(1)} - p^{(2)}) \cdot (m^{[h],(1)} - m^{[h],(2)}) < 0$

$\Rightarrow$

(6.5) $(p^{(1)} - p^{(2)}) \cdot (M^{(1)} - M^{(2)}) < 0$

Thus, the aggregated mean consumption vectors satisfy the generalized law of demand. This concludes the proof.

Aggregates with respect to composite price vectors

As mentioned in a subsection of *section 3*, very commonly firms obtain price flexibility by shifting the weights between an ordinary price and a discount or bargain price.

The price vector p* and the price vector p** are the ordinary price vector and the bargain price vector respectively. The corresponding mean consumption vectors are $m^* = m\{ x \mid p^* \}$ and $m^{**} = m\{ x \mid p^{**} \}$ respectively.

The firm / firms charge p* in a fraction λ_i of a time period and p** in the fraction $(1 - \lambda_i)$ of the time period in question. Alternatively, the firm / firms charge p* to a fraction of λ_i of the consumers and p** to the fraction $(1 - \lambda_i)$ of the consumers.

The *composite price vector* varies with the firm's / firms' choice of λ_i

(6.6) $$p^{(i)} = \lambda_i \cdot p^* + (1 - \lambda_i) \cdot p^{**}$$

The corresponding *aggregated mean consumption* vector becomes

(6.7) $$M^{(i)} = \lambda_i \cdot m^* + (1 - \lambda_i) \cdot m^{**}$$

Theorem
The aggregated mean consumption vectors satisfy the generalized law of demand with respect to the composite price vectors. Normally, the average prices obtained by firms differ from the composite prices.

Proof
From (6.6) follows

(6.8) $$p^{(1)} - p^{(2)} = (\lambda_1 - \lambda_2) \cdot p^* - (\lambda_1 - \lambda_2) \cdot p^{**}$$

From (6.7) follows

(6.9) $$M^{(1)} - M^{(2)} = (\lambda_1 - \lambda_2) \cdot m^* - (\lambda_1 - \lambda_2) \cdot m^{**}$$

From (6.8) and (6.9)

(6.10) $(p^{(1)} - p^{(2)}) \cdot (M^{(1)} - M^{(2)}) = (\lambda_1 - \lambda_2)^2 \cdot (p^* - p^{**}) \cdot (m^* - m^{**})$

The generalized law of demand implies

(6.11) $(p^* - p^{**}) \cdot (m^* - m^{**}) < 0$

Since $(\lambda_1 - \lambda_2)^2 > 0$, it follows from (6.10) and (6.11) that

(6.12) $(p^{(1)} - p^{(2)}) \cdot (M^{(1)} - M^{(2)}) < 0$

Thus the aggregated mean consumption vectors $M^{(1)}$ and $M^{(2)}$ satisfy the requirement of the generalized law of demand with respect to the composite prices. This concludes the proof.

7 Price Formation; Stability Conditional on Experienced Suppliers' Behaviour

In the short run, we assume the suppliers (producers or firms) adjust separately the supply prices of the different goods. This applies to the short run fine tuning of prices only.

y_j, p_j is the stipulated quantity and the stipulated supply price of commodity or service no. j. In the short run adjustment process, y_j, is fixed, whereas the supply price p_j is adjusted according to market conditions. There is only one producer or seller of each commodity or service.

This producer faces the demand of a large number of consumers. M_j is the aggregated mean demand for commodity or service no. j.

The aggregated mean consumption vector $M = (M_j)$ is an element of the set of consumption vectors, that is, $M \in \chi$.

On the assumption, the probabilities $b_x\{ p \cdot x \}$ are differentiable functions of $p \cdot x$, the coordinates M_j of the aggregated mean consumption vector M are differentiable functions of $p \cdot x$, see (4.18) and (6.1).

As assumed above, $y = (y_j)$ is fixed in the short run, whereas $p = (p_j)$ is assumed to be adjusted.

The aim of the present *section 7* is to investigate whether the adjustment of the prices moves the mean consumption vector (M_j) *towards* equilibrium with the stipulated supply of goods (y_j), which is fixed in the short run

(7.1) $(M_j) \rightarrow (y_j)$

As (M_j) eventually approaches (y_j), the adjustment of the prices comes to an end.

Theorem
On conditions of:
(i) economizing consumer behaviour satisfying The generalized law of demand,
(ii) on the part of the producers, competitive price adjustments at an appropriate level based on the producers' general market experience [(8)],
the price formation process is stable and narrows steadily the deviation between the aggregated mean consumption vector (M_j) and the stipulated supply vector (y_j).

Argumentation for the theorem
With respect to the adjustment of prices, we assume

(7.2) $\triangle p_j = g_j \cdot (M_j - y_j) / y_j$

The parameter $g_j > 0$ is specific for the adjustment of the supply price p_j of commodity or service no. j.

Change of scale

Assume the unit of y_j to be scaled up by a factor s. The measures of quantities and the price per unit change accordingly

(7.3) $y_j' = y_j / s$ and $M_j' = M_j / s$

(7.4) $p_j' = s \cdot p_j$

(8) The *concept of experienced suppliers' behaviour* might be seen as having some conceptual resemblance with Muth's rational expectations and Friedman's earlier concept of permanent income. With respect to all three concepts, one assumes some general insight of the economic actors in the working of the economy.

Consequently, with respect to the parameter g_j of (7.2)

(7.5) $\quad g_j' = s \cdot g_j$

Deviation measure

We define a measure of deviation D between (M_j) and (y_j)

(7.6) $\quad D = \sum_{(j)} (g_j / y_j) \cdot (M_j - y_j)^2$

Since we assume $g_j > 0$, we have $D \geq 0$.

D is scale invariant. By application of (7.3) - (7.6)

(7.7) $$D' = \sum_{(j)} (g_j' / y_j') \cdot (M_j' - y_j')^2$$
$$= \sum_{(j)} (s \cdot g_j / (y_j / s)) \cdot (M_j - y_j)^2 / s^2 = D$$

This shows the scale invariance of the deviation measure D.

D is a value measure. From (7.2) follows that the parameter g_j has the dimension of money units divided by quantity units (the dimension of a price).Next, it follows from (7.6) that the deviation measure D has the dimension of g_j times quantity units. Thus, D has the dimension of money units. That is, D is a value.

The change of D as result of price adjustments $\triangle p_j$

As mentioned above, the coordinates of (M_j) are differentiable functions of $p \cdot x$ on the assumption of differentiability of the b_x probability functions. From this follows with respect to the differences $\triangle D$ and $(\triangle M_j)$

(7.8) $\quad \triangle D = \sum_{(j)} (g_j / y_j) \cdot 2 \cdot (M_j - y_j) \cdot \triangle M_j + o(\triangle D)$

Remember, in the short run process (y_j) is fixed. In (7.8) the expression $o(\triangle D)$ is small of second order to $\triangle D$ due to the assumption of differentiability.

By application of (7.2)

(7.9) $\triangle D = \sum_{(j)} 2 \cdot \triangle p_j \cdot \triangle M_j + o(\triangle D)$

Since the mean consumption vectors satisfy the generalized law of demand, we have

(7.10) $\sum_{(j)} \triangle p_j \cdot \triangle M_j < 0$

On the condition that $\triangle D$ is appropriately small, from (7.9) and (7.10) follows

(7.11) $\triangle D < 0$

That is, the deviation measure decreases as result of the price adjustment process. As the deviation between (M_j) and (y_j) is reduced, the adjustment of prices slows down.

Stability of the price formation process conditional on experienced suppliers' behaviour

The price formation process may be seen as having a number of steps. Every step include, see *figure 5*:
Producers' perception of a deviation $(M_j) - (y_j) \rightarrow$
producers' reactions in the form of price adjustments $(\triangle p_j) \rightarrow$
change of $p \cdot x \rightarrow$
consumers' reactions in the form of $(\triangle M_j) \rightarrow$
change of $(M_j) - (y_j) \rightarrow$
change of D measured by $\triangle D$.

The overall outcome of the process depends on the magnitude of $\triangle D$:
(i) $\triangle D$ should be numerically so small that $\triangle D < 0$ can be implied from (7.9) and (7.10).
(ii) $\triangle D$ should be numerically so large that the process towards $D = 0$ does not take too long time.

The magnitude of $\triangle D$ depends on the magnitude of the producers' reactions in the form of price adjustments, see (7.2). The parameters (g_j) describe how strongly the producers react. Appropriate reactions are conditions for the two requirements (i) and (ii).

If one can assume that *experienced suppliers* neither *overreact* nor *react too little* on deviations between stipulated supply and actual demand, the two requirements are met. Then, the theorem is valid.

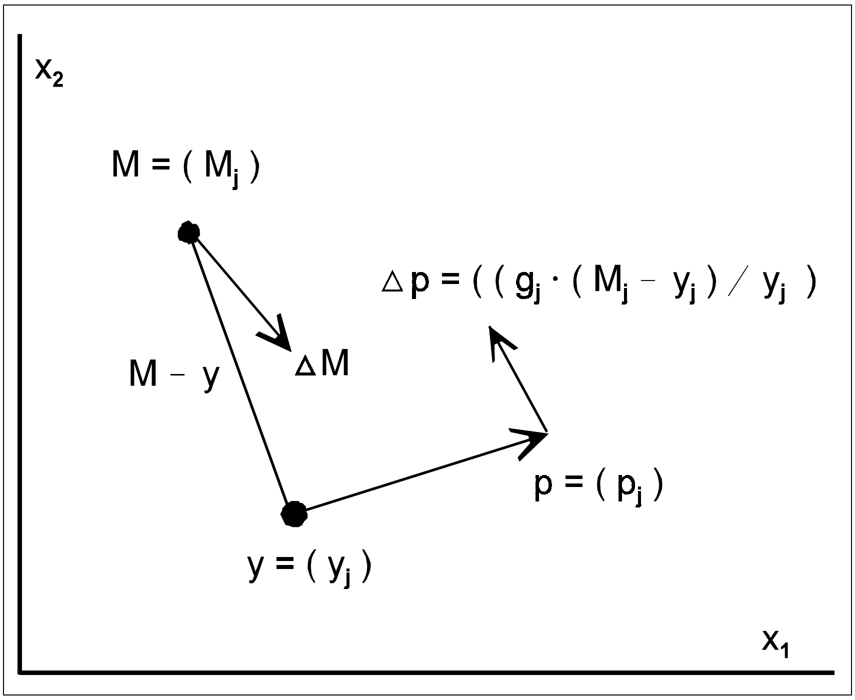

Figure 5 (referring to section 7). The producers perceive the deviation between the aggregated mean consumption vector M and the stipulated supply vector y. The producers react in the form of a change of the price vector Δp. The consumers' response is ΔM. The process may narrow the deviation between M and y relatively fast on the condition that the producers neither overreact nor react too little on the perceived deviations. As M approaches y, the price changes Δp becomes small.

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