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Stone Age Economics:
The Origins of Agriculture and
the Emergence of Non-Food Specialists

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Stone Age Economics:
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Abstract
This paper examines the prehistoric shift from hunting and gathering to agriculture. Among hunters and gatherers, all community members were engaged in food provision. Agricultural societies, in contrast, avail themselves of non-food specialists. This paper argues that the adoption of agriculture necessitated the introduction of non-food specialists. Since the release of labour from food generating activities stimulates economic development, this implies that the shift to agriculture literally bore the seeds of later economic growth. The model shows, in accordance with archaeological evidence, that hunters and gatherers, faced with redistribution costs arising from division of labour, delay the adoption of agricultural techniques for a period of time, after which a large step forward in food procurement technology—a 'Neolithic revolution'—is associated with the shift to farming.

Keywords: hunting-gathering, leisure time, neolithic revolution, non-food specialists, transition.

JEL codes: N50, O33, Q18

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"The first revolution that transformed human economy gave man control over his own food supply. Man began to plant, cultivate, and improve by selection edible grasses, roots, and trees. And he succeeded in taming and firmly attaching to his person certain species of animals in return for the fodder he was able to offer, the protection he could afford, and the forethought he could exercise."

V. Gordon Childe, *Man Makes Himself*, 1936

1 Introduction

This paper examines the transition from hunting and gathering to agriculture, which appeared for the first time some 10,000 years ago. The emergence of agriculture is unquestionably one of the most important events in human cultural history. For most of the time since the line of man that led to modern humans departed from that of the great apes around 7 million years ago, all humans fed themselves exclusively by hunting wild animals and gathering wild plants. The introduction of agriculture, as recognized by the eminent archeologist V. Gordon Childe in the quotation above, meant that humans began to cultivate plants and domesticate animals and so gained an unprecedented control over their food supply.

Radical changes in organizational structures accompanied the adoption of agriculture; an economy of non-food specialists—craftsmen, chiefs, bureaucrats, and priests—arose, founded upon the surplus that early farmers were able to create. The presence of non-food specialists led to countless innovations such as writing, metallurgy, cities, and scientific principles, elements of what today is known as 'civilization'.

Surprisingly though, archaeological evidence indicates that there were thousands of years between the discovery of agricultural techniques and their adoption. Considering the fact that cultivation techniques are time-costly, meaning that hunters and gatherers, contrary to common belief, worked less than early farmers, and that the transition to agriculture involved little or no increase in standards of living, the reluctance to take up farming is hardly surprising.

This leaves a compelling question: What were the factors that eventually tipped the competitive advantage away from hunting-gathering and towards agriculture? This issue is subject to intense debate within the archaeological and anthropological communities. It has been suggested that agriculture was adopted in response to external pressure arising from demographic or climatic changes, or, in the absence of outside stress, as the result of changes in cultural behaviour. But no universally applicable theory seems to have been found yet.

This paper provides a new view on the emergence of agriculture. The work is related to the branch of growth literature that investigates the interplay between economic and demographic variables associated with the historical tran-
sition from stagnation to growth (e.g., Galor and Weil, 2000, Galor and Moav, 2002; Goodfriend and McDermott, 1995; Hansen and Prescott, 2002; Jones, 2001; Kögel and Prskawetz, 2001; Lagerlöf, 2003; Lucas, 2002; Tamura, 2002; Weisdorf, 2003a). Despite their long-run perspective, none of these papers focus directly on the rise of Neolithic agriculture. Some, however, recognise the importance of the event: Galor and Moav (2002) argue that the shift from the tribal family structure of hunters and gatherers to the household level family organisation of agricultural societies enhanced the manifestation of the potential evolutionary advantage of individuals with a quality-bias that favoured economic growth; Lagerlöf (2002), who investigates the institution of serfdom, suggests that indicates that the birth of farming may have led to an era dominated by slavery; and Olsson and Hibbs (2003) show that the timing and the location of the transition to agriculture is strongly correlated with the distribution of wealth among today’s countries.

A small but growing number of papers deal specifically with the shift from foraging to farming. Smith (1975) examines the hypothesis that the extinction of large herding animals, due to ‘overkill’ by Paleolithic hunters, led to the rise of agriculture. North and Thomas (1977) argue that population pressure, together with the shift from common to exclusive communal property rights, altered man’s incentive sufficiently to encourage the application of cultivation and domestication techniques. Locay (1989) studies the implications of nomadism versus sedentarism in relation to the rise of agriculture. More recently, Morand (2002) has presented a model that discusses the family’s resource-allocation behaviour in relation to the shift to farming, while Olsson (2003), in a framework that is able to compare a number of archaeological explanations, supports the theory that environmental factors, along with genetic changes in the species suitable for domestication, paved the way for agriculture.2

The factors that eventually tipped the comparative advantage in favour of agriculture in this paper differ essentially from those presented in the existing literature. First and foremost, we challenge the idea that external pressure is needed to have hunters and gatherers embarking upon time-costly agricultural techniques. Instead, we argue that organizational change in the form of a division of the labour force into food and non-food specialists, driven by exogenous improvements in food procurement technology, was the prime factor behind the shift. The reason is that the goods produced by the non-food specialists compensated for the loss of leisure associated with the adoption of more productive but also more time-costly food procurement methods. The argument rests upon two empirically well-established elements: One is that the individual’s learning time associated with the application of a given food procurement method increases with the method’s productivity. The other is that the demand for food is income-inelastic and that individuals, once their food needs are fulfilled, care about leisure and non-food goods, which they consider to be substitute.

We also show that redistribution costs arising from the division of labour will delay the adoption of agricultural techniques, despite the existence of the

2See Weisdorf (2003b) for a more detailed survey of these papers.
knowledge about how to use them. Under such circumstances, a period of technical stagnation is followed by a shift to considerably more productive food procurement methods, resembling the perception of a ‘Neolithic revolution’.

The ideas presented in this paper carries an important message with regards to the emergence of economic growth. The process of industrialisation, which has been particularly fast in the western world during the past two centuries, and which has been the impetus for modern economic growth, relies on the release of labour from food into non-food generating activities. If the emergence of non-food specialists is closely linked to the adoption of agriculture, the shift from foraging to farming in the Stone Age therefore literally bears the seeds of the later process of industrialisation.

The paper continues as follows. Section 2 outlines the archaeological evidence upon which the model is based; section 3 presents the model, section 4 performs the analysis, and section 5 concludes.

2 Archaeological Evidence

A tenet common among archaeologists and anthropologists is that the knowledge about how to practise agricultural techniques was present for a long time prior to their adoption (e.g., Harlan, 1995; Diamond, 1997; Milthen, 1996; Redman, 1978). Following Milthen (1996, p. 218), ”the origins of agriculture 10,000 years ago are not to be sought in a sudden breakthrough in technology, or the crossing of a threshold in botanical knowledge”. Along similar lines, Harlan (1995, p. 15) asserts that ”[i]f the hunter-gatherers chose to grow a plant from seed, they would do so and no new knowledge was required”. He claims (loc.cit.) that ”[h]unter-gatherers knew all they needed to know to take up agriculture at any time and, within economically suitable limits, at any place they chose”. But according to Redman (1978), the advantages of agriculture were not obvious at the outset (pp. 91-92), ... ”it seems that there were from several hundred to several thousand years between the first domestication of plants and animals in any given area and a heavy reliance by the people in that area on agriculture for food”. Thus in actuality, the shift to agriculture is regarded as innovation rather than invention.3

The apparent reluctance towards agricultural techniques is believed to have been partly due to the fact that farming has greater labour costs than hunting and gathering. Time budget studies of present-day primitive societies show, to most peoples’ surprise, that hunters and gatherers spent fewer rather than more hours per day at work compared to early farmers (Boserup, 1965; Lee and DeVore, 1968; Sahlins, 1974). In fact, Sahlins (1974) argues convincingly that hunters and gatherers were the most leisured people in history. Based on the studies that he refers to, he notes (p. 14) that, among hunter-gatherers, “rather

3 Delaying the introduction of agriculture is not confined to the first people who adopted it. There is a famous example from northern Germany where hunters and gatherers, situated no more than 125 miles north of farming societies, did not adopt agriculture until 1,300 years after the techniques were introduced by their southern neighbours (e.g., Diamond, 1997).
than continuous travail, the food quest is intermittent, leisure abundant, and there is a greater amount of sleep in the daytime per capita per year than in any other society”.

In support of Sahlins’ statement, the studies found in Lee and DeVore (1968) leave one with the impression that early farming was indeed back-breaking, time consuming, and labour intensive, which makes it difficult to believe that agriculture was adopted in order to save time or labour. Or as Fernandez-Armesto (2001) puts it (p. 93), “if labour-saving was the purpose behind the strategy, crop domestication must be reckoned a failure. In practice, it seems always to have cost early farmers more trouble than it saved”.

These issues add doubt to whether the implementation of agriculture is in fact compatible with the principle of least effort.

It seems perfectly sound to argue that hunters and gatherers would not embark upon time-costly methods of food procurement unless there was good reason to do so. Thus, some sort of imbalance must have generated the shift. Archaeologists and anthropologists have speculated extensively about the origins of such an imbalance.4 One can roughly divide the many theories into those that have external changes as the cause of the imbalance and those where the changes are internal, resulting from the actions taken by the societies themselves.

One of the major theories concerning external factors builds on ideas proposed by Boserup (1965). Boserup attributes the development of intensive agriculture to the impetus given by a growing population. In response to the pressure that a larger population puts on its resources, farmers increase their labour input to produce more food per unit of land. As the increasingly intensive land use supposedly leads to a drop in labour productivity, methods that raise productivity, such as ploughing and fertilization, are developed.

The introduction of such methods, however, belongs to a period in time where agriculture had already gained a foothold. In order to include development before the rise of agriculture, Cohen (1977) therefore extended the theories of Boserup. According to his version, communities of hunters and gatherers continually evolved towards higher and higher carrying capacity in order to accommodate recurrent population pressure. Cohen, along with fellow archaeologists such as Binford (1968) and Flannery (1973), thus believes that the increasing need to increase food supplies paved the way for agriculture.

The population pressure theory is challenged by theories that attribute the rise of agriculture to environmental changes. It is argued that population pressure could not have been an important factor because there is no evidence of a food crisis prior to the rise of farming (see, e.g., Harlan, 1995 and Milthen, 1996). Instead, it is assumed that the communities remain in equilibrium with carrying capacity, unless disturbed by factors in the environment. According to this view, agriculture was adopted in the early Holocene as a consequence of a period of unusual environmental change (e.g., Bar-Yosef and Benfer-Cohen, 1992; Byrne, 1987; Childe, 1935).

The weakness of theories that deal with external pressure is that agriculture

4See Weisdorf (2003b) for a brief survey.
seems to have been introduced regardless of whether or not such changes were present (e.g., Fernandez-Armesto, 2001). This has led to a number of theories that emphasise internal changes, based on alterations in cultural behaviour. Hayden (1990), for example, regards food as a source of social prestige, and envisions the rise of agriculture as resulting from what he calls 'competitive feasting'. He argues that early domestication took place in order to create delicacies for families or individuals who wanted to improve their social status. There also seems to be interest in the idea that agriculture was introduced as a response to religion (e.g., Cauvin, 2000; Harlan, 1992, 1995) or due to changes in the capacity of the modern man’s mind (Milthen, 1996).

Though many of the theories presented in the archaeological and anthropological literature fit well on a regional level, no single explanation seems to be universally applicable (e.g., Harlan, 1995; Smith, 1995; Fernandez-Armesto, 2001). However, the fact that agriculture was adopted even in the absence of external pressure makes a general theory more likely to rely on changes within the societies themselves; changes that are based upon incidents that are universally observable.

One of the world-wide fundamental changes that accompanied the adoption of agriculture was the appearance of non-food specialists (e.g., Diamond, 1997; Redman, 1978). According to Diamond (1997, p. 55), hunters and gatherers did not produce food surpluses and so did not support and feed non-hunting craft specialists, bureaucrats, and chiefs. But with agriculture, Diamond says (p. 261), "it became possible, for the first time in human evolution, to develop economically specialized societies consisting of non-food producing specialists fed by food-producing peasants".

Below we promote the idea that the introduction of non-food specialists made possible the adoption of time-intensive agricultural methods. We also argue that the costs of having a division of labour arising, for example, from the collection and redistribution of food and other goods between sectors and individuals, delayed the adoption of agricultural methods despite their accessibility.

3 The Model

In this section a simple economic model of primitive societies, which offers a framework for analysing the issue discussed above, is presented.

In contrast to the models found in, for example, Olsson (2003) and Morand (2002), in which hunting-gathering and agriculture are characterized as two distinctly different food procurement methods, the model presented in this paper has no such boundary distinction. Instead, in line with widely accepted ideas among archaeologists, we imagine a ladder of technical steps ranging from foraging to farming (e.g., Smith, 2001). Each step implies a higher availability of

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5 In Olsson’s model there are decreasing returns to labour among hunter-gatherers but constant returns to agriculture. In Morand’s model, human capital can be accumulated by farmers, an option that does not exist for hunter-gatherers.
food per acre of land, but simultaneously implies an increase in the amount of time or energy required for its use.

The traditional way of characterising foraging techniques—the one which we do not use here—is to consider the source of food on which hunters and gatherers subsist as constant. From this traditional perspective, moving one step up the technical ladder means that food can be obtained with greater certainty, or with less time or energy invested; but the size of the supply of food, which depends, so to speak, on nature’s rather than man’s decision, remains the same.

In the more contemporary approach—the one that our model builds on—hunters and gatherers are in fact able to influence the size of their food supply.6 This means that the line between the techniques that belong to foraging and those that belong to farming becomes somewhat blurred. Consider, for example, a hunter-gatherer community that subsists on grasses such as wheat or barley. A move from merely finding and reaping wild grasses to removing competing vegetation surrounding it would constitute a step up the technical ladder towards farming. However, it is not normally considered as such. A step up the ladder into farming methods involves land clearing and cultivation where no grasses formerly grew.7 Likewise, hunter-gatherer communities, subsisting on, say, reindeers, can increase the reindeer population by eliminating its natural enemies (other than humans). Such interventions that, as with grasses, increase the supply of food, are a step towards domestication, but are normally not considered as such. Domestication requires that the populations of animals on which humans subsist, depend on human assistance for their survival.

Both the grass and the reindeer examples stress that in the contemporary view, hunter-gatherers were not exclusively dependent upon nature for the size of their food supply. The examples also indicate why moving one step up the technical ladder involves an increase in time or energy invested in applying the method, regardless of whether the method is considered to be foraging or farming.

It is generally agreed that when new and more intensive food procurement methods, for whatever reasons, occurred, Stone Age communities evaluated the results of deploying the different methods. Considering the time and energy aspects suggested above, foraging techniques may have been preferred despite the knowledge of more intensive methods. Below we try to formalise the process of evaluation.

3.1 The Community

Consider a population consisting of \(N\) identical individuals who form a single, egalitarian community. By egalitarian we mean that all members of the com-

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6Australian Aboriginals, for instance, who are classified as hunters and gatherers, burn off vegetation to encourage the growth of the species of grasses that they depend on for food at the expense of other plants (e.g., Smith, 1995).

7More precisely, a step up the ladder into farming methods requires the use of domesticated grasses. According to Smith’s (1995) definition, a ‘domesticate’ is the human creation of a new form of plant (or animal) that is identifiably different from its wild ancestor.
community work the same amount of time and that all goods produced are divided equally between the members. The community’s population is assumed to be constant over time and so is the amount of land controlled by the community. We consider a one-period, non-overlapping generations model, meaning that, at the beginning of each period, the old generation is replaced by a new one of a similar size.

Although demographic changes certainly seem to have occurred in relation to the rise of agriculture (see, e.g., Diamond, 1997), the purpose of this model is to show, in accordance with archaeological evidence, that agriculture could have been adopted even in the absence of demographic (or for that matter climatic) changes.⁸ For an exposition that deals specifically with demographic changes in relation to the rise of agriculture, see Olsson (2003).

### 3.2 Food Procurement

With \( N_t^F \leq N \) individuals in food procurement (superscript \( F \) for food), who each work for \( n_t \) hours, and with \( X \) units of land, the community’s total food output, which is subject to constant returns to land and labour, is

\[
Y_t^F = B_t (n_t N_t^F)^{\alpha} X^{1-\alpha} \equiv B_t (n_t N_t^F)^{\alpha}, \quad \alpha \in (0, 1), X \equiv 1. \tag{1}
\]

The variable \( B_t \) measures the total factor productivity of the food procurement method that is applied by the community in period \( t \). Note that having decided on a method of food procurement (see further below), thus holding \( B_t \) constant, the assumption that \( \alpha \in (0, 1) \) means that there are diminishing returns to labour, which is due to the fact that only a limited amount of land is available or accessible to the members of the community.

Suppose that new and more productive food procurement methods are gradually added to the stock of methods that the community can choose among.⁹ More specifically, we assume that the variable \( A_t \), which measures the total factor productivity of the cutting-edge technology, i.e., the most productive method available at the beginning of period \( t \), grows exogenously at a non-decreasing rate.¹⁰ In addition, we assume that the choice of food procurement method is a once in a life-time decision that is made at the beginning of each life-period.

As time passes, new generations of the community’s population are thus able to employ still more productive food procurement methods. But the familiarity

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⁸Milthen (1996) and Harlan (1995), for example, both argue that the idea of a food crises prior to the rise of agriculture, broad about by demographic or environmental pressure, is no longer convincing.

⁹Rather than being intentional, improvements in food procurement technology in the Stone Age were probably incidental; human latrines, for example, may have been an unconscious testing ground of the first crop breeders, as suggested by Diamond (1997).

¹⁰Olsson (2003), in a manner similar to ours, assumes that the productivity in food production among primitive societies is an increasing function of time. One could also imagine that improvements in food procurement technology arose from learning-by-doing. For example, the arrival of new technology could be an increasing function of the total food output, in which case improvements in technology would occur in an endogenous manner. Such a construction, however, would affect neither the qualitative nor the quantitative results of the model.
with methods that are more productive than the one used by the previous
generation does not necessarily lead to a technical replacement. The reason is
that the application of more productive methods requires an increase in time
spent learning how to practise the method (see below). Hence, the level of
applied technology, whose productivity is measured by the variable \( B_t \), is equal
to, or below, the productivity level of the cutting-edge technology, \( A_t \). That is,
\[
B_t \leq A_t. \tag{2}
\]

In other words, invention and innovation do not necessarily go hand in hand.\(^\text{11}\)

### 3.3 Time Consumption and leisure

Individual time spent in food procurement is divided into two types of activi-
ties. The first type of activity is learning, which involves time spent becoming
acquainted with a certain food procurement method. The second type of activity
concerns regular work, i.e., the time during which the acquired method is
applied in order to procure food, denoted \( n_t \) (see equation (1) above).

Anthropological studies of present-day primitive societies indicate that the
time required to become acquainted with a certain food procurement method
increases with the method’s productivity (e.g., Bird and Bird, 2002; Blok, 2002).
Suppose, therefore, that when applying a method with a total factor productivity
equal to \( B_t \), it requires a number of individual learning hours equal to
\[
e_t = \phi B_t^\gamma \equiv e (B_t), \quad \phi > 0, \quad \gamma \in (0, 1). \tag{3}
\]

Having \( \gamma \in (0, 1) \) indicates that learning time increases at a diminishing rate
with the method’s productivity. Note the indivisibility associated with the learning
process: If the time devoted to learning a given method is less than the time
required to master that methods, the method cannot be learnt and used; instead,
less productive methods, requiring less learning, are applied.\(^\text{12}\)

\(^{11}\)Mokyr (1990, 2002) provides examples of inventions that were made available during
the Middle Ages or even in the classical antiquity, that were not adopted until the time of
the Industrial Revolution. Note that the use of the term ‘invention’ not necessarily implies
that the knowledge of more productive methods exclusively stems from inventiveness. The
dispersion of information from community to community may also have been a source of new
knowledge. Therefore, the results of the model are not confined to the people who invented
agricultural techniques. I thank George Grantham for pointing this out.

\(^{12}\)Two coastal foraging techniques—shellfish collection and spearfishing—may illustrate the
point. Gathering shellfish is perceptually easy; children from a very early age are able to
participate in such a food quest. Spearfishing, on the other hand, which is a more produc-
tive method, i.e., generates a larger amount of food per unit of time once the method is
learnt, requires intensive, method-specific training. The learning process here is thus more
time-consuming than that needed to gather shellfish. Moreover, the methods are distinctly
different: one cannot upgrade one’s learning from shellfish collecting to spearfishing; the
shellfish-gatherer is practically a novice concerning spearfishing and must start his or her
learning from scratch. The relationship between learning time and the complexity of food
procurement methods among primitive people has been profoundly studied in the anthropo-
logical literature; for recent discussions and empirical observations, see Blok (2002) and Bird
and Bird (2002).
Having learnt how to practise a given food procurement method, each individual supplies a number of work hours, \( n_t \), in order to obtain a given amount of food. To procure \( a_t \equiv Y_t^F / N_t^F \) units of food, each individual, according to equation (1), must supply a number of regular work hours equal to

\[
 n_t = \left( \frac{N_t^F}{B_t} \right)^{1-\alpha} \equiv n \left( a_t, B_t, N_t^F \right). \tag{4}
\]

Due to the egalitarian nature of the community, and to the similarities in individual preferences (see below), all individuals engaged in food procurement end up learning and applying the same method.

The individual’s leisure time consists of the total number of hours available to the individual, symbolically denoted \( \pi \), minus his learning time, \( e_t \), and the time he spends in food procurement, \( n_t \). Leisure time, therefore, is

\[
 l_t = \pi - n \left( a_t, B_t, N_t^F \right) - e(B_t) \equiv l \left( a_t, B_t, N_t^F \right). \tag{5}
\]

### 3.4 Preferences

In accordance with Craft (1985), we assume that the demand for food, unlike other goods that we consider, is income-inelastic.\(^{13}\) More specifically, we assume that having consumed a number of food units equal to \( a \), utility is derived from leisure time, \( l_t \), as well as non-food goods, \( m_t \), as specified in the utility function:

\[
 u_t (l_t, m_t) = l_t^\beta (1 + m_t)^{1-\beta}, \quad \beta \in (0, 1). \tag{6}
\]

Non-food goods can be thought of as daily consumption goods such as houses, clothing, cooking tools, pottery for storage, etc. Or as more luxurious goods such as pearls and other kinds of ornaments; goods aimed at sweetening the value of the individual’s leisure time. Interpreted in a more broad sense, non-food goods may also consist of protection and salvation. As will become apparent below, non-food goods, in the case that these are consumed, are produced by non-food specialists.

Note that a positive level of utility can be obtained from consuming leisure exclusively. Note also that the consumption of the \( \pi \) units of food does not provide any utility but is necessary for survival.\(^{14}\)

### 3.5 Non-Food Production

In the analysis below, we will consider both the case where every member of the community is engaged in food provision, which characterises foraging societies, and the case where the community’s labour force is divided into food and non-food specialists, which characterises agricultural societies.

\[^{13}\text{Adam Smith (1776, p. 164) perhaps said it most precisely, noting that ”[t]he desire for food is limited in every man by the narrow capacities of the human stomach but the desire of the conveniences and ornaments of buildings, dress, equipage, and household furniture, seems to have no limit or certain boundary”.}

\[^{14}\text{A similar construction is found in Kögel and Prskawetz (2001).} \]
3.5.1 Non-Food Specialists

What does a division of the labour force into food and non-food specialists involve in terms of this model? The presence of non-food specialists means that some of the community members who would otherwise provide their own food are exclusively engaged in non-food production. Non-food specialists, therefore, depend upon those that are engaged in food procurement for the food that they need for survival. In exchange for this food, food producers thus require a compensation in terms of non-food goods for their loss of leisure associated with their additional food output.

We denote the number of non-food producers in period $t$ as $N_{NF}^t$ (superscript $NF$ for non-food), so that, per definition, the total population is

$$N = N_{F}^t + N_{NF}^t.$$  

Equilibrium in the market for food means the total food demand, $aN_{eq}$, equals the total food supply, $a_t N_{F}^t$. This implies that the number of food producers in relation to the entire population is $N_{F}^t = (\pi/a_t) \cdot N$. The number of non-food specialists, supported by the food producers, is thus

$$N_{t}^{NF} = (1 - \pi/a_t) \cdot N. \quad (7)$$

Note that when a food producer’s food output, $a_t$, exceeds his satiation level, $\pi$, the community is capable of supporting non-food specialists. This, however, is no guarantee that a division of labour is effectuated. We also need to make sure that the rate at which the food producer wishes to trade off leisure for non-food goods corresponds to the rate at which his food surplus, $a_t - \pi$, attained by decreasing his leisure time, is converted into non-food goods. We will return to this matter in the analysis below.

3.5.2 Non-Food Production Technology

Suppose that non-food goods are produced using constant returns to labour technology. The total number of non-food goods, when the community avail itself of non-food specialists, is thus

$$Y_{t}^{NF} = \nu (n_t + e_t) N_{t}^{NF}, \quad \nu > 0, \quad (8)$$

where $\nu$ measures non-food productivity. Since we abstract from the learning process associated with non-food goods production, and for individuals to be indifferent between employment in the food and in the non-food sector, non-food producers, as do their food producing colleagues, work for $n_t$ plus $e_t$ hours.

3.5.3 Costs of Redistribution

Whereas communities that divide their labour force into food and non-food specialists need a system of goods transfer, i.e., the collection and redistribution of food and other stuffs between sectors and individuals, societies in which all
members of the community participate in the food quest need no such system (see, e.g., Diamond, 1997, Ch. 14).

The collection and redistribution associated with the division of the labour force between food and non-food activities therefore calls for an amount of redistribution costs that societies with no division of labour are not exposed to. Accordingly, we will assume that the costs of redistribution associated with the existence of the two-sector economy are \( \pi > 0 \) units of non-food goods per individual per generation.

### 4 Analysis

Considering the shift to agriculture as a transition from low-productive into high-productive methods of food procurement, as is done in this paper, makes it difficult to distinguish those technologies that belong to foraging from those that belong to farming. In order to create a point of reference, we will apply two commonly accepted archaeological theories. First, as discussed on page 6, archaeological evidence indicates that non-food specialists are seldom found in societies that embark upon foraging techniques. Second, as discussed on page 4, archaeological evidence indicates that foragers were the most leisured people in history. In consequence, we therefore propose that the level of technology that maximises the individual’s leisure time when everyone is engaged in food provision will be characterised as a foraging method. Using this method, there are thus no non-food specialists in equilibrium.

The optimization problem for an individual engaged in food procurement is

\[
\min_{B_t} e_t + n_t = c(B_t) + n(a_t, B_t, N^F_t),
\]

subject to the food constraint that \( a_t \geq \tilde{a} \); to the occupational constraint that \( N^F_t \leq N \); to the time-budget constraint that \( n_t + e_t \leq \pi \), and to the technology constraint that \( B_t \leq A_t \). An interior solution implies that

\[
\frac{\partial e_t}{\partial B_t} = \frac{\partial n_t}{\partial B_t} \iff \phi \gamma B_t^{\gamma - 1} = \frac{1}{\alpha B_t} \left( \frac{a_t (N^F_t)^{1-\alpha}}{B_t} \right)^{1/\alpha}.
\]

Equation (10) says that the marginal utility obtained from embarking on a more productive method, thus being able to decrease the time it takes to acquire \( a_t \) units of food, in optimum equals the marginal utility forgone from spending more time learning how to practise this method.\(^{15}\)

From equation (10), it follows that if the method could be chosen freely, i.e., disregarding the technology constraint in equation (2), then the leisure-maximizing technology (marked with an asterisk) would be

\[
B^*_t = \left( a_t \left( N^F_t \right)^{1-\alpha} / (\alpha \gamma \phi)^{\alpha} \right)^{1/1+\alpha \gamma} \equiv B^* (a_t, N^F_t),
\]

\(^{15}\)The second-order condition indicates that the leisure-maximising level is a local optimum.
where \((B^*_a, B^*_N) > 0\). For foraging communities in which every member is engaged in food provision, the leisure-maximising technology is thus \(B^*_t = B^*(\pi, N)\).\(^{16}\)

Note that while the adoption of a more productive method below the leisure-maximizing level is leisure-increasing, the implementation of more productive methods above the leisure-maximizing level requires an unequivocal increase in the total time spend obtaining a given amount of food. That is, the increase in learning time associated with the adoption of a more intensive method eventually becomes so pronounced that it dominates the time-saving effect on the time it takes to procure a given amount of food once the method is learned. Adopting a more productive method above the leisure-maximizing level is, therefore, leisure-decreasing.

It thus immediately follows that when cutting-edge technology, \(A_t\), is below the leisure-maximizing level, cutting-edge technology is adopted at once (this is formalised in Corollary 1 below). At the same time, we see the cause of the reluctance among hunters and gatherers towards more productive food procurement methods, as their implementation decreases the individual’s leisure time. It is the latter fact that seems to puzzle archaeologists and anthropologists, and is the reason why it has been suggested that the adoption of time-costly agricultural techniques was a result of external pressure arising from demographic or climatic changes (see the introductionary section).

It appears, however, that archaeologists and anthropologists overlook the individual’s willingness to trade off leisure for non-food goods. The willingness to practise leisure-decreasing methods, in this model, requires that utility is somehow increased, or, at least, maintained. Hence, according to his preferences, the forager decreases his leisure time in order to adopt a methods that is more productive than the leisure-maximizing level, only if compensated for his loss of leisure in terms of non-food goods. With learning involved in the food procurement process, we thus arrive at the following conclusion.

**Lemma 1** If \(N_{NF} = 0\) when \(B_t = B^*(\pi, N)\), then \(B_t > B^*(\pi, N)\) requires \(N_{NF} > 0\) in equilibrium.

**Proof** Since \(B^*\) is the leisure-maximising method, it follows that for \(B > B^*\), \(l_B < 0\). Since \((u_l, u_m) > 0\), the goods of non-food specialists are required to compensate for the decline in utility from using a leisure-decreasing method. \(\square\).

\(^{16}\)The fact that the leisure-maximising method varies with the size of the community’s population implies, according to our characterisation, that relatively large communities in which everyone is in food provision use methods that are closer to those of agriculture than does communities with relatively small populations. The difficulties of drawing a sharp line between foraging and farming is also found in the archaeological literature (e.g., Smith, 2001, p. 1), in which the term ‘middle ground’ is used as "the definitional and developmental ‘no-man’s land’ that stretches between hunter-gatherer-foragers, with economies based exclusively on wild plants and animals, on one side, and agriculturalists, who strongly depend on domesticated species as food sources, on the other". The leisure-maximising method discussed in this model is considered to belong to this ‘middle ground’.
Lemma 1 provides a key statement in this paper. What the Lemma basically says is that since individuals care about leisure and non-food goods, then if foragers should ever accept the adoption of leisure-decreasing methods, it must be the case that a separate sector of non-food producers emerges in order to compensate for the longer work hours associated with these methods.\footnote{One could also imagine that individuals would agree to longer work hours in exchange for technologies that allow a more sedentary life-style. Since the nomadic life-style, with its constant or periodic travelling, does not complement the possession of (material) non-food goods (as mentioned by Sahlins, 1974), farmers should derive more utility from non-food goods than do nomadic foragers. I thank an anonymous referee for pointing this out. It is beyond the scope of this model, however, to consider the relationship between technologies and degrees of sedentariness. For an exposition that deals formally with this matter, see Locay (1989).} But under what circumstances will the community undergo such a reorganization? And what postpones the adoption of leisure-decreasing methods despite their accessibility? The remaining part of the paper deals with these questions.

4.1 The Rise of Agriculture

In accordance with archaeological evidence, our point of departure is a situation where everyone is engaged in food provision (i.e., $N^F = N$); where each individual, therefore, produces food for himself only (i.e., $a = \pi$); and where the most leisured method associated with the individual’s satiation of his food needs is applied (i.e., $B = B^* (\pi, N)$). In accordance with the evidence presented in Diamond (1997) and Sahlins (1974), we thus consider a community of hunters and gatherers.

In the following, we identify the levels of $A_t$, above the leisure-maximizing level, at which non-food specialists are present. In other word, we investigate under what circumstances the community of hunters and gatherers is capable of adopting methods that are more productive but also more time-costly than the leisure-maximizing level; methods that we consequently identify as being agricultural.

Keeping in mind that the costs of redistribution must be covered, the presence of non-food specialists in equilibrium requires that the rate at which food producers wish to trade off leisure for non-food goods exceeds the rate at which their food surplus, $a_t - \pi$, attained by decreasing their leisure, is converted into non-food goods. To simplify matters we divide the issue into two separate questions: First, how many non-food goods will an individual ask for in exchange for the loss of leisure associated with the adoption of a leisure-decreasing method used to produce a food surplus of a certain amount? And second, how much is the non-food goods sector, which is supported by the food sector’s surplus, then able to supply?

Consider, in regards to the former question, the following options: Either the individual applies the leisure-maximizing method, providing food for himself only and consuming no non-food goods (i.e., the individual is a forager). Or he produces a food surplus, $a_t > \pi$, applying a method with a higher productivity
than the leisure-maximizing method while consuming a positive amount of non-food goods (i.e., the individual is a farmer).

From the utility function, equation (6), we find that the individual is indifferent between the two alternatives when he receives an amount of non-food goods equal to (superscript $c$ for compensation)

\[ m^c_t = \left( \frac{1/(\pi, B^*(\pi, N_s), N_t)}{l(a_t, B_t, N^F_t)} \right)^{\beta/(1-\beta)} - 1 \equiv m^c(a_t, B_t, N), \]  

(12)

where, for $a_t > \pi$, $(m^s_t, m^a_t) > 0$; where, for $B_t > B^*$, $m^a_t > 0$; and where $m^s_N > 0$. Inserting equation (7) into (8), each individual, due to the egalitarian nature of the community, accordingly receives a number of non-food goods equal to (superscript $s$ for supply)

\[ m^s_t = \nu \left( \pi - l(a_t, B_t, N^F_t) \right) \left( 1 - \pi/a_t \right) \equiv m^s(a_t, B_t, N), \]  

(13)

where, for $a_t > \pi$, $(m^s_t, -m^a_t) > 0$; where, for $B_t > B^*$, $m^a_t > 0$; and where $m^s_N > 0$. Examining at what levels of $A_t$ above the leisure-maximizing level that non-food specialists are present in equilibrium thus corresponds to finding the levels of $A_t$ at which $m^s_t - \pi > m^c_t$, where $\pi$ is the individual’s costs of redistribution associated with the two-sector economy.

To be used below, define the function $\sigma$ such that

\[ \sigma(\xi) = (1/\pi)^{1/(1-\alpha)} \left( (\pi/\rho) - \xi \beta / (1 - \beta) \right)^{1+\alpha\gamma/(1-\alpha)}, \]

where $\rho \equiv \phi(\alpha\phi\gamma)^{-\alpha\gamma/(1+\alpha\gamma)} + (\alpha\phi\gamma)^{1/(1+\alpha\gamma)}$. Consider the following Lemma.

**Lemma 2** Assume that $N < \sigma(\xi)$, where $\xi \equiv (\alpha\gamma\phi)^{1/(1+\alpha\gamma)}/\nu^2$. Then there exists a level of food output per food producer, $a' > \pi$, where $\partial m^s(a', B^*(\pi, N), N)/\partial a_t = \partial m^c(a', B^*(\pi, N), N)/\partial a_t$, and a unique level of redistribution costs, $\bar{\pi} > 0$, such that for $\pi > \bar{\pi}$, $m^s(a', B^*(\pi, N), N) - \pi < m^c(a', B^*(\pi, N), N)$.

**Proof** See Appendix A. □.

The statement in Lemma 2 is illustrated in Figure 1. The Figure shows that with technology at the leisure-maximising level and redistribution costs equal to $\bar{\pi}$, the members of the community are indifferent between satisfying only their own food needs, having no non-food specialists, and generating an individual food surplus equal to $a' - \bar{\pi}$, supporting non-food specialists. Indifference arises because, by the definition of $\bar{\pi}$, non-food specialists are exactly able cover both the costs of redistribution and the individual’s compensation associated with the loss of leisure that arises from generating an amount of food equal to $a'$ rather

\[ \text{Note that implicitly we have used the fact that equilibrium in the food market implies that } N^F_t = (\pi/a_t) N. \text{ This is also the case in equation 13 below.} \]

\[ \text{For convenience, we assume that the } \pi \text{ units of non-food goods that it costs per individual to collect and redistribute the goods in the two-sector economy are 'shoeleather costs' in the sense that in order to collect and redistribute the goods, the } \pi \text{ units of non-food goods vanish.} \]
than $\pi$. It thus follows that when the leisure-maximising method is applied, communities that are subject to costs of redistribution that exceed $\pi$ will have no non-food specialists in equilibrium.

[Figure 1 about here: The Characterisation of $\pi$]

Figure 1 also shows that if there were no redistribution costs, non-food specialists would exist in the foraging community for all $a_t \in (\pi, a^*)$. This becomes evident when comparing the dashed $m^s$-curve to the solid $m^c$-curve. The existence of this interval of food surpluses, which allows a division of the labour force between food and non-food activities, is ensured by holding the population density below the limitation in the Lemma. When the density of the community’s population is sufficiently small, a decline in the leisure time arising from an increase in the food output per individual in food provision has a smaller impact on the individual’s demand for compensation than on its non-food supply. Implicitly, the reason is that when the community’s population is relatively small, individuals have a relatively large amount of leisure at their disposal. Due to a diminishing marginal rate of substitution, individuals therefore ask for a relatively small amount of non-food goods in exchange for one unit of leisure. This is why the slope of the compensation curve, as illustrated in Figure 1, is relatively flat when departing from $\pi$.

With Lemma 2 in mind, consider the following proposition.

**Proposition 1** Let $a'$ and $\pi$ (defined in Lemma 2) be given and let $\delta > 0$. Assume that $\sigma (\xi_2) < N$, where $\xi_2 = 1/\nu \rho$. Then there exists $\pi \in (\pi, \pi + \delta)$ and functions $\mu (\pi) > \varepsilon (\pi) > 0$ and where $\mu (\pi + \delta) = \varepsilon (\pi + \delta)$, such that for any $A_t \in (B^* (\pi, N) + \varepsilon (\pi), B^* (\pi, N) + \mu (\pi)), m^s (a', A_t, N) - \pi > m^c (a', A_t, N)$.

**Proof** See Appendix B.

Proposition 1 says that there are levels of redistribution costs that prevent the introduction of non-food specialists when using the leisure-maximising method (i.e., $\pi > \pi$) and hence also when using methods that are less productive than the leisure-maximising one, but allow their introduction within a given interval of leisure-decreasing methods somewhat above the leisure-maximising level.\(^{20}\)

The intuition behind the statement in Proposition 1 can be obtained from considering Figure 2. In Figure 1, technology was held constant at the leisure-maximising level while the food output per food producer varied. In Figure 2, the level of food output is held constant at $a' > \pi$, while the level of technology varies. Equivalent to Figure 1, Figure 2 shows that with $B_t = B^* (\pi, N)$; with $a' > \pi$, and with $\pi = \pi$, the supply of non-food goods minus redistribution costs exactly match the demand for compensation. This follows from comparing the dashed supply curve with the compensation curve. However, for redistribution

\(^{20}\)Note that by ‘adoptable’ methods is meant that compared to the leisure-maximising level, a particular method within the ‘adoptable’ interval yields at least the same level of utility.
costs that exceed $\pi$, the supply of non-food goods minus redistribution costs fall below the demand for compensation at $B^*(\pi, N)$, meaning that using this method there are no non-food specialists in equilibrium. This becomes evident when comparing the solid supply curve with the compensation curve.

The illustration in Figure 2 suggests that for redistribution costs, $\pi > \tilde{\pi}$, that are not too larger, the leisure-decreasing methods just above the method that maximises leisure, are unadoptable. Methods within this interval, though more productive than the leisure-maximising level, cannot generate a total food surplus large enough to support a non-food sector, which, in turn, is capable of covering both redistribution costs and individual compensation. Since for these methods division of labour is unattainable, then, following Lemma 1, the leisure-maximising method continues to be applied, i.e., there is technical stagnation. This, in accordance with archaeological evidence, means that the foraging community is familiar with agricultural methods that they nevertheless decide not to adopt.

[Figure 2 about here: Leisure-Decreasing Technologies with $a_t = a'$]

However, being exposed to more and more productive methods of food procurement, food producers, in case they decide to adopt the methods, become increasingly effective. For redistribution costs above $\pi$ that are not too larger (i.e., below $\tilde{\pi} + \delta$), an interval of leisure-decreasing methods eventually is reached, at which the food sector, by putting these methods into practise, is capable of supporting a non-food sector on a scale that is able to cover both the costs of redistribution and demand for compensation (see Figure 2). Using these methods, both sectors are so efficient, and individuals so moderate in their demand for compensation, that a pareto improvement area (the shaded zone in Figure 2) occurs, causing an increase in individual utility compared to that from applying the leisure-maximising method. For that reason, these methods are adoptable.

Note that at the moment the interval of adoptable methods is reached, there is a significant increase in the productivity of the applied method. Accordingly, after a period of technical stagnation, the community experiences a technical revolution, reflecting the perception among archaeologists of a 'Neolithic revolution'.

The dynamic properties of the model are summarised in the following corollary to Proposition 1.

**Corollary 1** Let $a', \pi \in (\tilde{\pi}, \tilde{\pi} + \delta), \varepsilon(\pi)$, and $\mu(\pi)$ (defined in Lemma 2 and Proposition 1) be given and let $A_t$ grow at a non-decreasing rate. Then,

(i) for $A_t \leq B^*(\pi, N)$, $B_t = A_t$, which implies that $B_t$ grows at the rate of $A_t$.

(ii) for $A_t \in (B^*(\pi, N), B^*(\pi, N) + \varepsilon(\pi))$, $B_t = B^*(\pi, N)$.

(iii) for $A_t \geq B^*(\pi, N) + \varepsilon(\pi)$, $B_t = A_t$, which implies that $B_t$ once again grows at the rate of $A_t$, as long as $m^*_t + 1(a', A_t, N) - m^*_t(a', A_t, N) \geq m^*_t + 1(a', A_t, N) - m^*_t(a', A_t, N)$. 

17
Corollary 1 follows from Lemma 2 and Proposition 1. Implicitly, item (i) in the Corollary says that a community of foragers increase their leisure time by adopting the more productive foraging methods the moment that these occur. Item (ii), however, states that a point will be reached at which the foragers continue to use a foraging technique, despite the knowledge about more productive but also more time-costly methods. Finally, item (iii) suggests that the foragers eventually abandon the leisurely foraging method in favour of agricultural methods that are more productive. This happens at a cost to leisure which is being compensated for by the goods produced in the simultaneously emerging non-food goods sector. Note that the restriction in (iii) implies that the applied method remains at the cutting-edge level, as long as the increase in the demand for compensation does not exceed the increase in the supply; that is, as long as utility increases as a result of the adoption of more productive methods.21

As mentioned above, Figure 2 can help illustrate the intuition behind the result. Since the leisure-maximising method increases with the food output per food producer, \( a_t \), it follows (see appendix B) that when \( a_t = a' > \overline{a} \), the level of technology that yields the most leisure is \( B^* (a', N^F_t) > B^* (\overline{a}, N) \). This means that as cutting-edge technology, departing from \( B^* (\overline{a}, N) \), improves over time, food producers, until \( B^* (a', N^F_t) \) is reached, increase their leisure time by putting these methods into practice. This in turn enables them to lower their demand for compensation, and is why the compensation curve in Figure 2, to begin with, slopes downward. Since, with these technologies, labour time is also lower in the non-food sector, the supply curve, to begin with, slopes downward as well. However, when the density of the community’s population is sufficiently large—that is, when it exceeds the limitation in Proposition 1—changes in the leisure time arising from changes in technology, have a smaller impact on the individual’s non-food supply than on the demand for compensation.22 More precisely, the impact on the individual’s non-food goods supply of an increase in leisure is independent of the size of the community’s population, while the impact on the individual’s demand for compensation of an increase in leisure grows with the size of the community’s population. Due to a diminishing marginal rate of substitution, this means that individuals in relatively large communities, having relatively little leisure time at their disposal, demand a relatively large compensation in exchange for a unit of leisure. Thus, departing from \( B^* (\overline{a}, N) \), the absolute slope of the compensation curve is larger than that of the supply curve. Hence, when more productive methods eventually become available, the

---

21 Due to a diminishing marginal rate of substitution, the loss of leisure becomes increasingly difficult to compensate. One can think of this as being the reason why time-saving implements, such as animal power, were eventually introduced (see, e.g., Boserup, 1965). As we abstract from the use of capital goods in the production of output, it is outside the scope of this model to consider such effects.

22 This argumentation is similar to that made on page 16, *mutatis mutandis*. 
scope for covering both redistribution costs and compensation increases.\textsuperscript{23}

Using a numerical example, which is based upon parameter values from Table 1, Figure 3 shows that there are more levels of food output per food producer than \(a'\) that are associated with the adoption of methods that are more productive than the one that maximises leisure time.\textsuperscript{24} In this Figure, the gray area indicates the intervals of technologies and food outputs at which the non-food sector covers more than compensation and redistribution costs.\textsuperscript{25} Figure 3 also reveals that technical stagnation is followed by a technical revolution.

\section{Conclusion}

This paper investigates the shift in food procurement technology from leisurely foraging methods to time-costly farming techniques. On the face of it, hunters and gatherers’ reluctance towards farming arises because individuals value leisure and so have no interest in spending more time in food procurement than is necessary in order to fulfill their food needs.

In the traditional view the adoption of agriculture is forced by external changes, such as climatic or demographic pressure. But evidence shows that agriculture was adopted even in the absence of such changes. The main theoretical result in this paper, therefore, is that neither climatic nor demographic changes are needed to have hunters and gatherers embarking upon leisure-decreasing methods of food procurement. Organizational changes leading to the introduction of non-food specialists, driven by exogenous improvements in food procurement technology, suffices to trigger the shift to farming.

This result carries an important message with regard to economic growth. If the adoption of more productive food procurement methods went hand in hand with the emergence of non-food specialists, the rise of agriculture bore the seeds for the later process of industrialisation and thus for economic growth.

Unfortunately, as the model does not allow for demographic changes, it is incapable of explaining the rise in population densities that appears to have

\textsuperscript{23}It should also be evident from Figure 2 that the size of the redistribution costs influences the outcome. Lower costs, corresponding to an upward movement of the \(m^*(a', A, N) - \pi\) curve, decrease the period of technical stagnation and widen the range of methods that are adoptable (i.e., \((-\mu_\pi, \epsilon_\pi) > 0\)). Thus, the smaller the costs of redistribution, the earlier is the introduction of non-food specialists. By contrast, higher costs of redistribution increase the period of technical stagnation and reduce the range of methods that are adoptable. In fact, redistribution costs may be so large that they completely prevent the introduction of leisure-decreasing methods. This is the case when the supply curve minus \(\pi\) is below the compensation curve at \textit{all levels of leisure-decreasing} methods (i.e., \(\pi > \bar{\pi} + \delta\)). Under such circumstances, the leisure-maximising method is the optimal choice, despite the presence of methods that are far more productive.

\textsuperscript{24}Figure 2, which holds \(a_t = a'\), is simply a cross section of Figure 3.

\textsuperscript{25}Note that the pareto improvement areas in Figures 2 and 3 (the shaded zones) represent a potential increase in the size of the community’s population (i.e., its population density), as these areas imply that units of food can be taken from the non-food sector in order to support a larger population at no cost to standards of living.
accompanied the adoption of agriculture in the Stone Age. Although the pareto improvement areas (the shaded areas) in Figures 2 and 3 indicate that units of food can be taken from the non-food sector in order to support a larger population at no cost to standards of living, endogenising population growth would significantly improve the model.\textsuperscript{26}

\textsuperscript{26}Olsson (2003), who’s model deals specifically with demographic changes in relation to the rise of agriculture, is able to show that once farming has been introduced, population growth accelerates and more and more people enter the sector of agriculture, even though this infers declining standards of living.
6 Proof of Lemma 1

With \((m^*_c, m^*_a) > 0\) and \((m^*_c, -m^*_a) > 0\), it follows that if, evaluated at \(a_t = \pi\), \(\partial m^*(a_t, B^*(\pi, N), N)/\partial a_t > \partial m^c(a_t, B^*(\pi, N), N)/\partial a_t\), then there exists a unique positive level of redistribution costs, \(\pi > 0\), that solves the problem \(\max_{a_t} m^*(a_t, B^*(\pi, N), N) - m^c(a_t, B^*(\pi, N), N)\), in which case there exists a unique level of redistribution costs, \(\pi > 0\), at which \(m^*(a'_t, B^*(\pi, N), N) - \pi = m^c(a'_t, B^*(\pi, N), N)\). (Figure 1 should provide the intuition.) The slope of the compensation curve, evaluated at \(a_t = \pi\), is

\[
\frac{\partial m^c(a_t, B^*(\pi, N), N)}{\partial a_t}\bigg|_{a_t = \pi} = \frac{\beta}{1-\beta} \left( \frac{(\pi N)^{1-\alpha}}{B^*(\pi, N)} \right)^{\frac{1}{\beta}} \frac{\pi - \pi}{\pi - \rho (\pi N^{1-\alpha})^{\frac{1}{\beta}}} - \phi(B^*(\pi, N))\gamma
\]

where \(\rho \equiv \phi(a\phi(1-\gamma) + (\alpha\phi\gamma)^{1/(1+\gamma)}\). The slope of the supply curve, evaluated at \(a_t = \pi\), is

\[
\frac{\partial m^s(a_t, B^*(\pi, N), N)}{\partial a_t}\bigg|_{a_t = \pi} = \frac{\nu}{\pi} \left( \frac{(\pi N)^{1-\alpha}}{B^*(\pi, N)} \right)^{\frac{1}{\beta}} + \phi(B^*(\pi, N))\gamma
\]

\[
= \frac{\nu}{\pi} \left( \frac{(\pi N^{1-\alpha})}{\pi - \rho (\pi N^{1-\alpha})^{\frac{1}{\beta}}} \right).
\]

Evaluated at \(a_t = \pi\), the slope of the supply curve exceeds that of the compensation curve when

\[
\left( \pi - \rho (\pi N^{1-\alpha})^{\frac{1}{\beta}} \right) \nu \left( \rho (\pi N^{1-\alpha})^{\frac{1}{\beta}} \right) > \frac{\beta}{1-\beta} (\pi N^{1-\alpha})^{\frac{1}{\beta}} (\alpha \gamma \phi)^{\frac{1}{\beta}},
\]

that is, when \(N < \sigma(\xi_1), \) where \(\xi_1 = (\alpha \gamma \phi)^{1/(1+\alpha \gamma)} / \nu \rho^2\), which we have assumed.

7 Proof of Proposition 1

It follows from equations (12) and (13) that \((m^*_c, m^*_a) < 0\). The effect on the supply of non-food goods, \(m^*_c\), and on demand for compensation, \(m^*_a\), of a change in the level of technology, \(B_t\), depends on the effect that this change has on the individual’s leisure time, \(l_t\). Since \(B^*\) is the leisure-maximising method, it follows that \(l_B \leq 0\) when \(B_t \geq B^*\). Since \((m^*_c, m^*_a) < 0\), it thus follows that \((m^*_c, m^*_a) \geq 0\) when \(B_t \geq B^*\). Note that using the food market equilibrium (i.e., the fact that \(a_t N^*_f = \pi N\), the leisure-maximising method, equation (11), can be written as \(B^*(a_t, N^*_f) = (a\phi(\pi N)^{1-\alpha} / (\alpha \gamma \phi)^{1/(1+\alpha \gamma}) \equiv B^*(a_t, N)\), where \((B^*_a, B^*_N) > 0\). Thus, for \(B_t < B^*(a'_t, N), (m^*_a(a'_t, B_t, N), m^*_a(a'_t, B_t, N)) < 0\).
It follows from Lemma 2 that $m^c (a', B^* (\pi, N), N) - \tilde{\pi} = m^c (a', B^* (\pi, N), N)$. Thus, if, evaluated at $B_1 = B^* (\pi, N) < B^* (a', N)$, $|\partial m^c (a', B_1, N) / \partial B_1| > |\partial m^c (a', B_1, N) / \partial B_1|$, then there must be some $A_1 > B^* (\pi, N)$, at which $m^c (a', A_1, N) - \tilde{\pi} > m^c (a', A_1, N)$. (Figure 2 should provide the intuition.) It thus follows that there exists a positive constant $\delta > 0$ and $\pi \in (\tilde{\pi}, \tilde{\pi} + \delta)$, and functions $\mu = \mu (\pi)$ with $\mu < 0$ and $e = e (\pi)$ with $e > 0$, where $\mu (\tilde{\pi} + \delta) = e (\tilde{\pi} + \delta)$, such that for $A_1 \in (B^* (\pi, N) + e (\tilde{\pi}), B^* (\pi, N) + \mu (\tilde{\pi}))$, $m^c (a', A_1, N) - \pi > m^c (a', A_1, N)$.

(See Figure 2.) For any $a_1 > \pi$ (and thus especially for $a_1 = a'$), the slope of the compensation curve, evaluated at $B_1 = B^* (\pi, N)$, is

$$
\left. \frac{\partial m^c (a_1, B_1, N)}{\partial B_1} \right|_{B_1 = B^* (\pi, N)} = -\frac{\beta}{1 - \beta} \left( \frac{\pi}{\alpha} \right)^{\alpha-1} \left( \frac{\pi}{\alpha} \right)^{\alpha-1} \phi (B^* (\pi, N))^{-1} (1 - \frac{\pi}{a_1})
$$

For any $a_1 > \pi$, the slope of the supply curve, evaluated at $B_1 = B^* (\pi, N)$, is

$$
\left. \frac{\partial m^c (a_1, B_1, N)}{\partial B_1} \right|_{B_1 = B^* (\pi, N)} = -\nu \left( \frac{a_1}{B_1} \right) \left( \frac{\pi}{\alpha} \right)^{\alpha-1} \left( \frac{\pi}{\alpha} \right)^{\alpha-1} \phi (B^* (\pi, N))^{-1} (1 - \frac{\pi}{a_1})
$$

Evaluated at $B_1 = B^* (\pi, N)$, the absolute slope of the compensation curve exceeds the absolute slope of the supply curve when

$$
\frac{\beta}{1 - \beta} \left( \frac{\pi}{\alpha} \right)^{\alpha-1} \left( \frac{\pi}{\alpha} \right)^{\alpha-1} \phi (B^* (\pi, N))^{-1} (1 - \frac{\pi}{a_1}) > \nu \left( \frac{a_1}{B_1} \right) \left( \frac{\pi}{\alpha} \right)^{\alpha-1} \left( \frac{\pi}{\alpha} \right)^{\alpha-1} \phi (B^* (\pi, N))^{-1} (1 - \frac{\pi}{a_1})
$$

For $a_1 > \pi$, it follows that $1 > (1 - \pi) / a_1$ and that $l (a_1, B^* (\pi, N), N) < l (\pi, B^* (\pi, N), N)$. Thus, evaluated at $B_1 = B^* (\pi, N)$, the absolute slope of the compensation curve exceeds that of the supply curve when

$$
\frac{\beta}{1 - \beta} \frac{1}{\pi - \rho (\pi N^{1-\alpha})} > \nu,
$$

that is, when $\sigma (\xi_2) < N$, where $\xi_2 = 1 / \nu \rho$, which we have assumed.
References


Figure 1: The Characterisation of $\pi$. 

Table 1. Parameter Values

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<th>Comment</th>
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<td>Individual food satiation level</td>
</tr>
<tr>
<td>$\Pi$</td>
<td>35</td>
<td>Individual time endowment</td>
</tr>
<tr>
<td>$N$</td>
<td>45</td>
<td>Size/density of the community’s population</td>
</tr>
</tbody>
</table>
Figure 2: Leisure-Decreasing Technologies with $a_t = a'$.  

Figure 3: Leisure-Decreasing Technologies.